

Two-subband system in quantizing magnetic field: probing many-body gap by non-equilibrium phonons

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We study the many-body effects in a two-subband quasi-two-dimensional electron system in a quantizing magnetic field at filling factor three. A manifestation of these effects in the phonon absorption spectroscopy is discussed. The electron system is mapped onto a two-level system with the separation between levels determined by the intersubband splitting and the cyclotron energy. The electron-electron interaction enhances the excitation gap, which exists at all values of the interlevel splitting. This results in a single-peak structure of the phonon absorption rate as a function of magnetic field, instead of the double-peak structure for non-interacting electrons.

I. INTRODUCTION

The behavior of a two-dimensional (2D) electron system in a quantizing magnetic field is completely determined by the interaction part of the Hamiltonian. This results in several interesting phenomena in such a system, for example, including the Fractional and Integer Quantum Hall Effects.¹ A wide range of different experimental techniques has been used to study this system and its excitations. One of these methods is the acoustic-phonon spectroscopy.²⁻⁵ The specific feature of the electron-phonon interaction in the 2D system in a magnetic field is a strong suppression of phonon induced transitions with the phonon momentum larger than the inverse magnetic length, $q_0 \approx 1/l$.⁶ Due to the energy conservation the single-phonon transitions are only possible if $E(q_0) \approx \hbar s q_0$, where s is the speed of sound and $E(q)$ is the dispersion of the neutral excitations of the electron system. By changing $E(q_0)$ one can switch off or on acoustic phonon effects in the magnetically-quantized 2D system.

At certain conditions a sharp magnetic field dependence of the electron-phonon interaction can be achieved in a two-subband system. When the energy splitting between the first and the second subbands becomes close to the cyclotron energy the second Landau level of the first subband has almost the same energy as the first Landau level of the second subband. An interesting feature of this system is that the separation between these close Landau levels, Δ , can be changed by changing the magnetic field. Under these conditions if there are electrons in the second Landau level of the first subband, i.e. the electron filling factor is greater than two, the phonon-induced effects become strong when $\Delta \approx \hbar s/l$. This results in a double-peak structure of the dissipative electron conductivity as a function of Δ or as a function of magnetic field.⁷ The increase of the electron density in this system increases the role of the electron-electron interaction which results in the renormalization of the excitation gap Δ and has a considerable effect on the electron-phonon coupling. In what follows we analyze the case when the total electron filling factor is equal to three. We show that in this case the excitation spectrum has a gap at all values of interlevel splitting Δ . As a result the acoustic-phonon absorption of the two-subband system has a single peak as a function of the magnetic field. By lowering the electron density one should observe the transformation of a single-peak dependence into a double-peak one, which corresponds to a non-interacting case.

We consider below the quasi-2D electron system confined in a single heterojunction. The effects related to the intersection of different Landau levels belonging to different subbands were studied both theoretically and experimentally for double quantum well structures.⁸ Unlike the double-well case the intersection of levels in the two-subband single heterojunction occurs at much higher magnetic field, which makes all field-dependent many-body effects more pronounced.

II. GROUND STATE AND EXCITATIONS

At filling factor $\nu_0 = 3$ the electrons completely occupy the lowest Landau level (with both spin directions) of the first subband. The only effect of the electrons in this level on the electrons in the higher levels is the exchange renormalization of the interlevel splitting Δ . Because this renormalization does not depend on the momentum of the electron in the next levels in what follows we do not take the background (completely occupied) level into account and consider Δ as the value, which already includes the renormalization due to the background electrons. As a result the

system becomes the two-level electron system with the effective filling factor $\nu = 1$ and the splitting between levels Δ , see Fig.1. These levels are the second Landau level ($n = 1$) of the first subband and the first Landau level ($n = 0$) of the second subband. Both levels have the same spin, e.g., $S_z = +1/2$ for a negative g -factor.

It should be mentioned that the mapping of the $\nu_0 = 3$ system onto the two-level problem is valid for any value of Zeeman splitting. Due to exchange interaction the ground state is a mixture of the states with the same spin only. Since we are interested in the phonon induced transitions, which conserve the electron spin, we should consider the excitations without spin reversal. As a result only the electron states with the same spin are relevant to the considered problem. Because both levels have the same spin we can treat electrons as spinless particles. For convenience the levels are numbered by the corresponding subband numbers, $\mu = 1$ or 2 . To simplify the expressions the energy is measured in the Coulomb units $\varepsilon_C = e^2/\kappa l$ and the length is in units of magnetic length l . Following the standard procedure⁹ of rewriting the interaction part of the Hamiltonian in the momentum representation, the total Hamiltonian of the two-level electron system can be written in the form:

$$H = \frac{\Delta}{2} \sum_{\mu\sigma k_y} (2\mu - 3) C_{k_y\mu\sigma}^+ C_{k_y\mu\sigma} + \frac{1}{2} \sum_{\{\mu\}} \sum_{q_x, q_y} \tilde{V}_{(\mu_1\mu_4\mu_3\mu_2)}^{(n_1n_4n_3n_2)}(\hat{q}) \sum_{k_1, k_2} e^{iq_x(k_1 - k_2)} C_{k_1+q_y, \mu_1}^+ C_{k_2, \mu_2}^+ C_{k_2+q_y, \mu_3} C_{k_1, \mu_4} \quad , \quad (1)$$

where the Landau level number is equal to $n_i = 1$ for $\mu_i = 1$ and $n_i = 0$ for $\mu_i = 2$, i.e. $n_i = 2 - \mu_i$. We use the Landau gauge with vector potential $\vec{A} = (0, Bx, 0)$ and $C_{k, \mu}^+$, $C_{k, \mu}$ are the creation and annihilation operators of the electron in the state $\psi_{n, k_y, \mu}$:

$$\psi_{n, k_y, \mu}(x, y, z) = \chi_\mu(z) \frac{e^{ik_y y}}{\sqrt{L_y}} \phi_n(x - k_y) \quad , \quad (2)$$

where $\chi_\mu(z)$ is the envelope function of the μ th subband; $\phi_n(x)$ is the n th harmonic oscillator function.

In Eq. (1) we introduced the notations:

$$\tilde{V}_{(\mu_1\mu_4\mu_3\mu_2)}^{(n_1n_4n_3n_2)}(\hat{q}) = \frac{1}{q} F_{\mu_1\mu_4\mu_3\mu_2}(q) G_{n_1n_4}(\hat{q}) G_{n_3n_2}(\hat{q}^*) \exp\left(-\frac{q^2}{2}\right) \quad , \quad (3)$$

where⁹

$$G_{n_1n_2}(\hat{q}) = \left(\frac{n_1!}{n_2!}\right)^{1/2} \left(\frac{-iq}{\sqrt{2}}\right)^{n_2 - n_1} L_{n_1}^{n_2 - n_1}\left(\frac{|q|^2}{2}\right) \quad , \quad (4)$$

with $\hat{q} = q_x + iq_y$, $q = |\hat{q}|$ and L_n^m being a generalized Laguerre polynomial. The only functions relevant to our problem are

$$G_{0,0}(\hat{q}) = \exp\left(-\frac{q^2}{4}\right) \quad , \quad G_{0,1}(\hat{q}) = \frac{\hat{q}}{\sqrt{2}} \exp\left(-\frac{q^2}{4}\right) \quad , \\ G_{1,1}(\hat{q}) = \left(1 - \frac{q^2}{2}\right) \exp\left(-\frac{q^2}{4}\right)$$

The modification of the Coulomb interaction due to the finite extent of the electron wave functions in z -direction is given by the expression:

$$F_{\mu_1\mu_4\mu_3\mu_2}(q) = \int_0^\infty \int_0^\infty dz_1 dz_2 e^{-q|z_1 - z_2|} \chi_{\mu_1}(z_1) \chi_{\mu_4}(z_1) \chi_{\mu_2}(z_2) \chi_{\mu_3}(z_2) \quad .$$

We use the Fang-Howard approximation¹¹ with the parameter b for the envelope functions of the first and second subbands:

$$\chi_1(z) = \sqrt{\frac{b^3}{2}} z \exp\left(-\frac{1}{2}bz\right) \quad , \\ \chi_2(z) = \sqrt{\frac{b^5}{6}} z \left(z - \frac{3}{b}\right) \exp\left(-\frac{1}{2}bz\right) \quad ,$$

where the function $\chi_2(z)$ is found from the condition that $\chi_1(z)$ and $\chi_2(z)$ are orthogonal.

The first problem we are interested in is the ground state of the system. It is clear that if the splitting Δ between the layers is larger than the characteristic inter-electron Coulomb interaction ($\Delta > 1$) then the ground state is a completely occupied level with $\mu = 1$. Similarly, if $\Delta < -1$, the ground state is a completely occupied level with $\mu = 2$. With changing Δ the transition between these two phases occurs. This transition can be either sharp or smooth and in the last case the transition occurs through the new intermediate phase. The example of sharp phase transition in the two-level electron system with the total filling factor $\nu = 1$ is given in Ref. 12, where the two levels correspond to different Landau levels of the same subband but with opposite spin. There is also an example of a smooth phase transition in the two-level system of composite fermions with filling factor $\nu = 1$.¹³ The two levels in Ref. 13 are the same as the levels in Ref. 12. The only difference from Ref. 12 is in the type of interaction between the particles. Our system is different from the systems considered in Refs. 12,13 in two aspects: the levels in our case correspond to different subbands and the spin for both levels is the same, i.e., we have an additional exchange interaction between the electrons in the different levels.

To find the ground state of the system in the intermediate range of Δ we follow the method of Ref. 12 and use the standard Hartree-Fock approximation. We assume the non-zero average values of $\langle C_{k,1}^+ C_{k,1} \rangle$, $\langle C_{k,2}^+ C_{k,2} \rangle$ and $\langle C_{k,1}^+ C_{k+Q,2} \rangle$ which do not depend on k . In general, the parameter Q which minimizes the Hartree-Fock Hamiltonian is not equal to zero. For each value of k we introduce the new wavefunctions which are the eigenfunctions of the Hartree-Fock Hamiltonian. The creation and annihilation operators corresponding to these functions are $a_{k,i}^+$, $a_{k,i}$, $i = 1, 2$. The new functions are related to the original ones by the rotation by angle θ :

$$\begin{aligned} C_{k,1} &= a_{k,1} \cos(\theta/2) + a_{k,2} \sin(\theta/2) \quad , \\ C_{k+Q,2} &= -a_{k,1} \sin(\theta/2) + a_{k,2} \cos(\theta/2) \quad . \end{aligned} \quad (5)$$

Assuming that $a_{k,1}$ corresponds to the lowest energy state we write the average: $\langle a_{k,i}^+ a_{k,j} \rangle = \delta_{i1} \delta_{j1}$. Substituting expression (5) into the Hamiltonian (1) we get the Hartree-Fock Hamiltonian as a function of θ and Q :

$$\begin{aligned} H^{HF} &= \frac{1}{2} [-\Delta - \epsilon_{11}^{11} \cos^2(\theta/2) - \epsilon_{12}^{01} \sin^2(\theta/2)] \cos^2(\theta/2) \\ &+ \frac{1}{2} [\Delta - \epsilon_{22}^{00} \sin^2(\theta/2) - \epsilon_{12}^{01} \cos^2(\theta/2)] \sin^2(\theta/2) \\ &+ \sin^2(\theta/2) \cos^2(\theta/2) V_{12}(Q) \quad , \end{aligned}$$

where $\epsilon_{j_1 j_2}^{i_1 i_2}$ is the exchange energy of an electron in the i_1 th Landau level of the j_1 th subband interacting with the electrons of the same spin in the filled i_2 th Landau level of the j_2 th subband:

$$\epsilon_{j_1 j_2}^{i_1 i_2} = \int_0^\infty dk F_{j_1 j_2 j_1 j_2}(k) |G_{i_1 i_2}(\hat{k})|^2 \exp\left(-\frac{k^2}{2}\right) \quad . \quad (6)$$

The potential $V_{12}(Q)$ is given by the expression:

$$V_{12}(Q) = \int_0^\infty dk \left[F_{1122}(k) G_{11}(\hat{k}) G_{00}(\hat{k}) J_0(kQ) - F_{1212}(k) |G_{12}(\hat{k})|^2 J_2(kQ) \right] \exp\left(-\frac{k^2}{2}\right) \quad .$$

Analyzing the Hartree-Fock Hamiltonian, one can show that the energy minimum can be achieved only at $\theta = 0$ or $\theta = \pi$, which corresponds to a completely occupied level $\mu = 1$ or $\mu = 2$, respectively. This means that there are only two different phases with a sharp transition between them. Comparing the Hartree-Fock energy at $\theta = 0$ and $\theta = \pi$, we obtain the inter-level splitting Δ^* at which the transition between two phases occurs:

$$\Delta^* = \frac{1}{2} (\epsilon_{22}^{00} - \epsilon_{11}^{11}) \quad . \quad (7)$$

For $\Delta > \Delta^*$ the ground state is the completely occupied level with $\mu = 1$ and for $\Delta < \Delta^*$ the ground state is the completely occupied level with $\mu = 2$.

To find the energy spectrum of neutral excitations we follow the method proposed in Ref. 14 for a double layer system at electron filling factor equal to 1. We introduce the normalized exciton operator, which creates an electron in the empty level and a hole in the filled level:

$$\mathcal{A}^+(\vec{q}) = \frac{1}{\sqrt{N}} \sum_k e^{iq_x k} C_{k+q_y/2,2}^+ C_{k-q_y/2,1} \quad , \quad (8)$$

where $\vec{q} = (q_x, q_y)$ is the momentum of the electron-hole pair, N is the number of electrons and we assume that the level with $\mu = 1$ is filled. Then we calculate the energy spectrum of such excitations as an average of the Hamiltonian (1) in the state $\mathcal{A}^+(\vec{q})|0\rangle$:

$$\begin{aligned} E(q) &= \langle \mathcal{A}(\vec{q}) H \mathcal{A}^+(\vec{q}) \rangle = \Delta + \epsilon_{11}^{11} - \epsilon_{12}^{01} + \frac{q}{2} F_{1212}(q) \exp(-q^2/2) \\ &\quad - \int_0^\infty dk \left(1 - \frac{q^2}{2}\right) J_0(kq) F_{1122}(k) \exp(-k^2/2) \\ &= (\Delta - \Delta^*) + \frac{1}{2} (\epsilon_{22}^{00} + \epsilon_{11}^{11}) - \epsilon_{12}^{01} + \frac{q}{2} F_{1212}(q) \exp(-q^2/2) \\ &\quad - \int_0^\infty dk \left(1 - \frac{q^2}{2}\right) J_0(kq) F_{1122}(k) \exp(-k^2/2) \quad , \end{aligned} \quad (9)$$

where we rewrite the $E(q)$ in terms of the difference $(\Delta - \Delta^*)$. Because the Hamiltonian (1) does not conserve the number of excitons we should take into account the processes which involve several excitons. Such processes are important at small values of the level splitting Δ . To take into account the multi-exciton states we approximate the Hamiltonian (1) by¹⁴

$$H = \sum_{\vec{q}} \left\{ E(q) \mathcal{A}^+(\vec{q}) \mathcal{A}(\vec{q}) + \frac{1}{2} [\lambda(\vec{q}) \mathcal{A}(\vec{q}) \mathcal{A}(-\vec{q}) + \lambda^*(\vec{q}) \mathcal{A}^+(\vec{q}) \mathcal{A}^+(-\vec{q})] \right\} \quad . \quad (10)$$

The function $\lambda(q)$ is defined by the equation:

$$\begin{aligned} \lambda(\vec{q}) &= \langle H \mathcal{A}^+(\vec{q}) \mathcal{A}^+(-\vec{q}) \rangle = e^{2i\phi} \frac{q}{2} F_{1212}(q) \exp(-q^2/2) \\ &\quad - e^{2i\phi} \int_0^\infty dk \frac{q^2}{2} J_2(kq) F_{1212}(k) \exp(-k^2/2) \quad , \end{aligned} \quad (11)$$

where $\phi = \arctan(q_y/q_x)$. After diagonalizing the Hamiltonian (10) using the Bogoliubov transformation¹⁶ we obtain the energy spectrum of the elementary neutral excitations:

$$E_0(q) = \sqrt{E^2(q) - |\lambda(\vec{q})|^2} \quad . \quad (12)$$

If the level $\mu = 2$ is occupied in the ground state, i.e. $\Delta < \Delta^*$, then the Eqs. (11) and (12) are the same and the only difference is in Eq. (8) where we should write $-(\Delta - \Delta^*) = |\Delta - \Delta^*|$ instead of $(\Delta - \Delta^*)$.

The numerical calculations show that the lowest energy excitation (Eq. (12)) has a zero momentum. The energy of this excitation can be considered as a renormalized splitting, Δ_r , between the levels of the double-level electron system. It follows from Eqs. (9), (11) and (12) that

$$\Delta_r = E_0(0) = |\Delta - \Delta^*| + \frac{1}{2} (\epsilon_{22}^{00} + \epsilon_{11}^{11}) - \epsilon_{12}^{01} - \int_0^\infty dk \left(1 - \frac{q^2}{2}\right) F_{1122}(k) \exp(-k^2/2) \quad . \quad (13)$$

The splitting Δ_r reaches its lowest value at $\Delta = \Delta^*$. The renormalization of the excitation energy in a 2D electron system (g -factor enhancement) due to the inter-electron interaction was well studied both theoretically and experimentally.¹⁷⁻²⁰ At the same time there are excitations of electron systems which are not altered by the electron-electron interaction. For example, Kohn theorem²¹ tells us that the energy of a cyclotron excitation does not depend on the electron density.

In Fig.2(a) the critical level splitting Δ^* is shown as a function of the Fang-Howard parameter b by a solid line. With increasing b the spreading of the electron wave function in z direction becomes smaller. This results in increasing Δ^* , which can be understood from the Eqs. (6) and (7). The first term in equation (6), ϵ_{22}^{00} , is determined by inter-electron interaction in the first Landau level of the second subband, while the second term ϵ_{11}^{11} is determined by the inter-electron interaction in the second Landau level of the first subband. Since the spreading of the electrons in the second Landau level is broader than the spreading of the electrons in the first Landau level, the 2D form-factor of the inter-electron interaction, which is given by $G_{i_1, i_2}(q)$ (Eq. (4)), is larger for the electrons in the first Landau level. Similarly, due to a different spreading of the wave functions in z -direction for the electrons in the different subbands the form-factor $F_{\mu_1 \mu_2 \mu_3 \mu_4}(q)$ is larger for the electron in the first subband. The competition between these two tendencies gives rise to the increase of Δ^* with increasing b . We can see from Fig.2a that at $b \approx 1.6$ the effects of the different spreading in z -direction and in the (x, y) -plane cancel each other, so that $\Delta^* = 0$.

The dotted line in Fig.2(a) shows the dependence of the renormalized splitting Δ_r on the parameter b for $\Delta = \Delta^*$. We can see again that with decreasing of the spreading of electron wave function in z direction the splitting Δ_r increases. This increase of Δ_r has the same reason as the increase of Δ^* .

In Fig.2(b) the dispersion curve $E_0(q)$ (Eq. (12)) is shown for $\Delta = \Delta^*$ and for different values of parameter b : $b = 1$ (solid line) and $b = 2$ (dotted line). We can see again that the excitation energy increases with increasing b . The effect becomes stronger for larger values of momentum q .

In Fig.2(c) the dispersion curve $E_0(q)$ is shown for $b = 1$ and for two different values of Δ : for $\Delta = \Delta^*$ (solid line) and for $\Delta = \Delta^* + 0.05$ (dotted line). The increase of Δ has no significant effect on the form of the dispersion curve due to a small value of the function $\lambda(\vec{q})$. As a result the dispersion curve $E_0(k)$ has an almost linear dependence on splitting Δ for all values of momentum q and can be approximated by $E_0(q) \approx E(q)$.

III. PHONON SPECTROSCOPY

In this section we study the interaction of the quasi-2D double subband electron system, discussed in the previous section, with acoustic phonons. We consider the case of the absorption phonon spectroscopy when the 2D electrons are subjected to an external beam of non-equilibrium acoustic phonons.²⁻⁵

Below we are using the isotropic Debye approximation in which the phonon frequency has a linear dependence on the wave vector:

$$\omega_j(K) = s_j K \quad ,$$

where s_j is the speed of sound, j is labeling the phonon modes, $j = 1$ for longitudinal mode and $j = 2, 3$ for two transverse modes. The wave vector K is the three dimensional (3D) vector, $K = |\vec{K}|$. For convenience we label the 3D vectors by the capital letters and their projections by the corresponding small letters, $\vec{K} = (\vec{k}, k_z)$.

The electron-phonon interaction Hamiltonian can be written in terms of exciton operators $\mathcal{A}^+, \mathcal{A}$ (Eq. (8)):

$$H_{e-ph} = - \sum_{j, \vec{Q}} \frac{M_j(\vec{Q})}{\sqrt{V}} Z(q_z) \left[\mathcal{A}(\vec{q}) \hat{d}_j^+(\vec{Q}) + \mathcal{A}^+(\vec{q}) \hat{d}_j(\vec{Q}) \right] \quad , \quad (14)$$

where \hat{d}_j^+ is the creation operator of a phonon in the j th mode, V is a normalization volume; $M_j(\vec{Q})$ are the matrix elements of electron-phonon interaction which are determined by the deformation potential and piezoelectric coupling. In GaAs the matrix elements $M_j(\vec{Q})$ are usually written in the form¹⁰:

$$M_j(\vec{Q}) = \sqrt{\frac{\hbar}{2\rho_0 s Q}} \left[-\beta \frac{Q_x Q_y \xi_{j,z} + Q_y Q_z \xi_{j,x} + Q_z Q_x \xi_{j,y}}{Q^2} - i \Xi_0 (\vec{\xi}_j \cdot \vec{Q}) \right] \quad , \quad (15)$$

where ρ_0 is the mass density of GaAs, β and Ξ_0 are the parameters of piezoelectric and deformation potential couplings,⁶ respectively; $\vec{\xi}_j$ is the polarization vector of the j th phonon mode.

The form-factor $Z(q_z)$ in Eq. (14) is determined by the electron spreading in z direction:

$$Z(q_z) = \int dz e^{iq_z z} \chi_1(z) \chi_2(z) \quad . \quad (16)$$

To illustrate the specific feature of the electron-phonon interaction in our two-subband system we study the total rate of absorption of phonons from an external pulse by the 2D electrons. At low enough temperature the rate of absorption is given by¹⁰

$$\omega_{abs} = \frac{2\pi}{\hbar} \sum_j \int \frac{d\vec{Q}}{(2\pi)^3} \delta(E_0(q) - sQ) n_j(\vec{Q}) \left| M_j(\vec{Q}) Z(q_z) \right|^2 R_{01}(q) \quad , \quad (17)$$

where $n_j(\vec{Q})$ is the phonon distribution function. The function $R_{01}(q)$ corresponding to the transition from the first to the second Landau level is given by

$$R_{01}(q) = |G_{0,1}(\hat{q})|^2 = \frac{q^2}{2} e^{-q^2/2} \quad . \quad (18)$$

In Eq. (17) we assume that the energy relaxation time of the 2D electron system is much shorter than the characteristic time of the phonon absorption, which as shown below is of the order of 10^{-10} s.

From the expression for $R_{01}(q)$ (Eq. (18)) one can see that the phonon absorption occurs mostly for the 2D momentum q being in the range of $q \sim 1 \div 2$. In this range of q the energy spectrum $E_0(q)$ is almost dispersionless (Fig.2(b)), $E_0(q) \sim 0.08 + |\Delta - \Delta^*|$ for $b = 1$. Energy conservation requires the energy of absorbed phonon to be equal to $E_0(q)$, i.e. $s\sqrt{q^2 + q_z^2} = E_0(q)$. Taking into account that the phonon momentum q_z is restricted by the parameter b ($|q_z| < b$) and that the speed of sound is about 0.03 for longitudinal and 0.02 for transverse phonons, we see that the phonon absorption is suppressed in the two-subband system. This suppression is completely due to the many-body effects, since the many-body gap, $E_0(q)\epsilon_C$, is greater than $\hbar s/l$.

Let us assume that the non-equilibrium phonon distribution function is isotropic. Then, substituting Eqs. (15) and (18) into the Eq. (17) and performing the integration, we get:

$$\begin{aligned} \frac{1}{\tau_{abs}} &= \frac{\omega_{abs}}{n(E_0(q_0))} \\ &= \frac{q_1^5}{s_1^2} \int_0^1 du(1-u^2) |Z(uq_1)|^2 \left[M_d + \frac{9}{8} \frac{M_p}{q_1^2} u^2(1-u^2) \right] \exp(-q_1^2(1-u^2)/2) \\ &\quad + \frac{q_2^3}{s_2} \frac{M_p}{8} \int_0^1 du(1-u^2)^2 (9u^4 - 2u^2 + 1) |Z(uq_2)|^2 \exp(-q_2^2(1-u^2)/2) \quad , \end{aligned} \quad (19)$$

where $q_1 = E_0(q_0)/s_1$, $q_2 = E_0(q_0)/s_2$ and q_0 is taken on the plateau of the dispersion curve $E_0(q)$ (Fig.2(b)). Here the constants M_d and M_p , characterizing the deformation and piezoelectric couplings, are:

$$\begin{aligned} M_d &= \frac{1}{4\pi} \left(\frac{\Xi}{\epsilon_C} \right)^2 \frac{1}{l^5 \rho_0} \quad , \\ M_p &= \frac{1}{4\pi} \left(\frac{\beta l}{\epsilon_C} \right)^2 \frac{1}{l^5 \rho_0} \quad . \end{aligned}$$

In Fig.3 the absorption rate is shown as a function of $\Delta - \Delta^*$ for the two values of parameter b : $b = 1$ (solid line) and $b = 1.5$ (dotted line). Both curves have a single-peak structure and show a weak dependence on b . The energy $E_0(q_0)$ is increasing with increasing b (Fig.2(b)). This results in the decrease of the phonon absorption rate. However, the increase of b is favorable for the absorption of phonons with larger q_z , because the form-factor $Z(q_z)$ effectively cuts off the phonon momentum q_z at $q_z = b$. These two factors counteract each other, which results in a weak absorption rate dependence on parameter b . For the focused phonon beam, the first effect becomes more important and the decrease of the absorption rate with increasing b should be observed.

The rate of the phonon absorption for the non-interaction electron system is shown in Fig.3 by the dashed line. The main difference between the interacting and non-interacting systems is in the shape of the dependencies of the absorption rate on the level splitting. These dependencies have the double-peak and single-peak structures for non-interacting and interacting systems, respectively. For the non-interaction system the excitation spectrum is just the bare gap $|\Delta|$ and the excitation energy has any value starting from 0. The spectrum becomes gapless at $\Delta = 0$. In this case the maximum of the absorption rate corresponds to the condition: $|\Delta| \sim s/l$. This results in a double-peak structure for the non-interaction system.⁷ For the interacting system there is an energy gap for any value of Δ . Thus, the absorption rate as a function of Δ has a single-peak structure.

If the electron filling factor ν is changed from $\nu = 0$ to $\nu = 1$ one should expect the transformation of the double-peak structure into a single-peak one, because at small filling factor the inter-electron interaction becomes less important. This transformation occurs as the merging of the two peaks into a single peak. Assuming that the interaction gap of the 2D electron system is proportional to ν , one can see that with increasing ν the separation between peaks decreases linearly, and the two peaks transform into a single peak at $\nu \approx 0.75$ for $b = 1$. The strength of the peaks increases linearly with increasing ν , because the phonon absorption rate is proportional to the electron density. The transformation of the double-peak into a single-peak structure with increasing electron filling factor can be observed experimentally. The splitting between the levels Δ is the difference between the intersubband splitting and cyclotron energy with some corrections due to exchange interaction with the background electrons. Therefore, this splitting can be changed by changing magnetic field.

At the same time the gate voltage should be adjusted to keep the electron filling factor constant. However, even for the changing filling factor at fixed electron density one should expect the single-peak structure of the phonon absorption curve as a function of magnetic field if the electron filling factor is close to $\nu = 1$. The double-peak structure appears when the electron filling factor is smaller than ≈ 0.7 . For a constant electron density the single-peak and double-peak structures are both asymmetric when the difference $\Delta - \Delta^*$ changes sign.

IV. SUMMARY

We have studied a two-subband electron system in a quantizing magnetic field at the total filling factor $\nu_0 = 3$. When the second Landau level of the first subband is close in energy to the first Landau level of the second subband the system becomes effectively a two-level electron system with filling factor $\nu = 1$. Depending on the value of splitting between the two levels the system can be in one of the two possible ground states: for $\Delta > \Delta^*$ all the electrons are in the first subband and for $\Delta < \Delta^*$ all the electrons are in the second subband. The inter-electron interaction renormalizes the critical splitting Δ^* which should be zero for a non-interacting system. Another effect related to the electron-electron interaction is a non-zero gap for any value of level splitting Δ . This effect should be important for the phonon spectroscopy experiments. We show that due to the non-zero many-body gap the phonon absorption rate has a single-peak structure as a function of the splitting between the levels or as a function of magnetic field. At small electron filling factor ν , when the system can be considered as a non-interacting system, the double-peak structure is expected. The transformation of the double-peak structure into a single-peak one occurs when the filling factor is changed from $\nu = 0$ to $\nu = 1$. Our calculations show that the two peaks merge at filling factor $\nu \approx 0.7$.

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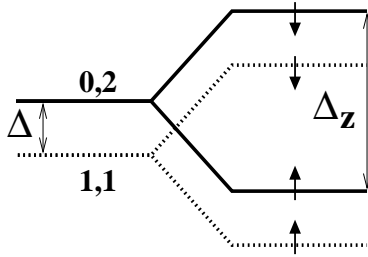


FIG. 1. Single-electron energy levels of the two-subband system. Each level is numerated by the pair of numbers: the Landau level number and the subband number. The arrows indicate spin directions. The interlevel splitting is Δ . Zeeman splitting is Δ_z .

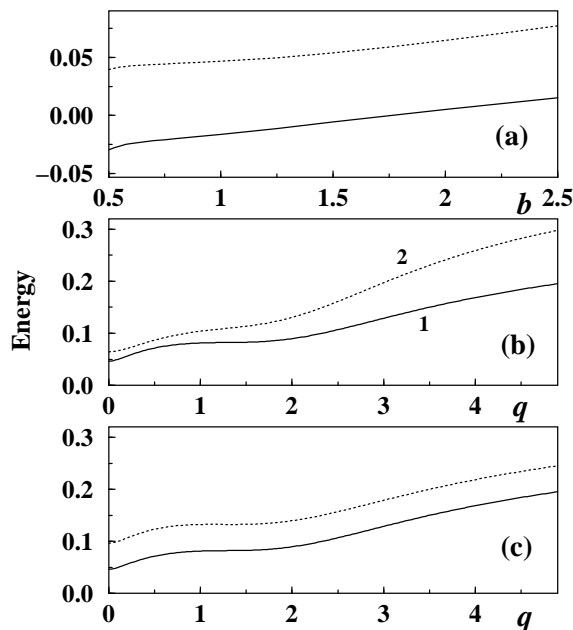


FIG. 2. (a) The critical level splitting Δ^* (Eq. (7)) and the renormalized splitting Δ_r at $\Delta = \Delta^*$ (Eq. (13)) are shown as the functions of the Fang-Howard parameter b by solid and dotted lines, respectively. (b) The dispersion curve $E_0(q)$ (Eq. (12)) is shown for $\Delta = \Delta^*$. The numbers near the lines are the values of the parameter b . (c) The dispersion curve $E_0(q)$ (Eq. (11)) is shown for $b = 1$ and different Δ : $\Delta = \Delta^*$ (solid line) and $\Delta = \Delta^* + 0.05$ (dotted line). Both q and b are in units of inverse magnetic length, $1/l$; the energy is in units of ε_C .

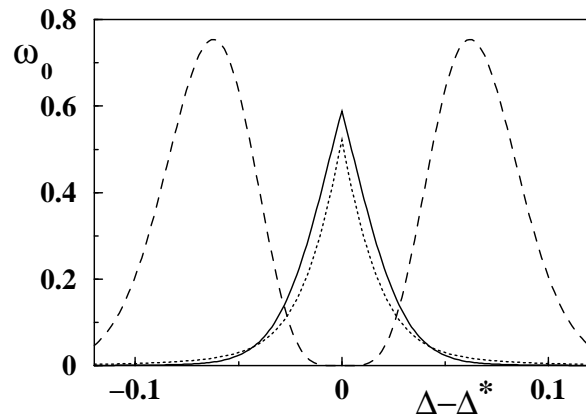


FIG. 3. The phonon absorption rate $\omega_0 = 1/\tau_{abs}$ (Eq. (19)) as a function of $\Delta - \Delta^*$ for $b = 1$ (solid line) and $b = 1.5$ (dotted line). The absorption rate for the non-interacting system is shown by the dashed line. The absorption rate ω_0 is in units of 10^{10} s^{-1} , $\Delta - \Delta^*$ is in units of ε_C .