



## Can Behavioral Finance Explain the Term Structure Puzzles?

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### Abstract

We test two well-known behavioral models using the expectations of the short yield that are implicit in the term structure of interest rates. We find evidence that is consistent with these models. To investigate whether these behavioral biases are sufficient to explain the scale of the empirical rejections of the expectations hypothesis, we simulate behaviorally biased expectations of the short yield and use these to construct long yields. We apply the conventional tests to the simulated data and are able to generate rejections of the expectations hypothesis that are similar to those observed in practice.

Keywords: Behavioral finance; Expectations hypothesis of the term structure of interest rates; Momentum; Return reversals.

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## Introduction

The expectations hypothesis of the term structure of interest rates states that the yield on a long bond is determined by the expectations of short yields over the life of the bond, plus a risk premium. The expectations hypothesis (henceforth EH) is usually tested by examining whether the market's expectations of future changes in long and short yields, which are implicit in the term structure, are unbiased. The evidence from a large number of studies for different countries, different time periods and different bond maturities is that the EH is overwhelmingly rejected.<sup>1</sup>

There has inevitably been a sustained effort to explain the empirical failure of the EH. Perhaps the most obvious limitation of tests of the EH is that they assume that the risk premium is constant. If instead the risk premium is time-varying then tests of the EH are potentially biased. A number of studies have explored this possibility, and indeed tests that allow for a time-varying risk premium have generally produced weaker rejections of the EH (see, for example, Fama, 1984; Evans and Lewis, 1994; Mankiw and Miron, 1996). However, the results of these studies are sensitive to the choice of proxy for the risk premium, the bond maturities considered and the sample period used.<sup>2</sup> On balance, it would appear that while the assumption of a constant risk premium might partially explain the empirical failure of the EH, the scale of the rejection is simply too large to be fully accounted for by a time-varying risk premium (see Backus et al., 1994; Dai and Singleton, 2000; Duffee, 2002).

Another potential explanation for the rejection of the EH is that there are statistical problems with the tests. For example, Stambaugh (1988) shows that measurement error in the long yield potentially bias tests of the REH in favour of rejecting it. However Campbell and Shiller (1991) show that the EH is strongly rejected even after allowance is made for such measurement error. Bekaert et al. (1997) identify a further small sample bias in tests

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<sup>1</sup> See, for example, Shiller (1979), Shiller et al. (1983), Campbell and Shiller (1984), Mankiw and Summers (1984), Mankiw (1986), Campbell and Shiller (1991) and Campbell (1995). Hardouvelis (1994) demonstrates that the rejection of the REH is not confined to the US.

<sup>2</sup> See also Shiller et al. (1983), Jones and Roley (1983), Backus et al. (1987), Simon (1989), Froot (1989), Tzavalis and Wickens (1997) and Harris (2001).

of the EH.<sup>3</sup> They show by Monte Carlo simulation that even in the relatively large samples that are typically used in empirical work, this bias remains significant.<sup>4</sup> However the direction of the bias is such that the empirical evidence actually represents unambiguously *stronger* evidence against the EH than asymptotic theory would imply.

In this paper we take a different approach. We accept the empirical failure of the EH, but ask how it might be explained. There are two distinct ways in which the EH might fail. It might be that long yields are not set as the expectation of future short yields, or it might be that while the market *does* set long yields as the expectation of future short yields, short yield expectations are not *rational* expectations. We investigate this second possibility in this paper and test whether the same behavioral models that can explain rejections of the efficient markets hypothesis in equity markets might also explain expectations formation in the bond market.

The bond market offers an interesting opportunity to directly test behavioral models because the market's expectational errors at any date can be directly measured. This is in contrast to equity markets, where market expectations about specific realizations can only be inferred indirectly from prices and returns. In the bond market, expectations formed at any specific date, for the short yield at any future date, can be inferred from the term structure of interest rates and matched to the corresponding realization. This allows us to directly test the predictions that the models of behavioral biases make for the time series properties of expectational errors, and for revisions in expectations. Indeed, the attraction of directly working with expectations data has led to the use of laboratory tests of behavioral models (see, for example, Bloomfield and Hales, 2002). The data on expectations that is readily available in the bond market offers a useful opportunity to undertake an even more direct test of these models.

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<sup>3</sup> This bias is related to the downward bias of the OLS estimator of the autoregressive coefficient in the short yield model (see Kendall, 1954).

<sup>4</sup> For further discussion of the statistical properties of tests of the EH, see also Bekaert and Hodrick (2001), Kool and Thornton (2004) and Thornton (2005).

We investigate two classes of behavioral models.<sup>5</sup> The first builds on experimental evidence that individuals over-extrapolate from short runs of data. This characteristic is sometimes known as the ‘law of small numbers’ (LSN) and is a type of ‘representativeness’ bias (see, for example, Kahneman and Tversky, 1971). The LSN describes the way in which individuals expect the moments of a population to be reflected even in short samples of data that are drawn from that population. The implication of this model in the bond market is that there should be positive short run serial correlation in the one-step ahead forecast errors of the short yield, and negative serial correlation in average expectation errors of the short yield at longer horizons.

The second class of behavioral model that we examine builds on the widespread finding that individuals tend to be too conservative when reacting to new information. In particular, agents attach too much weight to their prior beliefs about the true model that generates the data, and too little weight to recent public news. Daniel et al. (1998) show that overconfidence in prior judgments about stocks can lead investors to give too little weight to new public information, compared with the weights that are specified by Bayes’ rule. This leads to initial underreaction to public news but, over time, agents learn of their mistake and so there are subsequent revisions in expectations that are of the same sign as the initial response to the news announcement. This model implies that revisions in expectations about the short yield for some specific future date will be positively serially correlated at short horizons.

We take three approaches to evaluating these behavioral models in the bond market. We first test whether the stylized facts of short-term momentum and long-term reversals in returns, which are the hallmarks of behavioral models in the equity market, are also present in the bond market. We then test whether the expectational errors for the short yield, and revisions in expectations, exhibit the systematic properties implied by the representativeness and conservatism. Finally we examine whether behavioral biases could be *sufficient* to explain the observed rejections of the EH in empirical work. We undertake a Monte Carlo experiment in which we simulate short yield data from a first order

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<sup>5</sup> There are other behavioral models that have been developed within the context of the equity market that do not have clear implications for bond returns. For example models where ‘winner’ stocks are sold and ‘loser’ stocks are held (see, for example, DeBondt and Thaler, 1985; DeBondt and Thaler, 1987).

autoregressive model (which is calibrated from the data), but generate expectations of the short yield assuming that agents exhibit the representativeness bias. We then construct long yields, according to the EH, as the average of the behaviorally biased forecasts of the short yield and apply the conventional tests to this simulated data.

The evidence from all three approaches is consistent with a behavioral explanation for the empirical rejection of the EH. In particular, we find that there is significant positive serial correlation in short term excess holding period returns in the bond market, and significant negative serial correlation in long term excess holding period returns. We also find that the predictions of the representativeness the conservatism biases for systematic patterns in errors in short yield expectations, and revisions in expectations, are confirmed in the data. Finally we find that the EH is very strongly rejected in the simulated data where short yield expectations are constructed with these behavioural biases. For plausible parameterizations of the model, the pattern of rejection in the simulated data across different tests and different bond maturities is very similar to the pattern of rejection that is observed in empirical tests of the EH.

Tests of the EH are greatly simplified by the use of zero coupon bond data. Since there are only a limited number of traded zero coupon bonds in practice, one must rely on synthetic data on zero coupon bond yields that are imputed from the yields of coupon-paying bonds. Most of the studies cited above use the synthetic zero coupon bond yield data of McCulloch and Kwon (1993). In this paper, we extend this data set to December 2004 using data on coupon-paying bonds from the CRSP US Treasury Database. We update the evidence on the EH by applying the conventional tests to this extended sample and confirm that the EH is again strongly rejected.

The outline of this paper is as follows. In the following section, we describe the construction of the new dataset of zero-coupon bond yields that we use in the empirical sections of the paper. Section 2 presents the theory of the EH, and the empirical tests that have been widely used to test the EH. We replicate these tests using the extended dataset. In section 3, we describe the representativeness and conservatism biases, and derive the testable implications of these biases for the bond market. The results of the tests of these biases are reported in Section 4. In Section 5 we report the results of repeating the

empirical tests of the EH using the simulated data that is behaviorally biased by construction. Section 6 concludes.

## 1. Data on Bond Yields

In this paper we use monthly zero-coupon bond yields on US Treasury securities for the period January 1952 to December 2004.<sup>6</sup> Many of the empirical studies of the EH described in the following section make use of the McCulloch and Kwon (1993) monthly US term structure data set.<sup>7</sup> The data consists of a monthly time series of estimated zero-coupon yields, par bond yields and instantaneous forward rates (and their respective standard errors) from December 1946 to February 1991. The data are continuously compounded, recorded as annual percentages. Synthetic zero-coupon bond yields are available for 56 maturities from overnight to 40 years.

For the purpose of this paper, we have updated the McCulloch and Kwon (1993) dataset to December 2004. The data are constructed using the tax-adjusted cubic spline method of McCulloch (1975).<sup>8</sup> The raw data were obtained from the CRSP US Treasury Database and include all available quotations on US Treasury bills, notes and bonds.<sup>9</sup> Since the raw data that we use originate from a different source, it is important to check the integrity of the resulting estimated zero-coupon bond yields. We therefore computed zero-coupon bond yields over a six-year overlapping period, August 1985 to February 1991, and compared these with the corresponding yields reported in the McCulloch and Kwon (1993) dataset.<sup>10</sup>

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<sup>6</sup> Although data are available from December 1946, the quality of the estimated data improves significantly after the Treasury Accord of 1951 and so only data after this period are used, as recommended by McCulloch and Kwon (1993).

<sup>7</sup> The McCulloch and Kwon (1993) zero-coupon bond yield dataset can be found at [www.econ.ohio-state.edu/jhm/ts/mcckwon/mccull.htm](http://www.econ.ohio-state.edu/jhm/ts/mcckwon/mccull.htm).

<sup>8</sup> The authors are indebted to J. Huston McCulloch for kindly providing the FORTRAN program that fits the term structure of interest rates using the tax-adjusted cubic spline method and for his valuable help in resolving a number of problems associated with the construction of the data set.

<sup>9</sup> Data on tax rates are obtained from the Internal Revenue Service, US Department of Treasury ([www.irs.gov](http://www.irs.gov)).

<sup>10</sup> We choose the start date of August 1985 for the overlapping period on the grounds of convenience. Before this date, there are many more irregular bonds in the raw data which have to be manually deleted. Also, McCulloch and Kwon stopped using long-term callable bonds as from this date. For these reasons, it is easier to match the numbers in the original McCulloch and Kwon data set for August 1985-February 1991.

Panel A of Table 1 reports summary statistics for the two datasets for the ten bond maturities that we use in this paper. For all ten bond maturities, the correlation between the two datasets is in excess of 0.99, and for all except the one month maturity, the correlation is in excess of 0.999.

[Table 1]

Figure 1 plots the estimated yields for the ten maturities over the overlapping period. For maturities greater than one month, there is no discernable difference between the two datasets. For the one-month maturity, there are some very minor discrepancies that arise mainly from the use of a different raw data source. Panel B of Table 1 reports summary statistics for the period covered by the McCulloch and Kwon data (January 1952 to February 1991), for the extended period (March 1991 to December 2004), and for the combined full sample. It is the full sample that we employ in the empirical sections of this paper.

[Figure 1]

Figure 2 plots the estimated yields over the full sample. A striking feature of the extended dataset is the evident structural break in each of the time series around 1980-82. Prior to this, yields of all maturities secularly increased, while after this, they secularly decreased. This has important implications for the empirical tests in the following sections. Under the EH, rational expectations of this change in the long term trajectory of interest rates would be impounded in long bond yields, and so there is no need to explicitly accommodate the structural break under the null hypothesis that the EH holds. However, under the alternative hypothesis that the EH does not hold, we cannot rule out the possibility, suggested by visual inspection of the data, that there is a corresponding structural break in agents' expectation errors, which is linked to the break between the long upward and downward trends. It is important that this structural break is incorporated in the empirical tests of the behavioral models. Omitting the structural break can induce systematic patterns in time series correlations that vanish once the structural break is introduced.

In order to establish the precise location of the structural break, we employ the structural stability test of Hansen (1992). This indicates a breakpoint between June 1981 and June

1982, depending on the regression estimated. For consistency, we assume a common breakpoint at December 1981.<sup>11</sup> We explicitly allow for the structural break in all of the empirical tests by including a dummy variable in each regression that is set to one for the period after the breakpoint and zero otherwise.

[Figure 2]

## 2. The Expectations Hypothesis: Theory and Evidence

Consider an  $n$ -period zero coupon bond with unit face value, whose price at time  $t$  is  $P_{n,t}$ . The yield to maturity of the bond,  $Y_{n,t}$ , satisfies the relation

$$P_{n,t} = \frac{1}{(1 + Y_{n,t})^n} \quad (1)$$

or, in natural logarithms,

$$p_{n,t} = -ny_{n,t} \quad (2)$$

where  $p_{n,t} = \ln(P_{n,t})$  and  $y_{n,t} = \ln(1 + Y_{n,t})$ . If the bond is sold before maturity then the log  $m$ -period holding period return,  $r_{n,t+m}^m$ , where  $m < n$ , is defined as the change in log price,  $p_{n-m,t+m} - p_{n,t}$ , which using (2) can be written as

$$\begin{aligned} r_{n,t+m} &= p_{n-m,t+m} - p_{n,t} \\ &= ny_{n,t} - (n-m)y_{n-m,t+m} \end{aligned} \quad (3)$$

The expectations hypothesis states that the expected holding period return for bonds of different maturities should be equal, except for a risk premium. Combined with the rational

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<sup>11</sup> We tested the robustness of the analysis to the location of the breakpoint. In general, we found that the empirical results are not sensitive to the exact choice of breakpoint. Indeed, for most of the regressions, excluding the dummy variable does not alter the qualitative conclusions that we draw.



expectations hypothesis, the expectations hypothesis of the term structure has a number of important implications for the relationships between bond yields, and their movement over time. In particular, the expectations hypothesis states that the expected  $n$ -period return on an investment in a series of one-period bonds should be equal to the (certain)  $n$ -period return on an  $n$ -period bond, which implies that the  $n$ -period long yield should be an average of the expected short yield over the following  $n$  periods, plus a constant risk premium. That is

$$y_{n,t} = \frac{1}{n} \sum_{i=0}^{n-1} E_t(y_{1,t+i}) + \phi_n \quad (4)$$

where  $\phi_n$  is the risk premium and  $E_t(\cdot)$  is the expectation conditional on the time  $t$  information set. The relation given by (4) is known as the expectations hypothesis (EH).

The earliest tests of the EH focus on the predictive ability of the expectations of future spot yields that are implicit in the term structure of interest rates. By combining expression (4) for bonds of two different maturities, we can define the  $m$ -period ‘forward’ yield for an  $n$ -period bond as

$$\begin{aligned} f_{n,t}^m &= ((n+m)y_{n+m,t} - my_{m,t})/n \\ &= E_t(y_{n,t+m}) + ((n+m)\phi_{n+m,t} - m\phi_m)/n \end{aligned} \quad (5)$$

The earliest tests of the EH directly examined whether the forward rates that are implied by the term structure are unbiased predictors of future interest rates. This can be tested using a regression of the form

$$y_{n,t+m} - y_{n,t} = \alpha_1 + \beta_1(f_{n,t}^m - y_{n,t}) + \varepsilon_{1,t+m} \quad (6)$$

If forward rates are unbiased then the slope coefficient,  $\beta_1$ , should be equal to unity, while the constant risk premium differential is captured by the intercept,  $\alpha_1$ . This regression has been estimated for values of  $m$  of between one month and twenty years, and for values of  $n$  of between one month and five years. While forward yields clearly contain information that

is relevant for future spot yields, the estimated coefficient,  $\beta_1$ , is usually found to be significantly less than unity (see, for example, Fama, 1984, and Fama and Bliss, 1987). Here, we estimate this regression using the extended dataset, including a dummy variable that is set equal to one for the period after December 1981 and zero otherwise, to capture the structural break identified in the previous section.

$$y_{n,t+m} - y_{n,t} = \alpha_1 + \gamma_1 D_t + \beta_1 (f_{n,t}^m - y_{n,t}) + \varepsilon_{1,t+m} \quad (6a)$$

Table 2 reports the estimated parameters from this regression for two bond maturities and a range of forward horizons. In particular, Panel A reports results for  $n = 1$  month and  $m = 1, 3, 6, 9$  and 12 months and, while Panel B reports results for  $n = 12$  months and  $m = 12, 24, 36, 48, 60$  and 120 months. Panel A therefore replicates the results of Fama (1984) over the extended sample while Panel B replicates the results of Fama and Bliss (1987). Standard errors are reported in parentheses.

[Table 2]

For the one-month yield, the estimated slope coefficient is significantly less than unity for all horizons, at first declining with maturity and then rising with maturity. The coefficient on the dummy variable is highly significant in all cases, highlighting the importance of the structural break. For the 12-month yield, the estimated slope coefficient is significantly less than unity for the 12-month and 24-month horizons, but significantly greater than unity for longer horizons up to 60 months. For the 120-month horizon, the coefficient is greater than unity, but not significantly so. Again the coefficient on the dummy variable is highly significant in all cases. These results are very similar to those reported by Fama (1984) and Fama and Bliss (1987), and strongly reject the EH.

A second way to test the predictions of the EH is to focus on the predictive ability of the yield spread between long maturity and short maturity bonds, defined as  $s_{n,t} = y_{n,t} - y_{1,t}$ . In particular, rearranging equation (4) gives

$$\sum_{i=1}^{n-1} \frac{E_t y_{1,t+i}}{n-1} - y_{1,t} = \frac{n}{n-1} (y_{n,t} - y_{1,t}) + \phi_n \quad (7)$$

which states that the yield spread, when suitably scaled, should predict the cumulative expected change in the short yield over the life of the long bond. Alternatively, combining equation (4) for two adjacent bond maturities, and rearranging, gives.

$$E_t y_{n-1,t+1} - y_{nt} = \frac{1}{n-1}(y_{nt} - y_{1t}) + \phi_{n-1} - \frac{n}{n-1}\phi_n \quad (8)$$

which states that the yield spread, when suitably scaled, should predict the following period's expected change in the yield on the long bond. These two predictions of the EH can be tested with regressions of the form

$$\sum_{i=1}^{n-1} \frac{y_{1,t+i}}{n-1} - y_{1,t} = \alpha_2 + \beta_2 \frac{n}{n-1}(y_{n,t} - y_{1t}) + \varepsilon_{2,t+1} \quad (9)$$

$$y_{n-1,t+1} - y_{n,t} = \alpha_3 + \beta_3 \frac{1}{n-1}(y_{n,t} - y_{1t}) + \varepsilon_{3,t+1} \quad (10)$$

We call (9) the short yield regression and (10) the long yield regression. If the EH holds then the coefficients  $\beta_2$  and  $\beta_3$  should be equal to unity, while the intercepts  $\alpha_2$  and  $\alpha_3$  capture the constant risk premium terms. Estimating regression (10) generates a very significant rejection of the EH. The coefficient  $\beta_3$  is typically found to be significantly less than unity, and falls with the maturity of the long bond. For long maturity bonds, it is significantly less than zero. The coefficient  $\beta_2$  in equation (9), in contrast, is typically found to be significantly less than unity for short maturity bonds, but it rises with maturity. For long maturity bonds, it is often found to be significantly greater than unity. (see, for example, Campbell and Shiller, 1991; Bekaert et al., 1997; Bekaert and Hodrick, 2001).<sup>12</sup> The fact that regression (10) delivers a significant rejection of the EH but regression (9) does not, at least for some bond maturities, is ostensibly puzzling (see, for example,

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<sup>12</sup> Campbell and Shiller (1991) also test the REH using analogous regressions based on the yield spread between all possible pairs of bond maturities,  $s_{n,t} = y_{n,t} - y_{m,t}$ , for  $n$  between two months and 120 months and for  $m$  between one month and 60 months. The REH is strongly rejected for almost all pairs of bonds.

Campbell, 1996). However, Bekaert et al. (1998) show that while both regression (9) and regression (10) are subject to small sample biases, the bias is much greater for regression (9) than it is for regression (10). Once this small sample bias is allowed for, regression (9) also delivers a decisive rejection of the EH.

We estimate these regressions, again including a dummy variable that takes the value of unity for the period after December 1981.

$$\sum_{i=1}^{n-1} \frac{y_{1,t+i}}{n-1} - y_{1,t} = \alpha_2 + \gamma_2 D_t + \beta_2 \frac{n}{n-1} (y_{n,t} - y_{1t}) + \varepsilon_{2,t+1} \quad (9a)$$

$$y_{n-1,t+1} - y_{n,t} = \alpha_3 + \gamma_3 D_t + \beta_3 \frac{1}{n-1} (y_{n,t} - y_{1t}) + \varepsilon_{3,t+1} \quad (10a)$$

Table 3 reports the estimated parameters from regressions (9a) and (10a) using the extended dataset. For regression (9a), the standard errors are estimated using the Newey and West (1987) estimator to allow for the fact that the dependent variable is overlapping. The regressions are estimated for  $n = 3, 6, 9, 12, 24, 36, 48, 60$  and 120 months. For the short-yield regression (9a), the estimated slope coefficient is significantly lower than unity for short maturity bonds, and initially falls with maturity up to nine months, but then rises with maturity. For the 120-month bond, the coefficient is significantly greater than unity. For the long yield regression (10a), the estimated slope coefficient is negative and significantly lower than unity in all cases, and falls monotonically with maturity. For all but the three-month bond, the coefficient is also significantly less than zero.

These results for the extended sample are consistent in all respects with those reported by Campbell and Shiller (1991) and Bekaert et al. (1997) for earlier periods. Like these studies, we find that the long yield regression rejects the EH decisively but evidence against the EH from the short yield regression is much weaker. We also find that the strength of the rejection varies systematically with the horizon of the long bond in both regressions, in the same way that has been reported in earlier work. Why the strength of the rejection should vary with the horizon of the long bond is an interesting and unexplained feature of the empirical evidence on the EH.

[Table 3]

A third way to test the EH is the vector autoregression (VAR) approach of Campbell and Shiller (1991). In particular, a  $p$ th-order VAR for the  $n$ -period spread,  $s_{n,t}$ , and the change in the short yield,  $\Delta y_{1,t}$ , can be written in companion form as

$$Z_{n,t} = AZ_{n,t-1} + \varepsilon_{4,t} \quad (11)$$

where  $Z_{n,t}$  is a  $(2p \times 1)$  vector comprising the current value and  $p - 1$  lags of  $s_{n,t}$  and the current value and  $p - 1$  lags of  $\Delta y_{1,t}$ ,  $A$  is a  $(2p \times 2p)$  matrix of parameters and  $\varepsilon_{4,t}$  is a  $(2p \times 1)$  vector of errors. Forecasts of the  $n$ -period spread and the change in the short yield are then given by  $\hat{Z}_{n,t+i} = A^i Z_{n,t}$ . Using the expectations hypothesis relation (7), we can then define the ‘theoretical’ spread as

$$\tilde{s}_{n,t} = e' A [I - (1/n)(I - A^n)(I - A)^{-1}] (I - A)^{-1} Z_{n,t} \quad (12)$$

where  $e$  is a  $(1 \times 2p)$  ‘selection’ vector, such that  $e' Z_{n,t} = s_{n,t}$  and  $I$  is the  $(2p \times 2p)$  identity matrix (see, for example, Campbell and Shiller, 1991). Since the conditioning information in the VAR includes the current  $n$ -period spread, which itself embodies the market’s expectations of future short yields over the life of the long bond, the theoretical spread should be equal to the actual spread. Campbell and Shiller (1991) suggest the following two tests of the EH. Firstly, the correlation between the theoretical spread and the actual spread should be equal to unity. Secondly, the ratio of the standard deviation of the theoretical spread to the standard deviation of the actual spread should be equal to unity. Using the McCulloch (1987) dataset, Campbell and Shiller (1991) find that while the correlation coefficient is indeed close to unity, the ratio of the standard deviations is typically around 0.5, thus strongly rejecting the EH.

Table 4 reports the correlation coefficient and standard deviation ratio for  $n = 3, 6, 12, 24, 36, 48, 60$  and 120 months for the extended sample. The VAR was specified with a lag

length of four, chosen on the basis of the Schwartz Bayesian criterion, and includes a dummy variable for the post-December 1981 period in each of the VAR equations. The dummy variable is also incorporated into the expression for the theoretical spread given by equation (12). The correlation coefficient rises with maturity, and for long maturity bonds, it is not significantly different from one. For short maturity bonds, the correlation coefficient is significantly less than one. The standard deviation ratio has a U-shaped relationship with maturity, but is significantly lower than one for all bond maturities. These results for the extended sample are consistent with the findings of Campbell and Shiller (1991), again leading to a rejection of the EH.

[Table 4]

This section has replicated the existing empirical tests of the EH for the extended sample of monthly US zero-coupon bond yields over the period January 1952 to December 2004. Taken together, these tests represent overwhelming evidence against the EH. In the following section, we turn to the question of how this rejection of the EH might be explained.

### **3. Behavioral Models**

In this section we describe in more detail the nature of the two behavioral biases that underpin the models that we test, explain how models have been built on these biases to explain anomalies in the equity market, and set out their testable implications for the bond market.

#### **3.1 Representativeness**

The ‘representativeness’ bias describes the belief that a randomly drawn sample of data will reflect the characteristics of the population from which it is drawn more closely than sampling theory would predict. The representativeness bias is related to two specific behavioral biases that have been documented in the psychology literature. The first is ‘base rate neglect’, which describes the finding that subjects put too little weight on the unconditional probability of observing a particular sample. The second is ‘sample size neglect’ or the ‘law of small numbers’, which describes the finding that subjects

overestimate the statistical relevance of information that is contained in the sample (see Tversky and Kahneman, 1971). Both base rate neglect and sample size neglect cause subjects to overweight (compared to a Bayesian) the importance of a given sample of data, when drawing inferences about the population from which it is drawn.

Barberis et al. (1998) and Rabin (2002) develop the implications of the representativeness bias for returns in equity markets. One consequence is sometimes described as the ‘gambler’s fallacy’, which describes the finding that when subjects are asked to forecast drawings, with replacement, from an urn with 50 percent red balls and 50 percent black balls, they tend to forecast a black ball with a probability greater than 50 percent if a red ball was drawn previously. Applied to a model of earnings with i.i.d. shocks, this implies that if one earnings shock is negative then investors will expect the following shock to be positive with probability greater than 50 percent. However, under the true model, the next period’s innovation is positive with 50 percent probability, and hence investors will experience a second negative surprise with more than 50 percent probability. Barberis et al. (1998) and Rabin (2002) show that models that have this structure result in momentum in abnormal returns in the short run, an empirical feature of equity returns that is well documented. The corresponding implication of these biases, which we test in the bond market, is that we should expect positive short run serial correlation in one-step ahead expectation errors for spot rates, and momentum in excess returns on long bonds in the short run.

The ‘gambler’s fallacy’ describes how subjects interpret sample data, *given their beliefs* about the model that generated the sample. But it also has implications for how subjects *revise their beliefs* about the model. In particular, subjects tend to employ ‘over-inference’, meaning that when they observe a *series* of observations that don’t accord with their original beliefs about the true model (perhaps a chance drawing of several red balls from the urn), they too readily interpret this as evidence that their original beliefs were incorrect, and update their estimate of the true data generating model too quickly relative to a Bayesian. In the context of the above example, following a chance drawing of several red balls, they infer too quickly that the urn contains more than 50 percent red balls.

Barberis et al. (1998) employ this idea to explain long-term return reversals in the stock market, another well documented feature of equity returns. In particular, they posit that

while earnings actually follow a random walk, investors believe that earnings are either drawn from a ‘mean-reverting’ regime, or from a ‘trending’ regime. Investors believe that the dynamic process governing earnings switches exogenously between these two regimes. By relying on recent earnings performance, investors ‘identify’ the current regime and forecast next period’s earnings accordingly. For example if investors experience a series of positive earnings surprises for a company (which results in a series of positive excess returns), they interpret this as evidence that the company is a ‘high growth’ company, and consequently predict that future earnings will also be high. In reality, however, this may just be an ‘average growth’ company that happened to have experienced a run of good earnings by chance. Therefore investors will subsequently, on average, experience a negative earnings surprise (relative to their expectations based on the revised model) leading to negative excess returns. Thus, over the longer run, stock returns will be negatively serially correlated (see, for example, DeBondt and Thaler, 1985; DeBondt and Thaler, 1987; Jegadeesh and Titman, 2001).

The implications of over-inference for the bond market is that investors will revise their model for the short yield after observing a series of positive shocks to the short yield, and forecast correspondingly higher values of the short yield into the future. In due course, it transpires that these expectations are too high, which results in a series of negative short yield surprises. In this way we should expect negative serial correlation in excess returns on long bonds at longer horizons. We therefore test the following hypotheses about excess returns in the bond market.

H<sub>1</sub>:  $r_{n,t+m} - my_{m,t}$  is positively serially correlated for small values of  $m$

H<sub>2</sub>:  $r_{n,t+m} - my_{m,t}$  is negatively serially correlated for large values of  $m$

These tests are analogous to the tests of momentum and return reversals that have been conducted in the equity market. The bond market, however, offers an opportunity for more direct tests of these behavioral models of expectational errors, since data on expectations of the short yield at any future date can be inferred from the term structure and matched to the corresponding realizations of the short yield. In particular, we are able to test whether expectation errors for the short yield are positively serially correlated at short horizons and



negatively serially correlated at longer horizons. This yields the following testable hypotheses.

H<sub>3</sub>:  $y_{1,t+1+m} - E_t y_{1,t+1+m}$  is positively serially correlated for small values of  $m$

H<sub>4</sub>:  $\frac{1}{n-1} \sum_{i=1}^{n-1} (y_{1,t+i} - E_t y_{1,t+i})$  is negatively serially correlated at lag  $n$  for large values of  $n$

### 3.2 Conservatism

When confronted with the possibility that the true model is changing, the representativeness bias implies that subjects are too quick to adopt a new model because, relative to a Bayesian, they overweight the importance of *the sample* that they observe. In contrast, ‘conservatism’ describes a subject’s response to a *single observation* of news.<sup>13</sup> It describes the observation that individuals are too *slow* to revise their beliefs, effectively attaching too much weight to their prior beliefs about the true model, and too little weight to new information. Daniel et al. (1998) build on the closely related ‘overconfidence bias’, which has similar testable implications. They show this bias can lead to underreaction to public news as agents’ expectations following the news are not immediately revised to the full extent that would be justified by Bayesian updating. However, over time agents learn of their initial underreaction, and so there are subsequent revisions in agents’ expectations that are of the same sign as the initial response to the news announcement. This process is consistent with evidence of momentum in returns and is further confirmed in the equity market with evidence of underreaction to public news, for example earnings announcements.

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<sup>13</sup> On the face of it, the conservatism bias (which implies that agents underreact to new information) is at odds with the representativeness bias (which assumes that agents overreact to new information). One resolution, suggested above, is that the conservatism bias describes individuals’ response to single observations of news, while the representativeness bias describes the response to samples of observations. Barberis (2003) offers another way of reconciling the two biases: If an observed sample is representative of a ‘salient’ model (i.e. one that concurs with the subject’s belief about the set of probable models), then subjects will overweight the data. Conversely, if the observed sample is not representative of a salient model, subjects will tend to underweight the data.

The existence of conservatism implies that investors should initially underreact to public news about future interest rates, so that *revisions in expectations* of future short yields will be of the same sign in the periods following public news, as they are in the month in which the news became public. Data on revisions in expectations can be inferred from the term structure and so this model can be tested directly in the bond market. This leads to our fifth testable hypothesis:

$$H_5: E_{t+1}y_{1,t+m} - E_t y_{1,t+m} \text{ is positively serially correlated for small values of } m$$

#### 4. Evidence for Behavioural Models

##### 4.1 Evidence of Momentum and Return Reversals in the Bond Market

We start by investigating whether the stylized features of short-term momentum and long-term return reversals that have been documented in the equity market, also exist in the bond market. In particular, we estimate the degree of serial correlation in excess holding period returns (hypotheses H<sub>1</sub> and H<sub>2</sub>) using the following regression.

$$r_{n,t+m}^m - my_{m,t} = \alpha_4 + \gamma_4 D_t + \beta_4 (r_{n,t}^m - my_{m,t-m}) + \varepsilon_{4,t+m} \quad (13)$$

where the  $m$ -period holding period return for an  $n$ -period bond,  $r_{n,t}^m$ , is defined by equation (3). In all the tests in this section we include a dummy variable that takes the value of unity for the period after December 1981 as explained above.

Table 5 reports the results of estimating regression (13) for  $n = 2, 3, 6, 9, 12, 24, 36, 48, 60$  and 120 months and  $m = 1, 2, 3, 6, 9, 12, 24, 36, 48$  and 60 months. The regression is estimated by OLS and standard errors are computed using the Newey and West (1987) estimator to allow for the fact that the dependent variable is overlapping. Table 5 reveals that patterns of short-run momentum and long-run return reversals, similar to those that have been documented in the equity market, are also present in the bond market. In particular, for the shortest holding period of one month, there is very significant positive serial correlation in excess holding period returns for all bond maturities. The degree of

serial correlation decreases with bond maturity, but remains marginally significant even for long maturity bonds.<sup>14</sup>

For longer holding periods – between 24 and 120 months – there is very significant negative serial correlation in excess holding period returns for longer maturity bonds, suggesting that there are return reversals in excess holding period returns in the bond market. This pattern of momentum and return reversals in the bond market is similar to that found in the equity market, although the horizon over which there is significant momentum in excess returns in the bond market is shorter than is typically found in the equity market (see, for example, Jegadeesh and Titman, 1993).

[Table 5]

#### 4.2 Evidence from Errors and Revisions in Expectations

We next test the specific predictions that the representativeness and conservatism models make for expectational errors. Table 6 reports the results of the test of the short term implications of the representativeness bias (hypothesis H<sub>3</sub>) which is that the one-step ahead expectation error of the short yield is positively serially correlated. In order to test this hypothesis, we report the results of estimating the following regression

$$y_{1,t+m} - E_{t+m-1}y_{1,t+m} = \alpha_5 + \gamma_5 D_t + \beta_5 (y_{1,t} - E_{t-1}y_{1,t}) + \varepsilon_{5,t} \quad (14)$$

where  $E_t y_{1,t+m} = m y_{m,t} - (m-1) y_{m-1,t}$  is the forecast of  $y_{1,t+m}$  that is implicit in the current term structure of interest rates. The regression is estimated for lags  $m = 1, 2, 3, 4, 5$  and 6 months.

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<sup>14</sup> Mankiw (1986) estimates the serial correlation in the excess holding period return for long term US Treasury bonds relative to the three month Treasury bill rate, using quarterly data over the period 1961-84, and finds that the first order autocorrelation coefficient is 0.02. The long bond yield used is an aggregate yield of bonds with maturities of 10 years or over. This is broadly consistent with the findings reported here.

There is very significant positive serial correlation in one-step ahead expectation errors for the short yield for horizons of one and two months. For longer horizons, there is no significant serial correlation. This is consistent with the short run predictions of the LSN.

[Table 6]

Table 7 reports the results of the test of the long term implications of the representativeness bias, (hypothesis  $H_4$ ) which is that the average (measured here over 6 or more months) expectation error of the short yield is negatively serially correlated. Note that from (4)

$$y_{n,t} = \frac{1}{n} \sum_{i=0}^{n-1} E_t(y_{1,t+i}) + \phi_n, \quad \text{and so} \quad \frac{1}{n-1} \sum_{i=1}^{n-1} (y_{1,t+i} - E_t y_{1,t+i}) = \frac{1}{n} \sum_{i=0}^{n-1} y_{1,t+i} - y_{n,t} - \phi_n. \quad \text{We}$$

therefore test  $H_2$  using the following regression

$$\frac{1}{n} \sum_{i=0}^{n-1} y_{1,t+n+i} - y_{n,t+n} = \alpha_6 + \gamma_6 D_t + \beta_6 \left( \frac{1}{n} \sum_{i=0}^{n-1} y_{1,t+i} - y_{n,t} \right) + \varepsilon_{6,t} \quad (15)$$

The regression is estimated for horizons  $n = 6, 12, 18, 24, 36, 48, 60$  and 120 months. The estimated slope coefficient is significantly negative for horizons of 18 months and more, rising at first and then falling. The strongest negative serial correlation is at a horizon of 48 months. This is consistent with the implication of the representativeness bias which implies agents will revise their model too much in response to a short run of surprises resulting in expectation errors that are negatively correlated.

[Table 7]

Table 8 reports the results of the test of the conservatism bias, hypothesis  $H_5$ , which is that expectation revisions are positively serially correlated at short lags. In order to test this hypothesis, we estimate the following regression

$$E_{t+1} y_{1,t+m+1} - E_t y_{1,t+m+1} = \alpha_7 + \gamma_7 D_t + \beta_7 (E_t y_{1,t+m+1} - E_{t-1} y_{1,t+m+1}) + \varepsilon_{7,t} \quad (16)$$

where  $E_t y_{1,t+m} = m y_{m,t} - (m-1) y_{m-1,t}$ . The regression is estimated for lags  $m = 1$  to 12 months.

Consistent with hypothesis  $H_5$ , there is positive serial correlation in the one-step ahead expectation revisions for the short yield at all lags considered. The pattern of serial correlation generally increases with autocorrelation lag up to about seven months, and then declines monotonically. For all cases except 12 months, the positive serial correlation in expectation revisions is statistically significant. Therefore, the prediction of the conservatism bias – that one-step ahead expectation revisions in the short yield are positively serially correlated – is strongly supported by the data.

[Table 8]

## 5. Monte Carlo Simulation

The empirical evidence reported in the previous section suggests that short yield expectations are biased in a way that is consistent with two specific well-known behavioral models. However are behavioral models *sufficient* to explain why the empirical failure of the EH? In order to address this issue, we undertake a Monte Carlo experiment. We generate simulated short yield data from a model that is calibrated using the actual yield data. We then simulate expectations of the short yield that are, by construction, subject to the representativeness bias. Finally we construct estimates of the long yield by inserting these short yield expectations into the EH relation described in equation (4). We then test the EH using the tests described in Section 2.

We simulate data for the short yield using the following first-order autoregressive (AR1) model.

$$y_{1,t} = 0.072 + 0.985y_{1,t-1} + v_t \quad \text{var}(v_t) = 0.231 \quad (17)$$

The model was calibrated by estimating it over the full sample of monthly short yields from January 1952 to December 2004. The lag length of one was chosen on the basis of the Schwartz Bayesian Criterion. The estimated parameters of the AR1 model were adjusted for the small sample bias of Kendall (1954). We use (17) to generate 636 observations of the short yield (which matches the empirical sample size used to test the EH in Section 2)

with  $v_t$  drawn from a normal distribution. For simplicity, we do not include a structural break in the data generating process, nor in the tests of the EH performed on the simulated data. Limited experimentation suggested that the qualitative conclusion drawn from the simulation experiments were independent of the inclusion of a structural break.

In order to simulate behaviorally biased expectations of the short yield, we construct a model of expectations based on the representativeness bias, incorporating both the ‘gambler’s fallacy’ element (which is relevant for short-run expectations) and the ‘over-inference’ element (which is relevant for long-run expectations). We assume that in forecasting the short yield, agents start with the ‘true’ model given by (17). However, in order to incorporate the short run and long run implications of the bias, we modify (17) in two ways. Firstly, we assume that when agents forecast the following period’s short yield, they predict that there will be a surprise that is opposite in sign to the current period’s surprise, although we allow for the possibility that they may predict it to be smaller in magnitude. Secondly, we assume that when agents experience a series of surprises that are non-zero on average, forecasts of the future short yield are adjusted as if agents had revised their expectations of the mean of the short yield model in the same direction as the average lagged surprise. Again, we allow for the possibility that the model revision is smaller in magnitude than the actual average forecast error. We assume that the horizon over which investors measure average forecast errors is 12 months. This lead to the following model for simulating expectations of the short yield.

$$\hat{y}_{1,t+1} = 0.072 + \theta \frac{1}{12} \sum_{i=1}^{12} \hat{v}_{t-i} + 0.985 y_{1,t} - c \hat{v}_t \quad (18)$$

$$\text{where } \hat{v}_t = y_t - \left( 0.072 + \theta \frac{1}{12} \sum_{i=1}^{12} \hat{v}_{t-i-1} + 0.985 y_{1,t-1} \right).$$

When  $\theta=0$  and  $c=0$ , the forecasting model (18) reduces to the ‘true’ model given by (17). Increasing  $\theta$  increases the importance of the short run component of the representativeness bias, while increasing  $c$  increases the importance of the long run component of the representativeness bias. To simulate data from this model, we must set values of  $c$  and  $\theta$ . Calibrating such a model is difficult, since we have no information

about the representativeness bias that would allow us to measure its quantitative importance in such a model. We therefore instead report results for a range of values of  $c$  and  $\theta$ . In particular, we set  $c = 0.00, 0.10$  and  $0.20$  and  $\theta = 0.00, 0.10, 0.20$  and  $0.30$ .

Once we have simulated the actual short yield data,  $y_{1,t}$ , and the behaviorally biased expectations of the short yield,  $\hat{y}_{1,t}$ , we construct simulated long yield data,  $y_{n,t}$ , using the expectations hypothesis relation (4), for bond maturities  $n = 3, 6, 9, 12, 24, 36, 48, 60$  and  $120$  months. We set the risk premium,  $\phi_n$ , equal to zero for all maturities. We then test the EH using (i) the forward yield regression given by (6), (ii) the yield spread regressions given by (9) and (10) and (iii) the VAR tests based on the theoretical spread given by (12). The simulation is performed using 1000 replications. In each case, we report the average estimated coefficient and the standard deviation of the estimated coefficient across the 1000 simulations. For reasons of brevity, we report only the estimated slope parameter in each regression.

Table 9 reports the results of estimating the forward yield regression given by (6) for  $n = 1$ , for the different values of  $c$  and  $\theta$ . For  $c = 0.00$  and  $\theta = 0.00$  (which corresponds to the rational expectations), the slope coefficient is significantly greater than one at all forward horizons. This reflects the small sample bias of Bekaert, Hodrick and Marshall (1997), which arises from the use of a highly persistent autoregressive process to generate the data. Holding  $c$  constant, increasing  $\theta$  leads to a reduction in the value of the slope coefficient, although for  $c = 0.00$ , the coefficient remains greater than one in all cases. The impact on the slope coefficient is independent of the forward horizon. As  $c$  increases, however, the coefficient rapidly declines, and for short horizons, the coefficient is lower than unity. For  $c = 0.20$  and  $\theta = 0.20$ , the slope coefficient is significantly lower than unity for the one, three and six month horizons, and marginally greater than unity for the nine and twelve month horizons. Table 10 reports the results of estimating the forward yield regression given by (6) for  $n = 12$ . In contrast with the case for  $n = 1$ , increasing  $c$  for a given value of  $\theta$  does not change the estimated slope coefficient very much, but increasing  $\theta$  for a given value of  $c$  leads to a reduction in the estimated slope coefficient, particularly for shorter horizons. For  $c = 0.20$  and  $\theta = 0.20$ , the estimated slope coefficient is marginally lower than one for the 12 month horizon and marginally greater than one for longer horizons. Comparing these simulation results with the corresponding

results using the actual yield data in Table 2, we can see that the simulated behavioral bias goes some way towards explaining the rejection of the EH that we observe in practice using the forward yield regression. In particular, it is able to generate estimated coefficients that are significantly lower than unity for short horizons and significantly greater than unity for long horizons.

[Tables 9 and 10]

Table 11 reports the results of estimating the short yield regression given by (9) for the different values of  $c$  and  $\theta$ . For  $c = 0.00$  and  $\theta = 0.00$ , the slope coefficient is significantly greater than one for all bond maturities, owing to small sample bias. The average estimated values for this case are consistent with those reported by Bekaert, Hodrick and Marshall (1997) in their simulation experiments. Increasing  $\theta$  for a given value of  $c$  leads to a reduction in the estimated slope coefficient, although it does not appear to be monotonic. Increasing  $c$  leads to a further small reduction in the estimated slope coefficient. For  $c = 0.20$  and  $\theta = 0.20$ , the estimated slope coefficient is marginally lower than unity for short maturity bonds and marginally higher than unity for long maturity bonds. Table 12 reports the results of estimating the long yield regression given by (10). Here, increasing  $c$  has little impact on the estimated slope coefficient, while increasing  $\theta$  has a substantial impact. For  $c = 0.10$  and  $c = 0.20$ , the slope coefficient is not only less than unity, but also less than zero for almost all values of  $\theta$ . The slope coefficient declines with bond maturity for all combinations of  $c$  and  $\theta$ .

Comparing these results with those reported in Table 3 using the actual yield data, we can again see that the simulated behavioral bias captures many of the features of the empirical rejections of the EH. In particular, for the short yield regression, for higher (but still modest) values of  $c$  and  $\theta$  the estimated slope coefficient is increasing in maturity, lower than one for short maturity bonds and higher than one for long maturity bonds. For the long yield regression, the slope coefficient is decreasing in maturity, lower than one for all bond maturities, and lower than zero for all but the shortest maturity bonds.

[Tables 11 and 12]



Table 13 report the results of estimating the correlation coefficient between the actual (simulated) yield spread and the theoretical (simulated) yield spread, given by equation (12), for the different values of  $c$  and  $\theta$ . Again, for  $c = 0.00$  and  $\theta = 0.00$ , the correlation coefficient is significantly greater than one for all bond maturities owing to the small sample bias of Bekaert, Hodrick and Marshall (1997). The correlation coefficient declines as either  $c$  or  $\theta$  increases, although for  $\theta$ , the relationship is not monotonic. For  $c = 0.10$  and  $c = 0.20$ , the correlation coefficient is significantly less than unity for short maturity bonds, but close to unity for longer maturity bonds. Table 14 reports the results of estimating the standard deviation ratio for the actual (simulated) yield spread and the theoretical (simulated) yield spread, given by equation (12). Again, increasing either  $c$  or  $\theta$  reduces the standard deviation ratio, particularly for short maturity bonds. For  $c = 0.20$  and  $\theta = 0.30$ , the estimated standard deviation ratio is significantly lower than one for all bond maturities.

Comparing these results for the simulated data with those reported in Table 4 once again suggests that the two behavioral biases can potentially explain the rejection of the EH. In particular, the correlation coefficient between the actual yield spread and the theoretical yield spread is lower than unity for short maturity bonds, rising with maturity and approximately equal to one for long maturity bonds. In contrast the standard deviation ratio is significantly lower than unity for all bond maturities, rising slowly with bond maturity. This pattern of results is replicated quite closely in the simulated data.

## 6. Conclusion

There is overwhelming evidence that the expectations hypothesis (EH) does not describe how long yields are determined in practice. We take this evidence at face value and ask how long yields might be set, if not by the EH. We explore the possibility that the EH fails because short yield expectations are subject to behavioral biases, rather than because the hypothesis that long yields are determined by expected short rates is false. To explore this idea, we draw on the well-established literature on behavioral finance that has been developed to explain the stylized features of short-term momentum and long-term return reversals in equity returns. We focus on two particular classes of behavioral models – those based on the representativeness bias and the conservatism bias – and derive the testable

implications of these models for expectations in the bond market. In contrast with the equity market – where the markets’ expectations of earnings are not observable – expectations of the short yield can be imputed from the term structure of interest rates. The bond market therefore offers a valuable opportunity to directly test the implications of behavioral models for expectational errors. We find that the predictions of these models are strongly supported by the data, suggesting that investors in the bond market are indeed subject to these behavioral biases.

To investigate whether these biases might be *sufficient* to explain the reported rejections of the EH we undertake a simulation experiment in which we generate expectations of the short yield that are subject to the same two biases. We then construct long yields from these short rate expectations as specified by the EH, but where the expectations are subject to the two behavioral biases. We test the EH using this synthetic data and find that the tests that the EH is strongly rejected using the same tests that have been applied to empirical data. The specific patterns of rejections across tests and bond maturities are very similar to those reported in the empirical literature. We infer that the same behavioral biases that have been documented in the equity market have the potential to explain the rejections of the EH in the bond market. The evidence that the same biases arise in both the bond and the equity markets, and can moreover explain a pre-existing puzzle in the bond market, provides further support for behavioral finance. In particular these results address the criticism of Fama (1998), that behavioral finance can only explain the puzzles that it was specifically designed to explain.

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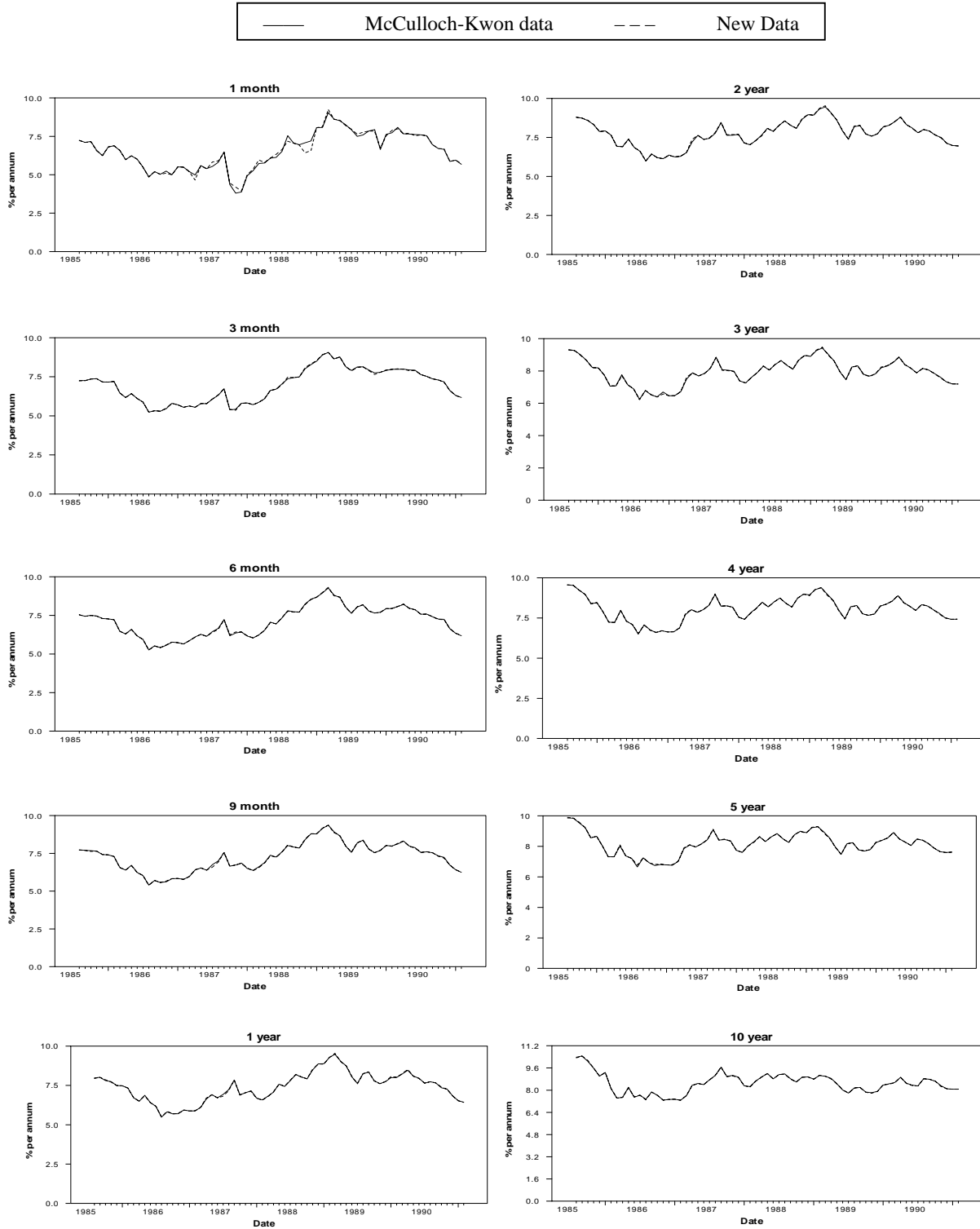
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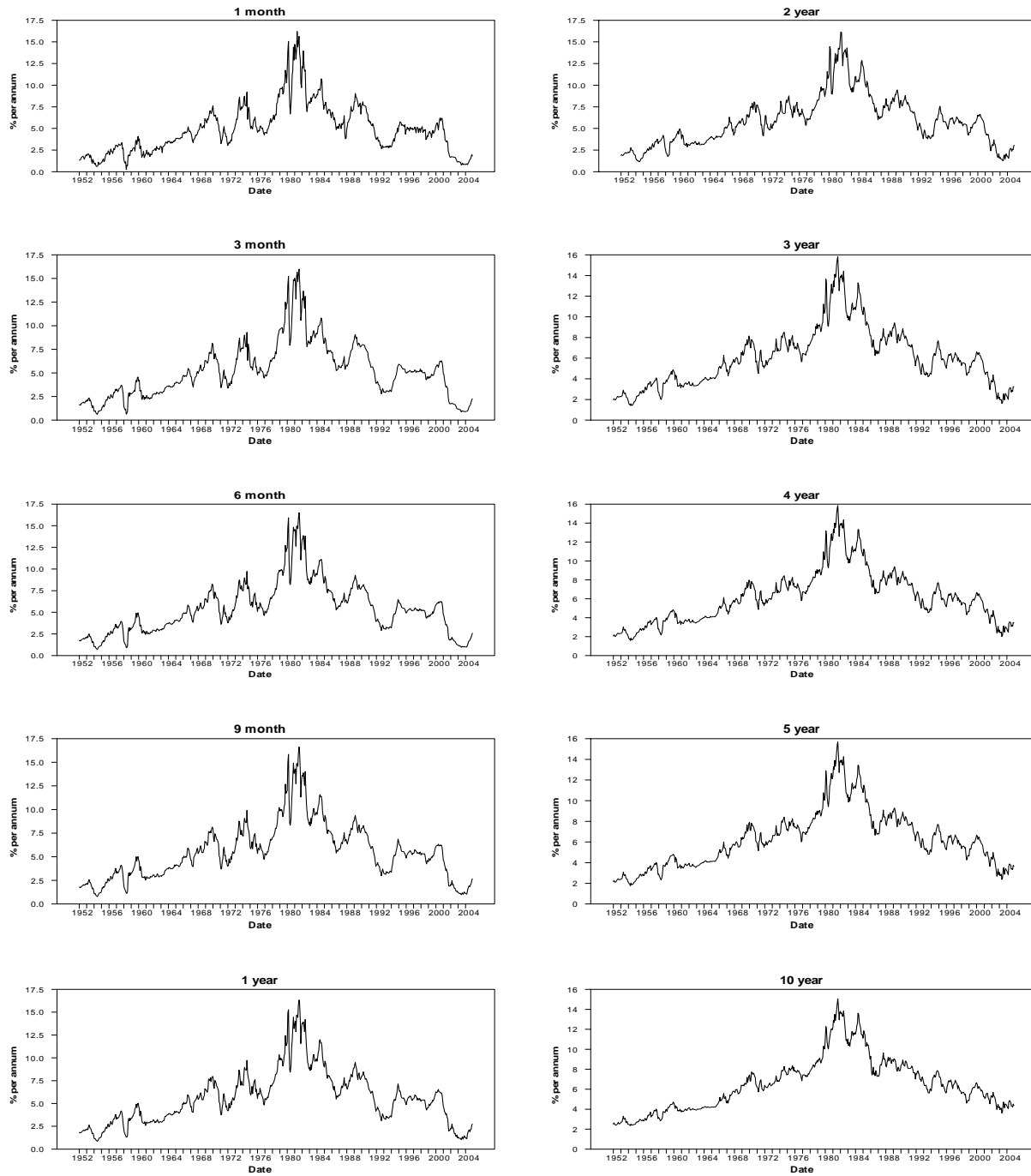
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**Figure 1 McCulloch-Kwon and New Zero-Coupon Bond Yields for 08/1985-02/1991**



Notes: The figure plots the McCulloch and Kwon (1993) and new zero-coupon bond yields over the overlapping period 08/1985-02/1991 for the ten bond maturities that are used in the paper.

**Figure 2 Zero-Coupon Bond Yields for the Full Sample 01/1952-12/2004**



Notes: The figure plots the zero-coupon bond yields for the full sample 01/1952-12/2004 for the ten bond maturities that are used in the paper.

**Table 1 Summary Statistics**

Panel A												
<i>n</i> (months)	McCulloch-Kwon data (Aug. 1985 - Feb. 1991)				New data (Aug. 1985 - Feb. 1991)				Correlation			
	Mean	Std Error	Minimum	Maximum	Mean	Std Error	Minimum	Maximum				
1	6.528	1.193	3.800	9.043	6.527	1.176	3.948	9.248				0.99125
3	6.936	1.057	5.242	9.053	6.935	1.058	5.243	9.056				0.99963
6	7.104	0.977	5.262	9.279	7.102	0.976	5.261	9.314				0.99975
12	7.388	0.927	5.485	9.490	7.390	0.931	5.488	9.545				0.99953
24	7.726	0.821	5.988	9.454	7.734	0.824	6.003	9.527				0.99938
36	7.918	0.775	6.247	9.417	7.915	0.780	6.218	9.470				0.99923
48	8.046	0.748	6.505	9.559	8.044	0.747	6.535	9.572				0.99967
60	8.149	0.743	6.648	9.859	8.155	0.738	6.775	9.899				0.99930
120	8.486	0.715	7.274	10.459	8.479	0.716	7.271	10.459				0.99957

Panel B												
<i>n</i> (months)	McCulloch-Kwon data (Jan. 1952 - Feb. 1991)				New data (Mar. 1991 - Dec. 2004)				Extended data (Jan. 1952 - Dec. 2004)			
	Mean	Std Error	Minimum	Maximum	Mean	Std Error	Minimum	Maximum	Mean	Std Error	Minimum	Maximum
1	5.314	3.064	0.249	16.210	3.716	1.584	0.773	6.210	4.896	2.842	0.249	16.210
3	5.640	3.143	0.615	15.999	3.908	1.650	0.865	6.291	5.188	2.929	0.615	15.999
6	5.884	3.178	0.685	16.511	4.033	1.668	0.955	6.456	5.401	2.974	0.685	16.511
12	6.079	3.168	0.847	16.345	4.275	1.686	1.034	7.142	5.608	2.963	0.847	16.345
24	6.272	3.124	1.149	16.145	4.672	1.610	1.271	7.569	5.854	2.894	1.149	16.145
36	6.386	3.087	1.412	15.825	4.968	1.487	1.616	7.684	6.016	2.829	1.412	15.825
48	6.467	3.069	1.595	15.847	5.221	1.396	2.017	7.712	6.142	2.786	1.595	15.847
60	6.531	3.056	1.770	15.696	5.387	1.323	2.359	7.911	6.232	2.758	1.770	15.696
120	6.683	3.013	2.341	15.065	5.957	1.118	3.608	8.325	6.493	2.670	2.341	15.065

Notes: The table reports summary statistics for the McCulloch and Kwon (1993) and new zero-coupon bond yield datasets for the ten bond maturities that are used in the paper. Panel A reports summary statistics for the overlapping period 08/1985-02/1991. Panel B reports summary statistics for the two sub-samples 01/1952-02/1991 and 03/1991-12/2004, and for the full sample.



**Table 2 Forward Yield Regressions**

<b>Panel A:</b> Forecasts of 1-month spot rates ( $n = 1$ )				<b>Panel B:</b> Forecasts of 1-year spot rates ( $n = 12$ )			
$m$	$\alpha_1$	$\gamma_1$	$\beta_1$	$m$	$\alpha_1$	$\gamma_1$	$\beta_1$
<b>1</b>	-0.155 (0.034)	-0.054 (0.043)	0.504 (0.053)	<b>12</b>	0.253 (0.087)	-1.068 (0.143)	0.413 (0.105)
<b>3</b>	-0.198 (0.068)	-0.163 (0.078)	0.434 (0.069)	<b>24</b>	0.310 (0.115)	-2.173 (0.190)	0.827 (0.098)
<b>6</b>	-0.122 (0.091)	-0.399 (0.103)	0.353 (0.073)	<b>36</b>	0.335 (0.113)	-3.547 (0.186)	1.353 (0.084)
<b>9</b>	-0.108 (0.096)	-0.718 (0.119)	0.444 (0.074)	<b>48</b>	0.378 (0.110)	-4.281 (0.180)	1.558 (0.075)
<b>12</b>	-0.178 (0.105)	-1.077 (0.141)	0.582 (0.074)	<b>60</b>	0.479 (0.117)	-4.860 (0.201)	1.494 (0.077)
				<b>120</b>	1.360 (0.146)	-6.939 (0.294)	1.094 (0.093)

Notes: The table reports the results of estimating the forward yield regression (6a) in the main text for the full sample 01/1952-12/2004, including a dummy variable that is set equal to one for the period after December 1981 and zero otherwise. Panel A reports results for forecasts of the 1-month yield at forward horizons of 3, 6, 9 and 12 months. Panel B reports results for forecasts of the 12-month yield at forward horizons of 12, 24, 36, 48, 60 and 120 months. Standard errors are reported in parentheses.

**Table 3 Yield Spread Regressions**

	<b>Panel A: Short Yield Regression (9a)</b>			<b>Panel B: Long Yield Regression (10a)</b>		
<i>n</i>	$\alpha_2$	$\gamma_2$	$\beta_2$	$\alpha_3$	$\gamma_3$	$\beta_3$
<b>3</b>	-0.114 (0.040)	-0.064 (0.038)	0.490 (0.110)	-0.074 (0.034)	-0.056 (0.041)	-0.098 (0.141)
<b>6</b>	-0.128 (0.073)	-0.156 (0.088)	0.387 (0.132)	0.022 (0.036)	-0.051 (0.040)	-0.565 (0.234)
<b>9</b>	-0.119 (0.096)	-0.273 (0.129)	0.376 (0.132)	0.065 (0.036)	-0.083 (0.040)	-0.783 (0.318)
<b>12</b>	-0.143 (0.123)	-0.408 (0.166)	0.439 (0.160)	0.068 (0.035)	-0.090 (0.039)	-0.875 (0.381)
<b>24</b>	-0.234 (0.252)	-0.946 (0.326)	0.639 (0.186)	0.059 (0.030)	-0.046 (0.037)	-0.886 (0.563)
<b>36</b>	-0.288 (0.335)	-1.477 (0.425)	0.791 (0.193)	0.061 (0.027)	-0.040 (0.035)	-1.307 (0.691)
<b>48</b>	-0.345 (0.320)	-1.957 (0.472)	0.934 (0.191)	0.060 (0.026)	-0.036 (0.034)	-1.602 (0.796)
<b>60</b>	-0.332 (0.281)	-2.279 (0.524)	0.984 (0.201)	0.059 (0.024)	-0.035 (0.032)	-1.823 (0.877)
<b>120</b>	-0.369 (0.364)	-4.276 (0.532)	1.280 (0.152)	0.054 (0.019)	-0.033 (0.027)	-2.713 (1.227)

Notes: The table reports in Panel A the results of estimating the regression of changes in short yields on the scaled spread, equation (9a), and the regression of changes in long yields on the scaled spread, equation (10a), is reported in Panel B for the full sample 01/1952-12/2004 for bond maturities 3, 6, 9, 12, 24, 36, 48, 60 and 120 months. Standard errors are reported in parentheses.

**Table 4 VAR Correlation and Standard Deviation Ratio**

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<b><i>n</i></b>	<b>Correlation</b>	<b>SD ratio</b>
<b>3</b>	0.872	0.564
<b>6</b>	0.799	0.493
<b>9</b>	0.820	0.454
<b>12</b>	0.867	0.462
<b>24</b>	0.945	0.499
<b>36</b>	0.973	0.543
<b>48</b>	0.983	0.576
<b>60</b>	0.988	0.600
<b>120</b>	0.996	0.708

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Notes: The table reports the correlation coefficient and standard deviation ratio between the actual yield spread and the theoretical yield spread given by equation (12), for bond maturities 3, 6, 9, 12, 24, 36, 48, 60 and 120 months. Standard errors are reported in parentheses

**Table 5 Momentum and Return Reversals**

<i>n</i>		<i>m</i>																				
		1	2	3	6	9	12	24	36	48	60	120										
<b>2</b>	$\alpha_4$	0.243	(0.032)																			
	$\gamma_4$	0.041	(0.045)																			
	$\beta_4$	0.266	(0.038)																			
<b>3</b>	$\alpha_4$	0.379	(0.059)	0.395	(0.072)																	
	$\gamma_4$	0.099	(0.085)	0.104	(0.075)																	
	$\beta_4$	0.182	(0.039)	0.148	(0.078)																	
<b>6</b>	$\alpha_4$	0.559	(0.137)	0.953	(0.284)	1.105	(0.315)															
	$\gamma_4$	0.194	(0.204)	0.310	(0.294)	0.321	(0.334)															
	$\beta_4$	0.186	(0.039)	0.063	(0.092)	0.022	(0.118)															
<b>9</b>	$\alpha_4$	0.451	(0.217)	0.819	(0.467)	1.085	(0.561)	1.113	(0.438)													
	$\gamma_4$	0.640	(0.328)	1.279	(0.491)	1.668	(0.632)	1.614	(0.616)													
	$\beta_4$	0.168	(0.039)	-0.001	(0.108)	-0.067	(0.123)	-0.055	(0.113)													
<b>12</b>	$\alpha_4$	0.351	(0.286)	0.556	(0.608)	0.638	(0.768)	0.573	(0.844)	0.673	(0.575)											
	$\gamma_4$	1.118	(0.436)	2.439	(0.690)	3.508	(0.953)	4.754	(1.277)	2.716	(0.850)											
	$\beta_4$	0.175	(0.039)	0.004	(0.107)	-0.059	(0.123)	-0.054	(0.101)	0.114	(0.073)											
<b>24</b>	$\alpha_4$	0.020	(0.540)	-0.263	(1.049)	-0.146	(1.429)	-1.133	(2.132)	-1.508	(2.511)											
	$\gamma_4$	1.768	(0.821)	4.185	(1.381)	7.336	(2.016)	13.085	(3.495)	15.092	(4.180)											
	$\beta_4$	0.140	(0.039)	-0.050	(0.095)	-0.084	(0.110)	-0.045	(0.085)	0.071	(0.072)											
<b>36</b>	$\alpha_4$	-0.253	(0.761)	-0.937	(1.389)	-1.763	(1.844)	-2.946	(3.084)	-5.826	(3.709)											
	$\gamma_4$	2.744	(1.159)	6.395	(2.007)	9.958	(2.894)	20.917	(5.370)	23.368	(6.445)											
	$\beta_4$	0.109	(0.040)	-0.054	(0.085)	-0.106	(0.096)	-0.035	(0.079)	-0.023	(0.071)											
<b>48</b>	$\alpha_4$	-0.558	(0.960)	-1.657	(1.693)	-2.853	(2.255)	-6.120	(3.691)	-8.845	(4.961)											
	$\gamma_4$	3.677	(1.463)	8.526	(2.574)	13.114	(3.715)	23.875	(6.658)	32.462	(8.597)											
	$\beta_4$	0.095	(0.040)	-0.063	(0.078)	-0.098	(0.089)	-0.049	(0.077)	-0.034	(0.077)											
<b>60</b>	$\alpha_4$	-0.869	(1.136)	-2.376	(1.941)	-3.915	(2.592)	-8.101	(4.358)	-11.884	(6.000)											
	$\gamma_4$	4.537	(1.731)	10.438	(3.071)	15.868	(4.443)	29.079	(8.010)	40.626	(10.487)											
	$\beta_4$	0.086	(0.040)	-0.064	(0.073)	-0.086	(0.085)	-0.036	(0.076)	-0.034	(0.081)											
<b>120</b>	$\alpha_4$	-2.432	(1.881)	-5.963	(2.917)	-9.348	(3.935)	-18.205	(7.024)	-27.775	1(0.227)											
	$\gamma_4$	8.317	(2.866)	18.815	(5.192)	28.404	(7.574)	52.357	1(3.792)	78.007	1(8.737)											
	$\beta_4$	0.068	(0.040)	-0.056	(0.065)	-0.064	(0.077)	-0.010	(0.076)	-0.051	(0.090)											

Notes: The table reports the results of estimating regression (13) for the full sample 01/1952-12/2004 for bond maturities 3, 6, 9, 12, 24, 36, 48, 60 and 120 months, and holding periods 1, 2, 3, 6, 9, 12, 24, 36, 48 and 60 months.

**Table 5 Momentum and Return Reversals (Continued)**

<i>n</i>	<i>m</i>										
	12	24	36	48	60	120	120	120	120	120	
<b>2</b>	$\alpha_4$										
	$\gamma_4$										
	$\beta_4$										
<b>3</b>	$\alpha_4$										
	$\gamma_4$										
	$\beta_4$										
<b>6</b>	$\alpha_4$										
	$\gamma_4$										
	$\beta_4$										
<b>9</b>	$\alpha_4$										
	$\gamma_4$										
	$\beta_4$										
<b>12</b>	$\alpha_4$										
	$\gamma_4$										
	$\beta_4$										
<b>24</b>	$\alpha_4$	-1.474	(2.795)								
	$\gamma_4$	16.975	(4.497)								
	$\beta_4$	0.006	(0.107)								
<b>36</b>	$\alpha_4$	-4.809	(4.909)	-5.089	(4.928)						
	$\gamma_4$	31.837	(7.942)	35.770	(6.969)						
	$\beta_4$	-0.046	(0.110)	-0.257	(0.135)						
<b>48</b>	$\alpha_4$	-8.746	(6.656)	-12.829	(8.661)	-9.112	(6.042)				
	$\gamma_4$	46.387	1(0.759)	65.989	1(2.746)	53.720	(8.790)				
	$\beta_4$	-0.091	(0.112)	-0.271	(0.122)	-0.456	(0.068)				
<b>60</b>	$\alpha_4$	-13.167	(8.199)	-21.685	(11.717)	-21.942	(11.240)	-12.637	(6.627)		
	$\gamma_4$	58.504	(13.223)	92.872	(17.666)	97.559	(16.073)	64.405	(9.424)		
	$\beta_4$	-0.124	(0.117)	-0.288	(0.110)	-0.434	(0.066)	-0.505	(0.105)		
<b>120</b>	$\alpha_4$	-38.521	(14.162)	-73.040	(24.566)	-99.777	(33.003)	-113.646	(33.179)	-106.109	(30.256)
	$\gamma_4$	118.637	(22.499)	210.297	(38.753)	279.988	(44.332)	319.132	(39.181)	306.259	(38.683)
	$\beta_4$	-0.200	(0.129)	-0.307	(0.101)	-0.387	(0.080)	-0.471	(0.077)	-0.464	(0.114)

Notes: The table reports the results of estimating regression (13) for the full sample 01/1952-12/2004 for bond maturities 3, 6, 9, 12, 24, 36, 48, 60 and 120 months, and holding periods 1, 2, 3, 6, 9, 12, 24, 36, 48 and 60 months.

**Table 6 Short Term Predictions of the LSN**

---

<i>m</i>	$\alpha_5$	$\gamma_5$	$\beta_5$
<b>1</b>	-0.243 (0.032)	-0.041 (0.045)	0.266 (0.038)
<b>2</b>	-0.302 (0.033)	-0.056 (0.046)	0.080 (0.040)
<b>3</b>	-0.308 (0.033)	-0.053 (0.046)	0.069 (0.040)
<b>4</b>	-0.326 (0.033)	-0.068 (0.046)	0.001 (0.040)
<b>5</b>	-0.315 (0.033)	-0.060 (0.046)	0.043 (0.040)
<b>6</b>	-0.336 (0.033)	-0.059 (0.047)	-0.011 (0.040)

---

Notes: The table reports the results of estimating regression (14) for horizons 1, 2, 3, 4, 5, and 6 months. Standard errors are reported in parentheses.

**Table 7 Long Term Predictions of the LSN**

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<i>m</i>	$\alpha_6$	$\gamma_6$	$\beta_6$
<b>6</b>	-0.472 (0.042)	-0.125 (0.059)	-0.038 (0.040)
<b>12</b>	-0.506 (0.056)	-0.449 (0.083)	0.036 (0.041)
<b>18</b>	-0.572 (0.066)	-0.980 (0.102)	-0.133 (0.041)
<b>24</b>	-0.585 (0.073)	-1.395 (0.116)	-0.222 (0.041)
<b>36</b>	-0.606 (0.080)	-2.046 (0.130)	-0.323 (0.039)
<b>48</b>	-0.580 (0.081)	-2.641 (0.133)	-0.404 (0.037)
<b>60</b>	-0.491 (0.081)	-2.981 (0.135)	-0.393 (0.035)
<b>120</b>	0.066 (0.099)	-4.239 (0.166)	-0.171 (0.038)

---

Notes: The table reports the results of estimating regression (15) for holding periods 6, 12, 18, 24, 36, 48, 60 and 120 months. Standard errors are reported in parentheses

**Table 8 Conservatism**

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<i>m</i>	$\alpha_7$	$\gamma_7$	$\beta_7$
<b>1</b>	-0.124 (0.031)	-0.063 (0.047)	0.091 (0.041)
<b>2</b>	-0.090 (0.030)	-0.007 (0.045)	0.127 (0.040)
<b>3</b>	-0.090 (0.030)	-0.014 (0.045)	0.073 (0.038)
<b>4</b>	-0.031 (0.031)	-0.082 (0.047)	0.034 (0.039)
<b>5</b>	0.026 (0.032)	-0.138 (0.049)	0.066 (0.041)
<b>6</b>	0.049 (0.031)	-0.159 (0.048)	0.114 (0.041)
<b>7</b>	0.049 (0.030)	-0.165 (0.046)	0.144 (0.041)
<b>8</b>	0.043 (0.028)	-0.176 (0.044)	0.138 (0.041)
<b>9</b>	0.036 (0.028)	-0.174 (0.042)	0.116 (0.041)
<b>10</b>	0.025 (0.027)	-0.165 (0.041)	0.099 (0.041)
<b>11</b>	0.017 (0.026)	-0.152 (0.040)	0.087 (0.040)
<b>12</b>	0.014 (0.026)	-0.131 (0.039)	0.073 (0.040)

---

Notes: The table reports the results of estimating regression (16) for horizons 1-12 months. Standard errors are reported in parentheses



**Table 9 Forward Yield Regressions Using Simulated Data ( $n = 1$ )**

$m$	$c = 0.00$				$c = 0.10$				$c = 0.20$			
	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$
<b>1</b>	1.461 (0.644)	1.615 (0.700)	1.136 (0.506)	0.632 (0.429)	0.484 (0.243)	0.454 (0.260)	0.356 (0.263)	0.270 (0.273)	0.201 (0.158)	0.176 (0.170)	0.147 (0.161)	0.114 (0.165)
<b>3</b>	1.450 (0.623)	1.616 (0.681)	1.131 (0.473)	0.638 (0.402)	1.089 (0.377)	1.138 (0.382)	0.842 (0.323)	0.527 (0.323)	0.712 (0.211)	0.682 (0.195)	0.535 (0.225)	0.365 (0.234)
<b>6</b>	1.440 (0.604)	1.613 (0.666)	1.130 (0.453)	0.640 (0.385)	1.298 (0.494)	1.429 (0.533)	1.021 (0.389)	0.595 (0.344)	1.088 (0.362)	1.110 (0.343)	0.842 (0.312)	0.518 (0.293)
<b>9</b>	1.433 (0.588)	1.611 (0.658)	1.127 (0.443)	0.636 (0.374)	1.350 (0.521)	1.516 (0.592)	1.067 (0.404)	0.606 (0.346)	1.231 (0.440)	1.293 (0.441)	0.960 (0.349)	0.570 (0.312)
<b>12</b>	1.426 (0.572)	1.602 (0.646)	1.120 (0.426)	0.634 (0.356)	1.369 (0.522)	1.550 (0.613)	1.088 (0.404)	0.612 (0.338)	1.295 (0.472)	1.378 (0.492)	1.013 (0.361)	0.593 (0.309)

Notes: The table reports the results of estimating the forward yield regression (6) in the main text for the simulated data for  $n = 1$  and  $m = 1, 3, 6, 9$  and 12 months using the simulated data. Standard deviations are reported in parentheses.

**Table 10 Forward Yield Regressions Using Simulated Data ( $n = 12$ )**

$m$	$c = 0.00$				$c = 0.10$				$c = 0.20$			
	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$
<b>12</b>	1.426 (0.572)	1.651 (0.664)	0.999 (0.434)	0.305 (0.415)	1.398 (0.551)	1.664 (0.702)	0.990 (0.451)	0.270 (0.425)	1.385 (0.552)	1.575 (0.647)	0.957 (0.448)	0.255 (0.424)
<b>24</b>	1.406 (0.522)	1.597 (0.612)	1.051 (0.407)	0.483 (0.332)	1.383 (0.496)	1.615 (0.634)	1.063 (0.425)	0.468 (0.346)	1.393 (0.524)	1.560 (0.612)	1.035 (0.422)	0.456 (0.354)
<b>36</b>	1.386 (0.479)	1.555 (0.560)	1.042 (0.394)	0.516 (0.320)	1.365 (0.455)	1.568 (0.572)	1.055 (0.407)	0.509 (0.333)	1.383 (0.485)	1.531 (0.566)	1.030 (0.405)	0.487 (0.336)
<b>48</b>	1.371 (0.442)	1.523 (0.513)	1.028 (0.390)	0.525 (0.319)	1.349 (0.426)	1.530 (0.518)	1.045 (0.390)	0.526 (0.323)	1.362 (0.440)	1.503 (0.525)	1.017 (0.388)	0.502 (0.335)
<b>60</b>	1.352 (0.409)	1.498 (0.468)	1.019 (0.383)	0.527 (0.322)	1.331 (0.399)	1.494 (0.473)	1.026 (0.373)	0.536 (0.324)	1.346 (0.408)	1.477 (0.493)	1.003 (0.380)	0.518 (0.339)
<b>120</b>	1.266 (0.315)	1.411 (0.369)	0.968 (0.380)	0.514 (0.343)	1.277 (0.323)	1.382 (0.386)	0.969 (0.390)	0.512 (0.339)	1.286 (0.318)	1.380 (0.401)	0.967 (0.387)	0.511 (0.355)

Notes: The table reports the results of estimating the forward yield regression (6) in the main text for  $n = 12$  and  $m = 12, 24, 36, 48, 60$  and 120 months using the simulated data. Standard deviations are reported in parentheses.

**Table 11 Yield Spread Regressions Using Simulated Data (Short Yield Regression)**

<i>n</i>	<i>c</i> = 0.00				<i>c</i> = 0.10				<i>c</i> = 0.20			
	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$
<b>3</b>	1.458 (0.636)	1.618 (0.693)	1.139 (0.488)	0.635 (0.411)	0.714 (0.242)	0.688 (0.248)	0.538 (0.260)	0.374 (0.275)	0.344 (0.153)	0.309 (0.164)	0.251 (0.165)	0.191 (0.169)
<b>6</b>	1.449 (0.618)	1.618 (0.676)	1.131 (0.463)	0.639 (0.391)	1.086 (0.365)	1.132 (0.360)	0.842 (0.308)	0.525 (0.307)	0.712 (0.196)	0.678 (0.174)	0.533 (0.201)	0.366 (0.216)
<b>9</b>	1.445 (0.606)	1.617 (0.669)	1.131 (0.449)	0.638 (0.377)	1.228 (0.445)	1.326 (0.462)	0.960 (0.348)	0.573 (0.321)	0.946 (0.283)	0.937 (0.251)	0.720 (0.251)	0.463 (0.252)
<b>12</b>	1.440 (0.594)	1.614 (0.663)	1.129 (0.440)	0.637 (0.365)	1.293 (0.481)	1.424 (0.523)	1.016 (0.367)	0.592 (0.324)	1.086 (0.350)	1.104 (0.327)	0.835 (0.287)	0.517 (0.270)
<b>24</b>	1.426 (0.552)	1.596 (0.634)	1.115 (0.403)	0.631 (0.317)	1.363 (0.500)	1.540 (0.594)	1.087 (0.385)	0.614 (0.304)	1.292 (0.456)	1.371 (0.480)	1.002 (0.346)	0.586 (0.279)
<b>36</b>	1.416 (0.519)	1.578 (0.601)	1.106 (0.385)	0.628 (0.295)	1.371 (0.479)	1.553 (0.587)	1.097 (0.380)	0.619 (0.291)	1.345 (0.473)	1.443 (0.517)	1.043 (0.359)	0.601 (0.276)
<b>48</b>	1.408 (0.490)	1.562 (0.565)	1.096 (0.372)	0.623 (0.285)	1.369 (0.457)	1.546 (0.561)	1.096 (0.370)	0.620 (0.281)	1.360 (0.461)	1.467 (0.516)	1.052 (0.356)	0.602 (0.277)
<b>60</b>	1.399 (0.465)	1.548 (0.529)	1.087 (0.359)	0.618 (0.281)	1.364 (0.436)	1.535 (0.528)	1.090 (0.356)	0.617 (0.278)	1.364 (0.441)	1.475 (0.500)	1.051 (0.348)	0.603 (0.278)
<b>120</b>	1.358 (0.348)	1.510 (0.383)	1.047 (0.312)	0.592 (0.284)	1.335 (0.342)	1.474 (0.390)	1.051 (0.307)	0.599 (0.281)	1.353 (0.336)	1.451 (0.398)	1.036 (0.306)	0.593 (0.289)

Notes: The table reports the results of estimating the short yield regression (9) for bond maturities 3, 6, 9, 12, 24, 36, 48, 60 and 120 months using the simulated data. Standard deviations are reported in parentheses.

**Table 12 Yield Spread Regressions Using Simulated Data (Long Yield Regression)**

<i>n</i>	<i>c</i> = 0.00				<i>c</i> = 0.10				<i>c</i> = 0.20			
	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$
<b>3</b>	1.919 (1.286)	2.201 (1.386)	1.193 (1.001)	0.148 (0.867)	-0.041 (0.519)	-0.218 (0.576)	-0.565 (0.607)	-0.846 (0.628)	-1.051 (0.357)	-1.216 (0.404)	-1.352 (0.391)	-1.452 (0.399)
<b>6</b>	1.912 (1.276)	2.116 (1.344)	0.956 (0.970)	-0.197 (0.898)	0.281 (0.586)	-0.116 (0.628)	-0.879 (0.745)	-1.447 (0.780)	-1.443 (0.403)	-2.031 (0.560)	-2.485 (0.606)	-2.692 (0.623)
<b>9</b>	1.906 (1.267)	2.031 (1.303)	0.718 (0.948)	-0.543 (0.938)	0.540 (0.658)	0.040 (0.646)	-1.036 (0.824)	-1.825 (0.877)	-1.276 (0.378)	-2.184 (0.659)	-2.959 (0.805)	-3.305 (0.825)
<b>12</b>	1.899 (1.258)	1.946 (1.262)	0.481 (0.937)	-0.888 (0.985)	0.706 (0.714)	0.131 (0.659)	-1.193 (0.886)	-2.164 (0.957)	-1.021 (0.344)	-2.136 (0.710)	-3.200 (0.964)	-3.707 (0.986)
<b>24</b>	1.874 (1.223)	1.608 (1.109)	-0.468 (0.997)	-2.271 (1.225)	0.992 (0.815)	0.139 (0.663)	-1.945 (1.129)	-3.496 (1.258)	-0.300 (0.370)	-1.832 (0.778)	-3.817 (1.387)	-4.953 (1.439)
<b>36</b>	1.850 (1.190)	1.272 (0.973)	-1.417 (1.195)	-3.654 (1.518)	1.090 (0.838)	-0.055 (0.647)	-2.787 (1.409)	-4.844 (1.572)	0.044 (0.447)	-1.767 (0.845)	-4.497 (1.721)	-6.189 (1.811)
<b>48</b>	1.828 (1.159)	0.939 (0.863)	-2.364 (1.476)	-5.038 (1.839)	1.137 (0.839)	-0.310 (0.660)	-3.660 (1.717)	-6.204 (1.901)	0.235 (0.494)	-1.843 (0.938)	-5.256 (2.047)	-7.462 (2.170)
<b>60</b>	1.808 (1.130)	0.607 (0.786)	-3.311 (1.801)	-6.422 (2.176)	1.162 (0.832)	-0.591 (0.715)	-4.549 (2.042)	-7.571 (2.239)	0.355 (0.520)	-1.992 (1.050)	-6.061 (2.377)	-8.760 (2.529)
<b>120</b>	1.722 (1.010)	-1.027 (1.029)	-8.038 (3.638)	-13.355 (3.955)	1.196 (0.768)	-2.112 (1.382)	-9.075 (3.772)	-14.461 (3.996)	0.609 (0.539)	-3.129 (1.752)	-10.361 (4.079)	-15.427 (4.347)

Notes: The table reports the results of estimating the long yield regression (10) for bond maturities 3, 6, 9, 12, 24, 36, 48, 60 and 120 months using the simulated data. Standard deviations are reported in parentheses.

**Table 13 VAR Correlation Coefficient Using Simulated Data**

<i>n</i>	<i>c</i> = 0.00				<i>c</i> = 0.10				<i>c</i> = 0.20			
	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$
<b>3</b>	0.867 (0.099)	0.777 (0.113)	0.669 (0.170)	0.503 (0.282)	0.670 (0.165)	0.561 (0.167)	0.500 (0.203)	0.417 (0.282)	0.504 (0.210)	0.409 (0.209)	0.373 (0.236)	0.321 (0.274)
<b>6</b>	0.943 (0.064)	0.900 (0.071)	0.818 (0.135)	0.647 (0.307)	0.862 (0.084)	0.789 (0.089)	0.732 (0.137)	0.608 (0.281)	0.735 (0.119)	0.652 (0.128)	0.610 (0.163)	0.531 (0.253)
<b>9</b>	0.968 (0.043)	0.942 (0.045)	0.878 (0.112)	0.710 (0.325)	0.928 (0.051)	0.872 (0.058)	0.816 (0.118)	0.675 (0.306)	0.846 (0.073)	0.770 (0.090)	0.719 (0.135)	0.615 (0.272)
<b>12</b>	0.980 (0.030)	0.961 (0.031)	0.906 (0.101)	0.740 (0.339)	0.956 (0.034)	0.911 (0.042)	0.857 (0.110)	0.706 (0.328)	0.902 (0.047)	0.835 (0.067)	0.780 (0.124)	0.658 (0.295)
<b>24</b>	0.995 (0.011)	0.983 (0.012)	0.940 (0.089)	0.775 (0.371)	0.988 (0.017)	0.960 (0.020)	0.913 (0.105)	0.747 (0.376)	0.973 (0.014)	0.929 (0.032)	0.874 (0.119)	0.723 (0.351)
<b>36</b>	0.998 (0.005)	0.987 (0.008)	0.948 (0.090)	0.782 (0.384)	0.995 (0.013)	0.973 (0.014)	0.929 (0.106)	0.757 (0.395)	0.988 (0.006)	0.955 (0.021)	0.904 (0.122)	0.743 (0.375)
<b>48</b>	0.999 (0.003)	0.988 (0.007)	0.950 (0.091)	0.785 (0.390)	0.997 (0.010)	0.978 (0.011)	0.936 (0.108)	0.762 (0.405)	0.994 (0.003)	0.967 (0.016)	0.917 (0.124)	0.752 (0.387)
<b>60</b>	0.999 (0.002)	0.989 (0.006)	0.951 (0.092)	0.786 (0.394)	0.998 (0.008)	0.981 (0.010)	0.940 (0.109)	0.764 (0.411)	0.996 (0.002)	0.973 (0.014)	0.925 (0.126)	0.757 (0.395)
<b>120</b>	1.000 (0.000)	0.989 (0.006)	0.953 (0.096)	0.788 (0.401)	1.000 (0.004)	0.986 (0.008)	0.947 (0.112)	0.769 (0.422)	0.999 (0.000)	0.983 (0.009)	0.938 (0.131)	0.766 (0.409)

Notes: The table reports the correlation coefficient between the actual yield spread and the theoretical yield spread given by equation (12), for bond maturities 3, 6, 9, 12, 24, 36, 48, 60 and 120 months using the simulated data. Standard deviations are reported in parentheses.

**Table 14 VAR Standard Deviation Ratio Using Simulated Data**

<i>n</i>	<i>c</i> = 0.00				<i>c</i> = 0.10				<i>c</i> = 0.20			
	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$	$\theta = 0.00$	$\theta = 0.10$	$\theta = 0.20$	$\theta = 0.30$
<b>3</b>	1.666 (0.663)	2.052 (0.755)	1.691 (0.534)	1.257 (0.369)	1.087 (0.311)	1.249 (0.322)	1.076 (0.264)	0.868 (0.244)	0.703 (0.168)	0.777 (0.168)	0.679 (0.168)	0.575 (0.153)
<b>6</b>	1.526 (0.623)	1.775 (0.692)	1.354 (0.460)	0.934 (0.317)	1.272 (0.448)	1.447 (0.492)	1.127 (0.345)	0.796 (0.275)	0.992 (0.300)	1.064 (0.299)	0.859 (0.269)	0.636 (0.217)
<b>9</b>	1.478 (0.604)	1.690 (0.672)	1.225 (0.436)	0.797 (0.303)	1.330 (0.498)	1.514 (0.568)	1.110 (0.377)	0.722 (0.282)	1.136 (0.381)	1.219 (0.394)	0.932 (0.327)	0.627 (0.248)
<b>12</b>	1.451 (0.587)	1.653 (0.660)	1.156 (0.425)	0.719 (0.300)	1.351 (0.514)	1.546 (0.603)	1.088 (0.391)	0.669 (0.286)	1.211 (0.424)	1.311 (0.455)	0.962 (0.359)	0.607 (0.264)
<b>24</b>	1.386 (0.520)	1.607 (0.621)	1.019 (0.392)	0.560 (0.278)	1.348 (0.491)	1.586 (0.624)	1.007 (0.390)	0.537 (0.272)	1.291 (0.452)	1.454 (0.537)	0.957 (0.383)	0.516 (0.266)
<b>36</b>	1.339 (0.461)	1.594 (0.586)	0.937 (0.364)	0.474 (0.253)	1.317 (0.444)	1.594 (0.600)	0.941 (0.370)	0.458 (0.248)	1.285 (0.422)	1.497 (0.540)	0.909 (0.368)	0.447 (0.247)
<b>48</b>	1.299 (0.412)	1.588 (0.555)	0.877 (0.342)	0.416 (0.231)	1.285 (0.399)	1.596 (0.571)	0.886 (0.351)	0.403 (0.227)	1.263 (0.385)	1.516 (0.525)	0.862 (0.350)	0.396 (0.228)
<b>60</b>	1.266 (0.371)	1.585 (0.527)	0.829 (0.324)	0.374 (0.214)	1.256 (0.361)	1.596 (0.543)	0.840 (0.335)	0.363 (0.210)	1.240 (0.350)	1.527 (0.505)	0.821 (0.334)	0.358 (0.212)
<b>120</b>	1.157 (0.242)	1.590 (0.425)	0.687 (0.276)	0.271 (0.164)	1.154 (0.236)	1.602 (0.436)	0.702 (0.287)	0.264 (0.161)	1.149 (0.233)	1.558 (0.417)	0.688 (0.285)	0.263 (0.164)

Notes: The table reports the standard deviation ratio between the actual yield spread and the theoretical yield spread given by equation (12), for bond maturities 3, 6, 9, 12, 24, 36, 48, 60 and 120 months using the simulated data. Standard deviations are reported in parentheses.