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DIGITAL COMPUTER STUDY  
OF  
A BRUSHLESS EXCITED  
SYNCHRONOUS MACHINE

BY  
DHARMENDRA KUMAR SHARMA

A Thesis

Submitted to the Faculty of Graduate Studies Through the  
Department of Electrical Engineering in Partial Fulfillment  
of the Requirements for the Degree of  
Master of Applied Science at  
University of Windsor

Windsor, Ontario, Canada.

1968

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Approved

*A. Raouf*

Prof. A. Raouf

*P.A.V. Thomas*

Dr. P.A.V. Thomas

*O.P. Malik*

Dr. O.P. Malik (Chairman)

210100



ABSTRACT

A mathematical model has been developed for the simulation of a synchronous machine with brushless excitation. The model also includes the simulation of Turbine - Governor connected to the synchronous machine. The model consists of a set of simultaneous non - linear first order differential equations in the generalised form suitable for the study of transient and steady state operation of a synchronous machine.

Using the mathematical model, the behaviour of a synchronous machine with brushless excitation, during asynchronous operation and resynchronization has been studied on a digital computer. Necessary digital programs for the study have been developed.

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LIST OF SYMBOLS

Most commonly used symbols are defined below. Any symbol not defined below, will be explained where used in the text. Unless otherwise stated, all quantities are expressed in per - unit.

Main Symbols :

$p$	Operator $d/dt$ or no. of phases in the rectifier.
$p^2$	Operator $d^2/dt^2$ .
$e$	Instantaneous value of voltage.
$r$	Resistance.
$L$	Total self inductance.
$l$	Leakage inductance.
$\omega$	Instantaneous Angular Velocity of rotor in radian per second.
$i$	Instantaneous value of current.
$\delta$	Load Angle in radians, of synchronous machine.
$E$	Constant or Phasor value of voltage.
$F$	Function.
$V_{se}$	Effective value of a.c. voltage input to rectifier.
$t$	Time in seconds.
$T$	Single Time Constant of the Exciter - Alternator.
$T_s$	Synchronous Torque.
$T_{as}$	Asynchronous Torque.

h	Step Size.
S	Step input to a.c. exciter field
K	A constant in the Transfer function for the Exciter - Alternator.
P	Power.
$T_g$	Governor Mechanism Time Constant.
$T_t$	Turbine Time Constant.
s	Slip in Radians per Second.
TP	Steady Power that the turbine is set to deliver.
$f_t$	Instantaneous applied torque.
$\omega$	Synchronous Speed in Radians per Second.
H	Machine Inertia Constant in KW sec/KVA.
$\theta$	Defined on Page 11.
$\Psi$	Flux linkage.

Subscripts :

d	Direct Axis
q	Quadrature Axis
f	Field winding
k	Damper winding
m	Mechanical or Maximum
md	Direct Axis mutual
mq	Quadrature Axis mutual
a	Armature

## CHAPTER I

### INTRODUCTION

#### 1.1 General.

The conventional method of exciting a synchronous machine is through a D.C. Exciter mounted on the same shaft as the synchronous machine.

In recent years, however, a major step<sup>1,2</sup> has been taken by developing brushless excitation system whereby the need of the collector, the collector brushes, the commutator and the commutator brushes has been eliminated. This system consists of an a.c. exciter with a rotating rectifier mounted on the same shaft as the synchronous machine. The output from the rotating armature of the a.c. exciter is fed along the hollow shaft to the rectifier whose output is fed to the D.C. field winding of the synchronous machine.

Selenium Rectifiers were first tried for the purpose but their use was not successful<sup>1,2</sup>. Silicon Diode Rectifiers capable of operating at relatively high temperature were then tried and they have been found to be satisfactory. So far the brushless excitation system has mainly been applied to aircraft generators and salient pole machines up to about 20 MVA<sup>3</sup>. The system has been proved<sup>2</sup> to offer the advantages of

1. improved reliability 2. less maintenance and 3. improved performance.

### 1.2 Present Work :

The behaviour of a synchronous machine, having conventional excitation, during asynchronous operation and the process of resynchronization has previously been studied<sup>4</sup>, using a general-purpose analog computer.

In this thesis, the behaviour of the synchronous machine, with brushless excitation, during asynchronous operation and the process of resynchronization has been studied using digital computer. The digital simulation program developed also takes into account the speed Governor-Turbine characteristics.

This is believed to be the first time that a digital program has been developed to study the asynchronous operation and resynchronization of a synchronous machine with brushless excitation system.

#### 1.2.1 Assumptions :

To study the behaviour of the synchronous machine during asynchronous operation and resynchronization, most authors have adopted one or more of the following simplifying assumptions<sup>4</sup>:

- a) The voltage behind the transient reactance is constant.
- b) The variation in speed during the transient process is small in comparison with the steady-state speed.
- c) The effect of all such terms as  $p\psi_d$ ,  $p\psi_q$  etc. are

neglected.

d) The stator resistance of the synchronous machine is negligible.

The validity of these assumptions, particularly under asynchronous conditions, is questionable, as the machine speed can alter considerably from synchronous speed.

In the present thesis, the simulation of the synchronous machine is by means of the two - reaction theory of synchronous machines as put forward by Park<sup>5</sup> and presented in a more comprehensive form by Adkins<sup>6</sup>. The digital simulation of the synchronous machine, as done in the thesis, makes no further assumptions except those inherent in the two-reaction theory itself. These assumptions are :

- a) The machine is ideal, as defined by Park<sup>7</sup>.
- b) Symmetrical conditions exist in the system, i.e., zero-sequence quantities are equal to zero.
- c) Damping circuits can be represented by a single coil on each axis.

### 1.2.2 Sign Convention<sup>6</sup> :

The voltage impressed on the coil from an external source is taken as positive.

The positive current is the one measured in the same direction as positive voltage.



With the above convention, positive power flows into the circuit from outside, if both voltage and current are positive.

The convention adopted above corresponds directly to motor operation and introduces negative quantities for generator operation.

### 1.2.3 Per - Unit System :

The per-unit system adopted in this thesis is  
as defined by Adkins <sup>6</sup>.

The advantage of per-unit system is that it becomes possible to compare machines of different physical dimensions. Number of turns do not enter the General Machine Equations. Further, the three mutual inductances on the direct axis can be assumed to be equal, which makes the General Machine Equations much easier to deal with.

CHAPTER II

THEORY

2.1 Synchronous Machine :

The general machine equations of a synchronous machine are :

$$\begin{bmatrix} e_f \\ e_{kd} \\ e_{kq} \\ e_d \\ e_q \end{bmatrix} = \begin{bmatrix} r_f + (L_{md} + l_f)p & L_{md}p & 0 & L_{md}p \\ L_{md}p & r_{kd} + (L_{md} + l_{kd})p & 0 & L_{md}p \\ 0 & 0 & r_{kq} + (L_{mq} + l_{kq})p & 0 \\ L_{md}p & L_{md}p & L_{mq}v & r_a + (L_{md} + l_a)p \\ -L_{md}v & -L_{md}v & L_{mq}p & -(L_{md} + l_a)v \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ L_{mq}p \\ (L_{mq} + l_a)v \\ r_a + (L_{mq} + l_a)p \end{bmatrix} + \begin{bmatrix} i_f \\ i_{kd} \\ i_{kq} \\ i_d \\ i_q \end{bmatrix} \quad \dots ( 2.1 )$$

Eq. 2.1 can be rearranged as

$$\begin{bmatrix} e_f \\ e_{kd} \\ e_{kq} \\ e_d \\ e_q \end{bmatrix} = \begin{bmatrix} r_f & 0 & 0 & 0 & 0 \\ 0 & r_{kd} & 0 & 0 & 0 \\ 0 & 0 & r_{kq} & 0 & 0 \\ 0 & 0 & L_{mq} \nu & r_a & (L_{mq} + l_a) \nu \\ -L_{md} \nu & -L_{md} \nu & 0 & -(L_{md} + l_a) \nu & r_a \end{bmatrix} \begin{bmatrix} i_f \\ i_{kd} \\ i_{kq} \\ i_d \\ i_q \end{bmatrix}$$

$$+ \begin{bmatrix} (L_{md} + l_f) & L_{md} & 0 & L_{md} & 0 \\ L_{md} & (L_{md} + l_{kd}) & 0 & L_{md} & 0 \\ 0 & 0 & (L_{mq} + l_{kq}) & 0 & L_{mq} \\ L_{md} & L_{md} & 0 & (L_{md} + l_a) & 0 \\ 0 & 0 & L_{mq} & 0 & (L_{mq} + l_a) \end{bmatrix} \begin{bmatrix} p i_f \\ p i_{kd} \\ p i_{kq} \\ p i_d \\ p i_q \end{bmatrix}$$

In symbolic notation, the above can be written as

$$[V] = [R] [I] + [Z] [pI] \quad \dots \quad (2.2)$$

where

$$[V] = \begin{bmatrix} e_f \\ e_{kd} \\ e_{kq} \\ e_d \\ e_q \end{bmatrix}$$

$$[I] = \begin{bmatrix} i_f \\ i_{kd} \\ i_{kq} \\ i_d \\ i_q \end{bmatrix}$$

$$[R] = \begin{bmatrix} r_f & 0 & 0 & 0 & 0 \\ 0 & r_{kd} & 0 & 0 & 0 \\ 0 & 0 & r_{kq} & 0 & 0 \\ 0 & 0 & L_{mq}^v & r_a & (L_{mq}+l_a)^v \\ -L_{md}^v & -L_{md}^v & 0 & -(L_{md}+l_a)^v & r_a \end{bmatrix}$$

$$[Z] = \begin{bmatrix} (L_{md}+l_f) & L_{md} & 0 & L_{md} & 0 \\ L_{md} & (L_{md}+l_{kd}) & 0 & L_{md} & 0 \\ 0 & 0 & (L_{mq}+l_{kq}) & 0 & L_{mq} \\ L_{md} & L_{md} & 0 & (L_{md}+l_a) & 0 \\ 0 & 0 & L_{mq} & 0 & (L_{mq}+l_a) \end{bmatrix}$$

$$[pI] = \begin{bmatrix} p i_f \\ p i_{kd} \\ p i_{kq} \\ p i_d \\ p i_q \end{bmatrix}$$

Re-arranging the terms, Eqn. 2.2 can be written as

$$[Z] [pI] = [V] - [R] [I]$$

$$\text{or } [pI] = [Z]^{-1} [V] - [Z]^{-1} [R] [I] \quad \dots (2.3)$$

The terms of  $[Z]^{-1}$  matrix are denoted by  $a_{ij}$  and the terms of  $[Z]^{-1} [R]$  matrix by  $b_{ij}$ , so that Eqn. 2.3 becomes

$$[pI] = [a_{ij}] [V] - [b_{ij}] [I]$$

The actual equation is

$$\begin{bmatrix} pI_f \\ pI_{kd} \\ pI_{kq} \\ pI_d \\ pI_q \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} e_f \\ e_{kd} \\ e_{kq} \\ e_d \\ e_q \end{bmatrix} - \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} \end{bmatrix} \begin{bmatrix} I_f \\ I_{kd} \\ I_{kq} \\ I_d \\ I_q \end{bmatrix}$$

Assuming  $e_{kd} = e_{kq} = 0$  ( Sect. 1.2.1 ),  
the following five first order differential equations are  
obtained :

$$\begin{aligned}
 p i_f &= a_{11} e_f + a_{14} e_d + a_{15} e_q - ( b_{11} i_f + b_{12} i_{kd} + b_{13} i_{kq} \\
 &\quad + b_{14} i_d + b_{15} i_q ) \\
 p i_{kd} &= a_{21} e_f + a_{24} e_d + a_{25} e_q - ( b_{21} i_f + b_{22} i_{kd} + b_{23} i_{kq} \\
 &\quad + b_{24} i_d + b_{25} i_q ) \\
 p i_{kq} &= a_{31} e_f + a_{34} e_d + a_{35} e_q - ( b_{31} i_f + b_{32} i_{kd} + b_{33} i_{kq} \\
 &\quad + b_{34} i_d + b_{35} i_q ) \quad .(2.4) \\
 p i_d &= a_{41} e_f + a_{44} e_d + a_{45} e_q - ( b_{41} i_f + b_{42} i_{kd} + b_{43} i_{kq} \\
 &\quad + b_{44} i_d + b_{45} i_q ) \\
 p i_q &= a_{51} e_f + a_{54} e_d + a_{55} e_q - ( b_{51} i_f + b_{52} i_{kd} + b_{53} i_{kq} \\
 &\quad + b_{54} i_d + b_{55} i_q )
 \end{aligned}$$

In the thesis, the synchronous machine will be  
considered connected directly to an infinite bus. For this  
case, the following two equations can be written<sup>6</sup>.

$$\begin{aligned}
 e_d &= E_m \sin \delta \\
 e_q &= E_m \cos \delta \quad \dots\dots ( 2.5 )
 \end{aligned}$$

Eqns. 2.4 together with Eqns. 2.5 are the basic  
equations used to study the asynchronous operation and  
resynchronization of the synchronous machine.

## 2.2 Exciter-Alternator and Rectifier :

The rectified voltage of a rectifier may be represented<sup>8</sup> by the Fourier Series

$$\begin{aligned}
 F(\theta) = & A_0 + A_1 \cos p\theta + A_2 \cos 2p\theta + A_3 \cos 3p\theta + \dots \\
 & + A_m \cos mp\theta + \dots + B_1 \sin p\theta + B_2 \sin 2p\theta \\
 & + B_3 \sin 3p\theta + \dots + B_m \sin mp\theta \quad \dots \quad ( 2.6 )
 \end{aligned}$$

$A_0$  represents the mean output steady D.C. voltage  $V_d$  of the rectifier.

It can be shown<sup>8</sup> that

$$\begin{aligned}
 A_n &= 2V_d / ( n^2 - 1 ) \\
 \& \quad B_n &= 0
 \end{aligned}$$

where  $n = mp$ ,  $m$  is an integer and  $p$  is the number of phases in the rectifier. The coefficient  $A_n$  thus gives the amplitude of the harmonic of frequency  $n = mp$ .  $A_n$  has a positive or negative sign depending upon the instant of time in the process of rectification.

The output voltage ripple contains only harmonics having frequencies which are multiples of both the supply frequency and the number of rectifier phases.

It can be shown<sup>8</sup> that

$$A_0 = V_{se} / \sqrt{2} ( p / \pi ) \sin ( \pi / p )$$

A half wave 3 phase rectifier is taken in the present studies. For this case, the following can be derived.

$$V_d = A_0 = 1.17 V_{se}$$

Since three phase rectification is being considered, the harmonics present in the D.C. output will be 3rd and its multiples. Therefore,

$$A_3 = 0.25 V_d$$

$$A_6 = 0.0571 V_d$$

$$A_9 = 0.025 V_d \quad \text{and so on.}$$

Harmonics upto the 9th only have been considered and higher harmonics are neglected since their amplitude is very small.

In view of the above, Eqn. 2.6 can be written as:

$$\begin{aligned} F(\theta) &= V_d + 0.25V_d \cos 3\theta + 0.0571V_d \cos 6\theta + 0.025V_d \cos 9\theta \\ &= V_d ( 1 + 0.25 \cos 3\theta + 0.0571 \cos 6\theta + 0.025 \cos 9\theta ) \end{aligned}$$

Assuming that the a.c. supply to the rectifier is at a constant frequency of 50 cycles per second,

$$\theta = \omega t = 314t. \quad \text{Therefore,}$$

$$\begin{aligned} F(\theta) &= V_d ( 1 + 0.25 \cos 942t + 0.0571 \cos 1884t + 0.025 \cos 2826t ) \\ &= 1.17V_{se} ( 1 + 0.25 \cos 942t + 0.0571 \cos 1884t + 0.025 \cos 2826t ) \end{aligned}$$



Assuming for the exciter-alternator a single time constant transfer function of  $( K/1+\tau p )$ ,

$$V_{se} = \frac{K}{1 + \tau p} ( S )$$

where S is the step input to the exciter-alternator. Therefore,

$$F(\theta) = \frac{1.17(K)(S)( 1+0.25\cos 942t+0.0571\cos 1884t+0.025\cos 2826t )}{1 + \tau p}$$

Now,  $F(\theta) = e_f$ , since  $e_f$  is the voltage applied to the field of the main alternator. Therefore,

$$e_f = \frac{1.17(K)(S)( 1+0.25\cos 942t+0.0571\cos 1884t+0.025\cos 2826t )}{1 + \tau p}$$

The above expression, after simplification, can be written as

$$pe_f = -\frac{1}{\tau} e_f + \frac{1.17(K)(S)(1+0.25\cos 942t+0.0571\cos 1884t+0.025\cos 2826t)}{\tau} \dots\dots\dots ( 2.7 )$$

Eqn. 2.7 is a differential equation of the first order, the solution of which gives the voltage applied to the field of the main alternator, from the exciter-alternator and rectifier combination.

### 2.3 Turbine - Governor :

The Turbine - Governor can be represented<sup>9</sup> by

$$P_m = \frac{F(s)}{(1 + T_g p)(1 + T_t p)} \quad \dots\dots\dots (2.8)$$

where  $F(s)$  is the steady state relation between slip  $s$  and  $P_m$ .

$F(s)$  can be written as

$$F(s) = \frac{s}{314} + TP \quad \text{Therefore,}$$

$$P_m = \frac{\frac{s}{314} + TP}{(1 + T_g p)(1 + T_t p)}$$

The above, after rearrangement, becomes

$$T_g T_t p^2 P_m + (T_g + T_t) p P_m + P_m = (s/314) + TP$$

Putting  $p P_m = P_m'$  and then simplifying,

$$p P_m' = - \frac{(T_g + T_t)}{T_g T_t} P_m' - \frac{P_m}{T_g T_t} + \frac{s}{314} + TP \quad \dots (2.9)$$

$$\text{Where } p P_m = P_m' \quad \dots\dots\dots (2.10)$$

Further, the relation between  $P_m$  and the instantaneous applied torque  $f_t$  can be derived as<sup>6</sup>,

$$P_m = (v/\omega) f_t$$

$$\text{or } f_t = \frac{\omega}{\nu} P_m \dots\dots\dots ( 2.11 )$$

The solution of the second order differential equation represented by Eqns. 2.9 and 2.10 gives  $P_m$ . Eqn. 2.11 gives a relation between  $f_t$  and  $P_m$ .

2.4 Synchronous Machine -

Equations for Slip and Load Angle :

The torque developed by the synchronous machine differs from the externally applied torque, if the speed varies, because of the inertia of the machine. In other words, Input torque = Output torque + Accelerating torque.

On the above basis, the following relation can be derived<sup>6</sup>.

$$f_t = \frac{\omega}{2} (\psi_d i_q - \psi_q i_d) - \frac{2H}{\omega} p^2 \delta$$

where  $\nu = \omega - p \delta$

Now,  $\psi_d = L_{md} i_f + L_{md} i_{kd} + (L_{md} + l_a) i_d$

&  $\psi_q = L_{mq} i_{kq} + (L_{mq} + l_a) i_q$

Substituting the values of  $\psi_d$  and  $\psi_q$  in the expression for  $f_t$ , and simplifying,

$$f_t = \frac{\omega}{2} ( L_{md} i_q i_f + L_{md} i_{kd} i_q + L_{md} i_d i_q - L_{mq} i_d i_{kq} - L_{mq} i_d i_q ) - \frac{2H}{\omega} p^2 \delta$$

Therefore,

$$p^2 \delta = \frac{\omega^2}{4H} ( L_{md} i_q i_f + L_{md} i_{kd} i_q + L_{md} i_d i_q - L_{mq} i_d i_{kq} - L_{mq} i_d i_q ) - \frac{\omega}{2H} ( \frac{\omega}{\nu} P_m ) \dots ( 2.12 )$$

Eqn. 2.12 is a second order differential equation, the solution of which gives slip and load angle.

### 2.5 Equations for Watts, Vars and Line Current :

The following equations were used for calculating Watts, Vars of the synchronous machine and the Line Current.

$$\text{Watts} = - \frac{\omega}{2} ( \Psi_d i_q - \Psi_q i_d ) \dots ( 2.13 )$$

$$\text{Vars} = \frac{\omega}{2} ( \Psi_d i_d + \Psi_q i_q ) \dots ( 2.14 )$$

where  $\Psi_d$  and  $\Psi_q$  are as defined in Sect. 2.4.

$$\text{Line Current} = \sqrt{\frac{i_d^2 + i_q^2}{2}} \dots ( 2.15 )$$

2.6 Mathematical Model of  
the Brushless Excited Synchronous Machine :

Eqns. 2.4, 2.5, 2.7, 2.9, 2.10 and 2.12 set up for simultaneous solution on a digital computer form a mathematical model for the study of asynchronous operation and resynchronization of a synchronous machine. It may be pointed out that these differential equations are non-linear, since the instantaneous speed  $\nu$  is not constant.

## CHAPTER III

### DIGITAL COMPUTER SIMULATION

#### 3.1 Choice of the computer technique :

The Computer Program developed has to find out the solution of the simultaneous non-linear differential equations mentioned in Sect. 2.6. The solution of differential equations on a digital computer is a field in which considerable research is still underway and a single, best method applicable to all types of differential equation problems has yet to be discovered. Two of the most commonly used techniques are the Runge Kutta and the predictor - corrector methods. Both the categories can further be subdivided depending upon the type of approximation algorithm used.

The Runge Kutta Methods were used in the present studies, for the following reasons<sup>10</sup>:

- 1) No special starting procedure is required, as the methods are self-starting.
- 2) Being self-starting, they permit an easy change in the step size.
- 3) A straight-forward computational procedure is repeated throughout the calculation.

4) No modification of the computation is necessary for non-linear equations or for systems of simultaneous equations.

### 3.2 Runge Kutta Methods used and Problems Encountered :

There are various Runge Kutta Methods available for the solution of differential equations. The methods initially considered for programming were<sup>10</sup>,

- |                              |                        |
|------------------------------|------------------------|
| 1) Basic Runge Kutta Method, | 2) Strachey's Process, |
| 3) Boulton's Process and     | 4) Gill's Process.     |

The Gill's method was chosen from the above for programming due to the following reasons<sup>10</sup> :

1) The method is generally more accurate than other methods described above.

2) The method results in the economy of computer storage space. Only three quantities have to be retained at any stage as against five for Basic Runge Kutta Method and four for the other two methods mentioned above.

The Gill's method was successfully programmed for use in the present studies.

For the present work, it is considered sufficient if the results are accurate upto the third decimal place.

With this in mind, and also to provide a further check on the

results, it was decided, at a later stage, to try to develop another program for the solution of the non-linear differential equations. The Basic Runge Kutta Method was chosen this time. The program was successfully developed and it was found that for the accuracy desired, the results agree with those obtained by the Gill's method. The agreement of the results by two different methods proves the results to be completely reliable.

In view of the above, it appears that, for the accuracy of results desired, Basic Runge Kutta Method is good enough for the present studies. However, Gill's method still has the edge since, if for some reason, the program has to be expanded and consequently there is shortage of computer storage space, Gill's method could provide the answer.

A brief description of the Basic Runge Kutta Method and Gill's method appears in Appendix I. A flow diagram of the digital programs is given in Appendix III.

The major problem faced, during programming, was solution instability. As is only too well known, this can be a perplexing problem in the numerical solution of differential equations.

The two major sources of error during numerical integration are 1) truncation error and 2) rounding error. Under unfavourable conditions, errors ( which are small initially ) may become magnified as the solution is carried out



for larger and larger values of the independent variable. Thus, divergence in solution will result.

According to McCracken and Dorn<sup>11</sup>, the errors can be controlled by a proper size of the integration step. However, it should be realized that a smaller integration step increases computation time. An optimum value of the integration step has, therefore, to be found such that both the error and the computation time are minimum. In most cases, it is difficult to determine the largest acceptable size of the integration step  $h$  theoretically, and a solution is considered reliable only when it agrees with that obtained at a smaller integration step<sup>10</sup>. This is the test of validity of results obtained during the present studies. Results are compared at intervals  $h$  and  $h/2$ . Only if they agree with each other to the pre-selected degree of accuracy, are the results accepted. A further proof of the validity of the results is that results obtained from two different Runge Kutta methods give the same results, within the prescribed degree of accuracy.

The feature of checking the results automatically with  $h$  and  $h/2$  has not been introduced in the computer program. This was done due to the following reason. The entire program was run with step  $h$  and results obtained. The program was then rerun with step  $h/2$ . The results compared well within the prescribed accuracy. The results were then compared with the results from the other Runge Kutta program and these checked

well. Computation time for each study was already running into many hours. This time would have been doubled if automatic checking with  $h/2$  had been introduced in the program. This was not considered necessary in view of the points mentioned above. All further studies were therefore, performed with step  $h$  and one Runge Kutta program only.

During the digital runs, it was found that the Basic Runge Kutta method took slightly less time than the Gill's method. That this can happen, in view of the nature of multiplications in the Gill's Process, is supported by Martin<sup>10</sup>. All further studies were, therefore, done by the Basic Runge Kutta method since the desired accuracy in results was being obtained in lesser computation time.

## CHAPTER IV

### DIGITAL SIMULATION RESULTS

#### 4.1 General :

During asynchronous operation, e.m.f. will be induced in the synchronous machine field circuit. Depending upon the design of the machine and magnitude of slip during asynchronous operation, these voltages could be quite high, and the rectifier may be damaged. Short circuiting devices are available which provide complete protection for the rectifier under these overvoltage transients<sup>12</sup>. This study assumes that the rectifier is short circuited during asynchronous operation.

The power and electrical torque developed in a synchronous machine, during asynchronous operation, depend not only on the load angle, but also on the time rate of change of load angle. Torque 'T' can then, as an approximation, be considered to be made up of a synchronous component and an asynchronous component.

$$T = T_s + T_{as}$$

The synchronous component will be present only when field excitation is present during asynchronous operation.

#### 4.2 Studies performed :

To study the effect of various parameters on machine behaviour ( the machine data is given in Appendix II ), during asynchronous operation and resynchronization, the following studies were performed :

Study 1) Field Resistance = 0.00446 p.u. No excitation during asynchronous operation, i.e., a.c. exciter field shorted on itself. Turbine output = 0.75 p.u.

Study 2) Field Resistance = 0.00089 p.u. No excitation during asynchronous operation. Turbine output = 0.75 p.u.

Study 3) Field Resistance = 0.00446 p.u. No excitation during asynchronous operation. Turbine-Governor output = 0.75 p.u.

A.C. Exciter-Rectifier combination is not included in this study, i.e., the machine has conventional excitation ( Sect. 1.1 ).

Study 4) Field Resistance = 0.00446 p.u. No excitation during asynchronous operation. Turbine-Governor output = 1.0 p.u.

Study 5) Field Resistance = 0.00089 p.u. No excitation during asynchronous operation. Turbine-Governor output = 1.0 p.u.

Study 6) Field Resistance = 0.00446 p.u. Field excited during asynchronous operation. Turbine-Governor output = 1.0 p.u.

#### 4.3 Pulsation in various quantities

##### during asynchronous operation :

Pulsations in various quantities for the studies

performed above are given in Figs. 4.1 to 4.24. It will be observed from these figures that, during operation with the a.c. exciter field excited, the quantities fluctuate much more violently than with field short circuited and unexcited.

#### 4.3.1 Slip Pulsations :

Figs. 4.1, 4.5, 4.9, 4.13, 4.17 and 4.21 give the variation of slip with respect to load angle, for the various studies performed. Figs. 4.2, 4.6, 4.10, 4.14, 4.18 and 4.22 give slip variation with respect to time.

From the figures mentioned above, it can be seen that in most of the cases studied, the machine went through brief intervals of synchronous motion during each slip cycle. At these intervals, the machine tended to lock, but after a brief interval of synchronous motion, went again into the next slip cycle.

From the graphs, it is found that the magnitude of slip pulsation is inversely proportional to the mean slip. Thus, at a mean slip of  $-0.765\%$ , the variation is between  $-1.57\%$  and  $0.048\%$ , Fig. 4.1, and at a mean slip of  $-0.605\%$ , the variation is between  $-1.9\%$  and  $0.706\%$ , Fig. 4.5.

The effect of reduction in field resistance, for the same p.u. power, is to increase the magnitude of slip pulsations and decrease mean slip.

This can be verified from Figs. 4.1 and 4.5 and also from Figs. 4.13 and 4.17. In the first instance, as the field resistance was changed from 0.00446 p.u. to 0.00089 p.u., the magnitude of slip pulsations changed from -1.57 % to -1.9 % and from 0.048 % to 0.706 %. The mean slip dropped from -0.765 % to -0.605 %. In the second instance, with the same change in the field resistance, the magnitude of slip pulsations changed from -2.04 % to -2.6 % and from -0.045 % to 0.65 %. The mean slip dropped from -1.05 % to -0.975 %. The increase in slip pulsations is due to the fact that with reduction in field resistance, the asynchronous torque component produced by the shorted field circuit is increased with a resultant decrease in mean slip. Due to the asymmetry of the rotor field circuit, this torque is not constant but pulsating. Thus, the higher proportion of pulsating torque increases the magnitude of slip pulsations.

A comparison of Figs. 4.1 and 4.9 shows that, with Exciter-Rectifier combination in or out, the slip pulsations are practically the same. Thus, slip pulsations are practically unchanged, whether there is conventional or brushless excitation.

It is also seen from the studies performed that mean slip is higher when turbine power is increased to meet the increased load demand of the machine. This has to be so from the torque-slip characteristics of an induction machine,

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as increased load demand can be supplied only by an increase in mean slip.

#### 4.3.2 Pulsation in Current :

Figs. 4.3, 4.7, 4.11, 4.15, 4.19 and 4.23 show the fluctuations in field current and line current under various modes of operation.

With reduction in field resistance, the amplitude of field current increases. This can be seen by comparison of Fig. 4.3 with Fig. 4.7 and of Fig. 4.15 with Fig. 4.19. With field excited, the amplitude is further increased and pulsations are at slip frequency. With field unexcited, they are at twice the slip frequency.

Due to the large reactive-power demand of the machine, the line current during asynchronous operation is high. The pulsations are more violent in the field excited case than the unexcited one. The magnitude of line current is also directly dependent on the power output.

#### 4.3.3 Pulsations in Watts and Vars :

Figs. 4.4, 4.8, 4.12, 4.16, 4.20 and 4.24 show the fluctuations in watts and vars during various conditions of asynchronous operation.

Pulsations in both the quantities follow the same pattern as that of slip. The magnitude of variation depends directly on the output of machine and slip. The field excited case produces more violent fluctuations than the unexcited one.

#### 4.4 Comparison of asynchronous operation with different field excitations :

In the present work, only two modes of field connections are considered :

- i. Field excited
- ii. Field unexcited and shorted.

In the case of field excited synchronous machine, the following points are observed when compared to the field unexcited case :

- a) The pulsations in various quantities are at slip frequency and are more violent than the unexcited case. Thus, there are greater chances of the entire system getting disturbed.
- b) For the same turbine power, the slip pulsations are higher. This is because magnitude of pulsation depends directly on the value of the synchronous component of the torque.
- c) Current and Var demands are also greater.



#### 4.5 Effect of Rectifier-Exciter combination on Asynchronous operation :

A comparison of study 1 and 3, Sect. 4.2, shows that pulsations in various quantities during asynchronous operation is the same whether brushless excitation or conventional excitation is there.

#### 4.6 Resynchronization :

With the synchronous machine operating in a steady asynchronous state, different values of excitation voltages were applied to the a.c. exciter field to find the minimum excitation required for resynchronization for a certain run. After obtaining the minimum magnitude of excitation required, the instant of voltage application was delayed so as to obtain the maximum value of the load angle at which the excitation must be applied or boosted to achieve synchronism.

Table 4.1 gives the values of minimum excitation for resynchronization and farthest load angle at which time the machine would still resynchronize, for the various studies performed. Column 4 of Table 4.1 gives the excitation voltage applied to the a.c. exciter field terminals. Column 5 gives the corresponding voltage that appears at the main machine field

Table 4.1

Study No.	Field Resistance p.u.	$P_m$ p.u.	Mini. Excit. (p.u.) at a.c. exciter terminals	Mini. Excit. (p.u.) at main machine terminals	Optimum load angle range in degrees	Mean slip %
1	0.00446	0.75	0.7	* 0.89	320 - 338	-0.76%
2	0.00089	0.75	0.8	1.00	315 - 330	-0.60%
3	0.00446	0.75	*	0.85	70 - 73	-0.76%
4	0.00446	1.00	1.2	1.46	303 - 320	-1.05%
5	0.00089	1.00	1.35	1.65	302 - 315	-0.97%
6	0.00446	1.00	1.10	1.35	72 - 88	-0.51%

\* Excitation is applied directly to Synchronous Machine field terminals. In all other cases, excitation is applied to a.c. exciter field terminals.

terminals. It takes some time for the voltage at machine terminals to build up to this final figure, due to the time constant of the a.c. exciter ( Sect. 2.2.). The table also gives the value of mean slip for each study. As mentioned earlier, study 3 ( Sect. 4.2 ) has conventional excitation ( Sect. 1.1 ) while all other studies have brushless excitation. Further, study 6 has field excited and the magnitude of excitation of the a.c. exciter during asynchronous operation is 0.9 p.u. All other studies have a.c. exciter field unexcited. It was possible to obtain only a range for the value of farthest load angle. This was because the digital computer prints output only at preselected machine time intervals. Thus, resynchronization can be performed, during digital study, only at the load angle values printed out.

#### 4.6.1 Effect of Field Resistance :

When the machine field resistance  $r_f$  is varied, the field time constant is changed. Due to the field time constant, the field current does not attain its full value immediately at the instant of switching. For a machine with larger field time constant, it will take more time to reach the full magnitude of field current. A larger time constant field would thus require the excitation to be switched in much earlier than the field with a smaller time constant. This can also be put in an alternative form, that, for the same instant of switching

field excitation required will be more for a larger field time constant machine than for a machine with smaller field time constant.

The above has been shown to be the case if study 1 and 2 are compared ( Table 4.1 ). Instant of switching of excitation is practically the same in both the studies. It is found that study 2, which has a larger time constant, requires 0.8 p.u. of excitation while study 1 required only 0.7 p.u. The same conclusions are obtained when study 4 and 5 are compared ( Table 4.1 ).

#### 4.6.2 Effect of A.C. Exciter-Rectifier Combination :

Study 1 and 3 are identical except that study 1 has brushless excitation while study 3 has conventional excitation. From the digital runs, it was found that when the machine is running with conventional excitation, the minimum excitation required to resynchronize is less than that for machine with brushless excitation. Referring to column 5 of Table 4.1, minimum excitation for study 1 ( with brushless excitation ) is 0.89 while it is 0.85 for study 3 ( with conventional excitation ). This happens because it takes some time for the voltage at the machine terminals to build up in the brushless excited case, due to the a.c. exciter time constant. There is no such delay when the machine is conventionally excited.

For the same reason, the farthest load angle is more in study 3. The difference is quite significant in this case.

#### 4.6.3 Mode of Field Connection :

In this thesis, two modes of field connection during asynchronous operation have been studied. i. Field shorted and unexcited ii. Field excited. Effects of these modes of operation on resynchronization are now discussed.

For this purpose, studies 4 and 6 are compared. For study 6, which has field excited during asynchronous operation, it is found that the minimum value of excitation required for resynchronization is less than that for the field unexcited case. This happens because the time required for the field to be fully effective is less in the excited case. Therefore, a lesser excitation suffices to bring the machine in synchronism. It should be noted that, in study 6, the field excitation during asynchronous operation is quite high. In fact, it is more than 80 % of the value required for resynchronization. Also, due to the reasons mentioned above, the optimum angle of resynchronization is significantly more in the field excited case.

#### 4.6.4 Effect of Power Output :

As the power output is increased, the magnitude of mean slip to produce the required induction torque is also increased. Thus, there is an increase in the additional stored kinetic energy in the rotor. The additional energy must be pumped out to establish synchronous conditions. This means that more excitation would be needed to bring the machine back into synchronism as the power output is increased. This is indeed the case as can be seen from the figures for studies 1 and 4 as well as 2 and 5 in Table 4.1.

## CHAPTER V

### CONCLUSIONS

#### 5.1 A review of results :

Asynchronous operation and resynchronization of synchronous machines has been studied by many investigators. Most of these authors have adopted one or more simplifying assumptions ( Sect. 1.2.1 ). These assumptions are not valid for modes of operation of the synchronous machine being studied. In the present work, no additional assumptions are made, except those inherent in the two - reaction theory itself<sup>5,6</sup>.

A mathematical model has been developed to study the behaviour during asynchronous operation and resynchronization, of a synchronous machine with brushless excitation system. Representation of a turbine - governor is also included in the mathematical model.

The mathematical model has been simulated on a digital computer. Runge Kutta Methods have been used for the simulation. It has been shown that, for an accuracy upto third decimal place, Basic Runge Kutta Method is good enough for the present studies.

Using the digital program developed, a number of studies were made to study the effect of various parameters on machine behaviour during asynchronous operation and resynchronization. A summary of the more important conclusions arrived at, is given below.

#### 5.1.1 Asynchronous operation :

1. Asynchronous operation with field unexcited produces less disturbance in the system than the field excited case.

2. Pulsation in various quantities is practically the same whether the machine has conventional excitation or brushless excitation.

#### 5.1.2 Resynchronization :

1. When the field is excited during asynchronous state, lesser excitation is needed to resynchronize the machine than that required when the field is unexcited.

2. When the machine is running with conventional excitation, the minimum excitation required to resynchronize is less than that for a machine with brushless excitation. Also, the optimum load angle at which the machine would still synchronize is more when the machine is conventionally excited.



## 5.2 Further Work :

### 5.2.1 Effect of Rectifier on asynchronous operation :

To protect the rectifier against overvoltage transients, short circuiting devices are available which short circuit the rectifier under asynchronous condition ( Sect. 4.1 ). In the present studies, it has been assumed that the rectifier is protected by the above device.

Such protection, against asynchronous operation, is not always required, as, for example, if the asynchronous operation is for not more than a few cycles<sup>12</sup>. If the rectifier is thus allowed to remain in circuit during asynchronous operation, asymmetry is introduced in the field circuit, due to the difference in forward and backward resistance of the rectifier. This may considerably affect the operational characteristics of the machine and would be worth further investigation.

### 5.2.2 Step length in Digital Program :

The digital computer studies were done on IBM 1620 computer. As this is only a medium sized computer, the time required for each run was in terms of many hours.

For this reason, the feature of automatically checking the results with half the previous step size was not incorporated in the program. In the present studies, this could be done for the reasons already mentioned in Section 3.2. However, it is suggested that this feature be incorporated in the digital program for further studies, if a faster digital computer is available.

FIG. 4-1  
STUDY I

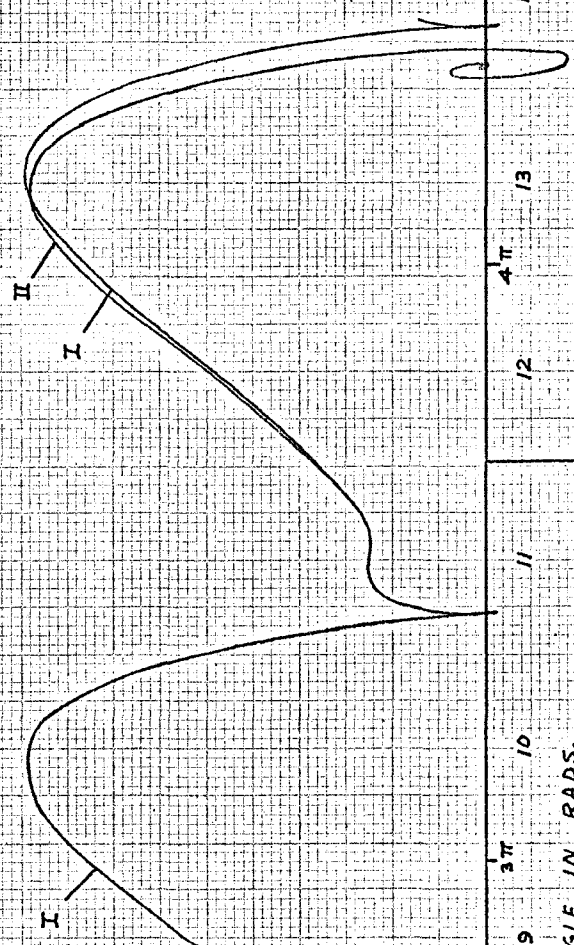
I - STEADY ASYNCHRONOUS OPERATION  
II - RESYNCHRONISATION RUN

SLIP IN RADS. PER SEC.

7 -6 -4 -2 0 2

8 9 10 11 12 13 14 15

LOAD ANGLE IN RADS.



S = 0.7 P.U.

ANGLE OF APPLICATION = 11.53 RADS.  
(301°)

FIG. 4-2  
STUDY 1

SLIP IN RADS. PER SEC.

TIME IN SECONDS

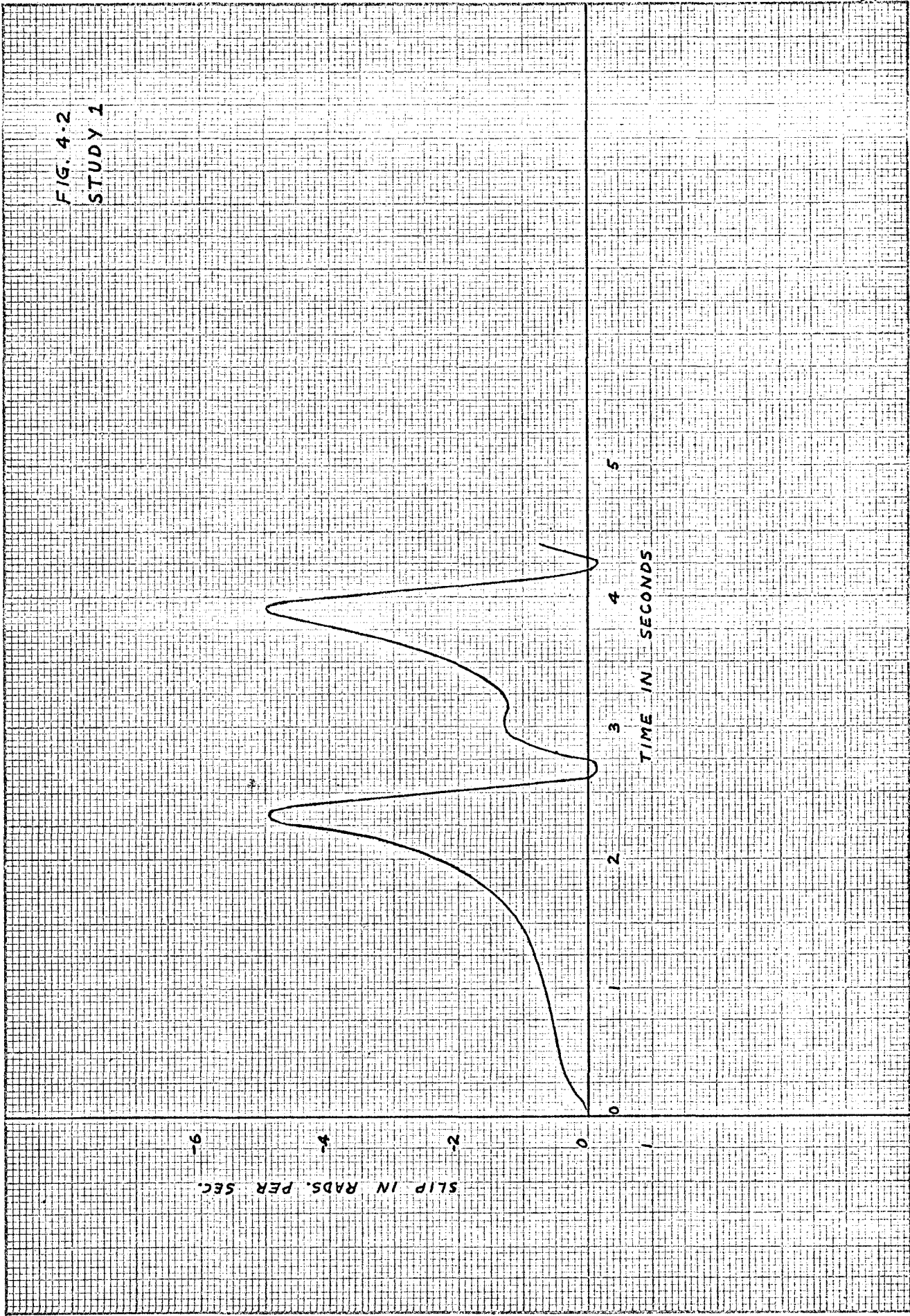


FIG. 4-3  
STUDY 1

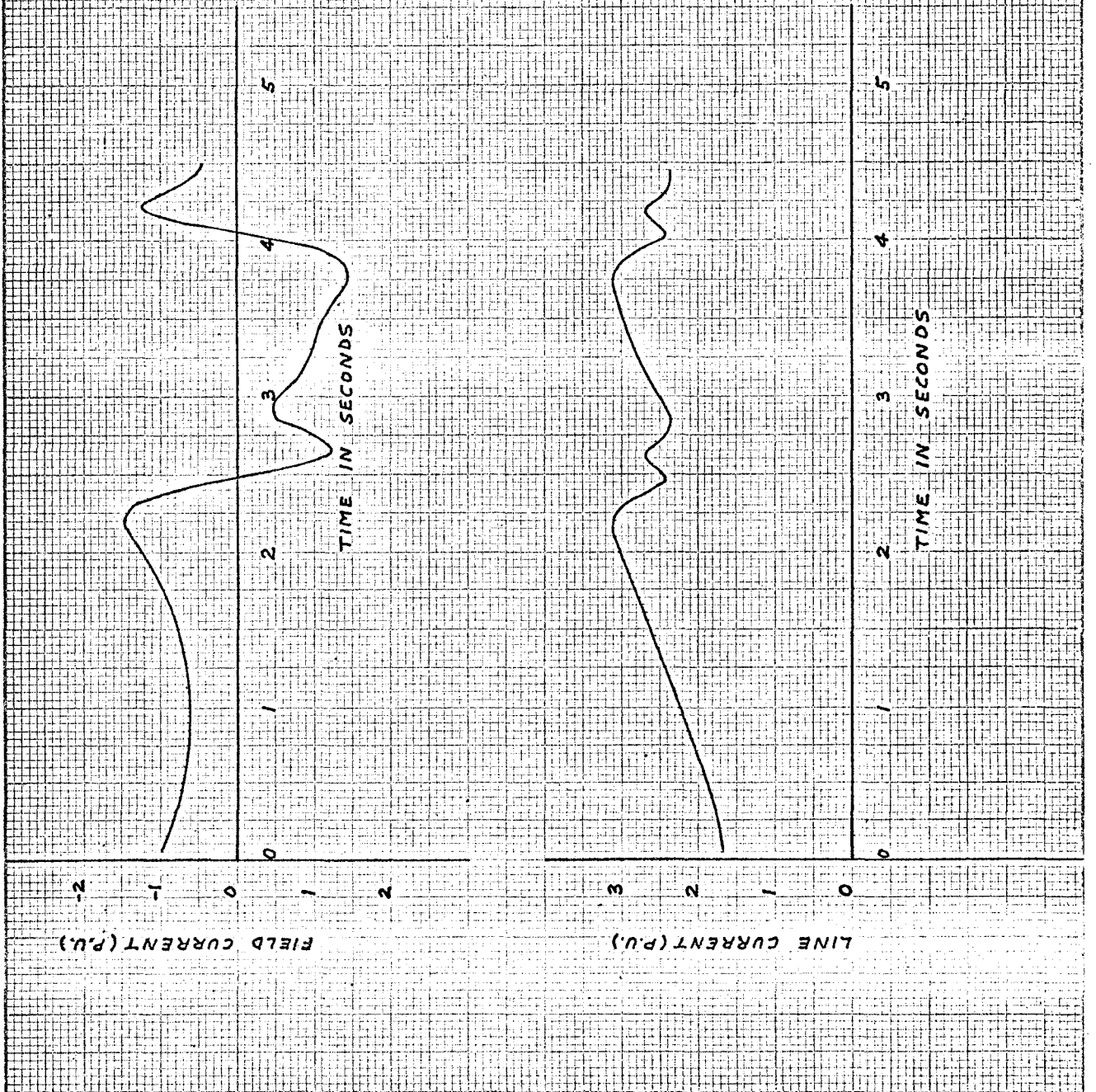




FIG. 4-4  
STUDY 1

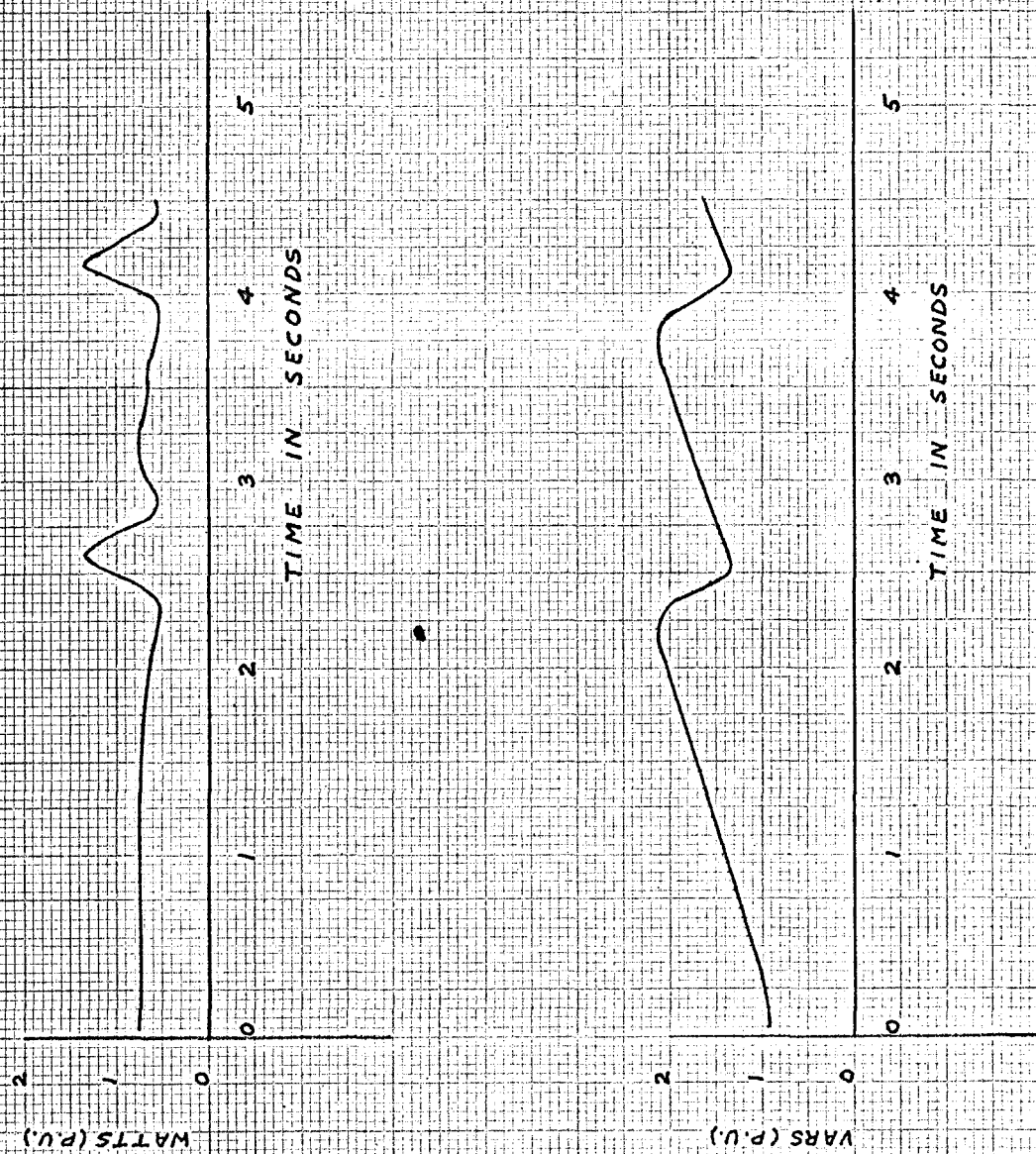
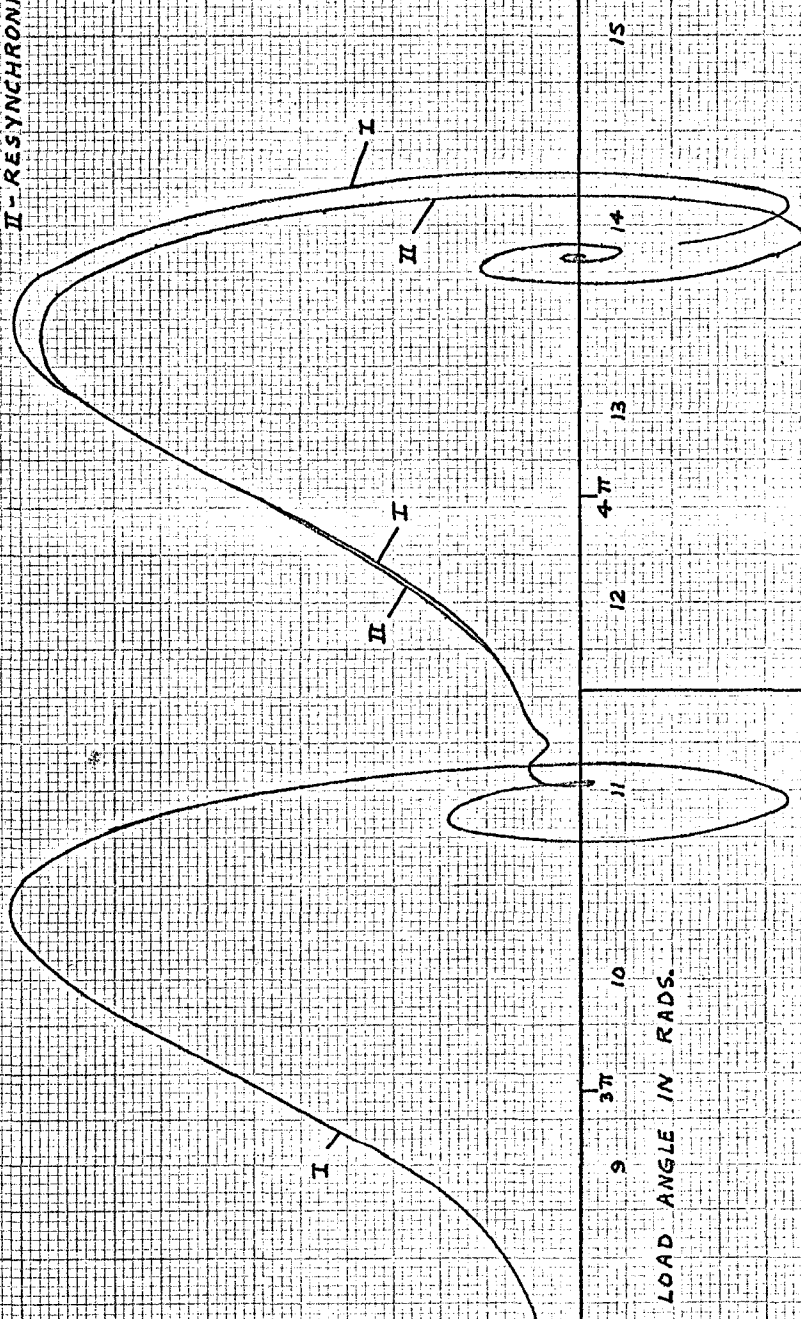


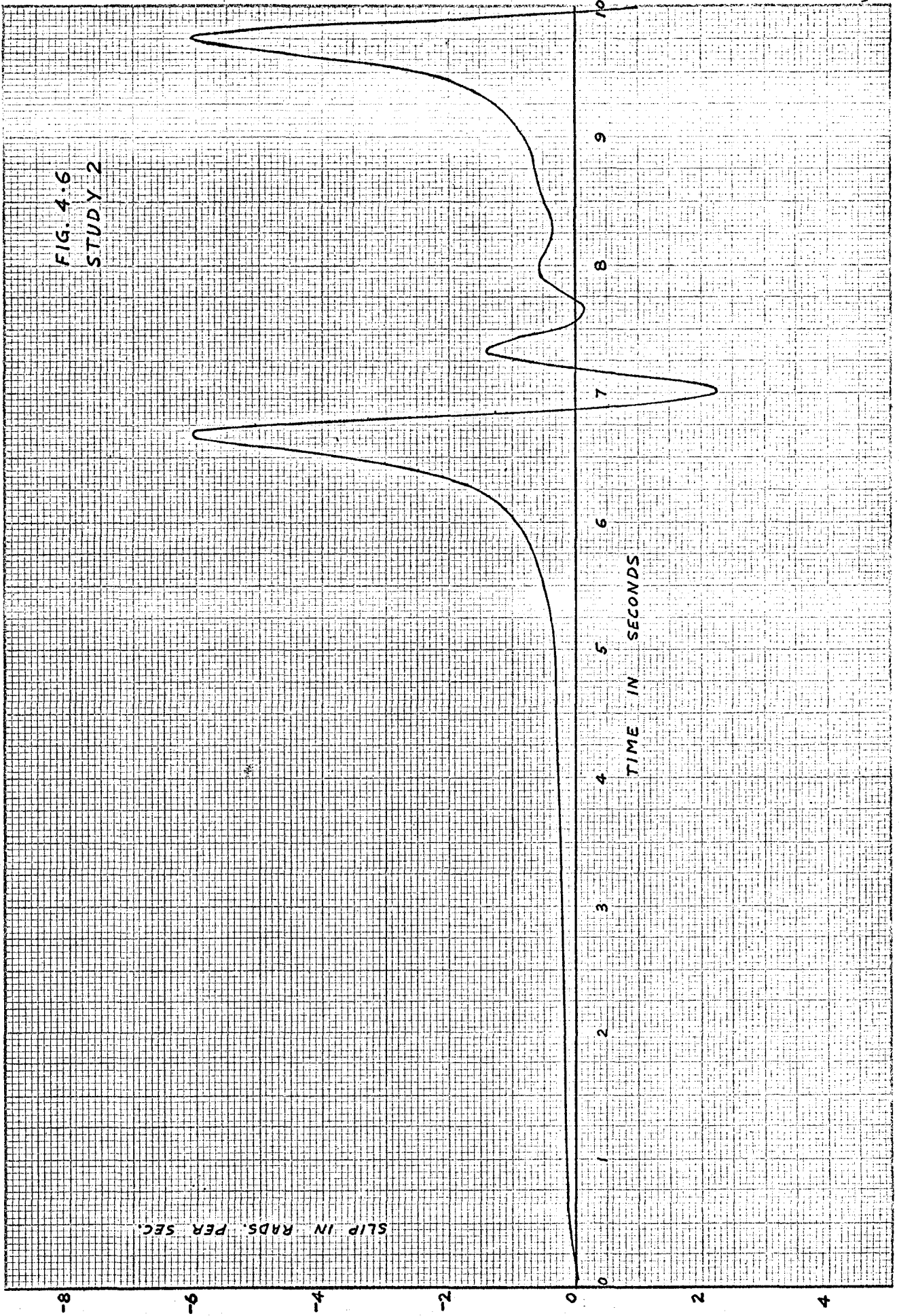
FIG. 4.5  
STUDY 2

I - STEADY ASYNCHRONOUS OPERATION  
II - RESYNCHRONISATION RUN



5 = 0.8 P.U.  
ANGLE OF APPLICATION = 11.53 RADS.  
(301°)

FIG. 4.6  
STUDY 2





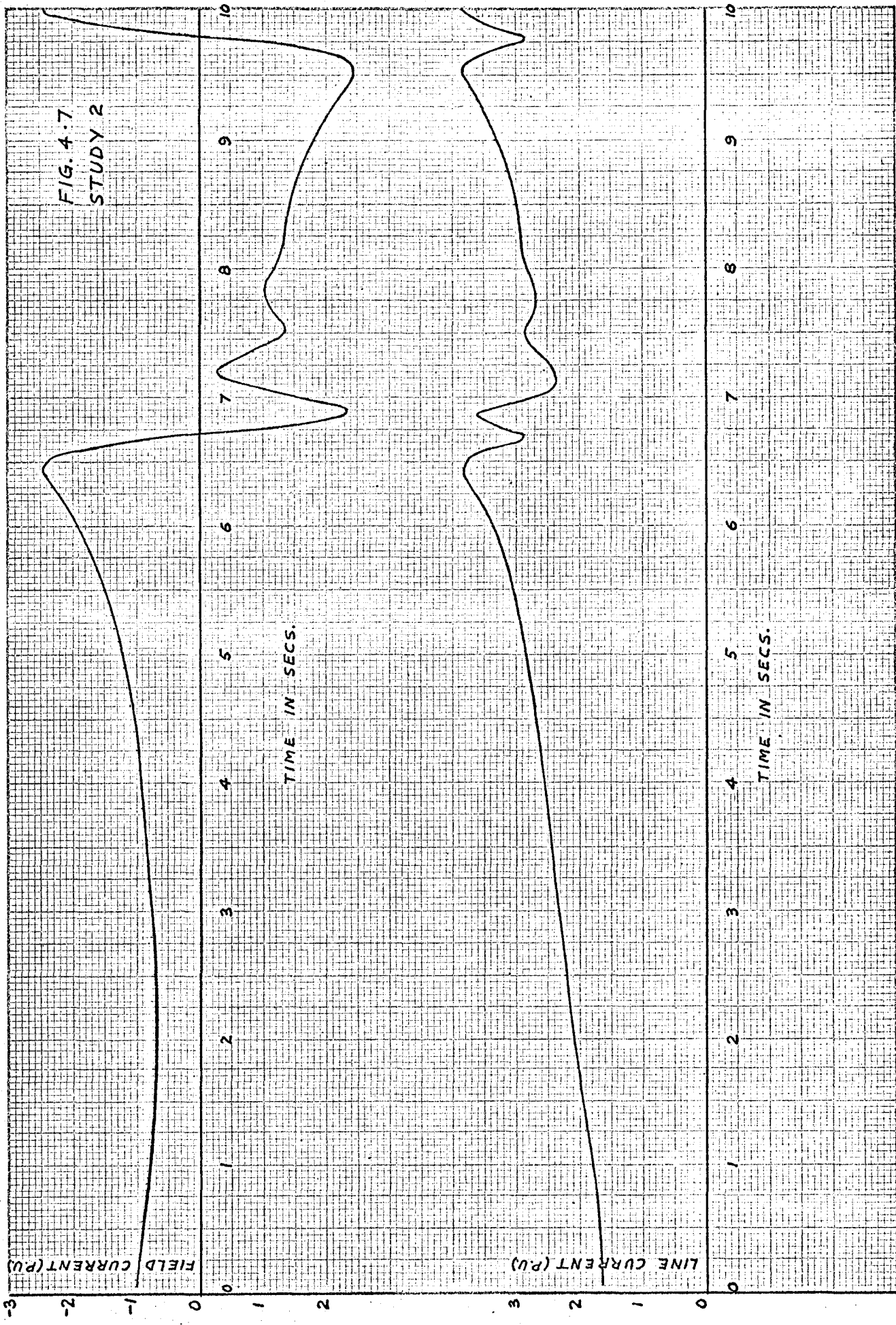


FIG. 4.7  
STUDY 2

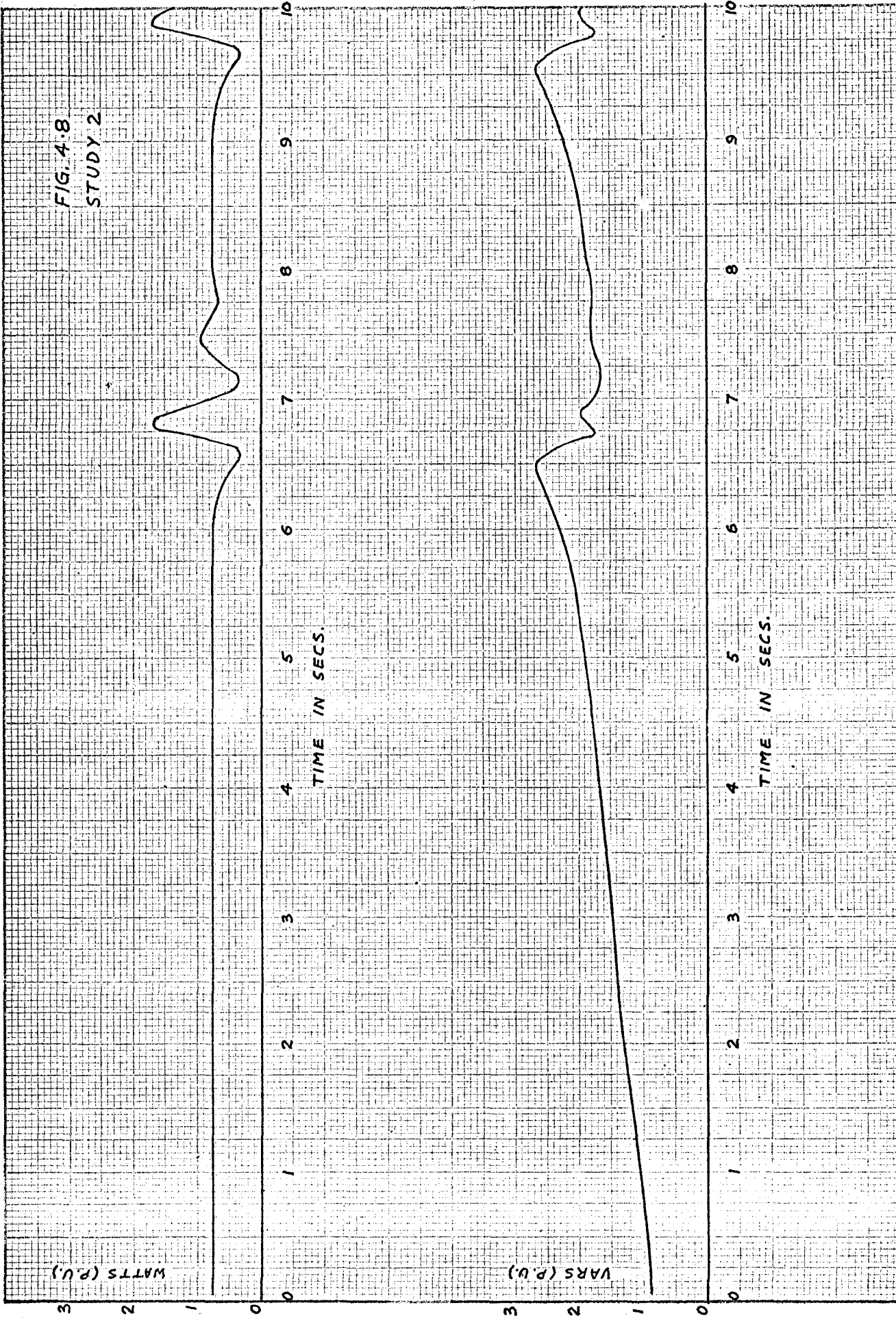


FIG. 4.9  
STUDY 3

I - STEADY ASYNCHRONOUS OPERATION  
II - RESYNCHRONISATION RUN

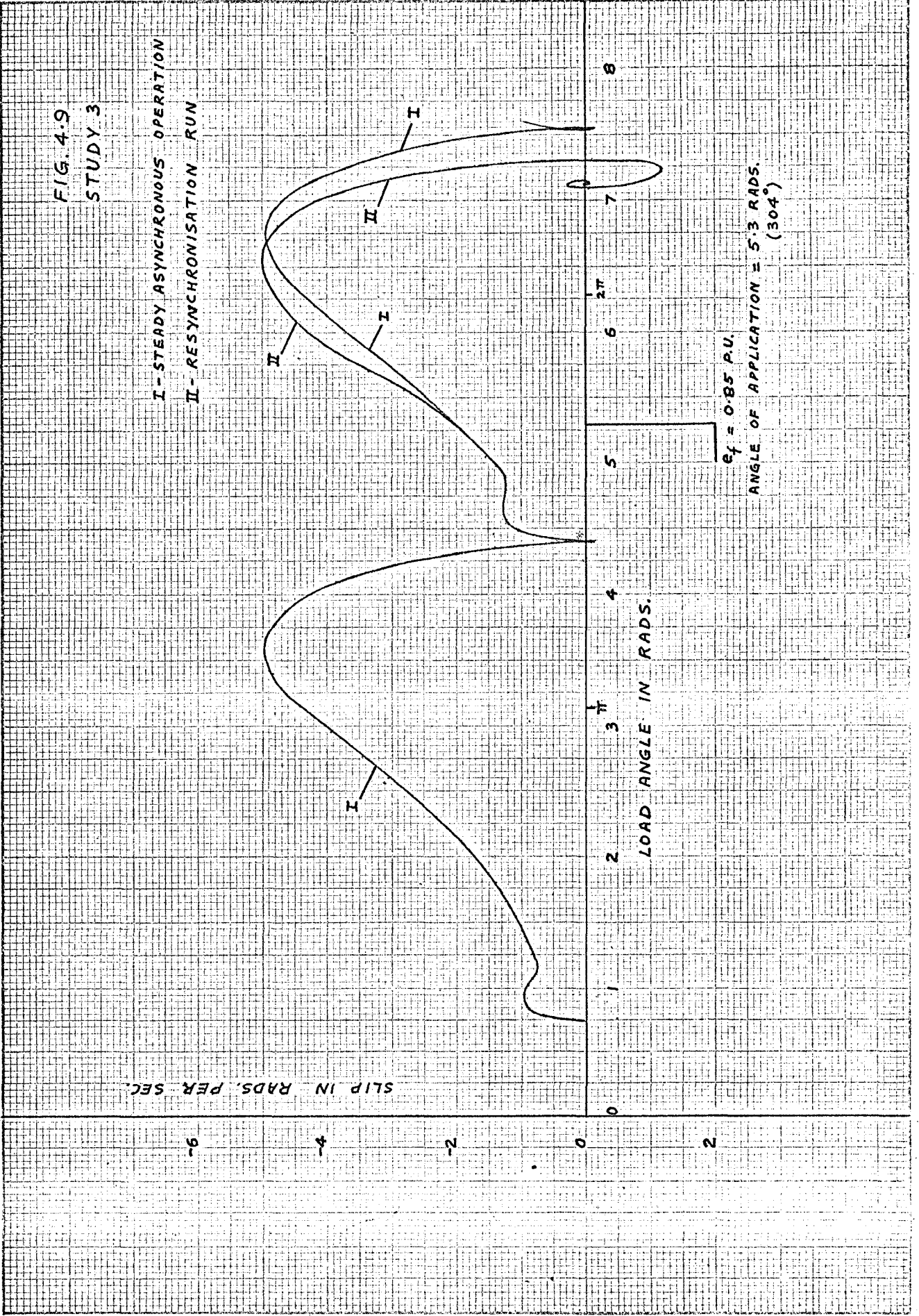


FIG. 4-10  
STUDY 3

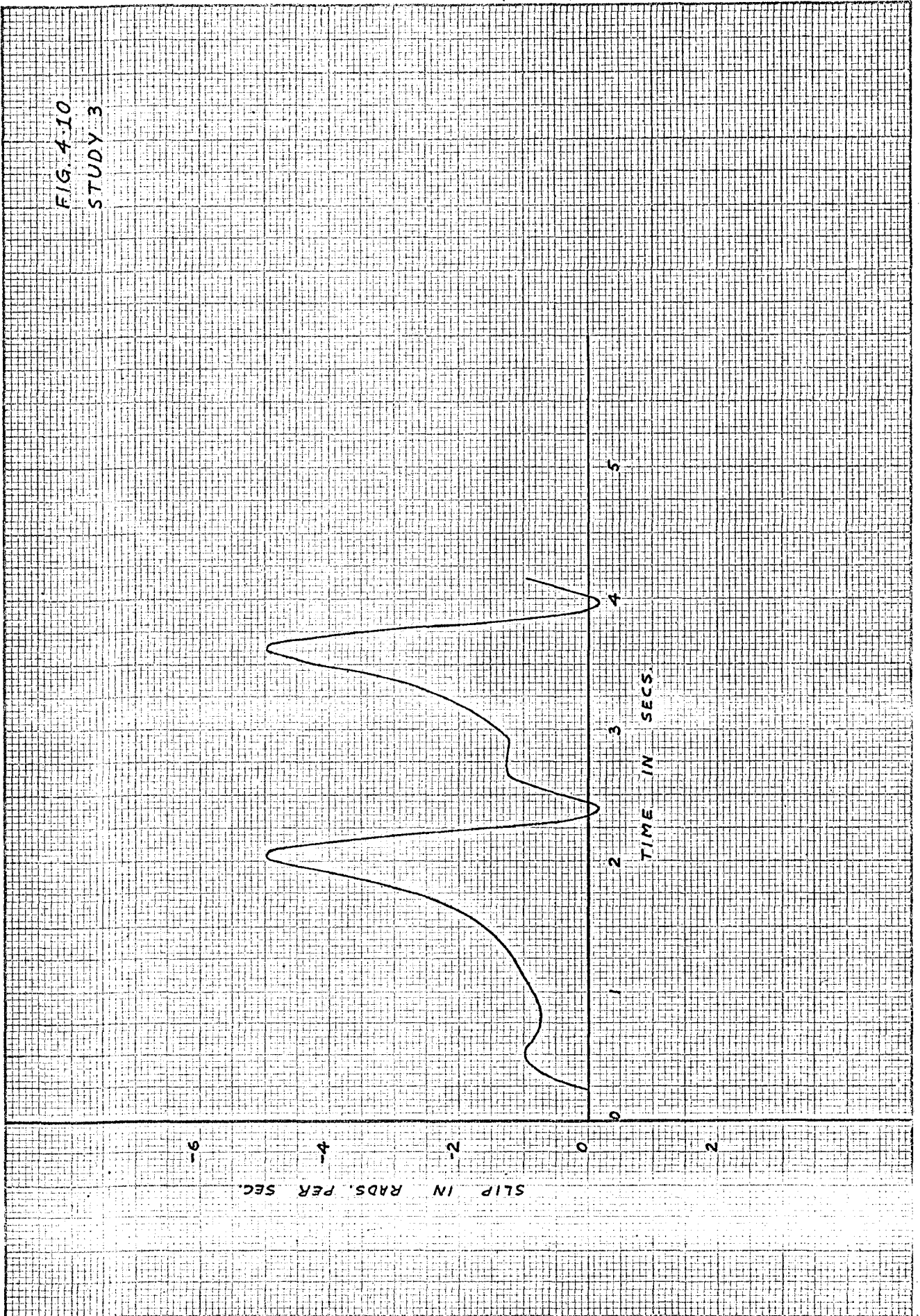




FIG. 4-11  
STUDY 3

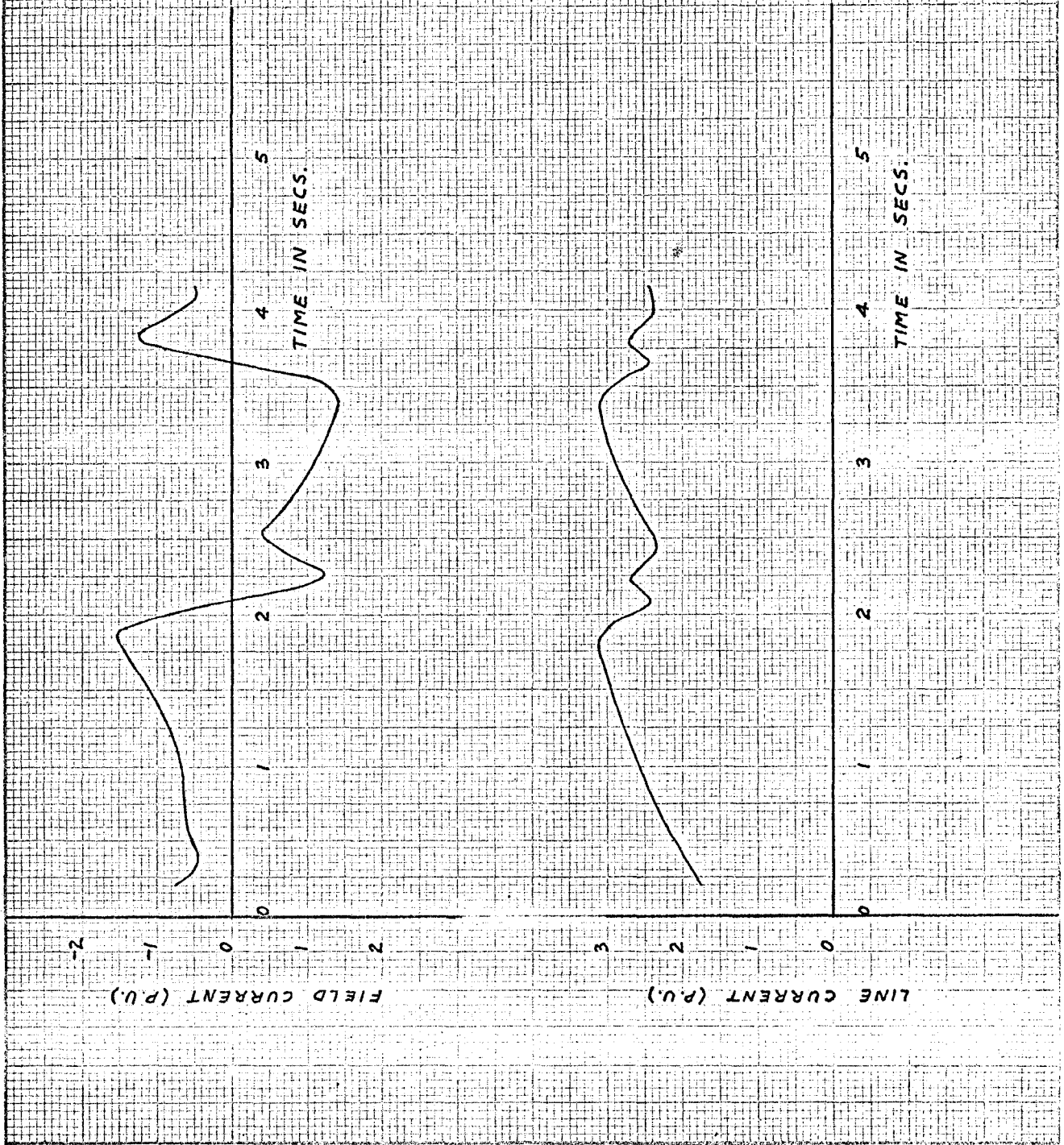


FIG. 4.12  
STUDY 3

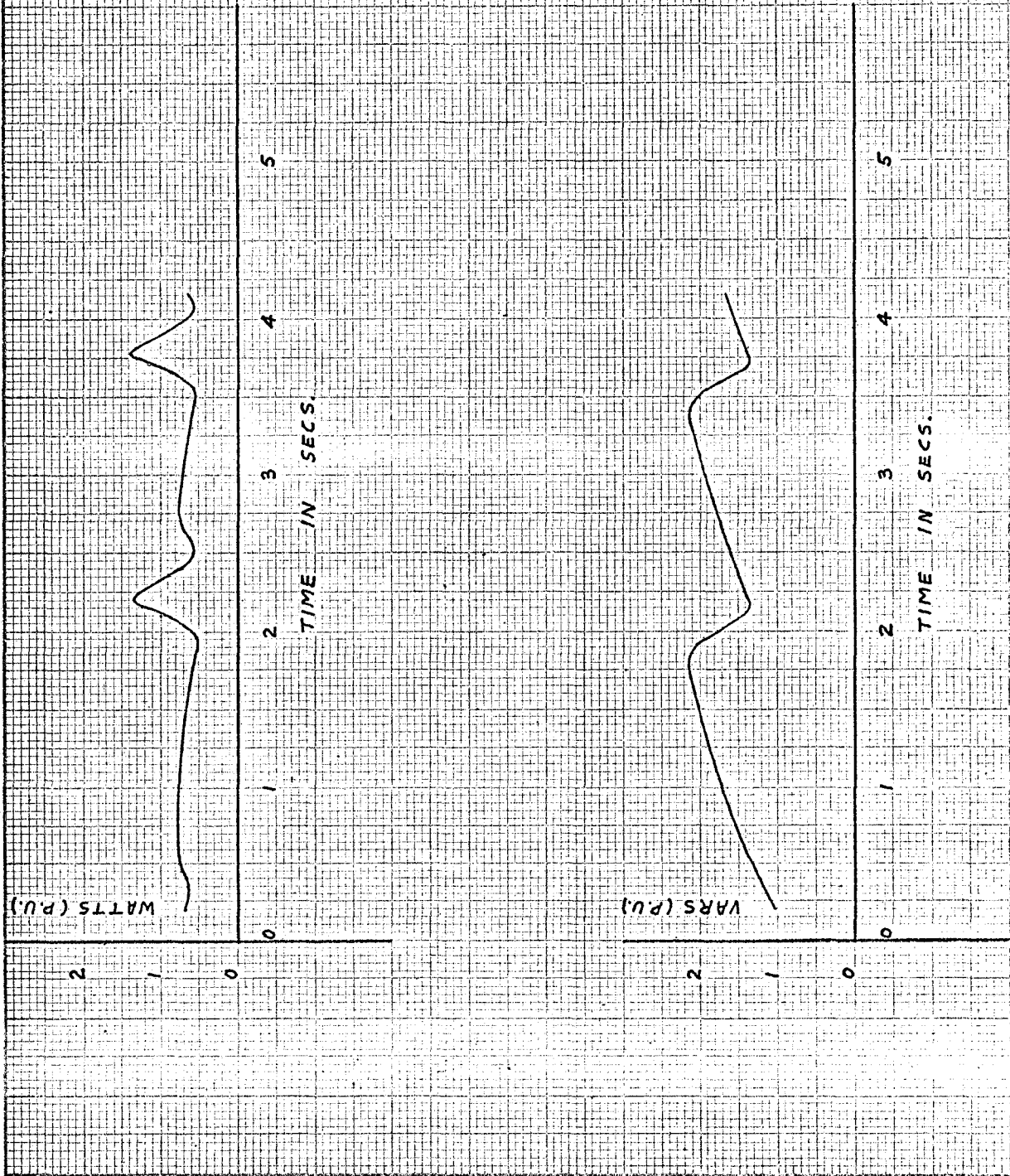
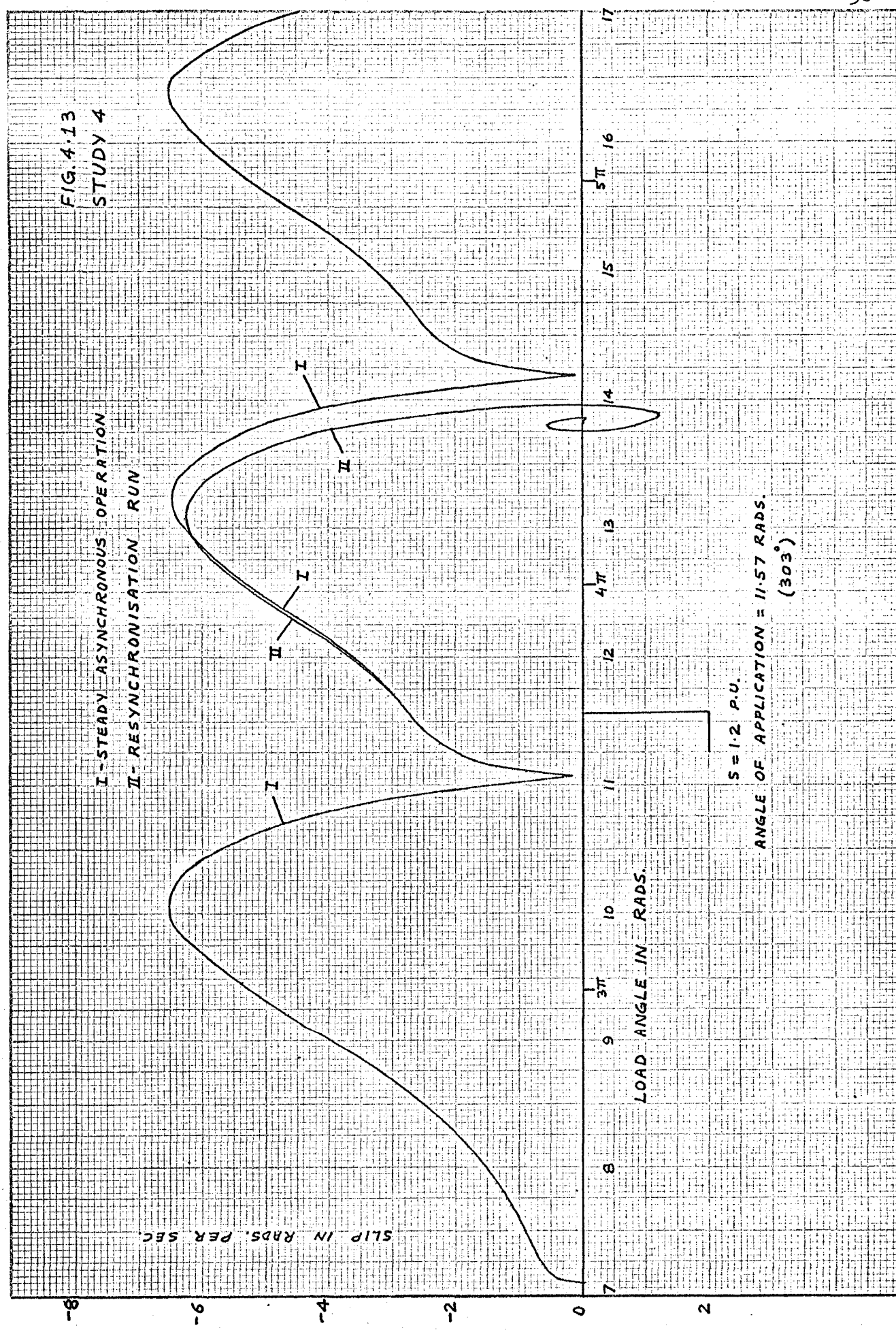


FIG. 4.13  
STUDY 4

I - STEADY ASYNCHRONOUS OPERATION  
II - RESYNCHRONISATION RUN



S = 1.2 P.U.  
ANGLE OF APPLICATION = 11.57 RADS.  
(303°)

FIG. 4-14  
STUDY 4

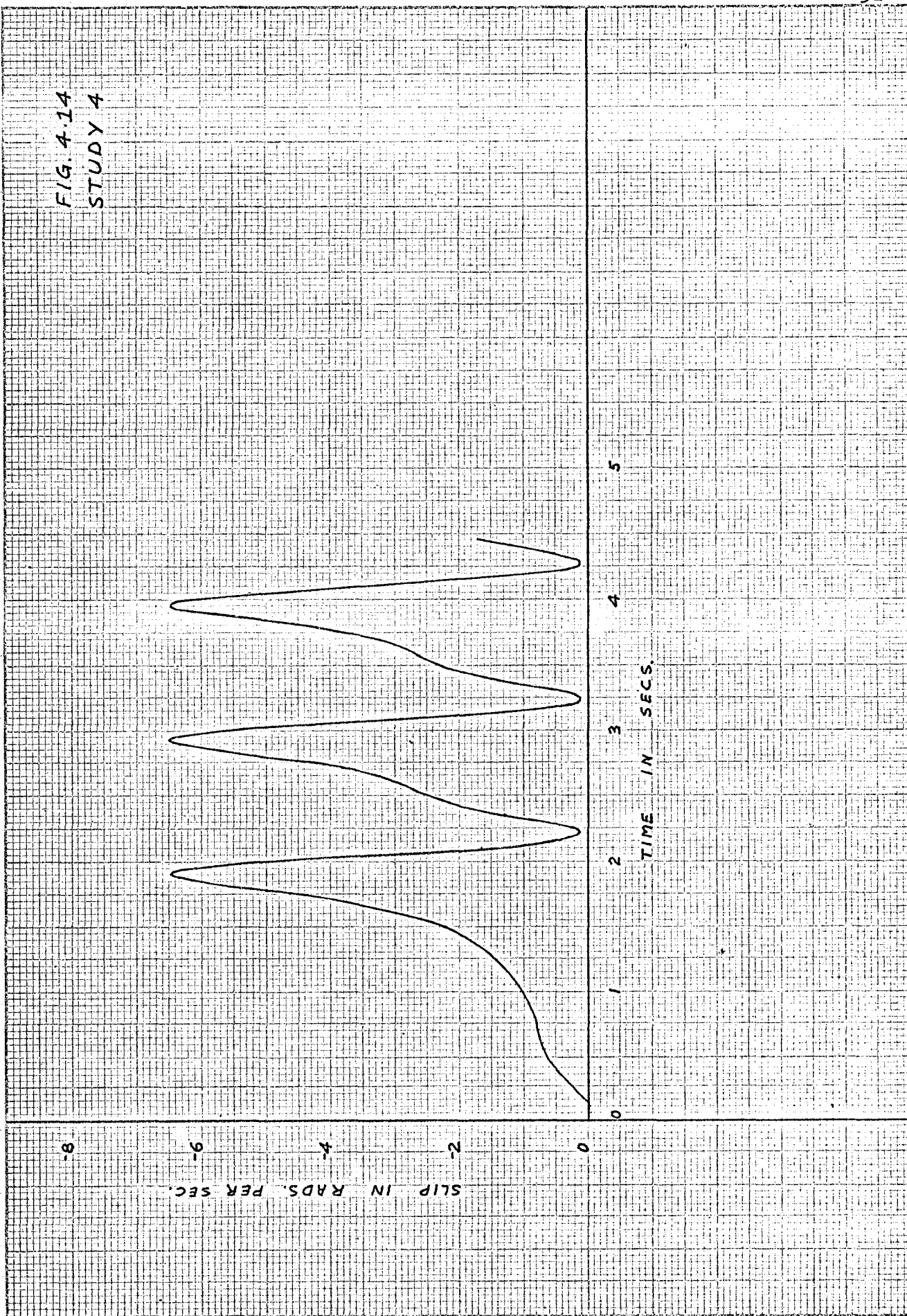




FIG. 4.15  
STUDY A

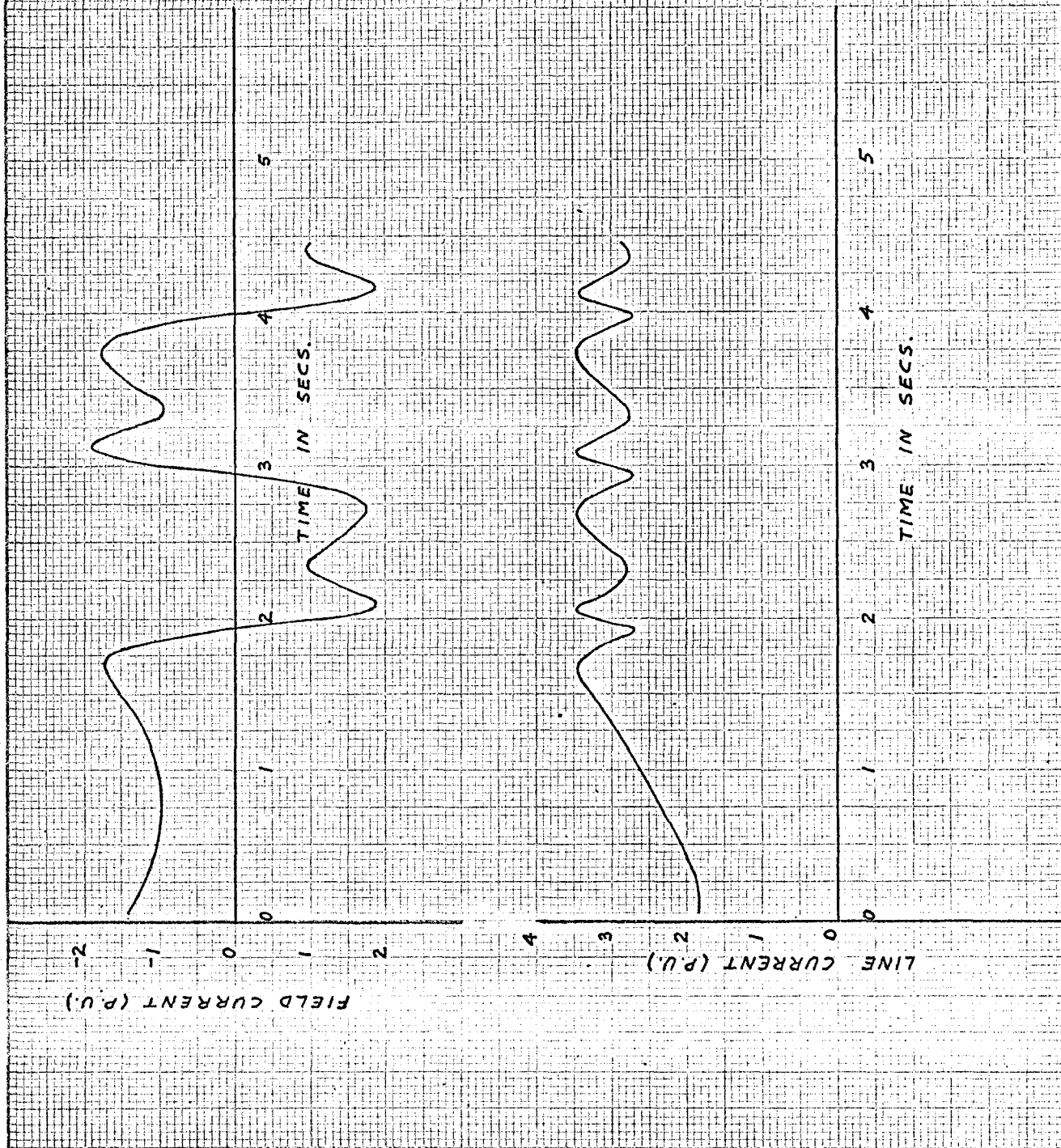


FIG. 4-16  
STUDY 4

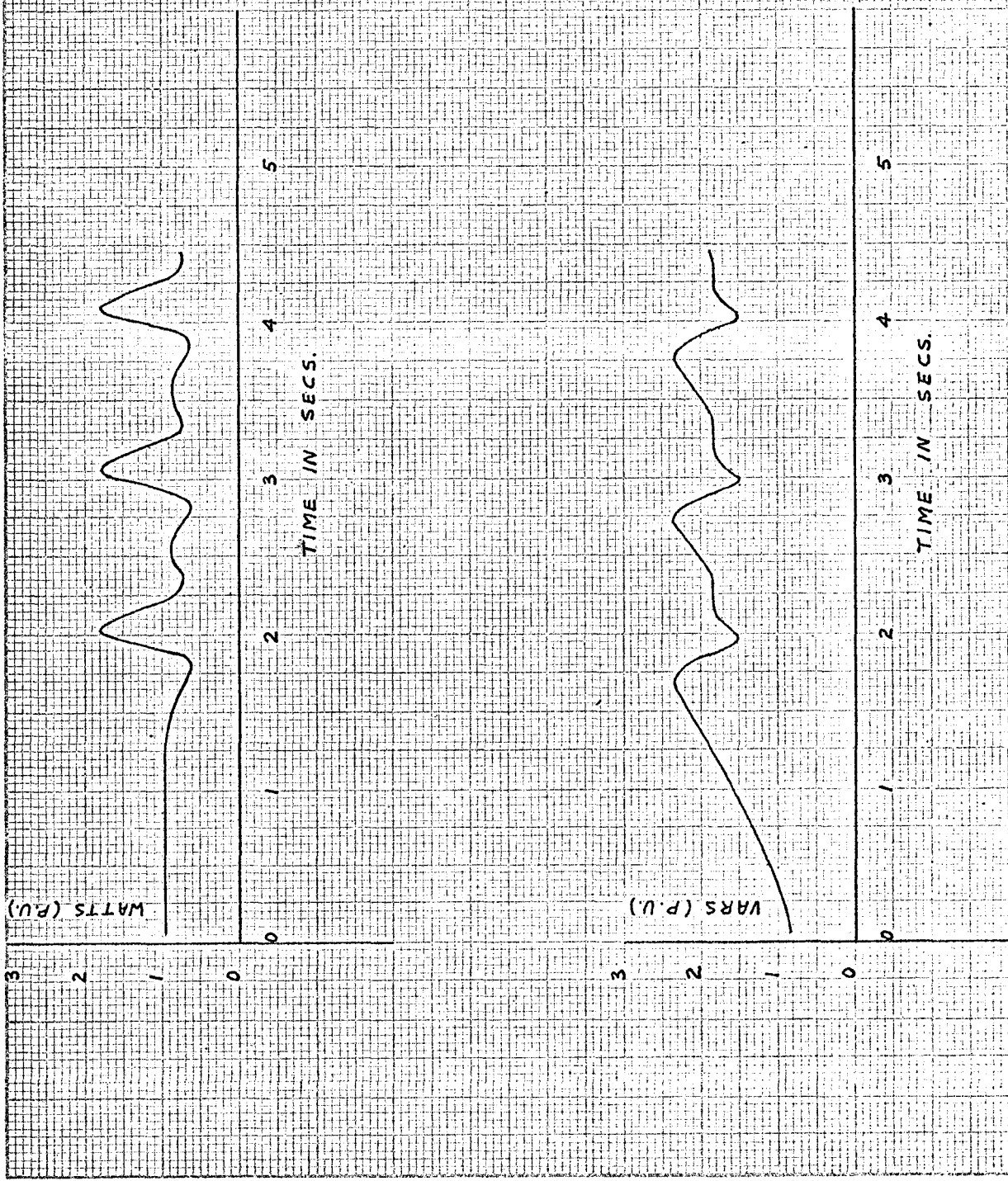


FIG. 4.17  
STUDY 5

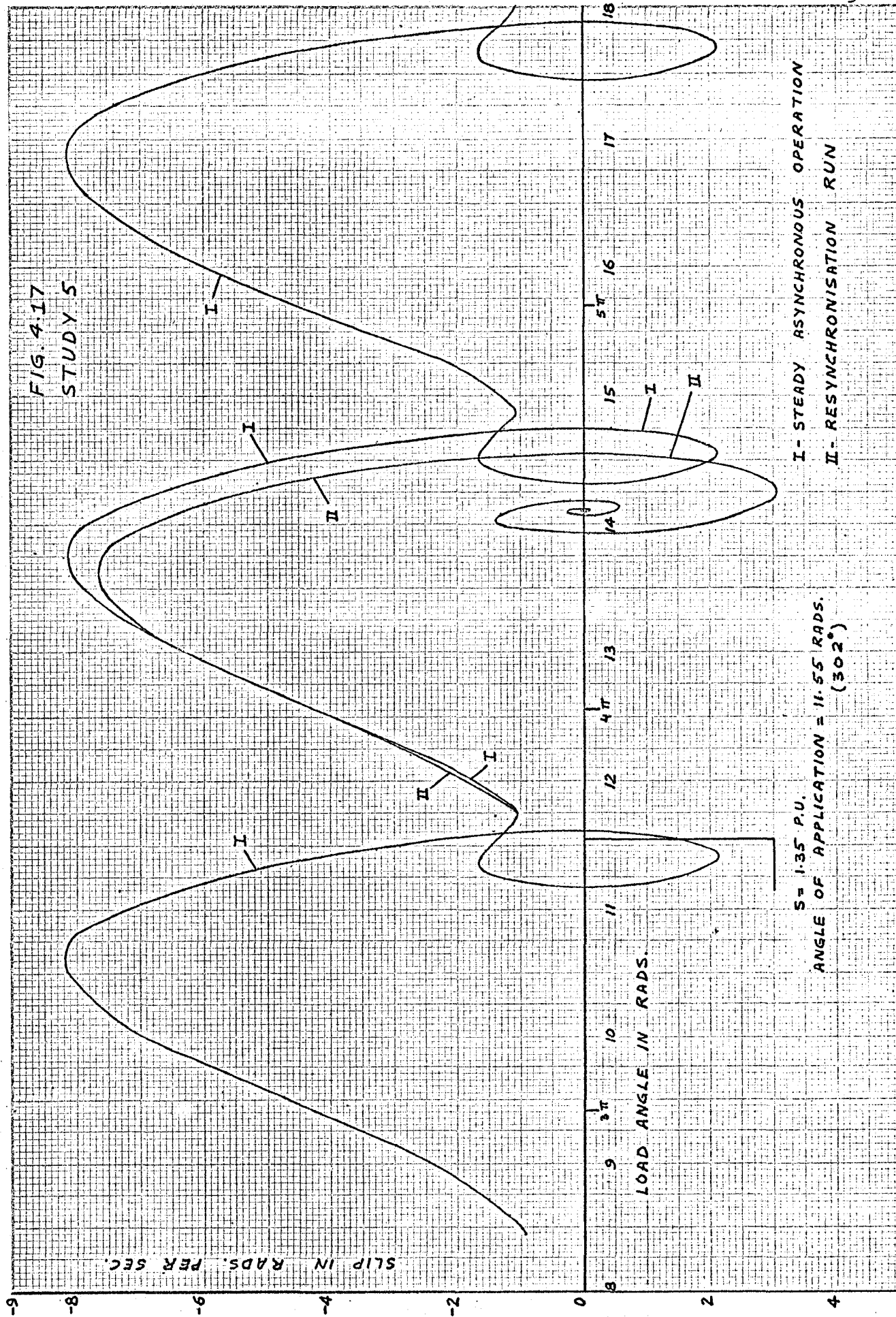


FIG. 4.18  
STUDY 5

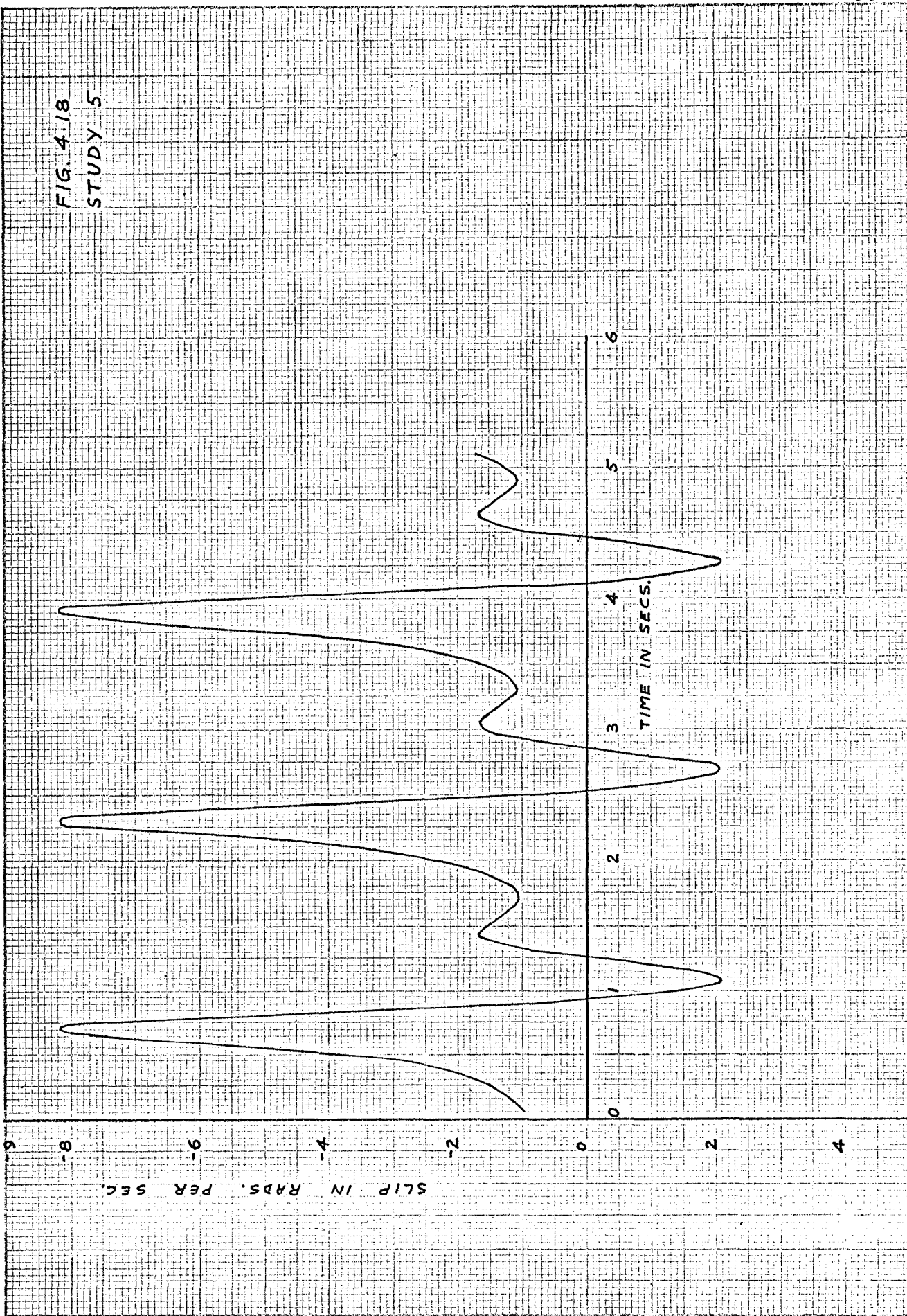




FIG. 4-19  
STUDY 5

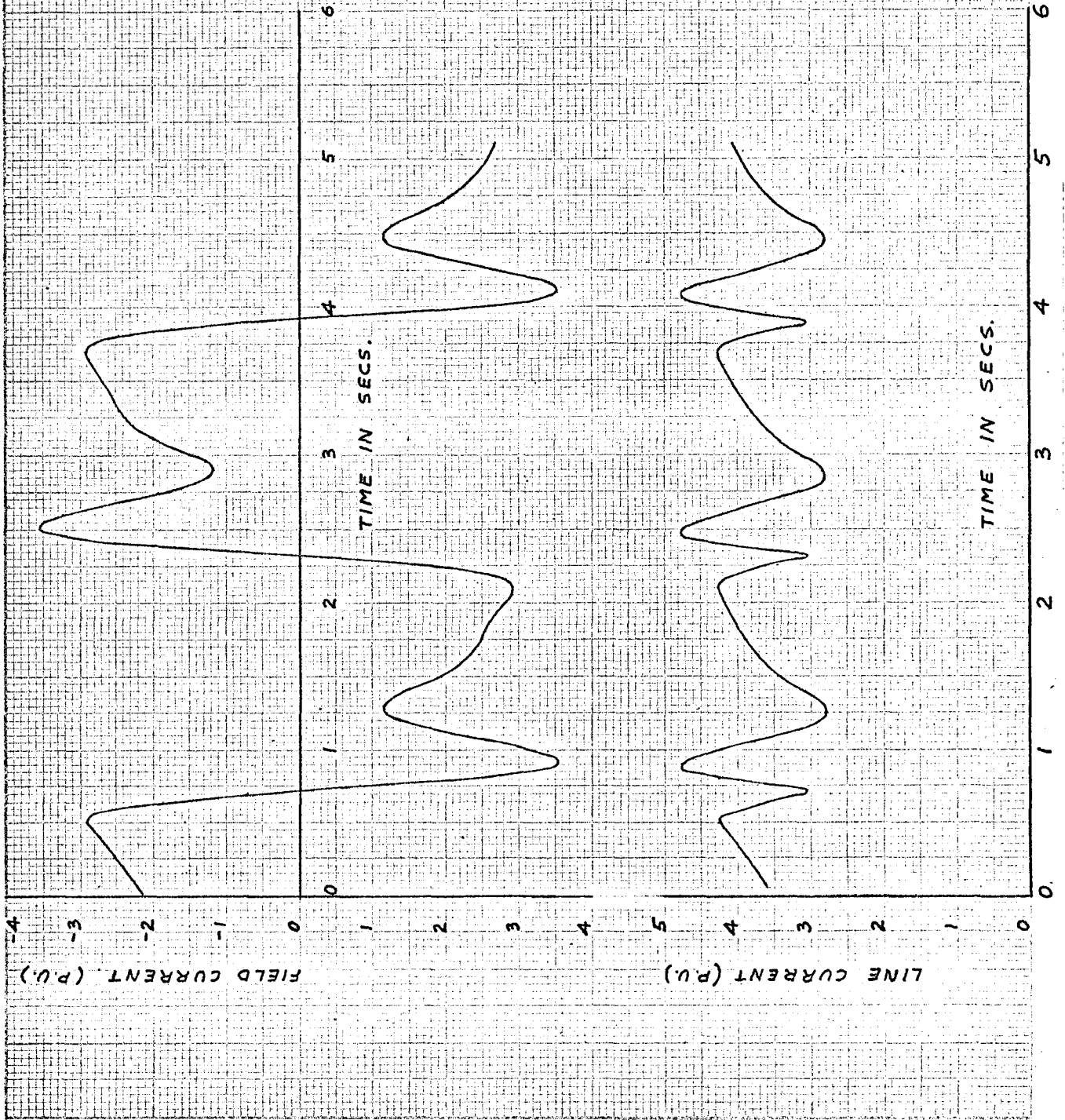
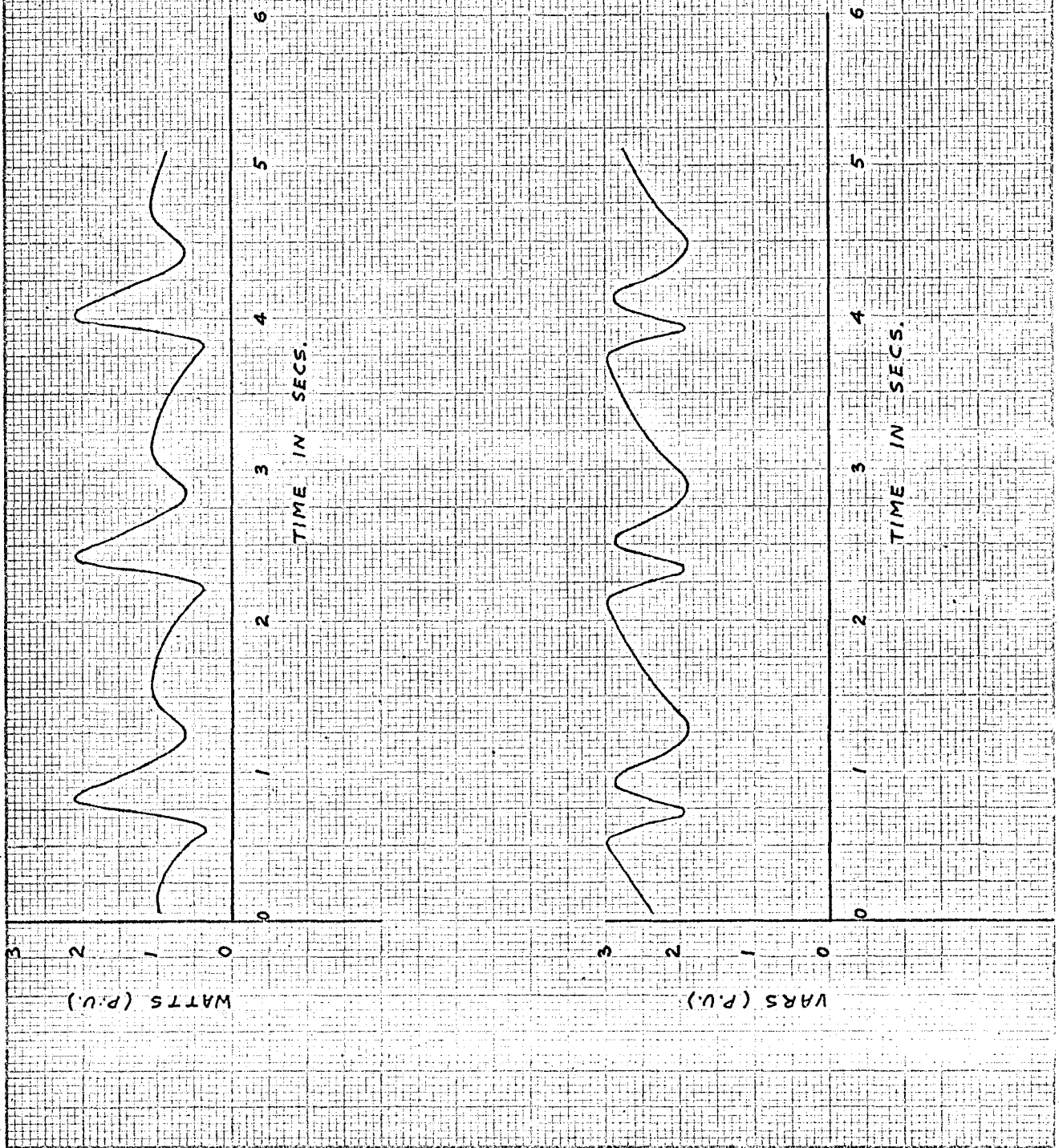
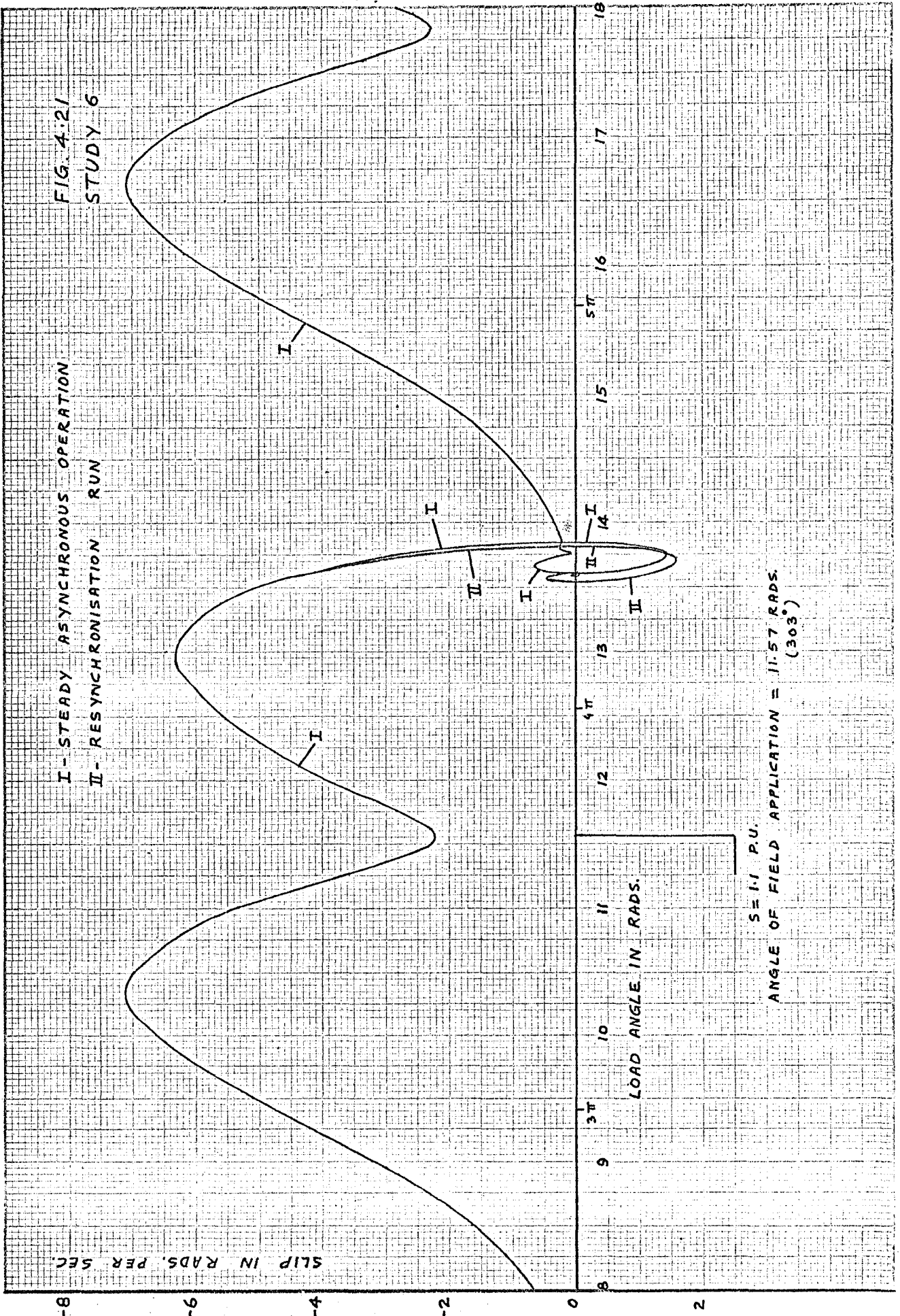
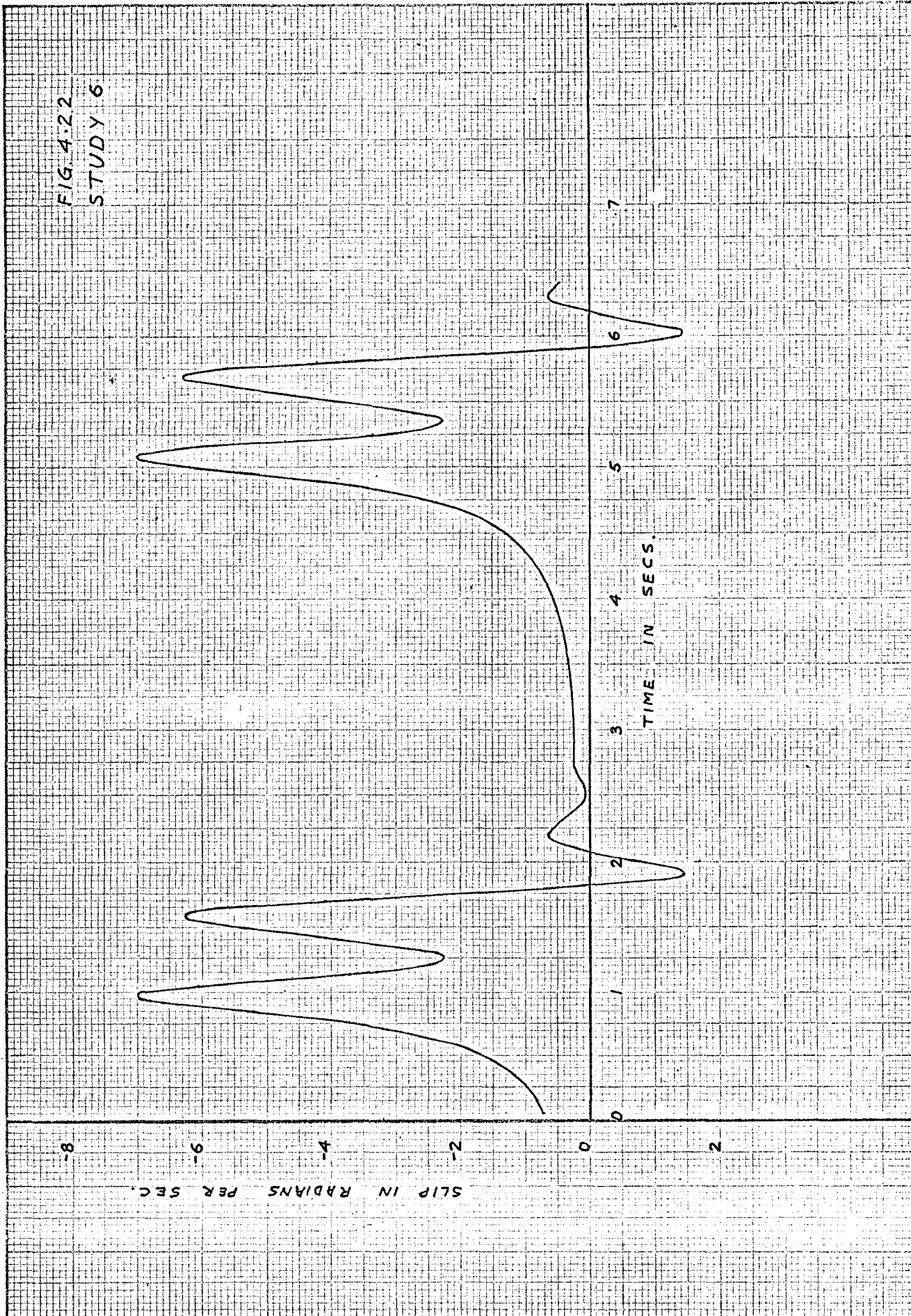


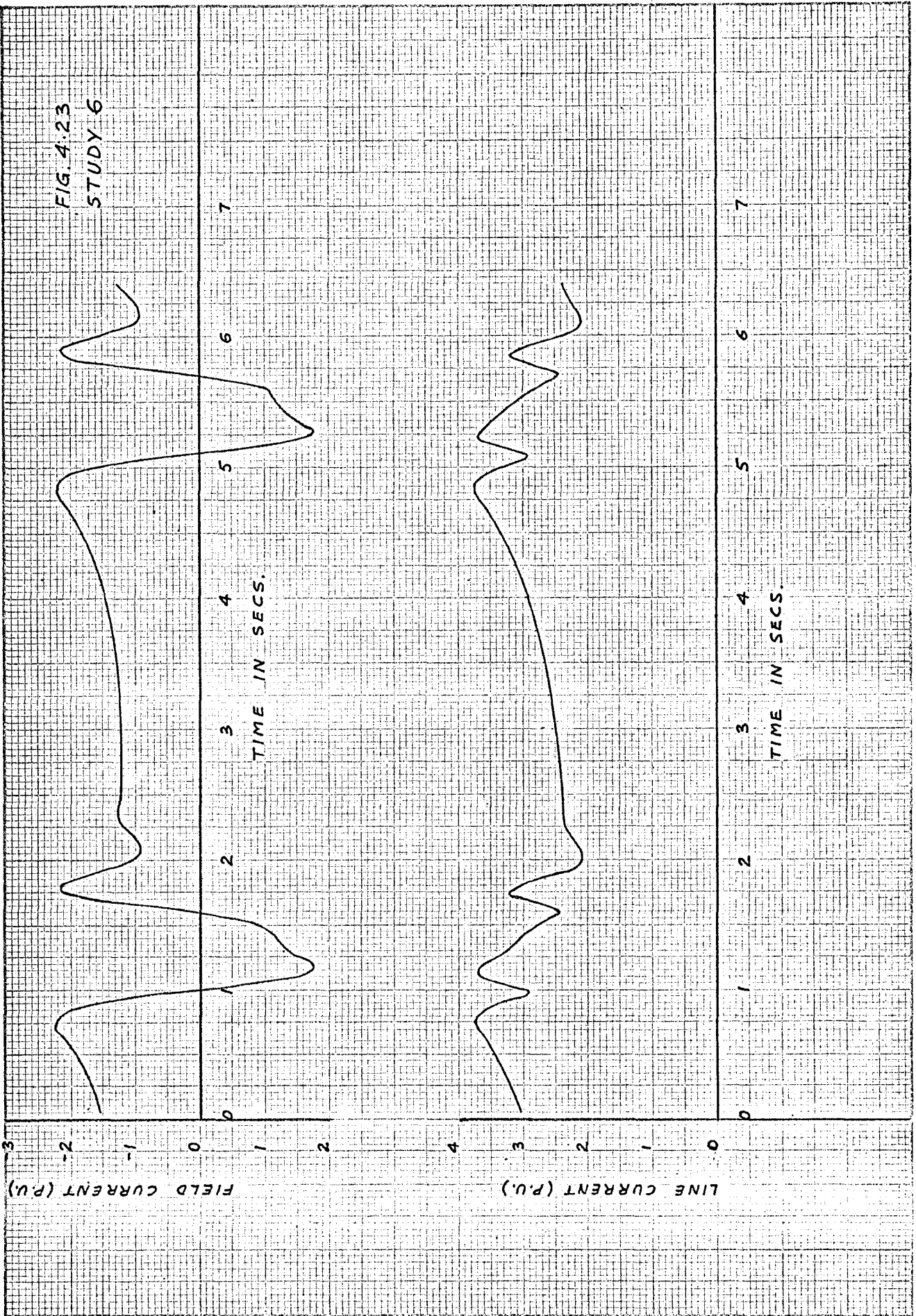
FIG. 4.20  
STUDY 5

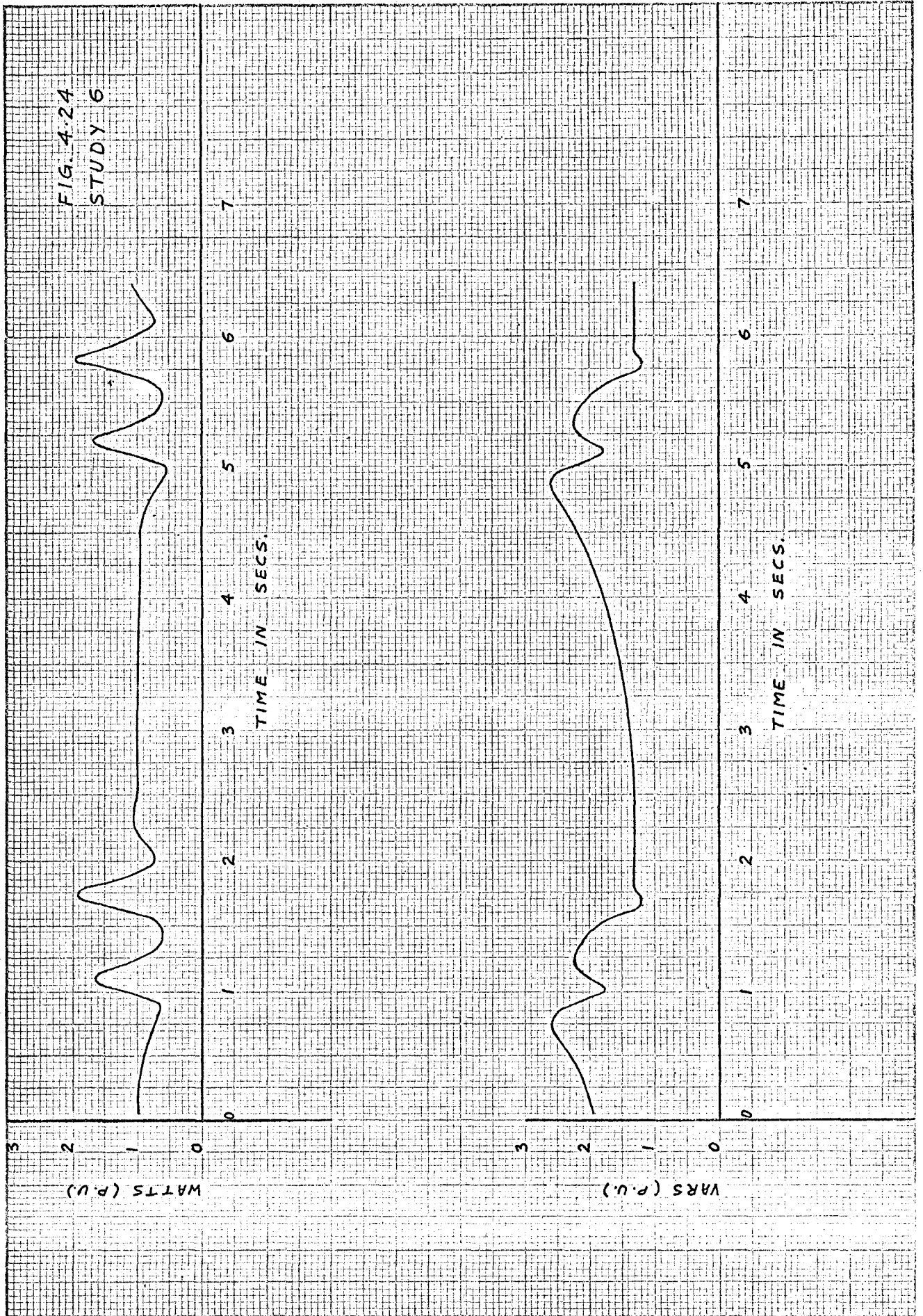












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APPENDIX IRUNGE KUTTA METHODSNomenclature :

$t_0$  = initial value of  $t$ .

$h$  = step size

$t_n$  =  $nh + t_0$

$m$  = the total number of first order simultaneous equations that must be solved.

$Y_n^{(i)}$  ( $i = 1, \dots, m$ ) = the value of any one of the dependent variables at  $t = t_n$ .

$f^{(i)}$  ( $t_n, Y_n^{(1)}, Y_n^{(2)}, \dots, Y_n^{(m)}$ ) =  $\frac{dY_n^{(i)}}{dt}$

Basic Runge Kutta Method<sup>10</sup> :

If the values of the dependent variables in the  $m$  first order differential equations are given at  $t = t_n$ , the following algorithm is used to find their numerical values at  $t_{n+1} = t_n + h$  :

$$Y_{n+1}^{(i)} = Y_n^{(i)} + \Delta Y_n^{(i)}$$

$$\text{where } \Delta Y_n^{(i)} = \frac{h}{6} ( k_0^{(i)} + 2k_1^{(i)} + 2k_2^{(i)} + k_3^{(i)} )$$

$$\text{and } k_0^{(i)} = f^{(i)}(t_n, Y_n^{(1)}, Y_n^{(2)}, \dots, Y_n^{(m)})$$

$$k_1^{(i)} = f^{(i)}\left(t_{n+(h/2)}, Y_n^{(1)} + \frac{k_0^{(1)}}{2} h, \dots, Y_n^{(m)} + \frac{k_0^{(m)}}{2} h\right)$$

$$k_2^{(i)} = f^{(i)}\left(t_{n+(h/2)}, Y_n^{(1)} + \frac{k_1^{(1)}}{2} h, \dots, Y_n^{(m)} + \frac{k_1^{(m)}}{2} h\right)$$

$$k_3^{(i)} = f^{(i)}(t_{n+h}, Y_n^{(1)} + k_2^{(1)} h, \dots, Y_n^{(m)} + k_2^{(m)} h)$$

and the superscript  $i = 1, 2, 3, \dots, m$  serves to indicate the  $m$  first order differential equations.

Gill Method<sup>10</sup> :

If the values of the dependent variables in the  $m$  first order differential equations are given at  $t = t_n$ , the following algorithm is used to find their numerical values at  $t_{n+1} = t_n + h$  :

$$Y_{n+1}^{(i)} = Y_n^{(i)} + \Delta Y_n^{(i)}$$

$$\text{where } \Delta Y_n^{(i)} = \frac{h}{6} \left[ k_0^{(i)} + 2(1-1/\sqrt{2})k_1^{(i)} + 2(1+1/\sqrt{2})k_2^{(i)} + k_3^{(i)} \right]$$

$$\text{and } k_0^{(i)} = f^{(i)}(t_n, Y_n^{(1)}, \dots, Y_n^{(m)})$$

$$k_1^{(i)} = f^{(i)}\left[t_{n+(h/2)}, Y_n^{(1)} + \frac{k_0^{(1)}}{2} h, \dots, Y_n^{(m)} + \frac{k_0^{(m)}}{2} h\right]$$

$$k_2^{(i)} = f^{(i)} \left[ t_{n+(h/2)}, Y_n^{(1)} + \left(-\frac{1}{2} + \frac{1}{\sqrt{2}}\right) k_0^{(1)} h + \left(1 - \frac{1}{\sqrt{2}}\right) k_1^{(1)} h, \dots \right. \\ \left. \dots \dots \dots, Y_n^{(m)} + \left(-\frac{1}{2} + \frac{1}{\sqrt{2}}\right) k_0^{(m)} h + \left(1 - \frac{1}{\sqrt{2}}\right) k_1^{(m)} h \right]$$

$$k_3^{(i)} = f^{(i)} \left[ t_{n+h}, Y_n^{(1)} + \left(-\frac{1}{\sqrt{2}}\right) k_1^{(1)} h + \left(1 + \frac{1}{\sqrt{2}}\right) k_2^{(1)} h, \dots \right. \\ \left. \dots \dots \dots, Y_n^{(m)} + \left(-\frac{1}{\sqrt{2}}\right) k_1^{(m)} h + \left(1 + \frac{1}{\sqrt{2}}\right) k_2^{(m)} h \right]$$

and the superscript  $i = 1, 2, 3, \dots, m$  serves to indicate the  $m$  first order differential equations.

APPENDIX IIMACHINE DATA1. Synchronous MachineLaminated rotor micro - machine :

Stator No.	334819
Rotor No.	334818

Base Quantities :

Voltamps	1525	VA
Voltage (phase)	127	Volts
Current (phase)	4	Amps
Impedance	31.75	Ohms
Field Current	0.519	Amps
Synchronous Speed	314	Radians/sec

Parameters in p.u. :

$r_a$	0.007
$r_f$	0.00446
H	3.64 KW sec/KVA
$l_f$	0.000677
$l_a$	0.00031
$l_{kd}$	0.000122
$l_{kq}$	0.000158



$L_{md}$	0.00268
$L_{mq}$	0.0015
$r_{kd}$	0.01463
$r_{kq}$	0.0176

2. Exciter - Alternator :

$$\tau = 0.5 \text{ sec}$$

$$K = 1$$

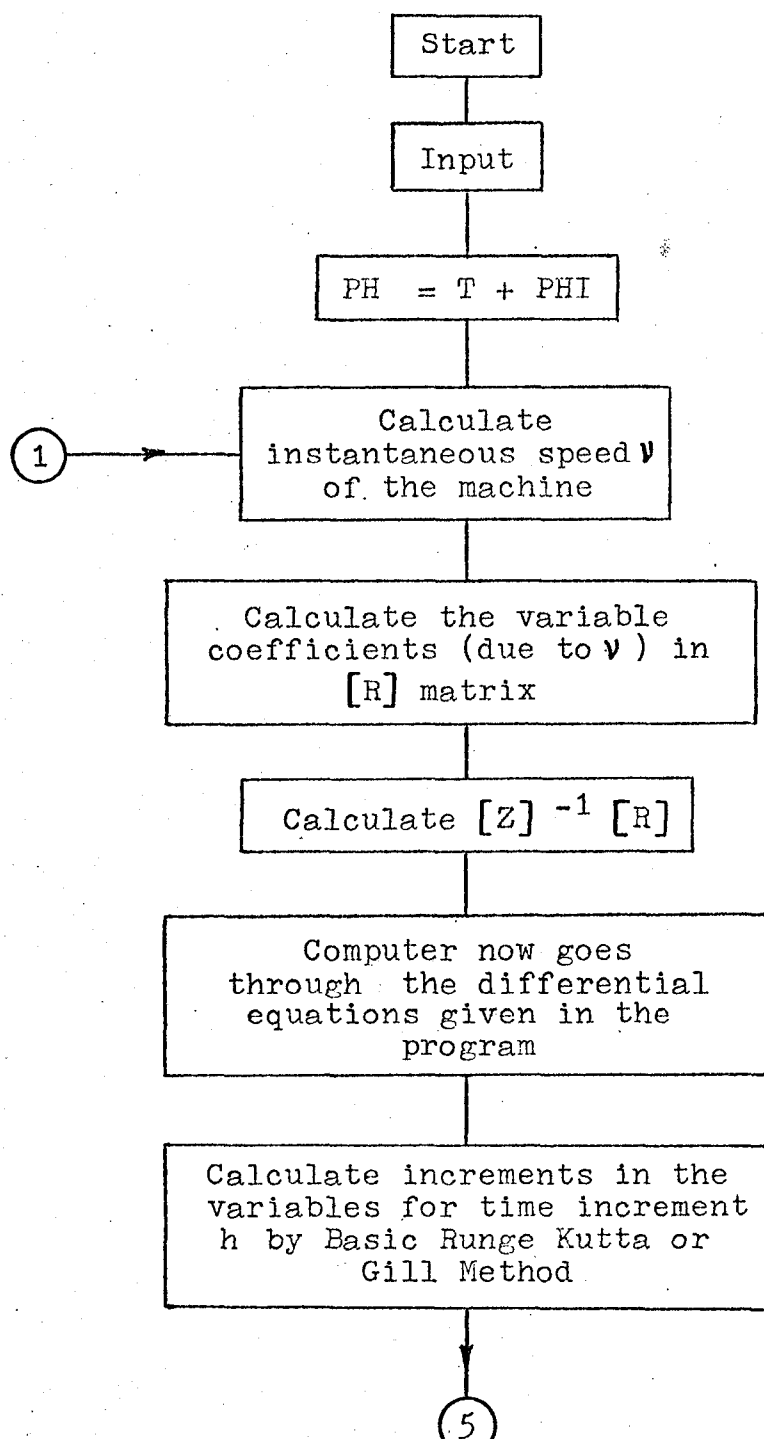
3. Turbine - Governor :

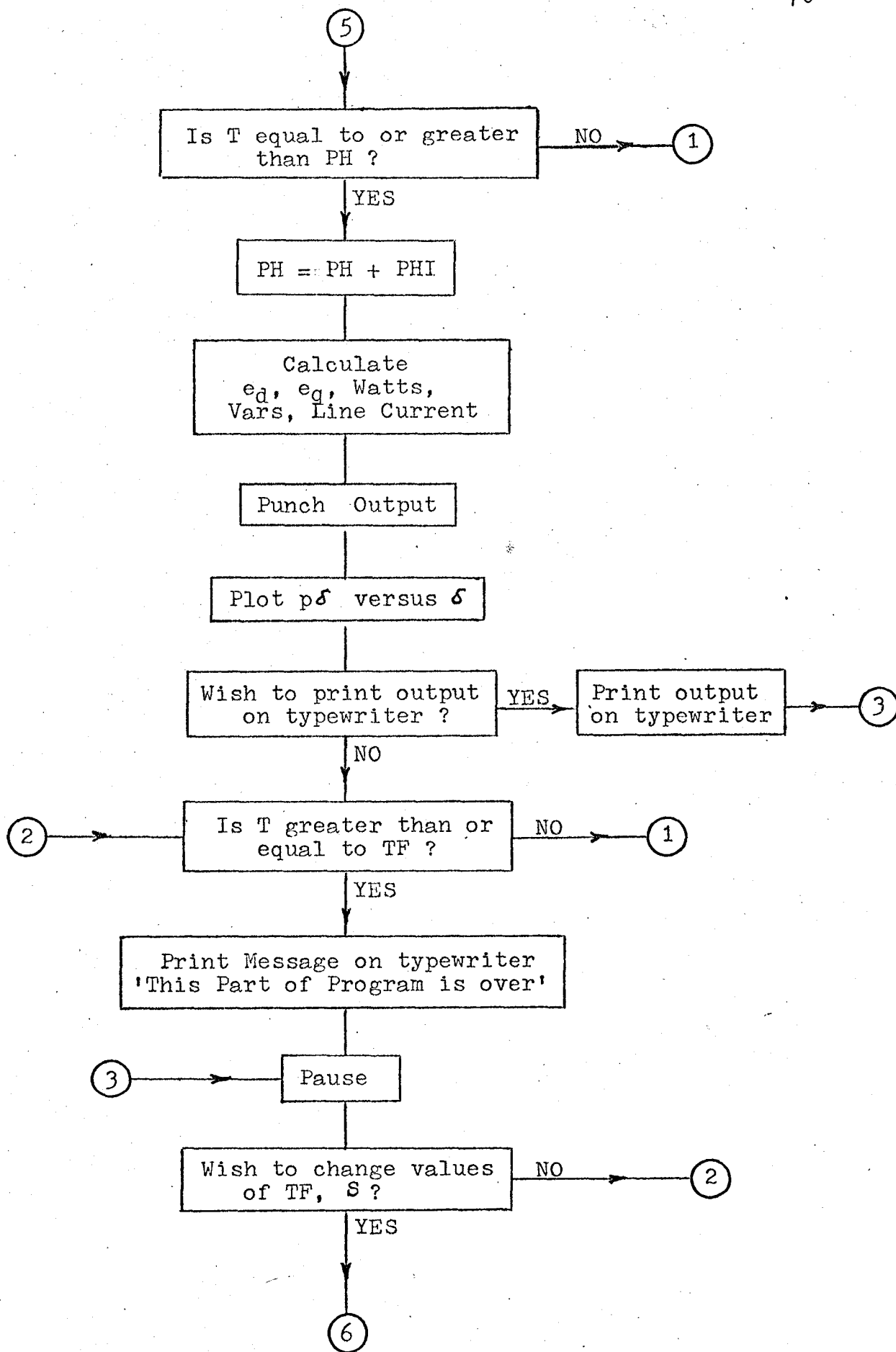
$$T_g = 0.6 \text{ sec}$$

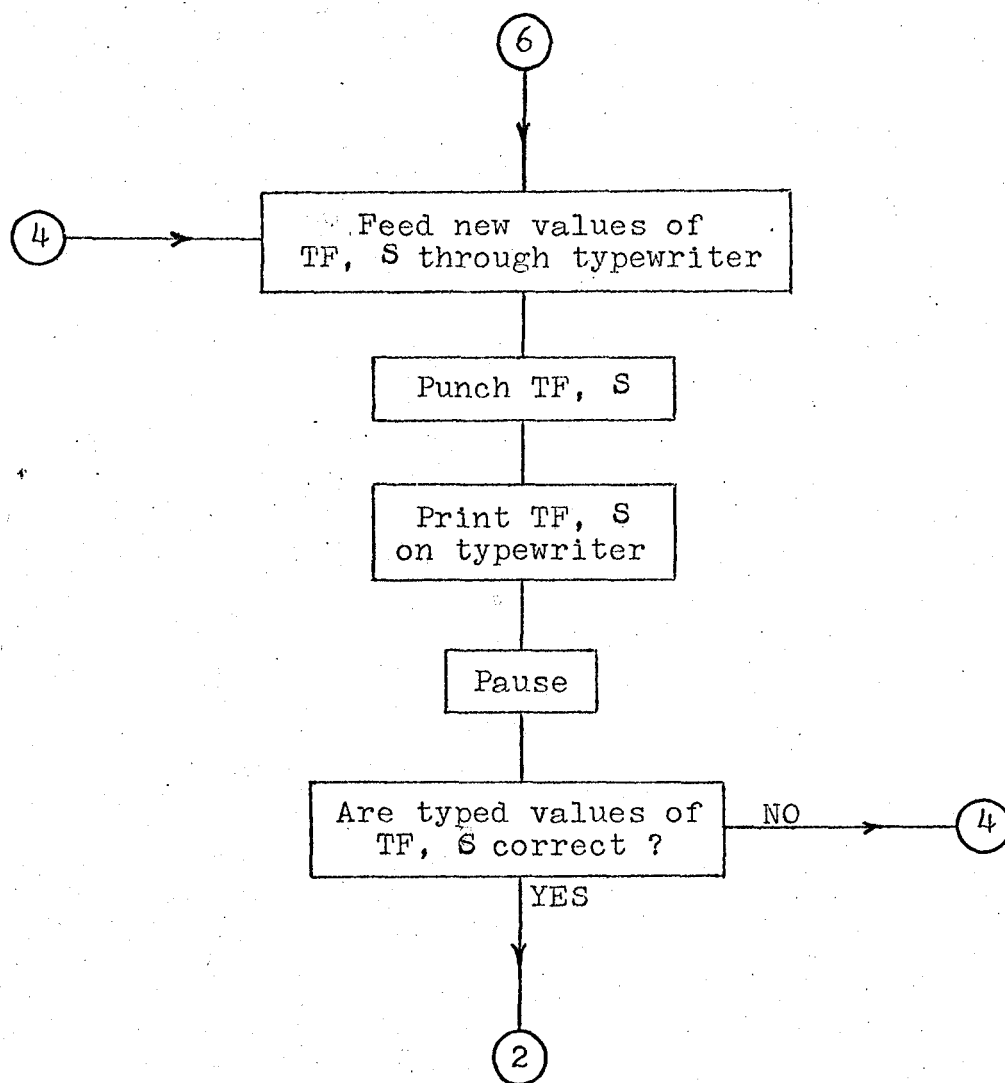
$$T_t = 0.2 \text{ sec}$$

APPENDIX III

FLOW DIAGRAM  
for  
The Digital Programs  
using  
Basic Runge Kutta Method or Gill Method.







NOTE :

1. The INPUT to the program consists of the following :
  - a) Initial values of variables  $i_f, i_{kd}, i_{kq}, i_d, i_q, p\delta, \delta, e_f, P_m'$  and  $P_m$ .
  - b) Coefficients of  $[Z]^{-1}$  and  $[R]$  matrices. The inversion of  $[Z]$  matrix is done earlier by another program.
  - c) Number of Differential Equations 'N'.
  - d) Initial Time 'T' and Final Time 'TF'.
  - e) Values of S and TP.
  - f) Various machine constants ( in per - unit ).
  - g) Step Size 'h' and Printing Interval 'PHI'.
  
2. The OUTPUT consists of the values of  $t, i_f, i_{kd}, i_{kq}, i_d, i_q, p\delta, \delta, e_f, P_m', P_m, e_d, e_q$ , Watts, Vars and Line Current.

VITA AUCTORIS

- 1939 Born on January 9, in Khurja, India.
- 1951 Completed High School education from M.M.H.V. Higher Secondary School, Ghaziabad, India.
- 1958 Graduated from Indian Institute of Technology, Kharagpur, India, with Bachelor of Technology(Honours) degree in Electrical Engineering.
- 1959 Joined Heavy Electricals (India) Ltd., Bhopal, India, as a Design Engineer.
- 1968 Candidate for M.A.Sc. degree in Electrical Engineering from University of Windsor.