Prism coupling to 'designer' surface plasmons

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Abstract: The excitation of 'designer' surface-plasmon-like modes on periodically perforated metals is demonstrated at microwave frequencies using the classical method of prism-coupling. In addition we provide a complete formalism for accurately determining the dispersion of these surface modes. Our findings fully validate the use of metamaterials to give surface plasmon-like behavior at frequencies below the visible.

OCIS codes: (240.6680) Surface Plasmons; (240.6690) Surface Waves; (350.4010) Microwaves; (050.1950) Diffraction gratings; (160.3918) Metamaterials; (050.6624) Subwavelength structures.

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1. Introduction

The excitation of electromagnetic (EM) surface waves confined at a conductor-insulator interface has been the subject of extensive research since the late 19th Century [1,2] including the 1902 observations by Wood of 'anomalous' dark bands in the TM- (transverse magnetic)

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polarized reflectivity spectra from ruled metal diffraction gratings [3]. Yet his results had to wait nearly forty years before theory [4] provided an explanation – the excitation of surface plasmons (SPs). A further twenty-five years passed before the seminal work of Otto. He used an attenuated total reflection technique [5] to couple visible light to a SP on a smooth metal film. In the visible domain, SP fields are strongly enhanced at the metal-dielectric interface and decay exponentially with distance into the media either side ($\sim 10^{-2}\lambda$ in the metal, and $\sim 10^{-1}\lambda$ in the dielectric) [6]. In the terahertz and microwave regimes, metals have conductivities close to their dc values, and therefore almost perfectly screen the incident radiation. Therefore the fields associated with the surface mode may extend for many hundreds of wavelengths above the interface. In this limit, the mode may be reasonably described as a grazing photon. However it has recently been demonstrated [7-10] that if one carefully structures metal-dielectric composites on a subwavelength scale ('metamaterials'), EM properties can be engineered for which there is no naturally occurring material equivalent.

The idea of structuring metal surfaces to gain new EM properties is clearly not a new one. Examples include the measurements of Palmer et al. [11], who recorded the reflectivity of radiation of 4 mm wavelength as a function of the angle of incidence from metal gratings of rectangular profile and observed dark anomalies that became wider as the groove depth was increased. These features occurred within a fraction of a degree from the angle at which a diffracted order becomes tangential to the surface (and ceases to propagate). They explained this phenomenon using a theory based on "surface wave modes which are supported by the rectangular periodic structure of the grating". A few years later, Ulrich and Tacke [12] determined the band structure of the surface modes supported by a thin metallic mesh, demonstrating that the surface mode supported only exhibits significant curvature from the light line in the vicinity of the Brillouin zone boundaries. It is important to realize that the surface waves studied by both Palmer et al., and Ulrich and Tacke only show dispersion due to diffraction effects (i.e., the photonic band gaps). The importance of our study is that the modes considered exhibit very strong dispersion and surface localization far away from the Brillouin zone boundaries.

Pendry et al. [9,10] originally reignited interest in low frequency surface modes, proposing that a simple, non-diffractive array of holes can provide the conditions required to confine a wave to the interface even in the limit of perfect conductivity. They showed that if the holes are sufficiently small to support only evanescent fields (i.e., below their cut off frequency), with a periodicity much smaller than the incident wavelength, then the surface waves supported are governed by an effective surface plasma frequency, ν_{SP}, dictated primarily by their cut off frequency, and have a frequency dependent permittivity similar in form to that of real (Drude) metals. Since these modes can be engineered to occur at almost any frequency, but are analogous in many ways to the conventional surface plasmons that propagate on metals in the visible, they are labelled *designer* surface plasmons.

The observation of these SP-like modes was initially corroborated by experiments in the microwave domain using grating coupling [13], rather like the observations of Wood's [3] in the visible domain. However the ultimate test of this new idea is to use, as did Otto [5] (and also Ulrich and Tacke [12]), simple prism coupling. By recording the largely unperturbed dispersion curve, Otto was able to compare for the first time his experimental dispersion with that derived analytically. Both of these two key issues are explored in this paper for our plasmonic-like metamaterial. Firstly we use a prism-coupling technique to experimentally determine the dispersion of these designer SP-like modes, having designed the sample and experiment to ensure that the mode is largely unperturbed by diffraction. Secondly, we compare experimental and numerical results to a new analytical model.

2. Method

A large wax prism, of refractive index n = 1.5, is used to couple collimated microwave radiation on to a metal surface perforated with an array of holes (Fig. 1). For TM-polarized incident radiation and a suitable tunnel barrier (established by spacing the prism some t millimeters from the sample), with the microwave beam incident beyond the critical angle of

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the prism, $\theta_{\text{int}} > \sin^{-1}(1/n)$, the evanescent fields beyond the prism will resonantly couple to the SP-like mode on the surface of the array, and a minimum will be recorded in the reflected spectrum. By varying the spacing between the prism and the sample, and recording the frequency of the reflectivity minimum, the in-plane wave vector of the mode, $k_{\|}$, may be extracted. Repetition of this measurement for different incident angles allows for a good approximation of the surface mode's dispersion to be derived.

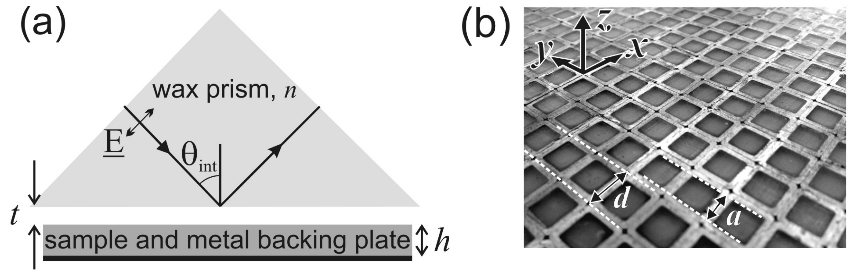

Fig. 1. (a) Schematic of the experimental set-up. (b) Photograph of sample surface and experimental coordinate system, where a = 6.96 mm and d = 9.53 mm.

For the case where the holes in a perfectly conducting surface are of infinite length, $h \to \infty$, the asymptotic limit is given by

$$\nu_{\text{SP}} = c/2a\varepsilon_{\text{h}}^{1/2} \tag{1}$$

where each tube has side length a and is filled with a dielectric of relative permittivity, ε_{h}. In order to calculate the dispersion of the surface modes, one may make the approximation that the waveguide modes are completely confined to the cavity [9,10]: a valid approximation if the fields in the region above the surface result entirely from specular reflection (i.e., far from the diffracting regime, periodicity $d \ll \lambda$). A further simplification can be made by assuming that the first wave guide mode (i.e., TE_{01}) will dominate in the cavity since it is the least strongly decaying. In these approximations Pendry et al. [9] showed that the in plane wave vector of the surface mode is given by

$$k_{\|} = k_0\left(1+\left[\frac{\beta\nu^2}{\nu_{\text{SP}}^2-\nu^2}\right]\right)^{1/2} \tag{2}$$

where $k_0 = 2\pi/\lambda$ and $\beta = 64a^4/\pi^4 d^4 \varepsilon_{\text{h}}$. Subsequently García de Abajo and Saenz [14] also provided an exact derivation of the dispersion relation for small, infinitely deep holes in the long wavelength limit by representing the holes by effective dipoles [15]. The coefficient has to be changed substantially to

$$\beta = \Gamma\left(4\pi^4 a^4/d^4\varepsilon_{\text{h}}\right) \tag{3}$$

where

$$\Gamma = \left(\left[\text{Re}\left(a^3\alpha_{\text{e}}^{-1}\right)\right]^{-1} + \left[\text{Re}\left(a^3\alpha_{\text{m}}^{-1}\right)\right]^{-1}\right)^2 \tag{4}$$

and α_{e} and α_{m} are the electric and magnetic polarizabilities of the effective dipoles.

Our 350×350 mm sample is a close-packed array of hollow, square-ended brass tubes of side d = 9.53 mm and length h = 15 mm, placed on a flat metal plate. The inner size of the tubes is a = 6.96 mm giving ν_{SP} ~ 22 GHz when unfilled, with the onset of diffraction at the Brillouin Zone boundary, $\nu_{\text{diff}\{10\}} = c/2d = 15.7$ GHz. In order to satisfactorily separate the diffractive effects (photonic band gap) from the plasmonic behavior, it is necessary for the effective surface plasma frequency ν_{SP} to be significantly suppressed below ν_{diff}. This is achieved by firstly filling the tubes with wax (ε_{h} = 2.29) to lower ν_{SP} to approximately 14

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GHz. We then rotate the plane of incidence by ϕ = 45º around the *z*-axis so that it lies along one diagonal axis of the square tubes (Fig. 1). Thereby we increase the onset of diffraction to $\nu_{\mathrm{diff}\{11\}} = c/\sqrt{2}d = 22.3$ GHz at the Brillouin zone boundary.

3. Results and discussion

A typical data set is illustrated in Fig. 2(a) (θ_{int} = 46.5º, ϕ = 45º, *t* = 10 mm) and is in excellent agreement with the finite element method (FEM) model predictions [16] using the parameters given, and setting the conductivity of brass to $1.5 \times 10^7\mathrm{Sm}^{-1}$. Note that the frequency of the surface mode has exceeded the infinite-length cut off frequency of the waveguides (solid vertical line), and appears to have approached that associated with the finite length tubes (broken line). In Fig. 2(b) we plot the EM fields associated with the resonance of the SP-like mode at 14.6 GHz. The electric field, plotted at a phase corresponding to maximal field enhancement, and the Poynting vector distributions are shown. Strong localization of the mode at the surface is clearly evident, with exponential decay of the fields away from the interface. Furthermore, the electric field-line loops are also highly reminiscent of SP-like character. However the non-zero power flow across the surface suggests that the mode has not yet reached its true asymptotic value (at which its group velocity $d\nu/dk = 0$), a result that becomes evident below when we determine the full dispersion curve.

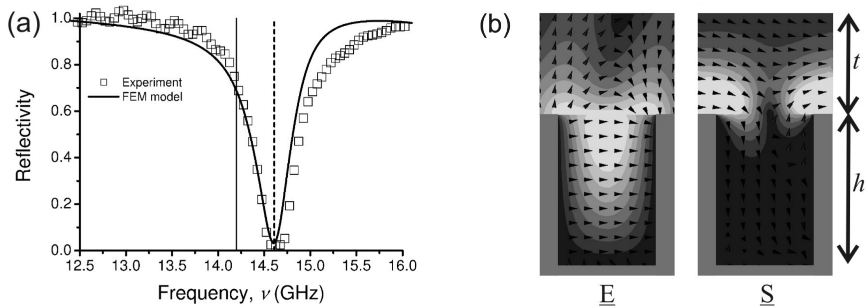

Fig. 2. (a) Reflection spectrum from the sample surface for θ_{int} = 46.5º and ϕ = 45º (where ϕ is the angle between the plane of incidence and the *xz*-plane) (squares). The predictions of the FEM model are also shown (line). The solid vertical line corresponds to the cut off frequency of an infinite length guide, Eq. (1), filled with ε_h = 2.29 and the dimensions previously described. The broken vertical line corresponds to the modified ν_{SP} associated with complete confinement of the waveguide mode to the truncated cavity. (b) EM field predictions on the plane of incidence of the mode at 14.6 GHz. The greyscale on the left shows the electric field at a phase corresponding to maximal field enhancement. The lightest shading indicates field intensities of at least five times the incident field. The plot on the right shows the Poynting vector (magnitude and direction) on resonance. Here the lightest shading corresponds to power enhancements of at least ten times.

Returning to our experiment, for a fixed angle of incidence θ_{int}, we record a series of reflectivity spectra for different tunnel gaps *t*. We determine the resonant frequency at the gap spacing for which coupling to the mode is strongest, and then repeat the procedure for a number of different angles, $\theta_{\mathrm{int}} > \sin^{-1}(n^{-1})$ (Fig. 3, circles). In the same manner we extract the optimum coupling frequencies from the reflectivity spectra predictions of the FEM model [15] (Fig. 3 inset: grey line). The circles (data) and grey line (FEM model) are in excellent agreement. At the largest angles, both experiment and modeled data exhibit a small decrease in the resonant frequency with increasing wave vector. These points correspond to the smallest tunnel gaps (typically only a few millimeters for θ ~ 55º) for which the penetration of the resonant fields into the high index prism becomes significant. Ideally, rather than recording the resonant frequency at the optimum coupling condition, one would increase the tunneling gap *t* and note the limiting frequency at which the influence of the prism is no

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longer significant. However coupling to the mode is often so weak in this limit that it makes accurate determination of the frequency impossible. Therefore we need to confirm the experimental validity of determining the dispersion via the optimum coupling condition. This is satisfactorily achieved by modeling the dispersion of the surface mode in the absence of the prism using the Eigen mode solver of the FEM software [15] (Fig. 3 inset: broken line), and comparing this to our experimental result. There is clearly very good agreement of the Eigen mode with experiment for $k_{\parallel} < 380\ \text{m}^{-1}$.

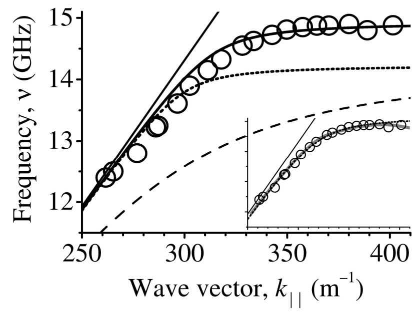

Fig. 3. Dispersion curves of the SP-like mode determined from experiment, and numerical and analytical models, when $\phi = 45°$. Frequency is plotted against wave vector along the surface of the sample. The data points derived from the experiment (optimum coupling condition) are shown as circles, and the straight line represents the air light line. The dashed and dotted curves correspond to the analytical solutions provided in Refs. 9 (and 10), and 12 respectively. The solid curve represents the solution of the analytical formalism provided in the present study, Eq. (7). The inset provides a comparison between the FEM modelled results on the same axes as the main diagram. The data points from the optimum coupling condition are shown as a solid grey line, whereas the broken line represents the Eigen mode solution in the absence of the prism.

4. Comparison to theory

Predictions of the surface mode dispersion from the analytical theories discussed above [9, 10, 14] are also plotted in the main part of Fig. 3. These analytical models clearly do not adequately describe our data. This is because, within their derivation, all three models contain inherent approximations which are invalid here. They are formulated to be exact in the long wavelength limit for narrow, deep holes (i.e., $a \ll d \ll \lambda \ll h$), whereas in the present study, a = 6.96 mm, d = 9.53 mm, h = 15 mm and $\lambda \sim 20$ mm. It is therefore clear that a more general formalism is required in order to describe the dispersion of these SP-like modes close to the asymptotic frequency. We achieve this here without the a priori assumptions intrinsic to Refs. 9, 10 and 14. In brief, we define general expressions for the EM fields in the regions either side of the interface and then match boundary conditions. In the semi-infinite vacuum region, the electric and magnetic fields are expressed as two dimensional Fourier-Floquet expansions for the set of evanescent diffracted orders. Each order is described by a wave vector component normal to the surface given by

$$(k_z^{m,n})^2 = k_0{}^2 - (k_x + 2m\pi / d)^2 - (k_y + 2n\pi / d)^2, \tag{5}$$

where m and n are integers and k_x and k_y describe the wave vector components parallel to the surface. Since we are interested in the dispersion of a *propagating* surface mode, we assume k_x and k_y to be real. Inside the finite depth cavities the fields are represented by a sum over the evanescent waveguide modes, where each mode is described by propagation constant

$$(q_z^{s,t})^2 = \varepsilon_h k_0{}^2 - (s\pi / a)^2 - (t\pi / a)^2, \tag{6}$$

where s and t are non-negative integers. The electric fields in vacuum (*vac*) and cavity (*cav*) regions may be then written as:

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$$
\begin{aligned}
E_x^{vac} &= \sum_{m,n} A_x^{m,n} \psi_1(x,y) \exp(-ik_z^{m,n} z), \\
E_y^{vac} &= \sum_{m,n} A_y^{m,n} \psi_1(x,y) \exp(-ik_z^{m,n} z), \\
E_x^{cav} &= \sum_{s,t} B_x^{s,t} \psi_2(x,y) \left[\exp\left(iq_z^{s,t} z\right) - \exp(iq_z^{s,t}(2h-z)) \right], \\
E_y^{cav} &= \sum_{s,t} B_y^{s,t} \psi_3(x,y) \left[\exp\left(iq_z^{s,t} z\right) - \exp(iq_z^{s,t}(2h-z)) \right],
\end{aligned} \tag{7}
$$

where

$$
\begin{aligned}
\psi_1(x,y) &= \exp\left(i\left(k_x + \frac{2m\pi}{d_1} \right) x \right) \cdot \exp\left(i\left(k_y + \frac{2n\pi}{d_2} \right) y \right), \\
\psi_2(x,y) &= \cos\left(\frac{s\pi x}{a_1} \right) \sin\left(\frac{t\pi y}{a_2} \right), \\
\psi_3(x,y) &= \sin\left(\frac{s\pi x}{a_1} \right) \cos\left(\frac{t\pi y}{a_2} \right).
\end{aligned} \tag{8}
$$

We can obtain the z components of the electric field inside and outside the cavity, and subsequently expressions for the magnetic field, through the free space Maxwell's relations. One can eliminate the set of unknowns $A_x^{m,n}, A_y^{m,n}, B_x^{s,t}$ and $B_y^{s,t}$ by applying the continuity at the vacuum-substrate interface (i.e. $z = 0$) of x and y components of the electric and magnetic fields over the entire unit cell over the aperture, respectively. Solving these coupled equations in the normal fashion yields the dispersion relation. In principle any number of waveguide modes and/or diffracted orders can be included in the calculation. In Refs. 9 and 10, the authors consider only the TE_{01} (i.e., s = 0, t = 1) waveguide mode. Furthermore, in the region of interest here, near the cut-off frequency of the TE_{01} mode, the higher order waveguide modes are very strongly evanescent (i.e., $q_z^{s,t}$ is large and imaginary) and have little effect on the surface mode dispersion. In Refs. 9 and 10 the authors also consider only the specular field outside the cavity (i.e., m = n = 0). This is a strict assumption, valid only in the long wavelength limit (i.e., $\lambda \gg d$), which is not the case for our sample. It is, however, straightforward to include higher order diffraction in our derivation of the dispersion relation, resulting in

$$
\sum_{m,n} \left(k_0^2 - \left(k_y + \frac{2n\pi}{d} \right)^2 \right) \frac{i \tan(hq_z^{0,1})(S^{m,n})^2}{k_z^{m,n} q_z^{0,1}} = 1, \tag{9}
$$

where

$$
S^{m,n} = \frac{4\pi\sqrt{2}}{a^2 d} \frac{\sin\left(a\left(k_x/2 + m\pi/d\right)\right)\cos\left(a\left(k_y/2 + n\pi/d\right)\right)}{\left(k_x + 2m\pi/d\right)\left(\left(\pi/a\right)^2 - \left(k_y + 2n\pi/d\right)^2\right)}. \tag{10}
$$

Here $S^{m,n}$ is termed the overlap integral between the TE_{01} waveguide mode inside the cavity and the (m,n) diffracted order outside the cavity.

For infinitely deep holes ($h = \infty$), the term $i\tan(hq_z^{0,1})$ in Eq. (9) tends to –1. For holes of finite depth, it is the $i\tan(hq_z^{0,1})$ term that modifies the asymptotic frequency ν_{SP}, accounting for the quantization of electric field in the z direction discussed qualitatively above. It should be noted that, in the limit where only the specular diffracted order (i.e., m = n = 0) is considered, together with infinitely deep holes and $k_y = 0$ (i.e., dispersion in the x direction), Eq. (9) simplifies to that found in Refs. 9 and 10. In practice, a more accurate dispersion relation is obtained by including first order diffraction by summing the terms in Eq. (9) for m and n from –1 to +1. Including first order diffraction and the finite hole depth, we calculate the dispersion predicted for our sample from Eq. (9) in the direction given by $\phi = 45°$, where $k_y = k_x$. The results from this analytical dispersion are plotted in the main part of Fig. 3 (solid

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line) and clearly give an excellent fit to the experimental data, accurately reproducing both the curvature and asymptote ($\nu_{SP} \approx 14.9$ GHz). It should be noted that the relation provided in Eq. (9) is very general, valid for square arrays of holes of *any* size and depth, near the cut off of the holes. Interestingly, the character of the metamaterial designs studied here limits the divergence of the mode's dispersion from the light line for frequencies well below the asymptote. As a result, their dispersion exhibits a much faster approach to the asymptotic frequency than an "intrinsic" SP supported by a flat, real metal [17].

5. Conclusions

In conclusion, we have provided a novel study of the dispersion of surface waves on plasmonic metamaterials close to their effective surface plasma frequency using the Otto prism coupling technique. Unlike previous studies, which at first sight may appear similar, the deviation of the surface mode's dispersion from the light line is not induced by photonic band gaps at the Brillouin zone boundaries. Importantly we have also employed two techniques to ensure that the mode's plasmonic-like dispersion is not perturbed by photonic effects. These microwave metamaterial surfaces are thus analogous in many ways to smooth metal surfaces in the visible and IR. As such, our unique microwave experimental techniques mirror the seminal prism-coupling work undertaken by Otto for conventional surface plasmons. By employing a modal matching method, we have developed a general formalism for determining the dispersion of the surface modes, which shows very good agreement with our measured dispersion.

The capability to confine electromagnetic energy at the interface of a conductor makes these "designer" SPs very promising for applications where efficient guiding of electromagnetic radiation on very small length scales is important. These may include sub-wavelength microscopy, non-linear optics, photolithography and sensing. Applications that involve the slowing or even storing of telecommunication signals (e.g., Ref. 18) will also benefit from these results.

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