AMBIGUITY AND PUBLIC GOOD PROVISION IN LARGE SOCIETIES[1]

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5th May 2004

Abstract: In this paper we consider the effect of ambiguity on the private provision of public goods. Equilibrium is shown to exist and be unique. We examine how provision of the public good changes as the size of the population increases. We show that when there is uncertainty there may be less free-riding in large societies.

Keywords: Ambiguity, public goods, Choquet expected utility.

JEL Classification: D81, H41.

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[1] For comments and discussion we would like to thank John Conley, Stephen King and participants in the Public Economic Theory Conference, University of Alabama 1998 and the Econometric Society meetings, Berlin, 1998 and the editor and referees of this journal.

1. Introduction

1.1 The Free Rider Problem

Conventional wisdom argues that voluntary provision of public goods is impossible in large societies due to free rider problems. In the absence of income effects, Nash equilibrium provision of a public good is independent of the size of society. In particular, a society of ten million will have the same level of public goods as a single individual living on a desert island. Despite this, some public goods are voluntarily provided in large societies. For instance, large metropolitan areas often support public broadcasting in this way. This is clearly vastly in excess of the level a single individual would choose. Hence it is difficult to explain the existence of public broadcasters by traditional models.

It has been common to explain the apparent low level of free riding by altruism, see for instance Sefton and Steinberg (1996). In an earlier paper, Eichberger and Kelsey (2002), we advanced an alternative explanation based on Knightian uncertainty (also known as ambiguity). In the present paper, we use a related model to investigate the relationship between population size and public good provision when there is ambiguity.

It is usual to model voluntary provision of a pure public good as a Nash equilibrium of a non-cooperative game. Such models have the following properties.

1. There is a continuum of equilibria. In all of these, total provision of the public good is the same. However the contribution of any given individual will typically vary between one equilibrium and another.
2. Total provision of the public good is independent of the number of individuals in society.

Both of these properties follow from the fact that any other individual's contribution is a perfect substitute for your own. In equilibrium, provision of public goods is determined where the marginal benefit of the public good equals the marginal cost of contributing. This condition is compatible with any allocation of the total contribution over different individuals, hence equilibria are non-unique. If all individuals have the same cost of contributing, total provision will be independent of the number of individuals.

1.2 Ambiguity

By ambiguity we mean uncertainty, which individuals do not analyse in terms of conventional probabilities. Many economists have believed that ambiguity has an important influence on economic decisions. A prominent example is Knight (1921). Following Schmeidler (1989), we model ambiguity by allowing individuals' beliefs to be possibly non-additive. This has the effect that individuals put more weight on bad outcomes than an expected utility maximiser would.

Ambiguity can arise in situations in which people find it difficult or impossible to assign subjective probabilities. They may be forced to make a decision in unfamiliar circumstances or they may believe that the model is an imperfect representation of reality and relevant features are omitted. We believe such situations are not uncommon in economics. In the context of single person decisions, Mukerji (1997) shows that such considerations can lead to preferences of the Schmeidler form. Kelsey and Milne (1999) provide an alternative justification of these preferences as a response to a hidden moral hazard problem. Alternatively it may be the case that individuals are able to formulate probability estimates but have low confidence in them. They react to this by using decision-making procedures, which protect them against errors. Such behaviour could again be compatible with Schmeidler preferences.

Provision of public goods is a game rather than a single person decision. In this context, ambiguity concerns the behaviour of others. In our model a player has a belief about the behaviour of his/her opponents but is not completely confident in this belief. As a result (s)he adopts a cautious mode of behaviour. Most previous literature on games with ambiguity has focused on games with finite strategy spaces.[2] An innovation in the present paper is that we extend the solution concept proposed by Dow and Werlang (1994) to games with infinite strategy spaces.

1.3 Public Goods and Ambiguity

We believe that it is possible that individuals perceive ambiguity when public goods are provided by voluntary donations, since they may regard the contributions of others as ambiguous. In Eichberger and Kelsey (2002) we show that increases in ambiguity will result in higher provision of public goods. Ambiguity concerning the donations of others will cause an individual to have less confidence in their contributions. Thus the anticipated provision of the public good by

[2] See for instance Dow and Werlang (1994), Lo (1996) and Marinacci (2000).

others is lower. If the benefit of a donation is a concave function of total donations, then the perceived marginal benefit of a donation will be increased. This tends to increase individual donations and hence total provision of the public good.

In this paper we present a related model of public good provision when there is ambiguity. We obtain some stronger conclusions. This is achieved by restricting players to believe that their opponents act independently. In addition we study a continuous model, while the model in the earlier paper was discrete. We prove existence and uniqueness of equilibrium and investigate comparative statics. As already noted, without ambiguity public goods models have a continuum of Nash equilibria. If there is any ambiguity our model has a unique equilibrium. In the presence of ambiguity, other people's contributions are no longer a perfect substitute for one's own, since one can have more confidence in one's own contributions. Thus when there is ambiguity, the key factor causing indeterminacy of equilibrium is no longer present and hence it is possible to establish uniqueness.

We investigate how public good provision varies with the size of the population. It is common to argue that free rider problems get worse as the size of community increases. In our model, if there is no ambiguity, total provision of the public good is independent of the size of society. Consequently individual contributions to the public good are a decreasing function of the number of individuals. With ambiguity, total contributions may rise with increases in the size of society. The contribution for a very large society is always greater than that for a small society. (Recall that, without uncertainty, total contributions are independent of the number of individuals.) However the relationship between the size of society and total contribution can be non-monotonic. We find a sufficient condition for monotonicity, which is satisfied by many commonly used functions. Numerical simulations also show that the relationship is usually monotonic, however there are some counter-examples.

The intuition for these results is as follows. Ambiguity causes individuals to overweight bad outcomes, which in this context means others making low contributions. Given that the benefit of the public good is concave, this implies that ambiguity increases the perceived marginal benefit of contributions and hence causes any given individual to donate more. As the size of society increases, the fraction of the public good which is ambiguous increases, hence producing higher donations by the previous argument. These arguments suggest that public good provision should be a strictly increasing function of the number of individuals. However if individuals act independently, ambiguity concerning different individuals can cancel, hence the possibility that the relationship can be non-monotonic. Donations above the Nash equilibrium level could also be explained by other behavioural theories, which cause individuals to over-weight bad outcomes. This point is discussed further in the conclusion.

We do not suggest that ambiguity is present in all public good problems. We interpret our research as saying that when there is ambiguity, free-rider problems will be reduced. Ambiguity is likely to be higher when the public good has a one-shot nature and when under-provision could result in especially bad outcomes. Examples of such public goods would include fund-raising for disaster relief or an appeal to save a work of art for the nation.

Organisation of the Paper In the next section we introduce the Schmeidler model of ambiguity and its application to game theory. In section 3 we present our model of public good provision. The main results on the comparative statics can be found in section 4. The relevant experimental literature is reviewed in Section 5 and Section 6 concludes. Key proofs are grouped in the appendix. The remaining proofs and some relevant background material can be found in Bailey, Eichberger, and Kelsey (2003).

2. CEU Preferences and Games

2.1 Introduction to Choquet Expected Utility

In this paper we model voluntary contributions to the provision of a public good as a non-cooperative game. The traditional solution concept is Nash equilibrium, which assumes that players have expected utility preferences. We model public good provision in the presence of ambiguity by allowing players' beliefs to be non-additive.

Consider a game $\Gamma = \langle N, (S_i)(u_i) : 1 \leqslant i \leqslant n \rangle$ with pure strategy sets S_i for each player and payoff functions $u_i(s_i, s_{-i})$. The notation $s_{-i} = \langle s_1, ..., s_{i-1}, s_{i+1}, ..., s_n \rangle$ indicates a strategy combination for all players except i. The space of all such strategy profiles is denoted by $S_{-i} = \prod_{j \neq i} S_j$. We represent player's beliefs as capacities on S_{-i}. Capacities are non-additive subjective probabilities. Formally, capacities are defined as follows.

Definition 2.1 *A* capacity *on S_{-i} is a real-valued function $\nu : \mathcal{P}(S_{-i}) \rightarrow \mathbb{R}$, (where $\mathcal{P}(S_{-i})$ denotes the set of all subsets of S_{-i}), which satisfies:*

1. $\nu(\emptyset) = 0, \quad \nu(S_{-i}) = 1;$ ***2.*** $A \subseteq B \Rightarrow \nu(A) \leqslant \nu(B)$.

Definition 2.2 *A capacity ν is* convex*, if $\nu(A) + \nu(B) \leqslant \nu(A \cup B) + \nu(A \cap B)$.*

Wakker (2001) argues that convex capacities reflect pessimism. Henceforth we shall assume that all capacities are convex.

Definition 2.3 *A capacity ν is called* simple *if there exists an additive probability distribution π on S_{-i} and a real number $\gamma \in [0,1]$ such that for all events $E \subsetneqq S_{-i}, \nu(E) = \gamma\pi(E)$.*

Simple capacities are contractions of additive probability distributions. It is possible to interpret π as the individual's estimate of the true probability distribution. The parameter γ may be interpreted as the individual's confidence in this estimate. Uncertainty measured by γ, can be distinguished from the likelihood of a particular strategy combination s_{-i} represented by $\pi(s_{-i})$.[3] A general capacity over a set of n states is described by 2^n parameters. This introduces a large number of free parameters. It is desirable to reduce the number of parameters by restricting beliefs. Requiring beliefs to be simple capacities achieves this, since a simple capacity is described by only $n + 1$ parameters.

The expected payoff from a strategy s_i, is determined by the Choquet integral, defined below. Such preferences are known as Choquet expected utility (henceforth CEU).[4]

Definition 2.4 (Choquet Integral) *The* Choquet integral *of u with respect to the capacity ν is defined as*

$$\int u d\nu = \int_0^{\infty} \nu(\{x \in X | u(x) \geqslant t\}) dt + \int_{-\infty}^{0} \left[\nu(\{x \in X | \, u(x) \geqslant t\}) - 1\right] dt.$$

Player i has beliefs about his/her opponents' behaviour, represented by a capacity ν_i on S_{-i}. We shall assume below that players' beliefs about the behaviour of a single opponent j are given by simple capacities on S_j. Except where otherwise stated, we shall assume that beliefs over the set of possible strategy profiles S_{-i} are given by an independent product of such capacities.

3 Simple capacities have been axiomatically justified in Eichberger and Kelsey (1999).
4 They have been axiomatised by Gilboa (1987), Sarin and Wakker (1992) and Schmeidler (1989).

The multiple priors model provides an alternative way to represent ambiguity, see Gilboa and Schmeidler (1989). In multiple priors theory, a decision-maker's beliefs are represented by a set $\mathcal{C}$ of conventional additive probability distributions on S. Preferences over acts have the following representation: $a \succcurlyeq b \Leftrightarrow \min_{p \in \mathcal{C}} \mathbf{E}_p u(a) \geqslant \min_{p \in \mathcal{C}} \mathbf{E}_p u(b)$, where $\mathbf{E}$ denotes a conventional expectation with respect to the additive probability distribution p. This formula can be interpreted as follows. When faced with ambiguity the decision-maker does not know the true probability distribution (if this concept is meaningful). Instead (s)he considers a number of probability distributions to be possible. The decision-maker adopts a cautious response to ambiguity. Each act is evaluated by the least favourable of these probability distributions.

The Proposition in Schmeidler (1989) shows that if beliefs are represented by a convex capacity ν, they can also be represented in the multiple priors form. Under these assumptions, there exists a closed convex set $\mathcal{C}$ of probability distributions on S, such that: $\int u(a(s))d\nu(s) = \min_{p \in \mathcal{C}(\nu)} \mathbf{E}_p u(a)$. The set $\mathcal{C}(\nu)$ is the core of the capacity ν, which is defined by, $\mathcal{C}(\nu) = \{p \in \Delta(S) : \forall B \subseteq S, p(B) \geqslant \nu(B)\}$, where $\Delta(S)$ denotes the set of all (additive) probability distributions on S. Both simple capacities and independent products of simple capacities are convex. Hence the preferences we consider have both a CEU and multiple priors representation.

We aim to state precisely that any individual believes that his/her opponents act independently. In Nash equilibrium this is formally modelled by requiring the mixed strategies of the opponents to be statistically independent. Unfortunately it is not straightforward to apply notions of independence developed for probabilities to capacities. Let ν_1 and ν_2 be capacities defined on sets S_1 and S_2 respectively. We wish to define the independent product ν of ν_1 and ν_2 on $S_1 \times S_2$. Clearly we require that if $A \subset S_1$ and $B \subset S_2$, then $\nu(A \times B) = \nu_1(A)\nu_2(B)$. This defines a product capacity on those subsets of $S_1 \times S_2$ which are Cartesian sets. For additive probabilities there is a unique extension of this to the whole of $S_1 \times S_2$. Unfortunately this is not sufficient to define a product capacity on $S_1 \times S_2$ if either ν_1 or ν_2 is non-additive.[5] Ghirardato (1997) argues in favour of an independent product of capacities known as the Möbius independent product. (For a formal statement see Definition A.3 of Bailey, Eichberger, and Kelsey (2003).) We shall use this notion of independence.

Definition 2.5 *Individual i's beliefs, ν_i, about his/her opponents' behaviour are* independent *if there exist simple capacities $\nu_i^j = \gamma_i^j \pi_i^j$ on S_j for all $j \neq i$, such that $\nu_i = \otimes_{j \neq i} \nu_i^j$, where $\otimes$*

[5] For further discussion see Hendon, Jacobsen, Sloth, and Tranæs (1993) and Ghirardato (1997).

denotes the Möbius independent product.

Assumption 2.1 (Independence) *All players have independent beliefs.*

Assumption 2.1 states formally that each player believes that his/her opponents act independently. The implications of this assumption are illustrated by Example 1 below.

Assumption 2.2 (Consistency) *We shall assume that player have consistent beliefs in the sense that for all* $k \neq i, j$, $\nu_i^k = \nu_j^k$.

Consistency implies that any two individuals will have the same beliefs over any third player. This assumption has been commonly used in game theory and we believe this extension to games with ambiguity is uncontroversial. If beliefs of all players are independent and consistent, then there exist beliefs ν^k on S_k for all players $k \in I$, such that $\nu_i(E) = \otimes_{j \neq i} \nu^j$ for $1 \leqslant i \leqslant n$.[6]

Assumption 2.3 *For* $0 \leqslant k \leqslant n$, ν^k *is a simple capacity.*

This does not imply that the beliefs of individual i, ν_i are a simple capacity, since the independent product of simple capacities is not itself a simple capacity. We view this as a simplifying assumption, analogous to assuming that a production function is Cobb-Douglas.

Assumption 2.4 *All of the capacities* ν^k *have the same degree of ambiguity* γ.

Given consistency, all players have the same beliefs concerning the behaviour of any given opponent. This assumption says that there is the same ambiguity concerning the behaviour of all players. Assumptions 2.3 and 2.4 are simplifying assumptions. A goal of the present paper is to investigate the effect of adding additional individuals to society. One needs to specify the ambiguity concerning the behaviour of the new individuals. The most natural assumption is that it is the same as the ambiguity concerning the behaviour of the current population.

Definition 2.6 *Let* $\nu^k = \otimes_{j \neq k} \gamma \pi_j$ *be an independent product of simple capacities, where* $0 < \gamma < 1$ *and* π_j *has finite support for* $1 \leq j \leq n$. *We define the* support *of* ν^k, $\operatorname{supp} \nu^k$, *by* $\operatorname{supp} \nu^k = \times_{j \neq k} \operatorname{supp} \pi_j$.[7]

[6] A related model where beliefs are not restricted in this way is studied in Eichberger and Kelsey (2002).

[7] This is justified by Lemma A.1 of Bailey, Eichberger, and Kelsey (2003), which implies that for a finite space S, the support of an independent product of simple capacities has this form.

2.2 Equilibrium in Games

We wish to develop a theory of public good provision with ambiguity. Hence it is necessary to modify the concept of Nash equilibrium to allow for ambiguity. A possible interpretation of a mixed Nash equilibrium is that it is an equilibrium in beliefs. Let π^1 be the equilibrium probability distribution over player 1's strategy space. Then π^1 describes the subjective beliefs of 1's opponents about what (s)he will do, rather than the objective randomisation used by player 1. We extend the concept of an equilibrium in beliefs by allowing these beliefs to be non-additive. This enables us to define an equilibrium with ambiguity.

We model an equilibrium with ambiguity as a situation, where players believe that their opponents will choose best responses, however they are not completely confident in this belief. The definition of an *equilibrium with ambiguity* below states this formally. It assumes maximising behaviour of all players given their beliefs and their confidence in these beliefs. It is consistent in the sense that no player believes his/her opponents will choose actions that are not best responses.

The definition of equilibrium requires beliefs to be an independent product of simple capacities. Otherwise this definition is analogous to that given for games with finite strategy spaces in Dow and Werlang (1994).

Definition 2.7 *An* equilibrium with ambiguity *of the public goods model is an n-tuple $\langle \mu^1, ..., \mu^n \rangle$ where μ^j is a simple capacity on S^j for $1 \leqslant j \leqslant n$ with the property that, if we define $\nu_i = \bigotimes_{j \neq i} \mu^j$ then*

$$\operatorname{supp} \nu_i \subseteq \underset{j \neq i}{\times} \operatorname{argmax}_{s_j \in S_j} \int u_j \left(s_j, s_{-j}\right) d\nu_j \left(s_{-j}\right).$$

The support represents the profiles of strategies that an individual believes his/her opponents will play. However due to ambiguity (s)he is not completely confident in this belief and hence behaves cautiously. This definition is similar in spirit to Nash equilibrium, however the consistency requirement here is weaker. Equilibria under ambiguity are equilibria in beliefs. In general, they will not specify exactly which pure strategy will be chosen. Equilibrium beliefs determine the strategy which will be played, if the support of the belief for each player consists of a single strategy. This will be the case in all equilibria of the public goods model. Otherwise, any strategy combination contained in the Cartesian product of the supports of the players' beliefs may be played in equilibrium.[8]

3. Public Good Provision

We have chosen a relatively simple model. This enables us to focus on the effects of ambiguity on public good provision.

3.1 The Model

There are two goods, a public good y and a private good x. Society consists of n individuals. Each has utility function $u(x_i, y) = w(y) - x_i d$, where y denotes the level of public good provision and x_i denotes the individual's contribution to the public good, (in terms of private good). Contributions are restricted to lie in the set $X_i = \{x_i : 0 \leqslant x_i \leqslant m^*\}$. Individuals are assumed to have a sufficiently large endowment that they are able to contribute m^*. The level of public good provision is given by the production function: $y = F(\sum_{i=1}^{n} x_i)$.

Assumption 3.1 (Concavity) *The function G, defined by $G : [0, nm^*] \to \mathbb{R}$ by $G(x) = w(F(x))$is continuously differentiable, strictly concave and $G(0) = 0$.*

The function G, measures the benefit, in utility terms, of contributions to the public good. Provided w and F are concave, it will be strictly concave if either there is diminishing marginal utility of the public good or there are decreasing returns to scale in production. The implications of a non-concave G are considered in Eichberger and Kelsey (2002). The following example illustrates the assumption of independence in the public goods model.

Example 1 *Consider a society of three individuals. Suppose that individuals 2 and 3 each use the pure strategy of contributing m units to the public good. Assume that individual 1's beliefs about the behaviour of any individual player are represented by a simple capacity with degree of ambiguity γ. Assume for simplicity that player 1 makes no contribution to the public good. If player 1 believes that players 2 and 3 act independently then his/her Choquet expected utility is:*

$$(1-\gamma)^2 G(2m) + 2\gamma(1-\gamma) G(m) + \gamma^2 G(0). \tag{1}$$

For capacities the two marginal distributions do not uniquely determine a product distribution. Hence if independence is not assumed, player 1's Choquet expected utility could take a number of values depending on which product capacity is used. The following would be a possible value for player 1's (Choquet) expected utility:

$$(1-\gamma) G(2m) + \gamma G(0). \tag{2}$$

[8] Dow and Werlang (1994) have proposed a solution concept for games with ambiguity. It is not applicable in the current context since it requires the strategy sets to be finite. However we justify our solution concept by showing in Bailey, Eichberger, and Kelsey (2003) that a profile of beliefs is an equilibrium, in the above sense, if and only if it is a limit of Dow-Werlang equilibria for a modified game where the strategy set is finite.

This says that player 1's beliefs about player 2 are wrong precisely when player 1's beliefs about player 3 are wrong.

Equation (2) is somewhat counter-intuitive. It says that although player 1 regards player 2's behaviour as ambiguous and regards player 3's behaviour as ambiguous, (s)he is confident that it is impossible that player 2 will contribute but player 3 will not. When modelling ambiguity it is important to decide what is viewed as ambiguous. It could be that a player in a game perceives ambiguity about whether or not his/her opponents correlate their strategies. In such circumstances the independence assumption would not be appropriate. However we do not believe that such issues are relevant in the present paper and hence the independence assumption is justified.

3.2 Existence of Equilibrium

We assume players use pure strategies and do not randomise. There is ambiguity about the behaviour of opponents. This is represented by non-additive beliefs over their strategy sets. The next result demonstrates the existence of an equilibrium, which satisfies our assumptions.

Theorem 3.1 (Existence of equilibrium) *For any given degree of ambiguity, γ, there is an equilibrium with ambiguity, which satisfies Assumptions 2.1, 2.2, 2.3 and 2.4, in which ν^j has degree of ambiguity γ, for $1 \leqslant i, j \leqslant n$.*

Definition 3.1 *We say that an equilibrium with ambiguity is* pure *if* $\operatorname{supp} \nu_i$ *contains a single element of S_{-i} for $1 \leqslant i \leqslant n$. An equilibrium which is not pure is said to be* mixed.

Since we assume that players do not randomise, any mixing is subjective rather than objective. The following result shows that, in all equilibria with ambiguity, each individual has a unique best response. This is due to strict concavity of G, which implies that best responses are unique.

Proposition 3.1 *Under Assumptions 2.1, 2.2, 2.3 and 2.4, all equilibria with ambiguity are pure.*

Definition 3.2 *An equilibrium under ambiguity $\langle \mu^1, ..., \mu^n \rangle$ is said to be* symmetric *if $\mu^j = \mu^i$ for $1 \leqslant i, j \leqslant n$.*

A symmetric game always has a symmetric equilibrium. We shall demonstrate that the equilibrium of the public goods model is unique. Hence we only need to consider symmetric equilibria. There are a continuum of conventional Nash equilibria, since each individual views his/her rival's contribution to the public good as a perfect substitute for his/her own. Thus for any interior equilibrium, there is another equilibrium where individual i contributes δ units more to the public good and individual j contributes δ units less. Since this holds for all δ in an open set there are a continuum of equilibria. When there is ambiguity concerning the contributions of others, a given individual does not regard the contributions of others as a perfect substitute for his/her own. The individual does not have complete confidence in the contributions of others hence they yield a lower (Choquet) expected utility. The previous argument does not apply and we are able to establish uniqueness of equilibrium.

Theorem 3.2 (Uniqueness of Equilibrium) *If Assumptions 2.1, 2.2, 2.3 and 2.4 are satisfied then, for any given degree of ambiguity γ, the public goods model has a unique equilibrium, which is symmetric.*

Notation 3.1 Let $z_j^{(n-1)} = \Pr(Z_{n-1} = j)$, where Z_{j-1} is a random variable with the Binomial distribution, parameters $n-1$ and γ. Hence

$$z_j^{(n-1)} \equiv \begin{cases} \binom{n-1}{j}\gamma^j(1-\gamma)^{n-1-j}, & (0 \leqslant j \leqslant n-1), \\ 0, & (n \leqslant j). \end{cases}$$

The following proposition gives the first order condition for equilibrium. It is useful since it enables us to apply conventional calculus techniques.[9]

Proposition 3.2 *The equilibrium of the public goods model is characterised by the following equation:*

$$\sum_{j=0}^{n-1} z_j^{(n-1)} G'\left(m(j+1)\right) = d. \tag{3}$$

4. Comparative Statics

4.1 Number of Individuals

We shall now investigate how increases in the population affect both individual and total contributions in equilibrium.

4.1.1 Example

Equation (3) implicitly defines individual contributions, m, as a function of the total number

9 For a proof see Proposition 3.3 of Bailey, Eichberger, and Kelsey (2003).

of individuals n. Below is an example, which we are able to solve analytically.

Example 2 (G Quadratic) *Assume that* G *is quadratic:* $G(t) = a + bt - \frac{1}{2}ct^2$, *hence* $G'(t) = b - ct$. *In this case, the first order condition is:* $\sum_{j=0}^{n-1} z_j^{(n-1)} (b - c(jm + m)) = d$. *Since the binomial coefficients sum to 1 and the mean of the binomial distribution is* $\gamma(n-1)$, *this may be simplified to* $b - cm - cm\gamma(n-1) = d$. *Thus* $m = \frac{b-d}{c(1-\gamma+n\gamma)}$. *Total contributions* $T(n)$ *are given by:*

$$T(n) = \frac{b-d}{c\left(\gamma + \frac{1-\gamma}{n}\right)}.$$

In this example we see that individual contributions fall, while total contributions rise, when the number of individuals increases. (An extra pair of hands always helps. This is true however uncertainty is modelled.) In a similar model without ambiguity, total contributions do not change with the number of individuals, hence individual contributions are smaller in a larger society. (This may be seen by setting $\gamma = 1$ in the above formulae.) More ambiguity (decreasing γ) increases provision of the public good. In section 4.3 we show that this holds generally.

4.1.2 General Analysis

We shall now proceed to the comparative statics of changing the population size in the general case. The next proposition shows that, as in the case of no ambiguity, individual contributions decrease as the number of individuals increase.

Proposition 4.1 *If* $\gamma > 0$, m *is a decreasing function of* n.

The more individuals there are, the higher you expect their contributions to be. If there are decreasing returns, this lowers the marginal benefit you expect to get from your own donations and therefore reduces your contribution. This logic holds even when there is ambiguity (i.e. beliefs are non-additive). The following proposition shows that total contributions, $T(n) = nm$, are greater in a very large society than in a small society.

Proposition 4.2 *For all* $\gamma : 0 \leqslant \gamma < 1, T(1) < \lim_{n\to\infty} T(n)$.

Proof. Equation (3) may be rewritten as: $\sum_{j=0}^{n-1} z_j^{(n-1)} G'\left((j+1)\frac{T}{n}\right) = d$. Hence if we define $\bar{T} = \lim_{n\to\infty} T(n)$ then by Lemma C.3 of Bailey, Eichberger, and Kelsey (2003), $G'(\gamma\bar{T}) = d$. Since $T(1)$ satisfies $G'(T(1)) = d$ and G' is decreasing, the result follows. ■

Note that $T(1)$ is independent of γ, while $\bar{T} = G'(\gamma\bar{T})$ is a decreasing function of γ.

Hence the more ambiguity there is, the greater the increase in provision as the size of society increases.

While total provision is greater in a very large society, the relationship between population size and public good provision may be non-monotonic. This is demonstrated by the numerical results below. The following condition guarantees a monotonic relationship between population size and public good provision. It is satisfied by many common functions, such as the power and logarithmic functions.

Assumption 4.1 *The function G is said to satisfy* Condition A *if $H(t) \equiv tG'(t)$ is concave on* $(0, \infty)$.

Proposition 4.3 *Provided that Condition A is satisfied, total provision of the public good is an increasing function of population size.*[10]

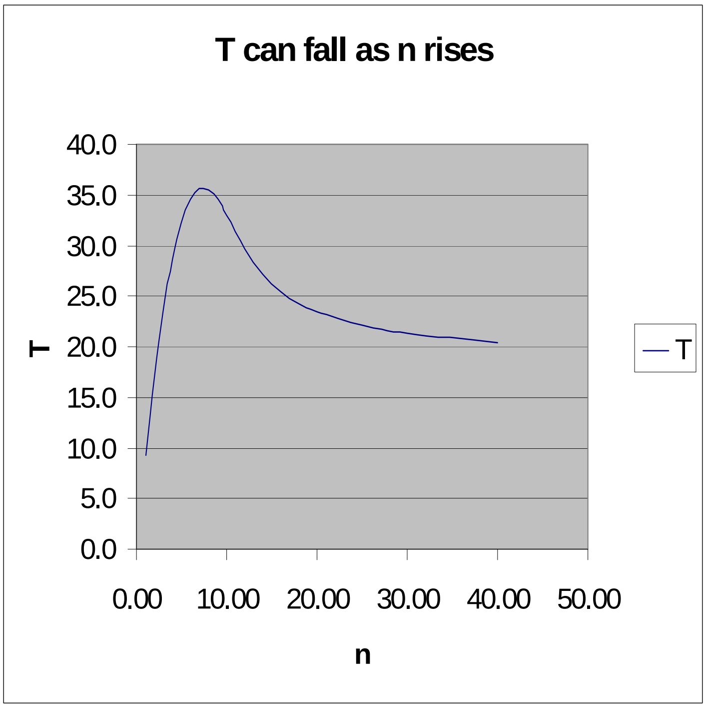

Figure 1:

4.2 Numerical Results

Application of condition A shows that for large classes of G functions, including the quadratic and logarithmic forms discussed earlier, as well as the CRRA form $G(t) = \frac{t^{1-r}}{(1-r)}$, for $0 \leqslant r < 1$, $T(n) = nm(n)$ is increasing in n. However this need not be true where condition A is violated.

[10] The terms 'increasing' and 'decreasing' are used in the non-strict sense.

An example occurs when G has the CARA form $G(t) = -\frac{e^{-at}}{a}$, $a > 0$, so marginal benefit is $G'(t) = e^{-at}$. This utility function does *not* satisfy Condition A, that $H(t) \equiv tG'(t) = te^{-at}$ should be everywhere concave. Thus we find $H'(t) = -ate^{-at} + e^{-at} = e^{-at}(1 - at)$; then $H''(t) = -a(1 - at)e^{-at} - ae^{-at} = a\,(at - 2)\,e^{-at}$, which fails to be negative for $t > \frac{2}{a}$.

Accordingly, Proposition 4.3 cannot, in this case, guarantee that total provision T of the public good is an increasing function of size n. We can show that T does eventually fall. Equation (3) becomes,

$$d = \sum_{j=0}^{n-1} \binom{n-1}{j} \gamma^j (1-\gamma)^{n-1-j} e^{-a(j+1)m} = e^{-am} \sum_{j=0}^{n-1} \binom{n-1}{j} \left(\gamma e^{-am}\right)^j (1-\gamma)^{n-1-j}$$
$$= e^{-am} \left(1 - \gamma + \gamma e^{-am}\right)^{n-1}.$$

This allows us to express n as an explicit function of m (though not vice versa): $n = 1 + \frac{\log d + am}{\log \Phi}$, where $\Phi(m) \equiv 1 - \gamma + \gamma e^{-am}$. Defining $T \equiv mn$, we are interested in $\frac{dT}{dn} = m + \frac{n}{dn/dm}$, where $\frac{dn}{dm} = \frac{a}{\log \Phi} - \frac{\Phi'(\log d + am)}{\Phi (\log \Phi)^2} = \frac{1}{\log \Phi}\left(a - (n-1)\frac{\Phi'}{\Phi}\right)$. Figure 1 shows a graph, where $T(n)$ is an initially increasing, but eventually decreasing, function of n.

4.3 Uncertainty

We can investigate the comparative statics of an increase in ambiguity by varying the parameter γ. This can be interpreted as varying the level of uncertainty, while holding other factors fixed.

Proposition 4.4 *A decrease in γ results in an increase in individual and hence total contributions to the public good.*

This result can be interpreted as saying that an exogenous increase in uncertainty will cause contributions to rise. A related result can be found in Eichberger and Kelsey (2002).

5. Experimental Evidence

There has been a large number of experimental studies on public goods. In general these have found that free riding is much less than predicted by conventional models. Previously these results have been explained by altruism.

A relevant experiment is reported by Andreoni (1988), who divides subjects into two groups called partners and strangers. The "partners" group plays a public goods game ten times with the same group of five players, i.e. partners play a conventional repeated game. The "strangers" group plays ten rounds of the same game, except any given subject is playing the game with different opponents in each round. In other words, strangers play a one-shot game

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ten times. The hypothesis being tested is that subjects build a reputation for cooperation in the repeated game along the lines described by Kreps, Milgrom, Roberts, and Wilson (1982). If this were correct, it would imply that partners are more likely to cooperate, since they have more scope for reputation building. In fact it was found that strangers were more likely to cooperate. This is compatible with our model. It seems reasonable to assume there is more ambiguity in the strangers group, since players were not able to get used to the behaviour of their opponents. In contrast, it does not seem likely that altruistic feelings should be stronger towards strangers than partners.[11]

Isaac, Walker, and Williams (1994) report experimental tests of the effect of changing population size on the provision of public goods. They find that free riding can be lower in larger societies. Our theory can explain these results in terms of ambiguity. As far as we are aware, there are few alternative explanations. An argument based on altruism has been proposed. If there are more other subjects, there are more people to feel altruistic towards, which causes subjects to donate more. However it is not clear that this is the best way to model altruism. It could alternatively be argued that a larger society is more anonymous. Hence people will empathise less with other members of society and make lower donations.

Experimental tests of other games have not produced very strong evidence of altruism. For instance in the ultimatum bargaining game there is a fixed sum of money to be divided between two players. Player 1 offers a proposed division which player 2 may accept or reject. In the event of rejection both players receive zero. In the sub-game perfect equilibrium, player 1 offers zero or close to zero. In experiments on the ultimatum game, however, player 1 often makes a much larger offer to player 2. (Half of the money is not uncommon.) This was originally explained by altruism.

The dictator and impunity games are similar to the ultimatum game, except that a rejection by player 2 does not reduce player 1's payoff. Experimental evidence suggests that subjects are not particularly altruistic in these games.[12] Many player 1's offered the least possible amount to player 2. This suggests that the high offers to player 2 in the ultimatum game are not due to altruism but to fear that player 2 might reject a low offer. (Many player

[11] Although Croson (1996) failed to replicate Andreoni's result, we believe that the subsequent experimental literature supports our conclusion that this result is due to ambiguity. A survey of this literature can be found in Andreoni and Croson (2004).

[12] Experiments on these games have been conducted by Bolton and Zwick (1995), Forsythe, Horowitz, Savin, and Sefton (1994) and Hofman, McCabe, Shacat, and Smith (1994).

2's do indeed reject low offers so such fears are not unjustified.)

In summary, if one looks at the combined evidence from experiments on public goods and bargaining it seems hard to support the view that subjects are motivated by altruism. For the above reasons, we believe that the available evidence favours ambiguity. However further experimental work will be needed.

6. Conclusion

6.1 Summary

In this paper we have shown that ambiguity can increase provision of public goods. Moreover increasing population size may increase provision of public goods in the presence of ambiguity. This provides an explanation of why appeals for voluntary donations for public goods in large societies sometimes succeed despite the free rider problem. (The fact that voluntary donations are often significantly above the theoretical prediction has been noticed as anomalous by previous researchers, see for instance Isaac, Walker, and Thomas (1984).) The main alternative explanation is altruism. We are not suggesting that voluntary donations are never influenced by altruism, merely that much of the available evidence can be explained by ambiguity. A more realistic account would allow for both altruism and ambiguity. It could be that the two effects are complementary and ambiguity acts to reinforce altruism. We believe that we have found a novel explanation of the low observed levels of free riding, which is worthy of further study. In particular experimental testing of these results may be desirable.

6.2 Extensions

We shall now consider two possible extensions of the basic model. Firstly a number of interesting issues concerning public good provision can only be studied in a dynamic context. To study these issues it would be useful to have a dynamic extension of the model in the present paper. Dynamic models present a new problem. Many non-expected utility theories fail to be dynamically consistent in multi period models. However these problems do not arise with the model considered in the present paper, since Epstein and Schneider (2003) have shown that the multiple priors model can be given a dynamically consistent extension know as recursive multiple priors.

Secondly there are some other behavioural theories which may have similar implications for public good provision. Consider two individuals, person 1 and person 2. Suppose person i can either get a high benefit from the public good or a low benefit from the public good, depending on the realisation of some exogenous random variable.[13] Suppose that individuals are pessimistic in the sense that they over-estimate the chance that their opponent gets a low benefit from the public good. Then, compared to Nash equilibrium, player 1 will anticipate a lower donation by player 2. If the production function for public goods is concave this will raise his/her marginal benefit of contributing and hence cause him/her to donate more. This will tend to increase the equilibrium contribution.

Such pessimism could be caused by ambiguity, however there are a number of other plausible behavioural explanations, for instance disappointment Gul (1991) or rank dependent expected utility, Quiggin (1982), Yaari (1987) (provided the probability transformation function is convex). This suggests there is considerable scope for further research on behavioural theories of public good provision. Although related, the model in the present paper based on ambiguity is distinct, since it is not an essential part of our explanation that the utility function can be influenced by random factors..

We believe the theory based on ambiguity is intuitively appealing since uncertainty about provision is frequently emphasised in appeals for donations. Voluntary provisions of public goods is often effective in situations where there is uncertainty, e.g. fund-raising for disaster relief or an appeal to save a work of art for the nation.

Appendix A. Selected Proofs

This appendix contains proofs of selected results. The proofs of other results can be found in Bailey, Eichberger, and Kelsey (2003).

Proof of Theorem 3.1 **Existence of equilibrium** For $1 \leqslant j \leqslant n$ define $\psi_j(x_1, ..., x_n)$ by:

$$\psi_j(x_1, ..., x_n) = \sum_{r=0}^{n-1} \gamma^r (1-\gamma)^{n-1-r} \sum_{\{I \subset N \backslash j : |I| = r\}} G\left(x_j + \sum_{i \in I} \hat{x}_i\right) - x_j d. \tag{4}$$

Consider the game Γ, where player j has strategy set $[0, m^*]$, and utility function ψ_j. Note that ψ_j is continuous, symmetric and strictly concave in x_j. By Nash's theorem for symmetric games (see Moulin (1986) p.115), there exists $\hat{x}$ such that the profile $\langle \hat{x}, ..., \hat{x} \rangle$ constitutes a

[13] This argument requires that both total and marginal benefit be greater for the high type.

symmetric Nash equilibrium of Γ.

Consider individual j, define a simple capacity μ^j on S_j by $\mu^j(S_j) = 1, \mu^j(A) = \gamma$, if $\hat{x} \in A \subsetneqq S_j, \mu^j(A) = 0$ otherwise. Let $\nu_i = \otimes_{j \neq i} \mu^j$. We assert that $\langle \nu_1, ..., \nu_n \rangle$ is an equilibrium under ambiguity of the public goods model.

The support of ν_i is $\langle \hat{x}, ..., \hat{x} \rangle$. Thus it is sufficient to show that $\hat{x}$ is a best response for individual i, for $1 \leqslant i \leqslant n$. Suppose not. Then there exists j and $\bar{x}$ such that:

$$\int G(\bar{x} + x) \, d\nu_j(x) - \bar{x}d > \int G(\hat{x} + x) \, d\nu_j(x) - \hat{x}d. \tag{5}$$

Let $V^i(x_1)$ denote i's (Choquet) expected utility given that (s)he contributes x_i units to the public good. We may show:

$$V^i(x_i) = \sum_{r=0}^{n-1} z_r^{(n-1)} G(x_i + r\hat{x}) - dx_i. \tag{6}$$

If (5) is satisfied $V^j(\bar{x}) > V^j(\hat{x})$. The similarity of equations (4) and (6) shows that $\psi_j(\bar{x}, x_{-j}) > \psi_j(\hat{x}, x_{-j})$, which contradicts the fact that $\langle \hat{x}, ..., \hat{x} \rangle$ is a Nash equilibrium of the game Γ. This completes the proof. ■

Proof of Theorem 3.2 (Uniqueness) Let $\hat{x}_i$ be the amount contributed in equilibrium by individual i. Consider individual 1, the first order condition for maximising his/her utility may be rearranged to yield:

$$(1-\gamma)^{n-1} G'(\hat{x}_1) + \sum_{r=1}^{n-1} \gamma^r (1-\gamma)^{n-1-r} \sum_{\{2 \in I \subset N : |I| = r\}} G'\left(\hat{x}_1 + \sum_{i \in I} \hat{x}_i\right)$$
$$+ \sum_{r=1}^{n-1} \gamma^r (1-\gamma)^{n-1-r} \sum_{\{2 \notin I \subset N : |I| = r\}} G'\left(\hat{x}_1 + \sum_{i \in I} \hat{x}_i\right) = d.$$

Hence $(1-\gamma)^{n-1} G'(\hat{x}_1) + \phi(\hat{x}_1 + \hat{x}_2) + \psi(\hat{x}_1) = d$, where
$\psi(x_1) = \sum_{r=1}^{n-1} \gamma^r (1-\gamma)^{n-1-r} \sum_{\{2 \notin I \subset N : |I| = r\}} G'\left(\hat{x}_1 + \sum_{i \in I} \hat{x}_i\right)$ and
$\phi(x_1 + x_2) = \sum_{r=1}^{n-1} \gamma^r (1-\gamma)^{n-1-r} \sum_{\{2 \in I \subset N : |I| = r\}} G'\left(x_1 + x_2 + \sum_{i \in I \setminus 2} \hat{x}_i\right)$. It can be seen that ϕ and ψ are strictly decreasing functions.

By similar reasoning the first order condition for individual 2's utility maximisation problem is $(1-\gamma)^{n-1} G'(\hat{x}_2) + \phi(\hat{x}_1 + \hat{x}_2) + \psi(\hat{x}_2) = d$. Since ϕ, ψ and G' are strictly decreasing, $\hat{x}_1 = \hat{x}_2$. By similar reasoning we may show that $\hat{x}_i = \hat{x}_j$ for any $i, j : 1 \leqslant i, j \leqslant n$. This establishes uniqueness of equilibrium. ■

Proof of Proposition 4.4 The following equation is satisfied in equilibrium:

$$V(m,n,\gamma) = G'(m(b+1)) - \sum_{j=0}^{b-1} \Pr(Z_{n-1} \leqslant j) \left[G'(m(j+2)) - G'(m(j+1))\right],$$

where Z_{n-1} is a random variable with the Binomial distribution, parameters $n-1$ and γ. Since G is concave $G'(m(j+2)) - G'(m(j+1)) \leqslant 0$. A decrease in γ will reduce $\Pr(Z_{n-1} \leqslant j)$ and hence $V(m,n,\gamma)$. Since $V(m,n,\gamma)$ is decreasing in m, m must also decrease for the equilibrium condition, $V(m,n,\gamma) = d$, to be satisfied. ■

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