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Behaviour of reinforced and prestressed waffle slabs.

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BEHAVIOUR OF REINFORCED AND PRESTRESSED WAFFLE SLABS

by

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To my wife and parents

ABSTRACT

Reinforced concrete waffle slabs have been used quite often in buildings and other structures, resulting in a reduced dead weight and material cost. The use of prestressed concrete waffle slabs for rectangular and skew decks of short and medium span bridges can also lead to further economics in dead weight and material.

In this investigation, a series solution for the analysis of rectangular and skew concrete waffle slabs by the orthotropic plate theory is presented. Single spans of reinforced and prestressed concrete deck bridge models are investigated. The deflection function of the slab is assumed in the form of a Fourier series so as to satisfy the governing differential equation of equilibrium. The arbitrary constants in the deflection function are chosen to satisfy the appropriate boundary conditions. The in-plane prestressing force along the edges and the resulting edge moment are represented by a Fourier series using a graphical technique.

Available computer program for solving orthotropic plates or slabs subjected to lateral loads is modified to compute the stresses and deflection due to the prestressing force. This computer program can be used for reinforced and prestressed concrete waffle slabs of rectangular and skew shapes subjected to uniform as well as concentrated

transverse loads.

The experimental study is carried out on three one-eighth models of reinforced concrete waffle slabs and two one-eighth models of prestressed concrete waffle slabs. The first slab is tested under uniformly distributed load applied by means of air pressure while the remaining slabs are tested under concentrated loads only at various positions; the tests were carried in the elastic domain as bridge slabs and finally to collapse. The effect of concrete cracking on the rigidities of the slabs is studied. The strains and deflections obtained from the tests are found to be in satisfactory agreement with the theoretical solution. Furthermore, theoretical studies are carried out on two types of structures, the waffle-type and the slab-type with a uniform thickness, both structures having the same volume of concrete and reinforcing steel. The comparison of stresses and deflection for the two types of structures show that the waffle type exhibits much smaller deflections and lower stresses, especially for prestressed concrete waffle slabs.

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LIST OF ABBREVIATIONS

$A_{j1} - A_{j8}$	Definite integrals (Appendix A)
A_n	Arbitrary function in n (Equation 5.1)
A_s (A_s')	Area of tension steel in longitudinal (transverse) rib
a	Skew semi-width of the slab
$B_{j1} - B_{j2}$	Definite integrals defined (Appendix A)
B_n	Arbitrary function in n (Equation 5.1)
b	Half span length of the slab
b_y (b_x)	Width of longitudinal (transverse) rib
$C_{1n} - C_{16n}$	Arbitrary functions dependent on n
$C_{17} - C_{24}$	Arbitrary constants
c	$\cos\theta$
D	Flexural rigidity of the flange plate with respect to its middle plane
D_x, D_y	Flexural rigidities of the slab per unit width and length respectively
D_{xy}, D_{yx}	Transverse and longitudinal torsional rigidities respectively
D_1, D_2	Coupling rigidities - contribution of bending to torsional rigidity of the slab
d" (d')	Concrete cover to centre of longitudinal (transverse) reinforcement
EI	Flexural rigidity of the edge beam
E	Modulus of elasticity of concrete

e_x (e_y)	Depths of neutral plane from top fibre for bending in the x - (y) directions
f_c'	28-days compressive strength of concrete
G_{xy}, G_{yx}	Shear modulii of the orthotropic slab
GJ	Torsional rigidity of the edge beam
H	Effective torsional rigidity of the slab
h	Thickness of flange plate
I_x' (I_y')	Moment of inertia of transverse (longitudinal) ribs
I_{xy}, I_{yx}	Torsional constants
M_x, M_y, M_{xy}	Bending and torsional moments associated with the x and y axis
m	Integer > 1, number of harmonics
n	Integer > 1, number of harmonics
Q_x	Shear force per unit width of the plate
$q(u,v)$	Load function
S_x (S_y)	Spacing of ribs in transverse (longitudinal) direction
W	Deflection function
W_c	Complementary solution
W_p	Particular solution
x,y,z	Rectangular axes
μ	Poission's ratio of concrete
θ	Skew angle

α_m	$= m\pi/a$
α_n	$= n\pi/a$
β_m	$= m\pi/b$
β_n	$= n\pi/b$
ε_x, ε_y	Strain in x and y directions
τ_x, τ_y	Plane stress per unit width (length)

CHAPTER I

INTRODUCTION

1.1 General

In recent years reinforced and prestressed concrete waffle slabs have become quite popular in buildings and deck bridges. Orthotropic plate structures are often required as component parts of large scale structures as floor system in buildings, auditoriums, aircrafts, ship bottoms, tunnels, highway structures, etc. The trend towards high-rise buildings, modern highway interchanges and the commercial availability of high strength light-weight concrete have re-focused attention on concrete waffle slabs all over the world. The use of prestressed concrete waffle slabs for rectangular and skew decks of short and medium span bridges can also lead to economical design by having a crack-free concrete wearing surface, lower maintenance costs and better live load distribution.

1.2 Objective

The primary objective of this investigation is to determine the behaviour of reinforced and prestressed concrete waffle slabs over the total range of loading up to the point of collapse, from the standpoint of deformation, stresses, cracking and ultimate strength capacity. A waffle slab can be classified as geometrically orthotropic; as such its orthotropic flexural and twisting rigidities must be accurately predicted before and after cracking of the concrete in order to reliably estimate its performance under working and collapse loads. In general, the deflection of a concrete waffle slab is rather small in comparison with the thickness of the slab so that for a proper design of such structures a linear (small deflection) analysis is sufficient. An experimental investigation and a theoretical study based on a series solution are undertaken; this solution is found by superimposing three solutions due to three different loadings, namely: the transverse loads (uniform as well as concentrated load at various locations), in-plane edge loads and edge moments, the latter two being due to in-plane prestressing.

1.3 Scope

The test of a structural concrete one-eighth scale "direct" model can simulate the behaviour of the prototype both before and after cracking of the concrete. This investigation covers reinforced and prestressed concrete waffle slabs, including both rectangular and skew plan forms and subjected to uniformly distributed, as well as concentrated lateral loads.

A review of the theoretical and experimental studies of orthotropic structures, reinforced and prestressed concrete waffle slabs are presented together with simple expressions for estimating the orthotropic rigidities. Mathematical formulation of the problem using a Fourier series for lateral and in-plane forces are derived.

Analysis and discussion of the theoretical and experimental results from three reinforced and two prestressed concrete waffle slabs are presented in this work. An approximate method is proposed to solve the in-plane stress problem due to the prestressing force along the four sides of the slab.

The experimental work comprises the following two groups:

1) Group A, includes three reinforced concrete waffle slabs. The first and second slabs were rectangular in plan with identical geometry and subjected to uniform and concentrated lateral loads, respectively. The third slab had a skew of 45°, and was subjected to a concentrated lateral load.

2) Group B, includes two prestressed concrete waffle slabs, one was rectangular in plan and the other had a skew of 45°, and subjected to concentrated lateral loads.

All the slab models are analysed as bridge slabs having two opposite edges simply supported and other two edges free (Group A) or elastically supported (Group B).

CHAPTER II

HISTORICAL REVIEW

2.1 Review of Literature

The study of theory of plates goes back to the French mathematician, Sophie Germain (1816), who obtained a differential equation for vibration of plates, but she neglected the work done by warping of the middle surface. The first corrected differential equation for the free vibration of plates was used by Lagrange by adding the missing term in Sophie's equation. This work, which was improved by researchers such as Navier, Poisson and Kirchoff, is considered to be the basis for the classical thin plate theory. Solutions of many problems in plates of circular, rectangular, skew, triangular shapes are available (see Timoshenko (31) and Szilard (29,30)). The exact solution to a plate problem should satisfy the boundary conditions as well as the governing differential equation of equilibrium or minimize the potential energy of the plate. The deflection function could be in the form of an infinite series and its sum to infinity gives any deflection pattern of the plate which satisfies the imposed geometrical conditions by suitable choice of values for the infinite number of constants of integration.

The development of the modern aircraft industry provided another strong impetus toward more rigorous analytical investigations of plate problems. Plates subjected to in-plane forces, postbuckling loads, stiffened plates, etc., were analyzed by various scientists and engineers. Most recently, the invention of high-speed electronic computers exerted a considerable influence on the static and dynamic analysis of plates. Probably the first approach used in computer analysis was the finite difference method.

Considerable attention has been given recently to developing methods of designing more economical concrete bridges. Various methods of estimating the load distribution in concrete bridge decks have been proposed to date. In all these methods values of the effective flexural and torsional rigidities of the deck system are required before the analysis can proceed. Little information was available as to how these rigidities might be assessed for many of the bridge-deck systems.

In 1956, Huffington (10) investigated theoretically and experimentally the method for the determination of rigidities for metallic rib-reinforced deck structures. It was applied to the case of equally spaced stiffeners, of rectangular cross section, and symmetrically placed with respect to its middle plane.

Methods of analysing rectangular and skew deck plates with simple boundary conditions have been recently investigated by Kennedy et al. (12,14,15,16). They solved the problem of skew plate under uniform load by means of variational techniques and a series solution were presented for rib stiffened plates under uniform and concentrated loads; the results were verified with experiments. They observed that critical stresses often occur in obtuse corners of such skew plates.

In 1968 Jackson (11) proposed a method to estimate the torsional rigidities of concrete bridge decks, using the membrane analogy and the estimation of the junction effect. The effect of the continuity of the slab on the flange plate was not accounted for.

In 1972, Perry and Heins (21) studied a series of equations for preliminary design of transverse floor beams in orthotropic deck bridges. The applied loads were represented in the form of a Fourier series. This method is not exact since it neglected the effects of contributions of bending to the torsional rigidities of the plate. Cardens et al (4,5) investigated the in-plane and flexural stiffnesses of isotropically and nonisotropically reinforced concrete plates. Results indicated that the stiffness of such plates was related quantitatively to the relative orientation of the reinforcement with respect to the applied forces.

Mathematical analysis of grid systems with particular regard to bridge type was given by Bares and Massonnet (2) and Rowe (24) together with practical applications.

2.2 Prestressed Concrete Slabs

Possibly, Guyon, in the early 1950's was the first to realize that slabs, prestressed in two directions behaved analogously to the two-way arch action of thin shell structures. In the late 1950's several prestressed slab research projects were undertaken in the United States. Scordelis et al, 1960, (26,27) studied the ultimate strength of continuous prestressed slabs and proposed several design recommendations. They investigated the load distribution between the column and the middle strips and the following conclusions were obtained:

1. The elastic plate theory may be used satisfactorily to predict the behaviour of a prestressed concrete slab loaded within the elastic limit.
2. The slab can sustain a large increase in load before widespread cracking takes place.

Possibly the largest stride in the design of prestressed slabs was taken by Lin (18) who was the first to introduce the Load Balancing Method. It was soon made apparent that the tendon profiles could be designed so that the upward cable force neutralized the vertical downward load. Between 1957 and 1969, many researchers developed the method and studied the load distribution in bridge slabs (Sherman (28), Rowe (24) and Wang (32)). In 1969, Muspratt (20) used the load balancing method on prestressed concrete waffle slabs. For other applications of this method in U.S.A. see references (19) and (23). Another investigation was made by Hondros and Smith (9) on a post-tensioned diagrid flat plate, simply supported on four edges using the link force method of analysis. The force method applies compatability but disregards the influence of torsion on the plate.

Burns and Hemakom in 1977 (3), investigated the problem of post-tension flat plate using frame analysis; furthermore, the cracks were predicted and the stresses compared with that obtained by ACI code (318-71), (1).

Although an appreciable amount of work has been done on the analysis of prestressed flat slabs, very little work is available on prestressed concrete waffle slabs. It is beyond the scope of this thesis to give a comprehensive survey of the entire literature in this field. However, a study of the literature shows that no exact solution is available for prestressed concrete waffle slabs under a general type of lateral load and in-plane prestressing force.

CHAPTER III

FORMULATION

3.1 General

With reinforced concrete structures cast in-situ, it is common practice to cast the slab and the supporting beam grid at the same time. The grid elements may be either reinforced or prestressed concrete beam or steel girders. Since a reinforced concrete slab has to withstand the local effects of heavy concentrated loads, the slab thickness is usually substantial, and composite action of the grid-and-slab system must be taken into account. Due to the action of concentrated loads, the transverse direction becomes important and the transverse strength of the structures has to be considered carefully. In dealing, herein with orthotropic concrete structures, it is assumed that orthotropy is a result of geometry and not of material, see Figure 3.1.

3.2 Assumptions

The analytical approach to the problem of the orthotropic plate has to be based on some simplifying assumptions related to the form and material of the plate and to the state of strain induced by the external loading. The assumptions for orthotropic plates are based on the same assumptions used in the analysis of isotropic plates, and they are as follows:

a) The material of the plate is elastic, i.e., the stress-strain relationship is given by Hooke's law.

b) The material of the plate is considered to be homogeneous, by transforming the steel area into an equivalent area of concrete.

c) The thickness of the plate is uniform and small in comparison to the other lateral dimensions of the plate. Thus the shearing and normal stresses to the plane of symmetry are small and can be neglected.

d) Straight lines normal to the middle plane of the plate remain straight and normal to the middle plane of the plate after bending.

e) The deflections of the plate are small in comparison with its thickness; and are such that there is no normal strain in planes tangent to the middle plane.

It should be mentioned that the theory of orthotropy is applicable for structures which have a small ratio of stiffness spacing relative to any lateral length of the structure.

3.3 Governing Differential Equation For The Lateral Loads And Edge Moments

It is assumed that the material of the plate has three planes of symmetry with respect to its elastic properties. Taking these planes as the coordinate planes, the relations between the moments and deflections are:

$$\left.\begin{aligned} M_x &= -(D_x W,_{xx} + D_1 W,_{yy}) \\ M_y &= -(D_y W,_{yy} + D_2 W,_{xx}) \\ M_{xy} &= D_{xy} W,_{xy} \end{aligned}\right\} \quad (3.1)$$

where,

D_x, D_y = flexural rigidities of the plate per unit width in x and y directions respectively

D_1, D_2 = coupling rigidities, measuring the contribution of bending to torsional rigidities of the plate.

D_{xy}, D_{yx} = torsional rigidities of the plate and ribs

Substituting expressions 3.1 in the following general differential equation of equilibrium, given by Timoshenko (31),

$$M_{x,xx} + M_{y,yy} - zM_{xy,xy} = -q(x,y) \quad (3.2)$$

The following fourth order differential equation governing the deflection of the orthotropic plate is obtained in rectangular coordinate:

$$D_x W,_{xxxx} + 2HW,_{xxyy} + D_y W,_{yyyy} = q(x,y) \quad (3.3)$$

where,

$$H = (D_1 + D_2 + D_{xy} + D_{yx})/2$$

and known as the effective torsional rigidity of the plate and characterizes the resistance of the plate element to twisting.

To generalize the solution from the rectangular slab to a skew slab the following transformation is used:

$$\begin{aligned} u &= x/\cos\theta \\ v &= y - x\tan\theta \end{aligned} \qquad (3.4)$$

where, θ is the skew angle.

Equation 3.3, known as Huber's equation, becomes,

$$D_x W_{,uuuu} - E_1 W_{,uuuv} + E_2 W_{,uuvv} - E_3 W_{,uvvv} + E_4 W_{,vvvv} = c^4 q(u,v) \qquad (3.5)$$

where,

$$E_1 = 4sD_x$$

$$E_2 = 2(3D_x s^2 + Hc^2)$$

$$E_3 = 4s(D_x s^2 + Hc^2)$$

$$E_4 = D_x s^4 + 2Hs^2c^2 + D_y c^4$$

$$c = \cos\theta \quad \text{and} \quad s = \sin\theta$$

3.4 Relation of Stress and Strain for Bending Action

Solving equations 3.1 and 3.3 to find $W_{,xx}$ and $W_{,yy}$ and substituting these two terms in the following formulae (31) relating strain to curvature,

$$\left.\begin{aligned} \varepsilon_x &= -zW_{,xx} \\ \varepsilon_y &= -zW_{,yy} \end{aligned}\right\} \qquad (3.6)$$

yields the strains in x and y directions in terms of the rigidities and moments:

$$\left.\begin{aligned} \varepsilon_x &= z(M_x D_y - M_y D_1)/(D_x D_y - D_1 D_2) \\ \varepsilon_y &= z(M_y D_x - M_x D_2)/(D_x D_y - D_1 D_2) \end{aligned}\right\} \quad (3.7)$$

z is the depth of the neutral axis from the top fibre.

3.5 Elastic Properties of the Waffle Slab Model

As mentioned earlier, the condition of orthotropy for the slabs treated herein, was mainly due to geometry and steel reinforcement. The problem was idealized by assuming that the slab is made of a homogeneous material with different elastic properties in two mutually perpendicular directions. Waffle slab construction considered as a "composite system" consists of two parts, the grid, and the plate. This "composite system" displays a high degree of torsional rigidity, especially for skew slabs. The composite system may be arranged in a sequence of structural forms; the sequence may consist of limiting case having a simple grid with no plate and the other extreme case of a true orthotropic slab; various types of composite systems fall between these two limits.

In addition to the basic assumptions in deriving the governing differential equation for an orthotropic plate, the following assumptions are made with respect to waffle slab construction:

1. The number of ribs in both directions is large enough for the real structure to be replaced by an idealized one with continuous properties.

2. The neutral plane in each of the two orthogonal directions coincides with the centre of gravity of the total section in the corresponding direction.
3. The area of the flange plate is magnified by the factor $1/(1 - \mu^2)$ due to the effect of Poisson's ratio μ.

Since the thickness of the slab is constant and the slab material is continuous, as assumed before, the different elastic properties in two principal directions must be due to different moments of inertia per unit width of the slab. The modulus of elasticity in two perpendicular directions are equal ($E_x = E_y = E$) as well as Poisson's ratio ($\mu_x = \mu_y = \mu$) and the torsional rigidities D_{xy} and D_{yx} are equal.

There is no difficulty in determining the flexural rigidities D_x and D_y of the slab models, but the difficulty is to find an accurate value for the torsional rigidities. Various methods of estimating the load distribution in concrete bridge decks (2,13,26) have been proposed to date. In all of these methods, values of flexural and torsional rigidities of the deck structures are required before the analysis can proceed. Some of the methods used for estimating the torsional rigidities are very limited in their application and can lead to appreciable errors in the value of the torsional parameter, unless their limitations are recognized. A method of determination of rigidities was investigated by Huffington (10).

3.6 Rigidities of Uncracked Sections

3.6.1 Flexural Rigidities

It is assumed that the neutral planes in each of the two coordinate directions coincides with the centre of gravity of the total section. This is an approximation only, since it can be shown that the location of the neutral surfaces is a function of the deflection as well as the geometry of the section.

Figure 3.2 shows a typical section of a waffle-type slab. Based on the assumptions made before, the orthotropic flexural rigidities D_x and D_y, as well as the coupling rigidities D_1 and D_2 due to the Poisson's effect (13) can be put in the form,

$$\left.\begin{aligned} D_x &= D + \left\{Eh(e_x - h/2)^2/(1 - \mu^2)\right\} + EI_x'/S_x \\ D_y &= D + \left\{Eh(e_y - h/2)^2/(1 - \mu^2)\right\} + EI_y'/S_y \\ D_1 &= \mu D_x' \\ D_2 &= \mu D_y' \end{aligned}\right\} \quad (3.8)$$

where,

D = the flexural rigidity of the flange plate with respect to its middle plane, $Eh^3/12(1 - \mu^2)$.

E = modulus of elasticity of the concrete

$= 57000\sqrt{f_c'}$ (1)

f_c' = 28 day concrete cylinder strength in psi

h = thickness of the flange plate

μ = Poisson's ratio of concrete

$= \sqrt{f'_c}/350$ (8,12)

S_y = spacing of longitudinal ribs

S_x = spacing of transverse ribs

e_x = depth of neutral plane from top fibre for bending in the x direction

e_y = depth of neutral plane from top fibre for bending in the y direction, i.e.,

$$\left.\begin{aligned} e_x &= \left\{ b_x d_x (h + d_x/2) + (n - 1) A_s (h + d_x - d') + S_x h^2/ 2(1 - \mu^2) \right\} / \left\{ b_x d_x + (n - 1) A'_s + S_x h/(1 - \mu^2) \right\} \\ e_y &= \left\{ b_y d_y (h + d_y/2) + (n - 1) A_s (h + d_y - d'') + S_y h^2/ 2(1 - \mu^2) \right\} / \left\{ b_y d_y + (n - 1) A_s + S_y h/(1 - \mu^2) \right\} \end{aligned}\right\} \quad (3.9)$$

I'_x = moment of inertia of transverse rib with respect to the assumed neutral axis

I'_y = moment of inertia of longitudinal rib with respect to the assumed neutral axis, i.e.,

$$\begin{aligned} I'_x &= b_x d_x \left\{ (h + d_x/2) - e_x \right\}^2 + (n - 1) A'_s \left\{ (h + d_x - d') - e_x \right\}^2 + b_x d_x^3/12 \\ I'_y &= b_y d_y \left\{ (h + d_y/2) - e_y \right\}^2 + (n - 1) A_s \left\{ (h + d_y - d'') - e_y \right\}^2 + b_y d_y^3/12 \end{aligned} \quad (3.10)$$

in which,

n = modular ratio

= E_s/E_c

b_y = width of longitudinal rib

b_x = width of transverse rib

d_y = depth of the longitudinal rib

d_x = depth of the transverse rib

d'' = concrete cover to the centre of the longitudinal reinforcements

d' = concrete cover to the centre of the transverse reinforcements

A_s = area of reinforcement steel in the longitudinal direction

A_s' = area of reinforcement steel in the transverse direction

D_x' = flexural rigidity of the flange plate with respect to the neutral plane of the gross-section associated with bending in the x direction

D_y' = flexural rigidities of the flange plate with respect to the neutral plane of the gross-section associated with bending in the y direction

Since it is assumed that the number of ribs in both directions is large, the effective width of the flange plate acting with one rib is taken as the distance between two adjacent ribs. Most of the investigators who have taken the same effective width have obtained very good results.

3.6.2 Torsional Rigidities

Reliable information on the estimation of the torsional rigidities of a bridge deck are very limited. The effective torsional rigidities H, in Eq. 3.3 is given by,

$$H = (D_{xy} + D_{yx} + D_1 + D_2)/2 \qquad (3.11)$$

For the analysis of reinforced concrete slabs with different reinforcements in the two prependicular directions, Huber recommended the expression, $H = \sqrt{D_x D_y}$; such an expression gives a too high an estimate for T-beam section.

The main problem lies in finding the values of D_{xy} and D_{yx}, which are given by

$$\left.\begin{aligned} D_{xy} &= G_{xy}\, I_{xy} \\ D_{yx} &= G_{yx}\, I_{yx} \end{aligned}\right\} \qquad (3.12)$$

where,

$G_{xy} = G_{yx}$

$= \text{shear modulus}$

$= E/2(1 + \mu)$

I_{xy} and I_{yx} are the torsional constants

A number of investigators have obtained the torsional constants for structural steel and aluminum-alloy sections, using membrane analogy and/or numerical methods. Massonet and Rowe (2,25) have determined the torsional constants by dividing open section into a number of rectangular areas. Thus the torsional constant is given by,

$$I_{xy}S_x \text{ or } I_{yx}S_y = (\tfrac{1}{3}k_1\ a_1^3\ b_1 + \sum_{1=2}^{n} K_i\ a_i^3\ b_i) \qquad (3.13)$$

where,

S_x, S_y = spacing of transverse (longitudinal) ribs

a_1 = the smaller dimension of the cut area

b_1 = the larger dimension of the cut area

K = factor depend on the ratio a_i/b_i (31)

Jackson (11) has considered the value mentioned above as not accurate enough since it neglects the junction effect on the torsional rigidity. The twisting rigidity of the uncracked section of concrete waffle-type slab is estimated by means of the membrane analogy method (31) taking into account the stiffening effect afforded by the ribs in the orthogonal direction to the one under consideration. Considering the geometry of the deflected membrane, the torsional constant for the rectangular sections 1, 2 and 3 as shown in Figure 3.3 are calculated to give the total torsional constant, I_{xy}.

$$I_{xy} = I_{xy_1} + I_{xy_2} + I_{xy_3} \qquad (3.14)$$

in which,

$$I_{xy_1} = \tfrac{1}{2}k_1\ S_y\ h^3$$

$$I_{xy_2} = k_1\, d_y\, b_y^3 \quad (\text{if } d_y > b_y)$$

$$= k_1\, b_y\, d_y^3 \quad (\text{if } b_y > d_y) \qquad (3.15)$$

$$I_{xy_3} = 4k_1(n - 1)(A_s')^2/\pi \quad (13)$$

where,

I_{xy_1} is reduced by factor ½ which accounts for the continuity of the flange plate. The torsional constant I_{xy_1} is modified by (Kennedy and Bali (13)) taking into account the effect of transverse ribs in both directions. It is assumed that the presence of the transverse rib will increase the torsional constant of the slab as:

$$I_{xy_1}(\text{modified}) = I_{xy_1}(I_{xy_{(S+W)}}/I_{xy_{(S)}}) \qquad (3.16)$$

in which,

$I_{xy_{(S+W)}}$ = torsional rigidity of slab and rib of the transverse or longitudinal ribs

Equation 3.14 becomes

$$I_{xy} = I_{xy_1}(\text{modified}) + I_{xy_2} + I_{xy_3} \qquad (3.17)$$

in which,

I_{xy_3} = torsional rigidity of transformed section of steel (13)

Similarly, the torsional rigidities D_{yx} can be calculated in the same way.

3.7 Plane Stress Problem Due To Prestressing Force

3.7.1 Elastic Constants For Membrane Action

Assuming the corresponding strains in the flange plate element (deck) and the ribs are equal, and that shearing action is resisted by the flange plate alone, the following relations for equivalent elastic constants can be developed. The stress resultants per unit length in an isotropic flange plate element, shown in Figure 3.2, are:

$$\left.\begin{aligned}(\tau_x)_1 &= \sigma_x h = h(E_x/(1-\mu^2))(\varepsilon_x + \mu\varepsilon_y)\\ (\tau_y)_1 &= \sigma_y h = h(E_y/(1-\mu^2))(\varepsilon_y + \mu\varepsilon_x)\\ (\tau_{xy})_1 &= \tau_{xy} h = hG\gamma_{xy}\end{aligned}\right\} \qquad (3.18)$$

where E, G and μ are the material properties of the concrete.

For the same strains in the rib, the stress resultants per unit length, are:

$$\left.\begin{aligned}(\tau_x)_2 &= E_x\varepsilon_x A_x^*\\ (\tau_y)_2 &= E_y\varepsilon_y A_y^*\\ (\tau_{xy})_2 &= 0\end{aligned}\right\} \qquad (3.19)$$

where A_x^* and A_y^* are the cross-sectional areas per unit length of the ribs in the x and y directions, respectively. The total stress resultants per unit length of an element are:

$$\begin{Bmatrix} \tau_x \\ \tau_y \\ \tau_{xy} \end{Bmatrix} = h \begin{vmatrix} E_x \; 1/(1-\mu^2)+A_x^*/h & \mu E_y/(1-\mu^2) & 0 \\ \mu E_x/(1-\mu^2) & E_y \; 1/(1-\mu^2)+A_y^*/h & 0 \\ 0 & 0 & G \end{vmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$= h \begin{vmatrix} E_x' & E_1 & 0 \\ E_2 & E_y' & 0 \\ 0 & 0 & G \end{vmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad (3.20)$$

where,

$$E_x' = E_x(1/(1-\mu^2) + A_x^*/h)$$

$$E_1 = \mu E_x/(1-\mu^2)$$

$$E_2 = \mu E_y/(1-\mu^2)$$

$$E_y' = E_y(1/(1-\mu^2) + A_y^*/h)$$

Based on the directions of the axes, x and y, as shown in Figure 3.2,

$$A_x^* = (b_x)(d \quad - h)/S_x$$

$$A_y^* = (b_y)(d \quad - h)/S_y$$

solving equations 3.20 to find the strains in both directions yields,

$$\left.\begin{aligned} \varepsilon_x &= ((\tau_x/h)E_y' - (\tau_y/h)E_2)/(E_x'E_y' - E_1E_2) \\ \varepsilon_y &= ((\tau_y/h)E_x' - (\tau_x/h)E_1)/(E_x'E_y' - E_1E_2) \end{aligned}\right\} \qquad (3.21)$$

in which ε_x and ε_y are the strains in the x and y directions due to the in-plane prestressing.

It should be noted that the deflection due to in-plane stresses is very small and can be neglected. Therefore the total deflection in the slab can be considered to be due to the lateral loads and the edge moments. On the other hand, the total strain in the slab is due to the lateral loads, in-plane forces and edge moments.

3.8 Boundary Conditions

A solution for the deflection function $W_{(x,y)}$ in cartesian coordinates or for $W_{(u,v)}$ in oblique coordinates to the plate problem must be consistent with the conditions at the edges of the plate. The kind of support is theoretically defined by three "boundary conditions" along each edge. The boundary conditions have to be re-formulated first in terms of the deflection, if the solution is to be based on the deflection. Thus rectangular and skew slabs have 12 boundary conditions which are to be satisfied by the solution of the partial differential equation governing the problem. However, this governing equation is of the fourth order in the variables x and y or u and v, and its solution involves only 8 arbitrary constants. These constants can be made to fit only 8 boundary conditions, two for each edge, so that the three conditions mentioned above must be reduced to two conditions. The boundary conditions for single span waffle slabs are presented below.

3.8.1 Bridge Slabs

The simple bridge type shown in Figures 3.4, 3.5 is simply supported along two opposite edges ($v = \pm b$) and free or elastically supported at the remaining two edges ($u = \pm a$).

At the simply supported edges, there is no vertical deflection, and the bending moments about these edge-lines equal the edge moments due to the prestressing. These two boundary conditions can be formulated as follows:

1. $W_{(v=\pm b)} = 0 \quad \text{for } -a < u < a$ (3.22)

2. $$M_{n\,(v=\pm b)} = M_x s^2 + M_y c^2 + (M_{xy} - M_{yx})\,sc \quad {}_{(v=\pm b)}$$

$$= M_{\text{external}\,(v=\pm b)}$$

or

$$R_1 W,_{uv} + R_2 W,_{vv} = M_{\text{external}} \text{ at } v=\pm b \text{ for } -a < u < a \qquad (3.23)$$

where

$$R_1 = s(2D_x s^2/c^2 + 2D_2 + D_{xy} + D_{yx})$$

$$R_2 = -s^2(D_x s\ /c^2 + 2H) - c^2 D_y$$

It should be noted that $W,_{uu} = 0$ and $W = W,_u = 0$ on the simple support

3. According to Timoshenko (31), combining the shear force along the edge with forces replaced by the twisting couples and equating this to the pressure transmitted from the plate to the supporting edge beam, lead to the following

equation:

$$-(Q_x - M_{xy,y}) = EI\ W,_{yyyy}$$

i.e.,

$$D_x W,_{xxx} + (D_1 + D_{xy} + D_{yx})W,_{xy} = EI\ W,_{yyyy}$$

or

$$R_3 W,_{uuu} + R_4 W,_{uuv} + R_5 W,_{uvv} + R_6 W,_{vvv} - EI\ W,_{vvvv} = 0$$

$$@\ u = \pm a \text{ for } -b < v < b \qquad (3.24)$$

where,

$R_3 = D_x/c^3$

$R_4 = -3D_x s/c^3$

$R_5 = (3D_x s^2/c^2 + D_1 + D_{xy} + D_{yx})/c$

$R_6 = -s(D_x s^2/c^2 + D_1 + D_{xy} + D_{yx})/c$

EI = flexural rigidities of the edge beam

4. Along the elastically supported edge ($u = \pm a$), equating the external moment due to prestressing force combined with the change in twisting moment in the edge beam to the plate internal moment parallel to the x-axis, yield:

$$-M_x = -GJ(W,_{xy,y}) - M_{ext.(u=\pm a)}$$

i.e.,

$$D_x W,_{xx} + D_1 W,_{yy} = -GJ(W,_{xyy}) - M_{ext.(u=\pm a)}$$

or

$$R_7 W,_{uu} + R_8 W,_{uv} + R_9 W,_{vv} + GJW,_{uvv} + R_{10} W,_{vvv} = -cM_{ext.}$$

$$@ \quad u = \pm a \quad \text{for} \quad -b < v < b \qquad (3.25)$$

where,

$R_7 = D_x/c$

$R_9 = -2D_x s/c$

$R_9 = (D_x s^2 + D_1 c^2)/c$

$R_{10} = -sGJ$

GJ = torsional rigidity of the edge beam

The boundary conditions for the bridge slab where the two edges are free are obtained by putting the rigidities for the edge beams EI and GJ equal to zero.

CHAPTER IV

ANALYTICAL SOLUTION

4.1 General

The mathematical solution comprises of finding a suitable complementary function which satisfies the governing equation (Eq. 3.5), and a particular solution which satisfies all the given boundary conditions. Combining the two solutions, a complete solution for the slab is obtained as follows:

$$W = W_c + W_p \tag{4.1}$$

where,

W = The total deflection at a point on the plate

W_c = Deflection found from the complementary solution of the homogeneous equation (Eq. 3.5)

W_p = Deflection from the particular solution of the non-homogenous equation.

4.2 Complementary Solution

According to Levy's solution, the deflection of the plate surface is assumed in the form of a Fourier series as

$$W_c = \sum_{n=1}^{\infty} e^{\lambda_n u}(P_n \sin \beta_n v + Q_n \cos \beta_n v) \tag{4.2}$$

where,

λ_n, P_n and Q_n are basic functions of the elastic properties and the geometry of the plate; n equals number of harmonics chosen to make the series closely convergent; and, $\beta_n = n\pi/b$

Solving equations 4.1 and 3.5 for q(u,v) equals zero, and equating the coefficients of $\sin \beta_n v$ and $\cos \beta_n v$, yields two equations in P_n and Q_n. According to Gupta (8) the final expression for λ_n is

$$\lambda_n = \left\{\pm\left[(H \pm \sqrt{H^2 - D_xD_y})/D_x\right]^{0.5} c \pm is\right\}\beta_n \qquad (4.3)$$

It can be observed, from equation 4.3, that there are eight possible values for λ_n which give rise to eight possible solutions.

For the slab model considered herein, it is assumed that the slab is flexurally stiff and torsionally weak, i.e., $(H^2<D_xD_y)$. This case includes all T-beam and open rib deck bridges.

4.2.1 Waffle Slab Which is Flexurally Strong and Torsionally Weak

The solution of a waffle slab can be taken as

$$\begin{aligned} W_{cl} = \sum_{n=1}^{\infty} & (C_{1n}\cosh k_3\beta_n u + C_{2n}\sinh k_3\beta_n u)\cos(k_4 u + v)\beta_n \\ & + (C_{3n}\cosh k_3\beta_n u + C_{4n}\sinh k_3\beta_n u)\sin(k_4 u + v)\beta_n \\ & + (C_{5n}\cosh k_3\beta_n u + C_{6n}\sinh k_3\beta_n u)\cos(k_5 u - v)\beta_n \\ & + (C_{7n}\cosh k_3\beta_n u + C_{8n}\sinh k_3\beta_n u)\sin(k_5 u - v)\beta_n \qquad (4.4) \end{aligned}$$

in which λ_n for this case can be written in the form

$$\lambda_n = \pm(k_1 \pm is)\beta_n \quad \text{or} \quad \pm(k_2 \pm is)\beta_n \qquad (4.5)$$

where,

$$k_1 = (c(H + \sqrt{H^2 - D_xD_y})/D_x)^{0.5}$$

$$k_2 = (c(H - \sqrt{H^2 - D_xD_y})/D_x)^{0.5}$$

C_{1n} to C_{8n} are arbitrary constants dependent on n and adjusted to satisfy the boundary condition. See references (7,8) and Appendix (A) for a more general representation of the deflection function W. Another possible complementary solution of W can be written as;

$$W_{c2} =$$

$$\sum_{n=1}^{\infty}(C_{9n}\cosh x_2k_3\alpha_nv + C_{10n}\sinh x_xk_3\alpha_nv)\cos(x_2k_4v + u)\alpha_n$$

$$+ (C_{11n}\cosh x_2k_3\alpha_nv + C_{12n}\sinh x_2k_3\alpha_nv)\sin(x_2k_4v + u)\alpha_n$$

$$+ (C_{13n}\cosh y_2k_3\alpha_nv + C_{14n}\sinh y_2k_3\alpha_nv)\cos(y_2k_5v - u)\alpha_n$$

$$+ (C_{15n}\cosh y_2k_3\alpha_nv + C_{16n}\sinh y_2k_3\alpha_nv)\sin(y_2k_5v - u)\alpha_n \tag{4.6}$$

where,

$$\alpha_n = n\pi/a$$

$$x_1 = 1/(k_1^2 + s^2)$$

$$x_2 = 1/(k_2^2 + s^2)$$

The boundary conditions used for the slab are eight in number (two along each edge) and the deflection function is made to satisfy these boundary conditions. Expanding each boundary condition in a Fourier series, will yield three equations and hence 24 equations for the eight boundary conditions.

To use the same number of arbitrary constants in the deflection function, the following polynomial function is assumed and added:

$$W_{c3} = \Big\{C_{17} + C_{18}u/a + C_{19}v/b + C_{20}u^2/a^2 + C_{21}v^2/b^2 + C_{22}u^3/a^3 + C_{23}v^3/b^3 + C_{24}(Tu^4 - v^4)/b^4\Big\} \tag{4.7}$$

in which,

$$T = (s^4D_x + 2s^2c^2H + c^4D_y)/D_x$$

C_{17} to C_{24} are arbitrary constants. Thus the total complementary solution becomes,

$$W_c = W_{c1} + W_{c2} + W_{c3} \tag{4.8}$$

4.3 Particular Solution

The particular solution has to be determined and added to the complementary solution and must satisfy the boundary conditions. The uniform load or concentrated load acting on a limited area as shown in Figure 4.1 is expanded into double Fourier series over the entire area of the slab. Following Gupta (8) the particular solution can be taken as:

$$W_p = \Big\{(a_oc^4/4E_4)(v^4/24 - v^2b^2/4 + 5b^4/24) + \sum_1^\infty (T_{5m}\cos\alpha_m u + T_{7m}\sin\alpha_m u) + \sum_1^\infty (T_{6n}\cos\beta_n v + T_{8n}\sin\beta_n v)$$

$$+ \sum_1^\infty \sum_1^\infty (K_{mn} \cos\alpha_m u \cos\beta_n v + P_{mn} \sin\alpha_m u \cos\beta_n v$$

$$\left. + \quad Q_{mn} \cos\alpha_m u \sin\beta_n v + L_{mn} \sin\alpha_m u \sin\beta_n v) \right\} \qquad (4.9)$$

4.4 Symmetric and Anti-Symmetric Loading

The loading on the structure is divided into symmetric and anti-symmetric components. This division is applied to the lateral load and the prestressing force along the edges of the slab. For symmetric loading odd terms vanish, i.e.,

$$C_{2n} = C_{3n} = C_{6n} = C_{7n} = C_{10n} = C_{11n} = C_{14n} = C_{15n} =$$

$$C_{18} = C_{19} = C_{22} = C_{23} = Q_{7m} = Q_{8n} = Q_{3mn} = Q_{4mn} = 0 \qquad (4.10)$$

For the anti-symmetric loading, all even terms vanish, i.e.,

$$C_{1n} = C_{4n} = C_{5n} = C_{8n} = C_{9n} = C_{12n} = C_{13n} = C_{16n} = C_{17} =$$

$$C_{20} = C_{21} = C_{24} = a_o = Q_{5m} = Q_{6n} = Q_{1mn} = Q_{2mn} = 0 \qquad (4.11)$$

By superposition, the results for any lateral loading can be obtained with the added advantage that the number of boundary conditions is reduced to four for each loading component.

4.5 Expansion of the In-Plane Prestressing Force In a Fourier Series by Graphical Method

When the equation for a periodic function is not known but a graph of the waveform is available (such as a variable prestressing force along the edges of a slab bridge), it is possible to obtain an approximate solution for the Fourier coefficients by means of graphical techniques (17,33). Actually this method represents a concept of graphical integration. This is accomplished by dividing one cycle into m equal divisions. The dependent-variable value (donated by Y_k's) in Figure 4.2 is obtained at the mid-point of each of these intervals. The coefficients of the Fourier series representing the in-plane prestressing force can be shown to be, (See Appendix C),

$$\left.\begin{aligned} a_o &= (2/m) \sum_{k=1}^{m} Y_k \\ a_n &= (2/m) \sum_{k=1}^{m} (-1)^n Y_k \cos(2n\pi/m)(k - \tfrac{1}{2}) \\ b_n &= (2/m) \sum_{k=1}^{m} (-1)^n Y_k \sin(2n\pi/m)(k - \tfrac{1}{2}) \end{aligned}\right\} \qquad (4.12)$$

CHAPTER V

SATISFACTION OF BOUNDARY CONDITIONS

To obtain the matrix equations, the deflection function must satisfy the boundary conditions for the slab subjected to any arbitrary lateral load. This is accomplished by dividing such a load into symmetric and antisymmetric loads.

5.1 Symmetric Load

1) The deflection must be zero at the edge $v = \pm b$. Substituting equations 3.22, 4.4, 4.6, 4.7, 4.8, 4.9 and 4.10 in equation 4.1, the following equation is obtained (8).

$$f_o(u) + \sum_1^\infty (f_n(u) + A_n \cos\alpha_n u + B_n \sin\alpha_n u)$$

$$= -\sum_1^\infty T_{5m}\cos\alpha_m u - \sum_1^\infty (-1)^n T_{6n} - \sum_1^\infty \sum_1^\infty (-1)^n K_{mn}\cos\alpha_m u \qquad (5.1)$$

in which

$$f_o(u) = C_{17} + C_{20}\, u^2/a^2 + C_{21} + C_{24}[(T\, u^4/b^4) - 1]$$

$$f_n(u) = (-1)^n[C_{1n}\cosh u_{3n}\cos u_{4n} + C_{4n}\sinh u_{3n}\sin u_{4n}$$

$$+ C_{5n}\cosh u_{3n}\cos u_{5n} + C_{8n}\sinh u_{3n}\sin u_{5n}]$$

$$A_n = C_{9n}K_{1n} + C_{12n}K_{2n} + C_{13n}L_{1n} + C_{16n}L_{2n}$$

and

$$B_n = -C_{9n}K_{4n} + C_{12n}K_{3n} + C_{13n}L_{4n} - C_{16n}L_{3n}$$

In which, u_{3n}, u_{4n}, u_{5n}, K_{1n} to K_{8n} and L_{1n} to L_{8n} are defined in Appendix (A).

The function $f_o(u)$ and $f_n(u)$ must be expanded in Fourier series to satisfy the boundary condition along the entire length of the edge, thus:

$$\left.\begin{aligned} f_o(u) &= a_{oo} + \sum_1^{\infty}(a_{om}\cos\alpha_m u + b_{om}\sin\alpha_m u) \\ f_n(u) &= c_{no} + \sum_1^{\infty}(c_{nm}\cos\alpha_m u + d_{nm}\sin\alpha_m u) \end{aligned}\right\} \quad (5.2)$$

where, a_{oo}, a_{om}, b_{om}, c_{no}, c_{nm} and d_{nm} are Fourier coefficient and defined in Appendix (B).

Substituting equation 5.2 in equation 5.1 and equating the coefficients of $\sin\alpha_m u$ and $\cos\alpha_m u$ and the constant term to zero, the following three equations are obtained:

$$\left.\begin{aligned} a_{oo} + \sum_1^{\infty} c_{no} &= -\sum_1^{\infty}(-1)^n T_{6n} \\ a_{nm} + \sum_1^{\infty} c_{nm} + A_m &= -T_{5m} - \sum_1^{\infty}(-1)^n K_{mn} \\ B_m &= 0 \qquad \text{for each } m. \end{aligned}\right\} \quad (5.3)$$

Substituting the Fourier coefficients in equation 5.3, yields three equations:

$$\begin{aligned} & C_{17} + C_{20/3} + C_{21} + C_{24}[(T\, a^4/5b^4) - 1] \\ + \quad & \sum_1^{\infty}(-1)^n(C_{1n}W_{1n} + C_{4n}W_{2n} + C_{5n}W_{3n} + C_{8n}W_{4n}) \\ = \quad & -\sum_1^{\infty}(-1)^n T_{6n} \end{aligned} \quad (5.4)$$

$$C_{20}I_{1m} + C_{24}T\ I_{3m}\ a^4/b^4$$

$$+{}_1\Sigma^{\infty}(-1)^n(C_{1n}A_{j1} + C_{4n}A_{j2} + C_{4n}A_{j2} + C_{5n}A_{j3} + C_{8n}A_{j4})$$

$$+C_{9m}K_{1m} + C_{12m}K_{2m} + C_{13m}L_{1m} + C_{16m}L_{2m}$$

$$=-T_{5m} - {}_1\Sigma^{\infty}(-1)^nK_{mn} \tag{5.5}$$

$$-C_{9n}K_{4n} + C_{12n}K_{3n} + C_{13n}L_{4n} - C_{16n}L_{3n} = 0 \tag{5.6}$$

2) Boundary condition 3.23 relates the moment normal to the support to the deflection function. Thus,

$$f_o(u) + {}_1\Sigma^{\infty}(f_n(u) + A_n\cos\alpha_n u + B_n\sin\alpha_n u)$$

$$=\sum_{n=1}^{\infty}(-1)^n\beta_n^2T_{6n}R_2 + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(-1)^n\beta_n(R_2K_{mn}\beta_n - R_1L_{mn}\alpha_m)\cos\alpha_m u$$

$$+(1/m_u)\sum_{k=1}^{m_u} m_k + (2/m_u)\sum_{k=1}^{m_u}\sum_{n=1}^{n}(-1)^n m_k\cos[n\pi(k - \tfrac{1}{2})/m_u] \tag{5.7}$$

where,

$$f_o(u) = 2C_{21}R_2/b^2 - 12C_{24}R_2/b^2$$

$$\begin{aligned} f_n(u) = (-1)^n\beta_n^2\Big\{&C_{1n}(A_7\cosh u_{3n}\cos u_{4n} - R_1K_3\sinh u_{3n}\sin u_{4n}) \\ &+C_{4n}(R_1K_3\cosh u_{3n}\cos u_{4n} + A_7\sinh u_{3n}\sin u_{4n}) \\ &+C_{5n}(B_7\cosh u_{3n}\cos u_{5n} + R_1K_3\sinh u_{3n}\sin u_{5n}) \\ &+C_{8n}(-R_1K_3\cosh u_{3n}\cos u_{5n} + B_7\sinh u_{3n}\sin u_{5n})\Big\} \end{aligned}$$

$$\begin{aligned} A_n = \alpha_n^2\Big\{&C_{9n}(A_8K_{1n} + A_9K_{2n}) + C_{12n}(-A_9K_{1n} + A_8K_{2n}) \\ &+C_{13n}(B_8L_{1n} + B_9L_{2n}) + C_{6n}\ (-B_9L_{1n} + B_8L_{2n})\Big\} \end{aligned}$$

$$B_n = \alpha_n^2\{C_{9n}(-A_8K_{4n} + A_9K_{3n}) + C_{12n}(A_9K_{4n} + A_8K_{3n})$$
$$+C_{13n}(B_8L_{4n} - B_9L_{3n}) + C_{16n}(-B_9L_{4n} - B_8L_{3n})\}$$

the values of A_7, A_8, ...B_9 are defined in Appendix (A)

m_u = total number of concentrated edge moments (for $-a<u<a$) due to prestressing force (m_v is for $-b<v<+b$).

m_k = an edge moment at any point along the support.

The second set of three equations due to the boundary condition (Equation 3.23) is obtained using the same procedure. Hence,

$$2C_{21}R_2/b^2 - 12C_{24}R_2/b^2 + \sum_{n=1}^{\infty}(-1)^n\beta_n^2\{C_{1n}(A_7W_{1n} - R_1K_3W_{2n})$$
$$+C_{4n}(R_1K_3W_{1n} + A_7W_{2n}) + C_{5n}(B_7W_{3n} + R_1K_3W_{4n})$$
$$+C_{8n}(-R_1K_3W_{3n} + B_7W_{4n})\} = \sum_{1}^{\infty}(-1)^n\beta_n^2T_{6n}R_2 + (1/m_u)\sum_{k=1}^{m_u}m_k \tag{5.8}$$

$$\sum_{n=1}^{\infty}(-1)^n\beta_n^2\{C_{1n}(A_7A_{j1} - R_1K_3A_{j2}) + C_{4n}(R_1K_3A_{j1} + A_7A_{j2})$$
$$+C_{5n}(B_7A_{j3} + R_1K_3A_{j4}) + C_{8n}(-R_1K_3A_{j3} + B_7A_{j4})\}$$
$$+\alpha_m^2\{C_{9m}(A_8K_{1m} + A_9K_{2m}) + C_{12m}(-A_9K_{1m} + A_8K_{2m})$$
$$+C_{13m}(B_8L_{1m} + B_9L_{2m}) + C_{16m}(-B_9L_{1m} + B_8L_{2m})\}$$
$$=\sum_{n=1}^{\infty}(-1)^n\beta_n(R_2\beta_nK_{mn} - R_1\alpha_mL_{mn}) + (2/m_u)$$
$$\sum_{k=1}^{m_u}(-1)^n m_k\cos[2n\pi(k - \tfrac{1}{2})/m_u] \quad \text{for } m\geq 1 \tag{5.9}$$

$$\alpha_n^2\Big\{C_{9n}(-A_8K_{4n} + A_9K_{3n}) + C_{12n}(A_9K_{4n} + A_8K_{3n})$$

$$+C_{13n}(B_8L_{4n} - B_9L_{3n}) + C_{16n}(-B_9L_{4n} - B_8L_{3n})\Big\} = 0 \qquad (5.10)$$

$$\text{for } n\geq 1$$

3) The third boundary condition (Equation 3.24 for $-b<v<b$) yields the following three equations:

$$24C_{24}(R_3T_a + EI)/b^4 + {}_1\Sigma^{\infty}(-1)^n\alpha_n^4\Big\{C_{9n}(A_5W_{5n} + A_6W_{6n})$$

$$+C_{12n}(-A_6W_{5n} + A_5W_{6n}) + C_{13n}(B_5W_{7n} + B_6W_{8n})$$

$$+C_{16n}(-B_6W_{7n} + B_5W_{8n})\Big\} = c^4a_oEI/4E_4 \qquad (5.11)$$

$$\sum_{m=1}^{\infty}(-1)^m\alpha_m^4\Big\{C_{9m}(A_5A_{j5} + A_6A_{j6}) + C_{12m}(-A_6A_{j5} + A_5A_{j6})$$

$$+C_{13m}(B_5A_{j7} + B_6A_{j8}) + C_{16m}(-B_6A_{j7} + B_5A_{j8})\Big\}$$

$$+\beta_n^3\Big\{C_{1n}(-EI\beta_nK_{5n} + A_{10}K_{7n} + A_{11}K_{8n})$$

$$+C_{4n}(-EI\beta_nK_{6n} - A_{11}K_{7n} + A_{10}K_{8n})$$

$$+C_{5n}(-EI\beta_nL_{5n} + B_{10}L_{7n} + B_{11}L_{8n})$$

$$+C_{8n}(-EI\beta_nL_{6n} - B_{11}L_{7n} + B_{10}L_{8n})\Big\}$$

$$= {}_1\Sigma^{\infty}(-1)^mK_{mn}\beta_n^4EI + T_{6n}\beta_n^4EI\ , \quad \text{for } n\geq 1 \qquad (5.12)$$

$$-24C_{24}R_6I_{5n}/b^3 + {}_1\Sigma^{\infty}(-1)^m\alpha_m^3\Big\{C_{9m}(A_{12}B_{j5} + A_{13}B_{j6})$$

$$+C_{12m}(-A_{13}B_{j5} + A_{12}B_{j6}) + C_{13m}(B_{12}B_{j7} + B_{13}B_{j8})$$

$$+C_{16m}(-B_{13}B_{j7} + B_{12}B_{j8})\Big\}$$

$$+\beta_n^3\Big\{C_{1n}(A_{11}K_{5n} - A_{10}K_{6n} + EI\beta_n K_{8n})$$

$$+C_{4n}(A_{10}K_{5n} + A_{11}K_{6n} - EI\beta_n K_{7n})$$

$$+C_{5n}(-B_{11}L_{5n} + B_{10}L_{6n} - EI\beta_n L_{8n})$$

$$+C_{8n}(-B_{10}L_{5n} - B_{11}L_{6n} + EI\beta_n L_{7n})\Big\}$$

$$= {}_1\Sigma^{\infty}(-1)^m\Big\{-\beta_n(R_4\alpha_m^2 + R_6\beta_n^2)K_{mn} + \alpha_n(R_3\alpha_m^2 + R_5\beta_n^2)L_{mn}\Big\}$$

$$-R_6c^4a_obI_{5n}/4E_4 - T_{6n}\beta_n^3R_6 \tag{5.13}$$

4) The fourth boundary condition (Equation 3.25) for $-b<v<b$) gives the following equations:

$$2C_{20}R_7/a^2 + 2C_{21}R_9/b^2 + 12C_{24}(R_7Ta^2 - R_9b^2/3)/b^4$$

$$+{}_1\Sigma^{\infty}(-1)^n\alpha_n^2\Big\{C_{9n}(A_{17}W_{5n} + A_{18}W_{6n}) + C_{12n}(-A_{18}W_{5n} + A_{17}W_{6n})$$

$$+C_{13n}(B_{17}W_{7n} + B_{18}W_{8n}) + C_{16n}(-B_{18}W_{7n} + B_{17}W_{8n})\Big\}$$

$$=-R_7\Big\{{}_1\Sigma^{\infty}(-1)^m(-T_{5m}\alpha_m^2)\Big\} + a_oc^4b^2R_9/12E_4 - c\ (1/m_v)\sum_{k=1}^{m}u_{m_k} \tag{5.14}$$

$$-12C_{24}R_9I_{1n}/b^2 + {}_1\Sigma^{\infty}(-1)^m\alpha_m^2\Big\{C_{9m}(A_{17}A_{j5} + A_{18}A_{j6})$$

$$+C_{12m}(-A_{18}A_{j5} + A_{17}A_{j6}) + C_{13m}(B_{17}A_{j7} + B_{18}A_{j8})$$

$$+C_{16m}(-B_{18}A_{j7} + B_{17}A_{j8})\Big\}$$

$$+\beta_n^2\Big\{C_{1n}\{A_{14}K_{5n} + A_{15}K_{6n} + \beta_n(-GJK_3K_{7n} + A_{16}K_{8n})\}$$

$$+C_{4n}\{-A_{15}K_{5n} + A_{14}K_{6n} + \beta_n(-A_{16}K_{7n} - GJK_3K_{8n})\}$$

$$+C_{5n}\{B_{14}L_{5n} + B_{15}L_{6n} + \beta_n(-GJK_3L_{7n} + B_{16}L_{8n})\}$$

$$+C_{8n}\{-B_{15}L_{5n} + B_{14}L_{6n} + \beta_n(-B_{16}L_{7n} - GJK_3L_{8n})\}\Big\}$$

$$= -c^4 a_o b^2 R_9 I_{1n}/8E_4 + T_{6n}\beta_n^2 R_9$$

$$+{}_1\Sigma^{\infty}(-1)^m\Big\{(\alpha_m^2 R_7 + \beta_n^2 R_9)K_{mn} - L_{mn}\alpha_m\beta_n R_8\Big\}$$

$$-c\Big\{(2/m_v)\ {}_1\Sigma^{m_v}\ (-1)^n m_k \cos 2n\pi(k - \tfrac{1}{2})/m_v\Big\}, \text{ for } n \geq 1 \tag{5.15}$$

$$-24C_{24}R_{10}I_{5n}/b^3 + {}_1\Sigma^{\infty}(-1)^m\alpha_m^3\Big\{C_{9m}(A_{19}B_{j5} + A_{20}B_{j6})$$

$$+C_{12m}(-A_{20}B_{j5} + A_{19}B_{j6}) + C_{13m}(B_{19}B_{j7} + B_{20}B_{j8})$$

$$+C_{16m}(-B_{20}B_{j7} + B_{19}B_{j8})\Big\} + \beta_n^2\Big\{C_{1n}\{\beta_n(A_{16}K_{5n} + GJK_3K_{6n})$$

$$+A_{15}K_{7n} - A_{14}K_{8n}\} + C_{4n}\{\beta_n(-GJK_3K_{5n} + A_{16}K_{6n})$$

$$+A_{14}K_{7n} + A_{15}K_{1n}\} + C_{5n}\{\beta_n(-B_{16}L_{5n} - GJK_3L_{6n})$$

$$-B_{15}L_{7n} + B_{14}L_{8n}\} + C_{8n}\{\beta_n(GJK_3L_{5n} - B_{16}L_{6n})$$

$$-B_{14}L_{7n} - B_{15}L_{8n}\}\Big\} = -c^4 a_o b R_{10} I_{5n}/4E_4 - T_{6n}\beta_n^3 R_{10}$$

$$-{}_1\Sigma^{\infty}(-1)^m\beta_n^2(R_{10}\beta_n K_{mn} - GJ\alpha_m L_{mn}) \quad \text{for } n \geq 1 \tag{5.16}$$

5.2 Anti-Symmetric Load

1) Proceeding as mentioned before for symmetric loads, the first boundary condition (Equation 3.22) gives the following three equations:

$$C_{19} + C_{23} = 0 \tag{5.17}$$

$$C_{10n}K_{3n} + C_{11n}K_{4n} + C_{14n}L_{3n} + C_{15n}L_{4n} = 0 \text{ for } n \geq 1 \tag{5.18}$$

$$C_{18}I_{5n}/a + C_{22}I_{7n}/a^3 + {}_1\Sigma^{\infty}(-1)^m$$

$$(C_{2m}B_{j1} + C_{3m}B_{j2} + C_{6m}B_{j3} + C_{7m}B_{j4})$$

$$-C_{10n}K_{2n} + C_{11n}K_{1n} + C_{14}L_{2n} - C_{15n}L_{1n}$$

$$= -T_{7n} - {}_1\Sigma^{\infty}(-1)^m P_{nm} \quad \text{for } n \geq 1 \tag{5.19}$$

2) The second boundary condition (Equation 3.23) along the support gives the following equations:

$$6C_{23}R_2/b^2 = 0 \tag{5.20}$$

$$\alpha_n^2\Big\{C_{10n}(A_8K_{3n} + A_9K_{4n}) + C_{11n}(-A_9K_{3n} + A_8K_{4n})$$

$$+C_{14n}(B_8L_{3n} + B_9L_{4n}) + C_{15n}(-B_9L_{3n} + B_8L_{4n})\Big\} = 0$$

$$\text{for } n>1 \tag{5.21}$$

$${}_1\Sigma^{\infty}(-1)^m\beta_m^2\Big\{C_{2m}(A_7B_{j1} - R_1K_3B_{j2}) + C_{3m}(R_1K_3B_{j1} + A_7B_{j2})$$

$$+C_{6m}(B_7B_{j3} + R_1K_3B_{j4}) + C_{7m}(-R_1K_3B_{j3} + B_7B_{j4})\Big\}$$

$$+\alpha_n^2\Big\{C_{10n}(A_9K_{1n} - A_8K_{2n}) + C_{11n}(A_8K_{1n} + A_9K_{2n})$$

$$+C_{14n}(-B_9L_{1n} + B_8L_{2n}) + C_{15n}(-B_8L_{1n} - B_9L_{2n})\Big\}$$

$$= {}_1\Sigma^{\infty}(-1)^m\beta_m(R_2\beta_m P_{nm} + R_1\alpha_n Q_{nm})$$

$$+(2/m_u) \sum_{k=1}^{m_u} (-1)^n \sin 2n\pi(k - \tfrac{1}{2})/m_u \quad \text{for } n \geq 1 \tag{5.22}$$

3) The following three equations result from the third boundary condition (Equation 3.24):

$$6C_{22}R_3/a_3^3 + 6C_{23}R_6/b^3 + \sum_{n=1}^{\infty}(-1)^n\alpha_n^3\Big\{C_{10n}(A_{12}W_{5n} + A_{13}W_{6n})$$

$$+C_{11n}(-A_{13}W_{5n} + A_{12}W_{6n}) + C_{14n}(B_{12}W_{7n} + B_{13}W_{8n})$$

$$+C_{15n}(-B_{13}W_{7n} + B_{12}W_{8n})\Big\} = \sum_{m=1}^{\infty}(-1)^m T_{7m}\alpha_m R_3 \qquad (5.23)$$

$$\sum_1^{\infty}(-1)^m\alpha_n^3\Big\{C_{10m}(A_{12}A_{j5} + A_{13}A_{j6}) + C_{11m}(-A_{13}A_{j5} + A_{12}A_{j6})$$

$$+C_{14m}(B_{12}A_{j7} + B_{13}A_{j8}) +$$

$$+C_{15m}(-B_{13}A_{j7} + B_{12}A_{j8})\Big\}$$

$$+\beta_n^3\Big\{C_{2n}(A_{10}K_{5n} + A_{11}K_{6n} - EI\beta_n K_{7n})$$

$$+C_{3n}(-A_{11}K_{5n} + A_{10}K_{6n} - EI\beta_n K_{8n})$$

$$+C_{6n}(B_{10}L_{5n} + B_{11}L_{6n} - EI\beta_n L_{7n})$$

$$+C_{7n}(-B_{11}L_{5n} + B_{10}L_{6n} - EI\beta_n L_{8n})\Big\}$$

$$=T_{8n}\beta_n^3 R_6 + \sum_{m=1}^{\infty}(-1)^m\Big\{\alpha_m(\alpha_m^2 R_3 + \beta_n^2 R_5)P_{mn}$$

$$+\beta_n(\alpha_m^2 R_4 + \beta_n^2 R_6)Q_{mn}\Big\} \quad \text{for } n\geq 1 \qquad (5.24)$$

$$\sum_1^{\infty}(-1)^m\alpha_m^4\Big\{C_{10m}(A_5B_{j5} + A_6B_{j6}) + C_{11m}(-A_6B_{j5} + A_5B_{j6})$$

$$+C_{14m}(B_5B_{j7} + B_6B_{j8}) + C_{15m}(-B_6B_{j7} + B_5B_{j8})\Big\}$$

$$+\beta_n^3\Big\{C_{2n}(EI\beta_n K_{6n} + A_{11}K_{7n} - A_{10}K_{8n})$$

$$+C_{3n}(-EI\beta_n K_{5n} + A_{10}K_{7n} + A_{11}K_{8n})$$

$$+C_{6n}(-EI\beta_n L_{6n} - B_{11}L_{7n} + B_{10}L_{8n})$$

$$+C_{7n}(-EI\beta_n L_{5n} - B_{10}L_{7n} - B_{11}L_{8n})\Big\}$$

$$=\beta_n^4 T_{8n} + {}_1\Sigma^\infty EI\beta_n^4(-1)^m Q_{mn} \qquad \text{for } n \geq 1 \qquad (5.25)$$

4) Finally, the following three equations are obtained from the fourth boundary condition (Equation 3.25):

$$6R_7C_{22}/a^2 + 6R_{10}C_{23}/b^3 + {}_1\Sigma^\infty(-1)^n\alpha_n^3\, C_{10n}(A_{19}W_{5n} + A_{20}W_{6n})$$

$$+C_{11n}(-A_{20}W_{5n} + A_{19}W_{6n}) + C_{14n}(B_{19}W_{7n} + B_{20}W_{8n})$$

$$+C_{15n}(-B_{20}W_{7n} + B_{19}W_{8n}) = 0 \qquad (5.26)$$

$$\sum_{m=1}^{\infty}(-1)^m\alpha_m^3\Big\{C_{10m}(A_{19}A_{j5} + A_{20}A_{j6}) + C_{11m}(-A_{20}A_{j5} + A_{19}A_{j6})$$

$$+C_{14m}(B_{19}A_{j7} + B_{20}A_{j8}) + C_{15m}(-B_{20}A_{j7} + B_{19}A_{j8})\Big\}$$

$$+\beta_n^2\Big\{C_{2n}\ \beta_n(-GJK_3K_{5n} + A_{16}K_{6n}) + A_{14}K_{7n} + A_{15}K_{8n}$$

$$+C_{3n}\ \beta_n(-A_{16}K_{5n} - GJK_3K_{6n}) - A_{15}K_{7n} + A_{14}K_{8n}$$

$$+C_{6n}\ \beta_n(-GJK_3L_{5n} + B_{16}L_{6n}) + B_{14}L_{7n} + B_{15}L_{8n}$$

$$+C_{7n}\ \beta_n(-B_{16}L_{5n} - GJK_3L_{6n}) - B_{15}L_{7n} + B_{14}L_{8n}\Big\}$$

$$=T_{8n}\beta_n^3R_{10} + {}_1\Sigma^\infty(-1)^m\beta_n^2(GJ\alpha_m P_{mn} + R_{10}\beta_n Q_{mn}),$$

$$n \geq 1 \qquad (5.27)$$

$$6R_9C_{23}I_{6n}/b^3 + {}_1\Sigma^\infty(-1)^m\alpha_m^2\Big\{C_{10m}(A_{17}B_{j5} + A_{18}B_{j6})$$

$$+C_{11m}(-A_{18}B_{j5} + A_{17}B_{j6}) + C_{14m}(B_{17}B_{j7} + B_{18}B_{j8})$$

$$+C_{15m}(-B_{18}B_{j7} + B_{17}B_{j8})\Big\}$$

$$+\beta_n^2\Big\{C_{2n}\ A_{15}K_{5n} - A_{14}K_{6n} + \beta_n(A_{16}K_{7n} + GJK_3K_{8n})$$

$$+C_{3n}\ A_{14}K_{5n} - A_{15}K_{6n} + \beta_n(-GJK_3K_{7n} + A_{16}K_{8n})$$

$$+C_{6n}\ -B_{15}L_{5n} + B_{14}L_{6n} + \beta_n(-B_{16}L_{7n} - GJK_3K_{8n})$$

$$+C_{7n}\ -B_{14}L_{5n} - B_{15}L_{6n} + \beta_n(GJK_3L_{7n} - B_{16}L_{8n}) \Big\}$$

$$=\beta_n^2T_{8n}R_9 + {}_1\Sigma^{\infty}(-1)^m\Big\{R_8\alpha_m\beta_nP_{mn} + (R_7\alpha_m^2 + R_9\beta_n^2)Q_{mn}\Big\}$$

$$- c\Big\{(2/m_v)\sum_{k=1}^{m_v}(-1)^n\sin[2n\pi(k - \tfrac{1}{2})/m_v]\Big\} \quad n\geq 1 \qquad (5.28)$$

It should be noted that for one boundary condition, (2n+1) equations are obtained. For the slab bridge with four boundary conditions, (8n+4) equations are obtained, so that for each case of loading (symmetric or anti-symmetric) the size of the matrix will be (8n+4).

Once the matrix equation is formulated and solved for the unknown constants, the deflection function is known over the entire area of the slab. The moments and strains can be computed readily as follows:

$$M_x = (-D_x/c^2)W,_{uu} + (2D_xs/c^2)W,_{uv} - (D_xs^2/c^2 + D_1)W,_{vv} \qquad (5.29)$$

$$M_y = (-D_2/c^2)W,_{uu} + (2D_2s/c^2)W,_{uv} - (D_2s^2/c^2 + D_y)W,_{vv} \qquad (5.30)$$

$$M_{xy} = (0)W,_{uu} + (D_{xy}/c)W,_{uv} - (D_{xy}s/c)W,_{vv} \qquad (5.31)$$

The strain can be obtained from Equation 3.7 by substituting for M_x and M_y.

CHAPTER VI

EXPERIMENTAL PROGRAM

6.1 Scope of the Experimental Program

To verify the analytical approach proposed in Chapters III, IV and V, tests were carried out on two groups of concrete waffle-type slabs: reinforced and prestressed. The waffle slabs tested were one-to-eight scale models of concrete bridge decks. The tests were aimed at obtaining the deflections, stresses and bending moments at various points and determining the cracking and ultimate loads.

The first group consisted of three reinforced concrete waffle slabs, two were rectangular and the third had a 45° skew. One rectangular slab was tested for a uniformly distributed load by means of air pressure and a rubber membrane while the other two slabs were under concentrated loads only. The second group consisted of two post-tensioned rectangular and 45°-skew prestressed concrete waffle slabs, each subjected to edge prestressing forces and external concentrated loads applied transversely at various points.

6.2 Materials

6.2.1 Concrete

High Early Strength Portland Cement (CSA) manufactured by Canada Cement Company was used in all slab models. This type of cement provides high strength within a week.

A clean sand free of impurities was used. The maximum size of aggregate was restricted to 0.25 inch (6 mm) since the narrowest dimension between the sides of the formwork was equal to 1.25 inch (32 mm) and the concrete cover to the reinforcing wires was 0.375 inch (10 mm). The combined aggregate was prepared according to the ACI code (1) by mixing 40 to 60% fine aggregates of the total aggregates. This combination gave a well-graded aggregate mix with a fineness modulus equal to 2.50. The coarse aggregates used were crushed stones with hard, clean and durable properties. Natural water having no impurities was used to obtain different concrete pastes and varying maximum strength for the concrete specimens.

Five trial mixes of air-entrained concrete of medium consistency and different water cement ratio varying between 40% to 70% were examined. Mixing was done in an Eerich Counter Current Mixer, Model EA2(2W) with five cu. ft. charging capacity and manually operated. Two batches of concrete mix were required for each slab model, each weighing 700 lbs. The compressive strength of all specimens were measured after seven, fourteen and twenty-eight days as shown in Appendix (D).

6.3 Steel

6.3.1 Mild Steel For Reinforced Concrete Slabs

To fulfill the code requirements (1), and using the minimum area of steel, three one-eighth inch (3.2 mm) in diameter mild steel wires were used. The wires were straightened and twisted in the laboratory, keeping the cross-sectional area the same along the whole length by providing the wires with constant number of pitches per unit length. The wires were cut into different lengths, hooked at both ends and cleaned from rust. The stress-strain relationship for the reinforcing wire is shown in Appendix (E); the yield strength was found to be 33,000 psi (227.7 MN/m^2). The modulus of elasticity was determined from the calibration test using strain gauges mounted on the wire and was found to be 30,000 ksi (207 GN/m^2).

6.3.2 High Tensile Steel for the Prestressed Concrete Slabs

High tensile steel wire of 0.196 inch (5 mm) in diameter were used for the prestressed concrete waffle slabs. Tensile tests on the wire indicated an ultimate strength of 225,000 psi (1.553 GN/m^2), and a yield stress of 29,700 psi (204.9 MN/m^2); see Appendix (E). This wire gave good resistance against slippage from the grips. Figure 6.1 shows the twisted reinforced steel wire and the high tensile steel wire used in the slab models.

6.4 Formwork

Two forms were made from plywood, 3/4 inch (19 mm) thick, and wood joists, 2 x 4 inches (51 x 102 mm) cross-sectional dimension, were used as stiffeners. The first form was rectangular in shape and used for casting three slabs, two reinforced concrete and the third being the prestressed concrete slab. The second form was used for casting the reinforced and prestressed concrete slabs of 45° skew as shown in Figure 6.2. Styrafoam cubes were used for producing the waffle shape; styrafoam plates of 3 inch (76 mm) thickness were cut into cubes of 3 x 4 x 4 inches (76 x 102 x 102 mm) dimensions, to fit the clearance between the ribs.

6.5 Experimental Equipment

6.5.1 Prestressing Equipment

The prestressing equipment used in prestressing the wires, were manufactured by Cable Covers Ltd., England. A hydraulic jack of twenty kips (89 KN) capacity was used for post-tensioning as shown in Figure 6.3. The mechanical gripping devices of the open grip type and washers shown in Figure 6.1, were very simple and quick to use. Black wax lubricant was applied to the wedges to make it easier to release the grips after completing the prestressing operation.

6.5.2 End Bearing Plate

Fifty-eight end bearing plates, 0.25 inch (6.4 mm) thick, with holes of 0.25 inch (6.4 mm) diameter and of 1.5 x 3.0 inches (38 x 76 mm) dimensions, were used to distribute the prestressing force at each end of the rib. Twenty-six steel blocks of 1.5 x 1.5 x 3.0 inches (38 x 38 x 19 mm) were used for the prestressed skew slab.

Each steel block was provided with two perpendicular eccentric holes to fit the wires in two directions. Figures 6.1 and 6.4 show the end bearing plates and the grooves in the concrete skew slab to accommodate the steel blocks.

6.5.3 The Steel Frame For Producing Uniformly Distributed Load

The purpose of the steel loading frame, Figures 6.5 and 6.7, was to apply a uniformly distributed load by means of an air chamber and a rubber membrane. The top of the air chamber was made of a stiffened steel plate, 0.25 inch (6.4 mm) thick and, 104.25 x 87.50 inches (2.65 x 2.22 mm) in plan. The bottom of the air chamber was a rubber membrane, 0.09 inch (2.4 mm) thick and 134.25 x 117.5 inches (3.41 x 2.98 m) in plan. The steel cover plate was placed on the top of the rectangular steel frame made of two steel channels as shown in Figures 6.5 and 6.6. Figures 6.7 and 6.8 show the longitudinal and transverse sections in the steel loading frame. Two steel rods of 1.25 inch (31.8 mm) diameter, were placed on the top flange of the beams to be used as simple supports for the slab. Styrafoam strips were placed inside the channel of the air chamber to avoid any damage to the rubber membrane from the sharp corners of the steel frame.

All supporting beams were tack-welded onto the floor to avoid any sliding or uplift during the experiment. Two steel rods, 0.25 inch (6.4 mm) in diameter were tack-welded onto the flange plate around the supporting steel rods to prevent sliding and to allow rotation. Three pressure gauges, two valves and a regulator were fixed on the steel plate to measure and control the air pressure as shown in Figure 6.6. Heavy steel clamps were used to clamp the steel plate and the rubber membrane with the steel frame as shown in Figure 6.5.

6.6 The Construction of the Slab Models

Figures 3.2 and 3.3 show the dimension of the cross-sections of the five one-eighth scale model slabs. Figures 6.9 to 6.13 show the layout of the five slabs. The reinforced concrete slabs (Group A) are denoted by A.1, A.2 and A.3, while the prestressed concrete slabs (Group B) are referred to as B.1 and B.2. The required form was prepared as shown in Figure 6.2 and then painted with grease material, Vitrea Oil 150, for easy form release after the concrete has set. The twisted steel wires for group A were placed in the form; they were instrumented with electric strain gauges. The bottom steel layer was supported on steel wire chairs which provided a clear cover of 0.5 inch (12.7 mm) from the bottom. The second steel layer was supported directly over the first layer and fixed with thin wires. All reinforcing wires were hooked at both ends.

For the group B slabs, rubber hoses having an inner diameter of 0.25 inch (6.4 mm) and 0.062 inch (1.6 mm) thick were used to cover the steel wires during casting of the concrete. The first layer of the steel wires was placed at a distance of one inch from the bottom, and supported on thin steel wires fixed between the styrafoam cubes. The second layer was placed directly over the first layer.

To determine the compressive strength of the concrete, six 3 x 6 inches (76 x 152 mm) cylinders were cast with each model.

All concrete in the slab models was vibrated with a high frequency vibrator. Care was taken during casting and vibrating to ensure that no segregation occurred. The top surface of the slab models was troweled smooth after casting. The slab models and the cylinder specimens were cured in water for 14 and 28 days and then were allowed to dry at least four days before mounting the strain gauges.

6.7 Instrumentation

6.7.1 Strain Gauges on the Reinforcement

The longitudinal and transverse wires were instrumented with electric strain gauges as shown in Figures 6.10 and 6.11. The gauges used were metal gauges of type EA-06-062AP-120. The surface of the reinforcing wire was prepared by cleaning it using fine silicon carbide paper and acetone. The gauge was mounted using Eastman M-Bond 200 adhesive with 200 catalyst as bonding agent according to the manufacturer's recommendations. The lead wires were then soldered to the gauges and water-proofed with a plasticised epoxy resin system, Bean Gagekote #3. After curing for 24 hours at room temperature a layer of wax was applied on the gauge and plastic tape for further protection from the wet concrete.

6.7.2 Strain Gauges On The Concrete

To measure the strain on the top and the bottom surfaces of the slab models, electric strain gauges of type EA-06-500 BH-120 were used. Three-legged rosette gauges were used also on the deck surface of the skew model at the obtuse corner. All gauges had a nominal gauge length of 0.50 inch (12.7 mm). The locations of the strain gauges are shown in Figures 6.9 to 6.13.

The concrete surfaces at the locations of the gauges were smoothed using sandpaper, all dust was removed and then the surfaces were cleaned with acetone. Surface cavities were then filled by applying an epoxy of high strength (RTC). This epoxy was mixed by one volume activator B, and the same volume of resin A. After the surface was dry, it was again smoothed and the gauge was mounted, soldered with lead wires and covered by coating epoxy Bean Gagekote #3. The gauges were then connected to a strain indicator, Strainsert model TN 20C.

6.7.3 Mechanical Dial Gauges

The deflections were measured using mechanical dial gauges having 0.001 inch (.025 mm) travel sensitivity. The locations of the dial gauges are shown in

Figures 6.9 to 6.13, and are seen in position in Figure 6.14. In group A slabs, the dial gauges were placed at the bottom surface of the slab. In group B slabs, the dial gauges were placed at the top surface of the slab and supported by a light steel frame.

6.7.4 Load Cells

a) Universal Flat Load Cell

Two universal flat load cells, having capacities of 5 kips (22.25 KN), and 25 kips (111.3 KN), were used in reinforced and prestressed concrete slabs respectively to determine the value of the applied concentrated load through the hydraulic jack as shown in Figure 6.15. The calibrations of these load cells are given in Appendix (E)

b) Cylindrical Load Cell

Thirty-eight cylindrical load cells were used to measure the prestressing force in the wires. The calibration of these load cells is given also in Appendix (E). Figure 6.4 shows the cylindrical load cells in position.

6.8 Experimental Setup and Test Procedure

Due to the heavy weight of each slab model and the difficulty in handling, it was decided to cast the slabs near the portal loading frame and in the vicinity of a crane.

6.8.1 Reinforced Concrete Waffle Slabs, Group A

a) Slab A.1

Reinforced concrete slab A.1 was subjected to a uniform load, applied by pumping compressed air into a chamber formed by a rubber membrane and a steel frame as shown in Figures 6.7 and 6.8. The setup was designed and fabricated. Care was taken to insure that the rubber membrane was in contact with the slab's surface, without allowing any tension in the membrane, which might affect the results. Shims were provided along the support rods so that uniform contact was maintained between the supports and the slabs. Clearance of 0.125 inch (3.15 mm) was allowed between the edges of the slab and the edge of the steel frame to allow rotation of the slab during the test. To ensure that the air chamber is totally air tight, heavy steel clamps were used every ten inches along the steel plate.

To measure the air pressure inside the air chamber three pressure gauges of 0.15 lb/in^2 (107 KN/m^2) accuracy were used at various points as shown in Figure 6.6. A regulator valve was used to control the air pressure and for loading and unloading as shown in Figure 6.6. Forty-two strain gauges were mounted on the concrete, at top of the deck and bottom surface of the longitudinal and transverse ribs and at mid-depth of the rib to observe the strain gradient through the depth of the rib. Nine dial gauges were used to measure the deflections as shown in Figure 6.9.

b) Slab A.2

Slab A.2 was identical in geometric shape and amount of steel to slab A.1, but was tested under transverse concentrated loads, applied through a rigid portal frame supporting a cross I beam and a hydraulic jack of 20,000 lbs. (89 KN) capacity. The arrangement was such that the concentrated load could be applied anywhere without moving the slab; see Figure 6.16. The transfer of the load from the hydraulic jack to the slab was made through a small rectangular steel plate, 1 inch (25.4 mm) thick and 5 x 6 inches (127 x 152 mm) in plan. Grooves of 0.187 inch (4.8 mm) thickness and 0.75 inch (19 mm) width were made on the bottom of the steel loading plate to avoid contact with the strain gauge located at loading positions. Loading and unloading were applied three times before starting the test, to minimize the residual strains and for proper seating of the slab on the supports. Figure 3.4 shows the loading points: points 1 to 6 for the working load stage, and point 7 up to failure of the model.

The strains were measured by thirty-five electric strain gauges mounted on the concrete and steel. Deflections were measured by sixteen mechanical dial gauges as shown in Figure 6.10. The readings from loading and unloading were recorded and averaged.

c) Slab A.3

The dimensions of this slab are shown in Figure 6.11 and its loading system is shown in Figure 6.17. The loading plate was one inch (25.4 mm) thick and 6x7 inches (152 x 179 mm) in plan as shown in Figures 4.1 and 6.15. The slab was tested under concentrated load at eight locations, Figure 3.4. Twenty-three electric strain gauges were used to measure the strains in the concrete. and steel. Ten mechanical dial gauges were used to measure the deflections; see Figure 6.11.

6.8.2 Prestressed Concrete Waffle Slabs, Group B

This group consisted of two post-tensioned waffle slabs, a rectangular slab B.1 and a 45^{o} skew B.2.

a) Slab B.1

Forty-five strain gauges were used to measure the strain on the concrete surfaces and eleven mechanical dial gauges were used to measure the deflections as shown in Figure 6.12. The slab was prestressed by twenty-nine high tensile steel wires, each one connected to a cylindrical load cell to measure the prestressing force.

All care and adequate precautions were taken during the prestressing process. To avoid any distortion in the edge beam and/or local failure, the wires were tensioned in a sequence starting with the odd-numbered wires and followed by tensioning the even-numbered wires. Figures 6.18 and 6.20 show the loading system and the final amount of prestressing force.

The slab was tested under concentrated load through a universal load cell of twenty-five kips capacity. Figure 3.5 shows the loading points: points 1 and 2 for the working load stage, and point 3 up to failure of the model.

b) Slab B.2

The slab was mounted with twenty-five strain gauges to measure the strain on the concrete surfaces and eleven dial gauges to measure the deflections as shown in Figure 6.13. The slab was prestressed by thirty-eight high tensile steel wires in both directions, thirteen wires in the longitudinal direction (parallel to the traffic) and twenty-five wires were perpendicular to the edge beams. To prestress the wires along the skew supports, steel end blocks were placed in ninety degree grooves formed before casting the concrete; see Figure 6.4. The wires were tensioned in the sequence mentioned earlier for slab B.1.

The loading system used is shown in Figure 6.19. The slab was tested at four points; Figure 3.5 shows the loading points: points 1,2,3 and 4 for the working load stage, and again at point 1 up to the failure of the model.

CHAPTER VII

DISCUSSION OF RESULTS

7.1 General

The theoretical and experimental results for deflections and strains of reinforced concrete waffle slabs are compared in Figures 7.3 to 7.8, Figures 7.11 to 7.17 and Figures 7.20 to 7.25. The upward deflections (camber) and strains of group B slabs due to prestressing force are presented in Figures 7.28, 7.29, 7.30, 7.39, 7.40 and 7.41.

The comparisons between experimental and theoretical results for stresses and deflections for group B slabs, are given in Figures 7.31 to 7.34 and Figures 7.42 to 7.46.

Finally, comparisons between the results for the waffle slab type and those for the slab type with uniform thickness and having the same volume of concrete and amount of steel, are given in Figures 7.47 to 7.56.

7.2 Reinforced Concrete Waffle Slabs (Group A)

7.2.1 Rectangular Slab Under Uniform Load (Slab A.1)

Figures 7.3 and 7.4 show the comparison between the theoretical and experimental results for the load-deflection relationships at various points on the centre line and on the free edge of the slab. Figures 7.5 and 7.6 show the comparison between the theoretical and experimental deflections for a uniform load of 0.3 lb/in^2 (2 KN/m^2) before cracking. Close agreement is observed in the results before reaching the cracking load, with a maximum difference between the theoretical and experimental results of varying from 5 to 7%.

Load-strain relationships for points at the centre and at the free edge are plotted in Figures 7.7 and 7.8, showing the strains at the top and bottom fibres in the x and y directions. Close agreement between the theoretical and experimental results is observed before cracking of the concrete. Considering the maximum moments due to self-weight of the slab, the microcracking occurs at a strain of approximately 100×10^{-6} in/in, which agrees with the results formed by Evans (6).

Figure 7.2 shows the crack progression during loading of slab A.1, and finally the failure of the slab as shown in Figure 7.1. It can be observed that the cracks appear in the longitudinal ribs between the supports with no cracks in the transverse ribs. Figure 7.8 shows that the maximum strain occurs in the longitudinal ribs, while the strain in the transverse ribs is very small. The final failure of the slab occurs when the maximum tensile stress due to bending is reached at the centre of the slab. The microcracking loads starts at a load equal to one-third the ultimate load.

7.2.2 Rectangular Slab Under Concentrated Load (Slab A.2)

Figure 7.11 shows the load-deflection curves up to the ultimate load due to a concentrated load at the centre of the slab. A very good agreement is observed between the theoretical and experimental results before microcracking. The microcracking occurs at 0.70 of the experimental cracking load and 0.50 of the ultimate load.

Since the slab was tested first up to 0.60 of the yield stress of the steel for various locations, local cracks occurred near the loading point. It is found that these local cracks reduce the rigidities of the slab by a ratio of 65 to 75%. Figure 7.12 shows the load-deflection curves up to 0.60 yield stress in the steel due to a concentrated load at the central point on the free edge. For this loading case if one accounts for the presence of local cracks by multiplying the experimental deflections by 0.70 (due to reduced rigidity of the slab) the adjusted experimental results become much closer to the theoretical ones.

Figures 7.13 and 7.14 show the theoretical and experimental deflection patterns for a one-kip (4.45 KN) concentrated load at various positions. Curve #1 represents the deflection due to a concentrated load at the centre of the slab and shows good agreement with the theoretical results. Curves #2 and #3 show the theoretical and experimental deflections due to a one-kip load at points 2 and 3; it is observed that 70% of the experimental deflections, gives good agreement with the theoretical deflections; as explained before, this percentage reduction is applied because of the reduced rigidities at points 1 and 2 due to local cracking resulting from prior loading of the slab at these points.

Figures 7.15 and 7.16 show the load-strain relationships for concentrated loading at the centre and at the free edge. There is better agreement between the theoretical and experimental results for a concentrated load at the centre; comparing the results in Figure 7.15 with the results for uniform loading, Figure 7.7, the orthotropic slab subjected to a concentrated load is more efficient than one loaded by a uniform load. In the case of concentrated load, the transverse ribs are working effectively together with the longitudinal ribs to transmit the load to the supports; which in the case of uniform load the slab acts as a wide beam. The differences in the strains in the transverse ribs for both slabs can also be noted. Figure 7.10 shows the cracks in the bottom fibres of the slab A.2, before failure. It is observed that the majority of the cracks occur in the longitudinal ribs, while only haircracks appear in the transverse ribs.

The strain patterns at the bottom fibre due to a one-kip load at the centre of the slab are shown in Figure 7.17. It can be observed that the strains in the transverse ribs at the loading point equal about 50% of the strains in the longitudinal ribs at the same point. The recorded ultimate load for this slab was 2.25 kips (10 KN) with the corresponding deflection of 2.50 inches (64 mm),

Figure 7.9.

7.2.3 45° Skew Slab Under Concentrated Load (Slab A.3)

Figures 7.20, 7.21 and 7.22 show the load-deflection results before and after cracking and the deflection pattern along the transverse direction. A very good agreement between the theoretical and experimental results is obtained before cracking as shown in Figure 7.20. It is observed that the experimental results in Figures 7.21 and 7.22 when scaled down by 70% (due to the reduced rigidities) are in close agreement with the theoretical ones.

Figures 7.23, 7.24 and 7.25 show the load-strain relationships and the strain pattern in both directions and in the top and bottom fibres. A study of Figure 7.23 reveals good correspondence between the theoretical and experimental results and demonstrates that in skew slabs, the transverse ribs work effectively together with the longitudinal ribs to transmit the load to the supports.

Figures 7.18 and 7.19 show the diagonal cracks in the bottom fibre at the free edge. Referring to Figures 7.10 and 7.19 and comparing the two patterns of cracks, it is observed that the cracks in the rectangular slab is due to flexure while the cracks in the skew slab is due predominantly to twisting.

7.3 Prestressed Concrete Waffle Slabs (Group B)

7.3.1 Rectangular Slab Under Concentrated Load (Slab B.1)

Figure 7.28 shows the upward deflection due to prestressing force and edge moment, while Figures 7.29 and 7.30 show the strain pattern in the top and bottom fibres of the slab due to prestressing force in both directions. The variation in the strain along the edge beam is due to the variation in the prestressing force.

Figures 7.31 and 7.32 show the load-deflection relationship for a concentrated load at the centre and on the edge beam respectively. It can be observed that there is very good agreement between the theoretical and experimental results before reaching the cracking load with a percentage difference of about 1%.

Load-strain relationships are shown in Figures 7.33 and 7.34 for concentrated loading at the centre of the slab and at the centre of the edge beam. It can be observed that there is close agreement between the theoretical and experimental results. From Figure 7.34 it can be noted that the cracks start due to flexure under the concentrated load, followed by another crack developing in the top fibre leading to the formation of a yield line as shown in Figure 7.26. Figure 7.27 shows the final collapse due to punching shear around the loading plate.

7.3.2 45° Skew Slab Under Concentrated Load (Slab B.2)

Figures 7.39, 7.40 and 7.41 show the deflection pattern and strain distribution in the top and bottom fibres over the entire area of the slab. The differences between the theoretical and experimental deflections (camber) and strains vary between 4 to 6%.

Figures 7.42, 7.43 and 7.44 show the load deflection curves for the theoretical and experimental results; very good agreement is obtained before reaching the cracking load, the difference between the results being less than 2%. Figures 7.45 and 7.46 show the load-strain relationships for concentrated loading at the centre and at the free edge. Point 3 located near the obtuse corner, Figure 7.46, appears to be the critical point on the slab as indicated by the magnitude of strains in the bottom and top fibres. For concentrated loading at the centre of the slab, the cracks start first at the bottom fibre under the load, followed by cracks appearing close to the obtuse corner as shown in Figure 6.4. Figures 7.35 and 7.36 show the development of these cracks. The cracking load for the concentrated load at the centre is 6 kips (26.7 KN) and the slab continues to carry load up to 11 kips (49 KN) and finally fails in punching shear as shown in Figure 7.37. Figure 7.38 shows the number and direction of cracks in the bottom fibre for the same slab under load.

7.4 Comparison Between the Behaviours of a Waffle Slab and a Uniform Thickness Slab Having the Same Volume of Concrete and Reinforcement

An analytical study, using the computer program and the proposed formulae to estimate the rigidities [13] is made on two types of orthotropic slabs. The first type is a waffle slab structure and the second is a slab structure with uniform thickness, both structures having the same volume of concrete and reinforcing steel. The study includes prestressed concrete waffle slabs as well as reinforced concrete waffle slabs before cracking.

According to the proposed formulae for flexural and torsional rigidities, the values of the flexural rigidities of the orthotropic slab of uniform thickness are approximately one-half of the investigated waffle slabs in both directions; however, the torsional rigidity of the former is four times that of the latter. Figures 7.47 to 7.56 show the results for deflection and strain patterns over the entire area of the slab due to uniform load as well as concentrated load.

Rectangular and skew slabs are considered in this study. The figures show the comparison for deflections and strain between the waffle slab and the slab with uniform thickness. It can be observed from these figures that the waffle slab exhibits much lower stress. For example, the reinforced concrete waffle slabs of rectangular shape and subjected to uniform load have deflections at the centre and centre of the free edge of 43% and 51%, respectively, of those obtained in a slab of uniform thickness. These ratios increase for the skew type to 88% and 94%. The higher flexural rigidities of the waffle slabs produce much smaller deflections and lower stresses, particularly for the rectangular slab. It should also be mentioned that increase in skew leads to a decrease in the advantages cited above as well as when the concentrated load moves away from the centre of the slab.

One advantage of waffle slabs over slabs of uniform thickness is in prestressed construction. The efficiency of the prestressed waffle slabs in carrying load is much higher than that of slabs of uniform thickness of the same volume of concrete and amount of steel. This is due to the presence of the ribs which contribute to increased eccentricities in waffle slabs for prestressing. Figure 7.56 shows a comparison for the deflection pattern between the two types.

7.5 Sources of Error

The discrepancies between the experimental and theoretical results can be attributed to several sources of error such as:

1. The assumptions made in the theory.
2. Estimates of Poisson's ratio and the modulus of elasticity of concrete by means of empirical formulas.
3. Approximation of the plane stress problem encountered in the prestressed waffle slab.
4. Estimating the strength of the concrete from tests on 3 x 6 inches cylinder.
5. Distortion in the formwork due to effect of water, and lack of complete contact between the support and the test model.
6. Positioning of reinforced and prestress wires.

7. The calibration of load cell, sensitivity and drag in mechanical dial gauges, the stability of strain gauges and the strain gauge measuring device.

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

The overall objective of this study was to obtain a better understanding of the behaviour of reinforced and prestressed concrete waffle slabs, before and after cracking of the concrete. A Fourier series method of analysing single span rectangular and skew waffle slabs by orthotropic plate theory was presented; uniform load as well as concentrated loads were considered. Proposed theoretical formulae for calculation of the various orthotropic rigidities were used. The theoretical solutions were supported by the experimental results obtained from tests on prestressed and reinforced concrete models. An analytical study and comparison between the behaviour of a waffle slab and a uniform thickness slab having the same volume of concrete and reinforcement were made.

Based on the results obtained from the theoretical and experimental studies the following conclusions are drawn:

1. The good agreement between the experimental and theoretical results supports the reliability of the proposed formulae for estimating the orthotropic rigidities.
2. The theoretical solution gives good convergence in the results for uniform and concentrated loads at the centre of rectangular

slabs. Such degree of convergency decreases as the concentrated load moves away from the centre of the slab or when the skew angle increases.

3. The discrepancies between the theoretical and the experimental results for reinforced concrete slabs are due to microcracking.
4. Local cracks produced near the concentrated load in reinforced concrete slabs can reduce the stiffness of the slab by as much as 30%.
5. The torsional rigidity of the slab plays an important part on the behaviour of a skew slab as well as when a concentrated load is close to the unsupported edges.
6. The behaviour of prestressed waffle slabs can be much better predicted than that for reinforced concrete waffle slabs; this is due to absence of local cracking and micro-cracking.
7. The presence of the ribs in waffle slab construction makes it possible to accommodate the prestressing steel more readily and at varying levels of eccentricity, and therefore the realization of its use for longer span, resulting in substantial economy.
8. In general reinforced and prestressed waffle slabs are structurally more efficient than

slabs with uniform thickness.

8.2 Suggestions For Future Research

The following suggestions are recommended for future research:

1. An exact solution for the plane stress problem for rectangular and skew prestressed concrete waffle slabs is required for better predictions.
2. The after cracking behaviour of reinforced and prestressed concrete waffle slabs up to failure of the slab should be investigated.
3. The effect of edge beam with different cross sections is of practical interest.
4. An analysis based on a finite element method or finite difference technique should be established to deal with the problem of varying rigidities of the slab due to cracking of the concrete at various locations.

FIGURES

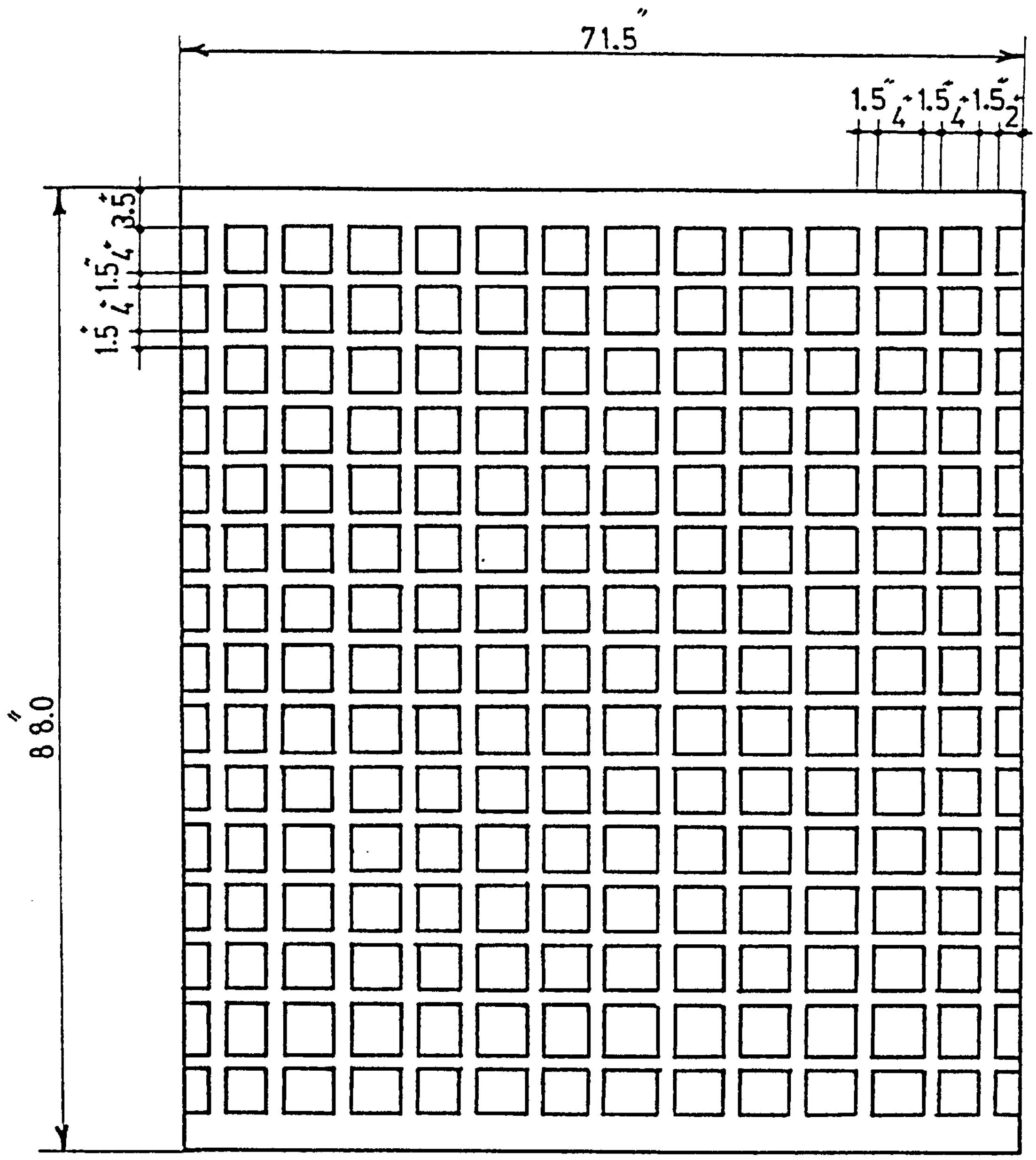


FIGURE 3.1 BOTTOM PLAN LAYOUT WAFFLE SLABS A_1 AND A_2.

(1 in. = 25.4 mm)

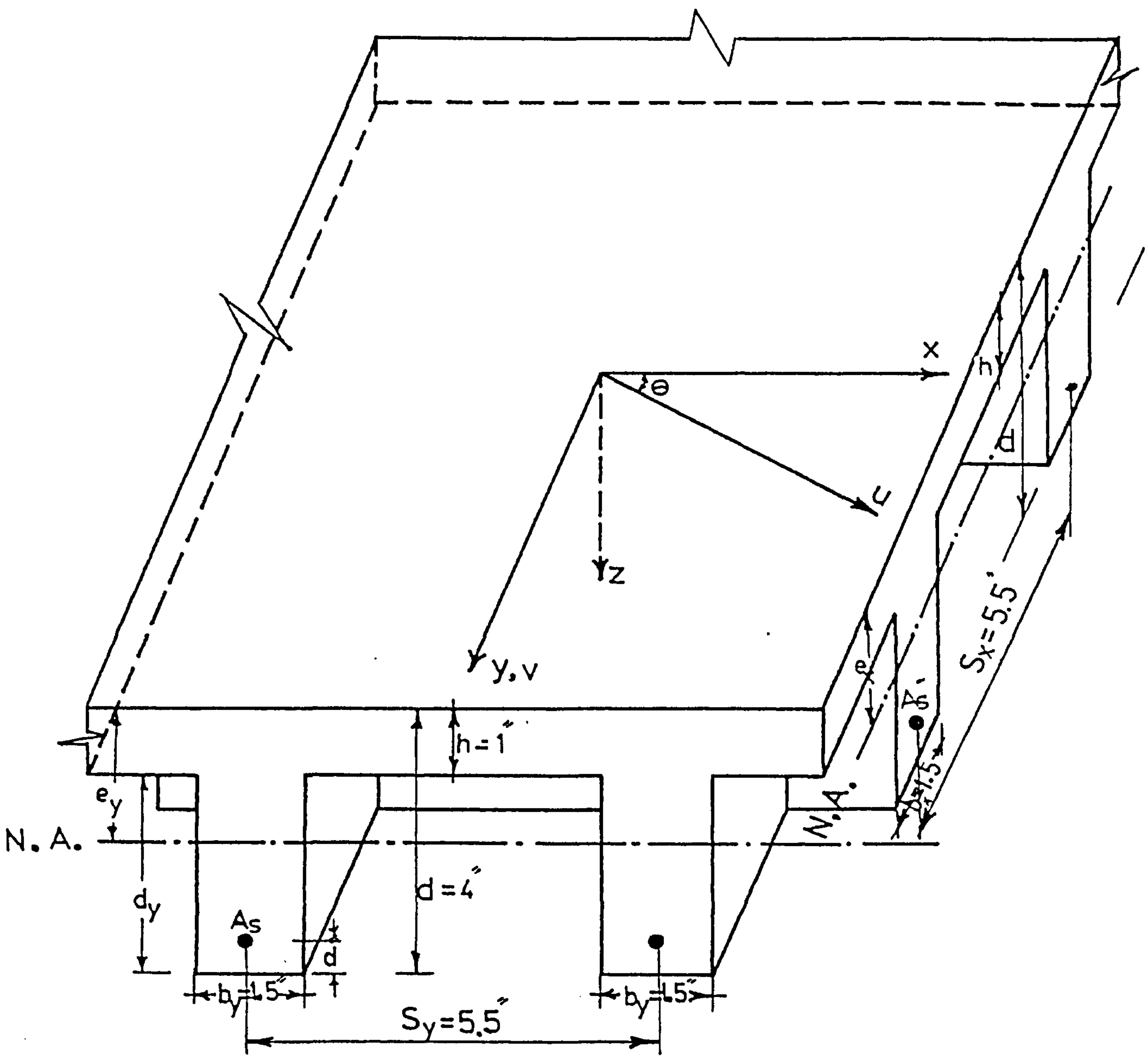
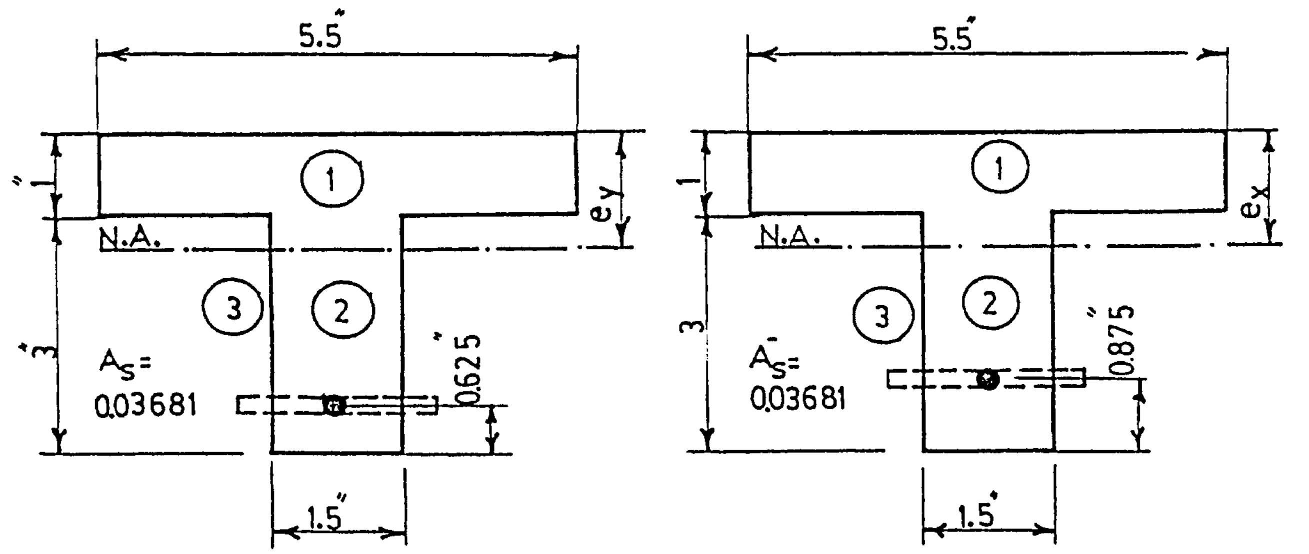


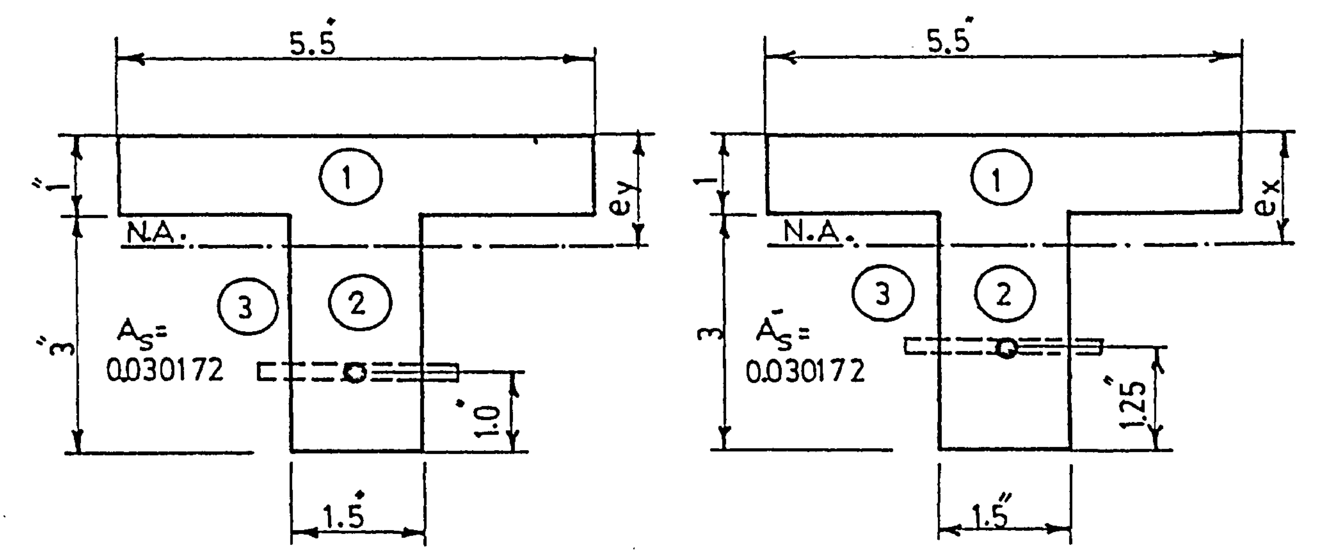
FIGURE 3.2 GEOMETRIC SHAPE OF THE RIBS AND THE PLATE FOR WAFFLE SLAB.

(1 in. = 25.4 mm)



LONGITUDINAL SECTION (y AXIS)   TRANSVERSE SECTION (x AXIS)

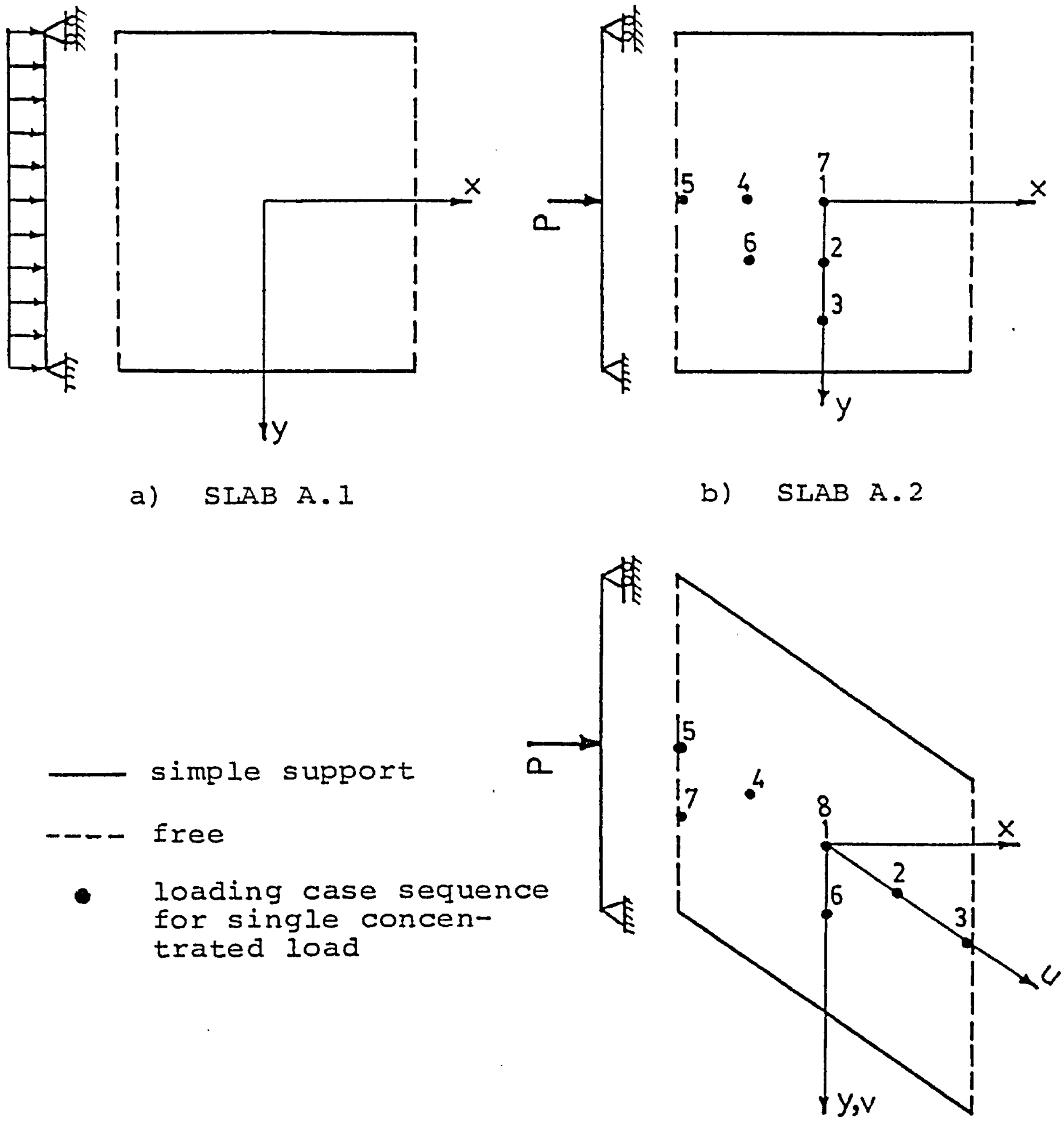
a) REINFORCED CONCRETE (GROUP A)



LONGITUDINAL SECTION (y AXIS)   TRANSVERSE SECTION (x AXIS)

b) PRESTRESSED CONCRETE (GROUP B)

FIGURE 3.3 ORTHOTROPIC RIGIDITIES OF WAFFLE SLAB.
(1 in. = 25.4 mm)

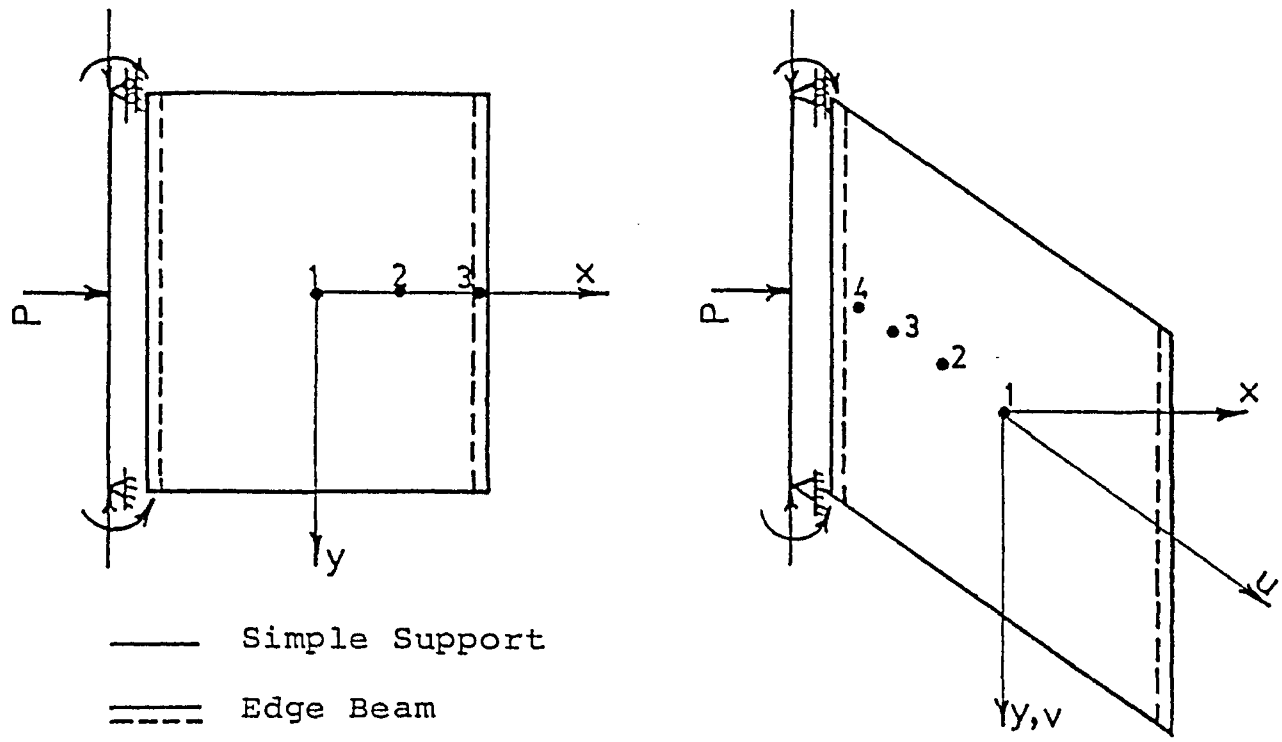


a) SLAB A.1

b) SLAB A.2

c) SLAB A.3

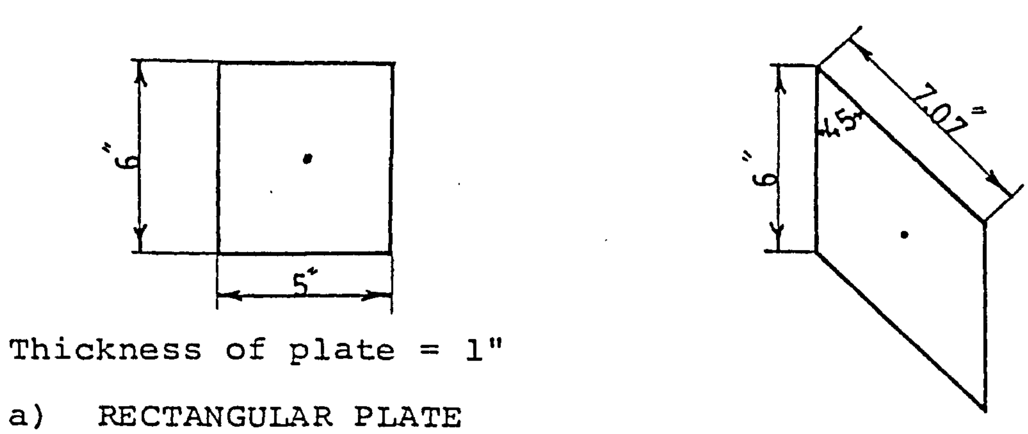
FIGURE 3.4 LATERAL LOADING CASES FOR GROUP A SLABS



a) SLAB B.1

b) SLAB B.2

FIGURE 3.5 LATERAL LOADING CASES FOR GROUP B SLABS



a) RECTANGULAR PLATE

b) SKEW PLATE

FIGURE 4.1 STEEL LOADING PLATES

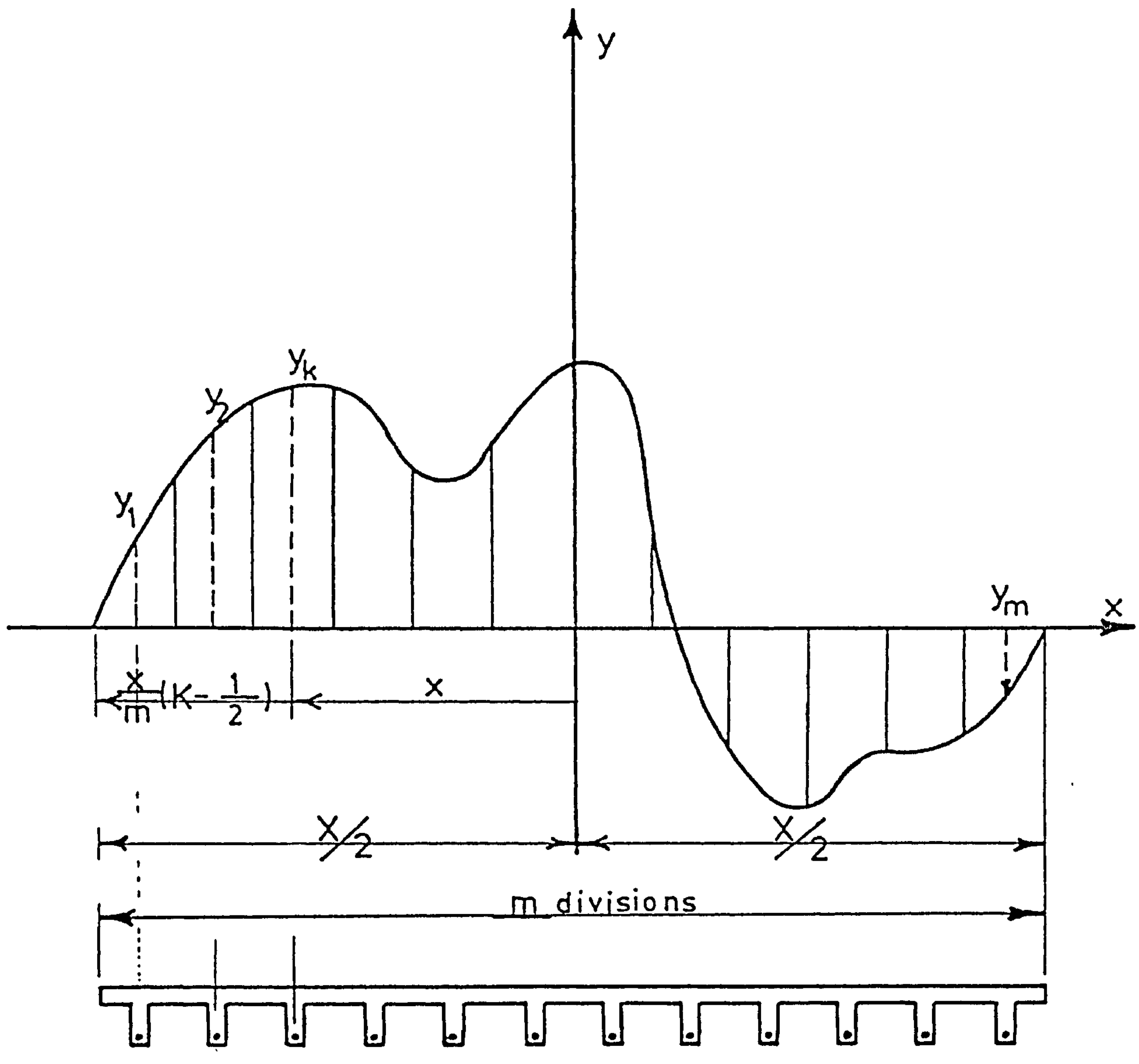


FIGURE 4.2 EXPANSION OF THE LOAD IN FOURIER SERIES.

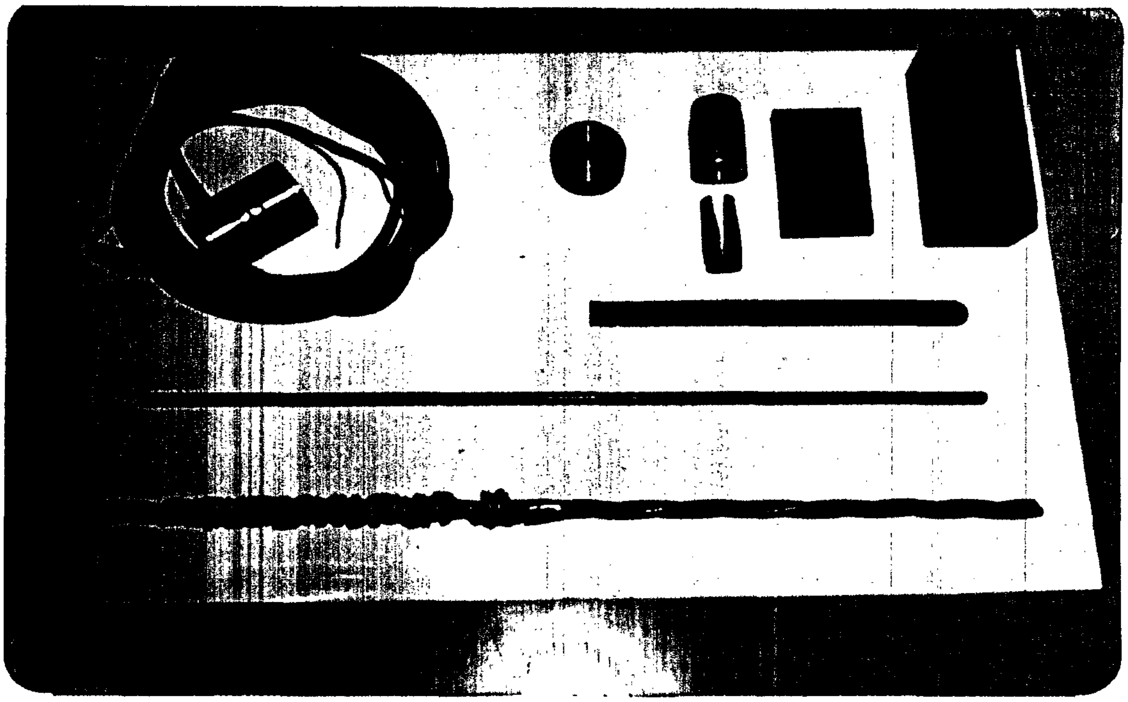


FIGURE 6.1 THE REINFORCING STEEL AND ANCHORAGE SYSTEM.

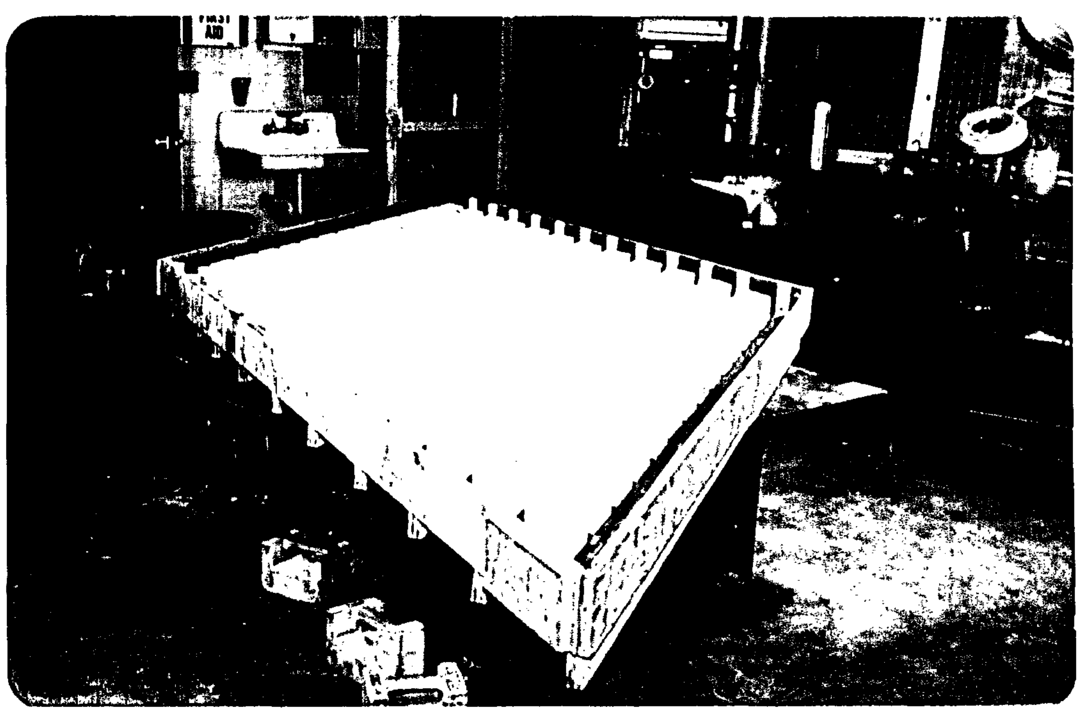


FIGURE 6.2 FORM BEFORE CASTING THE CONCRETE.

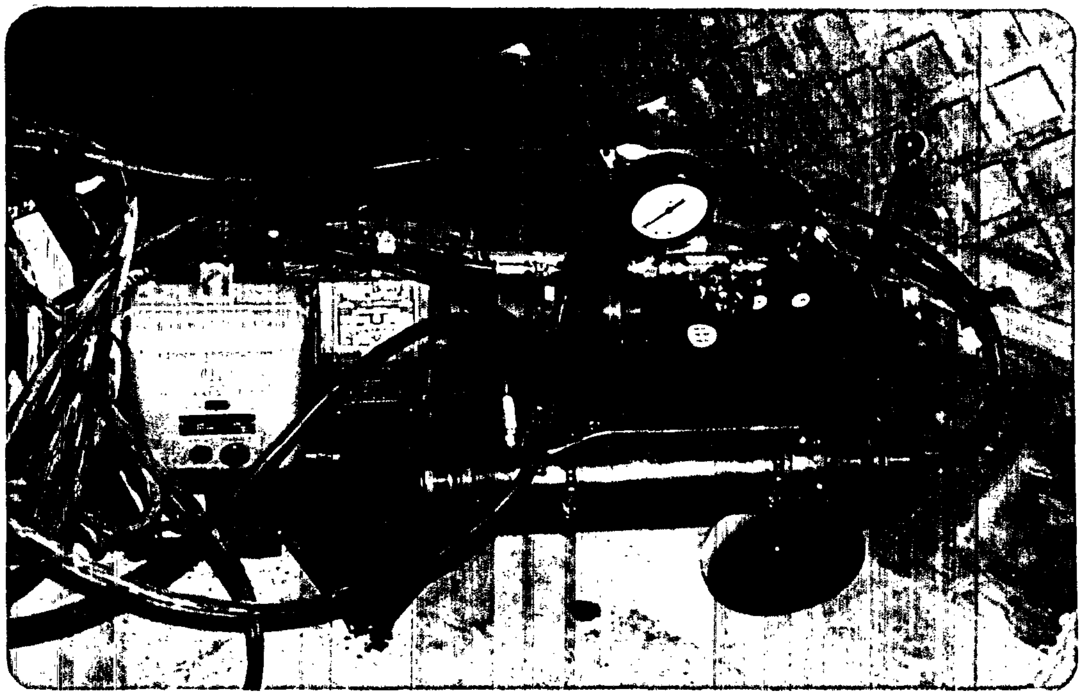


FIGURE 6.3 HYDRAULIC 20 KIP PRESTRESSING JACK.

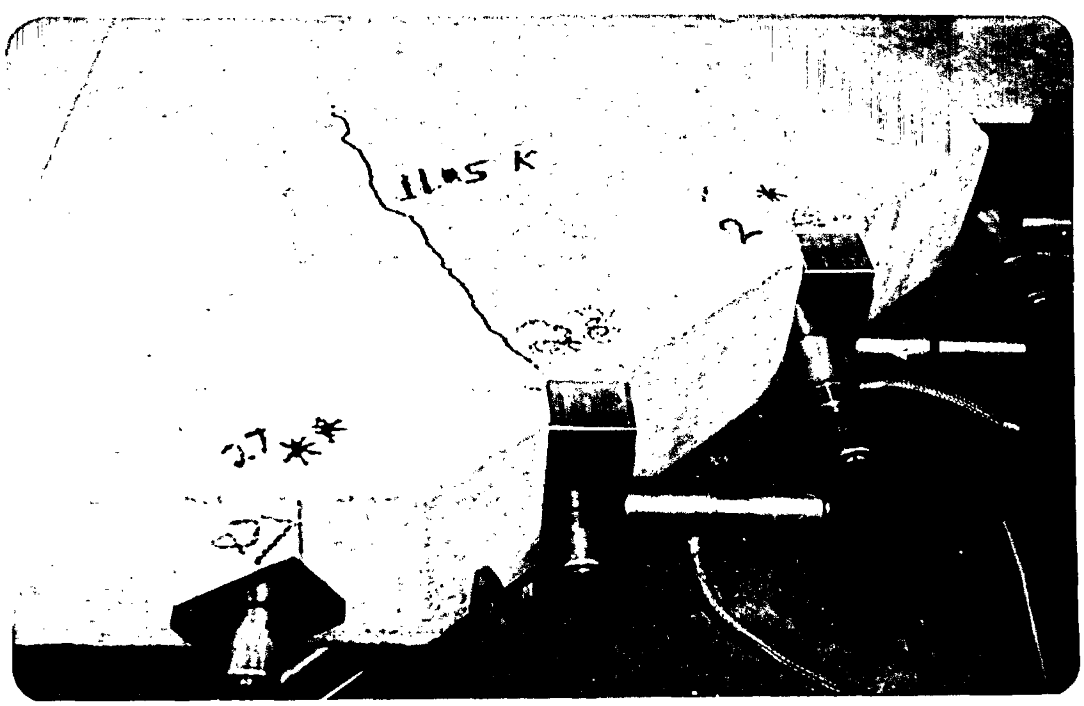


FIGURE 6.4 END BLOCKS AND LOAD CELLS FOR PRESTRESSED SKEW SLAB.

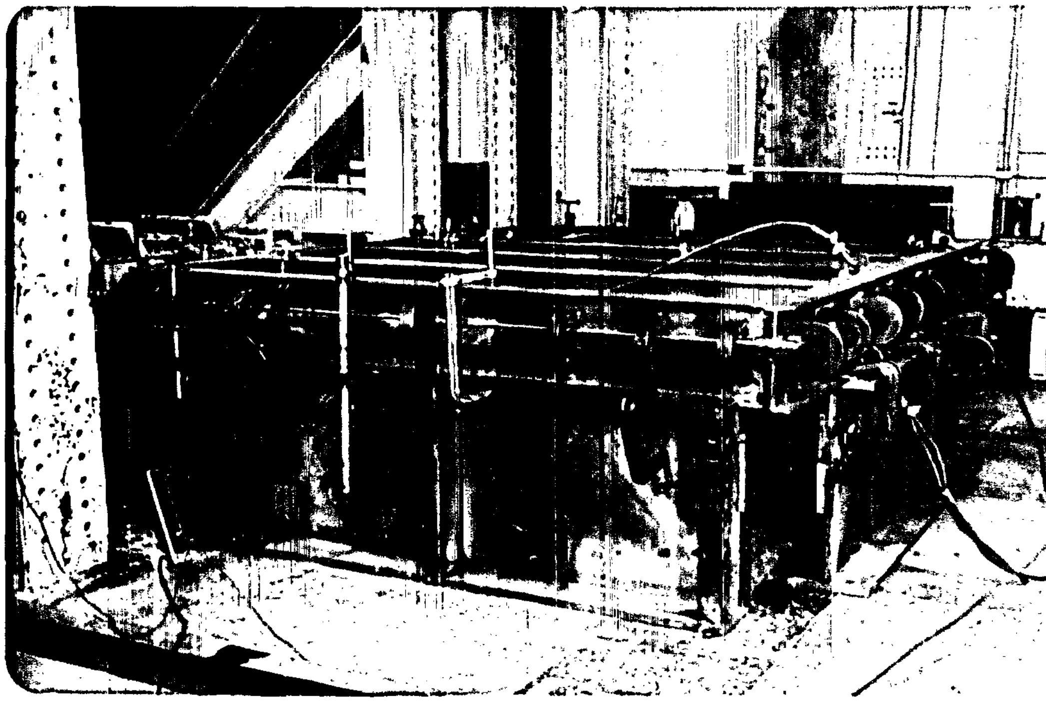


FIGURE 6.5 LOADING SYSTEM FOR UNIFORM LOAD.

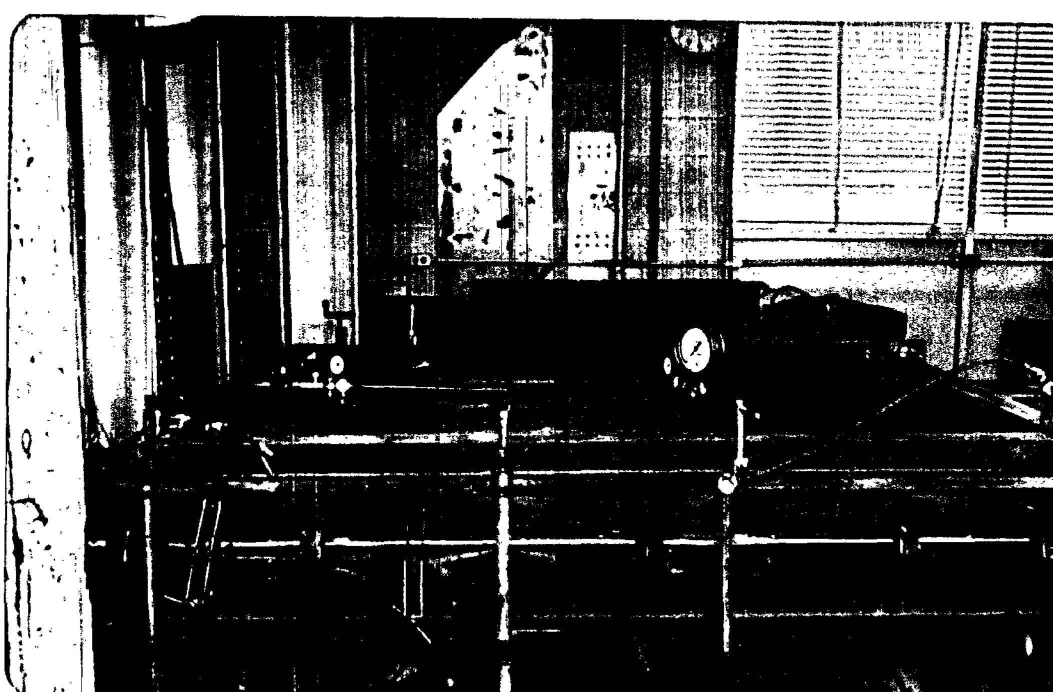


FIGURE 6.6 REGULATOR SYSTEM AND PRESSURE GAUGES.

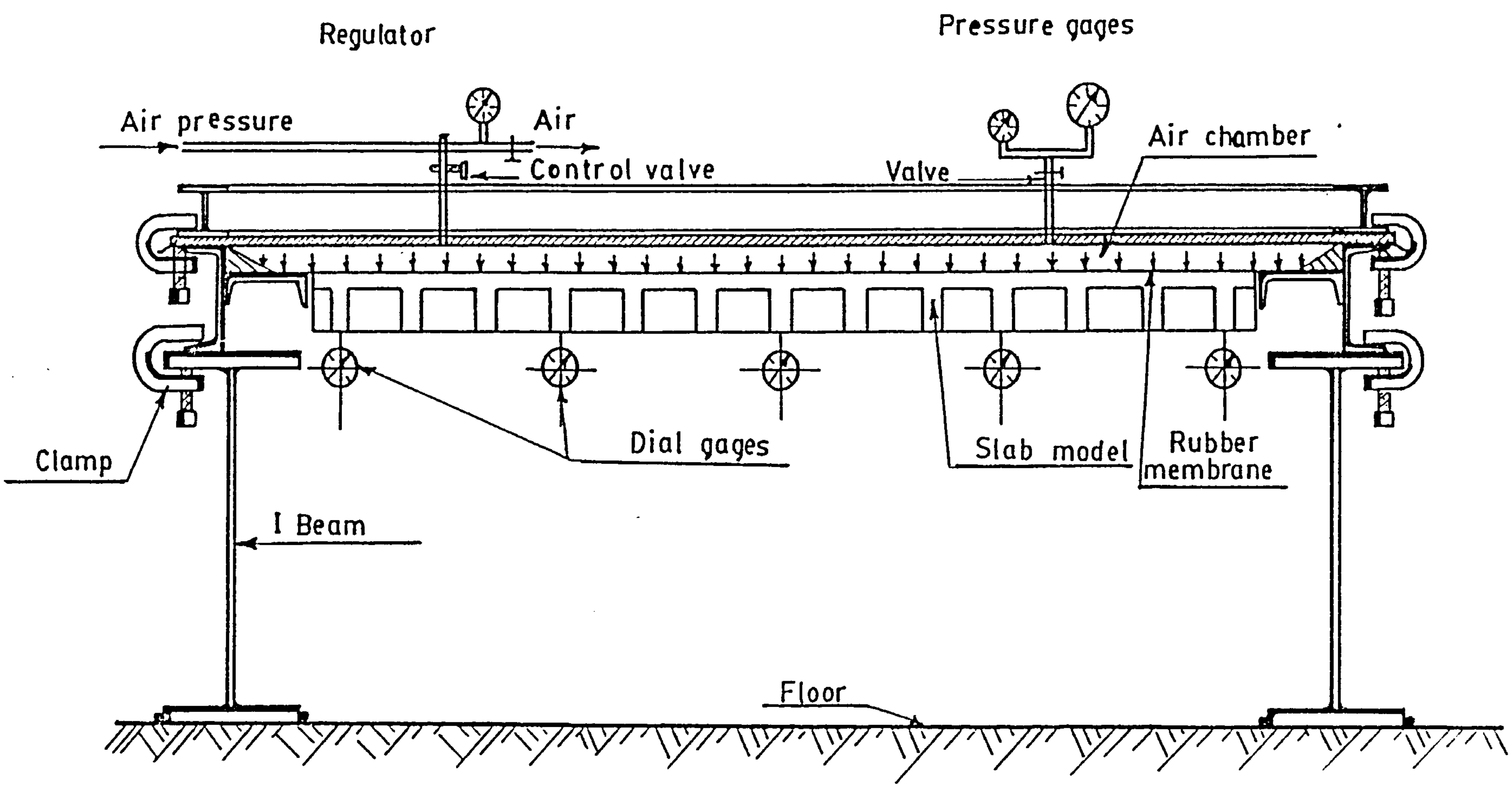


FIGURE 6.7 TRANSVERSE SECTION FOR THE LOADING FRAME.

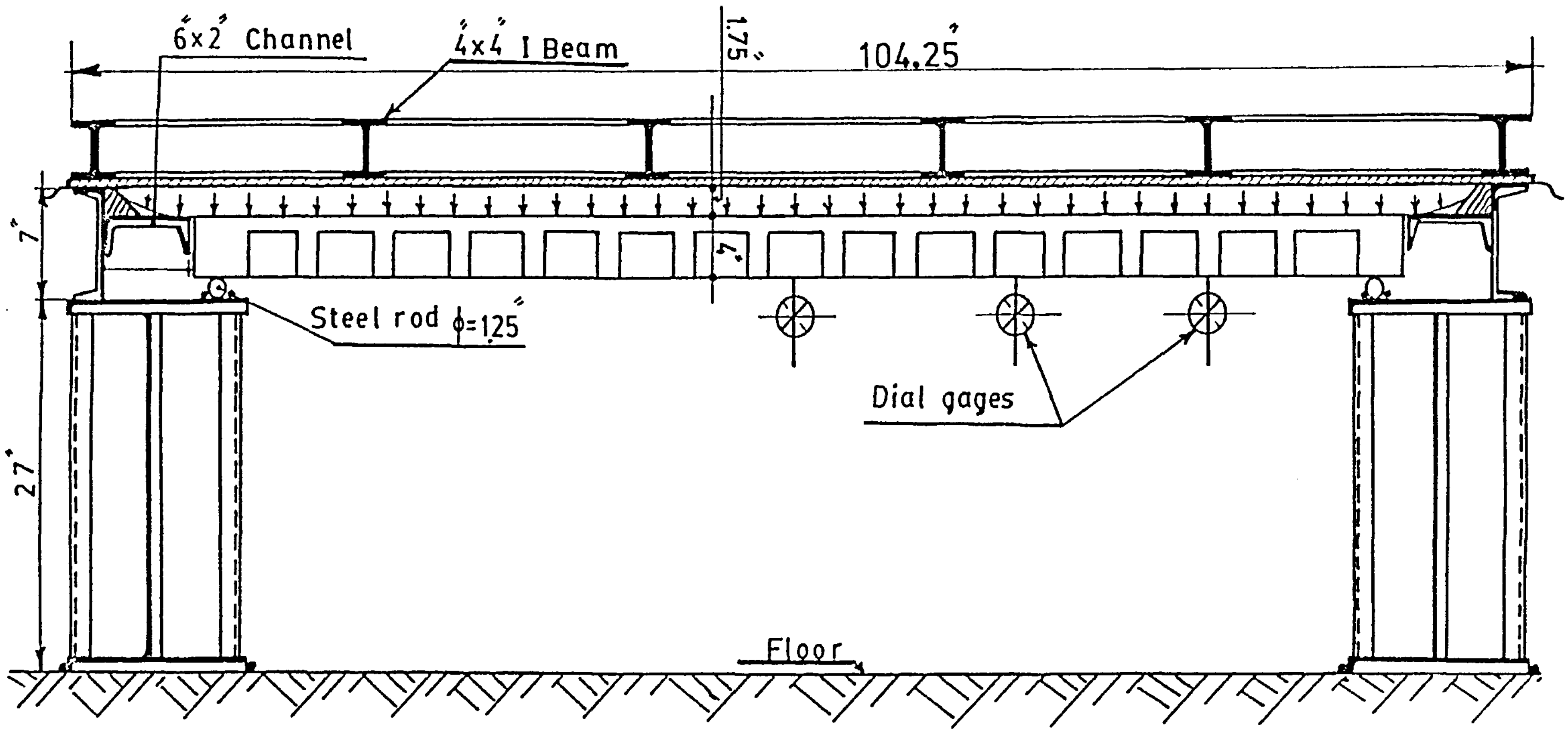


FIGURE 6.8 LONGITUDINAL SECTION FOR THE LOADING FRAME.

(1 in. = 25.4 mm)

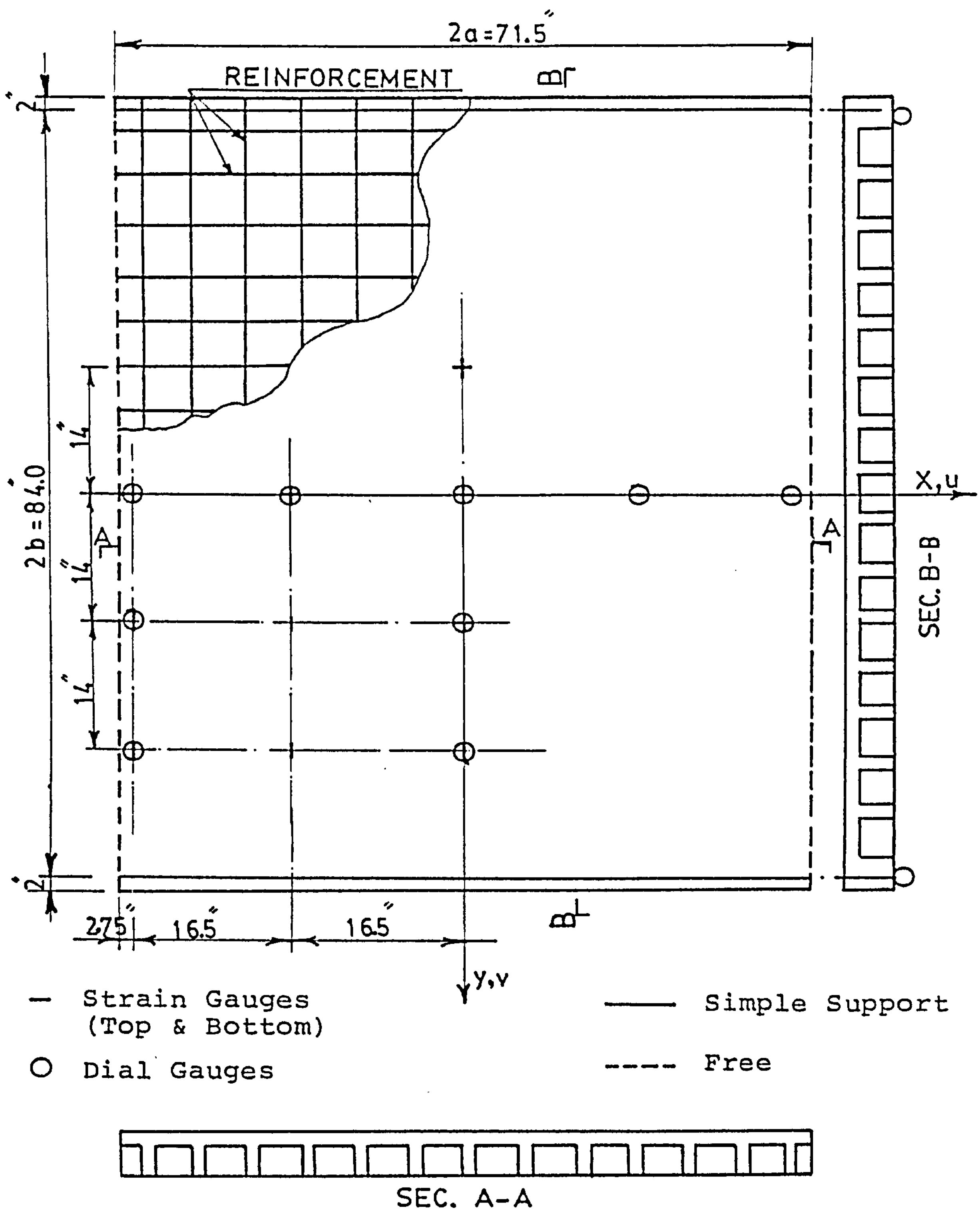
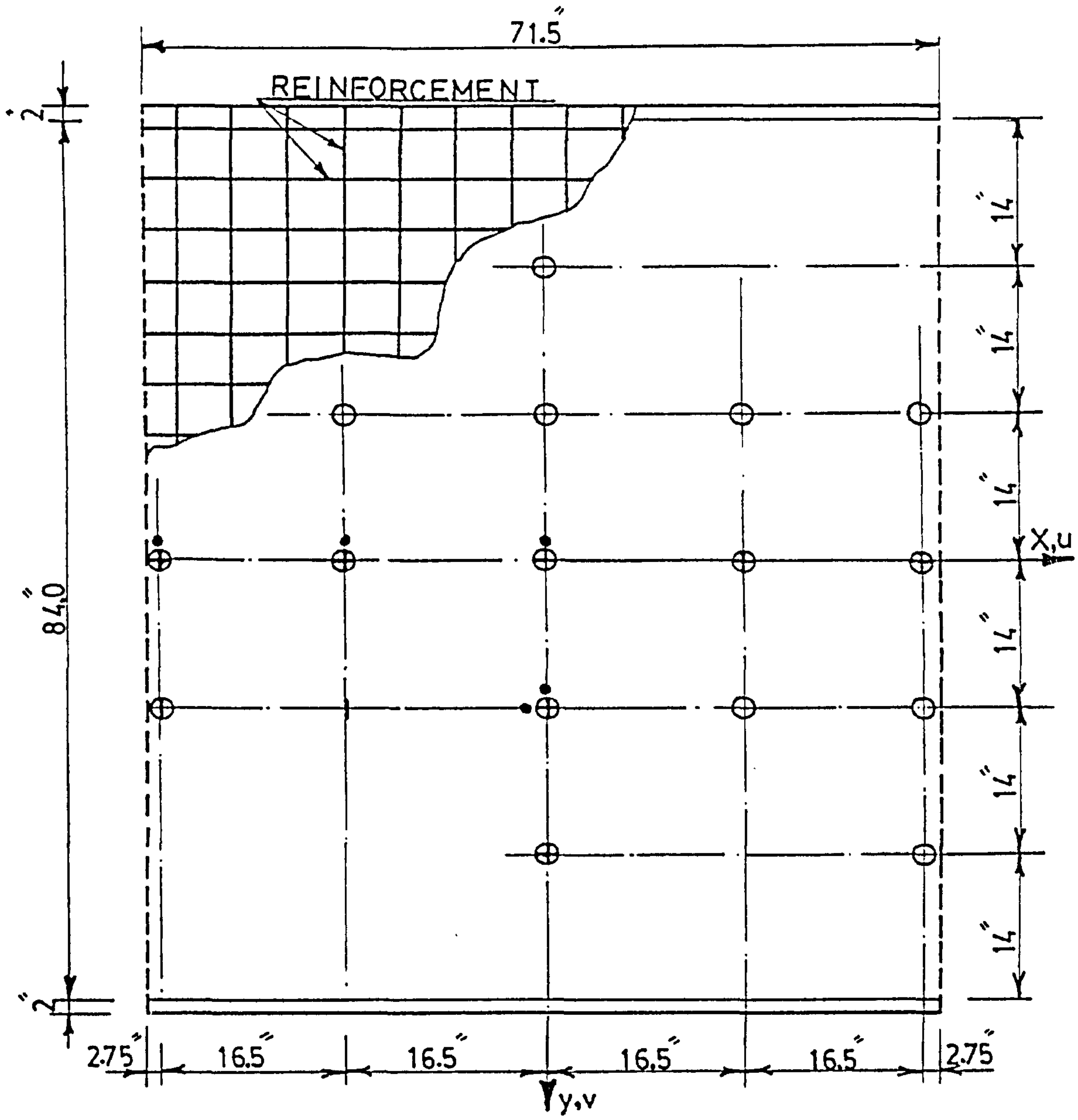


FIGURE 6.9 REINFORCED CONCRETE WAFFLE SLAB A.1

(1 in. = 25.4 mm)



——— Simple Support

---- Free

O Dial Gauges

— Strain Gauges (Top & Bottom)

● Strain Gauges on Steel

FIGURE 6.10 REINFORCED CONCRETE WAFFLE SLAB A.2.

(1 in. = 25.4 mm)

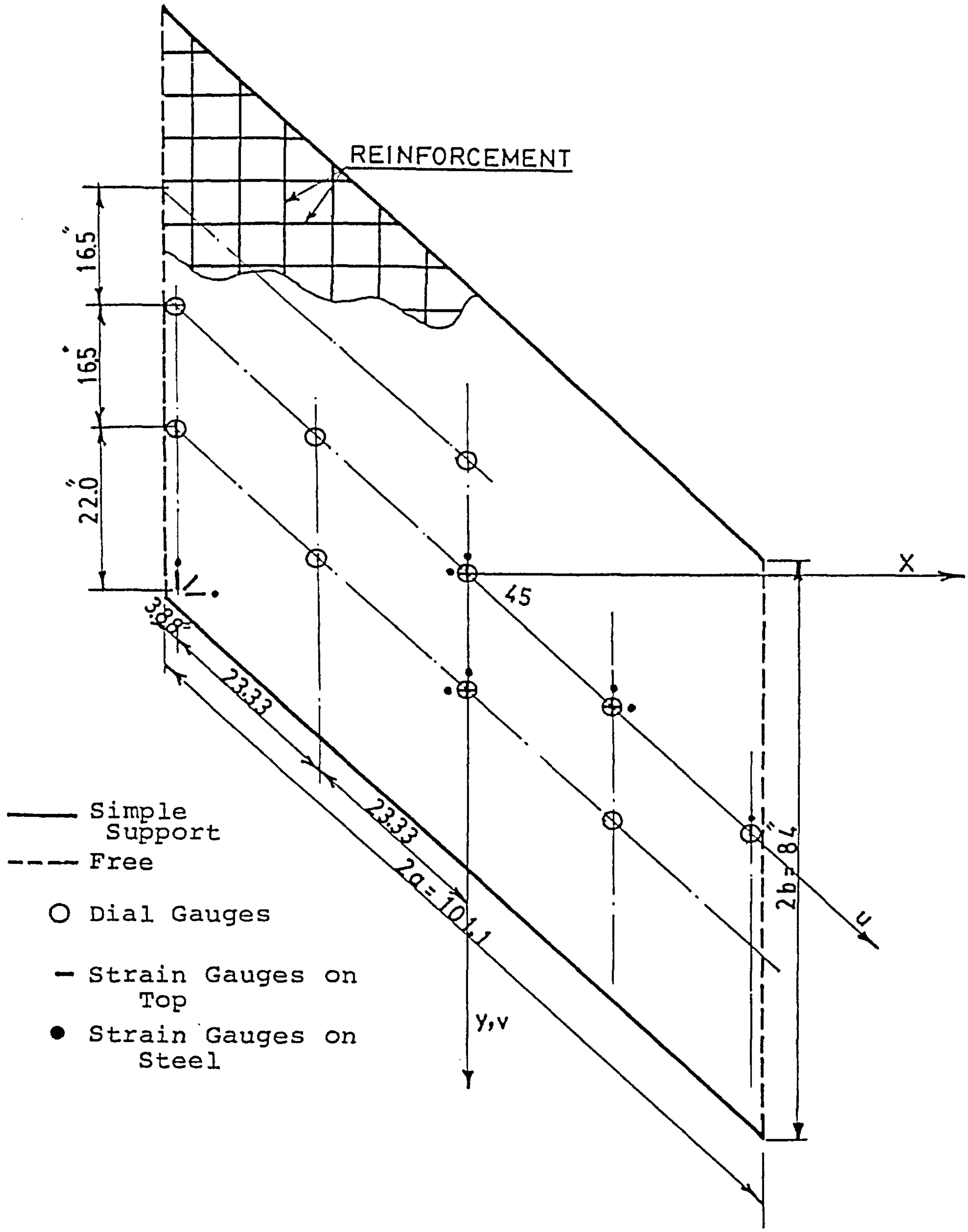
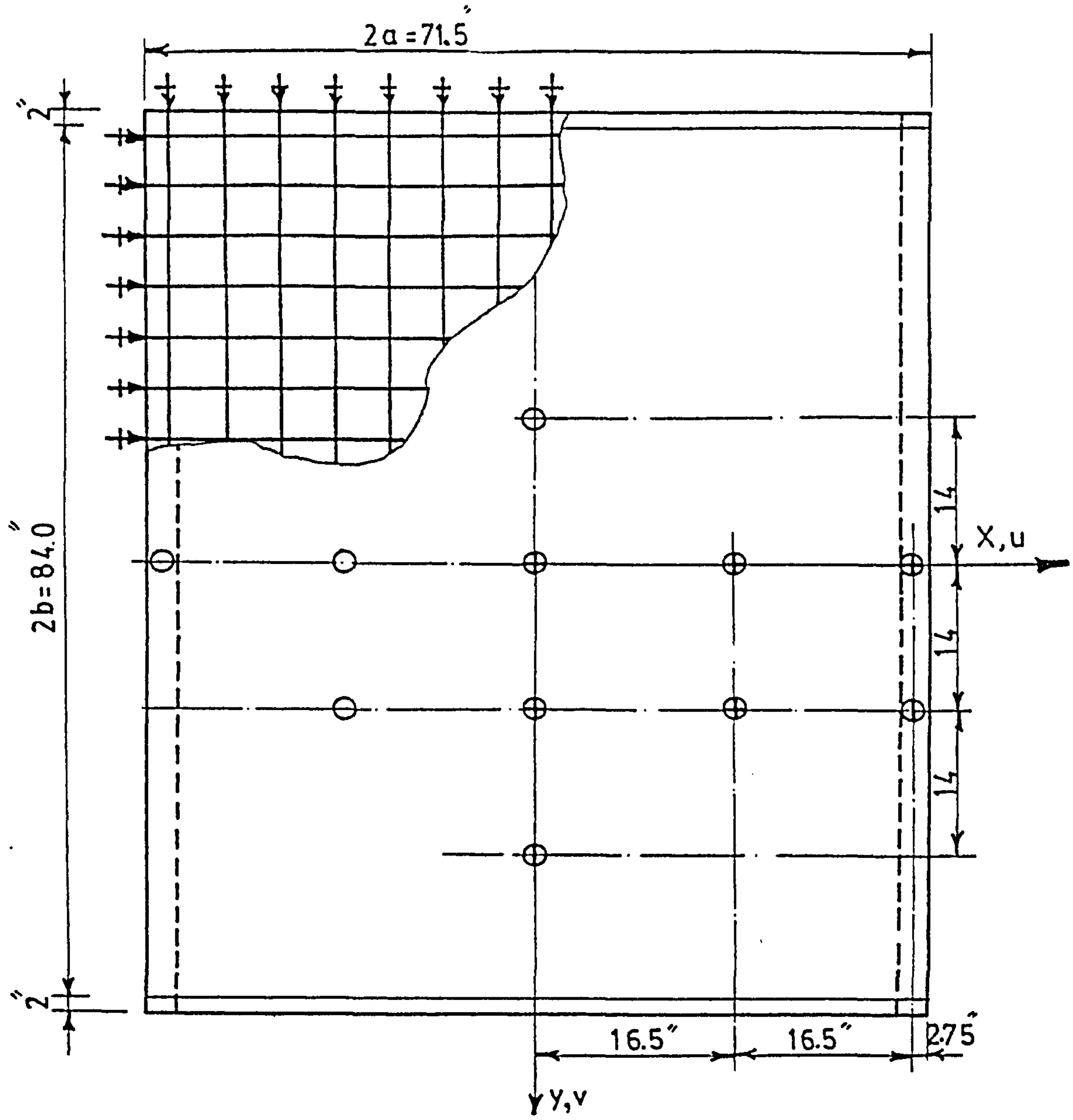


FIGURE 6.11 REINFORCED CONCRETE SKEW WAFFLE SLAB A.3, SKEW OF 45°.

(1 in. = 25.4 mm)



— Simple Support

Elastic Support (Edge Beam)

O Dial Gauges

– Strain Gauges on Top

FIGURE 6.12 PRESTRESSED CONCRETE WAFFLE SLAB B.1.

(1 in. = 25.4 mm)

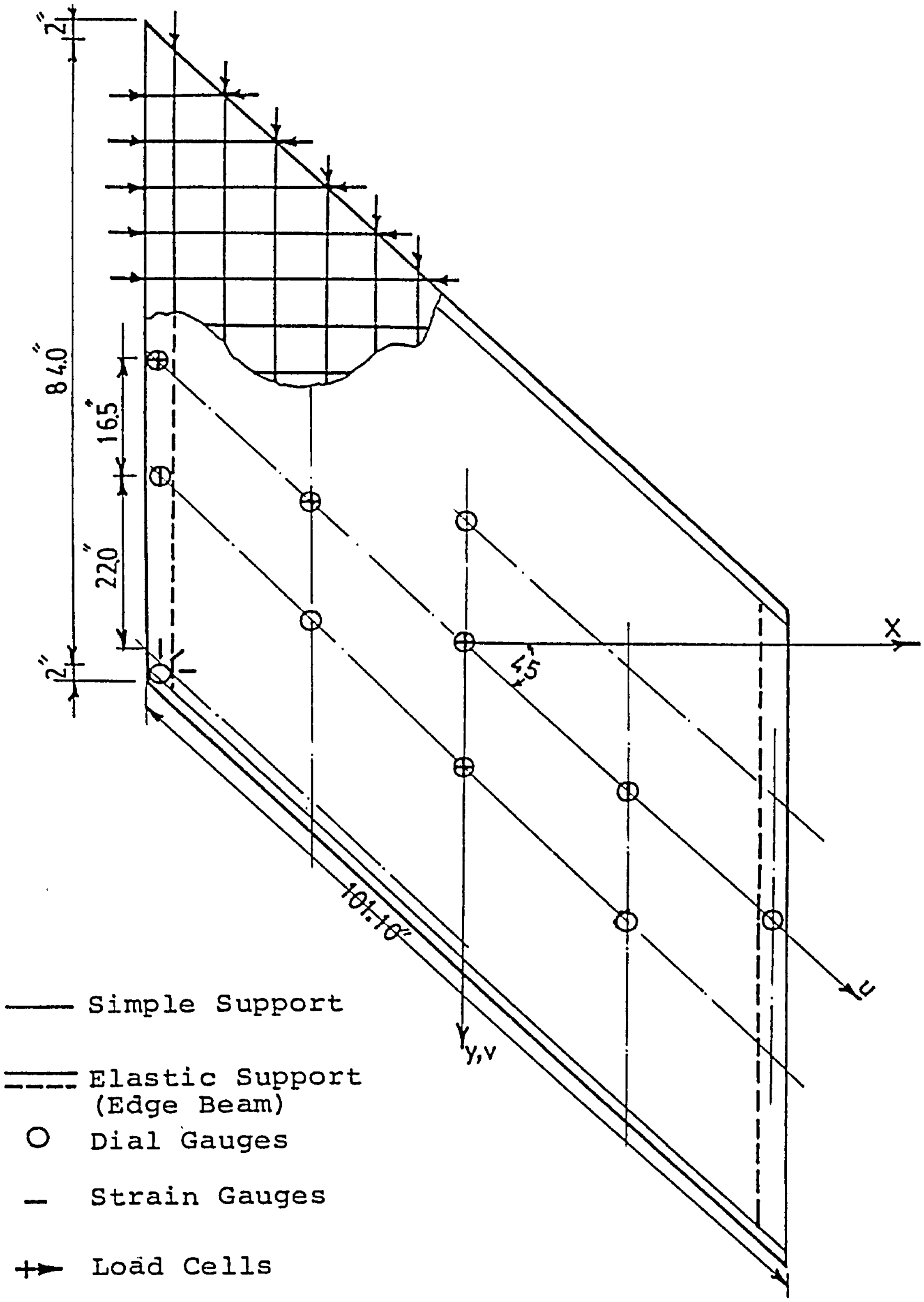


FIGURE 6.13 PRESTRESSED CONCRETE WAFFLE SLAB B.2.

(1 in. = 25.4 mm)

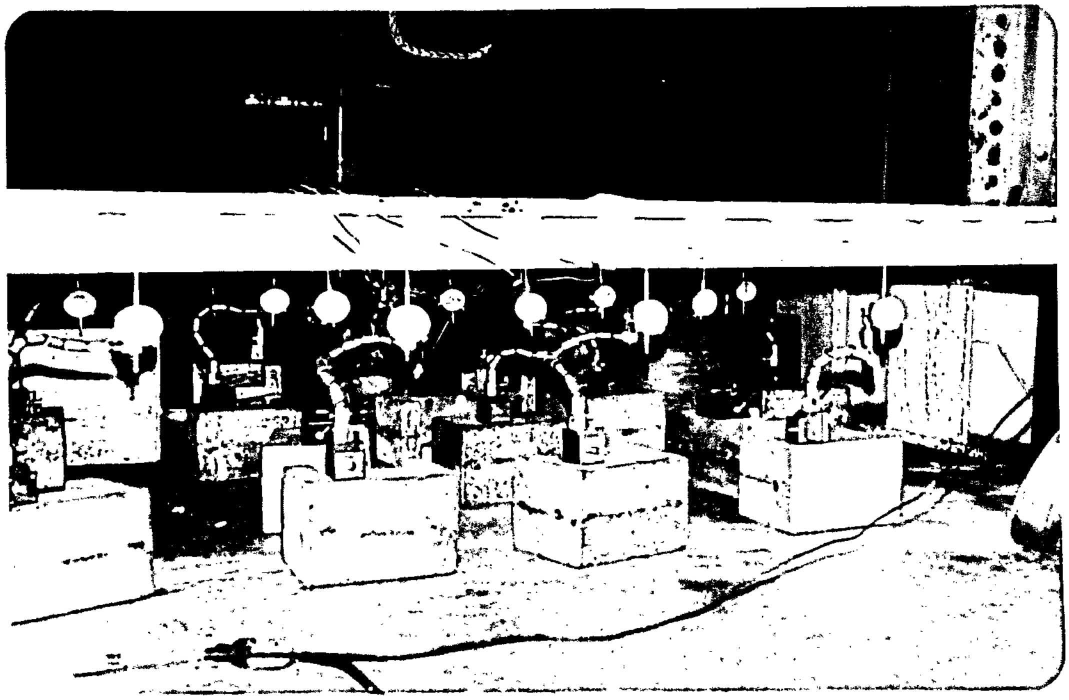


FIGURE 6.14 DIAL GAUGES IN POSITION.

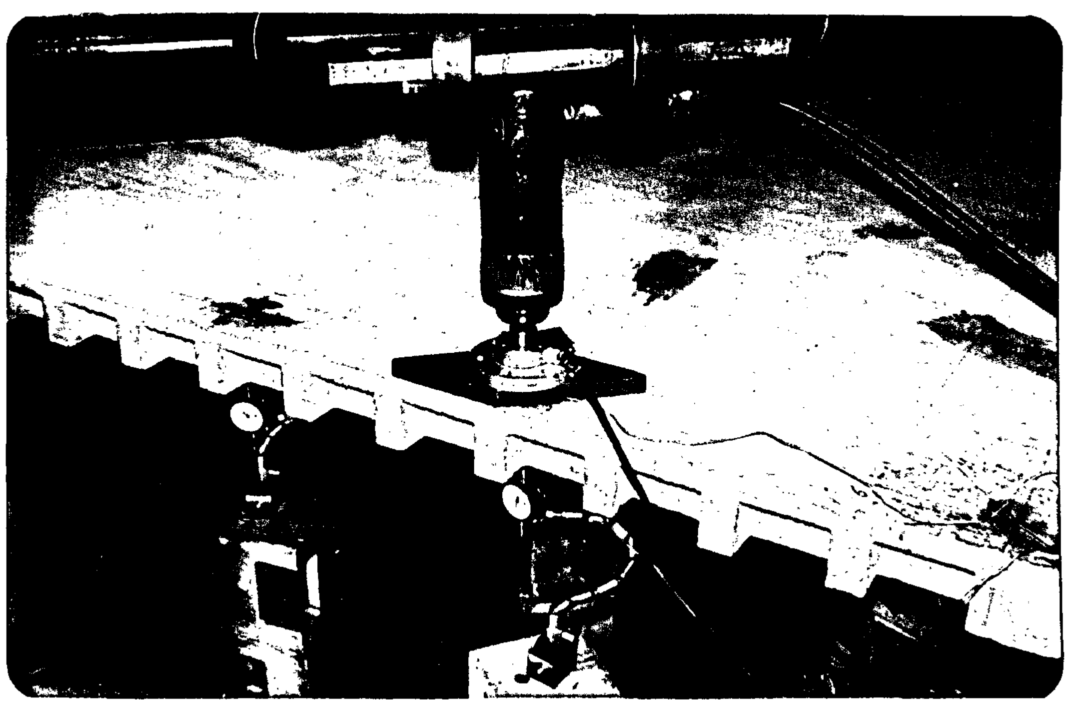


FIGURE 6.15 POINT LOADING SYSTEM.

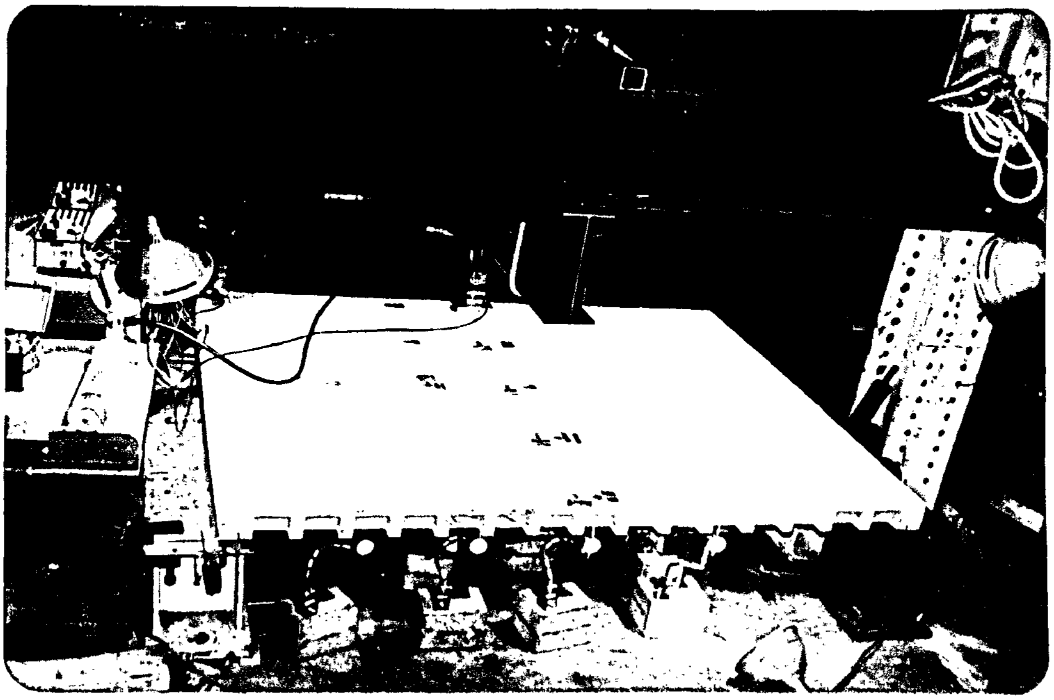


FIGURE 6.16 LOADING SYSTEM FOR SLAB A.2.

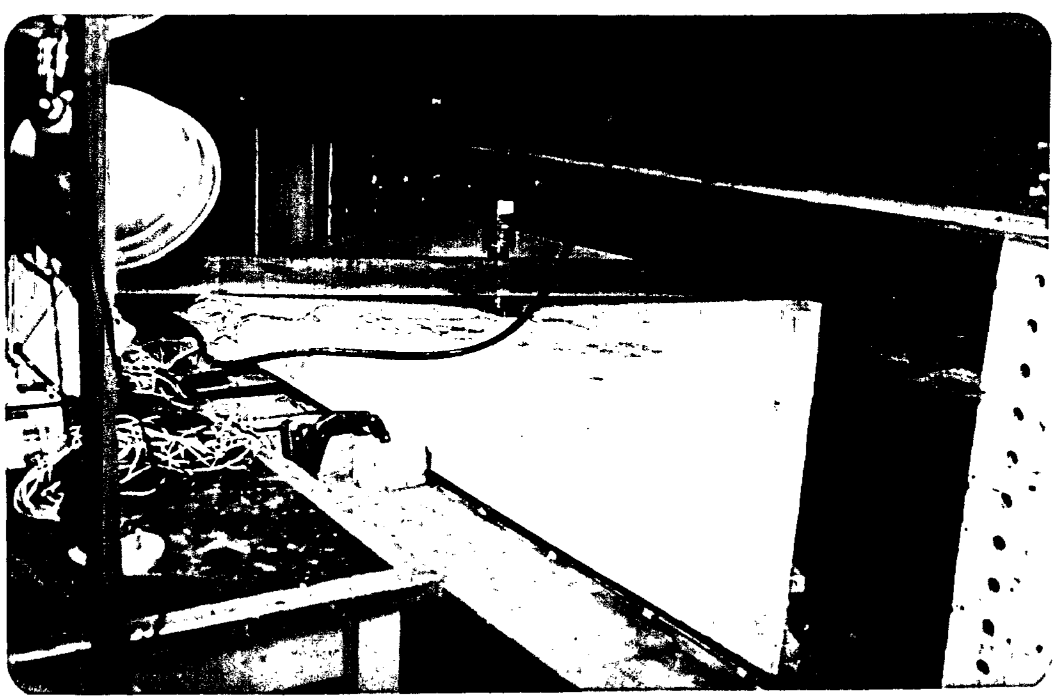


FIGURE 6.17 LOADING SYSTEM FOR THE SKEW SLAB A.3.

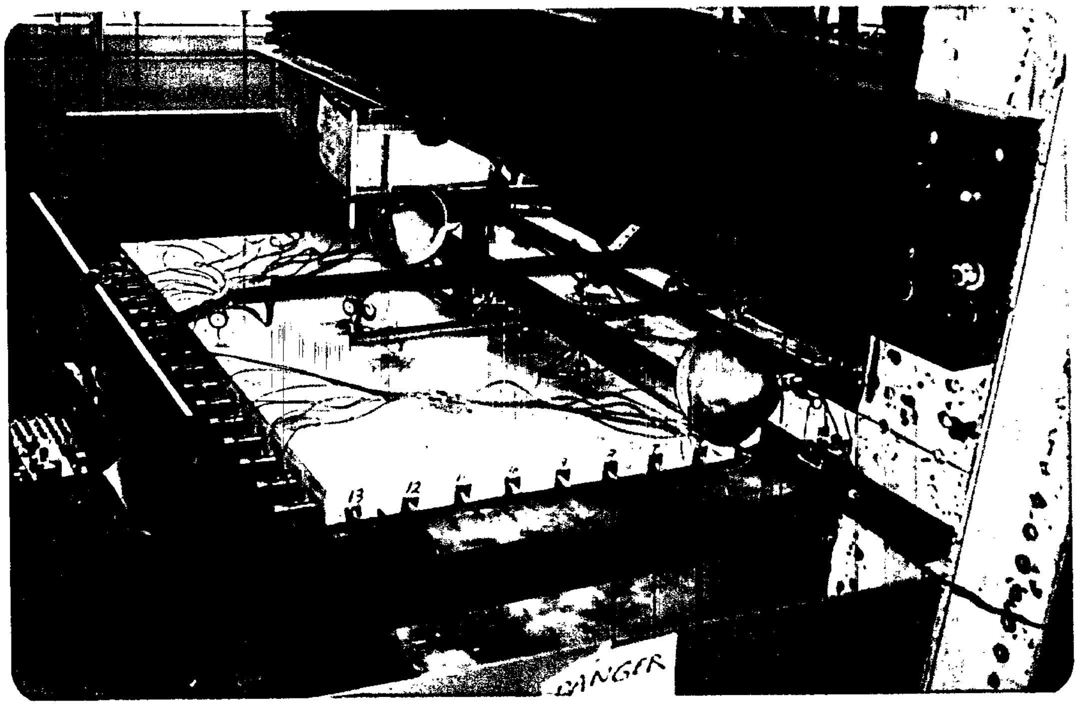


FIGURE 6.18 LOADING SYSTEM FOR PRESTRESSED SLAB B.1.

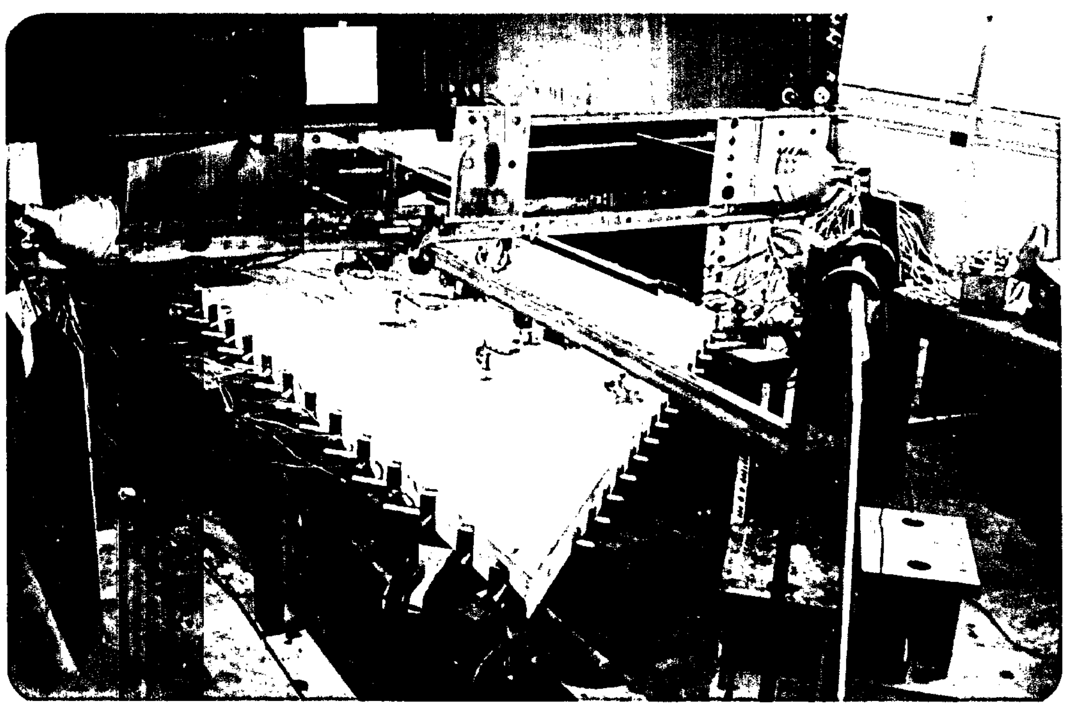


FIGURE 6.19 LOADING SYSTEM FOR PRESTRESSED SKEW SLAB B.2.

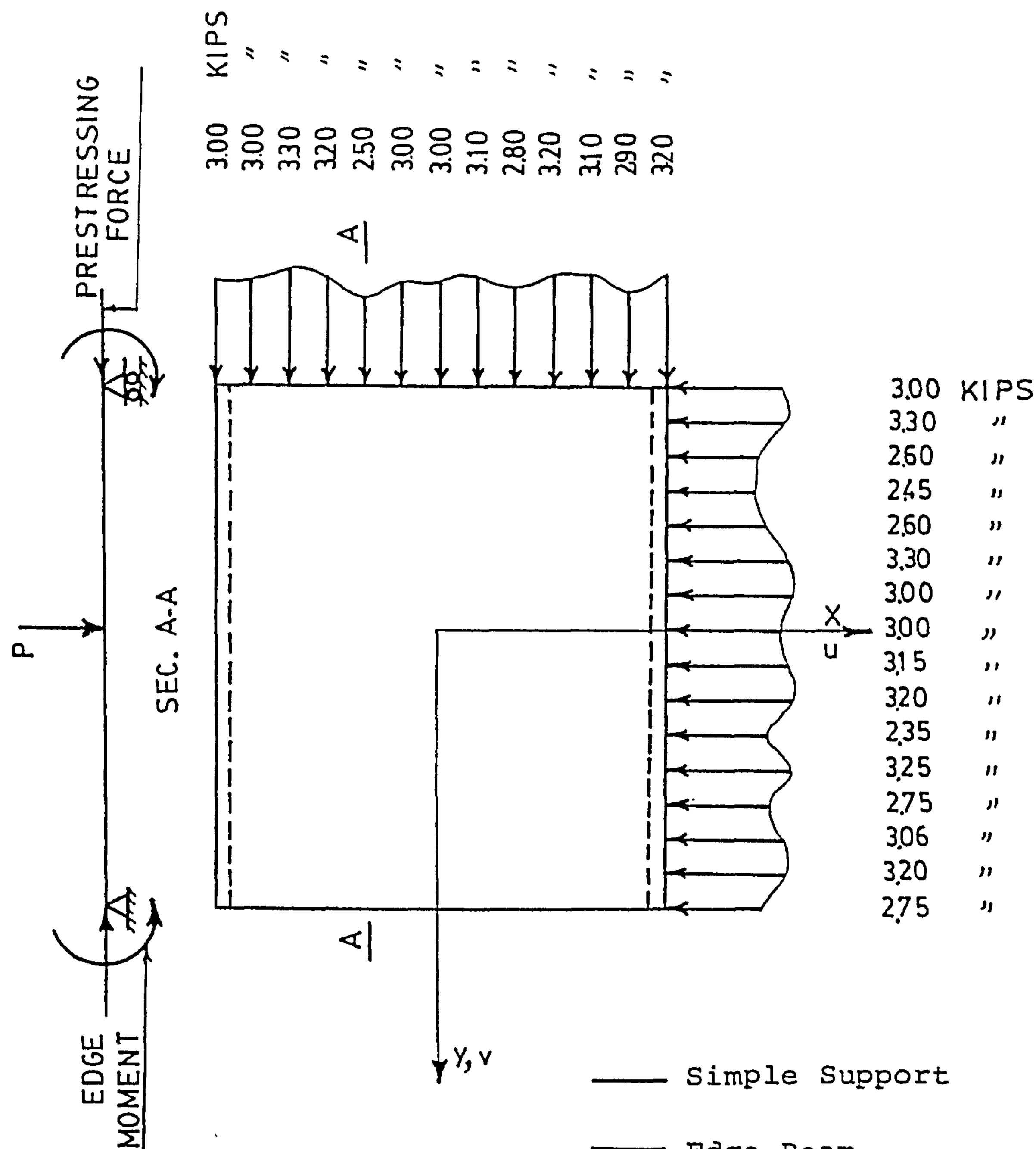


FIGURE 6.20 PRESTRESSING FORCE AND EDGE MOMENTS FOR SLAB B.1.

(1 KIP = 4.45 KN)

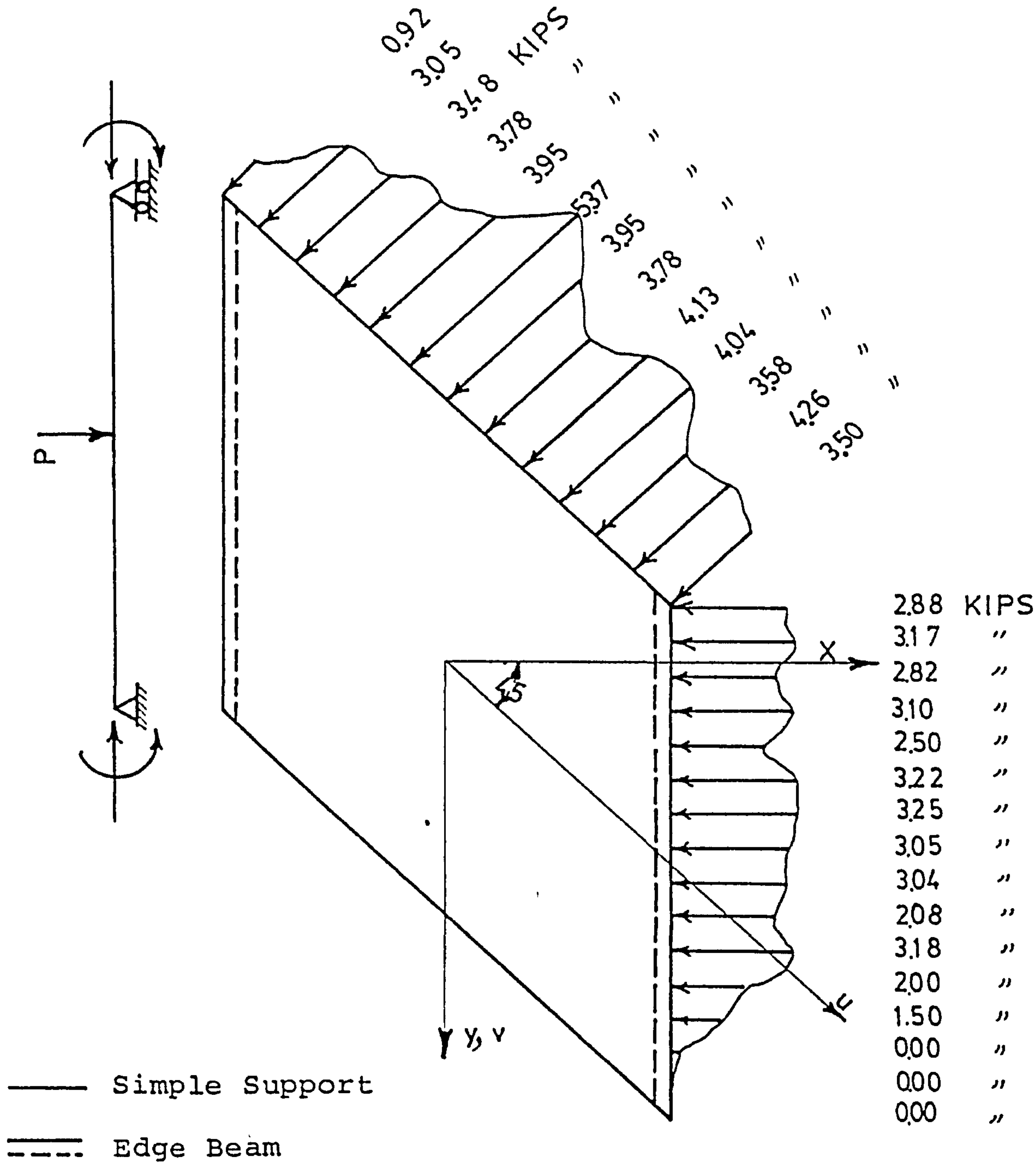


FIGURE 6.21 PRESTRESSING FORCE AND EDGE MOMENTS FOR SKEW SLAB B.2.

(1 KIP = 4.45 KN)

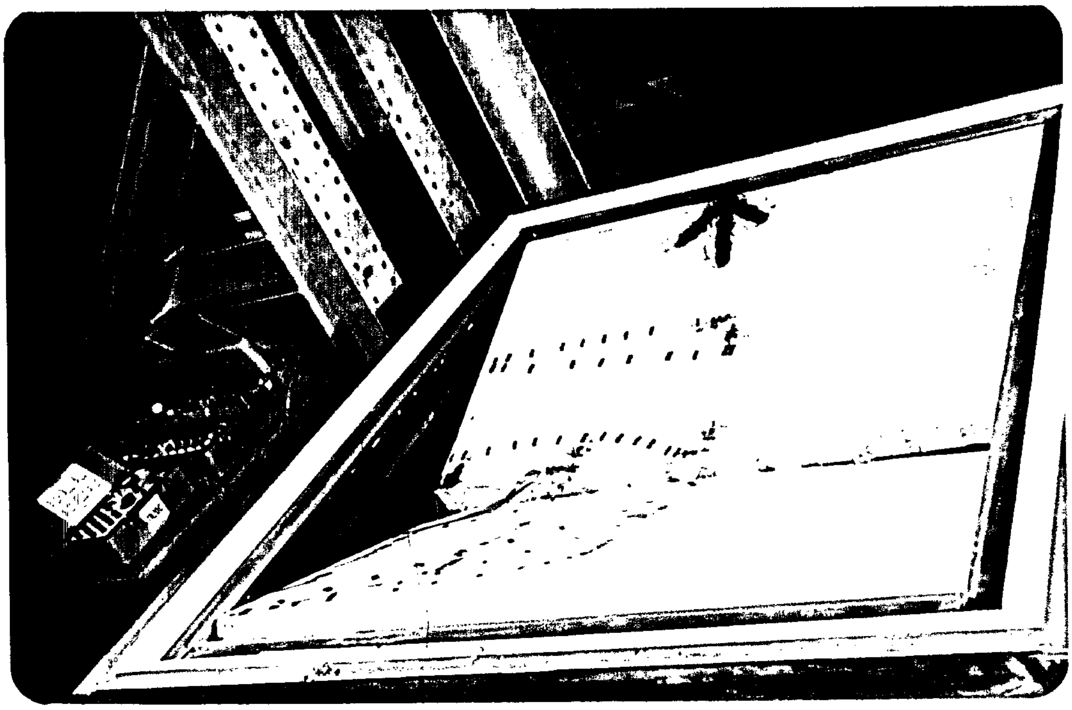


FIGURE 7.1 SLAB A.1 FAILURE.

FIGURE 7.2 SLAB A.1 CRACKS.

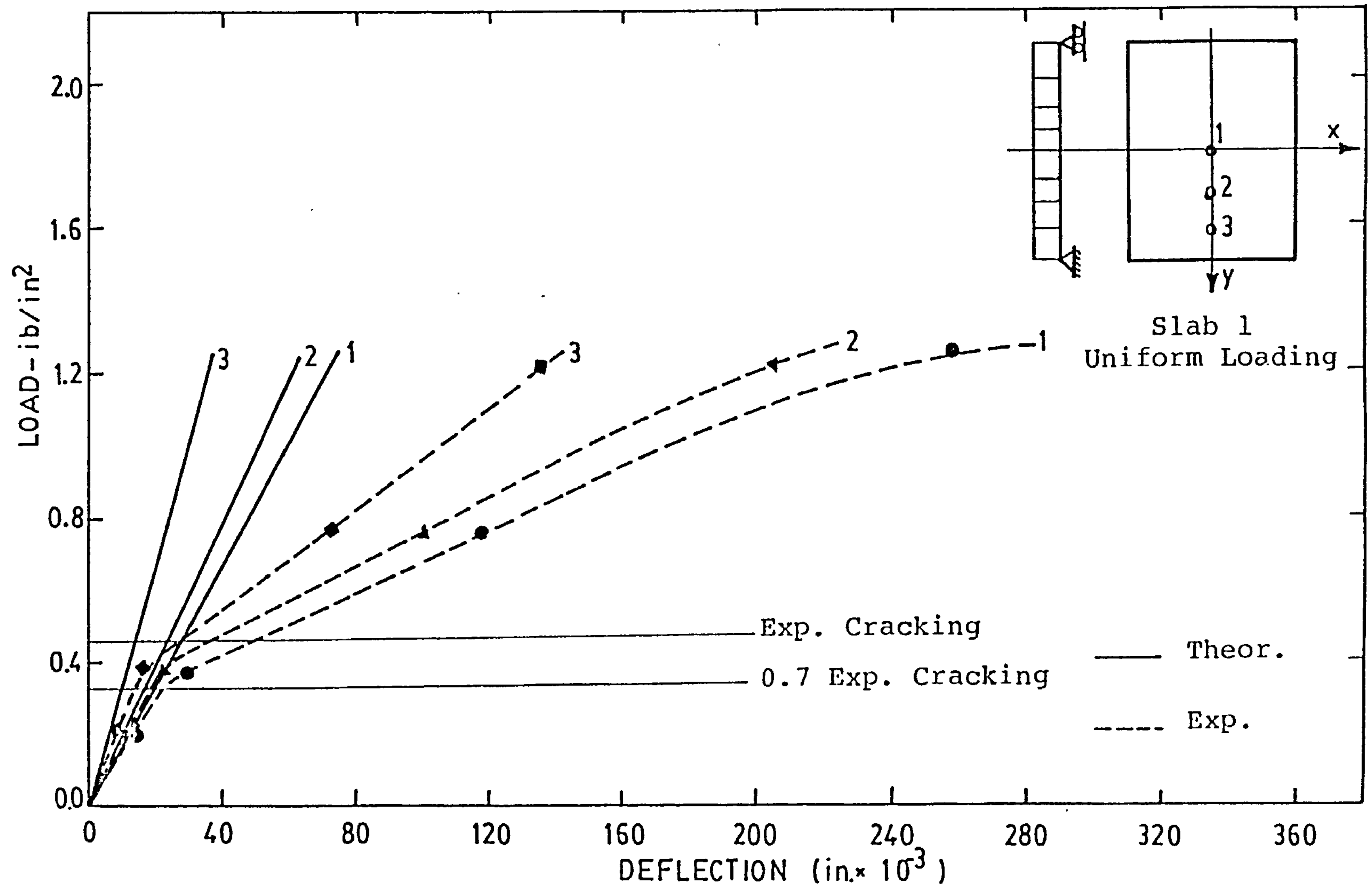


FIGURE 7.3 LOAD DEFLECTION RELATIONSHIP FOR SLAB A.1.

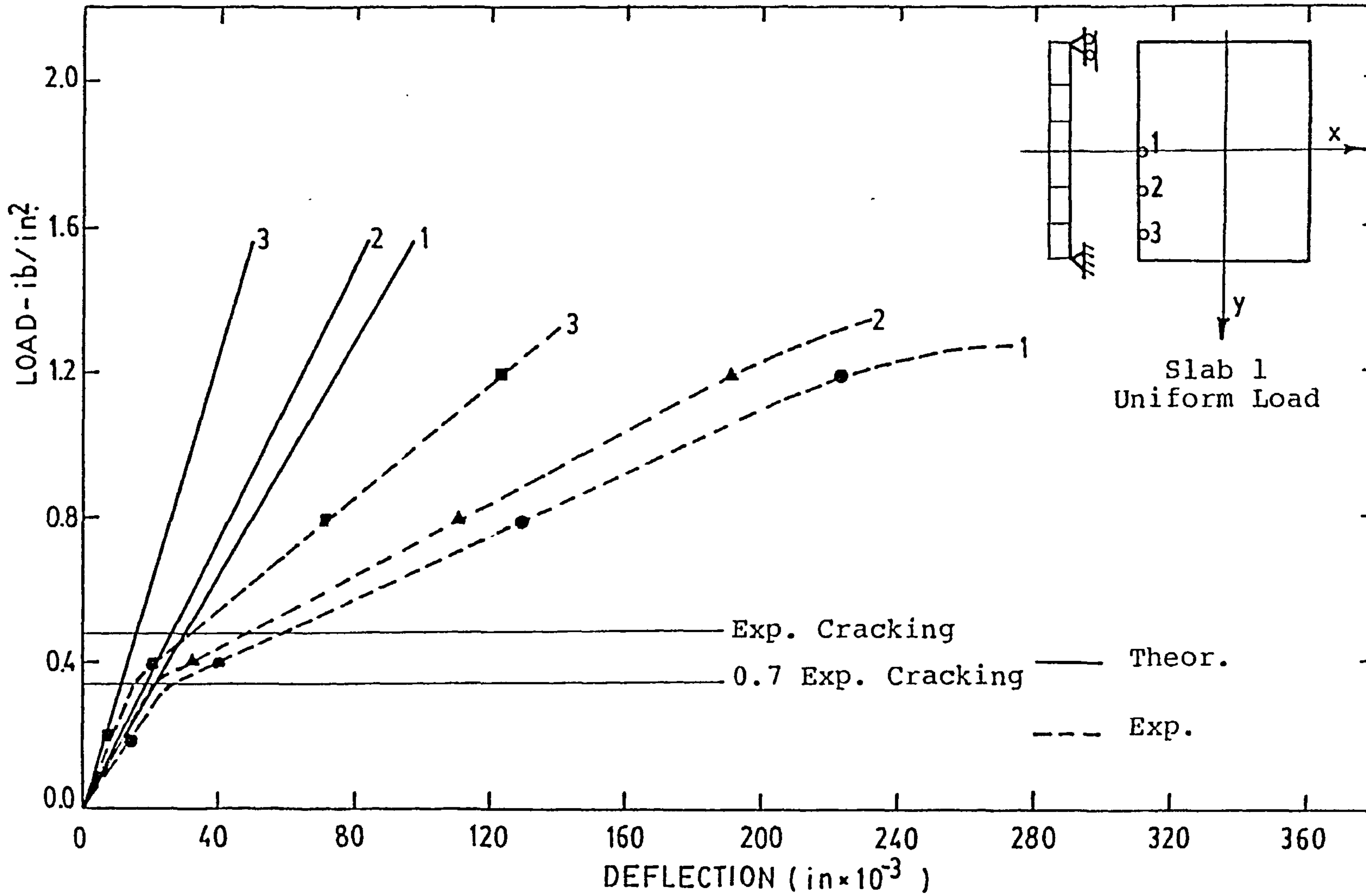


FIGURE 7.4 LOAD DEFLECTION RELATIONSHIP FOR SLAB A.1.

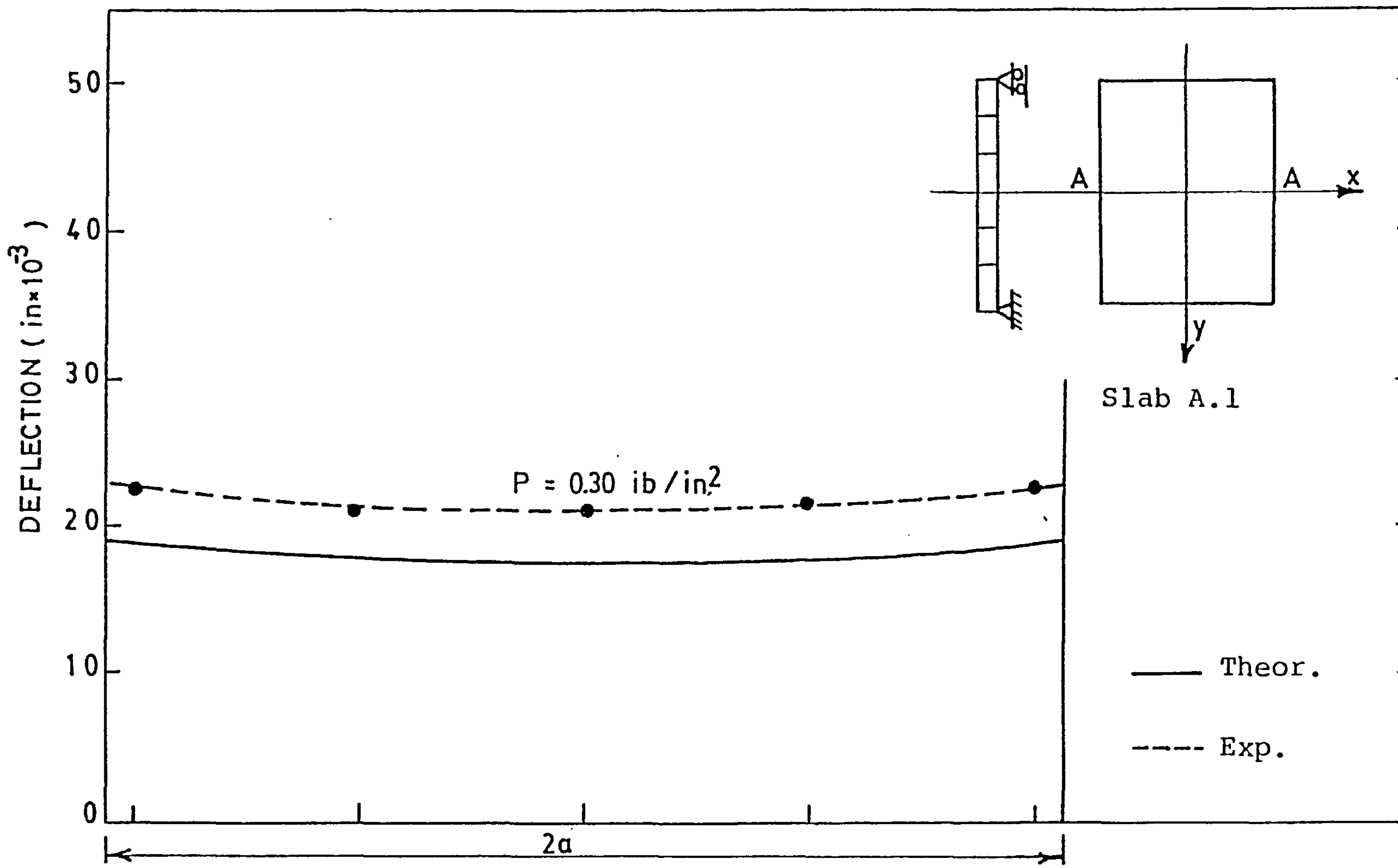


FIGURE 7.5 TRANSVERSE DEFLECTION.

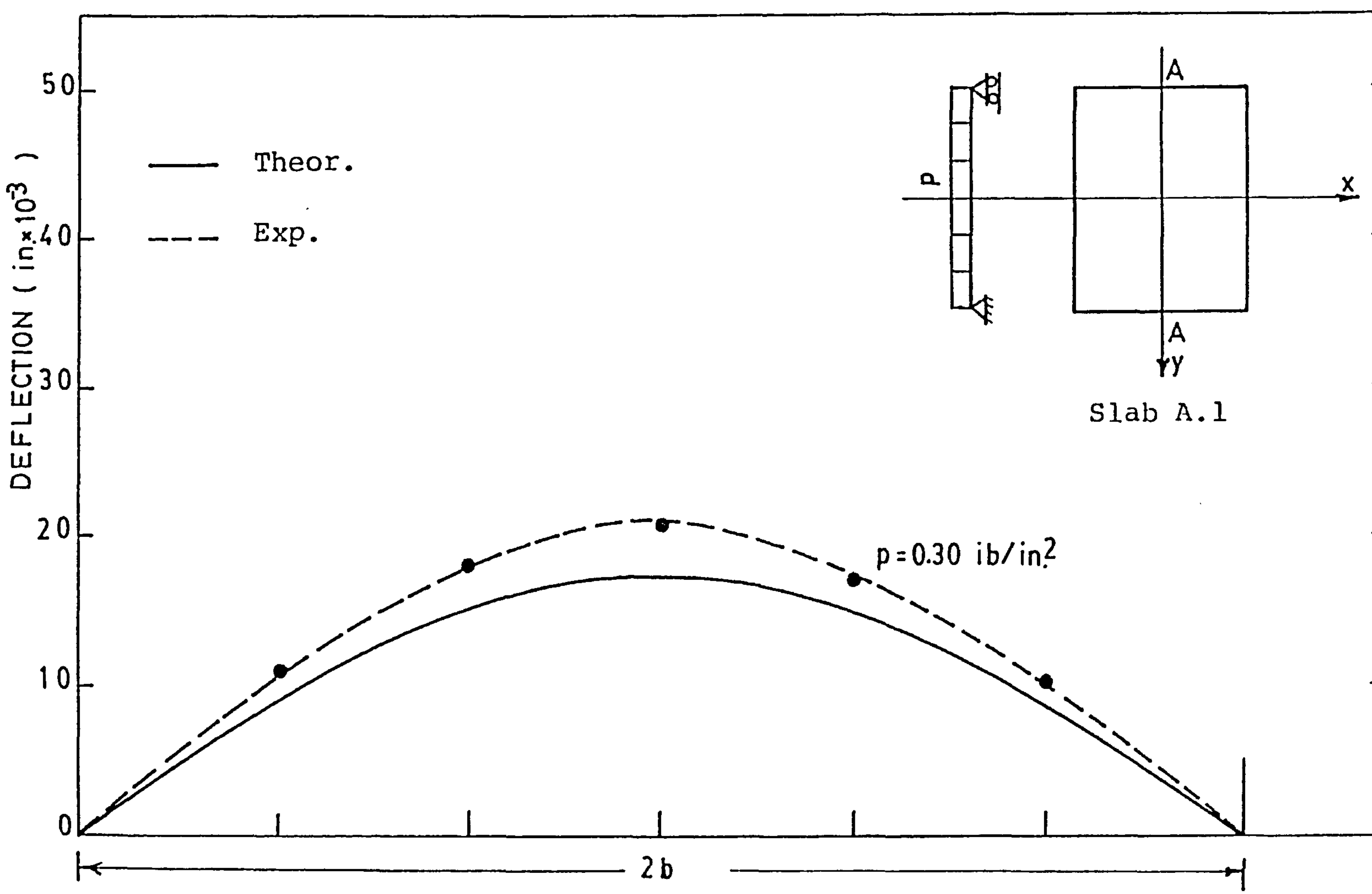


FIGURE 7.6 LONGITUDINAL DEFLECTION.

(1 in. = 25.4 mm)

(1 KIP = 4.45 KN)

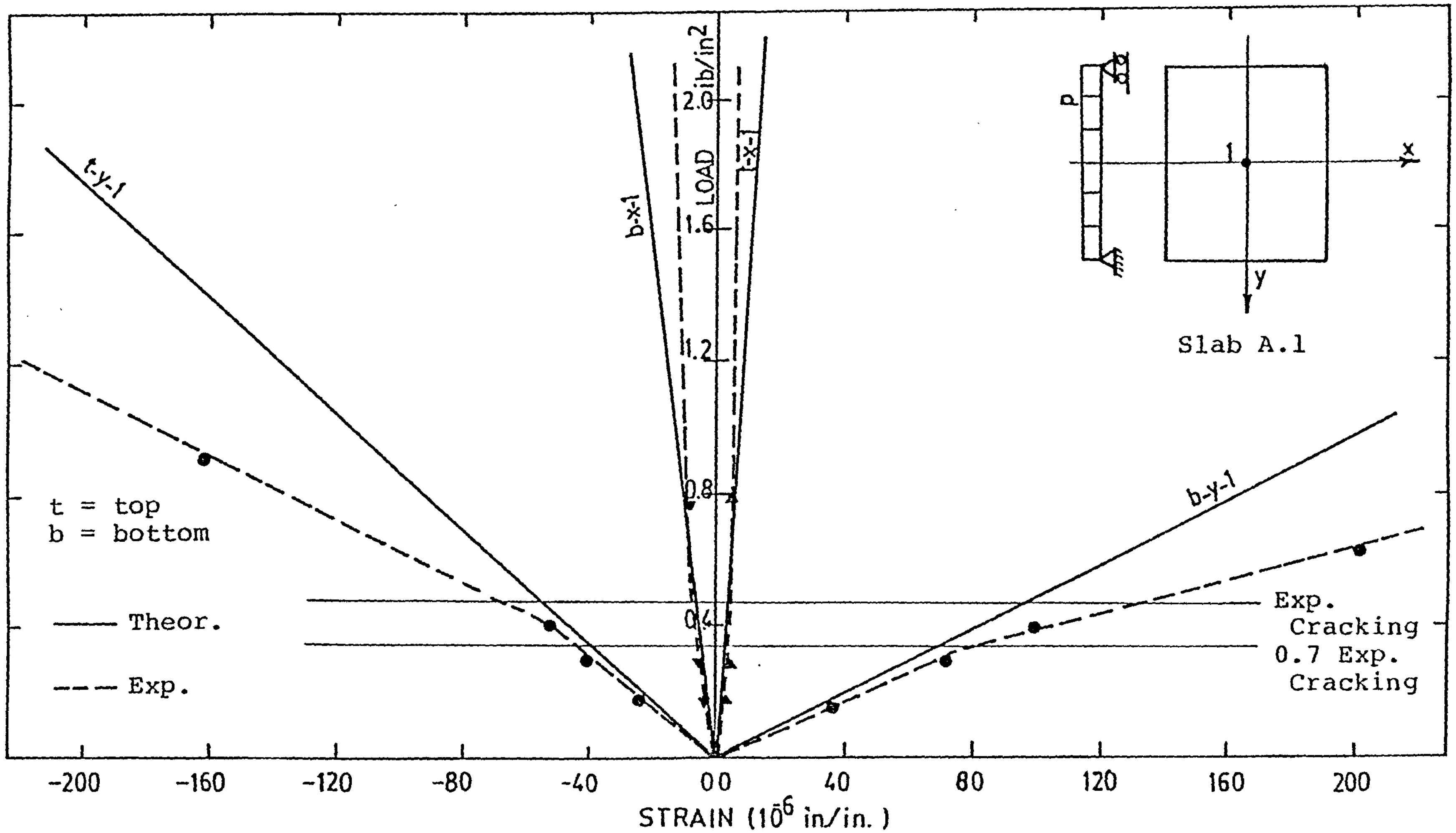


FIGURE 7.7 LOAD STRAIN RELATIONSHIP FOR SLAB A.1 AT CENTRE.

(1 KIP = 4.45 KN)

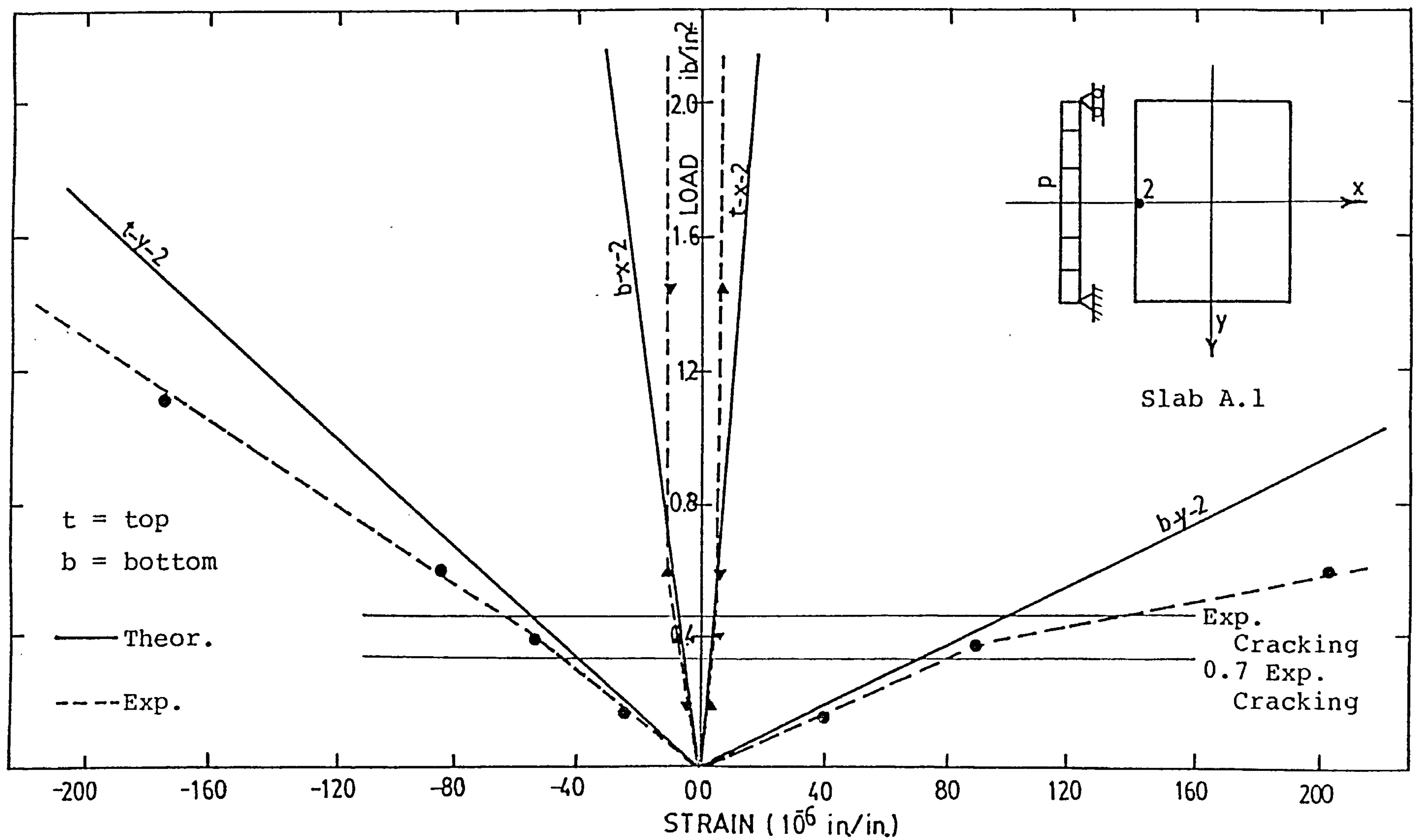


FIGURE 7.8 LOAD STRAIN RELATIONSHIP FOR SLAB A.1 AT THE FREE EDGE.
(1 KIP = 4.45 KN)

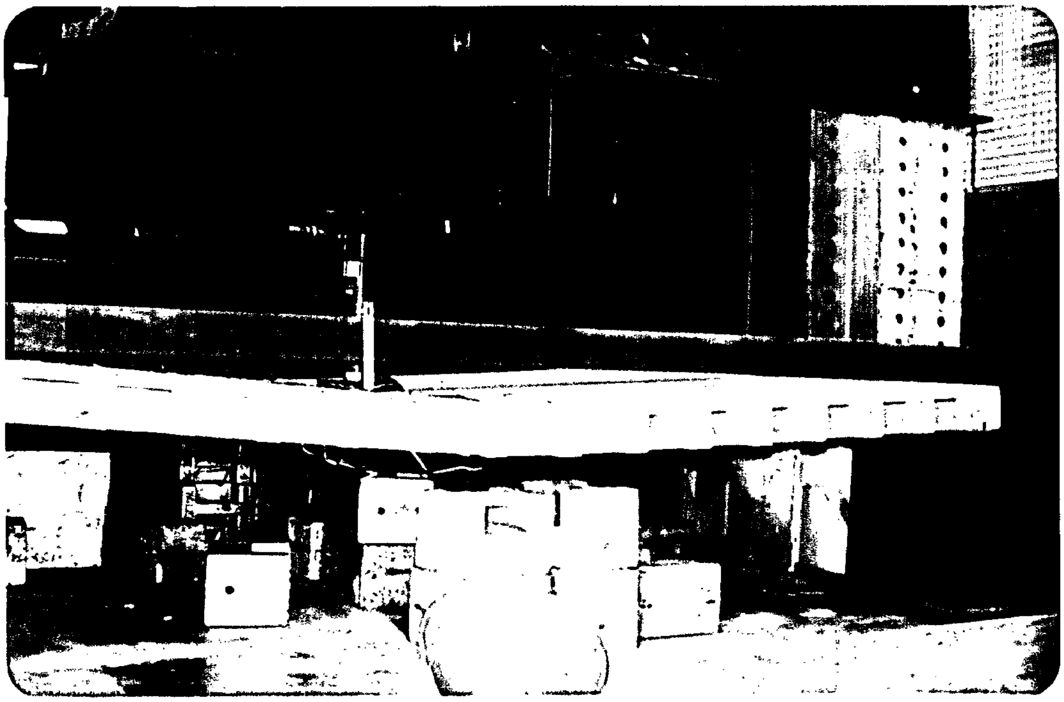


FIGURE 7.9 DEFLECTION SHAPE FOR SLAB A.2.

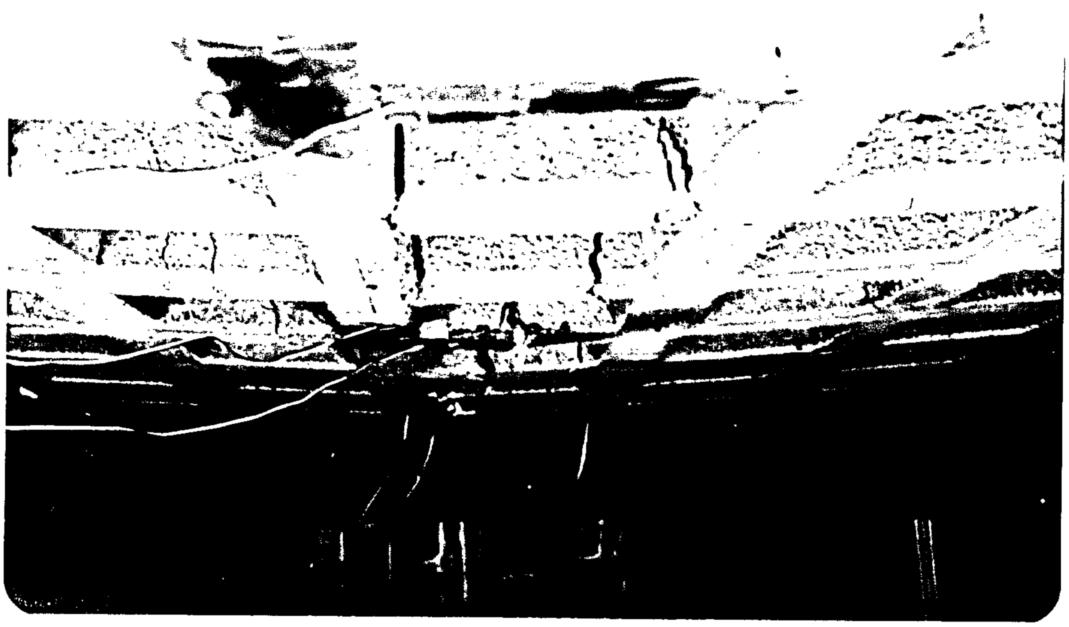


FIGURE 7.10 SLAB A.2 CRACKS.

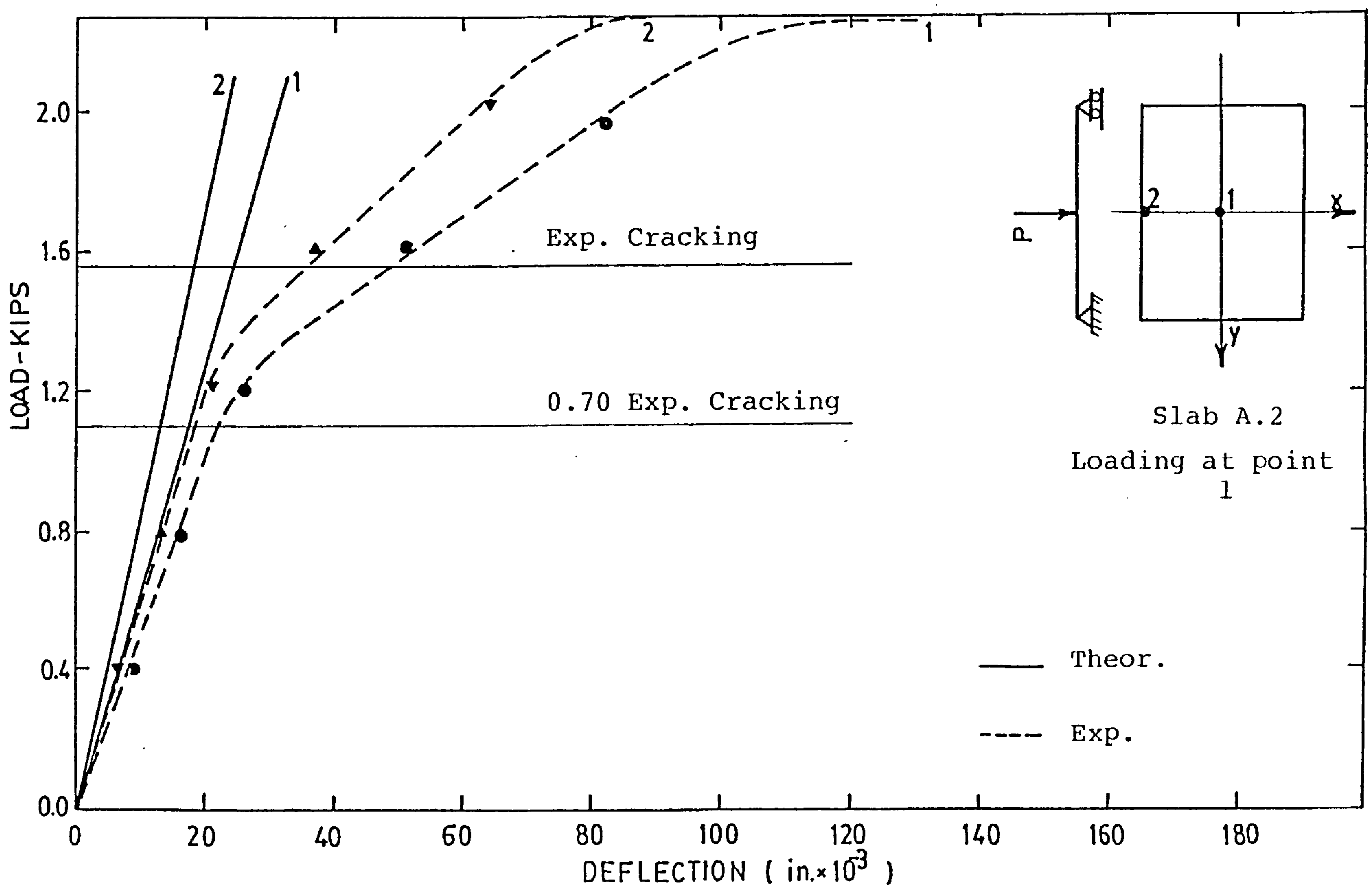


FIGURE 7.11 LOAD DEFLECTION RELATIONSHIP FOR SLAB A.2.

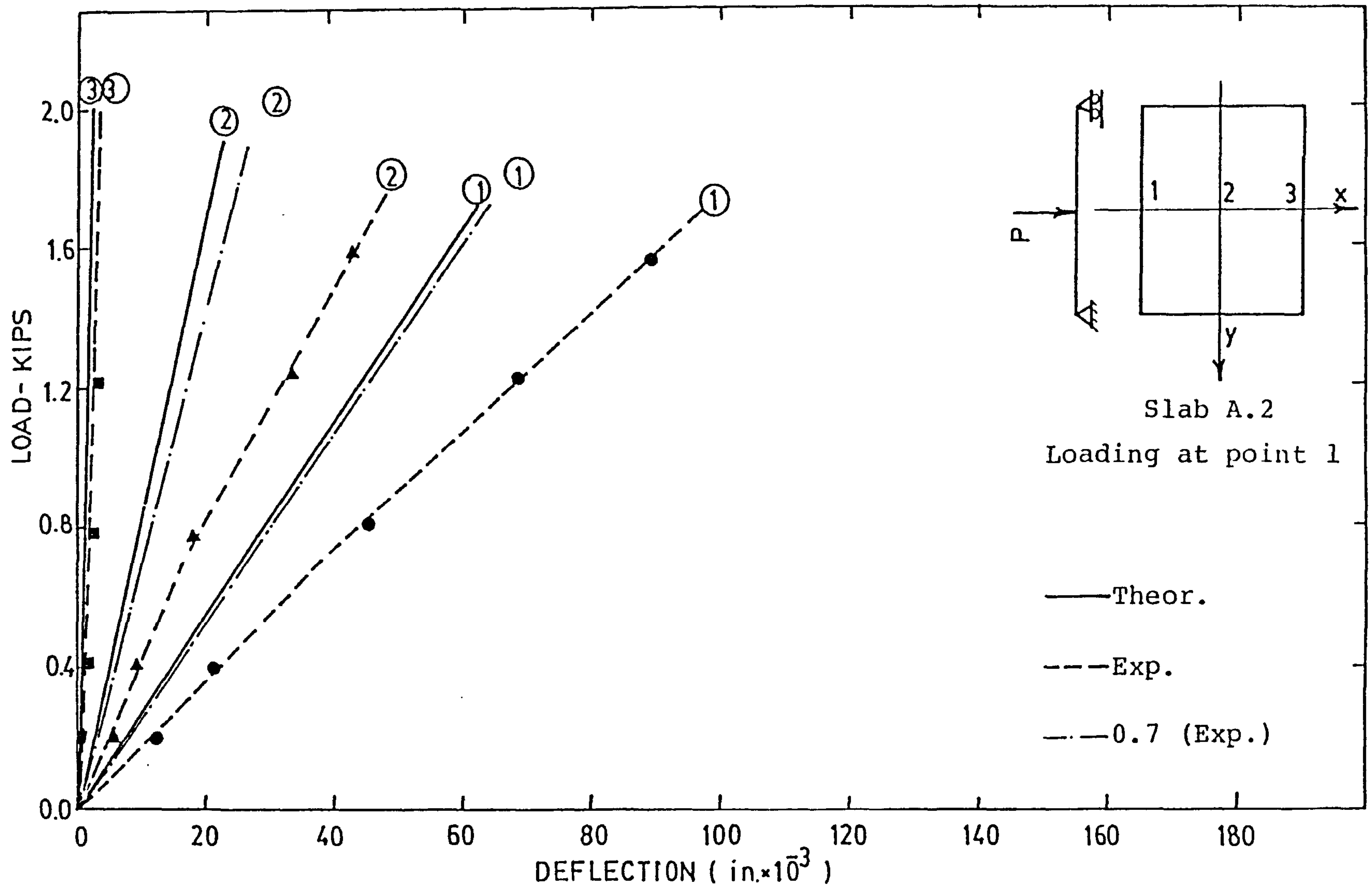


FIGURE 7.12 LOAD DEFLECTION RELATIONSHIP FOR SLAB A.2, LOAD AT THE EDGE.

(1 KIP = 4.45 KN)

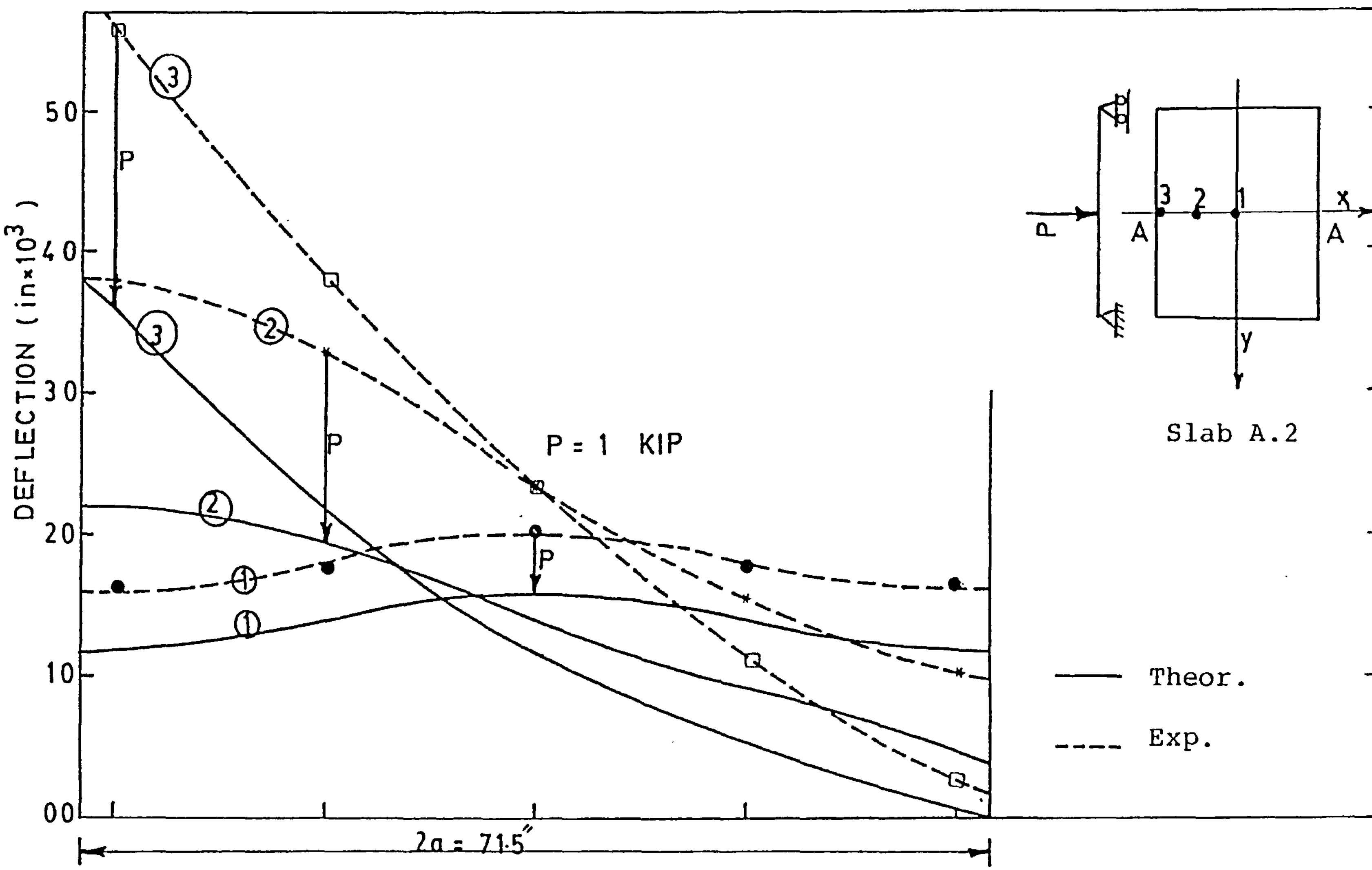


FIGURE 7.13 TRANSVERSE DEFLECTION FOR SLAB A.2 (p = 1 KIP)

(1 in = 25.4 mm)

(1 KIP = 4.45 KN)

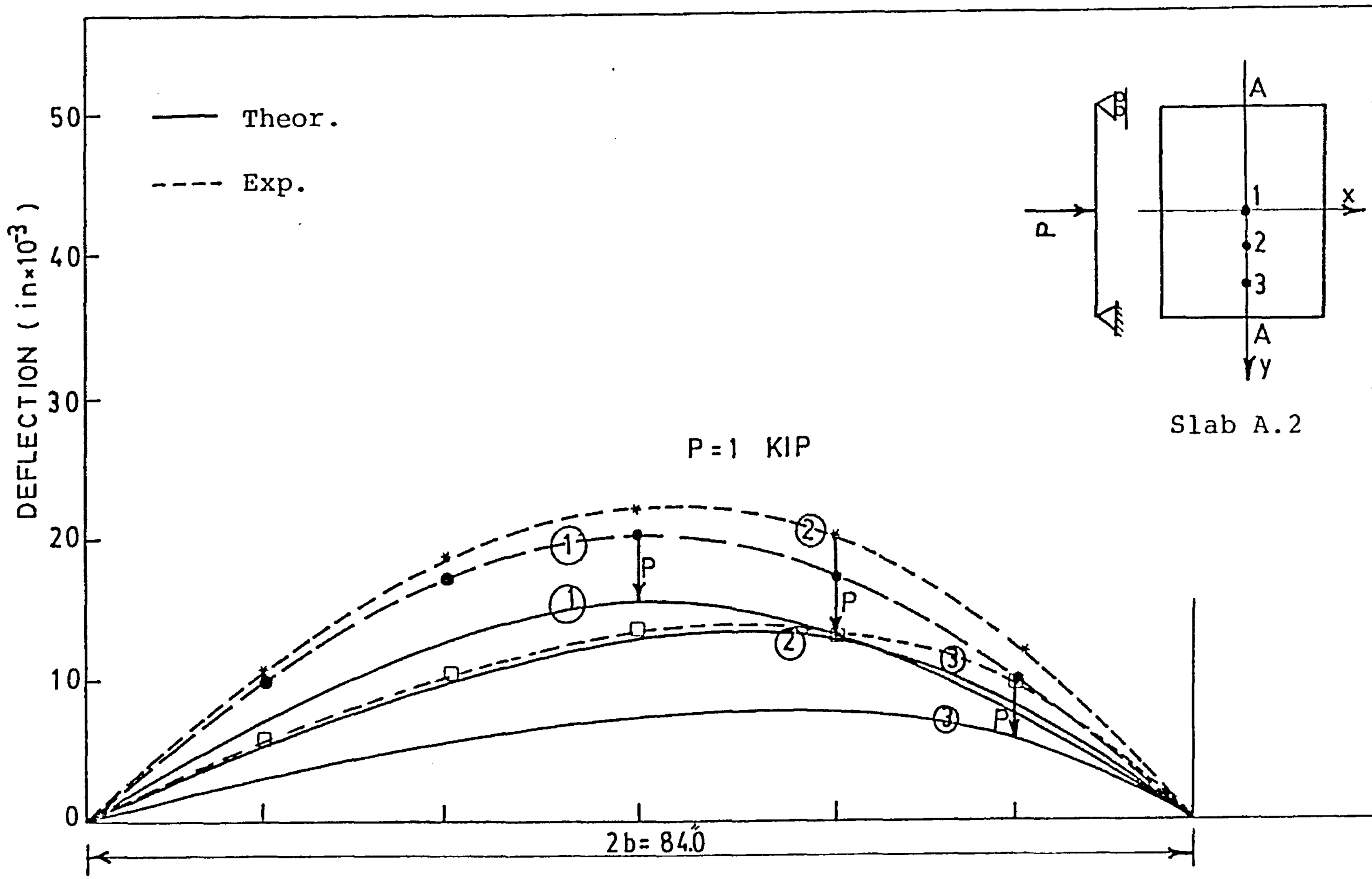


FIGURE 7.14 LONGITUDINAL DEFLECTION ALONG LINE A-A (p = 1 KIP)

(1 in = 25.4 mm)

(1 KIP = 4.45 KN)

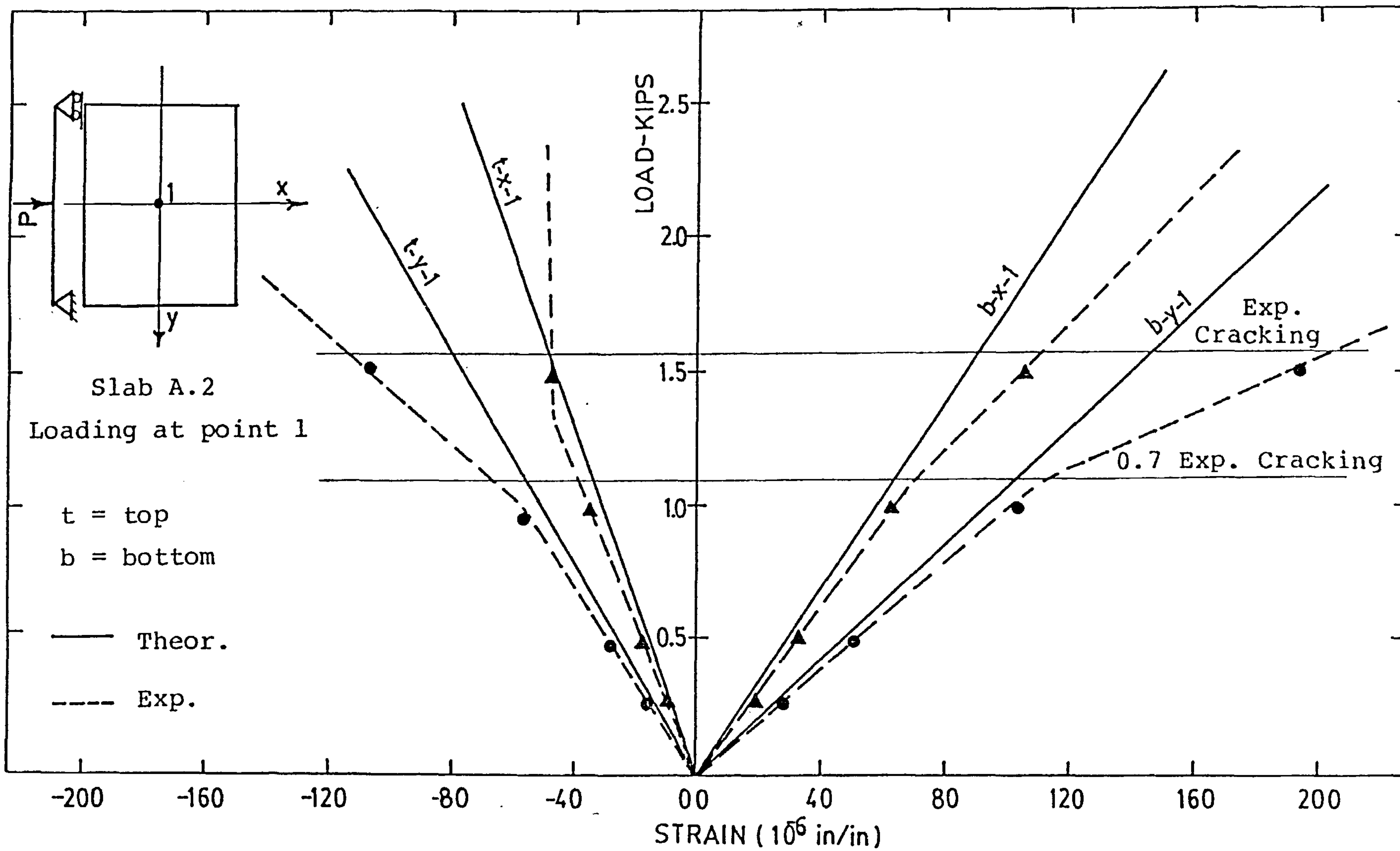


FIGURE 7.15 LOAD STRAIN RELATIONSHIP FOR SLAB A.2, LOAD AT CENTRE OF SLAB.

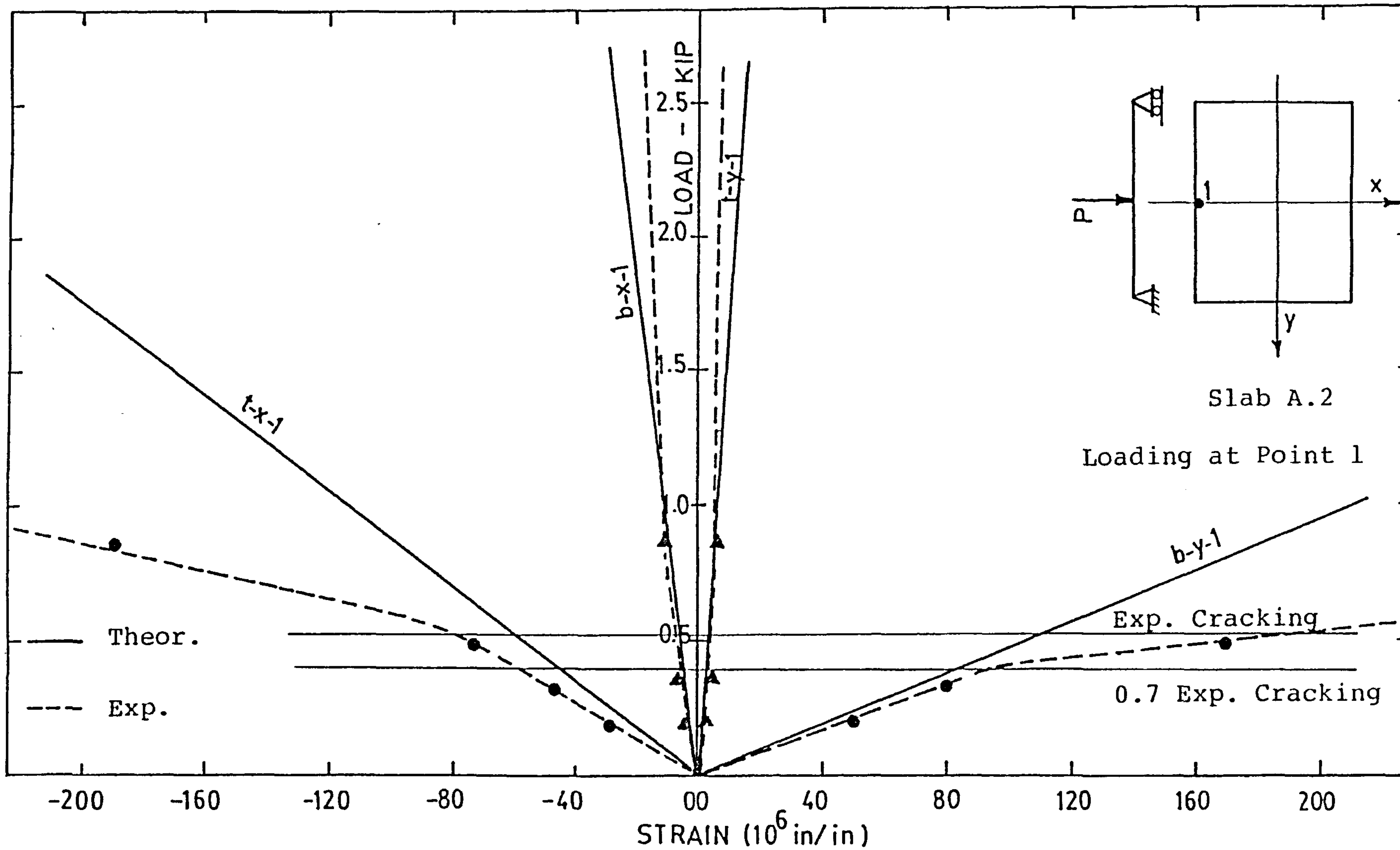


FIGURE 7.16 LOAD STRAIN RELATIONSHIP FOR SLAB A.2, LOAD AT THE FREE EDGE

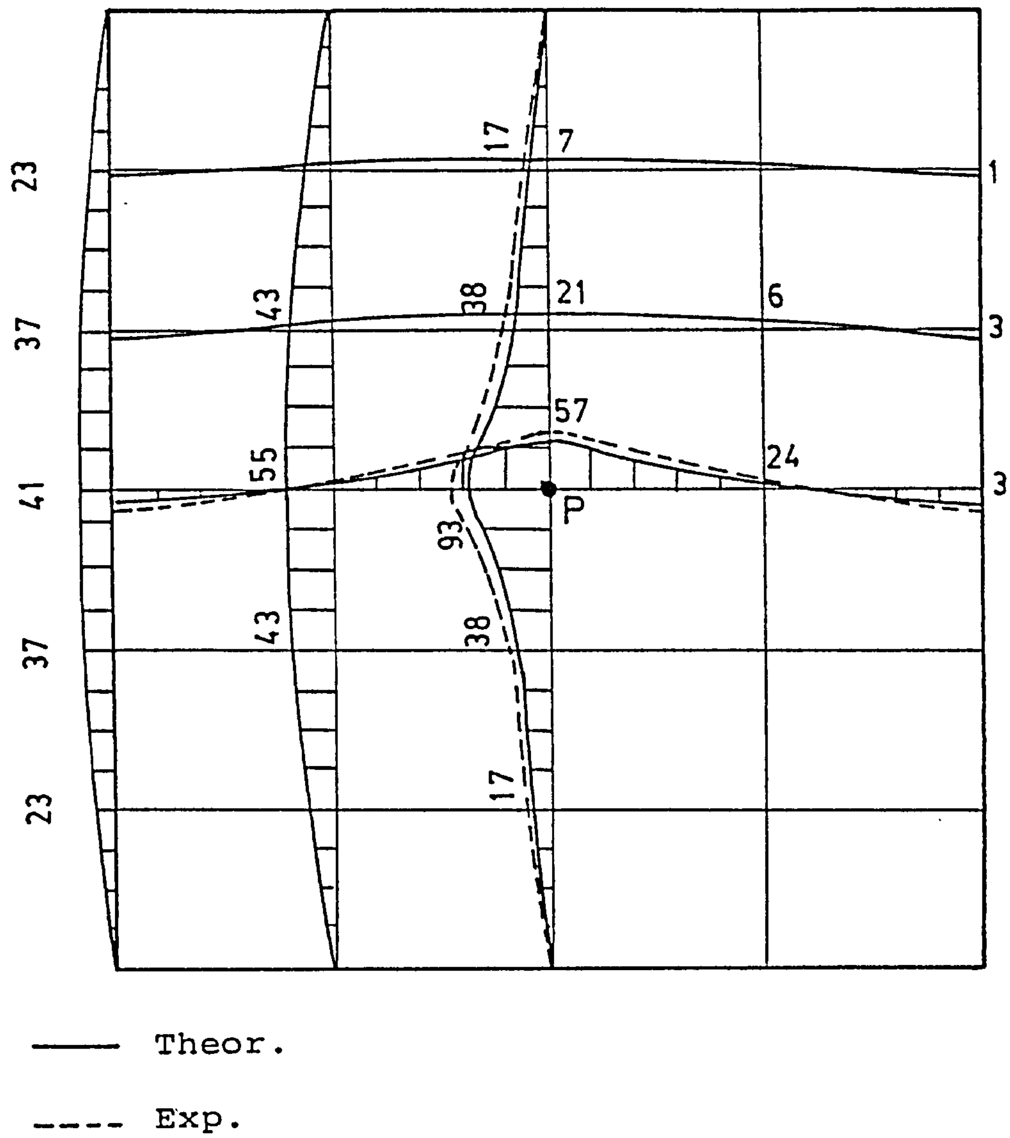


FIGURE 7.17 STRAIN DISTRIBUTION FOR SLAB A.2 AT BOTTOM FIBRE DUE TO 1 KIP AT CENTRE.

FIGURE 7.18 CRACK DEVELOPMENT IN SLAB A.3.

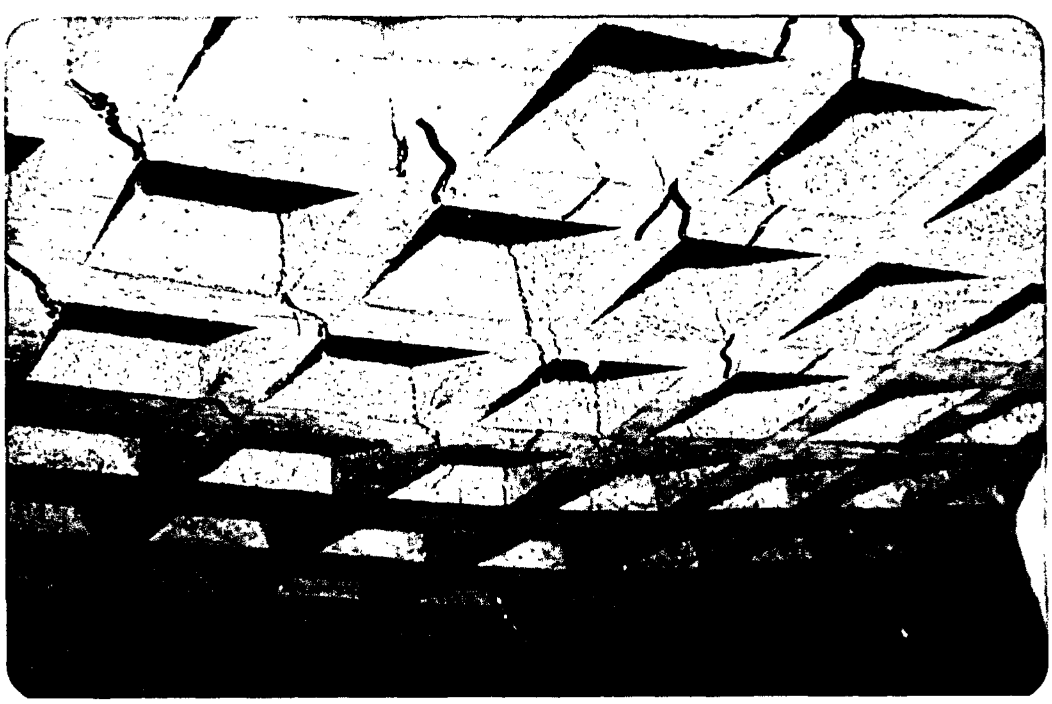


FIGURE 7.19 BOTTOM CRACKS FOR SLAB A.3.

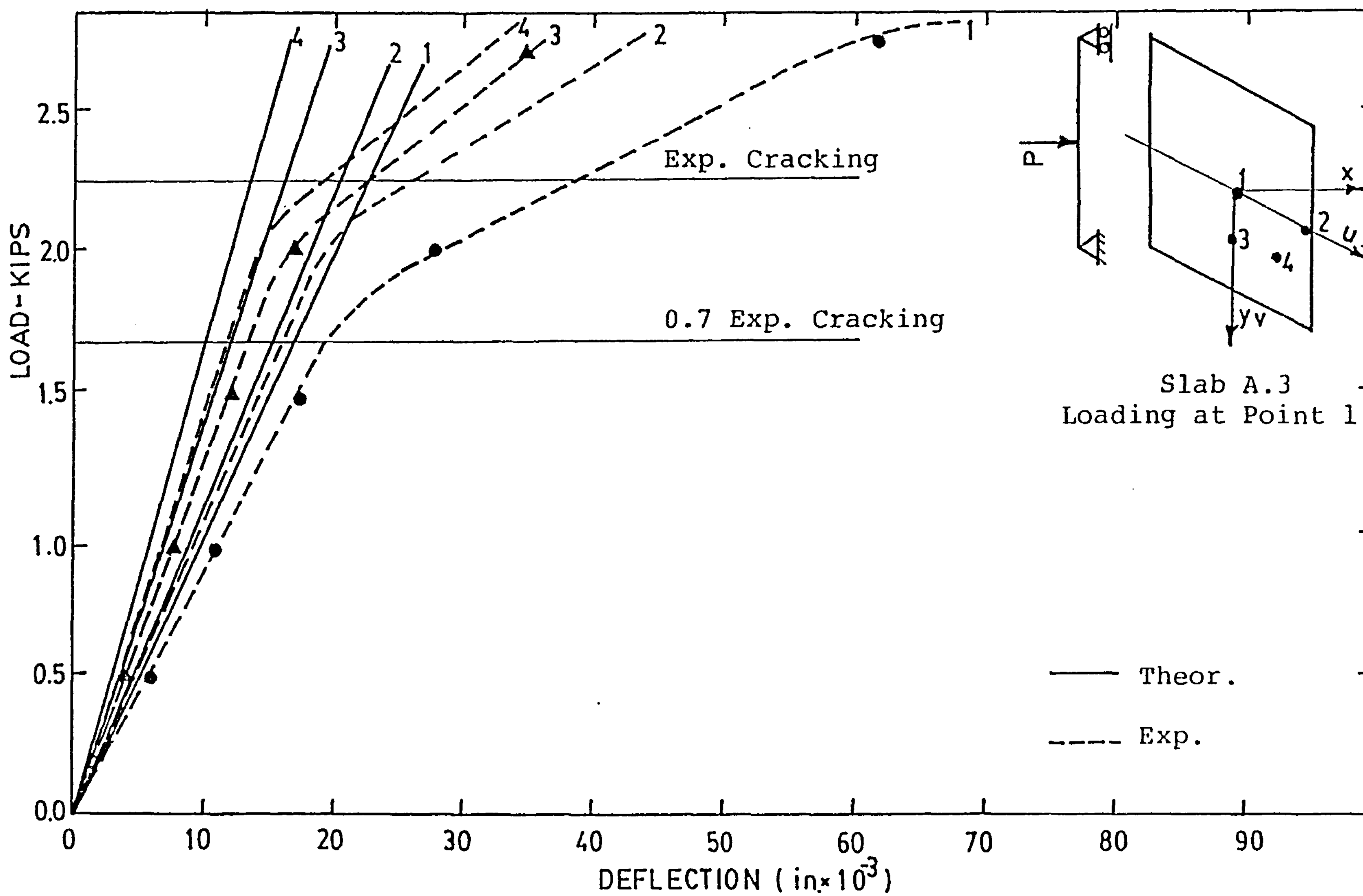


FIGURE 7.20 LOAD DEFLECTION RELATIONSHIP FOR SKEW SLAB A.3.

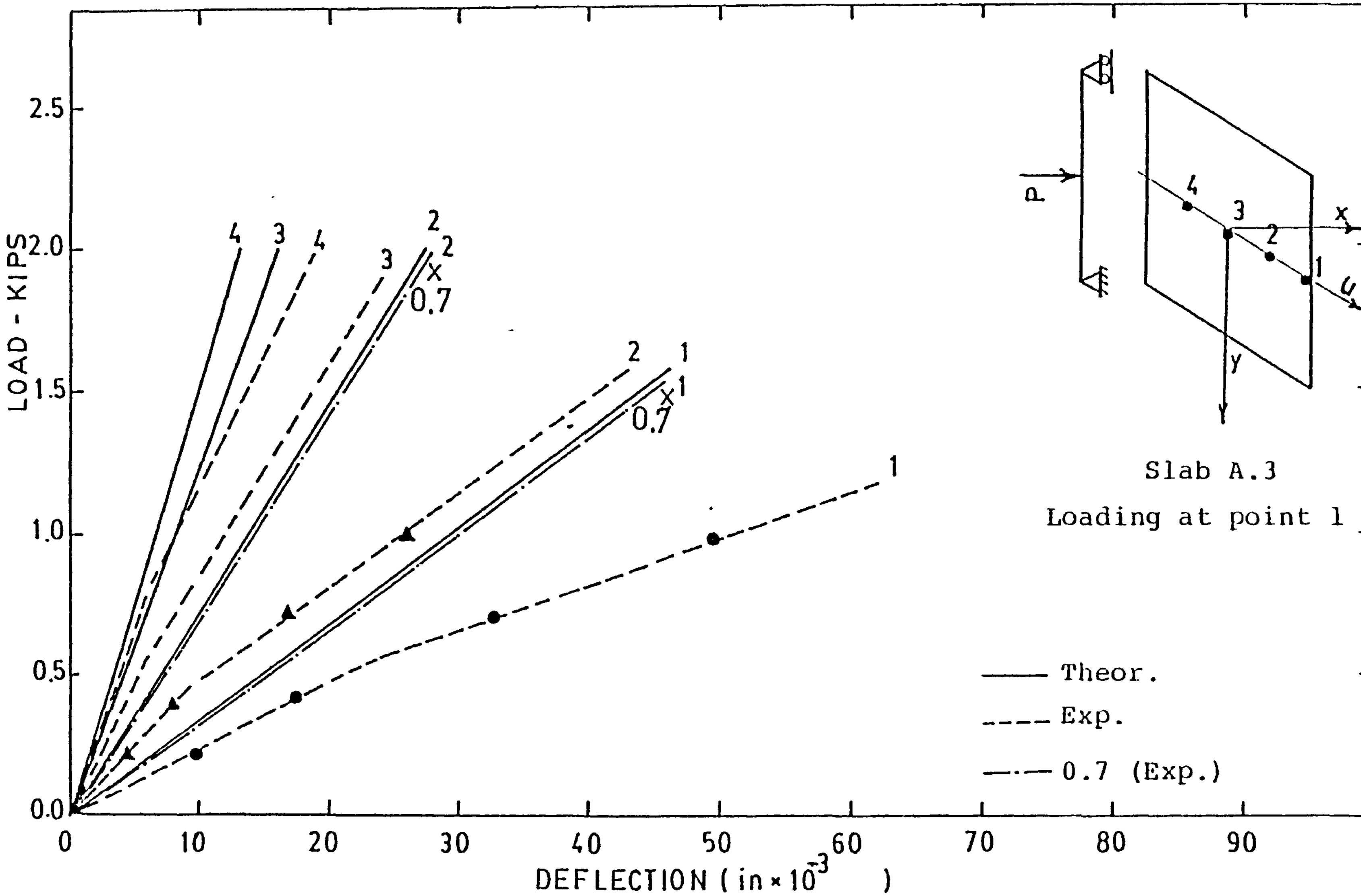


FIGURE 7.21 LOAD DEFLECTION RELATIONSHIP FOR SKEW SLAB A.3.

(1 in = 25.4 mm)

(1 KIP = 4.45 KN)

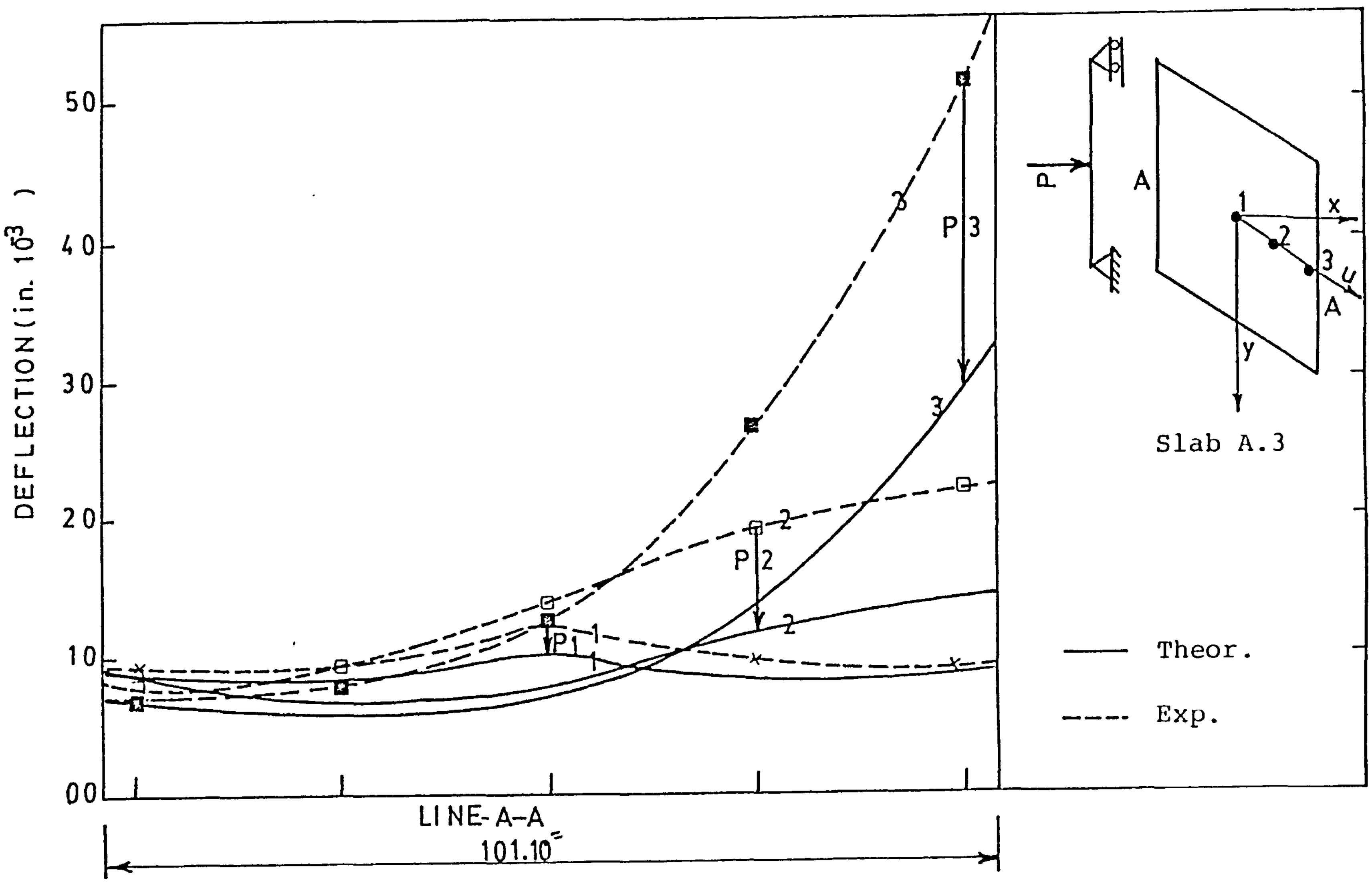


FIGURE 7.22 DEFLECTIONS ALONG LINE A-A FOR SKEW SLAB A.3 ($p = 1$ KIP)

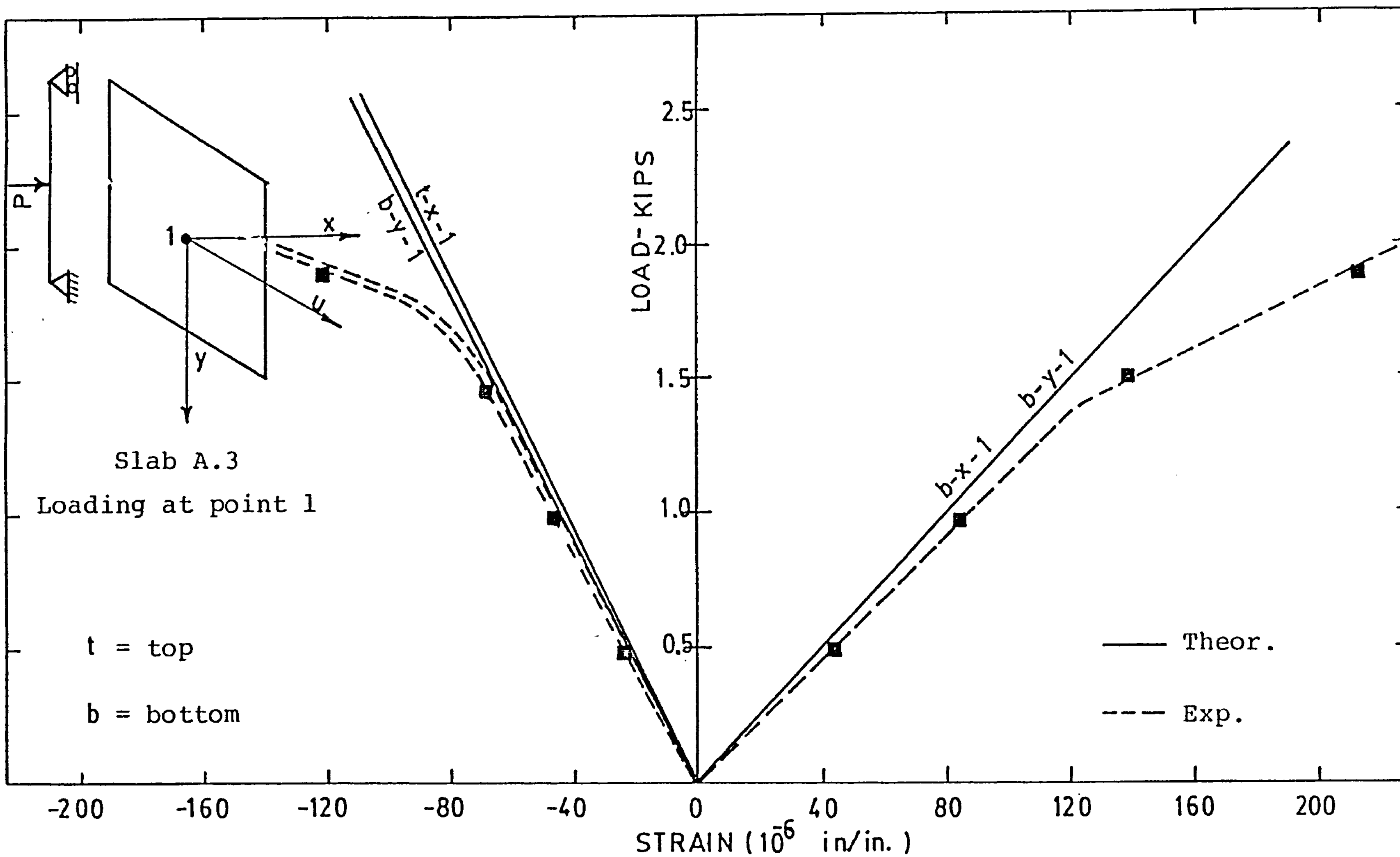


FIGURE 7.23 LOAD STRAIN RELATIONSHIP FOR SKEW SLAB A.3 AT POINT 1.

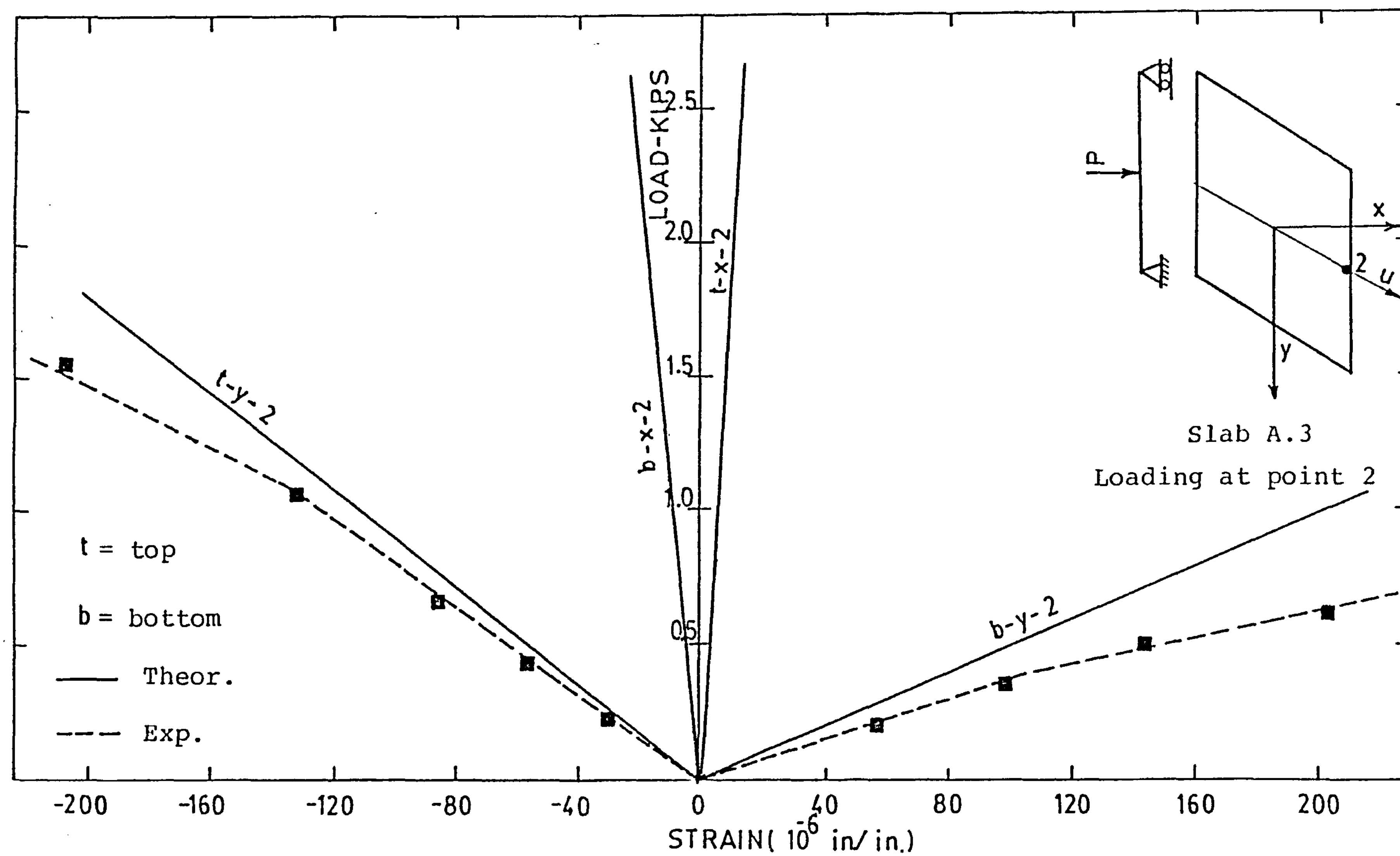


FIGURE 7.24 LOAD STRAIN RELATIONSHIP FOR SKEW SLAB A.3 AT POINT 2.

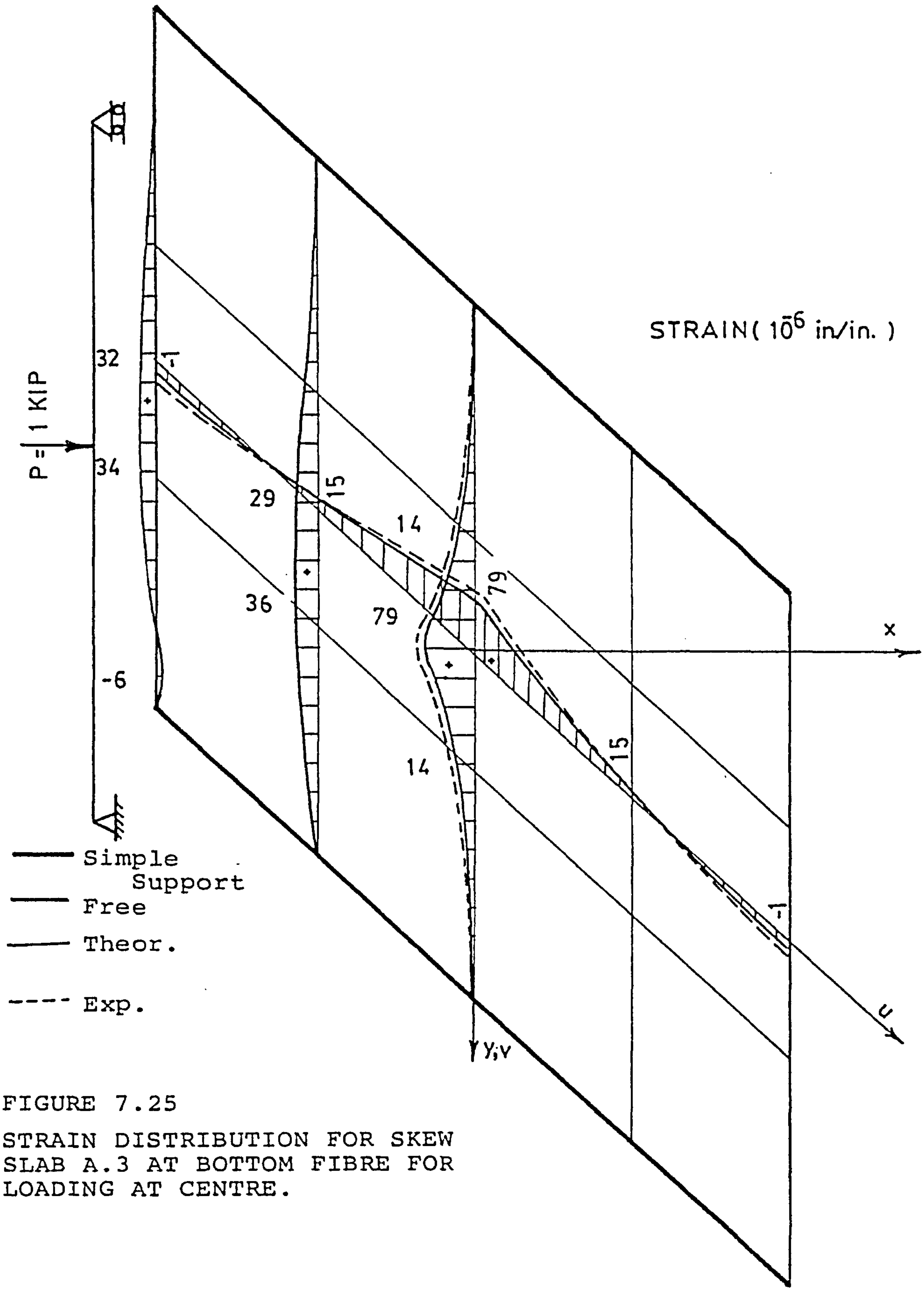


FIGURE 7.25

STRAIN DISTRIBUTION FOR SKEW SLAB A.3 AT BOTTOM FIBRE FOR LOADING AT CENTRE.

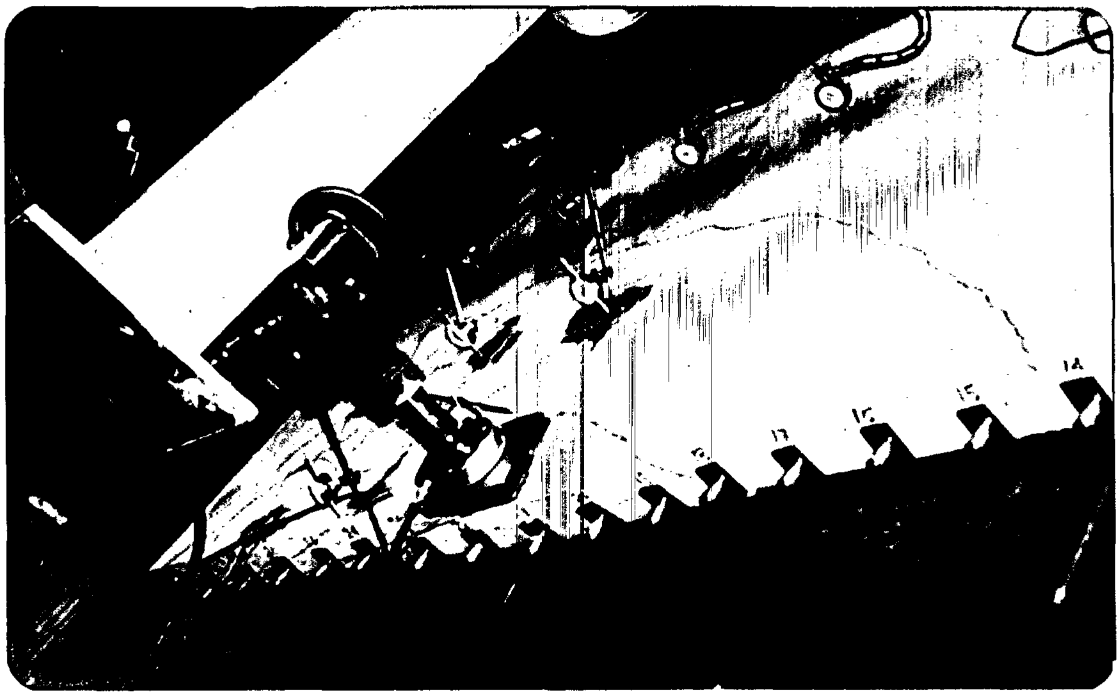


FIGURE 7.26 CRACKS DEVELOPMENT FOR SLAB B.1.

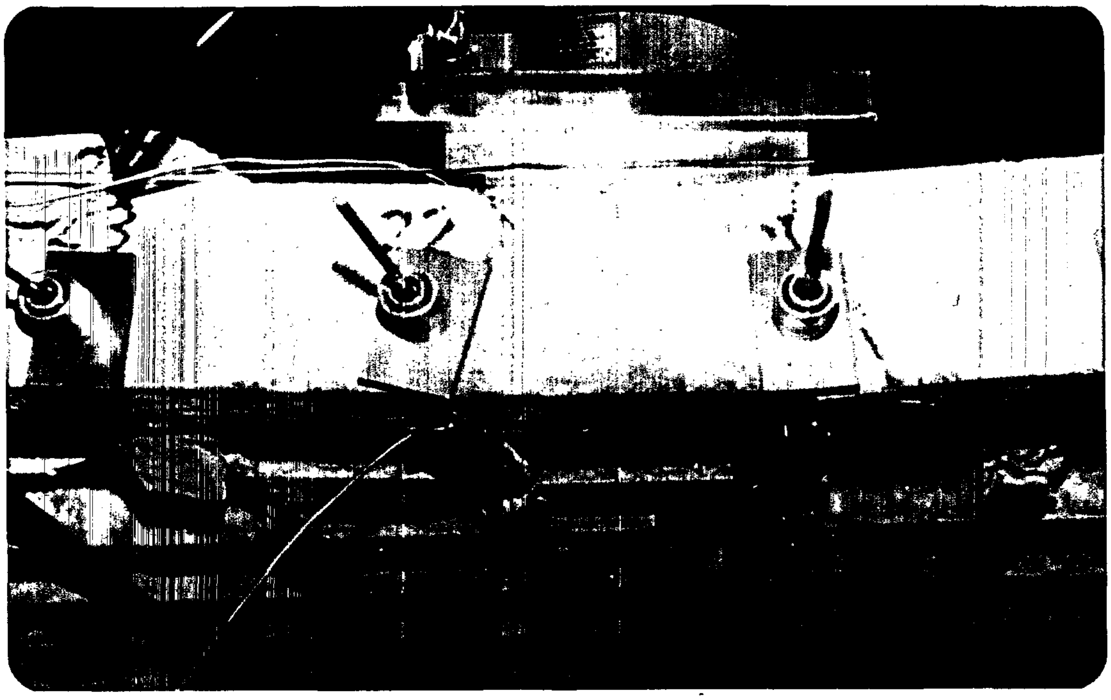
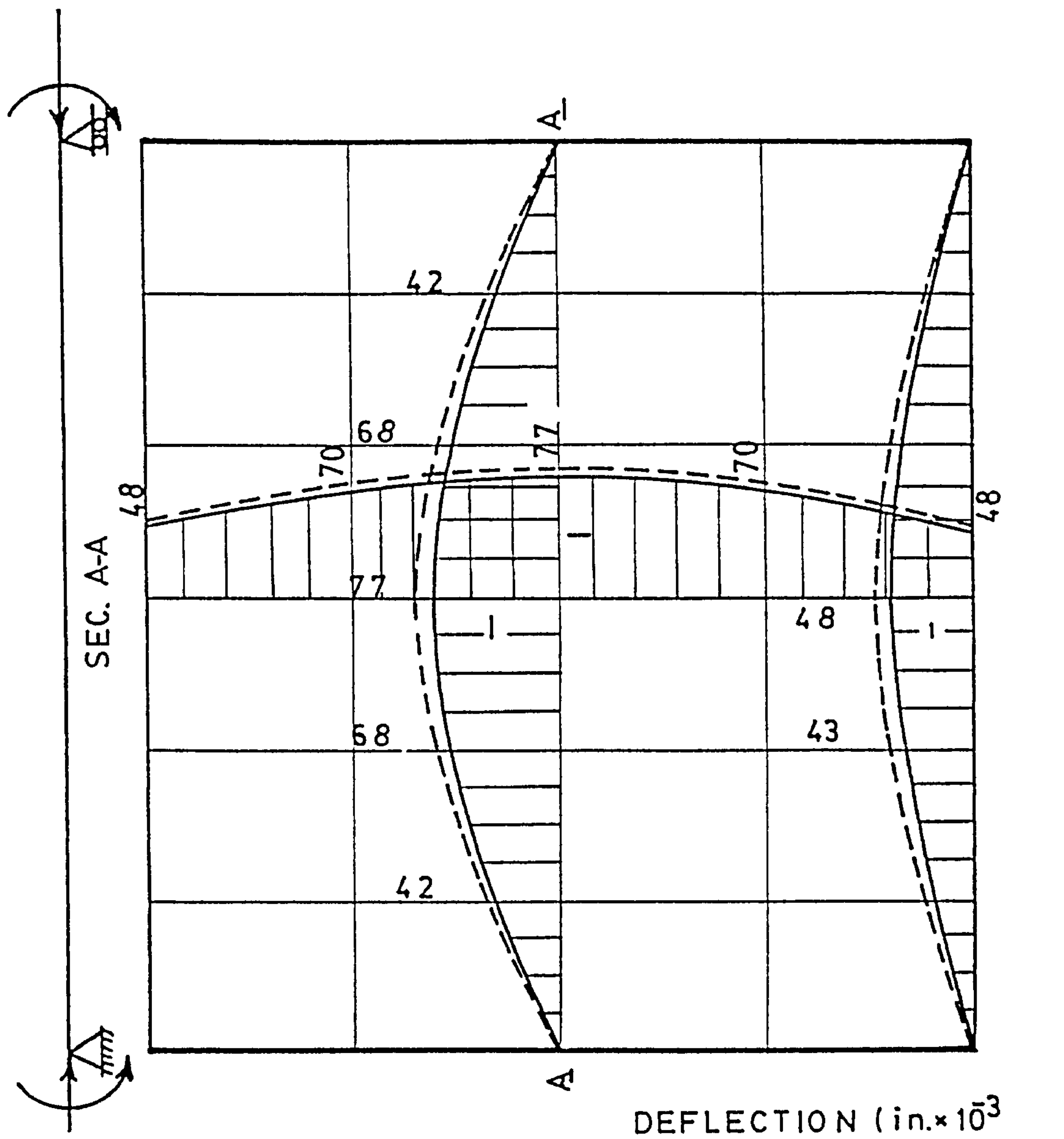


FIGURE 7.27 CRACKS FAILURE FOR SLAB B.1.



—— Theor
––– Exp.
Simple Support
Edge Beam

FIGURE 7.28 THE UPWARD DEFLECTION (CAMBER) IN SLAB B.1 DUE TO PRESTRESSING.

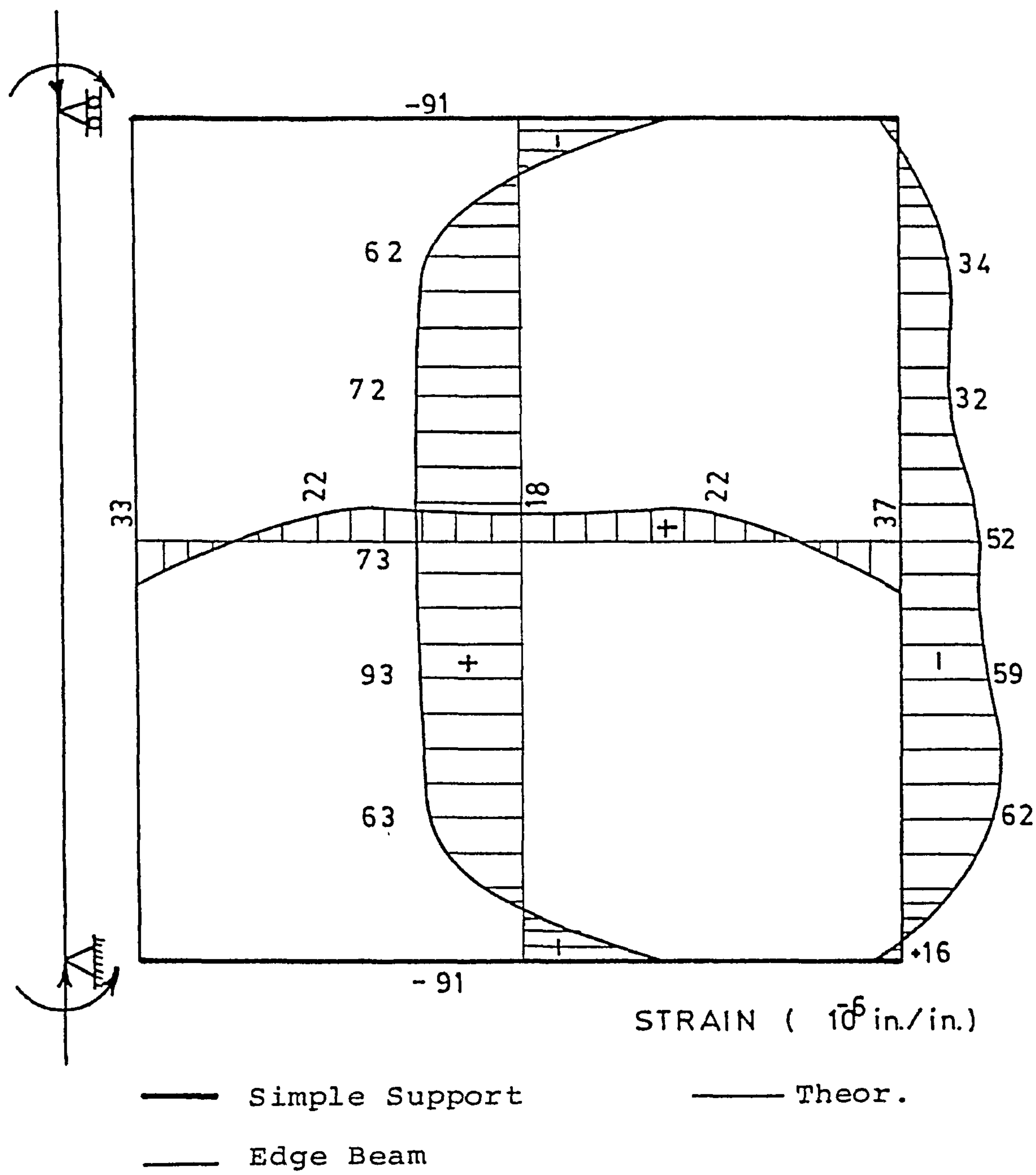


FIGURE 7.29 TOP FIBRE STRAIN DISTRIBUTION FOR SLAB B.1 DUE TO PRESTRESSING.

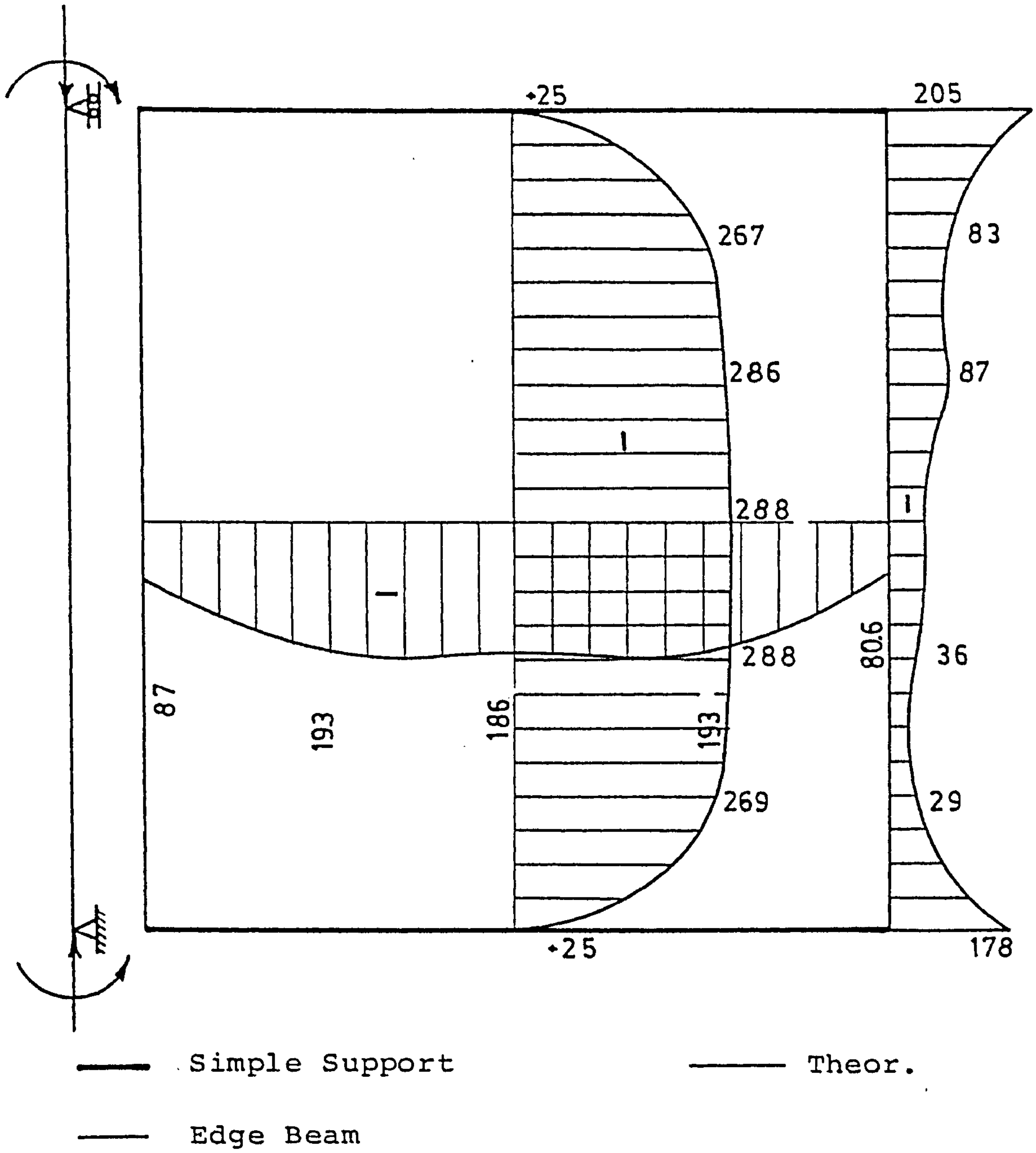


FIGURE 7.30 BOTTOM FIBRE STRAIN DISTRIBUTION FOR SLAB B.1 DUE TO PRESTRESSING.

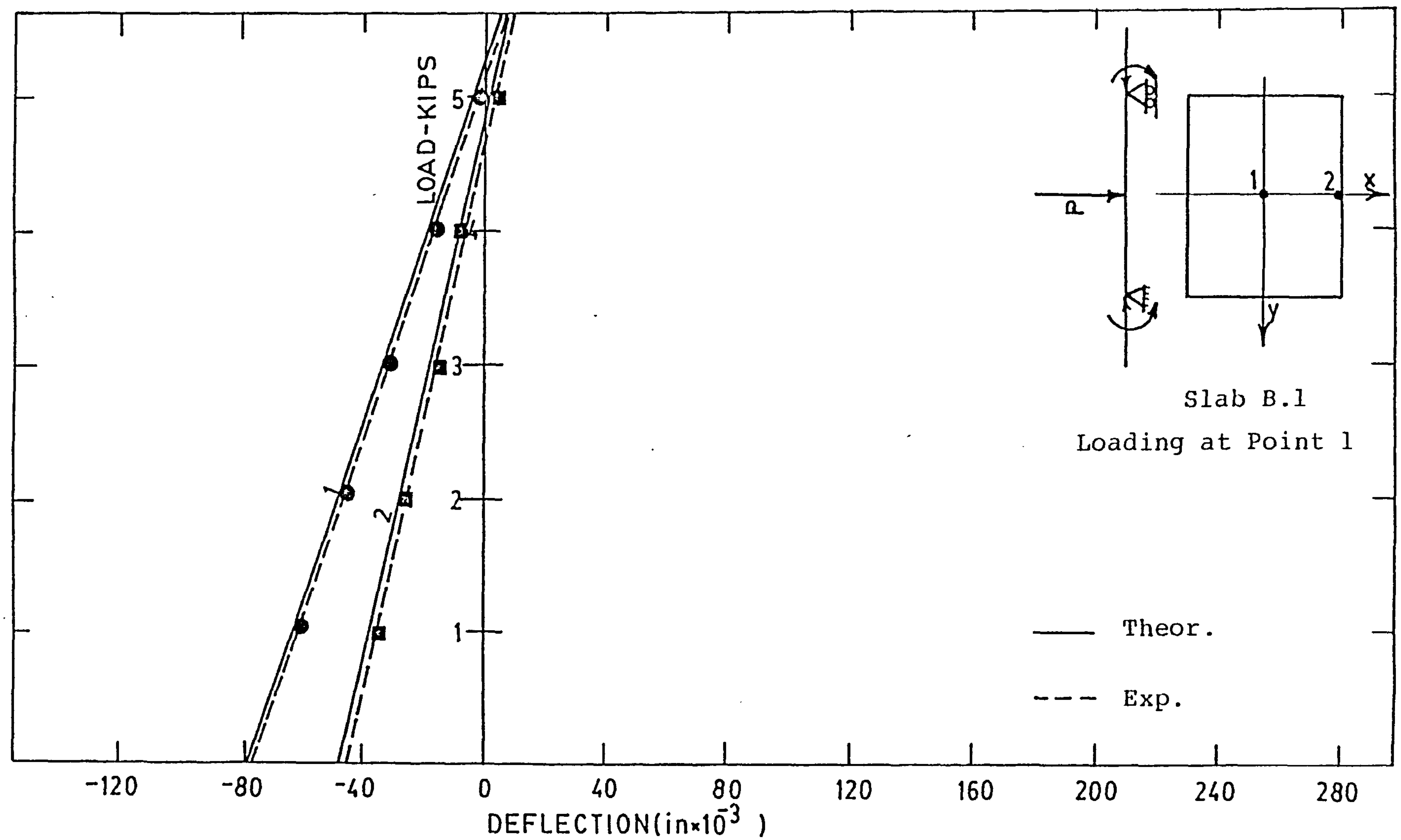


FIGURE 7.31 LOAD DEFLECTION RELATIONSHIP FOR SLAB B.1.

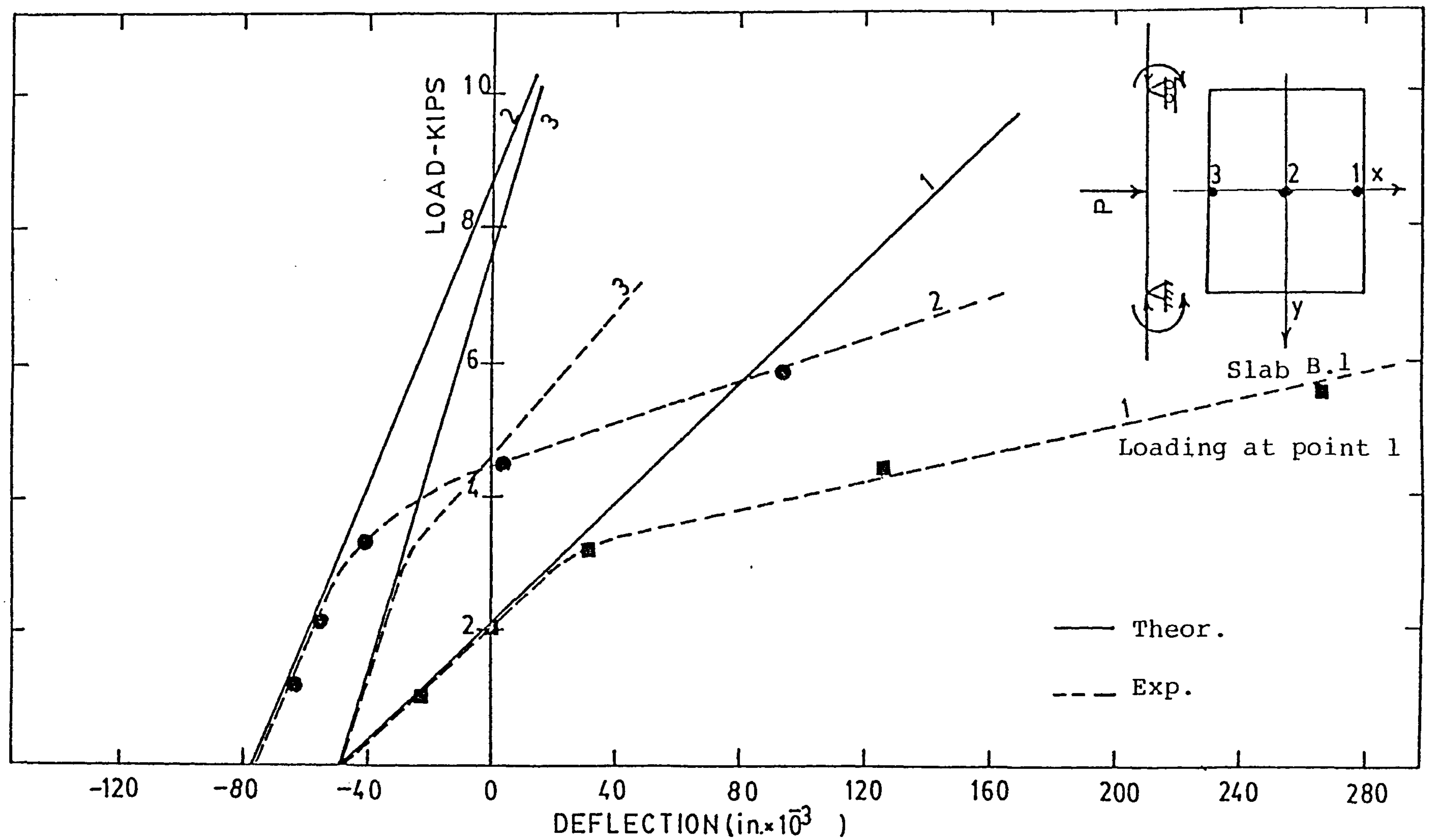


FIGURE 7.32 LOAD DEFLECTION RELATIONSHIP FOR SLAB B.1

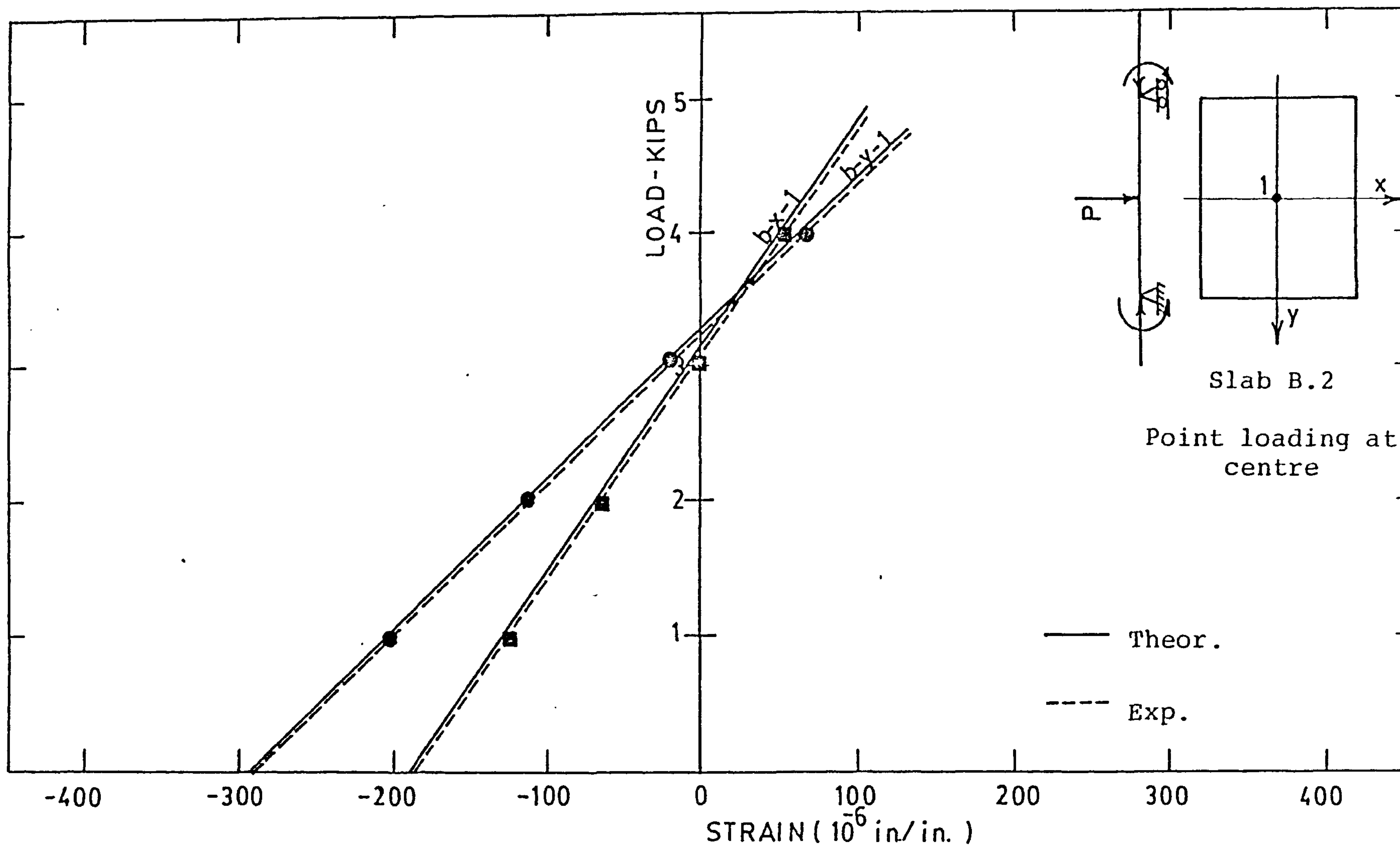


FIGURE 7.33 LOAD STRAIN RELATIONSHIP FOR SLAB B.1 AT CENTRE.

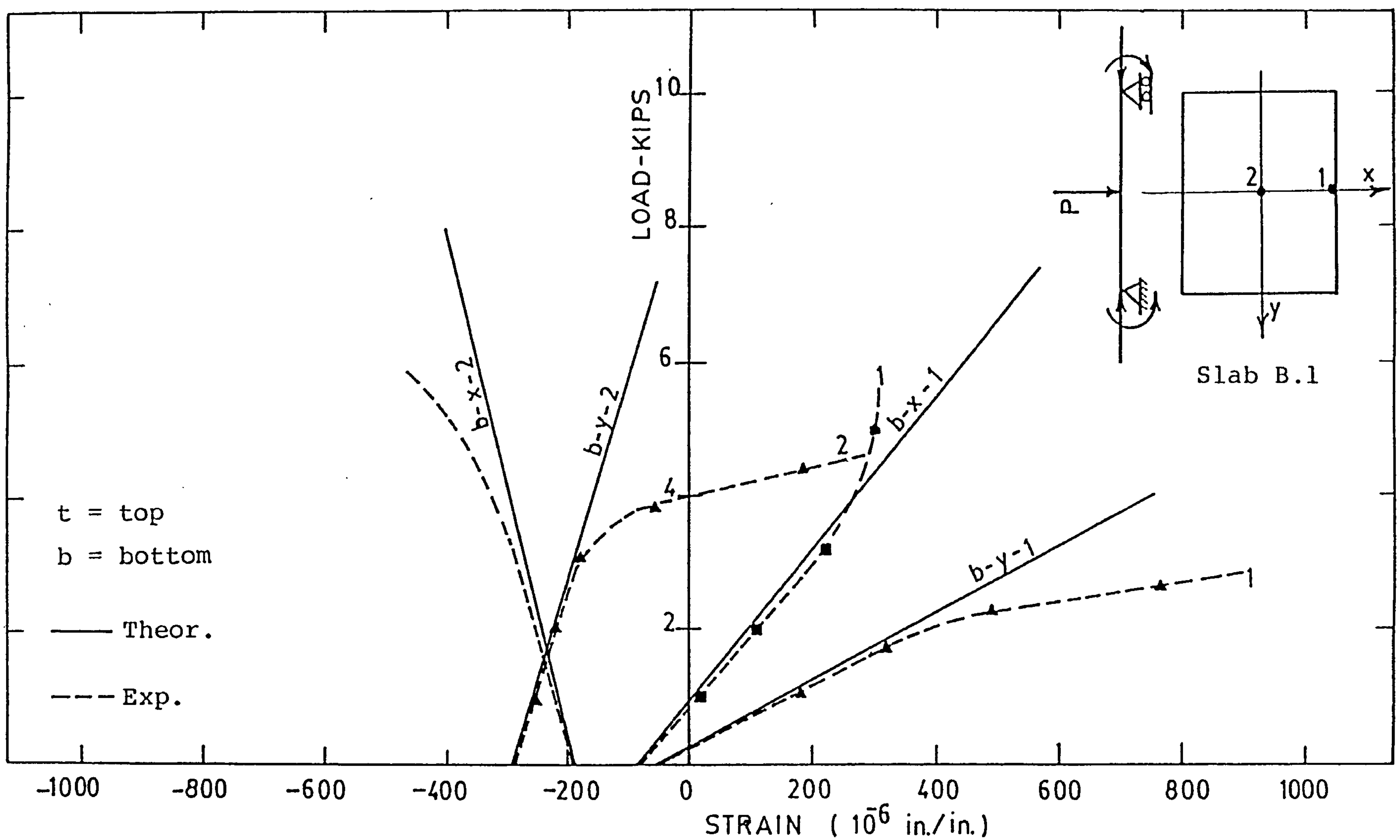


FIGURE 7.34 LOAD STRAIN RELATIONSHIP FOR SLAB B.1 (LOADING AT POINT 1).

FIGURE 7.35 CRACKS FAILURE FOR SLAB B.2.

FIGURE 7.36 CRACKS DEVELOPMENT FOR SLAB B.2.

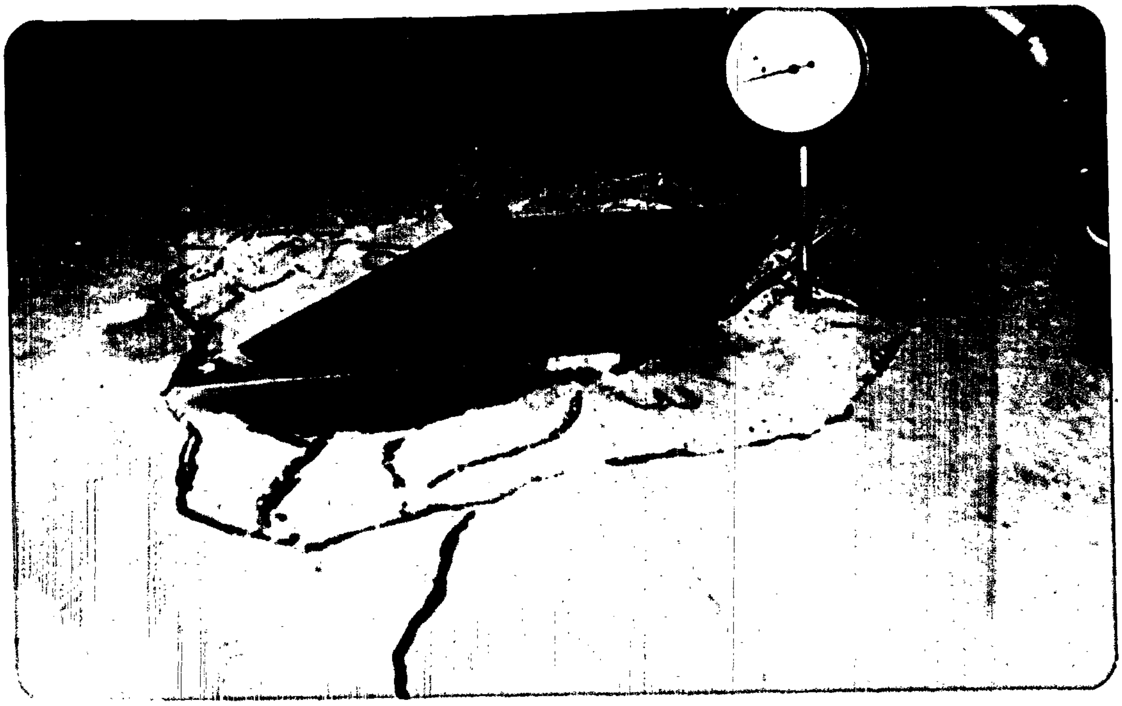


FIGURE 7.37 PUNCHING FAILURE OF SLAB B.2.

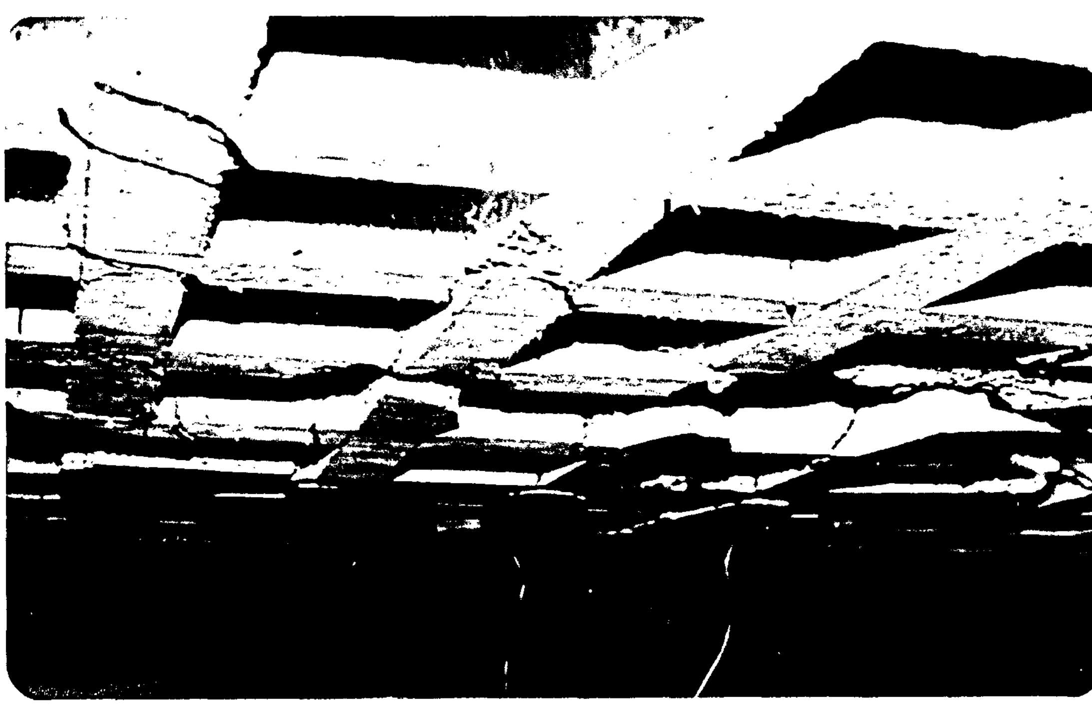


FIGURE 7.38 CRACKS FAILURE FOR SLAB B.2.

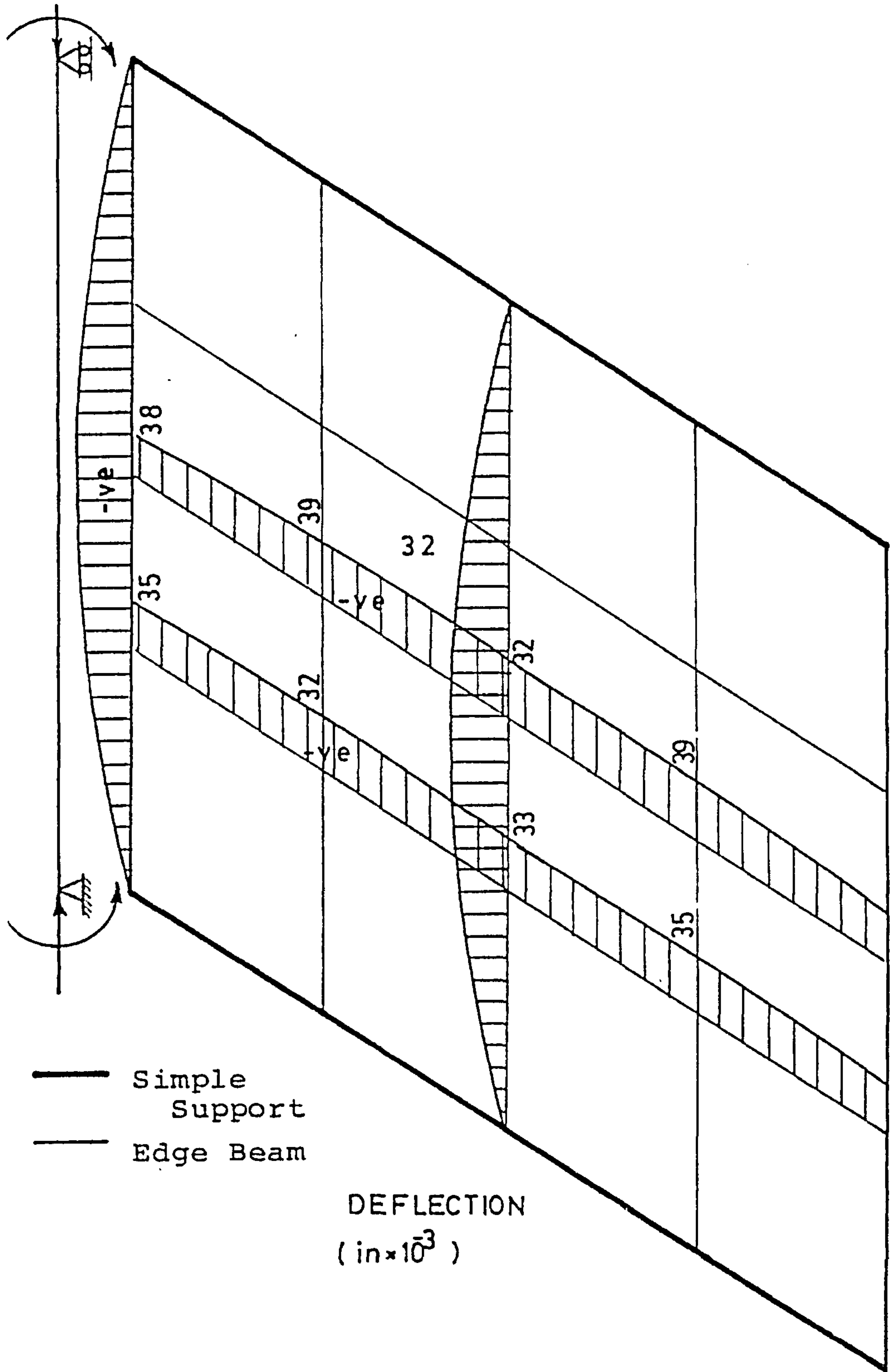


FIGURE 7.39 DEFLECTION DISTRIBUTION FOR SKEW SLAB B.2 DUE TO PRESTRESSING FORCE.

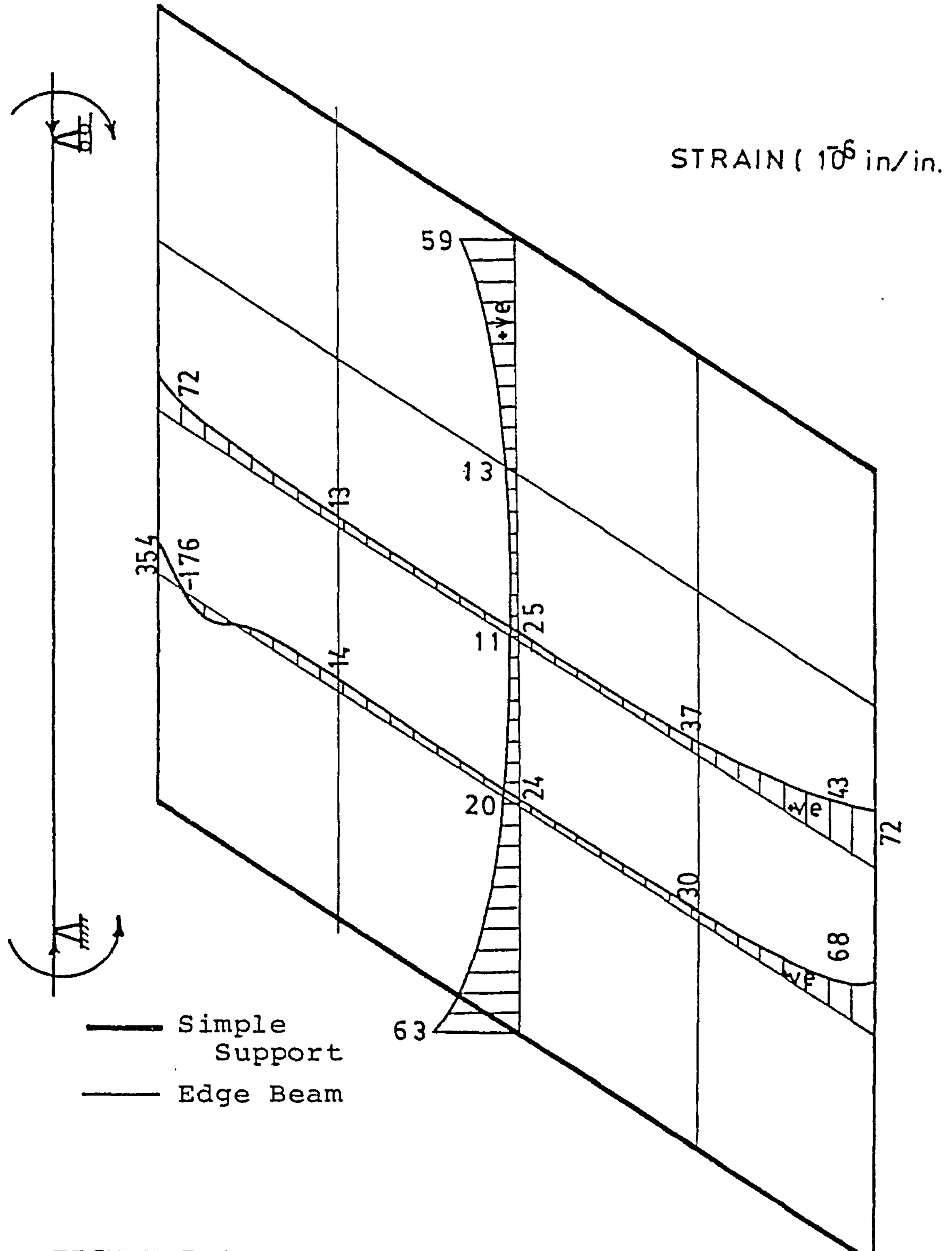


FIGURE 7.40 STRAIN DISTRIBUTION IN SLAB B.2 AT TOP FIBRE DUE TO PRE-STRESSING FORCE.

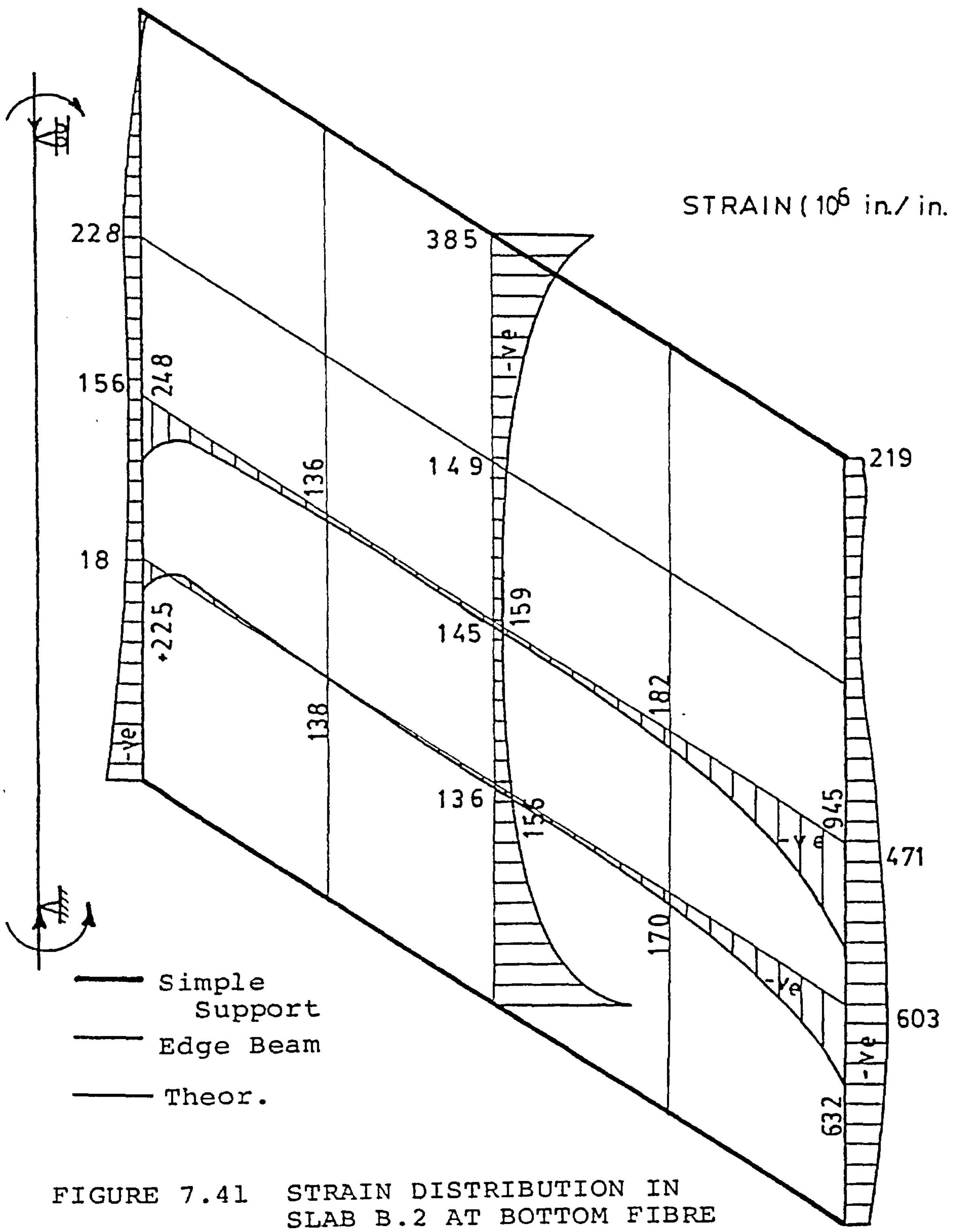


FIGURE 7.41 STRAIN DISTRIBUTION IN SLAB B.2 AT BOTTOM FIBRE DUE TO PRESTRESSING.

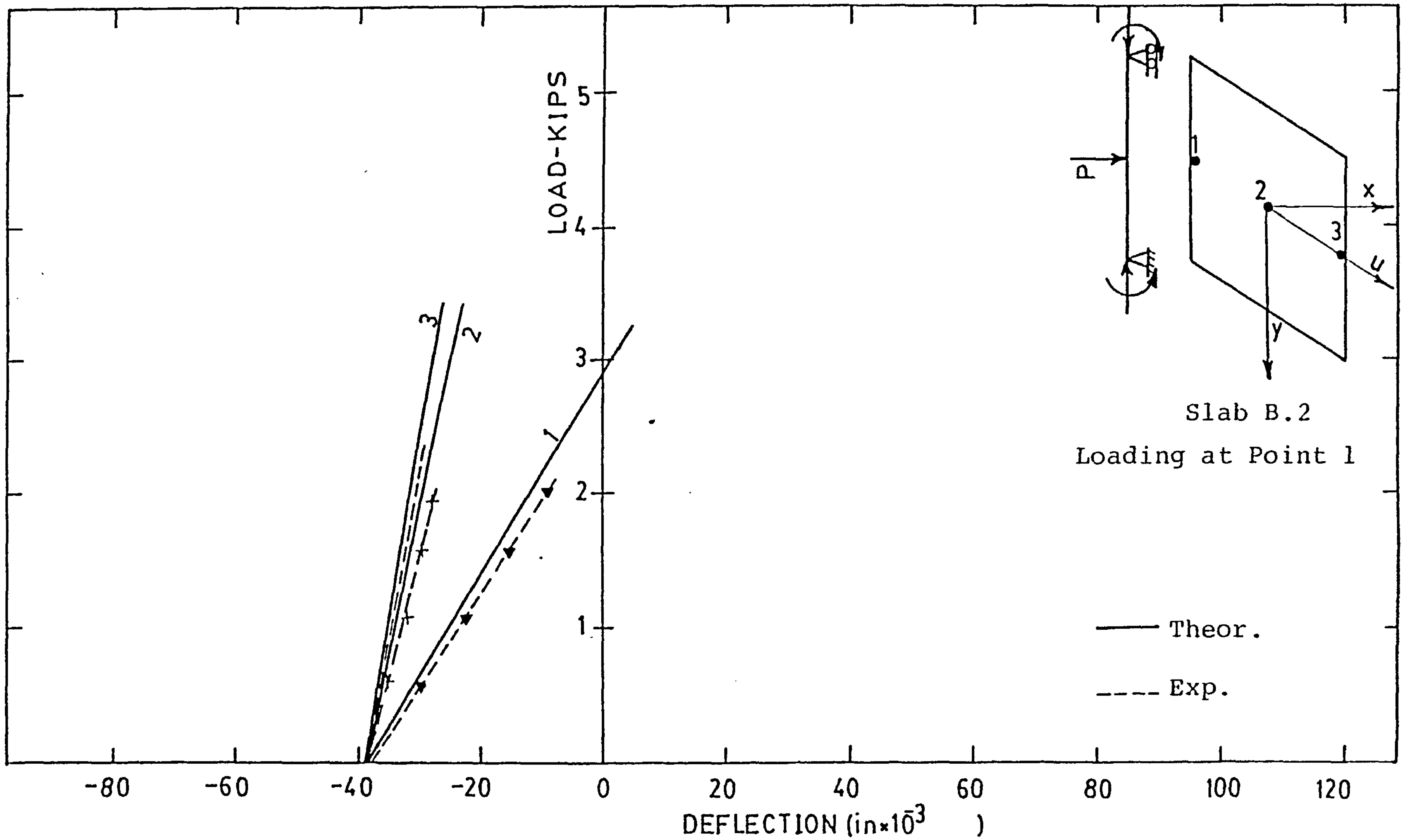


FIGURE 7.42 LOAD DEFLECTION RELATIONSHIP FOR SLAB B.2.

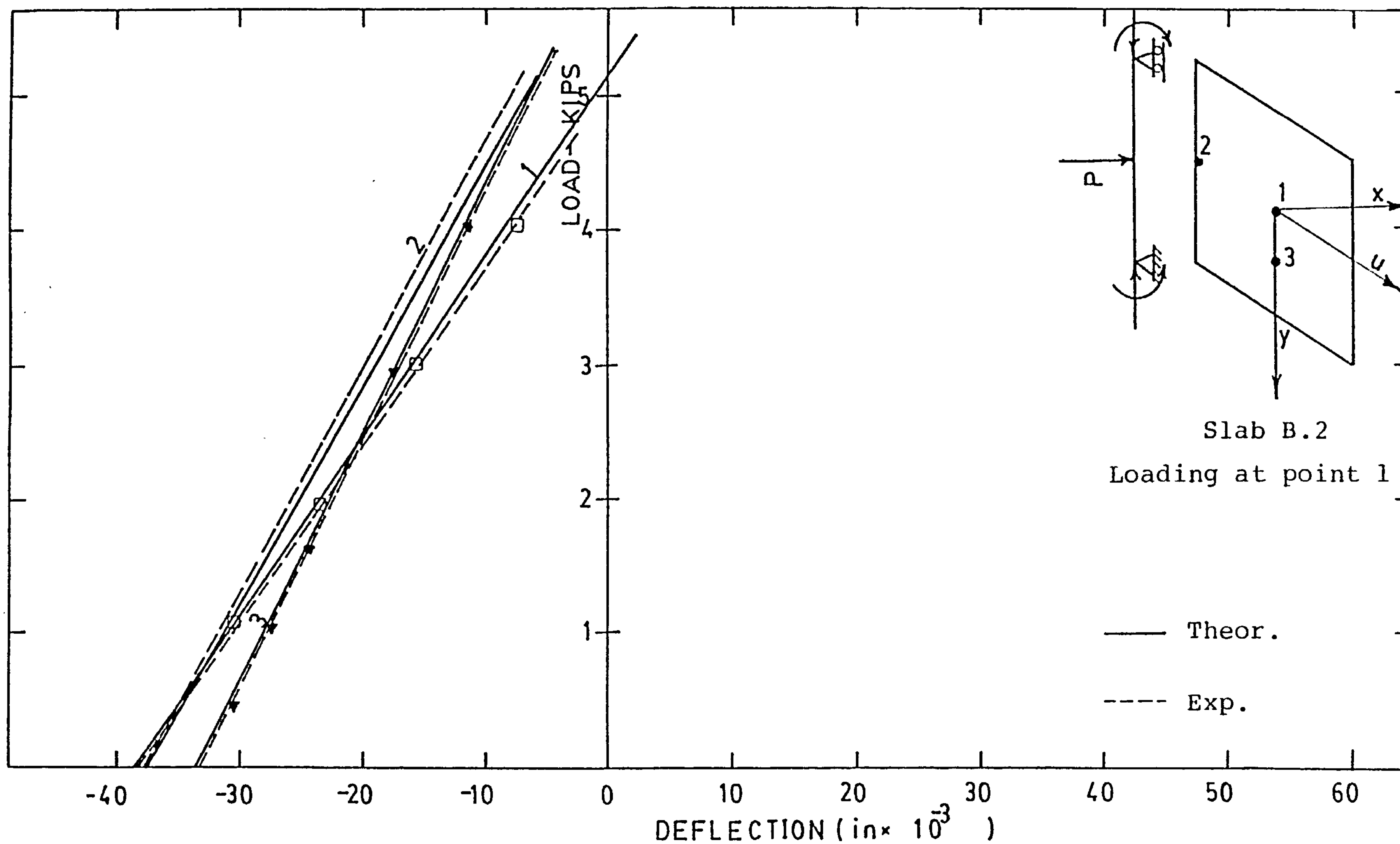


FIGURE 7.43 LOAD DEFLECTION RELATIONSHIP FOR SLAB B.2.

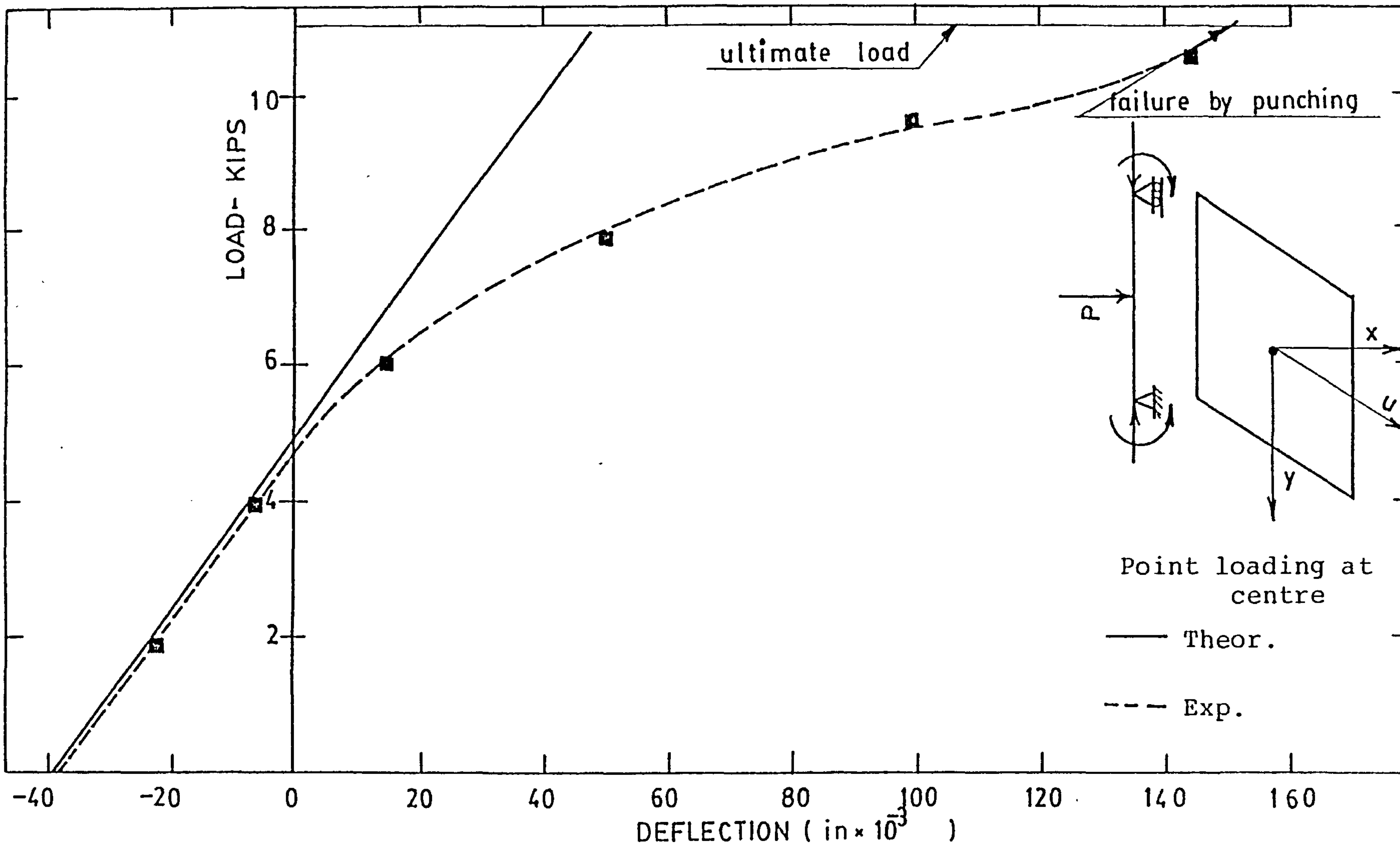


FIGURE 7.44 LOAD-DEFLECTION RELATIONSHIP FOR SLAB B.2 (ULTIMATE LOAD).

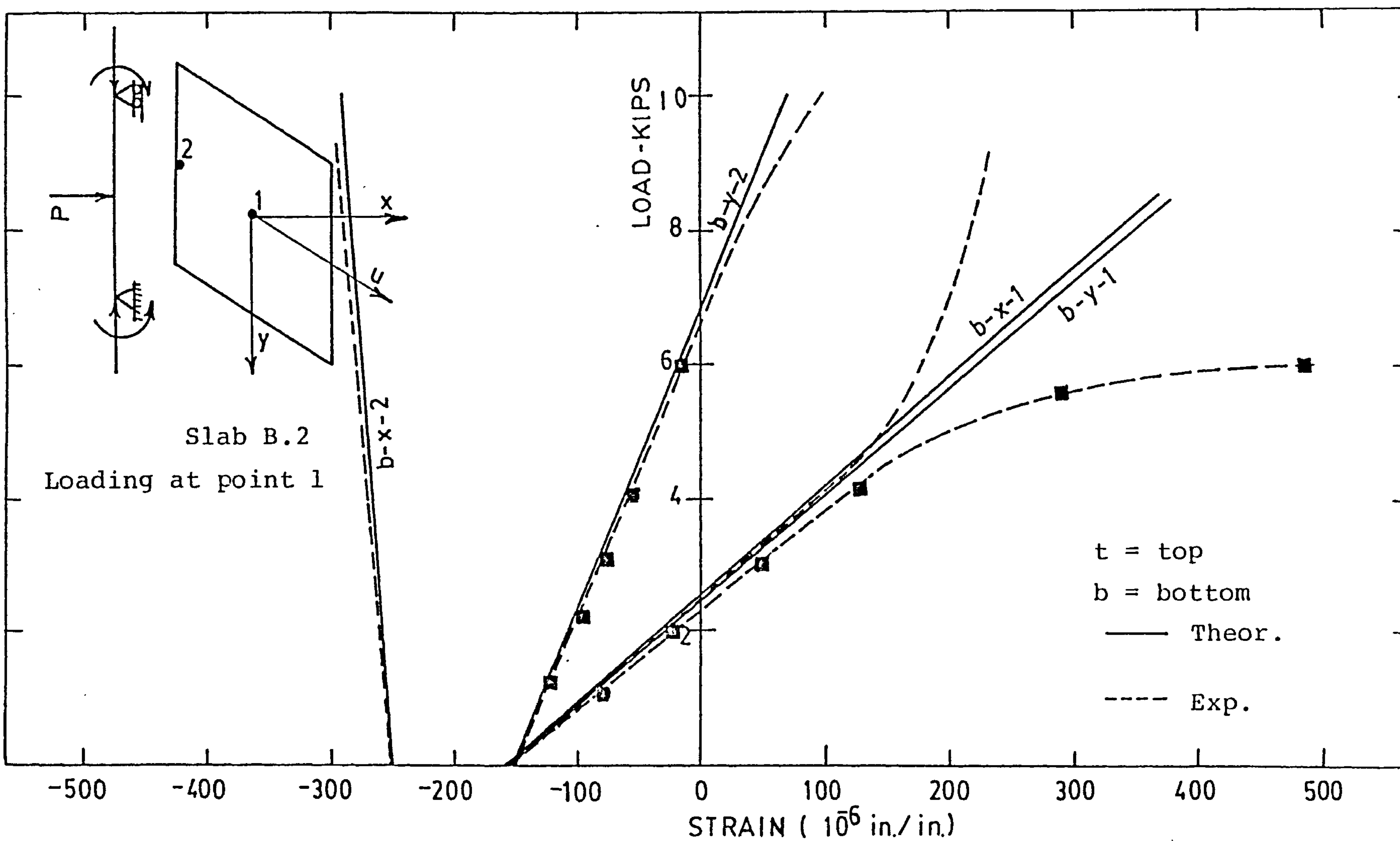


FIGURE 7.45 LOAD STRAIN RELATIONSHIP FOR SLAB B.2.

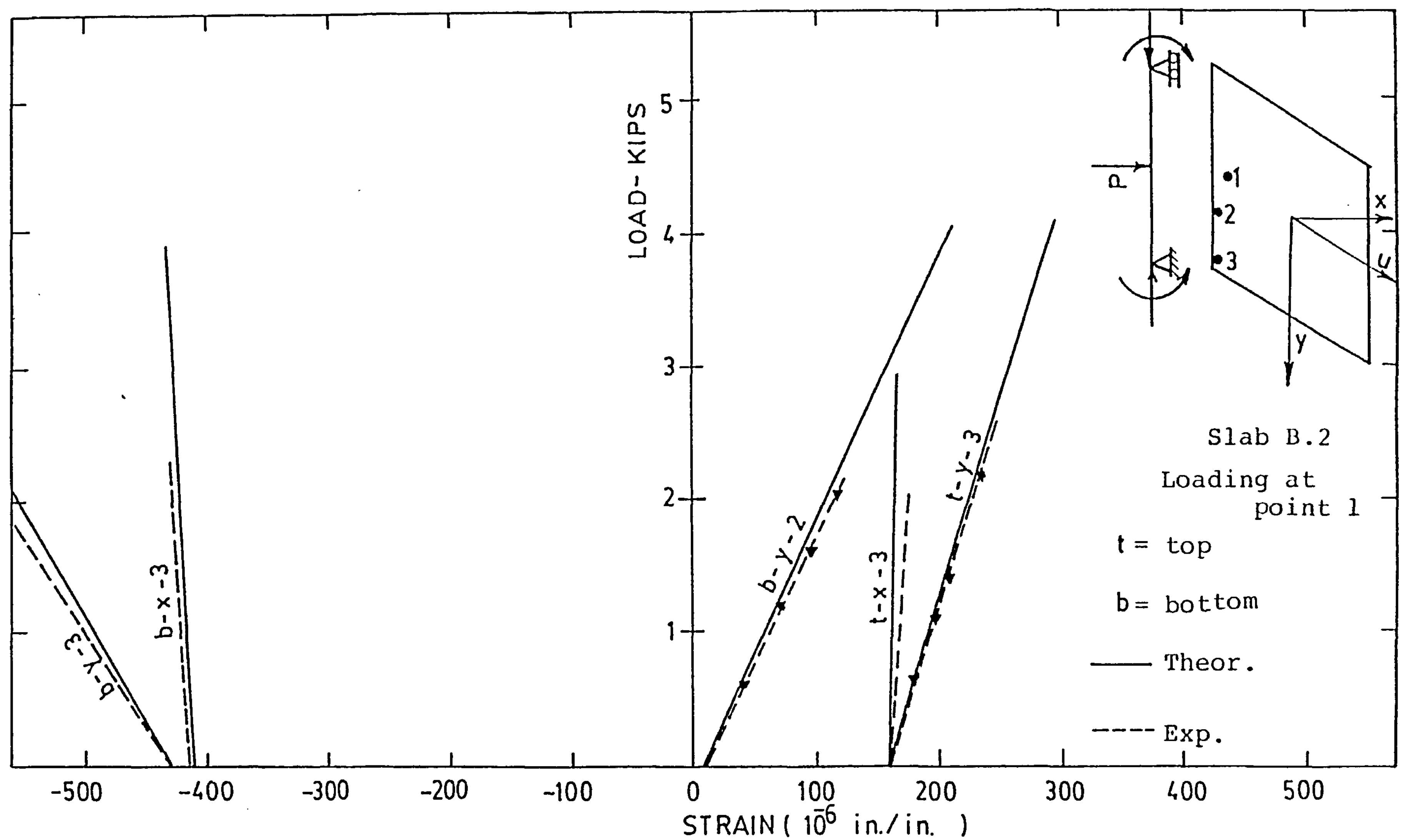


FIGURE 7.46 LOAD STRAIN RELATIONSHIP FOR SLAB B.2.

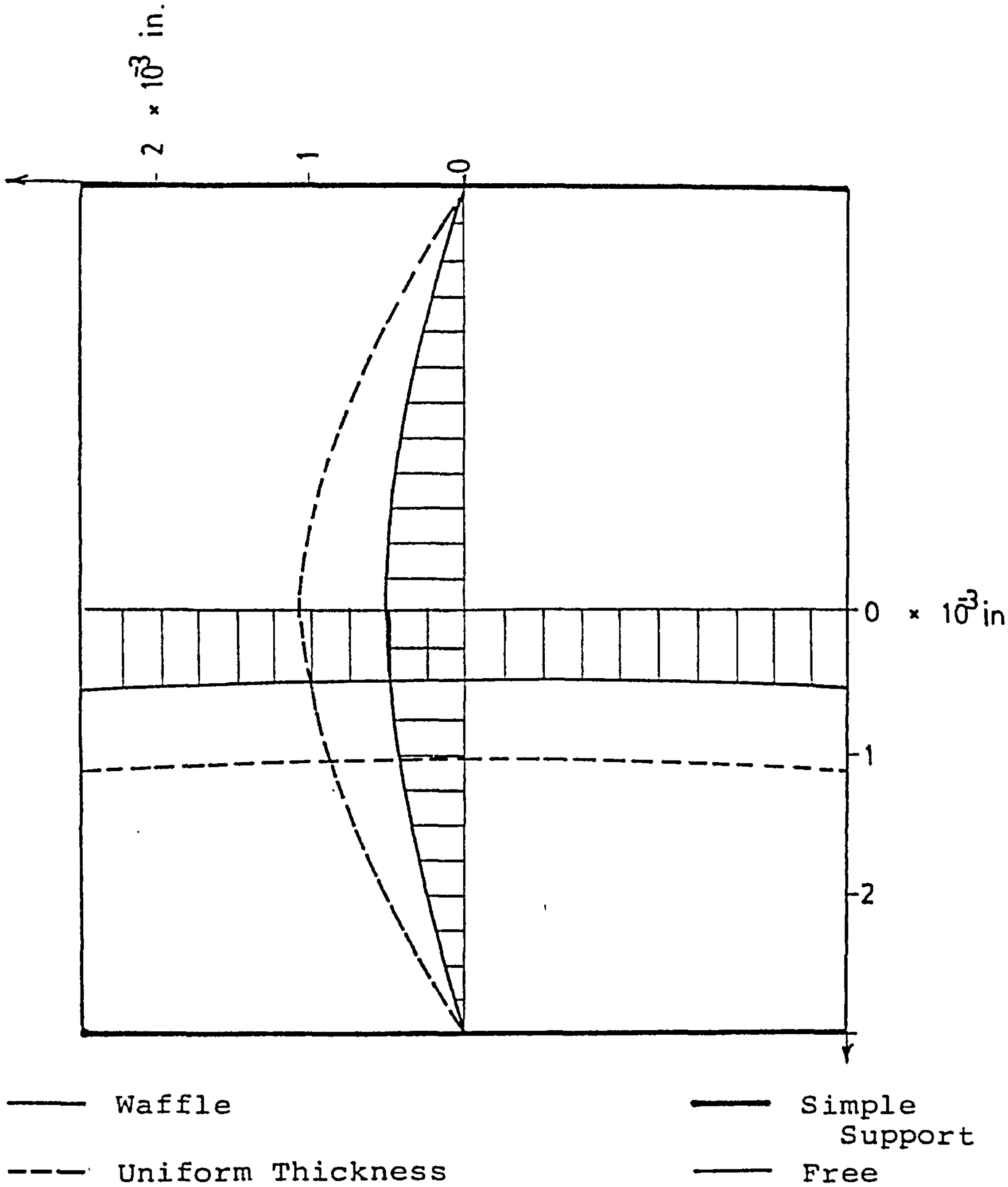
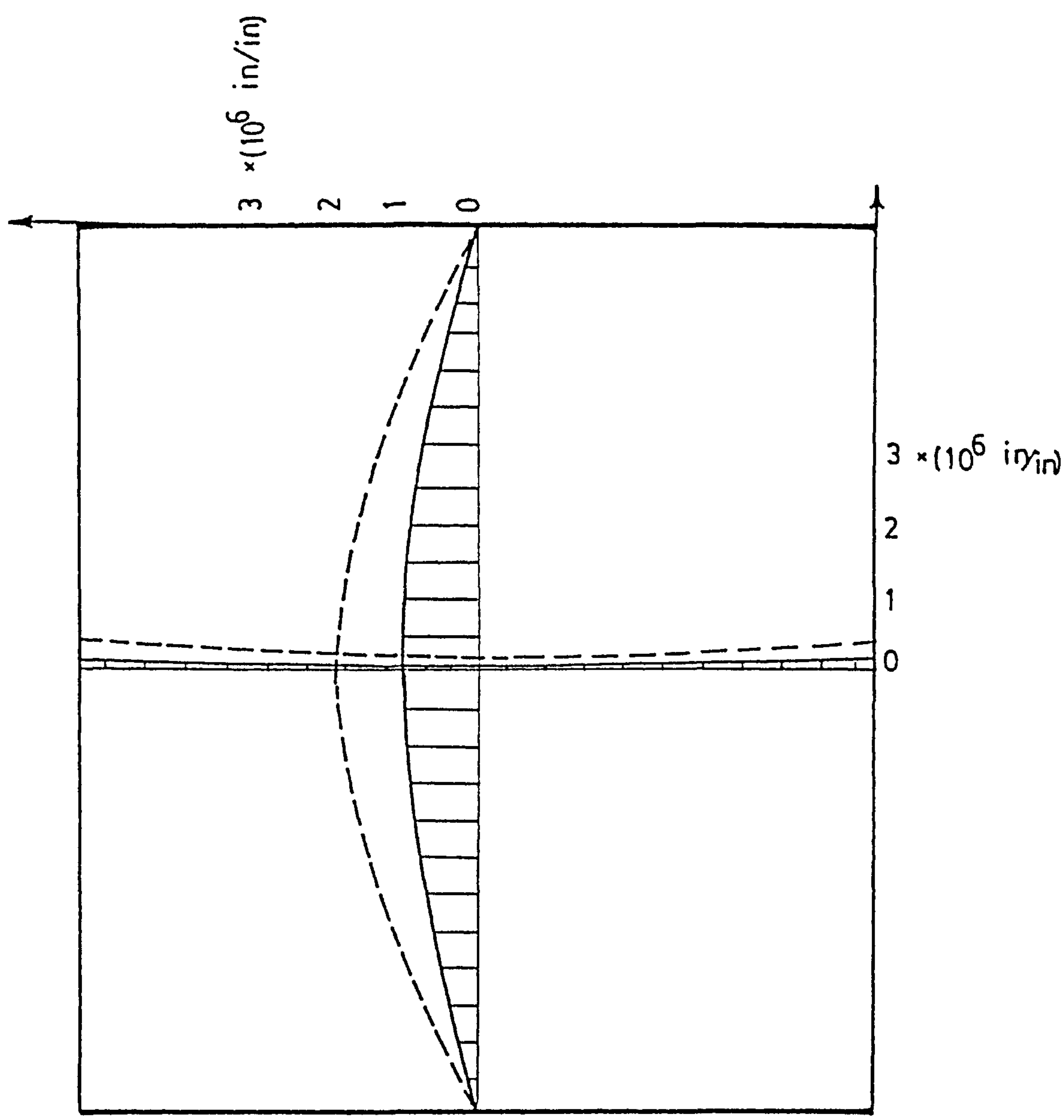


FIGURE 7.47 DEFLECTION PATTERN FOR RECTANGULAR SLAB DUE TO UNIFORM LOAD OF .01 lb/in^2.



—— Waffle

- - - Uniform Thickness

━━ Simple Support

—— Free

FIGURE 7.48 STRAIN RATTERN AT TOP FIBRE DUE TO UNIFORM LOAD OF .01 lb/in^2.

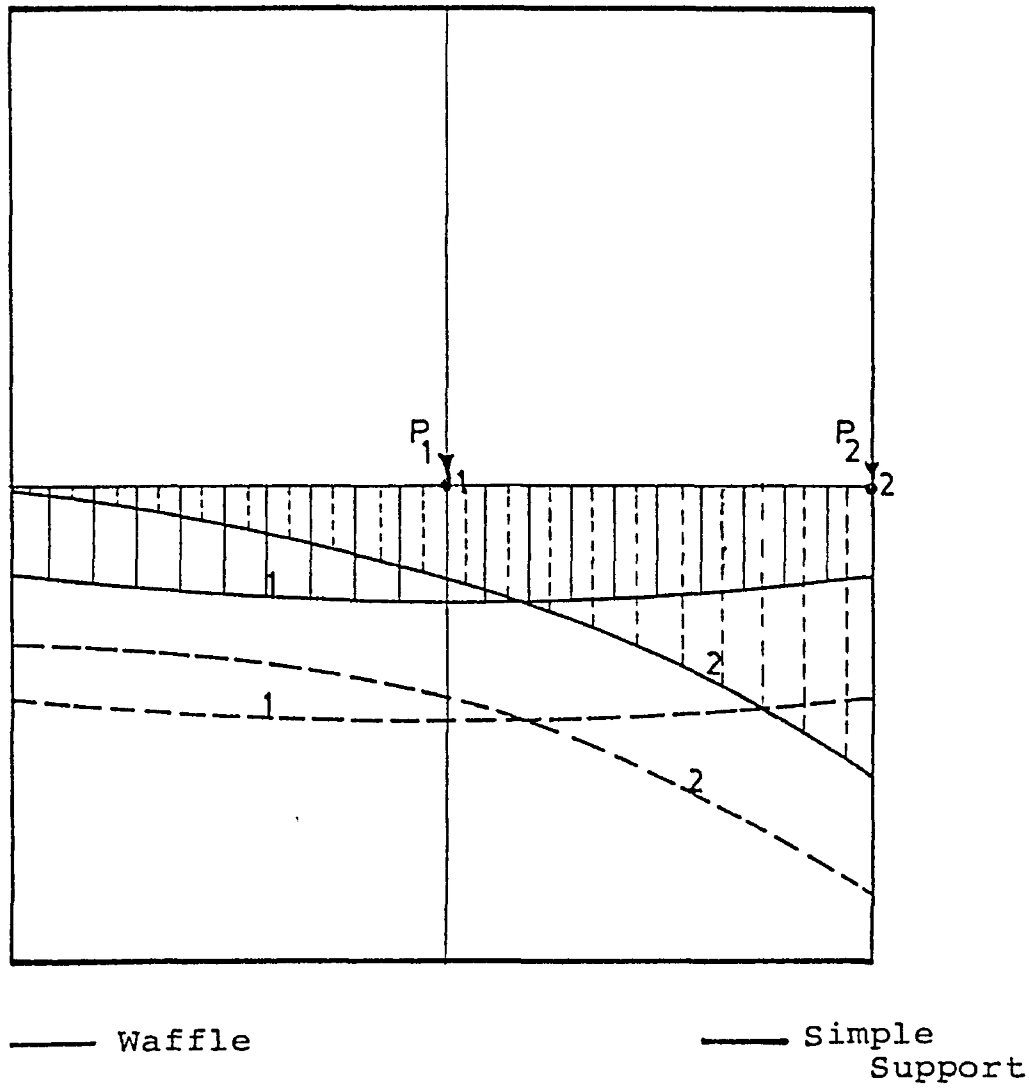
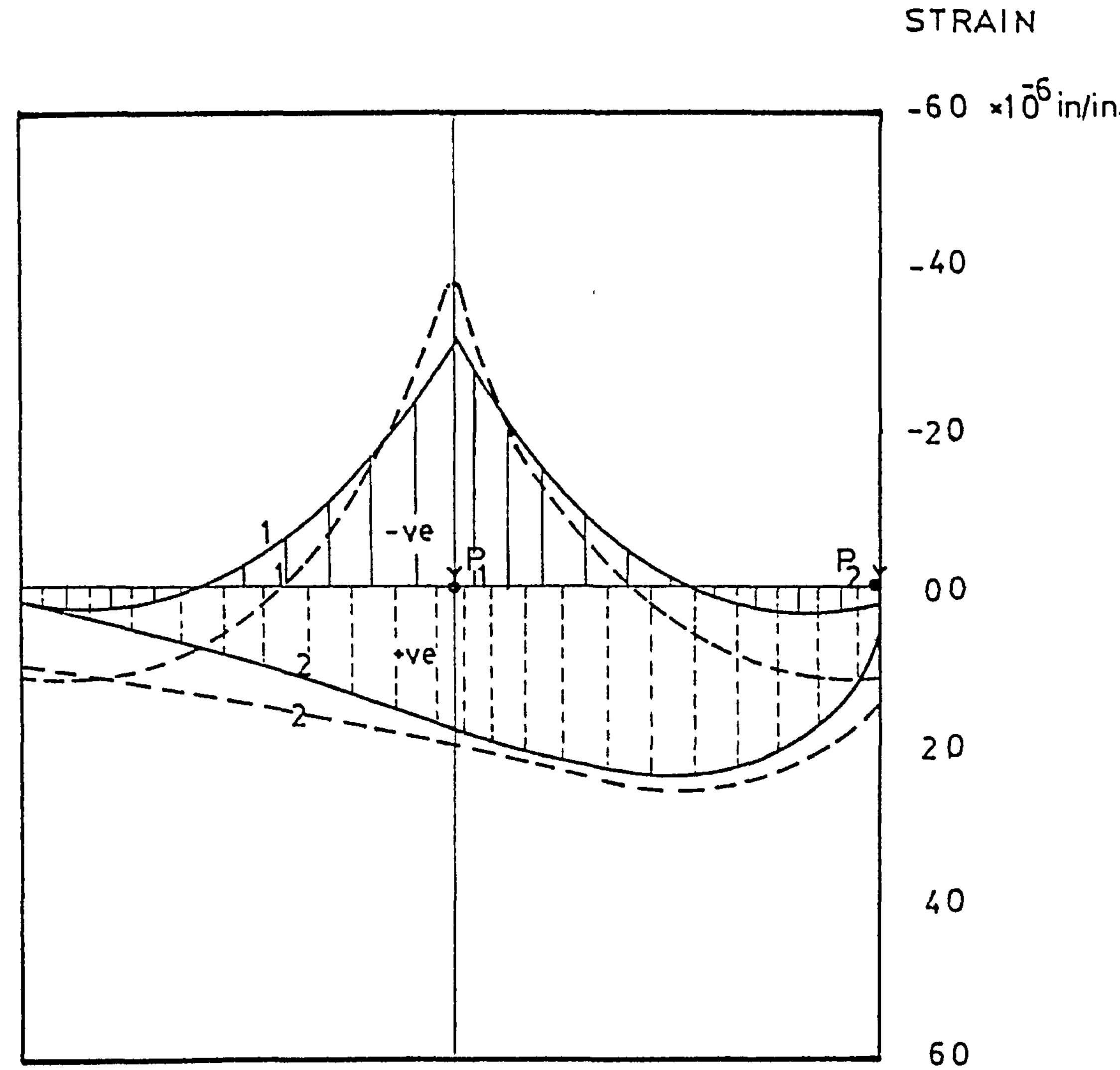


FIGURE 7.49 VARIATION OF DEFLECTIONS ALONG THE TRANSVERSE DIRECTION FOR POINT LOAD AT POINT 1 AND 2.



—— Waffle

---- Uniform Thickness

▬▬ Simple Support

—— Free

FIGURE 7.50 VARIATION OF STRAINS ALONG THE TRANSVERSE AXIS AT TOP FIBRE DUE TO CONCENTRATED LOAD 1 KIP AT POINT 1 AND 2.

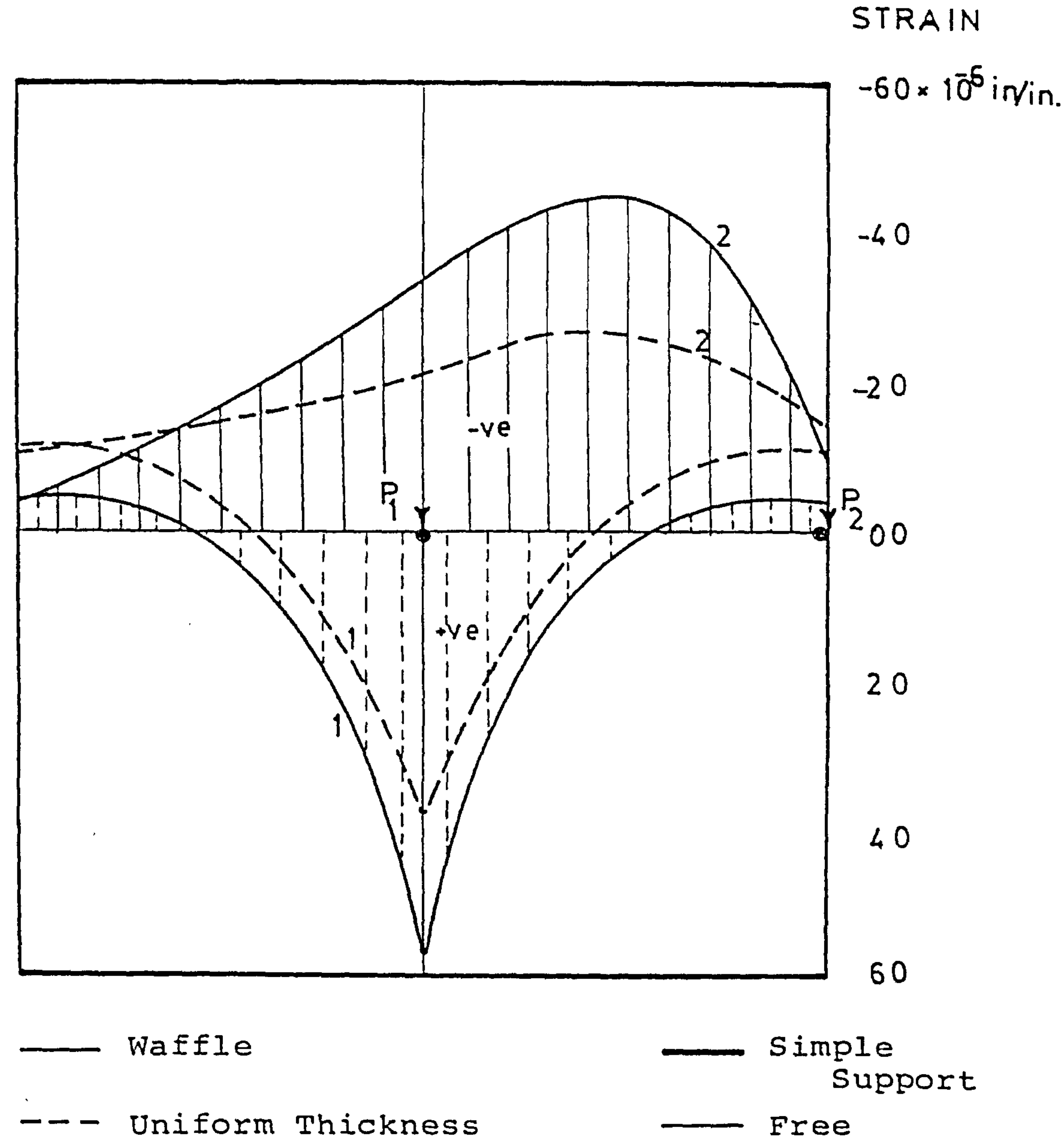


FIGURE 7.51 VARIATION OF STRAINS ALONG THE TRANSVERSE AXIS AT BOTTOM FIBRE DUE TO CONCENTRATED LOAD 1 KIP.

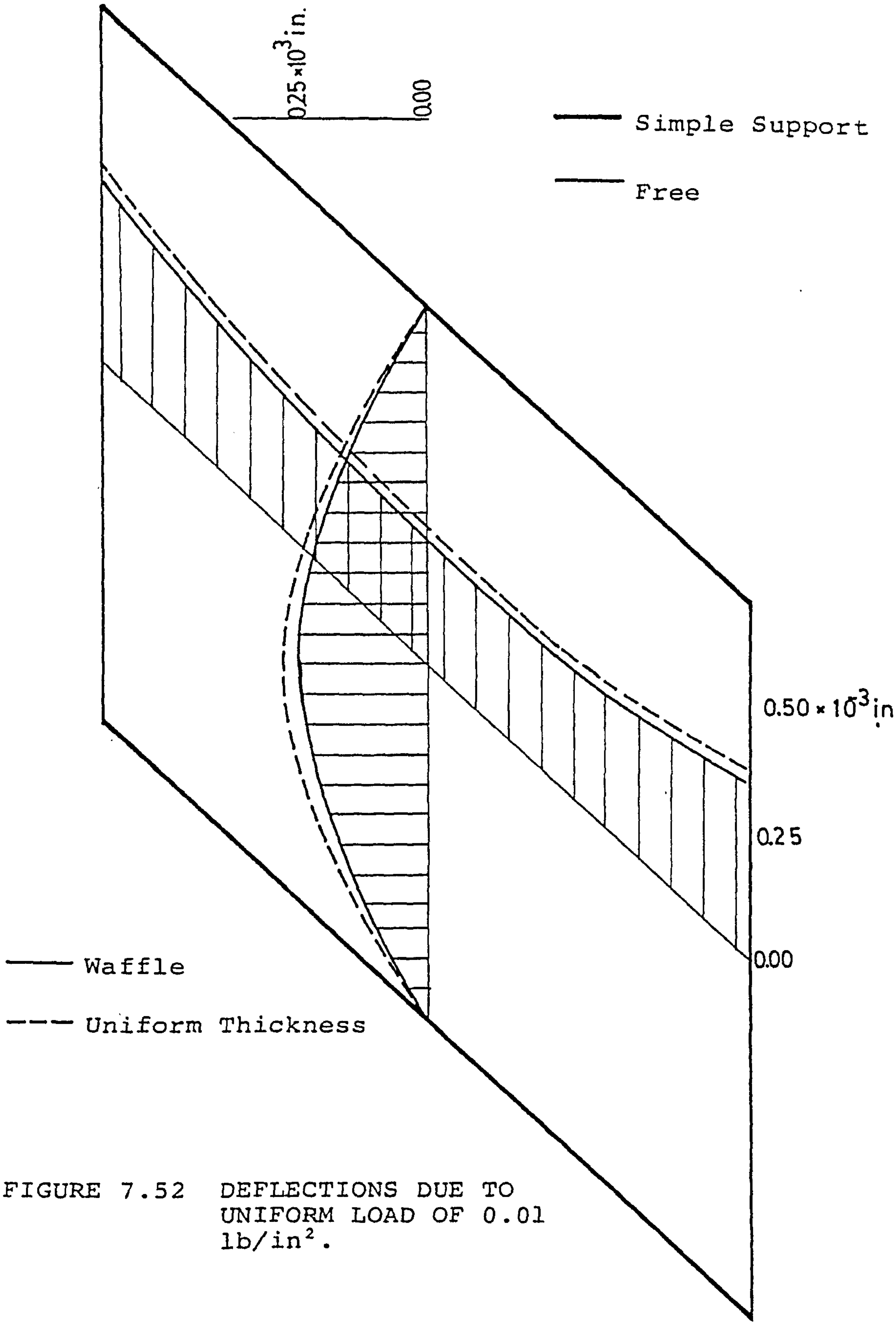


FIGURE 7.52 DEFLECTIONS DUE TO UNIFORM LOAD OF 0.01 lb/in^2.

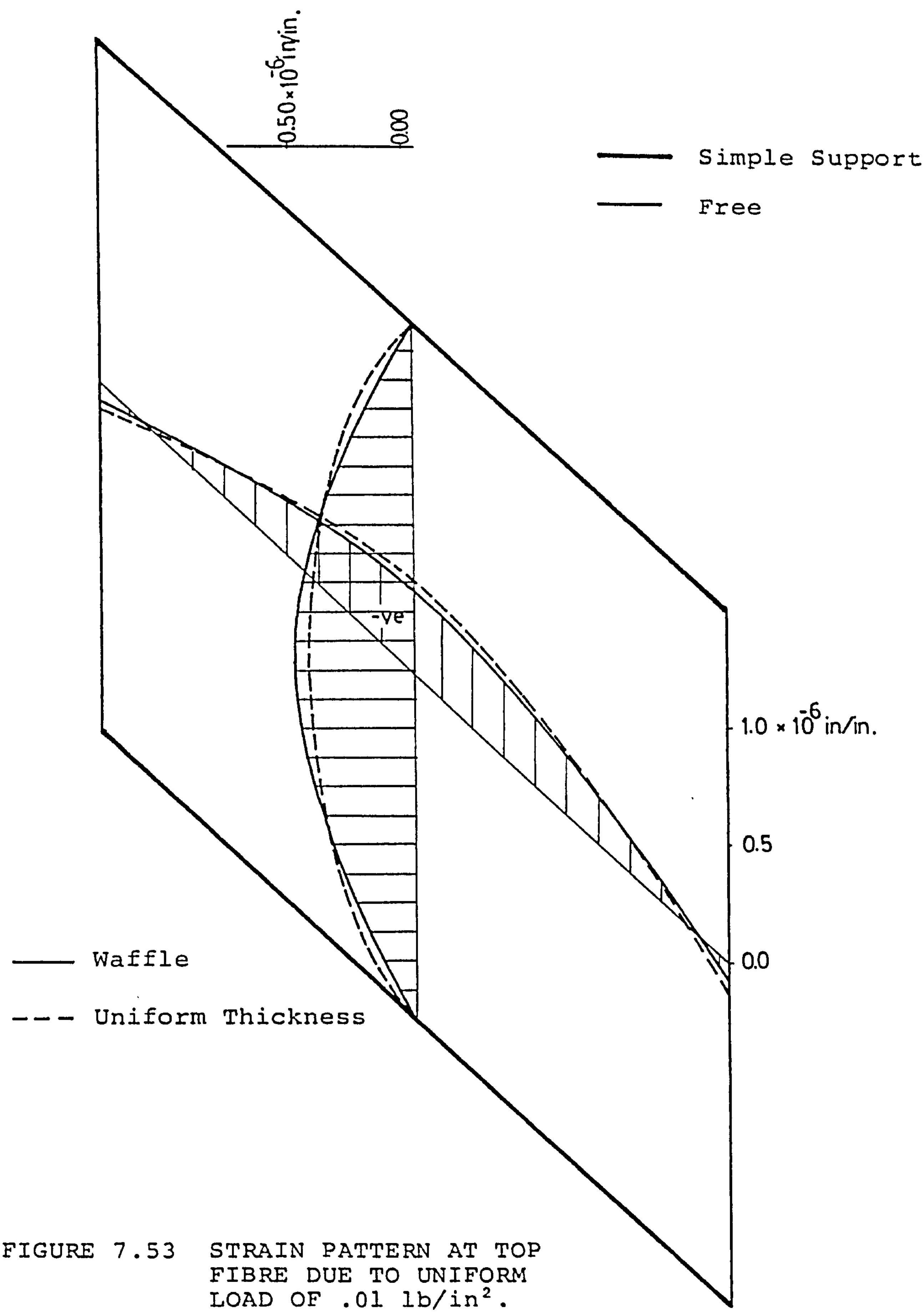


FIGURE 7.53 STRAIN PATTERN AT TOP FIBRE DUE TO UNIFORM LOAD OF .01 lb/in².

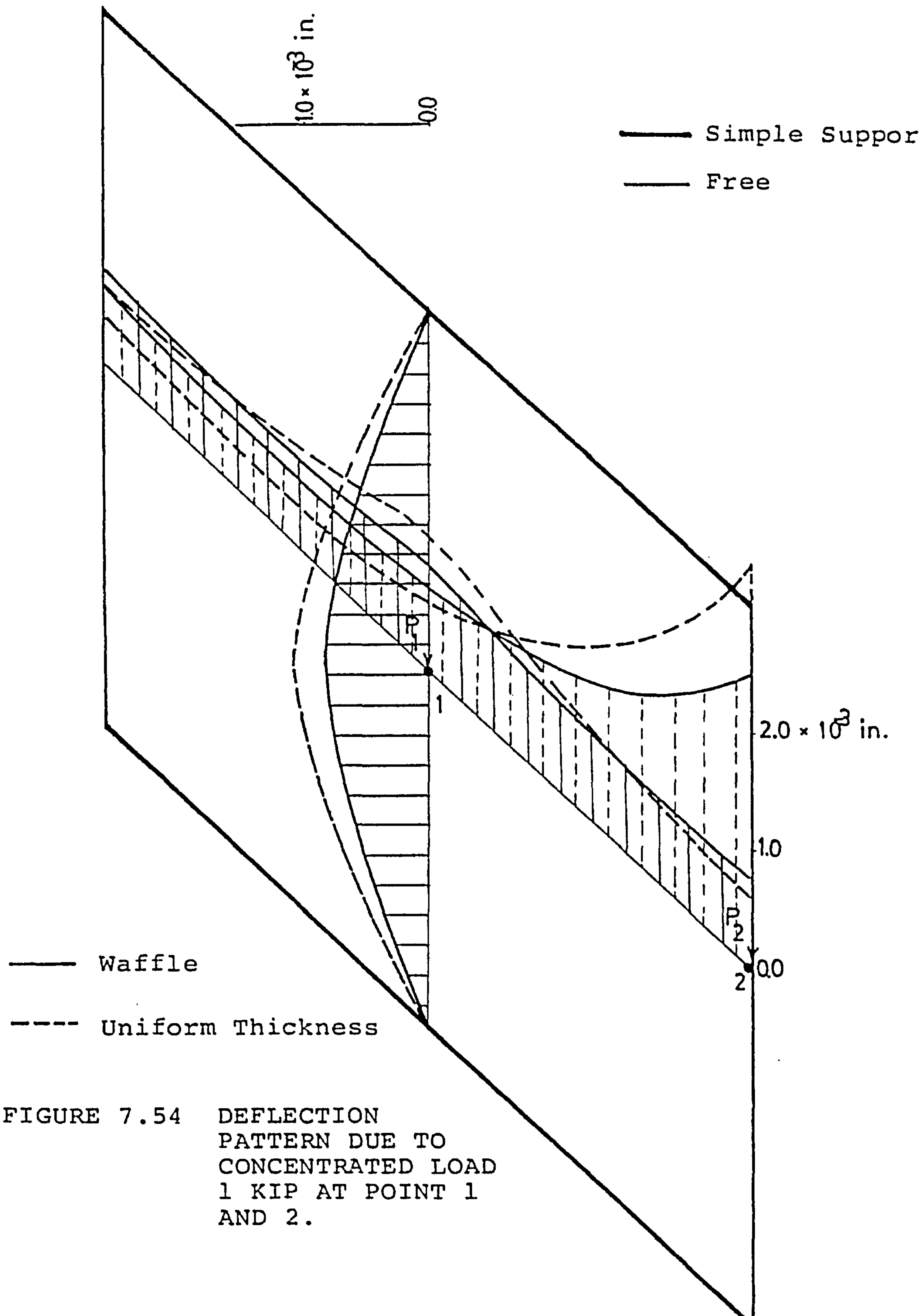


FIGURE 7.54 DEFLECTION PATTERN DUE TO CONCENTRATED LOAD 1 KIP AT POINT 1 AND 2.

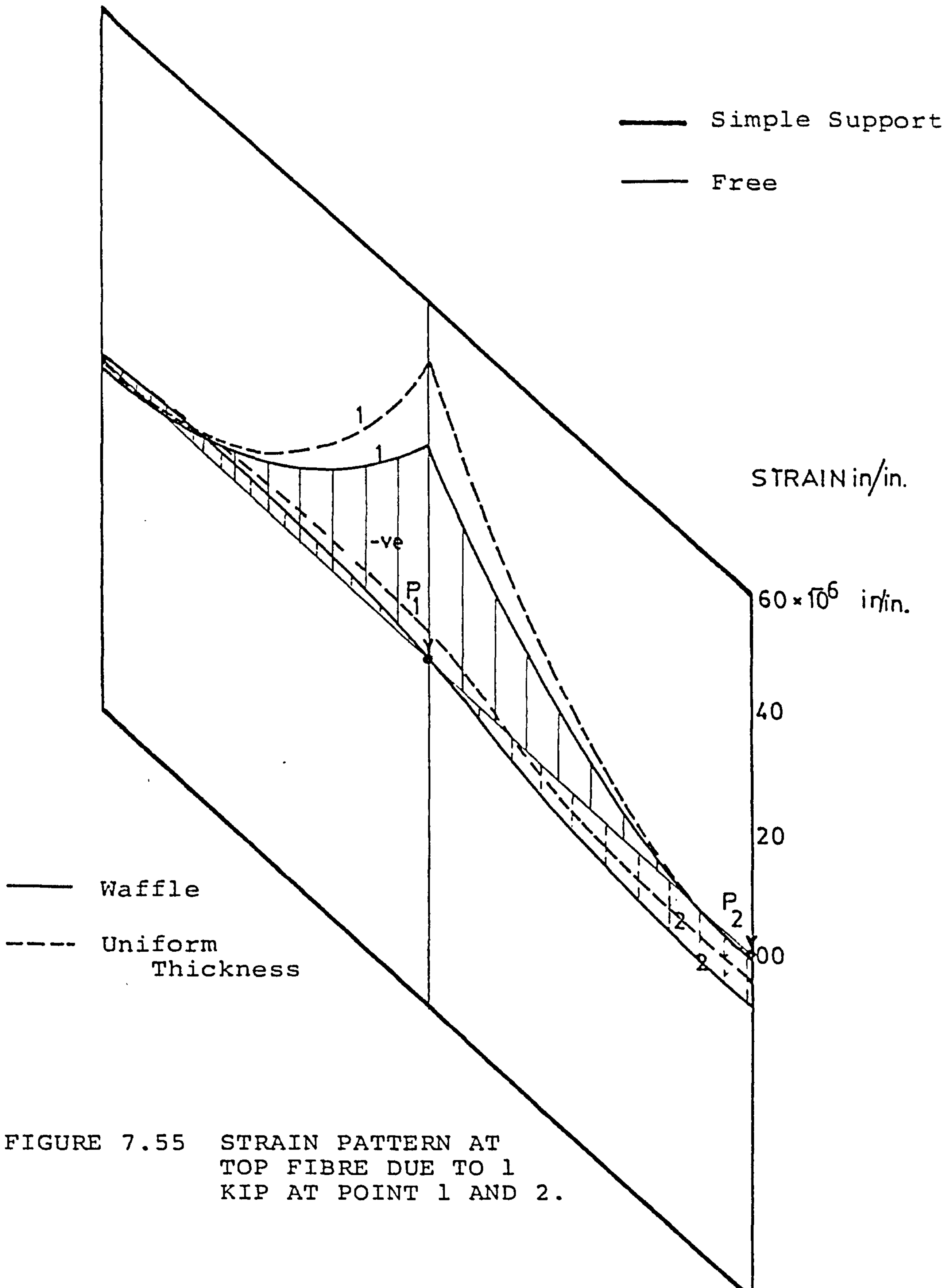
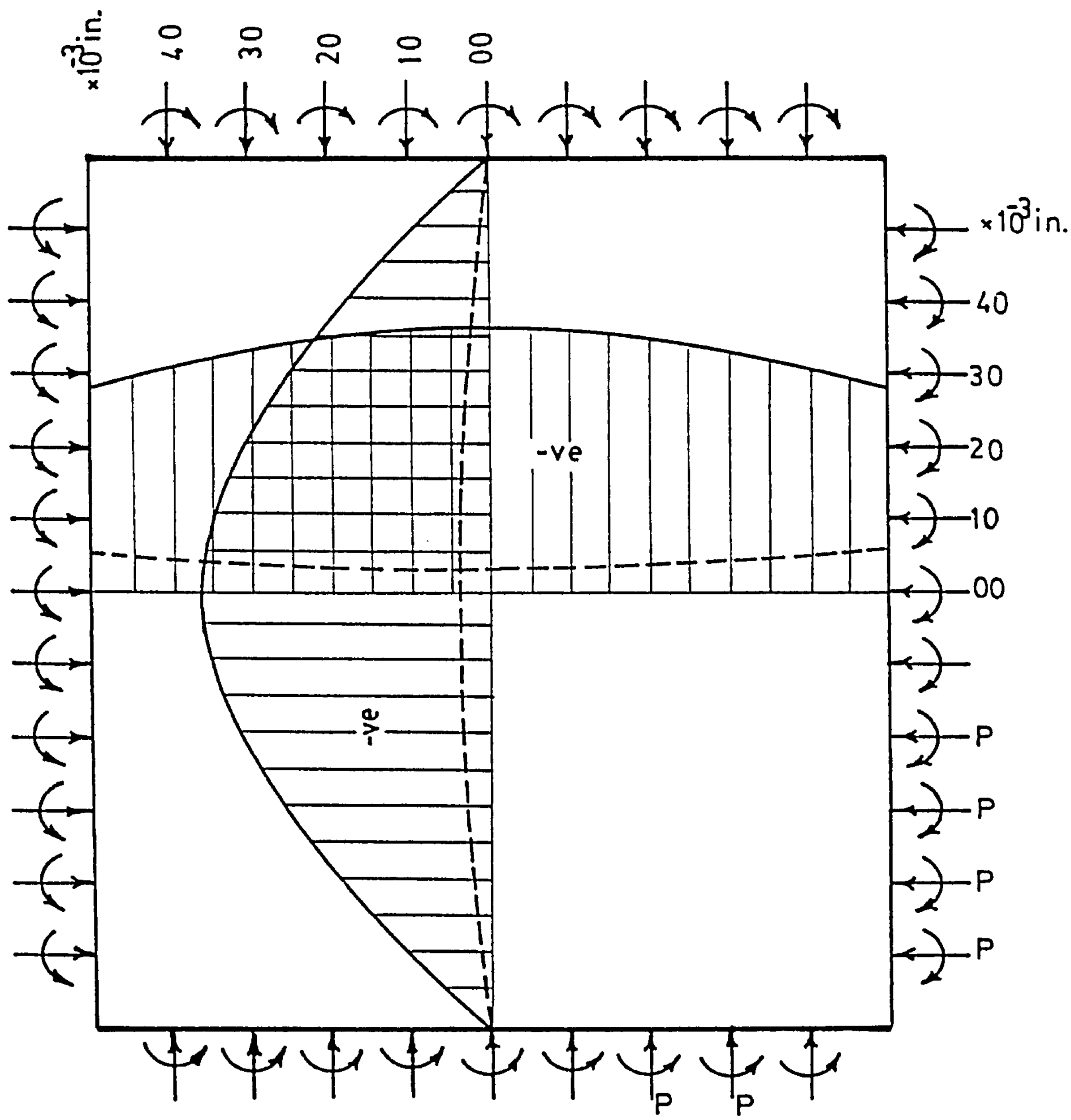


FIGURE 7.55 STRAIN PATTERN AT TOP FIBRE DUE TO 1 KIP AT POINT 1 AND 2.



—— Waffle

---- Uniform Thickness

▬▬ Simple Support

—— Free

FIGURE 7.56 DEFLECTION (CAMBER) DUE TO PRESTRESSING FORCE.

APPENDICES

APPENDIX (A)

Expressions For Matrix Elements

$A_1 = 2k_3k_4$

$A_2 = k_3^2 - k_4^2$

$A_3 = k_3(k_3^2 - 3k_4^2)$

$A_4 = k_4(3k_3^2 - k_4^2)$

$A_5 = -EI(A_2^2 - A_1^2)x_2^4$

$A_6 = 2EIx_2^4A_1A_2$

$A_7 = -R_1k_4 - R_2$

$A_8 = -x_2(R_1k_4 - R_2x_2A_2)$

$A_9 = -x_2(R_1k_3 + R_2x_2A_1)$

$A_{10} = R_3A_3 - R_4A_1 - R_5k_3$

$A_{11} = -R_3A_4 - R_4A_2 + R_5k_4 + R_6$

$A_{12} = -x_2(R_4k_3 + R_5x_2A_1 - R_6x_2^2A_3)$

$A_{13} = R_3 + R_4x_2k_4 - R_5x_2^2A_2 - R_6x_2^3A_4)$

$A_{14} = R_7A_2 - R_8k_4 - R_9$

$A_{15} = -R_7A_1 - R_8k_3$

$A_{16} = GJk_4 + R_{10}$

$A_{17} = -R_7 - R_8x_2k_4 + R_9x_2^2A_2)$

$A_{18} = -x_2(R_8k_3 + R_9x_2A_1)$

$A_{19} = -x_2^2(GJA_1 - R_{10}x_2A_3)$

$$A_{20} = -x_2^2(GJA_2 + R_{10}x_2A_4)$$

$$A_{21} = -T_nk_3$$

$$A_{22} = T_nk_4 - 1/c$$

$$A_{23} = x_2k_3/c$$

$$A_{24} = T_n - x_2k_4/c$$

$$A_{25} = k_3/c$$

$$A_{26} = T_n - k_4/c$$

$$A_{27} = -T_nk_3x_2$$

$$A_{28} = -1/c + T_nk_4x_2$$

$$B_1 = 2k_3k_5$$

$$B_2 = k_3^2 - k_5^2$$

$$B_3 = k_3(k_3^2 - 3k_5^2)$$

$$B_4 = k_5(3k_3^2 - k_5^2)$$

$$B_5 = -EIY_2^4(B_2^2 - B_1^2)$$

$$B_6 = 2EIY_2^4B_1B_2$$

$$B_7 = R_1k_5 - R_2$$

$$B_8 = Y_2(R_1k_5 + R_2Y_2B_2)$$

$$B_9 = Y_2(R_1k_3 - R_2Y_2B_1)$$

$$B_{10} = R_3B_3 + R_4B_1 - R_5k_3$$

$$B_{11} = -R_3B_4 + R_4B_2 + R_5k_5 - R_6$$

$$B_{12} = -Y_2(R_4k_3 - R_5Y_2B_1 - R_6Y_2^2B_3)$$

$$B_{13} = -R_3 + R_4Y_2k_5 + R_5Y_2^2B_2 - R_6Y_2^3B_4$$

$$B_{14} = R_7B_2 + R_8k_5 - R_9$$

$$B_{15} = -R_7B_1 + R_8k_3$$

$$B_{16} = GJk_5 - R_{10}$$

$$B_{17} = -R_7 + R_8y_2k_5 + R_9y_2^2B_2$$

$$B_{18} = y_2(R_8k_3 - R_9y_2B_1)$$

$$B_{19} = y_2^2(GJB_1 + R_{10}y_2B_3)$$

$$B_{20} = -y_2^2(GJB_2 - R_{10}y_2B_4)$$

$$B_{22} = T_nk_5 + 1/c$$

$$B_{23} = k_3y_2/c$$

$$B_{24} = -T_n - k_5y_2/c$$

$$B_{26} = -T_n - k_5/c$$

$$B_{27} = -T_nk_3y_2$$

$$B_{28} = 1/c + T_nk_5y_2$$

$$d_{11} = -D_x/c^2$$

$$d_{12} = 2D_xs/c^2$$

$$d_{13} = -(D_xs^2/c^2 + D_1)$$

$$d_{21} = -D_2/c^2$$

$$d_{22} = 2D_2s/c^2$$

$$d_{23} = -(D_2s^2/c^2 + D_y)$$

$$d_{31} = 0$$

$$d_{32} = D_{xy}/c$$

$d_{33} = -D_{xy}s/c$

$K_{1n} = \cosh R_{1n} \cos R_{r1}$

$K_{2n} = \sinh R_{1n} \cos R_{r1}$

$K_{3n} = \sinh R_{1n} \cos R_{r1}$

$K_{4n} = \cosh R_{1n} \sin R_{r1}$

$K_{5n} = \cosh R_{2n} \cos R_{r2}$

$K_{6n} = \sinh R_{2n} \sin R_{r2}$

$K_{7n} = \sinh R_{2n} \cos R_{r2}$

$K_{8n} = \cosh R_{2n} \sin R_{r2}$

$L_{1n} = \cosh S_{1n} \cos S_{s1}$

$L_{2n} = \sinh S_{1n} \sin S_{s1}$

$L_{3n} = \sinh S_{1n} \cos S_{s1}$

$L_{4n} = \cosh S_{1n} \sin S_{s1}$

$L_{5n} = \cosh R_{2n} \cos S_{s2}$

$L_{6n} = \sinh R_{2n} \sin S_{s2}$

$L_{7n} = \sinh R_{2n} \cos S_{s2}$

$L_{8n} = \cosh R_{2n} \sin S_{s2}$

$R_{r1} = x_2 k_4 \alpha_n b$

$R_{r2} = k_4 \beta_n a$

$R_{1n} = x_2 k_3 \alpha_n b$

$R_{2n} = k_3 \beta_n a$

$$S_{aa} = 2/(\beta_n^2 a^2(\beta_n^2 a^2/x_2^2 + 2A_2 m^2\pi^2) + m^4\pi^4)$$

$$S_{a1} = S_{aa} m\pi(-1)^m$$

$$S_{a2} = S_{aa}\beta_n a(-1)^m$$

$$S_{ba} = 2/(\beta_n^2 a^2(\beta_n a^2/y_2^2 + 2B_2 m^2\pi^2) + m^4\pi^4)$$

$$S_{b1} = S_{ba} m\pi(-1)^m$$

$$S_{b2} = S_{ba}\beta_n a(-1)^m$$

$$S_{s1} = y_2 k_5 \alpha_n b$$

$$S_{s2} = k_5 \beta_n a$$

$$S_{xa} = 2/(x_2^2\alpha_n^2 b^2(\alpha_n^2 b^2 + 2A_2 m^2\pi^2) + m^4\pi^4)$$

$$S_{x1} = S_{xa} m\pi(-1)^m$$

$$S_{x2} = S_{xa}\alpha_n b(-1)^m$$

$$S_{ya} = 2/(y_2^2\alpha_n^2 b^2(\alpha_n^2 b^2 + 2B_2 m^2\pi^2) + m^4\pi^4)$$

$$S_{y1} = S_{ya} m\pi(-1)^m$$

$$S_{y2} = S_{ya}\alpha_n b(-1)^m$$

$$S_{1n} = y_2 k_3 \alpha_n b$$

$$T_{a1} = A_1\beta_n^2 a^2$$

$$T_{a2} = A_2\beta_n^2 a^2 + m^2\pi^2$$

$$T_{a3} = k_3(\beta_n^2 a^2/x_2 + m^2\pi^2)$$

$$T_{a4} = k_4(\beta_n^2 a^2/x_2 - m^2\pi^2)$$

$$T_{b1} = B_1\beta_n^2 a^2$$

$$T_{b2} = B_2\beta_n^2 a^2/y_2 + m^2\pi^2$$

$$T_{b3} = k_3(\beta_n^2 a^2/y_2 + m^2\pi^2)$$

$$T_{b4} = k_5(\beta_n^2 a^2/y_2 - m^2\pi^2)$$

$$T_{x1} = A_1\alpha_n^2 b^2 x_2^2$$

$$T_{x2} = A_2\alpha_n^2 b^2 x_2^2 + m^2\pi^2$$

$$T_{x3} = x_2 k_3(x_2\alpha_n^2 b^2 + m^2\pi^2)$$

$$T_{x4} = x_2 k_4(x_2\alpha_n^2 b^2 - m^2\pi^2)$$

$$T_{y1} = B_1\alpha_n^2 b^2 y_2^2$$

$$T_{y2} = B_2 y_2^2\alpha_n^2 b^2 + m^2\pi^2$$

$$T_{y3} = k_3 y_2(y_2\alpha_n^2 b^2 + m^2\pi^2)$$

$$T_{y4} = y_2 k_5(y_2\alpha_n^2 b^2 - m^2\pi^2)$$

$$u_{3n} = k_3\beta_n u$$

$$u_{4n} = k_4\beta_n u$$

$$u_{5n} = k_5\beta_n u$$

$$v_{3n} = x_2 k_3\alpha_n v$$

$$\bar{v}_{3n} = y_2 k_3\alpha_n v$$

$$v_{4n} = x_2 k_4\alpha_n v$$

$$v_{5n} = y_2 k_5\alpha_n v$$

APPENDIX (B)

Fourier Coefficients For Lateral Load

The Fourier coefficients of Equation 5.2 are evaluated below:

$$a_{oo} = (1/2a)\int_{-a}^{a} f_o(u)\,du$$

$$= (1/2a)\int_{-a}^{a}\left\{C_{17} + C_{20}(u^2/a^2) + C_{21} + C_{24}\ (Tu^4/b^4 - 1)\right\}du$$

$$= C_{17} + 1/3C_{20} + C_{21} + C_{24}(Ta^4/5b^4 - 1)$$

$$a_{om} = (1/a)\int_{-a}^{a} f_o(u)\ \cos\alpha_m u\ du$$

$$= C_{20}I_{1m} + C_{24}T\ I_{3m}a^4/b^4$$

The integrals I_{1m}, I_{3m}, etc. are defined at the end of this appendix

$$b_{om} = (1/a)\int_{-a}^{a} f_o(u)\ \sin\alpha_m u\ du = 0$$

$$c_{no} = (1/2a)\int_{-a}^{a} f_n(u)\ du$$

$$= (C_{1n}W_{1n} + C_{4n}W_{2n} + C_{5n}W_{3n} + C_{8n}W_{4n})(-1)^n$$

$$c_{nm} = (1/a)\int_{-a}^{a} f_n(u)\ \cos\alpha_m u$$

$$= (C_{1n}A_{j1} + C_{4n}A_{j2} + C_{5n}A_{j3} + C_{8n}A_{j4})(-1)^n$$

$$d_{nm} = (1/a)\int_{-a}^{a} f(n)\ \sin\alpha_m u\ du = 0$$

<u>Fourier Expansions and Definite Integrals</u>

$$I_{1n} = (1/a)\int_{-a}^{a} (u^2/a^2)\cos\alpha_n u\ du$$

$$= (1/b)\int_{-b}^{b} (v^2/b^2)\cos\beta_n v\ dv$$

$$= 4(-1)^n/n^2\pi^2 \quad \text{and so on.}$$

$$I_{3n} = 8(-1)^n(1 - 6/n^2\pi^2)/n^2\pi^2$$

$$I_{5n} = -2(-1)^n/n\pi$$

$$I_{7n} = -2(-1)^n(1 - 6/n^2\pi^2)/n\pi$$

$$W_{1n} = (1/2a)\int_{-a}^{a} \cosh\ u_{3n}\cos\ u_{4n}\ du$$

$$= x_2(K_3K_{7n} + K_4K_{8n})/a\ \beta_n$$

$$W_{2n} = x_2(K_3K_{8n} - K_4K_{7n})/a\beta_n$$

$$W_{3n} = y_2(K_3L_{7n} + K_5L_{8n})/a\beta_n$$

$$W_{4n} = y_2(K_3L_{8n} - K_5L_{7n})/a\beta_n$$

$$W_{5n} = (K_3K_{3n} + K_4K_{4n})/b\alpha_n$$

$$W_{6n} = (K_3K_{4n} - K_4K_{3n})/b\alpha_n$$

$$W_{7n} = (K_3L_{3n} + K_5L_{4n})/b\alpha_n$$

$$W_{8n} = (K_3L_{4n} - K_5L_{3n})/b\alpha_n$$

$$A_{j1} = (1/a)\int_{-a}^{a} \cosh u_{3n} \cos u_{4n} \cos\alpha_m u \, du$$

$$= S_{a2}(T_{a3}K_{7n} + T_{a4}K_{8n})$$

$$A_{j2} = (S_{a2}(-T_{a4}K_{7n} + T_{a3}K_{8n})$$

$$A_{j3} = S_{b2}(T_{b3}L_{7n} + T_{b4}L_{8n})$$

$$A_{j4} = S_{b2}(-T_{b4}L_{7n} + T_{b3}L_{8n})$$

$$A_{j5} = S_{x2}(T_{x3}K_{3n} + T_{x4}K_{4n})$$

$$A_{j6} = S_{x2}(-T_{x4}K_{3n} + T_{x3}K_{4n})$$

$$A_{j7} = S_{y2}(T_{y3}L_{3n} + T_{y4}L_{4n})$$

$$A_{j8} = S_{y2}(T_{y4}L_{3n} - T_{y3}L_{4n})$$

$$B_{j1} = (1/a)\int_{-a}^{a} \sinh u_{3n} \cos u_{4n} \sin\alpha_m u \, du$$

$$= S_{a1}(-T_{a2}K_{7n} - T_{a1}K_{8n})$$

$$B_{j2} = S_{a1}(T_{a1}K_{7n} - T_{a2}K_{8n})$$

$$B_{j3} = S_{b1}(-T_{b2}L_{7n} - T_{b1}L_{8n})$$

$$B_{j4} = S_{b1}(T_{b1}L_{7n} - T_{b2}L_{8n})$$

$$B_{j5} = S_{x1}(-T_{x2}K_{3n} - T_{x1}K_{4n})$$

$$B_{j6} = S_{x1}(T_{x1}K_{3n} - T_{x2}K_{4n})$$

$$B_{j7} = S_{y1}(-T_{y2}L_{3n} - T_{y1}L_{4n})$$

$$B_{j8} = S_{y1}(T_{y1}L_{3n} - T_{y2}L_{4n})$$

APPENDIX (C)

The Fourier series is given by

$$y = f(x) = a_o/2 + \sum_{n=1}^{n=\infty}(a_n \cos 2n\pi x/X + b_n \sin 2n\pi x/X)$$

where X is the total length of the cycle.

The coefficients for Fourier series are obtained by evaluating the following definite integrals (33);

$$a_o = (2/X)\int_{-X/2}^{X/2} f(x)\,dx \qquad (C.1)$$

$$a_n = (2/X)\int_{-X/2}^{X/2} f(x)\cos(2n\pi x/X)\,dx \qquad (C.2)$$

$$b_n = (2/X)\int_{-X/2}^{X/2} f(x)\sin(2n\pi x/X)\,dx \qquad (C.3)$$

From Figure 4.2,

The width of one division = dx = X/m

$$x = \{(X/m)(k - ½) - X/2\}$$

$$x/X = \{(1/m)(k - ½) - ½\}$$

Substituting in Equation (C.1)

$$a_o = (2/X)\sum_{k=1}^{m} y_k(X/m)$$

$$= (2/m)\sum_{k=1}^{m} y_k \qquad (C.4)$$

Substituting in Equation (C.2)

$$a_n = (2/X) \sum_{k=1}^{m} y_k \left\{ \cos 2n\pi \left(1/m \ (k - \tfrac{1}{2}) - \tfrac{1}{2} \right) \right\} X/m$$

$$= (2/m) \sum_{k=1}^{m} (-1)^m y_k \cos \left\{ (2n\pi/m)(k - \tfrac{1}{2}) - n\pi \right\}$$

$$a_n = (2/m) \sum_{k=1}^{m} (-1)^m y_k \cos (2n\pi/m)(k - \tfrac{1}{2}) \qquad \text{(C.5)}$$

Substituting in Equation (C.3) and following the same procedures the final equation is obtained,

$$b_n = (2/m) \sum_{k=1}^{m} (-1)^m y_k \sin (2n\pi/m)(k - \tfrac{1}{2}) \qquad \text{(C.6)}$$

APPENDIX (D)

Design of Concrete Mix

Five trial mixes were made to meet the following conditions: concrete is required for reinforced and prestressed concrete waffle slab deck bridges;

Concrete mix must have: 3 to 4 in. range in slump, suitably vibrated and adequately cured.

Cement content: Different cement contents range from 660 to 750 lbs. per cu. yd. and air content of 5% were used in the concrete mixes.

Strength: Two different concrete mixes for strength equal to 4000 and 6000 psi were required for the reinforced and prestressed concrete slab models, respectively.

Aggregates: Maximum size of coarse aggregate was chosen as ¼ in. with a specific gravity equal to 2.65.

Assumptions: According to reference (22, Table 10), the amount of water required per cubic yard was 300 lb. The percentage of fine aggregate to the total aggregate was assumed between 40 to 70% for the different trial mixes.

Quantities per cubic yard: From the information given, the absolute volume occupied by the paste was calculated as:

Cement, abs. vol.	= 750/(3.15 x 62.4)	= 3.81	cu. ft.
Water, abs. vol.	= 300/(1.0 x 62.4)	= 4.81	cu. ft.
Air, abs. vol.	= 5 x 0.27	= 1.35	cu. ft.
Paste, total vol.	= 3.81 + 4.81 + 1.35	= 9.97	cu. ft.
Aggregate, abs. vol.	= 27 - 9.97	= 17.03	cu. ft.

Coarse aggregate, wt. = 17.6 x .40 x 2.65 x 62.4 = 1126 lbs.

Fine aggregate, wt. = 17.6 x .60 x 2.65 x 62.4 = 1689 lbs.

The above quantities per cubic yard were approximate, and used as a preliminary trial batch.

For 10 lbs. of cement, a trial batch consisted of:

Cement	....	=	10.00 lbs.
Water	.... 265 x 10/(750)	=	4.00 lbs.
Fine aggregate	.... 1746 x 10/(750)	=	22.50 lbs.
Coarse aggregate	.... 1164 x 10/(750)	=	15.00 lbs.

Based on the workability and maximum strength of this trial mix, four trial mixes with different water/cement ratio and fine aggregate content were made using Figure 32 in reference (22). The following table shows the proportions of the five mixes and their strength after 7, 14 and 28 days.

TABLE (D.1)

Mix No.	Ratio by Weight in lbs.				Strength in psi		
	Cement	Water	Sand	Gravel	7 days	14 days	28 days
1	1.0	0.70	3.00	2.00	4000	4670	5090
2	1.0	0.60	2.50	1.80	5580	5870	6270
3	1.0	0.50	2.00	1.50	6720	6580	7710
4	1.0	0.40	1.40	1.20	7045	7965	8275
5	1.0	0.40	2.25	1.50	6600	7400	7900

The total volume of the slab and cylinders

$$= 10 \text{ ft}^3$$

and the total weight of the concrete mix required

$$= 10 \times 146 = 1460 \text{ lbs.}$$

which was divided into two batches. Mix No. 1 was chosen for the reinforced concrete waffle slabs, and Mix No. 2 for the prestressed concrete waffle slabs.

TABLE D.2

	$E_x = E_y$ $\times 10^6$ psi	Poisson's Ratio μ	Depth of Neutral axis inch		Eccentricity of Prestressing Wire	
			e_x	e_y	$\bar{e}_x$	$\bar{e}_y$
GROUP	5.00	0.200	1.420	1.425	0	0
A	4.65	0.230	1.405	1.410	0	0
SLABS	3.90	0.195	1.415	1.420	0	0
GROUP B	5.00	0.250	1.375	1.380	1.400	1.620
SLABS	5.20	0.260	1.370	1.375	1.400	1.620

GEOMETRY AND PROPERTIES OF SLAB MODELS

TABLE D.3

	Slab	fc28 psi	Span Inch	Width Inch	Skew Angle	D_x	D_y	D_1	D_2	D_{xy}	Edge Beam x10^6 EI	Edge Beam x10^6 GJ
GROUP A	A.1	5000	84	71.5	0	10.73	10.88	0.78	0.79	1.65	0	0
	A.2	6635	84	71.5	0	12.47	12.48	1.03	1.02	1.85	0	0
	A.3	4690	84	101.1	45	10.34	10.48	0.74	0.73	1.50	0	0
GROUP B	B.1	7665	84	71.5	0	12.91	12.91	1.13	1.14	1.95	60.00	34.30
	B.2	8285	84	71.5	45	13.35	13.40	1.22	1.22	2.05	62.70	35.35

D_x, D_y, D_1, D_2, & D_{xy} in lb. in^2/in x 10^6

ORTHOTROPIC RIGIDITIES OF SLAB MODELS

APPENDIX E

CALIBRATION OF LOAD CELLS

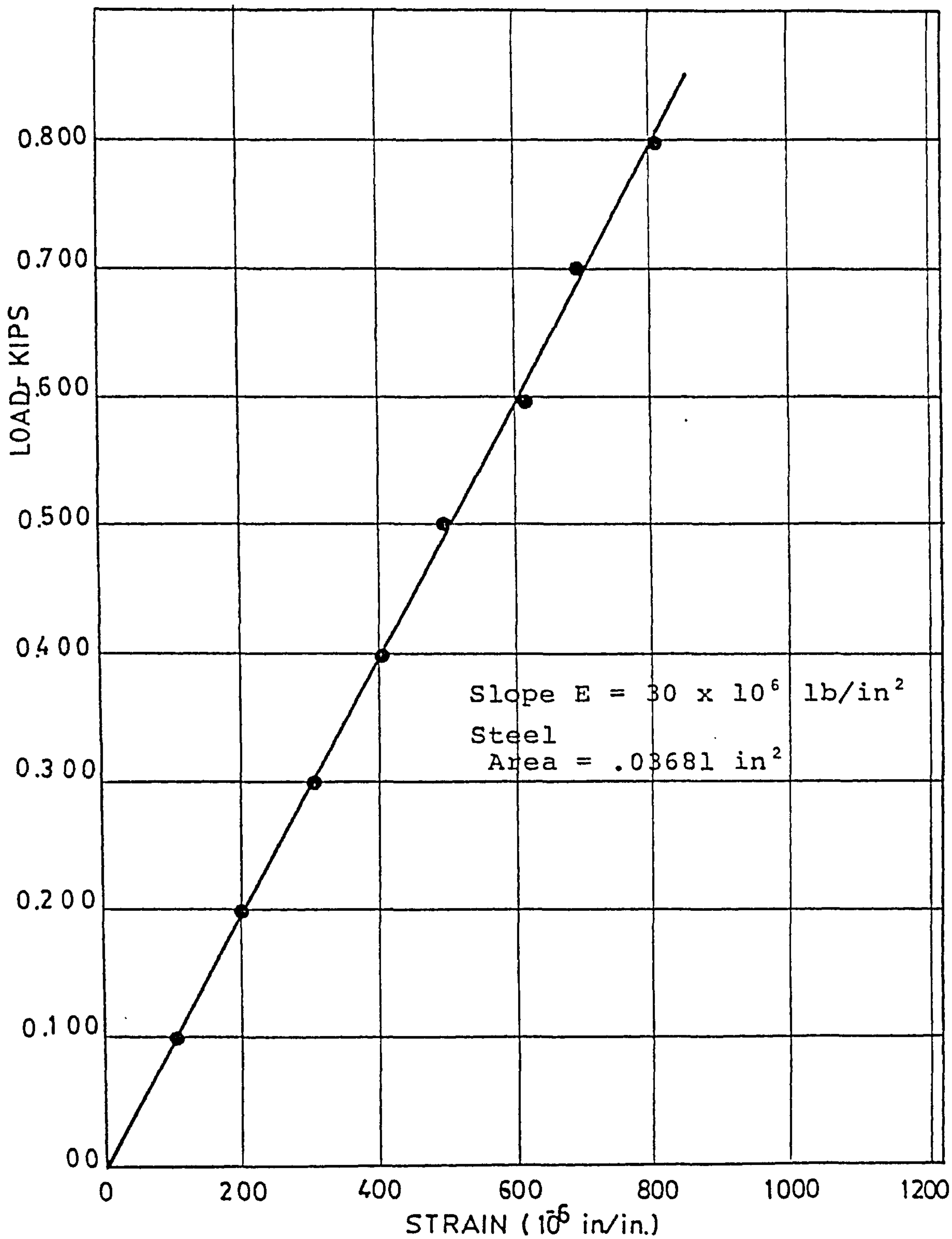


FIGURE (E1) CALIBRATION OF REINFORCING STEEL WIRES.

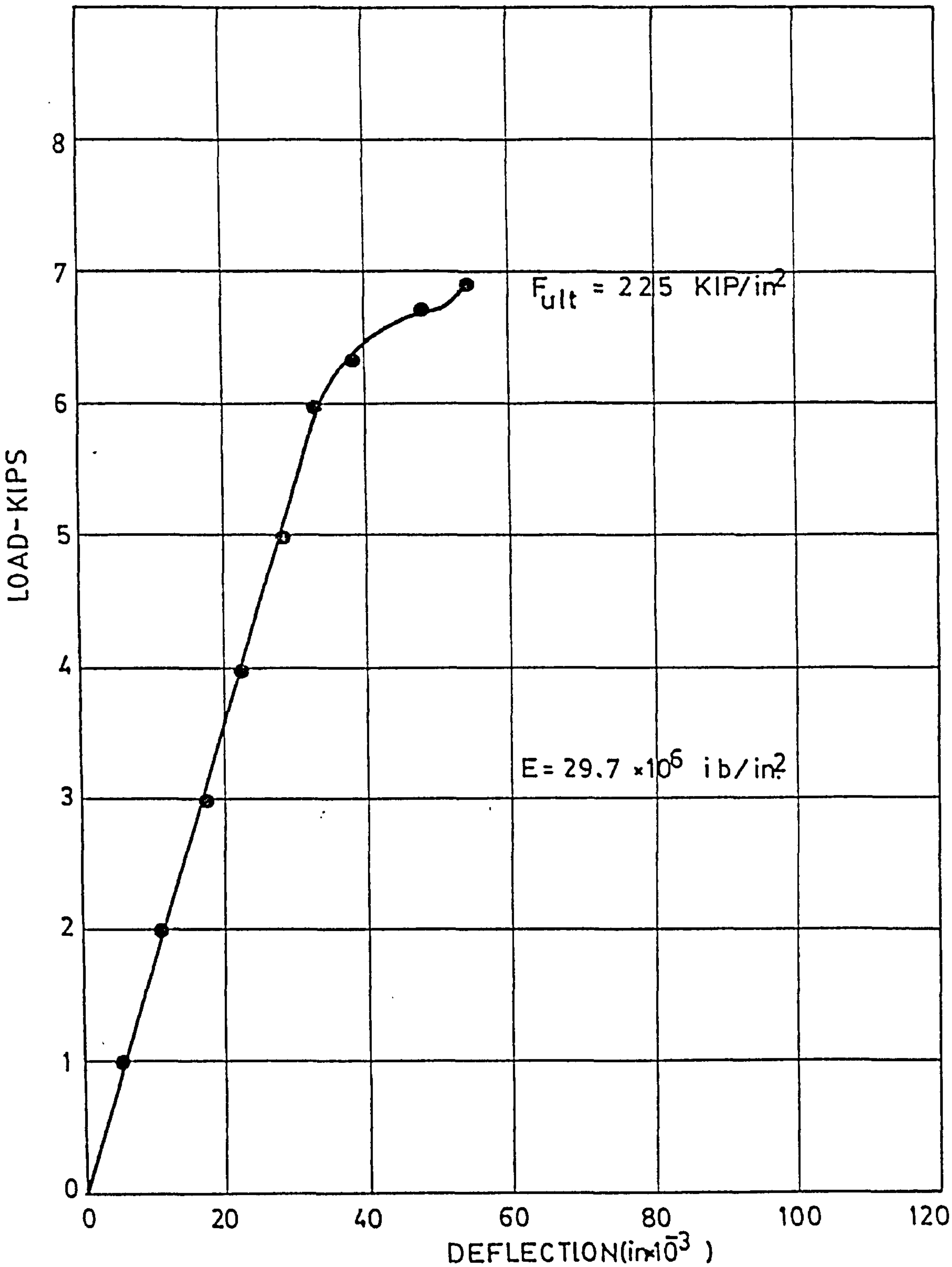


FIGURE (E2) LOAD DEFLECTION FOR HIGH TENSILE STEEL WIRE.

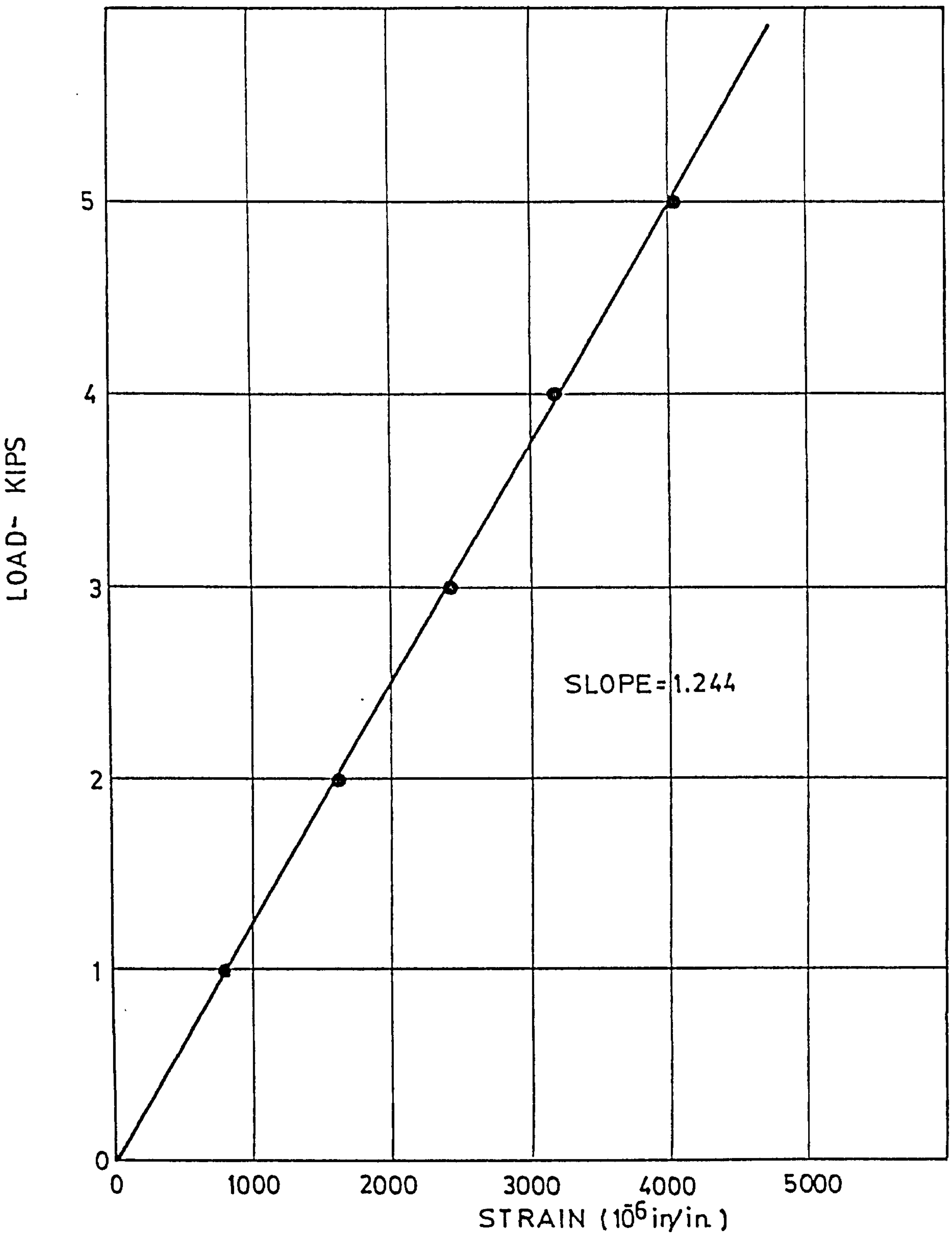


FIGURE (E3) LOAD STRAIN RELATIONSHIP (LOAD CELL 5 KIPS).

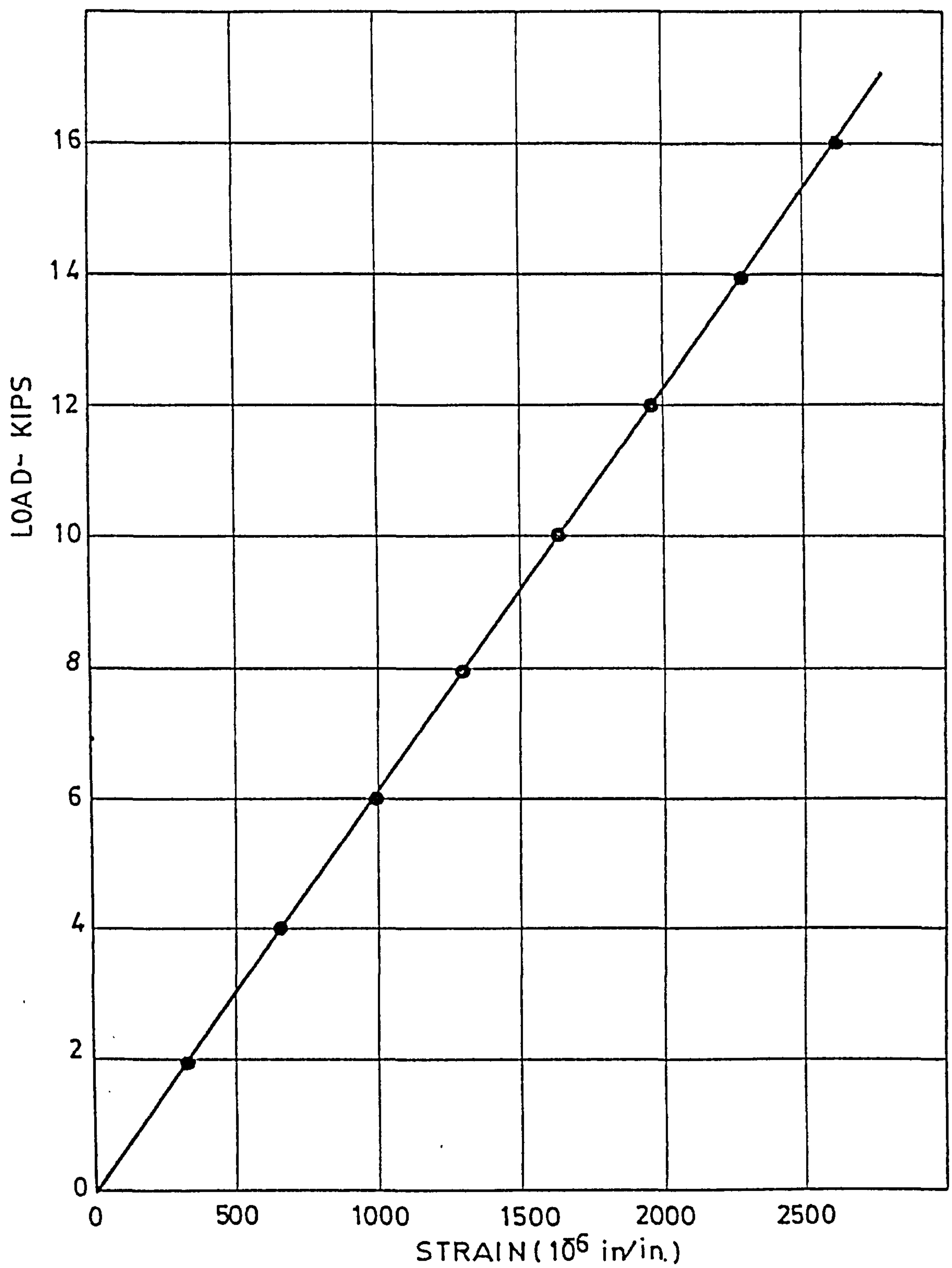


FIGURE (E4) LOAD STRAIN RELATIONSHIP (LOAD CELL 25 KIPS).

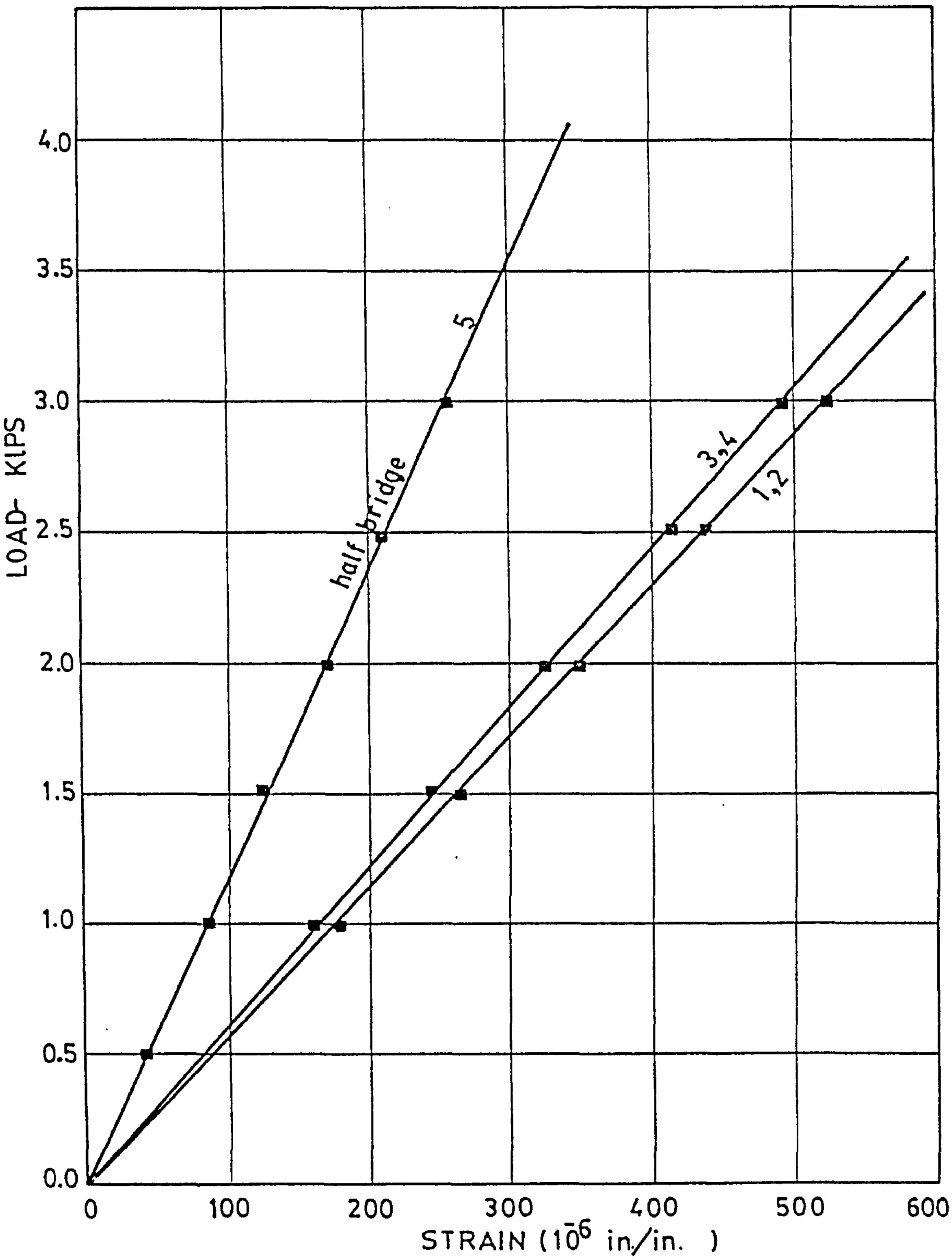


FIGURE (E5) LOAD STRAIN RELATIONSHIP (1-15 LOAD CELL).

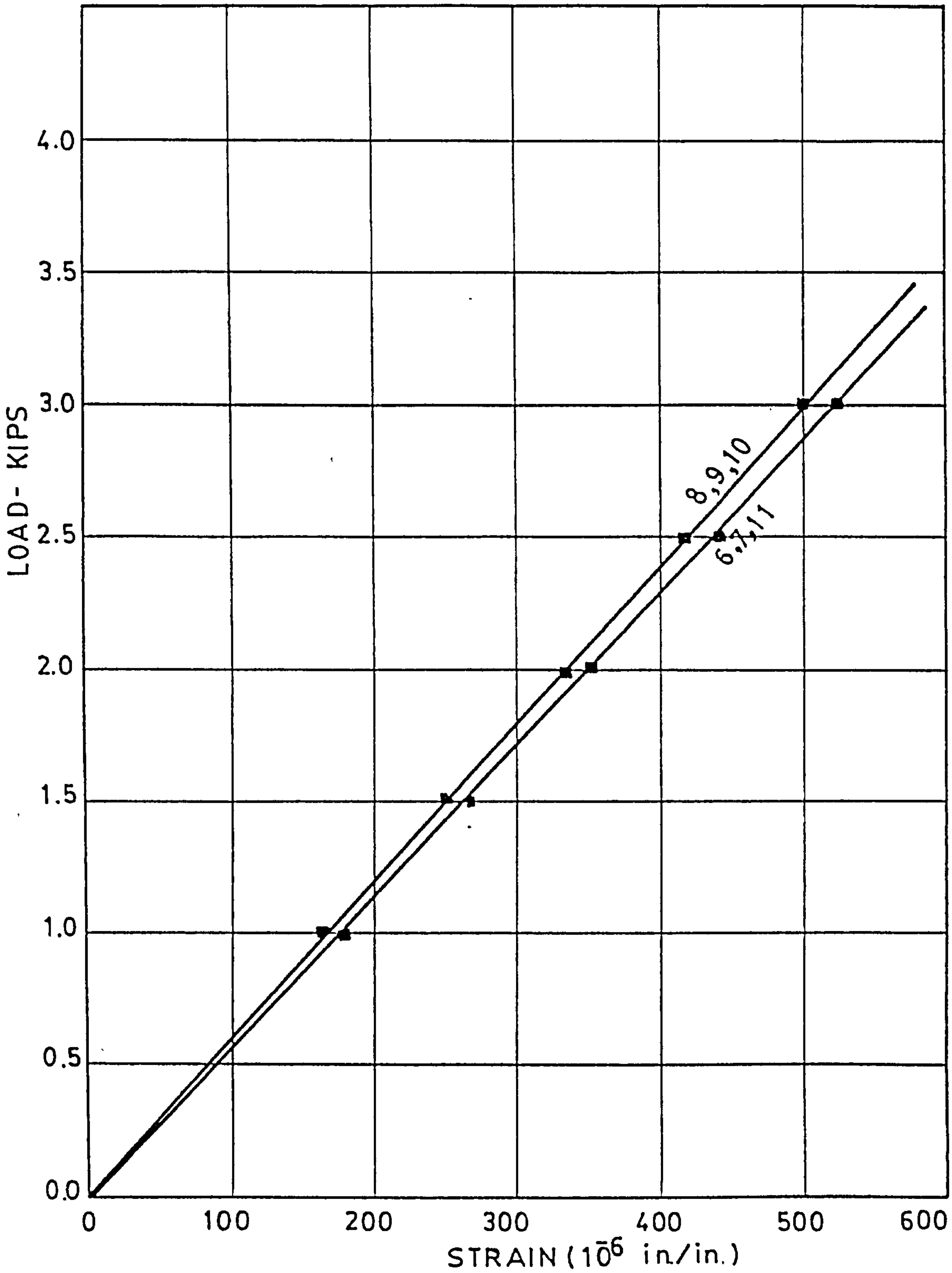


FIGURE (E6) LOAD STRAIN RELATIONSHIP
(6-11 LOAD CELL).

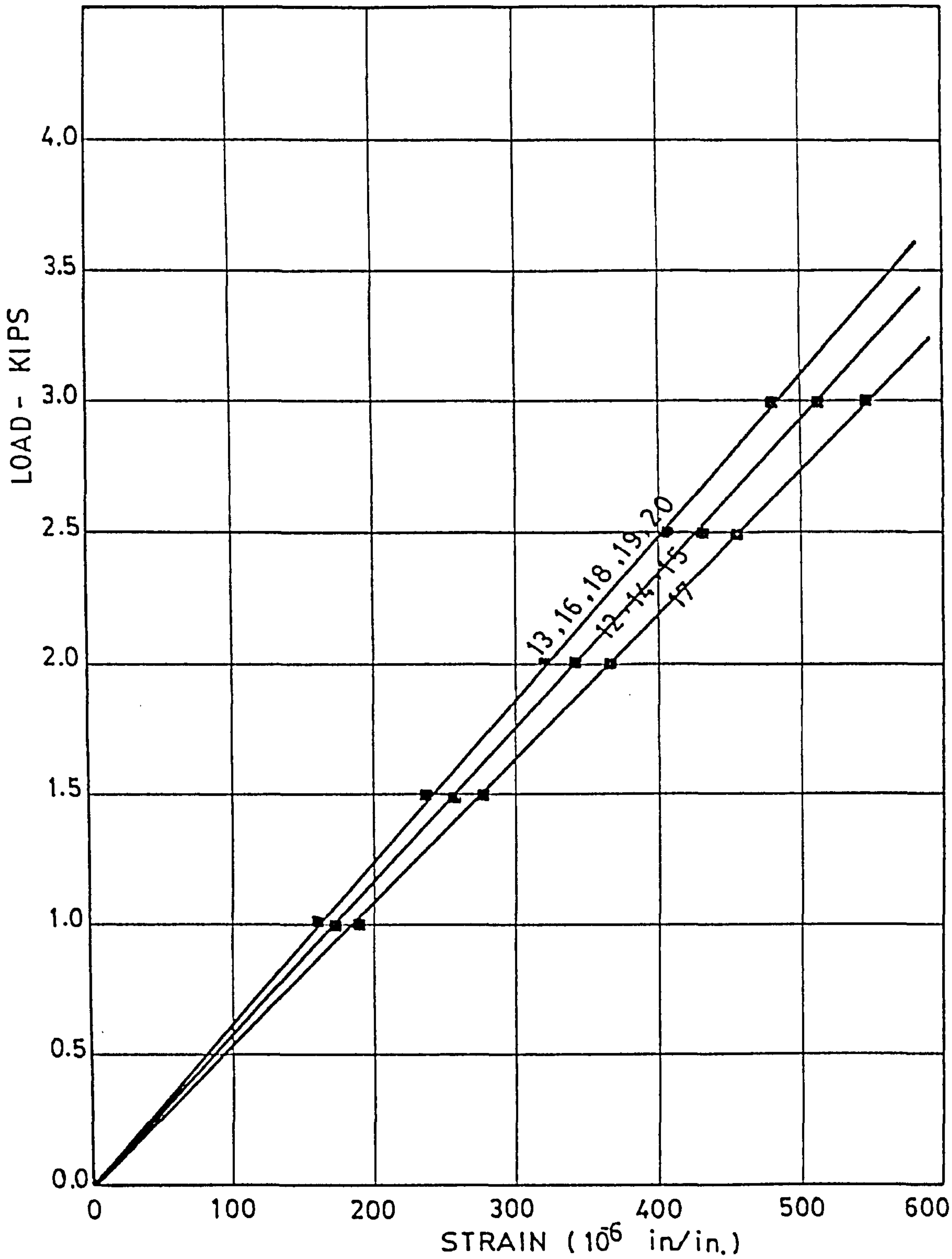


FIGURE (E7) LOAD STRAIN RELATIONSHIP (12-20 LOAD CELL).

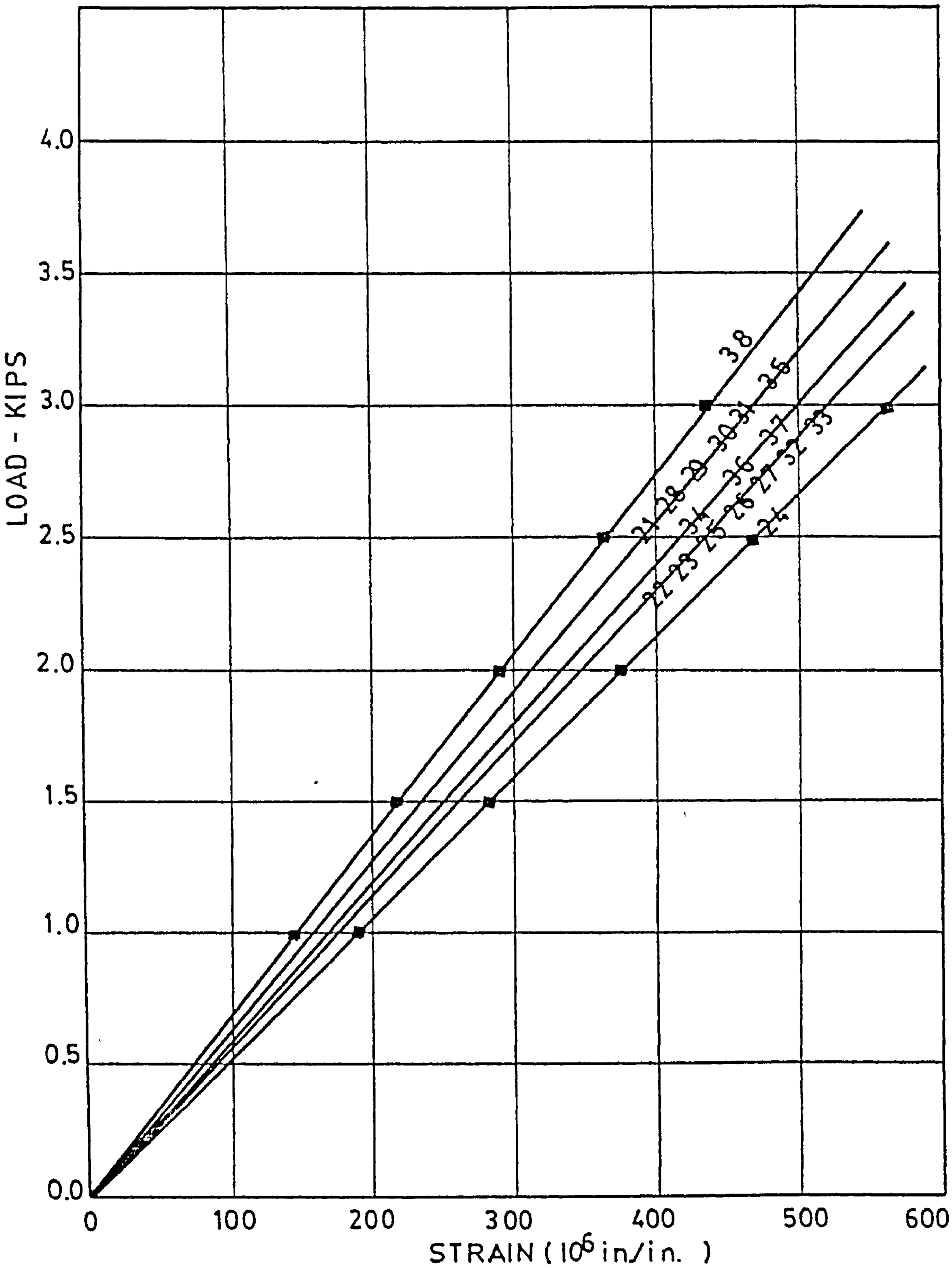


FIGURE (E8) LOAD STRAIN RELATIONSHIP.
(21-38 LOAD CELL).

APPENDIX F

COMPUTER PROGRAM

(WAFFLE)

```
WW        WW   AAAAAAAAAA   FFFFFFFFFFFF  FFFFFFFFFFFF  LL            EEEEEEEEEEEE
WW        WW  AAAAAAAAAAAA  FFFFFFFFFFFF  FFFFFFFFFFFF  LL            EEEEEEEEEEEE
WW        WW  AA        AA  FF            FF            LL            EE
WW        WW  AA        AA  FF            FF            LL            EE
WW        WW  AA        AA  FF            FF            LL            EE
WW        WW  AAAAAAAAAAAA  FFFFFFFF      FFFFFFFF      LL            EEEEEEEE
WW   WW   WW  AAAAAAAAAAAA  FFFFFFFF      FFFFFFFF      LL            EEEEEEEE
WW  WWWW  WW  AA        AA  FF            FF            LL            EE
WW WW  WW WW  AA        AA  FF            FF            LL            EE
WWWW    WWWW  AA        AA  FF            FF            LL            EE
WWW      WWW  AA        AA  FF            FF            LLLLLLLLLLLL  EEEEEEEEEEEE
WW        WW  AA        AA  FF            FF            LLLLLLLLLLLL  EEEEEEEEEEEE
```

```
//WAFFLE JOB (XXXXXXXXXX,9,5,,3257),'SEBAKHY',MSGLEVEL=(1,1),CLASS=B
C
C     ***************************************************************
C     *REINFORCED & PRESTRESSED CONCRETE WAFFLE SLAB DECK BRIDGES     *
C     ***************************************************************
C     ***************************************************************
C     *CD=TWO DIMENSIONAL ARRAY OF DIMENSION(NS,JS) TO STORE THE MATRIX EQUATION*
C     *DC=TWO DIMENSIONAL ARRAY OF DIMENSION(NS,LN) TO STORE THE SOL. VECTORS  *
C     *STOR=2 DIMENSIONAL ARRAY OF DIMENSION(NST1,NST2) TO STORE SOME OF THE   *
C     *ELEMENTS FOR ANTI-SYMMETRIC LOADING                                     *
C     *LI&MI=VECTORS OF DIMENSION (NS)                                         *
C     *VL=TWO DIMENSIONAL ARRAY OF DIMENSION(4,5).THE FIRST DIMENSION IS THE   *
C     *SERIAL NUMBER OF THE LOADING CASE AND THE SECOND ONE REPRESENTS U&V     *
C     *COORDINATES OF THE CENTRE OF PATCH LOAD,THE WIDTH AND THE LENGTH OF THE *
C     *PATCH LOAD AND THE TOTAL LOAD RESPECTIVELY.                             *
C     *NS=8N+4                                                                 *
C     *JS=NS+LN                                                                *
C     *NST1=8N+4                                                               *
C     *NST2=4N+4+LN                                                            *
C     *NSS=NS*NS                                                               *
C     *N=MAXIMUM NUMBER OF HARMONIC                                            *
C     *LN=MAXIMUM NUMBER OF LOADING CASES(4).                                  *
C     *Z=LESS THAN 3,IF ALL QUANTATIES ARE NEEDED AND GREATER THAN 3,IF INPUT & *
C     *FINAL RESULTS ONLY ARE NEEDED.                                          *
C     *ZA=0 OR +VE FOR SYMMETRICAL LOADING ONLY.                               *
C     *ZA=-VE FOR ANTI-SYMMETRICAL LOADING.                                    *
C     *ZB=0 FOR BRIDGE SLAB.                                                   *
C     *II=SLAB NUMBER.                                                         *
C     *A=SEMI WIDTH OF SLAB.                                                   *
C     *B=SEMI SPAN OF THE SLAB.                                                *
C     *SA=SKEW ANGLE IN DEGREE.                                                *
C     *NL=ACTUAL NUMBER OF LOADING CASES TO BE SOLVED.                         *
C     *MNH=FINAL NUMBER OF HARMONIC TO BE SOLVED.                              *
C     *NGC=NUMBER OF POINTS AT WHICH THE DEFLECTIONS AND MOMENTS ARE REQUIRED. *
C     *NGPB=THE SERIAL NUMBER OF THE GAGE FOR WHICH THESTRAIN IN THE BOTTOM    *
C     *FIBRE OF THE SLAB IS REUIRED.                                           *
C     *NGPB=0 ,STRAINS ARE NOT EVALUATED.                                      *
C     *YTP=DEPTH OF NEUTRAL AXIS OF BENDING FROM THE TOP FIBRE IN X-DIRECTION. *
C     *YTP=DEPTH OF NEUTRAL AXIS OF BENDING FROM THE TOP FIBRE IN Y-DIRECTION. *
C     *YTU=DEPTH OF NEUTRAL PLANE FOR TWISTING MOMENTS FROM THE TOP FIBRE      *
C     *THK=TOTAL SLAB THICKNESS.                                               *
C     *MU=NO. OF PRESRESSING FORCES ALONG THE SUPPORT.                         *
C     *MV=NO. OF PRESRESSING FORCES ALONG THE FREE EDGE.                       *
C     *ECU=ECCENTILITY OF PRESRESSING FORCE ALONG THE SUPPORTS.                *
C     *ECV=ECCENTILIITY OF PRESRESSING FORCE ALONG THE FREE EDGE.              *
C     *FU=PRESTRESSING FORCE ALONG THE SUPPORTS.                               *
C     *FV=PRESTRESSING FURCE ALONG THE FREE EDGE.                              *
C     *EXC=MODULUS OF ELASTICITY OF CONCRETE IN X-DIRECTION.                   *
C     *EYC=MODULUS OF ELASTICITY OF CONCRETE IN Y-DIRECTION.                   *
C     *AXX=AREA OF ONE RIB PER UNIT WIDTH OF RIB SPACING IN X-DIRECTION        *
C     *AYY=AREA OF ONE RIB PER UNIT WIDTH OF RIB SPACING IN Y-DIRECTION        *
C     *AMU=POISSON'S RATIO OF SLAB MATERIAL.                                   *
C     *TK=THICKNESS OF FLANGE ONLY.                                            *
C     ***************************************************************
C                        ****************************
C                        * HOW TO PUT THE INPUT DATA *
C                        ****************************
C     ***************************************************************
C     *Z,ZA,ZB,YTP,YTY,YTU,THK                        (1X,7F10.0)              *
C     *II,A,B,SA,DX,DY,D1                             (1X,A4,6F10.0)           *
C     *D2,DXY,DYX,EI,GJ,NL,1,NP,0                     (5F10.0,4I5)             *
C     *NGC       ,BOTTOM FIBRE(I5),IF BOTH OR TOP FIBRE  (2I5)                 *
C     *CENTRE COORDINATES OF THE PATCH LOAD,WIDTH,LENGTH AND TOTAL LOAD OF PATCH*
C     *                                               (12F6.0) 5.........      *
C     *EXC,EYC,AXX,AYY,AMU,TK                         (10X,6F10.3)             *
C     *MU,MV,ECU,ECV                                  (2I10,2F10.3)            *
C     *FU(I),I=1,MU&FV(I),I=1,MV                      (2X,12F6.2,6X)           *
C     *COORDINATES OF NGC(TOTAL POINTS)               (    12F6.0)             *
C     ***************************************************************
      DOUBLE PRECISION CH,DC                                             A   51
      COMMON A,PI,SA,C,S,TN,C4,AA,AA3,AA4,Z,SYAS,C2,C3,ST,SF,T,T3,MNH    A   52
      COMMON /BLO/ B,DX,DY,DXY,DYX,D1,D2,EI,GJ,H,VL(4,5),ZB              A   53
      COMMON/BLB/NHC,NB3,NC,JC,NH2,NH3,NH4,NH5,NH6,NH7,NH9,NB1,NB2,NH,NL
      COMMON /BLD/ FM2(3),F18(108),F38(71),F19(94),F20(125)              A   55
      COMMON /EXL/ UC(24),VC(24),NGC,YA(400,24),NR,LF(9),VR(3),NP(10)
      COMMON/PAT/FU(20),FV(20),MU,MV,FU1,FV1
      DIMENSION CD( 34, 39), DC( 34,4), STOR( 34,43), LI( 34), MI( 34),  A   57
```

```
      ICH( 7C55)
      DIMENSION FM1(3), FM5(15), FM6(15), FM7(8), F26(3), F29(9), F10(18
     1), F11(18), F13(36), F21(57), F22(4), F23(3), F24(5), F25(13), F2
     28(3), F33(33), F34(3), F35(3), F36(13), F37(3), F27(9), F16(25), F
     317(11), F30(9), F31(18), F32(13), F39(6), HA(104), HD(10), HE(15),
     4 X(9), Y(9), FM3(3), FM4(3), FM8(12), FM9(10), F12(36), F14(18), F
     515(36),VL5(4)
      DATA IBL/'    '/
      READ 56, FM1,FM2,FM3,FM4,FM5,FM6,FM7,FM8,FM9,F10,F11,F12,F13,F14,F
     115,F16,F17,F18,F19,F20,F21,F22,F23,F24,F25,F26,F27,F39,F28,F29,F30
     2,F31,F32,F33,F34,F35,F36,F38,F37
      WRITE (6,F37) FM1,FM2,FM3,FM4,FM5,FM6,FM7,FM8,FM9,F10,F11,F12,F13,
     1F14,F15,F16,F17,F18,F19,F20,F21,F22,F23,F24,F25,F26,F27,F39,F28,F2
     29,F30,F31,F32,F33,F34,F35,F36,F38,F37
      READ 56, (HD(I),I=1,3),HF,HG,HH,HI,HJ,HK
      WRITE(6,56)   (HD(I),I=1,3),HF,HG,HH,HI,HJ,HK
      READ 56, (HE(I),I=1,15)
      WRITE(6,56)   (HE(I),I=1,15)
      READ (5,FM1)  (HA(I),I=1,104)
      WRITE(6,FM1)  (HA(I),I=1,104)
      PI=3.14159265358?3
      PI9=90./PI
      N = 10
      LN = 4
      NS = 3*N + 4
      JS = NS + LN
      NST1 = NS
      NST2 = 4 * N + 4 + LN
      NSS = NS * NS
      CALL TINIT
1     READ (5,FM2)  Z,ZA,ZB,YTP,YTY,YTU,THK
      CALL TUSED (ICPU)
      CPU=ICPU/1000.
      WRITE (6,108) CPU
108   FORMAT (50X,'CPU USED =',F8.2,' SECS')
      CALL TINIT
      WRITE(6,FM2)  Z,ZA,ZB,YTP,YTY,YTU,THK
      IF (Z.EQ.0.)  GO TO 55
      READ (5,200)  II,A,B,SA,DX,DY,D1,D2,DXY,CYX,EI,GJ,NL,      MNH,
     1       (NP(I),I=1,MNH),NR,NGC,NTP,NGPY
      WRITE(6,200)  II,A,B,SA,DX,DY,D1,D2,DXY,CYX,EI,GJ,NL,      MNH,
     1       (NP(I),I=1,MNH),NR,NGC
      NRC = NR
      IF (NR.EQ.0)  NRC = 1
      READ (5,58 )  ((VL(L,J),J=1,5),L=1,NL),(VR(I),I=1,NRC)
      WRITE(6,58 )  ((VL(L,J),J=1,5),L=1,NL),(VR(I),I=1,NRC)
      THETA=SA*PI/180.
      C=COS(THETA)
      S=SIN(THETA)
      READ(5,76)    EXC,EYC,AXX,AYY,AMU,TK
      WRITE(6,76)   EXC,EYC,AXX,AYY,AMU,TK
   76 FORMAT(10X,6F10.3)
      EX=EXC*(1./(1.-AMU**2)+AXX/TK)
      EY=EYC*(1./(1.-AMU**2)+AYY/TK)
      E1=AMU*EXC/(1.-AMU**2)
      E2=AMU*EYC/(1.-AMU**2)
      READ(5,201)MU,MV,ECU,ECV
      WRITE(6,201)MU,MV,ECU,ECV
  201 FORMAT (2I10,2F10.3)
      READ(5,75) (FU(I),I=1,MU)
      WRITE(6,75)(FU(I),I=1,MU)
      READ(5,75) (FV(I),I=1,MV)
      WRITE(6,75)(FV(I),I=1,MV)
   75 FORMAT (2X,12F6.2,6X)
      FUT=0
      DO 215 I=1,MU
  215 FUT=FUT+FU(I)
      PAY=FUT    /(2.*A)
      FVT=0
      DO 216 I=1,MV
  216 FVT=FVT+FV(I)
      PAX=FVT/(2.*B)
      DXY1=EX*EY-E1*E2
      CXX=(PAX*EY/TK-PAY*E1)/DXY1
      CYY=(PAY*EX/TK-PAX*E2)/DXY1
      DO 202 I=1,MU
  202 FU(I)=FU(I)*ECU
      DO 203 I=1,MV
```

```
 203 FV(I)=FV(I)*ECV
     FU1=0
     DO 204 I=1,MU
 204 FU1=FU1+FU(I)
     FU1=FU1/(2.*A)
     FV1=0
     DO 205 I=1,MV
 205 FV1=FV1+FV(I)
     FV1=-C*FV1/(2.*B)
     NT = NNH*10
     LT=NT*NL                                                           A 112
     IF (NGC.NE.0) GO TO 104
     NGC = 12
     DO 100 I=1,5
     RI = I
     UC(I) = 0.
     UC(I+4) = (RI-1.)*A/4.
     VC(I) = (5-RI)*B/4.
 100 VC(I+4) = 0.
     UC(10) = 0.
     VC(10) = 0.9*B
     UC(11) = 0.9*A
     VC(11) = 0.
     UC(12) = 0.9*A
     VC(12) = 0.9*B
     GO TO 105
 104 READ 33,(UC(I),VC(I),I=1,NGC)
     WRITE(6,33)(UC(I),VC(I),I=1,NGC)
     DO 10 I=1,LT                                                       A 113
     DO 10 J=NGC,24
10   YA(I,J)=1000000000.                                                A 116
 105 NCTH = 1
     NTHE=4
     NCH = 0
     IF (DYX.EQ.0) GO TO 80
     NCH=3
     GO TO 83
80   NCH = NCH+1
     GO TO (82,81) , NCH
     DXY = 0
     DYX = 0
     D1 = 0
     D2 = 0
     GO TO 83
81   DXY = 0.5*DXY
     DYX = DXY
     D1 = 0.5*D1
     D2 = D1
     GO TO 83
82   DYX =DXY
     D2 = D1
83   IF (SA.GT.0) GO TO 88
     DO 91 L=1,NL
     IF (VL(L,3).EQ.2*A) VLS(L) = VL(L,5)
91   CONTINUE
     NCTH = 4
     NTHE = 1
11   GO TO (84,85,86) , NTHE
     SA = 60.
     GO TO 87
84   SA = 0.004999
     GO TO 87
85   SA = 30.0
     GO TO 87
86   SA = 45.0
87   DO 90 L=1,NL
     IF (VL(L,3).EQ.2*A) VL(L,5)=COS(SA*PI/180.)*VLS(L)
90   CONTINUE
88   CONTINUE
     IF (NR.EQ.0) GO TO 106
     DO 107 I=1,5
     RI = I
     UR(I) = (RI-5.)*A/4.
107  UR(I+4) = (RI-1.)*A/4.
     DO 101 I=1,NR
 101 VR(I) = (1.-VR(I))*B
106  WRITE (6,F11) II,A,B,SA,DX,DY,D1,D2,DXY,DYX,EI,GJ                  A  83
     IF (ZA) 3,2,2                                                      A  84
```

```
22    WRITE (6,67) IJ,(K,(CD(NJ,NH*(J-1)+K),J=1,8),CD(NJ,NH8+K),K=1,NN)
      GO TO (23,23,23,24), NH
      WRITE (6,F22) (K,(CD(NJ,NH*(J-1)+K),J=1,8),K=5,NH)
      GO TO 24
23    N1=NH+1
      WRITE (6,F23) (CD(NJ,NH8+K),K=N1,4)
24    WRITE (6,F24) (CD(NJ,NC+J),J=1,NL)
      WRITE (6,F44) (HF,I=1,32)
C
C     ************************************************************************
C          INVERSE THE MATRIX
C     ************************************************************************
C
25    CALL MINA (PI,NSS,CD,DC,STOR,NS,JS,LN,NST1,NST2,L1,M1,CH)
      IF (Z.GT.3) GO TO 32
      WRITE (6,F30)
      DO 31 L=1,NL
      IF (NH-4) 26,27,27
26    NN=NH
      GO TO 28
27    NN=4
28    WRITE (6,F12) L,(VL(L,I),I=1,5)
      WRITE (6,F33)
      WRITE (6,F44) (HF,I=1,32)
      WRITE (6,F23) (K,(DC(NH*(J-1)+K,L),J=1,8),DC(NH8+K,L),K=1,NN)
      IF (NH-4) 30,31,29
29    WRITE (6,F34) (K,(DC(NH*(J-1)+K,L),J=1,8),K=5,NH)
      GO TO 31
30    N1=NH+1
      WRITE (6,F35) (DC(NH8+K,L),K=N1,4)
31    WRITE (6,F44) (HF,I=1,32)
C
C     ************************************************************************
C          COMPUTE MOMENTS AND DEFLECTIONS
C     ************************************************************************
C
32    IF (SYAS) 34,33,33
33    CALL DS2A (CD,DC,STOR,NS,JS,LN,NST1,NST2)
      IF(NR.NE.0) CALL DSRA(CD,DC,STOR,NS,JS,LN,NST1,NST2)
      GO TO 35
34    CALL CA2A (CD,DC,STOR,NS,JS,LN,NST1,NST2)
      IF(NR.NE.0) CALL DARA(CD,DC,STOR,NS,JS,LN,NST1,NST2)
   35 IF (SYAS) 37,37,36
36    IF (ZA) 12,37,37
37    DO 40 L=1,NL
      LTX=(L-1)*NT+MNH+NHC
      LTY=LTX+MNH
      LXY=LTY+MNH
      LM1=LXY+MNH
      LM2=LM1+MNH
      LMT=LM2+MNH
      DO 40 I=1,NJC
      XPY=(YA(LTX,I)+YA(LTY,I))*0.5
      XMY=(YA(LTX,I)-YA(LTY,I))*0.5
      XY1=SQRT(XMY**2+(YA(LXY,I))**2)
      YA(LM1,I)=XPY+XY1
      YA(LM2,I)=XPY-XY1
      IF (XMY) 38,39,38
38    YA(LMT,I)=ATAN(YA(LXY,I)/XMY)*PI9
      GO TO 40
39    YA(LMT,I)=90.
40    CONTINUE
      IF (YTP) 41,46,41
41    DSTR=(DX*DY-D1*D2)
      DO 45 L=1,NL
      LTX=(L-1)*NT+MNH+NHC
      LTY=LTX+MNH
      LXY=LTY+MNH
      LSX=LTY+MNH*5
      LSY=LSX+MNH
      LSU=LSY+MNH
      DO 44 I=1,NJC
      IF (I-NTP) 42,42,43
42    YA(LSX,I)=YTP*(DY*YA(LTX,I)-D1*YA(LTY,I))/DSTR
      YA(LSY,I)=YTY*(DX*YA(LTY,I)-D2*YA(LTX,I))/DSTR
      YA(LSU,I)=(YA(LSX,I)+YA(LSY,I))*0.5-2.*YTU*YA(LXY,I)/(DXY+DYX)*1.E
     106
      YA(LSX,I)=YA(LSX,I)-EXX
```

```
2       PRINT 59
        GO TO 4
3       PRINT 60
4       WRITE (6,F12) (L,(VL(L,I),I=1,5),L=1,NL)
        IF (Z3) 5,6,7
5       PRINT 61
        GO TO 8
6       PRINT 62
        GO TO 8
7       PRINT 63
8       THETA=SA*PI/180.0
        C=COS(THETA)
        S=SIN(THETA)
        H=(D1+D2+DXY+DYX)/2.0
        PK=H**2-DX*DY
        IF (PK) 9,54,53
9       WRITE (6,FM3)
        YBM=THK+YTP
        YBY=THK+YTY
        YBU=THK+YTU
C       PRINT 64, (NP(I),I=1,MNH)
C
C       ************************************************************************
C            COMPUTE CONSTANTS OF THE SLAB
C       ************************************************************************
C
        CALL CONST (NL)
        DO 46 NHC=1,MNH
        NH = NP(NHC)
        NH2=NH+NH
        NH3=NH2+NH
        NH4=NH3+NH
        NH5=NH4+NH
        NH6=NH5+NH
        NH7=NH6+NH
        NH8=NH7+NH
        N81=NH8+1
        N82=NH8+2
        N83=NH8+3
        NC=NH8+4
        JC=NC+NL
C
C       ************************************************************************
C            COMPUTE THE MATRIX
C       ************************************************************************
C
        CALL SYLA (NS1,NSL,CD,DC,STOR,NS,JS,LN,NST1,NST2)
        GO TO 13
12      CALL ASLA (NS1,NSL,SYAS,CD,DC,STOR,NS,JS,LN,NST1,NST2)
13      IF (Z.GT.2.) GO TO 25
        IF (SYAS) 15,14,14
14      PRINT 65
        GO TO 16
15      PRINT 66
16      WRITE (6,F21) ((VL(L,I),L=1,2),I=1,2)
        WRITE (6,FM4) (HF,I=1,32)
        DO 20 I=1,8
        DO 20 N=1,NH
        IJ=NH*(I-1)+N
        GO TO (17,17,17), NH
        NN=4
        GO TO 18
17      NN=NH
18      WRITE (6,F23) I,N,(K,(CD(IJ,NH*(J-1)+K),J=1,8),CD(IJ,NH8+K),K=1,NN
       1)
        GO TO (19,19,19,20), NH
        WRITE (6,F22) (K,(CD(IJ,NH*(J-1)+K),J=1,8),K=5,NH)
        GO TO 20
19      N1=NH+1
        WRITE (6,F23) (CD(IJ,NH8+K),K=N1,4)
20      WRITE (6,F24) (CD(IJ,NC+J),J=1,NL)
        DO 24 N=1,4
        IJ=N+8
        NJ=NH8+N
        GO TO (21,21,21), NH
        NN=4
        GO TO 22
21      NN=NH
```

```
      YA(LSY,I)=YA(LSY,I)-EYY
      GO TO 44
43    YA(LSX,I)=YSM*(DY*YA(LTX,I)-D1*YA(LTY,I))/DSTR
      YA(LSY,I)=YSY*(DX*YA(LTY,I)-D2*YA(LTX,I))/DSTR
      YA(LSU,I)=(YA(LSX,I)+YA(LSY,I))*0.5-2.*YBU*YA(LXY,I)/(DXY+DYX)*1.E
     106
      YA(LSX,I)=YA(LSX,I)-EXX
      YA(LSY,I)=YA(LSY,I)-EYY
44    CONTINUE
45    CONTINUE
46    CONTINUE
C
C     ************************************************************************
C           SUMMARISE THE RESULTS
C     ************************************************************************
C
      IF (YTP) 49,48,49
48    KN=7
      GO TO 50
49    KN=10
      NTP1=NTP+1
      PRINT 68, YTP,YTY,YTU,THK,NTP,NTP1,NGC
50    DO103 L=1,NL
      LT=(L-1)*NT
      WRITE (6,F12) L,(VL(L,I),I=1,5)
      WRITE (6,69) (HI,I=1,32),HJ,HJ,(I,HJ,I=1,12)
      WRITE (6,70) (HI,I=1,32),HJ,HJ,(UC(I),HJ,I=1,12),HJ,HJ,(VC(I),HJ,I
     1=1,12),(HI,I=1,32)
      DO 52 K=1,KN
      K8=(K-1)*8
      LX=LT+(K-1)*MNH
      WRITE (6,FM3) (HA(K8+I),I=1,8)
      WRITE (6,71) (HJ,NP(N),
     1                   (HJ,YA(LK+N,I),I=1,12),HJ,IBL,N=1,MNH)
      IF (NGC-12) 52,52,51
51    WRITE (6,FM4) (HI,I=1,32)
      PRINT 72
      WRITE (6,69) (HI,I=1,32),HJ,HJ,(I,HJ,I=13,24)
      WRITE (6,70) (HI,I=1,32),HJ,HJ,(UC(I),HJ,I=13,24),HJ,HJ,(VC(I),HJ,
     1I=13,24),(HI,I=1,32)
      WRITE (6,71) (HJ,NP(N),
     1                   (HJ,YA(LK+N,I),I=1,12),HJ,IBL,N=1,MNH)
C 52  WRITE (6,F44) (HI,I=1,32)
52    CONTINUE
      IF (NR.EQ.0) GO TO 103
      PRINT 73
      DO 102 K=1,NR
      LX=LT+(K+8)*MNH
      WRITE (6,69) (HI,I=1,32),HJ,HJ,(I,HJ,I=1,12)
      WRITE (6,70) (HI,I=1,32),HJ,HJ,(UR(I),HJ,I=1,9 ),UR(1),HJ,UR(9),
     1      HJ,UR(3),HJ,HJ,HJ,(VR(K ),HJ,I=1,12),(HI,I=1,32)
      WRITE (6,71) (HJ,NP(N),
     1                   (HJ,YA(LK+N,I),I=1,12),HJ,IBL,N=1,MNH)
102   WRITE (6,F44) (HI,I=1,32)
  103 CONTINUE
      IF(NCTH.NE.1)GO TO 110
      IF(NCH.EQ.3) GO TO 1
      GO TO 50
  110 IF (SA.NE.60.0) GO TO 74
      DO 39 L=1,NL
      IF (VL(L,3).EQ.2*A) VL(L,5)=VLS(L)
39    CONTINUE
74    NTHE = NTHE+1
      IF (NTHE.LT.3) GO TO 11
      SA =-1.0
      IF (NCH.LT.3) GO TO 30
      GO TO 1
53    WRITE (6,FM5)
      GO TO 1
54    WRITE (6,F47)
      GO TO 1
55    STOP
C     ************************************************************************
C
56    FORMAT (13A4,8X)
57    FORMAT (16I5)
58    FORMAT (12F5.2,3X)
59    FORMAT (/54X'SYMMETRIC LOADING ONLY')
```

```
60    FORMAT (/42X'GENERAL LOADING(SYMMETRIC & ANTISYMMETRIC LOADS)')
61    FORMAT (/50X'THE SLAB IS CLAMPED ON ALL EDGES')
62    FORMAT (/40X'THE SLAB IS ELASTICALLY SUPPORTED ON TWO EDGES')
63    FORMAT (/45X'THE SLAB IS SIMPLY SUPPORTED ON ALL EDGES')
64    FORMAT (45X,'NUMBER OF HARMONICS ANALYSED',4XI2,10(',',I2)//)
65    FORMAT (/54X'SYMMETRIC LOAD SYSTEM')
66    FORMAT (/51X'ANTI-SYMMETRIC LOAD SYSTEM')
67    FORMAT (/I3,I4,9F14.8/(I7,9F14.8))
68    FORMAT (/45X'DEPTHS OF N.A. FROM TOP FIBRE',3F10.4/
     5           45X'SLAB THICKNESS =',F10.4,' INS'        /45X'TOP GAGES
     1 ',25X'1  THRU',I5/45X'BOTTOM GAGES',I25,'  THRU',I5/)
69    FORMAT (2X32A4/2XA1,6XA1,12(I5,3XA1))
70    FORMAT (2X32A4/2XA1,4X'U=',A1,12(F8.2,1X,A1)/2XA1,4X'V=',A1,12(F8.
     12,1X,A1)/2X32A4)
  71  FORMAT (2XA1,1X'NH=',I2,12(A1,F9.4),A1/(A2,A1,I6,12(A1,F9.4),A1))
72    FORMAT (50X'CONTINUED')
   73 FORMAT (/60X'REACTIONS'/)
200   FORMAT (1XA4,6E10.3/5E10.3,4I5,10X/14I5,10X)
      END
```

```
C     ***************************************************************************
      SUBROUTINE CONST (NL)
C     ***************************************************************************
      REAL K3,K4,K5
      COMMON A,PI,SA,C,S,TN,C4,AA,AA3,AA4,Z,SYAS,C2,C3,ST,SF,T,T3,MNH
      COMMON /BL0/ B,DX,DY,DXY,DYX,D1,D2,EI,GJ,H,VL(4,5),Z3
      COMMON /BL1/ P,CQ(4),COS(4),K3,K4,K5,BB,BB3,BB4,X2,Y2,A1,A2,A3,A4,
     1B1,B2,B3,B4,SCA,SCB,SCC,SCD,S1(4),S2(4),S3(4),S4(4),S5(4),S6(4),
     2     S7(4)
      COMMON /BL2/ R1,R2,R3,R4,R5,R6,R7,R8,R9,R10,R11,R12,R1K3,X22,X23,X
     124,Y22,Y23,Y24,A5,A6,A7,A8,A9,A10,A11,A12,A13,A14,A15,A16,A17,A19,
     2A20,A21,A22,A23,A24,AG,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,
     3B17,B18,B19,B20,B22,B23,B24,A18
      COMMON /BL3/ FM2(3),F18(108),F38(71),F19(94),F20(125)
      COMMON /SPL/ A25,A26,A27,A28,B26,B27,B28,CI,TNA,TNA3,TNB,TNB4,BC2,
     1BBC,BETN,R2J,R7A
      COMMON/PMT/FU(20),FV(20),MU,MV,FU1,FV1
      P=A/B
      C2=C*C
      C3=C2*C
      C4=C2*C2
      ST=S*S
      SF=ST*ST
      SCA=(3.0*ST*DX+C2*H)*2.0
      SCB=SF*DX+2.0*ST*C2*H+C4*DY
      SCC=4.0*S*DX
      SCD=(ST*DX+C2*H)*4.0*S
      DO 1 L=1,NL
      CQ(L)=C3/A/B*VL(L,5)
      COS(L)=CQ(L)/SCB
    1 CONTINUE
      AA=A*A
      AA3=AA*A
      AA4=AA*AA
      BB=B*B
      BB3=BB*B
      BB4=BB*B3
      DXDY=SQRT(DX*DY)
      DX2=DX*2.
      YA=(DXDY+H)/DX2
      PKK3=C2*YA
      K3=C*SQRT(YA)
      PKK1=C*SQRT((DXDY-H)/DX2)
      K4=PKK1+S
      K5=PKK1-S
      PKK4=K4**2
      PKK5=K5**2
      X2=1./(PKK3+PKK4)
      Y2=1./(PKK3+PKK5)
      X22=X2*X2
      X23=X22*X2
      X24=X23*X2
      Y22=Y2*Y2
      Y23=Y22*Y2
      Y24=Y23*Y2
      DD=D1+DXY+DYX
      TN=S/C
      DXT=DX*TN**2
      R1=S*(2.*DXT+2.*D2+DXY+DYX)
      R2=-(ST*(DXT+2.*H)+DY*C2)
      R3=DX/C3
      R4=-3.C*S*R3
      R5=(3.*DXT+DD)/C
      R6=-S*(DXT+DD)/C
      R7=DX/C
      R8=-2.0*S*R7
      R9=(ST+D1*C2/DX)*R7
      R10=-S*GJ
      R1K3=R1*K3
      R1K4=R1*K4
      R1K5=R1*K5
      A1=2.*K3*K4
      A2=(PKK3-PKK4)
      A3=K3*(PKK3-3.*PKK4)
      A4=K4*(3.*PKK3-PKK4)
      A5=-EI*(A2*A2-A1*A1)*X24
```

```
A6=2.*A1*A3*X24*EI
A7=-R1K4-R2
A8=-X2*(R1K4-R2*X2*A2)
A9=-(R1K3+R2*X2*A1)*X2
A10=R3*A3-R4*A1-R5*K3
A11=-R3*A4-R4*A2+R5*K4+R6
A12=-X2*(R4*K3+R5*X2*A1-R6*X22*A3)
A13=(R3+X2*R4*K4-R5*X22*A2-R6*X23*A4)
A14=R7*A2-R8*K4-R9
A15=-R7*A1-R8*K3
A16=K4*GJ+R10
A17=-R7-R8*X2*K4+R9*X22*A2
A18=-X2*(R8*K3+R9*X2*A1)
A19=-X22*(A1*GJ-R10*X2*A3)
A20=-(A2*GJ+R10*X2*A4)*X22
AG=K3*GJ
B1=2.*K3*K5
B2=(PKK3-PKK5)
B3=K3*(PKK3-3.*PKK5)
B4=K5*(3.*PKK3-PKK5)
B5=-EI*(B2*B2-B1*B1)*Y24
B6=2.*B1*B2*Y24*EI
B7=(R1K5-R2)
B8=(R1K3+R2*Y2*B2)*Y2
B9=(R1K3-R2*Y2*B1)*Y2
B10=R3*B3+R4*B1-R5*K3
B11=-R3*B4+R4*B2+R5*K5-R6
B12=-Y2*(R4*K3-R5*Y2*B1-R6*Y22*B3)
B13=-R3+R4*Y2*K5+R5*Y22*B2-R6*Y23*B4
B14=R7*B2+R8*K5-R9
B15=-R7*B1+R8*K3
B16=K5*GJ-R10
B17=-R7+R8*Y2*K5+R9*Y22*B2
B18=(R8*K3-R9*Y2*B1)*Y2
B19=(B1*GJ+R10*Y2*B3)*Y22
B20=(B2*GJ-R10*Y2*B4)*Y22
T=DCB/DX
CI=1./C
TNK4=TN*K4
TNK5=TN*K5
CK4=K4/C
CK5=K5/C
A21=-TN*K3
A22=TNK4-CI
A24=TN-CK4*X2
A25=K3/C
A23=A25*X2
A26=TN-CK4
A27=A21*X2
A28=-CI+TNK4*X2
B22=TNK5+CI
B23=A25*Y2
B24=-TN-CK5*Y2
B26=-TN-CK5
B27=A21*Y2
B28=CI+TNK5*Y2
TNA=-2.*TN/A
TNA3=-4.*TN*T*AA3/384
TNB=-2.*TN/B
TNB4=-2.*TN3
BBC=BB3/C/12.
BBTN=EB3*TN/24.
BC2=2./B/C
R2D=R2/BB*2.
R7A=R7/AA
S1(1)=-DX/C2
S2(1)=-2.0*S*S1(1)
S3(1)=-DXT-D1
S1(2)=-D2/C2
S2(2)=-2.0*S*S1(2)
S3(2)=-D2*T1**2-DY
S1(3)=0.0
S2(3)=DXY/C
S3(3)=-S*S2(3)
S4(1) = -        DX   /C3
S5(1) = -3.0*S*S4(1)
S6(1) = -(1.5*S2(1)*S        +D1+      DYX)/C
S7(1) =   (      -S3(1)         +      DYX)/C*S
```

```
      S4(2) = 0.0
      S5(2) = S1(2)-S2(3)/C
      S6(2) = -S3(2)*2.0*S
      S7(2) =-DY-(D2+     DXY)*(TN  )**2
      IF (DXY.EQ.0.0) GO TO 4
      DYXD = DYX/DXY
      GO TO 5
4     DYXD = 0.0
5     S1(4) = (S1(1)-S1(2))*C*S+S1(3)*(1.-ST+DYXD    *C2)
      S2(4) = (S2(1)-S2(2))*C*S+S2(3)*(1.-ST+DYXD    *C2)
      S3(4) = (S3(1)-S3(2))*C*S+S3(3)*(1.-ST+DYXD    *C2)
      S4(4) = (S1(1)-S1(2))*C*S+S1(3)*(   ST+DYXD    *C2)+S4(1)*S-S4(2)*C
      S5(4) = (S2(1)-S2(2))*C*S+S2(3)*(   ST+DYXD    *C2)+S5(1)*S-S5(2)*C
      S6(4) = (S3(1)-S3(2))*C*S+S3(3)*(   ST+DYXD    *C2)+S6(1)*S-S6(2)*C
      S7(4) =                                             S7(1)*S-S7(2)*C
      IF (Z-2) 2,2,3
2     WRITE (6,F13) C,S,P,C2,H,SCA,SCB,SCC,SCD,AA,BB,TN,X2,Y2,K3,K4,K5,R
     11,R2,R3,R4,R5,R6,R7,R8,R9,R10,R1K3,R1K4,R1K5,T,A1,A2,A3,A4,A5,A6,A
     27,A8
      WRITE (6,F33) A9,A10,A11,A12,A13,A14,A15,A16,A17,A18,A19,A20,AG,B1
     1,B2,B3,B4,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,B17,B18,B19,B
     220,(S1(I),S2(I),S3(I),I=1,3)
3     RETURN
      END
```

```
C     ***********************************************************************
      SUBROUTINE SYLA (NS1,NSL,CD,DC,STOR,NS,JS,LN,NST1,NST2)
C     ***********************************************************************
      DOUBLE PRECISION CH,DC
      REAL K1N,K2N,K3N,K4N,K5N,K6N,K7N,K8N,L1N,L2N,L3N,L4N,L5N,L6N,L7N,L
     18N,I1N,I3N,I5N,I7N,K3,K4,K5,KMN,LMN
      COMMON A,PI,SA,C,S,TN,C4,AA,AA3,AA4,Z,SYAS,C2,C3,ST,SF,T,T3,MNH
      COMMON /BL0/ B,DX,DY,DXY,DYX,D1,D2,EI,GJ,H,VL(4,5),Z3
      COMMON /BL1/ P,CQ(4),CQS(4),K3,K4,K5,BB,BB3,BB4,X2,Y2,A1,A2,A3,A4,
     1B1,B2,B3,B4,SCA,SCB,SCC,SCD,S1(4),S2(4),S3(4),S4(4),S5(4),S6(4),
     2    S7(4)
      COMMON /BL2/ R1,R2,R3,R4,R5,R6,R7,R8,R9,R10,R11,R12,R1K3,X22,X23,X
     124,Y22,Y23,Y24,A5,A6,A7,A8,A9,A10,A11,A12,A13,A14,A15,A16,A17,A19,
     2A20,A21,A22,A23,A24,AG,B5,B6,B7,B8,B9,B10,B11,B12,B13,B14,B15,B16,
     3B17,B18,B19,B20,B22,B23,B24,A18
      COMMON/BL5/NHC,NB3,NC,JC,NH2,NH3,NH4,NH5,NH6,NH7,NH8,NB1,NB2,NH,NL
      COMMON /BL9/ FM2(3),F18(108),F38(71),F19(94),F20(125)
      COMMON /SPL/ A25,A26,A27,A28,B26,B27,B28,CI,TNA,TNA3,TNB,TNB4,BC2,
     1DDC,BBTN,R28,R7A
      COMMON/PMT/FU(20),FV(20),MU,MV,FU1,FV1
      DIMENSION CD(NS,JS), DC(NS,LN), STOR(NST1,NST2)
      DIMENSION W1(8), AJ(8), BJ(8)
      DIMENSION UMC(4),VMC(4),UMC1(4),VMC1(4)
      NS1=NH4+1
      NS2=NH4+2
      NS3=NH4+3
      NS4=NH4+4
      NSL=NS4+NL
      DO 2 I=1,NC
      DO 1 J=1,NSL
      STOR(I,J)=0.
1     CONTINUE
      DO 2 J=1,JC
2     CD(I,J)=0.0
      CD(NB1,NB1)=1.0
      CD(NB1,NB2)=0.333333333333
      CD(NB1,NB3)=1.0
      CD(NB1,NC)=T*AA4/BB4/5.-1.
      STOR(NB3,NS2)=1.
      STOR(NB3,NS4)=1.
      IF (Z3) 3,5,3
3     CD(NB2,NB2)=2./A/C
      CD(NB2,NC)=4.*T*AA3/BB4/C
      CD(NB3,NB3)=BC2
      CD(NB3,NC)=-BC2*2.
      STOR(NB2,NS1)=1./A/C
      STOR(NB2,NS2)=-TN/B
      STOR(NB2,NS3)=3./A/C
      STOR(NB2,NS4)=-TN/B
      STOR(NC,NS1)=-TN/A
      STOR(NC,NS2)=1./B/C
      STOR(NC,NS3)=-TN/A
      STOR(NC,NS4)=3./B/C
4     CD(NC,NB1)=1.
      CD(NC,NB2)=1.
      CD(NC,NB3)=0.3333333333333
      CD(NC,NC)=T*AA4/BB4-0.2
      STOR(NB1,NS1)=1.
      STOR(NB1,NS3)=1.
      IF (Z3) 7,5,5
5     CD(NC,NC)=24.0*(R3*T*A+EI)/BB4
      STOR(NB1,NS3)=6.*R3/AA3
      STOR(NB1,NS4)=6.*R6/BB3
      STOR(NB2,NS4)=6.*R10/BB3
6     CD(NB2,NB2)=2.*R7A
      CD(NB2,NB3)=2.*R9/BB
      CD(NB2,NC)=-4.*R9/BB+12.*R7*T*AA/BB4
      CD(NB3,NB3)=R28
      CD(NB3,NC)=-R28*6.
      STOR(NB2,NS3)=6.*R7A
      STOR(NC,NS4)=R28*3.
      DO 210 I=1,4
      VMC(I)=0
      UMC(I)=0
      VMC1(I)=0
  210 UMC1(I)=0
```

```
7       UN=1.0
        DO 57 N=1,NM
        UN=-UN
        RN=N
        RNP=RN*PI
        AN=RNP/A
        AN2=AN**2
        OA1=UN*AN
        OA2=UN*AN2
        OA3=OA2*AN
        OA4=OA2*AN2
        BN=RNP/B
        BN2=BN**2
        BN3=BN2*BN
        BN4=BN2*BN2
        OB1=UN*BN
        OB2=UN*BN2
        BNA=BN*A
        ANB=AN*B
        XAN=X2*ANB
        YAN=Y2*ANB
        RR1=K4*XAN
        RR2=K4*BNA
        SS1=K5*YAN
        SS2=K5*BNA
        R1N=K3*XAN
        R2N=K3*BNA
        S1N=K3*YAN
        CRR1=COS(RR1)
        SRR1=SIN(RR1)
        CRR2=COS(RR2)
        SRR2=SIN(RR2)
        CSS1=COS(SS1)
        SSS1=SIN(SS1)
        CSS2=COS(SS2)
        SSS2=SIN(SS2)
        IF (R1N-50.0) 8,8,9
8       ER1=EXP(-2.0*R1N)
        CHR1=(1.0+ER1)/2.0
        SHR1=(1.0-ER1)/2.0
        GO TO 10
9       CHR1=0.5
        SHR1=0.5
10      IF (R2N-50.0) 11,11,12
11      ER2=EXP(-2.0*R2N)
        CHR2=(1.0+ER2)/2.0
        SHR2=(1.0-ER2)/2.0
        GO TO 13
12      CHR2=0.5
        SHR2=0.5
13      IF (S1N-50.0) 14,14,15
14      ES1=EXP(-2.0*S1N)
        CHS1=(1.0+ES1)/2.0
        SHS1=(1.0-ES1)/2.0
        GO TO 16
15      CHS1=0.5
        SHS1=0.5
16      K1N=CHR1*CRR1
        K2N=SHR1*SRR1
        K3N=SHR1*CRR1
        K4N=CHR1*SRR1
        K5N=CHR2*CRR2
        K6N=SHR2*SRR2
        K7N=SHR2*CRR2
        K8N=CHR2*SRR2
        L1N=CHS1*CSS1
        L2N=SHS1*SSS1
        L3N=SHS1*CSS1
        L4N=CHS1*SSS1
        L5N=CHR2*CSS2
        L6N=SHR2*SSS2
        L7N=SHR2*CSS2
        L8N=CHR2*SSS2
        RIN=1.-6./RNP/RNP
        I5N=-2./RNP*UN
        I1N=I5N*I5N/UN
        I3N=I1N*RIN*2.
        I7N=I5N*RIN
```

```
      BNAX=BNA/X2
      BNAY=BNA/Y2
      W1(1)=(K3*K7N+K4*K8N)/BNAX
      W1(2)=(K3*K8N-K4*K7N)/BNAX
      W1(3)=(K3*L7N+K5*L8N)/BNAY
      W1(4)=(K3*L8N-K5*L7N)/BNAY
      W1(5)=(K3*K3N+K4*K4N)/AN8
      W1(6)=(K3*K4N-K4*K3N)/AN8
      W1(7)=(K3*L3N+K5*L4N)/AN8
      W1(8)=(K3*L4N-K5*L3N)/AN8
      NHN=NH+N
      NH2N=NH2+N
      NH3N=NH3+N
      NH4N=NH4+N
      NH5N=NH5+N
      NH6N=NH6+N
      NH7N=NH7+N
      CD(NH4N,NH4N)=K1N
      CD(NH4N,NH5N)=K2N
      CD(NH4N,NH6N)=L1N
      CD(NH4N,NH7N)=L2N
      CD(NH4N,N82)=I1N
      CD(NH4N,NC)=T*I3N/B84*AA4
      CD(NH5N,NH4N)=-K4N
      CD(NH5N,NH5N)=K3N
      CD(NH5N,NH6N)=L4N
      CD(NH5N,NH7N)=-L3N
      CD(N81,N)=ON*W1(1)
      CD(N81,NHN)=ON*W1(2)
      CD(N81,NH2N)=ON*W1(3)
      CD(N81,NH3N)=ON*W1(4)
      STOR(NH5N,N81)=I5N
      STOR(NH5N,N83)=I7N
      IF (ZB) 17,19,18
17    CD(NH2N,N)=BN*(A25*K7N+A26*K8N)
      CD(NH2N,NHN)=BN*(-A26*K7N+A25*K8N)
      CD(NH2N,NH2N)=BN*(A25*L7N+B26*L8N)
      CD(NH2N,NH3N)=BN*(-B26*L7N+A25*L8N)
      CD(NH3N,N)=BN*(A26*K5N-A25*K6N)
      CD(NH3N,NHN)=BN*(A25*K5N+A26*K6N)
      CD(NH3N,NH2N)=BN*(-B25*L5N+A25*L6N)
      CD(NH3N,NH3N)=BN*(-A25*L5N-B26*L6N)
      CD(NH3N,N83)=TNB*I5N
      CD(NH3N,NC)=TNB4*I7N
      CD(NH6N,NH4N)=AN*(A23*K3N+A24*K4N)
      CD(NH6N,NH5N)=AN*(-A24*K3N+A23*K4N)
      CD(NH6N,NH6N)=AN*(B23*L3N+B24*L4N)
      CD(NH6N,NH7N)=AN*(-B24*L3N+B23*L4N)
      CD(NH7N,NH4N)=AN*(A24*K1N-A23*K2N)
      CD(NH7N,NH5N)=AN*(A23*K1N+A24*K2N)
      CD(NH7N,NH6N)=AN*(-B24*L1N+B23*L2N)
      CD(NH7N,NH7N)=AN*(-B23*L1N-B24*L2N)
      CD(NH7N,N82)=TNA*I5N
      CD(NH7N,NC)=TNA3*I7N
      STOR(NH2N,N84)=-3.*TN*I1N/B
      STOR(NH6N,N83)=-3.*TN*I1N/A
      STOR(N82,N)=OA1*(A27*W1(5)+A28*W1(6))
      STOR(N82,NHN)=OA1*(-A28*W1(5)+A27*W1(6))
      STOR(N82,NH2N)=OA1*(B27*W1(7)+B28*W1(8))
      STOR(N82,NH3N)=OA1*(-B28*W1(7)+B27*W1(8))
      STOR(NC,N)=OB1*(A21*W1(1)+A22*W1(2))
      STOR(NC,NHN)=OB1*(-A22*W1(1)+A21*W1(2))
      STOR(NC,NH2N)=OB1*(A21*W1(3)+B22*W1(4))
      STOR(NC,NH3N)=OB1*(-B22*W1(3)+A21*W1(4))
18    CD(N,N)=K5N
      CD(N,NHN)=K6N
      CD(N,NH2N)=L5N
      CD(N,NH3N)=L6N
      CD(N,N83)=I1N
      CD(N,NC)=-I3N
      CD(NHN,N)=-K8N
      CD(NHN,NHN)=K7N
      CD(NHN,NH2N)=L8N
      CD(NHN,NH3N)=-L7N
      CD(NC,NH4N)=ON*W1(5)
      CD(NC,NH5N)=ON*W1(6)
      CD(NC,NH6N)=ON*W1(7)
      CD(NC,NH7N)=ON*W1(8)
```

```
      STOR(NHN,N34)=I5N
      STOR(NHN,N54)=I7N
      IF (Z3) 22,19,20
19    EI3=-EI*BN
      CD(N,N)=BN3*(EI3*K5N+A10*K7N+A11*K3N)
      CD(N,NHN)=BN3*(EI3*K5N-A11*K7N+A10*K8N)
      CD(N,NH2N)=BN3*(EI3*L5N+B10*L7N+B11*L3N)
      CD(N,NH3N)=BN3*(EI3*L6N-B11*L7N+B10*L3N)
      CD(NHN,N)=BN3*(A11*K5N-A10*K6N-EI3*K3N)
      CD(NHN,NHN)=BN3*(A10*K5N+A11*K6N+EI3*K7N)
      CD(NHN,NH2N)=BN3*(-B11*L5N+B10*L6N+EI3*L3N)
      CD(NHN,NH3N)=BN3*(-B10*L5N-B11*L6N-EI3*L7N)
      CD(NHN,NC)=-24.0*R6*I5N/BB3
      CD(NH2N,N)=BN2*(A14*K5N+A15*K6N-AG*BN*K7N+A16*BN*K3N)
      CD(NH2N,NHN)=BN2*(-A15*K5N+A14*K6N-A16*BN*K7N-AG*BN*K8N)
      CD(NH2N,NH2N)=BN2*(B14*L5N+B15*L6N-AG*BN*L7N+B16*BN*L8N)
      CD(NH2N,NH3N)=BN2*(-B15*L5N+B14*L6N-B16*BN*L7N-AG*BN*L3N)
      CD(NH3N,N)=BN2*(A16*BN*K5N+AG*BN*K6N+A15*K7N-A14*K3N)
      CD(NH3N,NHN)=BN2*(-AG*BN*K5N+A16*BN*K6N+A14*K7N+A15*K8N)
      CD(NH3N,NH2N)=BN2*(-B16*BN*L5N-AG*BN*L6N-B15*L7N+B14*L8N)
      CD(NH3N,NH3N)=BN2*(AG*BN*L5N-B16*BN*L6N-B14*L7N-B15*L3N)
      CD(NH3N,NC)=-24.0*R10*I5N/BB3
      CD(NC,NH4N)=OA4*(A5*W1(5)+A6*W1(6))
      CD(NC,NH5N)=OA4*(-A6*W1(5)+A5*W1(6))
      CD(NC,NH6N)=OA4*(B5*W1(7)+B6*W1(8))
      CD(NC,NH7N)=OA4*(-B6*W1(7)-B5*W1(3))
      STOR(N81,N)=OA3*(A12*W1(5)+A13*W1(6))
      STOR(N81,NHN)=OA3*(-A13*W1(5)+A12*W1(6))
      STOR(N81,NH2N)=OA3*(B12*W1(7)+B13*W1(8))
      STOR(N81,NH3N)=OA3*(-B13*W1(7)+B12*W1(8))
      STOR(N82,N)=OA3*(A19*W1(5)+A20*W1(6))
      STOR(N82,NHN)=OA3*(-A20*W1(5)+A19*W1(6))
      STOR(N82,NH2N)=OA3*(B19*W1(7)+B20*W1(8))
      STOR(N82,NH3N)=OA3*(-B20*W1(7)+B19*W1(8))
      GO TO 21
20    CD(NH2N,N)=BN2*(A14*K5N+A15*K6N)
      CD(NH2N,NHN)=BN2*(-A15*K5N+A14*K6N)
      CD(NH2N,NH2N)=BN2*(B14*L5N+B15*L6N)
      CD(NH2N,NH3N)=BN2*(-B15*L5N+B14*L6N)
      CD(NH3N,N)=BN2*(A15*K7N-A14*K8N)
      CD(NH3N,NHN)=BN2*(A14*K7N+A15*K8N)
      CD(NH3N,NH2N)=BN2*(-B15*L7N+B14*L3N)
      CD(NH3N,NH3N)=BN2*(-B14*L7N-B15*L3N)
21    CD(NH2N,NC)=-R9*I1N/B8*12.0
      CD(NH6N,NH4N)=AN2*(A8*K1N+A9*K2N)
      CD(NH6N,NH5N)=AN2*(-A9*K1N+A8*K2N)
      CD(NH6N,NH6N)=AN2*(B8*L1N+B9*L2N)
      CD(NH6N,NH7N)=AN2*(-B9*L1N+B8*L2N)
      CD(NH7N,NH4N)=AN2*(-A8*K4N+A9*K3N)
      CD(NH7N,NH5N)=AN2*(A9*K4N+A8*K3N)
      CD(NH7N,NH6N)=AN2*(B8*L4N-B9*L3N)
      CD(NH7N,NH7N)=AN2*(-B9*L4N-B8*L3N)
      CD(N82,NH4N)=OA2*(A17*W1(5)+A18*W1(6))
      CD(N82,NH5N)=OA2*(-A18*W1(5)+A17*W1(6))
      CD(N82,NH6N)=OA2*(B17*W1(7)+B18*W1(8))
      CD(N82,NH7N)=OA2*(-B18*W1(7)+B17*W1(8))
      CD(N83,N)=OB2*(A7*W1(1)-R1K3*W1(2))
      CD(N83,NHN)=OB2*(R1K3*W1(1)+A7*W1(2))
      CD(N83,NH2N)=OB2*(B7*W1(3)+R1K3*W1(4))
      CD(N83,NH3N)=OB2*(-R1K3*W1(3)+B7*W1(4))
      STOR(NH3N,N54)=6.0*R9*I5N/B8
22    BNA2=BN2*AA
      BNX2=BNA2/X22
      BNY2=BNA2/Y22
      BNX=BNA2/X2
      BNY=BNA2/Y2
      ANB2=ANB*AN3
      ANX2=ANB2*X22
      ANY2=ANB2*Y22
      ANX=ANB2*X2
      ANY=ANB2*Y2
      A2AX=A2*ANX2
      B2AY=B2*ANY2
      A2BN=A2*BNA2
      B2BN=B2*BNA2
      TA1=A1*BNA2
      TB1=B1*BNA2
      TX1=ANX2*A1
```

```
      TY1=ANY2*B1
      IF (Z-1) 23,23,25
23    IF (NH-MNH) 25,24,25
24    WRITE (6,F19) NH,N,ON,AN,BN,AN2,BN2,AN3,BN3,AN4,I1N,I3N,I5N,R1N,R2
     1N,S1N,K1N,K2N,K3N,K4N,K5N,K6N,K7N,K8N,RR1,RR2,SS1,L1N,L2N,L3N,L4N,
     2L5N,L6N,L7N,L8N,RR2
      PRINT 58, RMP,RNPP,RNP5,BNA2,ANB2,BNX2,BNY2,ANX2,ANY2,A28N,B28N,A2
     1AX,B2AY,BNX,BNY,ANX,ANY,TA1,TB1,TX1,TY1,(I,I=1,8),(W1(I),I=1,8)
25    ON=1.0
      DO 57 M=1,NH
      ON=-ON
      RM=M
      RMP2=RM*PI
      RM22=RMP2*RMP2
      RM24=RM22*RM22
      SAA=2./(BNA2*(BNX2+2.*A2*RM22)+RM24)
      SBA=2./(BNA2*(BNY2+2.*B2*RM22)+RM24)
      SXA=2./(ANX2*(ANB2+2.*A2*RM22)+RM24)
      SYA=2./(ANY2*(ANB2+2.*B2*RM22)+RM24)
      SA1=SAA*RMP2*ON
      SB1=SBA*RMP2*ON
      SX1=SXA*RMP2*ON
      SY1=SYA*RMP2*ON
      SA2=SAA*BNA*ON
      SB2=SBA*BNA*ON
      SX2=SXA*ANB*ON
      SY2=SYA*ANB*ON
      TA2=A28N+RM22
      TB2=B28N+RM22
      TX2=A2AX+RM22
      TY2=B2AY+RM22
      TA3=(BNX+RM22)*K3
      TA4=(BNX-RM22)*K4
      TB3=(BNY+RM22)*K3
      TB4=(BNY-RM22)*K5
      TX3=(ANX+RM22)*K3*X2
      TX4=(ANX-RM22)*K4*X2
      TY3=(ANY+RM22)*K3*Y2
      TY4=(ANY-RM22)*K5*Y2
      AJ(1)=SA2*(TA3*K7N+TA4*K8N)
      AJ(2)=SA2*(-TA4*K7N+TA3*K8N)
      AJ(3)=SB2*(TB3*L7N+TB4*L8N)
      AJ(4)=SB2*(-TB4*L7N+TB3*L8N)
      AJ(5)=SX2*(TX3*K3N+TX4*K4N)
      AJ(6)=SX2*(-TX4*K3N+TX3*K4N)
      AJ(7)=SY2*(TY3*L3N+TY4*L4N)
      AJ(8)=SY2*(-TY4*L3N+TY3*L4N)
      BJ(1)=SA1*(-TA2*K7N-TA1*K8N)
      BJ(2)=SA1*(TA1*K7N-TA2*K8N)
      BJ(3)=SB1*(-TB2*L7N-TB1*L8N)
      BJ(4)=SB1*(TB1*L7N-TB2*L8N)
      BJ(5)=SX1*(-TX2*K3N-TX1*K4N)
      BJ(6)=SX1*(TX1*K3N-TX2*K4N)
      BJ(7)=SY1*(-TY2*L3N-TY1*L4N)
      BJ(8)=SY1*(TY1*L3N-TY2*L4N)
      IF (Z-1) 23,26,28
26    IF (NH-MNH) 28,27,28
27    WRITE (6,59) NH,N,M
      PRINT 60, RMP,RMPP,RMP2,RM24,RM25,CMR,SAA,SBA,SXA,SYA,SA1,SB1,SX1,
     1SY1,SA2,SB2,SX2,SY2,TA2,TB2,TX2,TY2,TA3,TB3,TX3,TY3,TA4,TB4,TX4,TY
     24
      WRITE (6,61) (I,I=1,8),(AJ(I),I=1,8),(BJ(I),I=1,8)
28    NHM=NH+M
      NH2M=NH2+M
      NH3M=NH3+M
      NH4M=NH4+M
      NH5M=NH5+M
      NH6M=NH5+M
      NH7M=NH7+M
      CD(NH4M,M)=ON*AJ(1)
      CD(NH4M,NHM)=ON*AJ(2)
      CD(NH4M,NH2M)=ON*AJ(3)
      CD(NH4M,NH3M)=ON*AJ(4)
      STOR(NH5M,M)=ON*BJ(1)
      STOR(NH5M,NHM)=ON*BJ(2)
      STOR(NH5M,NH2M)=ON*BJ(3)
      STOR(NH5M,NH3M)=ON*BJ(4)
      IF (ZB) 29,31,30
```

```
29    CD(NH3M,NH4N)=OA1*(A27*BJ(5)+A28*BJ(6))
      CD(NH3M,NH5N)=OA1*(-A28*BJ(5)+A27*BJ(6))
      CD(NH3M,NH6N)=OA1*(B27*BJ(7)+B28*BJ(8))
      CD(NH3M,NH7N)=OA1*(-B28*BJ(7)+B27*BJ(8))
      CD(NH7M,N)=OB1*(A21*BJ(1)+A22*BJ(2))
      CD(NH7M,NHN)=OB1*(-A22*BJ(1)+A21*BJ(2))
      CD(NH7M,NH2N)=OB1*(A21*BJ(3)+B22*BJ(4))
      CD(NH7M,NH3N)=OB1*(-B22*BJ(3)+A21*BJ(4))
      STOR(NH2M,N)=OA1*(A27*AJ(5)+A28*AJ(6))
      STOR(NH2M,NHN)=OA1*(-A28*AJ(5)+A27*AJ(6))
      STOR(NH2M,NH2N)=OA1*(B27*AJ(7)+B28*AJ(8))
      STOR(NH2M,NH3N)=OA1*(-B28*AJ(7)+B27*AJ(8))
      STOR(NH6M,N)=OB1*(A21*AJ(1)+A22*AJ(2))
      STOR(NH6M,NHN)=OB1*(-A22*AJ(1)+A21*AJ(2))
      STOR(NH6M,NH2N)=OB1*(A21*AJ(3)+B22*AJ(4))
      STOR(NH6M,NH3N)=OB1*(-B22*AJ(3)+A21*AJ(4))
30    CD(M,NH4N)=ON*AJ(5)
      CD(M,NH5N)=ON*AJ(6)
      CD(M,NH6N)=ON*AJ(7)
      CD(M,NH7N)=ON*AJ(8)
      STOR(NHM,N)=ON*BJ(5)
      STOR(NHM,NHN)=ON*BJ(6)
      STOR(NHM,NH2N)=ON*BJ(7)
      STOR(NHM,NH3N)=ON*BJ(8)
      IF (Z3) 33,31,32
31    CD(M,NH4N)=OA4*(A5*AJ(5)+A6*AJ(6))
      CD(M,NH5N)=OA4*(-A6*AJ(5)+A5*AJ(6))
      CD(M,NH6N)=OA4*(B5*AJ(7)+B6*AJ(8))
      CD(M,NH7N)=OA4*(-B6*AJ(7)+B5*AJ(8))
      CD(NHM,NH4N)=OA3*(A12*BJ(5)+A13*BJ(6))
      CD(NHM,NH5N)=OA3*(-A13*BJ(5)+A12*BJ(6))
      CD(NHM,NH6N)=OA3*(B12*BJ(7)+B13*BJ(8))
      CD(NHM,NH7N)=OA3*(-B13*BJ(7)+B12*BJ(8))
      CD(NH3M,NH4N)=OA3*(A19*BJ(5)+A20*BJ(6))
      CD(NH3M,NH5N)=OA3*(-A20*BJ(5)+A19*BJ(6))
      CD(NH3M,NH6N)=OA3*(B19*BJ(7)+B20*BJ(8))
      CD(NH3M,NH7N)=OA3*(-B20*BJ(7)+B19*BJ(8))
      STOR(M,N)=OA3*(A12*AJ(5)+A13*AJ(6))
      STOR(M,NHN)=OA3*(-A13*AJ(5)+A12*AJ(6))
      STOR(M,NH2N)=OA3*(B12*AJ(7)+B13*AJ(8))
      STOR(M,NH3N)=OA3*(-B13*AJ(7)+B12*AJ(8))
      STOR(NHM,N)=OA4*(A5*BJ(5)+A6*BJ(6))
      STOR(NHM,NHN)=OA4*(-A6*BJ(5)+A5*BJ(6))
      STOR(NHM,NH2N)=OA4*(B5*BJ(7)+B6*BJ(8))
      STOR(NHM,NH3N)=OA4*(-B6*BJ(7)+B5*BJ(8))
      STOR(NH2M,N)=OA3*(A19*AJ(5)+A20*AJ(6))
      STOR(NH2M,NHN)=OA3*(-A20*AJ(5)+A19*AJ(6))
      STOR(NH2M,NH2N)=OA3*(B19*AJ(7)+B20*AJ(8))
      STOR(NH2M,NH3N)=OA3*(-B20*AJ(7)+B19*AJ(8))
32    CD(NH2M,NH4N)=OA2*(A17*AJ(5)+A18*AJ(6))
      CD(NH2M,NH5N)=OA2*(-A18*AJ(5)+A17*AJ(6))
      CD(NH2M,NH6N)=OA2*(B17*AJ(7)+B18*AJ(8))
      CD(NH2M,NH7N)=OA2*(-B18*AJ(7)+B17*AJ(8))
      CD(NH6M,N)=OB2*(A7*AJ(1)-R1K3*AJ(2))
      CD(NH6M,NHN)=OB2*(R1K3*AJ(1)+A7*AJ(2))
      CD(NH6M,NH2N)=OB2*(B7*AJ(3)+R1K3*AJ(4))
      CD(NH6M,NH3N)=OB2*(-R1K3*AJ(3)+B7*AJ(4))
      STOR(NH3M,N)=OA2*(A17*BJ(5)+A18*BJ(6))
      STOR(NH3M,NHN)=OA2*(-A18*BJ(5)+A17*BJ(6))
      STOR(NH3M,NH2N)=OA2*(B17*BJ(7)+B18*BJ(8))
      STOR(NH3M,NH3N)=OA2*(-B18*BJ(7)+B17*BJ(8))
      STOR(NH7M,N)=OB2*(A7*BJ(1)-R1K3*BJ(2))
      STOR(NH7M,NHN)=OB2*(R1K3*BJ(1)+A7*BJ(2))
      STOR(NH7M,NH2N)=OB2*(B7*BJ(3)+R1K3*BJ(4))
      STOR(NH7M,NH3N)=OB2*(-R1K3*BJ(3)+B7*BJ(4))
33    AM=RMP2/A
      AM2=AM*AM
      AM3=AM2*AM
      AM4=AM2*AM2
      CNM=CN/CM
      DO 57 L=1,NL
      RMN=OX*AM4+SCA*AM2*BN2+SCB*BN4
      SMN=(SCC*AM2+SCD*BN2)*AM*BN
      BV1=BN*VL(L,2)
      CV1=COS(BV1)
      SV1=SIN(BV1)
      AU1=AM*VL(L,1)
      CU1=COS(AU1)
```

```
      SUI=SIN(AUI)                                                      C 459
      Q1MN=CUI*CVI                                                      C 460
      Q2MN=SUI*SVI                                                      C 461
      Q3MN=SUI*CVI                                                      C 462
      Q4MN=CUI*SVI                                                      C 463
      ALR=AM*VL(L,3)/2.                                                 C 464
      BLS=BN*VL(L,4)/2.                                                 C 465
      SAA=SIN(ALR)/ALR                                                  C 466
      SBB=SIN(BLS)/BLS                                                  C 467
      RS=CM/(RMN**2-SMN**2)*SAA*SBB*CQ(L)                               C 468
      RSR=RS*RMN                                                        C 469
      RSS=RS*SMN                                                        C 470
      KMN=Q1MN*RSR-Q2MN*RSS                                             C 471
      LMN=Q2MN*RSR-Q1MN*RSS                                             C 472
      PMN=Q3MN*RSR+Q4MN*RSS                                             C 473
      QMN=Q4MN*RSR+Q3MN*RSS                                             C 474
      NCL=NC+L                                                          C 475
      NAL=NS4+L                                                         C 476
      IF (M-1) 45,34,46                                                 C 477
34    IF (N-1) 40,35,40                                                 C 478
35    IF (Z8) 36,38,37                                                  C 479
36    CD(NC,NCL)=-COS(L)*BB4/180.                                       C 480
      GO TO 40                                                          C 482
37    CD(NC,NCL)=-COS(L)*BB4/30.                                        C 483
      IF (Z8) 40,38,39                                                  C 484
38    CD(NC,NCL)=EI*COS(L)/4.                                           C 485
      CD(NB1,NCL)=FUI
39    CD(NB2,NCL)=R9*COS(L)*BB/12.+FVI                                  C 486
40    CCD=COS(L)*BB/2.                                                  C 487
      T6N=CCD*CVI                                                       C 488
      T8N=CCD*SVI                                                       C 489
      CD(NB1,NCL)=CD(NB1,NCL)-T6N*ON/BN4                                C 490
      IF (Z8) 41,44,42                                                  C 491
41    CD(N,NCL)=-COS(L)*(I3N-2.*I1N)*BB4/96.-T6N/BN4                    C 492
      CD(NH3N,NCL)=BBTN*COS(L)*(I7N-I5N)-T6N/BN3*TN                     C 493
C     CD(NH3N,NCL)=                    BBTN*COS(L)*(I7N-2.*I5N)-T6N/BN3*TN C 494
      STOR(NH2N,NAL)=TN*T8N/BN3                                         C 495
      STOR(NC,NAL)=STOR(NC,NAL)-T8N*ON/BN3/C                            C 496
      GO TO 43                                                          C 497
42    CD(N,NCL)=-COS(L)*(I3N/6.-I1N)*BB4/16.-T6N/BN4                    C 498
43    STOR(NHN,NAL)=-T8N/BN4                                            C 499
      IF (Z8) 46,44,45                                                  C 500
44    SM1=-COS(L)*B*I5N/4.-T6N/BN                                       C 501
      CD(N,NCL)=EI*T6N                                                  C 502
      CD(NHN,NCL)=R5*SM1                                                C 503
      CD(NH3N,NCL)=R10*SM1                                              C 504
      STOR(N,NAL)=R6*T8N/BN                                             C 505
      STOR(NHN,NAL)=EI*T8N                                              C 506
      STOR(NH2N,NAL)=R10*T8N/BN                                         C 507
   45 DO 206 K=1,MV
  206 VMC(L)=VMC(L)+2.* FV(K)*((-1.)**N)*COS(2.*N*PI*(K-0.5)/MV)
      CD(NH2N,NCL)=-R9*(COS(L)*BB*I1N/8.-T6N/BN2)-C*VMC(L)/(2.*B)
      CD(NB3,NCL)=CD(NB3,NCL)+R2*T6N*ON/BN2                             C 509
      DO 209 K=1,MV
  209 VMC1(L)=VMC1(L)+2.*FV(K)*((-1.)**N)*SIN(2.*N*PI*(K-0.5)/MV)
      STOR(NH3N,NAL)=R9*T8N/BN2-C*VMC1(L)/(2.*B)
46    IF (N-1) 47,47,52                                                 C 511
47    CQD=CQ(L)*SAA/DX/2.                                               C 512
      T5M=CCD*CUI                                                       C 513
      T7M=CCD*SUI                                                       C 514
      CD(NH4M,NCL)=-T5M/AM4                                             C 515
      STOR(NH5M,NAL)=-T7M/AM4                                           C 516
      IF (Z8) 48,50,49                                                  C 517
48    CD(NH7M,NCL)=-T5M/AM3*TN                                          C 518
      STOR(NH6M,NAL)=T7M/AM3*TN                                         C 519
      STOR(NB2,NAL)=STOR(NB2,NAL)-T7M*OM/AM3/C                          C 520
49    CD(NC,NCL)=CD(NC,NCL)-T5M*OM/AM4                                  C 521
      IF (Z8) 52,50,51                                                  C 522
50    STOR(NB1,NAL)=STOR(NB1,NAL)+R3*T7M*OM/AM                          C 523
      DO 207 K=1,MU
  207 UMC(L)=UMC(L)+2.* FU(K)*((-1.)**N)*COS(2.*N*PI*(K-0.5)/MU)
      CD(NH6M,NCL)=UMC(L)/(2.*A)
      DO 208 K=1,MU
  208 UMC1(L)=UMC1(L)+2.*FU(K)*((-1.)**N)*SIN(2.*N*PI*(K-0.5)/MU)
      STOR(NH7M,NAL)=UMC1(L)/(2.*A)
51    CD(NB2,NCL)=CD(NB2,NCL)+R7*T5M*OM/AM2                             C 524
52    SK1=EI*BN4                                                        C 525
      SK2=-(R4*AM2+R6*BN2)*BN                                           C 526
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```
      SK3=(R3*AM2+R5*BN2)*AM
      CK1=R7*AM2+R9*BN2
      CK2=-R8*AM*BN
      CK3=-R10*BN3
      CK4=G1*AM*BN2
      CX5=R2*BN2*QNM
      CX6=-R1*AM*BN*QNM
      CD(NH4M,NCL)=CD(NH4M,NCL)-KMN*QNM
      STOR(NH5M,NAL)=STOR(NH5M,NAL)-PMN*QNM
      IF (Z3) 53,53,54
53    CD(NH3N,NCL)=CD(NH3N,NCL)-(TN*BN*KMN+AM/C*LMN)
      CD(NH7M,NCL)=CD(NH7M,NCL)-(TN*AM*KMN+BN/C*LMN)*QNM
      STOR(NH2N,NAL)=STOR(NH2N,NAL)-AM/C*PMN+TN*BN*QMN
      STOR(NH6M,NAL)=STOR(NH6M,NAL)+(TN*AM*PMN-BN/C*QMN)*QNM
54    CD(N,NCL)=CD(N,NCL)-KMN
      STOR(NHN,NAL)=STOR(NHN,NAL)-QMN
      IF (Z3) 57,55,56
55    CD(N,NCL)=CD(N,NCL)+SK1*KMN
      CD(NHN,NCL)=CD(NHN,NCL)+SK2*KMN+SK3*LMN
      CD(NH3N,NCL)=CD(NH3N,NCL)+CK3*KMN+CK4*LMN
      STOR(N,NAL)=STOR(N,NAL)+SK3*PMN-SK2*QMN
      STOR(NHN,NAL)=STOR(NHN,NAL)+SK1*QMN
      STOR(NH2N,NAL)=STOR(NH2N,NAL)+CK4*PMN-CK3*QMN
56    CD(NH2N,NCL)=CD(NH2N,NCL)+CK1*KMN+CK2*LMN
      CD(NH6M,NCL)=CD(NH6M,NCL)+CX5*KMN+CX6*LMN
      STOR(NH3N,NAL)=STOR(NH3N,NAL)-CK2*PMN+CK1*QMN
      STOR(NH7M,NAL)=STOR(NH7M,NAL)+CX5*PMN-CX6*QMN
57    CONTINUE
      SYAS=1.
      RETURN
C
58     FORMAT (3X,'RNP',13X,'RNPP',12X,'RNP5',12X,'BNA2',12X,'ANB2',/1X,5
     1F16.8/9X,'B1X2',12X,'BNY2',12X,'ANX2',12X,'ANY2',12X,'A2BN',8X,'B2
     2BN',12X,'A2AX',12X,'B2AY'/1X,8F16.8/3X,'BNX ',12X,'BNY ',12X,'ANX
     3',12X,'ANY ',12X,'TA1 ',12X,'TB1',12X'TX1',12X'TY1'/1X8F16.8/5X8I1
     45/6X'A1',5X8F15.8)
59     FORMAT (/30X'NH=',I5,5X,'N =',I5,5X,'M =',I5/)
60     FORMAT (3X,'RMP',13X,'RMPP',12X'RMP2',12X,'RM24',12X,'RM25',12X,'Q
     1MR'/1X,6F16.8/9X,'SAA',13X,'SBA',13X,'SXA',13X,'SYA',13X,'SA1',13X
     2,'SB1',13X,'SX1',13X,'SY1'/1X,8F16.8/9X,'SA2',13X,'SB2',13X,'SX2',
     313X,'SY2',13X,'TA2',13X,'TB2',13X,'TX2',13X,'TY2'/1X,8F16.8/9X,'TA
     43',13X,'TB3',13X,'TX3',13X,'TY3',13X,'TA4',13X,'TB4',13X,'TX4',13X
     5,'TY4'/1X,8F16.8)
61     FORMAT (5X,8I15/5X,'AJ ',5X,8F15.8/5X,'BJ',5X,8F15.8)
       END
```

```
C     ******************************************************************************
      SUBROUTINE ASLA (NS1,NSL,SYAS,CD,DC,STOR,NS,JS,LN,NST1,NST2)
C     ******************************************************************************
      DOUBLE PRECISION CH,DC
      DIMENSION CD(NS,JS), DC(NS,LN), STOR(NST1,NST2)
      COMMON/BL5/NHC,N83,NC,JC,NH2,NH3,NH4,NH5,NH6,NH7,NH8,N81,N82,NH,NL
      DIMENSION CDJ(4), CDK(4)
      DO 3 J=1,NH4
      NH4J=NH4+J
      DO 1 I=1,NH4
      CD(I,NH4J)=STOR(I,J)
1     CONTINUE
      NH41=NH4+1
      DO 2 I=NH41,NH8
      CD(I,J)=STOR(I,J)
2     CONTINUE
      CD(N81,J)=0.
      CD(N81,NH4J)=STOR(N81,J)
      CD(N82,J)=0.
      CD(N82,NH4J)=STOR(N82,J)
      CD(N83,J)=STOR(N83,J)
      CD(N83,NH4J)=0.
      CD(NC,J)=STOR(NC,J)
      CD(NC,NH4J)=0.
3     CONTINUE
      DO 5 N=1,NH
      NHN=NH+N
      NH2N=NH2+N
      NH3N=NH3+N
      NH4N=NH4+N
      NH5N=NH5+N
      NH6N=NH6+N
      NH7N=NH7+N
      DO 5 I=1,2
      INHN=NH*(2*I-1)+N
      INH1=INHN-NH
      INH2=INH1+NH4
      DO 4 J=1,4
      NJ=(J-1)*NH+N
      NK=(J+3)*NH+N
      CDJ(J)=CD(INH1,NJ)
      CDK(J)=CD(INH2,NK)
4     CONTINUE
      CD(INH1,N)=CD(INHN,NHN)
      CD(INH1,NHN)=-CD(INHN,N)
      CD(INH1,NH2N)=-CD(INHN,NH3N)
      CD(INH1,NH3N)=CD(INHN,NH2N)
      CD(INHN,N)=-CDJ(2)
      CD(INHN,NHN)=CDJ(1)
      CD(INHN,NH2N)=CDJ(4)
      CD(INHN,NH3N)=-CDJ(3)
      INHN=INHN+NH4
      CD(INH2,NH4N)=CD(INHN,NH5N)
      CD(INH2,NH5N)=-CD(INHN,NH4N)
      CD(INH2,NH6N)=-CD(INHN,NH7N)
      CD(INH2,NH7N)=CD(INHN,NH6N)
      CD(INHN,NH4N)=-CDK(2)
      CD(INHN,NH5N)=CDK(1)
      CD(INHN,NH6N)=CDK(4)
5     CD(INHN,NH7N)=-CDK(3)
      DO 6 J=NS1,NSL
      NH8J=NH4+J
      DO 6 I=1,NC
6     CD(I,NH8J)=STOR(I,J)
      SYAS=-1.
      RETURN
      END
```

```
C     ************************************************************************
      SUBROUTINE MINA (PI,NSS,CD,DC,STOR,NS,JS,LN,NST1,NST2,L1,M1,CH)
C     ************************************************************************
      DOUBLE PRECISION CH,DC
      REAL K3,K4,K5
      COMMON /BL1/ P,CQ(4),CDS(4),K3,K4,K5,B8,B83,B84,X2,Y2,A1,A2,A3,A4,
     1B1,B2,B3,B4,SCA,SCB,SCC,SCD,S1(4),S2(4),S3(4),S4(4),S5(4),S6(4),
     2    S7(4)
      COMMON/BL3/NHC,N83,NC,JC,NH2,NH3,NH4,NH5,NH6,NH7,NH8,N81,N82,NH,NL
      DIMENSION CD(NS,JS), DC(NS,LN), STOR(NST1,NST2), L1(NS), M1(NS), C
     1H(NSS)
      DO 1 J=1,NC
      NCJ=NC*(J-1)
      DO 1 I=1,NC
    1 CH(NCJ+I)=CD(I,J)
      NCC=NC*NC
      CALL MINV (CH,NC,D,L1,M1,NSS,NS)
      DO 2 I=1,NC
      DO 2 L=1,NL
      DC(I,L)=0.0
      DO 2 J=1,NC
    2 DC(I,L)=DC(I,L)+CH(NC*(J-1)+I)*CD(J,NC+L)
      DO 3 N=1,NH
      RN=N
      PN=PI*RN
      CKN=K3*PN
      PKE=CKN/P
      ER1=EXP(X2*PKE)
      ER2=EXP(CKN*P)
      ES1=EXP(Y2*PKE)
      NHN=NH+N
      NH2N=NH2+N
      NH3N=NH3+N
      NH4N=NH4+N
      NH5N=NH5+N
      NH6N=NH6+N
      NH7N=NH7+N
      DO 3 L=1,NL
      DC(N,L)=DC(N,L)/ER2
      DC(NHN,L)=DC(NHN,L)/ER2
      DC(NH2N,L)=DC(NH2N,L)/ER2
      DC(NH3N,L)=DC(NH3N,L)/ER2
      DC(NH4N,L)=DC(NH4N,L)/ER1
      DC(NH5N,L)=DC(NH5N,L)/ER1
      DC(NH6N,L)=DC(NH6N,L)/ES1
    3 DC(NH7N,L)=DC(NH7N,L)/ES1
      RETURN
      END
```

```
C       ************************************************************************
        SUBROUTINE MINV (A,N,D,L,M,NSS,NS)
C       ************************************************************************
        DOUBLE PRECISION A,DC,DABS,BIGA,HOLD
        DIMENSION A(NSS), L(NS), M(NS)
        D=1.0
        NK=-N
        DO 18 K=1,N
        NK=NK+N
        L(K)=K
        M(K)=K
        KK=NK+K
        BIGA=A(KK)
        DO 2 J=K,N
        IZ=N*(J-1)
        DO 2 I=K,N
        IJ=IZ+I
C       10 IF( ABS(BIGA)- ABS(A(IJ))) 15,20,20
        IF (DABS(BIGA)-DABS(A(IJ))) 1,2,2
1       BIGA=A(IJ)
        L(K)=I
        M(K)=J
2       CONTINUE
        J=L(K)
        IF (J-K) 5,5,3
3       KI=K-N
        DO 4 I=1,N
        KI=KI+N
        HOLD=-A(KI)
        JI=KI-K+J
        A(KI)=A(JI)
        A(JI)=HOLD
4       CONTINUE
5       I=M(K)
        IF (I-K) 8,8,6
6       JP=N*(I-1)
        DO 7 J=1,N
        JK=NK+J
        JI=JP+J
        HOLD=-A(JK)
        A(JK)=A(JI)
        A(JI)=HOLD
7       CONTINUE
8       IF (BIGA) 10,9,10
9       D=0.0
        PRINT 27
        RETURN
10      DO 12 I=1,N
        IF (I-K) 11,12,11
11      IK=NK+I
        A(IK)=A(IK)/(-BIGA)
12      CONTINUE
        DO 15 I=1,N
        IK=NK+I
        HOLD=A(IK)
        IJ=I-N
        DO 15 J=1,N
        IJ=IJ+N
        IF (I-K) 13,15,13
13      IF (J-K) 14,15,14
14      KJ=IJ-I+K
        A(IJ)=HOLD*A(KJ)+A(IJ)
15      CONTINUE
        KJ=K-N
        DO 17 J=1,N
        KJ=KJ+N
        IF (J-K) 16,17,16
16      A(KJ)=A(KJ)/BIGA
17      CONTINUE
C       D=D*BIGA
        A(KK)=1.0/BIGA
18      CONTINUE
        K=N
19      K=(K-1)
        IF (K) 25,25,20
20      I=L(K)
```

```
      IF (I-K) 23,23,21
21    JQ=N*(K-1)
      JR=N*(I-1)
      DO 22 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
      A(JI)=HOLD
22    CONTINUE
23    J=M(K)
      IF (J-K) 19,19,24
24    KI=K-N
      DO 25 I=1,N
      KI=KI+N
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
      A(JI)=HOLD
25    CONTINUE
      GO TO 19
26    RETURN
C
27    FORMAT (/10X'THE MATRIX IS SINGULAR')
      END
```

```
C     ***************************************************************************
      SUBROUTINE DS2A (CD,DC,STOR,NS,JS,LN,NST1,NST2)
C     ***************************************************************************
      DOUBLE PRECISION CH,DC
C
C        COMPUTE DEFLECTIONS AND MOMENTS  (OLD SERIES)
C
      REAL KMN,MK1,MK2,MK3,MK4,MK5,MK6,ML1,ML2,ML3,ML4,ML,K3,K4,K5,MF,LM
     1N
      COMMON A,PI,SA,C,S,TN,C4,AA,AA3,AA4,Z,SYAS,C2,C3,ST,SF,T,TS,MNH
      COMMON /BL0/ B,DX,DY,DXY,DYX,D1,D2,DI,GJ,H,VL(4,5),ZB
      COMMON /BL1/ P,CO(4),COS(4),K3,K4,K5,BB,BB3,BB4,X2,Y2,A1,A2,A3,A4,
     1B1,B2,B3,B4,SCA,SCB,SCC,SCD,S1(4),S2(4),S3(4),S4(4),S5(4),S6(4),
     2    S7(4)
      COMMON/BL5/NHC,NB3,NC,JC,NH2,NH3,NH4,NH5,NH6,NH7,NH8,NB1,NB2,NH,NL
      COMMON /EXL/ UC(24),VC(24),NGC,YA(400,24),NR,UR(9),VR(3),NP(10)
      DIMENSION CD(NS,JS), DC(NS,LN), STOR(NST1,NST2)
      DIMENSION P1(20), P2(20), P3(20), P4(20), P5(20), P6(20), P7(20),
     1P8(20), CC(20,20), SS(20,20)
      LT=MNH*10
      DO 14 I=1,NJC
      U=UC(I)
      V=VC(I)
      DO 1 N=1,NH
      RN=N
      RNP2=RN*PI
      AN=RNP2/A
      BN=RNP2/B
      VN=BN*V
      AU=AN*U
      BU=BN*U
      AV=AN*V
      AVK=AV*K3
      U3N=K3*BU
      V3N=AVK*X2
      V3Y=AVK*Y2
      U4V=K4*BU+VN
      U5V=K5*BU-VN
      V4U=K4*X2*AV+AU
      V5U=K5*Y2*AV-AU
      CHUN=COSH(U3N)
      SHUN=SINH(U3N)
      P1(N)=CHUN*COS(U4V)
      P2(N)=SHUN*SIN(U4V)
      P3(N)=CHUN*COS(U5V)
      P4(N)=SHUN*SIN(U5V)
      P5(N)=COSH(V3N)*COS(V4U)
      P6(N)=SINH(V3N)*SIN(V4U)
      P7(N)=COSH(V3Y)*COS(V5U)
      P8(N)=SINH(V3Y)*SIN(V5U)
      CV=COS(VN)
      SV=SIN(VN)
      DO 1 M=1,NH
      RM=M
      UM=RM*PI*U/A
      CC(M,N)=COS(UM)*CV
1     SS(M,N)=SIN(UM)*SV
      DO 14 L=1,NL
      LNW = NHC+(L-1)*LT
      CO4=COS(L)/4.
      WL=DC(NB1,L)+DC(NB2,L)*(U/A)**2+DC(NB3,L)*(V/B)**2+(T*U**4-V**4)/B
     1B4*DC(NC,L)
      IF (ZB) 3,2,2
2     WL=WL+CO4*(V**4-(V*B)**2*6.+5.*BB4)/24.
      GO TO 4
3     WL=WL+CO4*(V**4-(V*B)**2*2.+BB4)/24.
4     DO 13 K=1,3
      LNM=LNW+MNH*K
      ML=2.*S1(K)*DC(NB2,L)/AA+2.*S3(K)*DC(NB3,L)/BB+12.*(T*U**2*S1(K)-V
     1**2*S3(K))*DC(NC,L)/BB4
      IF (ZB) 6,5,5
5     ML=ML+CO4*(V*V-BB)/2.*S3(K)
      GO TO 7
6     ML=ML+CO4*(V*V-BB/3.)/2.*S3(K)
7     MK1=S1(K)*A2-S2(K)*K4-S3(K)
      MK2=S1(K)*A1+S2(K)*K3
```

```
      MK3=S1(K)+S2(K)*X2*K4-S3(K)*X2**2*A2
      MK4=(S2(K)*K3+S3(K)*X2*A1)*X2
      ML1=S1(K)*B2+S2(K)*K5-S3(K)
      ML2=S1(K)*B1-S2(K)*K3
      ML3=S1(K)-S2(K)*Y2*K5-S3(K)*Y2**2*B2
      ML4=(S2(K)*K3-S3(K)*Y2*B1)*Y2
      DO 12 N=1,NH
      RN=N
      RNP2=RN*PI
      AN=RNP2/A
      BN=RNP2/B
      AN2=AN**2
      BN2=BN**2
      BN4=BN2*BN2
      BV1=BN*VL(L,2)
      CV1=COS(BV1)
      SV1=SIN(BV1)
      BLS=BN*VL(L,4)/2.
      SBB=SIN(BLS)/BLS
      DO 10 M=1,NH
      RM=M
      AM=RM*PI/A
      AM2=AM*AM
      AM4=AM2*AM2
      AU1=AM*VL(L,1)
      CU1=COS(AU1)
      Q1MN=CU1*CV1
      Q2MN=SIN(AU1)*SV1
      RMN=DX*AM4+SCA*AM2*BN2+SCB*BN4
      SMN=(SCC*AM2+SCD*BN2)*AM*BN
      ALR=AM*VL(L,3)/2.
      SAA=SIN(ALR)/ALR
      RS=1./(RMN**2-SMN**2)*SAA*SBB*CQ(L)
      RSR=RS*RMN
      RSS=RS*SMN
      KMN=Q1MN*RSR-Q2MN*RSS
      LMN=Q2MN*RSR-Q1MN*RSS
      IF (N.GT.1) GO TO 8
      T5MB=CQ(L)*CU1*SAA/2./DX/AM2*COS(AM*U)
      ML=ML-T5MB*S1(K)
      IF (K.GT.1) GO TO 9
      WL=WL+T5MB/AM2
8     IF (K.GT.1) GO TO 9
      WL=WL+KMN*CC(M,N)+LMN*SS(M,N)
9     MK5=AM2*S1(K)+BN2*S3(K)
      MK6=BN*AM*S2(K)
      ML=ML-(MK5*KMN-MK6*LMN)*CC(M,N)+(MK6*KMN-MK5*LMN)*SS(M,N)
10    CONTINUE
      NHN=NH+N
      NH2N=NH2+N
      NH3N=NH3+N
      NH4N=NH4+N
      NH5N=NH5+N
      NH6N=NH6+N
      NH7N=NH7+N
      T6NB=COS(L)*CV1*SBB/2./BN2*COS(BN*V)
      IF (K.GT.1) GO TO 11
      WL=T6NB/BN2+DC(N,L)*P1(N)+DC(NHN,L)*P2(N)+DC(NH2N,L)*P3(N)+DC(NH3N
     1,L)*P4(N)+DC(NH4N,L)*P5(N)+DC(NH5N,L)*P6(N)+DC(NH6N,L)*P7(N)+DC(NH
     27N,L)*P8(N)+WL
11    ML=ML+AN2*((-MK3*P5(N)-MK4*P6(N))*DC(NH4N,L)+(MK4*P5(N)-MK3*P6(N))
     1*DC(NH5N,L)+(-ML3*P7(N)+ML4*P8(N))*DC(NH6N,L)+(-ML4*P7(N)-ML3*P8(N
     2))*DC(NH7N,L))
      ML=ML+BN2*((MK1*P1(N)-MK2*P2(N))*DC(N,L)+(MK2*P1(N)+MK1*P2(N))*DC(
     1NHN,L)+(ML1*P3(N)-ML2*P4(N))*DC(NH2N,L)+(ML2*P3(N)+ML1*P4(N))*DC(N
     2H3N,L))-T6NB*S3(K)
12    CONTINUE
      YA(LNM,I)=ML
13    CONTINUE
14    YA(LNW,I)=WL
      RETURN
      END
```

```
C     ****************************************************************
      SUBROUTINE DA2A (CD,DC,STOR,NS,JS,LN,NST1,NST2)
C     ****************************************************************
      DOUBLE PRECISION CD,DC
      REAL KMN,MK1,MK2,MK3,MK4,MK5,MK6,ML1,ML2,ML3,ML4,ML,K3,K4,K5,MF,LM
     1N
      COMMON A,PI,SA,C,S,TN,C4,AA,AA3,AA4,Z,SYAS,C2,C3,ST,SF,T,TB,MNH
      COMMON /BLD/ B,DX,DY,DXY,DYX,D1,D2,DI,GJ,H,VL(4,5),ZB
      COMMON /BL1/ P,CO(4),COS(4),K3,K4,K5,BB,BB3,BB4,X2,Y2,A1,A2,A3,A4,
     1B1,B2,B3,B4,SCA,SCB,SCC,SCD,S1(4),S2(4),S3(4),S4(4),S5(4),S6(4),
     2     S7(4)
      COMMON/BL5/ NHC,NB3,NC,JC,NH2,NH3,NH4,NH5,NH6,NH7,NH8,NB1,NB2,NH,NL
      COMMON /EXL/ UC(24),VC(24),NGC,YA(400,24),NR,UR(9),VR(3),NP(10)
      DIMENSION CD(NS,JS), DC(NS,LN), STOR(NST1,NST2)
      DIMENSION P9(20), P10(20), P11(20), P12(20), P13(20), P14(20), P15
     1(20), P16(20), SC(20,20), CS(20,20)
      LT=MNH*10
      DO 8 I=1,NGC
      U=UC(I)
      V=VC(I)
      DO 1 N=1,NH
      RN=N
      RNP2=RN*PI
      AN=RNP2/A
      BN=RNP2/B
      VN=BN*V
      AU=AN*U
      BU=BN*U
      AV=AN*V
      AVK=AV*K3
      U3N=K3*BU
      V3N=AVK*X2
      V3Y=AVK*Y2
      U4V=K4*BU+VN
      U5V=K5*BU-VN
      V4U=K4*X2*AV+AU
      V5U=K5*Y2*AV-AU
      CHUN=COSH(U3N)
      SHUN=SINH(U3N)
      P9(N)=SHUN*COS(U4V)
      P10(N)=CHUN*SIN(U4V)
      P11(N)=SHUN*COS(U5V)
      P12(N)=CHUN*SIN(U5V)
      P13(N)=SINH(V3N)*COS(V4U)
      P14(N)=COSH(V3N)*SIN(V4U)
      P15(N)=SINH(V3Y)*COS(V5U)
      P16(N)=COSH(V3Y)*SIN(V5U)
      CV=COS(VN)
      SV=SIN(VN)
      DO 1 M=1,NH
      RM=M
      UM=RM*PI*U/A
      SC(M,N)=SIN(UM)*CV
    1 CS(M,N)=COS(UM)*SV
      DO 8 L=1,NL
      LNW = NHC+(L-1)*LT
      WL=DC(NB1,L)*U/A+DC(NB3,L)*(U/A)**3+DC(NB2,L)*V/B+DC(NC,L)*(V/B)**
     13
      DO 7 K=1,3
      LNM=LNW+4NH*K
      ML=6.0*S1(K)*U/AA3*DC(NB3,L)+6.*S3(K)*V/BB3*DC(NC,L)
      MK1=S1(K)*A2-S2(K)*K4-S3(K)
      MK2=S1(K)*A1+S2(K)*K3
      MK3=S1(K)+S2(K)*X2*K4-S3(K)*X2**2*A2
      MK4=(S2(K)*K3+S3(K)*X2*A1)*X2
      ML1=S1(K)*B2+S2(K)*K5-S3(K)
      ML2=S1(K)*B1-S2(K)*K3
      ML3=S1(K)-S2(K)*Y2*K5-S3(K)*Y2**2*B2
      ML4=(S2(K)*K3-S3(K)*Y2*B1)*Y2
      DO 6 N=1,NH
      RN=N
      RNP2=RN*PI
      AN=RNP2/A
      BN=RNP2/B
      AN2=AN*AN
      BN2=BN*BN
```

```
      BN4=BN2*BN2
      BV1=BN*VL(L,2)
      CV1=COS(BV1)
      SV1=SIN(BV1)
      BLS=BN*VL(L,4)/2.
      SBB=SIN(BLS)/BLS
      DO 4 M=1,NM
      RM=M
      AM=RM*PI/A
      AM2=AM*AM
      AM4=AM2*AM2
      AU1=AM*VL(L,1)
      CU1=COS(AU1)
      SU1=SIN(AU1)
      O3MN=SU1*CV1
      O4MN=COS(AU1)*SV1
      RMN=DX*AM4+SCA*AM2*BN2+SCB*BN4
      SMN=(SCC*AM2+SCD*BN2)*AM*BN
      ALR=AM*VL(L,3)/2.
      SAA=SIN(ALR)/ALR
      RS=1./(RMN**2-SMN**2)*SAA*SBB*CQ(L)
      RSR=RS*RMN
      RSS=RS*SMN
      PMN=O3MN*RSR+O4MN*RSS
      OMN=O4MN*RSR+O3MN*RSS
      IF (N.GT.1) GO TO 2
      T7MB=CQ(L)*CU1*SAA/2./DX/AM2*SIN(AM*U)
      ML=ML-T7MB*S1(K)
      IF (K.GT.1) GO TO 3
      WL=WL+T7MB/AM2
2     IF (K.GT.1) GO TO 3
      WL=WL+PMN*SC(M,N)+OMN*CS(M,N)
3     MK5=AM2*S1(K)+BN2*S3(K)
      MK6=BN*AM*S2(K)
      ML=ML-(MK5*PMN+MK6*OMN)*SC(M,N)-(MK6*PMN+MK5*OMN)*CS(M,N)
4     CONTINUE
      NHN=NH+N
      NH2N=NH2+N
      NH3N=NH3+N
      NH4N=NH4+N
      NH5N=NH5+N
      NH6N=NH6+N
      NH7N=NH7+N
      T8NB=CQS(L)*SV1*SBB/2./BN2*SIN(BN*V)
      IF (K.GT.1) GO TO 5
      WL=WL+DC(N,L)*P9(N)+DC(NHN,L)*P10(N)+DC(NH2N,L)*P11(N)+DC(NH3N,L)*
     1P12(N)+DC(NH4N,L)*P13(N)+DC(NH5N,L)*P14(N)+DC(NH6N,L)*P15(N)+DC(NH
     27N,L)*P16(N)+T8NB/BN2
5     ML=ML+BN2*((MK1*P9(N)-MK2*P10(N))*DC(N,L)+(MK2*P9(N)+MK1*P10(N))*D
     1C(NHN,L)+(ML1*P11(N)-ML2*P12(N))*DC(NH2N,L)+(ML2*P11(N)+ML1*P12(N)
     2)*DC(NH3N,L))-T8NB*S3(K)
      ML=ML+AN2*((-MK3*P13(N)-MK4*P14(N))*DC(NH4N,L)+(MK4*P13(N)-MK3*P14
     1(N))*DC(NH5N,L)+(-ML3*P15(N)+ML4*P16(N))*DC(NH6N,L)+(-ML4*P15(N)-M
     2L3*P16(N))*DC(NH7N,L))
6     CONTINUE
      YA(LNM,I)=ML+YA(LNM,I)
7     CONTINUE
8     YA(LNW,I)=WL+YA(LNW,I)
      RETURN
      END
```

```
C     ****************************************************************************
      SUBROUTINE DSRA (CD,DC,STOR,NS,JS,LN,NST1,NST2)
C     ****************************************************************************
      DOUBLE PRECISION CH,DC
C
C        COMPUTE DEFLECTIONS AND MOMENTS   (OLD SERIES)
C
      REAL KMN,MK1,MK2,MK3,MK4,MK5,MK5,ML1,ML2,ML3,ML4,ML,K3,K4,K5,MF,LM
     1N
      COMMON A,PI,SA,C,S,TN,C4,AA,AA3,AA4,Z,SYAS,C2,C3,ST,SF,T,TD,MNH
      COMMON /BL0/ S,DX,DY,DXY,DYX,D1,D2,EI,GJ,H,VL(4,5),ZB
      COMMON /BL1/ P,CQ(4),CQS(4),K3,K4,K5,BB,BB3,BB4,X2,Y2,A1,A2,A3,A4,
     1B1,B2,B3,B4,SCA,SCB,SCC,SCD,S1(4),S2(4),S3(4),S4(4),S5(4),S6(4),
     2    S7(4)
      COMMON/BL5/NHC,NB3,NC,JC,NH2,NH3,NH4,NH5,NH6,NH7,NH8,NS1,NS2,NH,NL
      COMMON /EXL/ UC(24),VC(24),NGC,YA(400,24),NR,UR(9),VR(3),NP(10)
      DIMENSION CD(NS,JS), DC(NS,LN), STOR(NST1,NST2)
      DIMENSION P1(20), P2(20), P3(20), P4(20), P5(20), P6(20), P7(20),
     1P8(20), CC(20,20), SS(20,20)
      DIMENSION P9(20),P10(20),P11(20),P12(20),P13(20),P14(20),P15(20)
     1   ,P16(20),                          CS(20,20),SC(20,20)
      LT=MNH*10
      DO 14 J=1,NR
      V = VR(J)
      DO 14 I=1,9
      U = UR(I)
      DO 1 N=1,NH
      RN=N
      RNP2=RN*PI
      AN=RNP2/A
      BN=RNP2/B
      VN=BN*V
      AU=AN*U
      BU=BN*U
      AV=AN*V
      AVK=AV*K3
      U3N=K3*BU
      V3N=AVK*X2
      V3Y=AVK*Y2
      U4V=K4*BU+V.N
      U5V=K5*BU-V.N
      V4U=K4*X2*AV+AU
      V5U=K5*Y2*AV-AU
      CHUN=COSH(U3N)
      SHUN=SINH(U3N)
      CHVN = COSH(V3N)
      SHVN = SINH(V3N)
      CHVY = COSH(V3Y)
      SHVY = SINH(V3Y)
      CU4 = COS(U4V)
      SU4 = SIN(U4V)
      CU5 = COS(U5V)
      SU5 = SIN(U5V)
      CV4 = COS(V4U)
      SV4 = SIN(V4U)
      CV5 = COS(V5U)
      SV5 = SIN(V5U)
      P 1(N) = CHUN * CU4
      P 2(N) = SHUN * SU4
      P 3(N) = CHUN * CU5
      P 4(N) = SHUN * SU5
      P 5(N) = CHVN * CV4
      P 6(N) = SHVN * SV4
      P 7(N) = CHVY * CV5
      P 8(N) = SHVY * SV5
      P 9(N) = SHUN * CU4
      P10(N) = CHUN * SU4
      P11(N) = SHUN * CU5
      P12(N) = CHUN * SU5
      P13(N) = SHVN * CV4
      P14(N) = CHVN * SV4
      P15(N) = SHVY * CV5
      P16(N) = CHVY * SV5
      CV=COS(VN)
      SV=SIN(VN)
      DO 1 N=1,NH
```

```
      RM=M
      UM=RM*PI*U/A
      CU = COS(UM)
      SU = SIN(UM)
      CC(M,N) = CU * CV
      SS(M,N) = SU * SV
      SC(M,N) = SU * CV
    1 CS(M,N) = CU * SV
      DO 14 L=1,NL
      LNR =NHC+(L-1)*LT+(J+6)*MNH
      K = 4
      CO4=COS(L)/4.
      IF (.NOT.(I.EQ.1.OR.I.EQ.9)) GO TO 20
      ML=2.*S1(K)*DC(NB2,L)/AA+2.*S3(K)*DC(NB3,L)/BB+12.*(T*U**2*S1(K)-V
     1**2*S3(K))*DC(NC,L)/BB4+CO4*(V*V-BB)/2.*S3(K)
      IF (ZB) 6,5,5
    5 ML=ML+CO4*(V*V-BB)/2.*S3(K)
      GO TO 7
    6 ML=ML+CO4*(V*V-BB/3.)/2.*S3(K)
    7 MK1=S1(K)*A2-S2(K)*K4-S3(K)
      MK2=S1(K)*A1+S2(K)*K3
      MK3=S1(K)+S2(K)*X2*K4-S3(K)*X2**2*A2
      MK4=(S2(K)*K3+S3(K)*X2*A1)*X2
      ML1=S1(K)*B2+S2(K)*K5-S3(K)
      ML2=S1(K)*B1-S2(K)*K3
      ML3=S1(K)-S2(K)*Y2*K5-S3(K)*Y2**2*B2
      ML4=(S2(K)*K3-S3(K)*Y2*B1)*Y2
   20 QL = 24.*(S4(K)*T*U-S7(K)*V)/BB4*DC(NC,L)+CO4*V*S7(K)
      QK1 = S4(K)*A3-S5(K)*A1-S6(K)*K3
      QK2 = S4(K)*A4+S5(K)*A2-S6(K)*K4-S7(K)
      QK3 =       X2*(S5(K)*K3+S6(K)*X2    *A1-S7(K)*X2*X2*A3)
      QK4 = S4(K)+X2*S5(K)*K4-S6(K)*X2*X2*A2-S7(K)*X2**3*A4
      QL1 = S4(K)*B3+S5(K)*B1-S6(K)*K3
      QL2 = S4(K)*B4-S5(K)*B2-S6(K)*K5+S7(K)
      QL3 =       Y2*(S5(K)*K3-S6(K)*Y2    *B1-S7(K)*Y2*Y2*B3)
      QL4 = S4(K)-Y2*S5(K)*K5-S6(K)*Y2*Y2*B2+S7(K)*Y2**3*B4
      DO 12 N=1,NH
      RN=N
      RNP2=RN*PI
      AN=RNP2/A
      BN=RNP2/B
      AN2=AN**2
      BN2=BN**2
      AN3 = AN2*AN
      BN3 = BN2*BN
      BN4=BN2*BN2
      BV1=BN*VL(L,2)
      CV1=COS(BV1)
      SV1=SIN(BV1)
      BLS=BN*VL(L,4)/2.
      SBB=SIN(BLS)/BLS
      VN = BN*V
      DO 10 M=1,NH
      RM=M
      AM=RM*PI/A
      AM2=AM*AM
      AM4=AM2*AM2
      AU1=AM*VL(L,1)
      CU1=COS(AU1)
      SU1 = SIN (AU1)
      Q1MN = CU1*CV1
      Q2MN = SU1*SV1
      Q3MN = SU1*CV1
      Q4MN = CU1*SV1
      RMN=DX*AM4+SCA*AM2*BN2+SCB*BN4
      SMN=(SCC*AM2+SCD*BN2)*AM*BN
      ALR=AM*VL(L,3)/2.
      SAA=SIN(ALR)/ALR
      RS=1./(RMN**2-SMN**2)*SAA*SBB*CQ(L)
      RSR=RS*RMN
      RSS=RS*SMN
      KMN = Q1MN*RSR-Q2MN*RSS
      LMN = Q2MN*RSR-Q1MN*RSS
      PMN = Q3MN*RSR+Q4MN*RSS
      QMN = Q4MN*RSR+Q3MN*RSS
      UM = AM*U
      IF (N.GT.1) GO TO 8
      TSM3=CQ(L)*CU1*SAA/2./DX/AM
```

```
      IF (.NOT.(I.EQ.1.OR.I.EQ.9)) GO TO 21
      ML=ML-T5MB*S1(K)/AM*COS(UM)
   21 QL = QL+T5MB*S4(K)*SIN(UM)
    8 IF (.NOT.(I.EQ.1.OR.I.EQ.9)) GO TO 22
      MK5=AM2*S1(K)+BN2*S3(K)
      MK6=BN*AM*S2(K)
      ML=ML-(MK5*KMN-MK6*LMN)*CC(M,N)+(MK6*KMN-MK5*LMN)*SS(M,N)
22    QK5 = AM*(AM2*S4(K)+BN2*S6(K))
      QK6 = BN*(AM2*S5(K)+BN2*S7(K))
10    QL =QL+(QK5*KMN-QK6*LMN)*SC(M,N)+(QK6*KMN-QK5*LMN)*CS(M,N)
      NHN=NH+N
      NH2N=NH2+N
      NH3N=NH3+N
      NH4N=NH4+N
      NH5N=NH5+N
      NH6N=NH6+N
      NH7N=NH7+N
      T6NB=CCS(L)*CV1*SBB/2./BN
      IF (.NOT.(I.EQ.1.OR.I.EQ.9)) GO TO 23
11    ML=ML+AN2*((-MK3*P5(N)-MK4*P6(N))*DC(NH4N,L)+(MK4*P5(N)-MK3*P6(N))
     1*DC(NH5N,L)+(-ML3*P7(N)+ML4*P8(N))*DC(NH6N,L)+(-ML4*P7(N)-ML3*P8(N
     2))*DC(NH7N,L))
      ML=ML+BN2*((MK1*P1(N)-MK2*P2(N))*DC(N,L)+(MK2*P1(N)+MK1*P2(N))*DC(
     1NHN,L)+(ML1*P3(N)-ML2*P4(N))*DC(NH2N,L)+(ML2*P3(N)+ML1*P4(N))*DC(N
     2H3N,L))-T6NB*S3(K)/BN*COS(VN)
23    QL = QL+AN3*((-QK3*P13(N)+QK4*P14(N))*DC(NH4N,L)+
     1                 (-QK4*P13(N)-QK3*P14(N))*DC(NH5N,L)+
     2                 (-QL3*P15(N)-QL4*P16(N))*DC(NH6N,L)+
     3                 ( QL4*P15(N)-QL3*P16(N))*DC(NH7N,L))+T6NB*S7(K)
     4          *SIN(VN)
12    QL = QL+BN3*((QK1*P 9(N)-QK2*P10(N))*DC(   N,L)+
     1                 (QK2*P 9(N)+QK1*P10(N))*DC(NHN,L)+
     2                 (QL1*P11(N)-QL2*P12(N))*DC(NH2N,L)+
     3                 (QL2*P11(N)+QL1*P12(N))*DC(NH3N,L))
      YA(LNR,I) = QL
      IF (I.EQ.1) YA(LNR,10) =-ML
      IF (I.NE.9) GO TO 14
      YA(LNR,11) = ML
      YA(LNR,12) = (YA(LNR,1)+YA(LNR,9)+2.*(2.*(YA(LNR,2)+YA(LNR,4)+
     1YA(LNR,6 ) +  YA(LNR,8))+YA(LNR,3)+YA(LNR,5)+YA(LNR,7)))*A/12.
     2      +YA(LNR,10)+ML
   14 CONTINUE
      RETURN
      END
```

```
C     ************************************************************************
      SUBROUTINE DARA (CO,DC,STOR,NS,JS,LN,NST1,NST2)
C     ************************************************************************
      DOUBLE PRECISION CH,DC
      REAL KMN,MK1,MK2,MK3,MK4,MK5,MK6,ML1,ML2,ML3,ML4,ML,K3,K4,K5,MF,LM
     1N
      COMMON A,PI,SA,C,S,TN,C4,AA,AA3,AA4,Z,SYAS,C2,C3,ST,SF,T,TB,MNH
      COMMON /BL0/ B,DX,DY,DXY,DYX,D1,D2,EI,GJ,H,VL(4,5),ZB
      COMMON /BL1/ P,CO(4),COS(4),K3,K4,K5,BB,BB3,BB4,X2,Y2,A1,A2,A3,A4,
     1B1,B2,B3,B4,SCA,SCB,SCC,SCD,S1(4),S2(4),S3(4),S4(4),S5(4),S6(4),
     2     S7(4)
      COMMON/BLS/NHC,NB3,NC,JC,NH2,NH3,NH4,NH5,NH6,NH7,NH8,NB1,NB2,NH,NL
      COMMON /EXL/ UC(24),VC(24),NGC,YA(400,24),NR,UR(9),VR(3),NP(10)
      DIMENSION CO(NS,JS), DC(NS,LN), STOR(NST1,NST2)
      DIMENSION P1(20), P2(20), P3(20), P4(20), P5(20), P6(20), P7(20),
     1P8(20), CC(20,20), SS(20,20)
      DIMENSION P9(20), P10(20), P11(20), P12(20), P13(20), P14(20), P15
     1(20), P16(20), SC(20,20), CS(20,20)
      LT=MNH*10
      DO  8 J=1,NR
      V = VR(J)
      DO  8 I=1,9
      U = UR(I)
      DO 1 N=1,NH
      RN=N
      RNP2=RN*PI
      AN=RNP2/A
      BN=RNP2/B
      VN=BN*V
      AU=AN*U
      BU=BN*U
      AV=AN*V
      AVK=AV*K3
      U3N=K3*BU
      V3N=AVK*X2
      V3Y=AVK*Y2
      U4V=K4*BU+VN
      U5V=K5*BU-VN
      V4U=K4*X2*AV+AU
      V5U=K5*Y2*AV-AU
      CHUN=COSH(U3N)
      SHUN=SINH(U3N)
      CHVN = COSH(V3N)
      SHVN = SINH(V3N)
      CHVY = COSH(V3Y)
      SHVY = SINH(V3Y)
      CU4 = COS(U4V)
      SU4 = SIN(U4V)
      CU5 = COS(U5V)
      SU5 = SIN(U5V)
      CV4 = COS(V4U)
      SV4 = SIN(V4U)
      CV5 = COS(V5U)
      SV5 = SIN(V5U)
      P 1(N) = CHUN * CU4
      P 2(N) = SHUN * SU4
      P 3(N) = CHUN * CU5
      P 4(N) = SHUN * SU5
      P 5(N) = CHVN * CV4
      P 7(N) = CHVY * CV5
      P 8(N) = SHVY * SV5
      P 8(N) = SHVY * SV5
      P 9(N) = SHUN * CU4
      P10(N) = CHUN * SU4
      P11(N) = SHUN * CU5
      P12(N) = CHUN * SU5
      P13(N) = SHVN * CV4
      P14(N) = CHVN * SV4
      P15(N) = SHVY * CV5
      P16(N) = CHVY * SV5
      CV=COS(VN)
      SV=SIN(VN)
      DO 1 M=1,NH
      RM=M
      UM=RM*PI*U/A
      CU = COS(UM)
```

```
      SU = SIN(UM)
      CC(M,N) = CJ * CV
      SS(M,N) = SJ * SV
      SC(M,N) = SJ * CV
 1    CS(M,N) = CJ * SV
      DO 8 L=1,NL
      LNR =NHC+(L-1)*LT+(J+6)*MNH
      K = 4
      IF (.NOT.(I.EQ.1.OR.I.EQ.9)) GO TO 20
      ML=6.0*S1(K)*U/AA3*DC(NB3,L)+6.*S3(K)*V/BB3*DC(NC,L)
      MK1=S1(K)*A2-S2(K)*K4-S3(K)
      MK2=S1(K)*A1+S2(K)*K3
      MK3=S1(K)+S2(K)*X2*K4-S3(K)*X2**2*A2
      MK4=(S2(K)*K3+S3(K)*X2*A1)*X2
      ML1=S1(K)*B2+S2(K)*K5-S3(K)
      ML2=S1(K)*B1-S2(K)*K3
      ML3=S1(K)-S2(K)*Y2*K5-S3(K)*Y2**2*B2
      ML4=(S2(K)*K3-S3(K)*Y2*B1)*Y2
 20   QL = 6.*(S4(K)*DC(NB3,L)/AA3+S7(K)*DC(NC ,L)/BB3)
      OK1 = S4(K)*A3-S5(K)*A1-S6(K)*K3
      OK2 = S4(K)*A4+S5(K)*A2-S6(K)*K4-S7(K)
      OK3 =        X2*(S5(K)*K3+S6(K)*X2    *A1-S7(K)*X2*X2*A3)
      OK4 = S4(K)+X2*S5(K)*K4-S6(K)*X2*X2*A2-S7(K)*X2**3*A4
      QL1 = S4(K)*B3+S5(K)*B1-S6(K)*K3
      QL2 = S4(K)*B4-S5(K)*B2-S6(K)*K5+S7(K)
      QL3 =        Y2*(S5(K)*K3-S6(K)*Y2    *B1-S7(K)*Y2*Y2*B3)
      QL4 = S4(K)-Y2*S5(K)*K5-S6(K)*Y2*Y2*B2+S7(K)*Y2**3*B4
      DO 6 N=1,NM
      RN=N
      RNP2=RN*PI
      AN=RNP2/A
      BN=RNP2/B
      AN2=AN*AN
      BN2=BN*BN
      AN3 = AN2*AN
      BN3 = BN2*BN
      BN4=BN2*BN2
      BV1=BN*VL(L,2)
      CV1=COS(BV1)
      SV1=SIN(BV1)
      BLS=BN*VL(L,4)/2.
      SBB=SIN(BLS)/BLS
      VN = BN*V
      DO 4 M=1,NH
      RM=M
      AM=RM*PI/A
      AM2=AM*AM
      AM4=AM2*AM2
      AU1=AM*VL(L,1)
      CU1=COS(AU1)
      SU1 = SIN (AU1)
      Q1MN = CU1*CV1
      Q2MN = SU1*SV1
      Q3MN = SU1*CV1
      Q4MN = CU1*SV1
      RMN=DX*AM4+SCA*AM2*BN2+SCB*BN4
      SMN=(SCC*AM2+SCD*BN2)*AM*BN
      ALR=AM*VL(L,3)/2.
      SAA=SIN(ALR)/ALR
      RS=1./(RMN**2-SMN**2)*SAA*SBB*CO(L)
      RSR=RS*RMN
      RSS=RS*SMN
      KMN = Q1MN*RSR-Q2MN*RSS
      LMN = Q2MN*RSR-Q1MN*RSS
      PMN = Q3MN*RSR+Q4MN*RSS
      OMN = Q4MN*RSR+Q3MN*RSS
      UM = AN*J
      IF (N.GT.1) GO TO 2
      T7MB=CO(L)*SU1*SAA/2./DX/AM
      IF (.NOT.(I.EQ.1.OR.I.EQ.9)) GO TO 21
      ML=ML-T7MB*S1(K)/AM*SIN(UM)
 21   QL = QL-T7MB*S4(K)*COS(UM)
 2    IF (.NOT.(I.EQ.1.OR.I.EQ.9)) GO TO 22
      MK5=AM2*S1(K)+BN2*S3(K)
      MK6=BN*AM*S2(K)
      ML=ML-(MK5*PMN+MK6*OMN)*SC(M,N)-(MK6*PMN+MK5*OMN)*CS(M,N)
 22   OK5 = AM*(AM2*S4(K)+BN2*S6(K))
      OK6 = BN*(AM2*S5(K)+BN2*S7(K))
```

```
4       QL =QL-(QK5*PMN+QK6*QMN)*CC(M,N)+(QK6*PMN+QK5*QMN)*SS(M,N)
        NHN=NH+N
        NH2N=NH2+N
        NH3N=NH3+N
        NH4N=NH4+N
        NH5N=NH5+N
        NH6N=NH6+N
        NH7N=NH7+N
        TBNB=COS(L)*SV1*SBB/2./BN
        IF (.NOT.(I.EQ.1.OR.I.EQ.9)) GO TO 23
5       ML=ML+BN2*((MK1*P9(N)-MK2*P10(N))*DC(N,L)+(MK2*P9(N)+MK1*P10(N))*D
       1C(NHN,L)+(ML1*P11(N)-ML2*P12(N))*DC(NH2N,L)+(ML2*P11(N)+ML1*P12(N)
       2)*DC(NH3N,L))-TBNB*S3(K)/BN*SIN(VN)
        ML=ML+AN2*((-MK3*P13(N)-MK4*P14(N))*DC(NH4N,L)+(MK4*P13(N)-MK3*P14
       1(N))*DC(NH5N,L)+(-ML3*P15(N)+ML4*P16(N))*DC(NH6N,L)+(-ML4*P15(N)-M
       2L3*P16(N))*DC(NH7N,L))
23      QL = QL+AN3*((-QK3*P 5(N)+QK4*P 6(N))*DC(NH4N,L)+
       1                (-QK4*P 5(N)-QK3*P 6(N))*DC(NH5N,L)+
       2                (-QL3*P 7(N)-QL4*P 8(N))*DC(NH6N,L)+
       3                ( QL4*P 7(N)-QL3*P 8(N))*DC(NH7N,L))-TBNB*S7(K)
       4          *COS(VN)
6       QL = QL+BN3*((QK1*P 1(N)-QK2*P 2(N))*DC(   N,L)+
       1                (QK2*P 1(N)+QK1*P 2(N))*DC(NHN,L)+
       2                (QL1*P 3(N)-QL2*P 4(N))*DC(NH2N,L)+
       3                (QL2*P 3(N)+QL1*P 4(N))*DC(NH3N,L))
        YA(LNR,I) = QL +YA(LNR,I)
        IF (I.EQ.1) YA(LNR,10) =-ML+YA(LNR,10)
        IF (I.NE.9) GO TO 8
        YA(LNR,11) = ML+YA(LNR,11)
        YA(LNR,12) = (YA(LNR,1)+YA(LNR,9)+2.*(2.*(YA(LNR,2)+YA(LNR,4)+
       1YA(LNR,6 ) +  YA(LNR,8))+YA(LNR,3)+YA(LNR,5)+YA(LNR,7)))*A/12.
       2     +YA(LNR,10)+ML
    8   CONTINUE
        RETURN
        END
```

SLAB NUMBER   HALF

INPUT DATA

PROPERTIES OF SLAB

HALF-WIDTH OF SLAB	50.55
HALF-SPAN OF SLAB	42.00
SKEW ANGLE	45.00
DX	13.40
DY	13.35
D1	1.22
D2	1.22
DXY	1.17
DYX	1.17
EI	62.71
GJ	35.35

LOADING SYSTEM

GENERAL LOADING(SYMMETRIC & ANTISYMMETRIC LOADS)

LOADING CASE	J1	V1	WIDTH OF LOAD	BREADTH OF LOAD	TOTAL LOAD
1	0.0	0.0	7.07000	6.00000	100.00000
2	23.32999	0.0	7.07000	6.00000	100.00000
3	46.66899	0.0	7.07000	6.00000	100.00000
4	0.0	14.00000	7.07000	6.00000	100.00000

THE SLAB IS ELASTICALLY SUPPORTED ON TWO EDGES

THE SLAB IS FLEXURALLY STRONG AND TORSIONALLY WEAK

DEPTHS OF N.A. FROM TOP FIBRE -1.3730 -1.3740 -1.3740
SLAB THICKNESS = 4.0000 INS
TOP GAGES 1 THRU 12
BOTTOM GAGES 13 THRU 12

LOADING CASE	J1	V1	WIDTH OF LOAD	BREADTH OF LOAD	TOTAL LOAD
1	0.0	0.0	7.07000	6.00000	100.00000

	1	2	3	4	5	6	7	8	9	10	11	12
J=	50.50	46.67	23.33	0.0	-23.33	-46.67	46.67	23.13	0.0	-23.33	-46.67	-50.50
V=	0.0	0.0	0.0	0.0	0.0	0.0	16.50	16.50	16.50	16.50	16.50	16.50
DEFLECTION												
MM=10	647.3438	622.2480	607.0251	748.3926	607.0251	622.2480	471.2402	437.7173	537.5442	554.9309	539.4605	566.6519
MOMENT PARALLEL TO X-AXIS (MX)												
MM=10	3.6597	-1.1145	7.7766	34.4389	7.7766	-1.1145	0.9574	4.0256	13.5987	6.5119	-1.5301	-1.7687
MOMENT PARALLEL TO Y-AXIS (MY)												
MM=10	11.6535	11.4193	11.5901	34.4439	11.5901	11.4193	8.1257	5.0343	6.7190	15.3438	10.5554	12.9759
TWISTING MOMENT MXY												
MM=10	0.5564	0.8454	1.2372	1.0351	1.2372	0.8454	0.2407	0.7089	0.7007	0.0290	1.2873	0.8622
PRINCIPAL MOMENT M1												
MM=10	11.6930	11.4761	11.9563	35.4764	11.9563	11.4761	8.1337	5.4000	13.6693	15.4210	10.6909	13.0260
PRINCIPAL MOMENT M2												
MM=10	3.8202	-1.1713	7.4104	33.4063	7.4104	-1.1713	0.9493	3.6599	6.6484	6.4347	-1.6736	-1.8189
PRINCIPAL DIRECTION												
MM=10	-4.0627	-3.8416	-16.4802	-44.9301	-16.4802	-3.8416	-1.9208	-27.2859	5.7566	-5.3163	-6.0091	-3.1353
STRAIN IN X-AXIS												
MM=10	-0.2808	0.2231	-0.6942	-3.2339	-0.6942	0.2231	-0.0221	-0.3685	-1.3420	-0.5280	0.2588	0.3055
STRAIN IN Y-AXIS												
MM=10	-1.1728	-1.1956	-1.1291	-3.2480	-1.1291	-1.1956	-0.8342	-0.4843	-0.5684	-1.5306	-1.1100	-1.3634
STRAIN IN U-AXIS												
MM=10	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********

LOADING CASE	J1	V1	WIDTH OF LOAD	BREADTH OF LOAD	TOTAL LOAD
2	23.32999	0.0	7.07000	6.00000	100.00000

	1	2	3	4	5	6	7	8	9	10	11	12
U=	50.50	46.67	23.33	0.0	-23.33	-46.67	46.67	23.33	0.0	-23.33	-46.67	-53.50
V=	0.0	0.0	0.0	0.0	0.0	0.0	16.50	16.50	16.50	16.50	16.50	16.50
DEFLECTION												
NN=10	914.9275	881.1711	796.3682	543.4211	400.5630	447.5977	605.5188	603.6943	520.4634	373.6277	364.6731	384.5767
MOMENT PARALLEL TO X-AXIS (MX)												
NN=10	-30.2967	-16.6230	23.7979	9.0393	3.9278	16.9212	-9.0988	6.7596	3.4668	3.0619	-4.9301	51.0521
MOMENT PARALLEL TO Y-AXIS (MY)												
NN=10	23.0628	7.2133	35.1260	10.0072	6.3325	21.0310	1.4578	8.7869	14.8120	9.1505	3.4632	0.4819
TWISTING MOMENT MXY												
NN=10	-2.2237	1.2060	1.2415	1.0980	0.0299	0.4650	-0.2231	0.5799	0.6753	0.7308	0.6173	-1.5714
PRINCIPAL MOMENT M1												
NN=10	23.9539	7.2742	35.4018	11.3329	6.5911	21.0030	1.4625	8.9411	14.8521	9.2370	3.5083	53.1052
PRINCIPAL MOMENT M2												
NN=10	-30.3078	-16.6835	23.5229	8.5137	3.6692	16.8692	-9.1035	6.6055	3.4267	2.9754	-4.9752	0.4289
PRINCIPAL DIRECTION												
NN=10	2.3472	-2.0890	-12.4917	-25.5821	-17.3072	-6.3736	1.2101	-14.8874	-3.3942	-6.7498	-4.1836	-1.8192
STRAIN IN X-AXIS												
NN=10	3.3570	1.7863	-2.7478	-0.8321	-0.3461	-1.5501	0.9542	-0.6156	-0.2183	-0.2300	0.5423	-5.4788
STRAIN IN Y-AXIS												
NN=10	-2.7636	-0.9062	-1.3629	-1.0359	-0.6199	-2.0222	-0.2375	-0.8478	-1.5043	-0.9206	-0.4061	0.4529
STRAIN IN U-AXIS												
NN=10	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********

LOADING CASE	J1	V1	WIDTH OF LOAD	BREADTH OF LOAD	TOTAL LOAD
1	46.66899	0.0	7.07000	6.00000	100.00000

	1	2	3	4	5	6	7	8	9	10	11	12
U=	50.50	46.67	23.33	0.0	-23.33	-46.67	46.67	23.33	0.0	-23.33	-46.67	-50.50
V=	0.0	0.0	0.0	0.0	0.0	0.0	16.50	16.50	16.50	16.50	16.50	16.50
DEFLECTION												
NN=10	1044.8345	1683.3528	783.4900	447.0958	361.7495	482.5201	1189.1978	794.8938	430.5667	314.0842	355.7827	384.2104
MOMENT PARALLEL TO X-AXIS (MX)												
NN=10	-56.9419	-22.4782	-3.8676	4.3769	2.9114	29.0287	-12.6892	-14.7208	-2.5972	2.7774	-9.0404	97.0587
MOMENT PARALLEL TO Y-AXIS (MY)												
NN=10	67.5071	39.0759	11.9491	7.7043	5.5892	32.8005	3.5422	21.5336	9.4508	6.7459	-1.1255	-6.9383
TWISTING MOMENT MXY												
NN=10	-4.3872	1.2395	1.9347	0.9421	0.7684	0.4371	-1.6704	0.6674	0.8109	0.5940	0.3839	-3.7524
PRINCIPAL MOMENT M1												
NN=10	67.6616	39.1005	11.1663	7.9525	5.7940	32.8634	8.6728	21.5459	9.5051	6.8329	-1.1069	97.1938
PRINCIPAL MOMENT M2												
NN=10	-57.0963	-22.5031	-3.2847	4.1207	2.7066	29.7657	-12.8198	-14.7331	-2.6516	2.6904	-9.0589	-7.1234
PRINCIPAL DIRECTION												
NN=10	2.0165	-1.1531	-6.4052	-14.7613	-14.9253	-8.1968	4.4711	-1.0543	-3.8334	-8.3326	-2.7707	-2.0620
STRAIN IN X-AXIS												
NN=10	6.5236	2.6527	0.4489	-0.3796	-0.2481	-2.7729	1.3923	1.7251	0.3578	-0.2233	0.9238	-10.0979
STRAIN IN Y-AXIS												
NN=10	-7.5455	-4.2603	-1.1767	-0.7580	-0.5524	-3.1212	-1.0068	-2.3743	-1.0054	-0.6737	0.0311	1.6453
STRAIN IN U-AXIS												
NN=10	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********

LOADING CASE	JL	VL	WIDTH OF LOAD	BREADTH OF LOAD	TOTAL LOAD
4	0.0	14.00000	7.07000	6.00000	100.00000

	1	2	3	4	5	6	7	8	9	10	11	12
J=	50.50	46.67	23.33	0.0	-23.33	-46.67	46.67	23.33	0.0	-23.33	-46.67	-50.50
V=	0.0	0.0	0.0	0.0	0.0	0.0	16.50	16.50	16.50	16.50	16.50	16.50
DEFLECTION												
MM=10	614.1040	592.6538	591.4458	579.7393	444.9343	480.3147	455.0152	456.4854	571.8003	445.0633	424.8433	449.4434
MOMENT PARALLEL TO X-AXIS (MX)												
MM=10	-5.0759	-4.1380	5.3085	14.4306	5.1023	2.9623	-2.3042	6.2902	29.4116	6.8501	-2.9151	9.8662
MOMENT PARALLEL TO Y-AXIS (MY)												
MM=10	13.0614	8.0565	11.6073	9.7388	6.1934	11.1691	6.4397	7.6130	27.3654	12.3684	7.5054	9.0925
TWISTING MOMENT MXY												
MM=10	0.0330	0.9357	0.9392	0.7296	0.3707	0.5047	0.3041	0.9029	0.8528	1.0593	1.0053	0.0930
PRINCIPAL MOMENT M1												
MM=10	13.0614	8.9235	14.7065	14.5414	6.6753	11.2001	6.4503	8.0708	29.7204	12.5648	7.6807	9.0773
PRINCIPAL MOMENT M2												
MM=10	-5.0760	-4.2050	5.7093	9.6279	4.6204	2.9314	-2.3148	5.8324	27.0565	6.6537	-3.0107	9.0815
PRINCIPAL DIRECTION												
MM=10	-0.0997	-4.0974	-6.0257	8.6380	-28.9652	-3.5058	-1.9892	-26.8083	19.9059	-10.5013	-5.4194	6.7563
STRAIN IN X-AXIS												
MM=10	0.7308	0.5114	-0.4623	-1.3995	-0.4638	-0.2006	0.2990	-0.5782	-2.7812	-0.5911	0.3730	-0.9338
STRAIN IN Y-AXIS												
MM=10	-1.4112	-0.9503	-1.4608	-0.8739	-0.5944	-1.1310	-0.6901	-0.7304	-2.5611	-1.2106	-0.8148	-0.8501
STRAIN IN U-AXIS												
MM=10	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********	*********

CPU USED = 111.77 SECS

0.0 0.0 0.0 0.0 0.0 0.0 0.0

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