Noisy excitation of nonlinear oscillator arrays

MW Hamilton, T Hill and L Stamatescu,

Department of Physics and Mathematical Physics,
The University of Adelaide, Adelaide SA5005, Australia

Abstract: We measure the composition of collective mode resonances in the intensity fluctuations of a multimode laser by separating the laser modes with Fabry-Perot interferometers and applying transfer function and cross-spectrum techniques. Comparison of the transfer function with the power spectrum of the fluctuations in individual cavity modes shows features that are attributable to the difference between single frequency excitation and noisy excitation of the collective mode resonances. These data together with the cross-spectra of the modal intensity fluctuations, highlight a need for greater understanding of the mechanisms of noisy excitation of a nonlinear oscillator array.

Although this conference is about unsolved problems of noise, this paper is in many ways more about opportunities than problems. The two are really just opposite sides of the one coin. It is well known and not particularly surprising that noise has an important effect on the dynamics arrays of nonlinear oscillator arrays. To begin with, the nonlinearity implies frequency mixing effects whereby all components of the noise contribute to the excitation of any particular collective mode resonance of the array. In a different context the concept of attractor crowding [1] has been identified as making large arrays very sensitive to the presence of noise. However well-controlled experiments in which quantitative measurements are made of the dependence of array behaviour on the statistics of the noise are lacking. This is the main point that we wish to make in this paper.

In nonlinear dynamics experiments with noise fall into two broad categories; first is those experiments where intrinsic or ambient noise is known to be present, the noise being introduced and adjusted in the modelling to explain the data [2]. The second category is those experiments in which noise is deliberately added to the experiment and its effects noted. It is in the latter that the statistical properties of the noise can actually be changed experimentally and the response of the system to changing statistics checked. There have been some experiments of this latter type, but by and large they have been confined to analogue electronic circuits [3] with only a few exceptions such as in optical bistability [4]. This type of experiment has been used in studies of nonlinear spectroscopic processes with some success. For example, the different effects of random telegraph and Gaussian noise on two photon processes in the microwave [5] and optical regimes [6] have been measured. Optical processes lend themselves to this type of experiment because it is relatively easy to control the external influences. However there is a limitation in using optical processes in that one cannot follow the dynamical evolution directly because of the short timescales

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(~1fs) involved. In this paper we describe an experiment in which we apply this approach to a nonlinear oscillator array where the characteristic timescale is of order 1μs, so that dynamical changes can be directly recorded.

We use a continuous wave multi-mode neodymium laser as a nonlinear oscillator array to study the composition of the collective mode resonances; i.e. how much each cavity mode of the laser contributes to the amplitude of each collective mode. A laser cavity mode is defined essentially by $m\lambda/2 = L$, where m is an integer mode index, λ is the wavelength and L is the optical path length between the mirrors which constitute the cavity. (All modes of interest have the same transverse mode structure.) The cavity mode responds to a sudden change in its operating conditions by oscillating about its new operating point; these are relaxation oscillations. In a multi-mode laser the relaxation oscillations constitute an array coupled by gain sharing and in some cases by processes such as four wave mixing. For the laser to be considered useful these oscillations must be reasonably well damped so that the system relaxes quickly to its operating point. In many lasers (such as Nd^{3+} or CO_2) modulation of the input power, or some other parameter, near the natural frequency of the relaxation oscillation can drive the laser into limit cycle or chaotic operation [7].

Since the cavity modes have different wavelengths they can be separated with filters (Fabry-Perot interferometers in our experiment), which enables transfer function and cross-spectral techniques to be applied to the individual modes while they are still part of the array. We illustrate this idea in figure 1, where two Fabry-Perots are shown, each isolating a single laser mode. To obtain a transfer function we modulate the power of the pump (i.e. the diode laser which drives the Nd laser) at a frequency that is scanned from about 4 kHz to 55kHz. If we are willing to eschew the phase of the transfer function, the maximum frequency is about 100kHz. This modulation could be applied to a different parameter of the laser but the pump power is a very convenient choice. The response of either the total laser intensity, or one the modes extracted from the beam with a filter, is detected with a photodiode connected to a signal analyser (Hewlett-Packard HP35670A) which measures the strength of the response and its phase with respect to the modulation. It is this phase information which is crucial in being able to say whether, for a given collective mode, the relaxation oscillations of two laser modes constructively or destructively interfere.

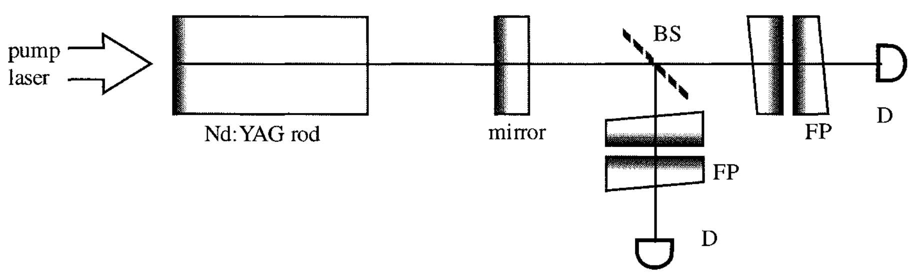

FIGURE 1 A schematic drawing of a multimode Nd laser, end pumped by a laser diode, illustrating the filtering of individual modes by Fabry-Perot interferometers (FP). The photodiode detectors (D) are shown but electrical connections are omitted. The shaded lines represent totally or partially reflective optical coatings. The cavity is defined by the coatings shown on the Nd:YAG rod and the mirror.

Destructive interference in this context, where the constituent oscillators are either in phase or 180° out of phase, is known as antiphasing. By fitting the measured complex transfer function to a sum of partial fractions, the pole-residue representation of the transfer functions, we are able to "unravel" the collective modes, determining how much each laser mode contributes to each collective mode and with what phase [8]. This is information that we cannot get from the intensity noise power spectra of the individual laser modes. Almost all experiments with multi-mode lasers have considered only power spectra [9] which by nature, or definition, lack phase information. One exception to that is an early work where the transfer function magnitude (but not phase) was presented [10].

Comparison of the power spectrum for a mode with the transfer function of the same mode shows intriguing differences. For one laser mode extracted from the beam during five mode operation, we illustrate this difference in Fig. 2. This mode is the

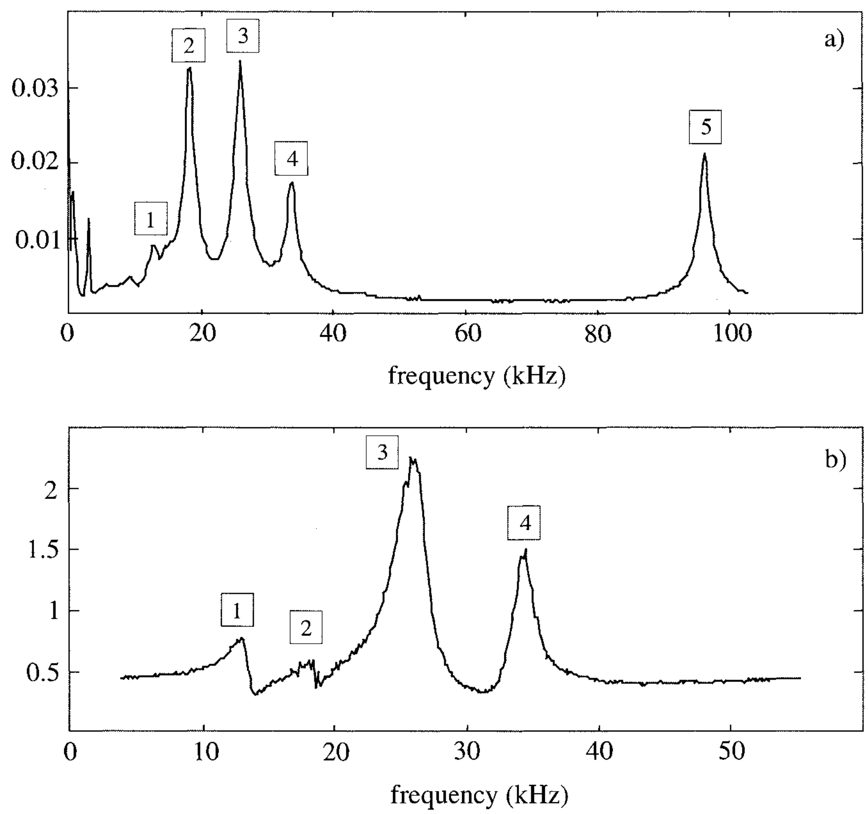

FIGURE 2. Collective mode resonances for a laser operating in five longitudinal modes. a) power spectrum of intensity for mode 3 (third to pass threshold) b) magnitude of transfer function for mode 3. Note the different horizontal scales between a) and b). The four peaks in b) correspond to the four labelled peaks of the same frequencies in a). The ordinates of each graph have a arbitrary linear scales.

third to reach threshold as the pump power is increased to the operating point. In Fig. 2a, we show the power spectrum of intensity noise. The five peaks between 10kHz and 100kHz are the collective mode resonances. The highest frequency one (labelled "5") is what is usually regarded as the relaxation oscillation peak; indeed if this were the power spectrum for the total intensity, this would be the only peak visible and only vestiges of the other four would survive because of destructive interference between the oscillators. For such a spectrum to be measurable, the array is excited by ambient noise, such as vibrations, and by intrinsic noise such as spontaneous emission. Figure 2b shows a measured transfer function for the same mode as in Fig. 2a, giving the magnitude of the complex response to a single-frequency perturbation of the pump strength. The phase of the transfer function is not shown. The scale is expanded to show only the four lowest frequency collective resonances. The heights of the four peaks in the group between 10kHz and 40kHz, with respect to each other, are dramatically different between Figs. 2a and 2b, especially for those peaks labelled "1" and "2". This is one instance of the problem to which this paper is directed, namely how noise affects the contributions of the constituent oscillators to the collective modes of an array. Before discussing how one might attack this question experimentally, we shall briefly examine another aspect of the problem.

A different technique which we have developed to look at antiphasing is that of measuring cross-spectra between the fluctuations of pairs of laser modes. The modes are separated by filters as in the transfer function technique, and the intensity fluctuations of two modes are recorded simultaneously with a fast analogue to digital converter. The cross spectrum, which is the Fourier transform of the cross-correlation function, is then calculated on a computer. This is a complex valued quantity. An example of a cross spectrum between two laser cavity modes is shown in figure 3. Peaks at the collective mode frequencies are seen in the magnitude of the cross spectrum, and the relative phases can be used to deduce whether the two laser modes are contributing to collective modes in phase or out of phase.

Given the cross spectra of the other cavity mode pairs, one can quite easily deduce the "pattern" of antiphasing; i.e. which cavity mode pairs exhibit constructive or destructive interference at a given frequency. However one would like also to be able to quantitatively measure the makeup of the collective modes. At first glance it might appear that this can be done from the peak heights in the magnitude of the cross spectrum, but in the light of Fig. 2, it is not clear whether this is true, since in the measurement of a cross spectrum the excitation of the array is by ambient noise.

How should we proceed from here? What we intend to do is to apply noise (random telegraph noise in the first instance) to the modulation of the pump intensity and thus provide a random component to the excitation of the array. Then by varying the strength and the statistics of the noise we will be able to measure the effect of noise on the system. However important experimental questions remain: To what variable should we apply noise?; are transfer functions and cross spectra the best quantities to measure?

Another worthwhile question to ask is, can these techniques be applied to other types of nonlinear oscillator array? What makes the Neodymium laser attractive for this work is the fact that its relaxation oscillation frequency is relatively low, so that

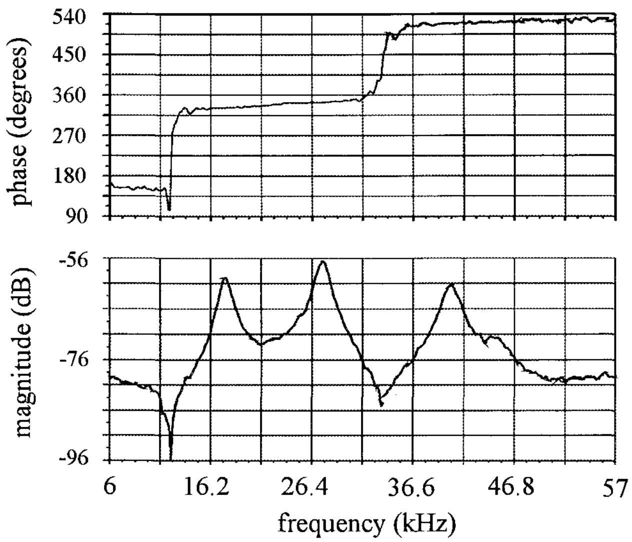

FIGURE 3. An example of the cross spectrum of the intensity fluctuations of two laser modes, in three mode operation of the laser. In this case the fluctuations in modes 1 and 3 are recorded. The numbering of the laser modes is as for Fig. 2. The three peaks in the magnitude correspond to the three collective mode resonances. The rightmost peak corresponds to the relaxation oscillation peak that would be seen in the power spectrum of the intensity fluctuations in the total power. The phase in the cross spectrum for this peak corresponds to constructive interference: the phase for the other two peaks differs by 180° and thus indicates that modes 1 and 3 destructively interfere at these two collective mode frequencies.

high resolution (12 - 16 bit) analogue to digital recorders can easily follow the dynamical evolution. Of great technological significance are diode lasers which are multi-longitudinal mode devices, even though only one mode may have any significant intensity. To account for the dynamical properties of these lasers it is essential to recognise that other longitudinal modes may become dynamically populated with photons. Even in continuous operation these other modes do have some photons and it is the antiphase dynamics of these modes that is one source of the interesting sub-Poissonian noise statistics of these lasers. A problem is that the characteristic frequency of the relaxation oscillations is ~2GHz, so that heterodyne techniques would be necessary to record phase information in the intensity fluctuations.

Another aspect of diode lasers that makes them interesting as nonlinear arrays is the technique of making high power devices by laying several low-power diodes side by side in a "bar". Several bars can then be stacked to make an even higher power device. The low-power diodes in such an array are coupled because of transverse light leakage from one diode junction to a neighbouring junction. Recent advances in the "fibre-pigtailing" of such arrays, where a bundle of fibres collects the light from the junctions could perhaps be extended so that an individual fibre would be associated with an individual junction to enable the separation of a single array element from the total output. (This is analogous to the technique of using filters to select particular longitudinal modes from the Nd laser output.) One would then be in a position to use the transfer function and cross-spectrum techniques as a diagnostic for the further development of these arrays. A third important nonlinear array in physics is the array of Josephson junctions. It was in studies of these arrays that the phenomenon of antiphasing was first noted [11] and other phase relationships between the oscillators, such as splay phase states, were identified. The characteristic frequencies involved in this case are very high, ~50GHz, so that heterodyne techniques to record the phase of a transfer function would again be necessary.

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