Dynamic Portfolio Construction and Portfolio Risk Measurement

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September 2011

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Submitted by Murat Mazibas to the University of Exeter as a thesis for the degree of Doctor of Philosophy in Finance in September 2011.

I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

Signature:…………………………………………………………….

Abstract

The research presented in this thesis addresses different aspects of dynamic portfolio construction and portfolio risk measurement. It brings the research on dynamic portfolio optimization, replicating portfolio construction, dynamic portfolio risk measurement and volatility forecast together. The overall aim of this research is threefold. First, it is aimed to examine the portfolio construction and risk measurement performance of a broad set of volatility forecast and portfolio optimization model. Second, in an effort to improve their forecast accuracy and portfolio construction performance, it is aimed to propose new models or new formulations to the available models. Third, in order to enhance the replication performance of hedge fund returns, it is aimed to introduce a replication approach that has the potential to be used in numerous applications, in investment management.

In order to achieve these aims, Chapter 2 addresses risk measurement in dynamic portfolio construction. In this chapter, further evidence on the use of multivariate conditional volatility models in hedge fund risk measurement and portfolio allocation is provided by using monthly returns of hedge fund strategy indices for the period 1990 to 2009. Building on Giamouridis and Vrontos (2007), a broad set of multivariate GARCH models, as well as, the simpler exponentially weighted moving average (EWMA) estimator of RiskMetrics (1996) are considered. It is found that, while multivariate GARCH models provide some improvements in portfolio performance over static models, they are generally dominated by the EWMA model. In particular, in addition to providing a better risk-adjusted performance, the EWMA model leads to dynamic allocation strategies that have a substantially lower turnover and could therefore be expected to involve lower transaction costs. Moreover, it is shown that these results are robust across the low - volatility and high-volatility sub-periods.

Chapter 3 addresses optimization in dynamic portfolio construction. In this chapter, the advantages of introducing alternative optimization frameworks over the mean-variance framework in constructing hedge fund portfolios for a fund of funds. Using monthly

return data of hedge fund strategy indices for the period 1990 to 2011, the standard mean-variance approach is compared with approaches based on CVaR, CDaR and Omega, for both conservative and aggressive hedge fund investors. In order to estimate portfolio CVaR, CDaR and Omega, a semi-parametric approach is proposed, in which first the marginal density of each hedge fund index is modelled using extreme value theory and the joint density of hedge fund index returns is constructed using a copula-based approach. Then hedge fund returns from this joint density are simulated in order to compute CVaR, CDaR and Omega. The semi-parametric approach is compared with the standard, non-parametric approach, in which the quantiles of the marginal density of portfolio returns are estimated empirically and used to compute CVaR, CDaR and Omega. Two main findings are reported. The first is that CVaR-, CDaR- and Omega-based optimization offers a significant improvement in terms of risk-adjusted portfolio performance over mean-variance optimization. The second is that, for all three risk measures, semi-parametric estimation of the optimal portfolio offers a very significant improvement over non-parametric estimation. The results are robust to as the choice of target return and the estimation period.

Chapter 4 searches for improvements in portfolio risk measurement by addressing volatility forecast. In this chapter, two new univariate Markov regime switching models based on intraday range are introduced. A regime switching conditional volatility model is combined with a robust measure of volatility based on intraday range, in a framework for volatility forecasting. This chapter proposes a one-factor and a two-factor model that combine useful properties of range, regime switching, nonlinear filtration, and GARCH frameworks. Any incremental improvement in the performance of volatility forecasting is searched for by employing regime switching in a conditional volatility setting with enhanced information content on true volatility. Weekly S&P500 index data for 1982-2010 is used. Models are evaluated by using a number of volatility proxies, which approximate true integrated volatility. Forecast performance of the proposed models is compared to renowned return-based and range-based models, namely EWMA of Riskmetrics, hybrid EWMA of Harris and Yilmaz (2009), GARCH of Bollerslev (1988), CARR of Chou (2005), FIGARCH of Baillie *et al*. (1996) and MRSGARCH of Klaassen (2002). It is found that the proposed models produce more accurate out of sample forecasts, contain more information about true volatility and exhibit similar or better performance when used for value at risk comparison.

Chapter 5 searches for improvements in risk measurement for a better dynamic portfolio construction. This chapter proposes multivariate versions of one and two factor MRSACR models introduced in the fourth chapter. In these models, useful properties of regime switching models, nonlinear filtration and range-based estimator are combined with a multivariate setting, based on static and dynamic correlation estimates. In comparing the out-of-sample forecast performance of these models, eminent return and range-based volatility models are employed as benchmark models. A hedge fund portfolio construction is conducted in order to investigate the out-of-sample portfolio performance of the proposed models. Also, the out-of-sample performance of each model is tested by using a number of statistical tests. In particular, a broad range of statistical tests and loss functions are utilized in evaluating the forecast performance of the variance covariance matrix of each portfolio. It is found that, in terms statistical test results, proposed models offer significant improvements in forecasting true volatility process, and, in terms of risk and return criteria employed, proposed models perform better than benchmark models. Proposed models construct hedge fund portfolios with higher risk-adjusted returns, lower tail risks, offer superior risk-return tradeoffs and better active management ratios. However, in most cases these improvements come at the expense of higher portfolio turnover and rebalancing expenses.

Chapter 6 addresses the dynamic portfolio construction for a better hedge fund return replication and proposes a new approach. In this chapter, a method for hedge fund replication is proposed that uses a factor-based model supplemented with a series of risk and return constraints that implicitly target all the moments of the hedge fund return distribution. The approach is used to replicate the monthly returns of ten broad hedge fund strategy indices, using long-only positions in ten equity, bond, foreign exchange, and commodity indices, all of which can be traded using liquid, investible instruments such as futures, options and exchange traded funds. In out-of-sample tests, proposed approach provides an improvement over the pure factor-based model, offering a closer match to both the return performance and risk characteristics of the hedge fund strategy indices.

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Abbreviations

AIC	:	Akaike Information Criteria
ACD	:	Autoregressive Conditional Duration
ARCH	:	Autoregressive Conditional Heteroskedasticity
BEKK	:	Baba, Engle, Kraft and Kroner model
CARR	:	Conditional Autoregressive Range model
CDaR	:	Conditional Drawdown-at-Risk
CSR	:	Conditional Sharpe Ratio
CVaR	:	Conditional Value-at-Risk
CCC	:	Constant Conditional Correlation
CA	:	Convertible Arbitrage strategy
CDF	:	Cumulative Distribution Function
DA	:	Directional Accuracy test
DS	:	Distressed Securities strategy
EM	:	Emerging Markets strategy
EH	:	Equity Hedge strategy
EMN	:	Equity Market Neutral strategy
ED	:	Event Driven strategy
EWMA	:	Exponentially Weighted Moving Average model
EVT	:	Extreme Value Theory
FLF	:	Firm Loss Function
FIGARCH	:	Fractionally Integrated Autoregressive Conditionally Heteroskedastic model
FOF	:	Fund of Hedge Funds strategy
GARCH	:	Generalized Autoregressive Conditional Heteroskedasticity model
GPD	:	Generalized Pareto Distribution
HFR	:	Hedge Fund Research
HMSE	:	Heteroskedasticity adjusted Mean Square Error
HFD	:	High-Frequency Data
HybEWMA	:	Hybrid EWMA model
LPM	:	Lower Partial Moment
MAC	:	Macro strategy
MRSACR	:	Markov Regime Switching Autoregressive Conditional Range model
MRSGARCH	:	Markov Regime Switching GARCH model
MDD	:	Maximum Drawdown
MAE	:	Mean Absolute Error
MV	:	Mean-Variance optimization model
MA	:	Mergers Arbitrage strategy
MDM	:	Modified Diebold-Mariano test

MEM	:	Multiplicative Error Model
OMG	:	Omega Ratio
QLIKE	:	Q-Likelihood Loss Function
RC	:	Reality Check test
RSDC	:	Regime Switching Dynamic Correlation model
RLF	:	Regulator Loss Function
RV	:	Relative Value Arbitrage strategy
RoMDD	:	Return on Maximum Drawdown
RMSE	:	Root Mean Square Error
BIC	:	Schwarz Information Criteria
SS	:	Short Selling strategy
SR	:	Success Ratio Test
SPA	:	Superior Predictive Ability test
SPA	:	Superior Predictive Ability test
TUFF	:	Time Until First Failure test
UPM	:	Upper Partial Moment
VaR	:	Value at Risk
VCV	:	Variance Covariance Matrix

Author's Declaration

I hereby declare that the research presented in this thesis incorporates published materials that are results of joint research conducted during my PhD studies, as follows:

Chapter 2 of the thesis is published in the Journal of International Review of Financial Analysis, with the reference of "Harris, R.D.F. and Mazibas, M. (2010). 'Dynamic Hedge Fund Portfolio Construction', *International Review of Financial Analysis* 19, 351–357."

Chapter 6 of the thesis is published as a chapter in the book titled 'Hedge Fund Replication', with the reference of "Harris, R.D.F. and Mazibas, M. (2011), 'Factor-Based Hedge Fund Replication with Risk Constraints', Gregoriou, G. N. and Maher, K. (eds)*, Hedge Fund Replication*, Palgrave-MacMillan (UK), 30–47."

Chapter 3 of the thesis is submitted to Journal of Banking and Finance for publication.

The published chapters are co-authored with my PhD supervisor Professor Richard D.F. Harris. Professor Harris provided editorial advice and guidance throughout the development of the models and the papers. I, Murat Mazibas, developed the models, carried out the analyses and wrote most of the papers.

I am aware of the University of Exeter's relevant regulations, and I certify that I have properly acknowledged the contribution of other researchers to my thesis, and have obtained permission from them to include the above materials in my thesis.

I certify that, with the above qualification, this thesis, and the research to which it refers, is the product of my own work.

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Acknowledgements

First of all, I am extremely grateful for the on-going support and extremely helpful guidance of my supervisor, Professor Richard D.F. Harris. With my humblest heart, I am full of appreciation for his guidance, patience, suggestions and every support for me over these years. His guidance helped me in all the time of research and writing of this thesis. I feel exceptionally proud to be his student, and it is a privilege to have this opportunity to learn from him and to make publications with him.

This thesis is dedicated to my wife, Annie, who was always available whenever I was in need of, and has always supported me through hard times. Without her support, patience, and endless efforts to keep me motivated and concentrated, this would not be possible. I am also deeply indebted to my parents, brothers and sisters, for their on-going support and patience over the years.

Last but not least, I would like to express my heartfelt gratitude to all my friends who always stand by me. In particular, my sincere gratitude goes to Professor C. Coskun Kucukozmen. I also would like to thank to Dooruj Rambaccussing and Felix Haller for stimulating discussions. All the cares given by them are deeply in my heart.

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Chapter 1

Introduction

The research presented in this thesis addresses different aspects of dynamic portfolio construction and portfolio risk measurement. It brings together dynamic portfolio optimization, replicating portfolio construction, dynamic portfolio risk measurement and volatility forecast for portfolio construction.

In particular, Chapter 2 and Chapter 3 address dynamic risk measurement and optimization in portfolio construction. Chapter 2 takes a step forward from the existing literature, and provides further evidence on the use of multivariate conditional volatility models in constructing dynamic hedge fund portfolios. Chapter 3 takes where Chapter 2 has left off, and addresses dynamic portfolio optimization in hedge fund portfolio construction by investigating the advantages of introducing alternative optimization frameworks over the mean-variance framework. This chapter proposes a new approach to eminent alternative optimization frameworks and provides evidence of the improvements that can be achieved in the performance of these frameworks.

Chapter 4 and Chapter 5 search for improvements in dynamic portfolio risk measurement through more accurate volatility forecast. In particular, Chapter 4 proposes two new univariate Markov regime switching models based on the intraday range in order to improve the accuracy of volatility forecasts for a better risk measurement. Chapter 5 complements Chapter 4 by introducing a multivariate version of the proposed univariate models. Chapter 5 provides evidence on possible improvements from the use of the proposed volatility models for dynamic risk measurement in portfolio construction. Chapter 6 takes a step forward from the available literature on hedge fund return replication, and proposes a new approach to constructing replicating portfolios that has several potential uses in portfolio management.

1.1. Aims, Objectives and Contributions

The overall aim of the research presented in this thesis is threefold. First, it aims to investigate dynamic portfolio construction performance of eminent risk measurement and optimization frameworks. Second, it aims to propose new models and new approaches to these models and frameworks in an effort to improve their performance of constructing dynamic portfolios. Finally, it aims to propose a (hedge fund) return replication approach that has the potential to be used in various applications, in investment management. The thesis is organized to best achieve the objectives and to address the different aspect of dynamic hedge fund portfolio construction and portfolio risk measurement.

The second chapter aims to examine the performance of multivariate volatility models in constructing dynamic hedge fund portfolios. At the same time, another aim is to provide further evidence on the use of multivariate conditional volatility models in dynamic measurement of hedge fund portfolio risk. The analysis in this chapter is built on the analysis of Giamouridis and Vrontos (2007) and considers a broader set of volatility models, including a number of additional multivariate GARCH models and the much simpler exponentially weighted moving average (EWMA) estimator of RiskMetrics (1996). In doing so, this chapter makes contribution to the empirical literature on portfolio risk measurement in dynamic hedge fund portfolio construction, by providing further evidence on the advantages of dynamic models over static models, and by establishing that the use of simple models over more sophisticated models produces better portfolio performance.

The third chapter aims to examine dynamic portfolio construction performance of alternative optimization frameworks, which are proposed against the mean-variance framework. Approaches based on CVaR, CDaR and Omega, are considered. At the same time, another aim of this chapter is to propose new formulations to these frameworks in order to improve their performance, and to make them better capture statistical and time series properties of the data. As there is an obvious literature gap in the use of alternative optimization frameworks in dynamic portfolio construction, this chapter also aims to show the potential uses of the alternative frameworks for investors with different risk tolerance.

This chapter contributes to the empirical literature on dynamic hedge fund portfolio optimization at three points. First, to the best knowledge of the author, this is the first study that implements EVT, copula and Monte Carlo simulation into alternative optimization frameworks and proposes a semi-parametric extension of the original frameworks for hedge fund portfolio construction. Second, although there are studies that compare CVaR to CDaR (Krokhmal et al., 2003), and Omega to CVaR (Hentati et al., 2010), this is the first study that compares CVaR, CDaR and Omega models together with a number of benchmark models including static and dynamic mean-variance model. Finally, this study contributes to the literature by considering different formulations of optimization frameworks based on an investor's risk tolerance.

The fourth chapter and fifth chapter complement each other. Following the renewed interest in using the intraday range, due to its attractive features as a variance measure, and the success of regime switching models in capturing abrupt changes in asset prices, the aim of the fourth chapter is to introduce an univariate volatility forecast framework that combines regime switching models with range-based variance estimator, hence, improves the accuracy of volatility forecasts, and fills the gap in the literature on range-based regime switching volatility modelling. In doing so, this chapter contributes to the empirical literature on conditional volatility models at two points. First, to the best knowledge of the author, this is the first study in applying range data to the regime switching models in a univariate context. Second, it is the first study that applies component structure to range data in a regime switching framework, and proposes one and two-component range-based regime switching models.

The fifth chapter mainly aims to extend the univariate models proposed in the fourth chapter into their multivariate equivalents. In doing so, this chapter contributes to the empirical literature on conditional volatility forecast and portfolio risk measurement in dynamic hedge fund portfolio construction. First, this chapter, for the best knowledge of the author, is the first study in applying range data to regime switching models in a multivariate context. Second, it is the first study which applies range-based models in hedge fund strategy indices through extracting high/low/open/close index values from daily strategy indices and comparing portfolio construction performance of range-based models to return based models.

The sixth chapter aims to provide a new approach to hedge fund return replication. In doing so, this chapter contributes to the literature on hedge fund return replication by introducing a composite approach that combines the factor and distribution-matching methodologies, and by empirically showing that the proposed approach represents an improvement over the out-of-sample performance of the factor and payoff distribution approaches reported by Jaeger and Wagner (2005) and Amenc et al. (2010).

1.2. Background and Motivation

Hedge funds have attracted much interest not only for their ability to generate relatively high average returns, but also for the large losses that they can incur, a risk that is exemplified by the rise and fall of Long Term Capital Management in the late 1990s. In spite of such risk, the hedge fund industry witnessed rapid growth in the 2000s, with assets under management reaching $1.93 trillion by early 2008. During the recent credit crisis, there was a significant reduction both in the number of hedge funds and assets under management, which resulted from a combination of trading losses and asset withdrawals by investors. However, by April 2011, it was estimated that hedge fund assets had recovered to their pre-crisis level (see Strasberg and Eder, 2011).

A critical factor contributing to the recent growth has been the availability of funds of hedge funds, which enable investors to access hedge fund alpha with lower risk, albeit at the expense of an additional layer of fees. Another contributing factor to the growth in the hedge fund industry was the launch in the early 2000s of investable hedge fund indices. These have generated further interest from small- and medium-sized investors, who would otherwise be precluded from investing in the hedge fund market. Central to both of these developments is the role of portfolio optimization in order to construct portfolios of individual hedge funds or hedge fund indices.

Another critical factor is the availability of the investment products, commonly known as ‘clones’, which replicate hedge fund returns and deliver the returns of hedge funds at lower cost and without the risks of direct hedge fund investments by employing statistical models or algorithmic trading strategies.

However, the optimal portfolio allocation across individual hedge funds and the replication of hedge fund returns is complicated by the fact that, owing to the strategies that hedge fund managers typically adopt, hedge fund returns are far from normally distributed, and they usually exhibit highly significant negative skewness and excess kurtosis (see, for example, Amin and Kat, 2001; Lo, 2001; Brooks and Kat, 2002; Fung and Hsieh, 1997a, 2001; Agarwal and Naik, 2004, Hudson *et al*., 2006; Wegener *et al*., 2010). These statistical properties of hedge fund returns defy the use of standard portfolio risk measurement and optimization methods, as well as, hedge fund replication approaches that ignore these properties. In the presence of these statistical properties, therefore, improving the risk measurement and optimization performance of hedge fund portfolios, and the replication performance of the clone portfolios creates a significant challenge for investors and the investment management companies who provide products to meet demand for these products.

1.2.1. Dynamic Portfolio Risk Measurement and Portfolio Optimization

In the literature, while non-normality of hedge fund returns is by now well-established, much less attention has been paid to the dynamic properties of hedge fund risk. In particular, the literature on hedge fund portfolio optimization has typically assumed a constant covariance structure of hedge fund returns, which leads to inaccuracies in the measurement of hedge fund risk and the optimization of hedge fund portfolios, particularly over shorter horizons, where time-variation in the covariance matrix of returns is most pronounced. Giamouridis and Vrontos (2007) is the first who show that the use of multivariate conditional volatility models improves the optimization of hedge fund portfolios and provides a more accurate tool for tail-risk measurement. They employ two static models (the sample covariance matrix and an implicit factor model) and three dynamic models (two implicit factor GARCH model and a regime switching dynamic correlations model). They compare the out-of-sample performance of optimised monthly and quarterly rebalanced portfolios of the Hedge Fund Research (HFR) indices for the period January 2002 to August 2005. They find that, in the mean-variance framework, the use of dynamic models generates portfolios of hedge fund indices with lower out-of-sample risk and higher realized returns.

As Giamouridis and Vrontos (2007) build their analysis on a few multivariate volatility models, and out-of-sample data belong to a relatively favourable period for hedge fund industry, the robustness of their results needs to be evaluated by considering a much broader set of the conditional covariance models and a longer out-of-sample evaluation period. The second chapter addresses these needs, and provides further evidence on the performance of a broader set of the conditional covariance models in constructing dynamic fund of hedge funds portfolios for a favourable, as well as, an unfavourable out-of-sample test period for the hedge fund industry.

While the use of multivariate conditional volatility models in the context of dynamic hedge fund risk measurement and portfolio allocation by employing mean-variance optimization framework is addressed by Giamouridis and Vrontos (2007) and the second chapter of this thesis, portfolio construction with optimization frameworks alternative to the mean-variance framework remains mostly untouched. Giamouridis and Vrontos (2007) compare mean-variance and mean-CVaR minimum risk portfolios with a return target constructed using HFR hedge fund strategy indices. Krokhmal *et al.* (2003) compare CVaR and CDaR approaches for a minimum risk portfolio of individual hedge funds, while Hentati *et al.* (2010) compare CVaR and Omega measures for portfolios including hedge funds for minimum risk portfolios. These alternative approaches rely on non-parametric estimation, in which the moments and quantiles of the density function of portfolio returns are estimated empirically, and these are used to compute the various risk measures used in the optimization process. The non-parametric approach, while straightforward to implement, relies on a large data sample to generate sufficiently accurate estimates of the various measures. Moreover, it does not readily lend itself to incorporating the well established dynamic characteristics of hedge fund returns, such as autocorrelation and volatility clustering. Therefore, optimization side of the dynamic hedge fund portfolio construction is called for further study. The third chapter of this thesis addresses the shortcomings of the non-parametric approach, proposes a semi-parametric approach to hedge fund portfolio optimization and extends the analysis for investors with different risk tolerances.

1.2.2. Portfolio Risk Measurement and Volatility Forecast

In recent years, there is a renewed interest in using the range as a variance measure. This is due to the attractiveness of the range-based volatility measures as an efficient, approximately normal and a robust measure to microstructure noise. The intraday range is defined as the difference between the highest and lowest log asset prices over the trading day. A significant practical advantage of the intraday range is that in contrast to intraday data, the range data for individual assets and indices are widely available over long historical spans.

Along with this renewed interest in the intraday range, financial markets also witness increased volatility and frequent changes in volatility states. This encourages using models which are able to capture the changes in volatility states. Regime switching models are well-known models that are able to capture the abrupt changes in asset prices due to financial crisis, government policy, business cycles or other economic fundamentals.

Considering the attractive features of the range as a volatility measure and established success of regime switching models in capturing shifts in volatility states, a new volatility model that combines the intraday range with regime switching, has the potential to improve volatility forecast in the presence of shifts in volatility regimes. The fourth chapter addresses this need and proposes two univariate models which combine useful properties of range, regime switching, nonlinear filtration and GARCH frameworks.

Although, at present, there is a renewed interest in range, most of the studies on the range are mainly in a univariate setting. Nonetheless, financial applications of volatility models require a multivariate framework, which includes not only time varying conditional variances but also conditional covariance of asset returns. However, the statistical property of range limits its use in estimating asset covariance, henceforth the multivariate volatility modelling using the range. Therefore, there are only a few studies on the application of the range in a multivariate setting (see Brandt and Diebold, 2006; Harris and Yilmaz, 2009; Chou *et al*., 2009 and Bannouh *et al*., 2009). To make the univariate models proposed in the fourth chapter useful for financial applications, including portfolio risk measurement in portfolio construction, developing their multivariate versions is called for. This need is addressed in the fifth chapter by introducing multivariate versions of the univariate models proposed in the fourth chapter.

1.2.3. Hedge Fund Return Replication

In parallel with the rapid growth in the hedge fund industry, there has been increased demand from investors for products that deliver the returns of hedge funds at lower cost, and without the risks, which are typically associated with hedge fund investment, such as illiquidity, lack of transparency and management specific risks. Investment banks and asset management firms have introduced investment products, commonly known as 'clones', in an effort to meet this growing demand. The clones seek to replicate hedge fund returns by employing statistical models or algorithmic trading strategies. Since replication of all hedge fund returns is not viable, the purpose of the replication is to capture a significant part of the systematic component with lower fees and better liquidity.

There are three broad approaches to hedge fund replication: the factor approach, the distribution-matching approach and the rule-based approach. Factor approaches are often able to generate a satisfactory fit to hedge fund returns in-sample, depending on the choice of factors and the time period, but are often found to have poor out-of-sample performance. The distribution-matching approach provides relatively better out-of-sample performance than factor approaches. However, it is considered more relevant to fund design than to return performance replication. The rule-based approach is often combined with factor-based replication, and it is proprietary in nature.

Distinctive statistical properties of hedge fund returns as a by-product of extremely complicated hedge fund strategies, and poor performance or incompatibility of the current approaches makes the replication of hedge fund returns more challenging. The need for a new approach that should consider systematic factors as well as the statistical properties of hedge fund returns, is addressed in the sixth chapter of this thesis.

1.3. Data

Along the thesis, depending on the requirements of the models employed in each chapter, different frequencies of the hedge fund return data is used. In the empirical analysis of the Chapters 2, Chapter 3 and Chapter 6, monthly return data of hedge fund strategy indices from Hedge Fund Research (HFR) database are used. However, in Chapter 5 weekly range and return data estimated from the daily strategy returns collected from the HFR database are used. In Chapter 2 and 5, to be comparable to Giamouridis and Vrontos (2007), eight hedge fund strategy indices, however, in Chapter 3 and 6, by including two additional strategy indices, a broader set of hedge fund strategies (ten strategy indices) are utilized. On the other hand, in Chapter 4, as the aim is to propose a new univariate volatility model, S&P500 stock index open/close/high/low prices are used. In this chapter, in an unreported work robustness of the proposed models is tested by using three foreign exchange data (i.e. USD, GBP and EUR) and another stock index (i.e. FTSE100) data. As the analysis provided in each chapter conducted at different times, an approach of using available dataset at the time of the analysis is adopted.

The empirical analyses conducted by using the hedge fund return data are performed by considering the certain limitations of the length and the quality of the available data. History of the hedge fund index datasets date back only very recently. Available index data at monthly frequency dates back to 1990's, while daily return data dates back only 2003. Therefore, it should be noted that the existing time series have a small number of observations and cover only the most recent past. Another crucial issue regarding the hedge fund data is the availability of certain bias in the series. Since hedge fund managers are not required to disclose information on the performance of their funds, currently available indices are constructed from individual databases collected on a voluntary basis, therefore their reliability is limited. All hedge fund indices mainly suffer from survivorship, selection and instant history biases (see Fung and Hsieh, 2001) and their performances are significantly different from each other. These biases often lead to available data to show evidence of the imprecise performance of hedge funds. These biases are common and unavoidable. For that reason, in this study other avoidable limitations are focused on and they are overcome by using the data provided by the databases that have received broad recognition in the sector and are commonly believed as precisely representing the characteristics of the sector.

1.4. Methodology

The methodology applied to the research presented in this thesis is carefully designed in each chapter to achieve the aims of the chapter and the overall aims of the thesis best.

1.4.1. Dynamic Portfolio Risk Measurement and Portfolio Optimization

The methodology applied to the research on dynamic portfolio risk measurement and portfolio construction is necessarily a hedge fund investment study[1]. In the investment

[1] In the investment study conducted in this thesis, hedging demands which are caused by the time-varying investment opportunity set are ignored. Therefore, this thesis does not consider a pure intertemporal asset allocation framework as described by Merton (1973).

study, portfolio construction and portfolio risk measurement of the models are evaluated by using a broad range of portfolio return, risk, risk-adjusted return and portfolio turnover criteria.

In the second chapter, the results of Giamouridis and Vrontos (2007) are extended to provide further evidence on the usefulness of multivariate conditional volatility models in hedge fund portfolio optimization. For this chapter, the methodology of Giamouridis and Vrontos (2007) is followed. In particular, monthly index return data from the HFR database for a longer period 1990 to 2009 of the eight investment strategies are used. Fund of funds portfolios are constructed by using the mean-variance optimization framework. Volatility forecasts are made by a broader set of volatility models, including a number of additional multivariate GARCH models and the much simpler exponentially weighted moving average (EWMA) estimator of RiskMetrics (1996). The models are applied to two sub-periods; one representing a bull market (2002 to 2005) and one representing a bear market (2006 to 2009), to test the robustness of the results. The bull market period corresponds to the entire out-of-sample period considered by Giamouridis and Vrontos (2007). Out-of-sample portfolio performance is evaluated by using the portfolio return, risk, risk-adjusted return and turnover criteria.

In the third chapter, alternative dynamic optimization frameworks for constructing portfolios of hedge funds are evaluated, and a semi-parametric approach to hedge fund portfolio optimization that addresses the shortcomings of the non-parametric approach (i.e. original CVaR, CDaR and Omega approaches), is proposed. In the semi-parametric approach, first the returns of each portfolio constituent are standardized in order to filter out the predictable dynamics related to autocorrelation and volatility clustering. Then the marginal density of each standardized return series is modelled by using a combination of extreme value theory (for the tails of the density) and a piecewise polynomial (for the centre of the density), and the joint density of hedge fund index returns is constructed by using a copula-based approach. Then hedge fund returns from this joint density are simulated in order to compute the relevant moments and quantiles required for portfolio optimization.

In the empirical analysis of the third chapter, monthly return data of ten hedge fund strategy index from the HFR database for the period 1990 to 2011 is used. Semi-parametric and non-parametric approaches are employed to obtain the CVaR-, CDaR-and Omega-based optimal portfolios. The performance of the different estimation approaches for conservative (minimum risk), as well as, aggressive (maximum return) investors is compared. For the aggressive investment strategy, three different formulations of the optimization problem are considered, each representing a different portfolio on the efficient frontier: minimization of risk subject to a target return, maximisation of return subject to a target risk, and maximization of return per unit of risk. The semi-parametrically and non-parametrically estimated CVaR, CDaR, and Omega models are also compared with a number of commonly used benchmarks, including an equally weighted portfolio, a constant volatility mean-variance model, a time-varying volatility mean-variance model, and a benchmark fund of hedge funds index. The out-of-sample portfolio performance of each model is evaluated with the help of a number of portfolio return, risk, risk-adjusted return and turnover criteria. Moreover, sensitivity to the risk limits and the robustness of the results to the choice of target return and estimation period is also tested.

1.4.2. Portfolio Risk Measurement and Volatility Forecast

The methodology applied to the research on better volatility forecast models is an evaluation of the in-sample goodness-of-fit and out-of-sample forecast performance of the proposed models in comparison to the benchmark models. In these analyses, the in-sample fit and out-of-sample forecast performance of the models are evaluated by using a comprehensive set of criteria based on statistical loss functions, statistical tests and risk management functions.

In Chapter 4, two univariate models which combine useful properties of range, regime switching, nonlinear filtration and GARCH frameworks are proposed. In this chapter, any improvement in the performance of volatility forecast is searched by employing regime switching in a conditional volatility setting with enhanced information content on true volatility. In this chapter, one and two component variants of a range-based regime switching model are proposed. These models nest regime switching conditional volatility models and eminent range-based models in a framework, which allows one to combine the desired properties of these two distinct models into the same framework.

In this chapter, the in-sample fit and out-of-sample forecast performance of the proposed univariate models are compared with return based and range-based counterparts. In particular, comparisons of the proposed models are made with the following return-based models; EWMA model of Riskmetrics, GARCH model of Bollerslev (1988), fractionally integrated GARCH (FIGARCH) model of Baillie *et al.* (1996), Markov Regime Switching GARCH (MRSGARCH) model of Klaassen (2002), and following range-based models; hybrid EWMA (HybEWMA) model of Harris and Yilmaz (2009) and CARR model of Chou (2005).

In the analysis, daily open, close, high and low prices of S&P500 index are used. The weekly data which covers the period of 03 January 1982 - 26 March 2010 (1460 observations) is estimated from daily return and range data. The full sample is divided into an initialization (i.e. in-sample estimation) period (1000 weeks) and out-of-sample forecast period (461 weeks). In comparing the forecast performance of the models, the four robust volatility proxy of the weekly integrated variance are used: the sum of squared daily returns, the sum of daily price range, the weekly squared return, and the weekly scaled range. In-sample fit and out-of-sample forecast performance of the models are evaluated by using a broad set of statistical and risk management loss functions. In particular, in statistical analysis mean error metrics, directional predictive ability tests, regressions of forecast evaluation, pair-wise and joint tests for model comparison are used. In analysing forecasts from a risk management perspective, coverage tests and risk management loss functions are utilized. The robustness of the results to different estimation and test periods, and to a different (daily) frequency is also checked.

In the fifth chapter, univariate volatility models proposed in the fourth chapter are extended to their multivariate setting. In particular, multivariate framework is based on a three step process. In the first step, conditional variances are estimated from the univariate models. In the second step, correlations are estimated from the constant conditional correlation (CCC) procedure of Bollerslev (1990) and dynamic conditional correlation (DCC) model of Engle (2002a). In the third step, conditional variances and conditional correlations are combined to estimate variance covariance matrix of portfolio assets. Four volatility forecast horizons (1, 4, 13 and 26 weeks) are considered.

The proposed multivariate models are compared to their return-based (EWMA, GARCH and MRSGARCH models), and range-based counterparts (HybEWMA and CARR models). In the analysis, daily index and return data of eight hedge fund strategy index obtained from HFR for the period 31 March 2003-29 January 2010 is used. From daily index values of each strategy, a weekly open/close/high/low index data set is constructed, and using the weekly open/close/high/low values, weekly range and returns are estimated. The daily return data is also used to construct a volatility proxy for weekly variance. Weekly range data set for hedge fund strategies is composed of 357 observations. The full sample is divided into an initialization period (150 observations) and an out-of-sample test period (207 observations).

Portfolios based on 1-, 4-, 13- and 26-week volatility forecasts of the models are constructed, and the performance of the portfolios is evaluated over the out-of-sample test period. In constructing portfolios of hedge fund strategies from the forecast covariance matrix, mean-variance optimization framework is used. A number of statistical tests and an investment study are used to evaluate the out-of-sample performance of the models. In statistical analysis, the forecasting performance of each model for each element of the variance covariance matrix of each portfolio is evaluated. In particular, mean error metrics, directional predictive ability tests, forecast evaluation regressions, and pair-wise and joint tests are used. In investment exercise, similar to second and third chapters, portfolio performance of each portfolio is evaluated by using a number of portfolio risk, return, risk-adjusted return and turnover metrics.

1.4.3. Hedge Fund Return Replication

The methodology, aimed for an approach to better replication of hedge fund returns, is designed to utilize the useful properties of factor and distribution matching approaches in an optimization framework. In so doing, this chapter proposes a composite approach that combines the factor and distribution-matching methodologies. In particular, a linear factor model is specified for hedge fund returns to capture their time series properties, but also a range of constraints is imposed to ensure that the replicating portfolio matches various risk measures of the hedge fund, including Conditional Value at Risk, Conditional Drawdown at Risk and the partial moments of returns. These risk measures are nonlinear functions of the higher moments of returns, and so this approach can be thought of as incorporating the distribution-matching approach. Also, a return constraint is imposed to ensure that the clone portfolio delivers the same absolute performance as that of the hedge fund.

The proposed approach is used to replicate the monthly returns of ten hedge fund strategy indices by using long-only positions in ten equity, interest rate, exchange rate and commodity indices, all of which can be traded using liquid, investible instruments such as futures, options and exchange traded funds. In the analysis, monthly data on ten hedge fund strategy indices obtained from HFR are used. The full sample, which covers the period June 1994 to January 2011 (200 observations), is divided into an initial estimation period, June 1994 to September 2002 (100 observations), and an out-of-sample evaluation period, October 2002 to January 2011 (100 observations).

The out-of-sample performance of the replicating portfolios is tested with different constraints over the out-of-sample test period. Portfolio performance is reported for a number of different specifications of the model. The out-of-sample performance of the replicating portfolios is evaluated using a number of statistical and economic measures. In particular, firstly, the regression result of realized hedge fund portfolio returns on the realized replicating portfolio returns, secondly, first four moments of the hedge fund portfolio and the replicating portfolio, and finally, Sharpe Ratio, maximum drawdown, and annualized CVaR and CDaR statistics for both the hedge fund portfolio and the replicating portfolio.

Chapter 2

Dynamic Hedge Fund Portfolio Construction: A Comparison of Alternative Volatility Forecast Approaches

2.1. Introduction

Hedge funds have attracted much interest not only for their ability to generate relatively high average returns, but also for the large losses that they can incur, a risk that is exemplified by the rise and fall of Long Term Capital Management in the late 1990s. In spite of such risk, the hedge fund industry witnessed rapid growth in the 2000s, with assets under management reaching $1.93 trillion by early 2008 and from as few as 300 funds in 1990 to about 9,000 funds today. During the recent credit crisis, there was a significant reduction both in the number of hedge funds and in assets under management, which resulted from a combination of trading losses and asset withdrawals by investors. However, by April 2011, it was estimated that hedge fund assets had recovered to their pre-crisis level (see Strasberg and Eder, 2011, and Stowell, 2010). Hedge fund managers are exempt from much of the regulation faced by other investment managers, and typically employ dynamic trading strategies with frequent rebalancing, and often make extensive use of derivatives, short positions and leverage.

An important contributing factor to this recent growth has been the availability of funds of hedge funds, which enable investors to access hedge fund alpha with lower risk, albeit at the expense of an additional layer of fees. Primarily aimed at institutional investors and high net worth individuals, hedge funds have recently become more widely accessible through the emergence of 'funds of funds,' which hold portfolios of hedge fund investments that are sold to a wider investor base. These funds provide a broad exposure to the hedge fund sector and diversify the risks associated with an investment in individual funds. Since their inception, these funds have become extremely popular. Indeed, by 2008, there were over two thousand funds of hedge funds listed in the Hedge Fund Research (HFR) database. Another contributing factor to the growth in the hedge fund industry was the launch in the early 2000s of investable hedge fund indices. These have generated further interest from small- and medium-sized investors, who would otherwise be precluded from investing in the hedge fund market. Central to both of these developments is the role of portfolio risk measurement and portfolio optimization in order to construct portfolios of individual hedge funds or hedge fund indices. While portfolio risk measurement is addressed in this chapter, portfolio optimization is addressed and a new measurement approach is proposed in Chapter 3.

In parallel with this rapid growth in the hedge fund industry, there has been increased demand from investors for products that deliver the returns of hedge funds at lower cost, and without the risks that are typically associated with hedge fund investment, such as illiquidity, lack of transparency and management specific risks. To meet this demand, investment banks and asset management firms have developed investment products, commonly known as 'clones', that seek to replicate hedge fund returns by employing statistical models or algorithmic trading strategies. Hedge fund return replication is addressed in detail and a new approach is proposed in Chapter 6.

Hedge fund returns are far from normally distributed, usually exhibiting very significant negative skewness and excess kurtosis (see, for example, Amin and Kat, 2001; Lo, 2001; Brooks and Kat, 2002; Fung and Hsieh, 1997a, 2001; Agarwal and Naik, 2001, Hudson *et al.*, 2006; Wegener *et al.*, 2010). While the non-normality of hedge fund returns is by now well-established, much less attention has been paid to the dynamic properties of hedge fund risk. In particular, the literature on hedge fund portfolio optimization has typically assumed a constant covariance structure of hedge fund returns. The assumption of a time-invariant variance-covariance matrix of hedge fund returns potentially leads to inaccuracies in the measurement of hedge fund risk and the optimization of hedge fund portfolios, particularly over shorter horizons where time-variation in the covariance matrix of returns is most pronounced. Indeed, Giamouridis and Vrontos (2007) show that the use of multivariate conditional volatility models improves the optimization of hedge fund portfolios, and provides a more accurate tool for tail-risk measurement. They employ two static models (the sample covariance matrix and an implicit factor model) and three dynamic models (two implicit factor GARCH models and a regime switching dynamic correlations model), and compare the out-of-sample performance of optimised monthly and quarterly rebalanced portfolios of the HFR indices for the period January 2002 to August 2005. In the mean-variance framework, the use of dynamic models generates portfolios of hedge fund indices with lower out-of-sample risk and higher realized returns. Using a mean-CVaR framework, which implicitly accommodates the non-normality of hedge fund returns, they show that dynamic optimization models are also more successful in reducing left tail risk.

In this chapter, the results of Giamouridis and Vrontos (2007) are extended to provide further evidence on the usefulness of multivariate conditional volatility models in hedge fund portfolio optimization using monthly index return data from the Hedge Fund Research (HFR) database for the period 1990 to 2009. In particular, a broader set of volatility models, including a number of additional multivariate GARCH models and the much simpler exponentially weighted moving average (EWMA) estimator of RiskMetrics (1996) are considered. It is found that while multivariate GARCH models provide some improvement in portfolio performance over static models, they are generally dominated by the EWMA model. In particular, in addition to providing better risk-adjusted performance, the EWMA model leads to dynamic allocation strategies that have a substantially lower turnover and could therefore be expected to involve lower transaction costs. To test the robustness of the results, the models are applied to two sub-periods, one representing a bull market (2002 to 2005) and one representing a bear market (2006 to 2009). The EWMA model appears to work well in both favourable and unfavourable market conditions.

The outline of the chapter is as follows. In the following section, the data used in the empirical analysis, the multivariate volatility models and the evaluation methodology are described. Section 3 reports the empirical results. Section 4 provides a summary and some concluding remarks.

2.2. Data and Methodology

2.2.1 Data

In the analysis monthly data on hedge fund index returns obtained from Hedge Fund Research (HFR[2]) is used. In line with Amenc and Martellini (2002), McFall and Lamm (2003), Agarwal and Naik (2004), Morton *et al*. (2006) and Giamouridis and Vrontos (2007), the indices are classified into the following investment strategies: convertible arbitrage (CA), distressed securities (DS), event driven (ED), equity hedge (EH), equity market neutral (EMN), mergers arbitrage (MA), macro (MAC) and relative value (RV). The data for the period January 1990 to September 2009 (237 observations) is used. This period covers a number of crises (e.g. the Mexican crisis, the Asian financial crisis, the default of the Russian government on its debt, the collapse of Long Term Capital Management, the collapse of the dotcom bubble and the most recent credit crisis). The initial estimation period covers the period January 1990 to December 2001 (144 observations). The out-of-sample forecast period is divided into two sub-periods: January 2002 to August 2005, representing relatively favourable market conditions (44 observations) and September 2005 to September 2009, representing more extreme market conditions (49 observations). Summary statistics for the hedge fund return series are reported in Table 2.1.

[Table 2.1]

Panel A reports various statistics for the different hedge fund investment strategies for the full sample of 237 observations. The characteristics of hedge fund returns are very heterogeneous. Some strategies (such as equity hedge and event driven) have relatively higher average returns and volatility. These are often thought of as return enhancers, used to substitute some fraction of the equity holdings in an investor's portfolio (see Amenc and Martellini, 2002). Other strategies (such as relative value arbitrage and equity market neutral) have a lower average return and volatility, and can be regarded as a substitute for some fraction of the fixed income or cash holdings in an investor's portfolio. All strategies except macro display negative skewness, and all are leptokurtic, particularly relative value arbitrage, convertible arbitrage and mergers arbitrage. The null hypothesis of normality is strongly rejected in all cases. Panel B reports the basic

[2] HFR construct investible indices (HFRX) as counterparts to these indices (HFRI). Details can be found on the HFR website: www.hfr.com.

time series properties of hedge fund returns. In particular, it reports the first five autocorrelation coefficients, the Ljung-Box portmanteau test for serial correlation up to 10 lags, the ARCH test of Engle (1982) and the DCC test of Engle and Sheppard (2001). All the hedge fund indices display highly significant positive autocorrelations and the ARCH test suggests that there is evidence of significant volatility clustering for all strategies except macro and mergers arbitrage. The DCC test, which tests the null hypothesis of constant correlation is tested against the alternative of dynamic conditional correlation, suggests that the data exhibit time-varying conditional correlations and hence motivates the use of dynamic conditional covariance models. Panel C reports the pair-wise correlations computed for the single strategies. Hedge fund returns exhibit correlations from 0.23 (between macro and convertible arbitrage) to 0.84 (between event driven and distressed securities). Generally, correlations between the different strategies are relatively moderate, which is a desirable property in the construction of funds of hedge funds.

2.2.2. Methodology

The first and second moments of m hedge fund returns, conditional on the information set Ω, are defined as follows

$$r_t = \mu + \varepsilon_t \tag{2.1a}$$

$$\varepsilon_t \mid \Omega_{t-1} \sim D(0, H_t) \tag{2.1b}$$

where r_t is the mx1 vector of hedge fund returns in period t with elements $r_{i,t}$, $i = 1,\dots, m$, μ is the mx1 vector of mean returns with elements μ_i, $i = 1,\dots, m$ and H_t is the mxm covariance matrix with diagonal elements $\sigma^2_{i,t}$ and off-diagonal elements $\sigma_{ij,t}$, $i, j = 1, \dots, m$. $D(.)$ is any location-scale family distribution. An investor in the m hedge funds who wishes to minimise the variance of portfolio returns in each period t subject to a minimum return constraint and short selling constraints is considered. The portfolio optimization problem can therefore be written as

$$\min_{\mathbf{x}} \Phi_p(\mathbf{x}) \tag{2.2}$$

$$\text{subject to } \mathbf{x} \geq 0, \ \mathbf{x}'1 = 1, \ E(r_{p,t}) \geq r_0 \tag{2.3}$$

where $r_{p,t}$ is the return of the hedge fund portfolio on day *t*, $\Phi_p(\mathbf{x})$ is the risk measure where $\Phi_p(\mathbf{x}) = \sigma(\mathbf{x}) = (\mathbf{x}'\mathbf{H}\,\mathbf{x})^{1/2}$ is the conditional portfolio standard deviation, $\mathbf{x}$ is the *mx1* vector of portfolio weights, $\mathbf{x} = [x_1, \ldots, x_m]'$, $E(r_{p,t})$ is the portfolio expected return and r_0 is the target portfolio return. In modelling the covariance matrix of hedge fund returns H_t, two static models and six dynamic models are employed. Each of these models is described below.

2.2.2.1. Sample Covariance Model

The simplest static model of the variance-covariance matrix of hedge fund returns is the sample covariance matrix of historical returns, given by

$$H^{SC} = \frac{1}{\tau - 1} \sum_{t=1}^{\tau} (r_t - \bar{r})(r_t - \bar{r})' \tag{2.4}$$

where $\bar{r} = (1/\tau)\sum_{t=1}^{\tau} r_t$ is the mx1 vector of sample mean returns and τ is the estimation sample size. The sample covariance model is perhaps the most commonly used estimator of the return covariance matrix (Amenc and Martellini, 2002).

2.2.2.2. Implicit Factor Model

The implicit factor model utilised by Fung and Hsieh (1997a), Amenc and Martellini (2002), Alexander and Dimitriu (2004) and Giamouridis and Vrontos (2007), assumes that returns are generated by a multifactor model. Under the implicit factor model, the covariance matrix of hedge fund returns is given by

$$H^{IF} = \Lambda \Sigma^{IF} \Lambda' + V \tag{2.5}$$

where Λ is the *m*x*K* matrix of factor loadings, Σ^{IF} is the *K*x*K* diagonal factor covariance matrix, V is a diagonal matrix with elements on the main diagonal $\sigma^2_{\varepsilon_i} = \mathrm{var}(u_{i,t})$, and $u_{i,t}$ is the idiosyncratic return with respect to the *K* factors.

2.2.2.3. Regime Switching Dynamic Correlation (RSDC) Model

Following Giamouridis and Vrontos (2007), the RSDC model of Pelletier (2006) is employed. The model is estimated using the two-step procedure described by Engle (2002). In the first step, a univariate volatility model for each hedge fund index is estimated, and in the second step, the parameters of the correlation matrix, conditional on the volatility estimates are estimated. It is assumed that there are two regimes and the GARCH (1,1) specification in modelling the conditional variances is employed. Under the RSDC model, the variance-covariance matrix of hedge fund returns is given as

$$H_t^{RSDC} = D_t P_t D_t \tag{2.6}$$

where $D_t = diag\left(h_{11,t}^{-1/2} ... h_{mm,t}^{-1/2}\right)$ is the mxm diagonal matrix of the inverse standard deviations of hedge fund returns and P_t is the mxm regime switching correlation matrix. The RSDC model stands somewhere between the Constant Conditional Correlation (CCC) model of Bollerslev (1990), in which conditional correlations are time-invariant, and the Dynamic Conditional Correlation (DCC) model of Engle (2002), in which conditional correlations are time-varying. Further details of the RSDC model can be found in Pelletier (2006).

2.2.2.4. Orthogonal GARCH Model

The Orthogonal GARCH model of Alexander (2001) is a generalization of the factor GARCH model introduced by Engle *et al*. (1990). For n factors the Orthogonal GARCH model is defined as

$$\varepsilon_t = V^{1/2} u_t = \Lambda_n f_t \tag{2.7}$$

where $V = diag(s_1^2,...,s_m^2)$ is a diagonal matrix comprising the variance of ε_{it}, $\Lambda_n = P_n diag\left(l_1^{1/2} ... l_n^{1/2}\right)$ is the (mxn) matrix of factor loadings, P_n is the matrix of mutually orthogonal eigenvectors and $l_1 \geq ... \geq l_n > 0$ are the n largest eigenvalues of the empirical correlation matrix of u_t. As f_t is defined as the random process with

$E_{t-1}(f_t)=0$ and $\mathrm{var}_{t-1}(f_t)=\Sigma_t=diag(\sigma^2_{f_{1t}},...,\sigma^2_{f_{nt}})$, the conditional variance-covariance matrix can be defined as

$$H_t^{ORTH}=V^{1/2}V_tV^{1/2} \tag{2.8}$$

where $V_t=\Lambda_n\Sigma_t\Lambda_n'$. Further details of Orthogonal GARCH model can be found in Alexander (2001) and Bauwens *et al.* (2006).

2.2.2.5. Dynamic Conditional Correlation (DCC) Model

The DCC model of Engle and Sheppard (2001) and Engle (2002) is defined as

$$H_t^{DCC}=D_tP_tD_t \tag{2.9}$$

where $D_t=diag(h_{11,t}^{-1/2}...h_{mm,t}^{-1/2})$ is the *m*x*m* diagonal matrix of the inverse standard deviations of returns and P_t is the *m*x*m* dynamic correlation matrix. There are also other versions of the DCC model proposed by Tse and Tsui (2002) and Christodoulakis and Satchell (2002).

2.2.2.6. Flexible Multivariate GARCH Model

The Flexible Multivariate GARCH model proposed by Ledoit *et al*. (2003) estimates the conditional covariance matrices within the framework of the diagonal vech GARCH model. The diagonal vech model is given by

$$H_t^{VEC}=C+Avech\left(H_{t-1}^{VEC}\right)+Bvech\left(\varepsilon_t\varepsilon_t'\right) \tag{2.10}$$

where *C* is an *m*(*m*+1)/2 x 1 vector and *A* and *B* are *m*(*m*+1)/2 x *m*(*m*+1)/2 matrices. The diagonal vech model has a number of well-documented shortcomings. Firstly, the number of parameters grows at the rate m^2, so for anything more than just a few assets, parameter estimation is infeasible. Secondly, the conditional covariance matrix is not guaranteed to be positive semi-definite. However, Ledoit *et al*. (2003) propose a two-step estimation method that ensures positive semi-definiteness. In the first step, a bivariate diagonal vech model is estimated for each pair of assets and the estimated parameters are stacked into matrices $\hat{C}$, $\hat{A}$ and $\hat{B}$. In the second step, $\hat{C}$, $\hat{A}$ and $\hat{B}$ are transformed in such a way that ensures that the resulting covariance matrix, H_t^{FLEX}, is positive semi-definite.

2.2.2.7. BEKK Model

The BEKK model of Engle and Kroner (1995) generalises the univariate GARCH model to the multivariate case. The BEKK (1, 1, *m*) specification is given by

$$H_t^{BEKK} = CC' + \sum_{i=1}^{m} A' H_{t-1} A + \sum_{i=1}^{m} B' \varepsilon_{t-1} \varepsilon'_{t-1} B \qquad (2.11)$$

where *A*, *B* and *C* are *m*x*m* parameter matrices and *C* is lower triangular. This model is also equivalent to a Factor GARCH(1, 1, *m*) model under certain conditions (see Bauwens *et al*., 2006).

2.2.2.8. Exponentially Weighted Moving Average (EWMA) Model

The EWMA model, popularised by its use in the estimation of Value at Risk by JP Morgan in their RiskMetrics software, is a special case of the diagonal vech GARCH (1,1) model. Under the EWMA model, the variance-covariance matrix of returns is given by

$$H_t^{EWMA} = \lambda R_{t-1} R'_{t-1} + (1-\lambda) H_{t-1} \qquad (2.12)$$

where λ is a decay factor, commonly set to a value of 0.94 for daily data. The mean return is assumed to be zero, as is common in practice.

Initially each of the six conditional volatility models is estimated using observations $t = 1, \ldots, \tau$ to generate one month-ahead out-of-sample forecast of the conditional covariance matrix for the month $t = \tau + 1$. Where appropriate, starting parameter values for this initial estimation were chosen on the basis of a grid-search procedure to maximize the likelihood function. The estimation sample is then rolled forward one month, and the models re-estimated and used to generate out-of-sample forecasts for month $t = \tau + 2$. The last iteration uses the sample $t = T - \tau - 1, \ldots, T - 1$ to generate forecast covariance matrix for the month $t = T$. At each iteration, the starting parameter values for each model were set to the values estimated in the previous iteration. The models were estimated by Quasi Maximum Likelihood function with a Gaussian conditional distribution, using the BHHH algorithm. Expected returns are estimated using the corresponding sample mean over the estimation sample.

2.2.3 Evaluation

Each month, the forecast conditional covariance matrix is used to estimate the weights of two optimal portfolios. The first is a 'conservative' portfolio, which does not have a target expected return, i.e. the minimum-variance portfolio. The second is an 'aggressive' portfolio with an annualised target expected return of either 15.5 percent or 13.5 percent, depending on the period analysed. In all cases, in order to force diversification, the maximum weight in any one index is constrained to be 70 percent. Results for a holding period of one month are reported. The results are qualitatively similar using longer holding periods of three and six months. Following Agarwal and Naik (2004) and Giamouridis and Vrontos (2007), for each portfolio and each holding period, a number of evaluation criteria, which are defined below are estimated.

(1) Return

$$R_{p,t+1} = x_t R_{t+1} \quad (2.13)$$

(2) Standard deviation

$$\sigma_{p,t+1} = x_t' H_{t+1} x_t \quad (2.14)$$

(3) Conditional Sharpe ratio

$$CSR_{P,t+1} = \frac{R_{P,t+1}}{\sigma_{P,t+1}} \quad (2.15)$$

(4) Portfolio Turnover

$$PT_p = \sum_{t=\tau}^{T-1} \sum_{k=1}^{m} \left| x_{k,t+1} - x_{k,t} \right| \tag{2.16}$$

This measures the fraction of the portfolio that must be liquidated and reinvested each month.

(5) Conditional value at risk

$$CVaR_{p,t+1}^{\alpha} = \zeta + \tfrac{1}{1-\alpha} \tfrac{1}{T-\tau} \sum_{t=\tau}^{T-1} \max\left\{ 0, -\sum_{k=1}^{m} x_{k,t} r_{k,t+1} - \zeta \right\} \tag{2.17}$$

where α is the CVaR confidence level and ζ is portfolio VaR. CVaR is estimated at the 90 percent, 95 percent and 99 percent confidence levels.

2.3. Empirical Results

The out-of-sample forecast performance of constant and dynamic conditional covariance models is tested in an investment exercise for the out-of-sample period January 2002 to September 2009. The out-of-sample test period is divided into two sub-periods. The first, which is from January 2002 to August 2005, is the same out-of-sample period analyzed in Giamouridis and Vrontos (2007), and is a relatively favourable period for hedge funds. The second, from September 2005 to September 2009, includes the recent financial crisis, and is hence a relatively volatile period for hedge funds. Empirical evidence for a conservative (i.e. minimum variance) portfolio and an aggressive (i.e. target return) portfolio is provided. In the first period, to be comparable to Giamouridis and Vrontos (2007), a target return for the aggressive portfolio of 15.5 percent is assumed. For the second period, when hedge fund returns were generally lower, a target return of 13.5 percent is assumed. By considering these two periods, the out-of-sample performance of the various conditional covariance models are being able to be assessed under both normal and extreme market conditions. For each of the eight models, both the mean and median values of the realized return, portfolio standard deviation, conditional Sharpe ratio, portfolio turnover and CVaR tail-risk measure are reported. The statistical significance of pair-wise differences in these measures between the eight models was tested using a t-test (for differences in mean values) and the Wilcoxon test (for differences in median values).

2.3.1. Period 1: Normal Market Conditions

Table 2.2 reports the mean and median values of the risk-return metrics for the estimated portfolios for Period 1. Realized returns are uniformly higher for the aggressive portfolio than for the conservative portfolio, but portfolio standard deviation is also uniformly higher. The Sharpe ratios are similar for the two portfolios. The static models (SC and IF) perform poorly in terms of risk-adjusted return for both portfolios, characterised by notably lower returns for the conservative portfolio and notably higher risk for the aggressive portfolio. As expected, however, they have the lowest turnover of almost all of the models, reflecting their static nature. The RSDC model offers a substantial improvement over the static models for both portfolios, offering higher returns for the conservative portfolio and lower risk for the aggressive portfolio, but has much higher turnover. In terms of risk-adjusted performance, the RSDC model generates a Sharpe ratio of 0.74 for the conservative portfolio and 0.75 for the aggressive portfolio, compared with 0.49 and 0.51, respectively for the SC model and 0.49 and 0.50, respectively, for the IF model. The superior performance of the RSDC model relative to the static model is statistically significant. These findings are consistent with Giamouridis and Vrontos (2007) who report the results for the same portfolios optimised with the SC, IF and RSDC models over the same period.

Of the remaining GARCH models, the DCC and FLEX models provide performance that is very similar to that of the RSDC model. In contrast, however, the ORTH and BEKK models perform notably worse, both generating significantly lower returns for the conservative portfolio and significantly higher risk for the aggressive portfolio. The best performing GARCH model in risk-adjusted return terms is the FLEX model for the conservative portfolio (with a Sharpe ratio of 0.78) and the RSDC model for the aggressive portfolio (with a Sharpe ratio of 0.75). However, for the conservative portfolio, the much simpler EWMA model generates the highest Sharpe ratio of 0.87. This improvement in risk-adjusted performance comes primarily from a reduction in risk. For the aggressive portfolio, the EWMA model offers a risk-adjusted performance, which is very similar to that of the RSDC, DCC and FLEX models. A notable feature of the EWMA model, however, is that it generates very low turnover for both the conservative and aggressive portfolios, and indeed, has similar turnover to that of the static models. The EWMA model could be expected, therefore, to generate lower transaction costs relative to the GARCH models that are considered here. The CVaR estimates re-enforce the conclusions about risk reached above. In particular, the EWMA model generates a conservative portfolio that is substantially less risky than any of the GARCH models, or indeed the two static models. For the aggressive portfolio, there is little difference in the CVaR among the best performing GARCH models.

2.3.2. Period 2: Extreme Market Conditions

Table 2.3 reports the mean and median values of the risk-return metrics for the estimated portfolios for Period 2. Realized returns are generally higher for the aggressive portfolio than for the conservative portfolio, although there are some exceptions (most notably the RSDC model), and portfolio standard deviation is significantly higher. Owing to the higher volatility of this period, risk-adjusted returns, as measured by the Sharpe ratio, are uniformly higher for the conservative portfolio than for the aggressive portfolio. In contrast with Period 1, the performance of the static models (SC and IF) in Period 2 is similar to that of the dynamic models in terms of realised return and standard deviation. In terms of risk-adjusted returns, the static models have lower mean Sharpe ratios, but the median values are similar to those of the other models, suggesting perhaps that the mean values in this case are driven by outliers. Similar to Period 1, the static models have much lower turnover owing to their relatively unresponsive nature. Among the dynamic models, the RSDC and EWMA models provide the best risk-adjusted return performance for the conservative portfolio and the RSDC, EWMA and DCC models provide the best performance for the aggressive portfolio.

Comparing the results for Period 1 and Period 2, it is seen that the deterioration in performance is greater for the aggressive portfolio than for the static portfolio, which is to be expected given the unfavourable conditions prevailing in Period 2. In particular, the standard deviation of returns is generally higher in Period 2 for the aggressive portfolio, while the level of returns is substantially lower. For the conservative portfolio, the standard deviation of returns is also generally higher in Period 2 than in Period 1, but the median level of returns is in many cases higher. The EWMA model is again notable for having the lowest turnover of the dynamic models, and for the aggressive portfolio, it is similar to that of the two static models. In terms of tail risk, the aggressive portfolios have significantly higher CVaR than the conservative portfolios, but there is not a systematic difference between the dynamic models and the static models. Of the dynamic models, the EWMA model and the FLEX model offer the lowest tail risk at all three CVaR levels for both the conservative and aggressive portfolios. A number of the dynamic models have substantially higher tail risk than the static models. This is especially true of the RSDC and ORTH and DCC models.

2.4. Conclusion

In this chapter, analysis are built on the analysis of Giamouridis and Vrontos (2007) and further evidence is provided on the performance of dynamic conditional covariance models for the optimization of funds of hedge funds. It is done so by considering a much broader set of conditional covariance models, and a longer out-of-sample evaluation period. In particular, not only additional GARCH models, but also the much simpler RiskMetrics EWMA model is considered. Moreover, evaluation periods of favourable and unfavourable conditions for the hedge fund industry are separately considered. In the first out-of-sample period, which is a relatively favourable period, the findings of Giamouridis and Vrontos (2007) are confirmed and it is shown that GARCH models are able to provide a substantial improvement in terms of risk-return trade-off over the static models, for the optimization of both conservative and aggressive portfolios. However, in most cases these improvements come at the expense of higher portfolio turnover and rebalancing expenses. In contrast, the EWMA model, which is the most parsimonious of the dynamic models, offers a superior risk-return trade-off and, moreover, does so with rebalancing costs those are no higher than those of the static models. In the second out-of-sample period, which is characterised by much greater volatility and generally unfavourable conditions for the hedge fund industry, dynamic models again tend to outperform static models, providing a superior risk-return trade-off, although the differences are less marked than during the favourable conditions of the first out-of-sample period. The best performing models in terms of risk-adjusted returns are the RSDC, EWMA and DCC models, but of these, the EWMA model is again notable for generating portfolios that have substantially lower turnover and less tail risk. The findings of the chapter therefore confirm the advantages of dynamic models over static models, but favour the use of simple models over more sophisticated models. This last result stands in contrast with other studies that question the effectiveness of the RiskMetrics framework in the univariate context (see, for example, McMillan and Kambouroudis, 2009).

Table 2.1

Summary Statistics and Time Series Properties of Hedge Fund Return Series

Panel A reports summary statistics for the monthly hedge fund return series over the period of January 1990 to September 2009. The statistics are estimated from monthly return series. The estimated kurtosis is excess kurtosis over the kurtosis of the Gaussian distribution. Panel B reports the autocorrelation, autoregressive conditional heteroskedasticity (ARCH) and dynamic conditional correlation (DCC) test results for the whole period. *,** and *** denote significance at 10, 5 and 1 percent levels, respectively. The Ljung-Box-Q test for autocorrelation of order up to 10 asymptotically distributed as a central Chi-square with 10 *d.o.f.* Under the null hypothesis, with 5 percent critical value is 18.307. ARCH(4) is Engle's LM test for autoregressive conditional heteroskedasticity, which is asymptotically distributed as a central Chi-square with 4 *d.o.f.* Under the null hypothesis with 5 percent critical value is 9.488. *p*-values are reported in parenthesis. DCC test statistics is the Chi-square with 13 *d.o.f.* The *p*-value (i.e. probability of a constant correlation) is in the bracket. Panel C reports correlations between hedge fund strategies over the period of January 1990 to September 2009.

Panel A: Summary Statistics								
	Mean	Median	SD	Skew	Kurt.	Min.	Max.	Jarque-Bera
Equity hedge	1.15	1.31	2.68	-0.24	4.91	-9.46	10.88	38.23 [0.00]
Macro	1.13	0.83	2.25	0.44	3.81	-6.40	7.88	14.03 [0.00]
Relative value arbitrage	0.85	0.91	1.31	-2.23	16.97	-8.03	5.72	2122.21 [0.00]
Event driven	1.00	1.29	2.02	-1.36	7.20	-8.90	5.13	246.92 [0.00]
Convertible arbitrage	0.74	1.00	1.97	-3.24	32.50	-16.01	9.74	9006.08 [0.00]
Distressed securities	1.02	1.12	1.94	-1.01	7.89	-8.50	7.06	276.58 [0.00]
Equity market neutral	0.63	0.58	0.94	-0.17	4.19	-2.87	3.59	15.16 [0.00]
Mergers arbitrage	0.76	0.96	1.23	-2.21	11.68	-6.46	3.12	936.70 [0.00]
Panel B: Basic Time Series Properties								
	ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)	LB-Q(10)	ARCH(4)	DCC test
								59.48 [0.00]
Equity hedge	0.26***	0.16***	0.09***	0.05***	-0.05***	29.53***	22.90***	
Macro	0.16**	-0.00**	0.01	0.11*	0.17***	22.03**	6.78	
Relative value arbitrage	0.46***	0.27***	0.12***	0.11***	0.03***	74.43***	44.74***	
Event driven	0.39***	0.17***	0.10***	0.06***	0.03***	48.13***	15.07***	
Convertible arbitrage	0.61***	0.32***	0.16***	0.12***	-0.03***	134.58***	55.29***	
Distressed securities	0.57***	0.28***	0.15***	0.14***	0.02***	108.66***	46.28***	
Equity market neutral	0.16***	0.18***	0.20***	0.19***	0.08***	72.98***	16.81***	
Mergers arbitrage	0.23***	0.08***	0.11***	0.00***	0.07***	26.92***	0.57	
Panel C: Correlation Structure of Individual Hedge Fund Strategies								
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Equity hedge	1.00							
Macro	0.56	1.00						
Relative value arbitrage	0.66	0.33	1.00					
Event driven	0.82	0.51	0.74	1.00				
Convertible arbitrage	0.56	0.23	0.78	0.63	1.00			
Distressed securities	0.68	0.42	0.77	0.84	0.63	1.00		
Equity market neutral	0.44	0.31	0.36	0.34	0.25	0.33	1.00	
Mergers arbitrage	0.58	0.32	0.54	0.75	0.46	0.56	0.30	1.00

Table 2.2
Out-of-Sample Evaluation Criteria of Monthly Rebalancing Portfolios (Period 1)

The table reports evaluation criteria for the out-of-sample monthly rebalancing conservative and aggressive portfolios of HFR indices in the period January 2002 to August 2005. Evaluation criteria include mean and median values of realized return (Return), portfolio standard deviation (Risk), Conditional Sharpe ratio (CSR), portfolio turnover (Turnover) and Conditional value at risk (CVaR) at the 90 (CVaR90), 95 (CVaR95) and 99 (CVaR99) percent confidence levels. Medians are reported in the brackets.

	Return	Risk	CSR	Turnover	CVaR90	CVaR95	CVaR99
Panel A: Conservative Portfolio							
SC	0.35 [0.40]	0.71 [0.71]	0.49 [0.56]	0.99 [0.63]	0.37 [0.36]	0.59 [0.58]	1.02 [1.00]
IF	0.33 [0.37]	0.67 [0.66]	0.49 [0.56]	0.62 [0.20]	0.32 [0.30]	0.52 [0.50]	0.92 [0.90]
RSDC	0.52 [0.59]	0.85 [0.75]	0.74 [0.84]	43.75 [36.86]	0.40 [0.21]	0.66 [0.44]	1.17 [0.90]
ORTH	0.35 [0.37]	0.76 [0.71]	0.49 [0.52]	7.85 [5.95]	0.46 [0.38]	0.69 [0.59]	1.15 [1.02]
DCC	0.53 [0.60]	0.88 [0.79]	0.73 [0.82]	43.37 [42.39]	0.45 [0.27]	0.72 [0.51]	1.25 [0.99]
FLEX	0.46 [0.53]	0.61 [0.59]	0.78 [0.82]	50.69 [46.54]	0.14 [0.10]	0.33 [0.28]	0.69 [0.64]
BEKK	0.39 [0.37]	0.70 [0.69]	0.57 [0.57]	20.56 [18.41]	0.35 [0.31]	0.57 [0.52]	0.99 [0.93]
EWMA	0.45 [0.44]	0.53 [0.51]	0.87 [0.87]	6.24 [3.41]	0.03 [0.02]	0.17 [0.17]	0.49 [0.48]
Panel B: Aggressive Portfolio with Target Return 15.5% p.a.							
SC	0.76 [0.87]	1.49 [1.50]	0.51 [0.58]	4.86 [4.35]	1.40 [1.42]	1.86 [1.89]	2.75 [2.75]
IF	0.73 [0.83]	1.44 [1.46]	0.50 [0.60]	7.06 [4.91]	1.32 [1.35]	1.77 [1.79]	2.63 [2.63]
RSDC	0.61 [0.67]	1.01 [0.90]	0.75 [0.87]	34.80 [22.53]	0.57 [0.38]	0.88 [0.65]	1.49 [1.49]
ORTH	0.74 [0.79]	1.47 [1.38]	0.55 [0.62]	22.27 [18.40]	1.37 [1.21]	1.82 [1.63]	2.70 [2.70]
DCC	0.61 [0.67]	1.04 [0.93]	0.73 [0.84]	36.77 [22.68]	0.62 [0.43]	0.95 [0.72]	1.57 [1.57]
FLEX	0.70 [0.80]	1.01 [0.98]	0.73 [0.78]	31.73 [23.97]	0.57 [0.50]	0.89 [0.81]	1.50 [1.50]
BEKK	0.76 [0.89]	1.35 [1.35]	0.57 [0.69]	17.21 [13.85]	1.15 [1.15]	1.57 [1.57]	2.38 [2.38]
EWMA	0.76 [0.87]	1.04 [1.03]	0.74 [0.81]	6.43 [4.11]	0.62 [0.61]	0.94 [0.92]	1.57 [1.57]

Table 2.3
Out-of-Sample Evaluation Criteria of Monthly Rebalancing Portfolios (Period 2)

The table reports evaluation criteria for the out-of-sample monthly rebalancing conservative and aggressive portfolios of HFR indices in the period September 2005 to September 2009. Evaluation criteria include mean and median values of realized return (Return), portfolio standard deviation (Risk), Conditional Sharpe ratio (CSR), portfolio turnover (Turnover) and Conditional value at risk (CVaR) at the 90 (CVaR90), 95 (CVaR95) and 99 (CVaR99) percent confidence levels. Medians are reported in the brackets.

	Return	Risk	CSR	Turnover	CVaR90	CVaR95	CVaR99
Panel A: Conservative Portfolio							
SC	0.20 [0.54]	0.74 [0.71]	0.32 [0.74]	0.99 [0.63]	0.54 [0.46]	0.77 [0.68]	1.22 [1.10]
IF	0.16 [0.55]	0.72 [0.69]	0.28 [0.74]	0.62 [0.20]	0.53 [0.45]	0.77 [0.68]	1.20 [1.08]
RSDC	0.21 [0.49]	1.10 [0.97]	0.69 [0.81]	43.75 [36.86]	0.97 [0.75]	1.32 [1.05]	1.99 [1.64]
ORTH	0.34 [0.62]	0.97 [0.75]	0.58 [0.70]	7.85 [5.95]	0.95 [0.59]	1.25 [0.83]	1.84 [1.29]
DCC	0.18 [0.51]	1.22 [1.04]	0.64 [0.62]	43.37 [42.39]	1.12 [0.84]	1.50 [1.20]	2.24 [1.83]
FLEX	0.29 [0.44]	0.73 [0.69]	0.46 [0.69]	50.69 [46.54]	0.48 [0.43]	0.71 [0.66]	1.16 [1.09]
BEKK	0.33 [0.52]	0.78 [0.74]	0.53 [0.70]	20.56 [18.41]	0.58 [0.56]	0.82 [0.80]	1.29 [1.26]
EWMA	0.31 [0.53]	0.68 [0.59]	0.69 [0.80]	6.24 [3.41]	0.43 [0.31]	0.65 [0.51]	1.08 [0.89]
Panel B: Aggressive Portfolio with Target Return 13.5% p.a.							
SC	0.23 [0.53]	1.30 [1.18]	0.26 [0.45]	5.29 [4.59]	1.30 [1.03]	1.72 [1.40]	2.53 [2.53]
IF	0.20 [0.50]	1.26 [1.13]	0.25 [0.43]	6.40 [4.65]	1.22 [0.95]	1.62 [1.31]	2.41 [2.41]
RSDC	0.11 [0.51]	1.32 [1.03]	0.64 [0.65]	30.94 [18.64]	1.33 [0.77]	1.76 [1.09]	2.58 [2.58]
ORTH	0.31 [0.50]	1.82 [1.31]	0.38 [0.25]	18.81 [14.47]	2.23 [1.48]	2.80 [1.92]	3.93 [3.93]
DCC	0.14 [0.54]	1.42 [1.10]	0.63 [0.59]	33.33 [19.09]	1.46 [0.85]	1.91 [1.22]	2.79 [2.79]
FLEX	0.27 [0.43]	0.96 [0.87]	0.45 [0.47]	31.44 [25.32]	0.72 [0.54]	1.04 [0.82]	1.65 [1.65]
BEKK	0.36 [0.55]	1.31 [1.14]	0.42 [0.41]	17.75 [14.49]	1.30 [1.04]	1.71 [1.40]	2.53 [2.53]
EWMA	0.35 [0.45]	0.98 [0.80]	0.61 [0.57]	6.85 [4.00]	0.80 [0.43]	1.12 [0.70]	1.76 [1.76]

Chapter 3

Dynamic Hedge Fund Portfolio Construction: A Semi-Parametric Approach to Optimization

3.1. Introduction

A number of studies have examined portfolio optimization in a hedge fund context. However, the optimal portfolio allocation across individual hedge funds is complicated by the fact that owing to the strategies that hedge fund managers typically adopt, hedge fund returns are far from normally distributed, usually exhibit very significant negative skewness and excess kurtosis (see related literature in Section 2.1). Portfolio optimization in the presence of such non-normality in hedge fund returns generally leads to very different portfolio allocations than those implied by mean-variance analysis (see, for example, McFall Lamm, 2003; Fung and Hsieh, 1997b; Cvitanic *et al.*, 2003; Terhaar *et al.*, 2003; Popova *et al*,. 2003; Glaffig, 2006; Wong *et al.*, 2008). Motivated by the well established volatility clustering in hedge fund returns, Giamouridis and Vrontos (2007) address the issue of hedge fund portfolio optimization in a dynamic context. They show that the use of multivariate conditional volatility models improves portfolio performance, and provides a more accurate tool for tail-risk measurement. Harris and Mazibas (2010) provide further evidence on the use of multivariate conditional volatility models in the context of dynamic hedge fund risk measurement and portfolio allocation, and show that simple volatility models, such as the RiskMetrics EWMA model of JP Morgan, provide the biggest improvements in performance. The non-normality in hedge fund returns has prompted the use of alternative measures of risk in the optimization framework. Agarwal and Naik (2004) and Giamouridis and Vrontos (2007) compare mean-variance and mean-CVaR portfolios constructed using HFR hedge fund strategy indices. Krokhmal *et al.* (2003) compare CVaR and CDaR approaches for minimum risk portfolios of individual hedge funds, while Hentati *et al.* (2010) compare CVaR and Omega measures for portfolios including hedge funds for minimum risk portfolios. These alternative approaches rely on non-parametric estimation, in which the moments and quantiles of the density function of portfolio returns are estimated empirically, and these are used to compute the various risk measures used in the optimization process. The non-parametric approach, while straightforward to implement, relies on a large data sample to generate sufficiently accurate estimates of the various measures. Moreover, it does not readily lend itself to incorporating the well established dynamic characteristics of hedge fund returns, such as autocorrelation and volatility clustering.

In this chapter, a semi-parametric approach to hedge fund portfolio optimization that addresses the shortcomings of the non-parametric approach is proposed. In the semi-parametric approach, first the returns of each portfolio constituent are standardized in order to filter out the predictable dynamics related to autocorrelation and volatility clustering. Then the marginal density of each standardized return series is modelled using a combination of extreme value theory (for the tails of the density) and a piecewise polynomial (for the centre of the density), and the joint density of hedge fund index returns is constructed by using a copula-based approach. Then hedge fund returns from this joint density are simulated in order to compute the relevant moments and quantiles required for portfolio optimization. Using monthly index return data from the HFR database for the period 1990 to 2011, the semi-parametric and non-parametric approaches are used to obtain the CVaR-, CDaR- and Omega-based optimal portfolios. The performance of the different estimation approaches for conservative (minimum risk) as well as aggressive (maximum return) investors are compared. For the aggressive investment strategy, three different formulations of the optimization problem are considered, each representing a different portfolio on the efficient frontier: minimization of risk subject to a target return, maximisation of return subject to a target risk, and maximization of return per unit of risk. The semi-parametrically and non-parametrically estimated CVaR, CDaR and Omega models are also compared with a number of commonly used benchmarks, including an equally weighted portfolio, a constant volatility mean-variance model, a time-varying volatility mean-variance model, and a benchmark fund of hedge funds index. Two main findings are reported. The first is that CVaR-, CDaR- and Omega-based optimization offers a significant improvement in terms of risk-adjusted portfolio performance over mean-variance optimization. The second is that, for all three risk measures, semi-parametric estimation of the optimal portfolio offers a very significant improvement over non-parametric estimation. The results are robust to as the choice of target return and the estimation period.

The outline of the chapter is as follows. Section 2 describes the optimization framework, the estimation methods and the evaluation criteria. Section 3 describes the data used in the empirical analysis. Section 4 reports the empirical results. Section 5 provides a summary and some concluding remarks.

3.2. Methodology

In this section, first the generic optimization problem is set out for two types of investor: conservative and aggressive, and the different measures of risk that are used to construct the objective function of the optimization problem are defined. Then the semi-parametric and non-parametric approaches to estimating the optimal portfolio are described. Finally the evaluation criteria that are used to compare the different approaches are defined.

3.2.1. Optimization Framework

Consider an investor who allocates her wealth among m individual hedge funds or hedge fund indices, with portfolio weight vector, $\mathbf{x} = [x_1, \ldots, x_m]'$. For a conservative investor, the portfolio optimization problem is given by

$$\min_{\mathbf{x}} \Phi_p(\mathbf{x}) \quad (3.1)$$
$$\text{subject to } \mathbf{x} \geq 0,\ \mathbf{x'1}_m = 1$$

where $\Phi_p(\mathbf{x})$ is the portfolio risk function to be minimized (i.e. standard deviation, CVaR, CDaR or lower partial moment) and $\mathbf{1}_m$ is an m-vector of ones. The budget constraint and non-negativity constraints yield an unleveraged long-only portfolio. For an aggressive investor, three separate formulations of the portfolio optimization problem are considered. The first minimizes portfolio risk subject to a target expected portfolio return, r_0:

$$\min_{\mathbf{x}} \Phi_p(\mathbf{x}) \quad (3.2)$$
$$\text{subject to } \mathbf{x} \geq 0,\ \mathbf{x'1} = 1,\ E(r_{p,t}) \geq r_0$$

where $\mathbf{r}_t = [r_{1,t},\ldots,r_{m,t}]'$ is the m-vector of hedge fund returns at time t and $r_{p,t} = \mathbf{x}'\mathbf{r}_t$ is the portfolio return at time t. The second formulation of the portfolio optimization problem for an aggressive investor maximizes portfolio return subject to a portfolio risk constraint:

$$\max_{\mathbf{x}} E(r_{p,t})$$

$$\text{subject to } \Phi_p(\mathbf{x}) \leq \omega C,\ \mathbf{x} \geq 0,\ \mathbf{x}'\mathbf{1} = 1 \tag{3.3}$$

where ω is the risk limit and C is the invested capital, which is set arbitrarily to 1. The third formulation of the portfolio optimization problem for an aggressive investor maximizes portfolio expected excess return per unit of portfolio risk:

$$\max_{\mathbf{x}} \frac{E(r_{p,t}) - r_f}{\Phi_p(\mathbf{x})}$$

$$\text{subject to } \mathbf{x} \geq 0,\ \mathbf{x}'\mathbf{1} = 1 \tag{3.4}$$

where r_f is risk-free rate of return. This formulation is a generalization of the tangency portfolio in expected return-risk space for the different risk measures.

3.2.2. Optimization Models

In this section, the different risk measures, Φ_p, that are used in the optimization problems described above, are defined.

3.2.2.1. Mean-Variance Optimization Model

As a benchmark, the standard mean-variance model is used, in which the risk measure is portfolio standard deviation, given by

$$\sigma(\mathbf{x}) = \left[\mathbf{x}'\mathbf{H}\,\mathbf{x}\right]^{1/2} \tag{3.5}$$

where **H** is the mxm covariance matrix of hedge fund index returns. Two versions of the mean-variance model are considered. In the static mean-variance model, **H** is estimated using the sample covariance matrix. In the dynamic mean-variance model, **H** is estimated using the multivariate RiskMetrics EWMA model of JP Morgan (1996). For the mean-variance optimization model, $\Phi(x) = \sigma(x)$ is set in the generic optimization problems (3.1)-(3.4).

3.2.2.2. Mean-CVaR Optimization Model

Conditional Value at Risk (CVaR) is a tail-risk measure derived from the Value at Risk (VaR) estimator, and is defined as the conditional expectation of losses exceeding VaR at specified confidence level and time horizon. CVaR can be defined as

$$CVaR_{\alpha}(\mathbf{x}, \zeta) = \zeta + \frac{1}{1-\alpha} E\left[-r_{p,t} - \zeta | r_{p,t} < \zeta\right] \tag{3.6}$$

where ζ is portfolio VaR measured at the α-confidence level, $F(\mathbf{x},\zeta) = P\left[-r_{p,t} \leq \zeta\right]$ is the cumulative distribution function, which is continuous and non-decreasing with respect to α, and $\zeta_{\alpha}(\mathbf{x}) = \min\{\zeta \mid F(\mathbf{x}, \zeta) \geq \alpha\}$. In equation (6), by minimizing CVaR, VaR is also minimized. Rockafeller & Uryasev (2002) show that CVaR satisfies all statistical axioms of a coherent measure of risk defined by Artzner *et al.* (1999) and is a convex function of portfolio positions (see also Rockafeller & Uryasev, 2000). For portfolio implementations of CVaR measure, see, for example, Krokhmal *et al.* (2002a, 2002b). For the mean-CVaR optimization model, $\Phi(x) = CVaR_{\alpha}^{P}(x, \zeta)$ is set in the generic optimization problems (3.1)-(3.4).

3.2.2.3. Mean-CDaR Optimization Model

Drawdown, also known as the underwater portfolio level, is defined as the drop in portfolio value from a previous highest level. The drawdown measure helps investors to construct portfolios that may enable them not to lose more than a fixed percentage of the maximum value of their wealth achieved up to that point in time. Chekhlov *et al.* (2000) propose Conditional Drawdown at Risk (CDaR), which combines the drawdown concept with CVaR approach. Similar to CVaR, CDaR is defined as the expectation of drawdowns that exceed a certain threshold $\zeta_{\alpha}(\mathbf{x})$ defined at an α-confidence level. However, unlike CVaR, CDaR accounts not only for the amount of losses over some period, but also for the sequence of those losses. For portfolio implementation of CDaR, see, Checkhlov *et al.* (2005).

Let the uncompounded cumulative portfolio value at time *t* be $w(\mathbf{x},t)$. The drawdown function at time *t* is given by

$$f(\mathbf{x},t) = \max_{0 \le j \le t}[w(\mathbf{x},j)] - w(\mathbf{x},t) \tag{3.7}$$

then $\alpha - CDaR$ is formulated as

$$CDaR_\alpha(\mathbf{x},\zeta) = \zeta + \frac{1}{1-\alpha} E[f(\mathbf{x},t) - \zeta \mid f(\mathbf{x},t) > \zeta] \tag{3.8}$$

The drawdown function satisfies Artzner's nonnegativity, positive homogeneity, translation invariance and subadditivity axioms (see Chekhlov *et al.*, 2005). Unlike CVaR, which is estimated for a one-month time horizon, CDaR is estimated for a one-year time horizon. For the mean-CDaR optimization model, $\Phi(\mathbf{x}) = CDaR_\alpha(\mathbf{x},\zeta)$ is set in the generic optimization problems (3.1)-(3.4).

3.2.2.4. Omega Optimization Model

Omega is a performance measure that was first introduced by Keating & Shadwick (2002a, 2002b) to overcome the shortcomings of classical measures such as the Sharpe ratio. In particular, Omega separately considers gains and losses without reference to a specific distribution for asset returns. For its implementation in portfolio construction see, for example, Avouyi-Dovi *et al.* (2004), Passow (2005), Gilli *et al.* (2006), Mausser *et al.* (2006) and Kane *et al.* (2009). Omega is defined for any portfolio return level as the probability weighted ratio of gains to losses relative to a threshold return defined by the investor, r_b.

In its simplest form, the Omega function can be expressed by the help of partitioning the portfolio return distribution into the upper partial moment of returns (gains) and the lower partial moment of returns (losses). Omega is then defined as

$$\Omega_p(r_b, \mathbf{x}) = \frac{E\lfloor r_{p,t} | r_{p,t} \geq r_b \rfloor - r_b}{r_b - E\lfloor r_{p,t} | r_{p,t} \leq r_b \rfloor} \tag{3.9}$$

where upper partial moments function is the conditional expectation of portfolio returns that exceed the threshold (nominator), and the lower partial moments function is the conditional expectation of returns below the threshold (denominator). For the Omega-based optimization model, three specifications based on (3.9) are considered. For the conservative investment strategy, (3.1), and the first formulation of the aggressive investment strategy, (3.2), the lower partial moment of returns is employed as the measure of risk and so $\Phi(\mathbf{x}) = r_b - E\lfloor r_{p,t} | r_{p,t} \leq r_b \rfloor$ is set, where the threshold return level, r_b is set to the risk-free rate of return. For the second formulation of the aggressive portfolio, (3.3), the upper partial moment, $g = E\lfloor r_{p,t} | r_{p,t} \geq r_b \rfloor - r_b$, is maximized with subject to a constraint on the lower partial moment, $l = r_b - E\left[r_{p,t} \mid r_{p,t} \leq r_b \right]$. For the third formulation of the aggressive investment strategy, the Omega ratio as defined in (3.9) is maximized.

3.2.3. Estimation Method

Hedge fund returns typically exhibit substantial non-normality, with significant negative skewness and excess kurtosis, and also often display significant autocorrelation. Portfolio optimization in the presence of these statistical properties generally leads to very different portfolio allocations than those implied by mean-variance analysis. This has prompted the use of alternative risk and performance measures, such as CVaR, CDaR and Omega. These approaches have invariably been implemented using a non-parametric approach, in which the unobserved moments and quantiles of the distribution of portfolio returns are estimated by their empirical counterparts. The non-parametric approach, while straightforward to implement, relies on a large data sample to generate sufficiently accurate estimates of the various measures. Moreover, it does not lend itself to incorporating the well established dynamic characteristics of hedge fund returns, such as autocorrelation and volatility clustering. In this chapter, a semi-parametric approach to optimization in the CVaR, CDaR and Omega frameworks is proposed that addresses these shortcomings of the non-parametric approach.

First, the returns for each hedge fund index is filtered with using an AR(1)-

EGARCH(1,1) model in order to generate an *i.i.d.* standardized return series. The autocorrelation functions (ACFs) of standardized returns are compared to the ACFs of the raw returns to make sure the standardized returns are approximately *i.i.d.* Second, Extreme Value Theory is applied to the standardized returns, by employing the Peaks-over-Threshold approach.[3] To do so, the upper and lower tails and the centre of the standardized return distribution are isolated, and a Generalized Pareto Distribution is fitted to the tails of the cumulative distribution (CDF) function of the standardized returns, and a piecewise polynomial to the centre of the distribution. The resulting piecewise distribution allows interpolation within the interior of the CDF and extrapolation in each tail. The ability to extrapolate the tails of the CDF allows one to estimate quantiles that lie outside the range of historical observations, and is hence highly suited to estimating tail-related risk measures. Third, a copula function is used to model the dependency structure of individual hedge fund return indices. The most commonly used copulae are the Gaussian copula for linear correlation, the Archimedean copula and *t*-copula for tail-dependence, and the Gumbel copula for extreme distributions (see, for example, Bouye *et al*. 2000). Since the AR(1)-EGARCH(1,1) filtered returns are *i.i.d.*, the *t*-copula is most suitable for modelling the dependency in returns. Both a constant correlation matrix and a dynamic correlation matrix that is obtained from the DCC-GARCH model of Engle and Sheppard (2001) and Engle (2002) are used in fitting the copula function. Fourth, the resulting multivariate return distribution is used to simulate portfolio returns[4]. Using the parameters of the fitted *t*-copula, dependent hedge fund index returns are jointly simulated by first simulating the corresponding dependent standardized returns. To do so, first dependent uniform variates are simulated, then are extrapolated into the GPD tails and interpolated into the smoothed interior. The uniform variates are transformed to the standardized residuals using the inversion of the semi-parametric marginal CDF of each index. Then using these simulated standardized returns as the *i.i.d.* input process, the autocorrelation and

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[3] In Extreme Value Theory, there are two approaches to modelling extreme values: Block-Maxima and Peaks-over-Threshold. In the Block-Maxima approach, the largest observations are collected from large samples of identically distributed observations, and the Generalized Extreme Value Distribution is fitted to the maxima of each block. In the Peaks-over-Threshold approach, a Generalized Pareto Distribution is fitted to all large observations that exceed a high threshold (i.e. the large losses in the tails of the loss distribution). Empirically, the Peaks-Over-Threshold approach appears to yield a more accurate description of the tails of the distribution for financial returns (see, for example, Embrecht *et al*., 1997; Reiss & Thomas, 1997).

[4] The alternative optimization frameworks employed here are very sensitive to the availability of enough observation at the tails of the return distribution. However, most of the time, realized (observed) tail observations are not enough to fully describe the tails of the distribution due to the unavailability of the probable but unrealized (unobserved) returns. Therefore, the purpose of the simulation, employed here, is to generate data enough to describe the theoretical return distribution to make sure that the returns that are probable but unobserved are considered within the portfolio construction process.

heteroskedasticity is reintroduced that is estimated using the AR(1)-EGARCH(1,1) model in the first step. Finally, the simulated returns of each index are used to construct the CVaR-, CDaR- and Omega-optimized portfolios[5]. Optimization results using the non-parametric, empirically-based approach are also reported.

Initially each optimization model is estimated using observations $t = 1,\ldots,\tau$ to generate one month-ahead out-of-sample forecast portfolio weights for the month $t = \tau + 1$. The estimation sample is then rolled forward one month, and the forecast portfolio weights for month $t = \tau + 2$ are generated. The last iteration uses the sample $t = T - \tau - 1,\ldots,T - 1$ to generate forecast portfolio weights for the month $t = T$. Expected returns are estimated using the corresponding sample mean over the estimation sample. The optimization models are estimated using the Matlab *fmincon* function.

3.2.4. Evaluation

Each portfolio is rebalanced at each month and the realized portfolio return is calculated at the rebalancing date. The performance of each portfolio is evaluated over the out-of-sample period $t = \tau + 1,\ldots,T$ using the following metrics:

(1) Average Realized Portfolio Return

$$\bar{r}_P = \frac{1}{T-\tau}\sum_{t=\tau}^{T-1}\sum_{k=1}^{m} x_{k,t} r_{k,t+1} \tag{3.10}$$

(2) Standard Deviation of Realized Portfolio Returns

$$\sigma_P = \sqrt{\frac{1}{T-\tau}\sum_{t=\tau+1}^{T}\sum_{k=1}^{m}(x_{k,t} r_{k,t+1} - \bar{r}_P)^2} \tag{3.11}$$

(3) Maximum Drawdown

[5] It should be noted that the simulated returns are used in "*constructing*" the CVaR-, CDaR- and Omega-optimal portfolios. However, in evaluating the out-of-sample forecast performance of these semi-parametric models, as it is done for all benchmark models, realized (observed) returns are used.

$$MDD_P = \max_{\tau \le t \le T}\left[\max_{\tau \le j \le T}\left(w_j\right) - w_t\right] \tag{3.12}$$

where w_t is the uncompounded cumulative portfolio value at date $t = \tau + 1, ..., T$, and at $t = \tau$ initial portfolio value is set equal to 1.

(4) CVaR

$$CVaR^{\alpha}_{p,t+1} = \zeta + \tfrac{1}{1-\alpha}\tfrac{1}{T-\tau}\sum_{t=\tau}^{T-1}\max\left\{0, -\sum_{k=1}^{m} x_{k,t} r_{k,t+1} - \zeta\right\} \tag{3.13}$$

where α is the CVaR confidence level and ζ is portfolio VaR.

(5) CDaR

$$\begin{aligned} CDaR^{\alpha}_{p,t+1} = \quad & \zeta + \frac{1}{(1-\alpha)(T-\tau)}\sum_{t=\tau}^{T-1}\max \\ & \left\{0, \max_{\tau \le j \le t}\left[\sum_{k=1}^{m}\left(\sum_{s=\tau+1}^{j} r_{k,s}\right)x_{k,t}\right] - \sum_{k=1}^{m}\left(\sum_{s=\tau+1}^{t} r_{k,s}\right)x_{k,t} - \zeta\right\} \end{aligned} \tag{3.14}$$

where ζ is the threshold drawdown level and $CDaR^{\alpha}_{p,t+1}$ is portfolio CDaR defined at the α – confidence level for the out-of-sample evaluation period.

(6) Sharpe Ratio

$$SR_p = \frac{\bar{r}_P - r_f}{\sigma_P} \tag{3.15}$$

(7) Omega Ratio

$$Omega_p = \frac{\sum_{t=\tau}^{T-1}\max\left(0, \sum_{k=1}^{m} x_{k,t} r_{k,t+1} - r_{b,t+1}\right)}{\sum_{t=\tau}^{T-1}\max\left(0, r_{b,t+1} - \sum_{k=1}^{m} x_{k,t} r_{k,t+1}\right)} \tag{3.16}$$

where $r_{b,t+1}$ is the realized benchmark return at *t+1*.

(8) Information Ratio

$$IR_{p,t+1} = \frac{\bar{r}_p - \bar{r}_b}{\sigma_{(r_p - r_b)}} \tag{3.17}$$

(9) Portfolio Turnover

$$PT_p = \sum_{t=\tau}^{T-1} \sum_{k=1}^{m} \left| x_{k,t+1} - x_{k,t} \right| \tag{3.18}$$

3.3. Data

In the analysis, monthly data on ten HFR indices are used. In addition to the hedge fund strategy indices used in Chapter 2 (see 2.2.1), the short selling (SS) and emerging markets (EM) indices are also considered. The fund of funds (FOF) strategy index is used as a benchmark. The sample is for the period January 1990 to January 2011 (253 observations). This period includes a number of crises (e.g. Mexican crisis, Asian financial crisis, the Russian government default, the collapse of Long Term Capital Management, the collapse of the dotcom bubble and the most recent credit crisis). The initial estimation period is January 1990 to June 2002 (150 observations). The out-of-sample forecast period is July 2002 to January 2011 (103 observations). As the length of the data is different from the data used in Chapter 2, the data analysis is performed for the new data set. Findings are in line with the findings presented in Chapter 2 and Chapter 6. Summary statistics for the hedge fund return series are reported in Table 3.1.

[Table 3.1]

Panel A reports various descriptive statistics for the different hedge fund strategies for the full sample of 253 observations. Unsurprisingly, hedge fund strategies exhibit quite diverse statistical characteristics. All strategies reveal positive mean return. Some strategies display relatively higher volatility (such as equity hedge, emerging markets and short bias), while some strategies have quite low volatility (such as equity market neutral, merger arbitrage, relative value). However, all strategies exhibit negative skewness (except short selling and macro), and all strategies are leptokurtic. The null hypothesis of normality is strongly rejected for all strategies. Panel B reports the basic time series properties of the strategy returns. In particular, it reports the first five autocorrelation coefficients, the Ljung-Box portmanteau test for serial correlation up to 10 lags and the ARCH test of Engle (1982). All hedge fund strategies, except short selling, exhibit highly significant autocorrelations. The ARCH test suggests that there is evidence of volatility clustering in all strategy returns except event driven, emerging markets and mergers arbitrage. Panel C reports pair-wise correlations between single hedge fund strategy returns. Correlations are as low as 0.21 (between emerging market and equity market neutral strategy) and as high as 0.85 (between event driven and distressed securities strategy). The short selling strategy is negatively correlated with all other strategies. Overall, pair-wise correlations are relatively moderate, a property that is clearly desirable in the construction of funds of hedge funds.

3.4. Empirical Results

In this section, the out-of-sample performance of the different optimization approaches is reported. In particular, the non-parametric CVaR, CDaR and Omega models are compared with the static and dynamic semi-parametric CVaR, CDaR and Omega approaches described in the previous section. The results for four benchmark portfolios are also reported: the static and dynamic mean-variance models, the naïve (i.e. equally weighted) portfolio, and the HFR fund of hedge funds index. The performance of these ten portfolios is reported for the conservative investment mandate and the three formulations of the aggressive investment mandate. For the first formulation of the aggressive strategy (which minimizes risk subject to a target return), the target return is set equal to 14 percent. For the second formulation of the aggressive investment strategy (which maximizes return subject to a risk limit), risk limits of ω=0.02 (monthly) for the CVaR portfolio, ω=0.10 (annual) for the CDaR portfolio, and ω=0.0005 for the Omega portfolio, are used. For each portfolio, end of period value (assuming an initial unit investment), annualized average return, standard deviation, maximum drawdown, CVaR, CDaR, Sharpe ratio, Omega, information ratio, and portfolio turnover are reported. A risk-free rate of 2.03 percent is used, which is the average long term US Government bond rate for the sample period. This rate is also used as the return threshold in the calculation of Omega for the optimization. In order to evaluate the sensitivity of the results to the various parameters, a range of alternative return targets, risk constraints and estimation sample sizes are considered.

3.4.1. Conservative Investment Strategy

Table 3.2 reports the results for the conservative investment strategy. Of the four benchmark portfolios, the naïve portfolio generates the highest return, but also has relatively high risk in terms of volatility, MDD, CVaR and CDaR. The dynamic MV portfolio has a slightly higher Sharpe ratio, but the Omega ratio is substantially higher for the naïve portfolio. The HFR index generates similar returns to the two MV portfolios, but it is much more volatile, and as a result, has a much lower Sharpe ratio. The non-parametric models generally underperform the four benchmarks, with none generating a return higher than that of the naïve portfolio, and only the non-parametric CVaR portfolio has comparable return performance. The Omega model, while generating the lowest returns of the three non-parametric portfolios (and lower than all four benchmark portfolios) has lower standard deviation, and very much lower MDD, CVaR and CDaR. However, it still underperforms in terms of risk-adjusted performance, with lower Sharpe and Omega ratios. A striking feature of the three non-parametric models is that they have substantially lower turnover than the static and dynamic MV benchmark portfolios, and so on balance would be preferred by investors. However, overall, the naïve portfolio appears to offer the best investment strategy. The static semi-parametric CVaR, CDaR and Omega models all offer returns that are higher than their non-parametric counterparts, and in all cases, higher also than the four benchmark portfolios. They are also less risky than the non-parametric models, and have higher Sharpe ratios and, for two of the three models, higher Omega ratios also. However, the static semi-parametric models have substantially higher turnover than both the non-parametric models and the two benchmark MV models, and so the superior performance of the semi-parametric models would have to be weighed against the transaction costs that they incur. Figure 3.1, which exhibits the course of portfolio values of different conservative portfolios during the out-of-sample period, also confirms the findings above. The performance of the semi-parametric models, especially during and after the credit crisis, is remarkable. Panel A of Table 3.6a reports the average portfolio weights of the different conservative portfolios. It is notable that the Omega-based portfolios exhibit a very different composition from the other portfolios, with a much higher allocation to risk-reducing strategies (e.g. equity market neutral, short bias, arbitrage strategies including merger, convertible and relative value arbitrage) and much lower allocation to return-enhancing strategies (e.g. event driven, distressed securities, equity hedge, emerging markets and macro).

[Table 3.2, Figure 3.1]

3.4.2. Aggressive Investment Strategy

Table 3.3 reports the results for the first formulation of the aggressive investment strategy, which minimizes portfolio risk subject to a target return. As expected, imposing a relatively high target return leads to portfolios that generate higher returns relative to the conservative investment strategy, but which are also significantly more risky in terms of standard deviation, MDD, CVaR and CDaR. For the benchmark MV portfolios and the non-parametric portfolios, the increase in risk outweighs the increase in return, and consequently imposing a target return reduces the Sharpe ratio. However, the effect on the Omega ratio is less clear, with a decrease for the two MV portfolios and the CVaR portfolio, but an increase for the CDaR and Omega portfolios. The semi-parametric CDaR portfolio performs worse in the presence of a target return, with lower return, higher risk and a much lower Sharpe ratio, but the semi-parametric CVaR and Omega portfolios generate similar performance relative to the conservative investment strategy, in both their static and dynamic variants. The semi-parametric models clearly dominate the non-parametric models for this formulation of the aggressive investment strategy. In particular, no non-parametric model is able to match the performance of any of the semi-parametric models, static or dynamic. The imposition of a target return substantially increases portfolio turnover, especially for the semi-parametric models. These findings are also clearly confirmed by Figure 3.2, which exhibits the course of portfolio values. Semi-parametric CVaR and Omega models clearly dominate all models in terms of portfolio values, while semi-parametric CDaR models display similar performance to benchmark and non-parametric models.

[Table 3.3, Figure 3.2]

Table 3.4 reports the results for the second formulation of the aggressive investment strategy, which maximizes portfolio return subject to a maximum risk constraint. For all portfolios, the returns are higher than in the minimum risk formulation of the aggressive investment strategy (Figure 3.3), although the resulting portfolio risk is similar in the two cases. In terms of risk-adjusted performance, there is some improvement in both the Sharpe ratio and the Omega ratio. Transaction costs are generally lower than in the previous case. Of the four benchmark portfolios, the static and dynamic MV portfolios offer the highest return, although also higher risk. Of the non-parametric models, only the Omega model is able to match the performance of the MV benchmark portfolios, although it displays a static portfolio allocation. As in the previous case, both the static and dynamic semi-parametric models offer a substantial improvement in performance over their non-parametric counterparts, both in absolute return terms and in terms of risk-adjusted performance. These findings are also confirmed by Figure 3.3, which exhibits the course of portfolio values. Semi-parametric models clearly dominate benchmark and non-parametric models. Exceptional performance of semi-parametric models, especially during and after the credit crisis, is noteworthy.

[Table 3.4, Figure 3.3]

Table 3.5 reports the results for the third formulation of the aggressive investment strategy, which maximizes the return per unit of risk. For the benchmark MV portfolios and the non-parametric models, the results are very similar to the conservative investment strategy, with slightly lower returns and slightly lower risk, and very similar Sharpe and Omega ratios. However, for the semi-parametric models, while returns are significantly higher, risk is lower, and so there is a significant improvement in risk-adjusted performance relative to the conservative case. Nevertheless, both the static and dynamic semi-parametric models comfortably outperform their non-parametric counterparts. Figure 3.4, which displays the course of portfolio values of the models, confirms these findings. Semi-parametric CVaR model clearly dominates all models in terms of portfolio values, while other semi-parametric models comfortably outperform non-parametric and benchmark models.

[Table 3.5, Figure 3.4]

Panel B of Table 3.6a, and Panel C and D of Table 3.6b report the average portfolio weights of the different aggressive portfolios. Semi-parametric models exhibit remarkably different portfolio composition than benchmark and non-parametric models for all formulations of the aggressive investment strategy. In particular, CVaR- and Omega-based portfolios make significantly higher allocations to risk-reducing strategies and much lower allocations to return-enhancing strategies. Although, semi-parametric CDaR-based portfolios display similar composition to their non-parametric counterpart for the first formulation, they make similar allocations to CVaR- and Omega-based portfolios in the second and third formulation. The portfolio compositions reveal that the superior performance of the semi-parametric models is due to their well-balanced portfolio allocations between risk-reducing and return-enhancing strategies.

[Table 3.6]

3.4.3. Sensitivity Analysis

The sensitivity of the model portfolios to the risk limits, return targets and estimation samples are examined in order to test the robustness of the main results provided in the previous section.

First, the sensitivity of the CVaR, CDaR, Omega and MV portfolios to the risk limits is examined in the second formulation of the aggressive investment strategy. Table 3.7 and Figure 3.5 report the results for CVaR models, while Table 3.8 and Figure 3.6 for CDaR models. For the CVaR and CDaR models, increasing the risk limit enhances portfolio returns as well as risk. In terms of risk-adjusted returns, increasing the risk limit increases the Sharpe ratio, information ratio and Omega ratio up to a certain level. Beyond this, portfolio risk increases faster than return, and so the risk-adjusted performance declines. The semi-parametric CVaR and CDaR models significantly outperform the non-parametric CVaR and CDaR models at every level of the risk limit. At lower risk limits, the non-parametric models yield portfolios with lower risk and return. Increasing the risk limit significantly enhances the return performance of the semi-parametric models, while portfolio risk remains lower than for the non-parametric models, resulting in a significant improvement in risk-adjusted return performance.

Table 3.9 and Figure 3.7 report the results for Omega models. In contrast with the CVaR and CDaR models, for Omega models increasing the risk limit, which implies higher risk tolerance, results in an allocation with more portfolio weights given to instruments with higher downside potential. In portfolios constructed by non-parametric Omega model, the portfolio allocation is insensitive to risk limit up to a very high level of the lower partial moment limit. However, semi-parametric Omega models are quite sensitive to risk limits. Semi-parametric models yield higher return, lower risk, hence better risk-adjusted return performance at low levels of the risk limit. Increasing the risk limit worsens the return performance while portfolio risk is consistently lower than the non-parametric model. Specifically, the dynamic model is more sensitive to risk limits than the static model. Both models significantly outperform the non-parametric model especially at low levels of risk limits. For the MV models, portfolio performance is insensitive to changes in risk limits up to a certain level. After this level, while static model is still insensitive to changes in risk limit, return performance of the dynamic model improves faster than increases in portfolio risks, hence displays better risk-adjusted return performance than static model at every level of risk limit.

Second, the sensitivity analysis is extended in an unreported work by incorporating five additional return targets; 13, 15, 16, 17 and 18 percent. These results are available from the author upon request. Use of lower/higher additional return targets in the first formulation of the aggressive investment strategy does not significantly alter the main findings. At lower return targets such as 13 percent MV-based models and non-parametric CDaR, CVaR and Omega models exhibit poorer return, risk and risk-adjusted return performance than naïve portfolio. Among the proposed semi-parametric models, CDaR model exhibits slightly better performance than its non-parametric counterpart. However, semi-parametric CVaR and Omega models significantly outperform the naïve portfolio at return, risk and risk-adjusted return grounds. These models also make quite balanced allocations between risk-reducing and return-enhancing strategies (allocate half of the portfolio to risk-reducing strategies), while other models allocate only a small fraction of the portfolio to risk-reducing strategies (e.g. on average 15, 3 and 1 percent for the 13, 15 and 16 percent target returns). At higher return targets while other models construct the portfolio with all return-enhancing strategies, semi-parametric CVaR and Omega models still allocate a significant portion of the portfolio to risk-reducing strategies to keep portfolio risks under control (e.g. on average 40, 34 and 30 percent for the 15, 16 and 17 percent target returns). This noteworthy difference in portfolio allocation is also reflected in portfolio performance. Semi-parametric CVaR and Omega models exhibit significantly higher portfolio return, lower portfolio risk and notably higher risk-adjusted return statistics. Opposite to other models, increasing the return target further improves their portfolio performance as they generate higher portfolio returns with similar or slightly higher portfolio risk, hence higher risk-adjusted return statistics.

Third, two shorter (i.e. 100 and 125 months) and one longer (i.e. 175 months) additional estimation periods are incorporated in an unreported work. These results are available from the author upon request. Use of longer/shorter estimation sample lengths does not change the main findings. In particular, for conservative and aggressive portfolios (except second formulation), using a shorter sample length increases the portfolio return and risk; hence slightly increase risk-adjusted return performance ratios. However, in maximum return formulation of the aggressive mandate longer estimation sample enhance return performance at the expense of higher risks, consequently display slight improvement in the risk-adjusted return performance ratios.

In summary, it is examined if certain parameter preferences, such as risk tolerance limits, target return, size of the estimation period, have any effects on the main findings of the previous section. It has been found that the main results, presented in the previous section, remain unchanged.

3.5. Conclusion

The non-parametric approach to CVaR-, CDaR- and Omega-based optimization, while straightforward to implement, relies on a large data sample to generate sufficiently accurate estimates of the various measures. Moreover, it does not readily lend itself to incorporating the well-established dynamic characteristics of hedge fund returns, such as autocorrelation and volatility clustering. In this chapter, a semi-parametric approach to hedge fund portfolio optimization that addresses the shortcomings of the non-parametric approach is proposed. The performance of the different estimation approaches for conservative (minimum risk), as well as, aggressive (maximum return) investors is compared. For the aggressive investment strategy, three different formulations of the optimization problem are considered, each representing a different portfolio on the efficient frontier: minimization of risk subject to a target return, maximization of return subject to a target risk, and maximization of return per unit of risk. The semi-parametrically and non-parametrically estimated CVaR, CDaR and Omega models are also compared with a number of commonly used benchmarks, including an equally weighted portfolio, a constant volatility mean-variance model, a time-varying volatility mean-variance model, and a benchmark fund of hedge funds index.

In this chapter, the potential advantages of employing alternative optimization frameworks to the mean-variance framework are investigated. First, portfolio construction performance of non-parametrically estimated CVaR, CDaR and Omega models are compared with the benchmark models and the benchmark index. Second, possible areas of improvement in the performance of the alternative optimization models are searched for. In the literature, in an effort to improve the performance of the CDaR model, Checklov *et al.* (2005) propose a 'multi-scenario' CDaR by using non-parametric bootstrapping in re-sampling historical observations. However, they demonstrate that, on average, their proposed solution is 20 to 30 percent worse than those predicted by historical returns, in terms of risk-adjusted returns. In this chapter, a different route is followed, and a semi-parametric estimation for CVaR, CDaR and Omega models is proposed. In the semi-parametric approach, first the returns of each portfolio constituent are standardized in order to filter out the predictable dynamics related to autocorrelation and volatility clustering. Second, the marginal density of each standardized return series is modelled using a combination of extreme value theory (for the tails of the density) and piecewise polynomial (for the centre of the density), and the joint density of hedge fund index returns is constructed by using a copula-based approach. Finally, hedge fund returns from this joint density are simulated in order to compute the relevant moments and quantiles required for portfolio optimization.

The analyses presented in this chapter report two main findings. The first finding is that CVaR-, CDaR- and Omega-based optimization offers a significant improvement in terms of risk-adjusted portfolio performance over the mean-variance optimization. The second finding is that for all three risk measures, semi-parametric estimation of the optimal portfolio offers a very significant improvement over non-parametric estimation. In particular, the optimal portfolios, estimated by using the proposed semi-parametric approach, display significantly higher portfolio return, lower risk, higher risk-adjusted return, better upside potential and higher active management ratios, at the expense of higher portfolio turnovers. Another remarkable finding is the demonstrated success of the semi-parametrically estimated models in predicting the market upturn and downturns. These models exhibit superior performance during the rising markets as well as during to, and recovery from the crisis. It is also examined if certain parameter preferences, such as risk tolerance limits, target return, size of the estimation period, have any effects on the main findings of the previous section. It has been found that the main results are robust to as the choice of target return and the estimation period.

Table 3.1
Summary Statistics and Time Series Properties of Hedge Fund Indices

Panel A reports summary statistics in percentages for the hedge fund strategy index series over the period of May 1998 to January 2011 (153 months). Panel B reports the autoregressive conditional heteroskedasticity (ARCH) and autocorrelation test results for the full period. The Ljung–Box-Q test for autocorrelation of order up to 10 asymptotically distributed as a central Chi-square with 10 d.o.f. Under the null hypothesis, with 5 percent critical value is 18.307. ARCH(4) is Engle's LM test for autoregressive conditional heteroskedasticity, which is asymptotically distributed as a central Chi-square with 4 d.o.f. Under the null hypothesis with 5 percent critical value is 9.488. *p*-values are also reported in the adjacent columns. Panel C reports HFR single strategy hedge fund index correlations.

Panel A: Summary Statistics									
Index	Mean	Median	Std. Dev.	Min.	Max.	Skew	Kurtosis	JB Stats	pval
Convertible Arbitrage	0.77	1.01	1.94	-16.01	9.74	-3.21	29.58	9657.61	0.00
Distressed Securities	1.02	1.13	1.90	-8.50	7.06	-1.03	5.00	308.06	0.00
Event Driven	1.00	1.29	1.99	-8.90	5.13	-1.36	4.25	268.98	0.00
Equity Hedge	1.13	1.30	2.65	-9.46	10.88	-0.24	1.87	39.36	0.00
Emerging Markets	1.19	1.59	4.17	-21.02	14.80	-0.89	3.73	180.22	0.00
Equity Market Neutral	0.60	0.58	0.92	-2.87	3.59	-0.13	1.17	15.09	0.01
Mergers Arbitrage	0.74	0.92	1.20	-6.46	3.12	-2.20	8.87	1032.68	0.00
Macro	1.09	0.81	2.22	-6.40	7.88	0.46	0.84	16.24	0.00
Relative Value	0.86	0.93	1.28	-8.03	5.72	-2.24	14.12	2311.63	0.00
Short Bias	0.17	-0.20	5.58	-21.21	22.84	0.18	1.93	40.78	0.00
Panel B: Basic Time Series Properties									
Index	ARCH	pval	LB-Q	pval	ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)
Convertible Arbitrage	50.96	0.00	132.11	0.00	0.59	0.29	0.17	0.12	-0.02
Distressed Securities	36.49	0.00	110.14	0.00	0.55	0.27	0.16	0.14	0.02
Event Driven	6.75	0.15	47.40	0.00	0.38	0.15	0.11	0.06	0.02
Equity Hedge	23.33	0.00	27.72	0.00	0.25	0.15	0.09	0.04	-0.05
Emerging Markets	4.08	0.40	42.63	0.00	0.34	0.15	0.08	0.05	0.01
Equity Market Neutral	19.54	0.00	83.31	0.00	0.16	0.19	0.21	0.19	0.09
Mergers Arbitrage	3.29	0.51	28.33	0.00	0.22	0.08	0.11	0.00	0.07
Macro	13.90	0.01	23.07	0.01	0.16	0.00	0.01	0.11	0.17
Relative Value	31.52	0.00	74.87	0.00	0.44	0.25	0.13	0.11	0.02
Short Bias	54.35	0.00	9.63	0.47	0.10	-0.06	0.01	-0.09	-0.09
Panel C: Correlations between single hedge fund strategy indices									
Index	CA	DS	ED	EH	EM	EMN	MA	MAC	RV
Convertible Arbitrage	1.00								
Distressed Securities	0.64	1.00							
Event Driven	0.64	0.85	1.00						
Equity Hedge	0.57	0.68	0.83	1.00					
Emerging Markets	0.51	0.68	0.74	0.72	1.00				
Equity Market Neutral	0.26	0.33	0.35	0.45	0.21	1.00			
Mergers Arbitrage	0.46	0.56	0.75	0.58	0.51	0.31	1.00		
Macro	0.23	0.42	0.51	0.57	0.57	0.32	0.33	1.00	
Relative Value	0.79	0.77	0.74	0.66	0.59	0.36	0.54	0.33	1.00
Short Bias	-0.31	-0.48	-0.62	-0.75	-0.58	-0.11	-0.41	-0.36	-0.39

Table 3.2
Risk, Return and Turnover Performance Measures of Conservative Portfolio

The table reports evaluation criteria for the out-of-sample monthly rebalancing conservative portfolio in the period July 2002 to January 2011 (103 months). Benchmark models are the HFR Fund of Fund Index, naive (equally-weighted) portfolio, static and dynamic mean-variance (MV) models. Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99 percent confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
Benchmark models										
HFR index	1.444	4.45	5.62	25.11	22.63	1.62	0.124	1.473	0.000	0.00
Naive	1.736	6.54	4.52	17.29	19.31	4.49	0.289	2.397	0.338	0.00
MV (static)	1.468	4.54	3.31	12.91	14.74	3.42	0.218	1.024	0.009	28.20
MV (dynamic)	1.515	4.88	2.69	5.43	11.85	1.49	0.306	1.083	0.028	24.03
Non-parametric models										
CVaR	1.698	6.27	4.22	15.88	17.65	4.15	0.290	1.994	0.266	1.04
CDaR	1.430	4.25	3.93	15.76	17.77	4.12	0.163	0.938	- 0.024	1.78
Omega	1.424	4.16	2.58	7.82	12.07	2.13	0.239	0.941	- 0.023	7.01
Semi-parametric models (static)										
CVaR	1.826	7.08	2.99	5.00	8.09	1.37	0.488	1.716	0.173	40.93
CDaR	2.019	8.30	4.08	8.31	15.15	2.25	0.444	4.521	0.382	19.18
Omega	1.926	7.72	3.37	5.10	7.57	1.40	0.487	1.898	0.202	58.87
Semi-parametric models (dynamic)										
CVaR	2.023	8.32	4.13	8.92	15.02	2.41	0.440	5.118	0.409	17.90
CDaR	1.842	7.22	3.98	9.72	10.60	2.62	0.376	2.031	0.249	63.99
Omega	1.941	7.83	4.03	7.34	14.76	2.00	0.416	2.146	0.255	57.89

Table 3.3
Risk, Return and Turnover Performance Measures of Minimum Risk Aggressive Portfolio (Formulation 1)

The table reports evaluation criteria for the out-of-sample monthly rebalancing aggressive portfolio with minimum risk formulation with an annual nominal target return 14 percent in the period July 2002 to January 2011 (103 months). Benchmark models are the HFR Fund of Fund Index, naive (equally-weighted) portfolio, static and dynamic mean-variance (MV) models. Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99 percent confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
Benchmark models										
HFR index	1.444	4.45	5.62	25.11	22.63	1.62	0.124	1.473	0.000	0.00
Naive	1.736	6.54	4.52	17.29	19.31	4.49	0.289	2.397	0.338	0.00
MV (static)	1.688	6.51	8.79	41.68	41.26	9.64	0.147	1.630	0.149	15.97
MV (dynamic)	1.685	6.48	8.79	41.68	41.26	9.64	0.146	1.601	0.146	16.33
Non-parametric models										
CVaR	1.796	7.25	9.00	41.48	41.26	9.60	0.168	1.851	0.200	12.93
CDaR	1.715	6.70	8.91	41.54	41.26	9.61	0.151	1.679	0.162	13.27
Omega	1.643	6.18	8.63	41.50	41.26	9.60	0.139	1.520	0.130	15.25
Semi-parametric models (static)										
CVaR	2.475	10.76	5.54	8.82	12.45	2.37	0.455	3.790	0.331	112.10
CDaR	1.811	7.35	8.97	41.48	41.26	9.60	0.171	1.901	0.207	18.53
Omega	2.550	11.10	5.41	8.93	12.44	2.40	0.484	4.309	0.362	112.65
Semi-parametric models (dynamic)										
CVaR	2.489	10.83	5.64	8.81	12.45	2.37	0.450	3.936	0.335	114.55
CDaR	1.809	7.34	8.98	41.49	41.26	9.60	0.171	1.887	0.206	16.93
Omega	2.529	11.02	5.63	9.18	12.45	2.47	0.461	4.052	0.344	112.32

Table 3.4
Risk, Return and Turnover Performance Measures of Maximum Return Aggressive Portfolio (Formulation 2)

The table reports evaluation criteria for the out-of-sample monthly rebalancing aggressive portfolio with maximum return formulation with a risk constraint in the period July 2002 to January 2011 (103 months). Benchmark models are the HFR Fund of Fund Index, naive (equally-weighted) portfolio, static and dynamic mean-variance (MV) models. Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99 percent confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
Benchmark models										
HFR index	1.444	4.45	5.62	25.11	22.63	1.62	0.124	1.473	0.000	0.00
Naive	1.736	6.54	4.52	17.29	19.31	4.49	0.289	2.397	0.338	0.00
MV (static)	1.819	7.43	9.30	41.57	41.18	9.62	0.168	1.768	0.197	13.63
MV (dynamic)	2.184	9.61	9.67	40.54	41.17	9.42	0.226	2.352	0.323	9.98
Non-parametric models										
CVaR	1.675	6.13	4.54	14.05	16.44	3.71	0.261	1.888	0.224	18.88
CDaR	1.818	7.42	9.33	41.63	41.26	9.63	0.167	1.757	0.195	14.00
Omega	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.222	0.302	0.00
Semi-parametric models (static)										
CVaR	3.363	14.40	6.28	6.14	8.33	1.68	0.569	9.055	0.563	99.95
CDaR	3.375	14.53	7.57	15.89	17.28	4.15	0.477	6.836	0.608	97.35
Omega	2.903	12.91	9.32	32.48	30.48	7.84	0.337	3.925	0.499	57.00
Semi-parametric models (dynamic)										
CVaR	3.187	13.74	5.83	5.60	8.37	1.54	0.580	13.850	0.698	98.42
CDaR	3.505	14.98	7.53	10.18	12.46	2.74	0.496	6.467	0.609	98.90
Omega	2.990	13.14	8.00	12.41	13.98	3.30	0.401	3.320	0.420	80.00

Table 3.5
Risk, Return and Turnover Performance Measures of Maximum Excess Return over Minimum Risk Aggressive Portfolio (Formulation 3)

The table reports evaluation criteria for the out-of-sample monthly rebalancing aggressive portfolio with maximum excess return over minimum risk formulation in the period July 2002 to January 2011 (103 months). Benchmark models are the HFR Fund of Fund Index, naive (equally-weighted) portfolio, static and dynamic mean-variance (MV) models. Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99 percent confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
Benchmark models										
HFR index	1.444	4.45	5.62	25.11	22.63	1.62	0.124	1.473	0.000	0.00
Naive	1.736	6.54	4.52	17.29	19.31	4.49	0.289	2.397	0.338	0.00
MV (static)	1.452	4.40	2.96	9.43	14.02	2.54	0.231	0.988	-0.005	10.28
MV (dynamic)	1.490	4.68	2.17	2.72	5.98	0.76	0.352	1.041	0.014	21.59
Non-parametric models										
CVaR	1.426	4.17	2.69	4.87	7.92	1.34	0.230	0.946	- 0.019	15.01
CDaR	1.462	4.47	2.78	6.22	9.13	1.70	0.254	1.006	0.002	11.26
Omega	1.464	4.49	3.06	9.95	14.16	2.68	0.232	1.012	0.005	10.21
Semi-parametric models (static)										
CVaR	2.027	8.32	3.40	4.26	5.45	1.17	0.534	2.135	0.230	70.00
CDaR	2.060	8.52	3.93	5.91	10.78	1.61	0.477	2.641	0.276	62.60
Omega	2.115	8.82	3.56	4.52	5.70	1.24	0.551	2.462	0.264	74.42
Semi-parametric models (dynamic)										
CVaR	2.332	9.98	4.01	5.16	6.93	1.41	0.573	3.440	0.352	66.43
CDaR	2.067	8.59	4.55	12.50	12.59	3.32	0.417	3.197	0.408	70.84
Omega	2.461	10.63	4.45	5.57	7.15	1.52	0.559	4.063	0.368	80.92

Table 3.6a
Portfolio Composition with 1-Month Rebalancing Portfolios

Panel A and Panel B report average portfolio weights of the ten single strategy hedge fund indices for conservative portfolio and aggressive portfolio with minimum risk formulation with an annual nominal target return 14 percent in the out-of-sample period July 2002 to January 2011. Maximum position for each asset is limited to 50 percent of the portfolio to force diversification. CVaR and CDaR portfolios are calculated at 95 percent confidence level.

	CA	DS	ED	EH	EM	EMN	MA	MAC	RV	SB
Panel A: Conservative Portfolio										
Benchmark models										
MV (static)	12.00	6.82	5.82	7.57	3.20	20.11	13.57	7.23	12.14	11.54
MV (dynamic)	8.19	7.01	6.27	6.27	3.60	16.57	13.43	7.36	12.22	19.08
Non-parametric models										
CVaR	9.89	9.73	9.71	9.80	9.03	10.31	10.12	10.02	10.03	11.37
CDaR	11.87	5.51	6.81	8.65	2.03	17.50	15.92	15.49	9.65	6.57
Omega	8.05	2.93	0.00	8.66	0.01	26.62	21.95	3.11	18.45	10.23
Semi-parametric models (static)										
CVaR	9.00	10.11	4.50	8.28	2.80	23.81	8.85	7.50	10.82	14.33
CDaR	9.97	10.70	8.03	10.30	5.89	14.36	11.50	12.28	11.76	5.21
Omega	8.34	10.48	1.47	7.08	1.45	31.80	6.83	6.25	15.64	10.67
Semi-parametric models (dynamic)										
CVaR	9.94	9.28	2.25	2.68	0.79	38.76	5.22	11.44	18.70	0.95
CDaR	9.94	10.70	8.97	10.03	5.89	14.77	11.01	12.81	11.86	4.03
Omega	9.36	10.25	1.26	0.64	0.00	38.81	5.80	7.68	26.20	0.00
Panel B: Minimum Risk Aggressive Portfolio (Formulation 1)										
Benchmark models										
MV (static)	0.58	5.02	18.66	46.46	17.37	0.15	0.37	5.92	1.67	3.81
MV (dynamic)	0.96	5.54	12.66	46.14	17.86	0.27	0.30	8.78	4.80	2.70
Non-parametric models										
CVaR	1.86	3.43	14.78	41.48	24.36	1.41	1.77	7.89	2.08	0.92
CDaR	2.34	3.07	15.81	42.63	19.50	1.76	2.14	9.79	2.40	0.57
Omega	0.09	5.67	16.73	48.23	15.78	0.05	0.00	6.66	2.61	4.18
Semi-parametric models (static)										
CVaR	11.97	17.30	6.85	15.38	7.89	5.50	6.23	12.81	6.17	9.89
CDaR	1.95	4.17	13.56	42.54	23.05	1.79	1.78	7.53	3.07	0.56
Omega	12.14	17.34	6.57	14.26	8.11	6.04	6.49	12.41	6.83	9.82
Semi-parametric models (dynamic)										
CVaR	11.30	19.46	6.73	10.71	6.04	9.34	5.56	12.73	13.17	4.97
CDaR	2.06	4.18	14.37	42.29	22.61	1.76	1.81	7.71	2.69	0.52
Omega	11.12	19.43	6.61	9.26	6.30	8.72	6.67	12.12	15.43	4.34

Table 3.6b
Portfolio Composition with 1-Month Rebalancing Portfolios (cont'd)

Panel C and Panel D report average portfolio weights of the ten single strategy hedge fund indices for maximum return aggressive portfolio with a risk limit portfolio (Formulation 2) and maximum excess return over minimum risk aggressive portfolio (Formulation 3) in the out-of-sample period July 2002 to January 2011. Maximum position for each asset is limited to 50 percent of the portfolio to force diversification. CVaR and CDaR portfolios are calculated at 95 percent confidence level.

	CA	DS	ED	EH	EM	EMN	MA	MAC	RV	SB
Panel C: Maximum Return Aggressive Portfolio (Formulation 2)										
Benchmark models										
MV (static)	0.03	0.13	12.19	49.29	25.71	0.02	0.03	12.55	0.04	0.01
MV (dynamic)	0.02	0.09	1.74	48.83	42.42	0.01	0.02	6.83	0.02	0.01
Non-parametric models										
CVaR	4.75	10.75	10.22	24.72	4.39	4.99	6.39	18.44	5.99	9.37
CDaR	0.00	0.01	12.15	49.50	25.72	0.00	0.00	12.61	0.00	0.00
Omega	0.00	0.00	0.00	50.00	50.00	0.00	0.00	0.00	0.00	0.00
Semi-parametric models (static)										
CVaR	12.53	21.49	4.04	15.43	17.98	2.70	4.15	14.06	3.65	3.98
CDaR	7.35	14.16	18.02	23.86	23.87	0.99	0.09	7.91	2.99	0.76
Omega	4.37	6.31	19.42	21.36	42.72	0.00	0.00	5.34	0.49	0.00
Semi-parametric models (dynamic)										
CVaR	12.23	23.34	4.74	13.40	13.32	7.10	4.09	15.25	5.85	0.67
CDaR	7.38	15.15	16.62	16.12	28.26	1.26	0.26	11.10	1.56	2.30
Omega	4.85	3.40	16.50	12.62	41.75	0.00	0.00	8.25	0.00	12.62
Panel D: Maximum Excess Return over Minimum Risk Aggressive Portfolio (Formulation 3)										
Benchmark models										
MV (static)	11.71	6.56	0.43	12.23	0.08	12.82	18.42	7.67	19.68	10.39
MV (dynamic)	3.30	8.39	2.21	5.98	0.06	16.69	20.51	5.72	23.02	14.13
Non-parametric models										
CVaR	14.07	0.00	0.32	23.96	0.00	16.74	17.15	10.64	1.39	15.23
CDaR	9.71	0.80	0.93	1.33	0.47	39.17	25.17	15.15	3.29	1.39
Omega	6.45	4.04	0.00	17.51	0.00	11.50	17.22	6.91	23.93	12.44
Semi-parametric models (static)										
CVaR	9.22	11.64	1.42	8.37	1.57	28.30	5.93	9.60	12.85	11.00
CDaR	12.17	9.96	1.66	8.37	0.25	34.42	7.25	8.62	13.96	1.52
Omega	9.61	11.62	2.47	10.45	1.56	24.18	6.73	8.59	13.26	11.53
Semi-parametric models (dynamic)										
CVaR	11.14	15.86	1.73	2.23	0.02	33.21	3.52	14.17	17.31	0.32
CDaR	13.64	14.97	2.49	8.97	0.30	28.88	5.16	9.74	12.05	0.33
Omega	12.10	14.89	2.54	3.23	0.89	24.25	6.04	10.10	25.44	0.52

Table 3.7
Risk Sensitivities of Maximum Return Aggressive Portfolios to CVaR Risk Limits

Table reports sensitivities of maximum return (formulation 2 for an aggressive investor) portfolios to CVaR risk limits, for the out-of-sample monthly rebalancing in the period July 2002 to January 2011 (103 months). Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99 percent confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
Non-parametric models										
CVaR-0.005	1.320	3.26	2.18	4.72	8.43	1.30	0.163	0.803	- 0.079	10.87
CVaR-0.01	1.523	4.95	2.81	5.75	10.29	1.58	0.299	1.124	0.038	13.25
CVaR-0.02	1.675	6.13	4.54	14.05	16.44	3.71	0.261	1.888	0.224	18.88
CVaR-0.05	1.815	7.27	7.77	34.24	32.29	8.20	0.195	2.187	0.278	28.36
CVaR-0.10	1.812	7.38	9.30	41.62	41.19	9.63	0.166	1.753	0.194	16.16
CVaR-0.20	1.815	7.40	9.30	41.59	41.19	9.62	0.167	1.762	0.196	13.53
Semi-parametric models (static)										
CVaR-0.005	2.302	9.84	4.27	5.11	7.64	1.40	0.528	3.456	0.352	85.78
CVaR-0.01	2.761	12.01	4.99	5.55	8.34	1.52	0.577	7.223	0.503	100.12
CVaR-0.02	3.363	14.40	6.28	6.14	8.33	1.68	0.569	9.055	0.563	99.95
CVaR-0.05	3.547	15.14	7.77	10.60	12.46	2.83	0.487	5.340	0.487	98.43
CVaR-0.10	3.542	15.12	7.78	10.66	12.46	2.84	0.486	5.305	0.486	99.69
CVaR-0.20	3.541	15.12	7.78	10.69	12.46	2.85	0.486	5.296	0.485	99.69
Semi-parametric models (dynamic)										
CVaR-0.005	1.966	7.98	3.99	6.54	10.52	1.79	0.430	2.185	0.252	61.75
CVaR-0.01	2.374	10.22	4.67	6.52	10.53	1.79	0.507	4.897	0.441	82.54
CVaR-0.02	3.187	13.74	5.83	5.60	8.37	1.54	0.580	13.850	0.698	98.42
CVaR-0.05	3.570	15.21	7.70	10.83	12.45	2.89	0.494	5.515	0.499	98.06
CVaR-0.10	3.511	15.02	7.79	10.99	12.46	2.93	0.481	5.067	0.478	100.15
CVaR-0.20	3.513	15.02	7.79	10.96	12.45	2.92	0.481	5.094	0.478	99.93

Table 3.8
Risk Sensitivities of Maximum Return Aggressive Portfolios to CDaR Risk Limits

Table reports sensitivities of maximum return (formulation 2 for an aggressive investor) portfolios to CDaR risk limits for the out-of-sample monthly rebalancing in the period July 2002 to January 2011 (103 months). Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99 percent confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
Non-parametric models										
CDaR-0.01	1.504	4.83	3.47	9.79	12.41	2.64	0.232	1.132	0.039	5.91
CDaR-0.05	1.683	6.40	8.02	36.95	35.46	8.73	0.157	1.652	0.179	15.33
CDaR-0.10	1.818	7.42	9.33	41.63	41.26	9.63	0.167	1.757	0.195	14.00
CDaR-0.20	1.818	7.42	9.33	41.63	41.26	9.63	0.167	1.757	0.195	13.99
CDaR-0.30	1.818	7.42	9.33	41.63	41.26	9.63	0.167	1.757	0.195	13.98
CDaR-0.50	1.818	7.42	9.33	41.63	41.26	9.63	0.167	1.757	0.195	14.00
Semi-parametric models (static)										
CDaR-0.01	2.282	9.74	4.25	5.57	8.98	1.52	0.523	5.004	0.374	66.37
CDaR-0.05	3.506	14.98	7.49	15.62	17.28	4.08	0.499	8.171	0.545	96.55
CDaR-0.10	3.375	14.53	7.57	15.89	17.28	4.15	0.477	6.836	0.608	97.35
CDaR-0.20	3.557	15.16	7.70	15.61	17.28	4.07	0.492	7.201	0.530	95.98
CDaR-0.30	3.484	14.93	7.78	15.60	17.28	4.07	0.479	6.492	0.512	96.95
CDaR-0.50	3.484	14.92	7.78	15.60	17.28	4.07	0.479	6.490	0.512	96.98
Semi-parametric models (dynamic)										
CDaR-0.01	2.040	8.42	4.21	7.83	12.43	2.13	0.438	3.047	0.325	64.23
CDaR-0.05	3.535	15.06	7.24	11.03	12.45	2.94	0.520	8.844	0.622	98.27
CDaR-0.10	3.505	14.98	7.53	10.18	12.46	2.74	0.496	6.467	0.609	98.90
CDaR-0.20	3.785	15.89	7.65	9.96	12.46	2.67	0.523	6.942	0.548	94.99
CDaR-0.30	3.785	15.89	7.65	9.96	12.46	2.67	0.523	6.942	0.548	94.99
CDaR-0.50	3.784	15.89	7.65	9.96	12.46	2.67	0.523	6.941	0.548	95.00

Table 3.9

Risk Sensitivities of Maximum Return Aggressive Portfolios to LPM Risk Limits

Table reports sensitivities of maximum return (formulation 2 for an aggressive investor) portfolios to lower partial moment (LPM) limit for Omega portfolios, in the out-of-sample test period July 2002 to January 2011 (103 months). Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99 percent confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
Non-parametric models										
OMG-0.0005	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.00
OMG-0.001	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.00
OMG-0.005	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.00
OMG-0.01	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.00
OMG-0.02	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.00
OMG-0.05	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.00
OMG-0.1	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.00
OMG-0.2	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.07
Semi-parametric models (static)										
OMG-0.0005	2.903	12.91	9.32	32.48	30.48	7.84	0.337	2.701	0.401	57.00
OMG-0.001	2.902	12.91	9.32	32.48	30.48	7.84	0.337	2.701	0.401	57.00
OMG-0.005	2.602	11.61	9.12	32.75	30.48	7.90	0.303	2.527	0.368	55.17
OMG-0.01	2.236	9.82	9.06	35.81	30.54	8.51	0.248	2.237	0.313	38.92
OMG-0.02	2.150	9.42	9.60	39.37	38.31	9.20	0.222	2.097	0.283	41.00
OMG-0.05	2.050	8.97	10.57	46.29	52.53	10.47	0.189	1.967	0.245	40.25
OMG-0.1	1.996	8.60	10.12	42.65	52.37	9.82	0.188	1.965	0.245	43.00
OMG-0.2	1.904	8.09	10.47	46.12	52.53	10.45	0.167	1.867	0.223	46.00
Semi-parametric models (dynamic)										
OMG-0.0005	2.990	13.14	8.00	12.41	13.98	3.30	0.401	3.219	0.475	80.00
OMG-0.001	2.787	12.32	8.09	14.42	13.98	3.79	0.367	2.956	0.440	81.00
OMG-0.005	2.389	10.48	7.74	15.82	28.16	4.14	0.315	2.797	0.391	72.87
OMG-0.01	1.621	5.83	6.25	15.10	28.09	3.96	0.176	2.159	0.268	49.69
OMG-0.02	1.140	1.64	4.80	14.88	11.91	3.91	-0.024	1.267	0.096	46.79
OMG-0.05	0.954	-0.29	7.12	32.70	51.39	7.89	-0.094	0.958	-0.013	47.48
OMG-0.1	0.885	-1.15	7.09	36.93	52.12	8.73	-0.130	0.852	-0.049	51.00
OMG-0.2	0.910	-0.82	7.17	33.41	52.12	8.03	-0.115	0.893	-0.035	49.00

Table 3.10
Risk Sensitivities of Maximum Return Aggressive Portfolios to Standard Deviation Risk Limits

Table reports sensitivities of maximum return (formulation 2 for an aggressive investor) portfolios to standard deviation (SD) limits for mean-variance portfolios in the out-of-sample test period July 2002 to January 2011 (103 months). Evaluation criteria include end of period portfolio value (EPV), annualized average return (AR), annualized standard deviation (SD), maximum drawdown (MDD), annualized conditional value at risk (CVaR) and conditional drawdown at risk (CDaR), Sharp Ratio (SR), Omega Ratio (OMG), Information Ratio (IR) and Portfolio Turnover (PT). CVaR and CDaR are estimated at 99 percent confidence level. AR, SD, MDD, CVaR and CDaR are in percentages.

	EPV	AR	SD	MDD	CVaR	CDaR	SR	OMG	IR	PT
MV (static)										
MV-0.005	1.818	7.42	9.30	41.57	41.17	9.62	0.167	1.852	0.230	13.76
MV-0.01	1.818	7.42	9.31	41.60	41.18	9.62	0.167	1.851	0.230	13.74
MV-0.015	1.818	7.42	9.31	41.61	41.26	9.62	0.167	1.851	0.230	13.93
MV-0.02	1.819	7.43	9.30	41.57	41.18	9.62	0.168	1.852	0.231	13.63
MV-0.03	2.189	9.66	9.91	42.78	41.17	9.84	0.222	2.050	0.282	4.42
MV-0.05	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.00
MV-0.1	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.00
MV-0.15	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.00
MV-0.2	2.275	10.14	10.22	43.56	41.26	9.98	0.229	2.071	0.287	0.00
MV (dynamic)										
MV-0.005	1.816	7.41	9.30	41.63	41.25	9.63	0.167	1.850	0.230	13.94
MV-0.01	1.817	7.42	9.30	41.59	41.25	9.62	0.167	1.851	0.230	13.91
MV-0.015	1.924	8.09	9.35	41.63	41.25	9.63	0.187	1.934	0.250	14.57
MV-0.02	2.184	9.61	9.67	40.54	41.17	9.42	0.226	2.098	0.287	9.98
MV-0.03	2.254	10.01	9.99	41.19	41.18	9.54	0.231	2.085	0.290	6.13
MV-0.05	2.450	11.07	10.70	43.97	52.43	10.06	0.244	2.217	0.299	5.51
MV-0.1	2.572	11.65	10.82	43.57	52.43	9.99	0.257	2.288	0.311	2.00
MV-0.15	2.599	11.77	10.78	42.55	52.43	9.80	0.261	2.318	0.315	4.00
MV-0.2	2.599	11.77	10.78	42.55	52.43	9.80	0.261	2.318	0.315	4.00

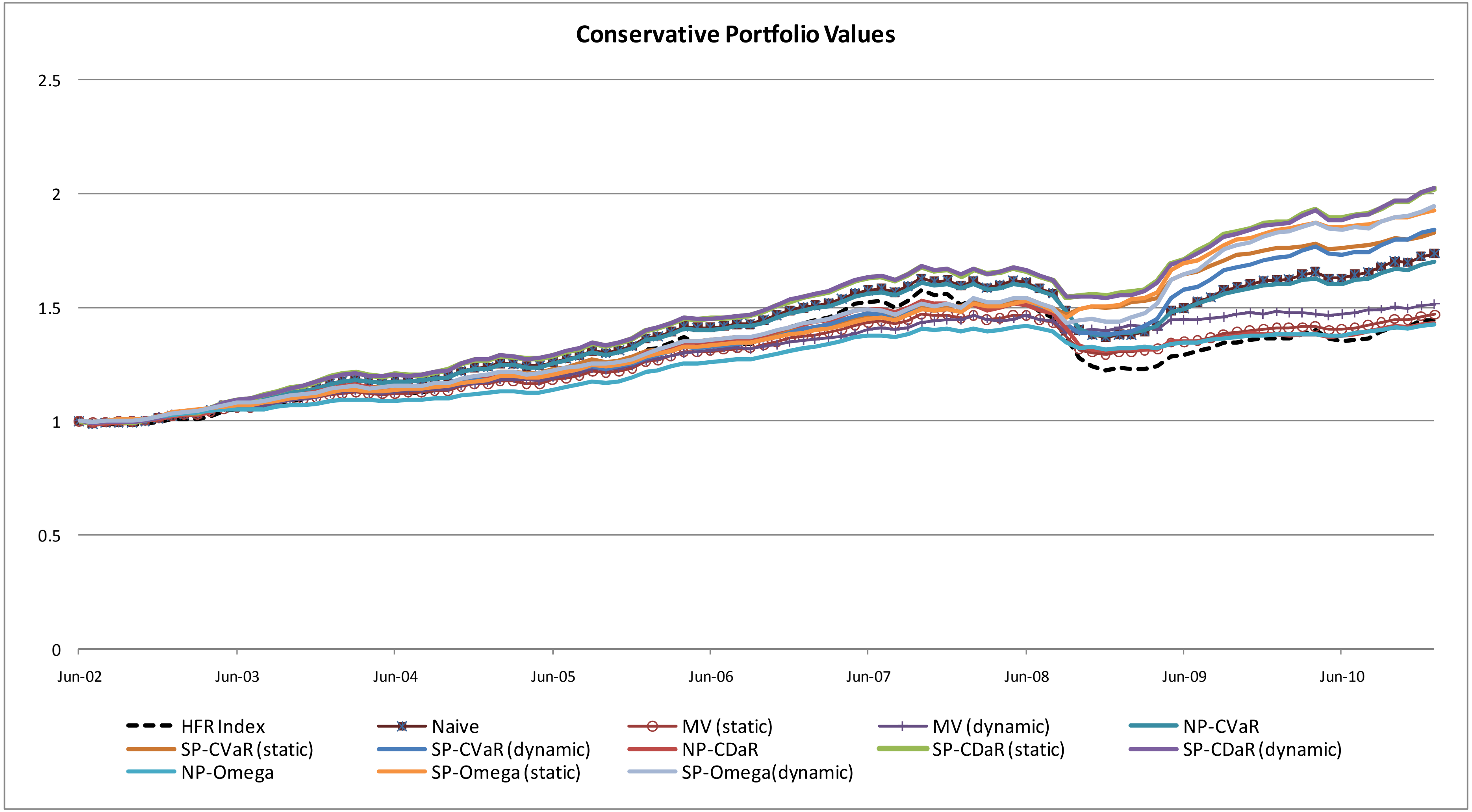


Figure 3.1: Portfolio Values of Conservative Portfolios. This figure displays portfolio value of conservative portfolios constructed by the models during the out-of-sample period July 2002 to January 2011 (103 months). NP stands for non-parametric, while SP stands for semi-parametric estimation method.

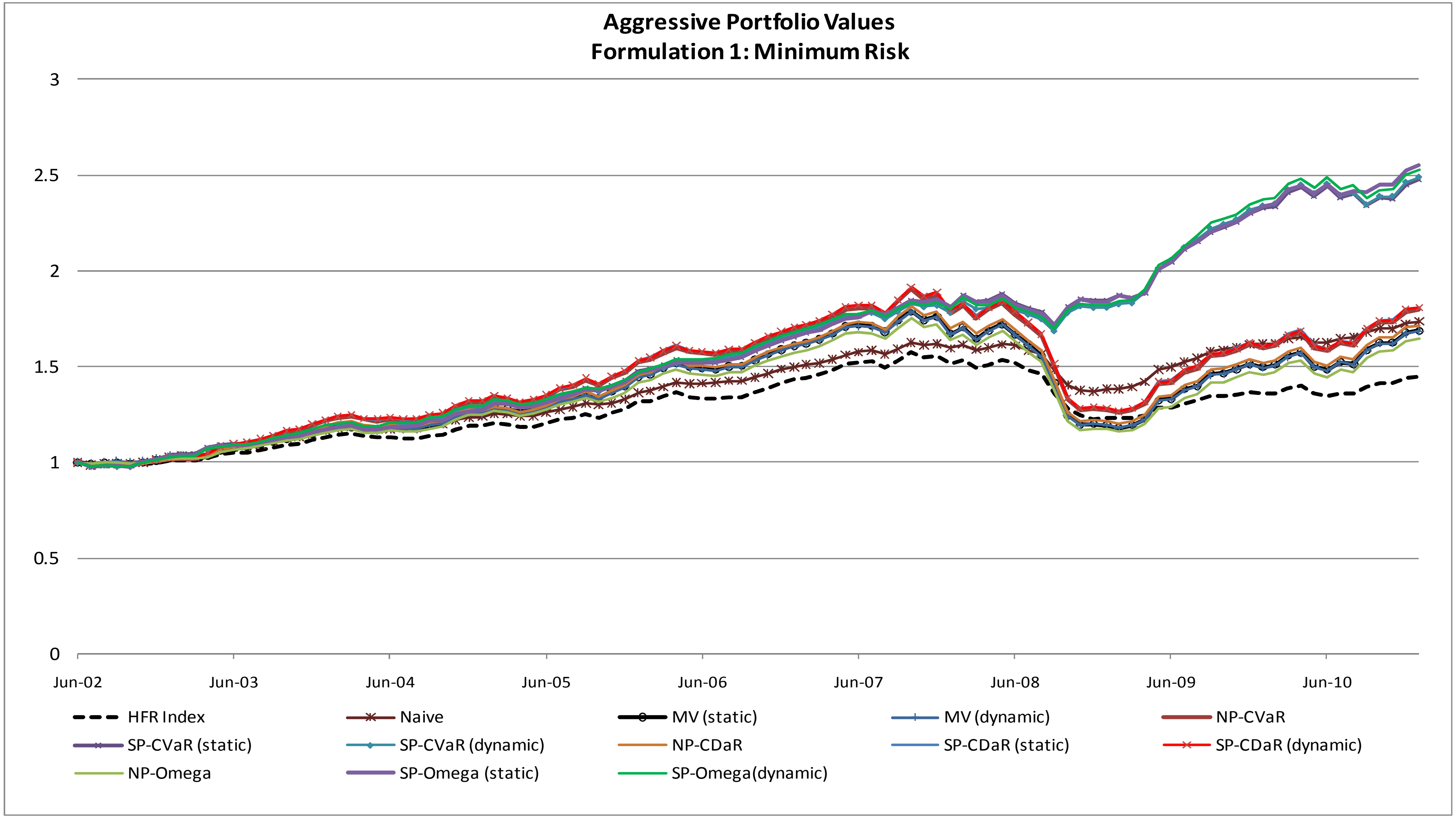


Figure 3.2: Portfolio Values of Minimum Risk (Formulation 1) Aggressive Portfolios. This figure displays portfolio value of minimum risk aggressive portfolios constructed by the models during the out-of-sample period July 2002 to January 2011 (103 months). Return target is annually 14 percent. NP stands for non-parametric, while SP stands for semi-parametric estimation method.

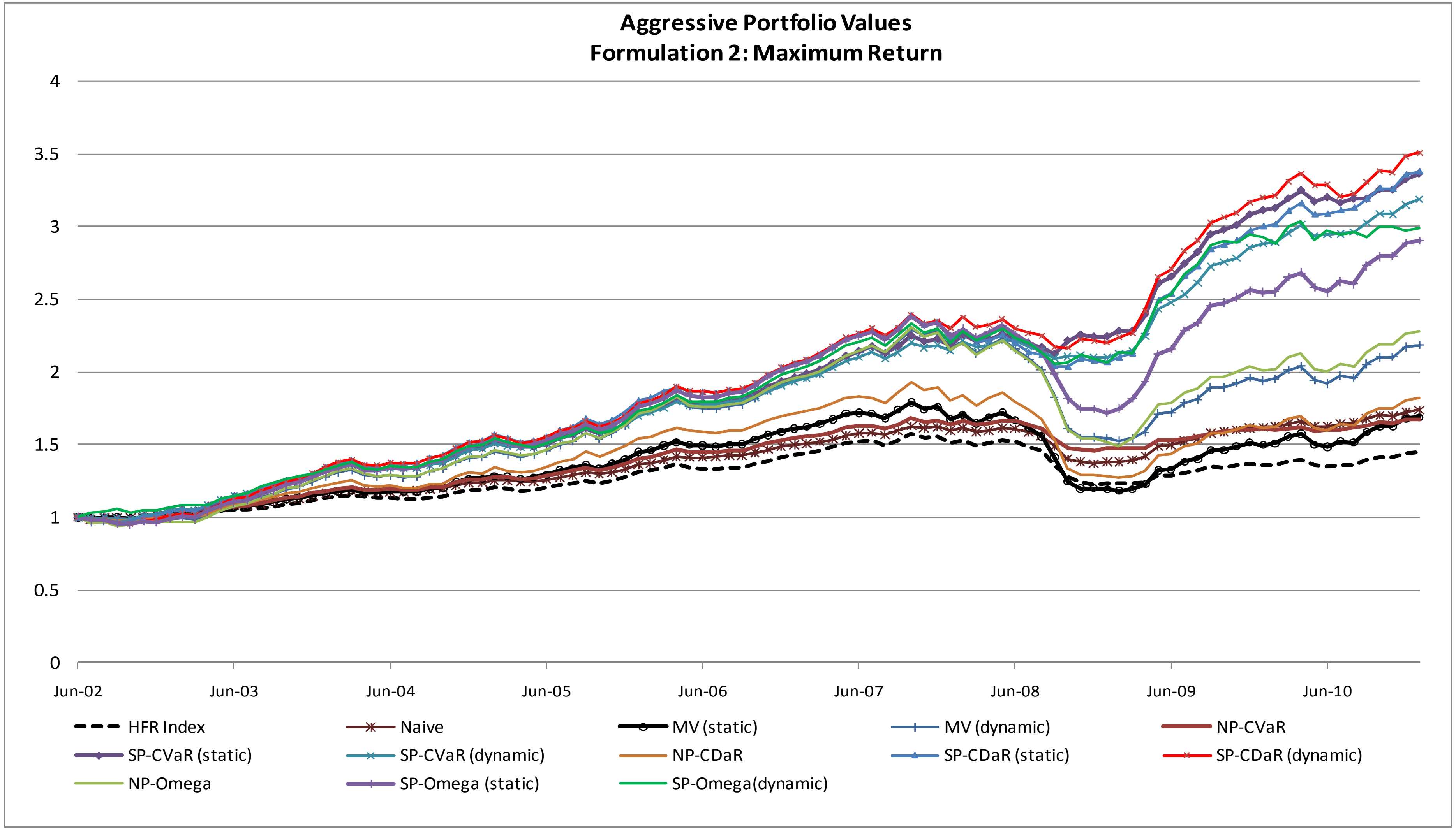


Figure 3.3: Portfolio Values of Maximum Return (Formulation 2) Aggressive Portfolios. This figure displays portfolio value of maximum return aggressive portfolios constructed by the models during the out-of-sample period July 2002 to January 2011 (103 months). NP stands for non-parametric, while SP stands for semi-parametric estimation method.

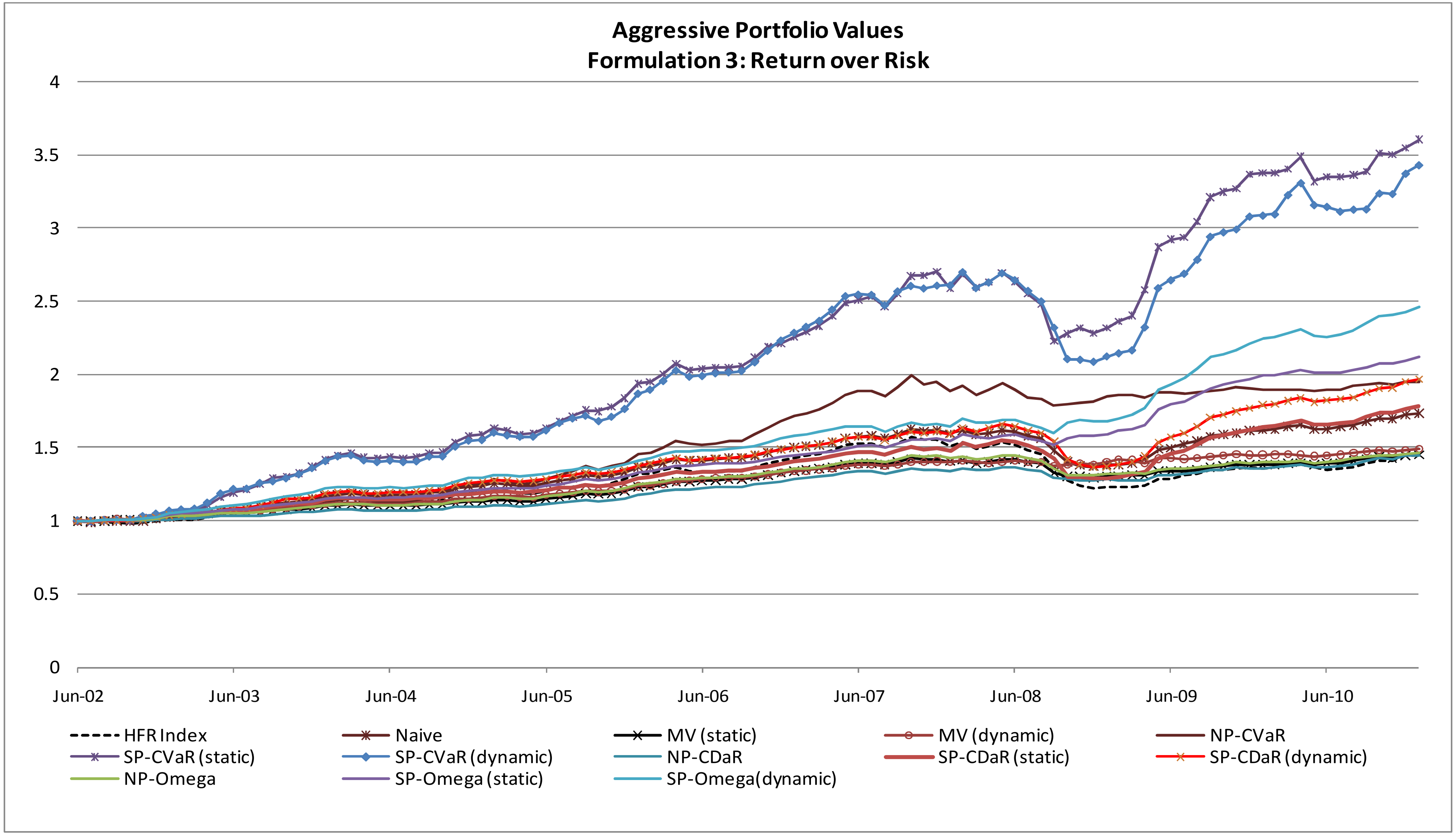
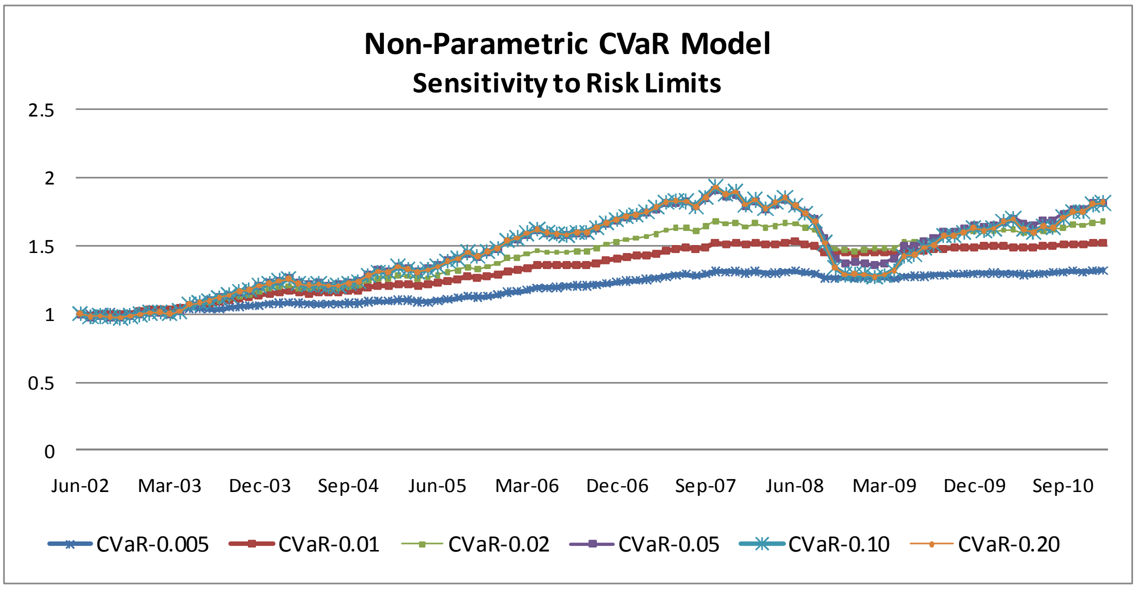
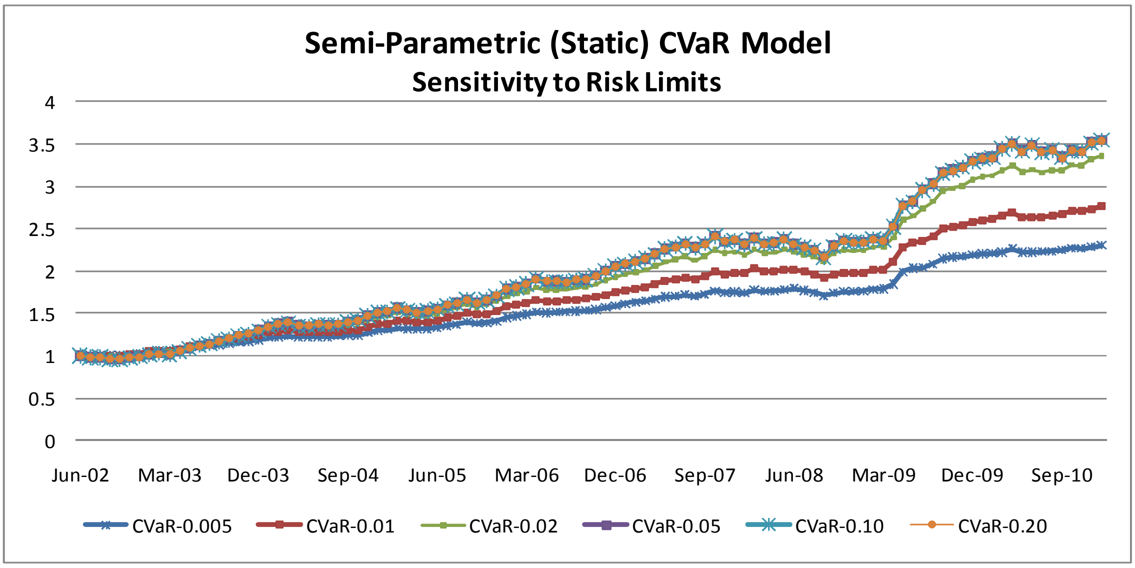


Figure 3.4: Portfolio Values of Return Over Risk (Formulation 3) Aggressive Portfolios. This figure displays portfolio value of return over risk aggressive portfolios constructed by the models during the out-of-sample period July 2002 to January 2011 (103 months). Return target is annually 14 percent. NP stands for non-parametric, while SP stands for semi-parametric estimation method.





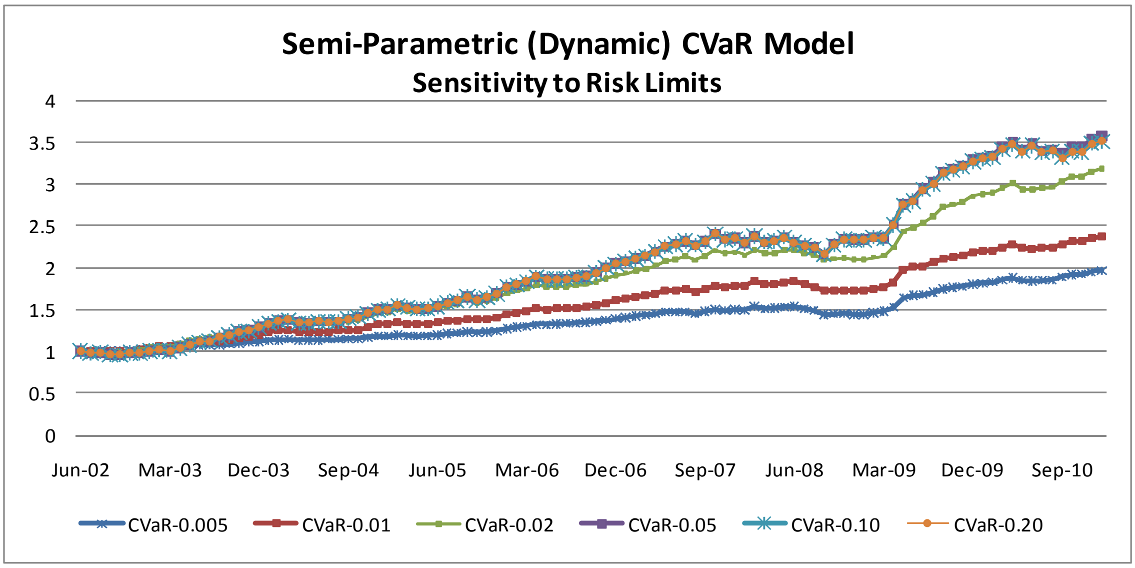


Figure 3.5: Sensitivity of CVaR Models to CVaR Risk Limits. This figure displays portfolio value of second formulation of aggressive portfolios constructed by CVaR models during the out-of-sample period July 2002 to January 2011 (103 months).

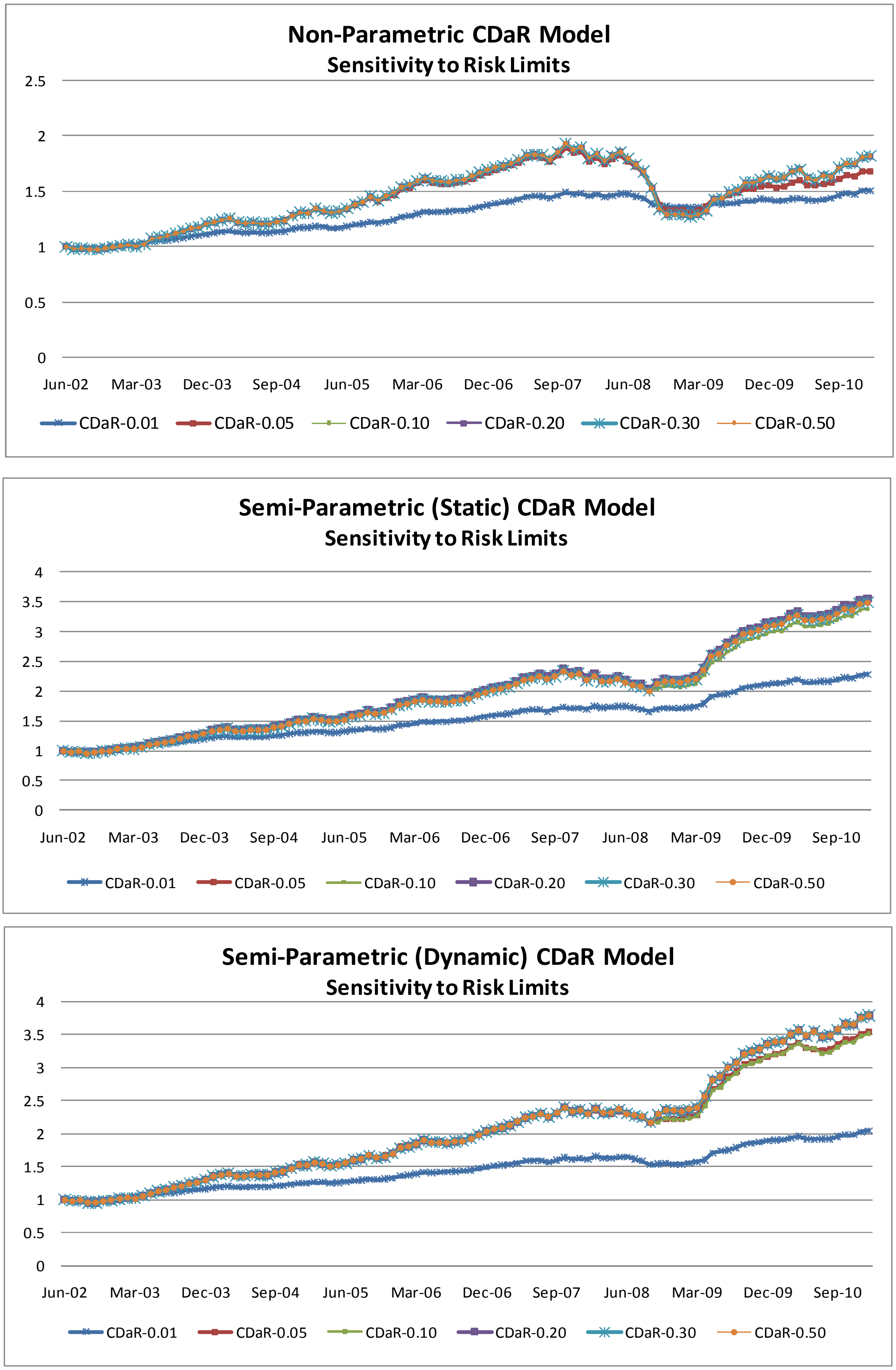


Figure 3.6: Sensitivity of CDaR Models to CDaR Risk Limits. This figure displays portfolio value of second formulation of aggressive portfolios constructed by CDaR models during the out-of-sample period July 2002 to January 2011 (103 months).

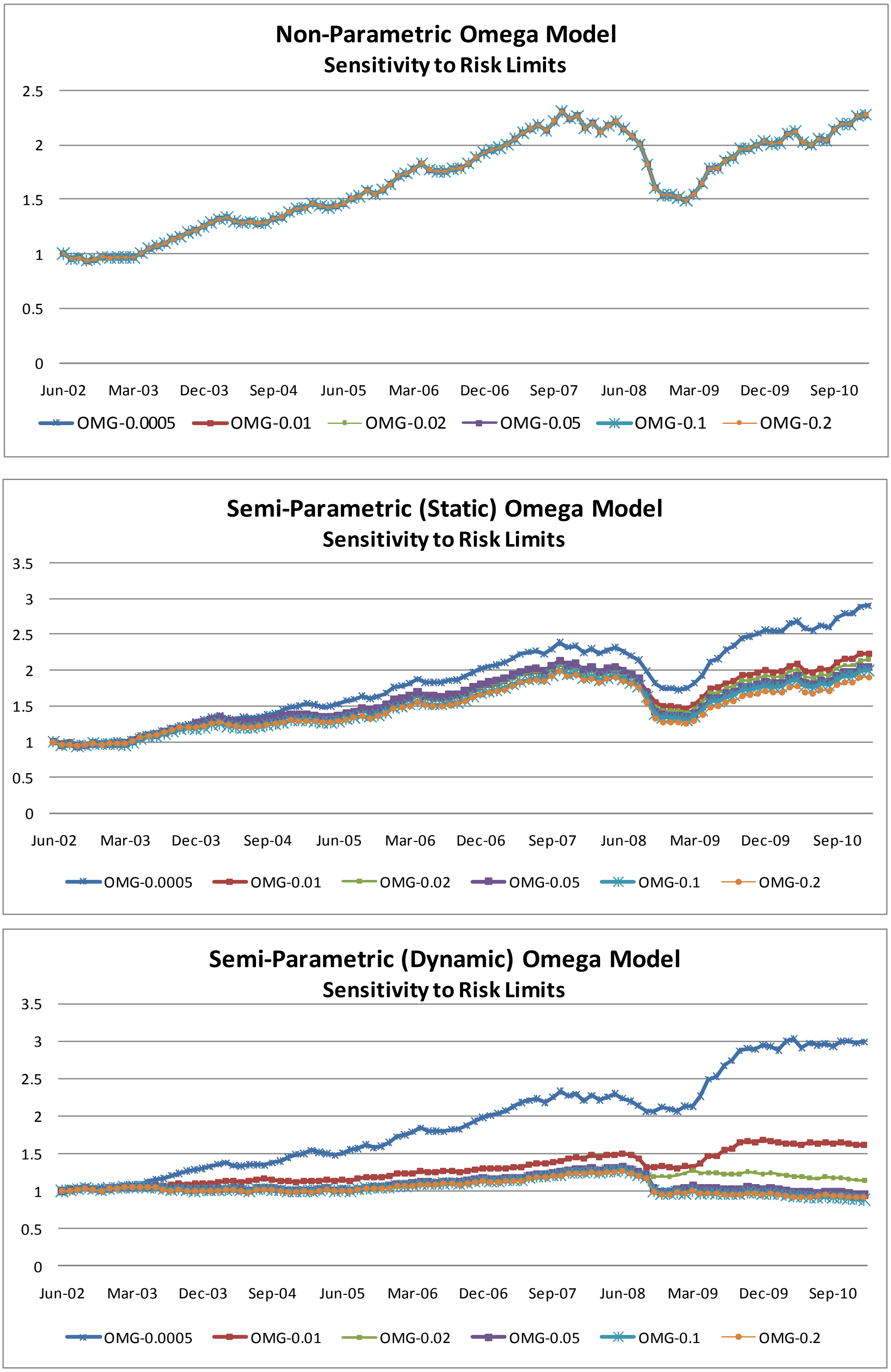


Figure 3.7: Sensitivity of Omega Models to LPM Limits. This figure displays portfolio value of second formulation of aggressive portfolios constructed by Omega models during the out-of-sample period July 2002 to January 2011 (103 months).

Chapter 4

Range-Based Regime Switching Models

4.1. Introduction

In recent years, there is a renewed interest in using the range as a variance measure. This is due to the attractiveness of the range-based volatility measures as an efficient, approximately normal and a robust measure to microstructure noise. Recent years also witnessed increased volatility and frequent changes in volatility states in financial markets. This encourages using models that are able to capture the changes in volatility states.

The aim of this chapter is to provide a volatility forecasting framework that combines regime switching models with data on daily range. The daily range is defined as the difference between the highest and lowest log asset prices over the trading day. A significant practical advantage of the intraday range is that, in contrast to with intraday data, the range data for individual assets and indices are widely available over long historical spans. Regime switching models capture the abrupt changes in asset prices due to financial crisis, government policy, business cycles or other economic fundamentals. Volatility prediction models should consider the stylized facts, financial time series exhibit, while forecasting the volatility in different volatility states (e.g. high, low).

Appearance of Markov switching models in econometrics starts with works of Hamilton (1988 and 1989). Markov switching models are popular both in financial and economic time series analysis. For the purpose of this study, the focus is on the financial applications of these models. In investigating regime switches in interest rates, Sola and Driffill (1994), Gray (1996), Ang and Bekaert (2002), Bansal *et al.* (2004) and Audrino (2006); in modelling stock returns, Turner *et al.* (1989), Pagan and Schwert (1990), Hamilton and Susmel (1994), Dueker (1997), Ryden *et al.* (1998), Susmel (2000), Billio and Pelizzon (2000), Maheu and McCurdy (2000), Perez-Quiros and Timmerman (2001) and Bhar and Hamori (2004); in foreign exchange rates, Engel and Hamilton (1990), Engel (1994), Vigfusson (1997), Bollen *et al.* (2000), Dewachter (2001), Klaassen (2002), Brunetti *et al.* (2003) and Beine *et al.* (2003) are prominent works among others.

Range is known as a viable measure of variability for a long time. Feller (1951) is the first to derive the distribution of the range of a driftless Brownian motion. Use of the range in finance dates back to Mandelbrot (1971) who employs the range in testing the long term dependence in asset prices. In search for a better variance estimator, under the assumption of a lognormal diffusion, Parkinson (1980) finds a variance estimator by using the high and low prices only. Extreme value theories imply that range is an efficient estimator of the local volatility due to Parkinson (1980). In this respect, he demonstrates the superiority of range as a volatility estimator as compared to conventional methods. Garman and Klass (1980) extend Parkinson's estimator to incorporate open/close/high/low prices. They introduce a minimum variance unbiased quadratic variance estimator for a driftless Brownian motion. Beckers (1983) extends Parkinson's analysis by incorporating time-varying drift. Ball and Torous (1984) derive a maximum likelihood equivalent of the estimator of Garman and Klass (1980). Rogers and Satchell (1991) also extend Parkinson's analysis, and find a better variance estimator using the high, low and closing prices. Kunitomo (1992) considers nonzero drift and derives a variance estimator based on the extremes of a constructed Brownian bridge motion. Recently, Yang and Zhang (2000) propose a new volatility estimator based on multiple periods of high, low, open and close prices, and demonstrate that the improvement of accuracy over the classical close-to-close estimator is dramatic. Andersen and Bollerslev (1998) report the favourable explanatory power of the range, and state that the daily range has approximately the same information content as sampling intraday returns for every four hours.

In search for a solution to high non-Gaussian measurement error in standard volatility processes, Alizadeh *et al.* (2002) incorporate the range into the equilibrium asset pricing models in a stochastic volatility framework. They demonstrate that range-based volatility proxies are not only highly efficient, but also approximately Gaussian, and robust to microstructure noise. Following the findings of Alizadeh *et al.* (2002) on the log range, Brandt and Jones (2006) combine EGARCH model with data on the intraday range, in an attempt to explain the dynamics of the conditional return volatility, and compare the range-based EGARCH model with return-based benchmark models. As a concurrent work, Chou (2005) proposes a conditional autoregressive range (CARR) model, which describes the dynamics of the conditional mean of the range and uses the level of the range.

There is an obvious literature gap between a regime switching volatility modelling and range. The aim of this chapter is to fill this gap. In this chapter, a regime switching conditional volatility model is combined with a robust measure of volatility based on the intraday range in one-factor and two-factor frameworks for volatility forecasting. This chapter proposes two univariate models that combine useful properties of range, regime switching, nonlinear filtration and GARCH frameworks. It is searched for any incremental improvement in the performance of volatility forecasting by employing regime switching in a conditional volatility setting with enhanced information content on true volatility. To the best knowledge of the author, this chapter contributes to the literature as the first study, which implements range-based data in volatility forecast with regime switching models. The proposed one and two component models nest regime switching conditional volatility models and eminent range-based models in a framework, which allows one to combine the desired properties of these two distinct models into the same framework.

The in-sample fit and the out-of-sample forecast performance of the proposed models are compared with eminent return-based and range-based counterparts. In particular, following return-based models are considered; exponentially weighted moving average (EWMA) model of Riskmetrics, generalized autoregressive conditional heteroskedasticity (GARCH) model of Bollerslev (1988), fractionally integrated autoregressive conditionally heteroskedastic (FIGARCH) model of Baillie *et al.* (1996), and Markov Regime Switching GARCH (MRSGARCH) model of Klaassen (2002). Proposed models are also compared with the well-known range-based counterparts; hybrid EWMA (HybEWMA) model of Harris and Yilmaz (2009), and CARR model of Chou (2005). In evaluating forecast comparison of the models, four different robust volatility proxy of weekly integrated variance are employed; the sum of squared daily returns, the sum of daily price range, the squared weekly return, and the scaled weekly range. In-sample goodness of fit and out-of-sample forecast performance of the models are evaluated by using a number of statistical and risk management loss functions. In particular, in statistical analysis mean error metrics, directional predictive ability tests, forecast evaluation regressions, pair-wise and joint tests are used for model comparison. In analysing forecasts from a risk management perspective, coverage tests and risk management loss functions are utilized. It is shown that the proposed models produce more accurate out of sample forecasts, contain more information about true volatility and exhibit similar or better performance when used for value at risk comparison. In particular, two-component model performs better than all other models.

The contribution of this chapter is twofold. First, to the best knowledge of the author, this is the first study in applying range data to regime switching models in a univariate context. Second, it is the first study which applies component structure to range data in a regime switching framework and proposes one and two-component range-based regime switching models.

In the second section, a comprehensive theoretical background of the range as a volatility proxy and regime switching GARCH models is provided, then proposed range-based models are introduced. In the third section, data and methodology are described. In the fourth section, empirical findings of in-sample and out of sample forecast comparisons are provided. Section 5 concludes.

4.2. Theoretical Background

4.2.1. Range as a Volatility Proxy

In this section, theoretical background of the range-based estimator is introduced. Before considering the range-based estimator, basic aspects of univariate volatility estimation should be given. First, consider a univariate stochastic volatility diffusion for the log of an asset price P_t, its drift μ , and instantaneous volatility σ_t is given by

$$\begin{aligned} dP_t &= \mu(P_t, \upsilon_t) + \sigma(P_t, \upsilon_t) dW_{P_t} \\ d\upsilon_t &= \alpha(P_t, \upsilon_t) + \beta(P_t, \upsilon_t) dW_{\upsilon_t} \end{aligned} \tag{4.1}$$

where W_{P_t} and W_{υ_t} are Wiener processes with correlation $dW_{P_t} dW_{\upsilon_t} = \theta(P_t, \upsilon_t)dt$. In line with Alizadeh *et al*. (2002), the streamlined version of the diffusion is defined as

$$\frac{dP_t}{P_t} = \mu dt + \sigma_t dW_{P_t}$$
$$d \ln \sigma_t = \alpha(\ln \overline{\sigma} - \ln \sigma_t)dt + \beta dW_{\upsilon_t} \tag{4.2}$$

where log volatility $\ln \sigma_t$ of returns dP_t / P_t is the latent state variable and evolves as a mean-reverting Ornstein-Uhlenbeck process. This process is discretised to approximate the continuous time model. Suppose that prices are observed at discrete intervals, $t = 1,...,T$. Here the volatility is assumed to be constant at the interval between *t* and *t+1*, and is stochastic from one interval to another. The security price process is assumed to follow a Brownian motion and discretised log volatility approximately has a conditional Gaussian distribution

$$\frac{dP_t}{P_t} = \mu dt + \sigma_t dW_{P_t}$$
$$\ln \sigma_{t+1} \mid \ln \sigma_t \sim N\left[\ln \overline{\sigma} + \rho(\ln \sigma_t - \ln \overline{\sigma}), \beta^2\right] \tag{4.3}$$

In volatility modelling, primarily, absolute and squared returns are used as volatility proxies. Given the continuously compounded return, $r_{t+1,t} = \ln P_{t+1} - \ln P_t$, over the interval *t* and *t+1*, traditional log volatility proxy is defined as (Alizadeh *et al.,* 2002).

$$\ln\left|f(P_{t+1,t})\right| = \gamma \ln\left|P_{t+1} - P_t\right| = \gamma \ln \sigma_t + \gamma \ln\left|P^*_{t+1} - P^*_t\right| \tag{4.4}$$

For absolute returns, $\gamma = 1$, and squared returns $\gamma = 2$. Since second part of the above equality requires that the log price process is a Martingale, it is homogeneous in volatility. This assumption is helpful from a statistical viewpoint. The log price can be defined as a Martingale process without a drift:

$$dP_s = \sigma_s dW_s \tag{4.5}$$

where σ_s is stochastic but assumed continuous and constant within each period. The discrete time process of integrated volatility h_t is defined as

$$h_t = \sqrt{Q_t - Q_{t-1}} \tag{4.6}$$

where Q_t denotes quadratic variation of log price . As Brandt and Jones (2006) point out, assuming zero drift causes only a very small bias, to the extent that the true drift differs slightly from 0. This reduces the mean squared error relative to a noisy drift estimator. It is well-established that the conditional distribution of log absolute and squared returns is different from a Gaussian distribution. As empirical findings of Jacquier *et al.* (1994), Andersen and Sorensen (1996), Kim *et al.* (1998) and Alizadeh *et al.* (2002) confirm that the quasi-maximum likelihood estimation with these traditional volatility proxies is *highly inefficient* and often *severely biased* in finite samples. In search for a better volatility proxy, the range provides a significant potential. Alizadeh *et al.* (2002) define log-range as the log volatility proxy which is the natural logarithm of the difference between highest and lowest log prices of a security;

$$R_t = \ln\left(\max_{s\in[t-1,t]} P_s - \min_{s\in[t-1,t]} P_s \right) \tag{4.7}$$

and is, to a very good approximation, distributed by a Gaussian distribution with mean of $.43 + \ln h_t$ and variance of $.29^2$. Thus, the log-range is a *noisy* linear proxy of log volatility $\ln h_t$ (see Brandt and Jones, 2006). Alizadeh *et al.* (2002) also relate the range data to return data. They show that the log absolute return has a mean of $-0.64 + \ln h_t$ and a variance of 1.11^2, and is far from Gaussian.

The main advantage of the range over integrated variance estimator that based on close-to-close prices is its extensive information content; the range contains information about all intraday movements of the price. In estimating the variance of returns, Parkinson (1980) proposes an estimator based on only high and low prices, while Garman and Klass (1980) propose an estimator based on high, low, opening and closing prices with certain assumptions on Brownian motion parameters (i.e. volatility and drift). The estimators, proposed by Rogers and Satchell (1991), Kunitomo (1992) and Yang and Zhang (2000), are mainly based on earlier estimators of Parkinson (1980) and Garman and Klass (1980). In this study, as the range estimator, due to Harris and Yilmaz (2009), an unbiased estimate of the variance of the daily close-to-close return, which uses a combination of the intraday range and the open-to-close return, is employed;

$$S_t^2 = \frac{1}{4\ln 2}\left(p_t^H - p_t^L\right)^2 + \left(p_t^O - p_{t-1}^C\right)^2 \qquad (4.8)$$

where p_t^O, p_{t-1}^C, p_t^H, p_t^L are the opening[6], closing, high and low prices, subsequently, and $1/(4\log 2)$ is the scaling factor and the second moment of range of a Brownian Motion.

Since Parkinson (1980) and as formalized by Andersen and Bollerslev (1998), the range is known as a much more efficient volatility proxy. Furthermore, Alizadeh *et al.* (2002) establish that the distribution of the log-range conditional on volatility is approximately Gaussian, which makes it highly effective volatility proxy. Basic properties of the range are its efficiency, normality and robustness to market microstructure noise. Alizadeh *et al.* (2002) state that, as a volatility proxy, log-range is superior to log-absolute and log-squared returns for two reasons: First, the variance of the measurement error are significantly smaller, thus range is more efficient. Second, it is remarkably well approximated as Gaussian. These findings make range an attractive volatility proxy for models that use Gaussian quasi-maximum likelihood estimations. In comparison to realized volatility, range is a less efficient volatility proxy under ideal conditions, but it proves superior to the realized volatility in biases caused by market microstructure noise.

Although range has many desirable properties, it suffers from discretisation bias. One source of this bias is the likely difference between the highest (lowest) asset price, observed at discrete points in time, and the true maximum (minimum) of the underlying diffusion process. This makes the observed range being a downward-biased estimate of the true range, thus, in turn, a noisy proxy of volatility. A second likely source of the bias is the infrequent trading of the component assets that may lead to a further downward bias of the range. Another source of bias might arise from conditional non-normality in returns (see Brandt and Jones, 2006).

4.2.2. Regime Switching Volatility Models

There have been some attempts to utilize the favourable properties of regime switching

[6] It should be noted that the opening prices are subject to the staleness problem identified by Stoll and Whalay (1990).

models, in capturing the effects of sudden dramatic economic and political events on the volatility, inherent in financial time series. Combining conditional variance models with regime switching models has the potential to explain latent switches between regimes, in addition to utilizing the potential of ARCH family models in explaining variation in financial time series.

First attempts of the use of regime switching in ARCH/GARCH models are the models proposed by Cai (1994) and Hamilton and Susmel (1994). Cai (1994) combines the Markov-switching model of Hamilton (1988) with the ARCH model of Engle (1982). He explicitly models the dynamics of the asymptotic variance in the switching regime ARCH as following a first-order Markov process. His model aims to maintain the volatility clustering property of the ARCH model and, in addition, capture the discrete shift in the intercept of the conditional variance that may cause spurious apparent persistence in the variance process. In a concurrent work, Hamilton and Susmel (1994) proposed another parameterization of the switching-regime ARCH (SWARCH) model. Hamilton and Susmel (1994) also extend their SWARCH model to SWARCH-L model with including an asymmetry part, which models leverage effects.

Although GARCH models are known better in describing volatility than higher order ARCH specifications, Cai (1994) and Hamilton and Susmel (1994) restricted their models to contain only ARCH parts, because of the inherent path dependence in GARCH models, and the difficulty in translating the GARCH model into a switching regime setting. Let the GARCH (1,1) model be given as

$$\begin{aligned} \varepsilon_t &= u_t \sqrt{h_t} \qquad u_t \overset{iid}{\sim} N(0,1) \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \end{aligned} \tag{4.9}$$

When the regime switching is introduced in the conditional variance equation in (4.9), it turns into

$$h_t^{(i)} = \gamma^{(i)} + \alpha^{(i)} \varepsilon_{t-1}^2 + \beta^{(i)} h_{t-1}^{(i)} \tag{4.10}$$

GARCH parameters are path dependent, and they depend on the entire history of ε_t and the regime indicator s_t, $i = 1,..,N$. Recursive estimation of the equation (4.10) requires the entire history of ε_t and $s_t.\{\varepsilon_{t-1},\varepsilon_{t-2},...,\varepsilon_0,s_t,s_{t-1},...,s_1\}$. Therefore, evaluation of the likelihood function for a sample of length T requires the integration over all N^T possible (unobserved) regime paths, rendering estimation of Equation (4.10) infeasible in practice (see Haas *et al.*, 2004). In order to avoid this problem, Gray (1996), Klaassen (2002) and Haas *et al.* (2004) proposed different formulations to generalize the ARCH specification into a GARCH setting.

4.2.3. A One Component Markov Regime Switching Autoregressive Conditional Range Model (MRSACR)

In this section, a range-based estimator of the conditional variance that captures the switches in volatility regimes is proposed. The estimator is mainly based on the multiplicative error models (MEM) of Engle and Russell (1998) and Engle and Gallo (2006), and the CARR model of Chou (2005) in specifying the conditional variance equation, and the MRSGARCH model of Klaassen (2002) in switching the conditional variance parameters.

After the rise of high-frequency data (HFD) in 1990's, a new class of models has been proposed by Engle and Russell (1998) for the analysis of data, which arrives at irregular intervals. The model, called autoregressive conditional duration (ACD) model that belongs to the MEM models of Engel (2002), focuses on the expected duration between events, and treats arrival times as random variables, which follow a point process. As suggested in Engle and Gallo (2006), the ACD model has the potential to be utilized in modelling daily high-low range, along with daily absolute returns and realized volatility. In the same way, Chou (2005) proposes CARR model, which is remarkably similar to ACD model. CARR model is proposed for the high-low range of asset prices within fixed time intervals.

Consider that P_t is the logarithmic asset price, $r_t = P_t - P_{t-1}$ is the logarithmic return, $R_t = \max\{P_t\} - \min\{P_t\}$ is the observed range, and the estimator given in equation (4.5) is an unbiased range-based estimator of variance. The regime switching autoregressive conditional range (MRSACR) model can be defined as

$$S_t = \sqrt{\lambda_t^{(i)}}\, v_t \quad \text{where} \quad v_t \mid \Omega_{t-1} \sim D(1, \xi_t) \tag{4.8a}$$

$$\lambda_t^{(i)} = \omega^{(i)} + \alpha_1^{(i)} S_{t-1}^2 + \beta_1^{(i)} E_{t-1}\left\{\lambda_{t-1}^{(i)} \mid s_t\right\} \tag{4.8b}$$

where S_t is the scaled range estimator[7] given in equation (4.5), v_t is a sequence of *i.i.d.* nonnegative random variable, and follows a positive support distribution (e.g. Exponential, Weibull, generalized error distribution). When the distribution of v_t is exponential, the model is called an *MRSACRE* (N, p, q) model. Similarly, when it is Weibull or GED, the resulting model is called an *MRSACRW* (N, p, q) or *MRSACRG* (N, p, q) model, subsequently. $\lambda_t^{(i)}$ is the conditional variance of the range, the coefficients $\left(\omega^{(i)}, \alpha^{(i)}, \beta^{(i)}\right)$ in the equation (4.8b) are all positive, and take different values in each state, *i*,, where $i = 1,2$. Parameters ω, α, β characterize the inherent uncertainty in range, and short term and long term effects (persistence) of shocks on the range in each state. In computing the expectation in equation (4.8b), due to path dependence and the resulting difficulty in estimating the state-dependent conditional variance estimator, $\lambda_t^{(i)}$, the estimation procedure of Klaassen (2002) is adapted as

$$E_{t-1}\left\{\lambda_{kt-1}^{(i)} \mid s_t\right\} = \sum_{j=1}^{2} \tilde{p}_{ji,t-1}\left(\lambda_{kt-1}^{(j)}\right) \tag{4.9}$$

where probabilities, $\tilde{p}_{ji,t}$, are calculated as

$$\tilde{p}_{ji,t} = \Pr\left(s_t = j \mid s_{t+1} = i, \Omega_{t-1}\right) = \frac{p_{ij} \Pr\left(s_t = j \mid \Omega_{t-1}\right)}{\Pr\left(s_{t+1} = i \mid \Omega_{t-1}\right)} = \frac{p_{ji} p_{j,t}}{p_{i,t+1}} \tag{4.10}$$

where $i, j = 1,2.$ The ex-ante transition probabilities of being in the first regime at t given the information at $t-1$ is estimated as

$$p_{j,t} = \Pr\left(s_t = j \mid \Omega_{t-1}\right) = \sum_{i=1}^{2} p_{ij}\left[\frac{f\left(S_{t-1} \mid s_t = i\right) p_{i,t-1}}{\sum_{k=1}^{2} f\left(S_{t-1} \mid s_t = k\right) p_{k,t-1}}\right] \tag{4.11}$$

where $f(\cdot)$ is the likelihood function of the conditional distributions (i.e. Exponential). Here, it is assumed that the latent regime indicator, s_t, is parameterized as a first-order

[7] Following Engle & Russel (1998), the proposed models are estimated by taking square root of S_t^2 in (4.5). For the proof, see Engle & Russel (1998, pp.1135).

Markov process by following Hamilton (1988, 1989 and 1990).

The multi-step-ahead volatility forecasts are estimated as a weighted average of the multi-step-ahead volatility forecasts in each regime

$$\widehat{\lambda}_{T,T+h} = \sum_{\kappa=1}^{h} \widehat{\lambda}_{T,T+\kappa} = \sum_{\kappa=1}^{h} \sum_{j=1}^{2} \Pr\left(s_{\kappa} = j \mid \Omega_{t-1}\right) \widehat{\lambda}_{T,T+\kappa}^{(i)} \tag{4.12}$$

where the weights are the predicted probabilities, and $\widehat{\lambda}_{T,T+\kappa}^{(i)}$ are the κ-step ahead volatility forecasts in each regime that can be recursively calculated as

$$\widehat{\lambda}_{T,T+\kappa}^{(i)} = \omega^{(i)} + \left(\alpha_1^{(i)} + \beta_1^{(i)}\right) E_T \left\{ \widehat{\lambda}_{T,T+\kappa-1}^{(i)} \mid s_{T+\kappa} \right\} \tag{4.13}$$

The model can also be extended to accommodate information carried out by log returns of prices, explicitly, if the estimator in equation (4.5) does not include return information. In doing so, return variable can enter into the conditional mean equation as an independent variable, in line with Engle and Gallo (2006).

4.2.3. A Two Component Markov Regime Switching Autoregressive Conditional Range Model (CMRSACR)

In this section, a range-based estimator of conditional volatility that captures both long run and short run dynamics is proposed. Following the authors who show that volatility can be estimated with a factor structure, it is assumed that the volatility process, which is represented by the square root of the intraday range, S_t here, is composed of two components; a long run trend component and a short run cyclical component (see Engle and Lee, 1999; Alizadeh *et al.*, 2002, Brand and Jones, 2006, Harris *et al.*, 2010).

$$S_t = q_t + \theta\left(S_{t-1} - q_{t-1}\right) + \varepsilon_t \tag{4.14}$$

where q_t is trend component, $S_{t-1} - q_{t-1}$ is the deviation of volatility from its long term trend, ε_t is a random error term with zero mean and constant variance. Engle and Lee (1999), Alizadeh *et al.* (2002), and Brand and Jones (2006) identify the dynamics of q_t as a GARCH-type model for return series or log-range series. However, Harris *et al.*

(2010) leave the precise dynamics unspecified, and utilize the filter of Hodrick and Prescott (1997) to decompose the long term trend component from the intraday range. In particular, in the first step, Harris *et al.* (2010) apply HP filter separately to intraday high and low prices, and then estimate long term trend of volatility, q_t, from these smoothed high and low prices, in a way similar to (4.15), and finally take the square root of scaled range. In the second step, they estimate an autoregressive model for the cyclical component. By using the estimated autoregressive parameter as a weight for the trend component and the intraday range, they make *h-step*-ahead forecasts for volatility.

The model given in (4.14) is estimated in two stages: in the first stage, similar to Harris *et al.* (2010), the precise dynamics of long term trend component is left unspecified. However, the long term nonlinear trend component is separated directly from the intraday range data by using the HP filter recursively. The smoothing parameter of the HP filter is set to 250,000 (i.e. $\lambda = 100 \times nf^2$, where assuming 50 trading weeks in a year, nf=50). With this, it is implicitly assumed that the long term component relates to the business cycle.

In the second step, a MRSGARCH model for the cyclical component is estimated. As the HP filter is used to obtain a smoothed nonlinear representation of the intraday range, it is more sensitive to long term shocks on the range. Therefore, observing more sensitivity to short term shocks in the cyclical component would be expected. To include these sensitivities in to volatility forecasts, a MRSGARCH model is implemented with zero mean in its variance equation, and regime switching parameters:

$$S_t - q_t = \sqrt{h_t^{(i)}} u_t \quad \text{where } u_t \mid \Omega_{t-1} \overset{iid}{\sim} N(0,1) \tag{4.15a}$$

$$h_t^{(i)} = \alpha_1^{(i)} (S_{t-1} - q_{t-1})^2 + \beta_1^{(i)} E_{t-1}\{h_{t-1}^{(i)} \mid s_t\} \tag{4.15b}$$

where $(S_t - q_t)$ is the cyclical component of volatility, u_t is *i.i.d*, zero mean, constant variance and approximately normally distributed. To estimate the expectation in (4.15b), formulas in (4.9), (4.10) and (4.11) are used.

In forecasting volatility with two component Markov regime switching range model, following Harris *et al.* (2010), it is assumed that the long run trend follows a random walk process over the forecast horizon. The θ parameter in (4.15) is used to forecast the cyclical component, and is defined as the sum of the persistence parameters in (4.15b). The persistence parameters are estimated using regime switching probabilities (4.10) as weights.

Therefore, *h-step*-ahead volatility for the two component Markov regime switching range model can be estimated using

$$\hat{S}_{t+h} = \left(1-\hat{\theta}^{h}\right)q_t + \hat{\theta}^{h}\left(\hat{S}_t\right) \tag{4.16}$$

where, as $h \to \infty$, volatility represented by using intraday range approaches its long term trend level, $\hat{S}_{t+h} \to q_t$.

4.3. Data and Methodology

4.3.1. Data

In the analysis, daily open, close, high and low prices of S&P500 index are used. The weekly data, which covers the period of 03 January 1982 - 26 March 2010 (1473 observations), is estimated from daily return and range data. The full sample is divided into an initialization (i.e. in-sample estimation) period, 8 January 1982 – 2 March 2001 (1000 weeks) and an out-of-sample forecast period, 9 March 2001 – 26 March 2010 (473 weeks). Models are estimated by using an initialization period of 1000 observations. Then models are tested out-of-sample for the remaining 473 weeks. An analysis with a shorter estimation period (i.e. 500 observations) and a longer out of sample test period (i.e. 973 observations) is also made. Models are also tested by using daily data for a shorter time span in order to check the robustness of the results. To save space, only the results of weekly analysis, for an estimation sample length of 1000 observations, are provided here.

[Table 4.1]

Summary statistics and the time series properties of the weekly range and return data are reported in Table 4.1. Panel A reports that S&P500 returns exhibit negative skewness, excess kurtosis and non-normality. Panel B reports the basic time series properties of the return series. In particular, it reports the first five autocorrelation coefficients, the Ljung-Box portmanteau test for serial correlation up to 10 lags and the ARCH test of Engle (1982). S&P500 returns display significant positive autocorrelations, and the ARCH test suggests that there is evidence of significant volatility clustering.

4.3.2. Methodology

In-sample fit and out-of-sample forecast performance of one and two component MRSACR models are compared with well-known return and range-based volatility models. In particular, the EWMA model of Riskmetrics with 0.94 decay factor, the GARCH model of Bollerslev (1986), the FIGARCH model of Baillie *et al.* (1996), and the Klaassen (2002)'s version of the MRSGARCH model are employed as return-based benchmarks. Similarly, the Hybrid EWMA model of Harris and Yilmaz (2009) and the CARR model of Chou (2005) are employed as range-based benchmarks. The models are estimated by assuming different error distributions. In particular, return-based models are estimated with Gaussian, Student's t and GED errors and range-based models with Exponential, Weibull and GED errors. To make comparisons on equal grounds and to save space, only the results for exponential/Gaussian error models are provided. The results of different error specifications are qualitatively similar. Potential improvements in model performances are investigated by extending the information content of the data with using daily high, low, open and close prices, and by introducing regime switches governed by a first degree Markov process.

Four measures of the ex-post volatility are used:

(1) Weekly return squared (RV1):

$$RV1_t = (r_t - \bar{r})^2 \tag{4.17}$$

(2) Weekly scaled range (RV2):

$$RV2_t = \frac{1}{4\ln 2}(p_t^H - p_t^L)^2 + (p_t^O - p_{t-1}^C)^2 \tag{4.18}$$

(3) Sum of squared daily returns (RV3):

$$RV3_t = \sum_{k=1}^{K} (r_{t,k})^2 \quad (4.19)$$

(4) Sum of daily price range (RV4):

$$RV4_t = \sum_{k=1}^{K} \frac{1}{4\log 2} \left(p_{t,k}^{H} - p_{t,k}^{L} \right)^2 \quad (4.20)$$

First proxy is the weekly return squared that is, although inefficient estimator of the true volatility, commonly used in the literature. Second proxy of the integrated volatility is the weekly scaled range that is the weekly range adjusted for opening and closing prices. Third and fourth volatility proxies are inspired from intraday return and intraday range volatility estimators, which are known as highly efficient estimators of integrated variance.

First, each of the eight conditional volatility models is estimated by using observations $t = 1, \ldots, \tau$ to make *h*-week ($h = 1,2,4,13$) ahead out-of-sample forecast of the conditional covariance matrix for the week $t = \tau + h$. The estimation sample is then rolled forward one week, and the models re-estimated and *h*-week out-of-sample forecasts are made for the week $t = \tau + 1 + h$. The last iteration uses the sample $t = T - \tau - h, \ldots, T - h$ to generate forecast covariance matrix for the week $t = T$. This yields a series of 461 out-of-sample forecasts for each model. Then, in-sample and out-of-sample forecast performance of the models are evaluated by using a variety of evaluation criteria.

Mainly two groups of evaluation criteria are used; statistical performance measures and risk management loss functions. First, in order to assess the statistical performance of the volatility forecasts, following evaluation measures are computed.

(1) Root mean square error (RMSE):

$$RMSE = \left[T^{-1} \sum_{t=1}^{T} \left(RV_{t+h} - VF_{t,t+h} \right)^2 \right]^{1/2} \quad (4.21)$$

(2) Mean absolute error (MAE):

$$MAE = T^{-1}\sum_{t=1}^{T}\left|RV_{t+h} - VF_{t,t+h}\right| \tag{4.22}$$

(3) Heteroskedasticity adjusted Mean Square Error (HMSE):

$$HMSE = T^{-1}\sum_{t=1}^{T}\left(RV_{t+h}VF_{t,t+h}^{-1} - 1\right)^2 \tag{4.23}$$

(4) Q-likelihood loss function (QLIKE):

$$QLIKE = T^{-1}\sum_{t=1}^{T}\left(\log VF_{t,t+h} + RV_{t+h}VF_{t,t+h}^{-1}\right) \tag{4.24}$$

(5) Success Ratio (SR):

$$SR = T^{-1}\sum_{j=0}^{T-1} I_{\left\{\overline{RV}_{t+h+j}\overline{VF}_{t+j,t+h+j} > 0\right\}} \tag{4.25}$$

(6) Directional accuracy (DA):

$$DA = \frac{\left(SR - SRI\right)}{\sqrt{\mathrm{var}\left(SR\right) - \mathrm{var}\left(SRI\right)}} \tag{4.26}$$

(7) Mincer-Zarnowitz regression:

$$RV_{t+h} = a + bVF_{t,t+h} + u_{t+h} \tag{4.27}$$

(8) Encompassing regression:

$$RV_{t+h} = a + b_1 VF_{t,t+h}^{Model1} + b_2 VF_{t,t+h}^{Model2} + \ldots + b_3 VF_{t,t+h}^{ModelN} + u_{t+h} \tag{4.28}$$

The error statistics, RMSE, MAE, HMSE, R2LOG and QLIKE, measure the forecast accuracy of each model in relation to the measured volatility estimates (i.e. volatility proxies). RMSE and MAE are commonly used error metrics. HMSE is heteroskedasticity adjusted MSE proposed by Bollerslev and Ghysels (1996). QLIKE is a loss function for a Gaussian likelihood proposed by Bollerslev *et al.* (1994). SR and DA are directional predictive ability tests proposed by Peseran and Timmerman (1992). In equation (4.25), $SRI = P\hat{P} + (1-P)(1-\hat{P})$, P is the proportion of time that $\overline{RV}_{t+h+j} > 0$, and $\hat{P}$ is the proportion of positive demeaned volatility forecasts. Mincer-Zarnowitz regression measures forecast bias and efficiency of each model from measured volatilities. In particular, the efficiency of each model is tested by employing a joint test of $H_0 \ : \ a = 0, b = 1$. If the model is efficient, the null hypothesis should not be able to be rejected. The R-square coefficient of the Mincer-Zarnowitz regression indicates the explanatory power of each model's forecast, without considering any bias or efficiency. Newey and West (1987) procedures are employed in computing heteroskedasticity-autocorrelation-consistent standard errors for the regressions. Finally, an encompassing regression is run to test whether the forecast of one model includes any incremental information over the forecasts of the other models. In particular, the null hypothesis of $H_0 \ : \ b_k = 0$ can be separately tested for each model $k = 1,...,N$.

Second, model forecasts are compared *pair-wise* by using Modified Diebold-Mariano (MDM) test of Harvey *et al.* (1997), and *jointly* by using Reality Check (RC) test of White (2000) and Superior Predictive Ability (SPA) test of Hansen (2005). In RC and SPA test, the null hypothesis that all models are no better than a particular model (i.e. benchmark model) is tested.

The second method of evaluating the forecasting performance of the individual conditional volatility models is to assess whether the models are adequate for risk management purposes. These tests use value-at-risk (VaR) based loss functions. In selecting the adequacy of the models for risk management purposes, primarily two groups of tests are used. The first group consists of the coverage tests; Time Until First Failure (TUFF) and proportion of failure (PF) tests of Brooks and Persand (2003). The TUFF test is based on the number of observations before the occurrence of the first exception, and test the hypothesis of $H_0 \ : \ \alpha = \alpha_0$ by using a LR test. TUFF is known as having low power of rejecting inadequate models. The PF is based upon the percentage of times that the calculated VaR is insufficient to cover the actual losses. The PF test is conducted as unconditional and conditional coverage tests. In PF test, a model is assessed if it's out-of-sample percentage of failure is close to the significance level (i.e. $\alpha = 0.01$ or $\alpha = 0.05$). The PF is also tested for the null of an independently distributed failure process by the coverage test proposed by Christoffersen (1998). This test combines the unconditional coverage test with an independence test. For the models which pass these tests, following Sarma *et al.* (2003), a second group of tests, the regulator loss function (RLF), $l_t^1 = \left(r_t - VaR_t^k\right)^2 I_{\left\{r_t < VaR_t^k\right\}}$, and the firm loss function (FLF), $-\delta VaR_t^k I_{\left\{r_t > VaR_t^k\right\}}$, of the surviving models are estimated. Finally, a sign test on the loss differentials between pairs of models for RLF and FLF estimates is conducted. It tests the null of a zero-median loss differential against the alternative of a negative median, with a studentized version of the sign test of Diebold and Mariano (1995). Rejection of the null implies that the first model is significantly better than the second model.

4.4. Empirical Findings

4.4.1. In-Sample Fit

In this section, goodness of fit of MRSACR and CMRSACR models is compared with the return-based models (EWMA, GARCH, FIGARCH and MRSGARCH) and the range-based models (HybEWMA and CARR). The in-sample parameter estimates of the eight models are presented in the Table 4.2. The standard errors are asymptotic standard errors. Following a common assumption in practice, the mean return is assumed to be zero for return-based models. As regime switching models have more parameters than single regime models, they are expected to fit data in sample better than more parsimonious single regime models. The parameters of the conditional variance and range estimates are all statistically significant.

[Table 4.2]

While long term persistence in the first regime, and constant and long term persistence in the second regime explain most of the volatility in the return-based MRSGARCH model, short and long term persistence explain it together in MRSACR model. On the overall, the persistence of shocks on the volatility is lower in range-based models than return-based models. The Akaike and Schwarz information criteria (AIC and BIC), which rank the models in terms of trade-off between precision and complexity, show that the CMRSACR model fits the data better than all others. Moreover, log likelihood estimates also show that the CMRSACR model fits the data better than all other models.

To conclude, the CMRSACR model shows better in-sample fit to the data.

4.4.2. Out-of-Sample Forecast Evaluation

The out-of-sample forecast performance of the selected models is evaluated by using statistical performance measures and risk management loss functions. Models are evaluated for the period 27 April 2001-26 March 2010 (461 observations). Forecasts for 1-, 2-, 4- and 13-week horizons are examined and the results are provided here. In addition to these results, the forecasting performance of the models is compared by employing different specifications of the error distributions (i.e. Student's t and GED for return-based models, Weibull and GED for the MRSACR model), and by adapting different forecast steps (1-, 4-, 8-, 13-week). The results are not qualitatively different from the results provided here. These additional results are available upon request.

The estimated out-of-sample evaluation criteria are provided in Table 4.3a and 4.3b. the MRSACR and CMRSACR model perform better than other single and two regime models. In particular, the CMRSACR model has the lowest forecast error statistics in all forecasts steps and for all volatility proxies, except the fourth step. In the fourth step, the MRSACR and MRSGARCH models perform better than all other models, and the CMRSACR follows them. The one factor MRSACR model also performs better than other competing models most of the time. All models exhibit quite high directional accuracy in their forecast: high SR and highly significant DA statistics. In particular, the CMRSACR model has the highest SR statistics for the first three forecast steps. At higher forecasts steps, it still performs better than all models except the MRSGARCH model. From the above results, the two factor CMRSACR model produces the lowest error predictions, predicts the direction of the volatility more accurate than return-based models at 1-, 2- and 4-week horizons, and still performs better at longer forecast horizons. The single factor MRSACR model also provides some improvement in terms of forecast accuracy over competing models.

[Table 4.3a and 4.3b]

In evaluating the significance of the pair-wise differences in prediction accuracy of the models, Modified Diebold-Mariano (MDM) test is employed. The null hypothesis that the mean square error of the proposed models is the same as that of each of the benchmark models is tested. Table 4.4 reports the pair-wise comparison of the eight models, as the MRSACR and CMRSACR models are being the benchmarks. The negative sign of the MDM statistics reveals that the loss of the proposed (MRSACR or CMRSACR) models is lower than the one implied by the benchmark models given the loss differential of ($MSE^{proposed} - MSE^{benchmark}$). MSE of one factor MRSACR model is slightly higher than the GARCH, FIGARCH and CMRSACR models at first three forecast steps. However, at the fourth forecast step, MSE of the MRSACR model is lower than all other models. MSE of the two-factor MRSACR (CMRSACR) model is lower than all other models at each forecast step except the fourth step. In general, forecasts of the MRSACR and CMRSACR models exhibit lower MSE than other models; however, the differences, most of the time, are not statistically significant. Overall, the CMRSACR model offers an increase in forecast accuracy at lower forecast horizons, and the MRSACR model offers increase at higher forecast horizons relative to other models. However, most of the time the differences are not statistically significant.

[Table 4.4]

In evaluating the significance of the joint prediction accuracy, Superior Predictive Ability (SPA) and Reality Check (RC) tests are used. The null hypothesis that the mean square error of all models is no better than a particular (benchmark) model is tested. Table 4.5 reports the p-values of joint prediction accuracy tests results. For the RV1 volatility proxy, the null is rejected for all models at the first forecast horizon, except the CMRSACR model. However, for other volatility proxies and forecast horizons, the null cannot be rejected for all models. Therefore, for the RV1 proxy at the first forecast horizon, the CMRSACR model generates forecasts with the lowest mean square errors, and outperforms all other models. However, for other volatility proxies and forecast steps, the tests do not give a clear comparison of forecast accuracy.

[Table 4.5]

In evaluating the significance of the forecast accuracy of each model relative to realized volatility, Mincer-Zarnowitz (MZ) regressions are estimated. Table 4.6 reports the estimated parameters, p-value of the hypothesis test, and the R-square statistic. Across all volatility proxies and at each forecast horizon, the CMRSACR model has the highest R-square statistics, which indicates information content about the true value of the realized volatility. The null hypothesis of efficiency is rejected for all models with some exceptions. In particular, null cannot be rejected for the EWMA and HybEWMA for RV3 at first three forecast horizons; for return-based models for RV3 at forecast horizon 3, the FIGARCH, MRSGARCH and MRSACR at fourth forecast step for all proxies except RV1. In Mincer-Zarnowitz regression, the value of slope coefficients indicates the efficiency of the models. The estimated coefficients of the MRSACR and CMRSACR models are close to unity most of the time. The coefficient of the CMRSACR model is close to one at lower forecast horizons, and provides efficient forecasts; however, it gets lower than unity at higher forecast horizons, and provides dispersed forecasts. The coefficient of the MRSACR model is slightly higher than or close to unity at lower forecast horizons, and gets closer to unity as the forecast horizon increases. In general, the CMRSACR model exhibits more efficient forecasts than all other models.

[Table 4.6]

In evaluating the forecast accuracy of the models together, encompassing regressions are estimated. Table 4.7 reports the estimation results of the encompassing regressions. In a number of cases, the CMRSACR model clearly encompasses all other models. In particular, the slope coefficient of the MRSACR and CMRSACR model is closer to unity than other models, and the CMRSACR model clearly encompasses all models for each volatility proxy at first three forecast steps (i.e. 1-, 2- and 4-week), while the MRSACR model encompasses at first and second forecast horizons for RV1. However, the GARCH and MRSGARCH models encompass all models at the fourth forecast horizon. Another way of evaluating Mincer-Zarnowitz and encompassing regression is to compare the R-square estimates of encompassing models, reported in Table 4.6, with the R-square estimates of the encompassing regression. At first three forecast horizon, the inclusion of all other return and range-based models only slightly increases the R-square of the encompassing regression. That means other models contain limited information over the CMRSACR model, and barely improve the performance of the CMRSACR model. In general, the CMRSACR model can explain more of the true integrated volatility, and it provides more accurate forecasts.

[Table 4.7]

The out-of-sample forecast performance of the models is also evaluated from a risk management perspective. Table 4.8 reports the results. The choice of the model in calculating the VaR is crucial. The model should not set too high or too low VaR. The VaR calculated too low could indicate that the bank does not have sufficient capital allocated to cover future losses. Equally, too high VaR could mean tying up too much capital and having higher opportunity costs. The risk management loss functions TUFF and PF, the LR tests of unconditional PF (LRuc), conditional PF (LRcc) and independence PF (LRind) examine the adequacy of the coverage of each model. The loss functions, RLF and FLF, incorporate the risk manager's preference into the model evaluation process. The loss functions are presented for 95 and 99 percent VaR levels (VaR95 and VaR99). The theoretical TUFF value at 5 and 1 percent are 20 and 100 weeks, respectively. TUFF estimates should be higher than these values. At the first forecast horizon, the CMRSACR model exhibits the lowest number of exceptions both at 95 and 99 percent VaR levels. The first failures of all models have happened before theoretical values for 1 week forecast horizons, except the CMRSACR and ECARR at 95 percent VaR level, and the CMRSACR, CARR and FIGARCH models at 99 percent VaR level. At higher forecast horizons, first exceptions occur later. Failure rates (PF) of the CMRSACR model are significantly lower than all other models at the first forecast horizon. As the duration of the forecast horizon is extended, PF of the CMRSACR increases, while PF of the MRSACR decreases.

[Table 4.8]

The hypothesis of having a correct unconditional coverage is rejected for most of the models at 95 percent VaR level. However, it cannot be rejected for some models at 99 percent VaR level. In particular, the hypothesis of having a true unconditional VaR coverage is rejected for all models at multistep ahead forecasts for 95 percent VaR estimates. However, it cannot be rejected for return and range-based EWMA and regime switching models at the first forecast step. At the 99 percent VaR level, conditional coverage is not rejected for return-based models for all forecast steps except the fourth step. At the second forecast step, all models pass the test except the CARR model. At the 95 percent VaR level, independence tests for all models are not rejected for all models at first and fourth forecast steps, except the GARCH at the first step and the FIGARCH model at the fourth step. In other steps, test cannot be rejected only for the CMRSACR and CARR models. Only the CMRSACR and CARR models pass the independence test for all steps at 95 percent VaR level. The hypothesis of having a correct conditional coverage is not rejected for return and range-based EWMA and regime switching models, for the first and fourth forecast horizons, at 95 percent VaR level. Opposite to unconditional test, at 99 percent VaR level, conditional coverage and independence tests are not rejected for range-based models for all forecast steps with some exceptions at the first forecast horizon. At the fourth forecast step, all models pass the test except the FIGARCH model. It is concluded that, at 95 percent VaR level the EWMA, HybEWMA, MRSGARCH and MRSACR models, and at 99 percent VaR level the HybEWMA, CARR, MRSACR and CMRSACR models pass the coverage tests.

The CMRSACR model passes only the independence test at 95 percent VaR level, while it passes all independence and conditional coverage tests at the 99 percent VaR level. However, the MRSACR model passes the conditional coverage tests for first and fourth forecast horizons at 95 percent VaR level, and all independence and conditional coverage tests at multistep ahead forecast horizons at 99 percent VaR level. Therefore, both proposed models have similar to or better coverage of VaR failure process than other models, and so far can be considered adequate for risk management purposes.

[Table 4.9]

Finally, model performances are compared with the help of the firm and regulatory loss functions (FLF and RLF). The estimated value of these functions at 95 and 99 percent VaR levels are provided in Table 4.9. A one-sided sign test to evaluate the null hypothesis that there is no difference between the models is also conducted for the models, which already passed the coverage tests. In terms of RLF estimates, the null is rejected in all cases. Hence, there is no significant difference between VaR estimates of the models from the regulatory perspective. In terms of FLF estimates, at 95 percent VaR level, the MRSACR model is significantly better than the EWMA and MRSGARCH models, but there is no significant difference between the MRSACR and HybEWMA models for the first three forecast horizons. At the 99 percent VaR level, the CMRSACR model performs better than the HybEWMA, CARR and MRSACR models for all forecast horizons.

4.5. Conclusion

In this chapter, two new Markov regime switching models based on intraday range are introduced. The one and two factor MRSACR models, proposed in this chapter, utilize enhanced information content, and estimate integrated volatility by using more information (daily open, close, high and low prices) than return-based counterparts, which utilize only close-close prices. The one factor MRSACR model shares many useful properties with Markov regime switching GARCH (MRSGARCH) models with enhanced information content. In addition to the MRSACR, the CMRSACR model utilize two volatility components (long term trend and short term cyclicality) and bring the useful properties of Markov regime switching models and component models with enhanced information content of intraday range.

It has been searched for if there is any incremental improvement with using these two models in volatility forecast. In-sample fit and out-of-sample forecast performance of the MRSACR and CMRSACR models are compared to return-based benchmark models: the single regime EWMA and GARCH, long memory FIGARCH, switching regime MRSGARCH, and range-based benchmark models: single regime hybEWMA and CARR. In evaluating the out-of-sample forecast performance of the models, model forecasts are compared with four volatility proxies; the sum of squared daily returns, the sum of daily range, the weekly squared returns, and the weekly scaled range. The out-of-sample performance of the models is tested in many grounds by using statistical (i.e. statistical loss functions, pair-wise and joint accuracy tests, Mincer-Zarnowitz regressions, encompassing regressions) and risk management performance measures (i.e. value at risk based loss functions, coverage and independence tests, firm and regulatory loss functions).

It has been shown that the proposed one and two-component models fit the data better than all other models. In particular, the CMRSACR model displays better fit than all other models. As the purpose of volatility estimation is to be able to forecast volatility, the performance of the MRSACR and CMRSACR models are tested in comparison to renowned return-based and range-based volatility models in an out-of-sample setting. In particular, the out-of-sample forecast performance is evaluated by using a number of statistical and risk management loss functions, at different forecast horizons and for different volatility proxies. It has been found that the MRSACR models, in general, and the CMRSACR model, in particular, generate more accurate volatility forecasts, contain more information about the true integrated volatility, and display similar or better performance than all competing models when used for risk management purposes.

The models offer a promising further research area. Application of the MRSACR models to different frequencies of range (e.g. daily, hourly, minutes or tick-by-tick data) might increase the accuracy of the volatility predictions. Moreover, application to different asset prices (fixed income, foreign currency, derivatives) or economic time series might improve forecast performance. Although, not having a multivariate analogue limits its usefulness, multivariate range models still offer a fruitful area for further research.

Table 4.1
Summary Statistics and Time Series Properties of S&P500 Return Series

Panel A reports summary statistics for the S&P 500 weekly return series over the period of 03/01/1982-19/02/2010. Panel B reports the autocorrelation, autoregressive conditional heteroskedasticity (ARCH) and the Ljung-Box-Q test for autocorrelation of order up to 10 asymptotically distributed as a central Chi-square with 10 d.o.f. under the null hypothesis with 5 percent critical value 18.307. ARCH (4) is Engle's LM test for autoregressive conditional heteroskedasticity, which is asymptotically distributed as a central Chi-square with four d.o.f. under the null hypothesis with 5 percent critical value 9.488. *p*-values are reported in parenthesis.

Panel A: Summary Statistics						
Mean (%)	Std. Dev. (%)	Skewness	Kurtosis	Maximum (%)	Minimum (%)	Jarque-Bera
0.15	2.34	-0.82	9.67	10.40	-20.08	2857.4 [0.00]

Panel B: Basic Time Series Properties						
ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)	LB-Q(10)	ARCH (4)
-0.051	0.066	-0.058	-0.005	0.003	27.04	146.94

Table 4.2
In-Sample Evaluation

Panel A reports in-sample parameter estimates of the models at each regime. Scripts indicate the regime. The parameters of the single regime models are exhibited at regime 1. Zero mean is assumed for the mean equation of the models. Asymptotic standard errors are provided in the row below each parameter. Panel B presents in-sample evaluation metrics: number of parameters (NP), persistence of shocks to the volatility (PERS), Akaike Information Criterion (AIC) and Schwarz Information Criterion (BIC) and negative log likelihood (LL) of the estimated models.

Panel A: Parameter Estimates										Panel B: Evaluation Metrics				
Model	ω(1)	ω(2)	α(1)	α(2)	d	β(1)	β(2)	p	q	NP	PERS	AIC	BIC	LL
EWMA	0.00		0.06			0.94				2	1	4.23	4.25	-2113
s.e	0.00		0.03			0.03								
HYBEWMA	0.00		0.06			0.94				2	1	4.23	4.25	-2113
s.e	0.00		0.02			0.02								
GARCH	0.17		0.11			0.85				3	0.96	4.23	4.25	-2110
s.e	0.04		0.04			0.04								
ECARR	0.26		0.26			0.65				3	0.91	3.90	3.92	-1944
s.e	0.09		0.00			0.04								
FIGARCH	0.30		0.27		0.46	0.60				4	0.87	4.22	4.24	-2105
s.e	0.13		0.08		0.10	0.12								
MRSGARCH	0.02	1.13	0.00	0.17		0.97	0.66	0.98	0.98	8	0.97	4.22	4.27	-2099
s.e	0.01	0.06	0.00	0.24		0.01	0.01	0.01	0.01					
MRSACR	1.18	0.13	0.47	0.16		0.36	0.80	0.99	0.99	8	0.95	4.11	4.16	-2044
s.e	0.03	0.01	0.03	0.03		0.02	0.01	0.01	0.00					
CMRSACR			0.60	0.03		0.37	0.97	0.94	0.99	8	1.00	2.33	2.38	-1157
s.e			0.02	0.01		0.01	0.03	0.03	0.01					

Table 4.3a
Out-of-Sample Estimation for Forecast Horizon 1 and 2

The table reports statistical loss functions Root mean square error (RMSE), Mean absolute error (MAE), Heteroskedasticity adjusted Mean Square Error (HMSE), Q-likelihood loss function (QLIKE) and directional accuracy test statistics of Success Ratio (SR) and Directional Accuracy (DA) tests are calculated for the out-of-sample period with respect to each of four volatility proxy (RV) and first two forecast horizons (1 and 4 weeks).

	Forecast Step 1 (1 week)						Forecast Step 2 (4 weeks)					
Model	RMSE	MAE	HMSE	QLIKE	SR	DA	RMSE	MAE	HMSE	QLIKE	SR	DA
Volatility Proxy RV1												
EWMA	1.84	1.34	0.42	1.55	0.69	7.78	3.06	2.13	0.28	2.26	0.70	8.22
HYBEWMA	1.86	1.34	0.45	1.56	0.66	6.80	3.03	2.10	0.28	2.26	0.69	7.84
GARCH	1.62	1.20	0.35	1.52	0.72	8.51	2.72	1.93	0.26	2.25	0.75	9.93
ECARR	2.50	1.96	0.41	1.69	0.66	5.33	4.40	3.52	0.32	2.39	0.68	6.45
FIGARCH	1.99	1.58	0.37	1.62	0.70	7.37	3.36	2.60	0.29	2.32	0.71	7.82
MRSGARCH	1.82	1.29	0.50	1.55	0.65	6.77	2.97	2.03	0.33	2.26	0.69	8.67
MRSACR	1.78	1.27	0.46	1.55	0.67	5.99	2.81	1.94	0.27	2.25	0.69	7.01
CMRSACR	1.31	0.92	0.26	1.42	0.80	11.98	2.27	1.50	0.23	2.19	0.79	11.48
Volatility Proxy RV2												
EWMA	1.21	0.82	0.16	1.69	0.77	11.29	2.29	1.55	0.14	2.39	0.78	12.06
HYBEWMA	1.19	0.79	0.15	1.68	0.78	11.85	2.22	1.46	0.13	2.38	0.79	12.64
GARCH	0.93	0.66	0.13	1.67	0.80	12.42	1.81	1.25	0.12	2.38	0.83	13.32
ECARR	1.73	1.42	0.21	1.77	0.78	11.15	3.29	2.76	0.19	2.47	0.80	12.14
FIGARCH	1.30	1.07	0.17	1.73	0.78	11.47	2.37	1.88	0.16	2.42	0.79	11.59
MRSGARCH	1.14	0.73	0.17	1.68	0.77	12.15	2.17	1.33	0.16	2.39	0.79	13.03
MRSACR	0.99	0.65	0.14	1.67	0.78	11.33	1.84	1.16	0.11	2.37	0.80	11.99
CMRSACR	0.01	0.00	0.00	1.60	1.00	21.47	1.09	0.72	0.07	2.35	0.88	15.87
Volatility Proxy RV3												
EWMA	1.33	0.85	0.20	1.80	0.74	10.04	2.44	1.54	0.17	2.50	0.78	11.75
HYBEWMA	1.29	0.82	0.19	1.79	0.77	11.43	2.34	1.46	0.15	2.49	0.79	12.32
GARCH	1.13	0.75	0.19	1.79	0.76	10.39	2.05	1.34	0.16	2.49	0.80	12.24
ECARR	1.47	1.19	0.17	1.84	0.79	11.52	2.72	2.25	0.15	2.54	0.83	13.50
FIGARCH	1.27	0.97	0.17	1.82	0.76	10.26	2.28	1.70	0.17	2.52	0.78	11.11
MRSGARCH	1.32	0.82	0.23	1.80	0.76	11.80	2.50	1.46	0.19	2.50	0.80	13.46
MRSACR	1.11	0.73	0.17	1.78	0.78	11.33	2.00	1.24	0.13	2.48	0.82	13.32
CMRSACR	0.89	0.57	0.12	1.75	0.85	14.69	1.66	1.12	0.18	2.49	0.85	14.58
Volatility Proxy RV4												
EWMA	1.04	0.71	0.11	1.67	0.80	12.69	2.04	1.39	0.11	2.37	0.80	12.67
HYBEWMA	1.00	0.67	0.10	1.67	0.81	13.46	1.94	1.30	0.10	2.37	0.80	13.07
GARCH	0.78	0.56	0.10	1.66	0.82	13.17	1.56	1.09	0.10	2.37	0.83	13.49
ECARR	1.63	1.39	0.19	1.76	0.85	14.53	3.17	2.73	0.18	2.46	0.86	14.97
FIGARCH	1.22	1.02	0.15	1.72	0.80	12.41	2.21	1.82	0.14	2.41	0.81	12.78
MRSGARCH	0.90	0.58	0.11	1.67	0.80	13.61	1.81	1.14	0.11	2.37	0.81	13.90
MRSACR	0.74	0.50	0.08	1.65	0.85	14.50	1.47	0.95	0.07	2.35	0.84	13.99
CMRSACR	0.58	0.38	0.05	1.63	0.87	15.62	1.33	0.89	0.09	2.35	0.85	14.43

Table 4.3b
Out-of-Sample Estimation for Forecast Horizon 3 and 4

The table reports statistical loss functions Root mean square error (RMSE), Mean absolute error (MAE), Heteroskedasticity adjusted Mean Square Error (HMSE), Q-likelihood loss function (QLIKE) and directional accuracy test statistics of Success Ratio (SR) and Directional Accuracy (DA) tests are calculated for the out-of-sample period with respect to each of four volatility proxy (RV) and for the third and fourth forecast horizons (8 and 13 weeks).

	Forecast Step 3 (8 weeks)						Forecast Step 4 (13 weeks)					
Model	RMSE	MAE	HMSE	QLIKE	SR	DA	RMSE	MAE	HMSE	QLIKE	SR	DA
Volatility Proxy RV1												
EWMA	5.32	3.76	0.22	2.97	0.76	10.89	15.25	10.89	0.17	4.17	0.80	12.68
HYBEWMA	5.14	3.63	0.20	2.96	0.75	10.75	14.52	10.57	0.14	4.16	0.80	12.77
GARCH	4.77	3.42	0.23	2.96	0.78	11.15	13.81	10.09	0.20	4.17	0.78	11.07
ECARR	7.96	6.64	0.28	3.09	0.73	8.90	21.67	19.57	0.25	4.28	0.81	12.60
FIGARCH	5.55	4.20	0.31	3.01	0.74	9.33	14.69	10.62	0.68	4.24	0.70	8.11
MRSGARCH	4.93	3.41	0.23	2.96	0.75	11.54	12.00	8.75	0.15	4.15	0.84	15.06
MRSACR	4.68	3.31	0.19	2.95	0.75	10.13	12.39	9.32	0.16	4.16	0.80	12.87
CMRSACR	4.19	2.88	0.17	2.92	0.80	12.28	15.40	9.50	0.19	4.15	0.80	12.36
Volatility Proxy RV2												
EWMA	4.55	3.05	0.16	3.10	0.79	12.08	15.14	9.86	0.19	4.30	0.78	11.70
HYBEWMA	4.34	2.86	0.13	3.09	0.78	12.14	14.41	9.58	0.15	4.29	0.78	11.80
GARCH	3.81	2.48	0.16	3.09	0.82	13.16	13.80	8.91	0.23	4.30	0.77	10.78
ECARR	6.35	5.45	0.19	3.17	0.82	12.77	19.16	16.87	0.20	4.37	0.79	11.70
FIGARCH	4.52	3.35	0.24	3.12	0.76	10.52	16.26	10.63	0.96	4.40	0.69	7.69
MRSGARCH	4.28	2.59	0.16	3.09	0.79	13.15	12.99	8.16	0.17	4.29	0.82	14.17
MRSACR	3.68	2.32	0.12	3.08	0.79	12.11	12.81	8.46	0.17	4.29	0.76	11.13
CMRSACR	3.06	1.94	0.12	3.07	0.85	14.50	14.73	9.03	0.24	4.31	0.80	12.30
Volatility Proxy RV3												
EWMA	4.87	2.96	0.19	3.20	0.77	11.18	16.19	9.30	0.25	4.41	0.75	10.32
HYBEWMA	4.63	2.77	0.15	3.19	0.77	11.63	15.41	8.89	0.20	4.40	0.75	10.45
GARCH	4.28	2.62	0.20	3.20	0.79	11.47	15.53	8.85	0.31	4.41	0.74	9.00
ECARR	5.46	4.50	0.15	3.24	0.82	13.12	17.98	14.66	0.18	4.45	0.78	10.96
FIGARCH	4.83	3.28	0.31	3.23	0.75	10.24	19.82	11.99	1.42	4.55	0.68	7.26
MRSGARCH	5.10	2.81	0.22	3.20	0.79	13.37	16.21	8.85	0.24	4.40	0.79	12.93
MRSACR	4.21	2.48	0.14	3.19	0.80	12.42	15.35	8.94	0.24	4.41	0.73	9.60
CMRSACR	3.58	2.35	0.20	3.20	0.83	13.40	15.74	9.73	0.35	4.44	0.78	11.22
Volatility Proxy RV4												
EWMA	4.18	2.81	0.13	3.08	0.78	11.88	14.49	9.34	0.16	4.28	0.78	11.53
HYBEWMA	3.93	2.63	0.10	3.07	0.78	12.14	13.71	9.03	0.13	4.27	0.78	11.67
GARCH	3.43	2.26	0.13	3.07	0.81	12.55	13.06	8.51	0.20	4.28	0.77	10.34
ECARR	6.21	5.41	0.18	3.16	0.83	13.59	18.93	16.87	0.20	4.35	0.80	11.90
FIGARCH	4.19	3.15	0.21	3.11	0.77	11.12	15.29	9.88	0.85	4.37	0.71	8.23
MRSGARCH	3.79	2.34	0.13	3.07	0.80	13.74	12.11	7.59	0.14	4.27	0.81	14.26
MRSACR	3.18	2.08	0.09	3.06	0.81	12.91	11.90	7.91	0.15	4.27	0.75	10.82
CMRSACR	3.10	1.96	0.12	3.06	0.83	13.48	14.39	8.76	0.21	4.29	0.80	12.17

Table 4.4
Modified Diebold-Mariano (MDM) Statistics

The table reports the Modified Diebold-Mariano statistics for the mean squared errors to test the null hypothesis that there is no difference between the MRSACR (CMRSACR) model and each of the other seven models. Results are reported for each of four volatility proxy at each of four forecast step. *,** and *** denote the MDM statistics for which null hypothesis of equal predictive accuracy at 10, 5 and 1 percent levels, respectively, can be rejected. The negative sign of the MDM statistics reveals that the loss of the proposed (MRSACR or CMRSACR) models is lower than the one implied by the benchmark models given the loss differential of ($MSE^{proposed} - MSE^{benchmark}$).

MRSACR	Volatility Proxy			
Models	RV1	RV2	RV3	RV4
Forecast step 1 (1-week)				
EWMA	0.53	0.06	-1.26	-2.01**
HYBEWMA	0.12	0.09	-1.19	-1.73*
GARCH	1.15	0.31	-0.78	0.62
ECARR	-1.69*	0.07	-1.31	-1.80*
FIGARCH	0.79	0.72	0.71	-1.02
MRSGARCH	-0.93	0.16	-1.33	-1.39
CMRSACR	1.21	1.32	1.48	0.75
Forecast step 2 (4-weeks)				
EWMA	0.04	-2.23**	-1.35	-2.61***
HYBEWMA	-0.24	-1.90*	-1.25	-2.19**
GARCH	1.11	0.95	0.57	0.71
ECARR	-1.65*	-1.76*	-0.97	-1.82*
FIGARCH	0.57	0.34	0.83	-0.42
MRSGARCH	-1.68*	-1.37	-1.36	-1.44
CMRSACR	1.47	1.3	1.13	-0.48
Forecast step 3 (8-weeks)				
EWMA	-0.39	-2.39**	-1.3	-2.82***
HYBEWMA	-0.08	-2.06**	-1.12	-2.47***
GARCH	0.93	0.74	0.88	0.52
ECARR	-1.58	-1.65*	-0.41	-1.82*
FIGARCH	0.4	0.03	-0.06	-0.51
MRSGARCH	-1.57	-1.29	-1.32	-1.36
CMRSACR	1.37	1.03	1.39	-1.16
Forecast step 4 (13-weeks)				
EWMA	-0.99	-1.96**	-0.51	-2.16**
HYBEWMA	-0.42	-1.69*	0.3	-1.92*
GARCH	-0.36	-0.61	0.9	-1.36
ECARR	-1.39	-1.46	0.16	-1.73*
FIGARCH	-1.73*	-1.61	-2.03**	-1.61
MRSGARCH	-0.92	-0.9	-1.19	-0.94
CMRSACR	-1.22	-1.27	-0.83	-1.3

CMRSACR	Volatility Proxy			
Models	RV1	RV2	RV3	RV4
Forecast step 1 (1-week)				
EWMA	-1.31	-1.46	-1.42	-1.8*
HYBEWMA	-1.31	-1.42	-1.36	-1.6
GARCH	-1.24	-1.45	-1.28	0.27
ECARR	-1.65*	-1.88*	-1.65*	-1.79*
FIGARCH	-1.52	-2.17**	-0.02	-1.44
MRSGARCH	-1.3	-1.39	-1.48	-1.28
MRSACR	-1.21	-1.32	-1.48	-0.75
Forecast step 2 (4-weeks)				
EWMA	-1.71*	-1.56	-1.32	-1.89*
HYBEWMA	-1.66*	-1.48	-1.25	-1.57
GARCH	-1.71*	-1.57	-1.19	1.03
ECARR	-1.91*	-1.94**	-2.05**	-1.82*
FIGARCH	-2.15**	-2.33**	-1.19	-0.3
MRSGARCH	-1.59	-1.38	-1.34	-1.12
MRSACR	-1.47	-1.3	-1.13	0.48
Forecast step 3 (8-weeks)				
EWMA	-1.47	-1.79*	-1.41	0.21
HYBEWMA	-1.42	-1.65*	-1.31	0.39
GARCH	-0.63	-0.33	-1.28	0.98
ECARR	-1.98**	-1.92*	-1.76*	-1.62
FIGARCH	-1.1	-0.98	-1.46	0.72
MRSGARCH	-1.57	-1.35	-1.4	0.32
MRSACR	-1.37	-1.03	-1.39	1.16
Forecast step 4 (13-weeks)				
EWMA	1.03	0.91	0.63	0.97
HYBEWMA	1.12	1.01	0.92	1.05
GARCH	1.14	1.14	0.89	1.19
ECARR	0.55	0.48	0.82	0.55
FIGARCH	0.98	1.03	-0.4	1.15
MRSGARCH	1.23	1.31	-0.11	1.33
MRSACR	1.22	1.27	0.83	1.3

Table 4.5a
p-values for Superior Predictive Ability (SPA) and Reality Check (RC) Tests

The table shows the p-values of the consistent (SPA (o, l) and lower bound (SPA (o, l)) Hansen's (2005) Superior Predictive Ability (SPA) test and the p-values of White's (2000) Reality Check test (RC) for the mean squared errors of the 1 and 4 weeks forecast horizons for all four measured volatilities. The rows of each model show the comparison of the benchmark versus all the other models. The null hypothesis that none of the models are better than the benchmark model is tested. The number of bootstrap replications for calculating the p-values is 2000. Windows size is 3.

		Forecast step 1 (1-week)				Forecast step 2 (4-weeks)			
		RV1	RV2	RV3	RV4	RV1	RV2	RV3	RV4
	SPA(o,l)	0	0	0	0	0	0	0	0
EWMA	SPA(o,c)	0	0.78	0.72	0.75	0.67	0.77	0.69	0.79
	RC	0	0.79	0.73	0.76	0.71	0.8	0.69	0.79
	SPA(o,l)	0	0	0	0	0	0	0	0
HYBEWMA	SPA(o,c)	0	0.8	0.67	0.83	0.55	0.74	0.63	0.79
	RC	0	0.82	0.68	0.83	0.6	0.78	0.63	0.79
	SPA(o,l)	0	0	0	0	0	0	0	0
GARCH	SPA(o,c)	0	0.76	0.73	0.76	0.7	0.76	0.68	0.79
	RC	0.16	0.77	0.74	0.76	0.71	0.78	0.68	0.79
	SPA(o,l)	0	0	0	0	0	0	0	0
ECARR	SPA(o,c)	0	0.69	0.75	0.9	0.57	0.74	0.72	0.78
	RC	0	0.7	0.75	0.91	0.59	0.76	0.72	0.78
	SPA(o,l)	0	0	0	0	0	0	0	0
FIGARCH	SPA(o,c)	0	0.81	0.7	0.85	0.6	0.75	0.78	0.89
	RC	0	0.81	0.7	0.85	0.62	0.81	0.78	0.89
	SPA(o,l)	0.01	0	0	0	0	0	0	0
MRSGARCH	SPA(o,c)	0.01	0.76	0.62	0.79	0.58	0.67	0.76	0.79
	RC	0.01	0.76	0.63	0.79	0.6	0.74	0.76	0.79
	SPA(o,l)	0.02	0	0	0	0	0	0	0
MRSACR	SPA(o,c)	0.02	0.75	0.7	0.76	0.66	0.79	0.72	0.76
	RC	0.02	0.78	0.71	0.76	0.69	0.81	0.72	0.76
	SPA(o,l)	0.54	0	0	0	0	0	0	0
CMRSACR	SPA(o,c)	1	0.78	0.73	0.76	0.59	0.77	0.68	0.79
	RC	1	0.81	0.73	0.76	0.65	0.79	0.68	0.79

Table 4.5b
p-values for Superior Predictive Ability (SPA) and Reality Check (RC) Tests

The table shows the p-values of the consistent (SPA (o, l) and lower bound (SPA (o, l)) Hansen's (2005) Superior Predictive Ability (SPA) test and the p-values of White's (2000) Reality Check test (RC) for the mean squared errors of the 8 and 13 weeks forecast horizons for all four measured volatilities. The rows of each model show the comparison of the benchmark versus all the other models. The null hypothesis that none of the models is better than the benchmark model is tested. The number of bootstrap replications for calculating the p-values is 2000. Windows size is 3.

		Forecast step 3 (8-weeks)				Forecast step 4 (13-weeks)			
		RV1	RV2	RV3	RV4	RV1	RV2	RV3	RV4
	SPA(o,l)	0	0	0	0	0	0	0	0
EWMA	SPA(o,c)	0.65	0.8	0.75	0.79	0.65	0.79	0.73	0.81
	RC	0.65	0.8	0.75	0.79	0.66	0.79	0.73	0.81
	SPA(o,l)	0	0	0	0	0	0	0	0
HYBEWMA	SPA(o,c)	0.59	0.83	0.68	0.73	0.66	0.77	0.67	0.78
	RC	0.59	0.83	0.68	0.73	0.69	0.77	0.67	0.78
	SPA(o,l)	0	0	0	0	0	0	0	0
GARCH	SPA(o,c)	0.67	0.74	0.73	0.78	0.64	0.76	0.64	0.8
	RC	0.67	0.74	0.73	0.78	0.64	0.76	0.64	0.8
	SPA(o,l)	0	0	0	0	0	0	0	0
ECARR	SPA(o,c)	0.68	0.79	0.57	0.82	0.72	0.82	0.62	0.77
	RC	0.68	0.79	0.57	0.82	0.74	0.82	0.62	0.77
	SPA(o,l)	0	0	0	0	0	0	0	0
FIGARCH	SPA(o,c)	0.69	0.79	0.71	0.79	0.66	0.75	0.72	0.8
	RC	0.69	0.79	0.71	0.79	0.7	0.75	0.72	0.8
	SPA(o,l)	0	0	0	0	0	0	0	0
MRSGARCH	SPA(o,c)	0.72	0.77	0.7	0.79	0.69	0.72	0.75	0.83
	RC	0.72	0.77	0.7	0.79	0.69	0.72	0.75	0.83
	SPA(o,l)	0	0	0	0	0	0	0	0
MRSACR	SPA(o,c)	0.68	0.79	0.67	0.8	0.6	0.66	0.68	0.82
	RC	0.68	0.79	0.67	0.8	0.62	0.66	0.68	0.82
	SPA(o,l)	0	0	0	0	0	0	0	0
CMRSACR	SPA(o,c)	0.65	0.81	0.63	0.76	0.67	0.75	0.77	0.79
	RC	0.65	0.81	0.63	0.76	0.68	0.75	0.77	0.79

Table 4.6a
Mincer-Zarnowitz Regression Results for Forecast Horizon 1 and 2

The table reports the results of the Mincer-Zarnowitz regression and hypothesis test for the efficiency of the parameters for each of four volatility proxies at first (1-week) and second (4-week) forecast horizons. The dependent variable in the regression is volatility proxies (RV), and the independent variable is volatility forecast of each model. In estimating heteroskedasticity-autocorrelation-consistent standard errors for the regressions, Newey and West (1987) estimation procedure is used. *Alpha* is constant parameter; *se* is standard error of parameter estimation; *beta* is slope parameter; *Stats* is Chi-square statistics; *pval* is p-value of Chi-square statistics, and R2 is R-square of the regression that shows the explanatory power of each model's forecasts.

	Forecast step 1 (1-week)							Forecast step 2 (4-weeks)						
Model	alpha	se	beta	se	Stats	pval	R2	alpha	se	beta	se	Stats	pval	R2
	Volatility Proxy RV1							Volatility Proxy RV1						
EWMA	-0.21	0.31	0.85	0.16	69.2	0.00	0.264	0.05	0.51	0.76	0.14	47.0	0.00	0.308
HYBEWMA	-0.07	0.28	0.80	0.14	49.3	0.00	0.238	0.09	0.49	0.76	0.13	40.0	0.00	0.311
GARCH	-0.77	0.28	1.12	0.14	136.2	0.00	0.426	-0.67	0.43	0.93	0.11	75.6	0.00	0.426
ECARR	-0.06	0.15	0.56	0.06	528.1	0.00	0.251	-0.14	0.38	0.57	0.07	486.3	0.00	0.342
FIGARCH	-0.67	0.27	0.84	0.11	387.3	0.00	0.365	-0.60	0.44	0.76	0.09	172.1	0.00	0.368
MRSGARCH	-0.47	0.25	1.05	0.14	42.2	0.00	0.222	-0.55	0.55	0.97	0.15	25.5	0.00	0.253
MRSACR	-0.16	0.15	0.89	0.07	35.1	0.00	0.271	-0.24	0.31	0.88	0.08	31.8	0.00	0.355
CMRSACR	-0.27	0.09	0.98	0.05	49.7	0.00	0.604	0.22	0.15	0.81	0.04	40.0	0.00	0.601
	Volatility Proxy RV2							Volatility Proxy RV2						
EWMA	-0.02	0.32	0.90	0.17	12.0	0.00	0.467	0.22	0.58	0.85	0.15	7.8	0.02	0.471
HYBEWMA	-0.02	0.30	0.91	0.16	10.1	0.01	0.482	0.18	0.54	0.86	0.15	6.8	0.03	0.497
GARCH	-0.49	0.24	1.13	0.12	25.4	0.00	0.687	-0.60	0.45	1.05	0.11	13.8	0.00	0.657
ECARR	-0.21	0.14	0.69	0.05	827.4	0.00	0.605	-0.42	0.32	0.69	0.06	540.0	0.00	0.632
FIGARCH	-0.48	0.26	0.88	0.10	175.2	0.00	0.635	-0.65	0.50	0.88	0.10	71.4	0.00	0.595
MRSGARCH	-0.66	0.33	1.27	0.17	10.3	0.01	0.516	-1.12	0.63	1.23	0.17	7.3	0.03	0.501
MRSACR	-0.27	0.11	1.08	0.06	12.5	0.00	0.625	-0.46	0.26	1.06	0.07	7.0	0.03	0.634
CMRSACR	-0.01	0.00	1.00	0.00	13.1	0.00	1.000	0.49	0.07	0.89	0.02	53.1	0.00	0.883
	Volatility Proxy RV3							Volatility Proxy RV3						
EWMA	-0.12	0.34	1.05	0.18	0.2	0.92	0.484	-0.05	0.66	1.02	0.17	0.0	1.00	0.509
HYBEWMA	-0.16	0.33	1.08	0.17	0.2	0.89	0.517	-0.15	0.63	1.05	0.17	0.1	0.94	0.549
GARCH	-0.55	0.19	1.27	0.10	8.0	0.02	0.657	-0.91	0.43	1.23	0.11	4.6	0.10	0.680
ECARR	-0.47	0.13	0.85	0.05	516.7	0.00	0.685	-0.99	0.28	0.86	0.06	368.0	0.00	0.726
FIGARCH	-0.58	0.25	1.00	0.10	56.8	0.00	0.622	-0.98	0.58	1.04	0.12	20.5	0.00	0.621
MRSGARCH	-0.99	0.38	1.54	0.20	7.1	0.03	0.573	-1.82	0.77	1.51	0.21	6.1	0.05	0.570
MRSACR	-0.52	0.11	1.31	0.06	27.4	0.00	0.692	-1.01	0.24	1.30	0.07	20.6	0.00	0.720
CMRSACR	0.22	0.09	1.03	0.05	53.3	0.00	0.792	0.67	0.15	0.98	0.04	59.0	0.00	0.800
	Volatility Proxy RV4							Volatility Proxy RV4						
EWMA	0.04	0.27	0.85	0.14	16.9	0.00	0.533	0.24	0.51	0.82	0.14	12.8	0.00	0.530
HYBEWMA	0.02	0.26	0.88	0.14	16.7	0.00	0.566	0.18	0.48	0.85	0.13	12.6	0.00	0.567
GARCH	-0.31	0.17	1.03	0.09	27.5	0.00	0.726	-0.45	0.33	1.00	0.09	19.0	0.00	0.709
ECARR	-0.21	0.11	0.68	0.04	1292.5	0.00	0.736	-0.45	0.23	0.69	0.04	840.4	0.00	0.735
FIGARCH	-0.34	0.20	0.82	0.08	202.1	0.00	0.693	-0.54	0.42	0.84	0.09	89.3	0.00	0.654
MRSGARCH	-0.66	0.29	1.25	0.15	18.9	0.00	0.631	-1.20	0.57	1.23	0.16	15.1	0.00	0.596
MRSACR	-0.25	0.09	1.05	0.05	21.0	0.00	0.742	-0.47	0.20	1.04	0.05	14.1	0.00	0.730
CMRSACR	0.32	0.05	0.84	0.03	35.8	0.00	0.872	0.83	0.11	0.79	0.03	60.6	0.00	0.829

Table 4.6b
Mincer-Zarnowitz regression results for forecast horizon 3 and 4

The table reports the results of the Mincer-Zarnowitz regression and hypothesis test for the efficiency of the parameters for each of four volatility proxies at third (8-week) and fourth (13-week) forecast horizons. The dependent variable in the regression is volatility proxies (RV), and the independent variable is volatility forecast of each model. In estimating heteroskedasticity-autocorrelation-consistent standard errors for the regressions, Newey and West (1987) estimation procedure is used. *Alpha* is constant parameter; *se* is standard error of parameter estimation; *beta* is slope parameter; *Stats* is Chi-square statistics; *pval* is p-value of Chi-square statistics, and R2 is R-square of the regression that shows the explanatory power of each model's forecasts.

	Forecast step 3 (8-weeks)							Forecast step 4 (13-weeks)						
Model	alpha	se	beta	se	Stats	pval	R2	alpha	se	beta	se	Stats	pval	R2
	Volatility Proxy RV1							Volatility Proxy RV1						
EWMA	0.85	0.89	0.68	0.12	31.4	0.00	0.351	7.02	2.15	0.55	0.07	52.1	0.00	0.379
HYBEWMA	0.68	0.86	0.70	0.12	31.5	0.00	0.382	6.22	1.71	0.58	0.06	49.1	0.00	0.429
GARCH	-0.15	0.85	0.81	0.11	35.0	0.00	0.429	4.54	2.49	0.65	0.07	54.8	0.00	0.369
ECARR	-0.24	0.94	0.57	0.08	371.3	0.00	0.406	-2.35	3.09	0.63	0.08	197.7	0.00	0.389
FIGARCH	0.10	1.00	0.72	0.11	46.2	0.00	0.332	13.00	5.49	0.44	0.20	9.5	0.01	0.081
MRSGARCH	-0.83	1.18	0.94	0.16	18.7	0.00	0.290	-4.21	2.96	1.01	0.13	18.7	0.00	0.366
MRSACR	-0.18	0.80	0.84	0.10	23.9	0.00	0.403	0.34	2.25	0.82	0.08	14.8	0.00	0.380
CMRSACR	1.45	0.31	0.69	0.05	45.5	0.00	0.614	10.67	1.25	0.48	0.04	167.3	0.00	0.478
	Volatility Proxy RV2							Volatility Proxy RV2						
EWMA	1.15	1.00	0.77	0.13	5.1	0.08	0.440	9.30	2.55	0.60	0.08	22.7	0.00	0.344
HYBEWMA	0.94	0.95	0.80	0.13	4.4	0.11	0.480	8.38	2.12	0.64	0.08	22.0	0.00	0.391
GARCH	-0.29	0.84	0.95	0.10	4.8	0.09	0.574	5.76	2.79	0.73	0.07	17.5	0.00	0.360
ECARR	-0.86	0.85	0.70	0.08	297.2	0.00	0.602	-3.02	3.52	0.74	0.09	86.9	0.00	0.405
FIGARCH	-0.19	1.17	0.86	0.12	13.0	0.00	0.466	13.77	6.80	0.56	0.24	4.2	0.12	0.100
MRSGARCH	-1.83	1.28	1.19	0.18	4.1	0.13	0.450	-5.25	3.48	1.19	0.16	3.0	0.22	0.382
MRSACR	-0.74	0.70	1.04	0.09	3.3	0.20	0.591	0.20	2.61	0.96	0.09	0.4	0.81	0.394
CMRSACR	1.84	0.23	0.79	0.03	64.9	0.00	0.769	12.27	1.38	0.56	0.04	128.6	0.00	0.495
	Volatility Proxy RV3							Volatility Proxy RV3						
EWMA	0.61	1.20	0.94	0.16	0.4	0.82	0.475	8.39	3.02	0.74	0.10	8.0	0.02	0.367
HYBEWMA	0.31	1.13	0.99	0.15	0.4	0.81	0.524	7.26	2.46	0.79	0.09	8.7	0.01	0.417
GARCH	-1.02	0.91	1.15	0.11	1.7	0.43	0.606	4.02	3.29	0.91	0.09	1.5	0.47	0.384
ECARR	-1.97	0.83	0.87	0.08	181.7	0.00	0.665	-7.25	4.11	0.93	0.11	37.5	0.00	0.441
FIGARCH	-0.86	1.53	1.04	0.16	1.2	0.55	0.488	14.26	8.49	0.69	0.30	7.0	0.03	0.103
MRSGARCH	-3.10	1.59	1.46	0.22	4.6	0.10	0.491	-8.83	4.41	1.44	0.20	5.0	0.08	0.392
MRSACR	-1.79	0.70	1.28	0.10	8.6	0.01	0.648	-2.89	3.04	1.19	0.11	3.7	0.16	0.420
CMRSACR	1.87	0.33	0.91	0.05	52.0	0.00	0.750	12.58	1.57	0.68	0.05	70.5	0.00	0.502
	Volatility Proxy RV4							Volatility Proxy RV4						
EWMA	1.09	0.91	0.76	0.12	8.4	0.02	0.490	8.69	2.37	0.60	0.08	25.1	0.00	0.377
HYBEWMA	0.86	0.85	0.79	0.12	8.0	0.02	0.538	7.78	1.95	0.64	0.07	24.3	0.00	0.429
GARCH	-0.21	0.66	0.92	0.08	8.2	0.02	0.622	5.22	2.59	0.73	0.07	20.7	0.00	0.393
ECARR	-0.90	0.64	0.70	0.06	431.5	0.00	0.672	-3.84	3.34	0.75	0.08	112.3	0.00	0.450
FIGARCH	-0.19	1.09	0.84	0.11	18.8	0.00	0.513	12.48	6.60	0.59	0.24	3.6	0.16	0.121
MRSGARCH	-1.95	1.15	1.18	0.16	8.7	0.01	0.511	-5.60	3.39	1.18	0.16	5.2	0.08	0.413
MRSACR	-0.75	0.53	1.02	0.07	6.9	0.03	0.655	-0.46	2.44	0.96	0.09	1.2	0.56	0.433
CMRSACR	2.14	0.24	0.73	0.03	81.2	0.00	0.765	12.13	1.27	0.55	0.04	143.6	0.00	0.513

Table 4.7
Encompassing Regression Results

The table reports the results of the encompassing regression. The dependent variable in the regression is volatility proxies (RV) and independent variables are volatility forecasts of all models. In estimating heteroskedasticity-autocorrelation-consistent standard errors for the regressions, Newey and West (1987) estimation procedure is used. *Alpha* is constant parameter; *se* is standard error of parameter estimation; *beta* is slope parameter, and R2 is R-square of the regression that shows the explanatory power of each regression. Beta coefficients (from beta1 to beta8) represents the regression coefficients of the following model forecasts subsequently; EWMA, HYBEWMA, GARCH, ECARR, FIGARCH, MRSGARCH, MRSACR, CMRSACR.

Proxy	alpha	se0	beta1	se1	beta2	se2	beta3	se3	beta4	se4	beta5	se5	beta6	se6	beta7	se7	beta8	se8	R2
Forecast step 1 (1-week)																			
RV1	0.34	0.22	1.23	0.75	-1.00	0.67	3.05	0.60	-1.20	0.38	-1.42	0.45	-1.01	0.28	1.12	0.55	0.85	0.10	0.74
RV2	0.00	0.00	0.00	0.01	-0.01	0.01	0.01	0.01	-0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.01	1.01	0.00	1.00
RV3	-0.35	0.15	0.02	0.36	0.17	0.26	-0.06	0.29	0.36	0.13	-0.54	0.29	0.32	0.24	0.19	0.25	0.80	0.06	0.85
RV4	-0.18	0.09	-0.30	0.20	0.29	0.15	-0.29	0.18	0.32	0.09	0.02	0.18	0.31	0.15	-0.13	0.13	0.66	0.05	0.92
Forecast step 2 (4-weeks)																			
RV1	1.36	0.51	0.77	0.68	-0.24	0.53	0.76	0.39	-0.42	0.34	-0.59	0.28	-1.07	0.32	0.88	0.34	0.73	0.08	0.64
RV2	0.25	0.13	0.12	0.24	0.04	0.20	-0.19	0.16	0.29	0.10	-0.11	0.13	-0.09	0.10	-0.06	0.13	0.82	0.06	0.90
RV3	-0.34	0.23	-0.05	0.27	0.27	0.25	-0.19	0.21	0.50	0.13	-0.41	0.20	0.04	0.16	0.13	0.18	0.72	0.08	0.88
RV4	-0.13	0.14	-0.17	0.21	0.26	0.19	-0.20	0.15	0.41	0.09	-0.14	0.14	0.15	0.10	-0.09	0.13	0.59	0.07	0.89
Forecast step 3 (8-weeks)																			
RV1	2.96	0.92	0.12	0.45	0.48	0.38	-0.22	0.39	0.12	0.28	-0.31	0.23	-0.61	0.26	0.21	0.29	0.68	0.12	0.66
RV2	1.02	0.53	-0.10	0.32	0.36	0.28	-0.46	0.26	0.46	0.18	-0.22	0.18	-0.02	0.14	-0.07	0.20	0.72	0.11	0.82
RV3	0.27	0.49	-0.26	0.33	0.60	0.30	-0.48	0.27	0.60	0.19	-0.38	0.21	-0.04	0.13	0.07	0.22	0.73	0.12	0.84
RV4	0.49	0.39	-0.30	0.27	0.53	0.24	-0.41	0.22	0.52	0.15	-0.19	0.16	0.08	0.09	-0.14	0.18	0.59	0.10	0.84
Forecast step 4 (13-weeks)																			
RV1	9.06	3.24	-1.11	0.54	1.32	0.51	-0.04	0.31	0.01	0.21	-0.58	0.23	0.42	0.28	0.09	0.27	0.37	0.10	0.65
RV2	3.16	3.42	-1.13	0.65	1.10	0.59	-0.28	0.36	0.25	0.22	-0.54	0.28	0.85	0.34	0.02	0.29	0.49	0.12	0.64
RV3	1.47	3.98	-1.26	0.75	1.33	0.66	-0.39	0.43	0.44	0.25	-0.69	0.34	0.84	0.41	0.01	0.32	0.55	0.15	0.66
RV4	1.60	3.12	-1.04	0.60	1.07	0.54	-0.34	0.35	0.37	0.20	-0.47	0.26	0.76	0.31	-0.07	0.25	0.45	0.12	0.67

Table 4.8a
Value-at-Risk Evaluation Results (95 percent)

The table reports number of exceptions (NE), Time Until First Failure (TUFF), Proportion of Failure (PF(%)), the likelihood ratio (LR) test of unconditional coverage (LRuc), conditional coverage (LRcc) and independence (LRind), and average regulator (RLF) and firm (FLF) function for the 95 percent VaR failure processes in 1-, 4-, 8- and 13-step ahead volatility forecasts. * indicates significance at the 5 percent level.

Models	NE	TUFF	PF	LRuc	LRind	LRcc	RLF	FLF
Forecast step 1 (1-week)								
EWMA	23	1	4.99	0	0.6	0.6	0.43	0.48
HYBEWMA	24	1	5.21	0	0.4	0.5	0.53	0.58
GARCH	18	1	3.91	1.3	4.8*	6.0*	0.17	0.23
ECARR	8	157	1.74	13.7*	0.3	14.0*	0.27	0.32
FIGARCH	9	1	1.95	11.6*	2	13.6*	0.21	0.27
MRSGARCH	28	1	6.07	1.1	0.9	2	0.03	0.16
MRSACR	31	1	6.73	2.6	0.4	3	1.33	1.35
CMRSACR	4	59	0.87	24.9*	0.1	25.0*	0.35	0.41
Forecast step 2 (4-weeks)								
EWMA	9	25	1.96	11.6*	6.9*	18.4*	0.81	0.93
HYBEWMA	9	25	1.96	11.6*	6.9*	18.4*	0.76	0.88
GARCH	9	0	1.96	11.6*	7.4*	18.9*	0.71	0.82
ECARR	1	395	0.22	38.8*	0	38.8*	0.75	0.87
FIGARCH	5	27	1.09	21.5*	4.3*	25.8*	0.73	0.84
MRSGARCH	12	1	2.61	6.7*	9.5*	16.2*	0.17	0.34
MRSACR	9	25	1.96	11.6*	6.9*	18.4*	1.44	1.52
CMRSACR	8	0	1.74	13.6*	2.7	16.3*	0.59	0.70
Forecast step 3 (8-weeks)								
EWMA	4	25	0.87	24.7*	14.6*	39.3*	1.12	1.36
HYBEWMA	4	25	0.87	24.7*	14.6*	39.3*	0.57	0.81
GARCH	5	25	1.09	21.3*	12.3*	33.6*	1.25	1.48
ECARR	1	393	0.22	38.6*	0	38.6*	1.28	1.51
FIGARCH	5	25	1.09	21.3*	22.7*	44.0*	1.26	1.49
MRSGARCH	5	25	1.09	21.3*	12.3*	33.6*	0.02	0.35
MRSACR	4	25	0.87	24.7*	14.6*	39.3*	2.13	2.29
CMRSACR	3	311	0.66	28.5*	0	28.5*	0.82	1.01
Forecast step 4 (13-weeks)								
EWMA	0	449	0.00	46.1*	0	0	0.00	0.78
HYBEWMA	0	449	0.00	46.1*	0	0	0.00	0.77
GARCH	1	390	0.22	37.7*	0	37.7*	0.00	0.76
ECARR	0	449	0.00	46.1*	0	0	0.00	0.75
FIGARCH	11	384	2.45	7.5*	82.4*	89.9*	0.00	0.75
MRSGARCH	0	449	0.00	46.1*	0	0	0.00	1.04
MRSACR	0	449	0.00	46.1*	0	0	0.40	0.85
CMRSACR	1	390	0.22	37.7*	0	37.7*	0.40	0.85

Table 4.8b
Value-at-Risk Evaluation Results (99 percent)

The table reports number of exceptions (NE), Time Until First Failure (TUFF), Proportion of Failure (PF(%)), the likelihood ratio (LR) test of unconditional coverage (LRuc), conditional coverage (LRcc) and independence (LRind), and average regulator (RLF) and firm (FLF) function for the 99 percent VaR failure processes in 1-, 4-, 8- and 13-step ahead volatility forecasts. * indicates significance at the 5 percent level.

Models	NE	TUFF	PF	LRuc	LRind	LRcc	RLF	FLF
Forecast step 1 (1-week)								
EWMA	8	1	1.74	2.1	7.9*	10.0*	0.13	0.21
HYBEWMA	10	1	2.17	4.8*	6.0*	10.8*	0.21	0.29
GARCH	3	28	0.65	0.6	0	0.7	0.01	0.09
ECARR	2	312	0.43	1.9	0	1.9	0.03	0.11
FIGARCH	1	395	0.22	4.2*	0	4.2	0.01	0.09
MRSGARCH	8	1	1.74	2.1	7.9*	10.0*	0.01	0.20
MRSACR	10	1	2.17	4.8*	1.6	6.4*	0.83	0.87
CMRSACR	0	461	0.00	9.3*	0	0	0.10	0.19
Forecast step 2 (4-weeks)								
EWMA	3	70	0.65	0.6	6.6*	7.3*	0.38	0.55
HYBEWMA	2	70	0.44	1.9	0	1.9	0.37	0.54
GARCH	3	27	0.65	0.6	6.6*	7.3*	0.29	0.45
ECARR	1	395	0.22	4.2*	0	4.2	0.32	0.49
FIGARCH	2	394	0.44	1.9	8.7*	10.6*	0.31	0.47
MRSGARCH	3	70	0.65	0.6	6.6*	7.3*	0.00	0.24
MRSACR	2	70	0.44	1.9	0	1.9	0.76	0.88
CMRSACR	3	311	0.65	0.6	0	0.7	0.24	0.40
Forecast step 3 (8-weeks)								
EWMA	2	393	0.44	1.9	8.7*	10.6*	0.34	0.68
HYBEWMA	1	393	0.22	4.1*	0	4.1	0.02	0.36
GARCH	2	393	0.44	1.9	8.7*	10.6*	0.52	0.85
ECARR	0	458	0.00	9.2*	0	0	0.52	0.85
FIGARCH	3	393	0.66	0.6	18.1*	18.7*	0.52	0.85
MRSGARCH	3	393	0.66	0.6	18.1*	18.7*	0.00	0.48
MRSACR	1	393	0.22	4.1*	0	4.1	0.88	1.10
CMRSACR	1	355	0.22	4.1*	0	4.1	0.15	0.43
Forecast step 4 (13-weeks)								
EWMA	0	449	0.00	9.0*	0	0	0.00	1.11
HYBEWMA	0	449	0.00	9.0*	0	0	0.00	1.09
GARCH	0	449	0.00	9.0*	0	0	0.00	1.07
ECARR	0	449	0.00	9.0*	0	0	0.00	1.06
FIGARCH	7	386	1.56	1.2	38.2*	39.4*	0.00	1.06
MRSGARCH	0	449	0.00	9.0*	0	0	0.00	1.47
MRSACR	0	449	0.00	9.0*	0	0	0.00	0.64
CMRSACR	0	449	0.00	9.0*	0	0	0.00	0.64

Table 4.9
Sign Tests of RLF and FLF

The table reports the sign test on loss differentials between pairs of models for each forecast horizon. It tests the null of a zero-median loss differential against the alternative of a negative median, with a studentized version of the sign test of Diebold and Mariano (1995). * indicates significance at 5 percent level. Rejection of the null implies that first model is significantly better than second model.

	RLF	FLF		RLF	FLF
Pairs of Models	VaR95	VaR95	Pairs of Models	VaR99	VaR99
Forecast step 1 (1-week)					
MRSACR vs EWMA	-19.3*	-4.1*	CMRSACR vs HYBEWMA	-21.5*	-9.1*
EWMA vs MRSACR	-20.6*	4.1	HYBEWMA vs CMRSACR	-20.5*	9.1
MRSACR vs HYBEWMA	-19.4*	0.7	CMRSACR vs ECARR	-21.5*	-18.5*
HYBEWMA vs MRSACR	-20.4*	-0.7	ECARR vs CMRSACR	-21.3*	18.5
MRSACR vs MRSGARCH	-19.8*	-3.0*	CMRSACR vs MRSACR	-21.5*	-8.1*
MRSGARCH vs MRSACR	-20.2*	3	MRSACR vs CMRSACR	-20.5*	8.1
EWMA vs MRSGARCH	-20.7*	3.3	HYBEWMA vs MRSACR	-21.0*	0
MRSGARCH vs EWMA	-19.4*	-3.3*	MRSACR vs HYBEWMA	-20.8*	0
Forecast step 2 (4-weeks)					
MRSACR vs EWMA	-21.1*	-4.3*	CMRSACR vs HYBEWMA	-21.3*	-8.5*
EWMA vs MRSACR	-20.9*	4.3	HYBEWMA vs CMRSACR	-21.3*	8.5
MRSACR vs HYBEWMA	-21.2*	0.3	CMRSACR vs ECARR	-21.2*	-18.2*
HYBEWMA vs MRSACR	-20.8*	-0.3	ECARR vs CMRSACR	-21.4*	18.2
MRSACR vs MRSGARCH	-21.1*	-3.3*	CMRSACR vs MRSACR	-21.3*	-6.9*
MRSGARCH vs MRSACR	-20.6*	3.3	MRSACR vs CMRSACR	-21.3*	6.9
EWMA vs MRSGARCH	-21.1*	2.6	HYBEWMA vs MRSACR	-21.3*	-0.5
MRSGARCH vs EWMA	-20.6*	-2.6*	MRSACR vs HYBEWMA	-21.4*	0.5
Forecast step 3 (8-weeks)					
MRSACR vs EWMA	-21.3*	-2.1*	CMRSACR vs HYBEWMA	-21.3*	-8.4*
EWMA vs MRSACR	-21.1*	2.1	HYBEWMA vs CMRSACR	-21.3*	8.4
MRSACR vs HYBEWMA	-21.2*	1.4	CMRSACR vs ECARR	-21.3*	-18.1*
HYBEWMA vs MRSACR	-21.2*	-1.4	ECARR vs CMRSACR	-21.4*	18.1
MRSACR vs MRSGARCH	-21.3*	-2.5*	CMRSACR vs MRSACR	-21.3*	-7.3*
MRSGARCH vs MRSACR	-21.0*	2.5	MRSACR vs CMRSACR	-21.3*	7.3
EWMA vs MRSGARCH	-21.2*	1.7	HYBEWMA vs MRSACR	-21.4*	-1.5
MRSGARCH vs EWMA	-21.1*	-1.7*	MRSACR vs HYBEWMA	-21.3*	1.5
Forecast step 4 (13-weeks)					
MRSACR vs EWMA	-21.2*	-0.1	CMRSACR vs HYBEWMA	-21.2*	-8.3*
EWMA vs MRSACR	-21.2*	0.1	HYBEWMA vs CMRSACR	-21.2*	8.3
MRSACR vs HYBEWMA	-21.2*	0.9	CMRSACR vs ECARR	-21.2*	-16.9*
HYBEWMA vs MRSACR	-21.2*	-0.9	ECARR vs CMRSACR	-21.2*	16.9
MRSACR vs MRSGARCH	-21.2*	0.1	CMRSACR vs MRSACR	-21.2*	-6.8*
MRSGARCH vs MRSACR	-21.2*	-0.1	MRSACR vs CMRSACR	-21.2*	6.8
EWMA vs MRSGARCH	-21.2*	0.7	HYBEWMA vs MRSACR	-21.2*	-0.9
MRSGARCH vs EWMA	-21.2*	-0.7	MRSACR vs HYBEWMA	-21.2*	0.9

Chapter 5

Hedge Fund Portfolio Construction: Range-Based Multivariate Regime Switching Models

5.1. Introduction

The variance covariance matrix is an essential ingredient in many applications in finance, including portfolio construction, hedging decision, risk management and derivative pricing. There are well-established methods to estimate the variance covariance matrix. In the literature, rolling window estimator (e.g. MA100), exponentially weighted moving average (EWMA) estimator and GARCH family estimators are typically used estimators. These estimators use log-return data estimated using close-close prices. They are remarkably inefficient though unbiased estimators of integrated volatility, as they use only one point in time information (e.g. Closing price) for each day.

This inefficiency can be overcome by using an estimator, which contains all the information of the asset price between measurement points. There are essentially two approaches for solving the problem; first, to estimate realized volatility by using intraday data, and second, to use intraday range. Although realized volatility estimate is favoured in this respect (for example, see, Andersen *et al*., 2004), unavailability of intraday data for many financial assets is the main limitation of this approach. It also suffers robustness from the market microstructure noise as time interval between measurements approaches to zero. In this respect, many financial assets have range data (defined as the difference between intraday high and low log prices) available for long historical spans, thus, using intraday range in measuring integrated volatility provides practical advantages over realized volatility. The practical advantages are also supported by the attractiveness of the range-based volatility measures as they are an efficient, approximately normal and a robust measure to microstructure noise (see Alizadeh *et al*., 2002). Andersen and Bollerslev (1998) and Brandt and Diebold (2006) report the favourable explanatory power of the range by showing that the range has approximately the same information content as sampling intraday returns every 3 to 6 hours.

Although known as a viable volatility measure for a long time, these advantages of the range provide a renewed interest only recently. Earlier works on range date back to Feller (1951) and Mandelbrot (1971). Recently, in search for a better variance estimator, Beckers (1983), Ball and Tourus (1984), Rogers and Satchell (1991), Kunitomo (1992) and Yang and Zhang (2000) propose extensions to the estimators of Parkinson (1980) and Garman and Klass (1980). In estimating conditional volatility, Brandt and Jones (2006) extend the EGARCH model of Nelson (1991) by using range instead of absolute returns. In a concurrent work, Chou (2005) proposes a conditional autoregressive range (CARR) model that is a special case of the conditional duration model of Engle and Russel (1998). All these studies are mainly in a univariate setting. Nonetheless, financial applications of volatility models require a multivariate framework, which includes not only time varying conditional variances, but also conditional covariance of asset returns. However, the statistical property of range limits its use in estimating asset covariance, hence, the multivariate volatility modelling using the range.

There are only a few studies on the application of the intraday range in a multivariate setting. Brandt and Diebold (2006) propose a range-based covariance estimator, which is based on no-arbitrage approach in a triangular relationship, and is limited only for the foreign currency market. Harris and Yilmaz (2009) propose a hybrid multivariate EWMA estimator for the variance-covariance matrix of returns which employs a range-based EWMA specification to estimate the conditional variances and a standard return-based EWMA specification to estimate the correlation between the returns. In another study, Chou *et al*. (2009) propose a range-based dynamic conditional correlation (DCC) model that combines the return-based DCC model for estimating correlations with the CARR model for conditional variances. Recently, Christensen and Podolskij (2007) and Martens and van Dijk (2007) propose realized range as a more efficient volatility estimator than the realized variance. Following the idea of realized range, Bannouh *et al.* (2009) propose co-range as an estimator of the daily covariance between asset returns based on the regularly spaced intraday high-low price ranges. Since realized range and realized variance both require intraday data, they are not useful for financial assets that do not have readily available intraday data.

On the other hand, use of regime switching models in capturing the effects of sudden substantial economic and political events on the volatility, inherent in economic time series, is mostly established. However, there is substantial potential in utilizing these models in financial applications. There are some applications of regime switching models in modelling financial time series. In investigating regime switches in interest rates; Sola and Driffill (1994), Gray (1996), Ang and Bekaert (2002), Bansal *et al.* (2004) and Audrino (2006), in stock returns; Turner *et al.* (1989), Pagan and Schwert (1990), Hamilton and Susmel (1994), Dueker (1997), Ryden *et al.* (1998), Susmel (2000), Billio and Pelizzon (2000), Maheu and McCurdy (2000), Perez-Quiros and Timmerman (2001) and Bhar and Hamori (2004), in foreign exchange rates; Engel and Hamilton (1990), Engel (1994), Vigfusson (1997), Bollen *et al.* (2000), Dewachter (2001), Klaassen (2002), Brunetti *et al.* (2003) and Beine *et al.* (2003) are prominent works among others. Combining conditional variance models with regime switching models has the potential to explain latent switches between regimes, and, in addition, to utilize useful properties of ARCH family models in modelling financial time series. The first attempts made by Cai (1994) and Hamilton and Susmel (1994) are considered first-order Markov switching for an ARCH specification of the conditional variance. Later, Gray (1996), Klaassen (2002) and Haas *et al.* (2004) extended the ARCH specification into the regime switching GARCH specification.

In this chapter, useful properties of regime switching models and nonlinear filtration are combined with range-based estimator in a multivariate framework in order to enhance the variance covariance estimation of asset returns. In so doing, one and two component Markov regime switching autoregressive conditional range models, introduced in Chapter 4, are employed in the estimation of conditional variances. In the estimation of the correlations, the constant conditional correlation (CCC) procedure of Bollerslev (1990) and the dynamic conditional correlation (DCC) model of Engle (2002a) are employed. The out-of-sample forecast performance of these models is compared with the following return and range-based benchmark models. The utilized benchmark return-based models are exponentially weighted moving average (EWMA) model of Riskmetrics, generalized autoregressive conditional heteroskedasticity (GARCH) model of Bollerslev (1988), and Markov Regime Switching GARCH (MRSGARCH) model of Klaassen (2002). The benchmark range-based models are hybrid EWMA (HybEWMA) model of Harris and Yilmaz (2009), and CARR model of Chou (2005) and Chou *et al.*

(2009). The out-of-sample forecast performance of the proposed models is evaluated in a hedge fund investment exercise. In the investment exercise, a fund of funds portfolio that is composed of individual hedge fund strategies is constructed for each model. Portfolios are constructed based on 1-, 4-, 13- and 26-week volatility forecasts, and performance of the portfolios are evaluated during the out-of-sample test period. The portfolios are constructed for each model, and their out-of-sample performances are tested by using a number of portfolio risk and return evaluation metrics. In particular, realized return, conditional Sharpe ratio, omega ratio, Sortino ratio, information ratio, maximum drawdown, return on Maximum drawdown, portfolio turnover ratio and conditional value at risk at 95 and 99 percent confidence levels are used. The out-of-sample forecast performance of each model is also tested in forecasting each element of the variance covariance matrix of each portfolio with the help of a number of statistical loss functions. In particular, mean error metrics, directional predictive ability tests, forecast evaluation regressions, and pair-wise and joint tests are used.

The out-of-sample analysis period covers the recent credit crisis and recovery after the crisis. During this period, hedge fund industry experienced high historical losses, and a number of funds are ceased to exist. This development motivates the use of a model, which is able to model high short term volatility and stable long term trend in volatility separately, as well as, able to capture the changes in the regimes. In the investment exercise, the performances of the models are tested in a period that is stylized as the high volatility and frequent changes in underlying volatility process for the hedge fund industry.

The contribution of this chapter is twofold. First, to the best knowledge of the author, this is the first study in applying range data to regime switching models in a multivariate context. Second, it is the first study that applies range-based models in hedge fund strategy indices through extracting high/low/open/close index values from daily strategy indices, and compares the portfolio construction performance of the range-based models with the return-based models.

The outline of the chapter is as follows. In the following section, analytical framework for volatility forecast is provided, and the proposed models are introduced. Section 3 describes the data and methodology used in the empirical analysis. Section 4 presents the empirical results. Section 5 provides a summary and some concluding remarks.

5.2. Theoretical Background

5.2.1. Range as a volatility proxy

Assume a $mx1$ vector of logarithmic prices, p_t, that follows a multivariate continuous time volatility diffusion given by

$$dp(t) = \mu(t)dt + \Omega(t)dW(t)$$

where $\mu(t)$ is the N-dimensional stationary instantaneous drift, $H(t)$ is the mxm dimensional strictly stationary diffusion matrix with the elements $\sigma_{jk,t}$, and $W(t)$ is a standard m-dimensional Brownian motion process with $\text{cov}(dW_{it}, \sigma_{jk,t}) = 0$ for all i, j, k. The stochastic process of the logarithmic return vector, defined as $r_t = p_t - p_{t-1}$, is given by

$$r_t = \mu_t + z_t H_t \tag{5.1}$$

where z_t is a $mx1$ vector of standard normally distributed and serially uncorrelated random variables, and Ω_t is an unobservable integrated variance-covariance matrix, approximated by its unbiased estimator realized volatility, given by

$$H_t^q = \sum_{s=1}^{1/q} r_{t-1+sq} r'_{t-1+sq} \tag{5.2}$$

Under ideal conditions (e.g. frictionless and arbitrage-free market), realized volatility, H_t^q, provides consistent and approximately unbiased true volatility estimates, and it converges, uniformly in probability, to H_t as $q \to 0$. There is a trade-off between increased efficiency and consistency: Although realized volatility is an unbiased estimator of the integrated volatility, due to the effects of market microstructure frictions at lower measurement intervals (e.g. 1, 5, 10 minutes), it no longer satisfies the consistency properties (see Andersen *et al.*, 2004). In addition to this setback, although intraday data are increasingly available for many securities, it may not be available for the securities of interest. Therefore, it is desirable to focus on volatility estimators that utilize the data readily available for a long time span, and are robust to microstructure noise. The intraday range rises as an excellent alternative. The log-range is defined as the natural logarithm of the difference between highest and lowest log prices of a security:

$$R_t = \ln\left(\max_{s \in [t-1, t]} P_s - \min_{s \in [t-1, t]} P_s \right) \tag{5.3}$$

The main advantage of the intraday range over the integrated variance estimator, which is based on close-to-close prices, is its comprehensive information content; range contains information about all intraday movements of the price. In estimating the variance of returns, Parkinson (1980) proposes an estimator based on only high and low prices, while Garman and Klass (1980) propose an estimator based on high, low, opening and closing prices with certain assumptions on Brownian motion parameters (i.e. volatility and drift). The estimators proposed by Rogers and Satchell (1991), Kunitomo (1992), and Yang and Zhang (2000) are mainly based on earlier estimators of Parkinson (1980) and Garman and Klass (1980). An unbiased estimate of the variance of the daily close-to-close return, due to Harris and Yilmaz (2009), is employed that uses a combination of the intraday range and the open-to-close return:

$$S_{i.t}^2 = \frac{1}{4 \ln 2}\left(p_{i,t}^H - p_{i,t}^L\right)^2 + \left(p_{i,t}^O - p_{i,t-1}^C\right)^2 \tag{5.4}$$

where $p_{i,t}^O, p_{i,t-1}^C, p_{i,t}^H,\ p_{i,t}^L$ are the opening, closing, high and low prices, subsequently. Parkinson (1980) shows that if the prices follow a Brownian motion process with a constant variance, log-range is five times more efficient than the squared daily close-to-close return. Likewise, Andersen and Bollerslev (1998) and Brandt and Diebold (2006) report that the efficiency of the daily range is between that of the realized range estimated using 3 and 6 hour returns. Alizadeh *et al.* (2002) and Brandt and Jones (2006) establish that the distribution of the log-range conditional on volatility is approximately Gaussian, and the log-range is robust to certain market microstructure effects (e.g. bid-ask bounce, asynchronous trading). These properties make the intraday range an attractive volatility proxy for models that employ Gaussian quasi-maximum likelihood (QMLE) estimations.

Although range has many desirable properties, it suffers from a discretisation bias. One source of this bias is the likely difference between the highest (lowest) stock price observed at discrete points in time and the true maximum (minimum) of the underlying diffusion process. This makes the observed range being a downward-biased estimate of the true range, thus, in turn, a noisy proxy of volatility. A second source of the bias is likely infrequent trading of the component instruments that may lead to an additional downward bias of the range. Another source of bias might arise from conditional non-normality in returns (see Brandt and Jones, 2006).

5.2.2. Multivariate Markov Regime Switching Autoregressive Range Model

Suppose that P_t is m-variate logarithmic asset price, $r_t = P_t - P_{t-1}$ is a $mx1$ vector of returns in the period t with elements $r_{k,t}$, $k = 1,...,m$, $R_t = \max\{P_t\} - \min\{P_t\}$ is observed range in the period t with elements $\Re_{k,t}$, $k = 1,...,m$, and Ω is the information set. The return series can be zero mean or a filtered process. The range series follow a zero mean and nonnegative stochastic process.

$$r_t = \mu + \varepsilon_t \quad \text{where} \, \varepsilon_t \mid \Omega_{t-1} \sim D(0, H_t) \tag{5.5a}$$

$$S_t = \sqrt{\lambda_t} v_t \qquad \text{where} \, v_t \mid \Omega_{t-1} \sim D(1, \xi_t) \tag{5.5b}$$

where μ is the $mx1$ vector of mean returns with elements μ_k, $k = 1,...,m$, H_t is the mxm covariance matrix with diagonal elements, $\sigma^2_{k,t}$ and off-diagonal elements, $\sigma^2_{kl,t}$, $k,l = 1,...,m$, $D(\cdot)$ is any location-scale family distribution, λ_t is the conditional expectation of the range as a function of all information available up to time $t-1$ [i.e. $\lambda_t = E(S_t^2 \mid \Omega_{t-1})$], v_t is a sequence of $i.i.d.$ nonnegative random variables with unit mean and time varying variance-covariance matrix, and v_t follows a positive support distribution (e.g. Exponential).

The time varying covariance matrix H_t can be decomposed into

$$H_t \equiv D_t \Gamma_t D_t \tag{5.6}$$

where D_t is a time-varying diagonal matrix composed of the standard deviations, $h_{k,t}$, $k=1,...,m$ and the Γ_t is a correlation matrix. Depending on the specification of the correlation model, correlation matrix can be constant (Γ) or time-varying (Γ_t). The conditional covariance is specified as the product of a diagonal matrix of conditional variances, produced by individual range-based models, and the conditional correlation matrix, estimated by using the return data. Henceforth, the estimation procedure is based on the estimate of the elements of this decomposition at each step. Proposed models are estimated in two steps: estimating the standard deviations and estimating the correlation matrix.

5.2.2.1. Estimating conditional standard deviations

In the first step, univariate models are estimated by employing the Markov regime switching autoregressive range models (MRSACR) introduced in Chapter 4. First, the one-factor MRSACR model is estimated to get the conditional standard deviation, D_t estimates:

$$S_{k,t} = \sqrt{\lambda_{k,t}^{(i)}}\, v_{k,t} \qquad \text{where } v_{k,t} \mid \Omega_{t-1} \sim D(1,\xi_t) \tag{5.7}$$

$$\lambda_{k,t}^{(i)} = \omega_k^{(i)} + \alpha_{1,k}^{(i)} S_{k,t-1}^2 + \beta_{1,k}^{(i)} E_{t-1}\left\{\lambda_{k,t-1}^{(i)} \mid s_t\right\} \tag{5.8}$$

where $\lambda_{k,t}^{(i)}$ is the conditional variance of the range for the series $k=1,...,m$, the coefficients $\left(\omega_i^{(i)}, \alpha_i^{(i)}, \beta_i^{(i)}\right)$ in the conditional mean equation are all positive, and take different values in each state, i, where $i=1,2$. Parameters ω, α, β characterize the inherent uncertainty in range, and short term and long term effects (persistence) of shocks on the range in each state. In the equation (5.8), following Klaassen (2002), the computation of the expectation is adapted as

$$E_{t-1}\left\{\lambda_{k,t-1}^{(i)} \mid s_t\right\} = \sum_{j=1}^{2} \tilde{p}_{ji,t-1}\left(\lambda_{k,t-1}^{(j)}\right) \tag{5.9}$$

where the probabilities, $\tilde{p}_{ji,t}$ are calculated as

$$\widetilde{p}_{ji,t} = \Pr\left(s_t = j \mid s_{t+1} = i, \Omega_{t-1}\right) = \frac{p_{ij} \Pr\left(s_t = j \mid \Omega_{t-1}\right)}{\Pr\left(s_{t+1} = i \mid \Omega_{t-1}\right)} = \frac{p_{ji} p_{j,t}}{p_{i,t+1}} \tag{5.10}$$

where $i, j = 1,2$. The ex-ante transition probabilities of being in the first regime at t given the information at $t-1$ is estimated as

$$p_{j,t} = \Pr\left(s_t = j \mid \Omega_{t-1}\right) = \sum_{i=1}^{2} p_{ij} \left[\frac{f\left(S_{t-1} \mid s_t = i\right) p_{i,t-1}}{\sum_{k=1}^{2} f\left(S_{t-1} \mid s_t = k\right) p_{k,t-1}} \right] \tag{5.11}$$

where $f(\cdot)$ is the likelihood function of the conditional distributions (i.e. Exponential). At this point, the latent regime indicator is assumed to be parameterized as a first-order Markov process by following Hamilton (1988, 1989 and 1990).

Multi-step-ahead volatility forecasts are computed as a weighted average of the multi-step-ahead volatility forecasts in each regime:

$$\widehat{\lambda}_{T,T+h} = \sum_{\kappa=1}^{h} \widehat{\lambda}_{T,T+\kappa} = \sum_{\kappa=1}^{h} \sum_{j=1}^{2} \Pr\left(s_\kappa = j \mid \Omega_{t-1}\right) \widehat{\lambda}^{(i)}_{T,T+\kappa} \tag{5.12}$$

where the weights are the predicted probabilities, and $\widehat{\lambda}^{(i)}_{T,T+\kappa}$ are the κ-step ahead volatility forecasts in each regime that can be recursively calculated as

$$\widehat{\lambda}^{(i)}_{T,T+\kappa} = \omega^{(i)} + \left(\alpha_1^{(i)} + \beta_1^{(i)}\right) E_T \left\{ \widehat{\lambda}^{(i)}_{T,T+\kappa-1} \mid s_{T+\kappa} \right\} \tag{5.13}$$

The second model is the two-component MRSACR model (CMRSACR) introduced in the fourth chapter. In the CMRSACR model, volatility process is represented by the square root of intraday range, S_t, and is composed of two components; a long run trend and a short run cyclical component:

$$S_{k,t} = q_{k,t} + \theta\left(S_{k,t-1} - q_{k,t-1}\right) + \varepsilon_{k,t} \tag{5.14}$$

where q_t is the trend component, ($S_{t-1} - q_{t-1}$) is the deviation of volatility from its long term trend, and ε_t is a random error term with zero mean and constant variance. The model is estimated in two steps: In the first step, the long term trend component is extracted from the intraday range by using Hodrick and Prescott (1997) filter (HP) recursively. As the weekly data set is used, the smoothing parameter of the HP filter is set to 250,000 (i.e. $\lambda = 100 \times nf^2$, where assuming 50 trading weeks in a year, frequency is *nf*=50). In the second step, a MRSGARCH model with zero mean in its variance equation and regime switching parameters is estimated.

$$S_{k,t} - q_{k,t} = \sqrt{h_{k,t}^{(i)}} u_{k,t} \quad \text{where } u_{k,t} \mid \Omega_{t-1} \overset{iid}{\sim} N(0,1) \tag{5.15a}$$

$$h_{k,t}^{(i)} = \alpha_{1,k}^{(i)} \left(S_{k,t-1} - q_{k,t-1}\right)^2 + \beta_{1,k}^{(i)} E_{t-1}\left\{h_{k,t-1}^{(i)} \mid s_t\right\} \tag{5.15b}$$

where $\left(S_t - q_t\right)$ is the cyclical component of volatility, u_t is *i.i.d.,* a zero mean, constant variance, and approximately normally distributed. To estimate the expectation in (5.15b), formulas in (5.9), (5.10) and (5.11) are used.

In forecasting volatility, it is assumed that the long run trend component follows a random walk process, and range approaches to its long term trend with a factor of θ parameter in (5.14). θ parameter is the sum of the persistence parameters in (5.15b), and estimated using the regime switching probabilities in (5.10) as weights. In the CMRSACR model, *h-step*-ahead volatility forecast is, therefore, estimated as

$$\hat{S}_{k,t+h} = \left(1 - \hat{\theta}_k^h\right) q_{k,t} + \hat{\theta}_k^h \left(\hat{S}_{k,t}\right) \tag{5.16}$$

5.2.2.2. Estimating conditional correlations

In the second step, the correlations in (5.6) are estimated. The correlation matrix is estimated by using the close-to-close return series. In estimating the correlations two procedures are adapted; the constant conditional correlation (CCC) model of Bollerslev (1990), and the dynamic conditional correlation (DCC) model of Engle (2002a). The time-varying correlation structure is:

$$\Gamma_t = diag\{Q_t\}^{-1/2} Q_t diag\{Q_t\}^{-1/2} \tag{5.17}$$

where Γ_t is time-varying positive-definite conditional correlation matrix, and Q_t is a matrix of dynamic conditional covariances that is estimated by

$$Q_t = C \circ (\iota\iota' - A - B) + A \circ Z_{t-1}Z_{t-1}' + B \circ Q_{t-1} \quad (5.18)$$

where C is the unconditional correlation matrix, $\circ$ is the Hadamard product (i.e. element-wise multiplication), ι is a vector of ones, A and B are diagonal parameter matrices, and Z_t is a matrix of standardized residuals given by, $Z_t = D_t^{-1} r_t$. Multi-step ahead correlations for the conditional correlation matrix are computed as

$$\Gamma_{t+h} = diag\{Q_{t+h}\}^{-1/2} Q_{t+h} diag\{Q_{t+h}\}^{-1/2} \quad (5.19)$$

by utilizing

$$Q_{t+m} = C \circ (\iota\iota' - A - B) + A \circ Z_{t+m-1}Z_{t+m-1}' + B \circ Q_{t+m-1} \quad (5.20)$$

In (5.20), when A and B are set to equal to zero, the model is turned into the constant correlation model (CCC).

5.3. Data and Methodology

5.3.1. Data

In the analysis, daily index and return data obtained from HFR of the same strategies utilized in Chapter 2 for the period 31 March 2003-29 January 2010 is used. By using daily index values of each strategy, a weekly open/close/high/low index data set is constructed. Using this data set weekly range (in equation 4) and return from weekly open/close/high/low values are estimated. The daily return data is also used to construct a volatility proxy for weekly variance. Weekly range data set for hedge fund strategies is composed of 357 observations. The full sample is divided into an initialization period 4 April 2003-3 March 2006 (150 observations) and an out-of-sample test period 10 February 2006-29 January 2010 (207 observations).

In the initial portfolio optimization, it was found that none of the models make allocations to event driven, equity hedge and macro strategies. Therefore, these strategies were taken out of the analysis and remaining five strategies are used in the empirical study. Summary statistics for the hedge fund return series are reported in Table 5.1.

[Table 5.1]

Panel A reports various statistics for the different hedge fund investment strategies for the full sample of 357 observations. The characteristics of hedge fund returns are particularly heterogeneous. Some strategies (such as equity hedge and event driven) have relatively higher average returns and volatility than other strategies. These strategies are often considered as return enhancers, used to substitute some fraction of the equity holdings in an investor's portfolio (see Amenc and Martellini, 2002). Other strategies (such as relative value arbitrage and equity market neutral) have a lower average return and volatility, therefore, can be thought of a substitute for some fraction of the fixed income or cash holdings in an investor's portfolio. The effect of credit crisis can also be seen on the returns of hedge fund strategies. Experiencing the high losses has changed statistical properties of return data. As general characteristics, all strategies display negative skewness, and all are leptokurtic, particularly convertible arbitrage, mergers arbitrage and relative value arbitrage. The null hypothesis of normality is strongly rejected in all cases. Panel B reports the basic time series properties of hedge fund returns. In particular, it reports the first five autocorrelation coefficients, the Ljung-Box portmanteau test for serial correlation up to 10 lags, the ARCH test of Engle (1982), and the DCC test of Engle and Sheppard (2001). All the hedge fund indices display significantly positive autocorrelations at some lags except macro strategy, and the ARCH test suggests that there is evidence of significant volatility clustering for all strategies. The DCC test, which tests the null hypothesis of constant correlation is tested against the alternative of dynamic conditional correlation, suggests that the data demonstrates time-varying conditional correlations, and hence motivates the use of dynamic conditional covariance models.

5.3.2. Methodology

In order to compare the statistical and portfolio construction performance of the proposed multivariate CMRSACR and MRSACR models, three return-based models (EWMA, GARCH and MRSGARCH) and two range-based models (HybEWMA and

CARR) are employed as benchmark models.

Multivariate versions of all models, including MRSACR models, are estimated by using a three-step approach. In the first step, univariate models are estimated for each hedge fund strategy, and volatility forecasts are made for 1-, 4-, 13- and 26-week horizons. In the second step, correlations are estimated by employing the CCC and DCC models as described in equations (5.17, 5.18, 5.19 and 5.20). In the third step, estimated conditional variances and correlations are brought together to form the variance covariance matrix described in equation (5.6).

Initially each conditional volatility model is estimated using observations $t = 1, \ldots, \tau$ to make *h*-week ($h = 1, 2, 4, 13$) ahead out-of-sample forecast of the conditional covariance matrix for the week $t = \tau + h$. The estimation sample is then rolled forward one week, and the models re-estimated and *h*-week out-of-sample forecasts are made for the week $t = \tau + 1 + h$. The last iteration uses the sample $t = T - \tau - h, \ldots, T - h$ to generate forecast covariance matrix for the week $t = T$. This yields a series of 208 one-step-ahead forecasts for each model. Then, in-sample and out-of-sample forecast performance of the models are evaluated by using a variety of evaluation criteria.

5.3.2.1. Forecast Evaluation

The forecast performance of each model is evaluated by using two approaches: statistical measures and economic measures. In the first approach, the statistical forecast performance of each model is assessed by comparing each element of the forecast variance-covariance (VCV) matrix, $\hat{\sigma}_{ij,t}$, with those of the benchmark VCV matrix, $r_{i,t} r_{j,t}$. Three measures of ex-post volatility (i.e. volatility proxy) are utilized as a benchmark VCV matrix, which are estimated from daily and weekly return, and weekly range data. Analyses are made by comparing the forecast VCV matrices with the benchmark ex-post VCV matrices for 1-, 4-, 13- and 26-week forecast horizons. However, for the sake of brevity, the results of statistical tests for the first forecast horizon, and the analyses made by using the volatility proxy calculated from weekly returns are reported. Unreported results are available upon request from the author.

Following statistical evaluation metrics are estimated:

(1) Root mean square error (RMSE)

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(r_{i,t}r_{j,t} - \hat{\sigma}_{ij,t-1}\right)^2} \tag{5.21}$$

(2) Mean absolute error (MAE)

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left|r_{i,t}r_{j,t} - \hat{\sigma}_{ij,t}\right| \tag{5.22}$$

(3) Mincer-Zarnowitz regression

$$r_{i,t}r_{j,t} = \alpha_{ij} + \beta_{ij}\hat{\sigma}_{ij,t} + \varepsilon_{ij,t} \tag{5.23}$$

(4) Encompassing regression

$$r_{i,t}r_{j,t} = \alpha_{ij} + \beta_{1,ij}\hat{\sigma}_{ij,t}^{Model\ 1} + \beta_{2,ij}\hat{\sigma}_{ij,t}^{Model\ 1} + \ldots + \beta_{N,ij}\hat{\sigma}_{ij,t}^{Model\,N} + \varepsilon_{ij,t} \tag{5.24}$$

The RMSE and MAE error statistics measure the forecast accuracy. The Mincer-Zarnowitz regression measures the bias and efficiency of the forecasts from each model. In particular, a hypothesis for each element of the VCV matrix of each model is tested. If the model is (weakly) efficient then the null hypothesis, $H_2\ :\ \alpha_{ij} = 0,\ \beta_{ij} = 1$, should not be able to be rejected. The R-square coefficient of each regression exhibits the explanatory power of the model's forecasts, without considering any bias or inefficiency. Finally, with an encompassing regression it is tested whether the forecasts of one model have any incremental information over the forecasts of the other models. In particular, the null hypotheses, $H_0\ :\ \beta_{k,ij} = 0$ for $k = 1,..,4$, is tested for each model individually.

The second approach to out-of-sample evaluation is using economic measures. In this approach, an investment exercise is conducted. In the investment exercise, a portfolio of hedge fund strategies is constructed by each model; volatility is forecast for 1-, 4-, 13- and 26- week horizons; each portfolio is rebalanced at the end of each horizon; finally, risk and return performance of the model portfolio are evaluated by using the criteria provided below.

Minimum variance portfolios of hedge fund strategies are constructed in 1-, 4-, 13- and 26-week maturities for each model. An investor in *m* hedge funds who wishes to minimize the variance of the portfolio returns on each day *t*, and is subject to a minimum return (target return) and a short selling constraint is considered. The portfolio optimization problem can, therefore, be written as

$$\min_{\mathbf{x}} \Phi_p(\mathbf{x}) \tag{5.25}$$

$$\text{subject to } \mathbf{x} \geq 0,\ \mathbf{x}'1 = 1,\ E(r_{p,t}) \geq r_0 \tag{5.26}$$

where $r_{p,t}$ is the return of the hedge fund portfolio on day t, $\Phi_p(\mathbf{x})$ is the risk measure where $\Phi_p(\mathbf{x}) = \sigma(\mathbf{x}) = (\mathbf{x}'\,\mathbf{H}\,\mathbf{x})^{1/2}$ is the conditional portfolio standard deviation, $\mathbf{x}$ is the *mx1* vector of portfolio weights, $\mathbf{x} = [x_1, \ldots, x_m]'$, $E(r_{p,t})$ is the portfolio expected return and r_0 is the target portfolio return. In modelling the covariance matrix of hedge fund returns, H_t, three return-based model and four range-based model, including the MRSACR and CMRSACR models, are employed.

Risk and return performance of each portfolio is evaluated by using the following criteria:

(1) Average Realized Portfolio Return

$$\bar{r}_P = \frac{1}{T-\tau} \sum_{t=\tau}^{T-1} \sum_{k=1}^{m} x_{k,t} r_{k,t+1} \tag{5.27}$$

(2) Conditional Sharpe ratio

$$CSR_{p,t+1} = \frac{r_{p,t+1}}{\sigma_{p,t+1}} \tag{5.28}$$

where $r_{p,t+1} = \mathbf{x}_t \mathbf{r}_{t+1}$ and $\sigma_{p,t+1} = \mathbf{x}_t' \hat{\mathbf{H}}_{t+1} \mathbf{x}_t$.

(3) Omega

$$Omega_p = \frac{\sum_{t=\tau}^{T-1} \max\left(0, \sum_{k=1}^{m} x_{k,t} r_{k,t+1} - r_f\right)}{\sum_{t=\tau}^{T-1} \max\left(0, r_f - \sum_{k=1}^{m} x_{k,t} r_{k,t+1}\right)} \qquad (5.29)$$

where r_f is the threshold return level set as the average long term US Government bond return of 4 percent over the analysis period.

(4) Sortino Ratio[8]

$$SORR_p(r_f) = \frac{\bar{r}_p - r_f}{\sqrt[2]{\sum_{t=\tau}^{T-1} \max\left(0, r_f - \sum_{k=1}^{m} x_{k,t} r_{k,t+1}\right)}} \qquad (5.30)$$

(5) Maximum Drawdown

$$MDD_P = \max_{\tau \le t \le T}\left[\max_{\tau \le j \le T}\left(w_j\right) - w_t\right] \qquad (5.31)$$

where w_t is the uncompounded cumulative portfolio value at date $t = \tau + 1, ..., T$, and at $t = \tau$ initial portfolio value is set equal to 1.

(6) Return on Maximum Drawdown

$$RoMDD_p = \frac{\bar{r}_p}{MDD_p} \qquad (5.32)$$

(7) Information Ratio

$$IR_{p,t+1} = \frac{\bar{r}_p - \bar{r}_b}{\sigma_{\left(r_p - r_b\right)}} \qquad (5.33)$$

---

[8] Sortino and Omega ratio are also defined as the special case of Kappa function $K_{p,n}(r_b) = \bar{r}_p - r_b \Big/ \sqrt[n]{LPM_n(r_b)}$ introduced by Kaplan and Knowles (2004), where r_b is the minimum acceptable or threshold return level and LPM is the lower partial moments with respect to threshold return level. When $n = 1$, Kappa function is equivalent to Omega ratio of Shadwick and Keating (2002), and when $n = 2$, it is equivalent to the Sortino ratio of Sortino and Messina (1997).

where $\bar{r}_p - \bar{r}_B$ is the active return of the portfolio over the benchmark return, r_B, and $\sigma_{(r_p - r_b)}$ is the active risk of the portfolio. As the benchmark, HFR Equal Weighted Strategies Index is used.

(8) Portfolio turnover

$$PT_p = \sum_{t=\tau}^{T-1} \sum_{k=1}^{m} \left| x_{k,t+1} - x_{k,t} \right| \tag{5.34}$$

(9) Conditional value at risk

$$CVaR_{p,t+1}^{\alpha} = \zeta + \frac{1}{1-\alpha} \frac{1}{T-\tau} \sum_{t=\tau}^{T-1} \max\left\{ 0, -\sum_{k=1}^{m} x_{k,t} r_{k,t+1} - \zeta \right\} \tag{5.35}$$

where α is the CVaR confidence level and ζ is portfolio VaR. CVaR is estimated at 95 percent and 99 percent confidence levels.

5.4. Results

5.4.1. Statistical Test Results

Table 5.2 reports the forecast error statistics (i.e. RMSE and MAE) and directional forecast accuracy test results for each element of the VCV matrix. Elements of VCV matrix are displayed in the rows, and error statistics of each model in the columns. The GARCH model yields the lowest RMSE for five elements, and the CMRSACR model for the four elements of VCV matrix. The CMRSACR model also yields second and third lowest RMSE for the four elements. The CMRSACR model exhibits the lowest MAE for ten across fifteen elements of VCV matrix. the MRSACR and CMRSACR models yields second lowest MAE for eight and five elements of VCV matrix, respectively. The CMRSACR model achieves the highest success rates for eleven elements of VCV. The CMRSACR model also displays the highest and significant directional accuracy test statistics for thirteen elements of VCV matrix. Therefore, in terms of out of sample test statistics the CMRSACR model displays the highest forecast accuracy of all models across fifteen elements of the VCV matrix.

[Table 5.2]

In establishing the significance of the differences, in terms of accuracy between the seven models, modified Diebold-Mariano statistics of Harvey *et al.* (1997) is estimated. Table 5.3 reports the estimated statistics. The null hypothesis that the mean square error of the MRSACR (CMRSACR) model is the same as that of each of the other six models is tested. The CMRSACR model displays statistically significant reduction in MSE relative to the standard EWMA model for fourteen elements, the HybEWMA model for eleven elements, the standard GARCH model for two elements, the ECARR model for seven elements, the MRSGARCH model for eleven elements, and the MRSACR model for five elements of the VCV. On the other hand, the MRSACR model yields statistically significant reduction in MSE relative to the standard EWMA for six elements, the HybEWMA model for three elements, the GARCH model for one element, the ECARR model for two elements, the MRSGARCH model for six elements, and the CMRSACR model for five elements of the VCV. Overall, the CMRSACR and MRSACR models offer large and statistically significant increases in forecast accuracy relative to all models.

[Table 5.3]

In assessing the significance of forecast accuracy of each model relative to realized volatility (i.e. ex-post volatility proxy), Mincer-Zarnowitz (MZ) regressions are estimated. Table 5.4 reports estimated intercept and slope coefficients, standard errors for each coefficient, R-square of the regression, test statistics and *p*-value of hypothesis test for the efficiency of the parameters for each element of the VCV matrix. In the table, each panel reports the MZ regression results for each of seven models. The CMRSACR model displays the highest R-square statistics for eleven across all elements of VCV matrix. The R-square statistics shows that the CMRSACR model contains more information about the true VCV matrix than any of all models. The null hypothesis of efficiency of the parameters is rejected at the conventional significance level for twelve elements of the VCV matrix for the CMRSACR, ECARR and GARCH models. Slope coefficients show that return-based models generate too dispersed forecasts while range-based models tend to produce too compressed forecasts, except CARR model. Overall, the CMRSACR model contains more information about the true volatility, and hypothesis test results yield that its parameters are more efficient.

[Table 5.4]

In assessing the forecast accuracy of the models together, encompassing regressions are estimated. Table 5.5 reports the estimation results. For eight out of fifteen cases, the CMRSACR model clearly encompasses all other models. The MRSACR model also encompasses all other models in four cases. Coefficients of the MRSACR and CMRSACR models are closer to one, and inclusion of other models into the regression does not significantly increase the R-square of the regression.

[Table 5.5]

In summary, statistical tests results conclude that the MRSACR models, in general, and the CMRSACR model, in particular, offer significant improvements in forecast accuracy of the true volatility of hedge fund return series.

5.4.2. Portfolio Construction Study

The out-of-sample forecast performance of the proposed models is tested in comparison to benchmark return and range-based models in an investment exercise. The portfolio construction study is conducted for the out-of-sample period 31 March 2003 to 29 January 2010 for five HFR hedge fund indices. The out-of-sample evaluation period covers favourable as well as unfavourable market conditions. The evaluation period includes the recent credit crisis and recovery after the crisis. In the recent crisis, hedge fund industry experienced high historical losses, and a number of funds ceased to exist. This motivates the use of models that can capture high short term volatility, along with the long term volatility trend and changes in the volatility regimes. In the investment exercise, the performance of the proposed models is tested in a period that is stylized as a high volatility and frequent changes in the underlying volatility process for the hedge fund industry.

The models are estimated by using two different multivariate specifications (i.e. CCC and DCC). The estimated portfolio evaluation metrics are provided in the Table 5.6 and 5.7. Realized return represents the average return/loss a portfolio has been experienced during the out-of-sample period. Regime switching models generate the highest average returns. In particular, the MRSACR model earns the highest return from the portfolios of 1-week horizon. However, the CMRSACR model provides the only positive average returns for the multi-step forecast horizons.

[Table 5.6 and Table 5.7]

Conditional Sharpe ratio (CSR) is a version of the original Sharpe ratio (also known as a reward to variability ratio), which is defined as the ratio of portfolio realized return to the total portfolio risk at each portfolio rebalancing date. The CSR is a measure of portfolio return per unit of portfolio risk defined as the standard deviation. In terms of CSR, range-based models perform better than return-based models, and the CMRSACR model generates the highest return per unit of risk undertaken.

Omega is a relative performance measure that assesses the return generation capacity in relation to loss generation capacity where risk is represented by the lower partial moments in equation (5.29b). Omega measure partitions returns into loss and gain above and below a given minimum acceptable (threshold) return level, and shows the probability of having a portfolio return by the probability of having a loss. The threshold return level is defined as the risk-free rate of return of 4 percent (annual). Omega ratio utilizes all moments of the distribution, while Sharpe ratio uses only two moments (i.e. mean and standard deviation), in estimating the risk adjusted return ratio. Omega criterion favours the regime switching models, which provide the highest Omega ratios. In particular, the MRSACR model displays the highest Omega ratio at all forecast horizons, and the MRSGARCH and CMRSACR models follow it.

Sortino ratio is a relative measure of excess return to risk where the risk is defined as the lower partial moments, and estimated by using only the returns below the minimum acceptable (threshold) return level in (5.30). Similar to Omega ratio, the threshold return level is set to be 4 percent annually. Sortino ratio is negative for all models, due to massive losses experienced by the hedge fund industry during the recent crisis. Sortino ratio favours the use of the MRSACR model in the hedge fund portfolio construction study. In particular, the MRSACR model provides the highest Sortino ratio at all forecast horizons.

Maximum Drawdown (MDD) is a measure of downside variation, which shows the largest drop in the value of the portfolio from its maximum on a given time interval.

The time interval is set to be the entire out-of-sample test period. During the out-of-sample analysis period, regime switching models experience the lowest maximum drawdown. This might be due to the ability of these models to successfully detect the switches between different volatility regimes. In particular, the MRSGARCH and CMRSACR models display lower drawdowns than all models.

Return on Maximum Drawdown (RoMDD) can be interpreted as the average portfolio return per unit of risk undertaken to earn that return, where risk is represented by MDD. Regime switching models provide higher RoMDD than all other models. In particular, the CMRSACR and MRSGARCH are the only models that have positive RoMDD at all forecast horizons. MRSACR models follow these models.

Information ratio is (IR) a performance measure for active portfolio management. IR shows the ratio of active return per unit of active risk undertaken. Active return is defined as the return difference of portfolio and a benchmark index. The HFR Equal Weighted Strategies Index is chosen as the benchmark portfolio, which represents a naïve (i.e. equally weighted) portfolio allocation with using hedge fund strategies. During the analysis period, regime switching models perform well and provide the highest information ratios. In particular, the CMRSACR model generates the highest active return per unit of active risk taken.

Portfolio turnover (PT) can be interpreted as the fraction of the portfolio that must be liquidated and reinvested at each rebalancing date. Capturing switches in volatility dynamics, and adjusting the portfolio accordingly come at a cost of changing the composition of the portfolio significantly at each rebalancing date. Regime switching models require more portfolio turnover than single regime models. In particular, as expected the CMRSACR model requires the highest portfolio turnover than all other models. The MRSGARCH and GARCH model follow. Interestingly, even performs well in terms of risk and return metrics, the MRSACR model requires less turnover than all these models.

CVaR is known as a coherent downside risk measure that measures the average loss a portfolio can experience above the portfolio VaR, for a given period of time, and at a certain significance level (5 and 1 percent in this analysis). Portfolio CVaR provides a clear picture of probability and severity of the loss. Range-based models improve CVaR estimates with their enhanced information content about the true volatility. In particular, the CMRSACR model provides significantly lower risk of mean loss above VaR at both 5 and 1 percent significance levels. Other range-based models, the MRSACR, HybEWMA and ECARR models follow.

Table 5.8 provides average weights of hedge fund portfolios constructed with different models. Panel A reports the portfolio weights of the models with CCC multivariate specification, while Panel B reports the weights of the models with DCC specification. The reported results are similar to each other; therefore, it can be concluded that using either of correlation specification does not alter the portfolio construction. Note that, in a preliminary portfolio analysis, it has been found that none of the models allocates weights to event driven, equity hedge and macro strategies. This finding is also in line with the results of Amenc and Martellini (2002) and Giamouridis and Vrontos (2007). The CMRSACR model makes allocations to the strategies, which exhibit high kurtosis (such as relative value arbitrage and merger arbitrage), more than other models in order to enhance returns, while it gives the lowest weights to the low risk strategies, such as equity market neutral strategy.

5.5. Conclusion

This chapter proposes multivariate versions of one and two factor MRSACR models, introduced in the fourth chapter. These models combine useful properties of regime switching models, nonlinear filtration and range-based estimation in two different multivariate frameworks, to enhance the variance covariance estimation of asset returns. Out-of-sample volatility forecast and portfolio construction performance of these models are compared with eminent return-based and range-based volatility models. In particular, a portfolio composed of hedge fund strategies for each model is constructed based on 1-, 4-, 13- and 26-week volatility forecasts, then the performance of the portfolios are evaluated over the out-of-sample period, by using a number of portfolio risk-return performance measures. The out-of-sample performance of each model in forecasting each element of the variance covariance matrix is also tested by using a number of statistical tests. In particular, forecast error functions, directional predictive ability tests, forecast evaluation regressions, and pair-wise and joint tests are utilized.

Statistical tests results conclude that MRSACR models, in general, and the CMRSACR model, in particular, offer significant improvements in forecast accuracy of the true volatility process of hedge fund return series. The investment study concludes that, in terms of risk and return criteria employed, proposed models perform better than benchmark models. The main purpose of a portfolio manager is either maximizing return at the determined level of risk tolerance or minimizing risk at the required level of return. Proposed models construct fund of hedge fund portfolios with higher risk-adjusted returns and lower tail risks, also offer superior risk-return tradeoffs and better active management ratios than benchmark return and range-based models. In particular, RR, CSR, IR and CVaR criteria favour the use of the CMRSACR model, while Omega and Sortino ratios favour the MRSACR model over all other models. For MDD and RoMDD criteria, the CMRSACR model is the second best model after the MRSGARCH model. However, in most cases these improvements come at the expense of higher portfolio turnover and rebalancing expenses. In particular, the CMRSACR model requires the highest portfolio turnover, while the MRSACR model requires a portfolio turnover similar to single regime models.

Multivariate range models offer a fruitful further research area. Developing the multivariate equivalents of univariate range models offers a promising further research area. Application of the MRSACR models to different frequencies of range (e.g. daily, hourly, minutes or tick-by-tick data) in a multivariate framework might increase the efficiency of portfolio construction. Moreover, application to different asset prices (fixed income, foreign currency, derivatives) or economic time series might also improve forecast performance.

Table 5.1

Summary Statistics and Time Series Properties of Hedge Fund Return Series

Panel A reports summary statistics for the constructed hedge fund return series over the period of 31 March 2003 to 29 January 2010. Panel B reports the autoregressive conditional heteroskedasticity (ARCH) and autocorrelation and dynamic conditional correlation (DCC) test results for the whole period. The Ljung–Box-Q test for autocorrelation of order up to 10 asymptotically distributed as a central Chi-square with 10 d.o.f. under the null hypothesis, with 5 percent critical value is 18.307. ARCH (4) is Engle's LM test for autoregressive conditional heteroskedasticity, which is asymptotically distributed as a central Chi-square with 4 d.o.f. under the null hypothesis with 5 percent critical value is 9.488. p-values are reported in parenthesis. DCC test statistics is the Chi-square with 13 d.o.f. The p-value (i.e. the probability of a constant correlation) is in the bracket.

Panel A: Summary Statistics								
HF Strategy	Mean	Variance	Std. Dev.	Min.	Max.	Skew	Kurtosis	JB Stats
Convertible arbitrage	-0.13	2.46	1.57	-17.80	2.52	-6.42	56.6	50040.7 [0.00]
Distressed securities	-0.01	0.51	0.72	-5.27	1.74	-2.74	14.5	3571.3 [0.00]
Event driven	0.09	0.69	0.83	-5.72	2.63	-2.09	10.5	1897.6 [0.00]
Equity hedge	0.03	1.18	1.09	-7.11	2.86	-1.49	6.4	747.8 [0.00]
Equity market neutral	-0.01	0.40	0.63	-4.97	2.37	-1.19	11.7	2133.4 [0.00]
Mergers arbitrage	0.10	0.55	0.74	-6.78	7.25	-0.20	46.7	32450.7 [0.00]
Macro	0.06	1.49	1.22	-7.37	3.34	-1.82	8.6	1298.4 [0.00]
Relative value arbitrage	0.03	0.89	0.94	-9.79	3.02	-4.17	36.4	20718.4 [0.00]

Panel B: Basic Time Series Properties								
HF Strategy	ARCH	LB-Q	ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)	DCC test
								48.09 [0.00]
Convertible arbitrage	117.0 [0.00]	445.8 [0.00]	0.70	0.50	0.40	0.33	0.29	
Distressed securities	46.6 [0.00]	351.9 [0.00]	0.31	0.35	0.42	0.38	0.27	
Event driven	107.8 [0.00]	51.2 [0.00]	0.24	0.13	0.12	0.08	0.06	
Equity hedge	45.8 [0.00]	24.5 [0.01]	0.11	0.09	0.08	0.10	0.08	
Equity market neutral	38.2 [0.00]	22.4 [0.01]	-0.07	-0.10	-0.07	0.00	0.12	
Mergers arbitrage	130.0 [0.00]	42.3 [0.00]	-0.29	0.13	-0.01	-0.01	-0.09	
Macro	40.2 [0.00]	14.8 [0.14]	0.08	0.09	0.05	0.02	-0.07	
Relative value arbitrage	42.6 [0.00]	237.4 [0.00]	0.32	0.32	0.41	0.17	0.24	

Table 5.2a
Out-of-Sample Forecast Statistics

Panel A reports statistical loss functions RMSE and MAE calculated for the out-of-sample period with respect to each element of variance-covariance matrix. Elements of VCV matrix are in the rows and loss functions of each model are in the columns. Numbers next to loss functions and directional accuracy tests (e.g. RMSE1, MAE1) show the number of models (i.e. 1: Standard EWMA, 2: Hybrid EWMA, 3: Standard GARCH, 4: Exponential CARR, 5: MRSGARCH, 6: MRSACR, 7: Two component CMRSACR.

Panel A: Forecast error statistics														
Item	RMSE1	RMSE2	RMSE3	RMSE4	RMSE5	RMSE6	RMSE7	MAE1	MAE2	MAE3	MAE4	MAE5	MAE6	MAE7
VAR1	22.97	24.47	10.87	136.39	18.33	22.97	20.05	5.62	4.06	2.00	27.33	3.54	3.57	3.13
VAR2	2.65	2.61	7.53	2.51	2.40	2.64	1.95	0.94	0.67	1.50	0.76	0.91	0.71	0.52
VAR3	2.41	1.92	11.43	3.17	1.51	1.75	1.25	0.86	0.57	2.21	1.00	0.65	0.53	0.42
VAR4	3.99	4.72	12.66	3.97	3.95	4.76	4.48	1.21	0.86	2.27	1.27	0.90	0.81	0.69
VAR5	5.76	6.85	7.40	24.03	10.52	6.92	6.13	1.62	1.30	1.67	4.41	4.28	1.22	1.07
COV21	3.93	4.42	2.13	2.79	4.26	4.56	4.21	1.18	0.95	0.67	0.95	0.93	0.93	0.87
COV31	1.87	1.71	2.16	2.28	1.72	1.68	1.70	0.57	0.54	0.61	0.62	0.54	0.54	0.54
COV41	9.27	9.84	8.26	11.60	10.21	9.89	9.41	1.65	1.40	1.16	2.04	1.44	1.36	1.30
COV51	12.40	12.87	8.28	23.27	12.38	12.94	12.00	2.45	2.04	1.42	4.96	2.05	2.01	1.86
COV32	0.88	0.70	2.86	0.86	0.68	0.69	0.67	0.40	0.32	0.65	0.37	0.33	0.32	0.31
COV42	1.42	1.66	2.97	1.51	1.75	1.74	1.64	0.55	0.42	0.65	0.47	0.45	0.41	0.38
COV52	1.92	2.38	2.72	1.68	2.31	2.52	2.29	0.75	0.67	0.74	0.59	0.70	0.67	0.62
COV43	1.07	1.08	4.75	1.52	1.01	1.08	1.02	0.48	0.37	0.98	0.53	0.40	0.35	0.32
COV53	1.60	1.33	5.48	3.50	1.28	1.32	1.26	0.70	0.51	1.24	0.97	0.57	0.50	0.48
COV54	4.02	4.66	3.41	4.38	4.39	4.74	4.41	1.02	0.79	0.95	1.20	0.73	0.76	0.69

Table 5.2b
Out-of-Sample Forecast Statistics (cont'd)

Panel B reports directional accuracy test statistics (SR and DA) calculated for the out-of-sample period with respect to each element of the variance - covariance matrix. Elements of VCV matrix are in the rows and loss functions of each model are in the columns. Numbers next to loss functions and directional accuracy tests (e.g. SR1, DA1) show the number of models (i.e. 1: Standard EWMA, 2: Hybrid EWMA, 3: Standard GARCH, 4: Exponential CARR, 5: MRSGARCH, 6: MRSACR, 7: Two component CMRSACR.

Panel B: Directional Forecast Accuracy Tests														
Item	SR1	SR2	SR3	SR4	SR5	SR6	SR7	DA1	DA2	DA3	DA4	DA5	DA6	DA7
VAR1	0.80	0.80	0.97	0.93	0.92	0.94	0.97	4.55	5.03	11.41	8.23	6.86	8.01	11.63
VAR2	0.81	0.79	0.84	0.80	0.85	0.76	0.93	6.77	6.47	3.63	5.64	7.60	4.61	10.74
VAR3	0.66	0.61	0.77	0.72	0.78	0.64	0.86	2.72	1.84	1.40	0.88	3.73	3.09	8.61
VAR4	0.79	0.75	0.90	0.86	0.76	0.90	0.93	4.46	5.08	4.93	4.35	4.79	7.40	9.59
VAR5	0.79	0.78	0.92	0.89	0.67	0.86	0.96	5.10	5.42	9.03	7.66	0.24	6.72	12.37
COV21	0.78	0.78	0.92	0.86	0.82	0.77	0.89	4.70	5.55	7.26	5.31	6.25	4.93	7.01
COV31	0.36	0.38	0.31	0.31	0.37	0.42	0.34	-0.94	-1.31	-1.54	-0.99	-1.31	-0.81	-1.91
COV41	0.77	0.78	0.92	0.88	0.84	0.88	0.93	2.79	3.75	7.32	5.29	3.31	5.05	7.65
COV51	0.77	0.77	0.95	0.92	0.90	0.89	0.96	4.37	5.20	9.67	8.24	6.96	6.75	11.30
COV32	0.65	0.66	0.68	0.67	0.64	0.59	0.69	2.03	2.26	1.04	2.16	2.00	1.01	3.44
COV42	0.58	0.58	0.70	0.65	0.55	0.56	0.70	-1.24	-1.32	0.43	-0.35	-2.42	-1.07	1.87
COV52	0.76	0.76	0.87	0.83	0.72	0.77	0.89	4.38	4.62	5.83	4.63	2.55	4.82	8.30
COV43	0.62	0.60	0.77	0.74	0.71	0.69	0.76	0.50	0.96	2.34	1.34	1.93	1.83	4.06
COV53	0.69	0.62	0.77	0.77	0.72	0.71	0.84	3.89	3.04	3.20	3.47	4.35	4.19	7.91
COV54	0.79	0.78	0.89	0.86	0.83	0.85	0.88	3.94	3.41	5.25	4.73	4.85	5.56	6.43

Table 5.3
Modified Diebold-Mariano Statistics for Mean Squared Errors

The table reports the Modified Diebold-Mariano statistics and p-values of each statistics for the mean squared errors to test the null hypothesis that there is no difference between the MRSACR model and each of the other seven models (Panel A), and CMRSACR model and each of the other seven models (Panel B). Results are reported for each element of the variance covariance (VCV) matrix. Elements of VCV matrix are in the columns and each model is in the rows.

VCV element	VAR1	VAR2	VAR3	VAR4	VAR5	COV21	COV31	COV41	COV51	COV32	COV42	COV52	COV43	COV53	COV54
Panel A: Benchmark Model is MRSACR															
EWMA	-1.93	-0.83	-1.82	-1.1	-0.12	-0.11	-2.08	-1.32	-0.24	-2.5	-1.33	-0.02	-3.79	-3.13	-0.69
pval	0.05	0.41	0.07	0.3	0.90	0.49	0.04	0.16	0.74	0.0	0.04	0.33	0.00	0.00	0.36
HybEWMA	-1.22	0.99	-1.32	-0.9	-0.27	0.83	2.34	-0.51	0.42	-1.5	0.69	0.91	-1.29	-1.55	-0.27
pval	0.22	0.32	0.19	0.4	0.79	0.33	0.02	0.55	0.63	0.1	0.30	0.33	0.05	0.00	0.61
GARCH	1.26	-1.38	-1.26	-2.0	0.46	1.25	-1.41	1.16	1.23	-1.4	-1.74	1.08	-1.49	-1.42	0.25
pval	0.21	0.17	0.21	0.0	0.65	0.21	0.15	0.24	0.22	0.2	0.08	0.23	0.14	0.15	0.68
ECARR	-1.66	0.28	-1.60	-1.2	-1.61	0.43	-1.24	-1.43	-1.61	-1.6	1.93	0.76	-2.15	-1.77	-0.94
pval	0.10	0.78	0.11	0.2	0.11	0.51	0.21	0.14	0.11	0.1	0.01	0.38	0.03	0.08	0.32
MRSGARCH	0.87	-0.27	-0.68	0.1	-3.03	0.96	2.40	1.23	1.29	2.7	-2.79	0.62	-3.33	-4.25	0.42
pval	0.39	0.78	0.50	0.9	0.00	0.30	0.01	0.22	0.19	0.0	0.01	0.22	0.00	0.00	0.45
CMRSACR	1.43	1.92	2.28	1.9	1.54	1.33	1.97	1.13	1.22	5.2	1.30	1.43	2.06	2.21	1.46
pval	0.15	0.06	0.02	0.1	0.13	0.18	0.04	0.26	0.22	0.0	0.17	0.15	0.03	0.03	0.14
Panel B: Benchmark Model is CMRSACR															
EWMA	-2.68	-2.90	-2.40	-2.2	-1.94	-1.68	-2.14	-3.12	-2.43	-3.0	-2.55	-1.93	-4.42	-4.00	-2.09
pval	0.01	0.00	0.02	0.0	0.05	0.06	0.03	0.00	0.02	0.0	0.01	0.02	0.00	0.00	0.03
HybEWMA	-1.35	-2.00	-1.88	-2.6	-1.97	-1.97	3.15	-1.37	-1.31	-3.7	-4.66	-2.62	-1.81	-1.88	-2.45
pval	0.18	0.05	0.06	0.0	0.05	0.03	0.00	0.16	0.19	0.0	0.00	0.01	0.04	0.03	0.01
GARCH	1.09	-1.61	-1.36	-2.1	-1.34	1.17	-1.33	1.25	1.32	-1.5	-1.81	-1.96	-1.59	-1.53	-0.62
pval	0.28	0.11	0.18	0.0	0.18	0.22	0.18	0.21	0.19	0.1	0.07	0.04	0.11	0.12	0.36
ECARR	-1.67	-2.24	-1.97	-1.9	-1.67	-0.52	-1.18	-1.42	-1.60	-2.2	-2.31	-0.54	-2.52	-1.97	-1.67
pval	0.10	0.03	0.05	0.1	0.10	0.25	0.23	0.15	0.11	0.0	0.02	0.02	0.01	0.05	0.10
MRSGARCH	-0.37	-3.00	-3.22	-1.1	-3.56	-1.60	-2.44	-1.31	-1.20	-3.7	-2.88	-3.85	-5.41	-9.37	-5.53
pval	0.71	0.00	0.00	0.3	0.00	0.02	0.01	0.19	0.23	0.0	0.00	0.00	0.00	0.00	0.00
MRSACR	-1.43	-1.92	-2.28	-1.9	-1.54	-1.33	-1.97	-1.13	-1.22	-5.2	-1.30	-1.43	-2.06	-2.21	-1.46
pval	0.15	0.06	0.02	0.1	0.13	0.18	0.04	0.26	0.22	0.0	0.17	0.15	0.03	0.03	0.14

Table 5.4
Mincer-Zarnowitz Regression Results

The table reports the results of the Mincer-Zarnowitz regression and hypothesis test for the efficiency of the parameters for each element of the variance covariance (VCV) matrix. Elements of VCV matrix are in the rows and each model is in the columns. The dependent variable in the regression is the values of the volatility proxy for that element of VCV matrix, and the independent variable is the volatility forecast of each model. In estimating heteroskedasticity-autocorrelation-consistent standard errors for the regressions, Newey and West (1987) estimation procedure is used.

	Panel A: Standard EWMA						
VCV	Intcpt	se	slope	se	Stats	pval	R2
VAR1	-1.03	0.87	1.28	0.75	2.1	0.35	0.16
VAR2	0.20	0.14	0.55	0.08	35.2	0.00	0.14
VAR3	0.56	0.17	0.05	0.07	215.9	0.00	0.00
VAR4	-0.83	0.45	1.63	0.61	4.5	0.11	0.42
VAR5	-1.24	0.87	1.72	0.76	2.9	0.24	0.42
COV21	-0.76	0.46	1.93	0.98	3.8	0.15	0.34
COV31	0.10	0.11	-1.05	1.00	7.1	0.03	0.08
COV41	-1.73	1.23	3.43	2.24	2.0	0.36	0.25
COV51	-0.77	0.67	1.92	1.18	1.7	0.44	0.12
COV32	0.09	0.05	-0.01	0.14	55.0	0.00	0.00
COV42	-0.43	0.21	1.42	0.66	6.6	0.04	0.40
COV52	-0.55	0.26	1.64	0.56	5.5	0.06	0.51
COV43	-0.04	0.10	0.62	0.20	5.5	0.06	0.15
COV53	0.15	0.11	0.12	0.16	33.2	0.00	0.01
COV54	-1.04	0.65	2.24	1.15	3.3	0.20	0.44

	Panel B: Hybrid EWMA						
VCV	Intcpt	se	slope	se	Stats	pval	R2
VAR1	-1.22	0.94	5.34	3.11	2.0	0.36	0.15
VAR2	0.06	0.09	2.43	0.37	15.7	0.00	0.15
VAR3	0.48	0.18	0.40	0.35	7.9	0.02	0.01
VAR4	-1.44	0.72	7.69	2.96	5.2	0.08	0.45
VAR5	-1.61	1.14	7.30	3.58	3.8	0.15	0.37
COV21	-0.74	0.42	6.83	3.32	3.3	0.19	0.30
COV31	0.15	0.13	-4.69	3.89	2.2	0.33	0.08
COV41	-0.86	0.76	8.97	7.25	1.3	0.53	0.07
COV51	-1.12	0.90	7.88	4.81	2.1	0.35	0.14
COV32	0.09	0.05	0.01	0.63	4.4	0.11	0.00
COV42	-0.44	0.22	5.95	2.98	3.9	0.15	0.29
COV52	-0.58	0.29	6.20	2.44	4.7	0.10	0.41
COV43	-0.05	0.10	1.89	0.87	1.1	0.59	0.07
COV53	0.11	0.13	0.70	0.81	0.9	0.63	0.01
COV54	-1.34	0.83	9.74	5.16	2.9	0.24	0.41

	Panel C: Standard GARCH						
VCV	Intcpt	se	slope	se	Stats	pval	R2
VAR1	-0.62	0.34	1.47	0.12	18.0	0.00	0.90
VAR2	0.59	0.20	0.11	0.02	2707.1	0.00	0.10
VAR3	0.59	0.17	0.00	0.01	27080.1	0.00	0.00
VAR4	0.17	0.12	0.25	0.02	1081.0	0.00	0.66
VAR5	0.22	0.14	0.49	0.04	138.7	0.00	0.84
COV21	0.02	0.10	1.16	0.07	6.0	0.05	0.80
COV31	-0.08	0.10	-0.06	0.22	25.3	0.00	0.00
COV41	-0.47	0.34	1.33	0.41	2.2	0.33	0.33
COV51	-0.28	0.23	1.55	0.25	5.4	0.07	0.68
COV32	0.10	0.04	-0.01	0.03	943.1	0.00	0.00
COV42	-0.14	0.08	0.34	0.03	559.4	0.00	0.64
COV52	0.06	0.08	0.48	0.02	473.0	0.00	0.70
COV43	0.11	0.09	0.06	0.02	2967.8	0.00	0.06
COV53	0.18	0.10	0.03	0.02	2524.6	0.00	0.01
COV54	-0.17	0.14	0.61	0.05	75.7	0.00	0.91

	Panel D: Exponential CARR						
VCV	Intcpt	se	slope	se	Stats	pval	R2
VAR1	-0.06	0.54	0.14	0.02	1727.5	0.00	0.71
VAR2	0.33	0.15	0.70	0.23	5.1	0.08	0.18
VAR3	0.58	0.17	0.02	0.03	1078.4	0.00	0.00
VAR4	-0.15	0.20	0.60	0.14	13.2	0.00	0.71
VAR5	0.33	0.13	0.21	0.03	940.8	0.00	0.77
COV21	-0.24	0.12	0.78	0.14	9.0	0.01	0.70
COV31	-0.07	0.10	-0.14	0.19	39.4	0.00	0.01
COV41	-0.09	0.20	0.30	0.25	45.1	0.00	0.08
COV51	-0.10	0.22	0.31	0.07	95.1	0.00	0.53
COV32	0.11	0.05	-0.17	0.29	16.6	0.00	0.01
COV42	-0.15	0.08	0.85	0.47	5.4	0.07	0.26
COV52	-0.05	0.09	0.75	0.16	4.7	0.10	0.64
COV43	0.07	0.09	0.26	0.10	54.7	0.00	0.12
COV53	0.18	0.10	0.04	0.04	523.1	0.00	0.01
COV54	-0.22	0.16	0.54	0.12	18.7	0.00	0.72

	Panel E: MRSGARCH						
VCV	Intcpt	se	slope	se	Stats	pval	R2
VAR1	0.77	0.89	1.00	0.23	0.8	0.68	0.46
VAR2	0.02	0.16	0.74	0.19	2.3	0.31	0.25
VAR3	0.07	0.12	0.77	0.24	1.2	0.56	0.40
VAR4	-0.62	0.15	2.12	0.42	18.6	0.00	0.51
VAR5	0.82	0.29	0.15	0.13	60.7	0.00	0.03
COV21	-0.18	0.18	2.76	0.59	12.6	0.00	0.24
COV31	0.11	0.10	-3.74	2.22	4.6	0.10	0.07
COV41	0.88	0.58	-1.48	1.09	6.0	0.05	0.04
COV51	0.83	0.70	1.02	0.61	1.6	0.45	0.10
COV32	0.00	0.05	0.79	0.58	0.6	0.75	0.04
COV42	0.02	0.05	0.36	0.94	0.5	0.79	0.00
COV52	-0.26	0.10	1.64	0.36	10.4	0.01	0.20
COV43	-0.15	0.05	1.63	0.38	8.8	0.01	0.20
COV53	-0.12	0.14	1.11	0.68	4.0	0.14	0.09
COV54	-0.13	0.18	1.58	0.36	3.4	0.19	0.20

	Panel F: MRSACR						
VCV	Intcpt	se	slope	se	Stats	pval	R2
VAR1	0.92	0.92	3.03	0.80	6.6	0.04	0.30
VAR2	0.14	0.21	2.02	0.66	4.0	0.14	0.10
VAR3	-0.67	0.31	6.27	1.85	13.2	0.00	0.63
VAR4	-0.54	0.29	5.54	2.07	5.2	0.08	0.29
VAR5	0.24	0.29	3.18	0.97	5.7	0.06	0.15
COV21	-0.35	0.15	10.34	3.94	7.9	0.02	0.14
COV31	0.03	0.10	-9.18	8.40	1.5	0.46	0.03
COV41	0.25	0.30	3.01	5.74	1.8	0.41	0.01
COV51	0.73	0.75	4.41	2.48	3.8	0.15	0.06
COV32	-0.03	0.04	3.10	1.23	3.0	0.23	0.04
COV42	0.09	0.14	-0.57	4.99	0.7	0.69	0.00
COV52	-0.05	0.10	4.84	2.77	1.9	0.38	0.06
COV43	-0.19	0.05	5.14	1.03	24.0	0.00	0.20
COV53	-0.19	0.17	5.35	3.31	1.8	0.40	0.12
COV54	-0.57	0.28	8.93	3.90	4.2	0.12	0.24

	Panel G: CMRSACR						
VCV	Intcpt	se	slope	se	Stats	pval	R2
VAR1	-0.75	0.27	4.76	0.13	852.0	0.00	0.98
VAR2	-0.03	0.03	3.11	0.24	94.4	0.00	0.95
VAR3	0.01	0.04	2.41	0.03	2424.3	0.00	0.91
VAR4	-0.21	0.09	4.20	0.84	14.7	0.00	0.48
VAR5	-0.64	0.33	5.26	1.07	24.9	0.00	0.81
COV21	-0.37	0.09	10.00	0.28	1047.5	0.00	0.92
COV31	0.10	0.08	-7.93	7.24	1.6	0.45	0.09
COV41	-0.68	0.45	10.06	1.23	55.1	0.00	0.52
COV51	-0.85	0.38	9.96	0.76	143.9	0.00	0.86
COV32	-0.10	0.04	7.08	1.78	12.8	0.00	0.29
COV42	-0.20	0.09	5.59	2.58	4.8	0.09	0.34
COV52	-0.35	0.11	7.36	0.89	52.8	0.00	0.78
COV43	-0.11	0.05	4.37	0.65	27.9	0.00	0.43
COV53	-0.20	0.12	5.28	2.28	3.5	0.17	0.35
COV54	-0.51	0.18	8.05	1.24	32.6	0.00	0.77

Table 5.5
Encompassing Regression Results

The table reports the results of the encompassing regression for each element of the variance covariance (VCV) matrix. Elements of VCV matrix are in the rows and each model is in the columns. The dependent variable in the regression is the values of the volatility proxy for that element of the VCV matrix, and the independent variables are volatility forecasts of all models for that element of VCV matrix. In estimating heteroskedasticity-autocorrelation-consistent standard errors for the regressions, Newey and West (1987) estimation procedure is used.

	intrcpt	se0	slope1	se1	slope2	se2	slope3	se3	slope4	se4	slope5	se5	slope6	se6	slope7	se7	R2
VAR1	-0.01	-0.26	1.08	1.78	0.0	-0.16	-1.68	1.14	0.23	1.06	4.6	0.27	0.01	0.02	0.20	0.62	1.00
VAR2	0.00	-0.13	0.14	0.01	0.1	0.02	-0.20	3.05	0.05	0.36	1.8	0.02	0.08	0.06	0.30	0.20	0.96
VAR3	-0.12	-1.23	3.54	0.11	-0.2	0.09	-0.19	2.33	0.05	0.83	2.3	0.10	0.27	0.06	0.30	0.07	0.93
VAR4	-0.84	-3.79	13.64	0.23	0.1	1.20	-1.49	1.05	0.37	1.37	6.0	0.06	0.19	0.48	0.85	1.30	0.88
VAR5	0.16	-0.81	3.39	0.33	0.0	-0.04	-2.08	2.93	0.40	1.52	6.3	0.04	0.04	0.03	0.51	0.43	0.95
COV21	0.12	4.89	-18.67	0.17	0.0	0.88	-4.75	8.43	0.04	0.72	3.2	0.10	0.04	0.33	1.00	0.64	0.97
COV31	-0.04	-7.30	18.33	4.56	-3.0	2.56	38.98	-31.76	0.09	3.64	12.6	1.07	0.86	4.87	13.28	11.55	0.52
COV41	0.44	6.93	-34.02	1.60	-0.4	-0.93	-6.44	8.61	0.12	0.65	3.7	0.67	0.21	0.66	1.78	2.89	0.98
COV51	0.36	1.93	-7.66	2.02	-0.1	-2.06	-0.19	5.18	0.13	0.74	2.8	0.25	0.08	0.48	1.06	1.88	0.99
COV32	-0.09	0.29	3.27	0.22	-3.0	0.07	-1.52	10.43	0.04	0.54	4.3	0.05	0.87	0.71	2.52	0.91	0.57
COV42	0.34	1.83	-5.19	0.37	-0.5	0.23	-14.12	1.05	0.10	0.66	3.4	0.10	0.14	1.75	8.34	1.49	0.86
COV52	0.12	3.33	-13.07	0.11	0.0	0.13	-2.70	5.12	0.06	0.83	2.1	0.07	0.15	0.45	2.77	1.56	0.87
COV43	0.10	2.87	-8.72	-0.03	-0.2	1.26	-5.77	5.34	0.05	0.65	2.5	0.04	0.23	0.84	4.03	1.12	0.72
COV53	0.03	-1.63	0.49	0.24	-0.1	0.09	3.01	5.52	0.07	1.39	3.8	0.25	0.33	0.25	2.88	2.58	0.45
COV54	0.10	-1.56	5.17	0.72	-0.1	0.11	-3.53	2.28	0.22	0.95	4.4	0.19	0.29	0.45	1.16	0.95	0.95

Table 5.6
Portfolio Performance Measures for Multivariate Models with CCC Specification

The table reports the portfolio risk-return performance measures for multivariate models with Constant Conditional Correlation (CCC) correlation specification. RR is the average realized return for the out of sample analysis period, CSR is the mean of conditional Sharpe ratios for the analysis period, OMG is Omega ratio, SORR is Sortino ratio, MDD is the maximum drawdown for the period, RoMDD is the return on maximum drawdown ratio, IR is the information ratio, PT is the portfolio turnover, CVaR is the portfolio conditional value at risk at 95 and 99 percent confidence levels.

Model	RR	CSR	OMEGA	SORR	MDD	RoMDD	IR	PT	CVaR95	CVaR99
Forecast step 1 (1-week)										
EWMA	-0.071	0.130	0.396	-0.240	295.2	-0.02	0.049	6.96	0.932	1.219
HybEWMA	-0.061	0.146	0.413	-0.242	98.1	-0.06	0.066	6.23	0.534	0.704
GARCH	-0.045	0.170	0.485	-0.195	166.5	-0.03	0.081	25.37	1.029	1.331
ECARR	-0.076	0.433	0.399	-0.302	336.6	-0.02	0.050	13.32	0.713	0.929
MRSGARCH	0.015	0.143	0.609	-0.187	17.8	0.08	0.226	34.09	0.723	0.943
MRSACR	0.019	0.261	0.647	-0.155	119.5	0.02	0.214	18.70	0.429	0.565
CMRSACR	0.013	0.434	0.509	-0.223	18.2	0.07	0.237	82.55	0.291	0.385
Forecast step 2 (4-weeks)										
EWMA	-0.071	0.065	0.396	-0.240	295.2	-0.02	0.049	6.96	1.757	2.333
HybEWMA	-0.061	0.073	0.413	-0.242	98.1	-0.06	0.066	6.23	0.969	1.309
GARCH	-0.051	0.072	0.476	-0.211	179.9	-0.03	0.082	18.25	2.010	2.595
ECARR	-0.073	0.198	0.406	-0.294	340.2	-0.02	0.049	10.09	1.186	1.565
MRSGARCH	-0.005	0.060	0.568	-0.211	-30.9	0.02	0.206	24.46	1.708	2.213
MRSACR	-0.003	0.079	0.612	-0.183	134.5	0.00	0.196	13.78	1.061	1.377
CMRSACR	0.013	0.215	0.510	-0.223	16.4	0.08	0.238	82.06	0.526	0.714
Forecast step 3 (13-weeks)										
EWMA	-0.071	0.036	0.396	-0.240	295.2	-0.02	0.049	6.96	2.858	3.896
HybEWMA	-0.061	0.041	0.413	-0.242	98.1	-0.06	0.066	6.23	1.455	2.069
GARCH	-0.071	0.025	0.445	-0.234	208.6	-0.03	0.080	12.62	3.455	4.435
ECARR	-0.075	0.095	0.406	-0.290	348.6	-0.02	0.048	7.98	1.738	2.342
MRSGARCH	-0.019	0.033	0.546	-0.221	-23.1	0.08	0.195	21.10	3.593	4.640
MRSACR	-0.021	0.029	0.573	-0.199	104.2	-0.02	0.183	11.01	2.304	2.956
CMRSACR	0.013	0.117	0.513	-0.223	14.7	0.09	0.238	81.55	0.774	1.115
Forecast step 4 (26-weeks)										
EWMA	-0.071	0.026	0.396	-0.240	295.2	-0.02	0.049	6.96	3.624	5.091
HybEWMA	-0.061	0.029	0.413	-0.242	98.1	-0.06	0.066	6.23	1.663	2.533
GARCH	-0.082	0.010	0.425	-0.246	217.2	-0.04	0.079	9.50	4.707	6.015
ECARR	-0.078	0.061	0.403	-0.290	351.1	-0.02	0.048	6.40	2.140	2.953
MRSGARCH	-0.026	0.024	0.536	-0.227	-24.3	0.11	0.188	19.99	5.510	7.112
MRSACR	-0.038	0.015	0.545	-0.199	95.1	-0.04	0.166	9.55	3.795	4.815
CMRSACR	0.013	0.083	0.516	-0.222	14.2	0.09	0.238	81.36	0.842	1.325

Table 5.7
Portfolio Performance Measures for Multivariate Models with DCC Specification

The table reports the portfolio risk-return performance measures for multivariate models with Dynamic Conditional Correlation (DCC) correlation specification. RR is the average realized return for the out of sample analysis period, CSR is the mean of conditional Sharpe ratios for the analysis period, OMG is Omega ratio, SORR is Sortino ratio, MDD is the maximum drawdown for the period, RoMDD is the return on maximum drawdown ratio, IR is the information ratio, PT is the portfolio turnover, CVaR is the portfolio conditional value at risk at 95 and 99 percent confidence levels.

Models	RR	CSR	OMEGA	SORR	MDD	RoMDD	IR	PT	CVaR95	CVaR99
Forecast step 1 (1-week)										
EWMA	-0.073	0.128	0.394	-0.239	306.6	-0.02	0.048	7.55	0.931	1.219
HybEWMA	-0.061	0.146	0.413	-0.242	98.1	-0.06	0.066	6.23	0.534	0.704
GARCH	-0.045	0.170	0.485	-0.195	166.5	-0.03	0.081	25.37	1.029	1.331
ECARR	-0.076	0.433	0.399	-0.302	336.6	-0.02	0.050	13.32	0.713	0.929
MRSGARCH	0.015	0.143	0.609	-0.187	17.8	0.08	0.226	34.09	0.723	0.943
MRSACR	0.019	0.261	0.647	-0.155	119.5	0.02	0.214	18.70	0.429	0.565
CMRSACR	0.013	0.434	0.509	-0.223	18.2	0.07	0.237	82.55	0.291	0.385
Forecast step 2 (4-weeks)										
EWMA	-0.073	0.064	0.394	-0.239	306.6	-0.02	0.048	7.55	1.757	2.332
HybEWMA	-0.061	0.073	0.413	-0.242	98.1	-0.06	0.066	6.23	0.969	1.309
GARCH	-0.051	0.072	0.476	-0.211	179.9	-0.03	0.082	18.25	2.010	2.595
ECARR	-0.073	0.198	0.406	-0.294	340.2	-0.02	0.049	10.09	1.186	1.565
MRSGARCH	-0.005	0.060	0.568	-0.211	-30.9	0.02	0.206	24.46	1.708	2.213
MRSACR	-0.003	0.079	0.612	-0.183	134.5	0.00	0.196	13.78	1.061	1.377
CMRSACR	0.013	0.215	0.510	-0.223	16.4	0.08	0.238	82.06	0.526	0.714
Forecast step 3 (13-weeks)										
EWMA	-0.073	0.035	0.394	-0.239	306.6	-0.02	0.048	7.55	2.858	3.894
HybEWMA	-0.061	0.041	0.413	-0.242	98.1	-0.06	0.066	6.23	1.455	2.069
GARCH	-0.071	0.025	0.445	-0.234	208.6	-0.03	0.080	12.62	3.455	4.435
ECARR	-0.075	0.095	0.406	-0.290	348.6	-0.02	0.048	7.98	1.738	2.342
MRSGARCH	-0.019	0.033	0.546	-0.221	-23.1	0.08	0.195	21.10	3.593	4.640
MRSACR	-0.021	0.029	0.573	-0.199	104.2	-0.02	0.183	11.01	2.304	2.956
CMRSACR	0.013	0.117	0.513	-0.223	14.7	0.09	0.238	81.55	0.774	1.115
Forecast step 4 (26-weeks)										
EWMA	-0.073	0.025	0.394	-0.239	306.6	-0.02	0.048	7.55	3.624	5.090
HybEWMA	-0.061	0.029	0.413	-0.242	98.1	-0.06	0.066	6.23	1.663	2.533
GARCH	-0.082	0.010	0.425	-0.246	217.2	-0.04	0.079	9.50	4.707	6.015
ECARR	-0.078	0.061	0.403	-0.290	351.1	-0.02	0.048	6.40	2.140	2.953
MRSGARCH	-0.026	0.024	0.536	-0.227	-24.3	0.11	0.188	19.99	5.510	7.112
MRSACR	-0.038	0.015	0.545	-0.199	95.1	-0.04	0.166	9.55	3.795	4.815
CMRSACR	0.013	0.083	0.516	-0.222	14.2	0.09	0.238	81.36	0.842	1.325

Table 5.8
Portfolio Weights for Multivariate Models

The table exhibits the mean weights of portfolios constructed by each multivariate model with Constant Conditional Correlation (CCC) correlation specification (Panel A) and Dynamic Conditional Correlation (DCC) correlation specification (Panel B). Portfolios are composed of hedge fund strategies: convertible arbitrage (CA), distressed securities (DS), equity market neutral (EMN), merger arbitrage (MAR) and relative value arbitrage (RVA). Portfolio weights are in percentages.

Panel A: Portfolio Weights for CCC Specification										
Model	CA	DS	EMN	MAR	RVA	CA	DS	EMN	MAR	RVA
	Forecast step 1 (1-week)					Forecast step 2 (4-weeks)				
EWMA	16.0	27.0	23.0	25.0	10.0	16.0	27.0	23.0	25.0	10.0
HybEWMA	16.0	28.0	23.0	23.0	10.0	16.0	28.0	23.0	23.0	10.0
GARCH	22.0	27.0	23.0	18.0	10.0	22.0	29.0	23.0	15.0	11.0
ECARR	12.0	30.0	25.0	15.0	17.0	12.0	31.0	24.0	15.0	18.0
MRSGARCH	20.0	26.0	24.0	23.0	7.0	25.0	29.0	27.0	11.0	9.0
MRSACR	16.0	24.0	23.0	26.0	11.0	21.0	26.0	28.0	10.0	16.0
CMRSACR	16.0	26.0	18.0	26.0	14.0	16.0	26.0	17.0	26.0	14.0
	Forecast step 3 (13-weeks)					Forecast step 4 (26-weeks)				
EWMA	16.0	27.0	23.0	25.0	10.0	16.0	27.0	23.0	25.0	10.0
HybEWMA	16.0	28.0	23.0	23.0	10.0	16.0	28.0	23.0	23.0	10.0
GARCH	22.0	31.0	23.0	12.0	11.0	22.0	33.0	22.0	11.0	11.0
ECARR	11.0	32.0	23.0	14.0	19.0	11.0	33.0	23.0	13.0	20.0
MRSGARCH	27.0	32.0	27.0	8.0	6.0	27.0	34.0	26.0	8.0	5.0
MRSACR	22.0	28.0	24.0	8.0	17.0	23.0	32.0	19.0	9.0	17.0
CMRSACR	16.0	27.0	17.0	26.0	14.0	16.0	27.0	16.0	26.0	14.0

Panel B: Portfolio Weights for DCC Specification										
Model	CA	DS	EMN	MAR	RVA	CA	DS	EMN	MAR	RVA
	Forecast step 1 (1-week)					Forecast step 2 (4-weeks)				
EWMA	16.0	27.0	23.0	25.0	10.0	16.0	27.0	23.0	25.0	10.0
HybEWMA	16.0	28.0	23.0	23.0	10.0	16.0	28.0	23.0	23.0	10.0
GARCH	22.0	27.0	23.0	18.0	10.0	22.0	29.0	23.0	15.0	11.0
ECARR	12.0	30.0	25.0	15.0	17.0	12.0	31.0	24.0	15.0	18.0
MRSGARCH	20.0	26.0	24.0	23.0	7.0	25.0	29.0	27.0	11.0	9.0
MRSACR	16.0	24.0	23.0	26.0	11.0	21.0	26.0	28.0	10.0	16.0
CMRSACR	16.0	26.0	18.0	26.0	14.0	16.0	26.0	17.0	26.0	14.0
	Forecast step 3 (13-weeks)					Forecast step 4 (26-weeks)				
EWMA	16.0	27.0	23.0	25.0	10.0	16.0	27.0	23.0	25.0	10.0
HybEWMA	16.0	28.0	23.0	23.0	10.0	16.0	28.0	23.0	23.0	10.0
GARCH	22.0	31.0	23.0	12.0	11.0	22.0	33.0	22.0	11.0	11.0
ECARR	11.0	32.0	23.0	14.0	19.0	11.0	33.0	23.0	13.0	20.0
MRSGARCH	27.0	32.0	27.0	8.0	6.0	27.0	34.0	26.0	8.0	5.0
MRSACR	22.0	28.0	24.0	8.0	17.0	23.0	32.0	19.0	9.0	17.0
CMRSACR	16.0	27.0	17.0	26.0	14.0	16.0	27.0	16.0	26.0	14.0

Chapter 6

Factor-Based Hedge Fund Replication with Risk Constraints

6.1. Introduction

The dynamics of hedge fund returns are relatively complex owing to the non-traditional investment strategies and tools that are commonly used by hedge fund managers, such as leverage, short selling, derivatives and dynamic trading. This results in a nonlinear relationship between hedge fund returns and the returns of the major asset classes. Moreover, it is well established that hedge fund returns are not normally distributed, with most strategies exhibiting high levels of negative skewness and excess kurtosis, and displaying positive autocorrelation as a result of holding illiquid assets (see for related literature in Section 2.1). It is therefore considerably more challenging to replicate hedge fund returns than it is to replicate, for example, mutual fund returns. Attempts to model hedge fund returns have searched for assets, styles or trading rules that can mimic the strategies that hedge fund managers employ (see, for example, Fung and Hsieh, 2001, Hasanhodzic and Lo, 2007, among others). Although most hedge fund managers claim that they achieve superior risk-adjusted performance, and are hence able to justify the high fees that they commonly charge, some studies estimate that up to 60-80 percent of hedge fund returns can be captured by systematic risk factors (see Jaeger and Wager, 2005; Fung and Hsieh, 2006 and 2007). The purpose of replication, therefore, is not to achieve exactly the same level of return performance, but to capture a significant part of it with lower fees and better liquidity.

There are three broad approaches to hedge fund replication: the factor approach, the distribution-matching approach and the rule-based approach. The factor approach projects hedge fund returns on to a set of investible factors, and uses linear regression (Jaeger and Wagner, 2005; Hasanhodzic and Lo, 2007) or nonlinear optimization (Amenc *et al.*, 2010) to minimise the tracking error between the hedge fund return and the weighted average return of the factors. Factor approaches are often able to generate a good fit to hedge fund returns in-sample, depending on the choice of factors and the time period considered, but are often found to have poor out-of-sample performance. In particular, it is commonly found that the replicating portfolio has lower average return and higher standard deviation (and hence higher risk) than the hedge fund portfolio that it is designed to track. This is potentially due to the dynamic nature of the investment strategies that hedge funds typically employ, which cannot be captured by the essentially backward looking factor approach. For a detailed summary of research in this area, see Fung and Hsieh (2004), Mitchell and Pulvino (2001), Tancar and Viebig (2008), Amenc *et al.* (2008), Takahashi and Yamamoto (2008), among others. The distribution-matching approach seeks to replicate the unconditional distribution of the payoffs of the hedge fund using an equivalent investment in the replicating assets (see Kat and Palaro, 2005, 2006; Papageorgiou *et al.* 2008; Takahashi and Yamamoto, 2010). In contrast with the factor approach, the distribution-matching approach is relatively robust out-of-sample in the sense that the higher moments (such as variance, skewness and kurtosis) of the replicating portfolio are similar to those of the hedge fund portfolio. However, as noted by Amenc *et al.* (2008), by focussing on the higher moments of hedge fund returns, rather than their time-series properties, there is nothing to guarantee the out-of-sample performance of the replicating portfolio since the first moment of returns is ignored. Indeed, the time-series correlation between the replicating portfolio and the hedge fund portfolio is often found to be extremely low. As such, the distribution-matching approach is more relevant to fund design than to performance replication (see, for example, Wallerstein *et al.*, 2010, Amenc *et al.*, 2008). The rule-based approach seeks to mimic well-known hedge fund strategies by implementing relatively simple trading algorithms that invest in liquid assets in such a way that generates a similar risk-return profile to the hedge fund being replicated. These algorithms are mainly proprietary in nature, and therefore there is little academic research concerning their performance. Mitchell and Pulvino (2001) and Duarte *et al.* (2007) investigate the risk-return characteristics of merger and fixed income arbitrage strategies and search for trading rules to mimic these strategies. In practice, the rule-based approach is often combined with factor-based replication.

In this chapter, a composite approach that combines the factor and distribution-matching methodologies is investigated. In particular, a linear factor model for hedge fund returns is specified to capture their time series properties, but a range of constraints are imposed to ensure that the replicating portfolio matches various risk measures of the hedge fund, including Conditional Value at Risk, Conditional Drawdown at Risk and the partial moments of returns. These risk measures are non-linear functions of the higher moments of returns, and so the proposed approach can be thought of as incorporating the distribution-matching approach. A return constraint is also imposed to ensure that the clone portfolio delivers the same absolute performance as that of the hedge fund. This approach is used to replicate the monthly returns of ten hedge fund strategy indices using long-only positions in ten equity, interest rate, exchange rate and commodity indices, all of which can be traded using liquid, investible instruments such as futures, options and exchange traded funds. Using out-of-sample evaluation, it is shown that the proposed composite approach yields replicating portfolios that better mimic both the risk-adjusted performance and distributional characteristics of the hedge fund indices that they are designed to track. On balance, proposed approach appears to represent an improvement over the out-of-sample performance of the factor and payoff distribution approaches reported by Jaeger and Wagner (2005) and Amenc *et al*. (2010).

The outline of the remainder of this chapter is as follows. In the following section, the characteristics of the data are described and the replication methodology is outlined. In Section 3, the results of out-of-sample tests are reported. Section 4 concludes.

6.2. Data and Methodology

6.2.1. Data

In the analysis, monthly data on ten hedge fund strategy indices obtained from HFR is used. In addition to the hedge fund strategy indices utilized in Chapter 2 (see 2.2.1), the emerging markets (EM) and fund of funds (FOF) indices are also considered. The full sample covers the period June 1994 to January 2011 (200 observations). The initial estimation period is June 1994 to September 2002 (100 observations), and the out-of-sample evaluation period is October 2002 to January 2011 (100 observations). Since the length of the dataset is different from the data used in Chapter 2 and Chapter 3, the data analysis is performed for the new data set. Findings are in line with the findings presented in Chapter 2 and Chapter 3. Summary statistics for the hedge fund return series over the full sample of 200 observations are reported in Table 6.1.

[Table 6.1]

Panel A of Table 6.1 reports various descriptive statistics for the monthly hedge fund strategy returns. Some strategies (such as emerging markets) exhibit relatively higher volatility than others (such as equity market neutral and mergers arbitrage). The returns for all ten hedge fund strategies are leptokurtic, and with the exception of the macro strategy, they all exhibit negative skewness. The null hypothesis of normality is strongly rejected in all cases. Panel B reports the first five autocorrelation coefficients of hedge fund strategy returns, the Ljung-Box portmanteau test for serial correlation up to 10 lags and the ARCH test of Engle (1982). With the exception of the macro strategy, all ten hedge fund strategies exhibit highly significant autocorrelations. The ARCH test suggests that there is evidence of volatility clustering in six of the ten strategies. The significant autocorrelation in hedge fund returns is largely due to the artificial smoothing of monthly returns that arises from time lags in the valuation of the securities held by the hedge fund, especially in less liquid strategies such as distressed securities. To correct for this autocorrelation, the method of Geltner (1991), originally proposed for smoothing appraisal-based returns of commercial real estate assets is used.

In order to replicate the performance of the hedge fund strategy indices, ten equity, bond, commodity and foreign exchange indices, which are taken from Datastream are used. These indices are listed in Table 6.2. Although not directly investible, they can be traded via a range of low cost, highly liquid instruments, such as futures, options and exchange traded funds.

[Table 6.2]

These indices are selected from a broad list of indices in five steps: Initially, 32 instruments are selected. These instruments are well known indices (they are highly traded by using the liquid instruments such as futures and ETFs) and they belong to the asset classes used in the replicating portfolio construction. Second, at each portfolio rebalancing date, a regression is run by using all 32 instruments in order to find out which instruments have statistically significant beta coefficients in explaining the replicated hedge fund strategies. The instruments which have statistically significant beta parameters are noted. This analysis is repeated for the whole out-of-sample test period. Third, each optimization model is run by considering all 32 instruments at each rebalancing date. This process is repeated for the whole out-of-sample test period. The instruments that all models give significant (>3%) average portfolio weights for the entire out-of-sample period are noted. Fourth, the factors employed by other authors are noted. Fifth, the size of the hedge fund industry investments in terms of strategies and markets are analyzed. For example, nearly in total 40 percent of hedge fund strategies are equity-based and operate in mainly medium-size, small-size and micro-size stock markets. Also about 15-20 percent invests in emerging market securities. The instruments which are related to these markets and strategies are noted. Finally, among the instruments noted in the earlier steps, 10 instruments which are believed to best replicate all considered hedge fund strategies are selected.

6.2.2. Methodology

The starting point is the factor-based approach to replicating hedge fund returns, but this is supplemented with a number of constraints on the return and risk of the replicating portfolio, and hence indirectly on its distributional characteristics. Specifically, the objective function is given by:

$$\min_{x} f(x) = \mathrm{var}(r_{hf,t} - r_{p,t}) \tag{6.1}$$

subject to

$$\sum_{i=1}^{m} x_i = 1, \quad i = 1,\dots,m \tag{6.2}$$

$$x_i \geq 0 \tag{6.3}$$

$$\sum_{i=1}^{m} \bar{r}_i \; x_i = \bar{r}_{hf} \tag{6.4}$$

$$CVaR_{p} = CVaR_{hf} \tag{6.5}$$

$$CDaR_{p} = CDaR_{hf} \tag{6.6}$$

$$UPM_{p} = UPM_{hf} \tag{6.7}$$

$$LPM_p = LPM_{hf} \tag{6.8}$$

where $r_{hf,t}$ is the return of the hedge fund index at time *t*, x_i , $i = 1,...,m$, is the weight of instrument *i* in the replicating portfolio, $r_{p,t} = \sum_{i=1}^{m} r_{i,t} x_i$ is the return of the replicating portfolio at time *t*. The budget constraint (6.2) represents full investment without leverage, while the positivity constraint (6.3) ensures long-only portfolio positions. The constraint (6.4) matches the mean return of the replicating portfolio with the mean return of the hedge fund index, which addresses the return component of the risk-adjusted performance of the replicating portfolio. While the CVaR (6.5) and the CDaR constraints match the tail risks, the Upper Partial Moments (UPM) and Lower Partial Moments (LPM) constraints in (6.7) and (6.8) implicitly match all moments of the return distributions by separately considering the right and left hand side of the distribution. The constraints from (6.5) to (6.8) concern the risk of the replicating portfolio, and are described in detail in the following sub-sections.

6.2.2.1. CVaR Constraint

Conditional Value at Risk (CVaR) is a risk measure derived from Value at Risk (VaR) and can be defined as the expected value of losses exceeding VaR over a specified time horizon at a specified confidence level (see Rockafeller and Uryasev, 2002). Let ζ be VaR with confidence level α. CVaR for the replicating portfolio is defined as

$$CVaR_{p,\alpha}(x) = \zeta + \tfrac{1}{1-\alpha}\tfrac{1}{N}\sum_{j=1}^{N}\max\left\{0, -\sum_{i=1}^{m} r_{ij}x_i - \zeta\right\} \tag{6.9}$$

and for the replicated hedge fund strategy index as

$$CVaR_{hf,\alpha} = \zeta + \tfrac{1}{1-\alpha}\tfrac{1}{N}\sum_{j=1}^{N}\max\left\{0, -r_{hf,j} - \zeta\right\} \tag{6.10}$$

where ζ is estimated from the returns of the hedge fund strategy and $CVaR_{p,\alpha}(x)$ is a convex function of portfolio positions with respect to α. The constraint in (5) therefore aims to match the CVaR of the replicating portfolio in (9) with that of the individual hedge fund strategy in (10).

6.2.2.2. CDaR Constraint

Drawdown, also known as the underwater portfolio level, is a commonly used performance indicator in portfolio management, and is defined as the reduction in portfolio value from a previous maximum. The drawdown concept helps investors construct portfolios in a way that avoids losses that exceed a fixed percentage of the maximum value of their wealth achieved up to that point in time. Chekhlov *et al.* (2000) propose the Conditional Drawdown at Risk (CDaR) measure that combines the drawdown concept with the CVaR approach. Analogous to CVaR, CDaR can be defined as the expectation of drawdowns that exceed a certain threshold drawdown level, ζ, which is defined at an α-confidence level similar to the way VaR is defined in the specification of CVaR. However, unlike CVaR, CDaR is a risk measure that accounts not only for the aggregate of losses over some period, but also for the sequence of those losses. For portfolio implementation of CDaR, see, for example, Chekhlov *et al.* (2005).

Let ζ be threshold drawdown level estimated at confidence level α. CDaR for the replicating portfolio is defined as

$$CDaR_{p,\alpha}(x) = \zeta + \tfrac{1}{1-\alpha}\tfrac{1}{N}\sum_{j=1}^{N}\max\left[0,\ \max_{0\le k\le j}\left[\sum_{i=1}^{m}\left(\sum_{s=1}^{k} r_{is}\right)x_i\right] - \sum_{i=1}^{m}\left(\sum_{s=1}^{j} r_{is}\right)x_i - \zeta\right] \qquad (6.11)$$

and for the hedge fund strategy index as

$$CDaR_{hf,\alpha}(x) = \zeta + \tfrac{1}{1-\alpha}\tfrac{1}{N}\sum_{j=1}^{N}\max\left[0,\ \max_{0\le k\le j}\left(\sum_{s=1}^{k} r_{hf,s}\right) - \sum_{s=1}^{j} r_{hf,s} - \zeta\right] \qquad (6.12)$$

where ζ is estimated from the drawdowns of the hedge fund strategy and $CDaR_{p,\alpha}(x)$ is convex function of portfolio positions with respect to α. The constraint in (6.6) therefore aims to match the CDaR of the replicating portfolio in (6.11) with that of the individual hedge fund strategy in (6.12).

6.2.2.3. Partial Moments Constraint

The constraints on the partial moments in (6.7) and (6.8) are motivated by the Omega performance measure first introduced by Keating and Shadwick (2002). Omega is a generalised measure of risk-adjusted return that implicitly utilises all moments of the distribution of portfolio returns, rather than focussing merely on the mean and the variance, and is defined as the upper partial moment of returns with respect to some threshold, divided by the lower partial moment of returns. Here the components of Omega – the upper and lower partial moments – are considered separately. In particular, upper and lower partial moments for the return r_p are defined as the probability weighted ratio of portfolio gains and losses relative to a threshold return defined by the investor:

$$UPM_p(r_b) = \int_{r_b}^{r_{\max}} (1 - F(y))dx = E_P[\max(0, r_p - r_b)] \tag{6.13}$$

$$LPM_p(r_b) = \int_{r_{\min}}^{r_b} F(y)dx = E_P[\max(0, r_b - r_p)] \tag{6.14}$$

where $F(.)$ is the cumulative probability distribution function of the portfolio returns, $F(y) = P\lfloor r_p \le y \rfloor$. Given the threshold return level r_b, the UPM and LPM functions of the replicating portfolio are defined in (6.7) and (6.8) as

$$UPM_p = E\lfloor r_p \mid r_p \ge r_b \rfloor - r_b \tag{6.15}$$

$$LPM_p = r_b - E\lfloor r_p \mid r_p \le r_b \rfloor \tag{6.16}$$

and the UPM and LPM functions of the replicated hedge fund strategy in (6.7) and (6.8) as follows

$$UPM_{hf} = E\lfloor r_{hf} \mid r_{hf} \ge r_b \rfloor - r_b \tag{6.17}$$

$$LPM_{hf} = r_b - E\lfloor r_{hf} \mid r_{hf} \le r_b \rfloor \tag{6.18}$$

where r_{hf} is the returns of individual hedge fund strategy.

6.2.3. Estimation and Evaluation

The out-of-sample performance of the replicating portfolios is tested with different constraints over the period October 2002 to January 2011. Portfolio performance is reported for a number of different specifications of the model. FM is the pure factor model, which imposes only the full investment constraint. RC is the pure factor model supplemented with the long only and average return constraints. CVaRC, CDaRC and PMC each have one additional constraint (on CVaR, CDaR or the upper and lower partial moments, respectively). ALLC imposes all of the constraints simultaneously. Initially the model is estimated by using the first 100 months, June 1994 to September 2002, to generate out-of-sample forecasts of the replicating portfolio weights for October 2002. The estimation sample is then rolled forward one month to forecast the portfolio weights for November 2002, and so on until the end of the sample is reached. The estimation window length is kept constant at 100 months. The model is estimated using the Matlab *fmincon* optimization function. In estimating CVaR and CDaR constraints 95 percent confidence level is used.

The out-of-sample performance of the replicating portfolios is evaluated using a number of statistical and economic measures. Firstly, a regression of realized hedge fund portfolio returns on the realized replicating portfolio returns over the out-of-sample period is estimated. For brevity, only the adjusted R-squared statistic and beta coefficient of this regression are provided. Secondly, the mean, standard deviation and skewness and kurtosis coefficients of the return distribution for both the hedge fund portfolio and the replicating portfolio are reported. Thirdly, the Sharpe Ratio, maximum drawdown, and annualized CVaR and CDaR statistics for both the hedge fund portfolio and the replicating portfolio are reported. In computing the Sharpe ratio, an annualised risk free rate of 2.03 percent, representing the average yield on US Treasury securities at a constant 3-month maturity over the out-of-sample period is used.

6.3. Empirical Results

The regression results are reported in columns 1-3 of Table 6.3a and 6.3b. Generally, the estimated beta coefficient is significantly greater than zero, and the adjusted R-squared values are relatively high. The factor-based model generates a beta value closer to one than all other models. For some strategies (such as emerging markets), the FM model is able to explain up to 85 percent of the variance in hedge fund returns. However, the highest R-squared statistic is generated by the PMC model in six out of ten cases, but by the FM model in only three cases. The CVaRC model generates the second highest adjusted R-squared statistic in seven out of ten strategies. In terms of statistical performance, therefore, the replicating portfolios in many cases display a significant improvement over the out-of-sample performance of the factor-based model. However, for some strategies, such as equity market neutral, the relatively low level of the systematic component is detrimental to the performance of the replicating portfolios.

[Table 6.3a and 6.3b]

The annualised mean and standard deviation of returns and the skewness and kurtosis coefficients are reported in columns 4-7 of Table 6.3a and 6.3b. In general, the composite model replicates the statistical properties of hedge fund returns in terms of their first four moments reasonably well. Composite models tend to offer slightly higher average returns relative to the hedge fund strategies, but also slightly higher standard deviation. In contrast, the benchmark FM model tends to offer a lower average returns than the hedge fund strategies, and lower standard deviation. Both the benchmark FM model and the composite models approximately match the skewness and kurtosis of hedge fund returns. The risk-adjusted performance of the models is given by the Sharpe ratio, reported in column 8 of Table 6.3a and 6.3b. The composite models, ALLC, CVaRC and CDaRC, offer the best replication of risk-adjusted performance. The FM model underperforms the hedge fund strategies in seven out of ten cases, and in two cases actually yields a negative Sharpe ratio. In contrast, the CDaRC model outperforms the hedge fund strategies in seven out of ten cases, and where it underperforms, the differences are relatively small. The success of the composite model over the pure factor-based model can be attributed to the different objective functions of the two models: the composite model implicitly considers all the moments of the return distribution through the return and risk constraints, while the factor-based model considers only the second moment. The risk characteristics of the replicating portfolios and hedge fund strategies are reported in columns 9-11 of Table 6.3a and 6.3b. In terms of CVaR and CDaR, the replication performance of the composite models is similar to the factor-based model, except for the PMC model.

Increasing the number of risk constraints reduces the out-of-sample explanatory power of the replicating portfolios, with the R-squared dropping significantly. However, this fall in explanatory power is not matched by a reduction in portfolio performance.

Indeed, the ALLC model generates higher returns, better risk-adjusted return and better replication of the higher moments than the factor-based model, and with similar (or better) risk, as measured by CVaR and CDaR. Among the composite models, although the PMC and CVaRC models explain more of the hedge fund return variance (i.e. they have a higher R-squared), the ALLC model produces the best overall replication performance. In particular, compared with the other composite models, the ALLC model provides a better match for the first four moments of hedge fund returns, better risk-adjusted return, better CVaR replication and similar CDaR replication.

The course of net asset values of the replicating portfolios and the hedge fund strategies over the out-of-sample test period is displayed in Figure 6.1. In general, the composite model portfolios provide a reasonable fit to the time series of hedge fund returns. For example, the ALLC model slightly underperforms EM and DS strategies and closely follows and outperforms other strategies. The PMC model generates the highest portfolio values and significantly outperforms all hedge fund strategies except EM and EH. On the other hand, the factor-based model closely follows the EM and FOF strategies but clearly underperforms other hedge fund strategies.

[Figure 6.1]

6.4. Conclusion

In principle, the ability to replicate hedge fund performance represents an attractive opportunity for investors to benefit from the high returns that hedge fund strategies are potentially able to offer, while avoiding the risks that such strategies involve. In practice, however, the effectiveness of existing hedge fund replication methods appears to be limited. Factor-based approaches work well in-sample, but are typically unable to maintain this performance out-of-sample. In contrast, payoff distribution matching approaches successfully replicate the unconditional distribution of hedge fund returns out-of-sample, but ignore the first moment of returns and hence are not able to deliver the absolute return performance associated with hedge fund strategies. In this chapter, an approach to hedge fund replication that combines the factor-based methodology with a series of risk and performance constraints is investigated. this approach is used to replicate the monthly returns of ten hedge fund strategy indices using long-only positions in a broad set of equity, interest rate, exchange rate and commodity indices, all of which can be traded using liquid, investible instruments such as futures, options and exchange traded funds. In out-of-sample tests, it is shown that the proposed composite approach yields replicating portfolios that are potentially able to mimic both the risk-adjusted performance and distributional characteristics of the hedge fund indices that they are designed to track. On balance, the proposed approach appears to represent an improvement over the out-of-sample performance of the factor and payoff distribution approaches reported by Jaeger and Wagner (2005) and Amenc *et al*. (2010).

Table 6.1
Summary Statistics and Time Series Properties of Hedge Fund Series

Panel A reports summary statistics in percentages for the replicated monthly Hedge Fund Research (HFRI) strategy indices over the period of June 1994 to January 2011. Panel B reports the autoregressive conditional heteroskedasticity (ARCH) and autocorrelation test results for the full period. The Ljung–Box-Q test for autocorrelation of order up to 10 asymptotically distributed as a central Chi-square with 10 d.o.f. Under the null hypothesis, with 5 percent critical value is 18.307. ARCH(4) is Engle's LM test for autoregressive conditional heteroskedasticity, which is asymptotically distributed as a central Chi-square with 4 d.o.f. Under the null hypothesis with 5 percent critical value is 9.488. *p*-values are also reported in the adjacent columns.

Panel A: Summary Statistics									
Index	Mean	Median	SD	Min.	Max.	Skew	E.Kurt.	JB Stats	pval
Convertible arbitrage	0.75	0.99	2.11	-16.01	9.74	-3.07	26.05	5971.0	0.00
Distressed securities	0.84	1.09	1.82	-8.50	5.55	-1.66	6.31	423.3	0.00
Event driven	0.92	1.27	2.00	-8.90	5.13	-1.39	4.58	238.6	0.00
Equity hedge	0.97	1.15	2.72	-9.46	10.88	-0.23	2.12	39.0	0.00
Emerging markets	0.91	1.51	4.14	-21.02	14.80	-1.00	4.29	187.0	0.00
Equity market neutral	0.51	0.51	0.92	-2.87	3.59	-0.11	1.49	19.0	0.00
Mergers arbitrage	0.70	0.84	1.08	-5.69	3.12	-1.71	6.31	428.8	0.00
Macro	0.81	0.64	1.86	-3.77	6.82	0.42	0.53	8.2	0.02
Relative value	0.73	0.83	1.27	-8.03	3.93	-3.08	17.03	2731.1	0.00
Fund of funds	0.53	0.74	1.76	-7.47	6.85	-0.75	4.05	155.2	0.00
Panel B: Basic Time Series Properties									
Index	ARCH	pval	LB-Q	pval	ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)
Convertible arbitrage	39.73	0.00	108.48	0.00	0.59	0.29	0.17	0.13	-0.03
Distressed securities	21.96	0.00	90.53	0.00	0.54	0.30	0.19	0.15	0.05
Event driven	5.12	0.28	42.05	0.00	0.39	0.17	0.12	0.09	0.04
Equity hedge	17.92	0.00	24.13	0.01	0.27	0.15	0.10	0.04	-0.06
Emerging markets	3.15	0.53	34.01	0.00	0.35	0.15	0.09	0.06	0.00
Equity market neutral	16.26	0.00	62.68	0.00	0.17	0.20	0.17	0.19	0.10
Mergers arbitrage	3.00	0.56	42.99	0.00	0.28	0.17	0.17	0.05	0.10
Macro	3.78	0.44	5.45	0.86	0.07	-0.04	-0.02	-0.01	0.04
Relative value	31.00	0.00	72.51	0.00	0.49	0.27	0.14	0.07	-0.03
Fund of funds	13.46	0.01	37.62	0.00	0.37	0.19	0.07	0.00	-0.05

Table 6.2
List of Assets Used in Replicating Portfolio Construction

The table lists stock, fixed income, commodity and foreign exchange rate assets to be used in constructing replicating portfolio. All indices are total return index and there are tradable highly liquid instruments on these indices (i.e. futures, ETFs, etc). All assets are traded in US Dollar.

Ticker	Asset
	Equity Indices
MSIEMF	MSCI Emerging Markets :Investable TR Index
WILDJMI	DJ US Micro Cap. Total Stock Market TR Index
WILDJMG	DJ US Medium Cap. Growth Total Stock Market TR Index
WILDJMV	DJ US Medium Cap. Value Total Stock Market TR Index
WILDJSV	DJ US Small Cap. Value Total Stock Market TR Index
	Foreign Exchange Futures
ICDCS	CME-Canadian Dollar Cont. Settlement Price
	Bonds Index
LHTBW3M	BARCLAYS US Treasury Bellwethers 3M
	Commodity Indices
GSCI	S&P GSCI Commodity TR Index
GSEN	S&P GSCI Energy TR Index
GSPM	S&P GSCI Precious Metal TR Index

Table 6.3a
Out-of-Sample Evaluation Criteria of Monthly Rebalancing Hedge Fund Return Replicating Portfolios

The table reports evaluation criteria for the out-of-sample monthly rebalancing replicating portfolios of HFRI indices in the period October 2002 to January 2011 (100 months). Evaluation criteria include regression results (i.e. beta, t statistics (tstat) of beta coefficient and adjusted R square (adjR2)), first four moments (i.e. annualized average return (AR), annualized standard deviation (SD), skewness (Skew), kurtosis (Kurt)), Sharp Ratio (SR) and risk measures (i.e. maximum drawdown (MDD), annualized conditional value at risk (CVaR) and annual conditional drawdown at risk (CDaR)). CVaR and CDaR statistics are estimated at 99 percent confidence level. Values of evaluation criteria of each replicated hedge fund index are also given to facilitate comparison to model results.

Contraint	Beta	tstat	adjR^2	AR	SD	Skew	Kurt	SR	MDD	CVaR	CDaR
Panel 1: Convertible arbitrage				0.07	0.19	-1.44	11.42	0.07	0.73	1.16	0.05
CVaRC	0.75	6.86	31.75%	0.12	0.14	-0.82	5.64	0.19	0.51	0.61	0.04
CDaRC	0.85	6.36	28.49%	0.12	0.12	-0.07	3.28	0.23	0.30	0.34	0.03
PMC	0.58	7.15	33.64%	0.14	0.19	-0.95	5.82	0.18	0.76	0.79	0.05
ALLC	0.57	4.91	18.95%	0.05	0.15	-0.38	5.02	0.06	0.63	0.44	0.05
RC	0.72	4.20	14.39%	0.11	0.10	-0.02	3.64	0.25	0.23	0.31	0.02
FM	1.11	5.70	24.12%	0.05	0.09	-2.09	15.61	0.09	0.31	0.56	0.03
Panel 2: Distressed securities				0.11	0.12	-0.87	4.99	0.21	0.39	0.49	0.03
CVaRC	0.60	9.61	48.01%	0.11	0.14	-1.73	11.43	0.18	0.47	0.74	0.04
CDaRC	0.58	8.67	42.82%	0.12	0.13	-1.28	9.03	0.22	0.38	0.67	0.03
PMC	0.45	11.71	57.89%	0.12	0.20	-1.37	8.81	0.15	0.79	1.01	0.05
ALLC	0.56	7.98	38.76%	0.09	0.13	-0.76	5.21	0.16	0.42	0.53	0.04
RC	0.61	9.81	49.03%	0.11	0.13	-1.53	10.60	0.19	0.43	0.71	0.04
FM	0.90	9.84	49.18%	0.06	0.09	-1.32	8.46	0.12	0.40	0.45	0.03
Panel 3: Event driven				0.10	0.10	-1.14	5.30	0.24	0.31	0.46	0.03
CVaRC	0.58	14.22	67.01%	0.11	0.14	-1.67	10.38	0.19	0.52	0.71	0.04
CDaRC	0.60	13.78	65.60%	0.10	0.13	-1.10	6.88	0.19	0.37	0.60	0.03
PMC	0.35	10.77	53.74%	0.12	0.20	-1.25	7.00	0.14	0.79	0.95	0.05
ALLC	0.62	12.00	59.11%	0.09	0.12	-1.07	7.13	0.17	0.38	0.54	0.03
RC	0.51	11.08	55.13%	0.11	0.14	-1.00	7.35	0.19	0.47	0.66	0.04
FM	0.84	16.37	72.94%	0.06	0.10	-1.60	9.57	0.11	0.44	0.52	0.04
Panel 4: Equity hedge				0.08	0.11	-0.84	4.29	0.15	0.39	0.46	0.03
CVaRC	0.68	15.77	71.44%	0.09	0.14	-1.33	7.63	0.14	0.54	0.66	0.04
CDaRC	0.67	15.11	69.65%	0.10	0.14	-0.74	5.10	0.17	0.40	0.56	0.03
PMC	0.39	11.84	58.45%	0.10	0.22	-1.05	5.97	0.11	0.89	1.06	0.06
ALLC	0.74	15.37	70.39%	0.09	0.13	-1.11	6.18	0.17	0.46	0.59	0.04
RC	0.39	7.83	37.83%	0.08	0.18	-0.59	7.45	0.10	0.47	0.74	0.04
FM	0.92	19.38	79.10%	0.06	0.11	-2.09	13.45	0.12	0.46	0.66	0.04
Panel 5: Emerging markets				0.15	0.17	-1.00	4.55	0.22	0.56	0.69	0.04
CVaRC	0.60	11.79	58.24%	0.13	0.22	-1.73	11.11	0.14	0.90	1.29	0.06
CDaRC	0.80	12.67	61.71%	0.16	0.17	-1.45	12.10	0.25	0.45	0.89	0.04
PMC	0.43	9.39	46.84%	0.05	0.27	-1.68	9.83	0.03	1.57	1.69	0.07
ALLC	0.73	10.47	52.33%	0.11	0.17	-1.13	8.17	0.16	0.60	0.84	0.05
RC	0.74	15.94	71.87%	0.17	0.19	-1.20	8.19	0.23	0.60	0.92	0.05
FM	0.88	23.44	84.71%	0.16	0.17	-1.45	7.86	0.23	0.70	0.88	0.05

Table 6.3b
Out-of-Sample Evaluation Criteria of Monthly Rebalancing Hedge Fund Return Replicating Portfolios

The table reports evaluation criteria for the out-of-sample monthly rebalancing replicating portfolios of HFRI indices in the period October 2002 to January 2011 (100 months). Evaluation criteria include regression results (i.e. beta, t statistics (tstat) of beta coefficient and adjusted R square (adjR2)), first four moments (i.e. annualized average return (AR), annualized standard deviation (SD), skewness (Skew), kurtosis (Kurt)), Sharp Ratio (SR) and risk measures (i.e. maximum drawdown (MDD), annualized conditional value at risk (CVaR) and annual conditional drawdown at risk (CDaR)). CVaR and CDaR statistics are estimated at 99 percent confidence level. Values of evaluation criteria of each replicated hedge fund index are also given to facilitate comparison to model results.

Contraint	Beta	tstat	adjR^2	AR	SD	Skew	Kurt	SR	MDD	CVaR	CDaR
Panel 6: Equity market neutral				0.03	0.03	-1.15	5.28	0.07	0.10	0.15	0.01
CVaRC	0.14	3.52	10.30%	0.06	0.07	-1.33	10.43	0.15	0.18	0.39	0.02
CDaRC	0.10	2.42	4.68%	0.08	0.08	-0.28	3.75	0.23	0.12	0.24	0.01
PMC	0.10	4.70	17.54%	0.10	0.13	-1.79	10.88	0.16	0.57	0.73	0.04
ALLC	0.13	2.64	5.70%	0.03	0.06	-0.83	5.91	0.05	0.18	0.27	0.02
RC	0.13	3.00	7.48%	0.07	0.07	-1.09	8.43	0.19	0.16	0.33	0.02
FM	0.19	1.32	0.74%	0.00	0.02	-3.07	22.95	-0.30	0.09	0.15	0.01
Panel 7: Mergers arbitrage				0.06	0.05	-0.82	4.37	0.26	0.10	0.14	0.01
CVaRC	0.34	8.29	40.63%	0.07	0.09	-1.53	10.15	0.18	0.22	0.44	0.02
CDaRC	0.30	6.76	31.12%	0.08	0.09	-0.69	5.86	0.21	0.17	0.36	0.02
PMC	0.19	9.04	44.91%	0.10	0.16	-1.60	8.58	0.15	0.68	0.82	0.05
ALLC	0.37	7.46	35.57%	0.05	0.07	-0.91	6.12	0.12	0.20	0.31	0.02
RC	0.36	8.73	43.16%	0.08	0.08	-1.02	7.54	0.21	0.20	0.39	0.02
FM	0.62	8.29	40.62%	0.02	0.05	-2.21	13.74	-0.02	0.20	0.31	0.02
Panel 8: Macro				0.08	0.06	0.26	3.16	0.30	0.05	0.18	0.01
CVaRC	0.28	6.23	27.63%	0.08	0.11	-1.82	11.67	0.16	0.31	0.61	0.03
CDaRC	0.21	4.52	16.43%	0.12	0.12	-0.27	5.20	0.25	0.18	0.43	0.02
PMC	0.18	6.17	27.27%	0.11	0.17	-2.03	12.48	0.15	0.59	0.96	0.05
ALLC	0.31	5.30	21.52%	0.08	0.09	-1.09	7.06	0.18	0.19	0.41	0.02
RC	0.29	6.61	30.12%	0.11	0.11	-1.43	10.16	0.23	0.28	0.58	0.03
FM	0.60	5.96	25.84%	0.04	0.05	-2.29	16.57	0.09	0.16	0.30	0.02
Panel 9: Relative value				0.07	0.08	-2.20	11.97	0.18	0.28	0.58	0.03
CVaRC	0.54	8.78	43.43%	0.09	0.10	-1.55	10.09	0.20	0.31	0.53	0.03
CDaRC	0.41	5.78	24.69%	0.10	0.10	-0.67	4.64	0.22	0.26	0.40	0.02
PMC	0.37	10.58	52.85%	0.12	0.16	-1.37	7.52	0.18	0.68	0.79	0.05
ALLC	0.46	6.46	29.17%	0.07	0.10	-0.80	4.84	0.14	0.36	0.38	0.03
RC	0.56	8.51	41.92%	0.09	0.10	-1.31	9.25	0.21	0.27	0.48	0.03
FM	0.97	8.69	42.93%	0.04	0.06	-1.87	11.85	0.08	0.23	0.33	0.02
Panel 10: Fund of funds				0.05	0.08	-1.22	5.52	0.10	0.28	0.40	0.03
CVaRC	0.59	12.40	60.67%	0.08	0.11	-1.76	10.87	0.15	0.43	0.58	0.04
CDaRC	0.62	9.86	49.29%	0.08	0.09	-0.96	7.19	0.19	0.27	0.42	0.03
PMC	0.39	12.88	62.48%	0.12	0.16	-1.64	9.37	0.17	0.69	0.92	0.05
ALLC	0.47	7.71	37.14%	0.06	0.11	-0.93	4.61	0.11	0.40	0.46	0.03
RC	0.73	12.00	59.09%	0.07	0.09	-1.37	9.37	0.16	0.30	0.44	0.03
FM	0.86	11.68	57.77%	0.04	0.07	-2.54	16.84	0.10	0.32	0.46	0.03

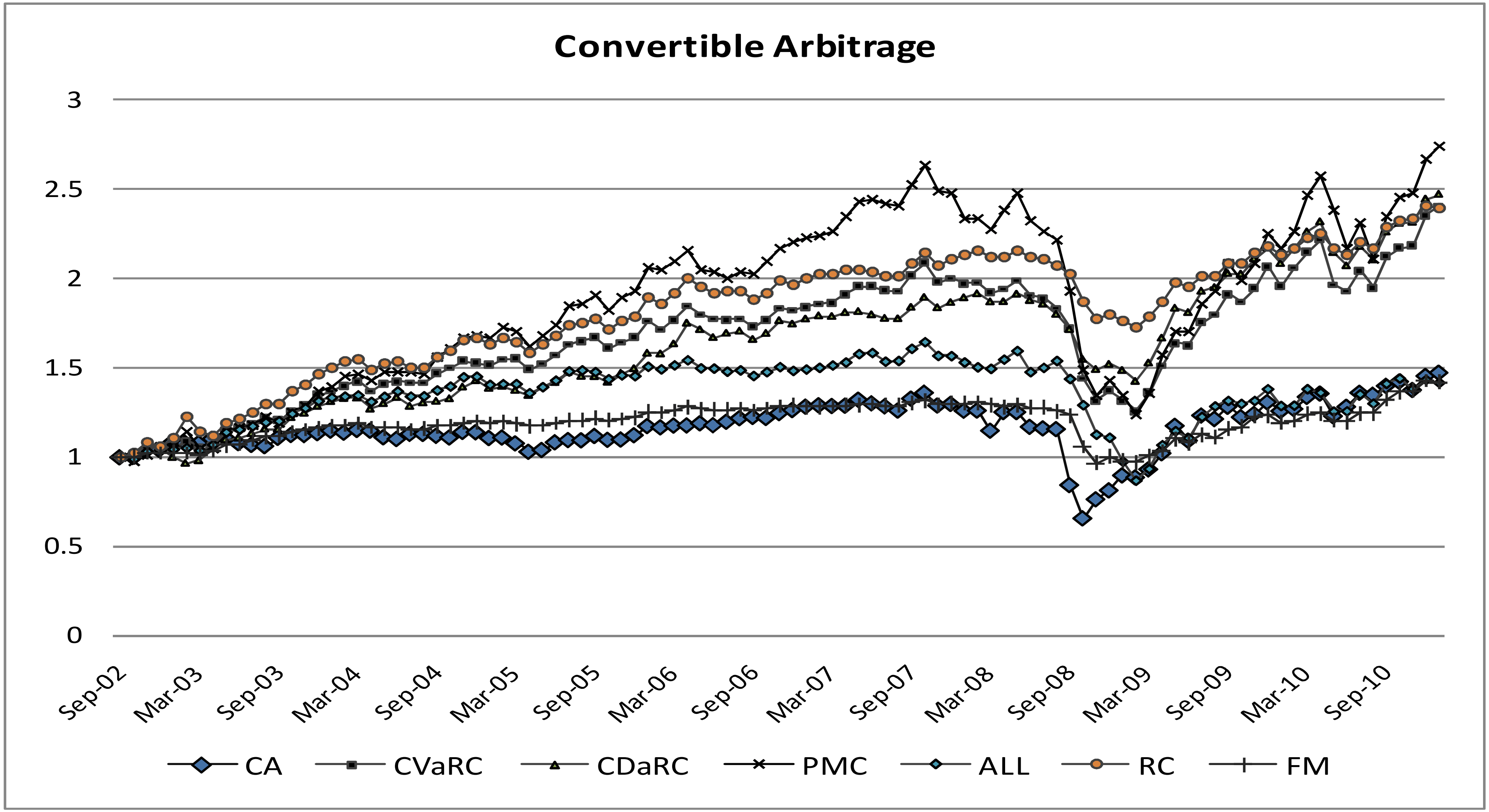


Figure 6.1. Net Asset Values of Replicating Model Portfolios and Replicated Convertible Arbitrage Strategy Index. This figure displays net asset values of replicating model portfolios in comparison to replicated Convertible Arbitrage hedge fund strategy index during the out-of-sample period October 2002 to January 2011 (100 months).

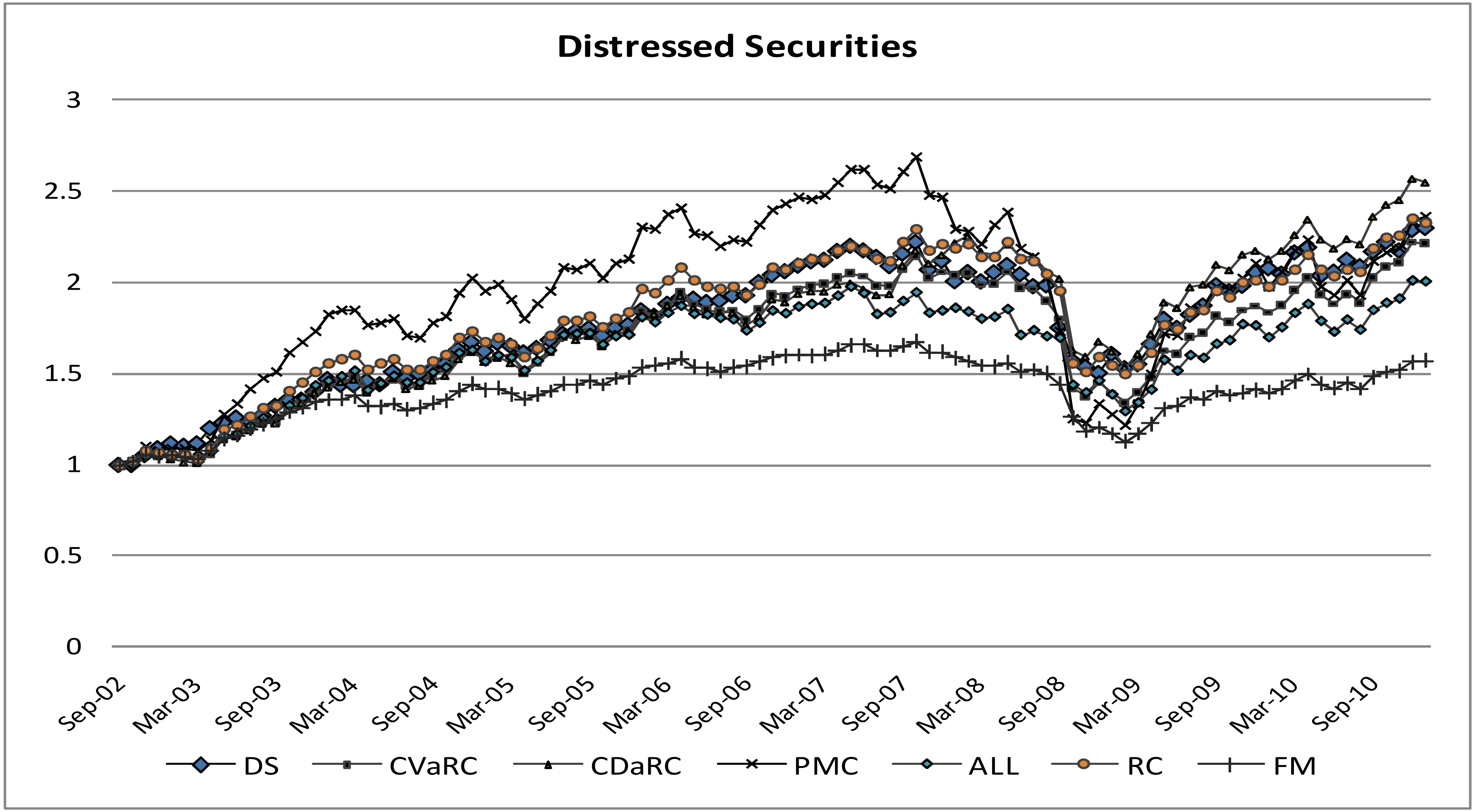


Figure 6.2. Net Asset Values of Replicating Model Portfolios and Replicated Distressed Securities Index. This figure displays net asset values of replicating model portfolios in comparison to replicated Distressed Securities hedge fund strategy index during the out-of-sample period October 2002 to January 2011 (100 months).

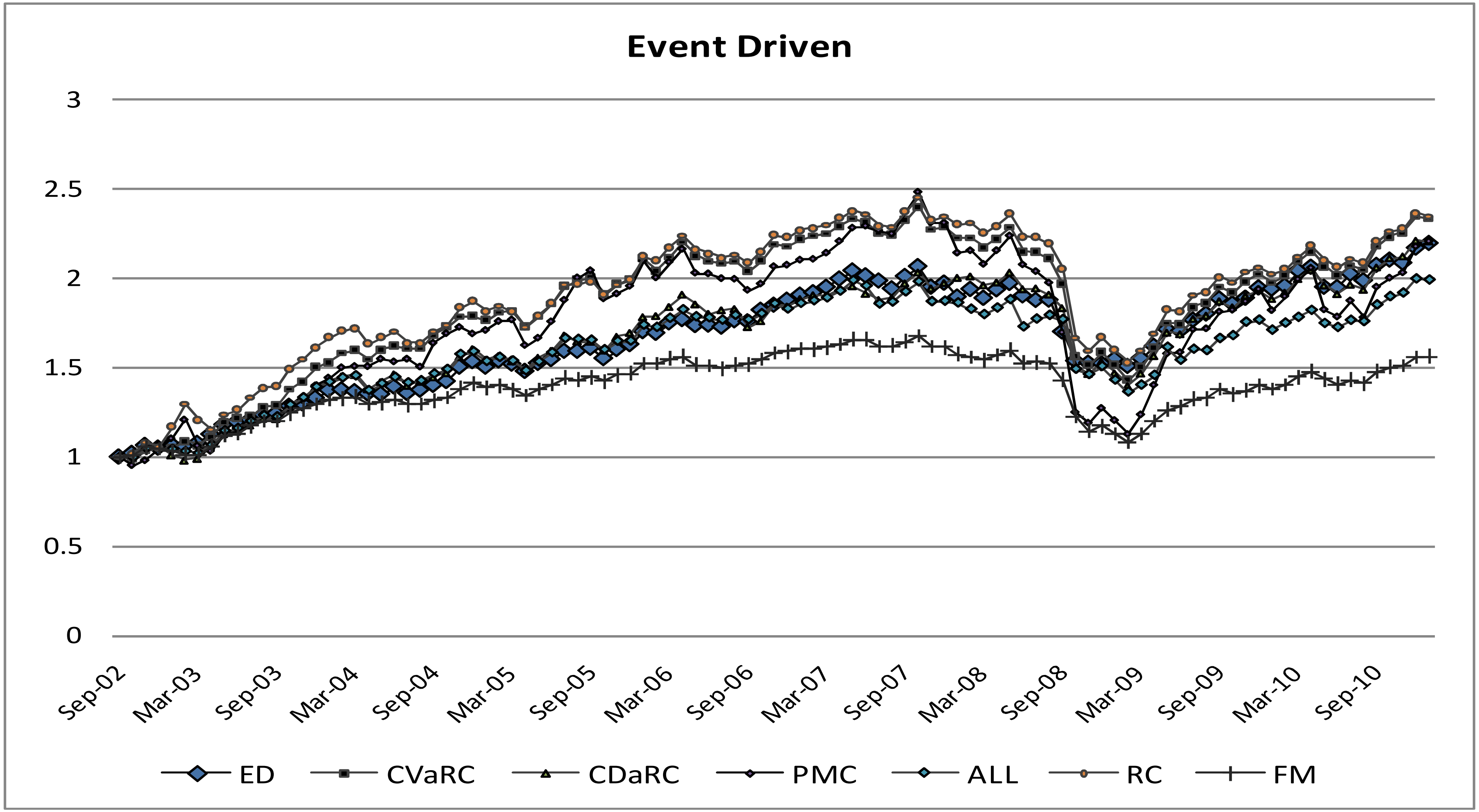


Figure 6.3. Net Asset Values of Replicating Model Portfolios and Replicated Event Driven Strategy Index. This figure displays net asset values of replicating model portfolios in comparison to replicated Event Driven hedge fund strategy index during the out-of-sample period October 2002 to January 2011 (100 months).

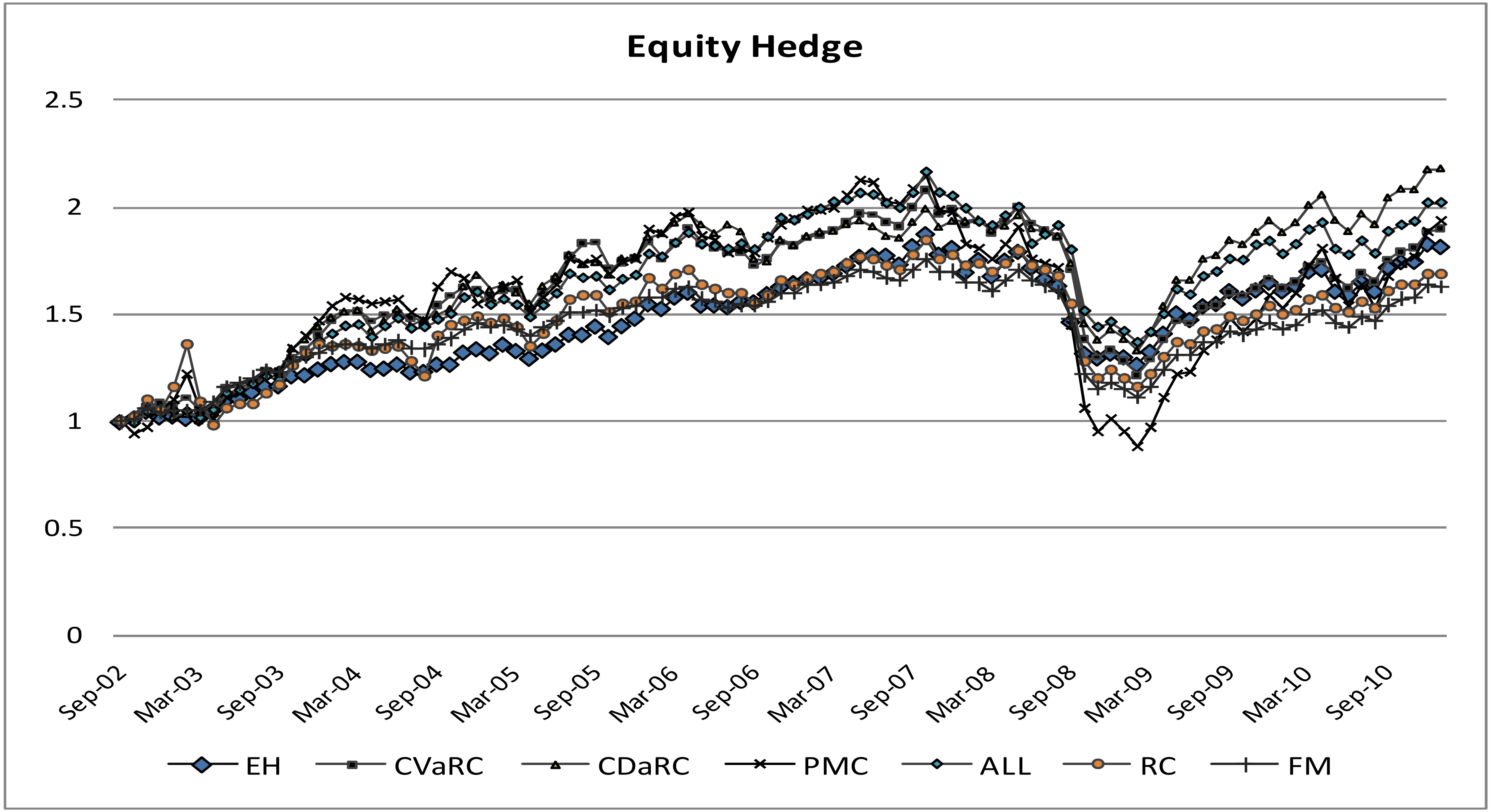


Figure 6.4. Net Asset Values of Replicating Model Portfolios and Replicated Equity Hedge Strategy Index. This figure displays net asset values of replicating model portfolios in comparison to replicated Equity Hedge hedge fund strategy index during the out-of-sample period October 2002 to January 2011 (100 months).

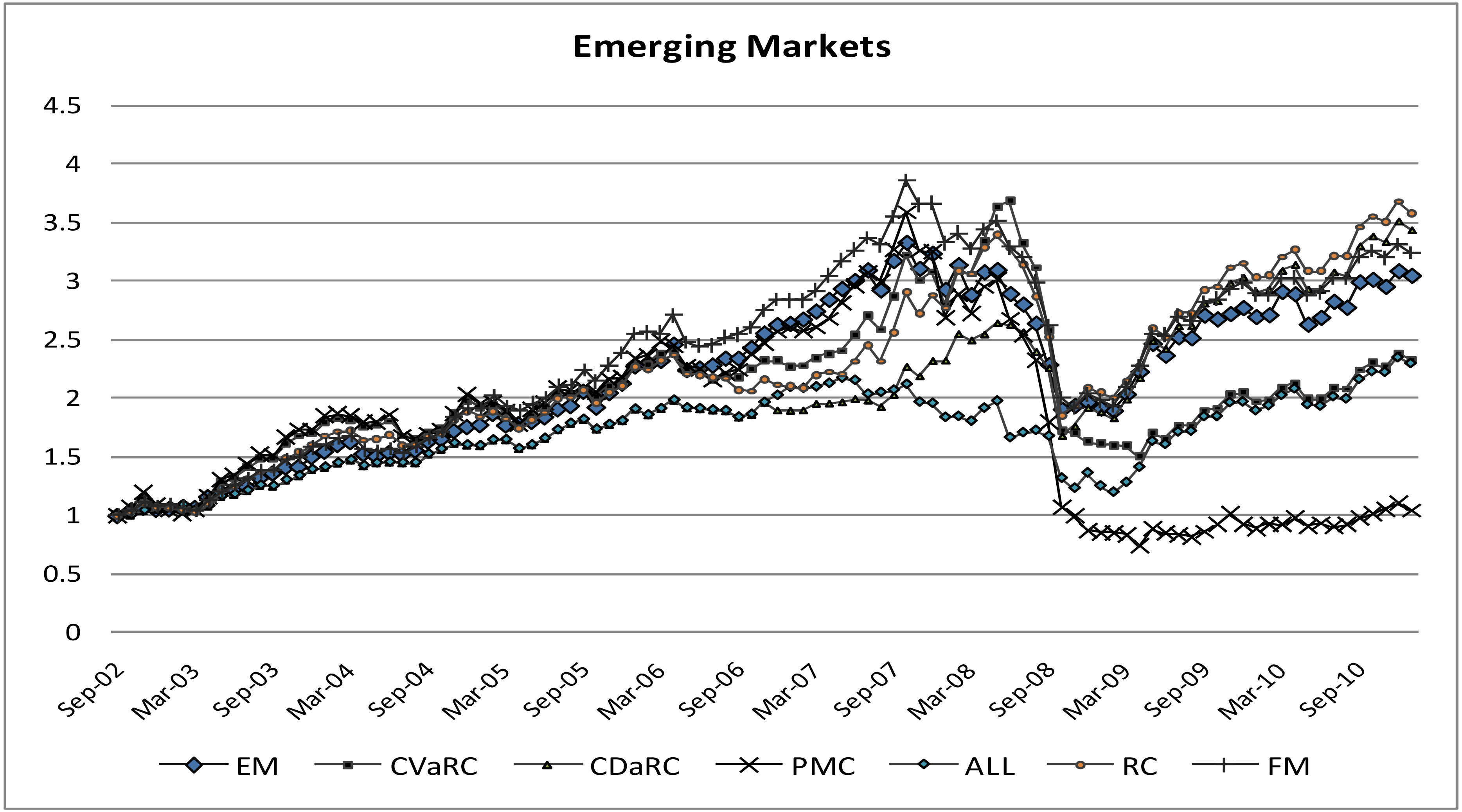


Figure 6.5. Net Asset Values of Replicating Model Portfolios and Replicated Emerging Markets Strategy Index. This figure displays net asset values of replicating model portfolios in comparison to replicated Emerging Markets hedge fund strategy index during the out-of-sample period October 2002 to January 2011 (100 months).

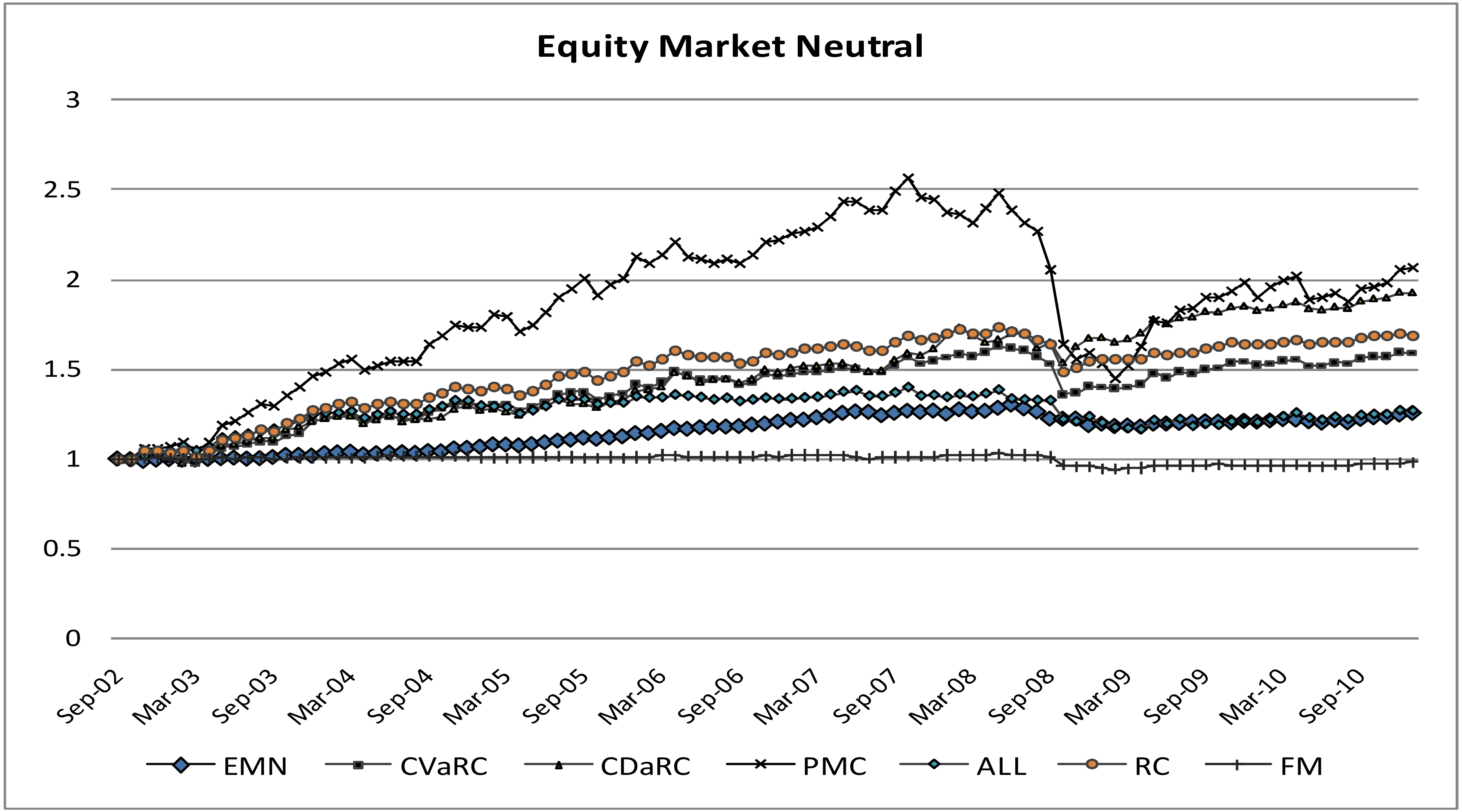


Figure 6.6. Net Asset Values of Replicating Model Portfolios and Replicated Equity Market Neutral Strategy Index. This figure displays net asset values of replicating model portfolios in comparison to replicated Equity Market Neutral hedge fund strategy index during the out-of-sample period October 2002 to January 2011 (100 months).

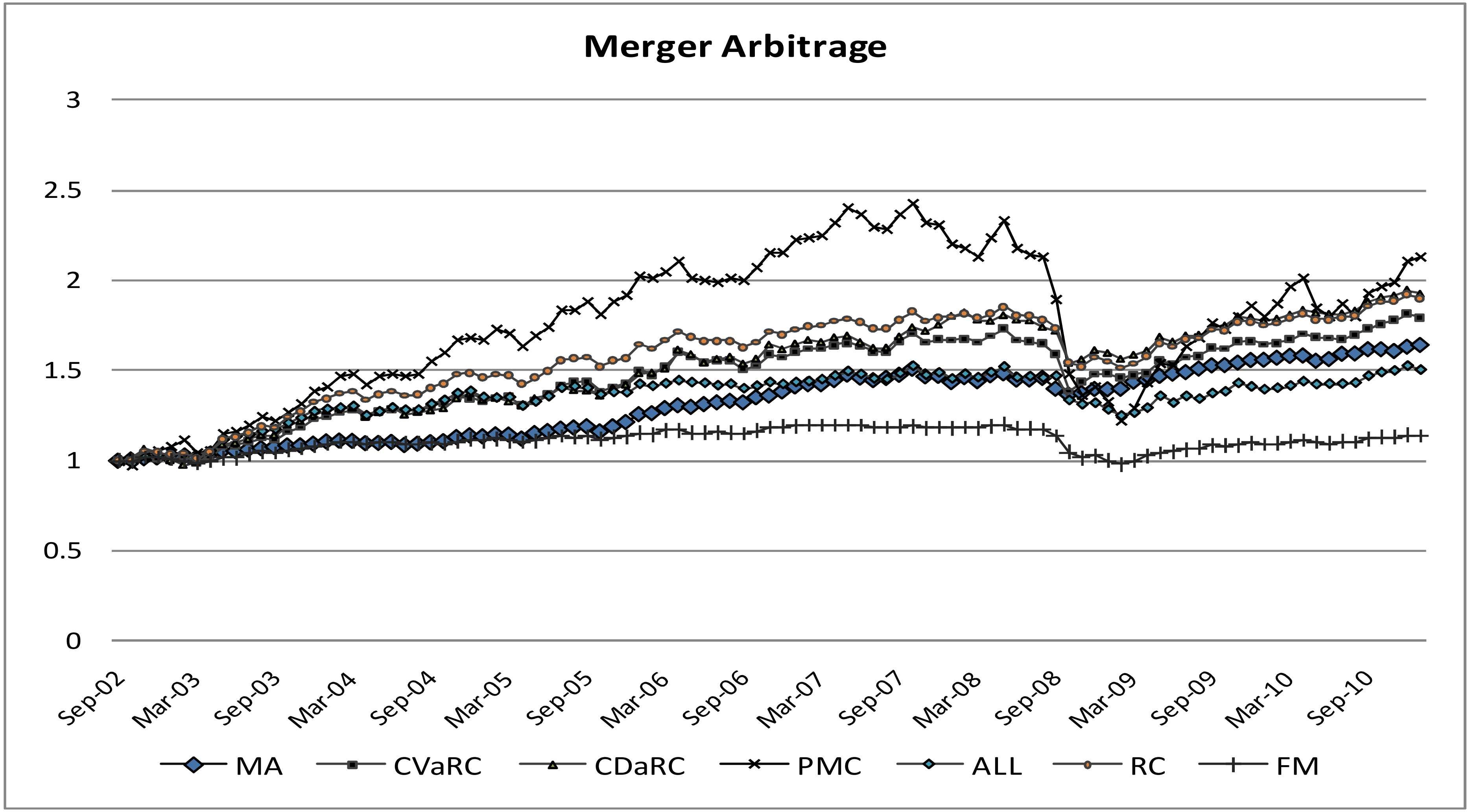


Figure 6.7. Net Asset Values of Replicating Model Portfolios and Replicated Merger Arbitrage Strategy Index. This figure displays net asset values of replicating model portfolios in comparison to replicated Merger Arbitrage hedge fund strategy index during the out-of-sample period October 2002 to January 2011 (100 months).

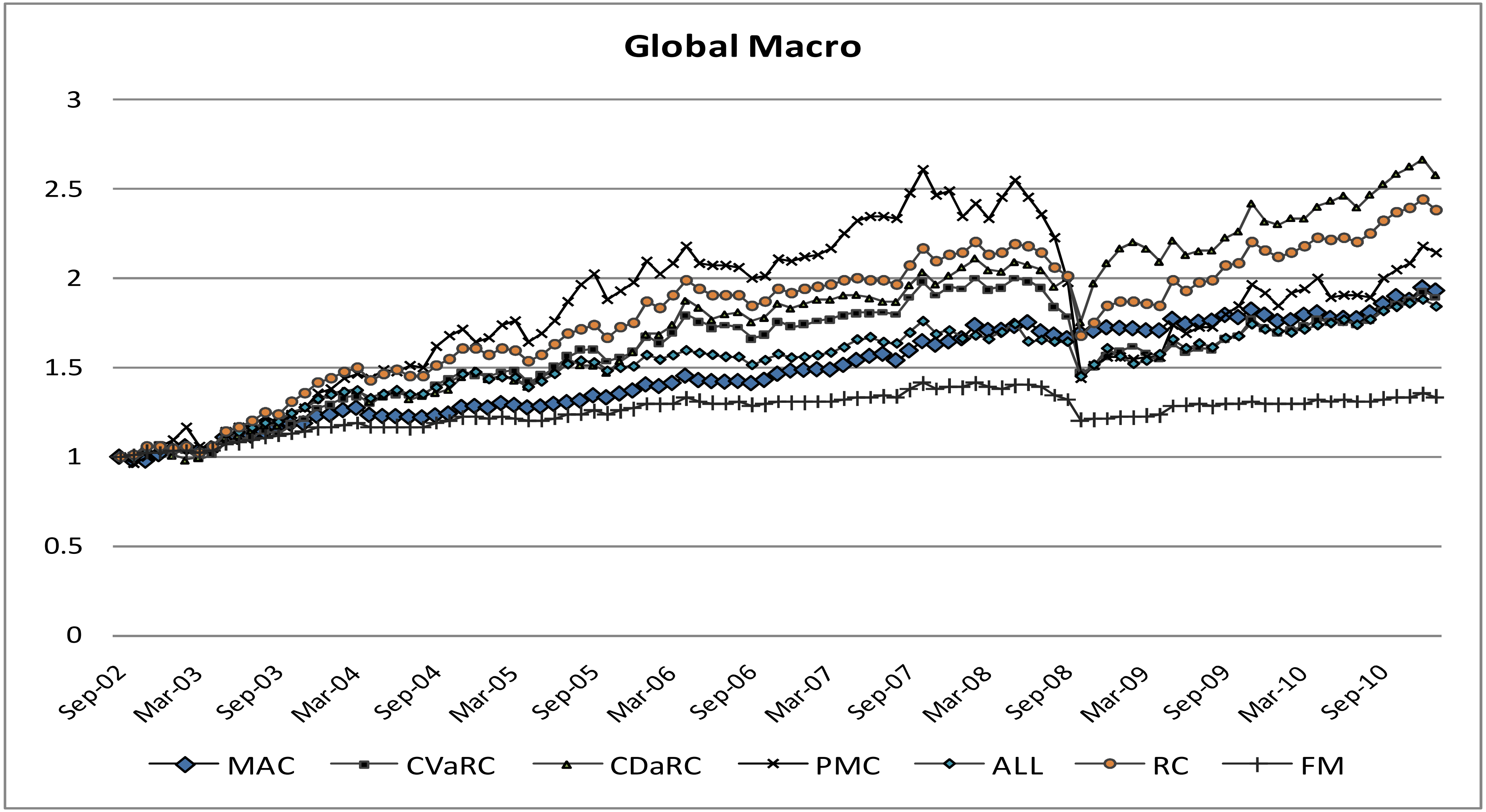


Figure 6.8. Net Asset Values of Replicating Model Portfolios and Replicated Global Macro Strategy Index. This figure displays net asset values of replicating model portfolios in comparison to replicated Global Macro hedge fund strategy index during the out-of-sample period October 2002 to January 2011 (100 months).

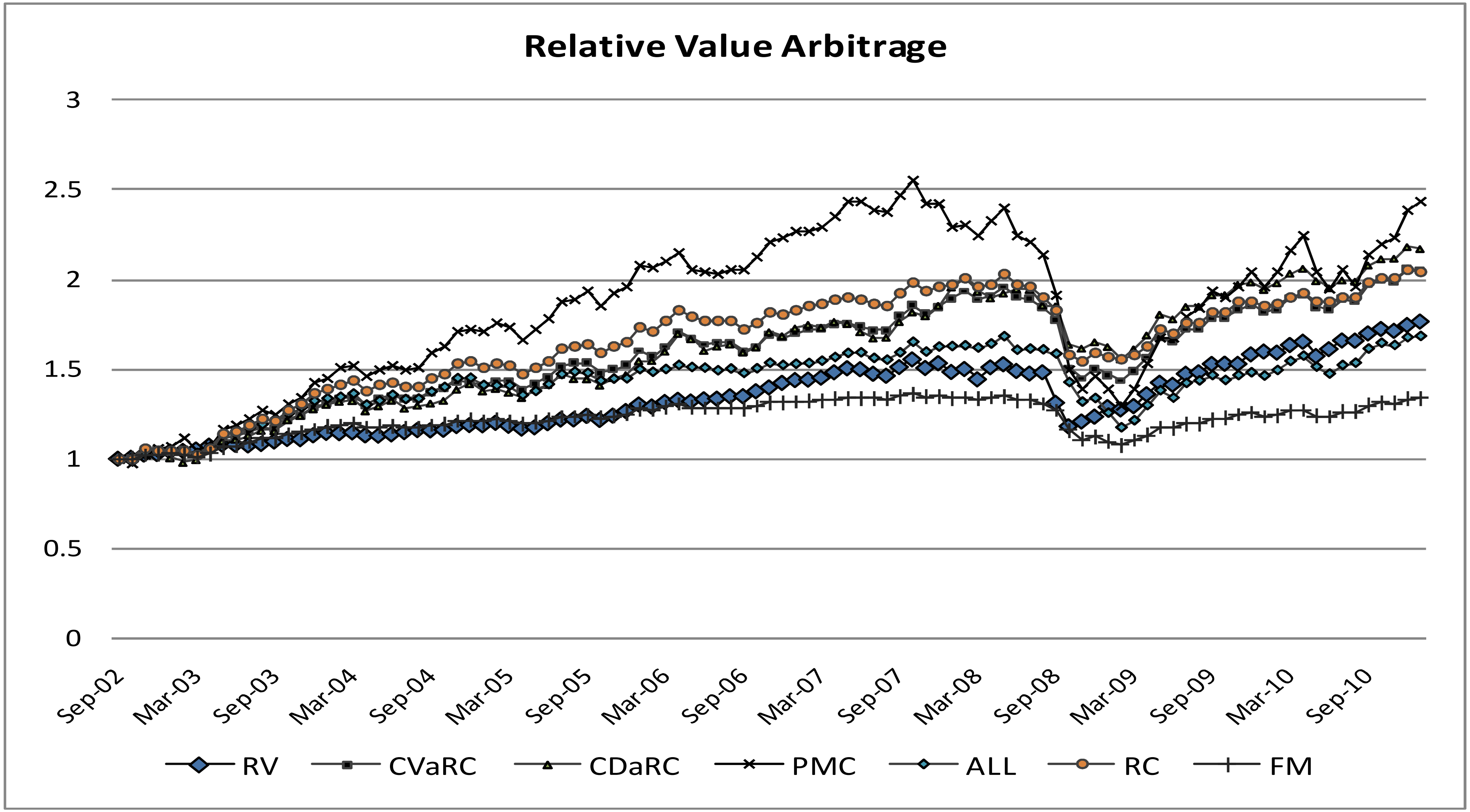


Figure 6.9. Net Asset Values of Replicating Model Portfolios and Replicated Relative Value Arbitrage Strategy Index. This figure displays net asset values of replicating model portfolios in comparison to replicated Relative Value Arbitrage hedge fund strategy index during the out-of-sample period October 2002 to January 2011 (100 months).

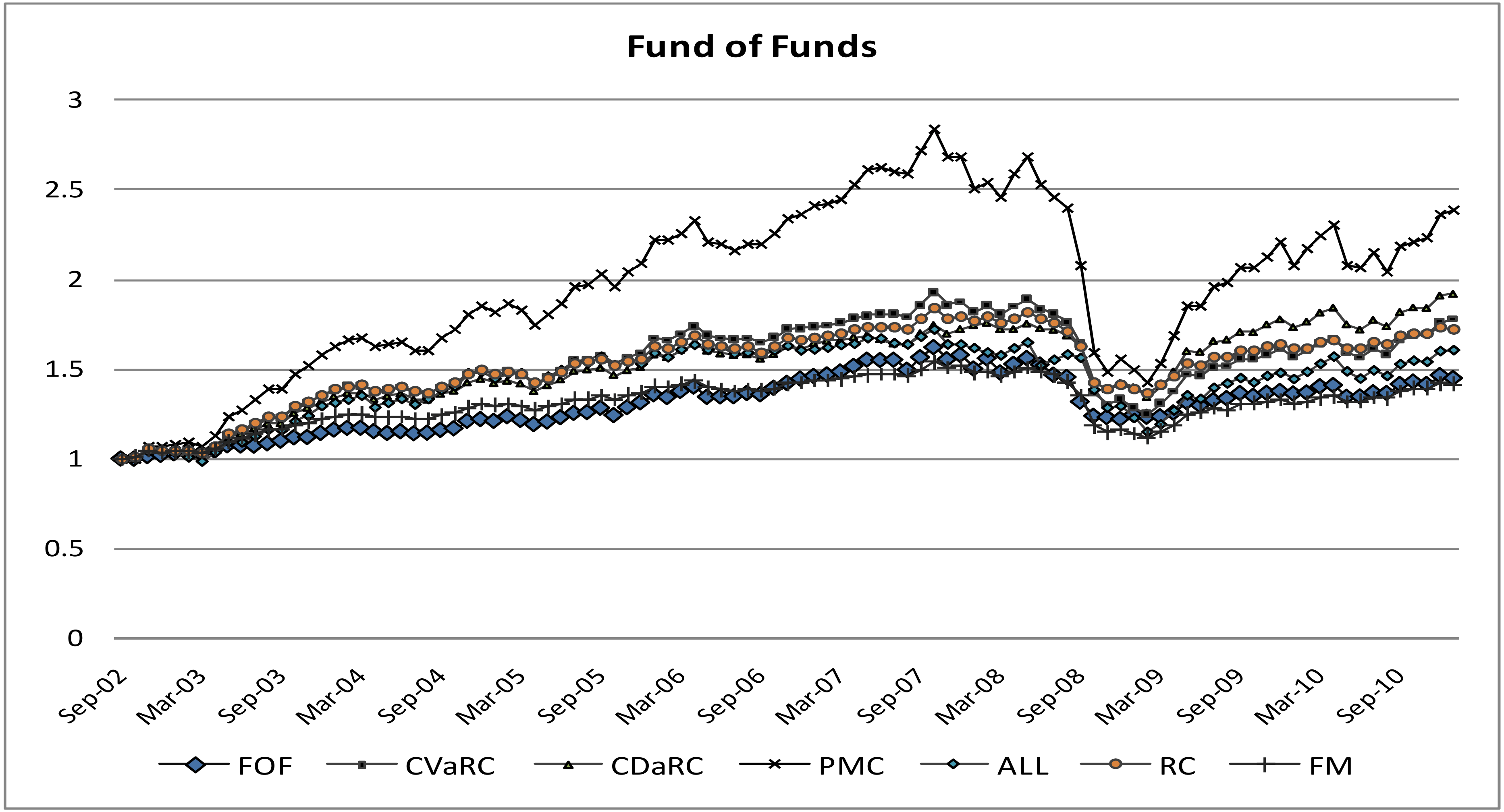


Figure 6.10. Net Asset Values of Replicating Model Portfolios and Replicated Fund of Hedge Funds Strategy Index. This figure displays net asset values of replicating model portfolios in comparison to replicated Fund of Hedge Funds strategy index during the out-of-sample period October 2002 to January 2011 (100 months).

Chapter 7

Conclusions

7.1. Conclusions

The research presented in this thesis had three overall aims: first, to examine portfolio construction and risk measurement performance of a broad set of volatility forecast and portfolio optimization models, second, in an effort to improve their forecast accuracy and portfolio construction performance, to propose new models or new formulations to these models, third, to introduce a (hedge fund) return replication approach that has the potential to be used in numerous applications in investment management. In achieving these overall aims and the aims specific to each chapter, set out in the first chapter, the following conclusions were reached.

In the second chapter, well-known multivariate volatility models were empirically examined, and further evidence on the use of these models in the dynamic measurement of hedge fund risk and portfolio allocation were provided. In particular, it is found that multivariate GARCH models provide significant improvement over static models in dynamic hedge fund portfolio construction and risk measurement. However, these models are generally dominated by the EWMA model. In addition to providing better risk-adjusted performance, the EWMA model leads to dynamic allocation strategies that have substantially lower turnover and could, therefore, be expected to involve lower transaction costs. Moreover, it was shown that these results are robust across the low - volatility and high-volatility sub-periods. Therefore, empirical evidence was provided for that highly complicated models may not necessarily be the best performing models.

In the third chapter, the potential advantages of employing alternative optimization frameworks to the mean-variance framework are investigated. First, portfolio construction performance of non-parametrically estimated CVaR, CDaR and Omega models are compared with the benchmark models and the benchmark index. Second, possible areas of improvement in the performance of the alternative optimization models are searched for. In so doing, a semi-parametric estimation for CVaR, CDaR and Omega models is proposed. The analyses presented in this chapter report two main findings. The first finding is that CVaR-, CDaR- and Omega-based optimization offers a significant improvement in terms of risk-adjusted portfolio performance over the mean-variance optimization. The second finding is that for all three risk measures, semi-parametric estimation of the optimal portfolio offers a very significant improvement over non-parametric estimation. In particular, the optimal portfolios, estimated by using the proposed semi-parametric approach, display significantly higher portfolio return, lower risk, higher risk-adjusted return, better upside potential and higher active management ratios, at the expense of higher portfolio turnovers. Another remarkable finding is the demonstrated success of the semi-parametrically estimated models in predicting the market upturn and downturns. These models exhibit superior performance during the rising markets as well as during to, and recovery from the crisis. It is also examined if certain parameter preferences, such as risk tolerance limits, target return, size of the estimation period, have any effects on the main findings of the previous section. It has been found that the main results are robust to as the choice of target return and the estimation period.

In the fourth chapter, in order to improve one's ability to measure risks dynamically, two univariate volatility models, which combine useful properties of range, regime switching, nonlinear filtration and GARCH frameworks, were proposed, and their in-sample fit and out-of-sample forecast performance were tested against eminent return and range-based models. Chapter 4 concluded that the proposed models produce more accurate out of sample forecasts, contain more information about true volatility and exhibit similar or better performance when used for value at risk comparison. In particular, two-component model performs better than all other models.

In the fifth chapter, the univariate models proposed in the fourth chapter were extended to their multivariate setting and tested for their out-of-sample risk forecast and portfolio construction performance in comparison to eminent return and range-based benchmark models. With the proposed extensions to the multivariate setting, the univariate models introduced in the fourth chapter were able to be used in improving the portfolio risk measurement and construction performance. Fifth chapter concluded that, in terms statistical test results, proposed models offer significant improvements in forecasting true volatility process, and, in terms of risk and return criteria employed, proposed models perform better than benchmark models. Proposed models construct hedge fund portfolios with higher risk-adjusted returns, lower tail risks, and they offer superior risk-return tradeoffs and better active management ratios. However, in most cases these improvements come at the expense of higher portfolio turnover and rebalancing expenses.

In the sixth chapter, a composite hedge fund return replication approach was proposed, and proofs of improvements in the replication performance of clone portfolios were provided in an empirical study. Chapter 6 concluded that proposed composite approach yields replicating portfolios that better mimic both the risk-adjusted performance and distributional characteristics of the hedge fund indices that they are designed to track in out-of-sample evaluation. On balance, proposed approach appears to represent an improvement over the out-of-sample performance of the factor and payoff distribution approaches reported by Jaeger and Wagner (2005) and Amenc *et al*. (2010).

7.2. Suggestions for Further Research

Volatility forecast models based on range offers a fruitful further research area. Applying the range data on asymmetric GARCH models or long memory models (e.g. FIGARCH) would be a possible route for further research in the area. Another significantly important area would be developing the multivariate extension of the univariate asymmetric and long memory range-based models. Also application of range based models to different frequencies of range (e.g. daily, hourly, minute data) and searching for possible market microstructure effects would be another area of further research. Moreover, application to different asset classes (fixed income, foreign currency, derivatives) or economic time series might also be considered.

There is significant need for further research in alternative portfolio optimization frameworks. A possible route would be full parametric estimation methods of these frameworks. Another possible route would be implementing the re-sampling techniques (e.g. bootstrapping) in an effort to improve the performance of non-parametric frameworks.

Hedge fund return replication area is quite a new area and offers possible further research in many directions. For example, developing factors for hedge fund returns similar to Fama-French factors, or considering jump-diffusion models in estimating distribution matching approach would be considered for further research.

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