PHYSICAL REVIEW C **87**, 015801 (2013)

Variations of nuclear binding with quark masses

M. E. Carrillo-Serrano
CSSM and ARC Centre of Excellence for Particle Physics at the Tera-Scale, and School of Chemistry and Physics, University of Adelaide, Adelaide, South Australia 5005, Australia

I. C. Cloët and K. Tsushima
CSSM, School of Chemistry and Physics, University of Adelaide, Adelaide, South Australia 5005, Australia

A. W. Thomas
CSSM and ARC Centre of Excellence for Particle Physics at the Tera-Scale, and School of Chemistry and Physics, University of Adelaide, Adelaide, South Australia 5005, Australia

I. R. Afnan
School of Chemical and Physical Sciences, Flinders University, GPO Box 2100, Adelaide, South Australia 5001, Australia
(Received 28 August 2012; published 9 January 2013)

We investigate the variation with light quark mass of the mass of the nucleon as well as the masses of the mesons commonly used in a one-boson-exchange model of the nucleon-nucleon force. Care is taken to evaluate the meson mass shifts at the kinematic point relevant to that problem. Using these results, we evaluate the corresponding changes in the energy of the $^1S_0$ antibound state and the binding energies of the deuteron, triton, and selected finite nuclei by using a one-boson exchange model. The results are discussed in the context of possible corrections to the standard scenario for Big Bang nucleosynthesis in the case where, as suggested by recent observations of quasar absorption spectra, the quark masses may have changed over the age of the Universe.

DOI: 10.1103/PhysRevC.87.015801 PACS number(s): 98.80.Cq, 06.20.Jr, 21.10.Dr, 26.35.+c

I. INTRODUCTION

In the last decade there has been considerable interest in the possibility that the fundamental "constants" of Nature may actually change with time [1]. Although it remains controversial, there is growing evidence that the fine structure constant α may have varied by an amount of the order of a few parts in 10^{-5} over a period of 5–10 billion years [2–5]. It has even been suggested that this variation may have a dipole structure as we look back in different directions [6]. Although this possible variation is quite small, within the framework of most attempts at grand unification, a variation of α implies considerably larger percentage changes in quantities such as the QCD scale Λ_{QCD} and in the quark masses m_q [7–9]. For example, in Ref. [7] it was shown that the variation $\delta m_q/m_q$ would be of order 38 times that of $\delta\alpha/\alpha$.

In the light of these developments it is very natural to ask what other signatures there may be for such changes. These may, for example, be consequent changes in hadron masses or magnetic moments [10–12]. Indeed, in some cases the level of precision possible in modern atomic, molecular, and optical physics means that it may even be feasible to detect the minute variations expected under the hypothesis of linear variation until the present day over a period as short as a year [13–15].

Another consequence of a variation in the parameters relevant to hadron structure is the possibility of observable consequences in Big Bang nucleosynthesis (BBN) or in other nuclear phenomena such as the composition of the ash of long extinct natural nuclear reactors [16–18]. Ideally one would like to have a direct solution for nuclear energy levels as a function of quark mass starting from QCD itself. Indeed, the effect of quark mass changes on the nucleon-nucleon force has been studied using QCD sum rules [19], in effective field theory [20,21], and most recently by including constraints from lattice QCD [22,23]. However, for the moment these lattice studies are at too high a quark mass to provide an accurate constraint [23,24]. Since the direct calculation within QCD is not currently feasible, one must rely on more traditional models of the nucleon-nucleon force. The latest work of Flambaum and Wiringa [25] on this topic involved the study of the variation of nuclear binding with quark mass using the Argonne potential and Schwinger-Dyson estimates of the variation of meson masses.

In this work we employ a one boson-exchange (OBE) model of the nuclear force to calculate the variation with changes in the quark mass of the binding energies of selected finite nuclei as well as the energy of the $^1S_0$ antibound state and the binding energies of the deuteron and triton. The OBE approach has deep roots in dispersion theory and all of the mesons required are found in the Particle Data Group (PDG) summary of particle properties [26]. Apart from its intrinsic interest, this approach complements the work of Ref. [25] and a comparison of the two provides one way to gauge the possible model dependence of the variations reported. The method used here involves a detailed study of the variation of the mass of each of the mesons usually employed in the OBE picture of the nucleon-nucleon (NN) force. Care is taken to estimate this variation at the relevant kinematic point, not just at the real, on-shell meson mass or its pole position. These changes are then introduced into the quark-meson coupling (QMC) model for some light nuclei and a typical OBE model for the two nucleon systems and a Faddeev calculation of the triton.

In Sec. II we examine the best available constraints on the variation with quark mass of the masses of the mesons

σ_0, σ_1, ω, ρ, π, and η, whose exchange between nucleons yields the OBE force. In each case we employ the best available constraints, whether from chiral symmetry or lattice QCD or, in the case of the σ meson, from the Roy equation. Section III presents the consequences of these mass changes for the binding energy of finite nuclei, while the two-nucleon system and triton are discussed in Sec. IV. The final section is reserved for some concluding remarks.

II. MESON MASSES

The variations of the π and η masses with light quark mass are well understood through the Gell-Mann–Oakes–Renner (GMOR) relation, which is a consequence of the chiral symmetry of QCD. For the other exchanged bosons, σ_0, $\sigma_1\rho$, and ω, we need more information. In the scalar-isoscalar case (σ_0) we introduce a description of the bare mass of the σ ($m_\sigma^{(0)}$) in terms of m_π using the Nambu-Jona-Lasinio (NJL) model [27,28]. Since this model respects the chiral symmetry of QCD there are powerful constraints between the properties of the π and the σ in this model. The self-energy of the σ is treated with an effective Lagrangian tuned to reproduce the model-independent pole position of the σ resonance in the complex energy plane found by Leutwyler *et al.* [29]. In the case of the ρ (and the closely related ω) we have excellent constraints from lattice QCD studies as a function of quark mass, supplemented with chiral effective field theory [30].

A. Variation in m_σ with m_q

In this work we choose to parametrize the intermediate-range attraction in the NN force in terms the exchange of a σ meson, following the traditional OBE approach. Earlier work on the effect of changes in quark masses by Flambaum and Wiringa [25] used explicit two-pion exchange for this purpose. Almost certainly the reality is somewhere in between these extremes and a comparison between our results and those of Ref. [25] should serve to pin down the uncertainties in this sector of the calculation.

The existence of the σ meson has been somewhat controversial, largely because its width is comparable with its mass. However, a careful dispersion relation treatment using the Roy equation has served to accurately locate a pole which can be unambiguously identified with the σ meson. Of course, because of the large imaginary part of the energy of this pole, one cannot easily relate the position of the pole to the position of a bump in the $\pi\pi$ cross section. When it comes to the mass of the virtual σ meson exchanged in a OBE NN potential, it is a third value that is of interest. Indeed, the invariant mass of a meson exchanged in a typical NN interaction is very near zero and so we actually need the σ mass for $p^2 \sim 0$. This is most readily found within an effective Lagrangian approach.

Any effective Lagrangian treatment of the σ meson involves a "bare" σ meson coupled to two pions. Since the "bare" σ is a zero-width state, the inclusion of pion loops ensures that the position and width of the pole in the second Riemann sheet are reproduced. When required, the variation of the mass of the bare state with quark mass will be calculated within the NJL model. As we remarked earlier this approach respects the symmetries of QCD and links the properties of the pion and kaon. For example, it guarantees that in the chiral limit (zero-mass pion) the σ mass is exactly twice the constituent quark mass. With the mass of the "bare" (two-quark) σ fixed this way, the propagator of the dressed σ is described as a bare scalar propagator plus an infinite series of contributions of the form shown in Fig. 1. By calling this self-energy $\Sigma^\sigma_{\pi\pi}$, the total propagator can be written as

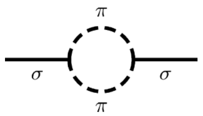

FIG. 1. Self-energy contributions for the σ meson.

$$\Delta_\sigma = \frac{i}{p^2 - \left(m_\sigma^{(0)}\right)^2} + \frac{i}{p^2 - \left(m_\sigma^{(0)}\right)^2}\left(i\Sigma^\sigma_{\pi\pi}\right)\frac{i}{p^2 - \left(m_\sigma^{(0)}\right)^2} + \cdots,$$

which resums to

$$\Delta_\sigma = \frac{i}{p^2 - \left(m_\sigma^{(0)}\right)^2 + \Sigma^\sigma_{\pi\pi}}. \tag{1}$$

The pole in Δ_σ is the mass of the σ resonance. This pole was calculated by Leutwyler *et al.* using the method of Roy equations, which is model independent [31]. They obtained a pole located in the complex second sheet for p at

$$p = m_\sigma - \frac{i}{2}\Gamma = 441^{+16}_{-8} - i272^{+9}_{-12.5}\ \text{MeV}. \tag{2}$$

The real part of the position of the resonance, $m_\sigma = 441$ MeV, is in the range 400–550 MeV given by the PDG [26], while its width, $\Gamma = 544$ MeV, is within the range 400–700 MeV, also from the PDG. A recent analysis by Pelaez *et al.* [32], based on the GPKY equations, yields a pole position of $457^{+14}_{-13} - i279^{+11}_{-7}$ MeV, which is also consistent with Eq. (2) within errors.

Having a reliable value for the σ pole we can find a relation that lets us fix $\Sigma^\sigma_{\pi\pi}$ such that

$$\sqrt{\left(m_\sigma^{(0)}\right)^2 - \Sigma^\sigma_{\pi\pi}\left(m_\sigma^2\right)} \simeq 441 - i272\ \text{MeV}. \tag{3}$$

With derivative coupling of the bare σ to two pions (consistent with chiral symmetry), the expression for the $\pi\pi$ self-energy is found to be

$$i\Sigma^\sigma_{\pi\pi} = \frac{3}{2}\gamma_0^2 \int \frac{d^4k}{(2\pi)^4}\frac{[k^\mu(p-k)_\mu]^2}{\left(k^2 - m_\pi^2\right)[(p-k)^2 - m_\pi]^2}, \tag{4}$$

where k represents the pion loop momentum, p the σ momentum, and γ the $\sigma\pi\pi$ coupling (where initially we took the value γ_0 from Harada, Sannino, and Schechter [33]). We are considering all these particles as elementary, so this is just an effective theory, and like any other effective theory it has to be regularized. The regularization scheme we choose is to impose a dipole cutoff (at each vertex) on the loop momentum with mass Λ:

$$\left[1 - \frac{\left(\frac{p}{2} - k\right)^2}{\Lambda^2}\right]^{-4}, \tag{5}$$

TABLE I. Parameters fixed to reproduce the position of the σ meson pole [$\gamma_0 = 6.416 \times 10^{-3}$ (MeV^{-1})]. Δm_σ is the deviation of the fitted from the empirical value.

γ ($\times\gamma_0$)	Λ (MeV)	$m_\sigma^{(0)}$ (MeV)	Δm_σ (MeV) $\times 10^{-5}$
4.56	320	563.5	$1.2 - 1.0i$
4.60	330	600.2	$1.1 - 0.5i$
4.70	340	639.8	$1.2 - 0.0i$
4.83	350	683.5	$1.2 - 0.0i$
5.02	360	732.0	$0.0 + 0.0i$
5.27	370	790.2	$0.9 - 1.2i$
5.61	380	859.2	$0.4 - 0.9i$
6.07	390	945.4	$0.9 - 0.9i$

which is sufficient to ensure convergence. This dipole regulator contains simple poles in k (after writing it in the form of derivatives with respect to Λ), which permits us to use contour integration over the time component.

For the remaining integral over the three-momentum we rotated $\vec{k}$ in the complex plane $|\vec{k}|e^{i\theta}$, with $-\frac{3\pi}{2} < \theta < 0$, to ensure that the imaginary part is located in the complex second Riemann sheet. We also performed a numerical integration with the help of the routine *NIntegral* of MATHEMATICA. The final value of $\Sigma^\sigma_{\pi\pi}(p^2)$ depends on two parameters: the regularization mass Λ and the coupling constant γ_0. We choose a range of values for Λ such that, after fixing γ_0 and $m_\sigma^{(0)}$ to reproduce the pole position [Eq. (2)], $m_\sigma^{(0)}$ varies from 560 to 950 MeV. The results are summarized in Table I, where Δm_σ represents the deviation of our result for the pole position from that of Leutwyler and collaborators.

We then define $m_\sigma^2(\text{OBE}) = (m_\sigma^{(0)})^2 - \Sigma^\sigma_{\pi\pi}(0)$, because in an OBE potential model for the NN interaction the exchanged boson has nearly zero momentum. $m_\sigma^2(\text{OBE})$ is a real value because $\Sigma^\sigma_{\pi\pi}(0)$ is real. Thus any variation on $m_\sigma(\text{OBE})$ with respect to m_π is given by variations in $m_\sigma^{(0)}$ and $\Sigma^\sigma_{\pi\pi}(0)$:

$$\frac{\delta m_\sigma^2\,(\text{OBE})}{\delta m_\pi^2} = \frac{m_\sigma^{(0)}}{m_\pi}\frac{\delta m_\sigma^{(0)}}{\delta m_\pi} - \frac{\delta \Sigma^\sigma_{\pi\pi}(0)}{\delta m_\pi^2}, \tag{6}$$

and using the GMOR relation [34], we get

$$\frac{\delta m_\sigma\,(\text{OBE})}{m_\sigma\,(\text{OBE})} = \nu_\sigma \frac{\delta m_q}{m_q} \tag{7}$$

with

$$\nu_\sigma = \frac{m_\pi^2}{2m_\sigma^2\,(\text{OBE})}\left[\frac{\delta\left(m_\sigma^{(0)}\right)^2}{\delta m_\pi^2} - \frac{\delta\Sigma^\sigma_{\pi\pi}(0)}{\delta m_\pi^2}\right]. \tag{8}$$

We change m_π near the physical value and find the variation $\frac{\delta\Sigma^\sigma_{\pi\pi}(0)}{\delta m_\pi^2}$, which is almost constant for a small change in m_π^2, so we only need to find the slope of the plot of $\Sigma^\sigma_{\pi\pi}(0)$ versus m_π^2. We also need the variation of $m_\sigma^{(0)}$ with m_π near 140 MeV. For this purpose we used the NJL model, which is known to respect the chiral behavior of QCD, including the GMOR relation. The results for all the cases in Table I are listed in Table II.

From Table II, we notice that as Λ increases (which occurs when $m_\sigma^{(0)}$ grows) the value of $\Sigma^\sigma_{\pi\pi}(0)$ also grows and its contribution of $\frac{\delta\Sigma(0)}{\delta m_\pi^2}$ to $\frac{\delta m_\sigma^2(\text{OBE})}{\delta m_\pi^2}$ increases. However, in practice the variation of the σ bare mass is much larger and, in total, the larger $m_\sigma^{(0)}$ the smaller the coefficient ν_σ. Moreover, calculations within the QMC model tend to favor values for m_σ (OBE) near 550 MeV (see Sec. IV).

TABLE II. Calculations for the coefficient ν_σ, which relates the fractional change of the mass of the σ meson relevant to the OBE potential model to the fractional change in the quark mass.

$m_\sigma^{(0)}$	$\frac{\delta\Sigma^\sigma_{\pi\pi}(0)}{\delta m_\pi^2}$	$\frac{\delta(m_\sigma^{(0)})^2}{\delta m_\pi^2}$	$\frac{\delta m_\sigma^2(\text{OBE})}{\delta m_\pi^2}$	ν_σ
563.5	−0.145	2.677	2.822	0.089
600.2	−0.164	2.632	2.796	0.078
639.8	−0.189	2.576	2.765	0.068
683.5	−0.220	2.546	2.766	0.060
732.0	−0.261	2.502	2.763	0.052
790.2	−0.314	2.451	2.765	0.045
859.2	−0.389	2.401	2.790	0.038
945.4	−0.495	2.344	2.839	0.032

B. Variations in m_ρ and m_ω with respect to m_q

In the case of the ρ meson one can provide a very reliable description of the variation of its mass with m_q because we have a good deal of data taken from lattice calculations in partially quenched QCD from the CP-PACS Collaboration. Armour *et al.* [30] used these data in an analysis that included the leading and next-to-leading nonanalytic chiral corrections to the self-energy to make an extrapolation of the mass m_ρ to the chiral limit ($m_\pi \approx 0$). At the physical value of m_π they found excellent agreement with the physical value.

The relevant self-energy diagrams for the ρ are given in Fig. 2.

These yield the following expressions:

$$\Sigma^\rho_{\pi\pi} = -\frac{f^2_{\rho\pi\pi}}{6\pi^2}\int_0^\infty \frac{k^4 u^2_{\pi\pi}(k)\,dk}{\omega_\pi(k)\left(\omega_\pi^2(k) - \frac{\mu_\rho^2}{4}\right)}, \tag{9}$$

$$\Sigma^\rho_{\pi\omega} = -\frac{g_{\omega\rho\pi}}{12\pi^2}\mu_\rho\int_0^\infty \frac{k^4 u^2_{\pi\omega}(k)\,dk}{\omega_\pi^2(k)}, \tag{10}$$

where $f_{\rho\pi\pi} = 6.028$ and $g_{\omega\rho\pi} = 0.016$ MeV^{-1}. The regularization functions used in the analysis are

$$u_{\pi\omega}(k) = \frac{\Lambda^4}{(\Lambda^2 + k^2)^2}, \tag{11}$$

$$u_{\pi\pi}(k) = \frac{\left(\Lambda^2 + \frac{\mu_\rho^2}{4} - \mu_\pi^2\right)^2}{(\Lambda^2 + k^2)^2}, \tag{12}$$

and we use the approximations $m_\pi \ll m_{\omega,\rho}$ and $m_\rho \approx m_\omega$.

The fit to the partially quenched lattice QCD data for the ρ meson involved a fit of the form

$$m_\rho = \sqrt{\left(a_0 + a_2 m_\pi^2 + a_4 m_\pi^4\right)^2 + \Sigma_{\text{TOT}}}, \tag{13}$$

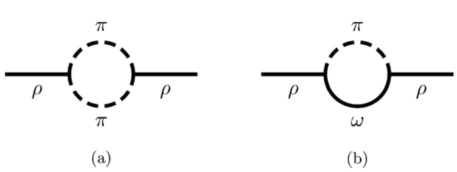

FIG. 2. Self-energy contributions for the ρ meson.

where $\Sigma_{\rm TOT} = \Sigma^{\rho}_{\pi\pi} + \Sigma^{\rho}_{\pi\omega}$, and the coefficients a_i are $a_0 = 832.00$ MeV, $a_2 = 4.94 \times 10^{-4}$ MeV^{-1}, $a_4 = -6.10 \times 10^{-11}$ MeV^{-3}, and $\Lambda = 655.00$ MeV (up to errors). At the physical pion mass (in full QCD) this yields a value of

$$m_\rho \approx 778 \text{ MeV}, \tag{14}$$

which shows remarkable agreement with the physical value, with a shift of only

$$m_\rho - m_\rho^{\rm phys} \sim 3.7 \text{ MeV}. \tag{15}$$

As in the case of the σ meson, we consider a OBE potential with almost zero momentum transfer, so that $\mu_\rho \sim 0$ in the propagator of Eq. (9) (not in the regulator, because the mass that appears there is the physical mass):

$$\Sigma^{\rho}_{\pi\pi} = -\frac{f^2_{\rho\pi\pi}}{6\pi^2}\int_0^\infty \frac{k^4 u^2_{\pi\pi}(k)\,dk}{\omega^3_\pi(k)}. \tag{16}$$

This of course changes the value of m_ρ, now denoted m_ρ(OBE), at the physical pion mass. Indeed, in this case it is near 762 MeV. The relation between m_ρ and m_π^2 near the physical value is almost linear and it presents a slope of $\frac{\delta m_\rho(\rm OBE)}{\delta m_\pi^2} = 0.001\,35$ MeV^{-1}. Following the analysis for m_σ we can relate this change with m_q in the following way:

$$\frac{\delta m_\rho(\rm OBE)}{m_\rho} = \left(\frac{m_\pi^2}{m_\rho(\rm OBE)}\frac{\delta m_\rho(\rm OBE)}{\delta m_\pi^2}\right)\frac{\delta m_q}{m_q}. \tag{17}$$

Using for m_ρ(OBE), henceforth simply written as m_ρ, the value 770 MeV, which is usually used in OBE models, we find

$$\frac{\delta m_\rho}{m_\rho} = 0.034\frac{\delta m_q}{m_q}. \tag{18}$$

The analysis for the ω meson is closely related to that of the ρ meson. However, the diagrams that contribute to the self-energy terms differ because there is no two-pion contribution because of G parity. In addition, $\Sigma^{\omega}_{\pi\rho}$ is $3\Sigma^{\rho}_{\omega\pi}$, because there are three possible ρ-π charge combinations. For the analytic terms in the expansion we use the same coefficients (a_i) as in the case of the ρ, because the mass difference between them is only of order 10 MeV. Then the variation of m_ω with respect to m_π^2 near the physical value gives $\frac{\delta m_\omega}{\delta m_\pi^2} = 0.000\,96$ MeV^{-1}, which leads to the relation

$$\frac{\delta m_\omega}{m_\omega} = 0.024\frac{\delta m_q}{m_q}, \tag{19}$$

where we used the physical mass for the ω, $m_\omega = 782$ MeV—again because that is the value typically used in a OBE potential. (The value obtained at zero momentum transfer would be 765 MeV.)

C. Summary of meson mass variation

For the η, like the pion, we use the GMOR relation to calculate the variation with respect to the u and d masses. In the case of the isovector scalar meson, σ_1, which has negative G parity and therefore does not couple to two pions, we use the NJL model (corresponding to the third column and second row of Table II) and Eq. (8) without the self-energy part. For convenience, in Table III we summarize the values of ν_i, defined as

$$\frac{\delta m_i}{m_i} = \nu_i\frac{\delta m_q}{m_q}, \tag{20}$$

which will be used below.

TABLE III. Coefficients ν_i summarizing the rate of variation of the masses of the mesons used in an OBE description of the NN force with respect to quark mass [see Eq. (20)].

Meson	ν (MeV)
π	0.5
η	0.012
σ_0	0.089
σ_1	0.072
ρ	0.034
ω	0.024

III. NUCLEON MASS

In order to compute the variation of nuclear binding energies with quark mass, we also need to know how the nucleon mass changes.

The variation with light quark mass is directly given by the so-called πN sigma commutator

$$\sigma_{\pi N} = m_q \langle N|\,\overline{q}q\,|N\rangle = m_q\frac{\delta m_N}{\delta m_q}, \tag{21}$$

where $\overline{q}q \equiv \overline{u}u + \overline{d}d$.

The last equality, which gives the information we need, follows from the Feynman-Hellmann theorem. A number of methods have been used to extract $\sigma_{\pi N}$ from pion-nucleon scattering data using dispersion relations, but the resulting value is still controversial.

Instead, the most reliable method seems to be to use fits to lattice QCD data for m_N as a function of m_q [35]. These fits, which build the constraints of chiral effective field theory, appear to yield very reliable values. We take the result of the latest analysis of PACS-CS data by Shanahan *et al.* [36], namely, $\sigma_{\pi N} = 45 \pm 6$ MeV. Thus we use

$$\frac{\delta m_N}{m_N} = 0.048\frac{\delta m_q}{m_q}. \tag{22}$$

IV. ^{7}Li, ^{12}C, AND ^{16}O NUCLEI

To study the effect of the quark mass variation on the single-particle energies of ^{7}Li, ^{12}C, and ^{16}O nuclei, it is highly desirable to use a nuclear model based on quark degrees of freedom. The QMC model, which originated with Guichon [37] as a description of nuclear matter, has been extended and improved to describe the properties of finite nuclei [38–40] and is ideal for this purpose. The successful features of the QMC model applied to various nuclear phenomena and hadronic properties in a nuclear medium are reviewed extensively in

TABLE IV. Single-particle energies (in MeV) for ^{7}Li, ^{12}C, and ^{16}O nuclei versus quark mass m_q (in MeV) calculated in the quark-meson coupling model [39]. E/nucleon stands for energy per nucleon. The standard value for the quark mass used in the QMC model is $m_q = 5.00$ MeV.

	States	m_q				
		4.90	4.95	5.00	5.05	5.10
^{7}Li						
p	$1s_{1/2}$	−19.02	−18.86	−18.71	−18.55	−18.39
	$1p_{3/2}$	−3.59	−3.51	−3.43	−3.34	−3.26
n	$1s_{1/2}$	−18.35	−18.21	−18.08	−17.94	−17.80
	$1p_{3/2}$	−3.24	−3.17	−3.10	−3.04	−2.97
	E/nucleon	−1.71	−1.66	−1.61	−1.57	−1.52
^{12}C						
p	$1s_{1/2}$	−26.26	−26.11	−25.96	−25.82	−25.67
	$1p_{3/2}$	−10.04	−9.94	−9.84	−9.74	−9.64
n	$1s_{1/2}$	−29.41	−29.26	−29.10	−28.96	−28.80
	$1p_{3/2}$	−12.94	−12.84	−12.73	−12.63	−12.52
	E/nucleon	−4.17	−4.11	−4.04	−3.97	−3.91
^{16}O						
p	$1s_{1/2}$	−29.07	−28.93	−28.78	−28.64	−28.49
	$1p_{3/2}$	−13.86	−13.75	−13.64	−13.53	−13.42
	$1p_{1/2}$	−12.06	−11.96	−11.87	−11.77	−11.67
n	$1s_{1/2}$	−33.09	−32.94	−32.78	−32.64	−32.49
	$1p_{3/2}$	−17.63	−17.51	−17.40	−17.28	−17.17
	$1p_{1/2}$	−15.82	−15.71	−15.61	−15.50	−15.40
	E/nucleon	−6.11	−6.04	−5.96	−5.89	−5.82

Ref. [41]. The model has been updated to study the properties of hypernuclei [42] and neutron star structure [43–45], where the quark structure of the nucleons and hyperons should play an important role at such high density.

We note that the energy functional derived from the QMC model [46] yields a density-dependent effective interaction of the Skyrme type that has recently been shown to be among the very few Skyrme forces that satisfies a selection of constraints derived from nuclear data by Dutra *et al.* [47]. In view of its origins at the quark level and its phenomenological success it seems appropriate to use this model to calculate the changes in binding energy of finite nuclei induced by small changes in quark mass. We calculate the change in the single-particle energies of these nuclei versus the current quark mass (m_q) and the mass of the nucleon (m_N) using the theory presented in Ref. [39] and the meson and nucleon mass changes calculated above. In future it would be worthwhile to extend this Hartree calculation to include Fock terms [48] or perhaps pursue a full relativistic Brueckner-Hartree-Fock treatment.

In Ref. [39] the standard values used to reproduce the nuclear matter saturation properties are $(m_N, m_\sigma, m_\omega, m_\rho) = (939, 550, 783, 770)$ MeV, with the current quark mass $m_q = 5.0$ MeV. For the calculation of finite nuclei, the ratio for the σ-N coupling constant and the mass, (g_σ^N/m_σ), was kept constant and fitted to the rms charge radius of ^{40}Ca, $r_{ch}(^{40}\mathrm{Ca}) = 3.48$ fm, by adjusting $m_\sigma \to \tilde{m}_\sigma = 418$ MeV. [Note that the variation of m_σ at fixed (g_σ^N/m_σ) has no effect on the nuclear matter properties.] To account for this, we calculate the shift in m_σ from 550 MeV for a given variation of the quark mass. This changes the ratio (g_σ^N/m_σ) and from that new value we deduce the corresponding shift in m_σ from 418 MeV, to be used in the finite nucleus calculation. First, with the nucleon mass fixed at $m_N = 939$ MeV and the variations of the meson masses, $\delta m_{\sigma,\omega,\rho}$, evaluated for quark mass variations of $\delta m_q = \pm 0.05$ and ± 0.1 MeV, we calculate the single-energies in ^{7}Li, ^{12}C, and ^{16}O. Note that the very small differences for the σ and ω meson mass values used to extract the relation in terms of δm_q in Secs. II A and II B, were neglected. In addition, we also calculate the energy per nucleon (E/nucleon). The results are given in Table IV.

From Table IV we see that the absolute values of the single-particle binding energies of each nucleus decrease as the quark mass increases. This is because an increase of the quark mass leads to a significant increase of the mass of the σ meson and this reduces the attraction arising from σ meson exchange by more than the repulsion associated with the ω decreases. It is interesting to point out that a small variation of the quark mass of 0.05 MeV is reflected in a change in the single-particle energies of the order of 0.1 MeV. That is, the impact is appreciable. Furthermore, we note that the binding energy per nucleon for each nucleus decreases linearly as the quark mass increases.

Next, we calculate the variation of the single-particle energies as the mass of the nucleon is varied. The results are given in Table V for the same nuclei as in Table IV. As the value of the nucleon mass increases the absolute values of the single-particle binding energies also increase. This seems to be natural, since the kinetic energy is suppressed.

It may be helpful to consider the binding energy per nucleon as a function of the quark mass. Based on the results given in Tables IV and V, and in Eq. (22), we get the following relations

TABLE V. Single-particle energies (in MeV) for ^{7}Li, ^{12}C, and ^{16}O nuclei versus nucleon mass m_N (in MeV), calculated in the quark-meson coupling model [39]. E/nucleon stands for energy per nucleon. The standard value for the nucleon mass used in the QMC model is $m_N = 939.0$ MeV.

	States	m_N				
		938.0	938.5	939.0	939.5	940.0
^{7}Li						
p	$1s_{1/2}$	−18.68	−18.69	−18.71	−18.72	−18.74
	$1p_{3/2}$	−3.40	−3.41	−3.43	−3.44	−3.45
n	$1s_{1/2}$	−18.05	−18.06	−18.08	−18.09	−18.10
	$1p_{3/2}$	−3.08	−3.09	−3.10	−3.11	−3.12
	E/nucleon	−1.60	−1.61	−1.61	−1.62	−1.63
^{12}C						
p	$1s_{1/2}$	−25.94	−25.95	−25.96	−25.97	−25.98
	$1p_{3/2}$	−9.82	−9.83	−9.84	−9.86	−9.87
n	$1s_{1/2}$	−29.08	−29.09	−29.10	−29.11	−29.12
	$1p_{3/2}$	−12.71	−12.72	−12.73	−12.74	−12.76
	E/nucleon	−4.02	−4.03	−4.04	−4.05	−4.06
^{16}O						
p	$1s_{1/2}$	−28.76	−28.77	−28.78	−28.79	−28.80
	$1p_{3/2}$	−13.62	−13.63	−13.64	−13.65	−13.66
		−11.84	−11.86	−11.87	−11.88	−11.89
n	$1s_{1/2}$	−32.77	−32.78	−32.78	−32.80	−32.80
	$1p_{3/2}$	−17.37	−17.38	−17.40	−17.41	−17.42
	$1p_{1/2}$	−15.58	−15.59	−15.61	−15.62	−15.63
	E/nucleon	−5.94	−5.95	−5.96	−5.97	−5.98

for each nucleus:

$$\frac{\delta\,|E_{^7\mathrm{Li}}|\,/\mathrm{nucleon}}{|E_{^7\mathrm{Li}}|\,/\mathrm{nucleon}} = -2.57\frac{\delta m_q}{m_q}, \tag{23}$$

$$\frac{\delta\,|E_{^{12}\mathrm{C}}|\,/\mathrm{nucleon}}{|E_{^{12}\mathrm{C}}|\,/\mathrm{nucleon}} = -1.44\frac{\delta m_q}{m_q}, \tag{24}$$

$$\frac{\delta\,|E_{^{16}\mathrm{O}}|\,/\mathrm{nucleon}}{|E_{^{16}\mathrm{O}}|\,/\mathrm{nucleon}} = -1.08\frac{\delta m_q}{m_q}. \tag{25}$$

The contributions to the previous coefficients from the variation of the exchanged mesons masses are found from Table IV, and from Table V we obtain the contribution from the nucleon mass. These calculations are summarized in the following equation:

$$\frac{\delta\,|E_i|\,/\mathrm{nucleon}}{|E_i|\,/\mathrm{nucleon}} = (\nu_{\mathrm{mesons}} + \nu_{\mathrm{nucleon}})\,\frac{\delta m_q}{m_q},$$

with i representing each of the three nuclei we are considering, ν_{mesons} being described by

$$\nu_{\mathrm{mesons}} = \frac{\delta\,|E_i|\,/\mathrm{nucleon}}{\delta m_q}\cdot\frac{m_q}{|E_i|\,/\mathrm{nucleon}}$$

and ν_{nucleon} by

$$\nu_{\mathrm{nucleon}} = \frac{\delta\,|E_i|\,/\mathrm{nucleon}}{\delta m_N}\cdot\frac{m_N}{|E_i|\,/\mathrm{nucleon}}\cdot 0.048.$$

V. VARIATION IN THE ENERGIES OF TWO- AND THREE-NUCLEON SYSTEMS WITH VARIATION IN THE MESON AND NUCLEON MASSES

To examine the variation in the binding energy of the deuteron and triton with changes in the meson and nucleon masses, we need to consider a purely OBE model for the nucleon-nucleon interaction. We choose to employ the OBE potential of Bryan and Scott (BS) [49], which includes the exchange of π, η, σ_0, σ_1, ρ, and ω mesons. To avoid the singular nature of this potential, BS introduced a monopole regularization scheme that ensured that the potential is finite at the origin. With a cutoff mass of 1500 MeV this regularization is shorter in range than the range of the heaviest of the bosons included in the potential. As a result, the medium-range interaction is dominated by the σ_0 and σ_1 followed by the ρ and ω exchanges.

Because of the nonlocal nature of the potential (term proportional to ∇^2), we have used the method of moments [50] to solve the Schrödinger equation for the binding energy of the deuteron and the $^1S_0$ amplitude. This entails expanding the radial wave function $\psi_\ell(r)$ for a given angular momentum ℓ as a linear combination of Yamaguchi [51] wave functions $\psi_\ell^{(Y)}(r;\beta_i)$ with different range parameters β_i, i.e.,

$$\psi_\ell(r) = \sum_{i=1}^{n} b_i^\ell\,\psi_\ell^{(Y)}(r;\beta_i), \tag{26}$$

where we have taken $n = 12$ and the β_i are multiples of the pion mass. The present choice for the variational wave function ensures that the correct long-range behavior of $\psi_\ell(r)$

is that defined by the asymptotic behavior of $\psi_\ell^{(Y)}(r)$. This in turn is determined by the binding energy of the deuteron or the $^1S_0$ antibound state. This procedure reduces the two-body Schrödinger equation to a set of $2n$ homogenous algebraic equations that give us the binding energy and the wave function for the deuteron to a very good approximation [50].

For the $^1S_0$, the pole in the scattering amplitude is on the second energy sheet, and the analytic continuation of the method of moments to the second energy sheet is not as simple, because the pole is along the negative imaginary momentum axis. However, since this antibound state pole is close to the zero energy ($E_P = -0.066$ MeV), we have chosen the zero-energy point to reduce the Schrödinger equation using the method of moments to a set of n algebraic equations. It has been demonstrated that this procedure gives a good representation of the original potential for the low-energy scattering parameters. As a result we use the effective range expansion to determine the position of the antibound state pole in the momentum or k plane; i.e.. we write the on-shell $^1S_0$ amplitude in terms of the phase shifts δ_0 as

$$t(k) = -\frac{\hbar^2}{\pi\mu}\,\frac{1}{k\cot\delta_0 - ik}, \tag{27}$$

where μ is the reduced mass, and make use of the effective range expansion

$$k\cot\delta_0 = -\frac{1}{a_s} + \frac{1}{2} r_s\, k^2 - P_s\, r_s^3\, k^4 + \cdots, \tag{28}$$

where P_s is the shape parameter, to analytically continue the amplitude onto the second energy sheet. Since the antibound state is close to zero energy ($k \approx -0.04\,i$), we can truncate the effective range expansion to include the k^4 term. To test the accuracy of this procedure, we compare the position of the pole on the second energy sheet for the BS potential by truncating at k^2 and k^4 terms with the result $E_P = -0.071\,153\,1$ and $-0.071\,154\,8$ MeV, respectively. As a result we have chosen to truncate the effective range expansion to include the k^4 term.

The use of the trial function in Eq. (26) has the added advantage of allowing us to construct an equivalent rank-one separable potential, often referred to as the unitary pole approximation (UPA), that has identically the same deuteron wave function as the original OBE potential [50]. After partial wave expansion, this is of the form

$$V^{\mathrm{UPA}}_{\ell;\ell'}(k,k') = g_\ell(k)\, C_{\ell;\ell'}\, g_{\ell'}(k'), \tag{29}$$

where the form factors are directly related to the radial wave function $\psi_\ell(r)$ and the strength of the potential is adjusted to ensure that the matrix element of the UPA and original OBE potential are identical at the energy of the pole in the amplitude. The same procedure is applied to the $^1S_0$ channel.

Having constructed a rank-one separable potential equivalent to the OBE potential, we can write the Faddeev equations as a set of coupled one-dimensional integral equations [52]. If one includes the $^1S_0$ and $^3S_1$-$^3D_1$ nucleon-nucleon partial waves only, then the number of coupled integral equations is reduced to five, and these can be solved for the binding energy and wave function of the triton [53].

To examine the variation in the binding energy with changes in the mass of the mesons and nucleon, we have calculated the slope of the binding energy as a function of the mass at the value of the mass used in the OBE potential. In Table VI we present this variation in the energy of the antibound state and the deuteron and triton binding energies with respect to the variation in the masses of the six bosons included in the OBE potential. We have also included the variation in the binding energies with changes in the nucleon mass m_N. Here, we note that the nucleon mass is present, not only in the kinetic energy of the two- and three-body equations but also in the definition of the BS OBE potential. For the one-pion exchange component, the strength of the potential is proportional to $(g_{\pi NN}/2M)^2$, which is equivalent to $(f_{\pi NN}/m_\pi)^2$ had BS used a pseudo-vector coupling in the Lagrangian. From the Goldberger-Treiman [54] relation we have that

$$\frac{g_{\pi NN}}{M} \propto \frac{g_A}{f_\pi}, \tag{30}$$

where f_π is the pion decay constant. Although g_A and f_π are dependent on the quark mass, the ratio to first order is not sensitive to variation in quark mass. This suggests that the strength of the one-pion exchange component of the BS potential should not change with changes in the nucleon mass. Since the η is part of the same SU(3) octet as the pion, one could apply the same argument for the η exchange component of the OBE potential. For the scalar (σ_0 and σ_1) and vector (ρ and ω) meson exchanges, the relative strength of the central, the spin-orbit, and the tensor components depend on the nucleon mass, and, to that extent, we have maintained the M dependence of the OBE potential for the scalar and vector exchanges. From Table VI we observe that the variation is largest for the σ_0 and ω, followed by the variation with the π, σ_1, ρ, and N masses, with the variation in the energy with the η mass being minimal.

TABLE VI. Variation in the position of the antibound state pole on the second energy sheet, the binding energy of the deuteron, and the binding energy of the triton with changes in hadron mass m_H. For the Bryan-Scott potential the position of the antibound state pole is $E_P = -7.1155 \times 10^{-2}$ MeV, the deuteron binding energy is $E_D = 2.183\,65$ MeV, while the triton binding energy is $E_T = 7.9131$ MeV in the UPA.

Hadron	m_H (MeV)	$\frac{\delta E_P}{\delta m_H}$	$\frac{\delta E_D}{\delta m_H}$	$\frac{\delta E_T}{\delta m_H}$
π	138.7	2.38×10^{-3}	−0.0201	−0.0146
η	548.7	-7.40×10^{-5}	0.0019	0.0034
σ_0	550.0	-1.99×10^{-3}	−0.1026	−0.3355
σ_1	600.0	-3.09×10^{-3}	0.0486	0.0790
ρ	763.0	1.27×10^{-4}	−0.0295	−0.0517
ω	782.8	1.34×10^{-2}	0.0923	0.2776
N	938.92	2.95×10^{-4}	0.0289	0.0527

From the detailed results given in Table VI and the earlier results for the variation of the meson and nucleon masses with quark mass, we can readily deduce the total variation of the deuteron and triton binding energies and the energy of the antibound state, E_P, with changes in the quark mass:

$$\frac{\delta E_D}{E_D} = -0.91\frac{\delta m_q}{m_q}, \tag{31}$$

$$\frac{\delta E_t}{E_t} = -0.98\frac{\delta m_q}{m_q}, \tag{32}$$

and

$$\frac{\delta E_P}{E_P} = -2.84 \frac{\delta m_q}{m_q}. \tag{33}$$

The details for these calculations are shown in the Appendix.

The variations of the deuteron and triton binding energies given in Eqs. (31) and (32), respectively, are completely compatible with those reported by Flambaum and Wiringa [25]. In particular, the coefficients on the right-hand side of those equations, namely, -0.91 for the deuteron and -0.98 for the triton, are very close to those reported in Ref. [25] for the AV14 potential, namely, -0.84 and -0.89.

On the other hand, for the $^1S_0$ antibound state, with energy E_P, there is a significant disagreement. The sign reported above for $\delta E_P/E_P$ is negative, whereas a positive value was reported in Ref. [25]. Since Dmitriev *et al.* [18] presented an apparently general argument relating the change in the deuteron binding to that in the energy of the antibound state, we rechecked every term in our calculation carefully. There is no doubt that our result is correct for the model used. We note that, from Table II of Flambaum and Wiringa [25], the individual pieces of the Argonne potential do not respect the supposedly general result of Dmitriev *et al.* and therefore it cannot be a model-independent result. We note, in particular, that the tensor force plays a significant role for the deuteron, whereas it is absent for the $^1S_0$ channel. Clearly, this difference for the $^1S_0$ antibound state will lead to significant changes when one computes the effect of a change in quark mass on the reaction rate for $n\,p \to d\,\gamma$.

VI. CONCLUSIONS

We have calculated the variation of the binding energy of the deuteron, the triton, and the $^1S_0$ antibound pole position, as well as the binding energy per nucleon for a number of light nuclei, with respect to variations in the light (average of u and d) quark mass. Although one would ideally like to make a first-principles calculation within QCD itself, that is not possible at present. Rather we have employed a physically motivated OBE model for the two- and three-body systems and the QMC model for finite nuclei. While our detailed results may not be the final word, we stress that the models chosen do offer the possibility to use state-of-the-art methods to calculate the variations of the input masses with light quark mass. The results, expressed in terms of a parameter K_A, defined by

$$\frac{\delta BE(A)}{BE(A)} = K_A \frac{\delta m_q}{m_q}, \tag{34}$$

are summarized in Table VII. In order to determine these coefficients, we first calculated the change with quark mass of the mesons used in a typical one-boson-exchange treatment of the nucleon-nucleon force. Those results were summarized in Table III. For each nucleus we calculated the rate of change of the binding energy with respect to the mass of each meson and the mass of the nucleon itself. The values of K_A were obtained by combining the latter with the results in Table III.

For the deuteron our result, $K_d = -0.91$, is very close to that reported by Flambaum and Wiringa [25] using the AV14 potential, namely, -0.84. Similarly for the triton, our value $K_t = -0.98$ is very close to their value of -0.89. The closeness of these results for two rather different treatments of the NN force lends considerable confidence in their reliability. However, for the position of the $^1S_0$ antibound state our calculation differs considerably from that of Ref. [25], taking the opposite sign. This suggests that this quantity may be rather more model dependent than has been realized hitherto.

TABLE VII. Coefficients K_A summarizing the rate of variation of the binding energies and the $^1S_0$ antibound state pole with respect to quark mass [see Eq. (34)].

Nucleus	K_A
D	-0.91
T	-0.98
E_P	-2.84
^{7}Li	-2.57
^{12}C	-1.44
^{16}O	-1.08

In the case of light nuclei, the binding energies reported here were calculated in the QMC model, a relativistic mean-field model that takes into account the self-consistent response of the internal structure of the nucleon to these mean fields. Through the self-consistency, the model yields many-body [55] or equivalently density-dependent interactions [46]. Indeed, the density-dependent Skyrme forces derived from QMC have proven remarkably realistic [47]. The values of K_A deduced in this way for ^{7}Li, ^{12}C, and ^{16}O are reported in Eqs. (24) and (25). It is interesting that the value obtained for ^{7}Li, namely, $K_{^7\mathrm{Li}} = -2.57$, is significantly larger than that reported in Ref. [25], namely, -1.03 (AV14) and -1.50 (AV18 + UIX). These authors did suggest that the uncertainty on the value of K could be as large as a factor of 2 and our value is consistent at that level. Clearly, this degree of variation calls for more investigation to see whether the model dependence can be reduced.

Our study of these variations of binding energies with quark mass is, of course, motivated by the possible effects on BBN. Among the many challenges there, the sizable discrepancy in the abundance of ^{7}Li with the latest photon-to-baryon ratio (post Wilkinson Microwave Anisotropy Probe) is of particular interest. Figure 3 illustrates the ^{7}Li abundance calculated using the BBN code of Kawano [56], if one allows *only* the binding energy of the deuteron and the energy of the virtual $^1S_0$ state to change with quark mass. The curves correspond to the values of K_d and K_P calculated here (solid line) as well as the values used by Berengut *et al.* [57] (dashed line). The substantial difference in slope means that while a 3% shift in $\delta m_q/m_q$ would suffice to reproduce the empirical abundance using the values of Berengut *et al.*, with our values this would require a huge change in quark mass. This simple example illustrates the importance of a complete study of the BBN problem including all of the consequences of a shift of quark mass within the current approach, which we leave for future work. Finally, we note that while the variation of the light quark masses should be most important, it will also be necessary to take into account the effect of a corresponding change in

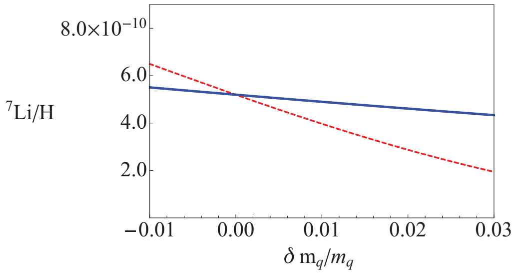

FIG. 3. (Color online) Abundance of ^{7}Li with respect to changes in the quark mass in $p(n,\gamma)d$ calculated in the same way as [57] (dashed red line) and using our results for K_D and K_{E_P} (solid blue line).

the strange quark mass, especially now that the strange quark sigma commutator seems to be under control [58].

ACKNOWLEDGMENTS

This work was supported by the Australian Research Council through the ARC Centre of Excellence in Particle Physics at the Tera-Scale and through the Australian Laureate Fellowship (FL0992247), as well as by the University of Adelaide.

APPENDIX

From Table VI we find the variations of the binding energies for the deuteron and triton (E_i with $i = D, T$) and the position of the pole for the ${}^{1}S_0$ antibound state, E_P; according to changes in the mass of the hadrons (m_H),

$$\frac{\delta E_i}{E_i} = \frac{1}{E_i}\sum_H \frac{\delta E_i}{\delta m_H}\delta m_H. \tag{A1}$$

We then relate the variation of the mass of each hadron to the variation of the quark mass, as given in Eq. (20):

$$\frac{\delta m_H}{m_H} = \nu_H \frac{\delta m_q}{m_q}, \tag{A2}$$

so that

$$\delta m_H = (\nu_H \cdot m_H)\frac{\delta m_q}{m_q}. \tag{A3}$$

Combining those results we finally obtain the formula that gives rise to the results in Eqs. (31)–(33):

$$\frac{\delta E_i}{E_i} = \frac{1}{E_i}\sum_H \frac{\delta E_i}{\delta m_H}(\nu_H \cdot m_H)\frac{\delta m_q}{m_q}. \tag{A4}$$

[1] J.-P. Uzan, Rev. Mod. Phys. **75**, 403 (2003).
[2] V. A. Dzuba, V. V. Flambaum, and J. K. Webb, Phys. Rev. Lett. **82**, 888 (1999).
[3] J. K. Webb, V. V. Flambaum, C. W. Churchill, M. J. Drinkwater, and J. D. Barrow, Phys. Rev. Lett. **82**, 884 (1999).
[4] M. T. Murphy, J. K. Webb, and V. V. Flambaum, Mon. Not. R. Astron. Soc. **345**, 609 (2003).
[5] H. B. Sandvik, J. D. Barrow, and J. Magueijo, Phys. Rev. Lett. **88**, 031302 (2002).
[6] J. K. Webb, J. A. King, M. T. Murphy, V. V. Flambaum, R. F. Carswell, and M. B. Bainbridge, Phys. Rev. Lett. **107**, 191101 (2011).
[7] W. J. Marciano, Phys. Rev. Lett. **52**, 489 (1984); X. Calmet and H. Fritzsch, Eur. Phys. J. C **24**, 639 (2002); P. Langacker, G. Segré, and M. J. Strassler, Phys. Lett. B **528**, 121 (2002).
[8] J. D. Bekenstein, Phys. Rev. D **25**, 1527 (1982).
[9] K. A. Olive, M. Pospelov, Y.-Z. Qian, A. Coc, M. Cassé, and E. Vangioni-Flam, Phys. Rev. D **66**, 045022 (2002); B. A. Campbell and K. A. Olive, Phys. Lett. B **345**, 429 (1995).
[10] V. V. Flambaum and E. V. Shuryak, Phys. Rev. D **67**, 083507 (2003).
[11] V. V. Flambaum, D. B. Leinweber, A. W. Thomas, and R. D. Young, Phys. Rev. D **69**, 115006 (2004).
[12] I. C. Cloet *et al.*, Few-Body Syst. **42**, 91 (2008).
[13] J. D. Prestage, R. L. Tjoelker, and L. Maleki, Phys. Rev. Lett. **74**, 3511 (1995).
[14] S. G. Karshenboim, Can. J. Phys. **78**, 639 (2000).
[15] H. Marion *et al.*, Phys. Rev. Lett. **90**, 150801 (2003).
[16] A. I. Shlyakhter, Nature (London) **264**, 340 (1976).
[17] T. Damour and F. Dyson, Nucl. Phys. B **480**, 37 (1996).
[18] V. V. Flambaum and E. V. Shuryak, Phys. Rev. D **65**, 103503 (2002); V. F. Dmitriev and V. V. Flambaum, *ibid.* **67**, 063513 (2003).
[19] A. Bulgac, G. A. Miller, and M. Strikman, Phys. Rev. C **56**, 3307 (1997).
[20] S. R. Beane and M. J. Savage, Nucl. Phys. A **713**, 148 (2003).
[21] E. Epelbaum, U.-G. Meissner, and W. Gloeckle, Nucl. Phys. A **714**, 535 (2003).
[22] S. R. Beane and M. J. Savage, Nucl. Phys. A **717**, 91 (2003).
[23] J. Soto and J. Tarrus, Phys. Rev. C **85**, 044001 (2012).
[24] R. D. Young, D. B. Leinweber, and A. W. Thomas, Prog. Part. Nucl. Phys. **50**, 399 (2003).
[25] V. V. Flambaum and R. B. Wiringa, Phys. Rev. C **76**, 054002 (2007).
[26] J. Beringer *et al.* (Particle Data Group Collaboration), Phys. Rev. D **86**, 010001 (2012).
[27] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
[28] W. Bentz and A. W. Thomas, Nucl. Phys. A **696**, 138 (2001).
[29] I. Caprini, G. Colangelo, and H. Leutwyler, Phys. Lett. **96**, 132001 (2006).
[30] W. Armour, C. R. Allton, D. B. Leinweber, A. W. Thomas, and R. D. Young, J. Phys. G **32**, 971 (2006); D. B. Leinweber, A. W. Thomas, K. Tsushima, and S. V. Wright, Phys. Rev. D **64**, 094502 (2001); C. R. Allton, W. Armour, A. W. Thomas, and R. D. Young, Phys. Lett. B **628**, 125 (2005).
[31] B. Ananthanarayan, G. Colangelo, J. Gasser, and H. Leutwyler, Phys. Rep. **353**, 207 (2001); S. M. Roy, Phys. Lett. B **36**, 353 (1971).
[32] J. R. Pelaez, R. Garcia Martin, R. Kaminski, and J. Ruiz de Elvira, PoS QNP **2012**, 070 (2012).
[33] M. Harada, F. Sannino, and J. Schechter, Phys. Rev. D **54**, 1991 (1996).
[34] P. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[35] R. D. Young, D. B. Leinweber, A. W. Thomas, and S. V. Wright, Phys. Rev. D **66**, 094507 (2002).
[36] P. E. Shanahan, A. W. Thomas, and R. D. Young, arXiv:1205.5365 [nucl-th].
[37] P. A. M. Guichon, Phys. Lett. B **200**, 235 (1988).
[38] P. A. M. Guichon, K. Saito, E. N. Rodionov, and A. W. Thomas, Nucl. Phys. A **601**, 349 (1996).
[39] K. Saito, K. Tsushima, and A. W. Thomas, Nucl. Phys. A **609**, 339 (1996).
[40] P. G. Blunden and G. A. Miller, Phys. Rev. C **54**, 359 (1996).
[41] K. Saito, K. Tsushima, and A. W. Thomas, Prog. Part. Nucl. Phys. **58**, 1 (2007).
[42] P. A. M. Guichon, A. W. Thomas, and K. Tsushima, Nucl. Phys. A **814**, 66 (2008); R. Shyam, K. Tsushima, and A. W. Thomas, Phys. Lett. B **676**, 51 (2009); Nucl. Phys. A **881**, 255 (2012).
[43] J. R. Stone, P. A. M. Guichon, and A. W. Thomas, arXiv:1012.2919 [nucl-th]; J. Rikovska-Stone, P. A. M. Guichon, H. H. Matevosyan, and A. W. Thomas, Nucl. Phys. A **792**, 341 (2007).
[44] D. L. Whittenbury, J. D. Carroll, A. W. Thomas, K. Tsushima, and J. R. Stone, arXiv:1204.2614 [nucl-th].
[45] T. Miyatsu, T. Katayama, and K. Saito, Phys. Lett. B **709**, 242 (2012).
[46] P. A. M. Guichon, H. H. Matevosyan, N. Sandulescu, and A. W. Thomas, Nucl. Phys. A **772**, 1 (2006).
[47] M. Dutra, O. Lourenco, J. S. Sa Martins, A. Delfino, J. R. Stone, and P. D. Stevenson, Phys. Rev. C **85**, 035201 (2012).
[48] G. Krein, A. W. Thomas, and K. Tsushima, Nucl. Phys. A **650**, 313 (1999).
[49] R. Bryan and B. L. Scott, Phys. Rev. **177**, 1435 (1969).
[50] I. R. Afnan and J. M. Read, Phys. Rev. C **12**, 293 (1975).
[51] Y. Yamaguchi and Y. Yamaguchi, Phys. Rev. **95**, 1635 (1954).
[52] I. R. Afnan and A. W. Thomas, in *Modern Three-Hadron Physics*, edited by A. W. Thomas (Springer-Verlag, Berlin, 1977), p. 1.
[53] I. R. Afnan and N. D. Birrell, Phys. Rev. C **16**, 823 (1977).
[54] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).
[55] P. A. M. Guichon and A. W. Thomas, Phys. Rev. Lett. **93**, 132502 (2004).
[56] L. Kawano, FERMILAB-PUB-92-004-A, 1992.
[57] J. C. Berengut, V. V. Flambaum, and V. F. Dmitriev, Phys. Lett. B **683**, 114 (2010).
[58] R. D. Young and A. W. Thomas, Nucl. Phys. A **844**, 266C (2010); Phys. Rev. D **81**, 014503 (2010).