The Modelling and Analysis of Command and Control Decision Processes using Extended Time Petri Nets

Fred David John Bowden
B.Sc. Hons. (Applied Mathematics)

Thesis submitted for the degree of

Doctor of Philosophy

at

The University of Adelaide

Department of Applied Mathematics

Faculty of Mathematical and Computer Sciences

June 2001

I dedicate this thesis to my wife Lesley.

Without whom this, along with many other things, would never have been possible. Thank you, for all your love, faith, patience, support and assistance.

CONTENTS

LIST OF FIGURES

LIST OF TABLES

DECLARATION

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text of the thesis.

I give consent to this thesis being made available for photocopying and loan if accepted for the award of the degree.

Fred David John Bowden

ACKNOWLEDGEMENTS

I would like to thank my family for never doubting that I would complete this thesis. They always asked by “when” I would complete rather than “if”.

I would like to acknowledge the Defence Science and Technology Organisation for its support during this research. In particular I would like to thank John Coleby, Mike Davies, David Fogg, Paul Gaertner, Stephen Hood and Darryn Reid for their support at various stages.

I would like to thank my co-supervisors Jonathan Billington, Mike Davies and Peter Taylor. In particular I thank Peter and Mike for their comments on my final draft. I would also like to thank Lesley, Lars Kristensen and Darryn for reading and commenting on certain portions of my thesis at varying stages of its readiness.

Above all I would like to thank my principal supervisor Charles Pearce. He was initially my teacher, then my mentor and is now my friend. No one could ask for more than Charles has given me and it has been my privilege to have him as my supervisor. I hope that in the future our lives continue to be entangled.

ABSTRACT

Effective command and control is crucial to both military and non-military environments. Accurate representations of the processes associated with the inter and intra activities of nodes or agencies of such systems is essential in the analysis of command and control. One of the most important things is to be able to model the decision processes. These are the parts of the system that make decisions and then guide the direction of other elements in the system overall.

This thesis uses a new type of extended time Petri net to model and analyse command and control decision processes. A comprehensive review of existing time Petri net structures is given. This concludes with the introduction of a time Petri net structure that incorporates the most commonly used time structures. This extended time Petri net structure is then used in the definition of the basic modelling blocks required to model command and control decision processes. This basic modelling block forms the basis of the direct analysis techniques that are introduced in the thesis.

Due to the transient nature of the systems being modelled and the measures of interest a new type of measure is introduced, the mean conditional first hitting reward. This measure does not currently appear to be part of the stochastic process literature. Explicit procedures are given to determine the hitting probabilities and mean conditional first hitting reward for decision process models and discrete, continuous and semi-Markov chains. Finally the some extensions of the decision process sub-class are considered.

GLOSSARY OF SYMBOLS

κ	100	Sub-set of states of interest in reaching the first time
Λ	106	Diagonal matrix containing elements a_i
$\Omega(\tau)$	127	Probability no transition fires before transition t
τ	25	Time
τ_{c_i}	135	Mean time spent in circuit C_i
τ_{s_k}	135	Mean holding time of absorbing transition R_k
ρ	26	Range of possible duration functions
μ'	123	Vector of mean times spent in i before going to j
$\mu_{i\,j}$	121	Mean time spent in i before going to j for a semi-Markov chain
A	89	Absorbing part of a decision process building block
a_i	100	Probability of reaching κ from state i
A_j	88	The j^{th} absorbing state of a decision process building block
${}^{A}M$	26	Available tokens marking in a time Petri net
B	100	Finite set of states with access to κ
C	20	Incidence matrix
C	88	Circuit part of a decision process building block
c_i	88	First transition in the i^{th} circuit of a decision process building block

D	27	Duration function
$^{E}D(t)(\tau)$	45	Enabling duration function of transition t
$E_{p\,t}(\tau)$	126	Enabling duration function for the arc from place p to transition t
^{E}M	32	Enabling marking in a time Petri net
$\lvert ^{E}M(t) \rvert$	36	Enabling degree of transition t
F	20	Firing sequence count vector
$^{H}D(t)(\tau)$	46	Holding duration function of transition t
$H_{t\,p}(\tau)$	126	Holding duration function for the arc from transition t to place p
I	88	Initially marked place of a decision process building block
$I_i(\tau)$	127	Probability that transition C_i fires before time τ
$J_j(\tau)$	127	Probability that transition S_j fires before time τ
L	88	Number of absorbing states in a decision process building block
M	8	A marking
M0	7	Initial marking
$[M\rangle$	13	Reachability set for the marking M
$M[t\rangle M'$	13	Marking M’ is the marking reached from M when transition t fires
$M[S\rangle M'$	13	M’ is the marking reach from marking M with firing sequence σ
N	88	Number of circuits in the a decision process building

		block
P	7	Set of places
$\mathbf{p}$	100	Transition probability matrix
$\mathbf{p'}$	106	Transition probability matrix containing only states in B
$p \succ p'$	162	Place p is upstream to place p'
p_{ij}	99	Single step transition probability from state i to state j
^{P}M	32	Token marking in a Petri net with enabling durations
$\mathbf{q}'$	117	Vector with q_i as its elements
$\mathbf{Q}$	112	Transition rate matrix
q_{ij}	112	Transition rate from state i to state j
q_i	112	$-\sum_{i \neq j} q_{ij}$
R_0^+	56	Nonnegative reals
R_i	103	Mean conditional first-passage reward from state i to κ
R_{ij}	103	Mean reward gained as a result of a transition form state i to state j
s_j	88	The j^{th} transition to an absorbing state of a decision process building block
T	7	Set of transitions
$\mathbf{T}$	108	Mean conditional first-passage time matrix
$t^{\bullet}$	10	Set of output place for transition t
$^{\bullet}t$	10	Set of input place for transition t
T_i	102	Mean conditional first-passage time from state i to κ
$\mathbf{U}$	103	Dual matrix
U_{ij}	103	Dual transition probabilities

^{U}M	26	Unavailable tokens marking in a time Petri net
$\mathbf{v}$	105	Mean conditional next step passage reward vector
v_i	104	Mean conditional next step passage reward from state i
W	8	Weighted flow function
$\lfloor x \rfloor$	37	Greatest integer less than or equal to x
Z^+	7	Positive integers
Z_0^+	7	Nonnegative integers