Nonlinear polarization bistability in optical nanowires

Wen Qi Zhang, M. A. Lohe, Tanya M. Monro, and Shahraam Afshar V.*

Institute for Photonics & Advanced Sensing (IPAS), The University of Adelaide, 5005, Australia

**Corresponding author: shahraam.afshar@adelaide.edu.au*

Received December 6, 2010; accepted January 7, 2011;
posted January 21, 2011 (Doc. ID 139229); published February 15, 2011

Using the full vectorial nonlinear Schrödinger equations that describe nonlinear processes in isotropic optical nanowires, we show that there exist structural anisotropic nonlinearities that lead to unstable polarization states that exhibit periodic bistable behavior. We analyze and solve the nonlinear equations for continuous waves by means of a Lagrangian formulation and show that the system has bistable states and also kink solitons that are limiting forms of the bistable states.

OCIS codes: 190.4370, 190.4360, 190.3270, 130.4310, 060.5530, 190.1450.

The nonlinear interactions of the two fundamental polarization modes of a waveguide lead to a host of nonlinear effects that have been studied extensively over the past 30 years (see [1], Chapter 6). Most previous work has used the weak guidance approximation according to which the modes of the waveguide are linearly polarized in the transverse plane, and are mutually orthogonal. It is usually assumed that the effective Kerr nonlinear coefficients of the two polarizations are equal for isotropic materials with Kerr nonlinearities. The weak guidance approximation does not, however, provide an accurate description for modal behavior, including nonlinear behavior, in waveguides with subwavelength dimensions and high index contrast, such as optical nanowires. In these waveguides there exists a large component of the electric field along the direction of propagation, which changes the orthogonality relation between the modes and contributes to nonlinear processes [2,3]. Full vectorial nonlinear Schrödinger (VNLS) equations have recently been developed that generalize the NLS equation for Kerr nonlinearity in all optical waveguides, including optical nanowires [2,4,5].

Here, we analyze an aspect of the nonlinear interactions of the two polarizations of a mode in optical nanowires that occurs within the VNLS model, which has not been previously explored. We reveal the existence of anisotropic nonlinear behavior with respect to the two polarizations of a mode that is structural in origin. This anisotropy originates from the structure of the waveguide in the subwavelength regime, not from the anisotropy of the waveguide materials and so differs from that reported in [5]. Furthermore, this anisotropy leads to periodic bistable polarization states (defined below), properties of which we describe here.

For waveguides with isotropic materials such as glass, the nonlinear interactions of the two polarizations are usually described by the coupled NLS equations [1]:

$$\frac{\partial A_j}{\partial z} + \sum_{n=1}^{\infty} \frac{i^{n-1}}{n!}\beta_{jn}\frac{\partial^n A_j}{\partial t^n} = i(\gamma_j|A_j|^2 + \gamma_c|A_k|^2)A_j + i\gamma_c' A_j^* A_k^2 \exp(-2iz\Delta\beta_{jk}), \quad (1)$$

where j, $k = 1$, $2 (j \neq k)$ are the two polarization modes, A_1, A_2 are the amplitudes of the corresponding fields, β_{jn} are the nth order propagation constants, $\Delta\beta_{jk} = -\Delta\beta_{kj}$ is the linear birefringence, γ_j, γ_c, and γ_c' are the effective nonlinear coefficients representing self phase modulation, cross phase modulation, and coherent coupling of the two polarization modes, respectively. The weak guidance approximation assumes that the effective mode areas of the two polarization modes are equal [1], leading to

$$\gamma_1 = \gamma_2 = 3\gamma_c/2 = 3\gamma_c'. \quad (2)$$

Weak guidance, and its related approximations, are no longer appropriate when considering light propagation in optical nanowires. The VNLS model shows that the propagating modes of a waveguide can have large z components (along the direction of propagation) [2]. We may derive a generalized form of Eq. (1) in the VNLS model by defining effective nonlinear vectorial coefficients γ_1, γ_2, γ_c, γ_c' which (in the notation of [2]) have the form

$$\gamma_j = \frac{2\pi\varepsilon_0}{3\mu_0\lambda}\int n^2(x,y)n_2(x,y)[2|\hat{\mathbf{e}}_j|^4 + |\hat{\mathbf{e}}_j^2|^2]\mathrm{d}A,$$
$$\gamma_c = \frac{4\pi\varepsilon_0}{3\mu_0\lambda}\int n^2(x,y)n_2(x,y)[|\hat{\mathbf{e}}_1|^2 + |\hat{\mathbf{e}}_2|^2]\mathrm{d}A,$$
$$\gamma_c' = \frac{2\pi\varepsilon_0}{3\mu_0\lambda}\int n^2(x,y)n_2(x,y)[\hat{\mathbf{e}}_1^2 + \hat{\mathbf{e}}_2^2]\mathrm{d}A. \quad (3)$$

Here $\hat{\mathbf{e}}_j = \mathbf{e}_j/\sqrt{N_j} (j = 1, 2)$ are the electric fields normalized by $N_j = \int|\mathbf{e}_j \times \mathbf{h}_j^* \cdot \hat{\mathbf{z}}|\mathrm{d}A$, and $n(x,y)$, $n_2(x,y)$ are the refractive and nonlinear refractive index distributions, respectively. In deriving Eq. (1) within the VNLS model we assume that terms containing $\hat{\mathbf{e}}_1 \cdot \hat{\mathbf{e}}_2$, although no longer zero, are nevertheless negligible by comparison with the other terms on the right-hand side of Eq. (1) (see Eq. (32) in [2]). We have recently confirmed experimentally [3] that the expression for γ_j in Eq. (3) is accurate for optical nanowires.

Figure 1 shows the γ coefficients calculated using Eq. (3), for elliptical waveguides surrounded by air, with chalcogenide glass ($n = 2.8$, $n_2 = 1.1 \times 10^{-17}\,\mathrm{m}^2/\mathrm{W}$ at $\lambda = 1.55\,\mu\mathrm{m}$) as the host material. Evidently the equalities (2) do not generally hold for these γ values. Figure 1 also shows that γ_1, γ_2 are asymmetric with respect to the diagonal line where the fiber is circular. This indicates that elliptical shapes have higher γ values than circular shapes, and that in elliptical waveguides the γ values of the modes polarized along the major/minor axes are different, similar to the γ values in waveguides with anisotropic materials. As a consequence, the nonlinear

0146-9592/11/040588-03$15.00/0

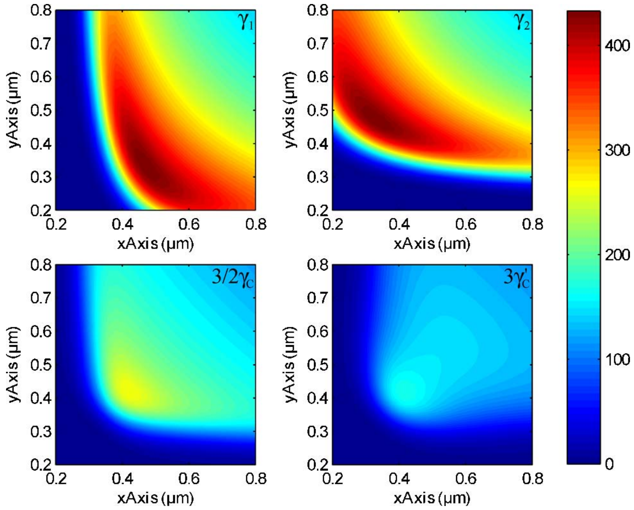

Fig. 1. (Color online) Contour plots of $\gamma_1, \gamma_2, 3\gamma_c/2, 3\gamma_c'$ in units of $(\mathrm{W}\cdot\mathrm{m})^{-1}$ as functions of the major/minor diameters for elliptical waveguides.

behavior in birefringent waveguides includes anisotropic properties, which we refer to as structurally induced anisotropic nonlinearity.

We now solve Eq. (1) in the static case for general values of the γ coefficients, where Eq. (2) are not necessarily satisfied, in particular for $\gamma_1 + \gamma_2 \neq 2(\gamma_c + \gamma_c')$. We substitute $A_j = \sqrt{P_j}e^{i\phi_j}$ for $j = 1, 2$ into Eq. (1), where P_j is the power of the field A_j with phase ϕ_j. For continuous waves we find that $P_1 + P_2 = P_0$ is constant in z. Define the following dimensionless variables:

$$v = \frac{P_1}{P_0}, \qquad \theta = 2\Delta\phi, \qquad \tau = 2\gamma_c' P_0 z,$$
$$a = -\frac{\Delta\beta_{12}}{\gamma_c' P_0} - \frac{\gamma_c - \gamma_2}{\gamma_c'}, \qquad b = \frac{\gamma_1 + \gamma_2 - 2\gamma_c}{2\gamma_c'}, \tag{4}$$

where $\Delta\phi = \phi_1 - \phi_2 + z\Delta\beta_{12}$ is the phase difference between the two fields. Evidently b depends only on the nanowire parameters, whereas a also depends on the total power P_0. From Eq. (1) we obtain

$$\dot{v} \equiv \frac{dv}{d\tau} = v(1-v)\sin\theta, \tag{5}$$

$$\dot{\theta} \equiv \frac{d\theta}{d\tau} = -a + 2bv + (1-2v)\cos\theta. \tag{6}$$

We choose initial values $\theta_0 = \theta(0)$, $v_0 = v(0)$ with $0 < v_0 < 1$, where we regard τ as a "time" variable, then it can be shown from Eq. (5) that $0 < v(\tau) < 1$ for all $\tau > 0$, i.e. v always remains within the physical region. We solve Eqs. (5) and (6), in terms of periodic elliptic functions by observing that $\Gamma = -av + bv^2 + v(1-v)\cos\theta$ is a constant of the motion, enabling us to write $\dot{v}^2 = Q(v)$ where Q is the fourth degree polynomial $Q(v) = v^2(1-v)^2 - (\Gamma + av - bv^2)^2$. The minimum and maximum values of v, denoted $v_{\min}$, $v_{\max}$ respectively, occur when $\dot{v} = 0$, i.e. at zeroes of Q. Since $Q(0)$, $Q(1) < 0$ and $Q(v_0) = v_0^2(1-v_0)^2\sin^2\theta_0 \geqslant 0$ we deduce that Q generally has at least two real zeroes in the interval $(0, 1)$. We integrate $\dot{v} = \sqrt{Q(v)}$ over the half-period in which v increases, in order to find τ as a function of v, and also the period T:

$$\int_{v_{\min}}^{v} \frac{du}{\sqrt{Q(u)}} = \tau - \tau_0, \qquad T = 2\int_{v_{\min}}^{v_{\max}} \frac{du}{\sqrt{Q(u)}}, \tag{7}$$

where $v_{\min} = v(\tau_0)$. These integrals may be evaluated in terms of elliptic integrals of the first kind, see for example [6] (Sections 3.145, 3.147). In particular, T is expressible in terms of the complete elliptic integral K, and so can be written as an explicit function of a, b, v_0, θ_0, i.e. as a function of the waveguide parameters and the initial power and phase of the input fields. The precise formulas depend on the relative location of the roots of Q.

We are interested in solutions which begin near the unstable steady states of Eqs. (5) and (6) because these lead to bistable solutions, where "bistable" refers to configurations which take values in adjacent unstable steady states. There are four classes of steady state solutions:

$$\cos\theta = 1, \qquad v = \frac{a-1}{2(b-1)}, \qquad (b \neq 1), \tag{8}$$

$$\cos\theta = -1, \qquad v = \frac{a+1}{2(b+1)}, \qquad (b \neq -1), \tag{9}$$

$$\cos\theta = a, \qquad v = 0, \qquad (|a| \leqslant 1), \tag{10}$$

$$\cos\theta = -a + 2b, \qquad v = 1, \qquad (|a - 2b| \leqslant 1). \tag{11}$$

Of these, (10) and (11) lie on the boundary of the physical region $0 < v < 1$, and (8) and (9) lie within the physical region for restricted values of a, b. We determine the stability of these steady states, and hence identify bistable states, by means of a Lagrangian formulation of Eqs. (5) and (6). By differentiating Eq. (6) and substituting for $\dot{v}$ and v in terms of θ, we obtain a second-order equation for θ that is precisely the equation of motion derived from the Lagrangian

$$L = T - V = \tfrac{1}{2}M(\theta)\dot{\theta}^2 - V(\theta), \tag{12}$$

where the "mass" M and potential V are given by

$$M(\theta) = \frac{2}{|b - \cos\theta|}, \qquad V(\theta) = -|b - \cos\theta| - \frac{(a-b)^2}{|b - \cos\theta|}. \tag{13}$$

The conserved energy $T + V$ is related to the conserved quantity Γ. Stability of each steady state solution, for which $V' = 0$, is determined by the sign of V'' at that solution, corresponding to either a local maximum or minimum of V. The shape of the potential, which depends on a, b but is always periodic, provides qualitative information on the properties of $\theta(\tau)$, for example small periodic oscillations occur when θ is near a local minimum, and bistable solutions and associated kink solitons appear when θ takes values near local maxima of V. θ can be

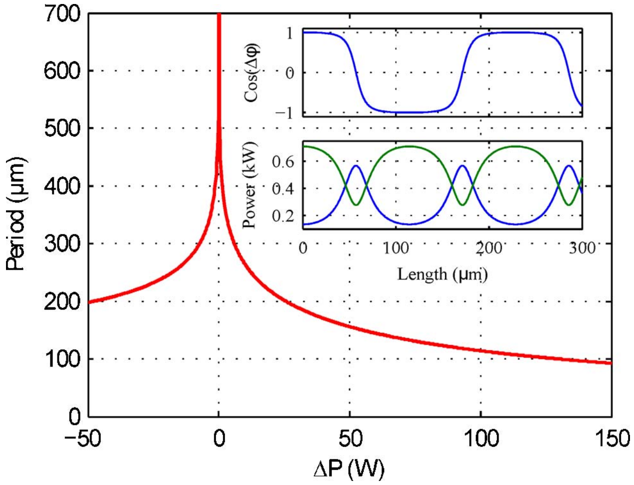

Fig. 2. (Color online) The period T as a function of ΔP. The insets show the periodic variation of the two polarization powers P_1, P_2, and $\cos\Delta\phi$, as the pulse (for $\Delta P = 100$ W) propagates along the fiber.

periodic, or an increasing or decreasing function of τ, although the corresponding power v is periodic.

We focus now on the first class of steady state solutions (8), which was not found in previous studies [1] (Chapter 6) for which Eq. (2) holds, leading to $b = 1$. We are interested in the case $b > 1$, which holds for a range of γ values calculated using Eq. (3). We require $1 < a < 2b - 1$ in order that the steady state solutions (8) satisfy $0 < v < 1$. Since $V'' = (a-1)(a-2b+1)/(b-1)^2$ is then negative these solutions are unstable. Small perturbations push the fields away from these steady states; however, the fields do not become chaotic, rather both the power v and $\cos\Delta\phi$ are periodic functions of τ. The period becomes arbitrarily large as the initial values v_0, θ_0 approach the unstable steady state, and $\cos\Delta\phi$ in particular shows bistability with abrupt transitions between the values ± 1 (an example is shown in the inset in Fig. 2). For some values of a, b this behavior is very sensitive to the proximity of v_0, θ_0 to the steady state, for example if $a = b$ the system behaves like a nonlinear pendulum, as is evident from the potential V in Eq. (13). For other values, such as $a = 1$ or $a = 2b - 1$, for which V', V'', V''' are all zero at the steady state, bistability is less sensitive to the values of v_0, θ_0.

In addition to periodic solutions, Eqs. (5) and (6) also have soliton solutions. These static kink solitons may be regarded as limits of the periodic bistable solutions, but with infinite period, and are found by solving the differential equation for θ with the boundary condition $\cos\theta \to 1$ as $|\tau| \to \infty$. An explicit exact solution is

$$\cos\theta = 1 + \frac{2\kappa}{1 - (\kappa+1)\cosh^2\sqrt{\kappa}(\tau - c)}, \qquad (14)$$

where $\kappa = (a-1)(-a+2b-1)/2(b-1)$ and c is any constant. We have $\kappa > 0$ for the values of a, b under consideration, namely, $1 < a < 2b - 1$; however, at $b = 1$ the solutions (14) do not exist. Given θ from Eq. (14), the power v is obtained from Eq. (6), and (5) is then also satisfied. The soliton (14) can propagate in time as a pulse over the length of the waveguide according to the evolution Eq. (1) and maintains its identity as a soliton provided that the boundary conditions remain intact.

As an example of bistable solutions, consider an elliptical waveguide made of chalcogenide glass with major/minor diameters equal to 640 and 620 nm, with γ values calculated from Eq. (3) giving $b = 2.4$. Consider initial values for the system that correspond to an unstable steady state, by setting $P_1(0) = p_1 + \Delta P$, $P_2(0) = p_2$ where $p_1 = 150.5$ W, $p_2 = 793.3$ W and ΔP (in units of watts) is a perturbation on p_1, together with $\theta_0 = 0$. We plot the period T as a function of ΔP in Fig. 2. At $\Delta P = 0$ we have $a = 1.3$ (hence $1 < a < 2b - 1$ is satisfied) and $v_0 = p_1/(p_1 + p_2) = (a-1)/2(b-1)$ corresponds to the unstable steady state solution (8), for which T is infinite. The insets in Fig. 2 show P_1, P_2, and $\cos\Delta\phi$ as functions of z, where the periodicity of the power functions and the bistability of $\cos\Delta\phi$ are evident. The polarization vector flips through an angle ≈20° over each period.

In conclusion, we have shown that within a full vectorial model of nonlinear processes in optical nanowires, we obtain γ coefficients that do not necessarily satisfy the relations (2). This results in structurally induced anisotropic nonlinearities for isotropic material-based linearly birefringent waveguides. The model allows continuous wave solutions of three types: steady state solutions, periodic (including bistable) solutions, and kink soliton solutions. The bistable states and the soliton solutions (14) exist only for the extended range of γ coefficients. Properties of the bistable states can in principle be utilized to construct photonic devices such as optical logical gates.

T. M. Monro acknowledges the support of an ARC Federation Fellowship, and M. Lohe acknowledges the support of the Larry Biedenharn Fellowship.

References

1. G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, 2007).
2. S. Afshar V. and T. M. Monro, Opt. Express **17**, 2298 (2009).
3. S. Afshar V., W. Q. Zhang, H. Ebendorff-Heidepriem, and T. M. Monro, Opt. Lett. **34**, 3577 (2009).
4. F. Biancalana, T. X. Tran, S. Stark, M. A. Schmidt, and P. St. J. Russell, Phys. Rev. Lett. **105**, 093904 (2010).
5. B. A. Daniel and G. P. Agrawal, J. Opt. Soc. Am. B **27**, 956 (2010).
6. I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products* (Academic, 1965).