Investment, Uncertainty and Pre-emption*

Robin Mason
University of Exeter and CEPR

Helen Weeds
University of Essex and CEPR

7 September 2009

Abstract

This paper examines irreversible investment in a project with uncertain returns, when there is an advantage to being the first to invest and externalities to investing when others also do so. We show that the possibility of pre-emption can have significant qualitative and quantitative effects on the relationship between uncertainty and investment. In a single-agent real options model, the trigger threshold for investment increases without bound as uncertainty grows. In contrast, the investment trigger of a leader faced with pre-emption is bounded above as uncertainty increases. In fact, we show that under certain parameter values, greater uncertainty can lead the leader to invest earlier. These findings reinforce the importance of extending real options analysis to include strategic interactions between players. Applications to industry situations are also discussed.

Keywords: Real options, investment valuation, uncertainty, pre-emption.

JEL classification: D81, G31, L20.

*We are grateful to In Ho Lee, David Newbery, Patrick Rey, Jean Tirole and Juuso Välimäki for comments. We thank also the editor and a referee for many helpful comments. Robin Mason acknowledges financial support from the ESRC Research Fellowship Award R000271265. `mason.robin@gmail.com`; `helen.weeds@gmail.com`.

1 Introduction

The literature on irreversible investment under uncertainty teaches three major lessons. First, the net present value (NPV) rule for investment is generally incorrect, since it considers only a now-or-never decision and fails to appreciate that the ability to delay investment may be important when future return is uncertain. Secondly, an option value is created by the fact that the return is bounded below by the payoff from not investing; the effect of this option value is to raise the threshold for a project to be undertaken, delaying investment relative to the NPV rule. Finally, the greater the degree of uncertainty, the higher this trigger: an increase in uncertainty increases the upside potential from investment, and so increases the value of the investment option.

The early literature on the 'real options' approach analyses investment decisions for a single agent in isolation. In many real world cases, however, investment takes place in a more competitive environment in which there are externalities and strategic interactions between investing agents. The purpose of this paper is to demonstrate that such interactions can have important consequences for irreversible, uncertain investments, with effects that run counter to the standard results given above. In particular, we study the role of uncertainty in an environment where the timing of investment is affected by the threat of pre-emption.

We analyse irreversible investment in a project with uncertain returns in a dynamic two-player model, with a general specification. Two types of strategic interactions are considered. The first is pre-emption: when there is some advantage to being the first to undertake an investment, there will be competition to be the first. In this situation, any benefit from delaying investment due to real option effects has to be balanced against the loss from being pre-empted. The second interaction arises when the value of an investment depends on the number of agents who have also invested. The interaction may affect value negatively: e.g., if it arises through a competitive effect; or it may have a positive effect, if there are complementarities between the agents' actions such as network externalities or demand expansion. In both cases, the value and timing of an agent's investment is influenced by the investment decisions of others.

The contribution of this paper is to investigate the relationship between investment and uncertainty when the threat of pre-emption is present. We show that under certain conditions, investment behaviour is significantly altered by the prospect of pre-emption. In a single-agent setting with irreversible investment under uncertainty, the degree of uncertainty has a major quantitative effect on investment behaviour. With a single state variable driving investment returns, the level at which the single agent invests increases without bound as the degree of uncertainty increases.

In contrast, we show that with two investors and the possibility of pre-emption, the effect can be much more muted, depending on the type of equilibrium that occurs. Two types are possible: either the agents invest sequentially (i.e., the 'leader' invests early while the 'follower' invests late), or they invest simultaneously. If equilibrium investment is always sequential, then the leader's investment point is insensitive, in relative terms, to the degree of uncertainty. Its investment point is bounded above as uncertainty increases; indeed, we show that with certain parameter values, greater uncertainty can lead the leader to invest *earlier*. (The follower, who, once the leader has invested, acts like a single agent, behaves in the standard way.)

If, however, equilibrium investment is simultaneous, then the standard results apply: the common investment point increases with uncertainty, and is unbounded as uncertainty grows. (Note, however, that there is always an equilibrium in which investment occurs sequentially.) We give the necessary and sufficient condition for simultaneous investment to be possible in equilibrium; and we show how this condition depends on the degree of uncertainty.

In summary: we show that strategic interactions can give rise to significant qualitative and quantitative effects that are omitted from the standard real options analysis of investment under uncertainty.

Two general strands of literature are related to this paper. Real options models have been used to explain delay and hysteresis arising in a wide range of contexts. McDonald and Siegel (1986) and Pindyck (1988) consider irreversible investment opportunities available to a single agent. Dixit (1989) and Dixit (1991) analyse product market entry and exit in monopolistic and perfectly competitive settings respectively. The second strand of literature concerns timing games of entry or exit in a deterministic setting. There are several types of paper within this strand. Papers analysing pre-emption games include Fudenberg, Gilbert, Stiglitz, and Tirole

(1983), Fudenberg and Tirole (1985), Katz and Shapiro (1987) and Lippman and Mamer (1993) . Wars of attrition have been modelled by e.g., Fudenberg and Tirole (1986).

A number of real options models incorporating strategic interactions now exist. Smets (1991) examines irreversible market entry in a duopoly facing stochastic demand. Simultaneous investment may arise only when the leadership role is exogenously pre-assigned. Consequently, he does not consider fully the pre-emption externality. Perotti and Rossetto (2001) analyse investments by a "platform" firm and a rival entrant. The two are distinguished by the cost of investment, which is lower for the platform. Consequently, pre-emption plays no part in their analysis, while it is central in ours. Weeds (2002) presents a model in which two firms may invest in competing research projects with uncertain returns. She does not impose an asymmetry between the firms, but allows the leader to emerge endogenously. She does not, however, include more general externalities. Other papers combining real options with game theory include Boyer, Lasserre, Mariotti, and Moreaux (2004), Huisman and Kort (1999), Lambrecht and Perraudin (2003) and Pawlina and Kort (2006); these, however, do not generate the comparative static result we find. The general specification of our model encompasses several of these contributions. Hoppe (2000) analyses a timing game of new technology investment in an uncertain environment. She considers second, rather than first, mover advantages and models uncertainty in a different way from our paper.

In a two-period model, Kulatilaka and Perotti (1998) find that greater uncertainty over market demand may increase cost-reducing investment undertaken in the first period. Their model is quite different from ours: there is an exogenous asymmetry between the firms—only firm 1 holds a strategic investment opportunity in the first period—and this firm exercises a subsequent option (over production) in the second period. Although their result has a superficial similarity to ours, it is driven by the strategic effect of first period investment in reducing the competitor's output in the second period (à la Cournot), or deterring entry altogether (as in Dixit (1980)), combined with optionality at the second stage. Since the first period investment is available to a single firm, there is no competition in exercising the option. In this paper, by contrast, our result is due to the effect of uncertainty on the equilibrium outcome of the timing game between the players. Dixit and Pindyck (1994) describe situations in which uncertainty can speed up investment, because investment itself reveals information about costs. We show that even in the absence of this 'shadow value', investment may be speeded up by uncertainty.

The rest of the paper is structured as follows. In the next section, we set up the model. Section 3 analyses the equilibria of the model, as well as deriving various benchmarks that can be used to assess the effect of combining the threat of pre-emption with the real option incentive to delay. Section 4 assesses when real options are important, asking: when are competitive interactions so strong that they undermine option effects, transforming the relationship between uncertainty and investment? Section 5 discusses the applicability of real options to various industry situations in light of our analysis. Section 6 concludes; the appendix contains lengthier derivations.

2 The model

This section develops a reduced-form model to capture the three effects that are the focus of this paper: (i) uncertainty, irreversibility and the possibility of delay in investment; (ii) investment externalities, where the return to investment depends on the number and sequence of investors; and (iii) pre-emption, where the early investor has an advantage, which may also be persistent.

Two risk neutral agents, labelled $i \in \{1, 2\}$ each can invest in a project. There is a cost $K > 0$ to doing so, which is the same for both agents. Investment is irreversible (the cost K is entirely sunk) and can be delayed indefinitely. Time is continuous and labelled by $t \in [0, \infty)$. The timing of investment is the main concern of the analysis. Investment by the two agents may occur sequentially—that is, the two agents invest at distinctly different times—or simultaneously.

Consider first the outcome when the agents invest sequentially. Call the first investor the 'leader' and the second investor the 'follower'. The leader's flow payoff at time t after investment, but before the follower has invested, is

$$\pi_L^I = \theta_t,$$

where θ_t is the stand-alone benefit from investment—the flow payoff received by an agent who has invested, when that agent is the sole investor. After the follower has invested, the leader's flow payoff becomes

$$\pi_L^{II} = (1 + \delta_L)\theta_t.$$

The follower's flow payoff at time t having invested is

$$\pi_2^{II} = (1 + \delta_F)\theta_t.$$

Now suppose that the agents invest simultaneously. The flow payoff at time t having invested is the same for both agents:

$$\pi^{III} = (1 + \delta_S)\theta_t.$$

We are interested in situations where pre-emption may occur; that is, where the relative payoff to being the first to invest is sufficiently large. We therefore make assumptions on the payoff parameters δ_L, δ_F and δ_S.

Assumption 1 (Payoffs) $-1 < \delta_F \leq \delta_L < 0$ *and* $-1 < \delta_S < 0$*: investment is profitable for the follower; there is an advantage to being the first-mover, which may be persistent; flow payoffs when both agents have invested are less than the monopoly payoff.*

We use reduced-form profit functions in order not to be tied to any particular model of competition.[1] Assumption 1 allows us to encompasses many related models. For example, in

[1] One model that is consistent with our set-up is as follows. Two agents decide when and where to enter in a horizontally differentiated Hotelling-style market. There is an entry cost; entry is irreversible (the cost is entirely sunk), and can be delayed indefinitely. Once an agent has entered, it can sell its product at zero marginal cost and compete on price. There are three possible locations at which the agents can enter: at

Fudenberg and Tirole (1985), when n firms have adopted the new technology, the payoff of a firm that has not adopted is $\pi_0(n)$, and of a firm that has adopted is $\pi_1(n)$. They assume that if $n' \geq n$, then $\pi_1(n') < \pi_1(n)$. A specific version of their payoffs can be represented in our model by supposing that $\pi_0(n) = 0 \ \forall \ n$, $\pi_1(1) = \theta$ and $\delta_L = \delta_F = \delta_S < 0$. Real options duopoly models such as Smets (1991), Weeds (2002) and Huisman and Kort (1999) employ functional forms equivalent to negative δ_L, δ_F and δ_S parameters. Similarly, some of the payoff structures used in Katz and Shapiro (1987) can be replicated within our model. What they term the 'stand-alone incentive' is measured by δ_L in this model; their 'pre-emption incentive' is measured by $\delta_L - \delta_F$; the degree of imitation that is possible can be captured by δ_F. Lippman and Mamer (1993) analyse a model in which the first firm to innovate spoils the market for its rival; in this case, $\delta_F = -1$. (While our analysis does not cover this exact case, since $\delta_F > -1$, we can come arbitrarily close to it.) Notice also that by setting $\delta_S = (\delta_L + \delta_F)/2$, we can allow for the possibility that, in the event of simultaneous adoption, the roles of leader and follower are assigned randomly between the two agents. The main restriction in the reduced-form payoffs is that they are not allowed to depend on the stochastic process driving the state variable (i.e., the parameters μ and σ introduced below do not appear in the payoffs). This is fine if the state variable is e.g., the size of the market; but may be more restrictive in other settings.

θ_t is assumed to be exogenous and stochastic, evolving according to a geometric Brownian motion (GBM) with drift:

$$d\theta_t = \mu\theta_t dt + \sigma\theta_t dW_t \tag{1}$$

where $\mu \in [0, r)$ is the drift parameter, measuring the expected growth rate of θ, r is the continuous-time discount rate,[2] $\sigma > 0$ is the instantaneous standard deviation or volatility

$x = 0$, $x = 0.5$ and $x = 1$ (for simplicity). A first-mover advantage arises because the early adopter can locate at $x = 0.5$ so as to attract more demand than the later adopter. If the agents enter simultaneously, then they locate at opposite ends of the line. This game can be solved and equilibrium prices computed. They are consistent with reduced-form parameter values such that $-1 < \delta_F < \delta_L < \delta_S < 0$. Details are available from the authors on request.

[2] The restriction that $\mu < r$ ensures that there is a positive opportunity cost to holding the 'option' to invest, and so that the option is not held indefinitely.

parameter, and dW is the increment of a standard Wiener process, $dW_t \sim N(0, dt)$. The parameters μ, σ and r are common knowledge and constant over time. The choice of continuous time and this representation of uncertainty is motivated by the analytical tractability of the value functions that result.

The strategies of the agents in the investment game are now defined. If agent i has not invested at any time $\tau < t$, its action set is $A_t^i = \{\text{invest, don't invest}\}$. If, on the other hand, agent i has invested at some $\tau < t$, then A_t^i is the null action 'don't move'. The agent therefore faces a control problem in which its only choice is when to choose the action 'invest'. After taking this action, the agent can make no further moves.

A strategy for agent i is a mapping from the history of the game H_t (the sample path of the stochastic variable θ and the actions of both agents up to time t) to the action set A_t^i. Agents are assumed to use stationary Markovian strategies: actions depend on only the current state and the strategy formulation itself does not vary with time. Since θ follows a Markov process, Markovian strategies incorporate all payoff-relevant factors in this game. Furthermore, if one player uses a Markovian strategy, then its rival has a best response that is Markovian as well. Hence, a Markovian equilibrium remains an equilibrium when history-dependent strategies are also permitted, although other non-Markovian equilibria may then also exist. (For further explanation see Maskin and Tirole (1988) and Fudenberg and Tirole (1991).)

The formulation of the agents' strategies is complicated by the use of a continuous-time model. Fudenberg and Tirole (1985) point out that there is a loss of information inherent in representing continuous-time equilibria as the limits of discrete time mixed strategy equilibria. To correct for this, they extend the strategy space to specify not only the cumulative probability that player i has invested, but also the 'intensity' with which each player invests at times 'just after' the probability has jumped to one.[3] Although this formulation uses mixed strategies, the outcomes in any symmetric equilibrium are equivalent to those in which agents

[3]In Fudenberg and Tirole (1985), an agent's strategy is a *collection* of *simple strategies* satisfying an *intertemporal consistency condition.* A simple strategy for agent i in a game starting at a positive level θ of the state variable is a pair of real-valued functions $(G_i(\theta), \epsilon_i(\theta)) : (0, \infty) \times (0, \infty) \rightarrow [0, 1] \times [0, 1]$ satisfying certain conditions (see definition 1 in their paper) ensuring that G_i is a cumulative distribution function, and that when $\epsilon_i > 0$, $G_i = 1$ (so that if the intensity of atoms in the interval $[\theta, \theta + d\theta]$ is positive, the agent is sure to invest by θ). A collection of simple strategies for agent i, $(G_i^\theta(.), \epsilon_i^\theta(.))$, is the set of simple strategies that satisfy intertemporal consistency conditions.

employ pure strategies. (See section 3 of Fudenberg and Tirole (1985).) Consequently, the analysis will proceed as if each agent uses a pure Markovian strategy, i.e., a stopping rule specifying a critical value or 'trigger point' for the exogenous variable θ at which the agent invests. Note, however, that this is for convenience only: underlying the analysis is an extended space with mixed strategies.

Our analysis focuses on trigger points of the stochastic variable θ. These could also be expressed in terms of expected stopping times; we do not, however, include this step. For our comparative static results it is sufficient to recall that, for a given time path of the stochastic variable, a lower trigger point corresponds to earlier investment.

The following assumptions are made:

Assumption 2 *Investment is irreversible.*

Assumption 3 $\mathbb{E}_0\left[\int_0^\infty \exp(-rt)\theta_t dt\right] - K < 0.$

Assumption 2 requires that if agent i has invested by date τ, it then remains active at all dates subsequent to τ. Assumption 3 states that the initial value of the project is sufficiently low that the expected return from investment is negative, thus ensuring that immediate investment is not worthwhile. (The operator $\mathbb{E}_0$ denotes expectations conditional on information available at time $t = 0$.)

3 Equilibrium

This section describes the two types of equilibrium, involving respectively sequential and simultaneous investment, and derives the necessary and sufficient condition for simultaneous investment to occur in equilibrium.

3.1 Sequential investment

Start by assuming that the agents invest at different points. As usual in dynamic games, the stopping time game is solved backwards; see e.g., Dixit (1989). Thus the first step is to consider the optimization problem of the follower who invests strictly later than the leader. Given that the leader has invested irreversibly, the follower's payoff on investing has two components: the flow payoff from the project, $(1+\delta_F)\theta_t$; and the cost of investment, $-K$.

The follower's value function $F(\theta)$ has two components, holding over different ranges of θ: one relating to the value of investment before the follower has invested, the other to the follower's value after investment. We derive these value functions in the appendix. We show there that the follower's value function is

$$F(\theta) = \begin{cases} b_F \theta^\beta & \theta < \theta_F, \\ \frac{(1+\delta_F)\theta}{r-\mu} - K & \theta \geq \theta_F \end{cases} \tag{2}$$

where β is a constant defined by

$$\beta \equiv \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1.$$

θ_F is the follower's optimally-chosen investment point. (The value function in equation (2) assumes that the leader invests at some level of θ less than θ_F. We verify below that this is the case in equilibrium.) By arbitrage, the critical value θ_F must satisfy a value-matching condition; optimality requires a second condition, known as 'smooth-pasting', to be satisfied. (See Dixit and Pindyck (1994) for an explanation.) These conditions give

$$\begin{aligned} \theta_F &= \left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\delta_F}\right)(r-\mu), \\ b_F &= \frac{(1+\delta_F)\theta_F^{-(\beta-1)}}{\beta(r-\mu)}. \end{aligned} \tag{3}$$

Equation (3) for the follower's trigger point can be interpreted as the effective flow cost of investment with an adjustment for uncertainty. The sunk investment cost is K, but this yields a flow payoff of $(1+\delta_F)\theta$; hence the effective sunk cost is $\frac{K}{1+\delta_F}$. With an effective interest rate of $r-\mu$ (i.e., the actual interest rate r minus the expected proportional growth in the flow payoff μ), this gives an instantaneous cost of $\left(\frac{K}{1+\delta_F}\right)(r-\mu)$. If a Marshallian rule were used for the investment decision, the trigger point would be simply this cost. But with uncertainty, irreversibility and the option to delay investment, the Marshallian trigger point must be adjusted upwards by the factor $\frac{\beta}{\beta-1} > 1$.

Now let the value after investing first be denoted $L(\theta)$; in the appendix, we show that it has the following form:

$$L(\theta) = \begin{cases} \frac{\theta}{r-\mu} - K + b_L\theta^\beta & \theta \in [0, \theta_F), \\ \frac{(1+\delta_S)\theta}{r-\mu} - K & \theta \geq \theta_F, \end{cases} \tag{4}$$

given investment by the follower at θ_F. The term $b_L\theta^\beta$ is an option-like effect, anticipating subsequent entry by the follower. We show in the appendix that

$$b_L = \frac{\delta_L \theta_F^{-(\beta-1)}}{r-\mu} < 0.$$

That is, investment by the follower lowers the leader's value (which is intuitive).

The next proposition describes the sequential equilibrium. The proof is in the appendix.

Proposition 1 (Sequential equilibrium) *Given assumptions 1–3, when equilibrium investment is sequential, the leader invests at θ_P and the follower at $\theta_F > \theta_P$. θ_P is the unique solution to the equation*

$$\frac{\theta_P - \theta_M}{\theta_M} = \frac{1}{\beta - 1}\left(\frac{1 + \delta_F - \beta\delta_L}{1 + \delta_F}\right)\left(\frac{\theta_P}{\theta_F}\right)^\beta \tag{5}$$

in the interval $(0, \theta_L)$, where $\theta_M \equiv K(r-\mu)$ is the Marshallian myopic trigger and $\theta_L \equiv (\beta/(\beta-1))K(r-\mu)$ is the single-agent trigger.

The explanation of the equilibrium follows Fudenberg and Tirole (1985): the leader's trigger point with pre-emption is determined by rent equalization. The leader cannot choose its investment point optimally, as the follower can. Instead, the first agent to invest does so at the point at which it prefers to lead rather than follow, not the point at which the benefits from leading are largest. Clearly, it cannot be that the first agent invests when the value from following is greater than the value from leading—if this were the case, the agent would do better by waiting. Likewise, it cannot be that the first agent invests when the value from leading is strictly greater than the value from following, since in this case without pre-assigned roles, the other agent could pre-empt it and still gain. Hence the investment point is determined by indifference between leading and following. The trigger point θ_P in the pre-emption model is given by indifference: $L(\theta_P) = F(\theta_P)$. This is in contrast to the trigger point of the follower, which is determined by value matching and smooth pasting, i.e., is chosen optimally.

3.2 Simultaneous investment

Now consider the alternative case, in which investment is simultaneous at the trigger point θ_S. Note, however, that even when such an equilibrium exists, there is always an equilibrium in which investment occurs sequentially; see Fudenberg and Tirole (1985).

The value function of each agent in the simultaneous investment equilibrium is

$$S(\theta) = \begin{cases} b_S\theta^\beta & \theta < \theta_S, \\ \frac{(1+\delta_S)\theta}{r-\mu} - K & \theta \geq \theta_S. \end{cases} \quad (6)$$

(This value function can be derived from the appropriate Bellman equation, following the steps shown in the appendix.) There is a continuum of simultaneous equilibria; it is straightforward to show that they can be Pareto ranked, with higher trigger points yielding higher value functions. In this case, it seems reasonable that the agents invest at the Pareto optimal point, given by both value matching and smooth pasting. So:

Proposition 2 (Simultaneous equilibrium) *The Pareto optimal trigger point for the simultaneous equilibrium is*

$$\theta_S = \left(\frac{\beta}{\beta-1}\right)\left(\frac{K}{1+\delta_S}\right)(r-\mu).$$

The coefficient in the value function is

$$b_S = \frac{(1+\delta_S)\theta_S^{-(\beta-1)}}{\beta(r-\mu)}.$$

The next proposition describes when simultaneous investment can occur in equilibrium. The proof is in the appendix.

Proposition 3 *Simultaneous investment occurs in equilibrium iff*

$$\lambda_E \equiv (1+\delta_S)^\beta - \left(1+\beta\delta_L(1+\delta_F)^{\beta-1}\right) \geq 0. \tag{7}$$

When this condition is satisfied, therefore, two classes of equilibria exist: those with sequential investment (characterized by proposition 1); and those with simultaneous investment (characterized by proposition 2).

Whether there is an equilibrium with simultaneous investment is determined by whether the leader wishes to invest before the follower, or at the same time (i.e., by the comparison of $L(\theta)$ and $S(\theta)$). The proposition shows the reasonable condition that, in order for simultaneous investment to occur in equilibrium, it must be the case that δ_S is sufficiently large and/or δ_L sufficiently small. Note that the simultaneous investment equilibrium, when it exists, Pareto dominates the sequential outcome; this is an immediate consequence of the condition for existence of the simultaneous investment equilibrium: $S(\theta) \geq L(\theta)$ for $\theta \in [0, \theta_S]$.

4 Investment under uncertainty with pre-emption

The standard lessons from the literature on irreversible investment under uncertainty can be seen readily in the case where the players' roles are pre-assigned. In this case, it is easy to show that the leader invests at the standard single-agent trigger given by

$$\theta_L = \left(\frac{\beta}{\beta-1}\right) K(r-\mu);$$

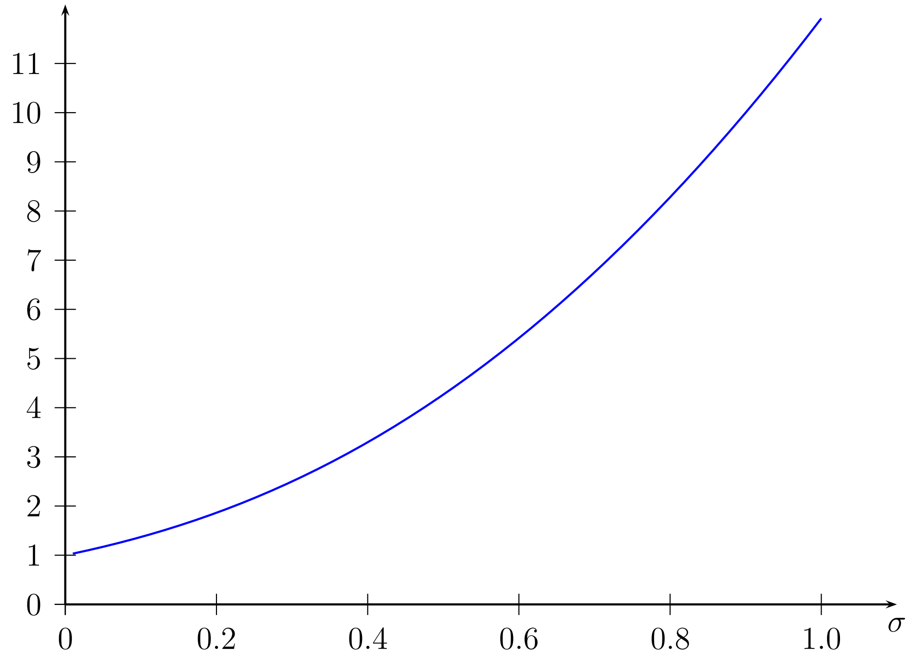

Figure 1: The standard real option effect: θ_L/θ_M against σ, with $\mu = 0$

and the follower invests at θ_F. Both of these triggers are larger than the myopic Marshallian trigger, θ_M (defined in Proposition 1). The size of the gap, due to real options effects, is driven by the factor $\beta/(\beta-1)$, since $\theta_L = \beta/(\beta-1) \times \theta_M$. Figure 1 plots θ_L as a function of σ. (Note that in this figure, μ is set to 0; this is why θ_L equals θ_M in the no-uncertainty limit as $\sigma \to 0$.[4]) The figure shows that, even at moderate levels of uncertainty, the real option effect can be large. For example, with a volatility of 0.2 (i.e., a variance for the process in equation (1) of 4%), the real options trigger θ_L is roughly a factor of 2 greater than the Marshallian trigger θ_M. If the variance increases to 25, this factor increases to 5. (The corresponding option values are also large.)

This is the standard story: see e.g., Dixit and Pindyck (1994). Our objective in this section is to examine how the relationship between trigger points, and the impact of uncertainty, is modified when pre-emption can occur.

[4] In the figure, $r = 5\%$.

4.1 The sequential equilibrium

We start by considering the sequential investment equilibrium. Clearly, the follower's investment behaviour is unaltered by pre-emption: it invests at θ_F regardless. Our analysis therefore focusses on the investment trigger of the leader, θ_P.

Proposition 4 *(i) In the limit as $\sigma \to \infty$, $\theta_P \to \theta_M/(\delta_L - \delta_F)$. (ii) θ_P is non-decreasing in σ. (iii) $\theta_P \leq \theta_M/(\delta_L - \delta_F)$.*

Proof. Part (i) follows immediately from equation (5), noting that as $\sigma \to \infty, \beta \to 1$. For the proof of part (ii), see the appendix. Part (iii) follows immediately from parts (i) and (ii). □

The proposition shows that θ_P is bounded above by $\theta_M/(\delta_L - \delta_F)$. This upper bound is large when δ_L is close to δ_F. In particular, if there is no persistent first-mover advantage, so that $\delta_L = \delta_F$, then as the degree of uncertainty grows, the pre-emption trigger increases without bound. But with a positive first-mover advantage, $\delta_L > \delta_F$, there is a finite upper bound for θ_P. This is in contrast to the follower's trigger θ_F, which increases without bound as σ becomes large. This is illustrated in figure 2, which shows the behaviour of θ_P and θ_L as σ increases. (Note that, as in figure 1, μ is set equal to 0, with the consequence that θ_P and θ_L both equal θ_M when $\sigma = 0$.)[5]

The analysis so far has highlighted that pre-emption lowers the trigger point of the first-mover when there is uncertainty. But we have shown that the effect of uncertainty on the pre-emption trigger is quantitatively different: the pre-emption trigger is bounded as the degree of uncertainty grows. This is in contrast to standard real options triggers, which grow without bound as uncertainty increases.

To emphasise the different effects that uncertainty has on investment triggers with and without pre-emption, we note the following possibility when assumption 1 is relaxed to allow $\delta_L > 0$. The proof is in the appendix.

Proposition 5 *Joint sufficient conditions for the leader's investment trigger θ_P to be decreas-*

[5] Also, in this figure, $r = 5\,\delta_L = -0.25, \delta_F = -0.5$ and $K = 10$. Hence $\theta_M = 0.5$.

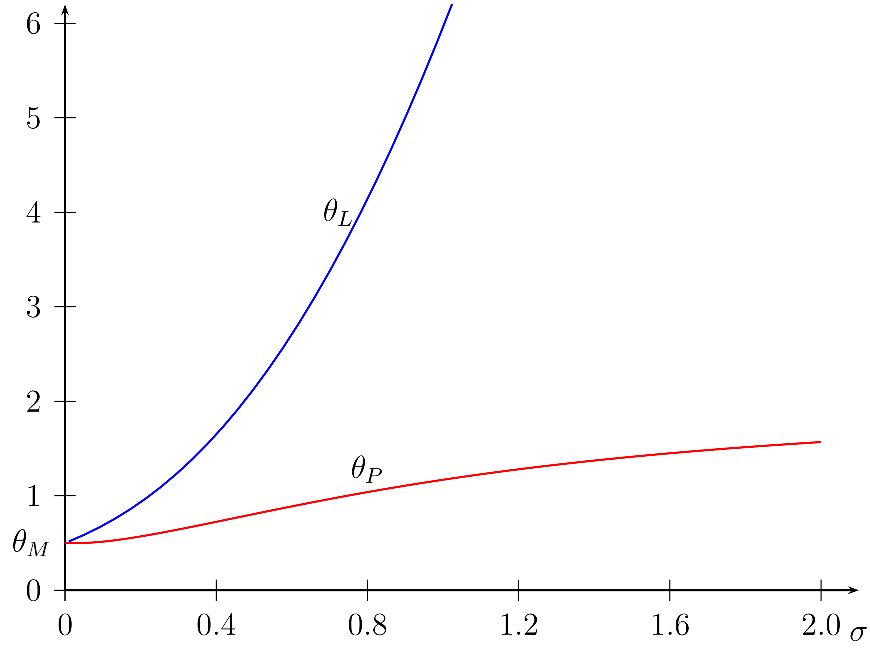

Figure 2: θ_P and θ_L against σ, with $\mu = 0$

ing in the volatility parameter σ are

$$1 + \beta \ln(1 + \delta_F) < 0 \quad \text{and} \quad 0 \leq \frac{(1 + \delta_F)\ln(1 + \delta_F)}{1 + \beta \ln(1 + \delta_F)} \leq \delta_L.$$

Proposition 5 raises the striking possibility that greater uncertainty lowers the leader's trigger point. The possibility is illustrated in figure 3, which plots θ_P/θ_M against σ when $\delta_L = 0.2, \delta_F = -0.5$ and $\mu = 0$; note that the latter parameter values ensures that $\theta_P = \theta_M$ when $\sigma = 0$.[6]

The possibility arises from the lack of optimality in the choice of the pre-emption trigger point. An optimal trigger point is such that the marginal benefit from delaying investment for a period equals the marginal cost. The marginal benefit is the interest saved on the investment cost plus the expected gain from the possibility that the flow payoff increases. The marginal cost is the flow payoff foregone by not investing. In this marginal calculation, the agent does not consider the effect of its delay on the investment decision of the other agent, since in the

[6]The other parameter values used in the figure are: $r = 5\%$ and $K = 10$.

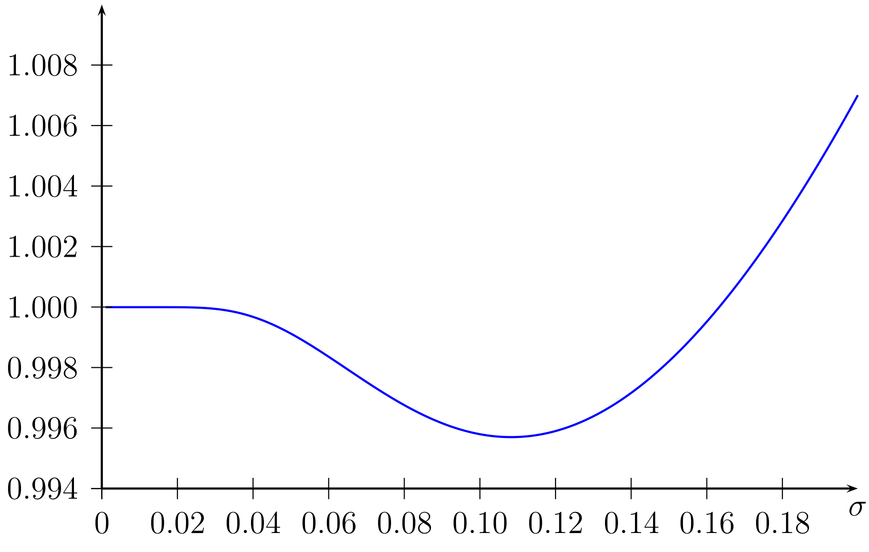

Figure 3: Non-monotonicity of θ_P against σ

models considered in this paper, each agent's trigger point (with the exception of θ_P) does not depend on the other's. Increased uncertainty raises the expected gain from delay, causing the (optimally chosen) trigger point to increase. This reasoning does not apply in the case of θ_P, however: it is not chosen according to a marginal equality, but an absolute equality between the value from leading and the value from following. The proposition shows that this difference in the determination of the trigger point can lead to θ_P decreasing as uncertainty increases.

Numerical analysis shows that the quantitative effect of this result is relatively small: θ_P does not fall much below θ_M for any parameter values that we have used. Nevertheless, the result reinforces the general message of this paper: competition for investment opportunities alters the qualitative and quantitative conclusions about investment behaviour.

4.2 The simultaneous equilibrium

We now turn to the analysis of the simultaneous investment equilibrium. First note that θ_S has the standard real options form. Its dependence on σ is driven by the factor $\beta/(\beta-1)$. The more difficult question is to determine when equilibrium involves simultaneous investment:

that is, when the necessary and sufficient condition

$$\lambda_E \equiv (1+\delta_S)^\beta - \left(1+\beta\delta_L(1+\delta_F)^{\beta-1}\right) \geq 0 \tag{8}$$

holds. We provide a partial analysis in the next proposition; the non-linearity of the expression for λ_E prevents a full characterization.

Proposition 6 *1.* $\lim_{\sigma\to 0}\lambda_E = -1$.

2. $\lim_{\sigma\to+\infty}\lambda_E = \delta_S - \delta_L$.

3. $\lim_{\sigma\to 0}\partial\lambda_E/\partial\sigma = 0$.

4. If $\delta_F \leq e^{-\frac{1}{\beta}} - 1$, *then* λ_E *is an increasing function of* σ.

The first part of the proposition tells us that simultaneous investment cannot occur in equilibrium in the limit as $\sigma \to 0$. Hence in the deterministic limit, only the sequential equilibrium exists. The last part of the proposition shows that if δ_F is less than around -0.63, then λ_E increases with σ. This sufficient condition is quite generous; numerical investigation indicates that λ_E is increasing in σ for almost parameter values that we have used. Figure 4 illustrates λ_E, for the parameter values $r = 5\%, \mu = 0, \delta_L = -0.25, \delta_S = -0.2$, and $\delta_F = -0.4$. Note that the value of δ_F used does not satisfy the sufficient condition in the proposition; nevertheless, λ_E is increasing in σ. For σ less than around 0.342, only the sequential investment equilibrium exists; for larger values of σ, both types of equilibrium exist.

5 Applications

Real options theory teaches that, under uncertainty, irreversible investments will be delayed compared with the traditional NPV rule. By contrast, the extensive literature on industrial organization suggests that, when relatively few firms compete, there is often an advantage to moving first. For example, the first investor may gain preferential access to scarce resources or key skills, or may benefit strategically from making an early commitment to the market. In many industry settings, then, a tension arises between pre-emption and delay, and the

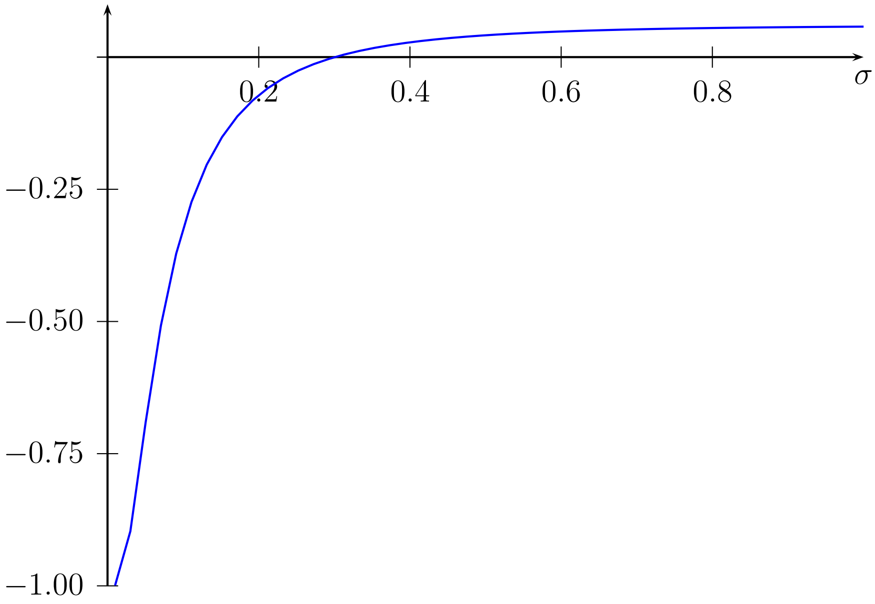

Figure 4: λ_E against σ

applicability of real options—at least in its usual, non-game theoretic form—becomes unclear. If first mover advantages are strong, the optimal response may be to invest pre-emptively at trigger point θ_P, relinquishing much of the option value of delay. In this situation, investment behaviour (at least for the first mover) will be much closer to the NPV rule, and, as we have demonstrated above, the impact of uncertainty will be mitigated.

The tension between real options and pre-emption has been recognised by a number of authors. In the closing chapter of their book on real options, Copeland and Antikarov (2001) point out the drawback of deferring investment in research when a firm is in a competitive race to develop a new drug. Howell, Stark, Newton, Paxson, Cavus, Pereira, and Patel (2001) refer to the danger of "[u]sing real options when we shouldn't"in game theoretic situations, stating that things are seldom clear cut in these cases. This limitation has also been discussed in policy settings such as regulation. Despite accepting in principle that allowance might be made for option values in estimating the cost of capital, Ofcom (2005) notes countervailing first mover advantages and sees the applicability of real options as being restricted to just a few cases.

The framework set out in this paper provides guidance on the applicability of real options to various industry situations. The crucial condition is inequality (8): when this holds, equilibria exist in which both firms delay investment and uncertainty affects behaviour as real options theory predicts; otherwise pre-emption occurs and the impact of uncertainty is much weaker. Proposition 6 suggests that the outcome depends on the relative strengths of investment complementarities (δ_S) and the persistent first mover advantage (δ_L).

If investment complementarities are strong, so that δ_S is large relative to δ_L, then λ_E is positive (for sufficiently large σ). Assuming the Pareto dominant equilibrium is achieved, both firms delay their investment and the prospect of pre-emption makes no difference to equilibrium behaviour. This outcome might be expected in settings where the return to investment is higher when other firms also invest: for example, network markets (e.g., telecommunications), industries where standard-setting is important (e.g., media recording formats such as CDs and DVDs), vertical relationships (e.g., manufacturers and suppliers) and in the presence of demand spillovers (e.g., advertising that stimulates demand for a product class, not just an individual product). In these situations, the predictions of real options theory may be expected to fit reasonably well.

If, on the other hand, there is a persistent first mover advantage such that δ_L is large relative to δ_S, then λ_E is negative in the limit as $\sigma \to \infty$ (our numerical investigations indicate that λ_E is then negative for all values of the volatility parameter). In this case, equilibrium investment is always sequential, with one firm forfeiting the option value of delay to pre-empt its rival. Several relevant situations come to mind. Patent races are characterised by lasting first mover advantage: the first to invent (or first to file) gains an exclusive right over the technology, which other firms must not infringe. Systems wars between incompatible technologies (e.g. Windows vs. Apple Mac, VHS vs. Betamax) are also instances where a first mover advantage tends to persist. Entry into industries with substantial economies of scale also tends to confer long-lasting benefit: incumbents are difficult to displace and typically earn high returns. In these situations, then, one might expect to observe pre-emptive investment, and relatively little sensitivity to uncertainty.

When equilibrium investment is sequential, the leader's investment timing (i.e. the level of its trigger, θ_P) then depends on the comparison between the leader's and the follower's payoffs, with an upper bound given by $\theta_M/(\delta_L - \delta_F)$. If the follower's payoff is relatively large, competition to be the first mover is weaker and investment can be delayed for a time. But if the follower's payoff is very low, pre-emption destroys most if not all of the leader's option value and its investment timing will be close to the NPV rule. This is likely to be the case in e.g., a patent race or entry into a natural monopoly industry.

6 Conclusions

This paper has analysed irreversible investment in a project with uncertain returns, when there may be a persistent advantage to being the first investor, and externalities to investing when others also invest. It therefore extends standard real options analysis to a setting where there are general strategic interactions and externalities between investing agents. This framework captures a variety of strategic situations and industry settings, and encompasses a number of earlier contributions.

We have shown that two distinct patterns of investment behaviour are possible in equilibrium. The relationship between investment and uncertainty depends on the type of equilibrium: while the simultaneous investment equilibrium displays standard real options properties, comparative statics of the pre-emption trigger in a sequential investment equilibrium are quite different. Thus, strategic interactions and externalities, omitted from standard real options analysis, can have important qualitative and quantitative effects on the relationship between investment and uncertainty.

In the light of our analysis, we have provided guidance on the applicability of real options to a number of industry situations. The general framework we employ allows for various types of strategic interactions between firms found in the industrial organization literature to be captured. Depending on comparisons between investment complementarities, persistent first mover advantage and follower's payoff, predictions can be made for the timing of investment—in particular, the incidence of pre-emption and delay—and the sensitivity of investment to uncertainty.

Appendix

A Value functions

Let the follower's value functions be denoted F_0 and F_1, before and after its investment respectively.

Prior to investment, the follower holds an option to invest but receives no flow payoff. In this 'continuation' region, in any short time interval dt starting at time t the follower experiences a capital gain or loss dF_0 . The Bellman equation for the value of the investment opportunity is therefore

$$F_0 = \exp(-rdt)\mathbb{E}_t\left[F_0 + dF_0\right]. \tag{A.9}$$

Itô's lemma and the GBM equation (1) gives the ordinary differential equation (ODE)

$$\frac{1}{2}\sigma^2\theta^2 F_0''(\theta) + \mu\theta F_0'(\theta) - rF_0(\theta) = 0. \tag{A.10}$$

From equation (1), it can be seen that if θ ever goes to zero, then it stays there forever. Therefore the option to invest has no value when $\theta = 0$, and must satisfy the boundary condition $F_0 = 0$. Solution of the differential equation subject to this boundary condition gives $F_0 = b_F\theta^\beta$, where b_F is a positive constant and $\beta > 1$ is the positive root of the quadratic equation $\mathcal{Q}(z) = \frac{1}{2}\sigma^2 z(z-1) + \mu z - r$; i.e., $\beta = \frac{1}{2}\left(1 - \frac{2\mu}{\sigma^2} + \sqrt{\left(1 - \frac{2\mu}{\sigma^2}\right)^2 + \frac{8r}{\sigma^2}}\right)$.

Now consider the value of the agent in the 'stopping' region, in which the value of θ is such that it is optimal to invest at once. Since investment is irreversible, the value of the agent in the stopping region is given by the expected value alone with no option value terms. There are two possibilities. In the first, the follower invests strictly after the leader. When the level at time t of the state variable is θ_t, the follower's value function in this case is

$$F_1(\theta_t) = \mathbb{E}_t\left[\int_t^\infty \exp(-r(\tau - t))(1+\delta_F)\theta_\tau d\tau - K\right].$$

θ is expected to grow at rate μ, so that

$$F_1(\theta_t) = \frac{(1+\delta_F)\theta_t}{r-\mu} - K. \tag{A.11}$$

In the second possibility, the follower invests at exactly the same point as the leader. In this case, the follower's value function is

$$F_2(\theta_t) = \mathbb{E}_t\left[\int_t^\infty \exp\left(-r(\tau-t)\right)(1+\delta_S)\theta_\tau d\tau - K\right] == \frac{(1+\delta_S)\theta_t}{r-\mu} - K. \tag{A.12}$$

The boundary between the continuation region and the stopping region is given by a trigger point θ_F of the stochastic process such that continued delay is optimal for $\theta < \theta_F$ and immediate investment is optimal for $\theta \geq \theta_F$. The optimal stopping time T_F is then defined as the first time that the stochastic process θ hits the interval $[\theta_F, \infty)$ from below.

Putting together the two regions gives the follower's value function:

$$F(\theta) = \begin{cases} b_F\theta^\beta & \theta < \theta_F, \\ \frac{(1+\delta_F)\theta}{r-\mu} - K & \theta \geq \theta_F, \end{cases} \tag{A.13}$$

given that the leader invests at some $\theta' < \theta_F$. If the leader invests at the same time as the follower, then $F(\theta) = (1+\delta_S)\theta/(r-\mu) - K$.

For the agent who invests first, there are two possibilities. The first is that it invests at some $\theta < \theta_F$ i.e., at some time $t < T_F$. In this case, there are two components to the agent's value function, holding over different ranges of θ. The first component L_1 holds after the leader has invested, but before the follower has done so; and the second component L_2, after the follower has invested. The second component is equivalent to that of the follower, determined previously. The first component is new, and so is derived in detail.

After the leader has invested, it has no further decision to take and its payoff is given by the expected value of its investment. This payoff is affected, however, by the action of the follower investing at the strictly later point θ_F. Taking account of subsequent investment by the follower, the leader's post-investment payoff is given by

$$L_1(\theta_t) = \mathbb{E}_t\left[\int_t^{T_F} \exp\left(-r(\tau - t)\right)\theta_\tau d\tau + \int_{T_F}^{\infty} \exp\left(-r(\tau - t)\right)(1+\delta_L)\theta_\tau d\tau - K\right]. \quad \text{(A.14)}$$

By standard calculations, this becomes

$$\frac{\theta_t}{r-\mu} - K + \left(\frac{\theta_t}{\theta_F}\right)\frac{\delta_L\theta_F}{r-\mu} \equiv \frac{\theta_t}{r-\mu} - K + b_L\theta_t^{\beta}.$$

The first part of the value function L_1 gives the expected value of investment before the follower invests, while the second is an option-like term reflecting the value to the leader of future investment by the follower.

The second possibility for the agent who invests first is that it invests at some $\theta \geq \theta_F$ i.e., some time $t \geq T_F$. In this case, the leader's post-investment payoff is given by

$$L_2(\theta_t) = \mathbb{E}_t\left[\int_t^{\infty} \exp\left(-r(\tau - t)\right)(1+\delta_S)\theta_\tau d\tau - K\right] = \frac{(1+\delta_S)\theta_t}{r-\mu}. \quad \text{(A.15)}$$

B Proof of Proposition 1

The follower's equilibrium investment point, θ_F, is given by equation (3). In this proof, we derive the leader's investment point and establish that it is given by rent equalization.

Define

$$\Delta(\theta) \equiv \frac{\theta}{r-\mu} - K - \left(\frac{\theta}{\theta_F}\right)^{\beta}\left(\frac{1-\beta\delta_L+\delta_F}{1+\delta_F}\right)\frac{K}{\beta-1} \quad \text{(B.16)}$$

i.e., $L(\theta) - F(\theta)$, where $L(\theta)$ is conditional on the leader having invested, and $F(\theta)$ is conditional on the leader having invested but not the follower. There are three possibilities: that there are (i) no, (ii) one or (iii) multiple roots of expression (B.16). We use the following facts: (i) $\Delta(\theta)$ is a continuously differentiable function of θ; (ii) $\Delta(0) = -K < 0$ (and $\Delta(\theta) < 0$ for all $\theta < \theta_0$, by assumption 3); (iii) $\Delta(\theta_L) > 0$; (iv) $\theta_L < \theta_F$.

Fact (iii) requires further proof. Using the definition of $\theta_L = (\beta/(\beta-1))K(r-\mu)$,

$$\Delta(\theta_L) = \frac{K}{(\beta-1)(1+\delta_F)}\left(\beta\delta_L\left(\frac{\theta_L}{\theta_F}\right)^{\beta} + (1+\delta_F)\left(1-\left(\frac{\theta_L}{\theta_F}\right)^{\beta}\right)\right).$$

Since $\delta_L \geq \delta_F > -1$ (by assumption 1),

$$\begin{aligned}\Delta(\theta_L) &\geq \frac{K}{(\beta-1)(1+\delta_F)}\left(\beta\delta_F\left(\frac{\theta_L}{\theta_F}\right)^{\beta} + (1+\delta_F)\left(1-\left(\frac{\theta_L}{\theta_F}\right)^{\beta}\right)\right)\\ &= \frac{K}{\beta-1}\big(\beta\delta_F(1+\delta_F)^{\beta-1} + 1 - (1+\delta_F)^{\beta}\big).\end{aligned}$$

Let $\phi(\delta_F) \equiv \beta\delta_F(1+\delta_F)^{\beta-1} + 1 - (1+\delta_F)^{\beta}$.[7] Note that $\phi(0) = 0$; and $\phi'(\delta_F) = \beta(\beta-1)\delta_F(1+\delta_F)^{\beta-2}$, which is ¡ (=) 0 when $-1 < \delta_F < (=)0$. Hence $\phi(\delta_F) > 0$ for all $\delta_F \in (-1, 0)$. Hence $\Delta(\theta_L) > 0$ for all $\delta_F \in (-1, 0)$.

Hence, by the intermediate value theorem, there exists a value $\theta_P < \theta_L$ such that $\Delta(\theta_P) = 0$, and $\Delta(\theta)$ is less (greater) than 0 for θ immediately less (greater) than θ_P. Further that there is only one such θ_P in the interval $(0, \theta_L)$. To see this, note that $1 - \beta\delta_L + \delta_F \geq 0$, by assumption 1, since $\delta_L \leq 0$ and $\delta_D > -1$. Hence $\Delta(\theta)$ is concave, with $\Delta(\theta_L) > 0$. Therefore $\Delta(\theta) = 0$ has a unique solution θ_P in $(0, \theta_L)$. Hence in the sequential equilibrium, no agent invests when $\theta \in [\theta_0, \theta_P)$. At $\theta = \theta_P < \theta_L$, the leader invests; at $\theta_F > \theta_P$, the follower invests.

C Proof of Proposition 3

Suppose that one agent follows the strategy: "invest at θ_S if the other agent has yet to invest; otherwise invest at the sequentially rational point in the continuation game". The sequentially rational point in the continuation game is θ_F if the other agent invests at some $\theta < \theta_F$; otherwise, it is immediate investment. The other agent then has three options: (i) invest at $\theta > \theta_S$; (ii) invest at $\theta = \theta_S$; (iii) invest at $\theta < \theta_S$. It is clear that option (i) is dominated by option (ii). Option (ii) has a value of $S(\theta) \equiv b_S\theta^{\beta}$ for $\theta \leq \theta_S$: see equation equation (6). Now consider option (iii). From equation (4), the value to the agent from

[7] We are grateful to the referee for suggesting the following argument.

investing at $\theta < \theta_S$ is

$$L(\theta) = \frac{\theta}{r-\mu} - K + b_L \theta^\beta.$$

The agent will choose option (ii) (i.e., investment will be simultaneous in equilibrium) iff $S(\theta) \geq L(\theta)$ for all $\theta \leq \theta_S$. We now establish parametric conditions so that $S(\theta) \geq L(\theta)$ for all $\theta \leq \theta_S$.

Note that $S(\theta)$ is an increasing and strictly convex function of θ, with $S(0) = S'(0) = 0$; while $L(\theta)$ is a concave function of θ, with $L(0) = -K$ and $L'(0) = 1/(r-\mu) > 0$. Hence there is a unique $\theta_0 > 0$ such that $S'(\theta_0) = L'(\theta_0)$. θ_0 is given by

$$\beta b_S \theta_0^{\beta-1} = \frac{1}{r-\mu} + \beta b_L \theta_0^{\beta-1}. \tag{C.17}$$

There are then three possibilities: (i) $\theta_0 \leq \theta_L$; (ii) $\theta_L < \theta_0 \leq \theta_S$; (iii) $\theta_S < \theta_0$. We examine each possibility in turn.

(i) $\theta_0 \leq \theta_L$.

Equation (C.17) can be rearranged to give

$$\theta_0^{\beta-1} = \frac{\theta_L^{\beta-1}}{(1+\delta_S)^\beta - \beta\delta_L(1+\delta_F)^{\beta-1}}.$$

Hence $\theta_0 < \theta_L$ iff

$$(1+\delta_S)^\beta - \beta\delta_L(1+\delta_F)^{\beta-1} > 1.$$

But also note that

$$\begin{aligned} S(\theta_0) = b_S\theta_0^\beta &= \frac{\theta_0}{\beta(r-\mu)} + b_L\theta_0^\beta \\ &\geq \frac{\theta_0}{r-\mu} - K + b_L\theta_0^\beta = L(\theta_0). \end{aligned}$$

(The inequality in the second line follows from $\theta_0 < \theta_L = (\beta/(\beta-1))K(r-\mu)$.) And since $S(\theta)$ is convex and $L(\theta)$ concave, this implies that $S(\theta) > L(\theta)$ for all $\theta \leq \theta_S$.

(ii) $\theta_L < \theta_0 \leq \theta_S$.

From case (i), it is clear that, since $\theta_0 > \theta_L$, $S(\theta_0) < L(\theta_0)$. Therefore there are no parametric conditions in this case such that $S(\theta) \geq L(\theta)$ for all $\theta \leq \theta_S$.

(iii) $\theta_S < \theta_0$.

In this last case, by previous arguments $S(\theta_0) < L(\theta_0)$. But since $\theta_0 > \theta_S$, it is still possible that $S(\theta) \geq L(\theta)$ for all $\theta \leq \theta_S$. We now show that, in fact, this is not the case. Rearranging equation (C.17) gives

$$\theta_0^{\beta-1} = \frac{(1+\delta_S)^{\beta-1}\theta_S^{\beta-1}}{(1+\delta_S)^{\beta} - \beta\delta_L(1+\delta_F)^{\beta-1}}.$$

Hence $\theta_0 > \theta_S$ iff

$$\frac{(1+\delta_S)^{\beta-1}}{(1+\delta_S)^{\beta} - \beta\delta_L(1+\delta_F)^{\beta-1}} > 1;$$

or

$$\delta_S(1+\delta_S)^{\beta-1} - \beta\delta_L(1+\delta_F)^{\beta-1} < 0. \tag{C.18}$$

If $S(\theta) \geq L(\theta)$ for all $\theta \leq \theta_S$, then (obviously) $S(\theta_S) \geq L(\theta_S)$. This requires that

$$b_S\theta_S^{\beta} = \frac{(1+\delta_S)\theta_S}{r-\mu} - K \geq \frac{\theta_S}{r-\mu} - K + b_L\theta_S^{\beta}$$

where the first equality is the value-matching condition for θ_S. Substituting for θ_S and b_L gives

$$\delta_S(1+\delta_S)^{\beta-1} - \delta_L(1+\delta_F)^{\beta-1} \geq 0. \tag{C.19}$$

Since $\delta_L < 0$ and $\beta \geq 1$, equation (C.18) implies that

$$\delta_S(1+\delta_S)^{\beta-1} - \beta\delta_L(1+\delta_F)^{\beta-1} > \delta_S(1+\delta_S)^{\beta-1} - \delta_L(1+\delta_F)^{\beta-1}.$$

Hence equations (C.18) and (C.19) cannot be satisfied simultaneously. Therefore it cannot be that $\theta_S < \theta_0$ and $S(\theta_S) \geq L(\theta_S)$.

In summary: $S(\theta) \geq L(\theta)$ for all $\theta \leq \theta_S$ iff $\theta_0 \leq \theta_L$ i.e., iff

$$(1+\delta_S)^\beta - \beta\delta_L(1+\delta_F)^{\beta-1} > 1.$$

Rearranging gives the necessary and sufficient condition of equation (7).

D Proof of part (ii) of Proposition 4

The difference between the values of the leader's option-like term and the follower's option associated with the follower's investment is

$$\Delta(\theta,\beta) \equiv (b_{L1} - b_F)\theta^\beta = \left(\frac{\beta\delta_L - (1+\delta_F)}{1+\delta_F}\right) F(\theta)$$

where $F(\theta) \equiv b_F\theta^\beta > 0$ for $\theta \in (\theta_P, \theta_F)$. The objective of the proof is to establish that $\partial\Delta(\theta_P,\beta)/\partial\beta \geq 0$, so that $\partial\Delta(\theta_P,\beta)/\partial\sigma \leq 0$, which means that the leader's value function increases by less than the follower's (evaluated at $\theta = \theta_P$) for a small increase in σ. If this is the case, then θ_P must increase in σ.

We start by evaluating the derivative of $\Delta(\theta,\beta)$ with respect to β:

$$\frac{\partial\Delta(\theta,\beta)}{\partial\beta} = \frac{\delta_L F(\theta) + (\beta\delta_L - (1+\delta_F))\frac{\partial F(\theta)}{\partial\beta}}{1+\delta_F}.$$

But

$$\frac{\partial F(\theta)}{\partial\beta} = F(\theta)\ln\left(\frac{\theta}{\theta_F}\right).$$

Hence

$$\frac{\partial\Delta(\theta,\beta)}{\partial\beta} = \frac{F(\theta)}{1+\delta_F}\left(\delta_L + (\beta\delta_L - (1+\delta_F))\ln\left(\frac{\theta}{\theta_F}\right)\right) \tag{D.20}$$

for $\theta \in [\theta_P, \theta_F]$.

Now note that $\theta_P \leq \theta_L$. Hence

$$\ln\left(\frac{\theta_P}{\theta_F}\right) \leq \ln(1+\delta_F).$$

We show, through contradiction, that $\partial\Delta(\theta_P, \beta)/\partial\beta > 0$. So, suppose not i.e., suppose that $\partial\Delta(\theta_P, \beta)/\partial\beta \leq 0$. In order for this inequality to hold, it must be that

$$\delta_L + (\beta\delta_L - (1+\delta_F))\ln\left(\frac{\theta_P}{\theta_F}\right) \leq 0. \tag{D.21}$$

A sufficient condition for this inequality to be satisfied is

$$\delta_L + (\beta\delta_L - (1+\delta_F))\ln(1+\delta_F) \leq 0.$$

(Here, we have used the fact that, from assumption 1, $\beta\delta_L - (1+\delta_F) \leq 0$.) Since $\delta_F < 0$, in order for this inequality to be satisfied for all values of β, it must be that

$$\phi(\delta_L) \equiv \delta_L + (\delta_L - (1+\delta_F))\ln(1+\delta_F) \leq 0.$$

Note that $\phi(0) = -(1+\delta_F)\ln(1+\delta_F) > 0$; and $\phi(\delta_F) = \delta_F - \ln(1+\delta_F) > 0$. (Both statements follow from assumption 1: $-1 < \delta_F < 0$.) Since $\phi(\delta_L)$ is linear in δ_L, this means that $\phi(\delta_L) > 0$: a contradiction. Therefore $\partial\Delta(\theta_P, \beta)/\partial\beta > 0$.

E Proof of Proposition 5

The proof follows the proof of proposition 4. $\delta_L \leq -(\beta\delta_F - (1+\delta_F))\ln(1+\delta_F)$ is a sufficient condition for $\partial\Delta(\theta_P, \beta)/\partial\beta \leq 0$. Re-arranging this inequality yields

$$\delta_L(1+\beta\ln(1+\delta_F)) \leq (1+\delta_F)\ln(1+\delta_F). \tag{E.22}$$

This inequality cannot be satisfied if $1+\beta\ln(1+\delta_F) > 0$ and assumption 1 holds (in particular, $\delta_L \geq \delta_F$). To see why, notice that equation (E.22) would require in this case that $\delta_L \leq \underline{\delta_L}$, where, as in the proposition,

$$\underline{\delta_L} \equiv \frac{(1+\delta_F)\ln(1+\delta_F)}{1+\beta\ln(1+\delta_F)}$$

and $\underline{\delta_L} \leq 0$. Assumption 1 then requires that $\underline{\delta_L} \geq \delta_F$. But this in turn requires that $(\beta-1)(1+\delta_F)\ln(1+\delta_F) - \beta\ln(1+\delta_F) + \delta_F \leq 0$. When $\beta = 1$, this inequality requires that $-\ln(1+\delta_F) + \delta_F \leq 0$, which is violated for all $\delta_F \in [-1, 0)$ and holds with equality only when $\delta_F = 0$. Since $(\beta-1)(1+\delta_F)\ln(1+\delta_F) - \beta\ln(1+\delta_F) + \delta_F$ is increasing in β , this means that $(\beta-1)(1+\delta_F)\ln(1+\delta_F) - \beta\ln(1+\delta_F) + \delta_F \geq 0$, with equality only when $\delta_F = 0$.

Hence inequality (E.22) requires that $1 + \beta\ln(1+\delta_F) < 0$; and hence that $\delta_L \geq \underline{\delta_L}$, where $\underline{\delta_L} \geq 0$.

F Proof of Proposition 6

The first three parts follow from straightforward calculation. To show the last part, differentiate λ_E with respect to σ:

$$\frac{\partial\lambda_E}{\partial\sigma} = \frac{\partial\lambda_E}{\partial\beta}\frac{\partial\beta}{\partial\sigma}.$$

Since β is decreasing in σ, $\partial\lambda_E/\partial\sigma$ has the opposite sign to $\partial\lambda_E/\partial\beta$. Differentiation gives

$$\frac{\partial\lambda_E}{\partial\beta} = (1+\delta_S)^{\beta}\ln(1+\delta_S) - \delta_L(1+\delta_F)^{\beta-1}(1+\beta\ln(1+\delta_F)). \tag{F.23}$$

It is sufficient for λ_E to be an decreasing function of β (and hence to be increasing in σ) that all terms in equation (F.23) be negative. Hence joint sufficient conditions are: (i) $\delta_S \leq 0$, so that $\ln(1+\delta_S) \leq 0$; (ii) $-\delta_L(1+\ln(1+\delta_F)) \leq 0$, which is satisfied when $\delta_L \leq 0$ and $1+\ln(1+\delta_F) \leq 0$, i.e., $\delta_F \leq e^{-1} - 1$.

References

BOYER, M., P. LASSERRE, T. MARIOTTI, AND M. MOREAUX (2004): "Preemption and Rent Dissipation under Price Competition," *International Journal of Industrial Organization*, 22, 309–328.

COPELAND, T., AND V. ANTIKAROV (2001): *Real options: A practitioner's guide*. New York: Texere Publishing.

DIXIT, A. K. (1980): "The role of investment in entry deterrence," *Economic Journal*, 90, 95–106.

——— (1989): "Entry and Exit Decisions under Uncertainty," *Journal of Political Economy*, 97, 620–638.

——— (1991): "Irreversible Investment with Price Ceilings," *Journal of Political Economy*, 99, 541–557.

DIXIT, A. K., AND R. S. PINDYCK (1994): *Investment under Uncertainty*. Princeton University Press, Princeton.

FUDENBERG, D., R. GILBERT, J. STIGLITZ, AND J. TIROLE (1983): "Pre-emption, Leapfrogging, and Competition in Patent Races," *European Economic Review*, 22, 3–31.

FUDENBERG, D., AND J. TIROLE (1985): "Pre-emption and Rent Equalization in the Adoption of New Technology," *Review of Economic Studies*, 52, 383–401.

——— (1986): "A Theory of Exit in Duopoly," *Econometrica*, 54, 943–960.

——— (1991): *Game Theory*. MIT Press, Cambridge.

HOPPE, H. (2000): "Second-mover Advantages in the Strategic Adoption of New Technology under Uncertainty," *International Journal of Industrial Organization*, 18, 315–338.

HOWELL, S., A. STARK, D. NEWTON, D. PAXSON, M. CAVUS, J. PEREIRA, AND K. PATEL (2001): *Real options: Evaluating corporate investment opportunities in a dynamic world*. Harlow, UK: Financial Times/Prentice Hall.

Huisman, K., and P. Kort (1999): "Effects of strategic interactions on the option value of waiting," CentER Discussion Paper 9992, CentER, Tilburg University.

Katz, M. L., and C. Shapiro (1987): "R&D Rivalry with Licensing or Imitation," *American Economic Review*, 77(3), 402–420.

Kulatilaka, N., and E. Perotti (1998): "Strategic growth options," *Management Science*, 44, 1021–1031.

Lambrecht, B. M., and W. Perraudin (2003): "Real Options and Pre-emption under Incomplete Information," *Journal of Economic Dynamics and Control*, 27(4), 619–643.

Lippman, S. A., and J. W. Mamer (1993): "Preemptive Innovation," *Journal of Economic Theory*, 61, 104–119.

Maskin, E., and J. Tirole (1988): "A theory of dynamic oligopoly, I: Overview and quantity competition with large fixed costs," *Econometrica*, 56(3), 549–569.

McDonald, R., and D. Siegel (1986): "The Value of Waiting to Invest," *Quarterly Journal of Economics*, 101, 707–728.

Ofcom (2005): "Ofcom's approach to risk in the assessment of the cost of capital—Final statement," Discussion paper, Ofcom.

Pawlina, G., and P. M. Kort (2006): "Real Options in an Asymmetric Duopoly: Who Benefits from Your Competitive Disadvantage?," *Journal of Economics and Management Strategy*, 15(1), 1–35.

Perotti, E., and S. Rossetto (2001): "Strategic Advantage and the Optimal Exercise of Entry Options," Discussion Paper 3061, CEPR Discussion Paper.

Pindyck, R. S. (1988): "Irreversible Investment, Capacity Choice, and the Value of the Firm," *American Economic Review*, 79, 969–985.

Smets, F. (1991): "Exporting versus Foreign Direct Investment: The Effect of Uncertainty, Irreversibilities and Strategic Interactions," Mimeo.

Weeds, H. F. (2002): "Strategic delay in a real options model of R&D competition," *Review of Economic Studies*, 69(3), 729–747.