Applications of the Octet Baryon Quark-Meson Coupling Model to Hybrid Stars

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Dedicated to my wife, Susanne;
for her love, sacrifice, and support of
a student for so many years.

Abstract

The study of matter at extreme densities has been a major focus in theoretical physics in the last half-century. The wide spectrum of information that the field produces provides an invaluable contribution to our knowledge of the world in which we live. Most fascinatingly, the insight into the world around us is provided from knowledge of the intangible, at both the smallest and largest scales in existence.

Through the study of nuclear physics we are able to investigate the fundamental construction of individual particles forming nuclei, and with further physics we can extrapolate to neutron stars. The models and concepts put forward by the study of nuclear matter help to solve the mystery of the most powerful interaction in the universe; the strong force.

In this study we have investigated a particular state-of-the-art model which is currently used to refine our knowledge of the workings of the strong interaction and the way that it is manifested in both neutron stars and heavy nuclei, although we have placed emphasis on the former for reasons of personal interest. The main body of this work has surrounded an effective field theory known as Quantum Hadrodynamics (QHD) and its variations, as well as an extension to this known as the Quark-Meson Coupling (QMC) model, and variations thereof. We further extend these frameworks to include the possibility of a phase transition from hadronic matter to deconfined quark matter to produce hybrid stars, using various models.

We have investigated these pre-existing models to deeply understand how they are justified, and given this information, we have expanded them to incorporate a modern understanding of how the strong interaction is manifest.

Statement of Originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution to Jonathan David Carroll and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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