Correlations and noise in magnetic recording media

M.M. Aziz, B.K. Middleton and J.J. Miles

Abstract: A theory is presented which predicts the role of interparticle correlations in determining the magnetisation power spectral densities within magnetic recording media in the absence of recorded signal magnetisation. Magnetic correlations are represented in terms of the probabilities of the changes of magnetisation direction between neighbouring particles and this feeds through into determining the shapes of the power spectral densities and the correlation lengths.

1 Introduction

To explain the observed replay spectra from erased longitudinal particulate magnetic media, theoretical models often assume for simplicity that the constituent particles are independent of one another [1]. This assumption is likely to be best represented in materials with small volume packing fractions of particles as is often the case in magnetic tapes where there is distance between particles and where there also exist passivation layers surrounding each particle. Theoretical treatments of noise based on this assumption usually predict noise power spectra which are less in magnitude than are measured, particularly at long wavelengths [1, 2]. This difference is caused by the presence of entities larger in volume than the individual particles which constitute the magnetic coatings. These can arise through spatial clumping or chaining of particles, by magnetic interactions causing large magnetic entities or by combinations of both. Consequently, chaining and clustering of particles have been investigated [2, 3] as a means of explaining the long-wavelength behaviour of the measured noise spectra and some success has been obtained. However, further developments are needed in the treatment of magnetic interparticle interactions to elucidate their influences on the observed noise spectra.

In a previous publication the current authors developed a theory of noise in particulate media which accounts for the presence of uncorrelated particles, particle size variations, particle chaining and nearest-neighbour interactions within particle chains [4]. This theory produced excellent agreement with experimental measurements, especially at long wavelengths, on high-density metal particle tapes [5]. In this paper a theory based on a single infinitely long chain of magnetic particles is developed to reveal the role of short- and long-range magnetic interactions on the replay spectra from magnetic recording media.

IEE Proceedings online no. 20030823

doi:10.1049/ip-smt:20030823

Paper received 11th February 2003

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2 Theory

This theory extends the previous work [4] on the influence of particle chains on the noise spectra emerging from particulate tapes. It assumes the special case of a single very long chain of particles as representative of a magnetic medium and considers the role of magnetic interactions within it.

Looking at Fig. 1 and considering nearest-neighbour correlations the autocorrelation function $R(x')$ of a single long chain of particles all having the same cross-sectional areas is [4]

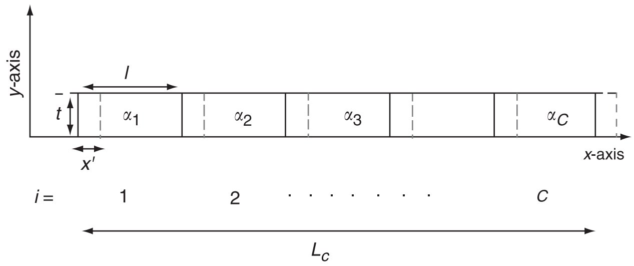

Fig. 1 *Chain of magnetic particles and their displaced images indicating correlation process*

$$R(x') = \lim_{V_L \to \infty} \frac{1}{V_L} \left\{ \sum_{i=1}^{C} (M_i^p)^2 \alpha_i^2 t_i w_i (l_i - x') + \sum_{i=1}^{C-1} (M_i^p)^2 \alpha_i \alpha_{i+1} t_i w_i x' \right\} \tag{1}$$

where M^p is the magnetisation within each particle, V_L the volume of the whole chain of C particles of total length L, $\alpha_i = \pm 1$ represents the orientation of magnetisation to the right or left within each particle, and t_i the height of the particle of width w_i and length l_i. Writing the summations in (1) as averages along the line of particles yields

$$R(x') = \lim_{V_L \to \infty} \frac{1}{V_L} \left\{ C\overline{(M^p)^2}\,\overline{\alpha^2}\,\bar{t}\bar{w}(\bar{l} - x') + (C-1)\overline{(M^p)^2}\,\overline{\alpha_i \alpha_{i+1}}\,\bar{t}\bar{w}x' \right\} \tag{2}$$

This can be rewritten in the form

$$R(x') = \frac{1}{\bar{l}} \lim_{V_L \to \infty} \frac{C\bar{t}\bar{w}\bar{l}}{V_L} \Big\{ \overline{(M^p)^2}\,\overline{\alpha^2}(\bar{l} - x') + (1 - 1/C)\overline{(M^p)^2}\,\overline{\alpha_i \alpha_{i+1}}\, x' \Big\} \tag{3}$$

which, with $C \to \infty$ and $C\bar{t}\bar{w}\bar{l} = V_L$, represents a continuous medium as

$$R(x') = \frac{\overline{(M^P)^2}}{\bar{l}} \{ \bar{l} - x'(1 - \overline{\alpha_i \alpha_{i+1}}) \} \tag{4}$$

The correlation length L_c is obtained from $R(L_c) = 0$ to give

$$L_c = \frac{\bar{l}}{1 - \overline{\alpha_i \alpha_{i+1}}} \tag{5}$$

Certain values of $\overline{\alpha_i \alpha_{i+1}}$ take on particular meanings and these are shown in Table 1 but, briefly, when $\overline{\alpha_i \alpha_{i+1}} = 0$ there are no correlations and the medium is AC-erased, when $0 < \overline{\alpha_i \alpha_{i+1}} < 1$ there is a net magnetisation, and when $\overline{\alpha_i \alpha_{i+1}} = 1$ the medium is DC-erased or saturated.

Table 1: State of medium magnetisation and probability of magnetisation transition between neighbouring particles

$\overline{\alpha_i \alpha_{i+1}}$	p_t	State of medium magnetisation
1	0	DC saturated or magnetized: all particle magnetisations are in same direction with α_i and α_{i+1} both having same sign such that product is unity
$0 < \overline{\alpha_i \alpha_{i+1}} < 1$	$0 < p_t < 0.5$	Net magnetised: particles may be magnetised in either direction but there is net value in one or other direction
0	0.5	AC erased or net demagnetised: adjacent particle magnetisations have equal probability of being +1 or −1 such that average is zero

Since $\overline{\alpha_i \alpha_{i+1}} = 1 - 2p_t$, where p_t is the probability of a magnetisation transition between two adjacent particles [4], the correlation length can also be represented by

$$L_c = \bar{l}/2p_t \tag{6}$$

Figure 2 shows the correlation length as a function of p_t and reveals, as would reasonably be expected, that the correlation length grows as the probability of changes of magnetisation level gets less. L_c can be thought of as a scale factor relating to the magnetic interactions and as a representation of the 'domain' sizes in the medium. The corresponding power spectral density $P(k)$ is obtained from the Fourier transform of the autocorrelation function (4) via

$$P(k) = 2 \int_{x'=0}^{L_c} R(x') \cos(kx') dx \tag{7}$$

where $k = 2\pi/\lambda$ is the wavenumber and λ is the wavelength. Evaluation of (7) gives

$$P(k) = \frac{4\overline{(M^p)^2}}{k^2 \bar{l}} (1 - \overline{\alpha_i \alpha_{i+1}}) \sin^2\left(\frac{k\bar{l}}{2(1 - \overline{\alpha_i \alpha_{i+1}})} \right) \tag{8a}$$

$$= (M^p)^2 L_c \frac{\sin^2(kL_c/2)}{(kL_c/2)^2} \tag{8b}$$

The form of (8*b*) shows how the correlation length influences the power spectrum through a scaling on the k-axis and also an enhancement of the DC content. The

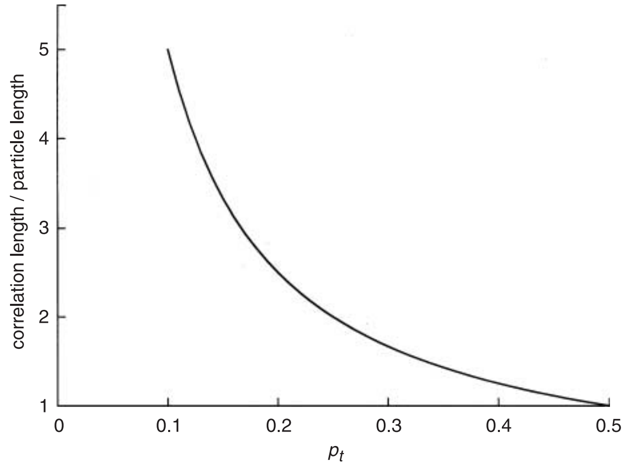

Fig. 2 *Correlation length as function of probability of there being magnetisation change between neighbouring particles*

power spectral density of (8*a*) can also be represented by

$$P(k) = \frac{8\overline{(M^p)^2}}{k^2 \bar{l}} p_t \sin^2\left(\frac{k\bar{l}}{4p_t} \right) \tag{9}$$

Figure 3 demonstrates these results with plots of the power spectral densities from (9), normalised by $\overline{(M^p)^2}$, as a function of wavenumber for three different values of p_t. It shows that as magnetic correlations increase, and p_t gets smaller, the DC content of the power spectrum rises in agreement with observations [5]. There will be a number of these chains forming the magnetic track and hence the power spectral density in (9) will be multiplied by the number of chains within the track width W.

When there are long-range correlations interactions additional to those of nearest neighbours have to be taken into account. These are represented by the different values of U in Fig. 4. Thus, following a similar line of analysis to

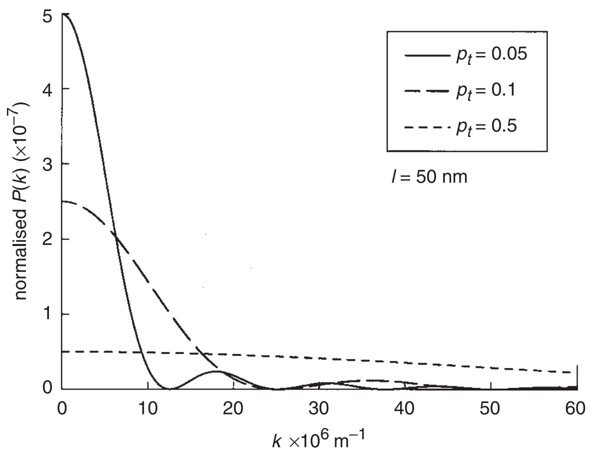

Fig. 3 *Power spectral density of infinitely long continous line of particles with different correlation strengths*

the foregoing, the overall autocorrelation function including short-range and long-range correlations was found to be

$$R(x') = \lim_{V_L \to \infty} \frac{1}{V_L} \sum_{j=0}^{U} \left\{ \sum_{i=1}^{C-j} (M_i^p)^2 \alpha_i \alpha_{i+j} t_i w_i (l_{i+j} - x') + \sum_{i=1}^{C-j-1} (M_i^p)^2 \alpha_i \alpha_{i+j+1} t_i w_i x' \right\} \tag{10}$$

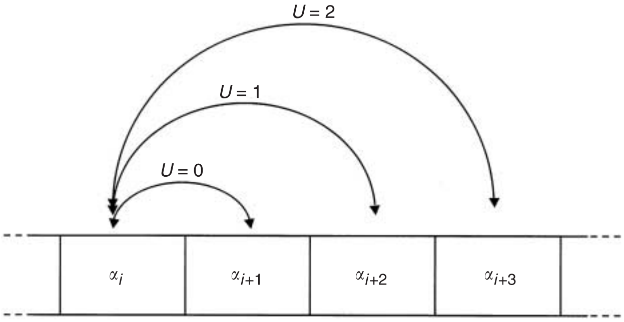

Fig. 4 *Chain of particles with different values of U representing different ranges of correlations*

Writing this in terms of averages gives

$$R(x') = \lim_{V_L \to \infty} \frac{\overline{(M^p)^2}}{V_L} \sum_{j=0}^{U} \{(C-j)\,\overline{\alpha_i \alpha_{i+j}}\,\bar{t}\bar{w}\,(\bar{l}-x') + (C-j-1)\,\overline{\alpha_i\alpha_{i+j+1}}\,\bar{t}\bar{w}x'\} \quad (11)$$

and rearranging after making $C \to \infty$ leads to

$$R(x') = \frac{\overline{(M^p)^2}}{\bar{l}} \sum_{j=0}^{U} \{\overline{\alpha_i\alpha_{i+j}}(\bar{l}-x') + \overline{\alpha_i\alpha_{i+j+1}}x'\} \quad (12)$$

Using (12), the correlation length determined from $R(L_c) = 0$ is found to be

$$L_c = \frac{\bar{l}}{1 - \frac{\sum_{j=0}^{U} \overline{\alpha_i\alpha_{i+j+1}}}{\sum_{j=0}^{U} \overline{\alpha_i\alpha_{i+j}}}} \quad (13)$$

which shows that long-range interactions further influence the correlation length. Comparison of (13) with (5) shows that the long-range interactions modify only the part of (5) involving interactions but not the overall form of it. To proceed further with (13) a knowledge of all the probabilities and conditions associated with the various ranges of interactions is needed. Probably this is best developed through studies of the micromagnetism of assemblies of particles and the interpretation of magnetisation distributions. Alternatively, experimental measurements of correlation length could be used to throw some light on their values. However, it is known that in a magnetic system of closely packed particles the short range interactions dominate and so will the terms of the order $\overline{\alpha_i\alpha_{i+1}}$.

3 Conclusions

The interactions depicted in this theory have all been represented by probabilities and, in the case of nearest neighbour interactions, this has led to power spectral densities and correlation lengths being represented by simple equations. The correlation lengths have been related to 'domain' sizes and these influence the shape of the power spectral densities by scaling the wavenumber axis and enhancing the DC content at low wavenumbers. The latter influence is confirmed in experimental observations [5].

The short-range correlations have been shown to have influences, revealed by the correlation length, which can be on scales much longer than individual particle sizes. The inclusion of long-range correlations complicates the picture and makes the correlation lengths and power spectral densities dependent on a range of interrelated probabilities whose values are not yet identifiable. These will, at some time, need to be determined, possibly from observed or predicted micromagnetic magnetisation distributions.

4 References

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