

Coulomb Drag, Mesoscopic Physics, and Electron-Electron Interaction.

Submitted by Adam S. Price to the University of Exeter as a thesis
for the degree of Doctor of Philosophy in Physics
October, 2008

This thesis is available for Library use on the understanding that it is copyright material and that no quotation from the thesis may be published without proper acknowledgement.

I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

A. S. Price
October, 2008

Abstract

The first part of this thesis deals with the study of mesoscopic fluctuations of the Coulomb drag resistance in double-layer GaAs/AlGaAs heterostructures, both in weak magnetic fields and strong magnetic fields. In the second part, measurements are made in a monolayer graphene structure, specifically of the quantum lifetime, and the mesoscopic resistance fluctuations at quantising magnetic fields.

In weak fields, we perform the first measurements of the fluctuations of the drag as a function of changing concentration and magnetic field. The amplitude of the fluctuations was seen to exceed the size of the average drag at low temperatures, resulting in random but reproducible changes in the sign of the drag with varying concentration and magnetic field. The variance and correlation magnetic field were studied as a function of temperature. Comparison was made to existing theories for drag fluctuations in the diffusive regime of the drag (where the electron mean free path is much smaller than the separation between layers). We observe a large enhancement of the magnitude of the fluctuations of four orders of magnitude compared to the theoretical expectation, as well as a different temperature dependence. Our results prompted further theoretical studies, extending the understanding of drag fluctuations into the ballistic regime, where the local properties of the layers become important. We compare our results to this theory and find good agreement.

We extend these measurements to the regime of strong magnetic fields where transport involves composite-fermion quasiparticles. We study the temperature dependence of the amplitude and correlation magnetic field of the fluctuations, and analyse our results using existing theoretical models for drag fluctuations in the diffusive regime of drag.

Much controversy exists over the dominant scattering mechanisms in graphene. We use measurements of the concentration and temperature dependence of the quantum lifetime and transport time in graphene to identify the dominant electron scattering mechanisms. We then perform measurements of resistance fluctuations in the integer quantum Hall effect regime of magnetic fields, and show our results to be well described by existing models developed for conventional 2D systems. Using these models we make estimates for the properties of the disorder in graphene.

Acknowledgements

My thanks must first of all go to my supervisor, Alex Savchenko. His interest and enthusiasm for the work described in this thesis has been a continual and important source of motivation throughout my project.

I would also like to thank Dave Horsell, who has been the post-doc in our research group for the entirety of my PhD study. He has always been willing to discuss any topic in physics, and his expertise has often been of great value. Thanks must also go to Sasha Mayorov, with whom I have had many useful discussions. My next thanks go to two former members of our group – Evgeniy Galaktionov and Andrei Kretinin, both of whom spared much time in helping me learn the experimental techniques that I relied on throughout the course of my project. Thanks also to Roman Gorbachev, Fedor Tikhonenko, both of whom were always willing to provide an extra hand when needed. Additionally, thanks to our two new students Alexei and Aleksey, who spent some of their first days in our lab with some data analysis whilst I was busy with putting the finishing touches on this thesis.

Last, but by no means least, I must thank Paul Wilkins, our lab technician, whom helped with countless mechanical difficulties as well as helped me move house on a number of occasions. Also, Dave Manning and Adam Woodgate, both of whom played the critical role of providing cryogens for our experiments, as well as maintaining much of the cryogenic equipment.

And of course, I appreciate the emotional support from my mother, Mary, brother and sister, Ben and Nichola, and friends during the last four years of study.

Contents

Abstract	2
Acknowledgements	3
Contents	4
List of Figures	7
List of Tables	18
Symbols and Abbreviations	19
List of publications	20
Introduction	21
1 Coulomb Drag	24
1.1 Scattering times	24
1.2 Shubnikov-de Haas effect	25
1.2.1 The Integer Quantum Hall Effect	27
1.3 Coulomb Drag	28
1.3.1 Phonon Drag	31
1.3.2 Plasmon enhancement of the drag	33
1.3.3 Drag In Disordered Systems	34
1.4 Drag at low electron densities	35
1.5 Samples and Techniques	36
1.6 Equipment used	39
1.6.1 Cryogenic apparatus	39
1.6.2 Magnetic field coils and power supplies	39

1.6.3	Voltage measurement, and noise reduction	40
1.7	Conductance and resistance measurements	41
1.7.1	Two-terminal conductance measurements	41
1.7.2	Four-terminal resistance measurements	42
1.8	Synchronous Offset Measurement	42
1.9	Characterisation of sample	43
1.10	Characterisation and Observation of Coulomb Drag	44
1.10.1	Detecting and reducing spurious signals	45
1.10.2	Carrier density dependence of the drag resistance	46
1.10.3	Evidence of phonon mediated contribution to the drag resistance	47
1.10.4	Temperature dependence of the drag resistance	48
2	Mesoscopic Fluctuations in Coulomb Drag	51
2.1	Mesoscopic Physics	51
2.1.1	Universal Conductance Fluctuations	52
2.1.2	UCF at finite temperatures	54
2.2	Autocorrelation functions	55
2.3	Fluctuations in diffusive drag (existing theory)	56
2.4	Drag fluctuations in the ballistic regime	59
2.5	Measurement of L_φ in single-layer	62
2.6	Macroscopic to mesoscopic transition	64
2.7	Fluctuations in drag resistance as a function of gate voltage and mag- netic field	65
2.8	Temperature dependence of variance and L_φ	67
2.9	Conclusion and further work	68
3	Fluctuations in Drag between Composite Fermions	71
3.1	Composite Fermions - review	71
3.1.1	Edge States	72
3.1.2	FQHE	73
3.1.3	Composite Fermions	74
3.2	UCF at $\nu = 1/2$ - existing work	76
3.3	Drag at $\nu = 1/2$ - existing work	78
3.4	Average drag behaviour	81

3.5	Non-linearity of Drag fluctuations	87
3.6	Conclusion and Further Work	89
4	Transport properties of graphene in strong magnetic fields	91
4.1	Introduction	91
4.2	Sample description	92
4.3	Graphene physics	93
4.4	Anomalous Hall effect	96
4.5	The Shubnikov-de Haas effect in graphene	97
4.6	Conductance fluctuations in strong magnetic fields	99
4.6.1	The case of modified diffusion	99
4.6.2	The case of tunnelling	102
4.7	Basic characterisation	107
4.8	Quantum lifetime in graphene	110
4.9	Resistance fluctuations in the IQHE regime	114
4.10	Fluctuations in the centre of the Landau level	116
4.11	Fluctuations in the flanks of the Landau level	119
4.12	Comparison with gate voltage fluctuations	121
4.13	Conclusion	124
5	Conclusion and suggestions for future work	126
	Bibliography	130
A	Appendix	135
A.1	Joule Heating - temperature of active and passive layers	135

List of Figures

1.1	The 0D DOS of a 2DEG system in the presence of a strong B -field where the LLs have been broadened by scattering. Near the centre of LLs the states are extended, whilst near the tails of the LLs the states are localised.	26
1.2	Measurements performed by von Klitzing of ρ_{xx} and ρ_{xy} as a function of B for a GaAs – Al _{0.3} Ga _{0.7} As heterojunction with concentration $n = 3.7 \times 10^{11} \text{cm}^{-2}$ demonstrating the key features of the IQHE [5]. .	27
1.3	The temperature dependence of the scaled frictional drag scattering rate τ_D^{-1}/T^2 for a sample with a 500 Å thick barrier. The dashed line is the expected drag scattering rate, calculated using Eq. 1.11. Inset: the same dependence plotted for several samples with different barrier thickness (500, 225, 175) after the calculated Coulomb drag using Eq. 1.11 has been subtracted. Taken from [16].	32
1.4	The scaled drag resistivity $\rho_t T^{-2}$ as a function of the reduced temperature T/T_F for several different matched carrier densities: $n = 1.37, 1.80, 2.23,$ and $2.66 \times 10^{11} \text{cm}^{-2}$. The dashed (solid) lines are the random phase approximation (Hubbard) calculations. Taken from [22].	34
1.5	Relative correction to the interlayer scattering rate τ_B/τ_Δ as a function of sample mobility for several different temperatures. Taken from [23].	35
1.6	Schematic of wafer used in double-layer samples. The 2DEG are formed in the two 200 Å thick GaAs layers. FFG, DFG, FBG, DBG are the full front and defining front gates, full back and defining back gates, respectively. All AlGaAs layers are actually Al _{0.33} Ga _{0.67} As. Adapted from [27].	36

1.7	Optical image of sample NA06. Contacts 8, 16, 21 and 26 are front defining gates; 4, 11, 12, 18 and 24 are back defining gates; 25/17 is the full front gate, 5 is the full back gate, whilst the other contacts are ohmic contacts to the Hall bar. Whether a contact is connected to the upper or lower layer is dependent upon which of the defining gates is charged.	37
1.8	Illustration of the selective depletion technique used to define the layers, such that there are two independently contacted 2DEG layers. In A , all defining gate voltages are zero and all connecting arms to the Hall bar are conducting: current flows through both layers. In B , a voltage is applied to the defining front gate such that the 2DEG in arms of the top layer becomes depleted: current can now only pass through the bottom layer. In C , voltages are applied to both the defining front and defining back gates, and the sample is now defined. In D , currents are put through both the top and bottom layers, demonstrating that the two layers are contacted independently. Taken from [27].	38
1.9	The conductance measured between contacts in the upper and lower layers as the sample is defined. The black curve is the conductance as a function of the front defining gate voltage whilst the back defining gate voltage is zero. The red curve is the conductance as a function of the back defining gate voltage whilst the front defining gate voltage is at 0.3 V. The actual defining-gate voltages required to deplete the arms to the Hall bar vary between cool-downs. Typical values are 0.3 V for the front defining gates, and 1.2 V for the back defining gates.	39
1.10	Two terminal conductance measurement. A constant voltage V_c is set using a potential divider formed by resistances R_1 and R_2 . The sample has a resistance R and contact resistances r . The current through the sample, I_m , is measured using a current pre-amp.	42
1.11	Four terminal resistance measurement. A constant current I is set using a large ballast resistance $R_B \gg R, r$. The sample has a resistance R between the probes and contact resistances r . The voltage drop V_m is measured across the sample using a voltage pre-amp.	43

1.12	The resistivity as a function of concentration for the upper layer in sample NA04; $T = 4.2$ K.	44
1.13	Drag voltage measurement circuit. The passive layer can be grounded at point A or B . Ideally, the voltage V_D measured in the passive layer is the drag voltage. However, in the case of current flowing between the layers due to tunnelling or capacitive coupling, there can be additional contributions to this voltage.	45
1.14	The voltage measured across the passive for different grounding points of the passive layer in sample NA04. The green and red curves are for points A and B grounded, respectively; $T = 4.2$ K. Inset: schematic representation of V_A and V_B measurements if there is a large leakage current from the active layer into the passive layer. The black curve is the average of the two measurements, $(V_A + V_B)/2$ and corresponds to the drag signal.	47
1.15	The drag resistance as a function of passive-layer carrier density for two different active-layer carrier densities, $n_1 = 2.75 \times 10^{11} \text{ cm}^{-2}$ (black) and $n_1 = 1 \times 10^{11} \text{ cm}^{-2}$ (red) in sample NA04. The dashed lines are best fits using $n_2^{-3/2}$ and the dotted lines are best fits n_2^{-2} ; $T = 4.2$ K.	48
1.16	Drag voltage as a function of passive-layer concentration for different fixed active-layer concentrations. Red circles denote points where the active- and passive-layer concentrations are matched in sample NA04. Note that a peak occurs in the drag resistance when the layer concentrations are matched, indicating a phonon-mediated drag contribution; $T = 4.2$ K.	49
1.17	The drag resistance as a function of temperature for matched carrier densities in the two layers, $n = 1 \times 10^{11} \text{ cm}^{-2}$. The solid line is the calculated Coulomb drag using Eq. 1.12, with a prefactor of 4 included (this factor has been seen by other groups, as discussed in main text).	50
2.1	Two different trajectories that an electron can follow when passing through a disordered two-terminal sample.	54

2.2	Two terminal sample that has been divided into coherent regions of L_φ^2 , the conductance of which display a variance of ΔG	55
2.3	Sketch of the expected T -dependence of the measured drag conductivity in the three different regimes described in [36]: $T > T_*$, $E_{Th}(L)/k_B < T < T_*$, and $T < E_{Th}(L)/k_B$ (dashed lines indicate approximate transition temperatures). T_* is the crossover temperature at which the size of mesoscopic drag resistance fluctuations is equal to the size of the average drag resistance.	58
2.4	Physical model of drag resistance fluctuations in the ballistic regime of drag. Δr is the distance an electron travels within a layer between e - e scattering events with electrons in the other layer.	60
2.5	A : Single-layer resistance as a function of magnetic field for different temperatures; temperatures as described in legend. A resistance of 480Ω has been subtracted at the point of measurement, as described in text. B : Single-layer resistance, after having background magnetoresistance removed, as a function of magnetic field for different temperatures. The concentration is held constant, $n_1 = 5.8 \times 10^{10} \text{ cm}^{-2}$; $I = 10 \text{ nA}$	62
2.6	Amplitude of the single-layer resistance as a function of temperatures. The solid line is a calculation of the variance expected using Eq. 2.5, multiplied by a factor of 4 for ease of comparison to measured data. Inset: τ_φ as a function of temperature for the single-layer UCF. The black and red lines are fits using the diffuse and ballistic expressions for τ_φ described in Eq. 2.1, respectively. The concentration is held constant, $n_1 = 6.1 \times 10^{10} \text{ cm}^{-2}$; $I = 10 \text{ nA}$	63
2.7	Drag resistivity as a function of passive-layer carrier concentration for different temperatures: $T = 5, 4, 3, 2, 1, 0.4$, and 0.24 K , from top to bottom. Active-layer concentration is held constant, $n_1 = 1.1 \times 10^{11} \text{ cm}^{-2}$; $I = 675 \text{ nA}$	64

2.8	<p>Drag resistance measured at low temperatures as a function of passive-layer concentration; $T = 1, 0.4,$ and 0.24 K, from top to bottom. Inset: ρ_D as a function of T for two values of n_2 denoted by the dotted lines in main panel; solid line is the expected T^2 dependence of the average drag.</p>	65
2.9	<p>Drag resistivity as a function of magnetic field for different temperatures; $T = 0.24, 0.3, 0.35, 0.4$ K. The graphs at higher T are offset by $25 \text{ m}\Omega$ for clarity. The single-layer concentrations are held constant, $n_1 = 1.1 \times 10^{11} \text{ cm}^{-2}, I = 700 \text{ nA}$.</p>	66
2.10	<p>Drag resistivity as a function of magnetic field demonstrating onset of Shubnikov-de Haas oscillations in the drag resistance. The single-layer concentrations are held constant, $n_{1,2} = 6 \times 10^{10} \text{ cm}^{-2}$. The top left inset is a zoomed in view of the region in which SdH oscillations are present. The top right inset shows the symmetry of drag resistivity fluctuations with changing the direction of magnetic field.</p>	67
2.11	<p>The ACF calculated for a set of drag conductance fluctuations (black line) and single-layer conductance fluctuations (red line). The single-layer concentrations are held constant, $n_{1,2} = 6 \times 10^{10} \text{ cm}^{-2}$. The left and right insets show histograms of the distribution of the single-layer and drag resistance fluctuations, respectively.</p>	68
2.12	<p>The variance of the drag conductance fluctuations (black squares) plotted as a function of T. The solid lines are plots of Eq. 2.13 using the diffusive (solid line) and ballistic (dashed) values for $\tau_\varphi(T)$. The values of $\tau_\varphi(T)$ are found from the single-layer conductance fluctuations and are shown in the inset. The single-layer concentrations are held constant, $n_{1,2} = 6 \times 10^{10} \text{ cm}^{-2}$.</p>	69
3.1	<p>Left: the energy of Landau levels as a function of position along the width of a Hall bar. The dots represent occupied states and the dashed line is the Fermi level. Right: the arrangement of edge states in a Hall bar when the Fermi level is between Landau levels. The N denotes the presence of N edge states, two of which are depicted. The direction of the electron's drift velocity is shown by the arrows.</p>	72

3.2	The longitudinal and Hall resistivities as a function of magnetic field for a high mobility sample at low temperatures that clearly shows the FQHE. Figure taken from [45]; $\mu = 1.3 \times 10^6 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$, $n = 3.0 \times 10^{11} \text{ cm}^{-2}$, and $T = 150 \text{ mK}$	73
3.3	The temperature dependence of the rms amplitude of resistance fluctuations at $\nu = 1/2$. The squares are taken from magnetoresistance fluctuations, and the crosses from fluctuations as a function of gate voltage. Figure taken from [55].	77
3.4	The drag resistance R_T as a function of magnetic field for three different T . The concentration of each layer is $n = 3.2 \times 10^{11} \text{ cm}^{-2}$ and the interlayer separation is $d = 30 \text{ nm}$. For comparison the single-layer resistance R_{xx} is also seen, and it is clear that there is a twin-peaked structure in the drag resistance associated with each LL peak in R_{xx} . Figure taken from [57].	79
3.5	The drag resistivity ρ_D and single-layer resistivity ρ_{xx} as a function of magnetic field; $T = 0.6 \text{ K}$. Figure taken from [61].	80
3.6	A comparison of the drag (red) and single-layer (black) resistivities as a function of magnetic field; $T = 5.6 \text{ K}$, $n_{1,2} = 5.6 \times 10^{10} \text{ cm}^{-2}$. The labels show the values of the filling factor; $\nu = 1, 2/3, 1/2$ are shown.	81
3.7	The drag resistance as a function of magnetic field, for different temperatures. $T = 0.05, 0.14, 0.2, 0.8, 5.6 \text{ K}$ from bottom to top; and $n_{1,2} = 1.45 \times 10^{11} \text{ cm}^{-2}$. The appearance of mesoscopic fluctuations is seen below 0.8 K , and dominate the drag resistance below 0.2 K . Plots are offset upwards by 50 Ohms from the lowest temperature. Inset: The drag resistivity as a function of temperature, taken at $\nu = 4.99$, as indicated by vertical dotted line in main figure. The solid line is a fit using Eq. 3.10.	82

3.8	The drag resistance as a function of gate voltage for a fixed $B = 11.45$ T and $T = 50$ mK. The top graph is for a fixed passive-layer gate voltage whilst the active-layer gate voltage is swept. The bottom graph is for fixed active-layer gate voltage whilst the passive-layer gate voltage is swept. Red and black lines are repeat measurements, demonstrating reproducibility of resistance fluctuations, which occur both as a function of active-layer and passive-layer concentration. The driving current is $I = 10$ nA.	83
3.9	The drag resistivity as a function of filling factor for $T = 50$ mK for varying carrier concentration and fixed magnetic field $B = 12$ T (black) and varying B and fixed concentration $n = 1.45 \times 10^{11}$ cm ⁻² (red). The driving current used is $I = 10$ nA.	84
3.10	Top: The drag resistance fluctuations, after the average background has been subtracted, as a function of filling factor for $T = 50$ mK for varying carrier concentration and fixed magnetic field $B = 12$ T ($\Delta\rho_D(n)$, red) and varying B and fixed concentration $n = 1.45 \times 10^{11}$ cm ⁻² ($\Delta\rho_D(B)$, black). Bottom: Comparison of ACF of the two sets of fluctuations in the top panel.	85
3.11	Main: The variance of drag resistivity fluctuations plotted against T ; $n = 1.45 \times 10^{11}$ cm ⁻² . The solid line is a theoretical fit using Eq. 3.12 and introducing a prefactor of 900. Inset: T dependence of L_ϕ^{cf} measured directly from the correlation concentration of the drag resistivity fluctuations. Solid line is a theoretical fit using $L_\phi^{cf} = 1.1\sqrt{D\hbar g_{cf}/k_B T \ln g_{cf}}$ [31].	87
3.12	The drag resistance as a function of filling factor for $T = 50$ mK (B varied, $T = 50$ mK and $n = 1.45 \times 10^{11}$ cm ⁻² are held constant) for different driving currents. Top left: $I = 3$ nA (black and red), 10 nA (green and olive), and 30 nA (blue). Bottom left: 100 pA (black), 300 pA (red), 1 nA (green), and 3 nA (blue). Top right: The current dependence of the variance of the fluctuations.	88

3.13 The single-layer resistivity as a function of filling factor (n varied, $B = 6$ T) for different currents $I = 0.1$ nA (black), 0.3 nA (red), and 1 nA (green); $T = 50$ mK. Joule heating is absent below 0.3 nA. Inset: Drag resistivity versus filling factor (n swept, $B = 12$ T held constant) for different driving currents; $T = 800$ mK. 89

4.1 Upper figure: Optical image of graphene sample G13DC3 with contacts, labels, and dimensions overlaid. The dashed line indicates the boundary of the mono-layer graphene flake. Lower figure: profile diagram of the sample – graphene flake is deposited on top of an insulating layer of silicon dioxide that covers the n^+ silicon substrate. By applying a voltage between the silicon layer and graphene flake the silicon layer acts as a gate, allowing the carrier concentration in the graphene flake to be controlled. 92

4.2 Left: the lattice structure of graphene, formed of two sub-lattices, A and B . The lattice unit vectors are a_1 and a_2 , whilst δ_i are the nearest neighbour vectors. Right: the Brillouin zone, where K and K' denote the Dirac points, and b_1 and b_2 are the reciprocal lattice vectors. Figure taken from [64]. 94

4.3 Energy spectrum with a zoomed-in view of energy bands around a Dirac point. The energies are plotted in units of 2.7 eV, the nearest-neighbour hopping energy. Figure taken from [64]. 94

4.4 Energy bands in graphene – a cross-section of $E(k_x)$ in Fig. 4.3 near the K and K' crossing points. The shaded strip is zoomed-in to show that the dispersion relation is linear for energies near the Dirac point. Depending upon the position of the Fermi level relative to the Dirac point, charge-carriers excitations are electron-like or hole-like. 96

4.5 The Hall conductivity and longitudinal resistivity as a function of concentration in graphene for $B = 14$ T and $T = 4$ K. Inset: the Hall conductivity in bi-layer graphene, demonstrating the conventional quantisation, but with the first step occurring at $n = 0$ unlike in usual 2D systems. Figure taken from [68]. 97

4.6	The Landau levels for normal electrons (left) and Dirac fermions (right). Figure taken from [72].	98
4.7	A disordered 2D conductor in a strong magnetic field, which was the model used in [77]. The circles represent the cyclotron orbits of real electrons, whilst the arrows represent the motion of the quasiparticles - the centres of the cyclotron-orbits.	101
4.8	Numerical calculations of [77] for the average conductance (solid circles) and amplitude of conductance fluctuations (open circles) for the $\nu = 2.5$ Landau level. B_N is the magnetic field at $\nu = 2.5$	102
4.9	Illustration of edge state configurations as a function of Fermi energy μ . The left column of figures shows the relative energies of μ and two Landau levels; the centre column - the corresponding edge state configurations (where shaded regions represent energies above μ and dashed lines represent tunnelling paths); and the right column the corresponding values of B . In (a) μ lies well between two LLs, corresponding to well-separated edge states and B in an R_{xx} minimum. In (b) μ is lower in energy, corresponding to islands in the Fermi sea and B at the high- B side of an R_{xx} minimum. In (c) μ is lower yet in energy, corresponding to Fermi lakes on insulating land, and B at the low- B side of an R_{xx} minimum. Transport in (c) occurs by tunnelling along the length of the channel. Taken from [53].	104
4.10	A simplified picture of a disorder-induced potential hill in a LL that lies below the Fermi level μ . This results in the existence of a localised edge state that allows tunnelling between edge states on opposite sides of the sample. Adapted from [53].	105
4.11	The resistivity as a function of back-gate voltage measured at 300 K (black line) and 5 K (red line). Inset: The conductivity as a function of concentration, measured at 5.6 K. Outside the vicinity of the Dirac point the conductivity varies linearly with concentration, reflecting a constant mobility. The mobilities of holes and electrons are nearly identical.	107

4.12	The longitudinal resistivity and Hall resistivity as a function of magnetic field for different electron concentrations: $n = 1$ (black), 1.5 (red), 2 (green), 3 (blue) $\times 10^{12}$ cm^{-2} ; $T = 5.6$ K.	108
4.13	The longitudinal (black and red) and Hall (green and blue) conductivities as a function of gate voltage, measured at 5.6 K with $B = 12.5$ T. The presence of a LL at zero energy is clear from the peak in σ_{xx} at zero gate voltage and the half-integer filling factor is apparent from the quantised values of σ_{xy}	109
4.14	Left panel: the longitudinal resistivity ρ_{xx} (upper graph) and its isolated oscillating component $\Delta\rho_{xx}$ (lower graph) as a function of magnetic field; $T = 5.5$ K, $n = 1.5 \times 10^{12}$ cm^{-2} . Right panel: the logarithm of the peaks of the SdH oscillations plotted against inverse magnetic field. The solid line is a linear fit, the gradient of which is proportional to $1/\tau_q$	111
4.15	Density dependence of τ_q for hole and electron regions for different temperatures. The solid lines are power-law fits: $\tau_q \propto n^{0.47}$ in the hole region, and $\tau_q \propto n^{0.65}$ in the electron region.	112
4.16	Comparison of τ_q (black squares) and τ_p (red circles) as a function of concentration. Solid lines are power-law fits. Measurements done at $T = 5.5$ K.	113
4.17	R_{xx} as a function of B for $n = 1.5 \times 10^{12}$ cm^{-2} ; $T = 50$ mK. The filling factor values are found from the position in B of centres of minima in R_{xx}	114
4.18	R_{xx} as a function of B for different temperatures: $T = 0.05, 0.4,$ and 1.36 K; $n = 1.5 \times 10^{12}$ cm^{-2} . Inset: the average background $R_{xx}(B)$ with the mesoscopic fluctuations removed by averaging of the data, as described in <i>S4.10</i>	115
4.19	R_{xx} as a function of gate voltage at $T = 50$ mK; $B = 7.3$ T, corresponding to the transition $\nu = 6 \rightarrow 10$. The different graphs are for different sweeps in the same direction, reflecting a tendency for charging to occur in the substrate. (Note that the resistance peak as a function of B was stable in Fig. 4.18 for a fixed V_G .)	116

4.20	The variance of the fluctuations in R_{xx} as a function of B for various T ; $n = 1.5 \times 10^{12} \text{ cm}^{-2}$, corresponding to the centre of the resistance peak in the transition $\nu = 6 \rightarrow 10$. Inset: The correlation magnetic field of the fluctuations in R_{xx} as a function of B for various T	117
4.21	The temperature dependence of the variance of the $R_{xx}(B)$ fluctuations near the centre of the resistance maximum; $n = 1.5 \times 10^{12} \text{ cm}^{-2}$. Inset: temperature dependence of the correlation magnetic field.	118
4.22	Top: colour-coded index of the division of the resistance peak fluctuations into regions prior to calculating the PSD. Bottom 3 panels: The PSD of the fluctuations. There is some suggestion of a dominant frequency (shaded) in the left flank of the LL, indicating a tunnelling process via a single localised edge state. The central region and right flank have a variety of frequency components.	119
4.23	The PSD of the fluctuations in the low-field flank of the fluctuations in $R_{xx}(B)$ for different temperatures; $T = 0.05, 0.1, 0.2, 0.4 \text{ K}$, $n = 1.5 \times 10^{12} \text{ cm}^{-2}$. Inset: temperature dependence of the peak in the PSD at the dominant magnetic frequency $f_B = 14 \text{ T}^{-1}$	120
4.24	The autocorrelation function of the fluctuations of R_{xx} near the centre of the resistance peak, measured by varying B (left) and V_G (right) . .	122
4.25	Top: The longitudinal resistance as a function of the energy after the average background has been subtracted; $T = 0.05 \text{ K}$, $B = 7.3 \text{ T}$. Bottom: the PSD of the fluctuations in different regions of the LL. . .	123
A.1	Variance of the SdH oscillations as a function of the temperature of the fridge thermometer for three different currents: $I = 1 \text{ nA}$ (black), 10 nA (red), and 100 nA (green); $T = 240 \text{ mK}$, $n_{1,2} = 6 \times 10^{10} \text{ cm}^{-2}$	135
A.2	The variance single-layer conductance as a function of temperature for two different currents: $I = 100 \text{ nA}$ (black) and 10 nA (red); $n_{1,2} = 6 \times 10^{10} \text{ cm}^{-2}$	136
A.3	Drag resistivity fluctuations (after an average background drag resistivity has been subtracted) as a function of magnetic field measured using two different currents: $I = 300 \text{ nA}$ (black) and 700 nA (red); $T = 240 \text{ mK}$, $n_{1,2} = 6.8 \times 10^{10} \text{ cm}^{-2}$	137

List of Tables

1.1	Transport properties of double-layer samples.	44
3.1	Properties of composite fermions for our samples when $n_{1,2} = 1.45 \times 10^{11} \text{ cm}^{-2}$. The effective mass is taken from [50], where $m_b = 0.067m_e$ is the band mass in GaAs.	83
4.1	Theoretical prediction for the n -dependence of τ_p for different sources of scattering in conventional 2D systems [83]. For the case of remote impurity scattering at high concentrations, a is the quantum well width of the 2DEG.	112
4.2	Expected ratio of τ_p to τ_q for different sources of scattering in conventional 2D systems [83]. For the case of background impurities, q_{TF} is the Thomas-Fermi screening wavenumber, and C_2 is a coefficient that depends on the quantum well width, q_{TF} and k_F . (The ratio τ_p/τ_q increases linearly with increasing n for the case of background impurities [83].) For the case of remote impurity scattering, z_i is the distance of the impurities from the 2DEG. For the case of roughness scattering, Λ is the correlation length of the fluctuations in the height of the SiO_2 interface.	113

Symbols and Abbreviations Used

ACF : autocorrelation function

CF : composite fermion

D : diffusion coefficient

DOS : density of states

$e-e$: electron-electron

E_{Th} : Thoules energy

g : dimensionless conductance, $\sigma/(e^2/h)$

$\varrho(E)$: The density of states

g_s : The spin degeneracy, taking values of 1 or 2

g_v : The valley degeneracy, taking values of 1 or 2

l : elastic mean free path

L : distance between voltage probes; equivalently, the sample size

LL : Landau level

L_φ : coherence length

L_T : thermal length

μ : mobility

n : carrier concentration

ν : Landau-level filling factor

PSD : power spectral density

ρ_D : drag resistivity

SdH : Shubnikov-de Haas

T : temperature

τ : momentum relaxation time.

τ_q : quantum lifetime.

τ_φ : coherence time

UCF : universal conductance fluctuations

ω_c : cyclotron frequency

W : sample width

List of publications

Publications

- A. S. Price, A. K. Savchenko, B. N. Narozhny, G. Allison, D. A. Ritchie, Giant Fluctuations of Coulomb Drag in a Bilayer System, *Science* **316**, 99-102 (2007).
- A. S. Price, A. K. Savchenko, G. Allison, B. N. Narozhny, Giant Fluctuations of Coulomb Drag, *Physica E: Low-dimensional Systems and Nanostructures* **40**, 961-966 (2008).

Conference presentations

- A. S. Price, A. K. Savchenko, B. N. Narozhny, G. Allison, D. A. Ritchie, Giant Fluctuations of Coulomb Drag in a Bilayer System, *Condensed Matter and Material Physics (CMMP 2007)* (Poster presentation).
- A. S. Price, A. K. Savchenko, G. Allison, B. N. Narozhny, Giant Fluctuations of Coulomb Drag, *International Conference on Electronic Properties of Two-dimensional Systems (EP2DS 17)* July, 2008 (Invited talk).