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# Study of lattice QCD form factors using the extended Gari-Krümpelmann model

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We explore the suitability of a modern vector meson dominance (VMD) model as a method for chiral extrapolation of nucleon electromagnetic form factor simulations in lattice QCD. It is found that the VMD fits to experimental data can be readily generalized to describe the lattice simulations. However, the converse is not true. That is, the VMD form is unsuitable as a method of extrapolation of lattice simulations at large quark mass to the physical regime.

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## I. INTRODUCTION

The electromagnetic form factors of the nucleon are a major source of information about its internal structure. On the experimental side, the unique capabilities of Jefferson Lab have recently led to a major revision of our knowledge of the proton electric form factor, with  $G_E/G_M$  unexpectedly decreasing quite rapidly with increasing  $Q^2$  [1,2]. Although various models of hadron structure have reasonable success in the low- $Q^2$  regime, only a few have any claim to also describing the  $Q^2$  dependence of  $G_E/G_M$  while reproducing the magnitude of  $G_M$ —see the review [3]. There is no consensus as to which model best represents QCD. However, at least one of them, the light front cloudy bag model (LFCBM) [4,5], did indeed anticipate the behavior found at JLab.

Lattice QCD has the great attraction of being able to give us the unambiguous consequences of nonperturbative QCD. Until now it has proven possible to calculate proton and neutron form factors to  $Q^2$  of order  $3 \text{ GeV}^2$  in quenched QCD [6]. While this datum is restricted to relatively large pion mass ( $m_\pi > 0.5 \text{ GeV}$ ), it is remarkable that a model like the LFCBM is able to not only describe the experimental data, but with relatively mild assumptions about the mass dependence of two parameters, it also produces an excellent description of the lattice QCD form factor data [7].

The latter finding works two ways. First it assures us that the model passes a test that any acceptable model should pass [8], namely, that it is consistent with the dependence of the quark mass found in QCD itself. Second, as there is no model independent way for chiral extrapolation of hadron properties at high- $Q^2$ , it suggests that the LFCBM presents a reasonable method whereby one can extrapolate hadron properties found at large light quark mass to the physical pion mass.

Given the considerable interest in the vector meson dominance (VMD) approach, in this paper we consider its suitability as a method of chiral extrapolation. There are many variations of the basic VMD, but we choose the implementation of Lomon [9], because it is phenomenologically extremely successful. We introduce the dependence of the vector meson masses in  $m_\pi$  found in earlier lattice studies [10,11] and parametrize the mass dependence of the corresponding

couplings in order to best describe the lattice QCD data. The major disappointment is that the functional form is so sensitive to the parameters, that it is meaningless to compare with extrapolated data any extrapolation of the form factors to the physical pion mass. This makes the VMD approach unsuitable as a method of chiral extrapolation. On the other hand, by fixing the parameters to the values given in Ref. [9] at the physical mass, it is possible to obtain a fit to all of the lattice QCD data of comparable quality to that found earlier using LFCBM.

## II. LATTICE DATA FITS AND RESULTS

We employ the extended Gari-Krümpelmann model (GKex) of Lomon [9] to fit lattice QCD calculated nucleon electric and magnetic form factors produced by the QCDSF Collaboration [6].

### A. Review of the GKex model

Here we briefly summarize the formulation of the GKex model from Ref. [9]. The extended Gari-Krümpelmann model exhibits the basic properties of a VMD model, and also phenomenologically incorporates the correct high- $Q^2$  behavior of the nucleon electromagnetic form factors as implied by PQCD. The model was successfully fit to the present experimental data sets available for the nucleon electromagnetic form factors. The particular interest in the model is increased by its ability to describe the fall-off of the proton ratio,  $G_E/G_M$ , vs  $Q^2$ , as measured recently in Refs. [1,2].

Our goal is to calculate the Dirac,  $F_1$ , and Pauli,  $F_2$ , form factors, defined through the nucleon electromagnetic current as

$$\langle N, \lambda' p' | J^\mu | N, \lambda p \rangle = \bar{u}_{\lambda'}(p') \left[ F_1(Q^2) \gamma^\mu + \frac{F_2(Q^2)}{2M_N} \times i \sigma^{\mu\nu} (p' - p)_\nu \right] u_\lambda(p). \quad (1)$$

The momentum transfer is  $q^\mu = (p' - p)^\mu$ ,  $Q^2 = -q^2$  and  $J^\mu$  is taken to be the electromagnetic current operator for

a nucleon. For  $Q^2 = 0$  the form factors  $F_1$  and  $F_2$  are, respectively, equal to the charge and the anomalous magnetic moment,  $\kappa$ , in units of  $e$  and  $e/(2M_N)$ , while the magnetic moment is  $\mu = F_1(0) + F_2(0)$ .

We are interested in the electric and magnetic Sachs form factors, which are defined as

$$G_E = F_1 - \frac{Q^2}{4M_N^2} F_2, \quad G_M = F_1 + F_2 \quad (2)$$

with normalization

$$\begin{aligned} G_E^p(0) &= 1; \\ G_M^p(0) &= \mu_p; \\ G_E^n(0) &= 0; \\ G_M^n(0) &= \mu_n. \end{aligned} \quad (3)$$

One can express the Pauli and Dirac form factors in terms of isoscalar and isovector form factors

$$\begin{aligned} 2F_i^p &= F_i^{\text{IS}} + F_i^{\text{IV}}; \\ 2F_i^n &= F_i^{\text{IS}} - F_i^{\text{IV}}; \end{aligned} \quad (4)$$

The isoscalar and isovector form factors were parametrized by Lomon as

$$\begin{aligned} F_1^{iv}(Q^2) &= N/2 \frac{1.0317 + 0.0875(1 + Q^2/0.3176)^{-2}}{(1 + Q^2/0.5496)} F_1^{\rho}(Q^2) \\ &\quad + \frac{g_{\rho'}}{f_{\rho'}} \frac{m_{\rho'}^2}{m_{\rho'}^2 + Q^2} F_1^{\rho}(Q^2) \\ &\quad + \left(1 - 1.1192 N/2 - \frac{g_{\rho'}}{f_{\rho'}}\right) F_1^D(Q^2), \\ F_2^{iv}(Q^2) &= N/2 \frac{5.7824 + 0.3907(1 + Q^2/0.1422)^{-1}}{(1 + Q^2/0.5362)} F_2^{\rho}(Q^2) \\ &\quad + \kappa_{\rho'} \frac{g_{\rho'}}{f_{\rho'}} \frac{m_{\rho'}^2}{m_{\rho'}^2 + Q^2} F_2^{\rho}(Q^2) \\ &\quad + \left(\kappa_v - 6.1731 N/2 - \kappa_{\rho'} \frac{g_{\rho'}}{f_{\rho'}}\right) F_2^D(Q^2), \quad (5) \\ F_1^{is}(Q^2) &= \frac{g_{\omega}}{f_{\omega}} \frac{m_{\omega}^2}{m_{\omega}^2 + Q^2} F_1^{\omega}(Q^2) + \frac{g_{\omega'}}{f_{\omega'}} \frac{m_{\omega'}^2}{m_{\omega'}^2 + Q^2} F_1^{\omega}(Q^2) \\ &\quad + \frac{g_{\phi}}{f_{\phi}} \frac{m_{\phi}^2}{m_{\phi}^2 + Q^2} F_1^{\phi}(Q^2) + \left(1 - \frac{g_{\omega}}{f_{\omega}} - \frac{g_{\omega'}}{f_{\omega'}}\right) F_1^D(Q^2), \\ F_2^{is}(Q^2) &= \kappa_{\omega} \frac{g_{\omega}}{f_{\omega}} \frac{m_{\omega}^2}{m_{\omega}^2 + Q^2} F_2^{\omega}(Q^2) + \kappa_{\omega'} \frac{g_{\omega'}}{f_{\omega'}} \frac{m_{\omega'}^2}{m_{\omega'}^2 + Q^2} \\ &\quad \times F_2^{\omega}(Q^2) + \kappa_{\phi} \frac{g_{\phi}}{f_{\phi}} \frac{m_{\phi}^2}{m_{\phi}^2 + Q^2} F_2^{\phi}(Q^2) \\ &\quad + \left(\kappa_s - \kappa_{\omega} \frac{g_{\omega}}{f_{\omega}} - \kappa_{\omega'} \frac{g_{\omega'}}{f_{\omega'}} - \kappa_{\phi} \frac{g_{\phi}}{f_{\phi}}\right) F_2^D(Q^2) \end{aligned}$$

with pole terms of the  $\omega(782)$ ,  $\phi(1020)$ ,  $\omega'(1420)$ ,  $\rho(770)$ , and  $\rho'(1450)$  mesons, and the  $F_i^D$  terms ensuring the correct asymptotic behavior as calculated in PQCD. The  $F_i^{\alpha}$ , with  $\alpha = \rho, \omega$ , or  $\phi$ , are the meson-nucleon form factors.

The following parametrization of these form factors is chosen for GKex:

$$\begin{aligned} F_1^{\alpha,D}(Q^2) &= \frac{\Lambda_{1,D}^2}{\Lambda_{1,D}^2 + \tilde{Q}^2} \frac{\Lambda_2^2}{\Lambda_2^2 + \tilde{Q}^2}, \\ F_2^{\alpha,D}(Q^2) &= \frac{\Lambda_{1,D}^2}{\Lambda_{1,D}^2 + \tilde{Q}^2} \left( \frac{\Lambda_2^2}{\Lambda_2^2 + \tilde{Q}^2} \right)^2, \end{aligned} \quad (6)$$

$$\begin{aligned} F_1^{\phi}(Q^2) &= F_1^{\alpha} \left( \frac{Q^2}{\Lambda_1^2 + Q^2} \right)^{1.5}, \quad F_1^{\phi}(0) = 0, \\ F_2^{\phi}(Q^2) &= F_2^{\alpha} \left( \frac{\Lambda_1^2 Q^2 + \mu_{\phi}^2}{\mu_{\phi}^2 \Lambda_1^2 + Q^2} \right)^{1.5}, \end{aligned} \quad (7)$$

$$\text{with} \quad \tilde{Q}^2 = Q^2 \frac{\ln[(\Lambda_D^2 + Q^2)/\Lambda_{\text{QCD}}^2]}{\ln(\Lambda_D^2/\Lambda_{\text{QCD}}^2)}.$$

With this formulation there are unknown eight meson coupling constants, four cutoff masses, one magnetic moment and a single normalization constant, all of which should be determined from the fits to the experimental data. Fits to the experimental data points were made using different sets of data, some of which excluded the controversial high  $G_E^p/G_M^p$  measured previously by Rosenbluth separation method. The fits with different data sets were labeled GKex(01), GKex(01-), GKex(02S), and GKex(02L). The values of the fitted parameters are listed in Table I of Ref. [9]. Figure 1 shows that the model GKex(02S) describes the fall-off of  $G_E^p/G_M^p$  with  $Q^2$ , in contrast with GKex(01) and GKex(01-), which stay almost flat in the considered range of the  $Q^2$ . In the present work we use all four models in our attempt to describe the lattice data.

Using the model to reproduce lattice data requires that we make extrapolations of some of the parameters that depend on the mass of the hadron constituents. We start by considering the normalizations of the isovector and isoscalar form factors that depend on the nucleon magnetic moments:

$$\begin{aligned} F_2^{\text{IV}}(0) &= \kappa_v = (\mu_p - 1 - \mu_n); \\ F_2^{\text{IS}}(0) &= \kappa_s = (\mu_p - 1 + \mu_n). \end{aligned} \quad (8)$$

TABLE I. GKex fitting parameters and  $\chi^2$  for lattice spacing  $a = 0.26 \text{ GeV}^{-1}$ .

	GKex(01)	GKex(01-)	GKex(02L)	GKex(02S)
$\chi^2$	185	103	671	217
$a_1^{\text{IV}}$	-1.80(16)	-2.35	-1.7(2)	-1.86(17)
$b_1^{\text{IV}}$	0.46(11)	0.81	0.54(14)	0.39(13)
$a_2^{\text{IV}}$	-11.9(5)	-55.2	-1(1)	-10.9(5)
$b_2^{\text{IV}}$	2.98(36)	18	-0.34(76)	2.34(38)
$a_1^{\text{IS}}$	-0.99(18)	-1.61	-0.39(9)	-0.58(1)
$b_1^{\text{IS}}$	-0.21(14)	0.28	0.06(6)	-0.078(73)
$a_2^{\text{IS}}$	8.6(18)	3.1	2.53(58)	-0.32(14)
$b_2^{\text{IS}}$	2.6(14)	-0.44	0.62(41)	0.1(1)
$a^{\Lambda}$	0.034(38)	-0.19	0.3(1)	0.035(42)
$b^{\Lambda}$	-0.10(3)	0.065(33)	-0.14(6)	-0.12(3)

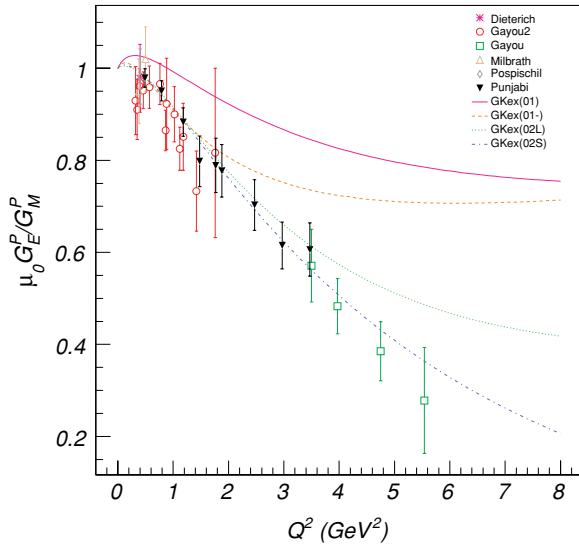


FIG. 1. (Color online)  $R_p$ , the ratio  $\mu_p G_p^E / G_p^M$ . Comparison of fits using the GKex model with the data. The experimental points used are taken from: Dieterich [12], Gayou2 [13], Gayou [2], Milbrath [14], Pospischil [15], and Punjabi [16].

The magnetic moments have nontrivial dependence upon the pion mass as a consequence of chiral symmetry. For example the leading dependence on the quark mass near the chiral limit is in fact nonanalytic (i.e., proportional to  $m_\pi \sim m_q^{1/2}$ ). As shown by Leinweber *et al.* in Ref. [10], to extrapolate the nucleon magnetic moments for the mass range accessible in lattice QCD to the physical mass scale, we use the Padé approximant derived in Ref. [10]

$$\mu_p(m_\pi) = \frac{3.31}{1 + 1.37 \cdot m_\pi + 0.452 \cdot m_\pi^2}, \quad (9)$$

$$\mu_n(m_\pi) = \frac{-2.39}{1 + 1.85 \cdot m_\pi + 0.271 \cdot m_\pi^2}.$$

The dependence of the masses of the vector mesons upon the pion mass was studied in the work by Leinweber *et al.* [11]. We use a linear extrapolation for the vector meson masses, which was shown in Ref. [11] to provide quite a good approximation to the full mass function including the LNA and NLNA behavior:

$$m_v(m_\pi) = c_0 + c_1 m_\pi^2;$$

$$m_v(m_\pi) = m_v^{\text{phys}} + c_1 (m_\pi^2 - (m_\pi^{\text{phys}})^2). \quad (10)$$

$$c_1 = 0.4273 \text{ GeV}^{-1};$$

The vector-meson nucleon effective coupling constants may also depend on the mass of the hadron constituents and to describe that we choose the following extrapolation forms:

$$g_i^\alpha(m_\pi^2) = g_{i0}^\alpha + a_i^{l_\alpha} (m_\pi^2 - (m_\pi^{\text{phys}})^2) + b_i^{l_\alpha} (m_\pi^4 - (m_\pi^{\text{phys}})^4), \quad (11)$$

where  $\alpha = \rho', \omega, \omega', \phi$ ;  $l_\alpha = \{\text{IV for } \alpha = \rho'; \text{IS for } \alpha = \omega, \omega', \phi\}$ ;  $i = 1, 2$ ; and  $g_{i0}^\alpha = \frac{g_\alpha}{f_\alpha}$ ,  $g_{20}^\alpha = \kappa_\omega \frac{g_\alpha}{f_\alpha}$  are the effective coupling constants at the physical  $m_\pi$ . These are taken from the fits to the physical data of Ref. [9].

We choose a similar ansatz for the extrapolation of the cut-off masses

$$\Lambda(m_\pi^2) = \Lambda_0 + a^\Lambda (m_\pi^2 - (m_\pi^{\text{phys}})^2) + b^\Lambda (m_\pi^4 - (m_\pi^{\text{phys}})^4), \quad (12)$$

where  $\Lambda = \Lambda_1, \Lambda_2, \Lambda_D, \Lambda_{\text{QCD}}$ , and  $\mu_\phi$ .

## B. Fitting procedure

Using the extrapolation forms given in Eqs. (9)–(12), we can fit the GKex form factors given by Eq. (6) to the lattice data by varying the coefficients  $a, b$  of relations (11) and (12). We performed the fits using the four different sets of physical GKex parameters reported in Ref. [9].

The form factor calculations in Ref. [6] were carried out using quenched, nonperturbatively  $O(a)$ -improved Wilson fermions (clover fermions), for three different values of the lattice spacing,  $a = \{0.47, 0.34, 0.26\} \text{ GeV}^{-1}$ . For each value of  $a$  several sets of pion (or equivalently nucleon) masses were considered. For each mass set Dirac and Pauli form factors for both the proton and neutron were calculated at several values of  $Q^2$ . The typical range for the pion mass used varied from 1.2 to 0.6 GeV, with the corresponding nucleon mass ranging from approximately 2 to 1.5 GeV. The typical range for  $Q^2$  was 0.6 to 2.3  $\text{GeV}^2$ . With the smallest lattice spacing being around 0.05 fm ( $\beta = 6.4$ ) and pion mass 580 MeV, these calculations represent the present state of the art.

We fitted the lattice data points for all three lattice spacings available using the Minuit package of CERN's ROOT framework [17]. The resulting fits for the smallest lattice spacing  $a = 0.26 \text{ GeV}^{-1}$  with 120 data points are shown in Figs. 2, 3, 4, and 5, where the corresponding fits using the LFCBM from our earlier work [7] are shown for comparison.

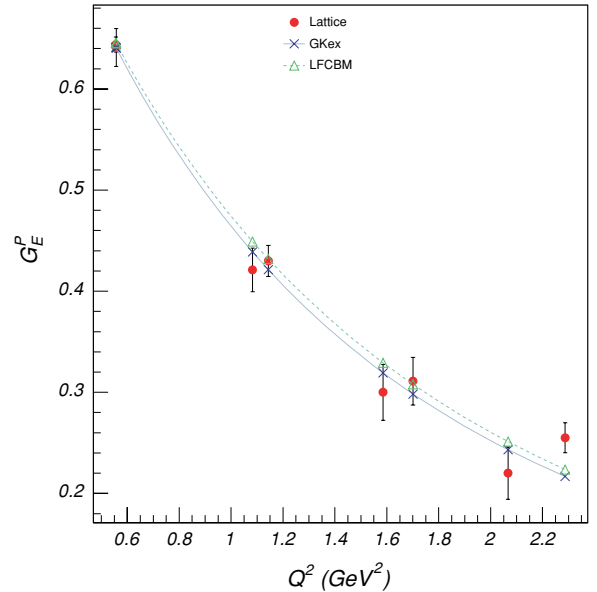


FIG. 2. (Color online) GKex(01-) fit (solid) to QCDSF data for  $G_p^E$  (in units of  $e$ ) for a lattice spacing  $a = 0.26 \text{ GeV}^{-1}$ ,  $M_N = 2.20 \text{ GeV}$ , and  $m_\pi = 1.24 \text{ GeV}$ . LFCBM fits (dashed) are also shown for comparison.

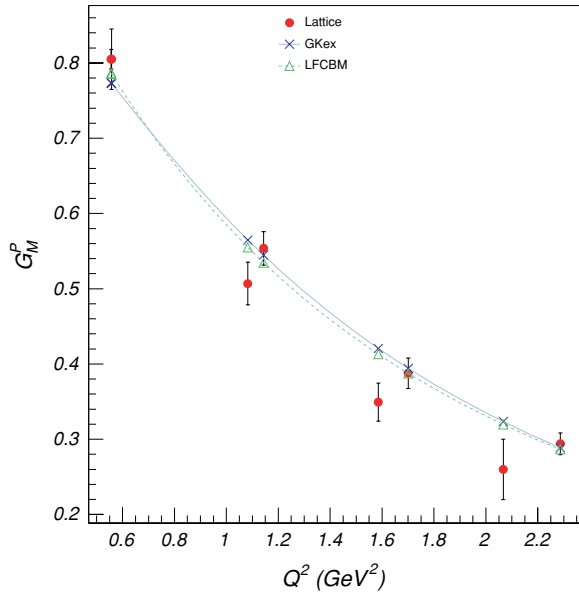


FIG. 3. (Color online) GKex(01-) fit (solid) to QCDSF data for  $G_M^P$  [in units of  $e/(2M_N^{\text{Physical}})$ ] for a lattice spacing  $a = 0.26 \text{ GeV}^{-1}$ ,  $M_N = 2.20 \text{ GeV}$ , and  $m_\pi = 1.24 \text{ GeV}$ . LFCBM fits (dashed) are also shown for comparison.

The resulting  $\chi^2$  and the fitting parameters for lattice spacing  $a = 0.26 \text{ GeV}^{-1}$  are summarized in Table I. For a comparison, the  $\chi^2 = 81$  for the LFCBM fit.

### C. Results

As one can see from Table I, the best fit to the data is obtained using the GKex(01-) model, even though one is

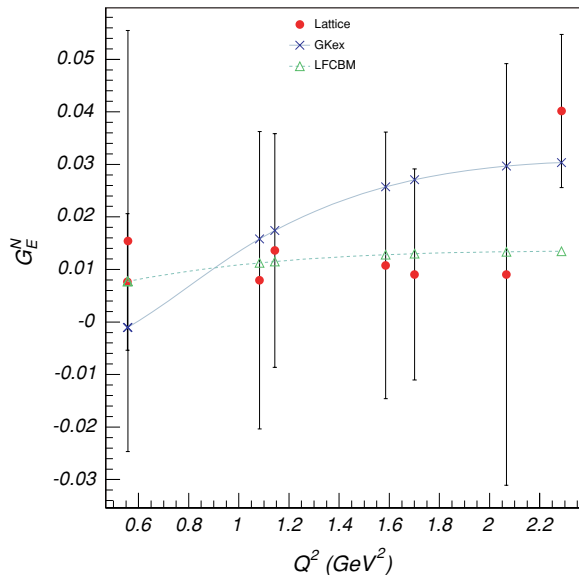


FIG. 4. (Color online) GKex(01-) fit (solid) to QCDSF data for  $G_E^N$  (in units of  $e$ ) for a lattice spacing  $a = 0.26 \text{ GeV}^{-1}$ ,  $M_N = 2.20 \text{ GeV}$ , and  $m_\pi = 1.24 \text{ GeV}$ . LFCBM fits (dashed) are also shown for comparison.

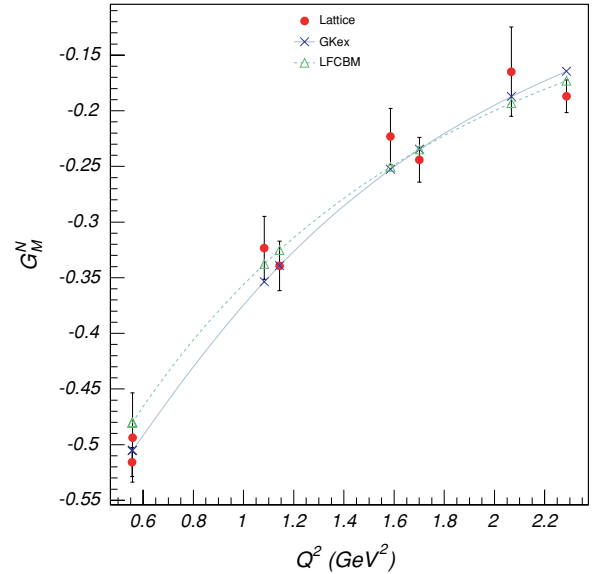


FIG. 5. (Color online) GKex(01-) fit (solid) to QCDSF data for  $G_M^N$  [in units of  $e/(2M_N^{\text{Physical}})$ ] for a lattice spacing  $a = 0.26 \text{ GeV}^{-1}$ ,  $M_N = 2.20 \text{ GeV}$ , and  $m_\pi = 1.24 \text{ GeV}$ . LFCBM fits (dashed) are also shown for comparison.

inclined to believe that GKex(02S) gives the best description of the nucleon structure, since it exhibits the rapid decrease with  $Q^2$  of the experimentally measured ratio,  $G_E^P/G_M^P$ . One can see this from our Fig. 1 as well as the original work of Ref. [9]. We also note that our attempts to fit the data using only the lowest order polynomial forms in  $m_\pi$  of the coupling constants (11), (12) did not yield satisfactory results. Indeed we had to include ten fitting parameters for successful extrapolations.

### III. CONCLUSION

We have explored the dependence of the nucleon electromagnetic form factors on quark mass, using recent lattice QCD simulations from the QCDSF group. Since the VMD approach has been widely used to describe the experimental data at high  $Q^2$  (a region of special phenomenological interest at the present time), we use a modern version of the VMD model, namely the Gari-Krümpelmann model as implemented by Lomon [9], and extend it in a natural way. Starting with the existing fit to the experimental data we find that it is possible to describe the lattice simulations quite well. However, it was necessary to allow some ten parameters to vary smoothly with the pion mass in order to do so. In comparison, the LFCBM produced a fit of similar quality with only two parameters varied. As a result we are led to the conclusion that VMD is not suitable as a method of chiral extrapolation.

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