# Inflation Conservatism and Monetary-Fiscal Policy Interactions<sup>\*</sup>

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This paper investigates the stabilization bias that arises in a model of monetary and fiscal policy stabilization of the economy, when monetary authority puts higher weight on inflation stabilization than society. We demonstrate that inflation conservatism unambiguously leads to social welfare losses if the fiscal authority acts strategically under discretion. Although the precise form of monetary-fiscal interactions depends on the leadership structure, the choice of fiscal instrument, and the level of steady-state debt, the assessment of gains is robust to these assumptions. We develop an algorithm that computes leadership equilibria in a general framework of LQ RE models with strategic agents.

JEL Codes: E31, E52, E58, E61, C61.

# 1. Introduction

There is a well-understood policy proposal that the agency charged with determining monetary policy, usually the central bank, should be "inflation conservative," by which it is meant that it should have a higher relative weight on the inflation stabilization objective than is socially optimal. The logic runs as follows. Suppose the policymaker

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must act under discretion. Also suppose, for some reason (and we nearly always assume some political economy issues here), it wants to have lower unemployment (or equivalently higher output). Such a policymaker will generate an excessively high equilibrium *rate* of inflation, a *level bias*. This is the now-textbook Barro and Gordon (1983) model. One possible resolution of this problem is to make the central bank both independent and inflation conservative, as famously shown by Rogoff (1985). This removes some of the incentive to reduce unemployment, and hence reduces the inflation bias. This analysis, and its myriad extensions, has been very influential: most central banks in developed countries are now operationally (or at least target) independent. Even so, central banks are often bestowed with the soubriquet "conservative" in the popular press, even if mitigating the inflationary bias is not quite what journalists have in mind.

There seems little doubt that major central banks do not pursue excessive output targets and none of them aims to stabilize inflation at too high a level. Contemporary policymakers demonstrate considerable agreement about what the *targets* of policy should be. Nonetheless, in *dynamic* models a *stabilization bias* might still arise. If a policymaker acts under discretion, then the *volatility* of welfarerelevant economic variables is necessarily higher than if it acts under commitment; see Currie and Levine (1993). Svensson (1997) calls the difference in the welfare "stabilization bias" and demonstrates, once again, that delegating policy to a conservative central bank could still lead to an improved outcome. Clarida, Galí, and Gertler (1999) show that an inflation-conservative monetary authority that acts under discretion can achieve the same level of welfare as under the optimal precommitment-to-rules policy. In other words, the stabilization bias can be reduced: making the central bank inflation conservative helps solve the problem of optimal delegation.<sup>1</sup> It seems reasonable to conclude that the conservative central bank proposal is a good one, as it deals with both the level and the stabilization bias.

However, it would be naive to assume that an independent inflation-conservative central bank *eliminates* any influence of

<sup>&</sup>lt;sup>1</sup>The optimal delegation is usually described as a possibility to distort targets of a discretionary policymaker so that it can achieve higher social welfare than it would achieve if it were benevolent.

government, which, after all, still controls the fiscal instruments of financial management. It may deviate from what is socially optimal and, say, have a lower unemployment target or prefer more stable growth. Even if it remains entirely benevolent, it may still behave *strategically*,<sup>2</sup> which can affect the resulting equilibrium. Both level and stabilization bias can still arise.

Indeed, Dixit and Lambertini (2003), using a static Barro-Gordon type model, demonstrate that a conflict of interests does make the outcome suboptimal in a model of monetary-fiscal interactions with a conservative central bank and a *benevolent* fiscal authority. The level bias still arises: the levels of inflation and unemployment in steady-state equilibrium are both higher than socially optimal. With an additional policymaker, it seems the conservative central bank proposal does not work out even in a *static* model.<sup>3</sup> Similarly, the conservative central bank solution to an optimal delegation problem in a *dynamic* stochastic environment could also be misleading if potentially strategic play by policymakers is ignored.<sup>4</sup>

In further related research, Adam and Billi (2008) examine the advantages of inflation conservatism using a non-linear dynamic model similar to ours and find that it can be beneficial. However, they mostly study the implications for the steady state, and limit their analysis of stabilization bias to the case of strategic fiscal leadership and a flexible-price version of the model. By contrast, our paper is concerned with the stabilization aspects of the problem and can be seen as a complement to their paper.

 $<sup>^{2}</sup>$ There is little doubt that the fiscal authorities can act strategically: An existing empirical literature on monetary-fiscal interactions suggests that fiscal policy does more than simply allow automatic stabilizers to operate; see, e.g., Auerbach (2003) and Favero and Monacelli (2005), who analyze fiscal policy in the United States. Moreover, since the onset of European monetary union, there are calls for greater fiscal flexibility, although how strategic such authorities should be is not explicitly discussed.

 $<sup>^{3}</sup>$ These results were further developed in Lambertini (2006) specifically for the conservative central bank proposal. See also Hughes Hallett, Libich, and Stehlik (2009).

<sup>&</sup>lt;sup>4</sup>Of course, most policy authorities (the central bank and the fiscal authority) would quite rightly deny that they "play games." However, by "playing a game" we mean that we model the ability of each authority to understand the other's priorities and reaction functions.

Making the model dynamic furthers the analysis on two accounts. First, we argue that there is now a wide consensus about appropriate level targets for the financial authorities, so the focus of the policy debate is often *how quickly* should they try to achieve these targets, rather than *which targets* to achieve. Second, modeling monetary-fiscal interactions in a dynamic setting allows us to study the effect of the government's solvency constraint. Debt stabilization issues can impose severe restrictions on the stabilization abilities of both authorities (see Leeper 1991), but it is very difficult to model these restrictions adequately in a static model.<sup>5</sup>

In this paper we explore the importance of both strategic behavior and inflation conservatism in a linear-quadratic rational expectations (LQ RE) infinite-horizon dynamic model of a kind widely used in policy analysis. We employ a conventional model with monopolistic competition and sticky prices in the goods markets (as in Woodford 2003a, ch. 6), extended to include fiscal policy and nominal government debt. We allow both authorities to act strategically and non-cooperatively in pursuit of their own objectives. We then delegate monetary policy to an inflation-conservative central bank. We demonstrate that even if the fiscal authority remains benevolent but acts strategically, delegating monetary policy in this way leads to welfare losses. The basic intuition is that if the authorities' objectives do not coincide, then one strategic policymaker can offset the policy of the other. We show that although a small degree of conservatism can be harmless, greater conservatism leads to substantial welfare losses.

As in Dixit and Lambertini (2003), the quantitative outcome of the game depends on the leadership structure. We study three possibilities: either the monetary or fiscal authorities lead or they move simultaneously (play a Nash game). This analysis is impossible to conduct without developing an appropriate modeling framework. Because an even moderately complicated dynamic model needs to be solved using numerical methods and these methods are neither readily available nor well articulated for models with rational

 $<sup>{}^{5}</sup>$ Beetsma and Bovenberg (2006) introduce public debt into a two-period model. Their results are not readily comparable with ours, as they focus their research on the dynamic of public debt and use an ad hoc welfare metric.

expectations,<sup>6</sup> we develop relevant solution methods for discretionary equilibria in dynamic linear rational expectations models where we make the role of leadership explicit. Although this is a key contribution of the paper and a necessary step in our analysis, we relegate detailed discussion to appendix 1.

This paper differs from Dixit and Lambertini (2003) and Adam and Billi (2008) in four important respects. First, we study the stabilization rather than the level bias, and focus on dynamics rather than the steady state. Second, we examine different leadership regimes, and provide suitable algorithms for calculating appropriate equilibria. In so doing we solve a rather general LQ RE model within a class of dynamic models commonly used by policymakers. Third, we explicitly account for the effects of potential fiscal insolvency. Finally, we show how the interaction of the two authorities depends on the steady-state *level* of debt.

Our results are in marked contrast to the received wisdom outlined above. We find that if the fiscal authority is benevolent but acts strategically, then delegating monetary policy to an inflation-conservative agency usually increases stabilization bias and so reduces social welfare. Any distortion to the social objectives can bring the two policymakers into conflict with each other in a way that nearly always reduces social welfare for our model. The message is clear: What works well in an economy with a single policymaker may not work at all in an economy with two strategic policymakers. Our assessment of the gains from delegation seems robust to the specification of the model and the choice of fiscal policy instrument. We conduct sensitivity analysis over the model, and show that whilst there are differences in the behavior of fiscal policy if we choose either distortionary taxes or spending, the qualitative results are the same. However, there is an issue about the steady-state level of debt. The low- and high-debt cases (defined below) have quite different qualitative effects.

The paper is organized as follows. In the next section we present the model and define the private-sector equilibrium. In section 3 we discuss policy design: the choice of instruments, policy objectives,

 $<sup>^{6}\</sup>mathrm{de}$  Zeeuw and van der Ploeg (1991) provide an excellent discussion of discrete dynamic games which can be compared with our analysis.

the degree of precommitment, and the leadership structure. Calibration of the model is explained in section 4. Section 5 presents the analysis of the benchmark regime with non-cooperative benevolent policymakers, where we compare the outcome of benevolent discretionary policymaking to a solution under commitment with full cooperation of the authorities. In section 6 we study the effect of an inflation conservatism of the monetary authorities on welfare, and we contrast this case to the case of cooperation of benevolent policymakers under discretion. The analysis in these sections assumes the fiscal authorities use spending as a fiscal instrument. Section 7 investigates how our results change if fiscal policy uses tax as an instrument instead or together with spending. Section 8 concludes.

#### 2. The Model

We consider the now-mainstream macro policy model, discussed in Woodford (2003a), modified to take account of the effects of fiscal policy. It is a closed-economy model with two policymakers—the fiscal and monetary authorities. Fiscal policy is allowed to support monetary policy in stabilization of the economy around the steady state.

#### 2.1 Private Sector

#### 2.1.1 Consumers

The economy is inhabited by a large number of households who specialize in the production of a differentiated good (indexed by z) and who spend h(z) of effort in its production. They consume a basket of goods C, and derive utility from per capita government consumption G. The household's maximization problem is

$$\max_{\{C_s,h_s\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} [u(C_s) + f(G_s) - v(h_s(z))].$$
(1)

The price of a differentiated good z is denoted by p(z), and the aggregate price level is P. All households consume the same basket of goods. Goods are aggregated into a Dixit and Stiglitz (1977) consumption index with the elasticity of substitution between any pair

of goods given by  $\epsilon_t > 1$  (which is a stochastic<sup>7</sup> elasticity with mean  $\epsilon$ ),  $C_t = \left[\int_0^1 c_t^{\frac{\epsilon_t - 1}{\epsilon_t}}(z) dz\right]^{\frac{\epsilon_t}{\epsilon_t - 1}}$ .

A household chooses consumption and work effort to maximize criterion (1) subject to the demand system and the intertemporal budget constraint:

$$\mathcal{E}_t \sum_{s=t}^{\infty} Q_{t,s} P_s C_s \le \mathcal{A}_t + \mathcal{E}_t \sum_{s=t}^{\infty} Q_{t,s} \{ (1 - \Upsilon_s) (W_s(z) h_s(z) + \Pi_s(z)) \} + TR \},$$

where  $P_t C_t = \int_0^1 p(z)c(z)dz$  is nominal consumption,  $\mathcal{A}_t$  are nominal financial assets of a household,  $\Pi_t$  is profit and TR is a *constant* lump-sum tax or subsidy. Here W is the wage rate, and  $\Upsilon_t$  is a tax rate on income.  $Q_{t,t+1}$  is the stochastic discount factor which determines the price in period t to the individual of being able to carry a state-contingent amount  $\mathcal{A}_{t+1}$  of wealth into period t+1. The riskless short-term nominal interest rate  $i_t$  is represented in terms of the stochastic discount factor as  $\mathcal{E}_t(Q_{t,t+1}) = (1+i_t)^{-1}$ .

We assume that the net present value of future income is bounded and that the nominal interest rate is positive at all times. Optimization for the household requires it to exhaust its intertemporal budget constraint, with wealth accumulation satisfying the no-Ponzi-game condition  $\lim_{s\to\infty} \mathcal{E}_t(Q_{t,s}\mathcal{A}_s) = 0$ . We assume the specific functional form for the utility-from-consumption component,  $u(C_s) = \frac{C_s^{1-1/\sigma}}{1-1/\sigma}$ , so household optimization leads to the following dynamic relationship for aggregate consumption:

$$C_t = \mathcal{E}_t \left( \left( \frac{1}{\beta} \frac{P_{t+1}}{P_t} Q_{t,t+1} \right)^\sigma C_{t+1} \right).$$
(2)

Additionally, aggregate (nominal) asset accumulation is given by

$$\mathcal{A}_{t+1} = (1+i_t)(\mathcal{A}_t + (1-\Upsilon_t)(W_t N_t + \Pi_t) - P_t C_t - TR),$$

where  $W_t$  and  $N_t$  are aggregate wage and employment.

We linearize equation (2) around the steady state (here and everywhere below for each variable  $X_t$  with steady-state value X,

<sup>&</sup>lt;sup>7</sup>We make this parameter stochastic to allow us to generate shocks to the markup of firms, as did, e.g., Beetsma and Jensen (2004).

we use the notation  $\hat{X}_t = \ln(X_t/X)$ ). Equation (2) leads to the following Euler equation (intertemporal IS curve):

$$\hat{C}_t = \mathcal{E}_t \hat{C}_{t+1} - \sigma(\hat{\imath}_t - \mathcal{E}_t \hat{\pi}_{t+1}).$$
(3)

Inflation is  $\pi_t = \frac{P_t}{P_{t-1}} - 1$ , and we ensure steady-state inflation is zero by appropriate transfers (Woodford 2003a).

#### 2.1.2 Firms

Price setting is based on Calvo contracting as set out in Woodford (2003a). Each period, firms are allowed to recalculate their prices with probability  $1 - \gamma$ , so that they remain fixed with probability  $\gamma$ . Following Woodford (2003a) we can derive the following Phillips curve for our economy<sup>8</sup>:

$$\hat{\pi}_t = \beta \mathcal{E}_t \hat{\pi}_{t+1} + \frac{(1 - \gamma \beta)(1 - \gamma)\psi}{\gamma(\psi + \epsilon)} \hat{s}_t, \tag{4}$$

where marginal cost is

$$\hat{s}_t = \frac{1}{\psi} \hat{Y}_t + \frac{1}{\sigma} \hat{C}_t + \frac{\tau}{(1-\tau)} \hat{\Upsilon}_t + \hat{\eta}_t.$$

The shock  $\hat{\eta}_t$  is a markup shock and parameter  $\psi = v_y/v_{yy}y$ . Parameter  $\tau$  is the steady-state level of tax rate.

Under flexible prices and in the steady state, the real wage is always equal to the monopolistic markup  $\mu_t = -(1 - \epsilon_t)/\epsilon_t$ . Optimization by consumers then implies

$$\frac{\mu^w}{\mu_t} = \frac{y_t^n(z)^{1/\psi}}{\left(1 - \hat{\Upsilon}_t^n\right) \left(C_t^n\right)^{1 - 1/\sigma}},\tag{5}$$

where superscript n denotes natural levels (see Woodford 2003a), and  $\mu^w$  is a steady-state employment subsidy which we discuss below. Linearization of (5) and aggregation yields

$$\hat{Y}_t^n \frac{1}{\psi} + \hat{C}_t^n \frac{1}{\sigma} + \frac{\tau}{(1-\tau)} \hat{\Upsilon}_t^n = 0.$$

<sup>&</sup>lt;sup>8</sup>The derivation is identical to the one in Woodford (2003a), amended by the introduction of markup shocks as in Beetsma and Jensen (2004).

# 2.2 Government

The government buys consumption goods  $G_t$ , taxes income with tax rate  $\Upsilon_t$ , raises lump-sum taxes T, pays an employment subsidy  $\mu^w$ , and issues nominal debt  $\mathcal{B}_t$ . Debt is assumed to be one period with a risk-free rate of return. The evolution of the nominal debt stock  $\mathcal{B}_t$  is

$$\mathcal{B}_{t+1} = (1+i_t)(\mathcal{B}_t + P_t G_t - \Upsilon_t P_t Y_t - T + \mu^w).$$
(6)

The lump-sum taxes (T) and employment subsidy  $(\mu^w = T)$  are constant and cannot be used to balance the budget or stabilize the economy.

Equation (6) can be linearized as follows:

$$\tilde{b}_{t+1} = \chi \hat{i}_t + \frac{1}{\beta} (\tilde{b}_t - \chi \hat{\pi}_t + (1 - \theta) \hat{G}_t - \tau (\hat{\Upsilon}_t + \hat{Y}_t)), \qquad (7)$$

where we defined a measure of real debt  $B_t = \mathcal{B}_t/P_{t-1}$  and denoted the steady-state ratio of debt to output as  $\chi$ . The steady-state level of debt is determined from the steady-state version of (6),  $\chi = \frac{\theta - (1-\tau)}{(1-\beta)}$ , and is a function of the steady-state tax rate,  $\tau$ , and of the steady-state ratio of consumption to output,  $\theta$ . We denote  $\tilde{b}_t = \chi \hat{B}_t$ , which allows us to use the same system if  $\chi = 0$ ; then  $\tilde{b}_t = B_t$ , and there are no first-order effects of either the interest rate or inflation on debt.

# 2.3 Market Clearing Conditions

Output is distributed as wages and profits:

$$P_t Y_t = W_t N_t + \Pi_t.$$

As government expenditures constitute part of demand, the national income identity is

$$Y_t = G_t + C_t \tag{8}$$

and in steady state  $G = (1 - \theta)Y$ . The linearized national income identity (or the resource constraint) is then

$$\hat{Y}_t = (1 - \theta)\hat{G}_t + \theta\hat{C}_t.$$
(9)

#### 2.4 Private-Sector Equilibrium

We simplify notation in equations (3), (4), (7), and (9) by using lowercase letters to denote "gap" variables, where the gap is the difference between actual levels and natural levels, i.e.,  $x_t = X_t - X_t^n$ . The final system of first-order conditions consists of an intertemporal IS curve (10), the Phillips curve (11), national income identity (12), and an equation explaining the evolution of debt (13):

$$c_t = \mathcal{E}_t c_{t+1} - \sigma(i_t - \mathcal{E}_t \pi_{t+1}), \tag{10}$$

$$\pi_t = \beta \mathcal{E}_t \pi_{t+1} + \frac{\kappa}{\sigma} c_t + \frac{\kappa}{\psi} y_t + \frac{\kappa \tau}{(1-\tau)} \tau_t + \eta_t, \qquad (11)$$

$$y_t = (1 - \theta)g_t + \theta c_t, \tag{12}$$

$$\tilde{b}_{t+1} = \chi i_t + \frac{1}{\beta} (\tilde{b}_t - \chi \pi_t + (1 - \theta)g_t - \tau(\tau_t + y_t)), \quad (13)$$

where parameter  $\kappa = \frac{(1-\gamma\beta)(1-\gamma)\psi}{\gamma(\psi+\epsilon)}$ . A private-sector rational expectations equilibrium consists of plan  $\{c_t, \pi_t, \tilde{b}_t, y_t\}$  satisfying equations (10)–(13), given the policies  $\{i_t, g_t, \tau_t\}$ , the exogenous process  $\{\eta_t\}$ , and initial conditions  $\tilde{b}_0$ .

#### 3. Monetary and Fiscal Policy Regimes

#### Welfare Criterion 3.1

We assume that both authorities set their instruments to maximize the aggregate utility function:

$$\frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{C_s^{1-1/\sigma}}{1-1/\sigma} + \xi \frac{G_s^{1-1/\sigma}}{1-1/\sigma} - \int_0^1 \frac{h_s^{1+1/\psi}(z)}{1+1/\psi} dz \right].$$
(14)

We show in appendix 2 that problem (14) implies the following optimization problem for a benevolent policymaker. A benevolent policymaker minimizes the discounted sum of all future losses:

$$L_t = \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} W_s^S,$$

where the one-period term is

$$W_s^S = \rho_c c_s^2 + \rho_g g_s^2 + \rho_y y_s^2 + \pi_s^2 + \mathcal{O}(\|\xi\|^3)$$
(15)

and  $\mathcal{O}(\|\xi\|^3)$  collects terms of higher order and terms independent of policy. We normalize the coefficient on inflation to one. This quadratic approximation to social welfare is obtained assuming that there is a steady-state production subsidy  $\mu^w = T$  that eliminates the distortion caused by monopolistic competition and income taxes.<sup>9</sup>

Note that expression (15) contains a quadratic term in government spending, g. This term enters the welfare expression because it is assumed in (1) that households derive utility from the consumption of public goods, and that the steady-state level of government spending reflects this. However, if we instead assumed that government spending was pure waste but the government still used g as a policy instrument, then changes in g would still influence social welfare through the national income identity, but it would not constitute an independent source of welfare loss.

Each policymaker minimizes its own loss functions. If it is benevolent, it adopts the social loss function. Note that when assigning social welfare (15) to the monetary authority, we do not eliminate quadratic terms in government spending. This term is not independent of policy, as monetary policy actions affect fiscal policy decisions.

# 3.2 The Choice of Policy Instruments

We assume that monetary policy uses short-term nominal interest rate as the policy instrument and that the fiscal authority uses government spending as its instrument. This follows current convention for the monetary policymaker, but the choice of fiscal instrument is more arbitrary. Our choice is determined by the following considerations. First, there is no well-established form of fiscal policy rule. Empirical estimates of fiscal policy reaction functions (see Taylor 2000, Auerbach 2003, and Favero and Monacelli 2005, for example) suggest that both government spending and taxes are varied by the fiscal authority. Second, from a methodological point of view, using spending is

 $<sup>^9{\</sup>rm This}$  derivation follows Woodford (2003a). Alternative ways of deriving social welfare (see, for example, Sutherland 2002 and Benigno and Woodford 2004) are inappropriate for discretionary policy.

a convenient starting point. We discuss in section 7 how our results change if we use taxes instead, so, effectively, we look at both possibilities. We shall refer to the income tax rate as "taxes" in what follows.

#### 3.3 The Benchmark Ramsey Allocation

The Ramsey allocation takes into account the presence of distortions, as summarized by the implementability constraints (10)-(11), the resource constraint (12), and the financial constraint (13). Specifically, the Ramsey allocation in the LQ framework solves

$$\min_{\{i_t, g_t, \tau_t\}} \frac{1}{2} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} W_s^S$$

subject to constraints (10)–(13) for all  $t \ge 0$ .

The Ramsey allocation requires commitment to policies and full cooperation between the authorities. In what follows we term this the *commitment solution*. We use the commitment solution as the benchmark case.

#### 3.4 Discretionary Policy and Non-Cooperative Regimes

We assume that both monetary and fiscal authorities act noncooperatively in order to stabilize the economy against shocks. Both authorities are assumed to act under discretion. We assume that the monetary authority chooses the interest rate to minimize the welfare loss with per-period metrics in the form of (15) subject to the system (10)-(13), and the benevolent fiscal authority chooses spending to minimize the welfare loss (15) subject to the same system.

Of course, if both authorities are benevolent, then they both use the per-period social loss as their objective function. If the monetary authority is inflation conservative, then it is assumed to deviate from the microfounded weight on inflation variability, so its per-period objective becomes

$$W_s^M = \rho_c c_s^2 + \rho_g g_s^2 + \rho_y y_s^2 + \rho_\pi \pi_s^2 + \mathcal{O}(\|\xi\|^3),$$
(16)

where  $\rho_{\pi} \geq 1$ .

Each of the policymakers solves an optimization problem every period. The resulting optimal policy reactions lead to stochastic equilibria that should be compared across a suitable metric. The obvious choice is the microfounded social loss. The sequence of events and actions within a period is as follows. At the beginning of every period t, the state  $\tilde{b}_t$  is known and shock  $\eta_t$  realizes. Then the two policymakers choose the value of  $i_t$ ,  $g_t$ , and  $\tau_t$ . There is a particular order of moves, and we shall study all possibilities: (i) one of the policymakers moves first, and the leader chooses the best point on the follower's reaction function or (ii) both policymakers move simultaneously and neither is able to exploit the reaction function of the other policymaker. In all cases, the policymakers know the state  $\tilde{b}_t$  and take the process by which private agents behave as given. After the policymakers have moved, in the next stage the private sector simultaneously adjusts its choice variables  $\pi_t$  and  $c_t$ . The optimal  $\pi_t$ ,  $c_t$  and policy  $i_t$ ,  $g_t$ , and  $\tau_t$  result in the new level of  $\tilde{b}_{t+1}$  by the beginning of the next period t + 1.

#### 4. Calibration

We take the model's frequency to be quarterly. To achieve a steadystate rate of interest of approximately 4 percent, we set the household discount rate  $\beta$  to 0.99. The remaining parameters of the utility function are typical of those used in the literature; see, e.g., Canzoneri et al. (2006). The elasticity of intertemporal substitution  $\sigma$  is taken as 1/1.5, the Calvo parameter  $\gamma$  is set at 0.75 so as to imply average contracts of about a year, the elasticity of demand is taken as  $\varepsilon = 7.0$  to achieve a 17 percent markup, and elasticity of labor demand is taken as  $\psi = 1/3$ .

The ratio of government consumption to output in the point of linearization,  $1 - \theta$ , is a function of relative weight on the utility-from-public-consumption term,  $\xi$ . We choose to calibrate  $\xi$  such that it would result in realistic  $1 - \theta = 0.25$ .

We shall demonstrate that different values of  $\chi$  can result in qualitatively different policy interactions. In what follows, we consider two values for the debt-to-GDP ratio. Our "high" debt level,  $\chi = 1.2$ , corresponds to 30 percent of annual output, which is still less than the level of debt in a number of European economies. However, we only consider one-period debt, so the figure of 30 percent is large enough to demonstrate *qualitative* difference with  $\chi = 0$ , which corresponds to 0 percent of annual output and which we treat as "low" debt level. The steady-state level of tax rate is a function of the steadystate debt level,  $\tau = \tau(\chi)$ . Of course, "high" and "low" level of debt corresponds to "low" and "high" tax rate.

We only study the effect of cost-push shocks, as they are the biggest potential source of social loss.<sup>10</sup> We calibrate the standard deviation of an i.i.d. cost-push shock as 0.005. In our baseline case, this generates a standard deviation for inflation of 0.0038, approximately the same order of magnitude as experienced in developed countries over the last couple of decades. This number is also not unreasonable given other academic studies. Ireland (2004) uses a cost-push shock, which is AR(1) with a standard deviation of 0.0044. Smets and Wouters (2003) report an i.i.d. cost-push shock with a much smaller standard deviation in the model with inflation persistence, while Rudebusch (2002) estimates a standard deviation of 0.01 for an i.i.d. cost-push shock.

#### 5. Benevolent Policymakers

We first look at how monetary and fiscal policy interact if policy objectives are not distorted. As we shall see, the qualitative results crucially depend on calibration of one particular parameter, the steady-state level of debt. We find it convenient to emphasize this difference from the very beginning, but we defer the remaining robustness analysis to section 7.

We assume that both authorities have *identical* intraperiod *social* objectives (15). We solve the optimization problem and find optimal policy and the incurred costs for jointly optimal policies under discretion and under commitment. Figure 1 plots the impulse responses to a unit cost-push shock for the low- and high-debt cases.<sup>11</sup> We report and discuss several results.

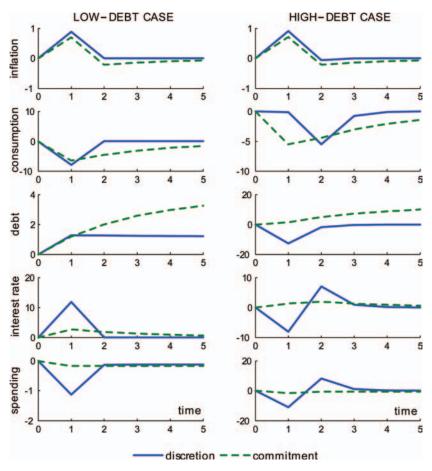
#### 5.1 Commitment vs. Discretion

Optimal policy under commitment, compared with discretionary policy, requires smaller and smoother movements in policy instruments.

<sup>&</sup>lt;sup>10</sup>We have examined what happens if we introduce taste or productivity shocks, and all quantitative results are virtually the same.

<sup>&</sup>lt;sup>11</sup>We only show several initial periods. With time, all variables (including debt) converge back to the steady state.

# Figure 1. Impulse Responses to a Unit Cost-Push Shock under Cooperative Optimization of Benevolent Authorities



Note: Fiscal policy uses spending as an instrument.

Commitment policy has control over all future states, including expectations. The policy is chosen once and the policymaker is able to adhere to it. As a result, the private sector sets expectations that help stabilization. By promising to keep the interest rate above the baseline in all future periods, the monetary policymaker is able to achieve an immediate fall in inflation and overall price stability. Also, only a small initial rise in the interest rate is sufficient for this stabilization. In contrast, the discretionary policymaker reoptimizes every period. The discretionary policymaker cannot promise to keep the interest rate above the baseline in future periods, as the private sector knows that all future policymakers will reoptimize. The policymaker, therefore, has no control over private-sector expectations along the whole future paths. It has to raise the interest rate by much more in the first period in order to move all variables sufficiently close to the steady state already after the first period.<sup>12</sup> This distinction in qualitative behavior of commitment and discretionary policies is well known and is frequently exploited in the design of delegation schemes—different from the one we consider here—that make discretionary policy smooth to replicate commitment policy and achieve higher social welfare.<sup>13</sup>

# 5.2 Non-Cooperative Discretionary Policy Regimes

All non-cooperative regimes with identical objectives deliver the same equilibrium. This result shows that strategic behavior can only be of importance if there is conflict in objectives. That the order of moves is of no importance is a direct consequence of policymakers having identical objectives *given* that the equilibrium is unique. In this setting, neither authority is trying to exploit the other in a pursuit of their own target—they internalize externalities.<sup>14</sup>

This result holds because there is a unique discretionary equilibrium. By uniqueness we mean that following a shock, the economy

 $<sup>^{12}</sup>$ The inability of discretionary policy to control expectations of the private sector about future policy and future states also implies that the choice of policy instruments might matter. In contrast to Benigno and Woodford (2004), we cannot use consumption or inflation as a policy instrument, as the discretionary policymaker has no control of their future values.

<sup>&</sup>lt;sup>13</sup>Å non-exhaustive list includes Svensson (1997), Walsh (2003), Woodford (2003b), and Vestin (2006).

<sup>&</sup>lt;sup>14</sup>Formally, it is straightforward to demonstrate that a Nash equilibrium with identical objectives coincides with the cooperative equilibrium. The system of first-order conditions for a single optimization problem with two instruments in cooperative equilibrium will be identical to the two systems of first-order conditions for the two separate maximization problems for each of the instruments. Similarly we can deal with Stackelberg equilibrium: the systems of first-order conditions will contain some additional terms, but they are all zero if the objectives are the same.

follows a unique path which satisfies the conditions of time consistency and optimality; see appendix 1. This is not an obvious result a priori given that a similar model with fiscal policy, which controls debt by means of an *exogenous* feedback-on-debt rule, is shown to have multiple discretionary equilibria in Blake and Kirsanova (2008). In that model, if the fiscal feedback on debt is sufficiently strong, so any debt displacement is corrected by fiscal means very quickly, monetary policy behaves in a conventional way. In response to a costpush shock, monetary policy raises the interest rate, which leads to an initial increase in debt and also slows down the debt stabilization process. The strong fiscal feedback ensures the debt stability. If fiscal feedback is zero, so fiscal policy does not control debt at all, monetary policy lowers the interest rate in response to a cost-push shock. As a result, the lower real rate reduces the level of debt below the steady state. The subsequent increase in the interest rate reduces output and inflation and also pushes debt back to the steady state. There is an intermediate case where fiscal policy controls debt only weakly, so both monetary regimes are possible.

The multiplicity of regimes is possible because of the complementarity between the decision variable of the private sector (inflation) and the instrument of the monetary policymaker (the interest rate) in their effects on debt, as defined in Cooper and John (1988). Namely, a too-low debt stock can be increased with either a high interest rate or with low inflation, and, crucially, an increase in the interest rate reduces inflation. This complementarity exists in our model too, but its strength depends on fiscal stance: when fiscal feedback is sufficiently strong, then debt is stabilized by fiscal means and effects of inflation and interest rate on the speed of debt stabilization are negligible. In contrast to Blake and Kirsanova (2008), the fiscal authority is a strategic player in our model in this paper, and it optimally chooses to reduce spending in a response to a higher interest rate, i.e., chooses sufficiently strong fiscal feedback to put the economy outside the area of multiple equilibria.<sup>15</sup>

 $<sup>^{15}</sup>$ It is easy to prove analytically that the discretionary equilibrium is unique in the "low-debt" case, as algebra can be substantially simplified as in Blake and Kirsanova (2008).

# 5.3 The Level and Dynamics of Government Debt

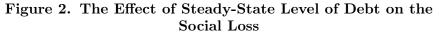
The policy mix very much depends on the *steady-state level* of debt,  $\chi$ . The substantial difference is in terms of the directions of optimal responses to the same shock.

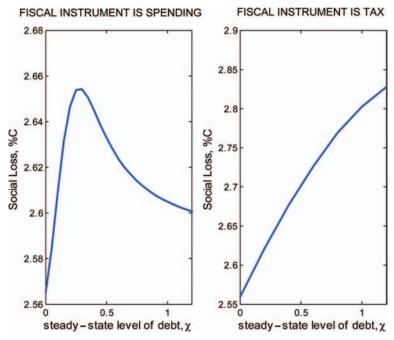
In the low-debt case, contractionary monetary policy is supported by contractionary fiscal policy in a response to a positive cost-push shock. This leads to a rise in debt. Fiscal policy is then used to bring debt back to the steady state: government spending stays at a reduced level for a long time.

In the high-debt case, the interest rate *falls* in the first period in a response to a positive cost-push shock. As the first-order effect of interest rates on debt is large, a fall in interest rates reduces the level of domestic debt. Moreover, the fiscal contraction reduces debt even further. This first-period response allows monetary policy to raise the interest rate in the second period and reduce inflation. (Note that *second-period* inflation overshoots the steady-state level. This helps to reduce inflation in the *first* period too, as the rational private sector sets prices lower in anticipation of this lower future price.) Fiscal policy also raises spending, as this not only stabilizes debt but also reduces recession caused by high interest rates.

The left panel in figure 2 demonstrates that the social loss is a non-monotonic function of  $\chi$ . This is consistent with a striking change in the way the stabilization policy works. When  $\chi$  is below some threshold level, then any further rise in  $\chi$  creates more problems for policymakers if monetary policy raises the interest rate in response to a cost-push shock. A high interest rate contributes to debt accumulation, and the effect is proportional to  $\chi$ . When  $\chi$  is below this threshold, the gains from inflation stabilization in the first periods after the shock outweigh the losses from the slow stabilization of the economy because of slow debt dynamics. When  $\chi$  is above this threshold, it becomes welfare improving to stabilize debt quicker. Therefore, it becomes optimal to lower the interest rate in the first period after the shock and raise it in the second period. This policy leads to faster debt stabilization and also curtails inflation. To emphasize the difference, we have chosen the low- and high-debt cases on either side of the "hump."

The *dynamic* process of debt accumulation plays a very important role for the stabilization policy mix. First, its presence imposes





**Notes:** The figure shows the joint monetary-fiscal optimization of benevolent policymakers under discretion. The fiscal policymaker uses different fiscal instruments.

the requirement on the policy mix to prevent an explosion of debt. Second, its presence alters the dynamics of stabilization. If there is no debt, then following a shock a discretionary policy stabilizes the economy within *one* period. Policy can only reduce the *amplitude* of reactions to shocks within the first period. If debt accumulation is present, then policy can also reduce the half-life of effects of shocks that have entered the system. In other words, it enables stabilization to be smoothed over many periods, which may or may not be welfare improving. With the presence of debt, the private sector's expectations affect the economy for more than one period, as the evolution of debt can be affected by the forecast of future policy. Expectations set in period t affect the whole future path of state variables and they affect policy decisions taken in period t + k,  $k \ge 1$ . The way

	Low-Debt Case			High-Debt Case		
$\textbf{Fiscal Instrument} \rightarrow$	G	Т	G&T	G	Т	G&T
Commitment Cooperation under Discretion Stabilization Bias	2.57	2.56	$0.22 \\ 2.54 \\ 2.33$	2.60	2.83	

Table 1. Social Loss for Different Scenarios with DifferentFiscal Instruments, % of Steady-State Consumption

monetary policy stabilizes inflation in the case of high debt is an example of how expectations of future policy can be exploited.

The stabilization bias is around 0.5 percent of the steady-state consumption level that should be given up in order to compensate for reduced volatility. We compute the stabilization bias as a difference between the social loss under discretion and the social loss under commitment (Ramsey allocation). Table 1 presents more detailed information. Together with the left panel in figure 2, the table suggests that the level of debt can affect the size of stabilization bias, and the order of the effect can be around 0.1 percent of the steady-state consumption level, which is a relatively big number. We shall use these numbers as a benchmark. All our future computations of losses will be relative to the loss under the *benevolent discretionary* policy, and to obtain the stabilization bias one has to add a corresponding number from table  $1.^{16}$ 

This section has established that with benevolent policymakers, the order of moves and information structure is of no consequence. However, there are issues that arise from the dynamic nature of the model through the steady-state level of debt. In what follows we need to investigate the importance of steady-state levels of debt together with the conservative central bank proposal.

#### 6. Conservative Central Bank

Suppose the fiscal authorities are benevolent, but we allow the monetary authorities to place a higher relative weight on inflation

 $<sup>^{16}\</sup>mathrm{Here}$  and everywhere else we measure the loss in percentage of steady-state consumption level.

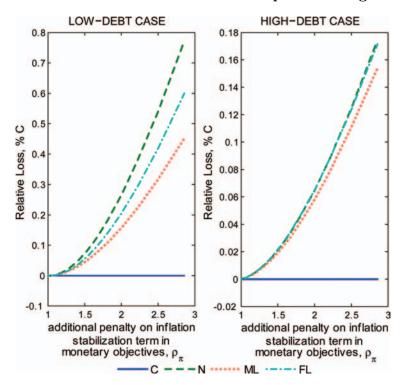


Figure 3. Social Welfare Loss as a Function of Monetary Conservatism for Different Non-Cooperative Regimes

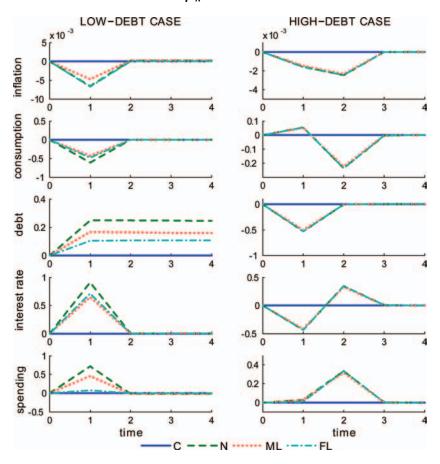
Note: Fiscal policy uses spending as an instrument.

stabilization, by adopting objective (16) with  $\rho_{\pi} > 1$ . How does this affect social welfare? We keep the fiscal authorities benevolent.

The left panel of figure 3 suggests that the loss quickly rises with the degree of monetary conservatism for all three non-cooperative regimes; i.e., we can double the stabilization bias very quickly. When the penalty is very close to one, and in the low-debt case, there is an extremely small social gain for both regimes of fiscal leadership and of simultaneous moves, but the monetary leadership regime is unambiguously worse than the cooperation of benevolent authorities. If the steady-state level of debt is large, then there is no social gain for any degree of inflation conservatism and the social loss rises quickly with the degree of inflation conservatism. To understand these results, it is instructive to look at impulse responses to a unit

# Figure 4. Impulse Responses to a Unit Cost-Push Shock for Different Strategic Regimes if the Monetary Authority Is Conservative with Small Degree of Conservatism,

 $\rho_{\pi} = 0.06$ 



**Notes:** Fiscal policy uses spending as an instrument. All responses are shown as relative to those under cooperation.

cost-push shock in figure 4. We plot differences with the cooperative solution, which itself is plotted in figure 1.

# 6.1 Simultaneous Policy Decisions

In the *low-debt* case, the monetary authority reacts more actively to a cost-push shock than if it were benevolent. It is more concerned

with inflation variability than society and is prepared to pay for gains with higher variability of the demand-related components. The monetary authority, therefore, raises the interest rate higher to eliminate inflation more aggressively. The benevolent fiscal authority tries to eliminate the resulting recession. It therefore contracts less (expands more) than if both authorities were benevolent, although inflation is still reduced. The reduction in the cost of inflation volatility only outweighs the cost of fiscal volatility if the degree of monetary conservatism is extremely small. With a higher degree of conservatism of the monetary authorities, the implied fiscal volatility becomes very costly.

In the *high-debt* case, it is optimal for the monetary authority to take into account the debt stabilization issues. It chooses to reduce the interest rate in the first period by more, but the resulting smaller fiscal contraction allows the monetary authorities to then raise the interest rate by more in the second period without having an adverse effect on debt. Inflation, thus, overshoots more (falls relative to the cooperative benevolent case) in the second period. It, therefore, rises less (or falls relative to the cooperative case) in the first period. Again, the reason for its reduction in the first period is the second-period overshooting and the rational expectations of price setters: A rational private sector perceives the second-period fall of prices and sets prices low in the first period.

The simultaneous-move regime leads to more aggressive policy than under cooperation, and this results in lower inflation but also in higher volatility for demand-related terms and instruments, and therefore, in a more costly equilibrium.

#### 6.2 Fiscal Leadership

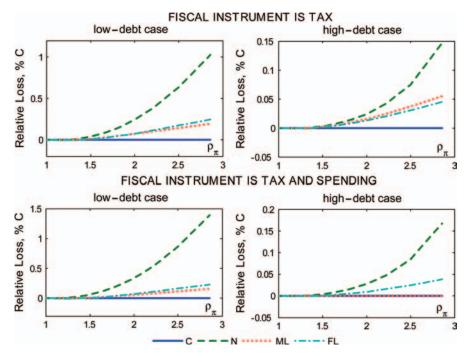
In the *low-debt* case, the fiscal authority knows that if it contracts less than in the cooperative scenario, then this will cause the monetary authority to contract more in order to fight excess inflation. Moreover, the monetary authority will contract even more due to its inherent conservatism. So the fiscal authority chooses to contract only slightly less than in the cooperative scenario. The monetary authority still contracts *more* than in the cooperative scenario, and overall this results in slightly lower inflation. Inflation falls nearly as much as under Nash, because fiscal authorities do not expand as much as under Nash. Debt rises only slightly higher than in the cooperative scenario, and less than if both authorities move simultaneously. With a higher degree of monetary conservatism, the monetary authorities have to pay too high an output and consumption cost for lower inflation, so the regime is unambiguously worse than full cooperation.

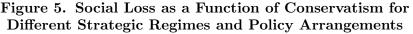
In the *high-debt* case, the authorities have more problems with debt stabilization. The fiscal authority contracts more in the first period, thus not allowing monetary policy to expand as much as under Nash. The monetary authority therefore contracts less in the second period. This creates less inflation overshooting than under Nash. The welfare loss is bigger than under Nash for a moderate degree of monetary conservatism. However, with a sufficiently high degree of conservatism, the monetary-fiscal interaction results in smaller consumption volatility than under Nash, and this improves welfare slightly. The regime becomes slightly better than the simultaneous-move regime, although the quantitative response is extremely close to it. The debt stabilization task dominates the concerns of both authorities and nearly equally constrains instrument movements in all three non-cooperative regimes.

# 6.3 Monetary Leadership

In the *low-debt* case, the leading monetary authority knows that raising the interest rate causes the fiscal authority to try to offset the resulting recession. This consideration would stop the *benevolent* monetary authority from raising interest rates. However, as the monetary authority is conservative, the cost of moving g becomes *relatively* less important, so it does raise the interest rate. But it raises the interest rate less than in the simultaneous-move regime. Lower pressure on debt allows fiscal policy to offset the effect more efficiently. Inflation falls less than under Nash and this determines the loss.

In the *high-debt* case, the monetary authority contracts more in the first period than under Nash and the fiscal authorities have to reduce spending by more. Debt itself does not fall as much as it does under Nash. The monetary authority is unable to achieve as large an inflation overshoot in the second period, and the consequent overall gain in inflation stabilization, as under Nash. It is able, however,





**Note:** Fiscal policy uses taxes as an instrument or it uses both instruments, taxes and spending.

to achieve smaller variability of demand-related terms than under Nash. Both effects taken together result in a smaller welfare loss than for the simultaneous-move case.

#### 7. Robustness: Using Tax as an Instrument

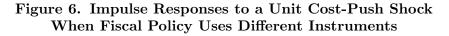
# 7.1 Tax as Fiscal Instrument

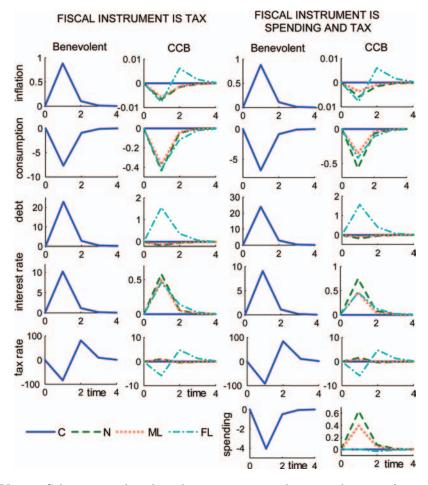
We have shown how our results change with the level of steady-state debt. It is also important to look at the choice of fiscal instrument. We rerun our simulations assuming that (i) fiscal policy uses tax as the instrument and (ii) fiscal policy uses two instruments, both income tax rate and spending. Figure 5, which repeats figure 3 but is plotted for taxes (the upper panels) and taxes and spending (the lower panels), suggests that our main conclusion remains valid: generally speaking, delegating monetary policy to a conservative central bank does not improve social welfare.

There are some important differences with the case where only spending is used. Consider using tax as a single fiscal instrument and suppose that authorities are *benevolent*. As taxes affect marginal cost directly, fiscal policy tries to offset any cost-push shocks immediately by lowering the tax rate. Indeed, the first column of plots in figure 6 suggests that in response to a unit cost-push shock, taxes fall, and this allows monetary policy to raise the interest rate without any solvency concerns.<sup>17</sup> The debt is also stabilized by taxes: the first-period reduction in the tax rate and high interest rates require higher taxes in subsequent periods; taxes rise and bring debt back to the steady state very quickly. As a result, the dynamics of debt is very different from the one in the case where fiscal policy can only use spending as an instrument: debt quickly converges to the steady state. Taxes have no direct effect on demand and consumption, so fiscal policy can eliminate debt displacement in consequent periods more efficiently with fewer externalities, and monetary policy can stabilize inflation.

Interestingly, the welfare gain from using taxes instead of spending is very small: the loss under the benevolent discretionary monetary-fiscal policy in the low-debt case is only 0.006 percent higher if fiscal policy uses spending instead of taxes as the only instrument. Moreover, in the high-debt case, using taxes is more costly than using spending. To understand these results, it is instructive to compare them with those under commitment of benevolent authorities which would deliver the highest welfare in the absence of dynamic lump-sum taxes. Benigno and Woodford (2004) demonstrate that fiscal policy under commitment is very successful in eliminating all effects of a cost-push shock on the economy. These results are consistent with ours; see the first line in table 1: the loss under the commitment policy falls substantially if taxes are used. Under

 $<sup>^{17}</sup>$ We have checked that impulse responses are qualitatively similar for all degrees of conservatism and for low- and high-debt scenarios. So we only present small conservatism and a low-debt case in figure 6.





**Notes:** Columns 2 and 4 plot relative responses, relative to the case of cooperation of benevolent authorities. The "low-debt" scenario and small degree of conservatism,  $\rho_{\pi} = 0.06$ , are assumed in all cases where applicable.

discretion, however, taxes cannot move as freely as under commitment, and they cannot efficiently offset the effect of cost-push shocks, although they still move in the right direction. The stabilization bias is large, around 2.5 percent, only because the commitment policy is so successful. The right panel in figure 2 suggests that with a higher level of steady-state debt,  $\chi$ , the social loss rises. The level of  $\chi$  determines the size of the first-order effect of the interest rate on debt accumulation. With higher  $\chi$ , the problem of debt stabilization becomes more difficult. Taxes become predominantly used in the debt stabilization task; they do not fall by much, and this makes the task of inflation stabilization more difficult for monetary policy.

The relative ranking of different leadership regimes barely changes. The simultaneous-move regime leads to most welfare losses. However, it is not the most aggressive regime. The second column of plots in figure 6 illustrates this. As before, we plot impulse responses relative to those under cooperation of benevolent policymakers. As all qualitative responses are very similar for different degrees of conservatism and for the low- and high-debt scenarios, we only plot the case with low conservatism and with a low level of debt.

The second column of plots in figure 6 demonstrates interactions for different leadership regimes in the case of a conservative central bank. Under the Nash regime, the monetary authority raises the interest rate more than if it was benevolent. It also reduces inflation by more and reduces consumption. The optimal response of the fiscal authority is to raise taxes. If monetary policy is not taken into account, then higher taxes would allow the cost-push shock to have a bigger effect on inflation, the real interest rate would fall, and consumption would rise. In the simultaneous-move regime, the fiscal authority, therefore, reduces taxes by less than it does under cooperation. Monetary policy raises the interest rate higher and so on. In equilibrium, inflation is reduced but consumption falls. For a small degree of conservatism, the gain of lower inflation outweighs the loss of lower consumption and output, but consumption volatility rises very fast with the degree of monetary conservatism, and the regime quickly becomes welfare inferior.

If the *monetary authority leads*, it knows that the fiscal authority will not reduce taxes as if it were benevolent, so it raises the interest rate by less and the fiscal authority reduces tax by less than under Nash. This leads to nearly the same outcome as under Nash if the degree of conservatism is small. With a higher degree of conservatism, policy aggressiveness leads to more volatility in consumption, which is still less than under Nash. So the monetary leadership regime is preferable over the Nash regime. If the *fiscal authority leads*, it knows that the monetary authority will raise interest rates, so it consequently reduces taxes by more. It explicitly exploits the monetary policy reaction function: the monetary authority raises the interest rate, but by less then if it were benevolent because the fiscal authority does part of the disinflation. As a result of these first-period movements, debt rises higher. Fiscal policy thus has to respond by raising taxes in the second period, which also results in higher inflation in the second period. If the degree of monetary conservatism is small, then the fiscal leadership regime delivers the lowest loss among the three non-cooperative regimes, because of the relatively large fall in inflation. The second-period inflation hike, however, dominates the large social welfare loss if the monetary authority has a higher degree of conservatism.

Neither of the non-cooperative regimes can be defined as the most aggressive here: the Nash regime results in higher variability in the interest rate while the fiscal leadership regime results in the biggest volatility of the fiscal instrument.

# 7.2 Tax and Spending as Fiscal Instruments

Adding government spending as the second fiscal instrument does not change any of the welfare results, as the two lower plots in figure 5 demonstrate. An inflation-conservative monetary authority, generally speaking, generates social welfare loss. If the authorities are benevolent, then the social loss is only marginally smaller than the loss from stabilization if only taxes are used; see table 1.

The similarity is also apparent from impulse responses shown in the last two columns of plots in figure 6. As debt is now stabilized by taxes, spending is optimally chosen to help monetary policy to reduce inflation if the authorities are benevolent. If there are distorted objectives, then interest rate/tax interactions are nearly the same as if tax were the only fiscal instrument. The possibility to use spending as well makes little difference: spending plays the same role (relative to the case of cooperation of benevolent policymakers) as in the scenario where it was the only fiscal instrument; one can compare the last column of plots in figure 6 with the first column of plots in figure 4.

#### 8. Summary of Results and Conclusions

This paper presents a detailed account of discretionary monetary and fiscal policy interactions in a fully specified intertemporal general equilibrium model with particular emphasis on non-cooperative interactions under inflation conservatism of the monetary authority.

We demonstrate that if the fiscal policymaker is benevolent but acts strategically, then delegating monetary policy to a policymaker that puts a higher than socially optimal weight on inflation stabilization generally increases the stabilization bias. Such a distortion of the otherwise social objectives of one policymaker brings the two policymakers into conflict with each other, and this is welfare destructive. This example demonstrates that what works well in an economy with one strategic policymaker may not work at all in an economy with two strategic policymakers, as the second strategic policymaker can offset all actions of the first one and vice versa.

Of course, this result does not imply that the problem of optimal delegation has no solution if there are several strategic policymakers. We have studied only one particular delegation scheme: the conservative central bank scenario. We have distorted the relative weights on social objectives, but we did not introduce any additional objectives in order to have a simple and clear experiment. Additional non-microfounded terms in policymakers' objectives (like instrument costs) or the use of different policy instruments (like VAT or a consumption tax) might have different effects on the ability and willingness of the policymakers to engage in conflict with each other. Different degrees of precommitment may affect the degree of conflict too. We leave these and similar issues for future research. Our result, however, suggests that making fiscal authorities flexible and strategic may have costs, and these should be taken into account. We also show how conflict arises if the weights on social objectives are changed but all objectives remain social.

To arrive at our main conclusions, we have investigated monetary and fiscal policy interactions under discretion in a fairly standard model with an explicit budget constraint for the fiscal authority. The analysis also yields some additional insights.

First, the choice of fiscal instrument is important, although not for assessing gains from delegation where there are losses whatever the fiscal instrument. The transmission mechanism differs considerably: (i) Taxes are most useful in stabilizing debt, and the way they are optimally used does not depend on the level of steadystate debt, and (ii) spending has more limited powers to stabilize debt, but has a large effect on domestic demand. If spending is the only fiscal instrument, then the size of steady-state debt has an important qualitative effect on policy interactions. If the steadystate level of debt is high, then monetary policy has to take an active part in debt stabilization.

Second, among the three non-cooperative regimes, the Nash one is unlikely to be welfare dominating. In most cases, this regime leads to large movements of the policy instruments, which typically implies increased volatility of the key economic variables. Most of our results suggest that monetary leadership is relatively better if the monetary authority is inflation conservative, but this hinges on the social welfare metric and requires that inflation stabilization has far greater weight than any other target variable.

Finally, this paper offers an additional contribution to the literature: we offer a methodological approach to solving a non-cooperative leadership equilibria in an LQ RE model with two policymakers. This approach can be easily modified to study different types of equilibria and interactions of many agents, say in a multi-country setting.

#### Appendix 1. Leadership Equilibria under Discretion

This section demonstrates how to solve non-cooperative dynamic games in the linear-quadratic rational expectations framework. Our definition of discretionary policy is conventional and is widely used in the monetary policy literature; see, e.g., Oudiz and Sachs (1985), Backus and Driffill (1986), Clarida, Galí, and Gertler (1999), and Woodford (2003a). Currie and Levine (1993) demonstrate how to solve Nash games. We therefore only describe leadership equilibria.<sup>18</sup>

#### Discretionary Policy

We assume a non-singular linear deterministic rational expectations model, augmented by a vector of control instruments.<sup>19</sup>

 $<sup>^{18}\</sup>mathrm{A}$  simultaneous-move regime will be a particular case of the leadership regime.

<sup>&</sup>lt;sup>19</sup>None of the results presented here depend on the deterministic setup outlined and the consequent assumption of perfect foresight. Shocks can be included into vector  $y_t$ ; see, e.g., Anderson et al. (1996) and Blake and Kirsanova (2008).

Specifically, the evolution of the economy is explained by the linear system

$$\begin{bmatrix} \mathsf{y}_{t+1} \\ \mathsf{x}_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \mathsf{y}_t \\ \mathsf{x}_t \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \mathsf{u}_t^L \\ \mathsf{u}_t^F \end{bmatrix}, \quad (17)$$

where  $y_t$  is an  $n_1$ -vector of predetermined variables with initial conditions  $y_0$  given,  $x_t$  is an  $n_2$ -vector of non-predetermined (or jump) variables with  $\lim_{t\to\infty} x_t = 0$ , and  $u_t^F$  and  $u_t^L$  are the two vectors of policy instruments of two policymakers, named F and L, of size  $k_F$  and  $k_L$ , respectively. For notational convenience we define the n-vector  $z_t = (y'_t, x'_t)'$ , where  $n = n_1 + n_2$ , and the k-vector of control variables  $u_t = (u_t^{L'}, u_t^{F'})'$ , where  $k = k_F + k_L$ . We assume the equations are ordered so that  $A_{22}$  is non-singular.

Typically, the second block of equations in this system represents an aggregation of the first-order conditions to the optimization problem of the private sector, which has decision variables  $x_t$ . Additionally, there is a first block of equations which explains the evolution of the predetermined state variables  $y_t$ . These two blocks describe the "evolution of the economy" as observed by policymakers.

The intertemporal welfare criterion of policy maker  $i, i \in \{L, F\}$ , is defined by the quadratic loss function

$$W_{t}^{i} = \frac{1}{2} \sum_{s=t}^{\infty} \beta^{s-t} \left( g_{s}^{i\prime} \mathcal{Q}^{i} g_{s}^{i} \right) = \frac{1}{2} \sum_{s=t}^{\infty} \beta^{s-t} \left( \mathsf{z}_{s}^{\prime} \mathcal{Q}^{i} \mathsf{z}_{s} + 2\mathsf{z}_{s}^{\prime} P^{i} \mathsf{u}_{s} + \mathsf{u}_{s}^{\prime} R^{i} \mathsf{u}_{s} \right).$$
(18)

The elements of vector  $g_s^i$  are the goal variables of policymaker i,  $g_s^i = C^i(\mathbf{z}'_s, \mathbf{u}'_s)'$ . Matrix  $Q^i$  is assumed to be symmetric and positive semi-definite.

ASSUMPTION 1. Suppose that at any time t, the private sector and policymakers respond only to the current state

$$\mathbf{u}_t^L = \mathcal{F}^L(\mathbf{y}_t) = -F^L \mathbf{y}_t, \tag{19}$$

$$\mathbf{u}_t^F = \mathcal{F}^F(\mathbf{y}_t) = -F^F \mathbf{y}_t, \tag{20}$$

$$\mathsf{x}_t = \mathcal{N}(\mathsf{y}_t) = -N\mathsf{y}_t. \tag{21}$$

This assumption rules out non-stationarity of policy and private-sector decisions—i.e., any time dependence from the more general formulation  $u_t^i = \mathcal{F}^i(t; y_t, y_{t-1}, ..., y_{t-k}, ...), x_t = \mathcal{N}(t; y_t, y_{t-1}, ..., y_{t-k}, ...)$ —and restricts policy decisions to memoryless feedback rules. We also assume that rules are *linear* in the state.

We define discretionary policy as satisfying several constraints. We want to assume that the policymaker can implement at each point of time its policy decision *before* the private sector selects its own action  $x_t$ .

ASSUMPTION 2. At each time t, the private sector observes the current decision  $u_t$  and expects that future policymakers at any time s > t will reoptimize, will apply the same decision process, and implement policy (19)–(20).

**PROPOSITION 1.** Given assumption 2, the current aggregate decision of the private sector can be written as a linear feedback function

$$\mathbf{x}_t = -J\mathbf{y}_t - K\mathbf{u}_t = -J\mathbf{y}_t - K^F\mathbf{u}_t^F - K^L\mathbf{u}_t^L, \qquad (22)$$

where

$$J = (A_{22} + NA_{12})^{-1}(A_{21} + NA_{11}),$$
(23)

$$K^F = (A_{22} + NA_{12})^{-1}(B_{22} + NB_{12}), (24)$$

$$K^{L} = (A_{22} + NA_{12})^{-1}(B_{21} + NB_{11}).$$
(25)

*Proof.* Relationship (21) can be taken with one lead forward, and  $y_{t+1}$  is substituted from the first equation (17). We obtain

$$\begin{aligned} \mathbf{x}_{t+1} &= -N\mathbf{y}_{t+1} = -N(A_{11}\mathbf{y}_t + A_{12}\mathbf{x}_t + B_{11}\mathbf{u}_t^L + B_{12}\mathbf{u}_t^F) \\ &= A_{21}\mathbf{y}_t + A_{22}\mathbf{x}_t + B_{21}\mathbf{u}_t^L + B_{22}\mathbf{u}_t^F, \end{aligned}$$

from where it follows

$$\mathbf{x}_{t} = -(A_{22} + NA_{12})^{-1} ((A_{21} + NA_{11})\mathbf{y}_{t} + (B_{22} + NB_{12})\mathbf{u}_{t}^{F} + (B_{21} + NB_{11})\mathbf{u}_{t}^{L}) = -J\mathbf{y}_{t} - K\mathbf{u}_{t},$$

where J and  $K = (K^F, K^L)$  are defined as in (23)–(25). Invertibility of  $A_{22}$  ensures invertibility of  $A_{22} + NA_{12}$  almost surely.

Proposition 1 implies that the policymakers, which move before the private sector, take into account their "instantaneous" influence on the choice of  $x_t$ , which is measured by -K.

ASSUMPTION 3. At each time t, policymaker F knows assumptions 1 and 2 and observes the current decision  $u_t^L$  of policymaker L.

PROPOSITION 2. Given assumption 3, the current decision of policymaker F can be written as a linear feedback function:

$$\mathbf{u}_t^F = -G\mathbf{y}_t - D\mathbf{u}_t^L. \tag{26}$$

*Proof.* We shall prove it as part of proof of proposition 3.

Proposition 2 implies that policymaker L, which moves before the private sector and before policymaker F, takes into account its "instantaneous" influence on the choice of  $u_t^F$ , which is measured by -D. It immediately follows that

$$F^F = G - DF^L. (27)$$

ASSUMPTION 4. At each point in time t, policymaker L knows assumptions 1, 2, and 3.

DEFINITION 1. Policies determined by (19)-(20) are discretionary (under intraperiod leadership of policymaker L) if each policymaker finds it optimal to continue to follow its policy in every period s > t, given assumptions 1–4.

**Policy of the Follower.** The policy of the follower,  $u_t^F$ , satisfies the following Bellman equation:

$$V_t^F(\mathbf{y}_t) = \min_{\mathbf{u}_t^F} \left( \frac{1}{2} \left( \mathbf{y}_s' \hat{Q} \mathbf{y}_t + 2 \mathbf{y}_t' \hat{P} \mathbf{u}_t + \mathbf{u}_t' \hat{R} \mathbf{u}_t \right) + \beta V_{t+1}^F (\hat{A} \mathbf{y}_t + \hat{B} \mathbf{u}_t) \right),$$
(28)

with

$$\begin{split} \hat{Q} &= Q_{11}^F - Q_{12}^F J - J' Q_{21}^F + J' Q_{22}^F J, \\ \hat{P}_1 &= J' Q_{22}^F K^L - Q_{12}^F K^L + P_{11}^F - J' P_{21}^F, \end{split}$$

$$\begin{split} \hat{P}_2 &= J'Q_{22}^FK^F - Q_{12}^FK^F + P_{12}^F - J'P_{22}^F, \\ \hat{R}_{11} &= K^{L\prime}Q_{22}^FK^L - K^{L\prime}P_{21}^F - P_{21}^{F\prime}K^L + R_{11}^F, \\ \hat{R}_{12} &= K^{L\prime}Q_{22}^FK^F - K^{L\prime}P_{22}^F - P_{21}^{F\prime}K^F + R_{12}^F, \\ \hat{R}_{22} &= K^{F\prime}Q_{22}^FK^F + R_{22}^F - K^{F\prime}P_{22} - P_{22}^{F\prime}K^F, \\ \hat{A} &= A_{11} - A_{12}J, \\ \hat{A}_1 &= B_{11} - A_{12}K^L, \\ \hat{B}_2 &= B_{12} - A_{12}K^F. \end{split}$$

Here we take the intraperiod leadership of the policymaker into account by substituting in constraint (22), but we treat  $u_t^L$  as given.

Because of the quadratic nature of the per-period objective in (18) and because policy and private-sector decisions are linear in the state, the discounted loss will necessarily have quadratic form in the state

$$V_t^F(\mathbf{y}_t) = \frac{1}{2} \mathbf{y}_t' S^F \mathbf{y}_t.$$

The Bellman equation characterizing discretionary policy of policymaker F, therefore, becomes

$$\mathbf{y}_{t}'S^{F}\mathbf{y}_{t} = \min_{\mathbf{u}_{t}^{F}} \left( \mathbf{y}_{t}'(\hat{Q} + \beta \hat{A}'S^{F}\hat{A})\mathbf{y}_{t} + 2\mathbf{y}_{t}'(\hat{P} + \beta \hat{A}'S^{F}\hat{B})\mathbf{u}_{t} + \mathbf{u}_{t}'(\hat{R} + \beta \hat{B}'S^{F}\hat{B})\mathbf{u}_{t} \right).$$

$$(29)$$

**Policy of the Leader.** For the leader, policy  $u_t^L$  satisfies the following Bellman equation:

$$V_t^L(\mathbf{y}_t) = \min_{\mathbf{u}_t^L} \left( \frac{1}{2} \left( \mathbf{y}_s' \tilde{Q} \mathbf{y}_t + 2 \mathbf{y}_t' \tilde{P} \mathbf{u}_t^L + \mathbf{u}_t^{L'} \tilde{R} \mathbf{u}_t^L \right) + \beta V_{t+1}^L \left( \tilde{A} \mathbf{y}_t + \tilde{B} \mathbf{u}_t^L \right) \right),$$
(30)

with

$$\begin{split} \tilde{Q} &= Q_{11}^L - P_{12}^L G - G' P_{12}^{L\prime} + G' R_{22}^L F^F - Q_{12}^L \tilde{J} - \tilde{J}' Q_{12}^{L\prime} \\ &+ G' P_{22}^{L\prime} \tilde{J} + \tilde{J}' P_{22}^L G + \tilde{J}' Q_{22}^L \tilde{J}, \end{split}$$

$$\begin{split} \tilde{P} &= P_{11}^{L} + \tilde{J}' Q_{22}^{L} \tilde{K} - Q_{12}^{L} \tilde{K} + G' P_{22}^{L'} \tilde{K} - P_{12}^{L} D - G' R_{21}^{L} \\ &+ G' R_{22}^{L} D - \tilde{J}' P_{21}^{L} + \tilde{J}' P_{22}^{L} D, \\ \tilde{R} &= R_{11}^{L} + \tilde{K}' Q_{22}^{L} \tilde{K} - R_{12}^{L} D - L' R_{21}^{L} + L' R_{22}^{L} D - \tilde{K}' P_{21}^{L} \\ &+ \tilde{K}' P_{22}^{L} D - P_{21}^{L'} \tilde{K} + D' P_{22}^{L'} \tilde{K}, \\ \tilde{J} &= J - K^{F} G, \\ \tilde{K} &= K^{L} - K^{F} D, \\ \tilde{A} &= A_{11} - B_{12} G - A_{12} \tilde{J}, \\ \tilde{B}_{1} &= B_{11} - B_{12} D - \tilde{A}_{12} \tilde{K}, \end{split}$$

Here we take the intraperiod leadership of the policymaker into account by substituting in constraints (22) and (26). Similarly,

$$V_t^L(\mathbf{y}_t) = \frac{1}{2} \mathbf{y}_t' S^L \mathbf{y}_t,$$

and the Bellman equation characterizing discretionary policy of policy maker  ${\cal L}$  becomes

$$\mathbf{y}_{t}^{\prime}S^{L}\mathbf{y}_{t} = \min_{\mathbf{u}_{t}^{L}} \left( \mathbf{y}_{s}^{\prime}(\tilde{Q} + \beta \tilde{A}^{\prime}S^{L}\tilde{A})\mathbf{y}_{t} + 2\mathbf{y}_{t}^{\prime}(\tilde{P} + \beta \tilde{A}^{\prime}S^{L}\tilde{B})\mathbf{u}_{t}^{L} + \mathbf{u}_{t}^{L\prime}(\tilde{R} + \beta \tilde{B}^{\prime}S^{L}\tilde{B})\mathbf{u}_{t}^{L} \right).$$

$$(31)$$

For a policy  $F^L$ ,  $F^F$  and the private-sector response N, the evolution of the state variable satisfies the following equation:

$$\mathbf{y}_{t+1} = M \mathbf{y}_t, \tag{32}$$

where  $M = A_{11} - A_{12}N - B_{11}F^L - B_{12}F^F$ .

#### Discretionary Equilibrium as a Matrix Sextuple

Given  $y_0$  and system matrices A and B, matrices N,  $F^L$ , G, and D define the trajectories  $\{y_s, x_s, u_s\}_{s=t}^{\infty}$  in a unique way and vice versa: if we know that  $\{y_s, x_s, u_s\}_{s=t}^{\infty}$  solve the discretionary optimization problem stated above, then, by construction, there are unique timeinvariant linear relationships between them which we label as N,  $F^L$ , G, and D. Matrices  $S^L$ ,  $S^F$  define the cost to go along a trajectory for each policymaker. Given the one-to-one mapping between equilibrium trajectories and  $\{y_s, x_s, u_s\}_{s=t}^{\infty}$  and the sextuple of matrices  $\mathcal{T} = \{N, F^L, G, D, S^L, S^F\}$ , it is convenient to continue with the definition of policy equilibrium in terms of  $\mathcal{T}$ , not trajectories.

The following proposition derives the first-order conditions for a discretionary optimization problem.

PROPOSITION 3 (First-Order Conditions). The first-order conditions to the optimization problem (17)–(18) can be written in the following form:

$$N = (A_{22} + NA_{12})^{-1} ((A_{21} - B_{22}F^F - B_{21}F^L + N(A_{11} - B_{12}F^F - B_{11}F^L)),$$
(33)

$$S^{F} = \hat{Q} - 2\hat{P}_{1}F^{L} - 2\hat{P}_{2}F^{F} + F^{L'}\hat{R}_{11}F^{L} + 2F^{L'}\hat{R}_{12}F^{F} + F^{F'}\hat{R}_{22}F^{F} + \beta\hat{A}'S^{F}\hat{A} - 2\beta\hat{A}'S^{F}(\hat{B}_{1}F^{L} + \hat{B}_{2}F^{F}) + \beta F_{1}^{L'}\hat{B}'S^{F}(\hat{B}_{1}F^{L} + \hat{B}_{2}F^{F}) + \beta F^{F'}\hat{B}_{2}'S^{F}\hat{B}_{2}F^{F}, \quad (34)$$

$$G = \left(\hat{R}_{22} + \beta \hat{B}_2' S^F \hat{B}_2\right)^{-1} \left(\hat{P}_2' + \beta \hat{B}_2' S^F \hat{A}\right),\tag{35}$$

$$D = (\hat{R}_{22} + \beta \hat{B}'_2 S^F \hat{B}_2)^{-1} (\hat{P}'_1 + \beta \hat{B}'_2 S^F \hat{B}_1), \qquad (36)$$
$$S^L = \tilde{Q} + \beta \tilde{A}' S^L \tilde{A} - 2(\tilde{P} + \beta \tilde{A}' S^L \tilde{B}) F^L$$

$$+ F^{L'}(\tilde{R} + \beta \tilde{B}' S^L \tilde{B}) F^L, \qquad (37)$$

$$F^{L} = (\tilde{R} + \beta \tilde{B}' S^{L} \tilde{B})^{-1} (\tilde{P}' + \beta \tilde{A}' S^{L} \tilde{B}), \qquad (38)$$

where matrices  $F^F$ ,  $\hat{Q}$ ,  $\hat{P}$ ,  $\hat{R}$ ,  $\hat{A}$ ,  $\hat{B}$ ,  $\tilde{Q}$ ,  $\tilde{P}$ ,  $\tilde{R}$ ,  $\tilde{A}$ , and  $\tilde{B}$  are defined by (28) and (30).

*Proof.* From relationships (21), (22), and (27), it immediately follows that

$$N = J - K^{F}G + K^{F}DF^{L} - K^{L}F^{L}.$$
(39)

A straightforward substitution of (23)–(25) and (27) into (39) leads to (33).

The optimal policy of policymaker F can be determined from (29) by differentiating the loss function with respect to  $u_t^F$ :

$$u_t^F = -(\hat{R}_{22} + \beta \hat{B}_2' S^F \hat{B}_2)^{-1} ((\hat{P}_2' + \beta \hat{B}_2' S^F \hat{A}) y_t + (\hat{P}_1' + \beta \hat{B}_2' S^F \hat{B}_1) u_t^L),$$

from where the policymaker's reaction function is defined by (35) and (36). We simultaneously proved proposition 2. We substitute the reaction rules (19), (26), (35), and (36) in (29) and obtain equation (34).

The optimal policy (38) of policymaker L can be determined from (31) by differentiating the loss function with respect to  $u_t^L$ . We substitute the solution into (31) and obtain (37) for  $S^L$ .

DEFINITION 2. The sextuple  $T = \{N, F^L, G, D, S^L, S^F\}$  is a discretionary equilibrium under intraperiod leadership of policymaker L if it satisfies the system of first-order conditions (33)–(38).

Definition 2 implicitly assumes that the first-order conditions are necessary and sufficient conditions of optimality. However, it is straightforward to demonstrate that under the assumption of symmetric positive semi-definite  $Q^i$ , the second-order conditions for the minimum are almost surely satisfied for each policymaker; see, e.g., Blake and Kirsanova (2008).

#### Numerical Solution

One way to search for discretionary equilibrium is to use an algorithm similar to the one in Oudiz and Sachs (1985) and Backus and Driffill (1986). We initialize the matrices  $\{N_0, S_0^L, S_0^F\}$  and then solve the non-linear system of first-order conditions (33)–(38) using an appropriate iterative scheme. This algorithm can be interpreted as search for equilibria that are "iterative-expectations stable" under joint learning; see Dennis and Kirsanova (2010). Alternatively, one can iterate between the solution to (33) that describes the response of the private-sector given policy, and the solution to (34)–(38) that describes the best policy given the response of the private sector; see Blake and Kirsanova (2008).

# Appendix 2. Social Welfare

The procedure to derive the appropriate welfare metric is standard and explained in Woodford (2003a). The one-period (flow) welfare in (14) is  $W_t$ :

$$\mathcal{W}_t = u(C_t) + f(G_t) - \int_0^1 v(y_t(z))dz.$$

Around the steady state, a quadratic approximation to this is

$$\mathcal{W}_{t} = Cu_{C}(C)\left(\hat{C}_{t} + \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\hat{C}_{t}^{2}\right) + Gf_{G}(G)\left(\hat{G}_{t} + \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\hat{G}_{t}^{2}\right) - Yv_{y}(Y)\left(\hat{Y}_{t} + \frac{1}{2}\left(1 + \frac{1}{\psi}\right)\hat{Y}_{t}^{2} + \frac{1}{2}\left(\frac{1}{\psi} + \frac{1}{\epsilon}\right)var_{z}\hat{y}_{t}(z)\right).$$
(40)

Further, a second-order approximation of aggregate demand (8) can be written as

$$\hat{C} = \frac{1}{\theta} \left( \hat{Y} - (1-\theta)\hat{G} - \theta \frac{1}{2}\hat{C}^2 - \frac{1}{2}(1-\theta)\hat{G}^2 + \frac{1}{2}\hat{Y}^2 \right),$$

so we can substitute consumption in (40) and obtain

$$\begin{aligned} \mathcal{W}_s &= \theta u_C \left( \left( 1 - \frac{v_y}{u_C} \right) \hat{Y}_s - (1 - \theta) \left( 1 - \frac{f_G}{u_C} \right) \hat{G}_s \\ &- \frac{(1 - \theta)}{2} \left( 1 + \frac{f_G}{u_C} \frac{(1 - \sigma)}{\sigma} \right) \hat{G}_s^2 - \frac{\theta}{2\sigma} \hat{C}^2 \\ &- \frac{1}{2} \left( \frac{v_y}{u_C} \frac{1 + \psi}{\psi} - 1 \right) \hat{Y}_s^2 - \frac{1}{2} \frac{v_y}{u_C} \frac{\psi + \epsilon}{\psi \epsilon} var_z \hat{y}_s(z) \end{aligned}$$

To transform this equation into a more convenient form that does not include linear terms, we proceed as follows (see Beetsma and Jensen 2005). If the government removes monopolistic distortions and distortions from income taxation in the steady state using a subsidy<sup>20</sup>  $\mu^w = \frac{\mu}{(1-\tau)}$ , then  $u_C/v_y = f_G/u_C = 1$  so the linear terms in (40) drop out. The final formula for social welfare is

$$\mathcal{W}_s = -\theta u_C \left( \frac{\theta}{2\sigma} c_s^2 + \frac{(1-\theta)}{2\sigma} g_s^2 + \frac{1}{2\psi} y_s^2 + \frac{1}{2} \left( \frac{1}{\psi} + \frac{1}{\epsilon} \right) var_z \hat{y}_s(z) \right).$$

Woodford (2003a) has shown that

$$\sum_{t=0}^{\infty} \beta^t var_z \hat{y}_s(z) = \sum_{t=0}^{\infty} \beta^t \frac{\gamma \epsilon^2}{(1-\gamma\beta)(1-\gamma)} \pi_t^2,$$

so, using a conventional notation for gap variables, we get the final formula for the social welfare function:

$$\begin{aligned} \mathcal{W}_s &= -\frac{\theta(\epsilon+\psi)\gamma\epsilon u_C}{2\psi(1-\gamma\beta)(1-\gamma)} \left(\frac{\kappa}{\epsilon} \left(\frac{\theta}{\sigma}c_t^2 + \frac{(1-\theta)}{\sigma}g_t^2 + \frac{1}{\psi}y_t^2\right) + \pi_t^2\right) \\ &= -\frac{\theta(\epsilon+\psi)\gamma\epsilon u_C}{2\psi(1-\gamma\beta)(1-\gamma)} \left(\rho_c c_s^2 + \rho_g g_s^2 + \rho_y y_s^2 + \pi_s^2\right),\end{aligned}$$

which, after normalization, is (15) in the main text with  $\rho_c = \frac{\kappa \theta}{\epsilon \sigma}$ ,  $\rho_g = \frac{\kappa (1-\theta)}{\epsilon \sigma}$ ,  $\rho_y = \frac{\kappa}{\psi \epsilon}$ .

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<sup>&</sup>lt;sup>20</sup>This subsidy is financed, of course, by lump-sum taxation,  $\mu^w = T$ .

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