

# Ant Colony Optimisation for Power Plant Maintenance Scheduling

by Wai Kuan Foong

A Thesis submitted for the Doctor of Philosophy Degree

The University of Adelaide School of Civil and Environmental Engineering

April 2007

To my loving parents, K im Lam Wong and K on Thoong Foong:

For what I have been taught and untaught.

### Abstract

Maintenance of power plants is aimed at extending the life and reducing the risk of sudden breakdown of power generating units. Traditionally, power generating units have been scheduled for maintenance in periods to ensure that the demand of the system is fully met and the reliability of the system is maximized. However, in a deregulated power industry, the pressure of maintaining generating units is also driven by the potential revenue received by participating in the electricity market. Ideally, hydropower generating units are required to operate during periods when electricity prices are high and to be able to be taken offline for maintenance when the price is low. Therefore, determination of the optimum time periods for maintenance of generating units in a power system has become an important task from both a system reliability and an economic point of view. Due to the extremely large number of potential maintenance schedules, a systematic approach is required to ensure that optimal or near-optimal maintenance schedules are obtained within an acceptable timeframe.

Metaheustics are high-level algorithmic frameworks that aim to solve combinatorial optimisation problems with a large search space in a reasonable computational run time. Inspired by the foraging behavior of ant colonies, Ant Colony Optimisation (ACO) is a relatively new metaheuristic for combinatorial optimisation. The application of ACO to a number of different applications has provided encouraging results when applied to scheduling, including the job-shop, flow-shop, machine tardiness and resource-constrained project scheduling problems.

In this thesis, a formulation is developed that enables ACO to be applied to the generalized power plant maintenance scheduling optimisation (PPMSO) problem. The formulation caters for all constraints generally encountered as part of real-world PPMSO problems, including system demands and reliability levels, precedence rules between maintenance tasks, public holidays and minimum outage durations in the case of shortening of maintenance tasks. As part of the formulation, a new heuristic and a new local search strategy have been developed. The new ACO-PPMSO formulation has been tested extensively on two benchmark PPMSO problems from the literature, including a 21-unit and a 22-unit problem. It was found that the ACO-PPMSO formulation resulted in significant improvements in performance for both case studies compared with the results obtained in previous studies. In addition, the new heuristic formulation was found to be useful in finding maintenance schedules that result in more evenly spread reserve capacity and resource allocations. When

tested using a modified version of the 21-unit and the 22-unit problems, the new local search strategy specifically designed for duration shortening was found to be effective in searching locally for maintenance schedules that require minimal shortening of outage duration. The ACO-PPMSO formulation was also successfully able to cater for all constraints as specified in both original and the modified versions of the two benchmark case studies.

In order to further test the ACO-PPMSO formulation developed, it was first applied to a scaled-down version of the Hydro Tasmania hydropower system (five power stations) and then to the full system (55 generating units). As part of the studies, the ACO-PPMSO formulation was linked with the simulation model used by Hydro Tasmania to assess the impact of various maintenance schedules on the total energy in storage of the system at the end of the planning horizon, the total thermal generation, the total number of days where the reliability level is not met, as well as the total unserved energy throughout the planning horizon. A number of constraints were considered, including the anticipated system demands, a 30% capacity reliability level, the minimum and maximum durations between related maintenance tasks, the precedence constraints and the minimum outage duration of each task in the case of shortening of maintenance tasks. The maintenance schedule was optimised for the maximum end-of-horizon total energy in storage, the minimum thermal generation and the minimum total outage durations shortened and deferred, under 77 different inflow conditions. The optimal maintenance schedule obtained compared favourably with that obtained by Hydro Tasmania over many years based on experience. Specifically, the ACO-PPMSO schedule results in higher end-ofhorizon total energy in storage and satisfies both hard and soft constraints, which overall equates to over \$0.5 million dollars of savings when compared to the schedule obtained using the practitioners' experience and engineering judgment. The ACO-PPMSO algorithm was also shown to be a useful decision-making tool for scheduling maintenance under different circumstances when tested with four scenarios commonly encountered in practical maintenance scheduling problems.

In conclusion, the ACO-PPMSO formulation developed, tested and applied as part of this thesis research provides a powerful and flexible means of obtaining optimal or near-optimal maintenance schedules for power plants.

## Declaration

I, **Wai Kuan Foong**, declare that the work presented in this thesis is, to the best of my knowledge and belief, original and my own work, except as acknowledged in the text, and that the material has not been submitted, either in whole or in part, for a degree at this or any other university.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

Signed:

Dated:

## Acknowledgements

I wish to thank my supervisors, Associate Professor Holger Robert Maier and Professor Angus Ross Simpson, for their enlightening guidance and endless support throughout the entire period of my postgraduate studies. I especially dedicate the completion of this thesis to the unconditional support and timely encouragement by Associate Professor Holger Robert Maier. Your support of students, both academically and personally, is very much appreciated.

I would also like to thank Dr. Michael Connarty, for this research would not have been realized without his proposal (with Associate Professor Holger Robert Maier) about finding a power plant maintenance scheduling optimisation tool for Hydro Tasmania. Throughout this research, Mr. Stephen Stolp's enthusiasm for facilitating this research, including the supply of the case study data and the critical assessment of results obtained, was invaluable.

The financial support provided by Hydro Tasmania, Eng Test and the Australian government are appreciated.

The staff of the School of Civil & Environmental Engineering at University of Adelaide are acknowledged for the help they have given me throughout my stay in the school. Particular thanks are given to Dr. Stephen Carr, who resolved all my computing-related problems promptly and precisely, regardless of my location. Robert May, Michael Leonard and Aaron Zecchin are also thanked for not only being nice and friendly colleagues, but their enthusiasm for research, which has been an encouragement to me.

A special thanks goes to my family including my husband, Ken for their patience and selfless love. Thank you all for just being there for me during the challenging times and sharing the happiness with me during the good times.

## **Table of Contents**

Table of List of A List of F List of T	tion ledgements contents appendices igures	i iii v vii viii xiii xv
CHAPT	ER 1 INTRODUCTION	1
1.1	Research background	1
1.2	Research objectives	2
1.3	Thesis layout	3
CHAPT	ER 2 LITERATURE REVIEW	5
2.1	Power plant maintenance scheduling optimisation	5
2.1.1	Objectives	6
2.1.2	Constraints	11
2.1.3	Deregulation of electricity market	13
2.2	Optimisation methods previously adopted for PPMSO	14
2.2.1	Design requirements for a maintenance scheduling tool	14
2.2.2	Heuristic approaches	16
2.2.3	Mathematical programming	17
2.2.4	Expert systems	20
2.2.5	Metaheuristics	20
2.2.6	Comparison of optimisation methods for PPMSO	36
2.3	Summary and conclusions	39
СНАРТ	ER 3 ANT COLONY OPTIMISATION METAHEURISTIC	40
3.1	From Real Ants to Artificial Ants	40
3.1.1	Foraging behavior of real ants	40
3.1.2	Artificial ants	45
3.2	ACO for Combinatorial Problems	46
3.2.1	Problem representation	46
3.2.2	The ACO metaheuristic: a general framework	47
3.2.3	Prerequisites of ACO implementation	49

3.3	Variants of Ant Colony Optimisation algorithms	51
3.3.1	Ant System (AS) and its direct successors	52
3.3.2	Non-AS-based ACO algorithms	57
3.4	Ant Colony Optimisation Applications	60
3.4.1	Benchmark scheduling optimisation problems	60
3.4.2	Real-world optimisation problems	64
3.5	Motivation for Applying ACO to PPMSO	68

70

# CHAPTER 4 PROPOSED APPROACH TO MAINTENANCE SCHEDULING OPTIMISATION

4.1	Definition of power plant maintenance scheduling optimisation (PPMSO)	70
4.2	Proposed ACO formulation for PPMSO	75
4.3	The ACO-PPMSO algorithm	
4.3.1	Initialization	77
4.3.2	Construction of a trial maintenance schedule	78
4.3.3	Evaluation of trial maintenance schedule	82
4.3.4	Local search	83
4.3.5	Pheromone updating	85
4.3.6	Termination of run	87
4.4	Constraint handling techniques in ACO-PPMSO	88
4.5	Software development	94
4.6	Summary	95

#### CHAPTER 5 TESTING ON BENCHMARK CASE STUDIES 97

5.1	Benchmark case studies	98
5.1.1	21-unit system	98
5.1.2	22-unit system	103
5.2	Modified case studies	107
5.2.1	Modified 21-unit case study	108
5.2.2	Modified 22-unit case study	112
5.3	Experimental Procedure	115
5.4	Results and analysis	116
5.5	Summary and conclusions	163

### CHAPTER 6 HYDROELECTRIC POWER CASE STUDIES 165

6.1 E	Background	165
-------	------------	-----

6.2	Five-Station Hydropower System	166
6.2.1	Problem specification	167
6.2.2	Problem formulation	170
6.2.3	Analysis conducted	172
6.2.4	Results and discussion	177
6.2.5	Summary	199
6.3	Full Hydro Tasmania Maintenance Scheduling Case Study	200
6.3.1	Problem specification	200
6.3.2	Problem formulation	209
6.3.3	Analysis conducted	210
6.3.4	Results and discussion	213
6.4	Summary and conclusions	250

#### CHAPTER 7 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

7.1	Summary & Conclusions	253
7.2	Recommendations for future work	255
7.3	Published and accepted papers	257

#### **CHAPTER 8 REFERENCES**

## **List of Appendices**

Appendix A: ACO-PPMSO source code and sample input files	273
Appendix B: 21-unit case study results	284
Appendix C: 22-unit case study results	313
Appendix D: 5-station hydropower system results	342

253

259

# **List of Figures**

violation of constraints associated with iteration-best schedules and infeasibility ratio during optimisation run).
Figures 5.7(a) & (b): Performance of Max-Min Ant System (MMAS) in solving the original 22-unit case study with and without heuristic (Comparison of the LVL-values associated with iteration-best schedules during optimisation run; Best-known LVL = 52.06 MW)
Figures 5.7(c) & (d): Performance of Max-Min Ant System (MMAS) in solving the original 22-unit case study with and without heuristic (Comparison of the violation of constraints associated with iteration-best schedules and infeasibility ratio during optimisation run)
Figures 5.8(a) & (b): Performance of Elitist-Ant System (EAS) in solving the modified 21-unit case study with and without heuristic (Comparison of the SSR- and total duration shortened values associated with iteration- best schedules during optimisation run; Best-known SSR = 2.62 x 106 MW2 with 5-week deferral)
Figures 5.8(c) & (d): Performance of Elitist-Ant System (EAS) in solving the modified 21-unit case study with and without heuristic (Comparison of the violation of constraints associated with iteration-best schedules and infeasibility ratio during optimisation run)
Figures 5.9(a) & (b): Performance of Max-Min Ant System (MMAS) in solving the modified 21-unit case study with and without heuristic (Comparison of the SSR- and total duration shortened values associated with iteration- best schedules during optimisation run; Best-known SSR = 2.62 x 106 MW2 with 5-week deferral)
Figures 5.9(c) & (d): Performance of Max-Min Ant System (MMAS) in solving the modified 21-unit case study with and without heuristic (Comparison of the violation of constraints associated with iteration-best schedules and infeasibility ratio during optimisation run)
Figures 5.10(a) & (b): Performance of Elitist-Ant System (EAS) in solving the modified 22-unit case study with and without heuristic (Comparison of the LVL- and total duration shortened values associated with iteration- best schedules during optimisation run; Best-known SSR = 101.791 MW with 8-week shortening)
Figures 5.10(c) & (d): Performance of Elitist-Ant System (EAS) in solving the modified 22-unit case study with and without heuristic (Comparison of the violation of constraints associated with iteration-best schedules and infeasibility ratio during optimisation run)
Figures 5.11(a) & (b): Performance of Max-Min Ant System (MMAS) in solving the modified 22-unit case study with and without heuristic (Comparison of the LVL- and total duration shortened values associated with iteration- best schedules during optimisation run; Best-known SSR = 101.791 MW with 8-week shortening)
Figures 5.11(c) & (d): Performance of Max-Min Ant System (MMAS) in solving the modified 22-unit case study with and without heuristic (Comparison of the violation of constraints associated with iteration-best schedules and infeasibility ratio during optimisation run)
Figure 5.12: Infeasible local solutions obtained using <i>PPMSO-2-opt</i> (original 21-unit case study using EAS)
Figure 5.13: Infeasible local solutions obtained using <i>PPMSO-2-opt</i> (original 21-unit case study using MMAS)

Figure 5.14: Infeasible local solutions using <i>PPMSO-2-opt</i> (original 22-unit case study using EAS)
Figure 5.15: Infeasible local solutions using <i>PPMSO-2-opt</i> (original 22-unit case study using MMAS)146
Figure 5.16: Infeasible local solutions using <i>PPMSO-2-opt</i> (modified 21-unit case study using EAS)
Figure 5.17: Infeasible local solutions using <i>PPMSO-2-opt</i> (modified 21-unit case study using MMAS)
Figure 5.18: Infeasible local solutions using <i>PPMSO-2-opt</i> (modified 22-unit case study using EAS)
Figure 5.19: Infeasible local solutions using <i>PPMSO-2-opt</i> (modified 22-unit case study using MMAS)
Figure 5.20: Comparison between results obtained using other optimisation methods (Aldridge <i>et al.</i> , 1999; Dahal <i>et al.</i> , 1999; Dahal <i>et al.</i> , 2000) and the ACO algorithms used in this thesis
Figure 5.21: The (a) maintenance schedule of the 21-unit case study best-found- SSR solution A, the associated (b) personpower allocation and (c) reserve capacity levels
Figure 5.22: The (a) maintenance schedule of the 21-unit case study best-found- SSR solution B, the associated (b) personpower allocation and (c) reserve capacity levels
Figure 5.23: Comparison of reserve levels obtained using ACO, implicit enumeration (Escudero <i>et al.</i> , 1980) and tabu search (El-Amin <i>et al.</i> , 2000)
Figure 5.24: Best-known (a) schedule and (b) the associated generation reserve levels for the 22-unit case study
Figure 5.25: A near best-known (a) schedule and (b) the associated generation reserve levels for the 22-unit case study
Figure 5.26: The (a) maintenance schedule of the modified 21-unit case study best- found-SSR solution A, the associated (b) personpower allocation and (c) reserve capacity levels
Figure 5.27: The (a) maintenance schedule of the modified 21-unit case study best- found-SSR solution B, the associated (b) personpower allocation and (c) reserve capacity levels
Figure 5.28: (a) Maintenance schedule A associated with the best-found OFC for the modified 22-unit case study and (b) the associated generation reserve levels
Figure 5.29: (a) Maintenance schedule B associated with the best-found OFC for the modified 22-unit case study and (b) the associated generation reserve levels
Figure 6.1: Geographical location of Tasmania166
Figure 6.2: Schematic diagram of the five-station hydropower system
Figure 6.3: Forecasted system demand for 2006169
Figure 6.4: Handling of constraints 1 and 2170
Figure 6.5: Dry year storage inflows174
Figure 6.6: Intermediate year storage inflows174
Figure 6.7: Wet year storage inflows174

Figure 6.8: Experimental procedure for the five-station hydropower plant maintenance scheduling optimisation case study	176
Figure 6.9: (a) The best-OFC schedule for wet inflow conditions and (b) the associated unserved energy and spillage conditions	179
Figure 6.10: (a) The best-OFC schedule for intermediate inflow conditions and (b) the associated unserved energy and spillage conditions	180
Figure 6.11: (a) The best-OFC schedule for dry inflow conditions and (b) the associated unserved energy and spillage conditions	181
Figure 6.12: (a) IB-OFC and (b) IB-ETFEIS associated with iteration-best schedules recorded throughout the ACO run that produced the schedule in Figure 6.9 (wet inflow condition)	182
Figure 6.13: (a) IB-OFC, (b) IB-EUE and (c) IB-ETFEIS associated with iteration-best schedules recorded throughout the ACO run that produced the schedule in Figure 6.10 (intermediate inflow condition)	184
Figure 6.14: (a) IB-OFC, (b) IB-EUE and (c) IB-ETFEIS associated with iteration-best schedules recorded throughout the ACO run that produced the schedule in Figure 6.11 (dry inflow condition)	186
Figure 6.15: Schedule obtained by engineering judgment	188
Figure 6.16: The best- <i>OFC</i> schedule for the wet year (EUE = 0 GWh; ETFEIS = 4718 GWh)	190
Figure 6.17: The best-OFC schedule for the intermediate year (EUE = 0 GWh; ETFEIS = 2539 GWh)	190
Figure 6.18: The best-OFC schedule for the dry year (EUE = 0 GWh; ETFEIS = 544 GWh)	191
Figure 6.19: (a) IB-OFC and (b) IB-ETFEIS associated with iteration-best schedules recorded throughout the ACO run that produced the maintenance schedule shown in Figure 6.16 (wet inflow condition)	192
Figure 6.20: (a) IB-OFC, (b) IB-ETFEIS and (c) IB-DurCuttot associated with iteration-best schedules recorded throughout the ACO run that produced the maintenance schedule shown in Figure 6.17 (intermediate inflow condition).	195
Figure 6.21: (a) IB-OFC, (b) IB-EUE, (c) IB-ETFEIS and (d) IB-DurCuttot associated with iteration-best schedules recorded throughout the ACO run that produced the maintenance schedule shown in Figure 6.18 (dry inflow condition)	
Figure 6.22: Forecasted 2006 daily average demand for Hydro Tasmania case study	
Figure 6.23: Historical total system inflows for 2001 ~ 2005 (monthly average)	207
Figure 6.24: Optimisation runs for Hydro Tasmania system	210
Figure 6.25: The ACO schedule-A (plotted)	227
Figure 6.26: The Hydro schedule (plotted)	230
Figure 6.27: Difference between Lake Gordon end-of-month energy in storage (%FSL) associated with ACO and Hydro schedules	237
Figure 6.28: Objective function cost associated with iteration-best schedules (a) Iterations 1 to 100 and (b) Iterations 41 to 100	239
Figure 6.29: Averaged expected unserved energy (IB-EUEavg) associated with iteration-best schedules	240

Figure 6.30: Averaged violation of reliability constraints (IB-ResVioavg) associated with iteration-best schedules	240
Figure 6.31: Averaged total reduction in maintenance duration due to shortening and deferral (IB-DurCutavg) associated with iteration-best schedules	241
Figure 6.32: Averaged expected total final energy in storage (IB-ETFEISavg) associated with iteration-best schedules	241
Figure 6.33: Infeasibility ratio for scenario-A runs	242
Figure 6.34: IB-OFC curve for scenario-B run	243
Figure 6.35: IB-ResVio and IB-ETFEIS for scenario-B run	243
Figure 6.36: IB-OFC for scenario-B run (considering shortening and deferral)	244
Figure 6.37: IB-ResVio and IB-DurCut for scenario-B run (considering shortening and deferral)	245
Figure 6.38: IB-ETFEIS for scenario-B run (considering shortening and deferral)	245
Figure 6.39: IB-ResVio and IB-ETFEIS for scenario-C run	247
Figure 6.40: IB-OFC for scenario-D run	249
Figure 6.41: IB-ResVio and IB-ETFEIS for scenario-D run	249
Figure 6.42: IB-dev for scenario-D run	250

## **List of Tables**

Table 2.1: Summary of optimisation methods for PPMSO
Table 3.1: Pheromone intensity updated on paths 44
Table 3.2: A summary of the distinguishing features of the ACO algorithms      discussed
Table 4.1: Adaptations for Eq. 4.9 in stages 1, 2 and 3 of the selection process
Table 5.1: Details of 21-unit system (Aldridge et al., 1999)
Table 5.2: Details of 22-unit system (Escudero et al., 1980) 104
Table 5.3: Weekly peak load of the 22-unit system (El-Amin et al., 2000) 105
Table 5.4: Personpower utilization for the modified 21-unit case study system
Table 5.5: Details of the modified 22-unit system
Table 5.6: Results for the 21-unit unit problem instance given by Elitist-Ant System (EAS) [deviation from best-known OFC of \$13.66M]
Table 5.7: Results for the 21-unit unit problem instance given by Max-Min Ant
System (MMAS)
[deviation from best-known OFC of \$13.66M]118
Table 5.8: Results for the 22-unit unit problem instance given by Elitist-Ant System (EAS)
[deviation from best-known OFC of \$52.06]
Table 5.9: Results for the 22-unit unit problem instance given by Max-Min Ant System (MMAS)
[deviation from best-known OFC of \$52.06M]119
Table 5.10: Results for the Modified 21-unit unit problem instance given by Elitist- Ant System (EAS) [deviation from best-known OFC of \$15.71M]
Table 5.11: Results for the Modified 21-unit problem instance given by Max-Min AntSystem (MMAS) [deviation from best-known OFC of \$15.71M]
Table 5.12: Results for the Modified 22-unit unit problem instance given by Elitist- Ant System (EAS) [deviation from best-known OFC of \$916.12]
Table 5.13: Results for the Modified 22-unit unit problem instance given by Max-Min Ant System (MMAS) [deviation from best-known OFC of \$916.12]
Table 5.14: Impact of the new heuristic formulation with and without using local search      search
Table 5.15: Impact of <i>PPMSO-2-opt</i> local search operator with and without heuristic 143
Table 5.16: Impact of Duration Extender local search operator with and without heuristic    143
Table 6.1: Power station and headwater data 167
Table 6.2: Details of maintenance tasks
Table 6.3: Results given by ACO-PPMSO for different inflow conditions investigated 177
Table 6.4: Comparison of results obtained by different methods
Table 6.5: Results obtained by different methods 199
Table 6.6: Fixed-date maintenance tasks 201

Table 6.7: Maintenance tasks that need to be scheduled
Table 6.8: Storage levels on 1 Jan 2006 208
Table 6.9: Parameters for optimisation runs 213
Table 6.10: Summary of results for the Full Hydro Tasmania case214
Table 6.11: The schedules proposed by ACO (^) and Hydro Tasmania (^^)215
Table 6.12: Objective values and constraint satisfaction associated with the ACO and the Hydro schedules    234
Table 6.13: Levels of major storages at the end of planning horizon associated with the ACO and Hydro schedules
Table 6.14: Affected maintenance tasks as a result of increased system demand246
Table 6.15: Summary of the utility of two different schedules obtained by ACO-      PPMSO for scenario C
Table 6.16: Revised start dates as a result of the late return of the Tungatinah      station

## **Glossary of Selected Acronyms and Notation**

#### Acronyms

ACO	Ant Colony Optimisation
ACS	Ant Colony System
AS	Ant System
AS <sub>rank</sub>	Rank-Based Ant System
DP	Dynamic programming
DurCut <sub>tot</sub>	total reduction in maintenance duration due to shortening and
	deferral
EAS	Elitist-Ant System
EFEIS	Expected final energy in storage of a lake
EUE	Expected unserved energy
ETFEIS	Expected total final energy in storage of a power system
GA	Genetic algorithm
GNGA	Generational genetic algorithm
GSP	Group-shop scheduling problem
HCF	Hyper-Cube Framework for ACO
IB	Iteration-best
IP	Integer programming
LOLP	Lost of load probability
LP	Linear programming
LVL	the summed deviation of generation reserve from the average
	reserve over the entire planning horizon
MIP	Mixed integer programming
MIP	Mixed integer programming

MMAS	Max-Min Ant System
OFC	Objective function cost
PPMSO RCPSP	Power plant maintenance scheduling optimisation Resource-constrained project scheduling problem
ResVio	violation of reserve constraints
SA	Simulated annealing
SA SMTWTP	Simulated annealing Single-machine total weighted tardiness scheduling problem
	Ũ
SMTWTP	Single-machine total weighted tardiness scheduling problem
SMTWTP SSGA	Single-machine total weighted tardiness scheduling problem Steady state genetic algorithm
SMTWTP SSGA SSR	Single-machine total weighted tardiness scheduling problem Steady state genetic algorithm Sum of squares of the reserve generation capacity in each week

### <u>Notation</u>

lpha and $eta$	relative importance of pheromone intensity and the heuristic,
	respectively
<i>chdur</i> <sub>n</sub>	chosen maintenance duration for task $d_n$
$D = \{d_1, d_2,, d_N\}$	finite set of $N$ maintenance tasks to be scheduled
$\Delta  au_*(t)$	amount of pheromone rewarded to pheromone trail $ au_*$ by the
	end of iteration t
ear <sub>n</sub>	the earliest time for maintenance task $d_n$ to begin
f	objective function which assigns an objective function value
	f(s) to each trial maintenance schedule s
$f_{ews}$	factor of load demand required for reserve
η	heuristic value

$L_t$	anticipated load for period t
$lat_n$	the latest time for maintenance task $d_n$ to end
m	size of an ant population
<i>NormDur<sub>n</sub></i>	normal (default) duration of maintenance task $d_n$
f(s) or $OFC(s)$	objective function cost associated with maintenance schedule $s$
Ω	set of constraints
$P_n$	loss of generating capacity associated with maintenance task
	$d_n$
$p_{best}$	probability that the paths of the current iteration-best-solution
	will be selected, given that non-iteration best-options have a
	pheromone level of $\tau_{min}$ and all iteration-best options have a
	pheromone level of $\tau_{max}$ .
$p_{n,opt}(t)$	probability that decision path opt is chosen for maintenance of
	task $d_n$ in iteration $t$
Ψ	infeasibility ratio
$ResAvai_t^r$	associated capacity of resource of type $r$ available at period $t$
$Res_{n,k}^r$	amount of resource of type $r$ available that is required by task
	$d_n$ at period k
S	set of all maintenance schedules
$S^*$	set of globally optimal maintenance schedules
start <sub>n</sub>	maintenance start time chosen for task $d_n$
<i>S</i> <sub>n</sub>	time step considered for maintenance duration shortening for
	task $d_n$
S	a trial maintenance schedule
$T_{plan}$	the planning horizon
$ au_0$	initial pheromone trail intensity
τ	pheromone trail intensity
$ au_{min,}$ $ au_{max}$	lower and upper limits of pheromone trail
$X_{n,t}$	Binary variables, which can take on values 0 or 1, are used to
	represent the state of a maintenance task in a given time period
	t