



UNREINFORCED MASONRY WALLS SUBJECTED TO
OUT-OF-PLANE SEISMIC ACTIONS

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Thesis submitted in fulfilment of the requirements for the degree of
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CONTENTS

CONTENTS	v
ABSTRACT	xi
STATEMENT OF ORIGINALITY	xiii
ACKNOWLEDGEMENT	xv
LIST OF PUBLICATIONS	xix
NOTATION	xxi
1 INTRODUCTION	1
1.1 Out-of-Plane Walls	1
1.2 Shortcomings of Current Design Methodology	4
1.3 Research Objectives, Scope and Outline	8
I EXPERIMENTAL WORK	11
2 QUASISTATIC CYCLIC TESTING	13
2.1 Introduction	14
2.2 Review of Past Experimental Research	16
2.3 Test Programme and Methodology	19
2.4 Load-Displacement Behaviour	33
2.5 Observed Damage and Crack Patterns	51
2.6 Wall Deformation Profiles	59
2.7 Conclusions	63
3 SHAKETABLE TESTING	65
3.1 Introduction	66
3.2 Test Programme and Methodology	67

3.3	Data Processing	78
3.4	Load-Displacement Behaviour	84
3.5	Observed Damage and Crack Patterns	100
3.6	Conclusions	108
II ANALYTICAL WORK 111		
4	ULTIMATE STRENGTH PREDICTION IN MORTAR-BONDED MASONRY WALLS	113
4.1	Introduction	114
4.2	Review of Plastic Analysis Methods	115
4.3	Moment Capacities	124
4.4	Walls with Openings	140
4.5	Analysis Results and Comparison with Experiment	144
4.6	Conclusions	150
5	PROBABILISTIC METHODOLOGY FOR HORIZONTAL BENDING	153
5.1	Introduction	154
5.2	Previous Use of Probabilistic Models	156
5.3	Random Variability in the Material Properties	157
5.4	Theoretical Basis	163
5.5	Reduction in the Ultimate Strength	166
5.6	Expected Likelihood of Each Failure Mode	176
5.7	Comparison with Experiment	181
5.8	Parametric Study	187
5.9	Conclusions	192
6	COLLAPSE LOAD PREDICTION IN DRY MASONRY WALLS	193
6.1	Introduction	194
6.2	Collapse Mechanisms	196
6.3	Boundary Conditions	199
6.4	General Method for Calculating the Collapse Load	208
6.5	Formulations for Type G, J, B, K1 and K2 Mechanisms	220
6.6	Comparison of Predictions with Experimental Results	234

6.7	Sensitivity of Predictions to the Choice of Mechanism	246
6.8	Conclusions	254
7	LOAD-DISPLACEMENT MODELLING	257
7.1	Introduction	258
7.2	Load-Displacement Capacity Model	259
7.3	Model Validation Using Experimental Data	275
7.4	Displacement-Based Seismic Assessment	289
7.5	Examples	298
7.6	Conclusions	305
8	CONCLUSION	307
8.1	Experimental Work	307
8.2	Analytical Work	308
8.3	Topics for Future Research	312
	REFERENCES	313
	III APPENDICES	329
A	MATERIAL TESTING	331
A.1	Introduction	331
A.2	Flexural Tensile Strength	332
A.3	Lateral Modulus of Rupture	342
A.4	Compression Tests	344
A.5	Coefficient of Friction	359
B	QUASISTATIC CYCLIC TESTING	363
B.1	Miscellaneous Technical Details	363
B.2	Analysis of Response During Initial Push	370
B.3	Analysis of Cyclic Response	375
B.4	Wall Deformation Profiles	396
B.5	Crack Pattern Photographs	400

C	SHAKETABLE TESTING	405
C.1	Test Run Nomenclature	405
C.2	Earthquake Input Motions	406
C.3	Standard Data Processing	412
C.4	Cyclic Response Analysis	430
C.5	Data Filtering	457
C.6	Load-Displacement Graphs	470
D	DIAGNOSIS OF SHAKETABLE IMPACTS	511
D.1	Introduction	511
D.2	Diagnosed Cause of the Impacts	517
D.3	Experimental Behaviour	519
D.4	Numerical Simulation of the Ram's Motion	521
D.5	Predicting the Onset of the Impacts	528
D.6	Diagnostic Experimental Tests	531
D.7	Conclusions and Recommendations	543
E	MOMENT CAPACITIES	545
E.1	AS 3700 Expressions for Ultimate Moment Capacities	545
E.2	Torsional Capacity of a Mortar-Bonded Section	548
E.3	Torsional Friction Capacity of a Dry Masonry Section	550
E.4	Biaxial Failure Criterion Model for Ultimate Moment Capacity	556
F	PROBABILISTIC METHODOLOGY FOR HORIZONTAL BENDING	563
F.1	Distribution Fitting to Material Properties	563
F.2	Comparison of Analytical Results to Test Data	563
G	COLLAPSE LOAD PREDICTION IN DRY MASONRY WALLS	569
G.1	Internal Work for an In-Plane Shear Panel	569
G.2	Formulations for Type G, J, B, K1 and K2 Mechanisms	573
G.3	General Formulation for Mechanism J	617
G.4	Worked Examples	633
G.5	Additional Parametric Study Results	642

H	LOAD-DISPLACEMENT MODELLING	645
H.1	Load-Displacement Capacity Based on Rocking	645
H.2	Influence of the Load Shape Function on the Load Capacity	663

ABSTRACT

During a seismic event, the walls within an unreinforced masonry (URM) building must possess sufficient capacity to withstand out-of-plane collapse. Traditionally, design against this type of failure has been performed using a force-based (FB) approach, in which the engineer must ensure that the force capacity of the wall is not exceeded during a design earthquake. In recent years, however, seismic design philosophy for ductile systems has experienced a move away from FB methods and toward displacement-based (DB) methods, where the aim is to ensure that structural deformations are kept within acceptable displacement limits.

URM walls subjected to out-of-plane actions make a prime candidate for the development of such methodology. This is particularly true for two-way spanning walls, which have significant displacement capacity as well as good energy dissipation capability during cyclic response—both highly favourable characteristics with respect to seismic performance.

This thesis documents research undertaken at the University of Adelaide into the seismic response of two-way URM walls subjected to out-of-plane actions. The aims of this work were to facilitate improvements to the presently-used FB design methods and to provide a basis for the development of a reliable DB design approach.

The following outcomes have been achieved:

- Characterisation of the load-displacement behaviour of two-way walls through quasistatic cyclic testing using airbags;
- Verification of this behaviour under true seismic loading conditions by means of dynamic shaketable tests;
- Improvements to the current state-of-the-art design approach for predicting the ultimate load capacity of walls possessing tensile bond strength;

- A probabilistic approach to deal with the different modes of possible failure in horizontal bending;
- Development of analytical methodology for predicting the load capacity of walls using the assumption of zero tensile bond strength;
- A proposed model for representing the nonlinear inelastic load-displacement behaviour of two-way walls; and finally,
- Implementation of the load-displacement model into a simple **DB** seismic assessment procedure.

It is anticipated that this research will eventually culminate in a multi-tiered seismic design procedure incorporating both the **FB** and **DB** components, with applicability toward the design of new buildings and assessment of existing buildings alike.

STATEMENT OF ORIGINALITY

I, Jaroslav Vaculik, hereby declare that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution. To the best of my knowledge and belief, it contains no material previously published or written by another person, except where due reference has been made in the text.

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Image in Figure 5.11	<i>Willis</i> [2004]	C. R. Willis
Images in Figure 6.2	<i>D'Ayala and Speranza</i> [2003]	Earthquake Engineering Research Institute (EERI)
Images in Figures 6.18, G.17, G.18, and Tables 6.4, 6.5	<i>Restrepo Vélez and Mageses</i> [2009]	Istituto Universitario di Studi Superiori (IUSS) Press, Pavia

*For full details of each source, see list of references.

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TO MY PARENTS
AND
TO KATRINA

LIST OF PUBLICATIONS

The following is a list of selected publications related to the work in this thesis published at the time of its completion (April, 2012). Papers 1–5 report various aspects of the experimental work presented in Chapter 2. Paper 7 reports early findings from the shaketable study in Chapter 3. Paper 9 describes a nonlinear time-history analysis based on an early version of the load-displacement model presented in Chapter 7. Paper 10 deals with certain aspects of the probabilistic methodology developed in Chapter 5. Papers 11 and 12 describe the collapse load prediction model reported in Chapter 6. Papers 6 and 8 provide a general overview of the overall research.

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 11. Vaculik, J., Griffith, M., and Magenes, G. (2010), Collapse load predictions for masonry walls in bending, in *Proc. 8th International Masonry Conference*, Dresden, Germany. [Available from <http://www.masonry.org.uk>]
 12. Vaculik, J., Griffith, M. C., and Magenes, G. (2012), Dry stone masonry walls in bending—Part II: Analysis, *International Journal of Architectural Heritage*, in press.

NOTATION

SYMBOLS

The following list contains frequently used variables, functions and operators appearing in this thesis. A list of commonly used subscripts and their meanings is also provided (p. xxvii), and where applicable, these are indicated for the respective symbols. The third column indicates the dimensions, whereby: L = length, t = time, M = mass, F = force, '-' = dimensionless, X = generic property.

a	acceleration (subscripts: amp , min , max)	$L t^{-2}$
a	dimensionless parameter related to mechanism shape	-
$a_{w.avg}$	wall average acceleration	$L t^{-2}$
$a_{w.cent}$	wall central acceleration	$L t^{-2}$
A	area	L^2
A_o	surface area of opening	L^2
A_w	surface area of wall	L^2
A'	virtual displaced area	L
$C\langle X \rangle$	coefficient of variation of X	-
$Char\langle X \rangle$	characteristic value of X	X
$const(X)$	constant component of X	X
d	vertical distance measured from top edge of the wall	L
dX	differential of X	X
D_n	Kolmogorov-Smirnov test statistic	-
e'	virtual work performed by OBL (subscripts: m , r)	F
E	Young's modulus of elasticity	$F L^{-2}$
E	external work (subscripts: m , r , W , O , tot)	$F L$
E_j	elasticity modulus of mortar joint	$F L^{-2}$
E_m	elasticity modulus of masonry	$F L^{-2}$

E_u	elasticity modulus of brick unit	$F L^{-2}$
E'	external virtual work (subscripts: see E , external work)	F
$\mathbb{E}\langle X \rangle$	expected value (mean) of X	X
f	frequency	t^{-1}
f_c	cutoff frequency	t^{-1}
f_e	effective frequency of linearised SDOF system	t^{-1}
f_{mc}	unconfined compressive strength of masonry	$F L^{-2}$
f_{mt}	flexural tensile strength of masonry	$F L^{-2}$
f_o	excitation frequency	t^{-1}
f_{ut}	modulus of rupture of brick unit	$F L^{-2}$
$f\langle \cdot \cdot \cdot \rangle$	stress capacity function, defined by Eq. (6.22)	$F L^{-2}$
F	force (subscripts: w , ult , amp , min , max , env)	F
F_{ht}	force resistance at $\Delta = \frac{1}{2}t_u$	F
F_o	generic force capacity under uniform acceleration loading	F
F_o^*	generic force capacity under modal acceleration loading	F
F_{ut}	ratio of mean f_{ut} and mean f_{mt}	–
g	acceleration due to gravity	$L t^{-2}$
G_h	geometric constant defined by Eq. (5.30)	–
G_n	natural slope of diagonal crack	–
h_e	element height	L
h_u	brick unit height	L
H	height	L
H_d	height of diagonal crack	L
H_e, H_{eff}	effective mechanism height	L
H_o	opening height	L
H_r	height of in-plane mechanism module, or short vertical crack	L
H_t, H_{tot}	total mechanism height	L
H_w	wall height	L
k_{be}	bed joint elastic torsion coefficient	–
k_{bp}	bed joint plastic torsion coefficient	–
k_{line}	geometric constant defined by Eq. (5.11)	–
k_{res}	geometric constant defined by Eq. (5.36)	–
k_{step}	geometric constant defined by Eq. (5.10)	–
K	stiffness	$F L^{-1}$
K_e	effective stiffness of linearised SDOF system	$F L^{-1}$
K_{ht}	effective secant stiffness at $\Delta = \frac{1}{2}t_u$	$F L^{-1}$

K_{ini}	initial uncracked stiffness	$F L^{-1}$
l_C	crack span	L
l_u	brick unit length	L
L	length	L
L_a	length of short horizontal crack	L
L_d	length of diagonal crack	L
L_e, L_{eff}	effective mechanism length	L
L_o	opening length	L
L_t, L_{tot}	total mechanism length	L
L_w	wall length	L
m	moment per single element (subscripts: $v, h, d, step, line, mix, ult, res$)	$F L$
M	mass	M
M	moment (subscripts: v, h, d, vy, vo)	$F L$
M_w	wall mass	M
M^*	effective mass of SDOF system	M
\bar{M}	moment per unit length of crack (subscripts: v, h, d, c)	F
$Med\langle X \rangle$	median value of X	X
n	number of samples in data set	–
n	flexure/torsion interaction exponent	–
n_{hs}, n_{vs}	number of horizontal and vertical supports	–
N	axial force	F
N_m, N_r	number of out-of-plane and in-plane modules participating in the mechanism (subscripts: m, r)	–
N_w	number of out-of-plane walls in a specimen	–
$p_X(\dots)$	probability density function of X	–
P_{step}	probability of stepped failure	–
$P_X(\dots)$	cumulative distribution function of X	–
$P_X^{-1}(\dots)$	inverse cumulative distribution function of X	–
$Pr(\dots)$	probability	–
q	pressure (subscripts: $w, ult, test, calc$)	$F L^{-2}$
r	coefficient of f_{mt} , in expression for τ_{um}	–
r	dimensionless parameter related to mechanism shape	–
r_h, r_v	ratios of applied moments at failure and their uniaxial moment capacities (subscripts: h, v)	–

r_o	bed joint overlap ratio, defined by Eq. (4.26)	–
$\text{rand}(X)$	random component of X	X
R_C	cycle centrality ratio	–
R_f	rotational restraint factor along vertical edge	–
$\mathcal{R}_K\langle \cdot \cdot \cdot \rangle$	moment derivative (dM/dx) equation defined for various mechanism cross sections	F
R_O	cycle overlap ratio	–
R_{ts}	rotational restraint factor for top edge (subscripts: m, w)	–
R_T	elastic spectrum reduction factor based on period	–
R_{vs}	rotational restraint factor for vertical edge	–
R_ξ	elastic spectrum reduction factor based on damping	–
s	shear slip	L
s_b	bed joint overlap length	L
s_e	element length	L
S_a	spectral acceleration	$L t^{-2}$
S_d	spectral displacement	L
$\$ \langle X \rangle$	standard deviation of X	X
t	time	t
t	thickness; wall thickness	L
t_j	mortar joint thickness	L
t_u	brick unit thickness or wall thickness in single leaf masonry	L
T	torsion	$F L$
T	period	t
T_e	effective period of linearised SDOF system	t
T_o	excitation period	t
u	displacement	L
u	effective displacement of SDOF system	L
u_d	effective displacement demand	L
u'	virtual displacement	–
\dot{u}	velocity	$L t^{-1}$
\ddot{u}	acceleration	$L t^{-2}$
U	energy; internal work (subscripts: $m, r, w, C, fs, vy, O, tot$)	$F L$
U_{box}	energy enclosed within F - Δ cycle bounding box	$F L$
U_{loop}	energy enclosed within F - Δ hysteresis loop over full cycle	$F L$
$U_{1/2\text{cyc}}$	energy enclosed within F - Δ hysteresis loop over half-cycle	$F L$
U'	internal virtual work	F

	(subscripts: see U , internal work)	
$U'_r\langle \dots \rangle$	internal virtual work function for in-plane panel, defined by Eq. (6.36)	FL
V	shear force	F
V	volume	L^3
V'	virtual displaced volume	L^2
$w\langle \dots \rangle$	load distribution function	FL^{-1}
W	weight	F
W_{eff}	effective mechanism weight	F
W_{ho}	horizontally acting component of overburden weight	F
W_{tot}	total mechanism weight	F
W_{vo}	vertically acting component of overburden weight	F
W_w	wall weight	F
x	spatial coordinate	L
y	spatial coordinate	L
Z	moment modulus	L^3
Z_{be}	elastic modulus over single element	L^3
\bar{Z}	moment modulus per unit length	L^2
	(subscripts: v, h)	
α	normalised effective aspect ratio, defined by Eq. (6.44)	–
α_s	aspect ratio parameter defined by Eq. (7.37)	–
β	effective aspect ratio, defined by Eq. (6.43)	–
γ	weight density	FL^{-3}
Γ	mode participation factor	–
δ	normalised displacement, defined by Eq. (2.2)	–
	(subscripts: r, h, s, f, u, y)	
Δ	displacement	L
	(subscripts: amp , min , max , peak , env , y, m, r, w)	
$\Delta_{\text{w.cent}}$	central wall displacement	L
$\Delta_{\text{w.cent0}}$	central wall displacement, zeroed at start of run	L
$\Delta_{0.8Fu}$	displacement range encompassing 80% of ultimate strength	L
$\Delta\langle \dots \rangle$	displacement shape function	L
Δ'	virtual displacement	–
	(subscripts: see Δ , displacement)	
ϵ	OBL eccentricity factor, defined in Figure 6.6	–
ϵ	strain	–
ζ	internal work contribution factor	–
η	orthogonal strength ratio, defined by Eq. (5.4)	–

	(subscripts: step , line , mix , res)	
η	OBL orthogonal factor, defined by Eq. (6.4) (subscripts: m , r , w)	–
θ	rotation; crack rotation (subscripts: v , h , d)	–
θ_κ	angle of applied moment	–
θ'	virtual rotation (subscripts: see θ , rotation)	L^{-1}
κ	non-dimensional stiffness, as λ/δ	–
κ	slope of applied moment	–
λ	lateral load multiplier, defined by Eq. (2.3) (subscripts: o , p , r , h , s , f)	–
λ_o	collapse load multiplier	–
μ	friction coefficient	–
μ	coefficient of σ_v , in expression for τ_{um}	–
μ_m	friction coefficient of masonry	–
μ_o	friction coefficient between wall and overburden load	–
μ_Δ	displacement ductility	–
ν	Poisson's ratio	–
ν_u	Poisson's ratio of the brick unit	–
ζ	viscous damping ratio	–
ζ_e	total effective viscous damping	–
ζ_{hyst}	equivalent viscous damping based on hysteresis	–
ζ_{nom}	nominal viscous damping	–
ω	slope of in-plane shear crack	–
ρ	mechanism cross sectional shape parameter	–
$\rho\langle \dots \rangle$	mass density function	ML^{-1}
σ	axial stress	FL^{-2}
σ_v	vertical compressive stress	FL^{-2}
σ_{vo}	vertical compressive stress applied at top of the wall (subscripts: m , r , w)	FL^{-2}
Σ_v	ratio of σ_v and mean f_{mt}	–
τ	shear stress	FL^{-2}
τ_f	frictional shear stress capacity of masonry bond	FL^{-2}
τ_{um}	ultimate shear stress capacity of masonry bond	FL^{-2}
ϕ	capacity reduction factor (subscripts: char , mean , med)	–
Φ	OBL degree-of-freedom factor, defined by Eq. (6.6)	–

	(subscripts: m, r, w)	
$\Phi(\dots)$	mode shape function	–
$\Phi_N(\dots)$	standard normal cumulative distribution function	–
φ	mode shape	–
φ	crack angle	–
φ_n	natural angle of diagonal crack	–
ψ	overburden weight ratio, defined by Eq. (6.1)	–
ω	angular frequency	t^{-1}
ω_e	effective angular frequency of linearised SDOF system	t^{-1}
\hat{X}	expected value (mean) of X	X
X'	virtual form of X	$X L^{-1}$

SUBSCRIPTS

amp	amplitude
avg	average
c	capacity
calc	from calculation
char	characteristic value
C	crack
d	diagonal bending
e, eff	effective
env	envelope
f	combined frictional F - Δ component
fs	frictional shear
h	horizontal bending
h	horizontal bending F - Δ component
ini	initial
line	line failure mode in horizontal bending
m	out-of-plane module
max	maximum
mean	mean value
med	median value
min	minimum
mix	mixed failure mode (stepped and line) in horizontal bending
nom	nominal
o	rigid body capacity

O	overburden load
p , peak	peak
r	in-plane module/wall
r	rocking F - Δ component
res	residual (post-cracking) capacity
s	overburden load sliding F - Δ component
step	stepped failure mode in horizontal bending
t , tot	total
test	from experimental test
u , ult	ultimate capacity
v	vertical bending
v_0	vertical bending along top edge
v_y	vertical bending along vertical crack
w	wall; out-of-plane wall
W	self-weight
y	yield

ABBREVIATIONS

BCRA	British Ceramic Research Association
CDF	cumulative distribution function
CoV	coefficient of variation
CS	capacity spectrum
DB	displacement-based
DOF	degree-of-freedom
DSM	dry-stack masonry
FB	force-based
FRP	fibre-reinforced polymer
KS	Kolmogorov-Smirnov
LVDT	linear variable differential transformer
MDOF	multi-degree-of-freedom

OBL	overburden load
PDF	probability density function
PGA	peak ground acceleration
PGD	peak ground displacement
PGV	peak ground velocity
PID	proportional-integral-derivative
PSA	peak spectral acceleration
PSD	peak spectral displacement
PSV	peak spectral velocity
SDOF	single-degree-of-freedom
StD	standard deviation
THA	time-history analysis
URM	unreinforced masonry
VW	virtual work

