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Possible $B\bar{K}$ molecular structure of $B^*_{s0}(5725)$ in the Bethe-Salpeter approach

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We interpret the scalar $B^*_{s0}(5725)$ as an S-wave $B\bar{K}$ molecular bound state in the Bethe-Salpeter approach. In the ladder and instantaneous approximation, with the kernel induced by ρ and ω exchanges, we establish the Bethe-Salpeter equation for $B^*_{s0}(5725)$. We find that the bound state of $B\bar{K}$ can exist. We also calculate the isospin-violating decay width of the process $B^*_{s0}(5725) \to B_s\pi^0$ through exchanging K^* and B^* mesons including the $\eta - \pi^0$ mixing effect. We hope that the obtained decay width is instructive for the forthcoming experiment.

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I. INTRODUCTION

The hadron state $D^*_{s0}(2317)^+$ was reported by *BABAR* [1] in the $D_s^+\pi^0$ invariant mass distribution. Its narrow decay width stimulates both experimental and theoretical interests. In order to understand the structure of this new state, various pictures for its structure are assumed such as the traditional $c\bar{s}$ state [2–7], the exotic meson state [8–10], and the DK bound state [11–16]. By analyzing the experimental data, it is suggested that the quantum numbers of $D^*_{s0}(2317)^+$ are $I(J^P) = 0(0^+)$. In our previous work [16], we analyzed the structure of $D^*_{s0}(2317)^+$ in the Bethe-Salpeter (BS) approach and found that DK can indeed form a bound state. We also calculated the isospin-violating decay width of the process $D^*_{s0}(2317)^+ \to D_s^+\pi^0$ including the $\eta - \pi^0$ mixing effect. We found that the decay width is in the range of the resolution of the experimental detector.

Recently, two bottom-strange mesons $B_{s1}(5830)(J^P = 1^+)$ [17] and $B_{s2}(5840)(J^P = 2^+)$ [17,18] have been detected. However, the b-partner of $D^*_{s0}(2317)$, B^*_{s0} has not been observed. Many physicists have studied it theoretically using different models. In Refs. [3,14], the mass spectrum and strong decay width were predicted. Using the heavy chiral Lagrangian and the unitarized coupled-channel, authors in Ref. [14] predicted the mass of B^*_{s0}, $M_{B^*_{s0}(5725)} = 5725 \pm 39$ MeV. We adopt the same notation, $B^*_{s0}(5725)$. In Refs. [14,19], the state $B^*_{s0}(5725)$ was considered as a $B\bar{K}$ bound state. The authors in Refs. [3,20–22] supposed that it can be a $b\bar{s}$ state. In Ref. [23], it was interpreted as a tetraquark state. The quantum numbers of $B^*_{s0}(5725)$ were suggested as $I(J^P) = 0(0^+)$ [3,14]. In this paper, we will analyze the state $B^*_{s0}(5725)$ in the BS approach which has been applied to many theoretical studies including heavy mesons and heavy baryons [24–29]. Because the mass of $B^*_{s0}(5725)$ lies below the $B\bar{K}$ threshold, we assume that it is an S-wave $B\bar{K}$ bound state. Besides investigating whether the $B\bar{K}$ bound state exists or not, we also study the isospin-violating decay $B^*_{s0}(5725) \to B_s\pi^0$. We find that $B\bar{K}$ can indeed form the bound state with reasonable binding energy and form factor cutoff.

In the remaining of this paper, we proceed as follows. In Sec. II, we establish the BS equation for a bound state containing two pseudoscalar mesons, then derive the BS equation for the state $B^*_{s0}(5725)$ and obtain the numerical result of the BS equation. In Sec. III, we calculate the decay width of $B^*_{s0}(5725) \to B_s\pi^0$ including the $\eta - \pi^0$ mixing effect. In Sec. IV, we present a short summary.

II. BS EQUATION FOR $B\bar{K}$ SYSTEM

A. The BS equation for a bound state containing two pseudoscalar mesons

In this section we will review the general formalism of the BS equation for the system composed of two pseudoscalar mesons. We start by defining the BS wave function for the bound state of two pseudoscalar mesons $|P\rangle$ as

$$\chi_P(x_1, x_2) = \langle 0|T\phi_1(x_1)\phi_2(x_2)|P\rangle = e^{-iPX}\chi_P(x), \quad (1)$$

and its conjugate

$$\bar{\chi}_P(x_1, x_2) = \langle P|T\phi_1^\dagger(x_1)\phi_2^\dagger(x_2)|0\rangle = e^{iPX}\bar{\chi}_P(x), \quad (2)$$

where $\phi_1(x_1)$ and $\phi_2(x_2)$ are the field operators of the two pseudoscalar mesons, respectively, P denotes the total momentum of the bound state, x and X are the relative coordinate and the center-of-mass coordinate which are, respectively, defined as

$$x = x_1 - x_2, \qquad X = \eta_1 x_1 + \eta_2 x_2, \quad (3)$$

where $\eta_1 = m_1/(m_1 + m_2)$, $\eta_2 = m_2/(m_1 + m_2)$, with m_1 and m_2 being the masses of the constituent particles.

We can obtain the equation for the BS wave function with the help of the four-point Green function which is defined as the following:

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$$G(x_1, x_2; y_1, y_2) = \langle 0|T\phi_1(x_1)\phi_2(x_2)(\phi_1(y_1)\phi_2(y_2))^\dagger|0\rangle. \tag{4}$$

From Eq. (4), we can express the four-point Green function in terms of the four-point truncated two-particle irreducible kernel, $\bar{K}$,

$$\begin{aligned} G(x_1, x_2; y_1, y_2) &= G_0(x_1, x_2; y_1, y_2) \\ &+ \int d^4u_1 d^4u_2 d^4v_1 d^4v_2 G_0(x_1, x_2; u_1, v_2) \\ &\times \bar{K}(u_1, u_2; v_2, v_1) G(v_1, v_2; y_2, y_1), \end{aligned} \tag{5}$$

where G_0 is related to the forward scattering disconnected four-point amplitude,

$$G_0(x_1, x_2; y_1, y_2) = \Delta_1(x_1, y_1)\Delta_2(x_2, y_2), \tag{6}$$

with $\Delta_i(x_i, y_i)$ being the complete propagator of the i-th constituent particle,

$$\begin{aligned} \Delta_i(x_i, y_i) &= \langle 0|T\phi_i(x)\phi_i(y)^\dagger|0\rangle \\ &= \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} \Delta_i(k, m_i). \end{aligned} \tag{7}$$

From Eqs. (1), (5), and (6) one can derive the following BS equation for the bound state of two pseudoscalar particles:

$$\begin{aligned} &\int d^4y_1 d^4y_2 G_0^{-1}(x_1, x_2; y_1, y_2)\chi_P(y_1, y_2) \\ &= \int d^4v_1 d^4v_2 \bar{K}(x_1, x_2; v_2, v_1)\chi_P(v_1, v_2). \end{aligned} \tag{8}$$

It is convenient to study a two-particle system with relative momentum between the two constituent particles, $p(p')$, which is defined as

$$p = \eta_2 p_1 - \eta_1 p_2, \qquad p' = \eta_2 p_1' - \eta_1 p_2', \tag{9}$$

where $p_1(p_1')$ and $p_2(p_2')$ are momenta of the two constituent particles, respectively. The total momentum of the bound state is $P = p_1 + p_2(P' = p_1' + p_2')$. Therefore, $p_1(p_1')$ and $p_2(p_2')$ can be expressed inversely as

$$\begin{aligned} p_1 &= \eta_1 P + p, \qquad p_2 = \eta_2 P - p, \\ p_1' &= \eta_1 P + p', \qquad p_2' = \eta_2 P - p'. \end{aligned} \tag{10}$$

Using Eq. (1), in momentum space, the BS wave function is obtained as

$$\begin{aligned} \chi_P(p_1, p_2) &= \int d^4x_1 d^4x_2 e^{ipx_1 + ipx_2}\chi_P(x_1, x_2) \\ &= (2\pi)^4\delta^4(p_1 + p_2 - P)\chi_P(p). \end{aligned} \tag{11}$$

The four-point Green functions in momentum and coordinate spaces are related by the following Fourier transformation:

$$\begin{aligned} G(x_1, x_2; y_1, y_2) &= \int \frac{d^4p_1 d^4p_2 d^4p_1' d^4p_2'}{(2\pi)^{16}} \\ &\times e^{-ip_1x_1 - ip_2x_2 + ip_1'y_1 + ip_2'y_2} \\ &\times G(p_1, p_2; p_1', p_2'). \end{aligned} \tag{12}$$

Similarly, for the irreducible kernel we have

$$\begin{aligned} \bar{K}(x_1, x_2; y_1, y_2) &= \int \frac{d^4p_1 d^4p_1' d^4p_2 d^4p_2'}{(2\pi)^{16}} \\ &\times e^{-ip_1x_1 - ip_2x_2 + ip_1'y_1 + ip_2'y_2} \\ &\times \bar{K}(p_1, p_2; p_1', p_2'). \end{aligned} \tag{13}$$

Because of momentum conservation, we have

$$\begin{aligned} \bar{K}(p_1, p_2; p_1', p_2') &= (2\pi)^4\delta^4(P - P')\bar{K}(p, p'). \\ \bar{G}(p_1, p_2; p_1', p_2') &= (2\pi)^4\delta^4(P - P')\bar{G}(p, p'). \end{aligned} \tag{14}$$

With Eq. (6) and (14), the homogeneous Eq. (8) in momentum space becomes

$$\Delta_1^{-1}(p_1, m_1)\Delta_2^{-1}(p_2, m_2)\chi_P(p) = \int \frac{d^4p'}{(2\pi)^4}\bar{K}(p, p')\chi_P(p'). \tag{15}$$

Because the irreducible kernel involves nonperturbative strong interactions, the BS equation becomes very complicated. To proceed we use the so-called instantaneous approximation for the kernel. Since the energy exchanged between the constituents in the $B\bar{K}$ binding system is small, it can be neglected, i.e. $\bar{K}(p, p') = \bar{K}(\mathbf{p}, \mathbf{p}')$. Furthermore, the propagators of the constituent particles are set to have the forms of the free ones. Then, the BS Eq. (15) becomes

$$-(p_1^2 - m_1^2)(p_2^2 - m_2^2)\chi_P(p) = \int \frac{d^4p'}{(2\pi)^4}\bar{K}_P(\mathbf{p}, \mathbf{p}')\chi_P(p'). \tag{16}$$

Dividing by the two propagators on both sides of Eq. (16) and performing the integration over p^0 and p'^0, we have

$$\frac{E^2 - (E_1 + E_2)^2}{(E_1 + E_2)/E_1E_2}\tilde{\chi}_P(\mathbf{p}) = \frac{i}{2}\int \frac{d^3p'}{(2\pi)^3}\bar{K}_P(\mathbf{p}, \mathbf{p}')\tilde{\chi}_P(\mathbf{p}'), \tag{17}$$

where $E_i \equiv \sqrt{\mathbf{p}^2 + m_i^2}$, $E = P^0$, and the equal-time wave function is defined as

$$\tilde{\chi}_P(\mathbf{p}) = \int \frac{dp^0}{2\pi}\chi_P(p^0, \mathbf{p}). \tag{18}$$

For convenience we define the following potential:

$$V(\mathbf{p}, \mathbf{p}') = \frac{i}{E_1E_2(E_1 + E_2)}\bar{K}_P(\mathbf{p}, \mathbf{p}'). \tag{19}$$

Then, the BS Eq. (17) can be written as

$$\left[\frac{E^2}{(E_1+E_2)^2}-1\right]\tilde{\chi}_P(\mathbf{p})=\frac{1}{2}\int\frac{d^3p'}{(2\pi)^3}V(\mathbf{p},\mathbf{p}')\tilde{\chi}_P(\mathbf{p}'). \tag{20}$$

From Eqs. (16), (19), and (20) we can express $\chi_P(p)$ in terms of $\tilde{\chi}_P(\mathbf{p})$,

$$\chi_P(p^0,\mathbf{p})=\frac{i}{\pi}\frac{1}{(p_1^2-m_1^2+i\epsilon)(p_2^2-m_2^2+i\epsilon)}\times\frac{E^2-(E_1+E_2)^2}{(E_1+E_2)/E_1E_2}\tilde{\chi}_P(\mathbf{p}), \tag{21}$$

where $p_1^2-m_1^2=(\eta_1E+p^0)^2-E_1^2$, $p_2^2-m_2^2=(\eta_2E-p^0)^2-E_2^2$.

B. Structure of $B_{s0}^*(5725)$

As stated in the Introduction, we assume that $B_{s0}^*(5725)$ is an S-wave $B\bar{K}$ bound state. Based on this picture, we use the field doublets $(B^0,\bar{B}^0)$, (B^+,B^-), (K^+,K^-), and $(K^0,\bar{K}^0)$ which have the following expansions:

$$\begin{aligned}
B_1&=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_{B^0}}}(a_{B^0}e^{-ipx}+a_{\bar{B}^0}^{\dagger}e^{ipx}),\\
B_2&=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_{B^\pm}}}(a_{B^+}e^{-ipx}+a_{B^-}^{\dagger}e^{ipx}),\\
K_1&=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_{K^\pm}}}(a_{K^+}e^{-ipx}+a_{K^-}^{\dagger}e^{ipx}),\\
K_2&=\int\frac{d^3p}{(2\pi)^3\sqrt{2E_{K^0}}}(a_{K^0}e^{-ipx}+a_{\bar{K}^0}^{\dagger}e^{ipx}).
\end{aligned} \tag{22}$$

For the $B\bar{K}$ system, the isoscalar bound state can be written as $|P\rangle_{(0,0)}=\frac{1}{\sqrt{2}}(|B^+K^-\rangle+|B^0\bar{K}^0\rangle)$, the subscript (0,0) refers to the isospin and its third component $(I,I_3)=(0,0)$. After projecting the bound states $|P\rangle_{(I,I_3)}$ on the field operators B_1, B_2, K_1, and K_2, we have

$$\langle 0|TB_i(x_1)K_j(x_2)^\dagger|P\rangle_{(I,I_3)}=C^{ij}_{(I,I_3)}\chi_P(x_1,x_2), \tag{23}$$

where $C^{ij}_{(I,I_3)}(i,j=1,2)$ is the isospin coefficient. For the isoscalar state only $i=j$ contributes and $C^{11}_{(0,0)}=C^{22}_{(0,0)}=\frac{1}{\sqrt{2}}$.

In order to obtain the interaction kernel for the $B\bar{K}$ system, we use the following PPV-type Lagrangian as in Refs. [30–34]:

$$\mathcal{L}_{PPV}=-\frac{1}{\sqrt{2}}iG_V\,Tr([P,\partial_\mu P]V^\mu), \tag{24}$$

where P and V stand for the fields of pseudoscalar and vector mesons, respectively, and they have the following forms:

$$P=\begin{pmatrix}\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & \pi^+ & K^+\\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & K^0\\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta\end{pmatrix}, \tag{25}$$

$$V=\begin{pmatrix}\frac{1}{\sqrt{2}}\rho^0+\frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+}\\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0+\frac{1}{\sqrt{2}}\omega & K^{*0}\\ K^{*-} & \bar{K}^{*0} & \phi\end{pmatrix}. \tag{26}$$

From expressions (24)–(26), we obtain the interactions corresponding to K meson

$$\mathcal{L}_{KK\rho}=ig_{KK\rho}[\bar{K}\vec{\tau}(\partial_\mu K)-(\partial_\mu\bar{K})\vec{\tau}K]\cdot\vec{\rho}^\mu, \tag{27}$$

$$\mathcal{L}_{KK\omega}=ig_{KK\omega}[\bar{K}(\partial_\mu K)-(\partial_\mu\bar{K})K]\omega^\mu, \tag{28}$$

$$\mathcal{L}_{KK\phi}=ig_{KK\phi}[\bar{K}(\partial_\mu K)-(\partial_\mu\bar{K})K]\phi^\mu, \tag{29}$$

where

$$K=\begin{pmatrix}K^+\\ K^0\end{pmatrix},\qquad \bar{K}=(K^-\quad \bar{K}^0), \tag{30}$$

with $\vec{\tau}$ are usual Pauli matrices and $\vec{\rho}^\mu$ represent the field operators of ρ mesons. In $SU(3)_f$ limit, coupling constants $g_{KK\rho}$, $g_{KK\omega}$, and $g_{KK\phi}$ are related to $g_{\rho\pi\pi}$, which is determined by the KSRF relation [35]: $g_{\rho\pi\pi}\simeq M_\rho/(\sqrt{2}f_\pi)\simeq 6$, where M_ρ and f_π denote the mass of ρ meson and the pion weak decay constant, respectively.

When considering the interactions related to B meson, the authors in Ref. [36] assume that the coupling of a vector meson with two pseudoscalar mesons originates from the coupling of quarks inside them. The vector mesons ρ and ω couple to u and d quarks inside the pseudoscalar mesons, while ϕ only couples to s quark. B meson has the same u and d quark content as K meson, and it does not contain s quark. Therefore, B meson only couples to ρ and ω mesons. We define the coupling constants as follows:

$$\frac{g_{BB\rho}}{m_B}=\frac{g_{KK\rho}}{m_K}=G_V,\qquad \frac{g_{BB\omega}}{m_B}=\frac{g_{KK\omega}}{m_K}=G_V. \tag{31}$$

Then the interactions about B meson take the following form:

$$\mathcal{L}_{BB\rho}=ig_{BB\rho}[B\vec{\tau}(\partial_\mu\bar{B})-(\partial_\mu B)\vec{\tau}\bar{B}]\cdot\vec{\rho}^\mu, \tag{32}$$

$$\mathcal{L}_{BB\omega}=ig_{BB\omega}[B(\partial_\mu\bar{B})-(\partial_\mu B)\bar{B}]\omega^\mu, \tag{33}$$

where

$$B=\begin{pmatrix}B^+\\ B^0\end{pmatrix},\qquad \bar{B}=(B^-\quad \bar{B}^0). \tag{34}$$

The kernel of the $B\bar{K}$ system induced by ρ and ω exchanges is depicted in Fig. 1. With the interactions listed in Eqs. (27), (28), (32), and (33), we have

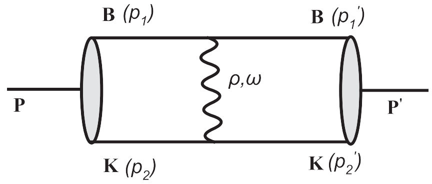

FIG. 1. One-particle exchange diagrams induced by ρ and ω mesons.

$$\bar{K}(p_1, p_2; p_1', p_2') = c_I G_V^2 (p_1 + p_1')^\mu (p_2 + p_2')^\nu \times \Delta_{\mu\nu}(p_1 - p_1', M_V)(2\pi)^4 \delta^4(P - P'), \quad (35)$$

where c_I is the isospin coefficient, $c_I = 3$ for ρ and $c_I = 1$ for ω, $\Delta_{\mu\nu}$ denotes the massive vector meson propagator

$$\Delta_{\mu\nu}(k, M_V) = \frac{-i}{k^2 - M_V^2}\left(g_{\mu\nu} - \frac{k^\mu k^\nu}{M_V^2}\right), \quad (36)$$

where $k = p_1 - p_1'$, M_V is the mass of the exchanged meson. With Eq. (19) one can write down the potential in the ladder approximation

$$\begin{aligned} V^{(I)}(\mathbf{p}, \mathbf{p}'; M_V) &= c_I U(\mathbf{p}, \mathbf{p}'; M_V) \\ &\equiv c_I \frac{-G_V^2}{E_1 E_2 (E_1 + E_2)} \frac{(\mathbf{p} + \mathbf{p}')^2 + 4\eta_1\eta_2 E^2 + (\mathbf{p} - \mathbf{p}')^2/M_V^2}{(\mathbf{p} - \mathbf{p}')^2 + M_V^2}. \end{aligned} \quad (37)$$

In order to reflect the effects of the nonpoint interaction of the hadrons, we introduce the following form factor at each vertex as in Ref. [30]:

$$F(\mathbf{k}) = \frac{2\Lambda^2 - M_V^2}{2\Lambda^2 + \mathbf{k}^2}, \qquad \mathbf{k} = \mathbf{p} - \mathbf{p}', \quad (38)$$

where Λ is a phenomenological cutoff which will be adjusted to give the solution of the BS equation. With the potential defined in Eq. (37), the BS Eq. (20) can be written in the form

$$\begin{aligned} \left[\frac{E^2}{(E_1 + E_2)^2} - 1\right]\tilde{\chi}_P(\mathbf{p}) &= \frac{1}{2}\int \frac{d^3 p'}{(2\pi)^3}[3U(\mathbf{p}, \mathbf{p}'; M_\rho) + U(\mathbf{p}, \mathbf{p}'; M_\omega)] \\ &\quad \times F(\mathbf{p} - \mathbf{p}')^2 \tilde{\chi}_P(\mathbf{p}'). \end{aligned} \quad (39)$$

Since the BS wave function $\tilde{\chi}_P(\mathbf{p})$ is a Lorentz scalar, it only depends on the norm of the three-momentum. After performing the azimuthal integration, the BS equation becomes a one-dimensional integral equation

$$\begin{aligned} \tilde{\chi}_P(|\mathbf{p}|) = \int d|\mathbf{p}'|[&3U(|\mathbf{p}|, |\mathbf{p}'|; M_\rho) \\ &+ U(|\mathbf{p}|, |\mathbf{p}'|; M_\omega)]\tilde{\chi}_P(|\mathbf{p}'|), \end{aligned} \quad (40)$$

where $U(|\mathbf{p}|, |\mathbf{p}'|; M_V)$ is the one-dimensional potential

$$\begin{aligned} U(|\mathbf{p}|, |\mathbf{p}'|; M_V) = \frac{-G_V^2}{4(2\pi)^2} &\frac{E_1 + E_2}{E_1 E_2[E^2 - (E_1 + E_2)^2]} \\ &\times \frac{|\mathbf{p}'|}{|\mathbf{p}|}(U_1 + U_2 + U_3), \end{aligned} \quad (41)$$

with

$$\begin{aligned} U_1 &= -4|\mathbf{p}||\mathbf{p}'|(2\Lambda^2 - M_V^2)\frac{2\Lambda^2 + 2(2\eta_1\eta_2 E^2 + |\mathbf{p}|^2 + |\mathbf{p}'|^2) + (|\mathbf{p}|^2 - |\mathbf{p}'|^2)^2/M_V^2}{[2\Lambda^2 + (|\mathbf{p}| + |\mathbf{p}'|)^2][2\Lambda^2 + (|\mathbf{p}| - |\mathbf{p}'|)^2]}, \\ U_2 &= [M_V^2 + 2(2\eta_1\eta_2 E^2 + |\mathbf{p}|^2 + |\mathbf{p}'|^2) + (|\mathbf{p}|^2 - |\mathbf{p}'|^2)^2/M_V^2]\ln\frac{M_V^2 + (|\mathbf{p}| + |\mathbf{p}'|)^2}{M_V^2 + (|\mathbf{p}| - |\mathbf{p}'|)^2}, \\ U_3 &= -[M_V^2 + 2(2\eta_1\eta_2 E^2 + |\mathbf{p}|^2 + |\mathbf{p}'|^2) + (|\mathbf{p}|^2 - |\mathbf{p}'|^2)^2/M_V^2]\ln\frac{2\Lambda^2 + (|\mathbf{p}| + |\mathbf{p}'|)^2}{2\Lambda^2 + (|\mathbf{p}| - |\mathbf{p}'|)^2}. \end{aligned} \quad (42)$$

C. Normalization and solution of the BS equation for $B_{s0}^*(5725)$

In order to see whether the $B\bar{K}$ bound state exists or not, we need to solve the homogeneous BS equation. When we calculate physical quantity such as the decay width, we need to normalize the BS wave function. Following Refs. [28,37], one can write down the normalization condition as

$$i\int \frac{d^4 p d^4 p'}{(2\pi)^8}\bar{\chi}(p)\frac{\partial}{\partial P^0}[I(p, p') + \bar{K}(p, p')]\chi(p') = 1, \quad (43)$$

where $P^0 = E$, $I(p, p') = -(2\pi)^4\delta^4(p - p')\times \Delta_1^{-1}(p_1, m_1)\Delta_2^{-1}(p_2, m_2)$, $\bar{K}(p, p')$ is the two-particle irreducible kernel. After we carry out p_0 and azimuthal integration, Eq. (43) takes the form

$$\begin{aligned} &-\frac{1}{2\pi^4}\int d|\mathbf{p}||\mathbf{p}|^2\tilde{\chi}_P(|\mathbf{p}|)^2 R \\ &\quad - \int \frac{\eta_1\eta_2 E}{8\pi^6} d|\mathbf{p}|d|\mathbf{p}'||\mathbf{p}||\mathbf{p}'|\tilde{\chi}_P(|\mathbf{p}|)\tilde{\chi}_P(|\mathbf{p}'|)T = 1, \end{aligned} \quad (44)$$

where

TABLE I. Values of E_b and corresponding Λ.

E_b(MeV)	−15	−20	−25	−30	−35	−40	−45	−50	−55	−60	−65	−70	−75	−80	−85
$\Lambda_{\min}$(GeV)	1.490	1.511	1.527	1.541	1.552	1.563	1.571	1.579	1.587	1.593	1.600	1.606	1.611	1.617	1.621
Λ(GeV)	2.432	2.443	2.452	2.460	2.465	2.471	2.476	2.481	2.486	2.490	2.494	2.497	2.500	2.503	2.506
Λ(GeV)	3.576	3.583	3.588	3.591	3.594	3.596	3.599	3.602	3.605	3.607	3.610	3.613	3.616	3.619	3.622

$$R = -E[-2E^2(E_1^2 - E_2^2)(E_1\eta_1 - E_2\eta_2) + E^4(E_1\eta_1 + E_2\eta_2) + (E_1^2 - E_2^2)(E_1^3\eta_1 + 3E_1E_2^2\eta_1 - 3E_1^2E_2\eta_2 - E_2^3\eta_2)]\{2E_1E_2[E^4 + (E_1^2 - E_2^2)^2 - 2E^2(E_1^2 + E_2^2)]^2\}^{-1}, \quad (45)$$

and $T = 3T(M_\rho) + T(M_\omega)$, which has the following explicit expression:

$$T(M_V) = G_V^2\Big[\frac{2\Lambda^2 - M_V^2}{2\Lambda^2 + (|\mathbf{p}| + |\mathbf{p}|')^2} - \frac{2\Lambda^2 - M_V^2}{2\Lambda^2 + (|\mathbf{p}| - |\mathbf{p}|')^2} + \ln\frac{2\Lambda^2 + (|\mathbf{p}| - |\mathbf{p}|')^2}{2\Lambda^2 + (|\mathbf{p}| + |\mathbf{p}|')^2} - \ln\frac{M_V^2 + (|\mathbf{p}| - |\mathbf{p}|')^2}{M_V^2 + (|\mathbf{p}| + |\mathbf{p}|')^2}\Big]. \quad (46)$$

The BS wave function satisfies a homogeneous equation. For the $B\bar{K}$ system, it can be solved numerically by discretizing the integration region $(0, \infty)$ into n pieces (n is large enough). After using the n-point Gauss quadrature rule to evaluate the integral, Eq. (40) becomes an eigenvalue equation. The numerical results for $\tilde{\chi}_P(|\mathbf{p}|)$ can be obtained by solving the eigenvalue equation. It can be seen from Eqs. (40)–(46) that there is only one parameter, the cutoff Λ, which contains the information about the nonpoint interaction due to the structure of hadrons. The value of Λ is a model dependent parameter. In Ref. [30] it is taken to be $\Lambda = 4.5$ GeV for the $KK\rho$ interaction, $\Lambda = 2.103$ GeV for the DK molecule bound state in the effective potential model [36], and in Ref. [28] Λ varies from 1.17 GeV to 4.5 GeV to form the $K\bar{K}$ bound state, we use $\Lambda_{\min} = 1.858$ GeV and $\Lambda_{\max} = 4.151$ GeV [16] to form the DK bound state. The values of Λ are about 1.27 GeV [25,26,38] in the quark-diquark picture for baryons. In the rest frame of the bound state $P = (E, 0)$, the binding energy of the $B\bar{K}$ system is defined as $E_b = E - (M_B + M_K)$. We vary E_b from −15 MeV to −85 MeV, the cutoff Λ from 1.0 GeV to 4.0 GeV. For a specific E_b, we find the solution of the eigenvalue equation corresponding to the BS Eq. (40) with the eigenvalue 1. There are several values of Λ corresponding to one value of E_b. We list the values of Λ and the corresponding E_b in Table I. The relation between $\Lambda_{\min}$ and E_b is depicted in Fig. 2. In our case, the binding energy $E_b = M_{B_{s0}^*(5725)} - (M_B + M_K) \simeq -50$ MeV, where we have used the mass $B_{s0}^*(5725)$ as 5725 MeV, and the averaged masses of B and K obtained from $M_{B^\pm} = 5279.15$ MeV, $M_{B^0} = 5279.53$ MeV, $M_{K^\pm} = 493.68$ MeV, and $M_{K^0} = 497.61$ MeV [39]. When $E_b = -50$ MeV, we find that there are three values of Λ, 1.579 GeV, 2.481 GeV, and 3.602 GeV, from which we can get solutions of the BS equation. The numerical result of the BS wave function $\tilde{\chi}_P(|\mathbf{p}|)$ is plotted in Fig. 3. It can be seen that the numerical solution of the BS wave function for different values of Λ are very close to each other.

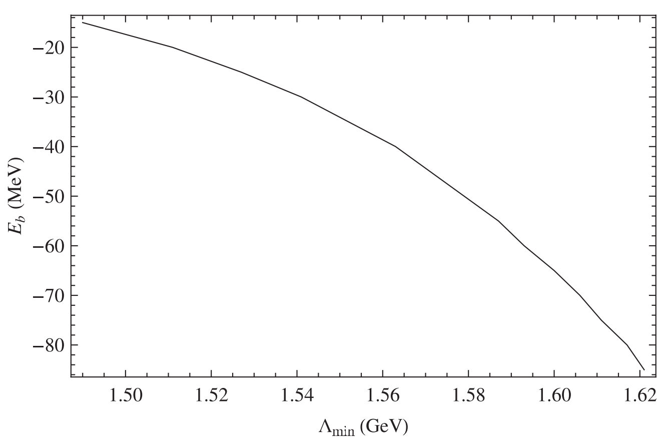

FIG. 2. Relation of the cutoff $\Lambda_{\min}$ and the binding energy E_b.

III. THE DECAY WIDTH FOR $B_{s0}^*(5725) \to B_s\pi^0$

In this section, we will calculate the decay width of the decay $B_{s0}^*(5725) \to B_s\pi^0$ through exchanging K^* and B^* mesons. Since this decay is an isospin-violating process,

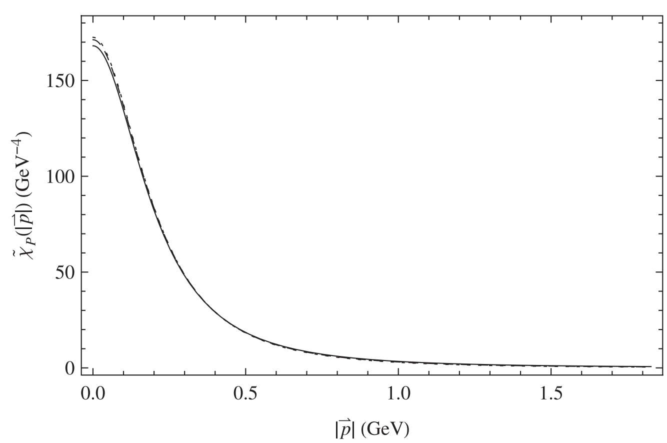

FIG. 3. Numerical result for the BS wave function $\tilde{\chi}_P(|\mathbf{p}|)$ for the bound state of $B\bar{K}$. The solid, dashed, and dotted lines correspond to $\Lambda = 1.579$ GeV, 2.481 GeV, and 3.602 GeV, respectively.

we also include the $\eta - \pi^0$ mixing effect. In Ref. [16], we calculated the decay width from two parts, one is originated directly from the transition $D_{s0}^*(2317)^+ \to D_s^+\pi^0$, while the other is originated from the $\eta - \pi^0$ mixing. The Lagrangian which gives mass to pseudoscalar octet is [40,41]

$$\mathcal{L}_{\text{mass}} = \frac{\mu f_\pi^2}{4}\,\text{Tr}(\xi m_q \xi + \xi^\dagger m_q \xi^\dagger), \tag{47}$$

where $\mu = \frac{m_\pi^2}{m_u + m_d}$ is related to the condensate parameter, $\xi = \exp(iM/f_\pi)$ with M representing the pseudoscalar octet matrix which has the form in Eq. (25); m_q refers to the light quark current mass matrix

$$m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}. \tag{48}$$

One can obtain the $\eta - \pi^0$ mixing from Eq. (47). On the other hand, the mixing effect can also be introduced by modifying the η and π^0 fields to a new form [41,42]

$$\pi^0 \to \pi^0 \cos\epsilon - \eta \sin\epsilon, \qquad \eta \to \pi^0 \sin\epsilon + \eta \cos\epsilon, \tag{49}$$

where ϵ is the mixing angle

$$\tan 2\epsilon = \frac{\sqrt{3}}{2}\frac{m_d - m_u}{m_s - \hat{m}}, \tag{50}$$

with $\hat{m} = (m_d + m_u)/2$.

As in Ref. [19], one can include the $\eta - \pi^0$ mixing by applying the pure π-coupling form with the modified flavor structure in Eq. (49). In other words, we replace $\pi^0\tau_3$ by $\pi_0(\tau_3 \cos\epsilon + I \sin\epsilon/\sqrt{3})$ for the coupling to BB^*, and by $\pi_0(\tau_3 \cos\epsilon + I \sin\epsilon\sqrt{3})$ for the coupling to KK^*. Then, the interactions among a vector meson and two pseudoscalar mesons are given by the following:

$$\begin{aligned}
\mathcal{L}_{B^*B\pi}(x) &= \frac{g_{B^*B\pi}}{2\sqrt{2}} B_\mu^{*\dagger}(x)\hat{\pi}_B i\overleftrightarrow{\partial}^\mu B(x) + \text{H.c.},\\
\mathcal{L}_{K^*K\pi}(x) &= \frac{g_{K^*K\pi}}{\sqrt{2}} K_\mu^{*\dagger}(x)\hat{\pi}_K i\overleftrightarrow{\partial}^\mu K(x) + \text{H.c.},\\
\mathcal{L}_{B^*B_sK}(x) &= g_{B^*B_sK} B_\mu^{\dagger}(x)K(x) i\overleftrightarrow{\partial}^\mu B_s^0(x) + \text{H.c.},\\
\mathcal{L}_{K^*B_sB}(x) &= g_{K^*B_sB} K_\mu^{\dagger}(x)B(x) i\overleftrightarrow{\partial}^\mu \bar{B}_s^0(x) + \text{H.c.},
\end{aligned} \tag{51}$$

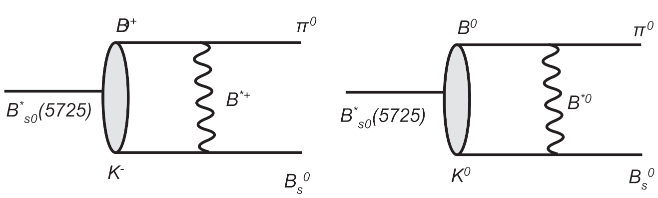

FIG. 4. Feynman diagrams for the strong decay $B_{s0}^*(5725) \to B_s\pi^0$ induced by B^* exchange.

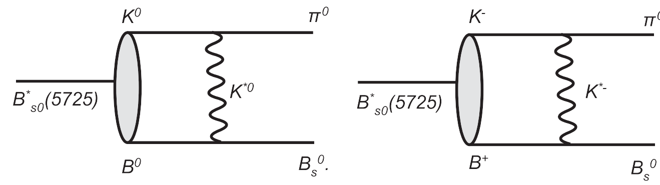

FIG. 5. Feynman diagrams for the strong decay $B_{s0}^*(5725) \to B_s\pi^0$ induced by K^* exchange.

where $\hat{\pi}_B = \pi_1\tau_1 + \pi_2\tau_2 + \pi_3(\tau_3 \cos\epsilon + I \sin\epsilon/\sqrt{3})$, $\hat{\pi}_K = \pi_1\tau_1 + \pi_2\tau_2 + \pi_3(\tau_3 \cos\epsilon + I \sin\epsilon\sqrt{3})$, π_3 refers to π^0, I is the identity matrix, and the doublets B^* and K^* have the following form:

$$B^* = \begin{pmatrix} B^{*+} \\ B^{*0} \end{pmatrix}, \qquad K^* = \begin{pmatrix} K^{*+} \\ K^{*0} \end{pmatrix}. \tag{52}$$

The coupling constant $g_{K^*K\pi}$ is 4.61 [39], and the coupling constants $g_{D^*D\pi} = 17.9$ [39]. $g_{D^*D_sK}(= 2.02)$ has been estimated using the light-cone QCD sum rules in Ref. [43], and $g_{DD_sK^*}(= 3.787)$ is related to the strong coupling of heavy mesons with ρ meson in terms of chiral and heavy quark symmetry [44]. With the help of heavy hadron chiral perturbation theory [41,45], the coupling constants corresponding to B meson are related to those of D meson, $g_{B^*B\pi} = g_{D^*D\pi}m_B/m_D$, $g_{B^*B_sK} = g_{D^*D_sK}m_B/m_D$, $g_{K^*B_sB} = g_{K^*D_sD}m_B/m_D$.

The decay $B_{s0}^*(5725) \to B_s\pi^0$ induced by B^* exchange is shown in Fig. 4; we can get the amplitude according to interactions in Eq. (51)

$$\begin{aligned}
\mathcal{M}_{B^*} = \frac{g_{B^*B\pi}g_{B^*B_sK}\sqrt{E}}{2}\int\frac{d^4p}{(2\pi)^4}C_{(0,0)}^{ij}F(|\mathbf{k}|)^2 \\
\times[(2\eta_1P + p + q)^\mu(2\eta_2P - p - q)^\nu] \\
\times(\tau_3\cos\epsilon + I\sin\epsilon/\sqrt{3})_{ji}\Delta_{\mu\nu}(k, M_{B^*})|_{k=p-q}(2\pi)^4 \\
\times\delta^4(p_1 + p_2 - P)\chi_P(p).
\end{aligned} \tag{53}$$

After azimuthal integration, the integration part in Eq. (53) becomes

$$\begin{aligned}
&\int\frac{d^4p}{(2\pi)^4}[(2\eta_1P + p + q)^\mu(2\eta_2P - p - q)^\nu] \\
&\quad\times\Delta_{\mu\nu}(k)|_{k=p-q}\chi_P(p) \\
&\quad = \frac{i}{(2\pi)^3}\int_{-\infty}^{+\infty}dp^0\int_0^{+\infty}d|\mathbf{p}||\mathbf{p}|^2f_1(p^0)\chi_P(p^0, |\mathbf{p}|),
\end{aligned} \tag{54}$$

where $q \equiv \eta_2q_1 - \eta_1q_2$, q_1 and q_2 are the four-momenta of π^0 and B_s^0, respectively; $f_1(p^0)$ takes the form

$$f_1(p^0) = \frac{(M_{B^*}^2 - 2\Lambda^2)^2}{(|\mathbf{p}|^2 - 2p^0q^0 + |\mathbf{q}|^2 + u_4 + 2\Lambda^2)^2} \times \Big[\frac{2(|\mathbf{p}|^2 - 2p^0q^0 + |\mathbf{q}|^2 + u_4 + 2\Lambda^2)(|\mathbf{p}|^2 - 2p^0q^0 + |\mathbf{q}|^2 - u_1 + u_2u_3/M_{B^*}^2 + 2\Lambda^2)}{(|\mathbf{p}|^2 - 2|\mathbf{p}||\mathbf{q}| + |\mathbf{q}|^2 + 2\Lambda^2)(|\mathbf{p}|^2 + 2|\mathbf{p}||\mathbf{q}| + |\mathbf{q}|^2 + 2\Lambda^2)} + \Big(\frac{u_1 + u_4 - u_2u_3/M_{B^*}^2}{2|\mathbf{p}||\mathbf{q}|}\Big)\ln\frac{2p^0q^0 + 2|\mathbf{p}||\mathbf{q}| - u_4}{2p^0q^0 - 2|\mathbf{p}||\mathbf{q}| - u_4}\frac{|\mathbf{p}|^2 - 2|\mathbf{p}||\mathbf{q}| + |\mathbf{q}|^2 + 2\Lambda^2}{|\mathbf{p}|^2 + 2|\mathbf{p}||\mathbf{q}| + |\mathbf{q}|^2 + 2\Lambda^2}\Big], \tag{55}$$

where

$$\begin{aligned} u_1 &= 2p\cdot P + 2q\cdot P - p^2 - q^2 - 4\eta_1 p\cdot P - 4\eta_1 q\cdot P - q^2 + 4P^2(1-\eta_1), \\ u_2 &= 2p\cdot P - 2q\cdot P - p^2 + q^2 - 2\eta_1 p\cdot P + 2\eta_1 q\cdot P, \\ u_3 &= p^2 - q^2 + 2\eta_1 p\cdot P - 2\eta_1 q\cdot P, \\ u_4 &= p^2 + q^2 - M_{B^*}^2. \end{aligned} \tag{56}$$

Similarly, the diagrams for $B_{s0}^*(5725) \to B_s\pi^0$ through exchanging K^* meson are shown in Fig. 5, and the amplitude can be written as

$$\begin{aligned} \mathcal{M}_{K^*} = {}& g_{K^*K\pi}g_{K^*B_sK}\sqrt{E}\int\frac{d^4p}{(2\pi)^4}C^{ij}_{(0,0)}F(|\mathbf{k}|)^2 \\ &\times[(2\eta_1P + p + q)^\mu(2\eta_2P - p - q)^\nu] \\ &\times(\tau_3\cos\epsilon + I\sin\epsilon\sqrt{3})_{ji} \\ &\times\Delta_{\mu\nu}(k, M_{K^*})|_{k=\eta_2P-\eta_1P-p-q}(2\pi)^4 \\ &\times\delta^4(p_1 + p_2 - P)\chi_P(p). \end{aligned} \tag{57}$$

Taking the same procedure as Eq. (53), we obtain

$$\begin{aligned} &\int\frac{d^4p}{(2\pi)^4}[(2\eta_1P + p + q)^\mu(2\eta_2P - p - q)^\nu] \\ &\quad\times\Delta_{\mu\nu}(k)|_{k=\eta_2P-\eta_1P-p-q}\chi_P(p) \\ &\quad= \frac{i}{(2\pi)^3}\int_{-\infty}^{+\infty}dp^0\int_0^{+\infty}d|\mathbf{p}||\mathbf{p}|^2f_2(p^0)\chi_P(p^0, |\mathbf{p}|), \end{aligned} \tag{58}$$

where $f_2(p^0)$ has the following expression:

$$f_2(p^0) = \frac{(M_{K^*}^2 - 2\Lambda^2)^2}{(|\mathbf{p}|^2 - 2p^0q^0 + |\mathbf{q}|^2 + u_4 + 2\Lambda^2)^2} \times \Big[\frac{2(|\mathbf{p}|^2 - 2p^0q^0 + |\mathbf{q}|^2 + u_4 + 2\Lambda^2)(|\mathbf{p}|^2 - 2p^0q^0 + |\mathbf{q}|^2 - u_1 + u_2u_3/M_{K^*}^2 + 2\Lambda^2)}{(|\mathbf{p}|^2 - 2|\mathbf{p}||\mathbf{q}| + |\mathbf{q}|^2 + 2\Lambda^2)(|\mathbf{p}|^2 + 2|\mathbf{p}||\mathbf{q}| + |\mathbf{q}|^2 + 2\Lambda^2)} + \Big(\frac{u_1 + u_4 - u_2u_3/M_{K^*}^2}{2|\mathbf{p}||\mathbf{q}|}\Big)\ln\frac{2p^0q^0 + 2|\mathbf{p}||\mathbf{q}| - u_4}{2p^0q^0 - 2|\mathbf{p}||\mathbf{q}| - u_4}\frac{|\mathbf{p}|^2 - 2|\mathbf{p}||\mathbf{q}| + |\mathbf{q}|^2 + 2\Lambda^2}{|\mathbf{p}|^2 + 2|\mathbf{p}||\mathbf{q}| + |\mathbf{q}|^2 + 2\Lambda^2}\Big], \tag{59}$$

with

$$\begin{aligned} u_1 &= p^2 + q^2 - P^2, \\ u_2 &= p^2 - q^2 - 2\eta_1 p\cdot P - 2\eta_1 q\cdot P - (2\eta_1 - 1)P^2 + 2p\cdot P, \\ u_3 &= p^2 - q^2 + 2\eta_1 p\cdot P - 2\eta_1 q\cdot P + (2\eta_1 - 1)P^2 + 2q\cdot P, \\ u_4 &= p^2 + q^2 - M_{K^*}^2 + (2\eta_1 - 1)^2P^2 - 4\eta_1 p\cdot P + 2p\cdot P + 4\eta_1 q\cdot P - 2q\cdot P. \end{aligned} \tag{60}$$

In the isospin limit ($m_u = m_d$), the mixing angle ϵ vanishes; the masses of B^0 and B^+, B^{*0} and B^{*+}, K^0 and K^+, K^{*0} and K^{*+} are equal to each other, respectively. The contribution from B^* exchange in the left panel of Fig. 4 cancels with that in the right one. Similar cancellation also occurs in Fig. 5. In the calculation, we use the physical mass values of $B(B^*)$ and $K(K^*)$.

In the numerical calculation, we use the following input parameters as in Ref. [39]: $M_{B^*} = 5325.0$ MeV, $M_{K^{*0}} = 896.00$ MeV, $M_{K^{*\pm}} = 891.66$ MeV, $M_{\pi^0} = 134.98$ MeV, $M_{B_s^0} = 5366.3$ MeV. After applying the numerical solution of the BS equation for $B_{s0}^*(5725)$ to the strong decay $B_{s0}^*(5725) \to B_s^0\pi^0$, we obtain the total decay width for different values of Λ

TABLE II. Decay width (in KeV) of $B_{s0}^*(5725) \to B_s\pi^0$ from various theoretical approaches.

Ref.	[3]	[14]	[19]	[46]
$\Gamma(B_{s0}^*(5725) \to B_s\pi^0)$	21.5	1.54	55.2–89.9	6.8–30.7

$$\begin{aligned} \Lambda &= 1.579\ \text{GeV}, & \Gamma &= 39.64\ \text{KeV}, \\ \Lambda &= 2.481\ \text{GeV}, & \Gamma &= 92.20\ \text{KeV}, \\ \Lambda &= 3.602\ \text{GeV}, & \Gamma &= 112.34\ \text{KeV}. \end{aligned} \tag{61}$$

When $E_b = -15$ MeV, the decay width for different values of Λ are as following:

$$\begin{aligned} \Lambda &= 1.490\ \text{GeV}, & \Gamma &= 2.98\ \text{KeV}, \\ \Lambda &= 2.432\ \text{GeV}, & \Gamma &= 5.89\ \text{KeV}, \\ \Lambda &= 3.576\ \text{GeV}, & \Gamma &= 7.16\ \text{KeV}. \end{aligned} \tag{62}$$

When $E_b = -85$ MeV, corresponding to different values of Λ, the decay width is

$$\begin{aligned} \Lambda &= 1.621\ \text{GeV}, & \Gamma &= 237.6\ \text{KeV}, \\ \Lambda &= 2.506\ \text{GeV}, & \Gamma &= 513.4\ \text{KeV}, \\ \Lambda &= 3.622\ \text{GeV}, & \Gamma &= 662.2\ \text{KeV}. \end{aligned} \tag{63}$$

We note that when $E_b \neq -50$ MeV, the bound state is not $B_{s0}^*(5725)$.

Because of the smaller binding energy the spatial extent of hadron molecules, which are characterized by their rms radius, is expected to be much larger than that of normal hadrons which are composed of compact quarks and antiquarks. With the numerical results for the BS wave functions in coordinate space which are obtained by Fourier transformation of the wave functions in momentum space, we calculate the rms radius for different values of E_b and Λ. When $E_b = -50$ MeV, $\sqrt{<r^2>}$ is 5.17 fm, 5.45 fm, and 5.55 fm corresponding to $\Lambda = 1.579$ GeV, 2.481 GeV, and 3.602 GeV, respectively. When $E_b = -15$ MeV, $\sqrt{<r^2>}$ is 7.53 fm, 7.60 fm, and 7.63 fm, while when $E_b = -85$ MeV, $\sqrt{<r^2>}$ is 4.17 fm, 4.21 fm, and 4.23 fm for corresponding values of Λ.

The strong decay width of $B_{s0}^*(5725) \to B_s\pi^0$ has also been calculated by several groups. In Table II, we list their results for comparison. In Ref. [3], the authors use the heavy hadron chiral perturbation theory. The result in Ref. [14] is based on the heavy chiral Lagrangian and the unitarized coupled-channel method. In Ref. [19], the effective Lagrangian approach is employed, and the result in Ref. [46] is from light-cone QCD sum rules.

IV. SUMMARY

The scalar charm-strange state $D_{s0}^*(2317)$ has been studied extensively in many papers. It is predicted that the b-partner of $D_{s0}^*(2317)$, $B_{s0}^*(5725)$, may exist. The state $B_{s0}^*(5725)$ has been studied theoretically, and the suggested quantum numbers of $B_{s0}^*(5725)$ are $I(J^P) = 0(0^+)$. Because the mass of $B_{s0}^*(5725)$ is below the threshold of $B\bar{K}$, we assume that $B_{s0}^*(5725)$ is an S-wave $B\bar{K}$ molecular bound state, just like the state $D_{s0}^*(2317)^+$ in which the mass is below the DK threshold. Based on this picture, we use the BS equation to analyze this possible structure of $B_{s0}^*(5725)$. We find that $B\bar{K}$ can form a bound state.

In addition, we also calculate the decay width of $B_{s0}^*(5725) \to B_s\pi^0$ including the $\eta - \pi^0$ mixing effect. We predict that the decay width is in the range 39.64–112.34 KeV. We expect forthcoming experimental results to test our model for the $B\bar{K}$ molecular structure of the state $B_{s0}^*(5725)$.

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[1] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **90**, 242001 (2003).
[2] S. Godfrey, Phys. Lett. B **568**, 254 (2003).
[3] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. D **68**, 054024 (2003).
[4] P. Colangelo and F. De. Fazio, Phys. Lett. B **570**, 180 (2003).
[5] X. Fayyazuddin and X. Riazuddin, Phys. Rev. D **69**, 114008 (2004).
[6] P. Colangelo, F. DeFazio, and A. Ozpineci, Phys. Rev. D **72**, 074004 (2005).
[7] J. Lu, W.-Z. Deng, X.-L. Chen, and S.-L. Zhu, Phys. Rev. D **73**, 054012 (2006).
[8] H.-Y. Cheng and W.-S. Hou, Phys. Lett. B **566**, 193 (2003).
[9] Y.-Q. Chen and X.-Q. Li, Phys. Rev. Lett. **93**, 232001 (2004).
[10] H. Kim and Y. Oh, Phys. Rev. D **72**, 074012 (2005).
[11] T. Barnes, F. E. Close, and H. J. Lipkin, Phys. Rev. D **68**, 054006 (2003).
[12] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B **582**, 39 (2004).

[13] J. Hofmann and M. F. M. Lutz, Nucl. Phys. **A733**, 142 (2004).
[14] F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, and B.-S. Zou, Phys. Lett. B **641**, 278 (2006).
[15] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y.-L. Ma, Phys. Rev. D **76**, 014005 (2007).
[16] Z.-X. Xie, G.-Q. Feng, and X.-H. Guo, Phys. Rev. D **81**, 036014 (2010).
[17] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **100**, 082001 (2008).
[18] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **100**, 082002 (2008).
[19] A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y.-L. Ma, Phys. Rev. D **77**, 114013 (2008); **76**, 114008 (2007).
[20] D. Ebert, V. O. Galkin, and R. N. Faustov, Phys. Rev. D **57**, 5663 (1998).
[21] S. Godfrey and R. Kokoski, Phys. Rev. D **43**, 1679 (1991).
[22] P. Colangelo, F. De Fazio, and R. Ferrandes, Nucl. Phys. B, Proc. Suppl. **163**, 177 (2007).
[23] M. E. Bracco and M. Nielsen, Phys. Rev. D **82**, 034012 (2010).
[24] H.-Y. Jin, C.-S. Huang, and Y.-B. Dai, Z. Phys. C **56**, 707 (1992).
[25] X.-H. Guo and T. Muta, Phys. Rev. D **54**, 4629 (1996).
[26] X.-H. Guo, A. W. Thomas, and A. G. Williams, Phys. Rev. D **59**, 116007 (1999).
[27] X.-H. Guo, Mod. Phys. Lett. A **13**, 2265 (1998); X.-H. Guo, A. W. Thomas, and A. G. Williams, Phys. Rev. D **61**, 116015 (2000); **64**, 096004 (2001).
[28] X.-H. Guo and X.-H. Wu, Phys. Rev. D **76**, 056004 (2007).
[29] C. S. Kim and G.-L. Wang, Phys. Lett. B **584**, 285 (2004); **634**, 564 (2006).
[30] D. Lohse, J. W. Dorso, K. Holinde, and J. Speth, Nucl. Phys. **A516**, 513 (1990).
[31] S. Krewald, R. H. Lemmer, and F. P. Sassen, Phys. Rev. D **69**, 016003 (2004).
[32] F.-Q. Wu, B.-S. Zou, L.-Li, and D. V. Bugg, Nucl. Phys. **A735**, 111 (2004).
[33] J. Hofmann and M. F. M. Lutz, Nucl. Phys. A **763**, 90 (2005).
[34] J. E. Palomar and E. Oset, Nucl. Phys. **A716**, 169 (2003).
[35] K. Kawaraayashi and M. Suzuki, Phys. Rev. Lett. **16**, 255 (1966); X. Riazuddin and X. Fayyazuddin, Phys. Rev. **147**, 1071 (1966).
[36] Y.-J. Zhang, H.-C. Chiang, P.-N. Shen, and B.-S. Zou, Phys. Rev. D **74**, 014013 (2006).
[37] D. Lurie, *Particle and Fields* (McGraw-Hill, New York, 1968).
[38] M. Anselmino, P. Kroll, and B. Pire, Z. Phys. C **36**, 89 (1987).
[39] C. Amsler *et al.*, Phys. Lett. B **667**, 1 (2008).
[40] P. Cho and M. B. Wise, Phys. Rev. D **49**, 6228 (1994).
[41] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
[42] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).
[43] Z.-G. Wang and S.-L. Wan, Phys. Rev. D **74**, 014017 (2006).
[44] H.-Y. Cheng, C.-K. Chua, and A. Soni, Phys. Rev. D **71**, 014030 (2005).
[45] M. B. Wise, Phys. Rev. D **45**, R2188 (1992).
[46] Z.-G. Wang, Eur. Phys. J. C **56**, 181 (2008).