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System identification by means of a digital computer method.

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SYSTEM IDENTIFICATION BY MEANS OF A
DIGITAL COMPUTER METHOD

by

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A Thesis
Submitted to the Faculty of Graduate Studies through the Department of
Electrical Engineering in Partial Fulfillment of the Requirements
for the Degree of Master of Applied Science
at the University of Windsor

Windsor, Ontario

1969

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ABSTRACT

System identification, that is, accurately obtaining the value of system parameters is an important problem in the simulation of many complex systems.

This thesis provides a method for obtaining the parameters of first and second order linear systems using Z-transform techniques and a digital computer. The accuracy of the identification exceeds that of an analog method referenced in the text. An extension to higher order systems is also proposed.

ACKNOWLEDGEMENTS

The author wishes to express his thanks to Dr. P.A.V. Thomas for his helpful suggestions and guidance.

Acknowledgement is also due to the National Research Council for its financial assistance.

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I. INTRODUCTION

The engineer is often faced with the problem of obtaining the characteristics of the differential equations representing a complicated system.

Simulation and design problems occurring in the determination of aerodynamic coefficients, dynamic characteristics of tires, automobile suspension systems and aircraft landing gears for example, require an accurate knowledge of the particular system to permit a meaningful simulation to be made. Recently a technique for 'System Identification By Means of a Implicit Synthesis Method' was presented by C.L. Sheng and M.Y. Wu [7]. The paper investigated the determination of the parameters in first order linear and non-linear systems and proposed a method for higher order systems. A general purpose EAI-TR-48 analog computer was used for the identification.

The purpose of this research was to try to equal or improve upon the accuracy of the Sheng-Wu identification of linear systems using a digital computer technique. The use of a digital computer would give the engineer a choice of tools, analog or digital computers, in the solution of a particular identification problem.

II. SAMPLED-DATA SYSTEM THEORY

2.1 Z-transform Theory

By the very nature of a digital computer, techniques for system identification require discrete information. Methods of analysis which use discrete data are called sampled-data system methods. A sampled-data system can be defined as one in which the flow of continuous information is transformed into a series of pulses or numbers. This sampling process is analogous to ideal switching. The discrete points or pulses can only relate to the continuous data at the sampling instants. That is, f(nT) can be completely determined from f(t) but f(t) can be known only at the discrete time intervals T,2T,3T, etc., when f(nT) is known. f(t) and f(nT) are the functions shown in Fig. 1. This train of pulses,

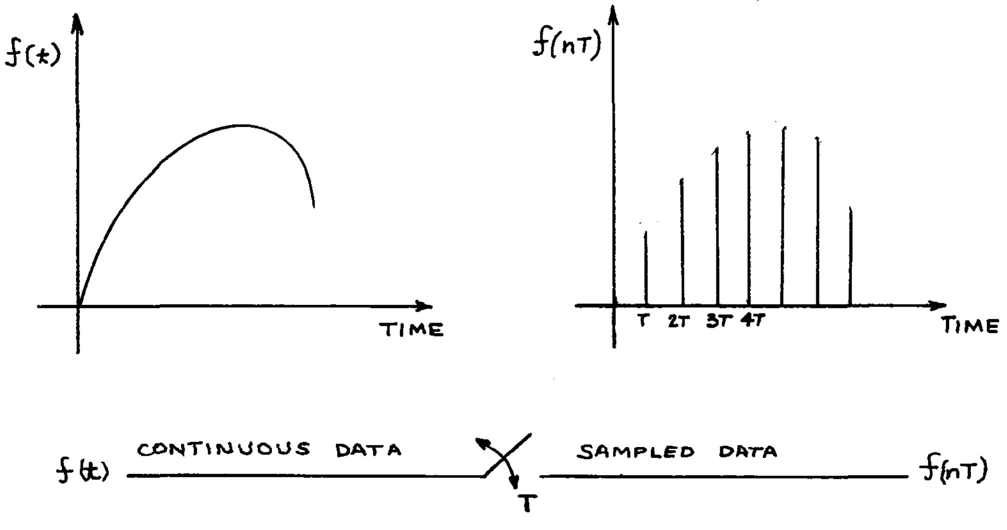

Fig. 1 Ideal Sampler, Sampling Period T.

$f(nT)$, can be analysed mathematically with the introduction of impulse modulation. For a continuous input function, $f(t)$, the sampled-data output function $f^*(t)$ consists of a series of impulses of areas $f(nT)$, i.e. mathematically,

$$f^*(t) = f(t)\ \delta^*(t) \qquad (2\text{-}1)$$

where $\delta^*(t)$ represents a unit impulse carrier train.

Expanding (2-1) we obtain

$$f^*(t) = f(t) \sum_{n=0}^{\infty} \delta(t - nT)$$

$$= \sum_{n=0}^{\infty} f(nT)\ \delta(t - nT) \qquad (2\text{-}2)$$

This becomes through standard Laplacian transformation

$$F^*(s) = \sum_{n=0}^{\infty} f(nT)\ e^{-nTs} \qquad (2\text{-}3)$$

Now, if for convenience we use the transformation $Z^{-n} = e^{-nTs}$ and use $F(Z)$ to represent the impulse modulated function, (2-3) becomes

$$F(Z) = \sum_{n=0}^{\infty} f(nT)\ Z^{-n}$$

$$= f(0) + f(T)\ Z^{-1} + \ldots\ldots + f(nT)\ Z^{-n} + \ldots \qquad (2\text{-}4)$$

Thus the Z-transform can be considered independently of a Laplace transform, with the exponent of Z being an ordering indicator. For an example of the calculation of a Z-transform, let $f(t) = e^{-at}$. Then modulating the unit impulse carrier with $f(t)$ we obtain

$$f^*(t) = e^{-0T} + e^{-aT}\ \delta(t-T) + e^{-2aT}\ \delta(t-2T) + \ldots$$

$$\ldots + e^{-anT}\ \delta(t-nT) + \ldots$$

i.e. $f^*(t) = 1 + e^{-aT}Z^{-1} + e^{-2aT}Z^{-2} + \ldots + e^{-anT} Z^{-n} + \ldots$

Writing this in closed form, it becomes

$$F(Z) = \frac{1}{1 - e^{-aT} Z^{-1}} \tag{2-5}$$

A table of standard Z-transforms can be found in Appendix A.

Assume now that F(Z) of (2-5) is a transfer function of the form $\frac{C}{R}(Z)$, where C(Z) is the output variable and R(Z) is the input variable. The numerical evaluation of C(Z) may be performed by several methods, two of which will be briefly described. One method utilizes conventional long division, but in this case the input R(Z) must be of a known form such as sine, ramp, step, etc.

$$C(Z) = \frac{R(Z)}{1 - e^{-aT} Z^{-1}}$$

If R(Z) is assumed to be a unit step function, then $R(Z) = \frac{1}{1 - Z^{-1}}$. C(Z) then becomes

$$C(Z) = \frac{1}{(1-Z^{-1})(1-e^{-aT}Z^{-1})}$$

$$= \frac{1}{1 - (1+e^{-aT})Z^{-1}+e^{-aT}Z^{-2}}$$

Evaluating by long division the output can be written

$$C(Z) = 1 + (1+e^{-aT})\ Z^{-1} + (1+e^{-aT} + e^{-2aT})\ Z^{-2} + \ldots$$

A more useful method and the one used exclusively in the identification technique to be described, is called the recursion method. It has the advantage that R(Z) can be a random input. Then

$$\frac{C}{R}(Z) = \frac{1}{1-e^{-aT}Z^{-1}}$$

i.e.

$$C(Z) - e^{-aT}\,C(Z)\,Z^{-1} = R(Z)$$

$$C(Z) = R(Z) + e^{-aT}\,C(Z)\,Z^{-1} \qquad (2\text{-}6)$$

which is equivalent to

$$C(nT) = R(nT) + e^{-aT}\,C[(n-1)T]$$

Now if R(Z) is a unit step function

then

$$C(1) = R(1) + e^{-aT}.C(0) = 1 \qquad \text{as } C(0) = 0$$

$$C(2) = R(2) + e^{-aT}.C(1) = 1 + e^{-aT}$$

$$C(3) = R(3) + e^{-aT}.C(2)$$

$$= 1 + e^{-aT} + e^{-2aT}$$

Thus the output C(Z) is the same by either method.

2.2 System Algebra

In addition to the Z-transform, a knowledge of the analysis of open loop transfer functions is necessary. Consider the two examples shown in Fig. 2.

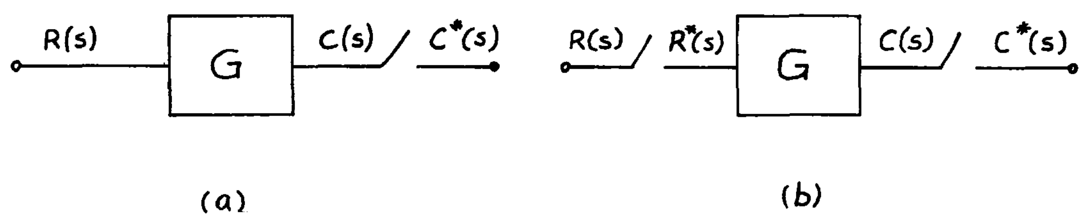

Fig. 2 (a) Continuous Input (b) Sampled Input

The output of the system in Fig. 2(a) is given by

$$C^*(s) = [R(s)G(s)]^* \tag{2-7}$$

while the output in 2(b) is

$$C^*(s) = [R^*(s)G(s)]^*$$

$$= R^*(s)G^*(s) \tag{2-8}$$

Now if, for example, $R(s) = \frac{1}{s}$ and $G(s) = \frac{a}{s+a}$ then (2-7) becomes

$$C^*(s) = [\frac{a}{s(s+a)}]^*$$

that is

$$C(Z) = \mathcal{Z}[\frac{a}{s(s+a)}]$$

where $\mathcal{Z}$ denotes the Z transform of the term in parenthesis.

$$C(Z) = \frac{Z(1 - e^{-aT})}{(Z-1)(Z-e^{-aT})}$$

Eq. (2-8) can be written

$$C(Z) = \frac{1}{Z-1} \cdot \mathcal{Z}[\frac{a}{s+a}]$$

$$= \frac{Z}{(Z-1)(Z-e^{-aT})}$$

Thus, the outputs $R^*(s)G^*(s)$ and $[R(s)G(s)]^*$ are not the same. The actual configuration of the system is therefore very important.

Cascaded elements are affected in the same manner. Two elements G_1 and G_2 separated by a sampling device would not give the same output as a system with G_1 and G_2 connected directly. The references [1], [2], and [3] go into this theory in much greater detail but this resumé is sufficient for an understanding of the applications to follow.

III. IDENTIFICATION

3.1 Method

It would be desirable to be able to identify the coefficients of a linear differential equation of any order which is of the form

$$\frac{d^nY}{dt^n} + A_1 \frac{d^{n-1}Y}{dt^{n-1}} + \ ..... + A_n Y = u(t) \qquad (3\text{-}1)$$

where $Y(t)$ = system output

$u(t)$ = system input

and A_1 through A_n are the unknown coefficients of the system and all initial conditions are zero.

This research will deal with first and second order systems although the method to be described may be extended to higher order systems.

This method depends upon a linear sampled-data model which assumes that the input and output samples of a given system are related by a Z-transform model of the following form

$$F(Z) = \frac{N(Z)}{D(Z)} \qquad (3\text{-}2)$$

where $N(Z) = \alpha_0 + \alpha_1 Z^{-1} + \ ..... \ \alpha_{n-1} Z^{-(n-1)}$

$D(Z) = 1 + \beta_1 Z^{-1} + \beta_2 Z^{-2} + \ .... \ \beta_n Z^{-n}$

and n indicates the order of the system.

Thus, if the coefficients of the Z-transform model can be identified, the time domain coefficients can be calculated using the known form of Z-transform for the order of system under study.

The input to the system to be identified can be of any form, but in most of the identifications, a step function of constant amplitude and starting at time zero was used. These step functions were sampled and the sampled values were used to drive the 'black box' system. The output of the system at each sampling instant was recorded and the corresponding values of input and output were listed together. This list of input and output samples is referred to as the input-output record, and contains all of the samples at the sampling instants until the input was removed. The input record can be expressed mathematically as the sum of the sampled values of input at the specific sampling time. Thus if the input is denoted by X(Z) then the total record of the input using normal Z-transform notation is

$$X(Z) = \sum_{n=0}^{m} X(nT)Z^{-n} \tag{3-3}$$

where m = number of samples

X(nT) indicates the value of the input at the sampling instant nT.

and Z^{-n} is the time ordering indicator.

Similarly the output W(Z) can be written as

$$W(Z) = \sum_{n=0}^{m} W(nT)\ Z^{-n}$$

where W(nT) indicates the value of the output at the sampling time nT.

These expressions for the input and output records can also be written as

$$X(Z) = \sum_{j=0}^{m} X_j Z^{-j}$$

$$W(Z) = \sum_{j=0}^{m} W_j Z^{-j}$$

where j indicates the time of sampling and W_j and X_j are the values of W(Z) and X(Z) at the time j, this being the form of notation used in later expressions.

If the same input is applied to the unknown system and the Z-transform model and the outputs of the two correspond exactly at each sampling instant, then the model is a true system model and the coefficients of the actual system are known. If the two outputs are not exactly the same however, the error between them indicates how close the model coefficients are to the true values. Thus if we were to minimize this error with respect to the coefficients of the model N(Z)/D(Z) we would obtain the best approximation to the system. The error determination is shown diagrammatically in Fig. 3.

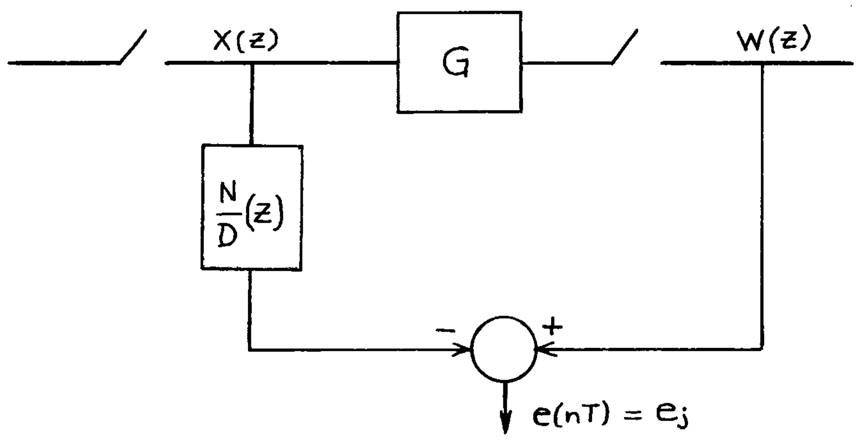

Fig. 3 True Model Error

The error at each sampling instant is e_j. The total effective error is found by summing the squares of the values of e_j over the record length. If we use the inversion integral to obtain the error, then the following error minimization is involved.

$$\Sigma\,(e_j)^2 = \frac{1}{2\pi j} \oint \left|X(Z)\cdot\frac{N(Z)}{D(Z)} - W(Z)\right|^2 \frac{dZ}{Z} = \text{minimum} \quad (3\text{-}5)$$

If this expression were minimized with respect to the coefficients α_i and β_i of the model, the result would give the best identification. This minimization however cannot be solved exactly [9].

A method suggested by Steiglitz and McBride [9] will now be developed to solve the above minimization problem. The minimization involved in Fig. 4, which has no physical interpretation, can be easily solved.

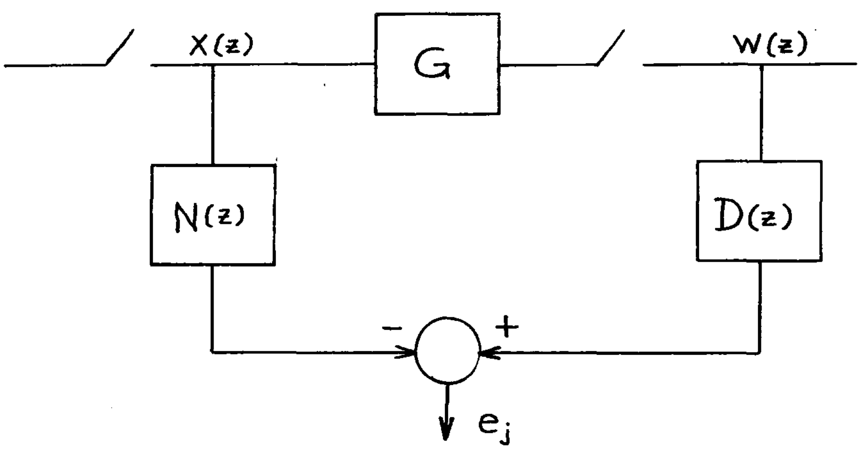

Fig. 4 Linear Regression Error

Again using the inversion integral the expression for the error is

$$\Sigma(e_j)^2 = \frac{1}{2\pi j} \oint |X(Z).N(Z)-W(Z).D(Z)|^2 \frac{dZ}{Z} = \text{minimum} \quad (3\text{-}6)$$

The error at any time j involves the values of input and output at time j plus several previous values. The number of previous values is dependent upon the order of the system. Thus the product at X(Z) and N(Z) can be written as

$$\left(\sum_{j=0}^{m} X_j Z^{-j}\right).(\alpha_0+\alpha_1 Z^{-1}+\alpha_2 Z^{-2} + \ldots + \alpha_{n-1} Z^{-(n-1)})$$

i.e. $\alpha_0 X_j + \alpha_1 X_{j-1} Z^{-1} + \ldots \alpha_{n-1} X_{j-n+1} Z^{-(n-1)}$

The effect of X(Z).N(Z) at time j is therefore

$$\sum_{i=0}^{n-1} \alpha_i X_{j-i}$$

Similarly the product of W(Z) and D(Z) at time j is $\sum_{i=0}^{n} \beta_i W_{j-i}$

The error at time j is then

$$e_j = \sum_{i=0}^{n-1} \alpha_i X_{j-i} - \sum_{i=0}^{n} \beta_i W_{j-i} \qquad (3\text{-}7)$$

or setting $\beta_0 = 1$ $\quad e_j = \sum_{i=0}^{n-1} \alpha_i X_{j-i} - \sum_{i=1}^{n} \beta_i W_{j-i} - W_j$

We can write this in matrix form if we let

$$\delta' = [\alpha_0, \alpha_1, \ldots \alpha_{n-1}, -\beta_1, -\beta_2, \ldots\ldots -\beta_n]$$

$$q'_j = [X_j, X_{j-1} \ldots\ldots X_{j-n+1}, W_{j-1}, \ldots\ldots W_{j-n}]$$

and δ and q_j be the transpose of δ' and q'_j.

Therefore

$$e_j = q'_j \delta - W_j$$

The total error can be obtained by squaring the sampled instant errors and summing over the record length.

$\Sigma(e_j)^2$ is the total error which we would like to minimize with respect to the coefficients α_i and β_i, i.e. with respect to the matrix δ. If we take the gradient of $\Sigma(e_j)^2$ with respect to δ we would obtain a minimization criterion

i.e. $\text{grad}\,(\Sigma(e_j)^2) = 2(\Sigma \frac{\partial e_j}{\partial \delta})\, e_j$

$$= 2\,\Sigma\, q_j\, e_j = 0$$

But $e_j = q'j\ \delta - W_j$

Therefore $\Sigma\, q_j q'_j\, \delta - \Sigma\, q_j W_j = 0$

let $Q = \Sigma\, q_j q'_j$ and $c = \Sigma\, q_j W_j$

Then the coefficients in δ are

$$\delta = Q^{-1} . c \tag{3-8}$$

for minimum error.

But this criterion does not mean anything as far as the original problem is concerned.

If the values of α_i and β_i and the corresponding values of N(Z) and D(Z) denoted as $N_1(Z)$ and $D_1(Z)$ which were found using the above technique are used to adjust the values of input and output such that

$$\frac{W_{new}(Z)}{W_{original}(Z)} = \frac{1}{1 + \beta_1 Z^{-1} + \ldots \beta_n Z^{-n}} = \frac{1}{D_1(Z)}$$

and

$$\frac{X_{new}(Z)}{X_{original}(Z)} = \frac{1}{1 + \beta_1 Z^{-1} + \ldots \beta_n Z^{-n}} = \frac{1}{D_1(Z)}$$

then new values of α and β are obtained. The new values of N(Z) and D(Z) then become $N_2(Z)$ and $D_2(Z)$ respectively. This procedure is continued several times. If we let i equal the number of iterations, that is, the number of times the original equation $\delta = Q^{-1}.c$ is solved then on the i^{th} iteration we solve for $N_i(Z)$ and $D_i(Z)$ with the original input and output records being filtered by $D_{i-1}(Z)$, the value of D(Z) found on the previous interation. The error minimization on this i^{th} iteration involves the equation (3-9) derived from the diagram shown in Fig. 5.

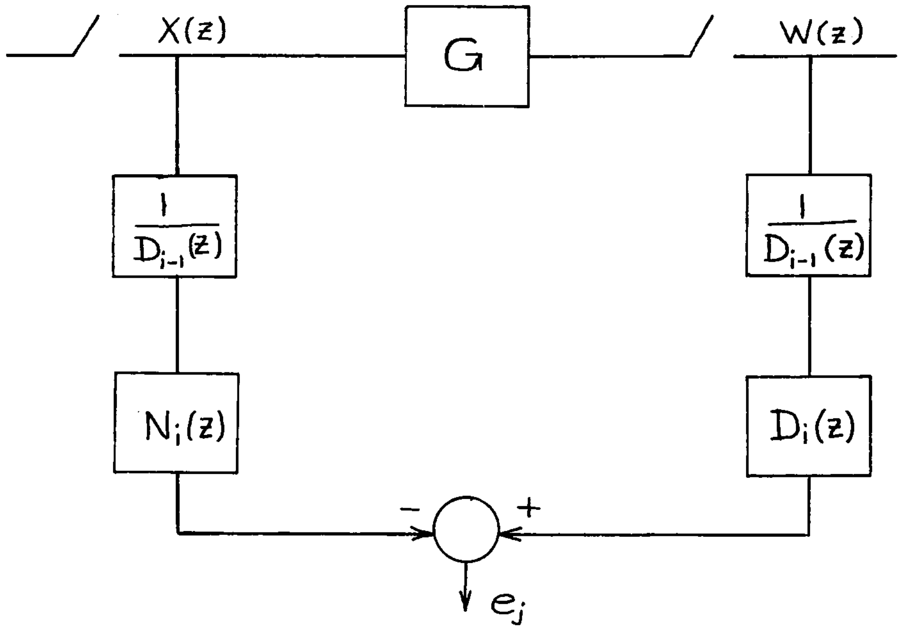

Fig. 5 Iterative Model Error

$$\Sigma(e_j)^2 = \frac{1}{2\pi j} \oint \left| X(Z) \frac{N_i(Z)}{D_{i-1}(Z)} - W(Z) \cdot \frac{D_i(Z)}{D_{i-1}(Z)} \right|^2 \frac{dZ}{Z} = \text{min} \tag{3-9}$$

If on the i^{th} iteration the values found for α and β are the same as the values found on the previous iteration, then $D_i(Z) = D_{i-1}(Z)$ and the error minimzation equation becomes

$$\oint \left| X(Z) \cdot \frac{N_i(Z)}{D_i(Z)} - W(Z) \right|^2 \left| \frac{D_i(Z)}{D_{i-1}(Z)} \right|^2 \frac{dZ}{Z} = \text{min.}$$

i.e.

$$\oint \left| X(Z) \cdot \frac{N_i}{D_i}(Z) - W(Z) \right|^2 \frac{dZ}{Z} = \text{min.} \tag{3-10}$$

Thus, the iterative procedure provides a solution to the original true model error minimization at convergence of the coefficients.

3.2 First Order Linear System

By using the iterative procedure described in Section 3.1, a first order system Z-transform model takes the form

$$\frac{N}{D}(Z) = \frac{\alpha_0}{1+\beta_1 Z^{-1}} = \frac{1}{1+\beta_1 Z^{-1}} \tag{3-11}$$

Having found β_1 we must now relate the Z-transform to the time domain coefficients of the equation

$$\dot{Y} + \lambda Y = u(t)$$

This equation may be written in a transfer function form using Laplace transformation as

$$\frac{Y}{u}(s) = \frac{1}{s+\lambda}$$

This equation can now be written in terms of Z-transforms as

$$\frac{Y}{u}(Z) = \frac{1}{1-e^{-\lambda T} Z^{-1}}$$

Therefore $\beta_1 = -e^{-\lambda T}$

i.e.

$$\lambda = \frac{-\ln(-\beta_1)}{T} \tag{3-12}$$

where ln = natural logarithm

T = sampling period

β_1 = coefficient of D(Z).

3.3 Second Order Systems

The Z-transform model for second order systems is

$$\frac{N}{D}(Z) = \frac{\alpha_0 + \alpha_1 Z^{-1}}{1 + \beta_1 Z^{-1} + \beta_2 Z^{-2}} \tag{3-13}$$

The coefficients $\alpha_0, \alpha_1, \beta_1$ and β_2 must now be equated to the time domain coefficients of the general second order system

$$\ddot{Y} + A\dot{Y} + BY = u(t) \tag{3-14}$$

Using Laplace transforms again, (3-14) becomes

$$s^2 Y(s) + S\ A\ Y(s) + B\ Y(s) = u(s)$$

or

$$\frac{Y}{u}(s) = \frac{1}{S^2 + As + B} = \frac{1}{(s+a_1)(s+a_2)} \tag{3-15}$$

where $s = -a_1, -a_2$ are the roots of the characteristic equation $s^2 + As + B$.

(3-14) becomes upon application of Z-transformation

$$\frac{Y}{u}(Z) = \frac{1}{a_2 - a_1} \cdot \frac{(e^{-a_1 T} - e^{-a_2 T})Z^{-1}}{1-(e^{-a_1 T} + e^{-a_2 T})Z^{-1} + e^{-(a_1+a_2)T}Z^{-2}} \tag{3-16}$$

Therefore equating coefficients of (3-13) and (3-16)

$$\beta_1 = -(e^{-a_1 T} + e^{-a_2 T})$$

$$\beta_2 = e^{-(a_1+a_2)T}$$

We can solve for a_1 and a_2

$$a_1 = \frac{-\ln}{T}\left(\frac{-\beta_1 + \sqrt{\beta_1^2 - 4\beta_2}}{2}\right)$$

$$a_2 = \frac{-\ln}{T}\left(\frac{-\beta_1 - \sqrt{\beta_1^2 - 4\beta_2}}{2}\right) \tag{3-17}$$

Then the coefficients of (3-14) become

$$A = a_1 + a_2$$

$$B = a_1 \cdot a_2$$

3.4 Higher Order Systems

Systems of a higher order than two may also be identified. If a look up table is developed to relate the time domain coefficients of such systems to the equivalent Z-transform coefficients, this table can then be stored in computer memory and the appropriate relationships can be retrieved when a particular order of system is to be identified.

A few examples will be shown to illustrate the usefulness of this technique.

3.5 Computer Solution

The input and output records used as data for the computer identification were generated by means of a recursion forumla of the appropriate order. This digital generation permitted many systems to be tested without any difficulty obtaining data. The number of samples obtained and the sampling period are known and are also input to the program. The identification of first and second order systems was accomplished by using two programs. These programs were written in Fortran and were run on an I.B.M. 1620. They are now described with reference to Fig. 6.

The values of input and output obtained from the recursion formula are first read into the computer along with T - the sampling period and N - the order of the system. Each value of X(Z) the input, and W(Z) the output, is stored as an element of vector XJ and WJ respectively for easy access. The solution of Eq.(3-8) is implemented. The matrices Q and c are labelled SQUE and SUMCI respectively. The matrix q_j is named QJ and its transpose q_j' is QJT. The parameters II -

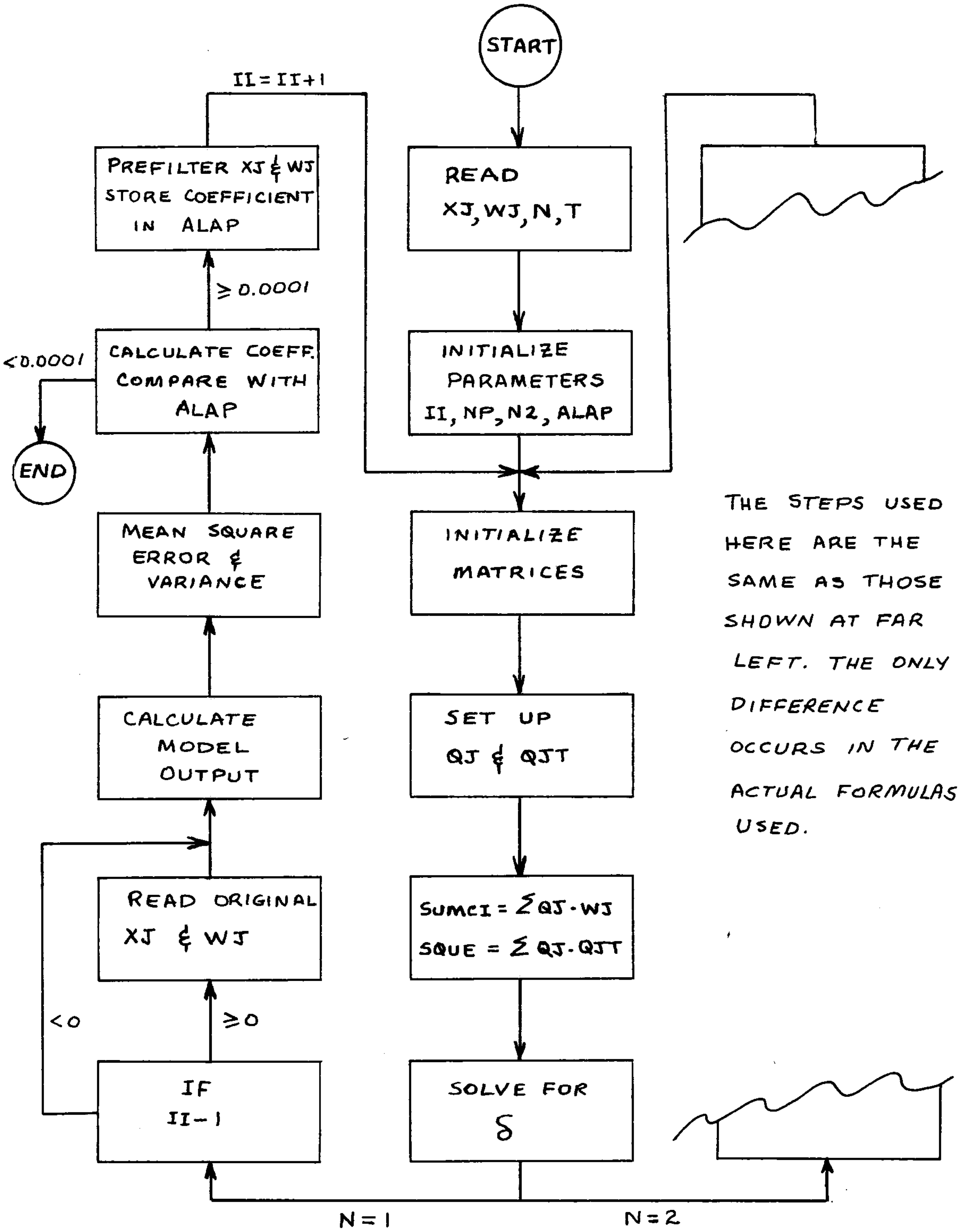

FIG. 6. PROGRAM FLOW CHART

the iteration counter, N2 and NP, and ALAP which contains the previous values of the time domain coefficients are set to their initial values. SQUE and SUMCI are initially set to zero. The appropriate values of XJ and WJ are stored in the matrices QJ and QJT. QJ is then used to form the matrix CI = QJ.WJ and CI is summed over the record length and the resultant stored in SUMCI. Also the product of QJ and QJT is summed and stored in SQUE where SQUE = Σ QJ.QJT. The matrix SQUE is inverted using a library subroutine and the resultant premultiplies SUMCI. This product is the coefficient matrix δ. Up to this point in the calculations the order of the system is immaterial as long as it is specified. Here the two programs begain to differ. The calculations have essentially the same form except that the actual formulas for model output, etc. are specifically for first or second order systems. The number of iterations already performed is checked by testing II. If II is other than zero the original input output record is reread. Using the coefficients of the δ matrix and the input record XJ the output of the model N(Z)/D(Z) is calculated, and this output is compared with the actual output WJ of the unknown system. The mean square error and the variance of this error are calculated and are used as the criterion for selecting the best identification if convergence of the coefficients is not obtained. The time domain coefficients λ for first order and A,B for second order systems are then calculated from the elements of the δ matrix. These values are compared with the coefficients which were calculated on the previous iteration and stored in ALAP. If they have converged to within a preset value (0.0001) the coefficients are punched out and the program ends. If convergence has not been obtained the original XJ and WJ are prefiltered by means of the digital filter $1/D_{II}$ as described in Section 3.1. The new coefficients replace the previous ones in ALAP, the counter II is increased by one, and the matrices SQUE and SUMCI are reinitialized. If convergence is not obtained after several iterations the program may be stopped and the error values can be used to select the best approximation to the coefficients.

IV. EXPERIMENTAL RESULTS

Several systems were identified using the method described. Various sampling periods and numbers of samples were used for some systems. Also various inputs were used. The results of these tests are listed in the following tables.

TABLE 1 First Order Systems

$\dot{Y} + \lambda Y = u(t)$ where $\lambda = 1.000$ and 200 samples

Tsec.	u(t)	λ	% Error
0.5	1.0	1.000134	+0.0134
0.2	1.0	0.9999923	-0.00077
0.1	1.0	1.0000045	+0.00045
0.05	1.0	1.0000088	+0.00088
0.02	1.0	0.99988	-0.012

TABLE 2 $\dot{Y} + \lambda Y = u(t)$ where $\lambda = 1.000$ and 150 samples

Tsec	u(t)	λ	% Error
0.5	1.0	1.0000068	+0.0068
0.2	1.0	0.9999923	-0.00077
0.1	1.0	0.999949	-0.0051
0.05	1.0	1.000029	+0.0029
0.02	1.0	0.99989	-0.011

TABLE 3 $\dot{Y} + \lambda Y = u(t)$ where $\lambda = 7.5320$ and 200 samples

Tsec	u(t)	λ	% Error
0.5	1.0	7.53671	+0.06
0.1	1.0	7.53364	+0.02
0.02	1.0	7.53300	+0.013
0.01	1.0	7.53207	+0.001
. .	. . .	. . .	. . .
0.1	5.0	7.53173	-0.004

TABLE 4 $\dot{Y} + \lambda Y = u(t)$ $\lambda = 20.150$ and 200 samples

Tsec	u(t)	λ	% Error
0.1	1.0	20.1111	-0.20

TABLE 5 $\dot{Y} + \lambda Y = u(t)$ $\lambda = 1.000$ and 200 samples

Tsec	u(t)	λ	% Error
0.1	3.0	0.999927	-0.0073
0.1	5.0	1.0000045	+0.00045
0.01	t	1.00019	+0.019

SECOND ORDER SYSTEMS

TABLE 6 $\ddot{Y} + A\dot{Y} + BY = u(t)$ and 200 samples

Tsec	ACTUAL VALUE		IDENTIFIED VALUE	
	A	B	A	B
0.1	35.0	300.0	35.0856	301.062
1.0	6.0	5.0	5.9914	4.9934
1.0	5.0	6.0	4.9908	5.9844
0.5	5.0	6.0	4.9850	5.9777

TABLE 7 SECOND ORDER IDENTIFICATION ERRORS

T	ACTUAL COEFFICIENTS		PERCENTAGE ERROR	
	A	B	A	B
0.1	35.0	300.0	+0.246	0.354
1.0	6.0	5.0	-0.143	-0.133
1.0	5.0	6.0	-0.184	-0.260
0.5	5.0	6.0	-0.52	-0.37

V. DISCUSSION OF RESULTS

The analog identification technique of Sheng and Wu [7] is based on the principle of steepest descent and utilizes an implicit synthesis method. It is used for first and second order linear systems and also nonlinear first order systems. A method to identify higher order linear systems using the same implicit synthesis circuits is also proposed. The systems to be identified were simulated using an analog computer circuit. The results of their method were compared to those of the digital method described.

In the digital method, the fact that the input was sampled; that is, interrupted, imposed a minor restriction. The system to be identified could not be run under normal operating conditions. However, since the major application of any identification technique is in the research field, this restriction is not felt to be of any great significance.

The input-output records were checked to make sure that the number of initializing zeros equalled the order of the system. If this condition was not true the data was misleading and caused erroneous identification.

The first order systems identified demonstrated the accuracy of this method. The system $\dot{Y} + Y = u(t)$, that is $\lambda = 1.000$ was identified using several different values of u(t), including a ramp function. All of the identified values of λ were within 0.02%. The same system was identified by the analog method with an accuracy of 0.1%. The first order system $\dot{Y} + 7.5320\ Y = u(t)$ was also identified very accurately.

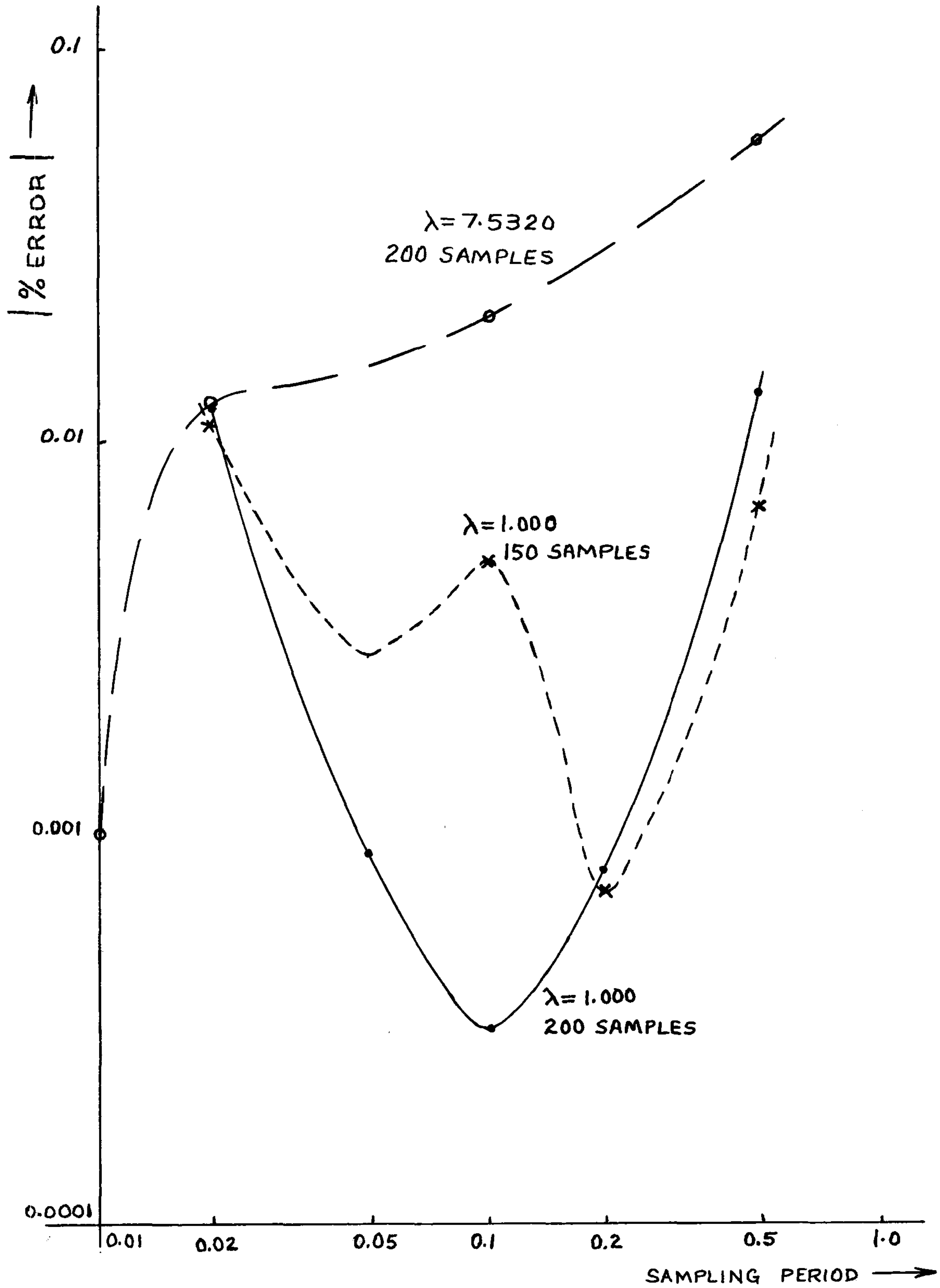

Fig. 7 |% Error| Versus Sampling Period

The parameter λ was found to be 7.5367 in the worst case and 7.53207 in the best case. A third system $\dot{Y} + 20.150\ Y = u(t)$ was also identified to within 0.2% but this value could be improved with the proper selection of sampling period. Tables 1,2,3 list the identified values and associated errors for two first order systems with several different sampling periods. Figure 7 shows that the best results are obtained when the total sampling time is equal to approximately 20-30 times the time constant of the system.

The selection of a sampling period is determined by two criteria. If the system response is as shown in Fig. 8 and the

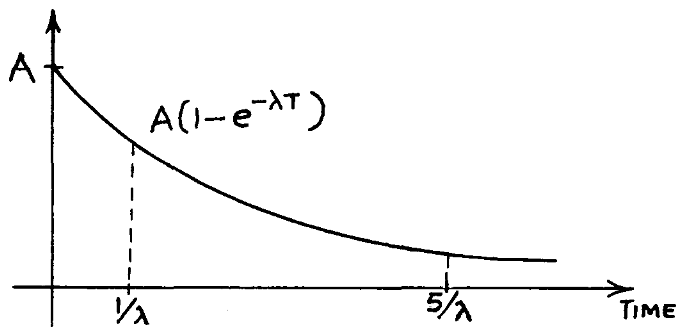

Fig. 8 Time Response of First Order System

sampling period T is large with respect to the time constant $1/\lambda$ the sampling is poor and therefore the error is higher. When nT, the total sampling time is small with respect to $5/\lambda$, only part of the system response is sampled and the error is higher. There is therefore an optimum sampling period which is small compared to $1/\lambda$ providing the total sampling time nT is large compared to the time constant of the system. For the system with $\lambda = 1.000$ the associated time constant is $\tau = 1/\lambda = 1.00$ sec. With two hundred samples and a period of 0.1 seconds the best identification of $\lambda = 1.000$ was obtained. The total time in this case is approximately twenty times the time constant and the sampling period is small compared with the time constant which satisfies the two criteria. Fig. 7 shows that as the sampling rate increases the absolute value of error increases and as the sampling period becomes quite large (the total sampling time is long) the absolute error again increases. Similar results were obtained for $\lambda = 1.00$ with 150 samples and $\lambda = 7.5320$ with 200 samples. Therefore once an approximation of the system is obtained, a proper value of T can be chosen to obtain the best identification.

The identification of second order systems although not as accurate as the first order systems was within 0.5% in even the worst case. For second order system the sampling period must be chosen to satisfy two criteria. According to the sampling theorem the period of sampling must be at least $T = 1/2W$ where W is the highest frequency present. Also the total sampling time must be sufficient to allow the system to settle which sets a lower limit on the period T. If a period of approximately 1/2 or 1/3 the upper limit is used, an approximation of the system can be found. This is possible if the bandwidth of the system can be approximated even roughly. The period can then be reduced until the error starts to rise again. For example the system $\ddot{Y} + 5\dot{Y} + 6Y = u(t)$ can be identified at $T = 1.0$ secs. with greater accuracy than at 0.5 secs. The maximum value of T in this case would be approximately $T = 2.0$ secs.

By applying the method suggested by McBride and Steiglitz with a sampled input instead of a continuous one a greater accuracy of identification has been achieved. Similar results could be expected for higher order systems.

VI. CONCLUSIONS

The digital method described in this research offers an alternative to analog identification for linear systems of any order. The look-up table for higher order systems would consist of relationships which would be of value to the particular user.

Both systems, the digital and analog, give acceptable results and the choice of one over the other would depend upon available equipment and the personnel involved.

The use of a digital computer technique for system identification makes available an additional tool to the engineer involved in design and simulation.

APPENDIX A

TABLE A.1 Z Transforms

f(t)	F(s)	F(Z)
1	$\frac{1}{s}$	$\frac{Z}{Z-1}$
t	$\frac{1}{s^2}$	$\frac{TZ}{(Z-1)^2}$
e^{-at}	$\frac{1}{s+a}$	$\frac{Z}{Z-e^{-aT}}$
$e^{-aT}-e^{-bT}$	$\frac{b-a}{(s+a)(s+b)}$	$\frac{Z(e^{-aT}-e^{-bT})}{(Z-e^{-aT})(Z-e^{-bT})}$
sin at	$\frac{a}{s^2+a^2}$	$\frac{Z \sin aT}{Z^2-2Z\cos aT+1}$
cos at	$\frac{s}{s^2+a^2}$	$\frac{Z(Z-\cos aT)}{Z^2-2Z\cos aT+1}$

APPENDIX C

FIRST ORDER SYSTEMS

```
      DIMENSION SUMCI(8,1),QJ(8),QJT(8),CI(8,1),DELTA(8,1)
      DIMENSION XJ(200),WJ(200)
      DIMENSION A(25,25),B(25,25),W(25),SQUE(8,8)
      DIMENSION AL(200)
  808 FORMAT(F12.7)
      READ 808,N
      READ 808,T
      READ 808,(XJ(J),WJ(J),J=1,200)
      N2=2*N
      XLAP=0.
      II=0
  800 SQUE(1,1)=0.000
      SUMCI(1,1)=0.
      DO 600 K=1,N2
      DO 600 L=1,N2
  600 SQUE(L,K)=0.0000
      DO 700 K=1,N2
      L=1
  700 SUMCI(K,L)=0.00000
      J=0
      NP=N+1
      DO 100 J=NP,200
      DO 27 I=1,N
      K=I-1
      L=J-K
      QJ(I)=XJ(L)
   27 QJT(I)=XJ(L)
      DO 28 I=NP,N2
      M=J-I+N
      QJ(I)=WJ(M)
   28 QJT(I)=WJ(M)
      CI(1,1)=0.
      DO 7 K=1,N2
      L=1
      X=QJ(K)
      Y=QJT(K)
      CI(K,L)=X*WJ(J)
      SUMCI(K,L)=SUMCI(K,L)+CI(K,L)
      A(K,L)=X
    7 B(L,K)=Y
      MA=N2
      NA=1
      MB=1
      NB=N2
      COMMON A,MA,NA,B,MB,NB,W
      CALL P[illegible]
```

```
      DO 6 K=1,N2
      DO 6 L=1,N2
    6 SQUE(K,L)=SQUE(K,L)+A(K,L)
  100 CONTINUE
      DO 8 K=1,N2
      DO 8 L=1,N2
    8 A(K,L)=SQUE(K,L)
      MA=N2
      NA=N2
      CALL P13(DETRM)
      DO 9 K=1,N2
      L=1
    9 B(K,L)=SUMCI(K,L)
      MA=N2
      NA=N2
      MB=N2
      NB=1
      CALL P09
      PUNCH 2,((A(I,J),J=1,1),I=1,N2)
      ALA=-LOGF(A(2,1))/T
      PUNCH 2,ALA
      IF(II-1)43,44,44
   44 PAUSE
      READ 551 ,(XJ(J),WJ(J),J=1,200)
  551 FORMAT(1X,F10.7,2X,F12.7)
   43 DO 801 K=1,N
      WJ(K)=0.
  801 XJ(K)=0.
      BWJ=0.
      AE(I)=0.0
      ERROR=0.00
      DO 887 K=NP,200
      AWJ=A(1,1)*XJ(K)+A(2,1)*BWJ
      AE(K)=(AWJ-WJ(K))**2
      ERROR=ERROR+AE(K)
  887 BWJ=AWJ
      ANP=200-NP
      ERROR=ERROR/ANP
      PUNCH 889,ERROR
  889 FORMAT(9HERROR IS ,E16.8)
      VARI=0.
      VART=0.
      DO 887 K=NP,200
      VARI=(ERROR-AE(K))**2
  887 VART=VART+VARI
      ANP=200-NP
      VART=VART/ANP
      PUNCH 886,VART
  886 FORMAT(21HVARIANCE OF ERROR IS ,E16.8)
      IF(ABSF(ALA-ALAP)-0.0001)99,99,837
  837 CONTINUE
```

```
      DO 302 K=4,200
      XJ(K)=XJ(K)+A(2,1)*XJ(K-1)
  302 WJ(K)=WJ(K)+A(2,1)*WJ(K-1)
      II=II+1
      PRINT 388,II
      PUNCH 388,II
      KL=KL+1
      ALAP=ALA
      GO TO 800
   99 PUNCH 23,ALA
   23 FORMAT(18HTAL COEFFICIENT IS,F12.7)
  388 FORMAT(5H II= I2)
    2 FORMAT(4E16.8)
  333 FORMAT(I1)
  334 FORMAT(1X,F12.7,2X,F12.7)
      CALL EXIT
      END
```

SECOND ORDER SYSTEMS

```
      DIMENSION SUMCI(8,1),QJ(8),QJT(8),CI(8,1),DELTA(8,1)
      DIMENSION XJ(200),WJ(200)
      DIMENSION A(25,25),B(25,25),W(25),SQUE(8,8)
      DIMENSION AL(200)
      READ 555,N
      READ 998,T
      READ 550,(XJ(J),WJ(J),J=1,200)
      A2P=0.0
      B2P=0.0
      N2=2*N
      II=0
  808 SQUE(1,1)=0.000
      SUMCI(1,1)=0.
      DO 600 K=1,N2
      DO 600 L=1,N2
  600 SQUE(L,K)=0.0000
      DO 700 K=1,N2
      L=1
  700 SUMCI(K,L)=0.00000
      J=0
      NP=N+1
      DO 100 J=NP,200
      DO 27 I=1,N
      K=I-1
      L=J-K
      QJ(I)=XJ(L)
   27 QJT(I)=XJ(L)
      DO 28 I=NP,N2
      M=J-I+N
      QJ(I)=WJ(M)
   28 QJT(I)=WJ(M)
      CI(1,1)=0.
      DO 7 K=1,N2
      L=1
      X=QJ(K)
      Y=QJT(K)
      CI(K,L)=X*WJ(J)
      SUMCI(K,L)=SUMCI(K,L)+CI(K,L)
      A(K,L)=X
    7 B(L,K)=Y
      MA=N2
      NA=1
      MB=1
      NB=N2
      COMMON A,MA,NA,B,MB,NB,W
      CALL P09
      DO 6 K=1,N2
      DO 6 L=1,N2
    6 SQUE(K,L)=SQUE(K,L)+A(K,L)
```

```
  100 CONTINUE
      DO 8 K=1,N2
      DO 8 L=1,N2
    8 A(K,L)=SQUE(K,L)
      MA=N2
      NA=N2
      CALL P13(DETRM)
      DO 9 K=1,N2
      L=1
    9 B(K,L)=SUMC1(K,L)
      MA=N2
      NA=N2
      MB=N2
      NB=1
      CALL P09
      PUNCH 2,((A(I,J),J=1,1),I=1,N2)
      IF(II-1)43,44,44
   44 PAUSE
      READ 551 ,(XJ(J),WJ(J),J=1,200)
  551 FORMAT(1X,F10.7,2X,F12.7)
   43 DO 301 K=1,N
      WJ(K)=0.
  301 XJ(K)=0.
      BWJ1=0.0
      BWJ=0.
      AE(1)=0.0
      ERROR=0.000
      DO 887 K=NP,200
      XY=A(1,1)*XJ(K)+A(2,1)*XJ(K-1)
      AWJ=XY+A(3,1)*BWJ+A(4,1)*BWJ1
      BWJ1=BWJ
      AE(K)=(AWJ-WJ(K))**2
      ERROR=ERROR+AE(K)
  887 BWJ=AWJ
      ANP=200-NP
      ERROR=ERROR/ANP
      PUNCH 889,ERROR
  889 FORMAT(20HERROR IS ,E16.8)
      VARI=0.
      VART=0.
      DO 557 K=NP,200
      VARI=(ERROR-AE(K))**2
  557 VART=VART+VARI
      ANP=200-NP
      VART=VART/ANP
      PUNCH 556,VART
  556 FORMAT(21HVARIANCE OF ERROR IS ,E16.8)
      FORAC=SQRTF((A(3,1))**2-4.*(-A(4,1)))
      A1=-LOGF((A(3,1)+FORAC)/2.)
      B1=-LOGF((A(3,1)-FORAC)/2.)
      A2=-(A1+B1)
      B2=A1*B1
      IF(ABSF(A2P-A2)-0.001)803,803,804
```

```
806 IF(ABSF(B2P-B2)-0.   1)  7,  7,804
807 CONTINUE
    A2=A2/T
    B2=B2/(T**2)
 99 PUNCH 23,A2,B2
    GO TO 9[illegible]
 23 FORMAT(21HTHE COEFFICIENTS ARE ,F12.7,3X,F12.7)
804 A2P=A2
    B2P=B2
    PUNCH 886,A2,B2
886 FORMAT(F12.7,2X,F12.7)
    DO 802 K=NP,4[illegible]
    WJ(K)=WJ(K)+A(3,1)*WJ(K-1)+A(4,1)*WJ(K-2)
802 XJ(K)=XJ(K)+A(3,1)*XJ(K-1)+A(4,1)*XJ(K-2)
    II=II+1
    PRINT 888,II
    PUNCH 888,II
    GO TO 808
888 FORMAT(5H II= I2)
998 FORMAT(F12.7)
  2 FORMAT(4E16.8)
555 FORMAT(I1)
556 FORMAT(1X,F12.7,2X,F12.7)
999 CONTINUE
    CALL EXIT
    END
```

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