UNIVERSITY OF EXETER

Economics Department Discussion Papers Series

ISSN 1473 – 3307

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Paper number 10/05

URL: http://business-school.exeter.ac.uk/economics/papers/

The Benefits of Costly Voting

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February 2010

Abstract

We present a costly voting model in which each voter has a private valuation for their preferred outcome of a vote. When there is a zero cost to voting, all voters vote and hence all values are counted equally regardless of how high they may be. By having a cost to voting, only those with high enough values would choose to incur this cost. Hence, the outcome will be determined by voters with higher valuations. We show that in such a case welfare may be enhanced. Such an effect occurs when there is both a large enough density of voters with low values and a high enough expected value.

1 Introduction

> "The object of our deliberations is to promote the good purposes for which elections have been instituted, and to prevent their inconveniencies." (Edmund Burke as cited in Lakeman and Lambert, 1959, p. 19)

Groups within society often have to make collective decisions. In order to reach correct social decisions, the valuations of all those affected by the decision should be aggregated. Leaving some out may reach an incorrect decision. For example, take a committee that must decide upon an issue at a meeting. Each member has a certain private value to the results of the decision reached by the committee. The committee's social value of the decision is the sum of the individual private

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values. In order to reach the correct outcome for the committee, the private values of all members should be aggregated. A common method to reach a decision is to have a vote and one may think that voting properly aggregates information. Since each member of the committee has information that is relevant to the decision, we would normally think that ensuring all participate in voting would improve the final outcome. In fact, many countries (including Argentina, Australia, Belgium, and Greece) have compulsory voting to ensure inclusion.[1] There is, however, significant difference between aggregating private values and ensuring full participation of all voters.

In social valuation, the strength of preference counts. In voting, the options for expressing preference for any particular alternative are limited to either voting for it, or not voting for it (that is, vote for an alternative or abstain).[2] This means with voting it is not possible to demonstrate intensity of preferences. One voter mildly in favor of an alternative exactly offsets another voter who is strongly opposed.

These observations suggest that there may be gains, in avoiding poor decisions, by ensuring that voters who have only mild feelings about the alternatives are excluded. In some sense, if their vote counts for more than their strength of feeling then they may change the voting outcome in a detrimental way. Indeed, in Australia where voting is mandatory, "donkey votes" (those that simply were cast by order of a ballot) give a 1% edge to those listed first (see Orr, 2002, and King and Leigh, 2009). One way to exclude such voters is to ensure that there is a cost to voting that deters participation by those without strong preferences (or are not well informed) and hence achieve a socially-better outcome. This intuition is contrary to the widely held view that costly voting is detrimental since it deters voting (and is a cost to those that do vote) leading to a paradox of why people vote (see Dhillon and Peralta, 2002, for an overview).

Let us look at a simple model that captures the main intuition of our paper. There are two options, A and B, and two voters: one prefers A and the other prefers B. There is an $1/2$ chance that a voter has utility of 1 for his preferred option (and -1 for the other) and $1/2$ chance of utility of 3 for his preferred option (and -3 for other). If the cost of voting is 0, everyone votes and the social surplus is 0 (ties are broken randomly). If the cost of voting is c and only those with a high

[1] Enforcement ranges from fines (Australia) to disenfranchisement (Belgium) or making it difficult to obtain a passport or driver's license (see Frankal, 2005).

[2] Among the rare exceptions are reality TV shows such as Pop Idol where individuals can vote more than once (and pay for each vote).

value will vote, then there is positive social surplus from the option when only one voter has a higher value and the other the lower value. This yields a gain of $3 - 1 = 2$ and occurs $1/2$ the time. So the net social benefits of voting is $1 - c$ (each voter votes half the time). Thus, as long as this is an equilibrium and $c < 1$, then costly voting can be socially beneficial. This is indeed an equilibrium if both voters find it in their individual interests to vote only with a high value. To see if this is true, a voter by voting improves the chance of his preferred option winning by $1/2$ no matter what the other voter chooses to do (since by voting one either goes from a tie to a win or losing to a tie). Hence, if one has a private value to an option of v, one votes if $v/2 > c$. Thus, for this to be an equilibrium we must have $3/2 \geq c \geq 1/2$ (only the high-value voter should vote).

In this toy model, we compare two situations, one with costly voting and one with free voting, and find that, in fact, costly voting is superior. Hence, adding a cost to voting can be welfare enhancing. This captures the intuition behind this phenomena, namely, with costly voting, only those with high enough values would choose to incur the cost to voting, while with free voting, all values are counted equally regardless of how valuable they may be.

Notice that there are two important components of this model that enable costly voting to be superior to free voting. First, the high value is large enough. If instead of 3, the value were 2, then costly voting would be inferior. (Gains from one voter having a high value and one a low value would be 1 occurring $1/2$ the time. This yields a net social gain of $1/2 - c$. Since $c \geq 1/2$, the social gain cannot be positive.) Second, the probability of having a low value should be sufficiently large as well. For instance, if instead of $1/2$ this was $1/4$, then again costly voting would be inferior.[3]

In this paper, we investigate the above phenomenon in two directions. First, we allow for each voter to have preferences over either candidate. In this initial model, we have discrete, privately-known preferences over both options. Here we find the distribution over values that yield superior costly voting. Again, there must be a sufficient chance that a voter has a low valuation for this superiority. We show this on a probability simplex. We follow this initial model with one that has a continuous range of values. This allows us to explore the case with a large number of voters more easily. Again, we show that increasing costs may be beneficial and in addition we determine the optimal cost c. Similar to our toy model, we find that whether costly voting is superior depends upon both the expected value and the density of lower value voters. We also show that in such a

[3] More generally, if p is the probability of having a low value and v is the high value, then there is a net gain to voting if $(v-1)p \geq c$. This would be an equilibrium if $v/2 \geq c \geq 1/2$.

model a government would never want to have mandatory voting by imposing fines or subsidizing voting but may wish to implement a poll tax (a charge for voting).

This analysis can be seen as the normative counterpart to the positive analyses of Bulkley et al. (2001) and Osborne et al. (2000). These papers establish that when voting is costly the outcome of the voting game will have an equilibrium in which only voters from the extremes will participate. Again, at first sight it might appear that this is a bad outcome since it excludes moderate opinion. What we show is that instead it can be efficient to have precisely such an equilibrium.

More recently, Borgers (2004) has a model that shows that an equilibrium with costly voting is superior to both mandatory voting (still with voting costs) and random selection of a winner (with no voting). In the Borgers model, there is uncertainty for which alternative a voter prefers, but no difference in intensity of preference for a particular alternative. Thus, mandatory voting has the benefit of including all the available information (everyone votes) and selecting the best alternative, but at the highest cost. Random selection has the lowest cost but uses no information. Voluntary voting is the superior tradeoff where only those with low costs of voting vote. Krasa and Polborn (2009) vary the Borgers model by allowing for ex-ante asymmetry of preferences over alternatives. They find that for a large enough number of voters, it is optimal to move towards mandatory voting from voluntary voting (by a penalty for not voting or a subsidy to voting).

In our paper, we add intensity on preferences over alternatives. Now in contrast to the Borgers model (and Krasa and Polborn), everyone voting no longer includes all information (it neglects intensity of preference) and thus sometimes does not select the best alternative. We find that even if we eliminate all costs to mandatory voting by setting the cost of voting to zero, it may not necessarily be superior. In fact, we find that the social planner should in addition to eliminating any penalties for not voting (as Borgers finds) the planner may in fact wish to increase the cost of voting even if this cost is completely wasteful.

While less related, the Condorcet Jury literature models voting by a group of individuals with a common value over two alternatives. Krishna and Morgan (2008) show that as the cost of voting goes to zero, voluntary voting is the optimal mechanism. Ghoshal and Lockwood (2009) have combined the common value in the Condercet Jury literature with the private value of alternatives in Borgers (2004). They find that if the voters put a high weight on personal preferences then there is an inefficient high turnout for voting and in the case voters care more about the common aspect then voter turnout is to low compared to the efficient outcome.[4]

In the next section we examine the discrete model. Then in section 3, we analyze the continuous model. We conclude in section 4.

2 Discrete Model

2.1 Description

There are n risk-neutral voters, who must choose between two options, A and B. The private value that voter i obtains from the outcome of the voting process is v_i if option B wins, $-v_i$ if option A wins, and 0 if there is a tie and neither A nor B is implemented.[5] It is assumed that the distribution of v_i is over $\mathbb{K} = \{-K, \ldots, -2, -1, 0, 1, 2, \ldots, K\}$. For all $k \in \mathbb{K}$, the probability that $v_i = k$ is p_k where $\sum_{k \in \mathbb{K}} p_k = 1$.

Each voter i chooses action α_i from three possibilities: $\{a, 0, b\}$, where 0 indicates not voting, a indicates voting for A, and b indicates voting for B. All voters have the same cost of voting, $c \geq 0$. A pure strategy, s_i, for each voter is a choice of action for each possible private value, $s_i : v_i \to \{a, 0, b\}$. Denote S_i as the set of possible strategies s_i.

An outcome for the voting process is from $W = \{A, \emptyset, B\}$, where $\emptyset$ denotes a tie. The outcome of the voting process is determined by the function $w(\alpha_1, \ldots, \alpha_n)$ on the actions of the voters. Denote $\#\{i : \alpha_i = x\}$ as the number of voters who choosing a particular action $x \in \{a, 0, b\}$. The winner is determined by the majority, hence $w(\cdot) = A$ if $\#\{i : \alpha_i = a\} > \#\{i : \alpha_i = b\}$, $w(\cdot) = B$ if $\#\{i : \alpha_i = a\} < \#\{i : \alpha_i = b\}$, and $w(\cdot) = \emptyset$ if $\#\{i : \alpha_i = a\} = \#\{i : \alpha_i = b\}$.

Given outcome function w, the utility for voter i, $u_i(s_1, \ldots, s_n, v_i)$ is given by

$$u_i(s_1, \ldots, s_n, v_i) = \begin{cases} v_i - c \text{ if } w(s_1(v_i), \ldots, s_n(v_i)) = B \text{ and } s_i(v_i) \in \{a, b\} \\ -v_i - c \text{ if } w(s_1(v_i), \ldots, s_n(v_i)) = A \text{ and } s_i(v_i) \in \{a, b\} \\ -c \text{ if } w(s_1(v_i), \ldots, s_n(v_i)) = \emptyset \text{ and } s_i(v_i) \in \{a, b\} \end{cases}$$

[4] The equivalent of costly voting has also been addressed in the Condorcet Jury literature by allowing voters to buy costly signals about the common feature of the alternatives (see Persico, 2004, Gerardi and Yariv, 2005).

[5] Equivalently, if there is a tie, A or B is chosen randomly.

$$u_i(s_1, \ldots, s_n, v_i) = \begin{cases} v_i \text{ if } w(s_1(v_i), \ldots, s_n(v_i)) = B \text{ and } s_i(v_i) = 0 \\ -v_i \text{ if } w(s_1(v_i), \ldots, s_n(v_i)) = A \text{ and } s_i(v_i) = 0 \\ 0 \text{ if } w(s_1(v_i), \ldots, s_n(v_i)) = \emptyset \text{ and } s_i(v_i) = 0 \end{cases}$$

Each voter i chooses a mixed-strategy σ_i over the possible pure strategies, $\sigma_i \in \Delta S_i$. Denote σ as $(\sigma_1, \ldots, \sigma_n)$ and $\pi_i(\sigma)$ as the expected utility of i over values and strategies. Using this notation, we now define the equilibrium of the voting game.

Definition 1 *A vector of strategies $\widehat{\sigma}$ forms a* ***Bayes-Nash equilibrium*** *if for all voters i, given the strategies of the other voters $\{\widehat{\sigma}_{-i}\}$, we have $\pi_i(\widehat{\sigma}_i, \widehat{\sigma}_{-i}) \geq \pi_i(\sigma_i, \widehat{\sigma}_{-i})$, for all $\sigma_i \in \Delta S_i$.*

Definition 2 *The* ***expected social surplus*** *in equilibrium $\widehat{\sigma}$ is $\sum_{i=1}^{n} \pi_i(\widehat{\sigma})$.*

In order to further analyze the equilibrium, we need to determine when a voter chooses to vote. In the following lemma, we describe the condition when the gains from voting exceeds the cost of voting.

Lemma 1 *Strategy $\widehat{s}_i(v_i) = a$ can be part of equilibrium $\widehat{\sigma}$ only if*

$$\left[\begin{array}{c} P(\#\{i : s_{-i}(v_i) = a\} = \#\{i : s_{-i}(v_i) = b\} | \widehat{\sigma}, v_i) + \\ P(\#\{i : s_{-i}(v_i) = a\} = \#\{i : s_{-i}(v_i) = b\} - 1) | \widehat{\sigma}, v_i) \end{array} \right] \frac{v_i}{2} \geq c$$

where $P(\cdot|\widehat{\sigma})$ is the probability of an event occurring in the equilibrium $\widehat{\sigma}$. Strategy $\widehat{s}_i(v_i) = b$ can be part of equilibrium $\widehat{\sigma}$ only if

$$\left[\begin{array}{c} P(\#\{i : s_{-i}(v_i) = a\} = \#\{i : s_{-i}(v_i) = b\} | \widehat{\sigma}, v_i) + \\ P(\#\{i : s_{-i}(v_i) = a\} - 1 = \#\{i : s_{-i}(v_i) = b\}) | \widehat{\sigma}, v_i) \end{array} \right] \frac{v_i}{2} \geq c.$$

Likewise, strategy $\widehat{s}_i(v_i) = 0$ can be part of equilibrium $\widehat{\sigma}$ only if

$$\left[\begin{array}{c} P(\#\{i : s_{-i}(v_i) = a\} = \#\{i : s_{-i}(v_i) = b\} | \widehat{\sigma}, v_i) + \\ P(\#\{i : s_{-i}(v_i) = a\} = \#\{i : s_{-i}(v_i) = b\} - 1) | \widehat{\sigma}, v_i) \end{array} \right] \frac{v_i}{2} \leq c$$

and

$$\left[\begin{array}{c} P(\#\{i : s_{-i}(v_i) = a\} = \#\{i : s_{-i}(v_i) = b\}|\widehat{\sigma}, v_i) + \\ P(\#\{i : s_{-i}(v_i) = a\} - 1 = \#\{i : s_{-i}(v_i) = b\})|\widehat{\sigma}, v_i) \end{array}\right] \frac{v_i}{2} \leq c.$$

Lemma 1 explains that one votes if the probability of one being pivotal multiplied by the gains from voting if one is pivotal is greater than the cost of voting. The probability of being pivotal consists of two parts, first, that the voting is tied without the voter, and second, the voting is tied including the voter. Note that the benefit from voting when one is pivotal is $v_i/2$ because if there is a tie, there is a fifty-percent chance of winning and hence from moving from losing to a tie increases the likelihood of winning by fifty percent (and moving from a tie to winning also increases the likelihood by fifty percent).

Lemma 2 *If $\widehat{s}_i(v_i) = a$ is part of equilibrium $\widehat{\sigma}$, then for all $\widetilde{v}_i \in \mathbb{K}$ such that $\widetilde{v}_i > v_i$, $\widehat{s}_i(\widetilde{v}_i) = a$ is part of equilibrium $\widehat{\sigma}$, while $\widehat{s}_i(\widetilde{v}_i) = b$ and $\widehat{s}_i(\widetilde{v}_i) = 0$ are not part of equilibrium $\widehat{\sigma}$. Likewise, if $\widehat{s}_i(v_i) = b$ is part of equilibrium $\widehat{\sigma}$, then for all $\widetilde{v}_i \in \mathbb{K}$ such that $\widetilde{v}_i > v_i$, $\widehat{s}_i(\widetilde{v}_i) = b$ is part of equilibrium $\widehat{\sigma}$, while $\widehat{s}_i(\widetilde{v}_i) = a$ and $\widehat{s}_i(\widetilde{v}_i) = 0$ are not part of equilibrium $\widehat{\sigma}$.*

It is clear that with donkey votes, increasing c from 0 to ε will increase the social surplus,[6] but it is not obvious that there is a benefit to increasing c beyond ε. In the following proposition, we that this may indeed be the case.

Proposition 1 *The social surplus can be increasing in c for $c > 0$.*

Proof. To show this, we will examine two examples in a symmetric environment where $n = 3$, $K = m \geq 2$, $p_i = p_{-i}$, and $p_j = p_{-j} = 0$ for all $m > j > 1$. Let us first look at the case there is a small cost.

Low cost equilibrium:

For small enough c, there is a unique equilibrium where everyone always votes for their preferred choice and those with value 0 do not vote. This can only be an equilibrium if it is worthwhile for a voter with value 1 or -1 to vote. From Lemma 1, this happens if for the voter, the expected gains of being pivotal is strictly greater than the cost, which implies $2(p_1 + p_m)^2(1/2) + (1 - 2p_1 -$

[6] For example, if $p_0 > 1/2$ and $c = 0$, all those with value 0 vote for A, then for large enough n the outcome will be A. This would be regardless of the valuations. If $p_1 = 1 - p_0$, then it would be efficient for B to win. Hence, there would be a social improvement by increasing c to ε.

$2p_m)^2(1/2) + 2(p_1 + p_m)(1 - 2p_1 - 2p_m)(1/2) = (p_1 + p_m)^2 + (1 - 2p_1 - 2p_m)/2 > c$. Note from Lemma 2, we need only check this one condition (and not an additional condition for a voter with absolute value of 2). Hence, if c is small enough each voter will vote for all $v_i \geq 1$.

We now proceed to compute the expected voter surplus of such an equilibrium. The possible states ex ante are (i) all three voters vote for the same option, (ii) two voters vote for the same option and the third votes for the other, (iii) one voter is indifferent and the other two prefer opposing options, (iv) one voter is indifferent and the other two voters prefer the same option, (v) two voters are indifferent and the third votes for his preferred option (vi) all three are indifferent. Therefore, we can compute the probability and the expected value of all the possible six states: (i) there is a $2(p_1 + p_m)^3$ chance that all three voters vote for the same option. In this case, the expected surplus is $3E[v_i|v_i > 0] - 3c = 3\frac{p_1+m \cdot p_m}{p_1+p_m} - 3c$. (ii) There is a $6(p_1 + p_m)^3$ chance that two voters vote for the same option and the other voter votes for the other option. In this case, the expected surplus is $E[v_i|v_i > 0] - 3c = \frac{p_1+m \cdot p_m}{p_1+p_m} - 3c$. (iii) There is a $6(p_1 + p_m)^2(1 - 2p_1 - 2p_m)$ chance that one voter is indifferent and the other two vote for opposing options. In this case, the expected surplus is $-2c$. (iv) There is a $6(p_1 + p_m)^2(1 - 2p_1 - 2p_m)$ chance that two voters vote for the same option and one is undecided. In this case, the expected surplus is $2E[v_i|v_i > 0] - 2c = 2\frac{p_1+m \cdot p_m}{p_1+p_m} - 2c$. (v) There is a $6(p_1 + p_m)(1 - 2p_1 - 2p_m)^2$ chance that two are undecided. In this case the expected surplus is $E[v_i|v_i > 0] - c = \frac{p_1+m \cdot p_m}{p_1+p_m} - c$. (vi) When all are indifferent, there is zero expected surplus.

Overall, the ex-ante surplus in the low equilibrium is

$$SSL(c) = 6(p_1 + mp_m)(1 + 2p_1^2 - 2(1 - p_m)p_m - 2p_1(1 - 2p_m)) - 6(p_1 + p_m)c.$$

High cost equilibrium:

In this equilibrium, when the cost of voting, c, is high, voters with value 1 or -1 will choose 0 (not vote) and only voters with values m and $-m$ will choose their respective option. The voter with absolute value m will vote if $(1-2p_m)^2 \cdot m \cdot (1/2) + 2 \cdot p_m \cdot (1-2p_m) \cdot m \cdot (1/2) + p_m^2 \cdot m \cdot (1/2) > c$ or $(m/2)\,(1 - p_m)^2 > c$. Also, the voter with absolute value 1 will not vote if his expected utility is smaller than the cost, that is, $(1 - 2p_m)^2 \cdot 1 \cdot (1/2) + 2 \cdot p_m \cdot (1 - 2p_m) \cdot 1 \cdot (1/2) + p_m^2 \cdot 1 \cdot (1/2) < c$ or $(1 - p_m)^2/2 < c$. Given that only the high-value voter, $\mid v_i \mid= 2$, will vote; (i) there is a $(2p_m)^3$ chance that all three voters have a value of 2 or -2. In such a case, the expected value is $(1/4) \cdot 3 \cdot 2 + (3/4) \cdot 2 - 3c = 3 - 3c$. (ii) There is a $3(2p_m)^2(1 - 2p_m)$ chance that two voters have values of 2 or -2. In such a case, there is only a benefit if both have the same sign. So the expected value in that case is $(1/2) \cdot 4 - 2c = 2 - 2c$. (iii) There is a $3(2p_m)(1 - 2p_m)^2$ chance that only one voter has a value of 2 or -2. In such a case, the benefit is $2 - c$. So the total surplus is

$$SSH(c) = 6m \cdot p_m(1 - 2(1 - p_m)p_m) - 6p_m c.$$

Comparison:

We can now compare free voting to costly voting in our example. If $(m/2)(1 - p_m)^2 > c > (1 - p_m)^2/2$, then it is an equilibrium for only voters with high values of m to vote. If in this case, $SSH(c) > SSL(0)$, then indeed costly voting is superior. Take for example $m = 10$ and $c = 1/2$. These parameters satisfy the equilibrium constraints $(m/2)(1 - p_m)^2 \geq c \geq (1 - p_m)^2/2$ for all $p_m \in (0, 0.6838)$. In Figure 1, we plotted the range of probabilities for which $SSH(c) > SSL(0)$. Notice that p_1 or p_m should not be too high or too low, while p_0 can be zero, but not too high. If p_m is too high, it simply means that there is a low probability of having values of 0, 1 or -1. This means that there is not much to be gained by excluding these voters compared with the cost. If p_m is too low, then by excluding voters with values of 1 or -1, one would be excluding a large amount of information and doing so is not worthwhile.

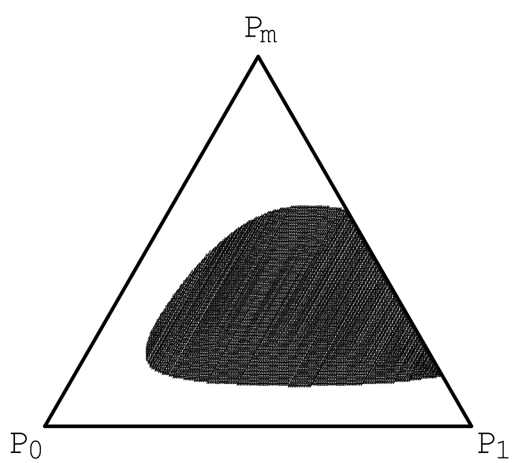

Figure 1: The probability simplex where probability is equal to the shortest distance to the side opposite the vertex. The shaded region is where costly voting is superior to free voting. This is for $c = 0.5$ and $m = 10$.

The optimal c in our example is either $c = (1-p_m)^2/2$ or $c = 0$. This depends whether or not $SSH(\frac{(1-p_m)^2}{2}) > SSL(0)$. This is because if c chosen to induce only the voters with values of m or $-m$ to vote, then c satisfies $(m/2)\,(1-p_m)^2 \geq c \geq (1-p_m)^2/2$. However, it is only these voters that voter for any c that satisfies those constraints. Since the voters that vote must expend c to vote. Subject to the constraints, this wasteful expense is minimized at $c = (1-p_m)^2/2$. ■

3 Continuous model

We now move to framework with a continuous range of values. This allows us to more effectively analyze a large number of voters, determine an optimal level of the cost of voting and determine which distributions lead to a strictly positive optimal cost. Unlike the discrete model (and like our toy model), we keep the number of voters preferring a particular option fixed and we vary only their preferences for how much they prefer that particular option. Here our model more appropriately represents a committee making a binary decision. For instance, a committee must decide where to build a casino. The casino will be built in either district A or district B. There is a representative for district A who values it being built in A as $v_a > 0$ (and being built in B at 0). There is a representative for district B who values it being built in B as $v_b > 0$ (and being built in A at 0). Each representative's value is private information.

If both (or neither) show up to the meeting, there is a 50% chance of either district being selected. If one shows up, then that representative's district wins. So, showing up improves the chance of having one's district selected by 50%; however, there is a common cost of showing up (cost of voting) which is c. This means representative i shows up if $v_i \geq 2c$.

More generally, assume that there are two types of voters and n voters of each type (overall there are $2n$ voters). Assume that each voter i has value $v_i \geq 0$ is drawn from distribution F. If $1 \leq i \leq n$, voter i is a type A voter who values a win by A at v_i and a win by B at 0. If $n+1 \leq i \leq 2n$, voter i is a type B voter who values a win by B at v_i and a win by A at 0.[7] Choice

[7] Note that A winning yields higher social surplus if $\sum_{i=1}^{n} v_i \geq \sum_{i=n+1}^{2n} v_i$. Rearranging terms yields $\sum_{i=1}^{n} \frac{v_i}{2} + \sum_{i=n+1}^{2n} -\frac{v_i}{2} \geq \sum_{i=n+1}^{2n} \frac{v_i}{2} + \sum_{i=1}^{n} -\frac{v_i}{2}$. Notice this is the condition for if the voter has utility $\frac{v_i}{2}$ for his preferred candidate winning and $-\frac{v_i}{2}$ for the other candidate winning. For simplicity, we frame the utility as only having a utility for one's preferred candidate.

A wins if the number of votes it receives, denoted by $\#_A$, is strictly greater the number of votes choice B receives, denoted by $\#_B$. (We apologize for the change of notation from the previous section.) Choice B wins $\#_B > \#_A$. There is a tie if $\#_B = \#_A$ and the winner is determined randomly with equal probability.

Denote $v^*(c)$ as a cutoff strategy such that a voter i votes if his value is above $v^*(c)$ and doesn't vote if his value is below $v^*(c)$. We denote $\Pr_{-i}(event|v^*)$ as the probability that *event* occurs given that all voters except voter i follows cutoff strategy $v^*(c)$ and voter i does not to vote.

A voter i where $1 \leq i \leq n$ (a type A) and with value v_i will vote if

$$V_i \left[\frac{1}{2} \Pr_{-i}(\#_A = \#_B|v^*) + \frac{1}{2} \Pr_{-i}(\#_A = \#_B - 1|v^*) \right] > c.$$

The first expression $\Pr_{-i}(\#_A = \#_B|v^*)$ represents the case when the voter will change the vote from a tie to a win by voting. The second expression $\Pr_{-i}(\#_A = \#_B - 1|v^*)$ represents the case when the voter will change the outcome of a vote from losing to a tie.

Lemma 3 *A cutoff $v^*(c)$ forms a Bayes-Nash equilibrium if*

$$v^* \sum_{i=0}^{n-1} \binom{n-1}{i} \binom{n}{i+1} F(v^*)^{2(n-i-1)} (1 - F(v^*))^{2i} [1 + iF(v^*)] = 2c. \tag{1}$$

Proof. The cutoff will be such that the value for voting equals the cost.

$$v^*(c) \cdot \left[\frac{1}{2} \Pr_{-i}(\#_A = \#_B|v^*) + \frac{1}{2} \Pr_{-i}(\#_A = \#_B - 1|v^*) \right] = c.$$

When the voter i prefers A ($i \leq n$), we can rewrite the probabilities in this equation as follows:

$$\begin{aligned} &\Pr_{-i}(\#_A = \#_B|v^*) \\ &\quad = \Pr_{-i}(\#_A = \#_B = 0|v^*) + \Pr_{-i}(\#_A = \#_B = 1|v^*) + ... + \Pr_{-i}(\#_A = \#_B = n-1|v^*) \\ &\qquad = \sum_{i=0}^{n-1} \binom{n-1}{i} \binom{n}{i} (1 - F(v^*))^{2i+1} F(v^*)^{2(n-i-1)} \end{aligned}$$

and

$$\begin{aligned}
&\Pr_{-i}(\#_A = \#_B - 1|v^*) \\
&= \Pr_{-i}(\#_A = 0, \#_B = 1|v^*) + \Pr_{-i}(\#_A = 1, \#_B = 2|v^*) + ... + \Pr_{-i}(\#_A = n-1, \#_B = n|v^*) \\
&= \sum_{i=0}^{n-1} \binom{n-1}{i}\binom{n}{i+1} (1 - F(v^*))^{2i+1} F(v^*)^{2(n-i-1)}.
\end{aligned}$$

Substituting these expressions into the cutoff value equation yields:

$$v^* \sum_{i=0}^{n-1} \left[\binom{n-1}{i}\binom{n}{i} (1 - F(v^*))^{2i} F(v^*)^{2(n-i)-1} + \binom{n-1}{i}\binom{n}{i+1} (1 - F(v^*))^{2i+1} F(v^*)^{2(n-i-1)} \right] = 2c.$$

This can be simplified as

$$v^* \sum_{i=0}^{n-1} \binom{n-1}{i}\binom{n}{i+1} F(v^*)^{2(n-i-1)} (1 - F(v^*))^{2i} [1 + iF(v^*)] = 2c$$

The same equation holds for voters preferring B $(i > n)$. ■

Lemma 4 *When there is a zero cost of voting, everyone votes,* $v^*(0) = 0$.

Proof. This follows directly from equation (1). ■

Lemma 5 *If* $n = 1$ *or* $\lim_{v \to 0} vF'(v) = 0$, *then* $\lim_{c \to 0} v_c^*(c) = 2$.

Proof. Equation (1) must hold for all c, so we can take the derivative w.r.t. c and take the limit as $c \to 0$. As $c \to 0$, we have $v \to 0$ (from Lemma 4), so $\lim_{v \to 0} (1 - F) = 1$, $\lim_{v \to 0} F = 0$. Notice that from this we need only worry about the term when $i = n - 1$. (For $n > 1$, the rest vanish.) This yields:

$$\left[1 - (n-1) \lim_{v* \to 0} v^* \cdot F'(v^*)\right] = \frac{2}{v_c^*(0)}.$$

Hence, $v_c^*(0) = 2$ when $n = 1$ or $\lim_{v \to 0} vF'(v) = 0$. ■

Example 1 *When* $F(v) = v^\alpha$ *where* $\alpha > 0$, *we have* $\lim_{v \to 0} vF'(v) = \lim_{v \to 0} \alpha v^\alpha = 0$.

Lemma 6 *The social surplus to voting is then the expected value of the winner minus the costs of voting:*

$$SSV(c) = \sum_{a=0}^{n}\sum_{b=0}^{n}\binom{n}{a}\binom{n}{b}F\left(v^*(c)\right)^{2n-a-b}\left(1-F\left(v^*(c)\right)\right)^{a+b}\begin{bmatrix}(n-\max\{a,b\})E[V_i|V_i<v^*(c)]+\\ \max\{a,b\}E[V_i|V_i>v^*(c)]\end{bmatrix} -2(1-F\left(v^*(c)\right))n\cdot c. \quad (2)$$

The expected value of the winner is computed by going through the possible number of voters for each candidate where a is the votes for candidate A and b is the votes for candidate B. The probability of each case is calculated and multiplied by the expected value of the voters that vote for the winner. The expression $2n(1-F\left(v^*(c)\right))c$ is the expected cost of the voters voting since $(1-F\left(v^*(c)\right))$ is the probability of each voter voting.

In the following example, we can examine how the social surplus to voting depends upon c.

Example 2 $F(v)=v^{\alpha}$ *on* $[0,1]$*,* $c<1/2$*,* $n=1$*.*

From (1), $v^*(c)=2c$. We can then write equation (2) as

$$\begin{aligned}(2c)^{2\alpha}E[V|V &< 2c]+(1-(2c)^{2\alpha})E[V|V>2c]-2(1-(2c)^{\alpha})c\\ &= \frac{\alpha-2c(1+\alpha)+(2c)^{\alpha}(\alpha+2c)}{1+\alpha}.\end{aligned}$$

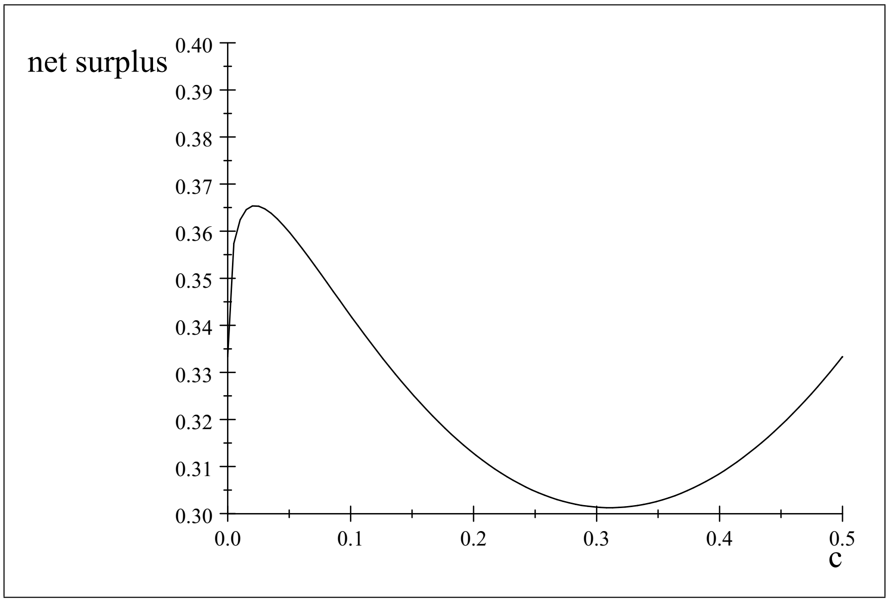

Figure 2. The social surplus net of voting costs versus the cost of voting c when $F(v) = v^{.5}$ and $n = 1$.

If $\alpha = 0.5$, the net surplus is plotted in Figure 2. As we see here, the ideal c is strictly positive. It reaches a maximum at $c = 0.0223291$. We can also examine the probability density function of v. This density is $0.5v^{-.5}$ and shown in Figure 3.

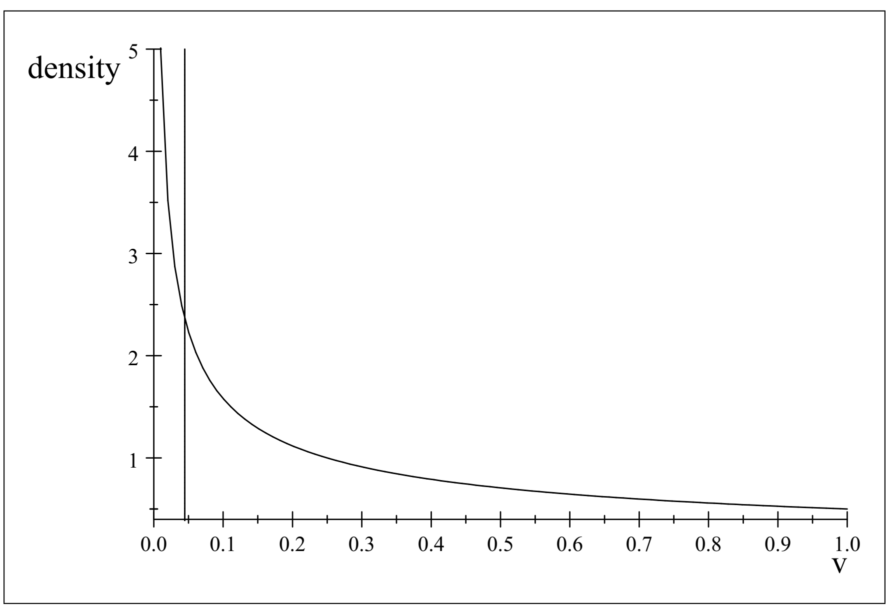

Figure 3. The graph of the density function of $f(v) = 0.5v^{-0.5}$ and a vertical line at $v = 2c$ at $c = .0223291$ (the optimal c).

In Figure 3, the voters to the left of the vertical line do not vote when c is at the optimal level. We now ask if in our example, for which α is there a gain in surplus to increasing the cost of voting. For $n = 1$, the slope of the surplus w.r.t. c is

$$SSV'(c) = -2 + \frac{2^{\alpha}c^{\alpha-1}(\alpha^2 + 2(1+\alpha)c)}{1+\alpha}.$$

For $\alpha > 1$, $\lim_{c\to 0} SSV'(c) = -2$. For $\alpha = 1$, $SSV'(c) = -1$. For $0 < \alpha < 1$, $\lim_{c\to 0} SSV'(c) = \infty$. Hence, when $0 < \alpha < 1$, the surplus improves by increasing the cost. It also turns out that for $\alpha \geq 1$, the surplus is at the highest when cost is zero. (When $\alpha \geq 1$, $SSV'(c)$ is increasing in c, hence $SSV'(c)$ can equal zero only once. Since $SSV(0) = SSV(1/2)$, no one votes in either case, and $SSV'(0) < 0$, that point must be a minimum.) The following is a proposition that examines the former possibility for general F.

Proposition 2 *If* $\lim_{v\to 0} F'(v)v = 0$, $\lim_{v\to 0} F'(v)F(v) = 0$ *and* $E[v]\cdot F'(0) > 1$, *then it is optimal to have* $c > 0$.

Proof. In this proof, we would like to show that $\lim_{c\to 0} SSV'(c) > 0$. In order to show this, we wish to show that the derivative of the expected value of the voters that prefer the winning candidate is higher than the derivative of the expected costs. Let us first examine the former.

Assuming $\lim_{v\to 0} F'(v)F(v) = 0$, the derivative of this expected value as c goes to 0 can be determined as follows. The probability of a voters voting for A and b voters voting for B is $\binom{n}{a}\binom{n}{b}F(v^*(c))^{2n-a-b}(1-F(v^*(c)))^{a+b}$. The limit of this term as $c \to 0$ is zero unless $a+b = 2n$. The derivative of this term goes to zero as $c \to 0$ unless $a+b = 2n$ or $a+b = 2n-1$. Note that there is one case when $a+b = 2n$ and there are $2n$ cases when $a+b = 2n-1$. When $a+b = 2n$, the derivative of $F(v^*(c))^{2n-a-b}(1-F(v^*(c)))^{a+b} = (1-F(v^*(c)))^{2n}$ goes to $-2nF'v_c$. When $a+b = 2n-1$, the derivative of $F(v^*(c))^{2n-a-b}(1-F(v^*(c)))^{a+b} = F(v^*(c))(1-F(v^*(c)))^{2n-1}$ goes to $F'v_c$. When $a+b = n$, we have $(n-\max\{a,b\})E[V_i|V_i < v^*(c)] + \max\{a,b\}E[V_i|V_i > v^*(c)] = nE[V_i|V_i > v^*(c)]$. When $a+b = 2n-1$, we also have $(n-\max\{a,b\})E[V_i|V_i < v^*(c)] + \max\{a,b\}E[V_i|V_i > v^*(c)] = nE[V_i|V_i > v^*(c)]$. Thus,

$$\lim_{c\to 0} SSV'(c) = \frac{ndE[V_i|V_i > v^*(c)]}{dc} - 2n + \lim_{c\to 0} 2F'(v^*(c))n \cdot 2 \cdot c.$$

Since $E[V|V > v^*(c)] = \frac{\int_{v^*(c)}^{\infty} vdF(v)}{1-F(v^*(c))}$, we have

$$\frac{dE[V|V > v^*(c)]}{dc} = \frac{-v_c^*(c)v^*(c)F'(v^*(c))}{(1 - F(v^*(c))} + \frac{F'(v^*(c))v_c^*(c)\int_{v^*(c)}^{\infty} vdF(v)}{(1 - F(v^*(c))^2}.$$

Hence, $\lim_{c \to 0} \frac{dE[V|V>v^*(c)]}{dc} = 2F'(v^*(c))\int_0^{\infty} vdF(v) = 2F'(v^*(c))E[v]$. Thus, if $F'(0)E[v] > 1$, $SSV'(0) > 0$ and it is optimal to increase costs. ■

It is interesting that the condition that ensures that the surplus is increasing in cost has two components that depend upon the distribution of v: the density at zero and the expected value. The combination of these two components must be large enough. Too low a value of the density at zero would mean that increasing cost does not eliminate enough low value votes. Too low an expected value would mean that the benefit to eliminating these low-value voters is not large enough. An alternative condition is to replace $2F'(0)E[v]$ by $\lim_{c \to 0} \frac{dE[V|V>v^*(c)]}{dc}$. This implies that the expected value of those voting is increasing in cost by a sufficient amount, 2. In other words, if one increases cost marginally by a dollar (at zero), then the expected value of those voting should go up by 2 in order for costly voting to be beneficial. This condition is also equivalent to $\lim_{v^* \to 0} \frac{dE[V|V>v^*]}{dv^*} > 1$.

3.1 What is the optimal level of voting?

We saw in the previous section that voting, even if it is costly, can have benefits to social surplus. However, in the previous section, who voted was determined by equilibrium conditions. In this section, we wish to ask what should be the correct level of voting for society. This is the equivalent of asking what is the critical level of value above which people should vote. Once this level is found, then it can be induced by either having penalties for not voting or adding a poll tax. These penalties and taxes are just transfers and thus do not affect overall welfare.[8]

Let us first examine this problem in example 2. The social surplus in equation (2) becomes

$$(v^*)^{2\alpha} E[V|V < v^*] + (1 - (v^*)^{2\alpha})E[V|V > v^*] - 2(1 - (v^*)^{\alpha})c =$$

[8] Another method would be to employ some criteria for voting that reflects values. For instance, Jefferson felt only the educated should vote (Padover 1952, page 43). In our model, they would have better information and hence higher values for certain candidates. There have also been literacy and property ownership as requirements. While literacy might have been used to disenfranchise certain minority groups, a property ownership requirement for the most part was to restrict voting to groups that had a stake in the country (high values).

$$\frac{1}{1+\alpha}\left(\alpha - 2c - 2\alpha c + v^{\alpha}(\alpha + 2c + 2\alpha c - \alpha v)\right).$$

From the first-order condition, the optimal cutoff $v^* = 2c + \frac{\alpha}{1+\alpha}$ while in equilibrium $v^* = 2c$. Thus, there is overvoting in the voting equilibrium. Hence, in our environment one would never want to impose a fine for not voting, but instead try to induce people to not vote.

Another interpretation is that in equilibrium one votes when $v \geq 2c$. However, this is not taking into the externality imposed upon the other voter (if you win, then the other voter loses). The other voter's expected value is $E[v]$ and the probability of him winning goes down by 1/2 if you vote. Hence, efficiency dictates that one should vote only if $\frac{1}{2}(v - E[v]) \geq c$ or $v \geq 2c + E[v]$. (Note that $E[v] = \frac{\alpha}{1+\alpha}$.) We generalize this intuition in the following proposition.

Proposition 3 *If there are n voters of each type with symmetric distributions of values, then (i) there is overvoting (ii) there should be no fines to encourage voting (no mandatory voting) (iii) there should be a poll tax to discourage voting.*

Proof. A voter's vote will be pivotal in two instances. Case (A): when there is a tie in votes without his vote. Case (B): when the other candidate leads by 1 without his vote. In case (A), say there are m voters that vote for each candidate. Hence, there will be m voters that favor his candidate with values above the cutoff v^* and $n-1-m$ voters that favor his candidate with values below the cutoff. Also, there will be m voters that favor the other candidate with values above the cutoff and $n-m$ voters that favor the other candidate with values below the cutoff. The net difference is one voter that prefers the other candidate with a value below the cutoff. Thus, in case (A), the externality imposed by a voter on others by voting is $E[v|v < v^*]$ (where v^* is the cutoff used by others). We can do a similar calculation for case (B). There, the externality imposed by a voter on others by voting is $E[v|v > v^*]$. In either case, this externality is negative and a voter doesn't take this into account. Thus, there should be less voting. Formalizing this logic, the optimal cutoff should then solve:

$$\sum_{i=0}^{n-1}\left[\begin{array}{c} \binom{n-1}{i}\binom{n}{i}\left(1-F\left(v^*\right)\right)^{2i}F\left(v^*\right)^{2(n-i)-1}\left(v^* - E[v|v<v^*]\right) + \\ \binom{n-1}{i}\binom{n}{i+1}\left(1-F\left(v^*\right)\right)^{2i+1}F\left(v^*\right)^{2(n-i-1)}\left(v^* - E[v|v>v^*]\right) \end{array}\right] = 2c. \tag{3}$$

From this equation, the equilibrium cutoff will then be lower than the optimal cutoff (the LHS will be smaller than $2c$ for all v^* less than the equilibrium cutoff). Since the optimal level of voting is lower than the equilibrium level. It is optimal for a government to charge for voting in order to implement the optimal cutoff. This would be change the c in equation (1) such that the v^* that solves that equation (1) matches the solution to the equation (3). ∎

Börgers (2004) has a model that shows that an equilibrium with costly voting is superior to both mandatory voting (still with voting costs). Our result in this section, agrees with that result but in our framework. The main difference in our continuous model is that we allow for different intensities of preferences. Because of this difference, we see that it is always optimal to charge a poll tax. This includes the case when there is no cost to voting when in fact in the Börgers model such free voting yields the optimal allocation. In our model, such free voting never does and it is optimal to charge a poll tax.

It is not clear that a poll tax is politically viable. This then leads to our previous section where it may be worthwhile to maintain a cost of voting. Doing so eliminates voters with little intensity for their preferred candidate.

4 Conclusion

Since the nineteenth century, political scientists have been in agreement that increasing the franchise will be beneficial to the society (Lakeman and Lambert, 1959, page 19). So over the last century in democracies, the right to vote has been given to most of the adult society and the requirements of registration to vote such as property qualifications have been removed. Social scientists have further asked the question whether or not it makes sense to require people to vote. In addition this requirement to vote also gets around the paradox that since each individual may find his vote negligible will choose not to vote if there is a cost to voting. We show that not only should one not require people to vote but there is a distinct benefit to having some people not vote and increasing the (wasteful) cost to voting may paradoxically be beneficial to society. (Note increasing a wasteful cost of voting may be politically more viable than imposing a poll tax.)

Since this is the first paper to show that increasing the cost to voting can be beneficial, there are many directions of future research where one can expand the result. One direction is to increase the number of alternatives on the ballot to more than two. With committee voting this seems quite logical. Furthermore, once this is done, one can introduce approval voting to ameliorate strategic voting (see Brams and Fishburn, 1978).

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