

## A DEMAND-BASED MODEL FOR THE ADVANCED AND SPOT PRICING OF SERVICES

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## A DEMAND-BASED MODEL FOR THE ADVANCED AND SPOT PRICING OF SERVICES

### **Abstract**

This paper models the probability of non-consumption of a service by advance buyers as well as the re-selling of the relinquished capacity, and investigates its effect on the pricing of advance and spot demand of services. The model shows that advance prices are always lower than spot prices, since advance buyers provide for higher potential revenue due to the non-consumption effect. Another significant finding of the model would be the conditions under which market failure occurs i.e. when the firm does not sell in advance nor at consumption time. It is also extended to show that providing a refund to advance buyers, as insurance against non-consumption, may be profitable.

*Key words: Services, Advance Selling, Revenue Management, Refund, Pricing*

### **INTRODUCTION**

The service sector now accounts for about 70% of aggregate production and employment in developed economies. However, many who have attempted to study the service industry have lamented on its diversity, for within the service economy lie a heterogeneous set of activities that include financial services, telecommunication, retail, transportation, entertainment, education, and also not-for-profit activities. Adding to the complexity of pricing is the fact that many services are bundled with goods, or may be a facilitating service between two or more services and/or goods e.g. a mobile payment service.

This paper aims to provide a theoretical insight into the demand behavior and the advance pricing of services, specifically those with high fixed costs, low variable costs and short-term capacity constraints, and that can be purchased in advance, such as hotels and airlines. It begins with the argument of why advance demand and spot demand would exist from a behavioral standpoint i.e. that the former consist of buyers who want certainty in availability of a service and the latter consist of buyers who want certainty in consumption. The paper also introduces the probability of advance buyers not being able to use the service at consumption time, and how the relinquished capacity can be re-sold. The study then proceeds to model the concepts. Unlike previous studies, this model captures consumers as heterogeneous through the use of demand functions, and also explicitly considers price sensitivity within each time period, as well as cross-time sensitivity.

The results show that advance prices are always lower than spot (consumption time) prices. This is because advance purchases are expected to contribute higher potential revenue, as a result of the non-consumption effect. This finding is significant, as no previous model has captured the non-consumption effect of services, and the model shows its impact on advance and spot prices and quantities. When the firm is constrained by capacity, the results show that it may not wish to sell in advance, since a service that is highly state-dependent leads to a price too low to sell in advance and thus the firm would prefer to sell at spot. This is consistent with Png (1989).

The study goes on to show the conditions of market failure; when prices are too low for the firm to sell in advance and too high for buyers to buy at spot. The model is then extended to show that providing a refund to advance buyers as insurance for non-consumption may result in higher profit. While previous models have shown that a refund may be optimal due to an increase in the price obtained from the offer, this study shows that the insurance may also be funded by an expanded advance demand and higher spot prices. Previous advance selling models have not fully integrated prices, quantities, heterogeneous demand functions and price/cross-time sensitivities and

the non-consumption effect. This paper also aims to provide a more realistic characterization of advance selling.

The rest of the paper is organized as follows. In §1, a review of literature is presented, highlighting the behavioral issues in advanced and spot demand. In §2, a theoretical model formulation is presented. The model is then extended to incorporate a refund offer in §3. In §4, asymmetric demand functions are considered. Discussion of the results follows in §5, together with managerial implications. The paper then concludes with some remarks and directions for future research.

## LITERATURE REVIEW

The importance of price has led researchers to describe price as the most flexible of the marketing mix, with relatively quick implementation characteristics compared to the other factors. As such, price is argued to be an important and powerful tool in businesses (Garda, 1991, Shipley and Jobber, 1981). More important, price is also known to be the only element in the marketing mix that generates revenue, whereas other components are associated with costs. Despite this importance however, price is still regarded as the most neglected area in marketing research compared to the other elements of the marketing mix (Nagle and Holden, 1995). One particular area of pricing research that is lacking considerably is the pricing of services (Schlissel and Chasin, 1991, Mitra and Capella, 1997).

Ng (2008) explains the lack of pricing research in services. She discusses the theoretical nature of services (perishability, intangibility and inseparability) as having key impact on pricing, and argues for a different treatment on how services should be fundamentally priced. Industry reports have supported this, with some claiming that “some of the new service industries may have special economic properties that do not fit well with the assumptions of conventional economic models.”<sup>[1]</sup>

Pricing researchers often research service pricing within the *context* of services (Danaher, 2002, Essegaier *et al.*, 2002, Dawes, 2004, Nunn and Sarvary, 2004), rather than trying to understand it conceptually. While some have attempted to adopt a more general approach to offer insights on common pricing practices and applications across industries (e.g. Guiltinan, 1987, Gotlieb, 1989, Schlissel and Chasin, 1991, Ansari *et al.*, 1996, Tung *et al.*, 1997, Bolton and Myers, 2003, Hu *et al.*, 2006, Avlonitis and Indounas, 2006), what is missing is a theoretical abstraction of service pricing that could provide knowledge across service industries so that one industry (e.g. airline) could learn from another (e.g. banking).

One implicit theoretical abstraction that has been reasonably well developed is the perishability of service at production/consumption resulting in the need to sell services in advance, for example, the case of flights and hotel rooms. This has led to a proliferation of research and practice in the field of revenue management (RM), advising optimal prices and capacity allocations for the firm before the point of perishability. RM is a collection of tools that makes up the practice of revenue management including inventory controls and allocation, and demand forecasting. RM practices are now applied to other service firms that face similar constraints as the airlines, such as perishable inventory, limited capacity and high fixed costs (Yeomen *et al.*, 1999; Kimes, 1989, 2003). There is clearly an undeniable relationship between RM practices and the price function, and RM researchers have also begun to investigate consumer behavior (Kimes and Thompson, 2004, Talluri and Van Ryzin, 2004, Kimes and Wirtz, 2003) and the dynamic pricing for services (Zhao and Zheng, 2000; see Elmaghraby and Keskinocak (2003) for a comprehensive review).

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However, RM research, often conducted by the OR (operations research) community generally assumes advanced demand to be probabilistic in nature.

Marketing researchers, in contrast to OR researchers, looked towards capturing primitive notions of advance demand behavior. Png (1989), for example, showed that costless reservations in advance is a profitable pricing strategy, as it induces truth revelation on the type of valuation that the consumer has for the service (which is private information). If the consumer has a high valuation i.e. ability to consume, s/he will exercise the reservation and pay a higher price. Otherwise, the consumer will not exercise. In another paper, Png (1991) weighed the strategy of charging a lower price for advance sales and attaching a price premium at consumption date, against one of charging a premium at advanced sale and promising a refund should consumption prices be lower than what was originally paid. Shugan and Xie (2000), on the other hand showed that advance selling is a marketing tool that does not need industry-specific characteristics but only requires the existence of buyer uncertainty about future valuations. Xie and Shugan (2001) then proposed when and how to advance sell in a variety of situations, including situations with limited capacity, second-period arrivals, refunds, buyer risk aversion, exogenous credibility, continuous preference distributions, and premium pricing. Lee and Ng (2001) described advance demand and suggested that it is optimal for firms to sell in advance.

Closely related is the economics literature on inter-temporal price discrimination, where buyers buy at different times and are charged different prices. Literature in this area suggests various reasons why such practices may exist. Lott and Roberts (1991) proposed that the higher price for late buyers include the opportunity costs of seats being perished. Gale (1993) argued that the difference between advanced and spot prices exists because buyers may be homogenous in advance and learn their preferences at spot, rendering the product to be differentiated at that time, thus allowing the firm to price discriminate. Dana (1998) showed that low and high-valuation customers could be separated if the cost of capacity is high, and where demand is uncertain.

Perhaps the most integrative work that brings together the above literature in economics, operations research and marketing is the work of Ng (2007, 2008), who described the advanced demand as a manifestation of behavioral risks. She proposed that buyers at the point of purchase trade-off between *valuation risk* (the risk of not valuing the service at the same level as at consumption if s/he buys too early) and *acquisition risk* (the risk of not being able to acquire the service if s/he buys too late). Given this trade-off, Ng argued that a market would exist for selling the service far in advance to buyers looking to ensure the availability of the service. Similarly, there would also be a market for selling at (close to) consumption time for buyers who would like to ensure that they are able to consume. This means that the *distribution* of demand across the selling period of the service becomes important in the firm's pricing decision. In addition, Ng proposed that advance buyers could relinquish their purchase (e.g. no-shows), thus allowing the firm to re-sell at consumption time.

The implications on the pricing decision are significant. Depending on the demand distribution across time, the level of non-consumption and capacity, firms might be prepared to manipulate advance and spot prices to optimize profits. This paper endeavors to formulate Ng's propositions mathematically, modeling demand behavior across two time frames – in advance and at the time of consumption. In modeling this phenomenon, it is important to highlight three primary differences between this study and previous ones. First, this investigation models the consumers as heterogeneous, through the use of demand functions. By modeling the consumers' price sensitivity, both the decision of buyers to buy or not to buy as well as the quantities of each

choice at a given price are captured in the demand function. Second, the investigation also models the substitutability between advance and spot demand, capturing the buyers' aggregate switching decisions as well as the quantities that will switch for a change in price. Finally, the model explicitly captures the probability of advance buyers who are not able to consume at the time of consumption, and how this impacts on pricing. Through this model, it is hoped that there will be greater applicability in the characterization of the phenomenon.

All proofs of propositions are found in the appendix.

## MODEL

The model will now be specified. The following is defined:

$P_A, (P_0)$	=	Price per unit of the service sold at advance time (spot)
$\pi$	=	Profit to the service firm
$q_A (q_0)$	=	Quantity of service demanded by the market at advance time (spot)
$K$	=	Capacity of the service firm and $K > 0$ and $t_A (t_0) =$ Advance (spot) time

A service sold in advance and at the time of consumption is not unlike two firms selling products differentiated only by the time of sale. The difference is that since there is only one service firm, the maximized profit is derived from demand at both times. Consequently, we can adapt product differentiation models derived from economics literature. Following Dixit (1979) and Singh and Vives (1984), we assume the following demand structure for selling the service at  $t_A$  and  $t_0$ :

$$q_0 = \alpha - \beta P_0 + \delta P_A \text{ and } q_A = \alpha - \beta P_A + \delta P_0 \text{ where } \beta > 0, \delta > 0 \text{ and } \beta > \delta$$

Forms of this demand curve have been used in marketing modeling literature e.g. McGuire and Staelin (1983) who modeled the decision of two manufacturers and their choice to intermediate when the demand faced by both are represented by linear demand functions similar to that modeled above; and Ingene and Parry (1995) who modeled two competing retailers also facing similar demand functions, and how a manufacturer would coordinate the channels.

### Capacity and State effect

The parameter  $\delta$  depicts the effect of increasing  $P_A$  on  $q_0$  and increasing  $P_0$  on  $q_A$ . The assumption  $\beta > \delta$  means that the effect of increasing  $P_0$  ( $P_A$ ) on  $q_0$  ( $q_A$ ) is larger than the effect of the same increase in  $P_A$  ( $P_0$ ) i.e. own time-price effect dominates the cross-time price effect.

This is a reasonable assumption because the price of a service is more sensitive to a change in the quantity at its own time than one across time. This could be due to several reasons: for instance, the services could be differentiated by time, which may create other uncertainties for the buyers and thereby result in a lower cross-time effect.

Note that the parameter  $\delta$ , in the context of advance selling of services, can be deemed to partially characterize buyers' valuation and acquisition risks. This means that a change in spot price would have an impact on advance demand, and the degree of impact is dependent on the magnitude of  $\delta$ . It is assumed, for convenience, that the demand functions are symmetric across time. This assumption will be relaxed later. Thus, if the acquisition and valuation risks are low,  $\delta$  may increase, implying that there is increased substitutability between buying in advance and at spot.

The probability that a buyer who buys in advance, but is unable to consume, is parameterized as  $\rho$  where  $0 < \rho < 1$ . Note that the portion of demand sold in advance that is

unable to be consumed at  $t_0$  can be equivalently depicted as  $\rho q_A$ . This capacity could be re-sold to buyers at spot, and at the spot price of  $P_0$ , yielding a revenue of  $P_0 \rho q_A$  (the assumption of the firm possessing full ability to re-sell relinquished capacity will be relaxed later in the study). Finally, the firm may be constrained by its overall capacity i.e.  $q_0 + q_A \leq K$ .

Given the situation described above, the objective function of the service firm becomes:

$Max_{P_A, P_0} [\pi]$  where  $\pi = P_A q_A + P_0 q_0 + P_0 \rho q_A$  for the unconstrained firm

and where  $q_0 + q_A \leq K$  for the constrained firm

The following are the model assumptions:

- While the study models the proportion of non-consuming buyers, it is assumed that this proportion, together with the consumer demand parameters, is common knowledge to the firm and the market i.e. there is perfect information.
- The marginal cost of providing the service is negligible, as service firms in general operate with high fixed costs. This is consistent with research in this area (e.g. Kimes, 1989; Desiraju and Shugan, 1999).
- The capacity has no salvage value after production/consumption.
- The service under study is a pure service (with no attributes of a good), so the consumer, after consumption, has no ownership of anything tangible.
- The firm will only sell if prices at spot and in advance are positive i.e.  $P_A, P_0 > 0$ .
- The firm can credibly commit to spot prices in advance (cf. Xie and Shugan, 2001)
- Buyers who buy in advance are guaranteed the availability of capacity at the time of consumption. As buyers are represented within demand functions, they are assumed to be heterogeneous (in terms of risk aversion and their preferences).
- Capacity relinquished by advance buyers can be fully re-sold at spot (this assumption is relaxed in the next section).
- The service is not transferable.
- The firm is a monopoly.

Furthermore, the model assumes a high congruence between what the seller sells and what the buyers believe they are buying. For example, a movie, a flight, a hotel room, a tow truck service, an annual auditing service can all be considered (almost) homogeneous units of services because what the buyer expects to consume is similar to what the seller expects to produce. Whilst it is acknowledged that services are heterogeneous, the heterogeneity is usually at the point of consumption and it is assumed that the degree of service heterogeneity expected by prospective buyers do not sufficiently influence the value they place on the service in advance of consumption. Hence, the heterogeneity in demand lies only in buyers' valuation of the service and not because of perceived differences in the service offering.

A few noteworthy comments on the firm's objective function are necessary at this juncture. First, it is assumed that by having perfect information, the firm maximizes its profit at one stage, despite the profit being derived across two times i.e. advance and spot. Since all parameters of demand at both times are common knowledge to the firm, it would be strategic in maximizing and manipulating its prices for both times. Certainly if information is not perfect, the model could be modeled in a variety of ways e.g. with two stages of strategic pricing through backward induction, or with myopic pricing where the firm maximizes profit in advance and then at spot (cf. Jagpal,

1998). The purpose here is to understand the phenomenon in its idealized form, without setting specific conditions.

The Xie-Shugan model depicts the phenomenon as a two-period process where homogeneous consumers arriving in period 1 can decide to buy or wait till after the firm announces their spot and advance prices. Consumers may also arrive in period 2. In reality, buyers are not merely heterogeneous in their valuation of the service (i.e. own-time price sensitivity). They are also heterogeneous in their willingness to switch between spot and advance time (cross-time sensitivity).

In Png's model, the advance buyer, should he choose to buy, knows how much he values the service only at the time of consumption. The probability of the buyer turning out to be a low or high valuation customer is depicted as  $\lambda$  (in Xie-Shugan, it is  $q$ ). This model incorporates this feature with  $\rho$ . A buyer is deemed to have a high valuation if he is able to consume, and a low valuation if he is unable to do so. However, a key difference is that Png assumes a low valuation customer is such, regardless of his ability to consume. Although this may not make a difference to the customer, his willingness to consume has a direct effect on the firm. If he does not consume, the capacity can be relinquished and re-sold. This ability to re-sell obviously impacts on the price of the service, both in advance and at spot. This ability had not been considered in all previous models, and this study has incorporated it here.

## ANALYSES

For the unconstrained firm, the study provides the following lemma as a benchmark:

*Lemma 1: When  $\rho = 0$  and the capacity constraint is non-binding,  $P_A^* = P_0^* = \frac{\alpha}{2(\beta - \delta)}$  which we*

*denote as  $P^*(0)$ , and  $q_A^* = q_0^* = \frac{\alpha}{2}$  which we denote as  $q^*(0)$  and  $\pi^* = \frac{\alpha^2}{2(\beta - \delta)}$  which we*

*denote as  $\pi^*(0)$ .*

If the fraction of non-consumption capacity is zero  $\rho = 0$ , i.e. all advance buyers are able to consume, prices and quantities sold at  $t_0$  and  $t_A$  will be the same, due to the symmetric demand functions. This is illustrated in Figure 1.

<< Take in Figure 1 >>

Since  $\rho$  will always take on a positive value, prices and quantities at  $t_0$  and  $t_A$  start diverging, as the following proposition shows:

*Proposition 1: When  $\rho > 0$ , the firm derives a higher profit by lowering advance price and increasing spot price such that*

$$P_A^* = P^*(0)(1 - S \cdot [2(\beta - \delta) + \rho(\beta - 2\delta)])$$

$$P_0^* = P^*(0)(1 + S \cdot [2(\beta - \delta) + \rho\beta])$$

*and obtains a higher advance demand but lower spot demand such that*

$$q_A^* = q^*(0)(1 + S \cdot [2(\beta + \delta) + \rho\beta])$$

$$q_0^* = q^*(0)(1 - S \cdot [2(\beta + \delta) + \rho(\beta + 2\delta)])$$

*where  $S = \frac{\beta\rho}{4(\beta^2 - \delta^2) - \beta^2\rho^2}$  and  $P_A^*, P_0^* > 0$  if and only if  $\beta > \delta \cdot \frac{2}{\sqrt{4 - \rho^2}}$ .*

As every unit of advance demand provides an opportunity for the firm to re-sell, the firm chooses to lower  $P_A^*$  to obtain a higher advance demand. Due to cross-time sensitivity, a lower  $P_A^*$  decreases spot demand. However, instead of compensating by lowering  $P_0^*$  to obtain higher spot demand, the firm chooses to increase  $P_0^*$  instead, since re-selling capacity can be done at a premium, and the marginal revenue from doing so (through a higher  $P_0^*$ ) is higher than marginal revenue derived from increasing spot demand through a lower  $P_0^*$ . A graphical representation of the above can be seen in Figure 1. Notice that the solutions are conditional on  $\beta > \delta \cdot \frac{2}{\sqrt{4 - \rho^2}}$ .

This means that the ability of a firm to obtain positive revenue from selling in advance is dependent on the degree of own-time price sensitivity vis-à-vis cross-time price sensitivity. Notice that  $S$  exists only when  $\rho > 0$ . Thus, while the terms proceeding  $S$  determine the level of increase or decrease in prices and quantities, we can intuitively label  $S$  as the *non-consumption effect* attributable to the advance purchase of services.

To highlight the impact on profit, the difference in expected profits (after some manipulation) can be written as

*Lemma 2:*

$$\pi^* - \pi^*(0) = (1) + (2) + (3) + (4) - (5)$$

$$\text{where (1) } \rho P^*(0) q^*(0) \quad (2) \quad \rho P^*(0) q^*(0) S \cdot [2(\beta - \delta) + \beta \rho] \quad (3) \quad \rho P^*(0) q^*(0) S \cdot [2(\beta - \delta) + \beta \rho] \\ (4) \quad \rho P^*(0) q^*(0) S \cdot [2(\beta + \delta) + \beta \rho] \quad (5) \quad \rho P^*(0) q^*(0) S^2 \cdot [2(\beta + \delta) + \beta \rho] \quad (6) \quad 2P^*(0) q^*(0) S^2 \cdot [4(\beta^2 - \delta^2) + 4\beta^2 \rho + \beta^2 \rho^2 - 4\delta^2 \rho]$$

The first term is the added profit due to double-selling the fraction of non-consuming capacity,  $\rho q(0)$ . The second shows that the capacity is double-sold with a price premium of  $S \cdot [2(\beta - \delta) + \beta \rho]$ . The third term shows that the advance demand also increases, amplified by the combination of both own-time and cross-time sensitivities i.e.  $S \cdot [2(\beta + \delta) + \beta \rho]$ . This amplification is due to advance demand being made higher through both a higher spot price and a lower advance price. The fourth term shows that the increase in advance demand also enjoys the same price premium. Finally, the fifth term captures the loss in revenue as a result of a lower advance price and a lower spot demand.

Lemma 1 is consistent with Xie-Shugan's model where it was shown that when marginal cost is low and capacity constraint is non-binding, advance and spot prices are the same. However, the findings here also show that advance prices may be lower even when marginal costs are zero, as the presence of  $\rho$  creates the divergence in advance and spot prices. Clearly, the potential revenue from one unit of advance sale is higher than that from spot sale. Therefore, advance price decreases to generate a higher advance demand. The cross-time effect of this is an even higher spot price.

As  $\rho$  increases, the firm has a greater incentive to price  $P_A$  lower to stimulate advance demand. This amplifies the decrease in spot demand, pushing  $P_0$  even higher.



*Proposition 2: The greater the probability of non-consumption, the higher (lower) the quantity sold in advance (at spot) and the lower (higher) the advance (spot) price i.e.  $\frac{\partial P_A^*}{\partial \rho} < 0$ ,  $\frac{\partial P_0^*}{\partial \rho} > 0$  and  $\frac{\partial q_A^*}{\partial \rho} >$ ,  $\frac{\partial q_0^*}{\partial \rho} < 0$ , and the higher the profit.*

In Png's model, a strategy of selling firm advance orders does not maximize profit because the advance buyer is unwilling to pay a higher price due to acquisition and valuation risks. Yet, a firm advance order usually guarantees availability and as the Xie-Shugan model has shown, firm advance orders can be optimal.

By modeling in non-consumption – and taking into account the fact that a buyer who purchases in advance has a non-zero probability of not consuming and that non-consumption frees up the capacity to be re-sold -- this study shows that the potential revenue from advance sales increases and that it may be optimal for the firm to sell in advance.

Png's model showed that the seller's revenue from spot sales is zero because buyers would prefer the non-contingent alternative in advance rather than wait till spot where the seller would extract all the consumer's surplus (i.e. high price). Where the market is heterogeneous in the form of a demand function, the optimal price at spot assumes not all surpluses are extracted from everyone. Consequently, there is also heterogeneity in the degree to which customers may be willing to wait till spot or buy in advance. This implies that both spot and advance demand would exist, with some degree of substitutability between buying at these two times, as modeled here. Accordingly, there is an optimal price at both times, as set out above in proposition 1.

Figure 1 also shows the same formulation, but with the firm constrained by capacity. Capacity constraint serves to push up advance and spot prices but reduce quantities in advance and at spot, with the result of a lower profit.

*Proposition 3: INTERMEDIATE CAPACITY*

*If the firm is constrained in capacity but the capacity is such that  $\frac{\alpha(\beta + \delta)}{\beta} < K < \frac{3\alpha(\beta + \delta)}{\beta + 2\delta}$ , the firm does not sell in advance when  $\rho' \leq \rho \leq 1$  AND  $K < \frac{3\alpha(\beta + \delta)}{\beta + 2\delta}$  whereby  $\rho' = \frac{2(K - 2\alpha)(\beta + \delta)}{K\beta - \alpha(\beta + \delta)}$  (NO ADVANCE SALE)*

*Proposition 4: LOW CAPACITY*

*If the firm is constrained in capacity such that  $K < \frac{\alpha(\beta + \delta)}{\beta}$ , the firm does not sell in advance nor at spot when  $0 \leq \rho' \leq \rho'' \leq \rho \leq 1$  whereby  $\rho'' = \frac{2K(\beta - \delta)}{K(\beta - 2\delta) + \alpha(\beta + \delta)}$  (MARKET FAILURE)*

Proposition 3 lays out the scenario for many services where the valuation risk (and therefore the non-consumption probability) is high and there is no positive price that would induce a buyer to buy in advance. Advance price is zero, and the firm will not sell in advance. This scenario also happens in the unconstrained case, although only at values of  $\rho$  higher than in the constrained case, since the constraint increases prices and lowers quantities, thereby resulting in  $q_0$  hitting the boundary faster with increasing  $\rho$ . This seems the case for many services that face high

valuation risks e.g. legal services, where buyers will only value the service at the time the need arises. Such services may therefore find no advance buyers.

Proposition 4 lays out the values of  $\rho$  where the firm isn't willing to sell in advance because the price is too low (i.e.  $P_A = 0$ ) and there are no buyers at spot either, because the spot demand is too low (i.e.  $q_0 = 0$ ) for the firm's price. This scenario of the firm's refusal to sell in advance and the market's refusal to buy at spot essentially demonstrates market failure. The propositions are diagrammatically depicted in Figure 2.

<< Take in Figure 2 >>

### MODEL EXTENSION: OFFERING A REFUND

Providing refunds for the buyer's inability to consume is widely practiced in the airline industry. Casual enquiries made by the author with airlines sales offices indicate that many airline tickets are sold with some refund value. Some airlines even provide a full refund, i.e. the purchased ticket can be returned to the airline for a complete reimbursement of the price at any time – even *after* the proposed travel date. This means that if the buyer cannot make a flight for *any* reason, the airline is fully prepared to return the price of the air ticket to the customer without any penalty fee, no questions asked. Furthermore, many airlines allow a refund on *non-utilized* sectors, e.g. if the consumer purchased a return ticket but only utilized one leg of the ticket.

According to Ng (2008), there is a fundamental difference between a full refund of this nature and those given out by retail shops for goods purchased or by service firms after the consumption of the service. In the latter, the refund is given (or promised) due to *firm's failure*; ie if the buyer perceives that the firm has failed to deliver the benefits. In the former -- and also the focal point of this study -- refunds are promised for *buyer failure* i.e. when the buyer fails to consume the service, through no fault of the firm. The use of the term 'buyer failure' is chosen for ease of explanation and is entirely from the firm's perspective; a firm's guarantee of capacity when selling in advance is usually in return for the buyer's guarantee to consume and if the buyer doesn't do so, he is deemed to have 'failed'. Buyers however, could argue that they have a right to demand for a refund since they have yet to consume the service. What this study aims to investigate, by extending the current model, is whether there is any benefit to the firm, revenue-wise, if it had to provide that refund.

Consequently, this study extends the model to investigate the circumstances where the firm may have to provide a refund,  $r$ , as a fraction of the advance price. The firm can be compelled to provide  $r$  due to reasons such as competition. It is assumed that there is a speculative market at  $t_A$ , such that it prohibits the firm from providing any refund above a full refund i.e.  $0 < r \leq 1$ . In addition, the offer of a refund expands advance demand, as this reduces valuation risk.

Definitions:

$P_{A,R}(P_{0,R}) =$  Price per unit of the service sold at advance time (spot), with a full refund offer

$\pi_R =$  Profit to the service firm when a refund is offered to advance buyers

$q_{A,R}(q_{0,R}) =$  Quantity of service demanded by the market at advance time (spot)

$r =$  Refund offered by the firm where  $0 < r \leq 1$

$\gamma =$  Advance demand shifting parameter due to refund offered.

The objective function is now  $\max_{P_A, P_0} \{(1-r\rho)P_A q_A + P_0 q_0 + P_0 \rho q_A\}$

subject to the system of demand  $q_0 = \alpha - \beta P_0 + \delta P_A$  and  $q_A = \alpha + \gamma r + \delta P_0 - \beta P_A$

We have the following constraints on probability of not consuming,  $\rho$ , and on the amount of refund,  $r$  which is  $0 \leq \rho \leq 1$  and  $0 \leq r \leq 1$  and also constraint on non-negativity of quantity of capacity sold, and the constraint that the number of customers who finally consume the service (whether they purchased in advance or at spot) does not exceed the capacity of the service firm i.e.  $q_0 + q_A \leq K$  and  $0 \leq q_0 \leq K$  and  $0 \leq q_A \leq K$

The resultant formulation for optimal refund cannot be described in a closed form, and is complex and not tractable. To understand optimal refund, the limiting cases cannot be seen from the formulae. Figure 3 shows the results from numerical analysis where each plot is for various values of  $\rho$ . Then for every demand-shifting factor,  $\gamma$ , there is an optimal value of refund,  $r$ , such that profit achieves maximum.

<< Take in Figure 3 >>

In order to understand Figure 3, suppose  $\rho$  is small (Figure 3: (2) or (3)). Then, if  $\gamma$  is small, the demand  $q_A$  is not sensitive to refund; the optimal refund is zero. If demand  $q_A$  is very sensitive to refund, then it is optimal to give a complete refund. It is clear that if the probability of not consuming is close to zero, the firm can then give a full refund even when  $q_A$  is not very sensitive to refund (Figure (3): (1)). Note that if probability  $\rho$  is not very large and if  $\gamma$  is large, then it is optimal to give a full refund.

Suppose  $\rho$  is close to one (above a certain threshold); basically almost everyone who buys in advance does not consume. Suppose the demand is sensitive to refund ( $\gamma > 0$ ). In this case, although it is not optimal to give a full refund, a refund offer is still made and the optimal refund falls with increasing probability  $\rho$  ((4), (5) and (6)). This optimal refund does not depend on  $\gamma$ . This is, however, misleading as at high values of  $\rho$ , advance price hits the boundary (i.e.  $P_A^* = 0$ ) as seen in Figure 4 which shows the effect of refund on optimal prices, quantities and profits, thereby resulting in the firm not selling in advance at all.

<< Take in Figure 4 >>

The intuition is as follows. First, at low values of  $\rho$  and  $\gamma$ , a full refund is optimal because the cost of providing it is low. However, as  $\rho$  increases while demand shift  $\gamma$  is constant (i.e. at X), optimal refund reduces as providing refunds become more costly. Since a refund offer increases advance demand and price as well as spot price (Figure 4), a refund offer may still be optimal even when the probability of not consuming is intermediately high (e.g. 0.6). Clearly, there is a balance between the benefit of higher demand and the cost of providing the refunds. Since the analysis is numerical, general conclusions are not possible.

*Proposition 5: There exists a scenario where  $\rho > 1$ ,  $\gamma > 1$ , and the firm sells both in advance and at spot i.e.  $P_A^*, q_A^* > 0$  and  $P_0^*, q_0^* > 0$  and the firm's profit is higher through advance prices and quantities and higher spot price (Figure 3) by (a) Providing no refund ( $X_1$ ) (b) Providing a positive refund such that  $0 < r < 1$  ( $X_2$ ) (c) Providing a full refund that is optimal ( $X_3$ )*

Xie-Shugan showed that firm advance orders with a refund offer may be optimal, as the firm is able to obtain a higher price in advance to compensate for the cost of refund, as well as derive greater profits from cost savings in not having to serve these customers.

The above proposition shows that the firm could derive higher revenue from higher prices, higher advance demand *as well as* higher spot prices when a refund is offered. This means that the

insurance from non-consumption is partially paid by an increase in advance price and partially borne by the expansion in advance demand.

## DISCUSSION

This study aims to offer a more applicable model of advance selling as a phenomenon. The theoretical model incorporated consumer price sensitivity, degree of cross-time substitutability, as well as the ability of the service firm to re-sell relinquished capacity at spot time to derive the optimal prices and quantities.

In proposition 1, the model found that advance prices are lower than spot prices even without binding capacity constraints or marginal costs, because the potential revenue from one unit of advance sale is higher than that from spot sale. Proposition 2 showed that as the probability of non-consumption increased, advance prices decreased while spot prices increased further.

Proposition 3 showed the conditions under which services are not able to sell in advance, where very low prices and capacity constraints mean only spot sales is optimal. Incidentally, emergency services would most likely fall into this category. Proposition 4 showed that market failure could occur when the firm refuses to sell in advance and there are no buyers at spot. This scenario could possibly illustrate the extreme case of selling a service where both acquisition and valuation risks (Ng, 2007) are high.

Proposition 5 showed that providing a refund could be optimal, resulting in increased advance prices and quantities, and with an effect on spot price. This proposition demonstrates the pareto optimality of such practices by service firms like airlines, where the offer of a refund is insurance against non-consumption for the buyer, but for the seller, the refund is funded in part by expanded demand and not merely by an increase in price, thus benefiting both buyers and seller.

The model can also be applied onto revenue management through a non-linear programming approach, where the firm can obtain the optimal price and quantity sold at each (continuous) point in time as:

$$P_t^* = \text{Arg max}_{P_t} \left\{ \pi \left| \int_n^0 q(t) dt \leq K \right. \right\} \text{ for } t \in \{0,1,2\dots n\} \text{ and } q_t^* = \alpha(t) - \beta(t)P_t^* + \delta(t)P_0^*$$

Where:  $q_t = \alpha(t) - \beta(t)P_t + \delta(t)P_0$  and  $q_0 = \alpha(0) - \beta(0)P_0 + \int_n^0 \delta_0(t)P_t dt$  and

$$\pi = \int_n^0 P_t q_t(P_t, t) dt + P_0 \int_n^0 \rho_t q_t(P_t, t) dt$$

Essentially, the firm faces a demand function at any point in advance that incorporates some degree of substitutability between the price at that time and the  $t_0$  price. Each demand parameter is time dependent, and the distribution of that parameter across time may be defined exogenously. The optimal price and quantity to be sold is generated from the objective function that multiplies the quantity sold at each time with the price at that time, cumulative across time. Each advance time will also yield a fraction that may not be consumed at  $t_0$  and that is accounted for in the objective function. Hence, the price and quantity relationship is captured explicitly, as is the substitutability between times of purchase. By using the demand function, a vast quantity of economic literature can be applied to the advance selling context to produce richer insights into the strategy of advance selling.

In the past, where pricing is often static, the above specification might have been difficult to implement. However, with the advent of the connected economy, where data can be obtained quickly, technological innovations have made complicated algorithms possible to implement. Thus, a dynamic pricing model such as the one suggested above is not impossible.

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## CONCLUSION

The study shows that without the possibility of influencing demand, services could end up with a service that has high state dependency, hence making it difficult for a firm to sell in advance. In the extreme case where a service has high state dependency characteristics and at the same time is not easily available, market failure occurs as the firm is unable to sell at all.

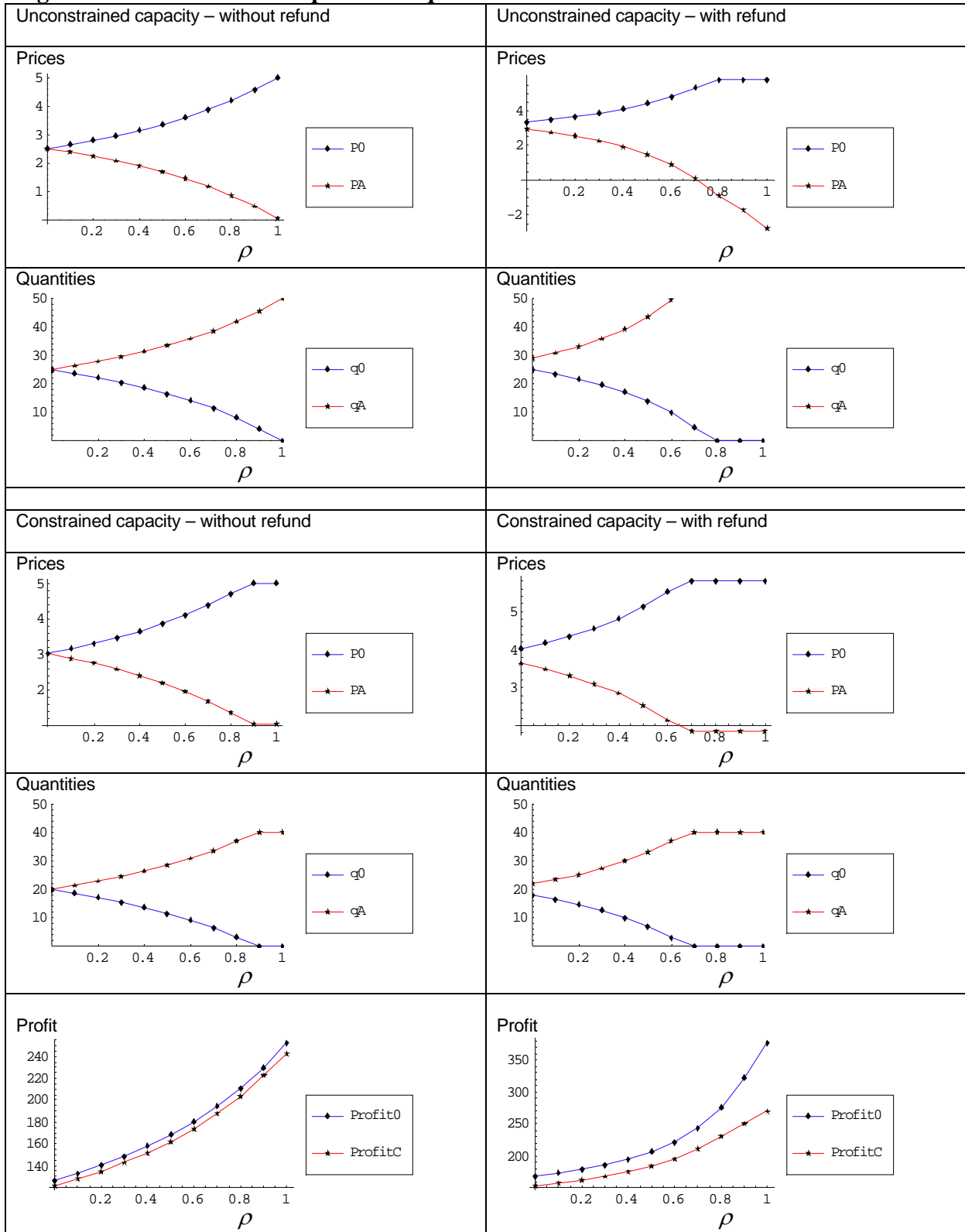
Our model also contributes to extant literature by demonstrating that when heterogeneous demand behavior is explicitly modeled with non-consumption and refunds, firms would then be able to understand where and how service revenue is being obtained (higher price or higher demand) and the impact of various sensitivities on revenue. The impact of capacity on the parameters modeled would also provide insight that could inform capacity investment decisions. Finally, as the model has demonstrated, the offer of refunds can increase profit for the firm. However, firms may interpret refunds as a form of costly differentiation and be tempted to drop the offer (as some low-cost airlines do). In doing so, they miss the opportunity to derive higher profits either through expanded advance demand or through higher advance prices.

It is unfortunate that the model extension is not mathematically tractable and therefore did not lend itself towards a closed form solution. Yet, we maintain that there is sufficient contribution from the results in propositions 1-4 and for the numerical scenarios presented in proposition 5 to warrant attention. The issue of services pricing has long been under-researched, and as service economies mature and become more competitive, it also becomes increasingly important.

The current study does not consider uncertainty in the spot price. Bringing in uncertainty could also allow for an investigation of a two-stage model that could be solved through backward induction. Further research can explore this as well as the costs of advance selling and the impact of competition on advance selling strategies. In addition, the limitation of the model which assumes a one-stage optimization could be relaxed into a multi-stage game where the service provider can change the spot prices after the advance demand has been realized. Finally, it would also be of use to investigate the case where the firm is not a monopoly. By abstracting the phenomenon of advance pricing in services through a stylized model, the study aims to provide a platform for a more thorough understanding of pricing in services across various service industries and the strategic levers to improve revenues.

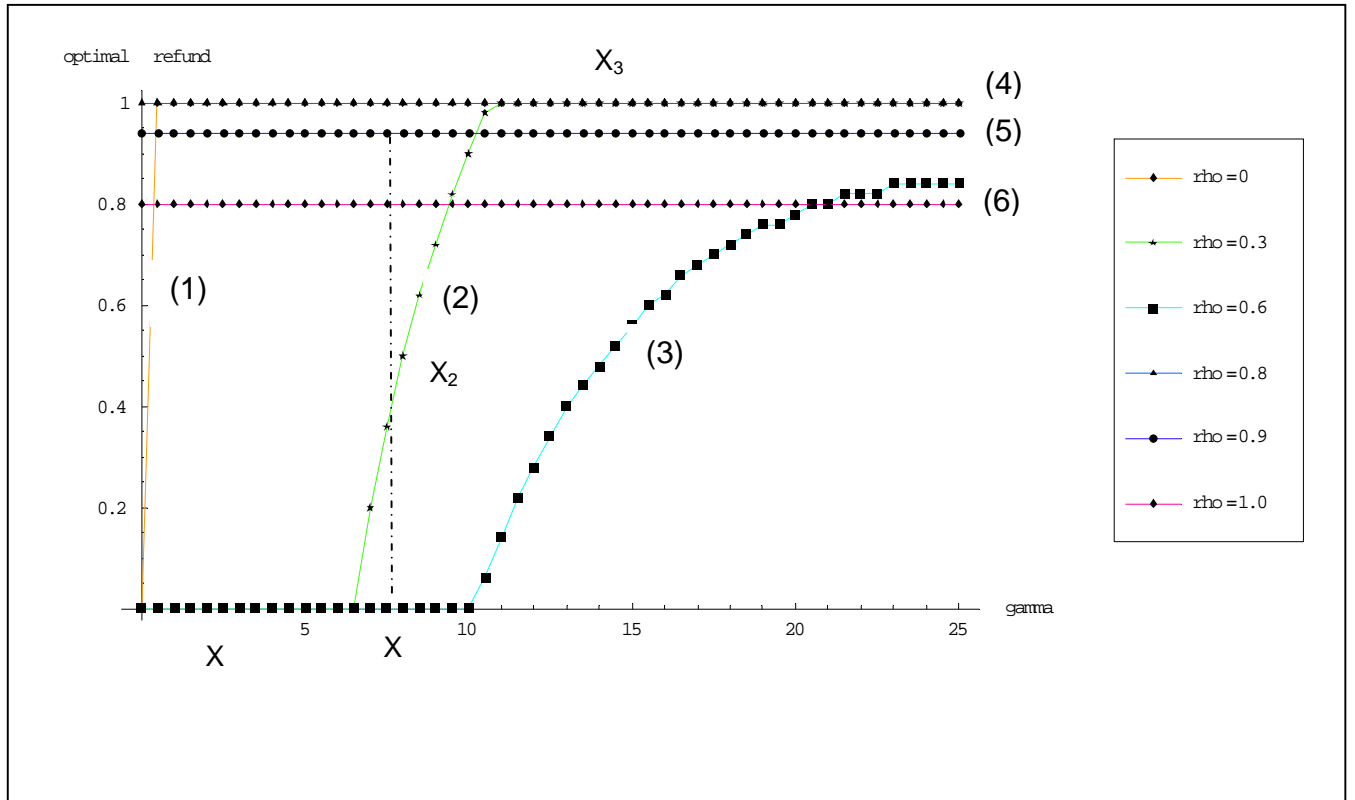


**Figure 3: Effect of refund on prices and quantities**



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**Figure 4: Optimal Refund with changing  $\gamma$  and  $\rho$**





APPENDIX (PROOFS)

**Lemma 1:**

$$(1) \quad q_A = \alpha - \beta P_A + \delta P_0$$

$$(2) \quad q_0 = \alpha - \beta P_0 + \delta P_A$$

and  $\pi = P_A q_A + P_0 q_0 + P_0 \rho q_A$

$Max_{P_A, P_0} \{\pi\}$  and solving for (1) and (2) results in

$$P_0^* = \frac{\alpha(2\delta + \beta(2 + \rho))}{4(\beta^2 - \delta^2) - \beta\rho^2} \quad ; \quad P_A^* = \frac{\alpha(2\delta + \beta(2 - \rho - \rho^2))}{4(\beta^2 - \delta^2) - \beta\rho^2}$$

Substituting back to the demand functions will lead to

$$q_0^* = \frac{\alpha[\beta^2(2 - \rho - \rho^2) - 2\delta^2 - \beta\delta\rho(1 + \rho)]}{4(\beta^2 - \delta^2) - \beta\rho^2} \quad ; \quad q_A^* = \frac{\alpha(\beta + \delta)(\beta(2 + \rho) - 2\delta)}{4(\beta^2 - \delta^2) - \beta\rho^2}$$

Thus when  $\rho = 0$ , the above yields

$$P_A^* = P_0^* = P^*(0) = \frac{\alpha}{2(\beta - \delta)}, \quad q_A^* = q_0^* = q^*(0) = \frac{\alpha}{2} \quad \text{and} \quad \pi^*(0) = \frac{\alpha^2}{2(\beta - \delta)}$$

**Proposition 1:**

From the above proof of lemma 1

$$P^*(0) - P_A^* \text{ results in } \Rightarrow P_A^* = P^*(0)[1 - S(2(\beta - \delta) + \rho(\beta - 2\delta))]$$

$$P_0^* - P^*(0) \text{ results in } \Rightarrow P_0^* = P^*(0)[1 + S(2(\beta - \delta) + \beta\rho)]$$

$$q^*(0) - q_0^* \text{ results in } \Rightarrow q_0^* = q^*(0)[1 - S(2(\beta + \delta) + \rho(\beta + 2\delta))]$$

$$q_A^* - q^*(0) \text{ results in } \Rightarrow q_A^* = q^*(0)[1 + S(2(\beta + \delta) + \beta\rho)]$$

Where  $S = \frac{\beta\rho}{4(\beta^2 - \delta^2) - \beta\rho^2}$  and  $S > 0$  if and only if  $\beta > \delta \cdot \frac{2}{\sqrt{4 - \rho^2}}$  and  $\frac{2}{\sqrt{4 - \rho^2}} > 1$  QED

**Lemma 2:**

From lemma 1 above, the optimal profit of the firm would be:

$$\pi^* = P_A^* q_A^* + P_0^* q_0^* + P_0^* \rho q_A^*$$

Replace the results of proposition 1 above (i.e. (9) – (12)) obtain the lemma after re-arranging (QED)

**Proposition 2:**

$$\frac{\partial S}{\partial \rho} = \frac{-4\beta\delta^2 + 4\beta^3(4 + \rho^2)}{(4(\beta^2 - \delta^2) - \beta\rho^2)^2} \quad \text{and} \quad \frac{\partial S}{\partial \rho} > 0 \text{ if } \beta > \delta \cdot \frac{2}{\sqrt{4 + \rho^2}} \text{ which is true since } \beta > \delta \text{ and}$$

$$\frac{2}{\sqrt{4 + \rho^2}} < 1. \text{ Hence,}$$

$$\frac{\partial P_A^*}{\partial \rho} = -2 \frac{\partial S}{\partial \rho} P^*(0)(\beta - \delta) - \frac{\partial S}{\partial \rho} P^*(0)\rho(\beta - 2\delta) - S P^*(0)(\beta - 2\delta) < 0,$$

$$\frac{\partial P_0^*}{\partial \rho} = 2 \frac{\partial S}{\partial \rho} P^*(0)(\beta - \delta) + \frac{\partial S}{\partial \rho} P^*(0)\rho\beta + S P^*(0)\beta > 0,$$

$$\frac{\partial q_0^*}{\partial \rho} = -2 \frac{\partial S}{\partial \rho} q^*(0)(\beta + \delta) - \frac{\partial S}{\partial \rho} q^*(0)\rho(\beta + 2\delta) - S q^*(0)(\beta + 2\delta) < 0$$

$$\frac{\partial q_A^*}{\partial \rho} = 2 \frac{\partial S}{\partial \rho} q^*(0)(\beta + \delta) + \frac{\partial S}{\partial \rho} q^*(0)\rho\beta + S q^*(0)\beta > 0$$

### Capacity Constraint and Refund Solution

It is convenient to solve the problem in quantities space, as all constraints are put on quantities. We can solve the demand system with respect to prices:

$$(13) \quad \begin{aligned} P_0 &= \frac{(\beta + \delta)\alpha + \delta\gamma r}{(\beta^2 - \delta^2)} - \frac{\beta}{(\beta^2 - \delta^2)} q_0 - \frac{\delta}{(\beta^2 - \delta^2)} q_A \\ P_A &= \frac{(\beta + \delta)\alpha + \beta\gamma r}{(\beta^2 - \delta^2)} - \frac{\delta}{(\beta^2 - \delta^2)} q_0 - \frac{\beta}{(\beta^2 - \delta^2)} q_A \end{aligned}$$

Note that if  $\beta > \delta$  as originally set, then intercept is always positive (provided that  $\alpha > 0$ ). If we try to solve the problem for  $\beta > \delta$  we need to calibrate  $\alpha < 0$ . Having inverted the system of demand, we can substitute prices as functions of quantities into the profit function to obtain:

$$(14) \quad \begin{aligned} &\max_{q_0, q_A} \left[ (1-r\rho) \left( \frac{(\beta + \delta)\alpha + \beta\gamma r}{(\beta^2 - \delta^2)} - \frac{\delta}{(\beta^2 - \delta^2)} q_0 - \frac{\beta}{(\beta^2 - \delta^2)} q_A \right) q_A \right. \\ &\quad \left. + \left( \frac{(\beta + \delta)\alpha + \delta\gamma r}{(\beta^2 - \delta^2)} - \frac{\beta}{(\beta^2 - \delta^2)} q_0 - \frac{\delta}{(\beta^2 - \delta^2)} q_A \right) (q_0 + \rho q_A) \right] \quad q_0 + q_A \leq K \\ &= q_0 \frac{r\gamma\delta + (\beta + \delta)\alpha}{(\beta^2 - \delta^2)} - \frac{\beta}{(\beta^2 - \delta^2)} q_0^2 + q_A \frac{(\rho + 1 - r\rho)(r\gamma\beta + (\beta + \delta)\alpha)}{(\beta^2 - \delta^2)} \quad \text{and } 0 \leq q_0 \leq K \\ &\quad - q_A^2 \frac{(1-r\rho)\beta + \rho\delta}{(\beta^2 - \delta^2)} + q_0 q_A \frac{(r\rho\delta - \rho\beta - 2\delta)}{(\beta^2 - \delta^2)} \quad 0 \leq q_A \leq K \end{aligned}$$

Note that we obtained a quadratic function with negative coefficients on  $q_0^2$  and  $q_A^2$ .

Note that we now maximize profit with respect to quantities.

These inequalities define us a triangular area in coordinates  $(q_0, q_A)$ .

3. If we want interpretation we can say that we are looking for a maximum of a paraboloid (think of an upside down bowl) underneath the triangular area. The maximum can be achieved either below the triangular area, in this case constraints are not binding, or outside. In the latter case, the maximum will be achieved on one of the boundaries of the triangular area, or in its vertexes.

4. This is what our algorithm does. We first find unconstrained maximum, then check if constraints are binding, and if they are, we look at the relevant constraint. Because all functions are quadratic, it is straightforward to do so.

5. The collection of formulae.

Unconstrained maximum is achieved at (partial derivatives of profit are zeros):

$$\begin{aligned} 0 &= \frac{(\rho + 1 - r\rho)(r\gamma\beta + (\beta + \delta)\alpha)}{(\beta^2 - \delta^2)} - 2q_A \frac{(1-r\rho)\beta + \rho\delta}{(\beta^2 - \delta^2)} + q_0 \frac{(r\rho\delta - \rho\beta - 2\delta)}{(\beta^2 - \delta^2)} \\ 0 &= \frac{r\gamma\delta + (\beta + \delta)\alpha}{(\beta^2 - \delta^2)} - 2 \frac{\beta}{(\beta^2 - \delta^2)} q_0 + q_A \frac{(r\rho\delta - \rho\beta - 2\delta)}{(\beta^2 - \delta^2)} \end{aligned}$$

from where:

$$q_A^* = \frac{(2(\beta - \delta) + \rho\beta + r\rho(\delta - 2\beta))(\beta + \delta)\alpha + \gamma r(2(\beta^2 - \delta^2) + \rho(2\beta^2 - \beta\delta - 2r\beta^2 + r\delta^2))}{((4 - \rho^2)\beta^2 - 4\delta^2 - \rho r(4(\beta^2 - \delta^2) - 2\rho\beta\delta + r\rho\delta^2))}$$

$$(15) \quad q_0^* = \frac{(2\beta(\rho + 1 - r\rho)(r\beta\gamma + \alpha(\beta + \delta)) + (r\gamma\delta + \alpha(\beta + \delta))(r\rho\delta - \rho\beta - 2\delta))}{((4 - \rho^2)\beta^2 - 4\delta^2 - \rho r(4(\beta^2 - \delta^2) - 2\rho\beta\delta + r\rho\delta^2))}$$

$$\times \frac{(r\rho\delta - \rho\beta - 2\delta)}{2\beta} + \frac{r\gamma\delta + \alpha(\beta + \delta)}{2\beta}$$

if capacity constraint is binding then:

$$(16) \quad q_A^* = \frac{(K(2(\beta - \delta) - \rho(\beta - r\delta)) + (\rho - r\rho + 1)(r\beta\gamma + \alpha(\beta + \delta)) - (r\gamma\delta + \alpha(\beta + \delta)))}{2(\delta - \beta)(\rho + r\rho - 2)}$$

$$q_0^* = K - q_A^*$$

if non-negativity ( $q_0=0$ ) constraint is binding:

$$(17) \quad q_A^* = \frac{(\rho + 1 - r\rho)(r\beta\gamma + \alpha(\beta + \delta))}{2(\beta - r\rho\beta + \rho\delta)}$$

$$q_0 = 0$$

if non-negativity ( $q_A=0$ ) constraint is binding:

$$(18) \quad q_0^* = \frac{r\delta\gamma + \alpha(\beta + \delta)}{2\beta} \quad \text{and} \quad (19) \quad P_0^* = \frac{(\beta + \delta)\alpha + \delta\gamma r}{(\beta^2 - \delta^2)} - \frac{\beta}{(\beta^2 - \delta^2)} q_0^* - \frac{\delta}{(\beta^2 - \delta^2)} q_A^*$$

$$q_A^* = 0 \quad P_A^* = \frac{(\beta + \delta)\alpha + \beta\gamma r}{(\beta^2 - \delta^2)} - \frac{\delta}{(\beta^2 - \delta^2)} q_0^* - \frac{\beta}{(\beta^2 - \delta^2)} q_A^*$$

and the profit is:

$$(20) \quad \pi^* = (1 - r\rho)P_A^*q_A^* + P_0^*q_0^* + P_0^*\rho q_A^*$$

The zero refund case (**proposition 3 and 4**)

Suppose there is no capacity constraint.  $K$  is much greater than  $\alpha$ . Then optimal quantities are determined by:

$$(21) \quad q_A^* = \frac{(2(\beta - \delta) + \rho\beta)(\beta + \delta)\alpha}{((4 - \rho^2)\beta^2 - 4\delta^2)}$$

$$q_0^* = \frac{(2(\beta - \delta) - \rho\beta(1 + \rho))(\beta + \delta)\alpha}{((4 - \rho^2)\beta^2 - 4\delta^2)} \frac{1}{2\beta}$$

Optimal prices are determined as:

$$(22) \quad P_0^* = \frac{(\beta + \delta)\alpha + \delta\gamma r}{(\beta^2 - \delta^2)} - \frac{\beta}{(\beta^2 - \delta^2)} q_0^* - \frac{\delta}{(\beta^2 - \delta^2)} q_A^*$$

$$P_A^* = \frac{(\beta + \delta)\alpha + \beta\gamma r}{(\beta^2 - \delta^2)} - \frac{\delta}{(\beta^2 - \delta^2)} q_0^* - \frac{\beta}{(\beta^2 - \delta^2)} q_A^*$$

Suppose there is a capacity constraint. Then optimal quantities are determined by:

$$(23) \quad q_A^* = \frac{K(2(\beta - \delta) - \rho\beta) + \rho\alpha(\beta + \delta)}{2(\beta - \delta)(2 - \rho)}$$

$$q_0^* = K - q_A^* = \frac{K(2(1 - \rho)(\beta - \delta) + \rho\beta) - \rho\alpha(\beta + \delta)}{2(\beta - \delta)(2 - \rho)}$$

As  $\rho$  approaches one,  $P_A^*$  approaches negative values. Hence, when  $P_A^* \leq 0$ , which is when  $\rho \leq \rho'$  whereby  $\rho' = \frac{2(K - 2\alpha)(\beta + \delta)}{K\beta - \alpha(\beta + \delta)}$  and  $\rho' < 1$  when  $K < \frac{3\alpha(\beta + \delta)}{\beta + 2\delta}$  therefore

No advance sale when:

$$(24) P_A^* = 0 \text{ when } \rho' \leq \rho \leq 1 \text{ AND } K < \frac{3\alpha(\beta + \delta)}{\beta + 2\delta}$$

Similarly, it is clear from the graphs in Figure 1 that if  $\rho$  approaches one,  $q_0$  becomes negative. The non-negativity constraint becomes binding i.e. when  $q_0^* \leq 0$ ,  $\rho > \rho''$  whereby  $\rho'' = \frac{2K(\beta - \delta)}{K(\beta - 2\delta) + \alpha(\beta + \delta)}$

and  $\rho'' < 1$  when  $K < \frac{\alpha(\beta + \delta)}{\beta}$  therefore:

No Spot sale when:

$$(25) q_0^* = 0 \text{ when } \rho'' \leq \rho \leq 1 \text{ AND } K < \frac{\alpha(\beta + \delta)}{\beta}$$

Hence, when:

$\rho' < \rho''$  if and only if:

$K < \frac{\alpha(\beta + \delta)}{\delta}$  and,  $K < \frac{\alpha(\beta + \delta)}{\beta}$  and  $K < \frac{\alpha(\beta + \delta)}{\beta - 2\delta}$  which are all true when  $K < \frac{\alpha(\beta + \delta)}{\beta}$  from (24) above.

Clearly, from (24) and (25), market failure occurs when  $0 \leq \rho'' \leq \rho \leq 1$

However, at intermediate capacity of  $\frac{\alpha(\beta + \delta)}{\beta} < K < \frac{3\alpha(\beta + \delta)}{\beta + 2\delta}$ , market failure does not occur although the firm is only able to sell in advance when  $0 \leq \rho' \leq \rho \leq 1$

### Proposition 5

This proposition is illustrated numerically from Figure 3 and 4, obtained by entering the following values into the formulae in (15) – (23) above:

$\alpha = 50$ ,  $\beta = 10$ ,  $\delta = 0.1$ ,  $\gamma = 16$ ,  $r = 0.5$  (in the case where a refund is offered),  $K = 40$  for the constrained firm,  $K = 100$  for the unconstrained firm

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<sup>1</sup> Inflation and Growth in a Service Economy", Bank of England Quarterly Bulletin: November 1998