

# **The Modelling and Analysis of Command and Control Decision Processes using Extended Time Petri Nets**

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*I dedicate this thesis to my wife Lesley.*

*Without whom this, along with many other things,  
would never have been possible. Thank you, for all  
your love, faith, patience, support and assistance.*

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# **DECLARATION**

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text of the thesis.

I give consent to this thesis being made available for photocopying and loan if accepted for the award of the degree.

Fred David John Bowden

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# ABSTRACT

Effective command and control is crucial to both military and non-military environments. Accurate representations of the processes associated with the inter and intra activities of nodes or agencies of such systems is essential in the analysis of command and control. One of the most important things is to be able to model the decision processes. These are the parts of the system that make decisions and then guide the direction of other elements in the system overall.

This thesis uses a new type of extended time Petri net to model and analyse command and control decision processes. A comprehensive review of existing time Petri net structures is given. This concludes with the introduction of a time Petri net structure that incorporates the most commonly used time structures. This extended time Petri net structure is then used in the definition of the basic modelling blocks required to model command and control decision processes. This basic modelling block forms the basis of the direct analysis techniques that are introduced in the thesis.

Due to the transient nature of the systems being modelled and the measures of interest a new type of measure is introduced, the mean conditional first hitting reward. This measure does not currently appear to be part of the stochastic process literature. Explicit procedures are given to determine the hitting probabilities and mean conditional first hitting reward for decision process models and discrete, continuous and semi-Markov chains. Finally the some extensions of the decision process sub-class are considered.

## GLOSSARY OF SYMBOLS

$\kappa$	100	Sub-set of states of interest in reaching the first time
$\Lambda$	106	Diagonal matrix containing elements $a_i$
$\Omega(\tau)$	127	Probability no transition fires before transition $t$
$\tau$	25	Time
$\tau_{c_i}$	135	Mean time spent in circuit $C_i$
$\tau_{s_k}$	135	Mean holding time of absorbing transition $R_k$
$\rho$	26	Range of possible duration functions
$\mu'$	123	Vector of mean times spent in $i$ before going to $j$
$\mu_{i,j}$	121	Mean time spent in $i$ before going to $j$ for a semi-Markov chain
$A$	89	Absorbing part of a decision process building block
$a_i$	100	Probability of reaching $\kappa$ from state $i$
$A_j$	88	The $j^{\text{th}}$ absorbing state of a decision process building block
${}^A M$	26	Available tokens marking in a time Petri net
$B$	100	Finite set of states with access to $\kappa$
$C$	20	Incidence matrix
$C$	88	Circuit part of a decision process building block
$c_i$	88	First transition in the $i^{\text{th}}$ circuit of a decision process building block

<b>D</b>	27	Duration function
$E^D(t)(\tau)$	45	Enabling duration function of transition $t$
$E_{p,t}(\tau)$	126	Enabling duration function for the arc from place $p$ to transition $t$
$E^M$	32	Enabling marking in a time Petri net
$ E^M(t) $	36	Enabling degree of transition $t$
<b>F</b>	20	Firing sequence count vector
$H^D(t)(\tau)$	46	Holding duration function of transition $t$
$H_{t,p}(\tau)$	126	Holding duration function for the arc from transition $t$ to place $p$
<b>I</b>	88	Initially marked place of a decision process building block
$I_i(\tau)$	127	Probability that transition $C_i$ fires before time $\tau$
$J_j(\tau)$	127	Probability that transition $S_j$ fires before time $\tau$
<b>L</b>	88	Number of absorbing states in a decision process building block
<b>M</b>	8	A marking
<b>M0</b>	7	Initial marking
$[M\rangle$	13	Reachability set for the marking $M$
$M[t\rangle M'$	13	Marking $M'$ is the marking reached from $M$ when transition $t$ fires
$M[S\rangle M'$	13	$M'$ is the marking reach from marking $M$ with firing sequence $\sigma$
<b>N</b>	88	Number of circuits in the a decision process building

		block
<b>P</b>	7	Set of places
<b>p</b>	100	Transition probability matrix
<b>p'</b>	106	Transition probability matrix containing only states in B
$p \succ p'$	162	Place p is upstream to place p'
$p_{ij}$	99	Single step transition probability from state i to state j
${}^pM$	32	Token marking in a Petri net with enabling durations
<b>q'</b>	117	Vector with $q_i$ as its elements
<b>Q</b>	112	Transition rate matrix
$q_{ij}$	112	Transition rate from state i to state j
$q_i$	112	$-\sum_{i \neq j} q_{ij}$
$\mathcal{R}_0^+$	56	Nonnegative reals
$R_i$	103	Mean conditional first-passage reward from state i to $\kappa$
$R_{ij}$	103	Mean reward gained as a result of a transition form state i to state j
$s_j$	88	The $j^{\text{th}}$ transition to an absorbing state of a decision process building block
<b>T</b>	7	Set of transitions
<b>T</b>	108	Mean conditional first-passage time matrix
$t^\bullet$	10	Set of output place for transition t
$\bullet t$	10	Set of input place for transition t
$T_i$	102	Mean conditional first-passage time from state i to $\kappa$
<b>U</b>	103	Dual matrix
$U_{ij}$	103	Dual transition probabilities

$U_M$	26	Unavailable tokens marking in a time Petri net
$\mathbf{v}$	105	Mean conditional next step passage reward vector
$v_i$	104	Mean conditional next step passage reward from state $i$
$W$	8	Weighted flow function
$\lfloor x \rfloor$	37	Greatest integer less than or equal to $x$
$Z^+$	7	Positive integers
$Z_0^+$	7	Nonnegative integers