Effects of electrical charging on the mechanical *Q* of a fused silica disk

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We report on the effects of an electrical charge on mechanical loss of a fused silica disk. A degradation of Q was seen that correlated with charge on the surface of the sample. We examine a number of models for charge damping, including eddy current damping, and loss due to polarization. We conclude that rubbing friction between the sample and a piece of dust attracted by the charged sample is the most likely explanation for the observed loss. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1619544]

I. INTRODUCTION

Experimental efforts to measure gravitational waves have been ongoing for over 40 years[1] and recently several interferometers[2–4] have begun taking science data; no gravitational wave signals have been observed to date. Estimates of the strength and rate of gravitational wave events are such that improvements in the sensitivity of detectors would be richly rewarded in terms of event rates and information extracted from the signals.[5] The sensitivity of these interferometers will be limited by fundamental noise sources, with thermal noise from the internal degrees of freedom in the mirrors setting the limit at the frequency of highest sensitivity. Any increase beyond what is currently anticipated for thermal noise, or additional noise sources beyond what is expected,[6] will reduce the sensitivity of advanced interferometers.

Thermal noise is generally studied in the laboratory indirectly, through application of the fluctuation-dissipation theorem,[7] which says that thermal noise can be predicted from the loss in the system. Studying the mechanical loss of optics is much simpler in the laboratory than direct measurement of thermal noise. Loss can be characterized by the quality factor, Q, of a resonant mode, with higher Q's resulting in lower thermal noise. A variety of mechanisms are thought to play a role in introducing mechanical loss in a suspended gravitational-wave detector optic: intrinsic losses in the mirror material, in the attachments to the suspension, in the dielectric optical coating,[8,9] and through interactions with the environment through electromagnetic couplings.

The buildup of electric charge on interferometer optics has been observed in LIGO. Fused silica optics are known to become charged and to increase their charge over many months.[10] Degradation in the Q of a fused silica suspension due to charging has also been observed in a pendulum[11] as well as a torsional[12] mode. The effect of charging on internal mode thermal noise has not been well studied.

We have investigated the effect of charging on the mechanical Q of a normal mode of a fused silica disk. We observed noticeable change in the ringdown of the mode when the optic is charged in particular circumstances. We present possible explanations for this change and discuss its relevance for interferometric gravitational wave detectors.

II. THEORY

There are a number of mechanisms by which charge on an optic could cause excess loss and thereby higher thermal noise. We examine some of these mechanisms to see which could cause excess mechanical loss in a laboratory setting. This list is not meant to be exhaustive.

A. Eddy-current damping

One possible source of excess loss from charging is eddy-current damping between the charged sample and a nearby ground plane. This is the mechanism which was suggested as an explanation for the excess loss seen in the charged pendulum.[11] We modeled the charged sample as a point mass placed near a ground plane infinite in extent. This ground plane could represent the metallic walls of a vacuum chamber, or a metal capacitor plate placed nearby the sample for exciting the normal modes.[13] The charge was assumed to oscillate back and forth relative to the ground plane at 3 kHz, which is a typical normal mode frequency for laboratory experiments on small silica samples. This oscillating charge creates an oscillating field which induces currents in the ground plane. These currents will suffer Ohmic losses in the metallic plane. This energy loss can be characterized by a limiting Q:

$$Q_{\mathrm{limit}} = 2\pi E/\Delta E_{\mathrm{cycle}}, \tag{1}$$

where $\Delta E_{\mathrm{cycle}}$ is the energy loss per cycle of oscillation, and E is the total elastic energy stored in the oscillation of the sample. The energy loss $\Delta E_{\mathrm{cycle}}$ can be calculated using the equations of electrodynamics.

The surface charge on the ground plane is found to be

$$\sigma(r,t) = -\frac{q[d + A\cos(2\pi f t)]}{2\pi(r^2+d^2)^{3/2}}, \tag{2}$$

where q is the charge of the oscillating point mass, d is the average distance between the point mass and the ground

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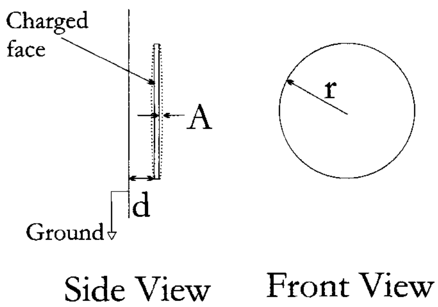

FIG. 1. Model used for calculation of loss due to charging. Dimension labels are shown for the sample radius *r*, distance from the sample to the ground plane *d*, and the amplitude of vibration *A*. The sample face which is charged and the ground plane are also indicated.

plane, A is the amplitude of the oscillation, and f is the oscillation frequency. Figure 1 shows the relative position of the sample and ground plane with the dimensions labeled. Since $\partial\sigma/\partial t = -\nabla \cdot J$, the surface current can be written:

$$J(r,t) = -\frac{1}{r}\int_0^r \frac{\partial}{\partial t}[r'\sigma(r',t)]dr'. \tag{3}$$

Taking the power loss per area to be $\partial P/\partial S = \rho/2\delta|J(r,t)^2|$, we get

$$\Delta E = \frac{\pi\rho}{\delta}\int_0^{1/f}\int_0^r r'[J(r',t)]^2 dr dt, \tag{4}$$

where ρ is the resistivity and δ is the skin depth of the ground plane.

For laboratory experiments, we used the following appropriate parameters; an aluminum ground plane with $\rho = 2.7\times10^{-8}\,\Omega$ m and $\delta = 1.5\times10^{-3}$ m, a charge $q = 1.6\times10^{-9}$ C, distance $d = 2\times10^{-3}$ m, $f = 3000$ Hz, and sample radius $r = 3.8\times10^{-2}$ m. With these numbers, Eq. (1) gives a limit to the Q of

$$Q_{\mathrm{limit}} \sim 10^{20}. \tag{5}$$

This Q is much higher than any modal Q seen in a material sample.[13–16] For normal mode ringdowns and interferometer thermal noise this mechanism was ruled out as a relevant loss mechanism.

A second model is similar to the first but with a resistive wire between the plate and ground. This could occur when the wires of the exciter are disconnected from the high voltage used during excitation and grounded with a grounding cap. The formula for surface charge on the on the plate is the same as Eq. (2). By integrating this surface charge over the area of the plate to get a total time-dependent charge, then differentiating with respect to time, a current that would flow through the wire, I, is obtained. The value $\Delta E_{\mathrm{cycle}}$ is found by using $P = I^2R$ for power loss and integrating P over one cycle. For similar laboratory values as above, with a wire resistance $R = 1\,\Omega$, a Q_{limit} of $\sim 10^{18}$ is obtained. This is similarly higher than any reasonable material Q, and thus we can conclude that loss due to induced currents flowing in a wire between a plate and ground also has a negligible effect on thermal noise.

B. Polarization losses

Another possible source of damping associated with surface charge is polarization loss. As the charge on a sample moves relative to a ground plane, the electric field inside the sample will change. This changing electric field inside the dielectric sample causes a changing polarization. This changing polarization can be thought of as a current of bound charges which undergoes loss as it flows. The time-dependent electric field from a point charge of the models in Sec. II A was examined for its effect on a fused silica sample. Since the sample is a dielectric medium, the changing electric field causes the polarization of the sample to oscillate, creating "bound currents,"

$$J = \epsilon_0 \chi_e \frac{\partial E}{\partial t}, \tag{6}$$

where E is the electric field, ϵ_0 is the premittivity of free space, and χ_e is the electric susceptibility of the sample material. The energy loss as a function of time is found by integrating J^2 over the volume of the sample and multiplying by the resistivity ρ.

The electric field used was the same as in Sec. II A, which assumed a point charge next to an infinite ground plane. Using those parameters for the point charge, a frequency of 3 kHz, a silica sample 3.8×10^{-2} m in radius, 2.5×10^{-3} m thick, and 2.6×10^{-2} kg in mass, a silica resistivity of $2\times10^{12}\,\Omega$ m and electric susceptibility of 2.8, and a distance between the sample and the ground plane of 5×10^{-3} m, a Q of 10^4 is obtained. This Q is a strong function of the distance between the charge and the ground plane, becoming 2×10^7 at 3 cm. With such a strong dependence, the approximations of the charge as a point and the ground plane as infinite break down for a realistic laboratory setting. The resistivity of bulk silica is also not that well characterized. This result does indicate that polarization loss in the body of a silica sample could be an important loss mechanism when a ground plane is close to a charged optic.

A closely related loss mechanism is the polarization loss in a surface layer of a silica sample. This is the mechanism that was suggested for the result in Ref. 12. Using the same model as above, but only integrating over a surface region with a resistivity of $2\times10^{10}\,\Omega$ m, the same susceptibility of 2.8, and a surface layers thickness of 10^{-5} m,[17] results in a Q of 2×10^{10}. This mechanism shares the strong dependence on distance with the volume polarization loss, but also suffers from uncertainties in the properties of the surface layer.

C. Electrostatic coupling to a lossy mechanical system

There could be an electrostatic coupling between surface charge on a sample and a nearby charged insulator. This could occur if the insulation on wires in the exciter becomes charged as well as the silica sample. If motion of the surface charges can cause motion in the insulator, loss can occur either from rubbing friction between parts of a mechanical structure or simply internal friction of the insulator material.

This can be modeled as a coupled oscillator in which the oscillation of the sample induces vibration of the insulator.

The equations of motion can be solved to determine the energy lost to the insulator. Using a charge on both the sample and the insulator of 1.6×10^{-9}, a sample mass of 0.26 kg, an insulator mass of 2.6×10^{-3} kg, a separation of 2×10^{-3} m, a frequency of 3000 Hz, a spring constant between the insulator and mechanical ground of 3.4×10^{7} N/m, and a loss angle for the insulator of 1×10^{-3} results in a Q of 2×10^{9}. Certain parameters for this model, notably the stiffness and loss angle of the insulator, are not well known for a laboratory setting. Despite this, the high predicted Q compared to sample internal friction suggest this should not be an important loss mechanism.

D. Rubbing with dust particles

Another possible source of loss is rubbing between a dust particle and the sample. This can be correlated with surface charge because a charged sample could attract a charged dust particle, making the chance contact between dust and sample far more likely when they are charged. The friction force between the sample and a material in contact with it can be written

$$F_f = -\mu N \mathbf{v}/|\mathbf{v}|, \tag{7}$$

where F_f is the frictional force, μ is the coefficient of friction, N is the normal force between the two materials, and $\mathbf{v}$ is the velocity. This velocity dependence results in a force of constant magnitude in the opposite direction of the relative velocity. Solving the equation of motion for a system with this force results in a sinusoidal oscillation with a linearly decaying amplitude:[18]

$$x(t) = [A_0 - \mu N t/(\pi^2 f_0 m)]\sin(2\pi f_0 t + \theta), \tag{8}$$

where f_0 is the frequency of the oscillation, A_0 is the initial amplitude of oscillation, m is the sample mass, and θ is an arbitrary phase. Linear amplitude decay of this type is known as Coulomb damping.[19] This distinctive form of decay allows Coulomb damping to be distinguished easily from other sources of loss. According to Eq. (8), the amplitude of vibrations in a Coulomb damped system will decrease by $\mu N/\pi^2 f_0^2 m$ per cycle. Thus the rate of decay should decrease linearly as the normal force is reduced.

III. EXPERIMENT

A. Method

To test these sources of loss, we measured the mechanical quality factor, Q, of a charged fused silica disk. The disk was 76.2 mm in diameter by 2.5 mm thick, made from Corning 7980, Grade 0-A silica. We found the frequency of a normal mode of vibration, excited this mode, and measured the ringdown. From the decay time of this ringdown we were able to determine the effect of the charge on loss. The normal mode measured was the $n=1$, $\ell=0$ mode, with a frequency of 4100 Hz.

The mode was excited using a comb capacitor[13,20] exciter. This consists of two wires wrapped side by side around an aluminum ground plane. This exciter was placed close to the sample, typically about 1 cm, but we were able to change this distance using a piezoelectric stepping motor. A dc voltage of 500 V was placed on one of the two wires while the other was held at ground. This creates a diverging electric field near the exciter and inside the glass dielectric. An ac field with peak amplitude 500 V at the normal mode frequency is then applied to the high voltage wire. This ac field couples to the polarization in the glass to give a force on the sample at the normal mode frequency. The electric field and the exciter can interact with any charge on the sample.

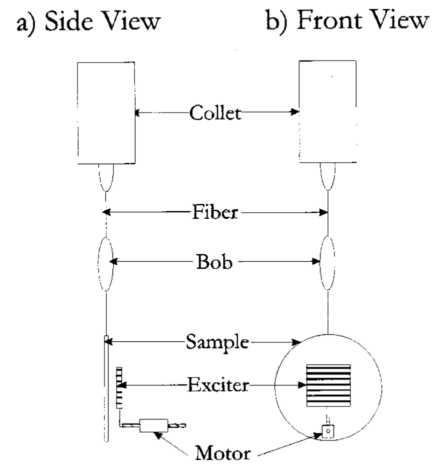

FIG. 2. Setup used for these experiments. The sample is suspended below a thin fiber of silica welded to a massive bob which itself is supended below a silica fiber welded to a silica mass held in a collet. Next to the sample is an exciter made from two wires wrapped around a ground plane. The exciter can be moved relative to the sample by the piezoelectric stepping motor. The oscillations in the sample are read out using stress polarimetry with a polarized HeNe laser as the probe.

The test sample was suspended by a monolithic, fused silica suspension of thin fibers and a single isolation bob.[13] The suspension is held on top by a collet attached to an aluminum stand. The monolithic fiber-bob suspension keeps excess loss from recoil damping or rubbing at interfaces from affecting the Q measurement. The entire setup is contained within a vacuum bell jar which is pumped down to at least 10^{-3} Pa and typically about 3×10^{-4} Pa to avoid loss from gas damping. The experimental setup is shown in Fig. 2. This experimental setup is similar to ones used in previous experiments and is more fully described there.[13,21,22]

The sample's normal mode amplitude is read out versus time using a stress polarimeter. A polarized HeNe laser is passed through the sample where stress induced birefringence changes the laser's polarization. After passing through a $\lambda/4$ plate, the beam's polarization oscillates at the mode frequency with an amplitude proportional to the mode amplitude. This signal is read out using a polarizing beamsplitter and two photodiodes. The signal is passed through a current-to-voltage amplifier and then heterodyned to about 0.3 Hz by a lock-in amplifier. Finally, the data are passed to an analog-to-digital converter and recorded on a PC.

The data stored on computer are typically of the form of a damped sinusoid. Most sources of loss cause an exponential decay to occur in the mode amplitude, so the data can be fit to

$$x(t) = e^{-t/\tau}\sin(2\pi f_{\mathrm{demod}} t + \theta), \tag{9}$$

where τ is the decay time, f_{demod} is the frequency after demodulation, and θ is an unimportant phase. From this fit, the decay time τ can be determined. This characterizes the loss, reported as the dimensionless value:

$$Q=\pi f_0 \tau, \tag{10}$$

where f_0 is the normal mode frequency. We also observed linear decay in the mode amplitude, where the data can be fit to

$$x(t)=(mt+A_0)\sin(2\pi f_{\mathrm{demod}} t+\theta). \tag{11}$$

This behavior is characteristic of Coulomb damping.[18]

To control the charge on the surface of the sample we used two techniques. First, an ionized-nitrogen spraygun, which could be used on the sample to either increase or decrease the charge. It is difficult to control exactly where on the sample's surface the charge ends up, so for studies involving charge distribution we used a small piece of silk cloth. Gently rubbing the sample with the silk allows a charge to be built up in the rubbed area. This allowed us to create a high charge density region in the center, for example, while leaving the edges of the sample with low charge. We also used the silk cloth to get greater charge densities across the entire face of the sample.

The distance between the exciter and the sample could be controlled very precisely using a piezoelectric stepping motor. The motor was measured to move at 1.7×10^{-5} m/s, about a millimeter per minute. Thus, by simply measuring the time the motor was engaged, we could tell how far the exciter had moved to within a few tens of microns. The zero position in distance was defined by when the exciter touched the sample. This could be determined by observing when the low frequency pendulum mode oscillations of the sample stopped due to contact with the exciter.

B. Trials

To test the effect of charge on thermal noise, we performed ringdown experiments. We changed various parameters, including surface charge density, surface charge distribution, and distance between the exciter and the sample, to determine which, if any, of the phenomenon described in Sec. II was affecting the Q.

To start, we collected Q data without any charge on the sample. A typical ringdown from these measurments is shown in Fig. 3. The result is an exponential ringdown with a quality factor of

$$Q=13.3\times 10^6. \tag{12}$$

This measured Q showed no dependence on the distance between the exciter and the sample, including down to separations below 300 μm (see below).

We measured the Q under a variety of charging conditions. The sample was charged using the ionized nitrogen spraygun, to give a uniform charge on both sides of the sample. To obtain an approximate value for the surface charge density we used an Ion Systems Model 775 Periodic Verification System electrostatic field meter. This gave an indication of the sign of the charge, as well as providing a method of verifying whether the charge density before and after the experiment remained the same. The charge density

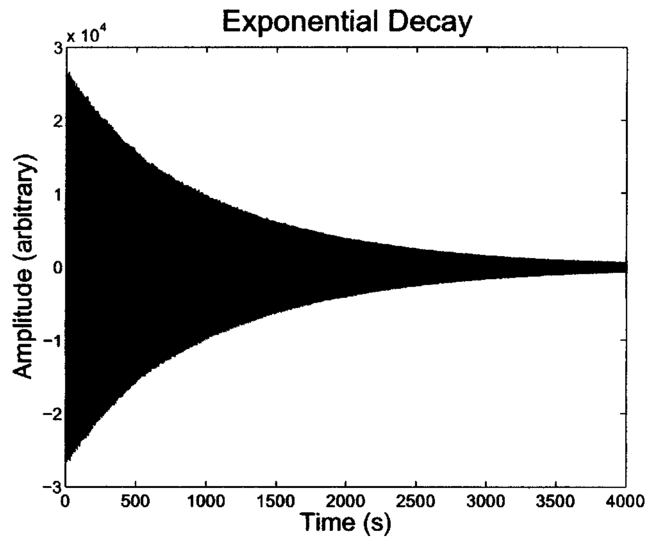

FIG. 3. Typical modal ringdown when the sample is limited by the internal friction of the silica. The envelope is exponential in shape.

attainable was very low and thus, we rubbed the surface of the sample using a piece of silk cloth. The electric field was measured to be 1.7×10^5 V/m at a distance of $\sim$2.5 cm from the sample surface, approximately 5 times higher than attainable with the ion spray. Assuming an infinite charge plane this converts to a minimum charge density of 1.5×10^{-5} C/m^2. The ion spray was subsequently used to discharge the surface. Next we varied the distribution of the charge on the surface. The center of the sample was charged with the silk cloth, to a minimum value of 9×10^{-6} C/m^2, again using the infinite plane approximation. We then gave the sample a uniform charge and measured the Q while the exciter was left with a dc voltage. This was to test the effect of a static force acting between the sample and the exciter structure.

Finally, we gave the sample a uniform charge and varied the distance between the exciter and sample. No change in the ringdown was observed until the exciter came very close

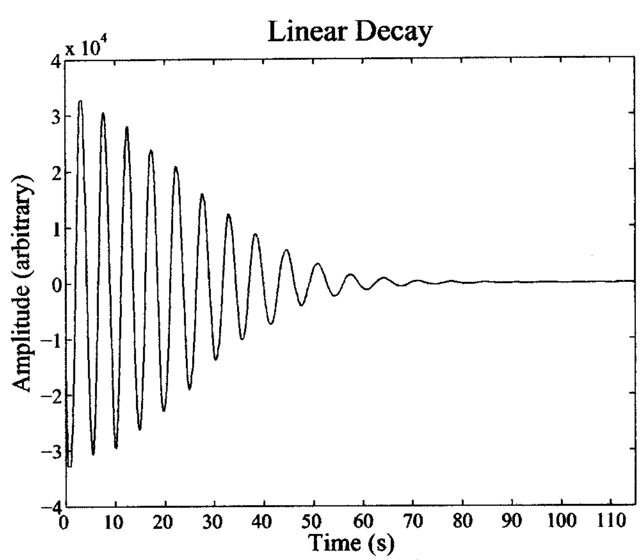

FIG. 4. Typical modal ringdown when the sample is limited by the charge-correlated loss mechanism. The envelope is linear in shape.

TABLE I. Measured Q's for varying surface charge densities on the silica disk. The charge density was calculated from a measured electric field.

Charge density (μC/m^2)	Measured Q
30	12.9×10^6
90	12.9×10^6
200	13.1×10^6

to the sample, within about 300 μm. Within this distance, two changes were observed; the ringdowns went from an exponential shape to linear, and the characteristic time for a ringdown dropped precipitously. A graph of a typical linear ringdown from when the exciter was very close to the sample is shown in Fig. 4. The change in ringdown versus distance between the sample and the exciter inside this close region was investigated.

C. Results

In order to investigate eddy current damping, polarization loss, and coupling to a lossy mechanical system, all discussed in Sec. II, we measured Q versus charge density. The results, shown in Table I, indicate that the mechanical loss in silica is unaffected directly by charge on the surface in our experimental setup.

To test the hypothesis that the charged sample was coupled to the lossy exciter structure, we tried varying the dc voltage on the exciter during the ringdown of a charged sample. The results shown in Table II indicate that this mechanism did not contribute to the loss in our setup.

The only trial that gave results different from the uncharged sample was with the exciter extremely close to the charged sample. Based on the linear shape of these ringdowns, Coulomb damping from dust rubbing against the sample, discussed in Sec. II D, is the best fit of all the models discussed in Sec. II. None of the other models predict this linear decay shape. All of these other models are consistent with no observable effect for some reasonable collection of parameters.

The Coulomb damping model predicts that the slope of the linear decay envelope should follow Eq. (8). This equation says that the slope gets steeper as the normal force between the sample and the exciter increases. The normal force between the dust and the sample is caused by either the spring constant of the dust itself, or of the pendulum suspension of the sample. As the exciter is moved closer to the sample, the normal force will increase in either case. The slope of the linear ringdowns versus distance between the exciter and the sample is plotted in Fig. 5. This figure suggests a correlation between low slope values and distance, in agreement with Eq. (8).

TABLE II. Measured Q's for varying dc voltages on the exciter while the silica disk was charged to 30 μC/m^2.

Exciter dc potential (V)	Measured Q
250	14.3×10^6
500	14.9×10^6
1000	14.5×10^6

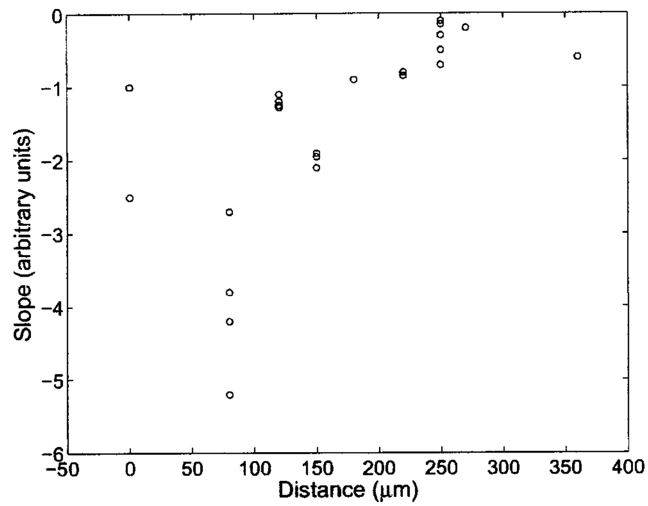

FIG. 5. Slope of the linear envelope of the decay vs distance from the sample. Zero distance is arbitrarily set as the closest point at which data were taken. There is a correlation between slopes closer to 0 and greater distance between the sample and the exciter.

IV. IMPLICATIONS

The silica test masses currently installed in the LIGO vacuum system are known to become charged. Measurement, control, and mechanical structures lie within several millimeters of the surface of the suspended optics. This combination of a charged sample and relatively close proximity to a separate body could allow for Coulomb damping from dust spanning the gap. In our laboratory experiments we never observed any Coulomb damping when the distance between the exciter and the sample was this large, however. The LIGO vacuum chambers are always surrounded by class 100 portable clean rooms whenever the chambers are opened. It is much less likely that dust could contaminate the LIGO optics than the sample we measured in an open laboratory. It is unlikely that conditions similar to what we experienced in the laboratory would allow for dust to cause Coulomb damping on the LIGO test masses.

The vacuum chamber walls and the metal support structures around installed LIGO optics could act as ground planes for charged optics. None of these conductors are close enough to any optic for the polarization losses discussed in Sec. II B to be important. The chamber walls are meters away while the support structure is tens of centimeters.

We did not investigate, either theoretically or experimentally, any noise sources beyond excess thermal noise that could be caused by charged optics. It is possible that patchy charge densities could have thermally driven fluctuations. Interactions with nearby ground planes or background electric fields could then cause noise in the interferometer. This possibility may require further study if charging of optics continues to be a problem.

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[19] Coulomb damping is named for Charles Augustin de Coulomb who first described it mathematically, although it is initially discussed in the literature by Aristotle. Coulomb, of course, also did work with electrostatics where his name is frequently seen as well. It is, however, a coincidence that the form of damping described here is both called Coulomb damping and is associated with electric charge.
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