Optical Dilution and Feedback Cooling of a Gram-Scale Oscillator to 6.9 mK

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We report on the use of a radiation pressure induced restoring force, the *optical spring effect*, to optically dilute the mechanical damping of a 1 g suspended mirror, which is then cooled by active feedback (cold damping). Optical dilution relaxes the limit on cooling imposed by mechanical losses, allowing the oscillator mode to reach a minimum temperature of 6.9 mK, a factor of ~40 000 below the environmental temperature. A further advantage of the optical spring effect is that it can increase the number of oscillations before decoherence by several orders of magnitude. In the present experiment we infer an increase in the dynamical lifetime of the state by a factor of ~200.

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To measure quantum effects in an oscillator, it is desirable to prepare the system in a low energy state, such that the number of quanta in the mode $N = E/\hbar\Omega_{\rm eff}$ is comparable to 1, where E is the energy of the mode and $\Omega_{\rm eff}$ is the resonant frequency. Typically a macroscopic system is maintained far above the quantum ground state by thermal fluctuations that enter through its mechanical coupling to the environment, and drive its motion. An oscillator of mass M and spring constant K undergoes motion at its resonant frequency $\Omega_{\rm eff} = \sqrt{K/M}$ that is related to its effective (or *noise*) temperature $T_{\rm eff}$ by

$$\frac{1}{2}Kx_{\rm rms}^2 = \frac{1}{2}k_B T_{\rm eff}. \tag{1}$$

Reduction of the root-mean-squared motion $x_{\rm rms}$, and hence $T_{\rm eff}$, may be achieved by a passive optical damping force (“cavity cooling”) [1–6], or by an active feedback force (“cold damping”) [7–10]. In either case, cooling is possible because such a force imposes a nonmechanical coupling with an external system that need not be in thermal equilibrium with the environment.

The limit of these techniques occurs when the oscillator is critically damped, placing an upper bound on the cooling factor at Q_M, the mechanical quality factor of the oscillator. However, in this Letter, we show that the constraint on cooling is relaxed when radiation pressure supplies the system's dominant restoring force, and demonstrate experimentally a cooling factor that is larger than the quality factor in the absence of radiation pressure.

In addition, it is desirable that a quantum state of the oscillator, once prepared, should survive for more than one oscillation period, enabling subsequent measurements to reveal quantum superpositions in macroscopic objects [11,12]. Interaction of the quantum system with its noisy environment typically acts to produce decoherence—departure from an ideal coherent quantum superposition. The thermal decoherence time of an oscillator subject to mechanical viscous damping is given by [7,13–15]

$$\frac{1}{\tau} = \frac{\Gamma_M k_B T_M}{2\pi\hbar\Omega_{\rm eff}}, \tag{2}$$

where Γ_M is the mechanical damping constant of the oscillator, and T_M is the ambient temperature of the environment. In practice, viscous mechanical damping and its associated thermal noise may not be the only cause of decoherence. For example, frequency and intensity fluctuations of a laser beam used to measure the position of the oscillator couple to its position, and could decohere the state. To include these effects generically, we extend Eq. (2):

$$\frac{1}{\tau} = \frac{1}{2\pi\hbar\Omega_{\rm eff}}\sum_i \Gamma_i E_i = \frac{\Gamma_{\rm eff} k_B T_{\rm eff}}{2\pi\hbar\Omega_{\rm eff}}. \tag{3}$$

The equation is written in terms of the characteristic energy E_i and coupling Γ_i of each noise source to emphasize that the noise need not be thermal in origin. The effective temperature used here is the same as in Eq. (1). We point out that in all cases $\Gamma_{\rm eff}T_{\rm eff} \geq \Gamma_M T_M$, such that the energy flowing into the mode is never decreased. So, in the best case, when no noise in addition to thermal noise is present, the equality is satisfied; otherwise, the inequality holds.

The average number of oscillations $n_{\rm osc}$ before decoherence is

$$n_{\rm osc} = \frac{\hbar\Omega_{\rm eff}}{k_B T_{\rm eff}}\frac{\Omega_{\rm eff}}{\Gamma_{\rm eff}}. \tag{4}$$

Unless $n_{\rm osc}$ exceeds unity, evidence of quantum superposition is quickly buried under environmental noise. This ordinarily precludes large objects from exhibiting such effects, since $\Omega_{\rm eff}$ tends to decrease for larger objects, due to their greater inertia. We also note that nonmechanical damping techniques reduce $T_{\rm eff}$ of the mode by increasing Γ_{eff}, while leaving Ω_{eff} and n_{osc} nearly unchanged. Therefore the mechanical oscillator must be fabricated so as to satisfy $n_{\mathrm{osc}} > 1$ initially; this poses a significant experimental challenge that increases with the size of the system.

Use of the optical spring effect in addition to nonmechanical damping should allow a system to exhibit quantum behavior even though its initial configuration does not satisfy $n_{\mathrm{osc}} > 1$. The new technique addresses two quantities of interest in measuring quantum states of macroscopic objects: (i) the average motion of the object [x_{rms} in Eq. (1)], which is related to *state preparation*; and (ii) the number of oscillations of the mode before the state decays [n_{osc} in Eq. (4)], relating to *state survival*.

In order to reduce thermal motion, the mirror must be as weakly coupled to the outside environment as possible, which in practice requires that the mirror should be suspended, with the stiffness of the suspension as soft as possible. A laser beam is used to create a potential well by generating an optical restoring force, commonly known as an optical spring [16–20]. This potential well creates a mode of oscillation with a natural frequency of up to a few kilohertz. In our experiment, the mode is dynamically unstable because of the delayed optical response, but can be stabilized by application of either electronic [18] or optical [21] feedback forces. The optical spring shifts the oscillator's resonant frequency while leaving its mechanical losses unchanged. The mechanical quality factor Q_M, as limited by those losses, is increased by the factor $\Omega_{\mathrm{eff}}/\Omega_M$, where Ω_M is the natural frequency of the free mechanical oscillator. We refer to this as "optical dilution," analogous to the phenomenon of "damping dilution" that accounts for the fact that the Q of the pendulum mode can be much higher than the mechanical Q of the material of which it is made [22,23]. This mitigation of intrinsic thermal noise is possible because a fraction of the energy is stored in the (noiseless) gravitational field. In the case of the pendulum, the dilution factor depends on the amount of elastic energy stored in the flexing wire compared to the energy stored in the gravitational field—approximated by the ratio of the gravitational spring constant to the mechanical spring constant. The optical dilution introduced here accounts for the fact that thermal noise in our mechanical oscillator is reduced due to energy stored in the optical field (the optical spring force acts similar to the gravitational force).

A further advantage of this scheme is that it does not conserve n_{osc}, because it changes the resonant frequency of the oscillator by orders of magnitude. This should allow quantum effects to become visible in a system that would not otherwise show them. We note again that the coupling of thermal energy ($\Gamma_M T_M$) into the oscillator is not decreased, but by raising the resonant frequency Ω_{eff}, the amount of energy in a single quantum increases, thereby increasing the decoherence time.

The experiment shown schematically in Fig. 1 was performed to demonstrate the optical dilution technique.

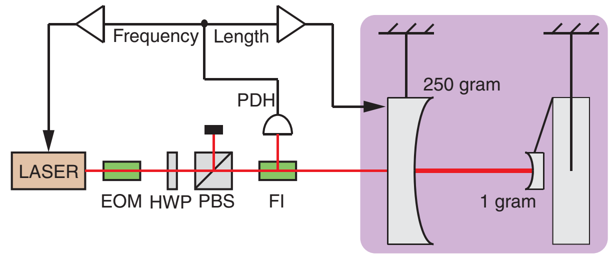

FIG. 1 (color online). Simplified schematic of the experiment. About 100 mW of $\lambda_0 = 1064$ nm Nd:YAG laser light passes through a Faraday isolator (FI) and a half-waveplate (HWP) and polarizing beam splitter (PBS) combination that allows control of the laser power, before being injected into the cavity, which is mounted on a seismic isolation platform in a vacuum chamber (denoted by the shaded box). A Pound-Drever-Hall (PDH) error signal derived from the light reflected from the cavity is used to lock it, with feedback to both the cavity length (actuated via magnets affixed to each suspended mirror), as well as the laser frequency.

The input mirror of the $L = 0.1$ m long cavity has mass of 0.25 kg and is suspended as a pendulum with oscillation frequency of 1 Hz for the longitudinal mode. The 10^{-3} kg end mirror is suspended by two optical fibers 300 μm in diameter; these fibers are attached to a stainless steel ring. The stainless steel ring is, in turn, suspended as a 1 Hz pendulum. The oscillation frequency of the longitudinal mode of the end mirror is $\Omega_M = 2\pi \times 12.7$ Hz, with quality factor $Q_M = 19950$ determined by measuring the ringdown time of the mode. The input mirror transmissivity is $\mathcal{T}_i = 800 \times 10^{-6}$, while that of the end mirror is 10^{-5}, and the laser wavelength is $\lambda_0 = 1.064 \times 10^{-6}$ m. On resonance, the intracavity power is enhanced relative to the incoming power by a resonant gain factor $4/\mathcal{T}_i \approx 5 \times 10^3$, and with resonant linewidth (HWHM) of $\gamma = \frac{\mathcal{T}_i c}{4L} \approx 2\pi \times 95$ kHz.

At zero detuning from resonance, the stored power in the cavity exerts a constant (dc) radiation pressure on each mirror. When the cavity is detuned, changes in its radiation pressure give rise to both a position-dependent restoring and a velocity-dependent damping force. For a cavity with detuning δ and input power I_0, and change of its length x, the radiation pressure force written in the frequency domain is

$$F = -Kx + M\Gamma(i\Omega x), \tag{5}$$

where the spring constant K and damping coefficient Γ at each frequency Ω are given by [21]

$$K(\Omega) = K_0 \frac{[1 + (\delta/\gamma)^2 - (\Omega/\gamma)^2]}{[1 + (\delta/\gamma)^2 - (\Omega/\gamma)^2]^2 + 4(\Omega/\gamma)^2}, \tag{6}$$

$$\Gamma(\Omega) = \frac{2K_0/(M\gamma)}{[1 + (\delta/\gamma)^2 - (\Omega/\gamma)^2]^2 + 4(\Omega/\gamma)^2}. \tag{7}$$

Here

$$K_0 = \frac{2}{c}\frac{dP}{dL} = \frac{128\pi I_0(\delta/\gamma)}{\mathcal{T}_i^2 c\lambda_0}\left[\frac{1}{1+(\delta/\gamma)^2}\right] \quad (8)$$

and M is the reduced mass of the two mirrors. The natural resonant frequency of the system is shifted to

$$\Omega_{\rm eff} = \sqrt{\Omega_M^2 + K(\Omega_{\rm eff})/M}. \quad (9)$$

The cavity is locked off-resonance by $\delta \approx 0.5\gamma$, to maximize the optical restoring force. The error signal for the locking servo, generated using the Pound-Drever-Hall technique [24], is split between a high bandwidth analog path fed back to the laser frequency, and a digital path fed back to the input mirror's magnet or coil actuators. The digital feedback is used at frequencies below 10 Hz to keep the cavity locked in its operating state. The analog feedback to the laser frequency is arranged so that it damps and cools the motion of the oscillator, a cold damping technique. The effective damping may be controlled by adjusting the gain of the feedback loop. Additional analog feedback is supplied to the magnet or coil actuators to damp a parametric instability of the input mirror at 28 kHz [18,25].

By comparison with our previous report [21], in the present experiment: (i) the cavity is shortened by a factor of 10 to reduce the effect of laser frequency noise; (ii) the end mirror suspension is reduced in stiffness by a factor of 180, and in mechanical loss by a factor of 80; (iii) active feedback to the laser frequency supplies the cold damping force, instead of a second detuned optical field; and (iv) only 100 mW of incoming laser power is used, which was necessary to avoid exciting a parametric instability of the 1 g mirror at 137 kHz.

Cooling.—The noise in our experiment remains dominated by frequency noise of the laser at $\Omega_{\rm eff}$. We estimate the effective temperature of the optomechanical mode, as determined by this noise, according to Eq. (1).

To determine $x_{\rm rms}$ in our experiment, we first find the resonant frequency and damping of the oscillator by measuring its frequency dependent response to a driving force, shown in Fig. 2. In the same configuration, we then measure the noise spectral density of the error signal from the cavity, calibrated by injecting a frequency modulation of known amplitude at 12 kHz. The measured displacement spectra, as the electronic damping was varied, are shown in Fig. 3. Since the optical spring resonance, given by Eq. (9), is at $\Omega \approx 2\pi \times 1000$ Hz, we integrate the spectrum from 850 Hz to 1100 Hz to obtain an estimate of the motion of the mirror. At other frequencies, sensing noise not present on the mirror itself is dominant. To correct for the finite integration band, we assume a thermally driven displacement noise spectrum for the oscillator, given by

$$\langle x^2\rangle = \frac{4k_B T_{\rm eff}\Gamma_{\rm eff}/M}{(\Omega_{\rm eff}^2 - \Omega^2)^2 + (\Omega_{\rm eff}\Omega/Q_{\rm eff})^2}, \quad (10)$$

and find $T_{\rm eff}$ by setting our measured spectrum integral equal to a thermal spectrum integrated over the same frequency band. The lowest temperature reached is 6.9 ± 1.4 mK. Thus the cooling factor from the ambient $T_M = 295$ K is $43\,000 \pm 11\,000$. Systematic error in the calibration dominates statistical error in these uncertainty estimates. We note that the mechanical quality factor was increased by a factor of about 80, from 19 950 to 1.6×10^6, by optical dilution. Without an optical spring, effective temperatures below 15 mK could not have been reached given the mechanical losses of the oscillator.

Lifetime.—In this experiment, we began with $\Omega_M = 2\pi \times 12.7$ Hz, $\Omega_M/\Gamma_M = 19950$, and $T_M = 295$ K, while

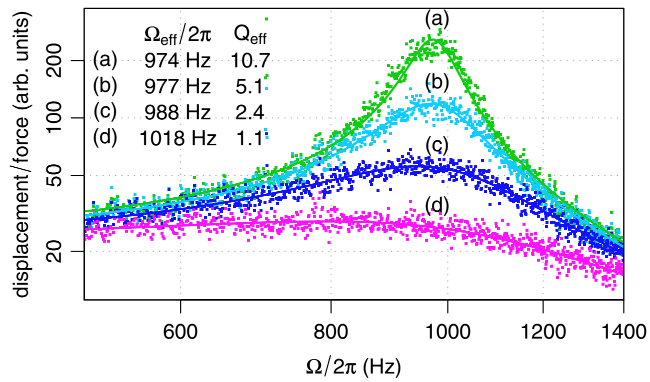

FIG. 2 (color online). The transfer function of an applied force to mirror motion, for increasing levels of damping [curves (a) to (d)]. The force is applied via the magnet or coil actuators, and the response is measured by the PDH error signal. The points are measured data, and the lines are fitted Lorentzians from which the resonant frequency and damping constant are derived for each configuration. Statistical errors in the fit parameters are on the order of 1%.

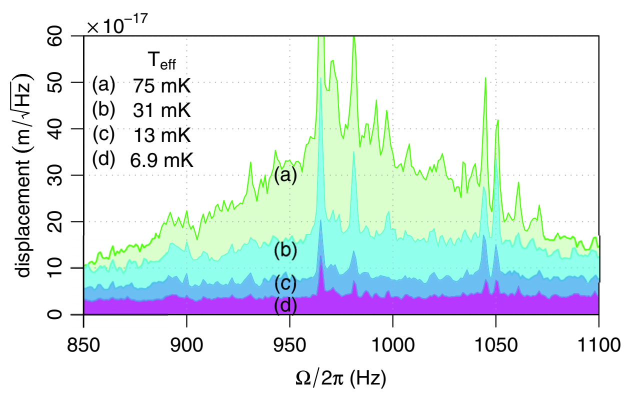

FIG. 3 (color online). The measured noise spectral density of the mirror displacement. The curves (a) to (d) correspond to increasing gain in the damping feedback loop; for each, the parameters of the resonance are measured and depicted in Fig. 2. The spectra are integrated from 850 to 1100 Hz, the frequency range where the mirror motion is the dominant signal, to obtain the rms motion of the mirror and its effective temperature. The broad limiting noise source is frequency noise of the laser. Narrow spectral features in addition to the main optical spring resonance are due to coupling of acoustically driven phase noise.

the coldest optical spring mode has $\Omega_{\rm eff} = 2\pi \times 1018$ Hz, $\Omega_{\rm eff}/\Gamma_{\rm eff} = 1.1$, and $T_{\rm eff} = 6.9 \times 10^{-3}$ K. This corresponds to a factor of 196 ± 40 increase in $n_{\rm osc}$; the error on this value comes from error estimates for measured values of temperature, frequency, and damping of the mode, with the temperature uncertainty dominating.

Laser frequency noise presently limits the achievable degree of cooling, but this noise source can be mitigated by placing two identical cavities into the arms of a Michelson interferometer [26]. The laser light reflected from each cavity interferes destructively at the beam splitter, allowing for rejection of laser noise at the antisymmetric output. The laser frequency may be further locked to the common motion of the two arms, providing additional stabilization of laser frequency noise. The remaining differential motion of the arm cavity mirrors becomes the oscillator degree of freedom to be placed in a quantum state.

The ultimate limit of optical cooling is expected to come from the vacuum noise of the optical field. The parameter regime required to achieve cooling to the ground state with nonmechanical damping techniques has been theoretically explored [27–30], although the regime in which a low occupation number may be reached with the aid of optical dilution has yet to be delineated. However, this will be the subject of another paper.

In conclusion, we have proposed a scheme that uses the optical spring effect to both reduce the occupation number *and* increase the dynamical lifetime of the mode of a 1 g mirror oscillator. We also provide an experimental demonstration showing that cooling factors that exceed the mechanical Q of the macroscopic oscillator can be achieved when damping dilution from the optical spring effect is used in conjunction with cold damping.

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