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Gravitino cosmology with a very light neutralino

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It has been shown that very light or even massless neutralinos are consistent with all current experiments, given nonuniversal gaugino masses. Furthermore, a very light neutralino is consistent with astrophysical bounds from supernovae and cosmological bounds on dark matter. Here we study the cosmological constraints on this scenario from big bang nucleosynthesis (taking gravitinos into account) and find that a very light neutralino is even favored by current observations.

I. INTRODUCTION

Within the minimal supersymmetric standard model (MSSM), the photon and the Z^0 boson, as well as the two neutral CP-even Higgs bosons have SUSY spin-1/2 partners that mix. The resulting mass eigenstates are denoted neutralinos χ_i^0 with $i = 1, \ldots, 4$ and are ordered by mass $m_{\chi_1^0} < \ldots < m_{\chi_4^0}$ [1]. The Particle Data Group quotes a lower mass bound on the lightest neutralino [2]

$$m_{\chi_1^0} > 46 \text{ GeV}, \tag{1}$$

which is derived from the LEP chargino search under the assumption of gaugino mass universality

$$M_1 = \tfrac{5}{3}\tan^2\theta_{\rm W} M_2. \tag{2}$$

Here $M_{1,2}$ are the supersymmetry (SUSY) breaking bino mass and wino mass, respectively, and $\theta_{\rm W}$ is the electroweak mixing angle. If we relax this latter assumption, the bound (1) no longer applies. In fact, for any value of M_2, μ, and $\tan\beta$ there is always a M_1

$$M_1 = \frac{M_2 M_Z^2 \sin(2\beta)\sin^2\theta_{\rm W}}{\mu M_2 - M_Z^2 \sin(2\beta)\cos^2\theta_{\rm W}} \tag{3}$$

$$\simeq 2.5 \text{ GeV}\left(\frac{10}{\tan\beta}\right)\left(\frac{150 \text{ GeV}}{\mu}\right), \tag{4}$$

such that the lightest neutralino is massless [3,4]. Here M_Z is the mass of the Z^0 boson, $\tan\beta$ is the ratio of the vacuum expectation values of the two CP-even neutral Higgs bosons in the MSSM, and μ is the Higgs mixing parameter of

the superpotential. A very light or massless neutralino is necessarily predominantly binolike since the experimental lower bound on the chargino mass sets lower limits on M_2 and μ [5,6]. Although Eq. (3) holds at tree-level, there is always a massless solution even after including quantum corrections to the neutralino mass [4].

Such a light or even massless neutralino is consistent with all laboratory data. The processes considered include the invisible width of the Z^0, electroweak precision observables, direct pair production, associated production, and rare meson decays. Note that a binolike neutralino does not couple directly to the Z^0. The other production processes, including the meson decays, thus necessarily involve virtual sleptons or squarks. If these have masses of $\mathcal{O}(200)$ GeV or heavier, then all bounds are evaded—for details on the individual analyses see Refs. [3–10]. The best possible laboratory mass measurement can be performed at a linear collider via selectron pair production with an accuracy of order 1GeV, depending on the selectron mass [11].

Light neutralinos can lead to rapid cooling of supernovae, so they are constrained by the broad agreement between the expected neutrino pulse from core collapse and observations of Supernova 1987A [12]. The neutralinos produced would interact via the exchange of virtual selectrons and squarks. For a massless neutralino, which "free-streams" out of the supernova, the selectron must be heavier than about 1.2 TeV and the squarks must be heavier than about 360 GeV. For light selectrons or squarks of mass $\sim$100–300 GeV, the neutralinos instead diffuse out of the supernova just as the neutrinos do and, thus, play an important role in the supernova dynamics. Hence, lacking a detailed simulation, which includes the effects of neutralino diffusion, no definitive statement can presently be made [10,12–14]. Recently, the luminosity function of

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white dwarfs has been determined to high precision [15,16] and this may imply interesting new bounds on light neutralinos, just as on axions.

If a neutralino is stable on cosmological time scales it can contribute to the dark matter (DM) of the Universe. If "cold", then its mass is constrained from below by the usual Lee-Weinberg bound [17], which depends only on the self-annihilation cross section. This limit has been widely discussed in the literature in the framework of the ΛCDM cosmology [18–21] and various values are quoted for a MSSM neutralino: $M_{\chi_1^0} > 12.6$ GeV [22,23] and $M_{\chi_1^0} > 9$ GeV [24,25]. The low mass range is particularly interesting because the DAMA [26] and CoGeNT [27] direct detection experiments have presented evidence for annual modulation signals suggestive of a DM particle with mass of $\mathcal{O}(10)$ GeV.

A light neutralino with a much smaller mass is also viable as "warm" or "hot" DM but this possibility has been less discussed. The observed DM density $\Omega_{\rm DM}h^2 \approx 0.11$ can, in principle, be entirely accounted for with warm dark matter in the form of neutralinos having a mass of a few keV [28]. However, the usual assumption of radiation domination and entropy conservation prior to big bang nucleosynthesis (BBN) then needs to be relaxed otherwise the relic neutralino density is nominally much larger than required. This scenario requires a (unspecified) late episode of entropy production or, equivalently, reheating after inflation to a rather low temperature of a few MeV. Although models of baryogenesis with such reheating temperatures exist [29,30], the necessary baryon number violating interactions would result in rapid decay of the proton to (the lighter) neutralinos. This makes such models very difficult to realize in this context, although the situation may be somewhat eased since the maximum temperature during reheating can be higher than the final thermalization temperature [31].

In this paper we focus on a light neutralino, which acts as hot dark matter (HDM),[1] i.e. can suppress cosmic density fluctuations on small scales through free-streaming. In order for its relic abundance to be small enough to be consistent with the observed small-scale structure, we require [4] following Ref. [35],

$$m_{\chi_1^0} \lesssim 0.7 \text{ eV.} \tag{5}$$

Such ultralight neutralinos affect BBN by contributing to the relativistic degrees of freedom and thus speeding up the expansion rate of the Universe; consequently, neutron-proton decoupling occurs earlier and the mass fraction of primordial ^{4}He is increased [36]. The resulting constraint on new relativistic degrees of freedom is usually presented as a limit on the number of additional effective $SU(2)$ doublet neutrinos

$$\Delta N_\nu^{\rm eff}(\chi_1^0) \equiv N_\nu^{\rm eff} - 3. \tag{6}$$

In Sec. II we calculate this number in detail and compare it with observational bounds on $\Delta N_\nu^{\rm eff}$ from BBN [37].

Until recently, the BBN prediction and the inferred primordial ^{4}He abundance implied, according to some authors [38,39],

$$\Delta N_\nu^{\rm eff} \lesssim 0. \tag{7}$$

This is however in tension with recent measurements of the cosmic microwave background (CMB) anisotropy by Wilkinson Microwave Anisotropy Probe WMAP, which suggests a larger value of [40,41][2]

$$\text{WMAP}: \Delta N_\nu^{\rm eff} = 1.34^{+0.86}_{-0.88}. \tag{8}$$

Recent measurements of the primordial ^{4}He abundance are also higher than reported earlier, implying [44,45]

$$\text{BBN}: \Delta N_\nu^{\rm eff} = 0.68^{+0.8}_{-0.7}. \tag{9}$$

Given these large uncertainties, a very light neutralino is easily accommodated, and even favored, by the BBN and CMB data. In the near future, the Planck mission [46] is foreseen to determine $N_\nu^{\rm eff}$ to a higher precision of about $\delta N_\nu^{\rm eff} = \pm 0.26$ [41], thus possibly constraining the light neutralino hypothesis.

Local SUSY models necessarily include a massive gravitino [47]. Depending on its mass, the gravitino can also contribute to $\Delta N_\nu^{\rm eff}$ as we discuss in Sec. III. This effect is only relevant for sub-eV mass gravitinos (for models see e.g. Ref. [48]). It is more common for the gravitino to have electroweak-scale mass and its decays into the light neutralino to result in photo-dissociation of light elements, in particular, ^{4}He [36]. The resulting (over) production of ^{2}H and ^{3}He is strongly constrained observationally, and we present the resulting bounds in Sec. IV. In Sec. V, we examine under which conditions the gravitino itself can be a viable DM candidate in the presence of a very light neutralino. Conclusions are presented in Sec. VI.

II. LIGHT NEUTRALINOS AND NUCLEOSYNTHESIS

In global SUSY models, or local SUSY models with a nonrelativistic gravitino, the sub-eV neutralino is the only relativistic particle present at the onset of nucleosynthesis apart from the usual photons, electrons, and three types of neutrinos.

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[1]Note that HDM cannot contribute more than a small fraction of the observed dark matter, so another particle is required to make up the cold dark matter (CDM). Potential candidates include the gravitino [32], the axion [33] or the axino [34].

[2]This preference for extra neutrino species comes mainly from their contribution to the expansion rate prior to recombination [42]; however it has been noted that the constraint depends strongly on the priors adopted in the analysis [43].

The contribution of the neutralino to the number of effective neutrino species is [36]:

$$\Delta N_\nu^{\rm eff}(\chi_1^0) = \frac{g_{\chi_1^0}}{2}\left(\frac{T_{\chi_1^0}}{T_\nu}\right)^4, \tag{10}$$

where $g_{\chi_1^0}$ is the number of internal degrees of freedom equal to 2 due to the Majorana character of the neutralino. The ratio of temperatures is given by

$$\frac{T_{\chi_1^0}}{T_\nu} = \left[\frac{g^*(T_{\rm fr}^\nu)}{g^*(T_{\rm fr}^{\chi_1^0})}\right]^{1/3}, \tag{11}$$

where $T_{\rm fr}^i$ is the freeze-out temperature of particle i and

$$g^*(T) = \sum_{\rm bosons} g_i \cdot \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{\rm fermions} g_i \cdot \left(\frac{T_i}{T}\right)^4 \tag{12}$$

with g_i being the internal relativistic degrees of freedom at temperature T. Usually T_i for a decoupled particle species i is lower than the photon temperature T because of subsequent entropy generation.

The freeze-out temperature of $SU(2)$ doublet neutrinos is $T_{\rm fr}^\nu \sim 2$ MeV [49]. The interaction rate $\Gamma_{\chi_1^0}$ of the lightest neutralino is suppressed relative to that of neutrinos [4] because the SUSY mass scale $m_{\rm SUSY} > M_W$, where $m_{\rm SUSY}$ denotes the relevant SUSY particle mass involved in the neutralino reactions. Hence the freeze-out temperature of the very light neutralino will generally be higher than $T_{\rm fr}^\nu$.

Estimating the thermally averaged neutralino annihilation cross section via an effective vertex, we obtain the approximate interaction rate

$$\Gamma_{\chi_1^0}(T) = 2\frac{3}{4}\frac{\zeta(3)}{\pi^2}G_{\rm SUSY}^2 T_{\chi_1^0}^5, \tag{13}$$

where $G_{\rm SUSY}/\sqrt{2} = g^2/(8m_{\rm SUSY}^2)$. Equating this to the Hubble expansion rate [36]

$$H(T) = \sqrt{\frac{4\pi^3 g^*(T)}{45}}\frac{T^2}{M_{\rm Pl}}, \tag{14}$$

where g^* counts the relativistic degrees of freedom, yields the approximate freeze-out temperature

$$T_{\rm fr}^{\chi_1^0} \approx 3\left(\frac{m_{\rm SUSY}}{200~\text{GeV}}\right)^{4/3}T_{\rm fr}^\nu. \tag{15}$$

Thus, for sparticle masses below ~ 3 TeV, the neutralinos freeze-out below the temperature at which muons annihilate [4].

We now calculate the freeze-out temperature of a pure binolike neutralino more carefully, considering all annihilation processes into leptons, which are present at the time of neutralino freeze-out

$$\chi_1^0\chi_1^0 \to \ell\bar{\ell}, \qquad \ell = e, \nu_e, \nu_\mu, \nu_\tau. \tag{16}$$

Assuming that sleptons and sneutrinos have a common mass scale $m_{\rm slepton}$, the following relations hold

$$\sigma(\chi_1^0\chi_1^0 \to \ell_R\bar{\ell}_L) = 16\sigma(\chi_1^0\chi_1^0 \to \ell_L\bar{\ell}_R)$$
$$= 16\sigma(\chi_1^0\chi_1^0 \to \nu\bar{\nu}), \tag{17}$$

so the total annihilation cross section into leptons is given by

$$\sigma(\chi_1^0\chi_1^0 \to \ell\bar{\ell}) = 20\sigma(\chi_1^0\chi_1^0 \to \ell_L\bar{\ell}_R), \tag{18}$$

where we have taken the electron to be massless. The thermally averaged cross section is then given by

$$\langle\sigma(\chi_1^0\chi_1^0 \to \ell\bar{\ell})v\rangle = \frac{20}{9\zeta(3)^2}\frac{2^5}{3}I(1)^2\hat{\sigma}T^2, \tag{19}$$

where

$$I(n) = \int_0^\infty \frac{y^{n+2}}{\exp(y)+1} \tag{20}$$

and

$$\hat{\sigma} = \frac{e^4}{8\pi\cos^4\theta_W}\frac{1}{m_{\rm slepton}^4} \tag{21}$$

for $m_{\rm slepton} \gg T$. In calculating the cross section (19), we have neglected the Pauli blocking factors in the final state statistics [50].

Relating the reaction rate (19) to the Hubble expansion rate (14), we can now obtain the freeze-out temperature for a binolike neutralino; shown in Fig. 1 as a function of the common mass scale $m_{\rm slepton}$. Note that for $m_{\rm slepton}$ below a few TeV, the neutralino decouples below the muon mass as noted earlier. Thus, neutrinos and neutralinos will have the same temperature
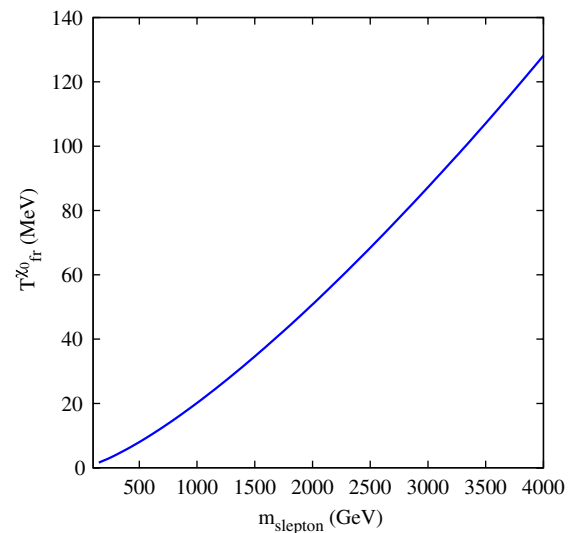
$$T_{\chi_1^0} = T_\nu, \tag{22}$$

hence during BBN

FIG. 1 (color online). Freeze-out temperature of the pure bino-like neutralino as a function of the common mass scale $m_{\rm slepton}$.

$$\Delta N_\nu^{\text{eff}}(\chi_1^0) = 1. \tag{23}$$

However, for slepton masses above a few TeV, the neutralino freeze-out temperature is close to the muon mass, and muon annihilation will influence the neutralino and neutrino temperature differently. For $T_{\text{fr}}^{\chi_1^0} \gtrsim m_\mu$ the neutrinos are heated by the muon annihilations; this affects the neutralinos only marginally. Therefore, $T_{\chi_1^0}/T_\nu$ is reduced due to the conservation of comoving entropy. The muons contribute to $g^*(T_{\chi_1^0})$, such that

$$\frac{T_{\chi_1^0}}{T_\nu} = \left[\frac{g_\gamma + \frac{7}{8}(g_e + 3g_\nu)}{g_\gamma + \frac{7}{8}(g_e + 3g_\nu + g_\mu)}\right]^{1/3} = \left(\frac{43}{57}\right)^{1/3}. \tag{24}$$

Thus, employing Eq. (10) we obtain

$$\Delta N_\nu^{\text{eff}}(\chi_1^0) = 0.69, \tag{25}$$

which is interestingly close to the observationally inferred central value of 0.68 in Eq. (9). The LHC already restricts the masses of strongly coupled SUSY particles (squarks and gluinos) to be above several hundred GeV [51–53], and the supernova cooling argument requires the selectron mass to be also above a TeV for a massless neutralino [10], so the picture is consistent.

Even for a neutralino freeze-out temperature somewhat below the muon mass, the effects from muon annihilation are notable. We now determine the equivalent number of neutrino species more carefully using the Boltzmann equation as in Refs. [49,54], in order to determine the effect for arbitrary slepton masses. Consider a fiducial relativistic fermion x, which is decoupled during $\mu\bar{\mu}$ annihilation so that its number density n_x satisfies

$$\dot{n}_x + \frac{3\dot{R}}{R}n_x = 0. \tag{26}$$

The Boltzmann equation controlling the number density of the lightest neutralino can then be written as

$$\frac{d}{dt}\left(\frac{n_{\chi_1^0}}{n_x}\right) = n_x\langle\sigma v\rangle\left[\left(\frac{n_\mu}{n_x}\right)^2 - f(T_{\chi_1^0})\left(\frac{n_{\chi_1^0}}{n_x}\right)^2\right], \tag{27}$$

where

$$f(T_{\chi_1^0}) = \left[\frac{n_\mu(T_{\chi_1^0})}{n_{\chi_1^0}(T_{\chi_1^0})}\right]^2_{\text{equilibrium}}. \tag{28}$$

The cross section $\mu\bar{\mu} \rightarrow \chi_1^0\chi_1^0$ is given by

$$16\pi s^2 \frac{\cos\theta_W^4}{e^4}\sigma(\mu_R\bar{\mu}_L \rightarrow \chi_1^0\chi_1^0) = \tag{29}$$

$$2(m_{\tilde{\mu}}^2 - m_\mu^2)\ln\left(\frac{2(m_{\tilde{\mu}}^2 - m_\mu^2) + s - \sqrt{s}\sqrt{s - 4m_\mu^2}}{2(m_{\tilde{\mu}}^2 - m_\mu^2) + s + \sqrt{s}\sqrt{s - 4m_\mu^2}}\right)$$

$$+ \sqrt{s}\sqrt{s - 4m_\mu^2}\frac{2(m_{\tilde{\mu}}^2 - m_\mu^2)^2 + m_{\tilde{\mu}}^2 s}{(m_{\tilde{\mu}}^2 - m_\mu^2)^2 + m_{\tilde{\mu}}^2 s}. \tag{30}$$

Since this involves a cancellation between the two terms, we Taylor expand to ensure numerical stability

$$16\pi\frac{\cos\theta_W^4}{e^4}\sigma(\mu_R\bar{\mu}_L \rightarrow \chi_1^0\chi_1^0) \approx \frac{\sqrt{1 - \frac{4m_\mu^2}{s}}(s - m_\mu^2)}{3(m_{\tilde{\mu}}^2 - m_\mu^2)^2}$$

then take the thermal average $\langle\sigma v\rangle$ following Ref. [55].

In order to reformulate Eq. (27) in terms of dimensionless quantities, we define

$$\delta \equiv \frac{T_{\chi_1^0} - T_x}{T_x}, \qquad \epsilon \equiv \frac{T_\gamma - T_x}{T_x}, \qquad y \equiv \frac{m_\mu}{T_\gamma}. \tag{31}$$

Here δ measures the temperature difference between the decoupled particle x and the lightest neutralino and thus quantifies the heating of the lightest neutralino due to $\mu\bar{\mu}$ annihilation. We now evaluate n_μ/n_x numerically and expand $n_{\chi_1^0}/n_x \approx 1 + 3\delta$ so Eq. (27) can be written as [49,54]

$$\frac{d\delta}{dy} \approx ay^{-2}(\epsilon - \delta) \tag{32}$$

for $\delta \ll 1$, i.e. for small temperature differences. The prefactor a depends on the size of the annihilation cross section, and thus on y and the slepton mass

$$a(y, m_{\tilde{l}}) = \frac{5.67 \times 10^{17}}{\sqrt{g^*}}\frac{\langle\sigma v\rangle}{\text{GeV}^{-2}}. \tag{33}$$

We approximate the drop in g^* when the muons become nonrelativistic by a step function with $g^*(y < 1) = 16$ and $g^*(y > 1) = 12.34$.

Now T_x and the photon temperature T_γ are related through entropy conservation [36]

$$\frac{T_x}{T_\gamma} = \left(\frac{43}{57}\right)^{1/3}[\zeta(y)]^{1/3}, \tag{34}$$

where

$$\zeta(y) = 1 + \frac{180}{43\pi^4}\int_0^\infty x^2 \frac{\sqrt{x^2 + y^2} + \frac{x^2}{3\sqrt{x^2 + y^2}}}{e^{\sqrt{x^2 + y^2}} + 1}\,dx. \tag{35}$$

We use Eqs. (34) and (35) to numerically evaluate $\epsilon(y)$ and then solve the differential Eq. (32) for $\delta(y, m_{\tilde{l}})$. The solution asymptotically approaches a limit [denoted by $\delta_{\max}(m_{\tilde{l}})$] for $y \gtrsim 10$ because for temperatures far below the muon mass there is no further heating of the neutralinos from muon annihilation. This improves our estimate (23) to
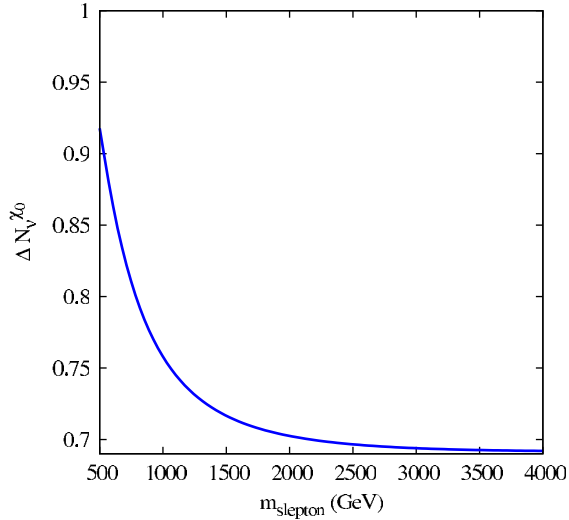
FIG. 2 (color online). Contribution of the pure binolike neutralino to the effective number of neutrinos versus the slepton mass.

$$\Delta N_\nu^{\mathrm{eff}}(\chi_1^0) = \left(\frac{T_{\chi_1^0}}{T_\nu}\right)^4 = 0.69[1 + \delta_{\max}(m_{\tilde{l}})]^4. \qquad (36)$$

In Fig. 2, we show $\Delta N_\nu^{\mathrm{eff}}(\chi_1^0)$ as a function of the common slepton mass m_{slepton}. We see that for slepton masses above 3 TeV, our previous result of 0.69 in Eq. (25) is not modified because if the interaction between the neutralinos and muons is too weak, then the neutralinos cannot stay in thermal contact with the muons. For slepton masses around 1 TeV, we get again one additional effective neutrino species. (Our numerical approximation is valid only for $\delta \ll 1$, so holds down to $m_{\mathrm{slepton}} = 0.5$ TeV when $\delta \simeq 0.1$.)

Summarizing, the neutralino contribution to the effective number of neutrinos lies between 0.69 and 1, depending on the slepton mass as seen in Fig. 2. Thus, a very light neutralino is easily accommodated by BBN and CMB data and is in fact favored by the recent observational indication (9) that $N_\nu \gtrsim 3$.

III. A VERY LIGHT NEUTRALINO AND A VERY LIGHT GRAVITINO

A very light gravitino (as realized e.g. in some models of gauge-mediated SUSY breaking) can constitute HDM. For its relic density to be small enough to be consistent with the observed small-scale structure requires [56]

$$m_{\tilde{G}} \lesssim 15\text{–}30 \text{ eV}. \qquad (37)$$

If the gravitino is heavier than the (very light) neutralino it will decay into the neutralino plus a photon with a lifetime $\gtrsim 10^{38}$ s [see Eq. (44) below], which is well above the age of the Universe $\sim 4 \times 10^{17}$ s. Conversely, if the gravitino is lighter than the neutralino, the latter will decay to a gravitino and a photon with lifetime [57]

$$\tau_{\chi_1^0} \simeq 7.3 \times 10^{41} \text{ s} \left(\frac{m_{\chi_1^0}}{1 \text{ eV}}\right)^{-5} \left(\frac{m_{\tilde{G}}}{0.1 \text{ eV}}\right)^2, \qquad (38)$$

assuming that there is no near-mass degeneracy between the neutralino and the gravitino. Again, the lifetime is well above the age of the Universe, therefore we can consider both the gravitino and the very light neutralino as effectively stable HDM.

The presence of a very light gravitino thus affects the primordial ^{4}He abundance analogously to a very light neutralino. However, the contribution of the gravitino to the expansion rate depends on its mass, since it couples to other particles predominantly via its helicity-$1/2$ components with the coupling strength $\Delta m^2/(m_{\tilde{G}} m_{\mathrm{Pl}})$, where Δm^2 is the squared mass splitting of the superpartners [58]. For a very light gravitino, the interaction cross section can be of the same order as the weak interaction cross section, leading to later decoupling. Hence it can have a sizeable effect on BBN.

The freeze-out temperature of a very light gravitino can be estimated from the conversion process with cross section [59]

$$\sigma(\tilde{G}e^\pm \to e^\pm \chi_1^0) = \frac{\alpha}{9} \frac{s}{m_{\mathrm{Pl}}^2 m_{\tilde{G}}^2}. \qquad (39)$$

We neglect self-annihilations $\tilde{G}\tilde{G} \to \ell\bar{\ell}$, $\gamma\gamma$ since the annihilation rate into photons is $\propto m_{\chi_1^0}^4$ [50,60], hence suppressed for a light neutralino, while the annihilation rate into leptons is $\propto T^6$ [50], which falls out of equilibrium much earlier than the conversions.

After thermal averaging of the conversion rate (39) as before, we find

$$T_{\mathrm{fr}}^{\mathrm{conversion}} \simeq 7.51 m_{\tilde{G}}^{2/3} m_{\mathrm{Pl}}^{1/3} g^{*1/6}$$

$$\approx 100 g^{*1/6} \left(\frac{m_{\tilde{G}}}{10^{-3} \text{ eV}}\right)^{2/3} \text{ MeV}. \qquad (40)$$

Since the Goldstino coupling is enhanced for decreasing gravitino mass, the freeze-out temperature of the gravitino increases with its mass. For a gravitino mass of 5.6×10^{-4} eV $(7.8 \times 10^{-4}$ eV) its freeze-out temperature equals the muon (pion) mass, so for heavier gravitinos the contribution to $\Delta N_\nu^{\mathrm{eff}}$ will decrease. We also consider the case $m_{\tilde{G}} = 10$ eV that gives a freeze-out temperature of $\mathcal{O}(100)$ GeV, thus a negligible effect on $\Delta N_\nu^{\mathrm{eff}}$. (Note, however, that $T_{\mathrm{fr}}^{\tilde{G}}$ will now depend on the SUSY mass spectrum because above temperatures of order GeV or other SUSY processes can also be in thermal equilibrium [61,61] and Eq. (40) may not apply.)

We can now evaluate the contribution of the gravitino, in conjunction with the very light neutralino, to the effective number of neutrino species. We need to keep in mind that the gravitino can affect neutralino decoupling since for very large slepton masses and/or very light gravitinos, the neutralino annihilation process $\chi_1^0\chi_1^0 \to \ell\bar{\ell}$ becomes

subdominant to the conversion process $\tilde{G}e^{\pm} \rightarrow e^{\pm}\chi_1^0$ and, therefore, neutralino freeze-out is also governed by Eq. (40).

In Fig. 3, we show contour lines for the ratio of the cross sections for neutralino annihiliation (16) and the conversion process (39) in the slepton-gravitino mass plane. For a ratio less than 0.1, the freeze-out temperature of both particles is determined via the conversion process (39) and $T_{\tilde{G}} = T_{\chi_1^0}$. Hence $\Delta N_\nu^{\text{eff}}(\tilde{G}, \chi_1^0) = 1/0.69/0.57$, the latter two cases corresponding to gravitino masses above 5.6×10^{-4} eV and 7.8×10^{-4} eV, respectively [corresponding to a freeze-out temperature below the muon and the pion mass, as determined from Eq. (40)]. The corresponding equivalent number of neutrino species is

$$\Delta N_\nu^{\text{total}} \equiv \Delta N_\nu^{\text{eff}}(\tilde{G}) + \Delta N_\nu^{\text{eff}}(\chi_1^0) = 2/1.38/1.14. \quad (41)$$

Thus a very light gravitino is strongly constrained by the BBN bound (9), a mass below 5.6×10^{-4} eV being excluded at 3σ. As the gravitino mass increases, $\Delta N_\nu^{\text{total}}$ decreases because the gravitino and neutralino freeze-out earlier, hence are colder than the neutrinos at the onset of BBN.

One can see from Fig. 3 that a further increase of the gravitino mass (or smaller slepton mass) accesses parameter regions where the neutralino annihilation process dominates over the conversion process. When the ratio of their rates exceeds ~ 10, the freeze-out of the neutralino and the gravitino is governed by the processes (16) and (39), respectively. For a slepton mass above ~ 3 TeV, the lightest neutralino decouples above the muon mass hence yielding $\Delta N_\nu^{\text{eff}}(\chi_1^0) = 0.69$. Figure 2 shows that with decreasing slepton mass increases to $\Delta N_\nu^{\text{eff}}(\chi_1^0) = 1$ as before. Hence

we obtain the same bounds on the gravitino mass for $\Delta N_\nu^{\text{eff}}(\tilde{G}) = 1/0.69/0.57$.

In summary, for a slepton mass below ~ 1 TeV

$$\Delta N_\nu^{\text{total}} = 2/1.69/1.57, \quad (42)$$

while for a slepton mass above ~ 3 TeV

$$\Delta N_\nu^{\text{total}} = 1.69/1.38/1.26; \quad (43)$$

for intermediate slepton masses there is a continuous transition between the two cases.

If the gravitino mass increases further, its effect on the expansion rate continues to decrease, e.g. for $m_{\tilde{G}} = 10$ eV (corresponding to $T_{\text{fr}}^{\tilde{G}} \approx 100$ GeV), we find $g^* = 395/4$ or $\Delta N_\nu^{\text{eff}}(\tilde{G}) \simeq 0.05$. Thus, gravitinos with mass $\gtrsim$ eV do not significantly affect the expansion rate.

Summarizing, $\Delta N_\nu^{\text{total}}$ is between 1.14 and 2 for scenarios with both a relativistic neutralino and a relativistic gravitino (when their freeze-out temperature lies between the freeze-out temperature of the neutrino and the pion mass). As before, we can use the Boltzmann equation if necessary to obtain exact values for $\Delta N_\nu^{\text{eff}}$ around the mass thresholds. From Eq. (9), $N_\nu^{\text{total}} > 4.9$ is excluded at 3σ, implying a lower bound on the gravitino mass of 5.6×10^{-4} eV (cf. Fig. 3). This bound is 2 orders of magnitude weaker than the one stated in Ref. [50], where a model with a very light gravitino but a heavy neutralino was considered. This is because the gravitino annihilation into diphotons or leptons is the relevant process when there is no light neutralino; Ref. [50] also assumed a more stringent BBN limit $N_\nu^{\text{total}} < 3.6$.

IV. DECAYING GRAVITINOS

So far we have considered the increase in the expansion rate caused by sub-eV neutralinos and gravitinos which are quasistable (cf. Sec. V). We now consider a gravitino with a mass above $\mathcal{O}(100 \text{ GeV})$ as would be the case in gravity-mediated SUSY breaking where the gravitino sets the mass scale of SUSY partners.

As the gravitino mass increases, the relative coupling strength of the helicity-1/2 components, $\Delta m^2/(m_{\tilde{G}} m_{\text{Pl}})$ decreases and the helicity 3/2 components come to dominate. These are however also suppressed by $1/m_{\text{Pl}}$, hence gravitinos decouple from thermal equilibrium very early. During reheating, gravitinos are produced thermally via two-body scattering processes (dominantly QCD interactions) and the gravitino abundance is proportional to the reheating temperature T_R [62]. The gravitino is unstable and will decay subsequently into the very light neutralino and a photon with lifetime [32,62–64],

$$\tau_{\tilde{G}} \simeq 4.9 \times 10^8 \left(\frac{m_{3/2}}{100 \text{ GeV}} \right)^{-3} \text{s}, \quad (44)$$

where we have assumed for simplicity that the gravitino is the next-to-lightest-SUSY particle while the neutralino is
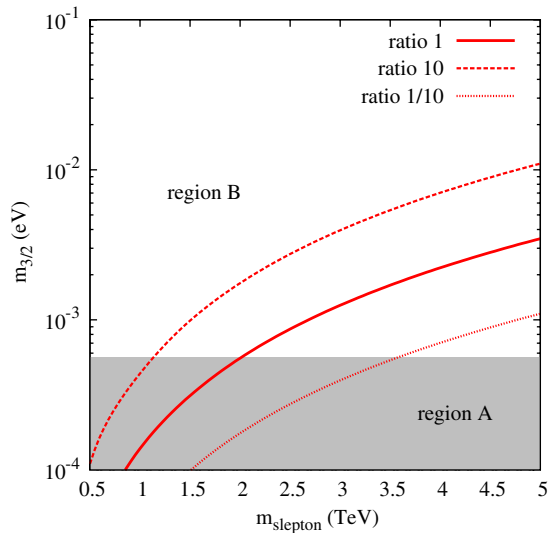

FIG. 3 (color online).   Contour lines for the ratio of cross sections for neutralino self-annihilation (16) and conversion (39) in the gravitino-slepton mass plane. The shaded area indicates where $\Delta N_\nu^{\text{total}} = 2$.

the lightest-SUSY particle. If the gravitino decays around or after BBN, the light element abundances are affected by the decay products whether photons or hadrons. In particular, there is potential overproduction of D and ^{3}He from photo-dissociation of (the much more abundant) ^{4}He [32,63], while for short lifetimes, decays into hadrons have more effect [64].

Therefore, the observationally inferred light element abundances constrain the number density of gravitinos. For a gravitino lifetime of $\mathcal{O}(10^8$ sec$)$ one obtains [65,66] a severe bound on the abundance $Y_{3/2} \equiv n_{3/2}/s$

$$Y_{3/2} \lesssim 10^{-14}\left(\frac{100 \text{ GeV}}{m_{\tilde{G}}}\right). \tag{45}$$

This is proportional to the reheating temperature through [32,62–64]

$$\left(\frac{T_{\text{R}}}{10^{10} \text{ GeV}}\right) \approx 3.0 \times 10^{11} Y_{3/2}, \tag{46}$$

hence the latter is constrained to be

$$T_{\text{R}} \lesssim 3.0 \times 10^7 \text{ GeV} \times \left(\frac{100 \text{ GeV}}{m_{3/2}}\right). \tag{47}$$

Note that a reheating temperature below $\mathcal{O}(10^8$ GeV$)$ is not consistent with thermal leptogenesis, which typically requires $T_{\text{R}} \sim 10^{10}$ GeV [67]. There are however other possible means to produce the baryon asymmetry of the Universe at lower temperature [29–31].

The contribution to the present neutralino relic density from gravitino decays is

$$\Omega_{\chi_1^0}^{\text{decay}} h^2 \approx 0.28 Y_{3/2}\left(\frac{m_{\chi_1^0}}{1 \text{ eV}}\right) \tag{48}$$

i.e. negligible, such that the Cowsik-McClelland bound on the neutralino mass is unaffected.

V. QUASI-STABLE GRAVITINOS

As mentioned in Sec. IV, when the gravitino mass is below ~ 100 MeV its lifetime is longer than the age of the Universe, so it is quasistable and can constitute warm dark matter. Decaying gravitino DM is constrained by limits on the diffuse γ-ray background. For a mass between ~ 100 keV and ~ 100 MeV the gravitino decays to a photon and a neutralino, and the photon spectrum is simply

$$\frac{dN_\gamma}{dE} = \delta\left(E - \frac{m_{\tilde{G}}}{2}\right). \tag{49}$$

The γ-flux from gravitions decaying in our Milky Way halo dominates [68,69] over the redshifted flux from gravitino decays at cosmological distances. Using a Navarro-Frenk-White profile for the distribution of DM in our galaxy, we obtain

$$E^2 \frac{\mathrm{d}J}{\mathrm{d}E}\bigg|_{\text{halo}} \equiv \frac{2E^2}{8\pi\tau_{\tilde{G}}m_{\tilde{G}}} \frac{\mathrm{d}N_\gamma}{\mathrm{d}E} \int_{\text{l.o.s}} \langle\rho_{\text{halo}}(\vec{\ell})d\vec{\ell}\rangle/\Delta\Omega$$
$$= 31.1\left(\frac{m_{\tilde{G}}}{1 \text{ MeV}}\right)^4 \delta\left(E - \frac{m_{\tilde{G}}}{2}\right)\frac{\text{MeV}}{\text{cm}^2 \text{ str s}}. \tag{50}$$

We compare this to the measurements of the γ-ray background by COMPTEL, EGRET and Fermi [70–72] and extract a conservative upper bound of 3×10^{-2} cm^{-2} str^{-1} s^{-1} MeV on the γ-ray flux from the inner Galaxy in the relevant mass region below ~ 100 MeV. This implies that gravitinos with mass above ~ 250 keV would generate a flux exceeding the observed galactic γ-ray emission. On the other hand, constraints from small-scale structure formation set a lower mass bound on warm dark matter of $\mathcal{O}(\text{keV})$ [73–75].

Now we consider the relic density of those gravitinos. Because of the presence of the very light neutralino, all sparticles will decay into the latter before the onset of BBN. Therefore, the gravitino will only be produced thermally with relic density [76]

$$\Omega_{3/2}h^2 \approx \left(\frac{1 \text{ keV}}{m_{\tilde{G}}}\right)\left(\frac{T_{\text{R}}}{10 \text{ TeV}}\right)\left(\frac{M_{\text{SUSY}}}{200 \text{ GeV}}\right)^2. \tag{51}$$

This further restricts the gravitino mass and/or the reheating temperature in order not to exceed the observed value $\Omega_{\text{DM}}h^2 \approx 0.11$. The least restrictive upper bound on the reheating temperature from Eq. (51) is $\mathcal{O}(10^5$ GeV$)$ for gravitino and gaugino masses of order 100 keV and 100 GeV, respectively. This could be alleviated if the gravitino density is diluted by the decay of particles (such as moduli fields [64] or the saxion from the axion multiplet [77,78]). In this context, there have been several detailed studies on gravitinos as light DM [79–83].

VI. SUMMARY

We studied the cosmology of the gravitino in the presence of a very light neutralino. Even a massless neutralino is compatible with all laboratory data, while the strictest astrophysical constraint is imposed by supernova cooling and requires selectrons to be heavy ($m_{\tilde{e}} \gtrsim 1$ TeV). Here we considered the effect of a stable, very light neutralino arising on the effective number of neutrino species during big bang nucleosynthesis. For slepton masses above ~ 3 TeV, $\Delta N_\nu^{\text{eff}}(\chi_1^0)$ is 0.69 and this increases as the slepton mass decreases, reaching 1 for slepton masses below ~ 0.5 TeV.

Next, we considered constraints on the gravitino mass in the context of local SUSY with a very light neutralino. A very light gravitino will affect the expansion rate of the Universe similarly to a light neutralino. We identified the mass range where a gravitino has a sizeable effect on the effective number of neutrino species as $\sim 10^{-4}$–10 eV. Within this range, we obtained values for $\Delta N_\nu^{\text{eff}}(\chi_1^0\&\tilde{G})$ between 0.74 and 1.69, depending on the gravitino and

slepton masses. Values around 0.7 are favored by recent BBN measurements. However, the uncertainties in the determination of ^{4}He are still sufficiently large that we need to await data from Planck to pin down the allowed gravitino and slepton mass.

If the gravitino is heavier than ~100 MeV, it decays to the neutralino and a photon with a lifetime smaller than the age of the Universe. This results in photodissociation of the light elements, which is strongly constrained observationally and translates into an upper bound on the reheating temperature of the Universe of $\sim10^7$ GeV for typical gravity-mediated SUSY breaking models. Note that neither the neutralino nor the gravitino can constitute *all* of the dark matter in the scenarios considered so far.

The mass range where the gravitino can constitute warm dark matter is constrained by bounds from the diffuse γ-ray background, from the formation of structure on small-scales, and from the observed DM abundance,

leaving a small window of allowed gravitino mass between 1 and 100 keV for a reheating temperature below 10^5 GeV.

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