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TRANSIENT ANALYSIS OF DYNAMIC SYSTEMS UNDER UNBALANCED OPERATIONS

by

WILBUR T. MILLER

A Dissertation
Presented to the Graduate Faculty of
Electrical Engineering, University of Windsor,
in partial fulfillment of the requirements
for the Degree of Master of Applied Science

University of Windsor

1966

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ABSTRACT

The primary object of this thesis is to make a comprehensive study on the transient behaviour of a dynamic system under unbalanced operations. Encountered in our study are a set of differential equations with coefficients containing periodic functions, and a method is proposed for the solution of these equations. Our study is made complete by investigating the system behaviour when the transients die out.

This dissertation contains four chapters. The first chapter deals with the performance of a synchronous machine. With the use of matrix algebra, the performance equations referred to moving reference axes, α and β, are derived. In chapter II, the short circuit currents, resulting from a single-phase short circuit of the synchronous machine, are solved for by a proposed approximation technique. In chapter III, the resulting phase quantities, open-phase voltages, sustained currents and voltages, and short-circuit torque are found. Chapter IV is the conclusion, discussing the merits and highlights of our transient analysis, especially those of the proposed solution to differential equations with periodic coefficients. Appendix A, following the conclusions, gives the Fourier series expansion formulas pertinent to this thesis, while Appendix B gives a list of symbols used throughout the thesis.

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SUMMARY

Any system can be represented by a set of differential equations. The system with which we are dealing is a power system -- in the form of a synchronous machine -- which we will represent by a set of performance equations. To arrive at our set of performance equations, we will cut out unnecessary labour by employing a strategic mathematical tool -- tensor analysis, in its grand concepts and broad generalizations.

Tensor analysis, first introduced by Gabriel Kron in 1935, has become one of the most powerful analytical tools and methods of analysis in modern engineering. Its unsurpassed supremacy as a means of generalization, its great unifying concepts, its beauty of expression, and the ease with which it may be learned and applied, establish it as perhaps the most powerful analytical tool at the disposal of the engineer.

More conspicuous in the first chapter of this thesis, will be the transformation tensor -- which expresses the relationship between old coordinates or variables and new coordinates or variables -- plus the use of matrix algebra which handles the routine operations involved in the applications of tensor analysis.

Now, we begin our transient analysis of an unbalanced dynamic system by considering as an example a line-to-neutral short circuit on our three-phase synchronous machine, and then solving the resulting equations due to the short circuit. Because there is no direct method available of solving these differential equations which have periodic coefficients, an approximate method is proposed, using successive approximations.

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The first approximation is to neglect all resistances in the differential equations to be solved. Having integrated both sides of each equation, the short circuit currents are solved for and expanded into harmonic series. We should note at this point that the currents just calculated must be initial short circuit currents, since neglecting resistances is the same as computing the initial values.

The second approximation is to account for the presence of resistances by multiplying each harmonic series type in each current expression by a decrement factor which is a function of time. Equations involving the decrement factors are found by substituting the new expressions for currents into the previously derived equations with periodic coefficients, expanding the trigonometric terms, and equating coefficients of corresponding trigonometric terms on both sides of the equations.

The third approximation is, in equating coefficients of trigonometric terms, to neglect the relatively small resistances in the coefficients of the harmonic terms, and to retain them in the d.c. terms.

The above process of approximation gives three independent equations with the three unknown decrement factors. Having solved for the decrement factors, they are substituted into the previously derived expressions for the short circuit currents. Hence the differential equations are solved completely.

Complete expressions can now be written for the short-circuit currents, and consequently the phase-currents and voltages, and short-circuit torque of the system can be found. A thorough analysis of the system is not completed until we have studied the sustained (or steady-state) currents, and voltages. The resulting electromagnetic action and inter-action involved leads to induced voltages and electromagnetic forces on particular elements of the system.

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CHAPTER I

PERFORMANCE OF A THREE-PHASE SYNCHRONOUS MACHINE

Coordinate Transformations:

First of all, we will make a transformation from 3-phase (a,b,c) to direct-quadrature-zero (i.e. d,q,0) axes, where the direct and quadrature axes are defined as pole and interpolar axes respectively.

From the elementary diagram of a 3-phase machine,

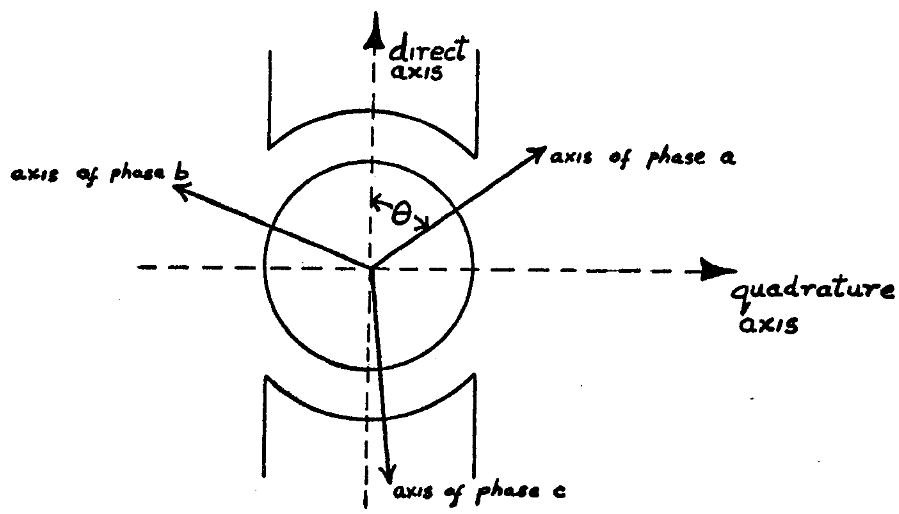

FIG.1

we have,

$$i_d = \sqrt{\frac{2}{3}} \left[i_a \cos\theta + i_b \cos(120 - \theta) + i_c \cos(240 - \theta) \right] \quad (1)$$

$$i_q = \sqrt{\frac{2}{3}} \left[i_a \sin\theta - i_b \sin(120 - \theta) - i_c \sin(240 - \theta) \right] \quad (2)$$

The factor, $\sqrt{\frac{2}{3}}$, was first introduced by G. Kron so that the armature circuit and the field will have reciprocal mutual inductance. This will be evidenced later.

In general, under transient or unbalanced conditions, i_d and i_q are functions of time.

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To complete the analysis and to allow for conditions where, during unbalance, there is current flow in the neutral of the armature, it is necessary to define a zero-sequence current which is equal to $\sqrt{3}$ times the conventional one of symmetrical components.

Hence,

$$i_o = \frac{1}{\sqrt{3}} \left(i_a + i_b + i_c\right) \tag{3}$$

The factor $1/\sqrt{3}$ is introduced so that, under this transformation, the power formula will remain both invariant in form and in magnitude.

In matrix form, the transformation from a,b,c to d,q,o, or vice versa, i.e. $[i^{dqo}] = [C^{dqo}_{abc}]\ [i^{abc}]$, is

$[C^{dqo}_{abc}]$ =

	a	b	c
d	$\sqrt{\frac{2}{3}}\cos\theta$	$\sqrt{\frac{2}{3}}\cos(\theta-120)$	$\sqrt{\frac{2}{3}}\cos(\theta-240)$
q	$\sqrt{\frac{2}{3}}\sin\theta$	$\sqrt{\frac{2}{3}}\sin(\theta-120)$	$\sqrt{\frac{2}{3}}\sin(\theta-240)$
o	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$

(4)

or conversely,

$[C^{abc}_{dqo}] = [C^{dqo}_{abc}]^{-1} =$

$\equiv [C^{dqo}_{abc}]_t$

	d	q	o
a	$\sqrt{\frac{2}{3}}\cos\theta$	$\sqrt{\frac{2}{3}}\sin\theta$	$\frac{1}{\sqrt{3}}$
b	$\sqrt{\frac{2}{3}}\cos(\theta-120)$	$\sqrt{\frac{2}{3}}\sin(\theta-120)$	$\frac{1}{\sqrt{3}}$
c	$\sqrt{\frac{2}{3}}\cos(\theta-240)$	$\sqrt{\frac{2}{3}}\sin(\theta-240)$	$\frac{1}{\sqrt{3}}$

(5)

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Note: $[e_{abc}] = [C^{dqo}_{abc}]_t [e_{dqo}]$

For unsymmetrical short circuit cases (or cases of unbalance), it is more convenient to use a set of orthogonal moving reference axes, (α,β). The α-axis is rigidly attached to phase a and the β-axis is the common axis of both phases b and c. Consequently, we will transform our stationary d-q-o axes to the moving α-β-o axes.

The elementary diagram of a 3-phase machine with moving reference axes is represented in the following manner:

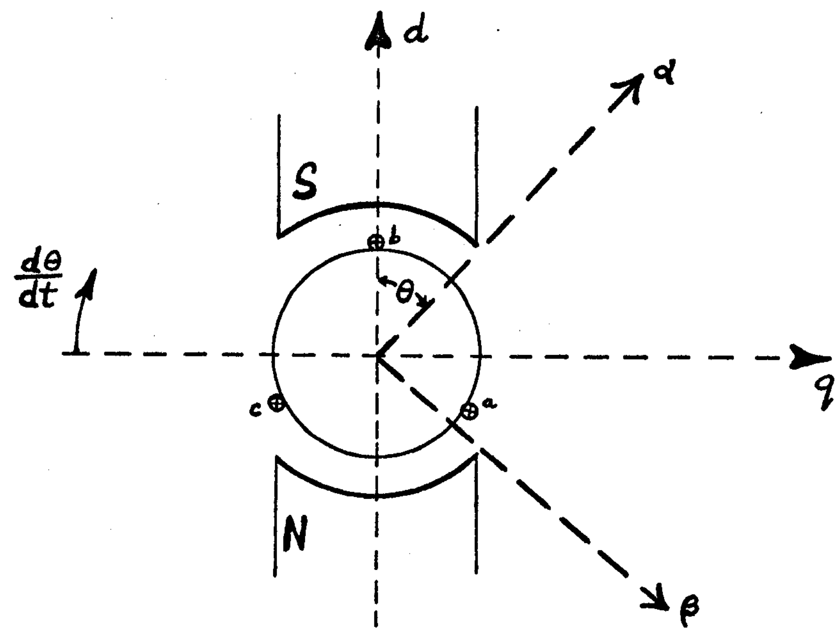

FIG.2

where the displacement of the new axes from the stationary axes is $\theta(t)$, a function of time.

From the diagram we find,

$$i_d = i_\alpha \cos\theta - i_\beta \sin\theta$$

$$i_q = i_\alpha \sin\theta + i_\beta \cos\theta \qquad (6)$$

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also $\quad i_o = i_o$ (i.e., remains unchanged)

In matrix notation, $[i^{dqo}] = [C^{dqo}_{\alpha\beta o}]\,[i^{\alpha\beta o}]$

and $[C^{dqo}_{\alpha\beta o}] =$ (7)

	α	β	o
d	$\cos\theta$	$-\sin\theta$	0
q	$\sin\theta$	$\cos\theta$	0
o	0	0	1

or conversely,

$[C^{\alpha\beta o}_{dqo}] = [C^{dqo}_{\alpha\beta o}]^{-1} =$

$\equiv [C^{dqo}_{\alpha\beta o}]_t$ (8)

	d	q	o
α	$\cos\theta$	$\sin\theta$	0
β	$-\sin\theta$	$\cos\theta$	0
o	0	0	1

Now, the transformation from α,β,o quantities back to phase (a,b,c) quantities and vice versa is readily found as follows:

$[C^{\alpha\beta o}_{abc}] = [C^{\alpha\beta o}_{dqo}]\,[C^{dqo}_{abc}]$

=

	d	q	o
α	$\cos\theta$	$\sin\theta$	0
β	$-\sin\theta$	$\cos\theta$	0
o	0	0	1

x

	a	b	c
d	$\sqrt{\frac{2}{3}}\cos\theta$	$\sqrt{\frac{2}{3}}\cos(\theta-120)$	$\sqrt{\frac{2}{3}}\cos(\theta-240)$
q	$\sqrt{\frac{2}{3}}\sin\theta$	$\sqrt{\frac{2}{3}}\sin(\theta-120)$	$\sqrt{\frac{2}{3}}\sin(\theta-240)$
o	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$

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$\therefore [C^{\alpha\beta o}_{abc}] =$

	a	b	c
α	$\sqrt{2/3}$	$-1/\sqrt{6}$	$-1/\sqrt{6}$
β	0	$-1/\sqrt{2}$	$1/\sqrt{2}$
o	$1/\sqrt{3}$	$1/\sqrt{3}$	$1/\sqrt{3}$

(9)

and conversely,

$[C^{abc}_{\alpha\beta o}] = [C^{\alpha\beta o}_{abc}]^{-1} =$

	α	β	o
a	$\sqrt{2/3}$	0	$1/\sqrt{3}$
b	$-1/\sqrt{6}$	$-1/\sqrt{2}$	$1/\sqrt{3}$
c	$-1/\sqrt{6}$	$1/\sqrt{2}$	$1/\sqrt{3}$

(10)

<u>Voltage-Current Relations in α-β-o</u>:

Now that we have obtained the tools (matrix tensors,[C]), we can develop the voltage-current relations in α,β,o components:

$$[e_{\alpha\beta of}] = [Z_{\alpha\beta of}]\,[i^{\alpha\beta of}] \qquad (11)$$

where $[Z_{\alpha\beta o}] = [C^{dqo}_{\alpha\beta o}]_t [Z_{dqo}]\,[C^{dqo}_{\alpha\beta o}]$ (12)

As we can see, we must first develop the voltage-current relations in d,q,o components, and then transform them in the above manner to α,β,o.

Now, with the general equations of induced voltage and armature reaction applied to our synchronous machine, we have an equivalent circuit[3] (excluding damper circuits):

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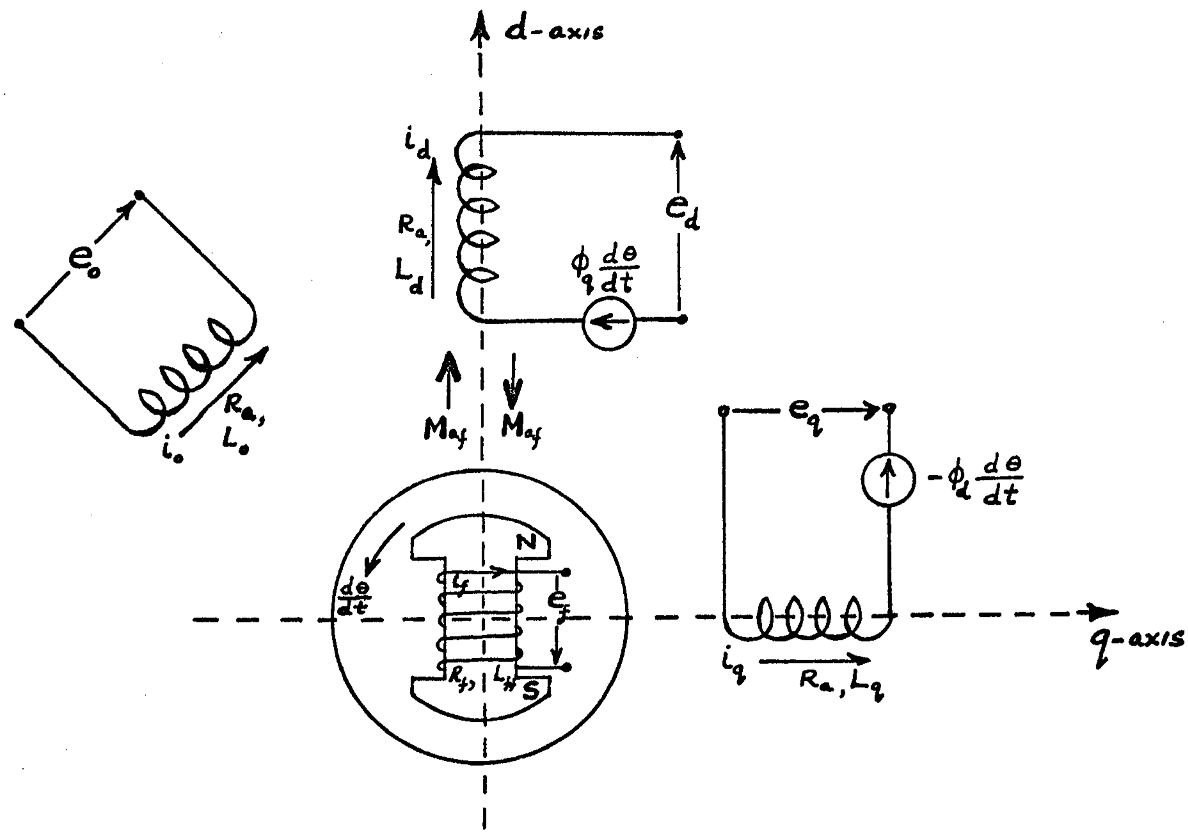

FIG.3

where, $\phi_d = M_{af}\, i_f + L_d\, i_d$

$$\phi_q = L_q\, i_q \tag{13}$$

$\phi_o = L_o\, i_o$

The voltage-current equations in matrix form is

$$[e_{dqof}] = [Z_{dqof}]\,[i^{dqof}] \tag{14}$$

where

$[Z_{dqof}]$ =

	d	q	o	f
d	$-(R_a + pL_d)$	$-pL_q\theta$	0	$-pM_{af}$
q	$pL_d\theta$	$-(R_a + pL_q)$	0	
o	0	0	$-(R_a + pL_o)$	
f	pM_{af}	0	0	$(R_f + pL_{ff})$

(15)

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where $p \equiv \frac{d}{dt}$.

It can be readily verified that the given equivalent circuits generally satisfy the machine performance equations and accurately represent the actual synchronous machine.

Now, for the voltage-current relations in α,β,o quantities:

$$[Z_{\alpha\beta of}] = [C^{dqof}_{\alpha\beta of}]_t \ [Z_{dqof}] \ [C^{dqof}_{\alpha\beta of}] \quad (16), \text{ as before;}$$

Utilizing our previously derived matrix tensors, [C],

$\therefore [Z_{\alpha\beta of}] =$

	d	q	o	f
α	$\cos\theta$	$\sin\theta$		0
β	$-\sin\theta$	$\cos\theta$		0
o	0	0	1	0
f	0	0	0	1

x

	d	q	o	f
d	$-(R_a + pL_d)$	$-pL_q\theta$	0	$-pM_{af}$
q	$pL_d\theta$	$-(R_a + pL_q)$	0	$pM_{af}\theta$
o	0	0	$-(R_a + pL_o)$	0
f	pM_{af}	0	0	$(R_f + pL_{ff})$

x

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x

	α	β	o	f
d	$\cos\theta$	$-\sin\theta$	0	0
q	$\sin\theta$	$\cos\theta$	0	0
o	0	0	1	0
f	0	0	0	1

=

	α	β	o	f
α	$[\sin\theta(pL_d\theta)$ $-\cos\theta(R_a+pL_d]\cos\theta$ $-[\cos\theta(pL_q\theta)$ $+\sin\theta(R_a+pL_q)]\sin\theta$	$-[\sin(pL_d\theta)$ $-\cos(R_a+pL_d)]\sin\theta$ $-[\cos(pL_q\theta)$ $+\sin(R_a+pL_q)]\cos\theta$	0	$\sin\theta(pM_{af}\theta)$ $-\cos\theta pM_{af}$
β	$[\sin\theta(R_a+pL_d)$ $+\cos\theta(pL_d\theta)]\cos\theta$ $+[\sin\theta(pL_q\theta)$ $-\cos\theta(R_a+pL_q)]\sin\theta$	$-[\sin\theta(R_a+pL_d)$ $+\cos\theta(pL_d\theta)]\sin\theta$ $+[\sin\theta(pL_q\theta)$ $-\cos\theta(R_a+pL_q)]\cos\theta$	0	$\sin\theta(pM_{af})$ $+\cos\theta(pM_{af}\theta)$
o	0	0	$-(R_a+pL_o)$	0
f	$pM_{af}\cos\theta$	$-pM_{af}\sin\theta$	0	(R_f+pL_{ff})

<u>Note</u>: $(\cos\theta\, p - \sin\theta\, p\theta)\, i = p\,(\cos\theta\, i)$

$(\sin\theta\, p + \cos\theta\, p\theta)\, i = p\,(\sin\theta\, i)$

amd $A = \dfrac{L_d + L_q}{2}$, $B = \dfrac{L_d - L_q}{2}$

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$\therefore [Z_{\alpha\beta of}] =$

	α	β	o	f
α	$-[R_a+p(A + B\cos 2\theta)]$	$p\ B\sin 2\theta$	0	$-pM_{af}\cos\theta$
β	$p\ B\sin 2\theta$	$-[R_a+(A-B\cos 2\theta)]$	0	$pM_{af}\sin\theta$
o	0	0	$-(R_a+pL_o)$	0
f	$pM_{af}\cos\theta$	$-pM_{af}\sin\theta$	0	(R_f+pL_{ff})

(17)

Now, $[e_{\alpha\beta of}] = [Z_{\alpha\beta of}]\ [i^{\alpha\beta of}]$ (11)

<u>Torque and Flux Expressions</u>:

The torque and flux expressions are found thus:

The impedance matrix of our machine may be broken down into three component matrices[1]:

(1) Resistance matrix, comprising all resistance terms

(2) Inductance matrix, comprising all terms having p as a coefficient

(3) Torque matrix, comprising all terms having pθ as coefficient

Hence $[Z_{\alpha\beta}] = [R_{\alpha\beta}] + [L_{\alpha\beta}]p + [G_{\alpha\beta}]p\theta$ (18)

The torque equation is: $T \propto [i^{\alpha\beta}]\ [G_{\alpha\beta}]\ [i^{\alpha\beta}]$ (19)

From the preceding page, we find

$[G_{\alpha\beta f}] =$

	α	β	f
α	$\sin\theta\cos\theta\ L_d$ $-\sin\theta\cos\theta\ L_q$	$-\sin^2\theta\ L_d$ $-\cos^2\theta\ L_q$	$M_{af}\sin\theta$
β	$\cos^2\theta\ L_d$ $+\sin^2\theta\ L_q$	$-\sin\theta\cos\theta\ L_d$ $+\sin\theta\cos\theta\ L_q$	$M_{af}\cos\theta$
f	$-M_{af}\sin\theta$	$-M_{af}\cos\theta$	0

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	α	β	f
= α	$B\sin 2\theta$	$-(A-B\cos 2\theta)$	$M_{af}\sin\theta$
β	$A+B\cos 2\theta$	$-B\sin 2\theta$	$M_{af}\cos\theta$
f	$-M_{af}\sin\theta$	$-M_{af}\cos\theta$	0

(20)

$\therefore\ [G_{\alpha\beta f}]\,[i^{\alpha\beta f}] = [\psi_{\alpha\beta f}] =$ (cross-flux vector)

α	$-[(A-B\cos 2\theta)i_\beta - B\sin 2\theta\, i_\alpha - M_{af}\sin\theta\, i_f]$
β	$[(A+B\cos 2\theta)i_\alpha - B\sin 2\theta\, i_\beta + M_{af}\cos\theta\, i_f]$
f	$-(M_{af}\sin\theta\, i_\alpha + M_{af}\cos\theta\, i_\beta)$

(21)

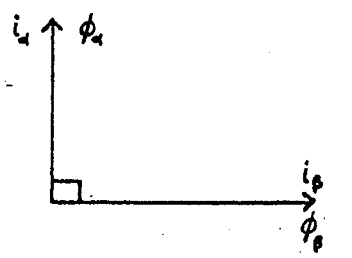
$$\therefore\ T \propto [i^{\alpha\beta}]\,[\psi_{\alpha\beta}]$$

$$\propto -\{(A-B\cos 2\theta)i_\beta - B\sin 2\theta\, i_\alpha - M_{af}\sin\theta\, i_f\}i_\alpha$$

$$+\{(A+B\cos 2\theta)i_\alpha - B\sin 2\theta\, i_\beta + M_{af}\cos\theta\, i_f\}i_\beta \qquad (22)$$

Now, consider the general torque equation,

$T \propto \vec{\phi} \times \vec{i}$, i.e. the vector product of flux-linkage and current



$$\therefore\ T \propto (\vec{\phi}_\alpha + \vec{\phi}_\beta) \times (\vec{i}_\alpha + \vec{i}_\beta)$$

$$\propto \phi_\beta i_\alpha - \phi_\alpha i_\beta$$

or, $$T = \frac{K3P}{4}\,[\phi_\beta i_\alpha - \phi_\alpha i_\beta] \qquad (23)$$

in which P = number of poles.

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If M.K.S. units were used, K is equal to unity when the unit of torque is the Newton-meter. If the torque is desired in pound-feet, K = 550/746.

Comparing equations (23) and (22),

$$\therefore \quad \phi_\beta, \text{ the } \beta\text{-axis flux linkage} = -\{(A - B\cos 2\theta)i_\beta - B\sin 2\theta\, i_\alpha - M_{af}\sin\theta\, i_f\} \quad (24)$$

$$\therefore \quad \phi_\alpha, \text{ the } \alpha\text{-axis flux linkage} = -\{(A + B\cos 2\theta)i_\alpha - B\sin 2\theta\, i_\beta + M_{af}\cos\theta\, i_f\} \quad (25)$$

Summary of Results:

Performance equations describing completely the physical facts involved in a synchronous machine have been derived in the preceding sections. Under any unbalanced operation conditions, having substituted equation (17) into equation (11) it is possible to solve equations (11) for the four unknown currents (i_α, i_β, i_o, i_f). Then by using equations (10), the phase currents (i_a, i_b, i_c) may be found. Substituting the solved currents (i_α, i_β, i_f) into equations (24) and (25), gives the β-axis and α-axis flux-linkages respectively. The torque can then be found by equation (23), after substituting the current values (i_α, i_β) and the values of equations (24) and (25). Finally the voltages can be found by simply differentiating the flux-linkage expressions with respect to t.

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CHAPTER II

SOLUTION OF SHORT-CIRCUIT CURRENTS

The single-phase short circuit of the synchronous machine which we are investigating is a line-to-neutral short circuit:

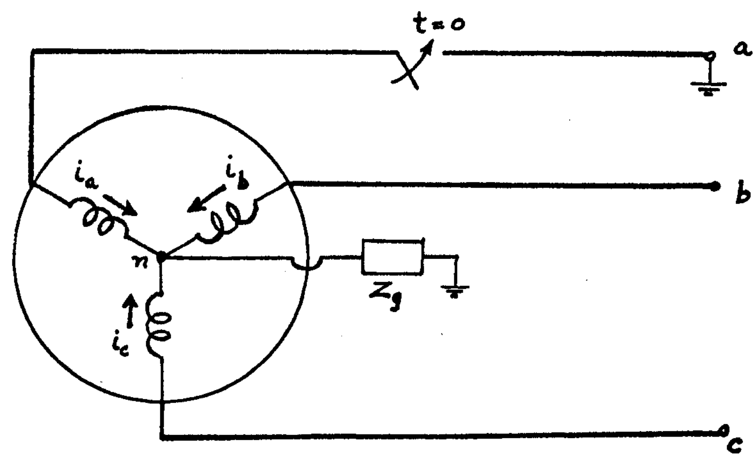

FIG.4

For such a case, as shown, with a short circuit on phase a, and the machine unloaded, we have

$$e_a = 0$$

$$i_b = i_c = 0 \tag{26}$$

In terms of α, β, o quantities defined by equations (9, 10), the conditions for the present case become:

$$i_\alpha = \sqrt{2/3}\, i_a \tag{27}$$

$$i_o = \frac{1}{\sqrt{3}}\, i_a = \frac{i_\alpha}{\sqrt{2}} \tag{28}$$

$$i_\beta = 0 \tag{29}$$

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$$e_\alpha = -\frac{e_o}{\sqrt{2}} \qquad (30)$$

Constant synchronous speed is assumed so that $\frac{d\theta}{dt} = \omega$; and the open circuit voltages before short circuit are:

$$v_a = E_f \sin(\omega t + \theta_o)$$

$$= E_f \sin\theta \qquad (31)$$

$$v_b = E_f \sin(\theta - 120) \qquad (32)$$

$$v_c = E_f \sin(\theta - 240) \qquad (33)$$

where i_{fo} = constant field current before short circuit

θ_o = angle between α- and d- axes at t = 0

By equation (9), the α-axis voltage before short circuit is,

$$e_{\alpha o} = \sqrt{\frac{3}{2}}\, E_f \sin\theta \qquad (34)$$

After short circuit on phase a, the α-axis voltage is, from equations (10) with short circuit condition, $e_a = 0$,

$$e_\alpha = -\frac{1}{\sqrt{2}}\, e_o \qquad (30)$$

$\therefore$ Δe_α, change in α-axis voltage $= -\frac{e_o}{\sqrt{2}} - \sqrt{\frac{3}{2}}\, E_f \sin\theta$ (35)

The effect of the short circuit on phase a is simulated by applying Δe_α to the armature with field voltage e_f equal to zero. To simplify the analysis we will assume that the machine has no damper circuits. Then by substituting $\Delta e_\alpha \equiv e_\alpha$, $i_\beta = 0$, $i_o = \frac{i_\alpha}{\sqrt{2}}$ and $e_f = 0$, the voltage-current differential equations (17) become

$$-\frac{e_o}{\sqrt{2}} - \sqrt{\frac{3}{2}}\, E_f \sin\theta = -\, pM_{af} \cos\theta\, i_f - [R_a + p(A + B\cos 2\theta)]\, i_\alpha \qquad (36)$$

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$$e_o = -(R_o + pL_o') i_o$$

$$= -(R_o + pL_o') \frac{i_\alpha}{\sqrt{2}} \qquad (37)$$

$$0 = (R_f + pL_{ff}) i_f + pM_{af} \cos\theta \; i_\alpha \qquad (38)$$

where $R_o = R_a + 3R_q$, $L_o' = L_o + 3L_q$,

and, R_q and L_q represent resistance and reactance from neutral to ground.

Simplifying above equations,

$$\sqrt{\frac{3}{2}} E_f \sin\theta = pM_{af} \cos\theta \; i_f + [r + p(A + \frac{L_o'}{2} + B\cos 2\theta)] \; i_\alpha \qquad (39)$$

where $r = R_a + \frac{R_o}{2}$

$$0 = (R_f + pL_{ff}) i_f + pM_{af} \cos\theta \; i_\alpha \qquad (38)$$

The short-circuit currents due to fault are the solutions of the above equations (38) and (39).

Approximate Solution:

The coefficients in equations (38, 39) are not constant and it is practically impossible to find an exact solution. However, an approximate solution can be obtained by successive approximations.

As a first approximation, we neglect all the resistances and integrate equations (38, 39) between the limits θ_o and θ to give

$$-\frac{\sqrt{3/2}\, E_f}{\omega} (\cos\theta - \cos\theta_o) = M_{af} \cos\theta \; i_f + (A + \frac{L_o'}{2} + B\cos 2\theta) i_\alpha \qquad (40)$$

$$0 = L_{ff} \; i_f + M_{af} \cos\theta \; i_\alpha \qquad (41)$$

$$\text{or } i_f = \frac{-M_{af}}{L_{ff}} \cos\theta \; i_\alpha \qquad (42)$$

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Solving above equations simultaneously, the short-circuit currents due to the fault are

$$i_\alpha = -\frac{\sqrt{6}\,E_f\,(\cos\theta - \cos\theta_o)}{(X_d' + X_q) + (X_d' - X_q)\cos 2\theta + X_o'} \tag{43}$$

$$\therefore \quad i_f = \sqrt{6}\,\frac{M_{af}}{L_{ff}}\,\frac{E_f\,(\cos\theta - \cos\theta_o)\cos\theta}{(X_d' + X_q) + (X_d' - X_q)\cos 2\theta + X_o'} \tag{44}$$

where $X_o' = \omega L_o'$

$$X_d' = \omega\left(L_d - \frac{M_{af}^2}{L_{ff}}\right)$$

$$X_q = \omega L_q$$

Re-writing above current expressions,

$$\therefore \quad i_\alpha = -\frac{\sqrt{6}\,E_f\,(\cos\theta - \cos\theta_o)}{(A' + B') + (A' - B')\cos 2\theta} \tag{45}$$

$$i_f = \sqrt{6}\,\frac{M_{af}}{L_{ff}}\,\frac{E_f\,(\cos\theta - \cos\theta_o)\cos\theta}{(A' + B') + (A' - B')\cos 2\theta} \tag{46}$$

where $A' = X_d' + \frac{X_o'}{2}$

$$B' = X_q' + \frac{X_o'}{2}$$

Applying the mathematical expressions (i.e. Fourier series expansion formulas) given in Appendix A, the currents of above equations (45, 46) may be resolved into the harmonic series:

$$i_\alpha = -\frac{\sqrt{6}\,E_f}{A' + A'B'}\left[\cos\theta + \sum_{n=1}^{\infty} b^n \cos(2n+1)\,\theta\right] + \frac{\sqrt{6}\,E_f\cos\theta_o}{\sqrt{A'B'}}\left[\frac{1}{2} + \sum_{n=1}^{\infty} b^n \cos 2n\theta\right] \tag{47}$$

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and

$$i_f = \frac{\sqrt{6}}{2} \frac{M_{af}}{L_{ff}} \frac{E_f}{A' + \sqrt{A'B'}} \left[1 + \frac{(1+b)}{b} \sum_{n=1}^{\infty} -b^n \cos 2n\,\theta \right]$$

$$- \frac{\sqrt{6}}{2} \frac{M_{af}}{L_{ff}} \frac{E_f \cos\theta_o}{\sqrt{A'B'}} (1+b) \left[\cos\theta + \sum_{n=1}^{\infty} b^n \cos(2n+1)\theta \right] \quad (48)$$

$$\text{where } b = \frac{\sqrt{B'} - \sqrt{A'}}{\sqrt{B'} + \sqrt{A'}} = \frac{\sqrt{A'B'} - A'}{\sqrt{A'B'} + A'} \quad (49)$$

Equations (47, 48) show that there is an unending series of reflection between armature and field. The odd-harmonic series for the armature current corresponds to an even-harmonic series for the field-current components, while the even harmonics series for the armature current corresponds to an odd-harmonic series of field current components.

The currents calculated above must be the initial short circuit currents, since neglecting resistances is the same as computing the initial values.

The second approximation is to make a correction for resistance by considering that the current components may be affected gradually by the effect of d.c. resistance drops. Hence the presence of resistance is effected by multiplying each harmonic series in each current expression by a decrement factor which is a function of time. In doing this, it is assumed that all harmonic terms of the same series are subject to a decrement factor with the same time constant.

In general form the currents may be thusly modified:

$$i_\alpha = -\frac{\sqrt{6}\,E_f}{A' + \sqrt{A'B'}} F_1(t) \left[\cos\theta + \sum_{n=1}^{\infty} b^n \cos(2n+1)\theta \right]$$

$$+ \frac{\sqrt{6}\,E_f \cos\theta_o}{\sqrt{A'B'}} F_2(t) \left[\frac{1}{2} + \sum_{n=1}^{\infty} b^n \cos 2n\,\theta \right] \quad (50)$$

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$$i_f = I_f(t) + \frac{\sqrt{6}}{2}\frac{M_{af}}{L_{ff}}\frac{E_f}{A' + \sqrt{A'B'}} F_1(t) \left[1 + \frac{(1+b)}{b}\sum_{n=1}^{\infty} b^n \cos 2n\,\theta\right]$$

$$- \frac{\sqrt{6}}{2}\frac{M_{af}}{L_{ff}}\frac{E_f \cos\theta_o}{\sqrt{A'B'}} (1+b)\, F_2(t) \left[\cos\theta + \sum_{n=1}^{\infty} b^n \cos(2n+1)\theta\right] \quad (51)$$

where $F_1(t)$ and $F_2(t)$ are decrement factors,

and $I_f(t)$ is a transient d.c. component introduced to include the effect of field resistance.

It is readily seen that the values of F_1, F_2 and I_f at $t = 0$ are unity, unity and zero respectively.

In 'closed form', the currents may be written as:

$$i_\alpha = -\frac{\sqrt{6}\,E_f\,(F_1 \cos\theta - F_2 \cos\theta_o)}{(A' + B') + (A' - B')\cos 2\theta} \quad (52)$$

$$i_f = I_f + \sqrt{6}\,\frac{M_{af}}{L_{ff}}\,\frac{E_f\,(F_1 \cos\theta - F_2 \cos\theta_o)\cos\theta}{(A' + B') + (A' - B')\cos 2\theta}$$

The three unknown factors can be found by substituting equations (50, 51) into the differential equations (38, 39), expanding the trigonometric expressions, and equating coefficients of corresponding terms on both sides of the equations.

In equating coefficients, we can make a third approximation -- that of neglecting the relatively small resistances in the coefficients of harmonic terms (i.e. $r \ll \omega L$), but retaining the resistances in the d.c. terms. This process results in many conditional equations for the three unknown current factors. However, it has been found that only three of the conditional equations are non-redundant -- hence independent, whereas the others are superfluous and furnish the same information.

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More explicitly:

First, on substitution of equations (50) and (51) into equation (38), and noting that equation (51) for i_f is more feasible in this form

$$i_f = I_f(t) - \frac{M_{af}}{L_{ff}} \cos\theta \, i_\alpha, \tag{54}$$

we get

$$\begin{aligned} 0 &= (R_f + pL_{ff})I_f - R_f \frac{M_{af}}{L_{ff}} \cos\theta \, i_\alpha \\ &= (R_f + pL_{ff})I_f + R_f \frac{M_{af}}{L_{ff}} \Bigg[\frac{\sqrt{3/2}\, E_f}{A' + \sqrt{A'B'}} F_1\{1 + (1 + b) \cos 2\theta \\ &\qquad + b(1 + b) \cos 4\theta + \ldots\} \\ &\qquad - \frac{\sqrt{3/2}\, E_f \cos\theta_o}{\sqrt{A' B'}} F_2 \{(1 + b) \cos\theta + b(1 + b) \cos 3\theta + \} \Bigg] \end{aligned} \tag{55}$$

Equating the coefficients of the constant terms on both sides of the above equation and remembering to neglect the resistances in the coefficients of the harmonic terms but retaining them in the d.c. terms, we have:

$$0 = (R_f + pL_{ff})I_f + R_f \frac{M_{af}}{L_{ff}} \frac{\sqrt{3/2}\, E_f}{A' + \sqrt{A'B'}} F_1 \tag{56}$$

Our second substitution, is that of equations (50) and (54) into equation (39), and the result is,

$$\begin{aligned} \sqrt{3/2}\, E_f \sin\theta &= pM_{af} \cos\theta \, I_f - \frac{pM_{af}^2}{L_{ff}} \cos^2\theta \\ &\qquad + [r + p(A + \frac{L_o'}{2} + B\cos 2\theta)]i_\alpha \\ &= pM_{af} \cos\theta \, I_f + \Bigg[r + p(A + \frac{L_o'}{2} + \{B - \frac{M_{af}^2}{2L_{ff}}\}\cos 2\theta \\ &\qquad - \frac{M_{af}^2)}{2L_{ff}}\Bigg] i_\alpha \end{aligned}$$

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Continuing,

$$\sqrt{\frac{3}{2}}\, E_f \sin\theta = pM_{af} \cos\theta\, I_f - \frac{r\sqrt{6}\, E_f}{A' + \sqrt{A'B'}} F_1 \left[\cos\theta + b\cos 3\theta + b^2\cos 5\theta + \ldots.\right]$$

$$+ \frac{r\sqrt{6}\, E_f \cos\theta_o}{\sqrt{A'B'}} F_2 \left[\frac{1}{2} + b\cos 2\theta + b^2 \cos 4\theta + \ldots.\right]$$

$$+ pA\sqrt{6}\, E_f \left\{\frac{-F_1}{A' + \sqrt{A'B'}} \left[\cos\theta + b\cos 3\theta + b^2\cos 5\theta + \ldots.\right]\right.$$

$$\left. + \frac{\cos\theta_o F_2}{\sqrt{A'B'}} \left[\frac{1}{2} + b\cos 2\theta + b^2\cos 4\theta + \ldots.\right]\right\}$$

$$+ \frac{pL_o}{2}\sqrt{6}\, E_f \left\{\frac{-F_1}{A' + \sqrt{A'B'}} \left[\cos\theta + b\cos 3\theta + b^2\cos 5\theta + \ldots.\right]\right.$$

$$\left. + \frac{\cos_o F_2}{\sqrt{A'B'}} \left[\frac{1}{2} + b\cos 2\theta + b^2\cos 4\theta + \ldots.\right]\right\}$$

$$- \frac{pM_{af}^2}{2L_{ff}}\sqrt{6}\, E_f \left\{-\frac{F_1}{A' + \sqrt{A'B'}} \left[\cos\theta + b\cos 3\theta + \ldots.\right]\right.$$

$$\left. + \frac{\cos\theta_o F_2}{\sqrt{A'B'}} \left[\frac{1}{2} + b\cos 2\theta + \ldots.\right]\right\}$$

$$+ pB\frac{\sqrt{6}E_f}{2} \left\{-\frac{F_1}{A' + \sqrt{A'B'}} \left[(1+b)\cos\theta + (1+b^2)\cos 3\theta\right.\right.$$

$$\left. + b(1+b^2)\cos 5\theta + \ldots.\right]$$

$$\left. + \frac{\cos_o F_2}{\sqrt{A'B'}} \left[b + (1+b^2)\cos 2\theta + b(1+b^2)\cos 4\theta + \ldots.\right]\right\}$$

$$- \frac{pM_{af}^2}{4L_{ff}}\sqrt{6}\, E_f \left\{-\frac{F_1}{A' + \sqrt{A'B'}} \left[(1+b)\cos\theta + \ldots.\right] + \right.$$

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$$+ \frac{\cos\theta_o F_2}{\sqrt{A\ B}} \left[b + (1 + b^2) \cos 2\theta + \ldots \right] \Big\} \qquad (57)$$

Being careful to note, that, for example, $pM_{af} \cos\theta I_f(t) \equiv -\omega M_{af} I_f \sin\theta + M_{af} \cos\theta\ pI_f$, we can proceed to equate coefficients of the $\sin\theta$ terms on both sides of equation (57).

Hence,

$$\sqrt{3/2}\ E_f = - M_{af}\ \omega\ I_f + \frac{\sqrt{6}\ A\ E_f}{A' + \sqrt{A'B'}}\ \omega\ F_1 + \frac{\sqrt{6}\ \frac{L_o}{2}\ E_f}{A' + \sqrt{A'B'}}\ \omega\ F_1$$

$$- \sqrt{6}\ \frac{M_{af}^2}{2L_{ff}}\ \frac{E_f}{A' + \sqrt{A'B'}}\ \omega\ F_1 + \sqrt{6}\ \frac{B}{2}\ \frac{E_f(1 + b)}{A' + \sqrt{A'B'}}\ \omega\ F_1$$

$$- \sqrt{6}\ \frac{M_{af}^2}{4L_{ff}}\ \frac{E_f\ (1 + b)}{A' + \sqrt{A'B'}}\ \omega\ F_1$$

or

$$- \sqrt{3/2}\ E_f = \omega M_{af}\ I_f - \frac{\omega\sqrt{6}\ E_f}{A' + \sqrt{A'B'}} \left(A + \frac{L_o'}{2} - \frac{M_{af}^2}{2L_{ff}} + \frac{B}{2}(1 + b) - \frac{M_{af}^2}{4L_{ff}}(1 + b) \right) F_1 \qquad (58)$$

Equating the coefficients of the constant terms on both sides of equation (57), we have:

$$0 = r\ \frac{\sqrt{6}\ E_f\ \cos\theta_o}{2\ \sqrt{A'B'}}\ F_2 + p\ F_2 \left[\frac{A\ \sqrt{6}\ E_f\ \cos\theta_o}{2\ \sqrt{A'B'}} + \frac{\frac{L_o'}{2}\ \sqrt{6}\ E_f\ \cos\theta_o}{2\ \sqrt{A'B'}} \right.$$

$$\left. - \frac{M_{af}^2}{2L_{ff}}\ \frac{\sqrt{6}\ E_f\ \cos\theta_o}{2\ \sqrt{A'B'}} + \frac{B\ \sqrt{6}\ E_f\ b\ \cos\theta_o}{2\ \sqrt{A'B'}} - \frac{M_{af}^2}{4L_{ff}}\ \frac{\sqrt{6}\ E_f\ b\ \cos\theta_o}{\sqrt{A'B'}} \right]$$

or

$$0 = r\ F_2 + p\ F_2 \left(A + \frac{L_o'}{2} - \frac{M_{af}^2(1 + b)}{2L_{ff}} + bB \right) \qquad (59)$$

The three equations (56), (58) and (59) just found, to determine the unknowns, are the independent equations, and are simplified to give:

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$$0 = (R_f + pL_{ff})\, I_f(t) + R_f \frac{M_{af}}{L_{ff}} \frac{\sqrt{3/2}\, E_f}{A' + \sqrt{A'B'}} F_1(t) \qquad (60)$$

$$-\sqrt{3/2}\; E_f = \omega\, M_{af}\, I_f(t) - \sqrt{3/2}\, E_f \; F_1(t) \qquad (61)$$

$$0 = r\, F_2(t) + p\, F_2(t) \; \frac{\sqrt{A'B'}}{\omega} \qquad (62)$$

Solving for $F_2(t)$, noting that $F_2(0) = 1$,

$$\therefore F_2(t) = e^{-t/\tau_a} \qquad (63)$$

where $\tau_a = \dfrac{\sqrt{A'B'}}{\omega r}$ = armature time constant

Define $\sqrt{A\ B} = X_2 + 0.5\, X_o'$.

Solving for $F_1(t)$ and $I_f(t)$ simultaneously, noting that $F_1(0) = 1$ and $I_f(0) = 0$,

$$\therefore F_1(t) = \frac{X_d' + X_o' + X_2}{X_d + X_o' + X_2} \left[\frac{X_d - X_d'}{X_d' + X_o' + X_2}\, e^{-t/\tau_d'} + 1 \right] \qquad (64)$$

and $\therefore I_f(t) = \dfrac{\sqrt{3/2}\, E_f}{\omega\, M_{af}} \dfrac{X_d - X_d'}{X_d + X_o' + X_2} \left[e^{-t/\tau_d'} - 1 \right]$ (65)

$$\text{or} \quad = \sqrt{3/2}\, E_f \frac{M_{af}}{L_{ff}} \frac{1}{X_d + X_o' + X_2} \left[e^{-t/\tau_d'} - 1 \right]$$

where $\tau_d' = \dfrac{\tau_{do}' (X_d' + X_o' + X_2}{(X_d + X_o' + X_2}$

= Field time constant

$\tau_{do}' = \dfrac{L_{ff}}{R_f}$ = open circuit field time constant

$$X_2 = \sqrt{\left(X_d' + \frac{X_o'}{2}\right)\left(X_q + \frac{X_o'}{2}\right)} - 0.5\, X_o' \qquad (66)$$

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The final approximate current expressions in 'open form' become:

$$i_\alpha = -\sqrt{6}\,E_f\left[\left(\frac{1}{X_d' + X_o' + X_2} - \frac{1}{X_d + X_o' + X_2}\right)e^{-t/\tau_d'} + \frac{1}{X_d + X_o' + X_2}\right]\left[\cos\theta + \sum_{n=1}^{\infty} b^n \cos(2n+1)\theta\right]$$

$$+ \frac{\sqrt{6}\,E_f \cos_o}{X_2 + 0.5\,X_o'}\left[0.5 + \sum_{n=1}^{\infty} b^n \cos 2n\theta\right] e^{-t/\tau_a} \qquad (67)$$

$$i_f = i_{fo} + \sqrt{3/2}\,\frac{M_{af}}{L_{ff}}\,\frac{E_f\left[e^{-t/\tau_d'} - 1\right]}{X_d + X_o' + X_2}$$

$$+ \sqrt{3/2}\,\frac{M_{af}}{L_{ff}}\,E_f\left[\left(\frac{1}{X_d' + X_o' + X_2} - \frac{1}{X_d + X_o' + X_2}\right)e^{-t/\tau_d'} + \frac{1}{X_d + X_o' + X_2}\right]\left[1 + \frac{(1+b)}{b}\sum_{n=1}^{\infty} b^n \cos 2n\theta\right]$$

$$- \sqrt{3/2}\,\frac{M_{af}}{L_{ff}}\,\frac{E_f \cos\theta_o}{X_2 + 0.5\,X_o'}\,(1+b)\left[\cos\theta + \sum_{n=1}^{\infty} b^n \cos(2n+1)\theta\right] e^{-t/\tau_a} \qquad (68)$$

where i_{fo} is initial d.c. value of field current.

Applying the formulas from Appendix A, the preceding expressions for current can be written in 'closed form' as:

$$i_\alpha = -\frac{\sqrt{6}\;E_f\,[F_1 \cos\theta - F_2 \cos\theta_o]}{X_d' + X_q + (X_d' - X_q)\cos 2\theta + X_o'} \qquad (69)$$

where $$F_1 = \frac{X_d' + X_o' + X_2}{X_d + X_o' + X_2}\left(\frac{X_d - X_d'}{X_d' + X_o' + X_2}\,e^{-t/\tau_d'} + 1\right)$$

$$= \frac{X_d' + X_o' + X_2}{X_d + X_o' + X_2} + \left(1 - \frac{X_d' - X_o' + X_2}{X_d + X_o' + X_2}\,e^{-t/\tau_d'}\right)$$

$$F_2 = e^{-t/\tau_a}$$

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$$i_f = i_{fo} + \sqrt{3/2}\,\frac{M_{af}}{L_{ff}}\;E_f\left\{\frac{\left(e^{-t/\tau_d'}-1\right)}{X_d + X_o' + X_2} + \frac{2\left(F_1\cos\theta - F_2\cos\theta_o\right)\cos\theta}{X_d' + X_q + (X_d' - X_q)\cos 2\theta + X_o'}\right\} \tag{70}$$

or

$$= i_{fo} + I_f - \frac{M_{af}}{L_{ff}}\cos\theta\; i_\alpha \tag{70 a}$$

Summary of Results:

The short circuit currents have been solved for in the foregoing sections. The complete expressions for the currents (i_α, i_f) are given by equations (67) and (68). The voltage-current differential equations resulting from the unbalance (i.e. short circuit), that were solved by an approximate method, are given by equations (38) and (39).

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CHAPTER III

PHASE CURRENTS, SHORT-CIRCUIT TORQUE AND OPEN CIRCUIT VOLTAGE

Phase Current:

All the phase currents for the line-to-neutral short circuit may be readily obtained by equations (9,10); since $\underline{i_b} = \underline{i_c} = 0$ and $i_o = \frac{i_\alpha}{\sqrt{2}}$

$$\therefore \quad i_a = \sqrt{\frac{3}{2}}\, i_\alpha = \text{phase} - \underline{a} \text{ current} \qquad (27)$$

From equation (69),

$$\therefore \quad \underline{i_a} = -3 E_f \left[\left(\frac{1}{X_d' + X_o' + X_2} - \frac{1}{X_d + X_o' + X_2}\right) e^{-t/\tau_d'} + \frac{1}{X_d + X_o' + X_2}\right]\left[\cos\theta + \sum_{n=1}^{\infty} b^n (2n+1)\theta\right] + \frac{3 E_f \cos\theta_o}{X_2 + 0.5 X_o'}\left[0.5 + \sum_{n=1}^{\infty} b^n \cos 2n\theta\right] e^{-t/\tau_a} \qquad (71)$$

or, in 'closed form',

$$\underline{i_a} = -\frac{3 E_f \left[F_1 \cos\theta - F_2 \cos\theta_o\right]}{X_d' + X_q + (X_d' - X_q)\cos 2\theta + X_o'} \qquad (72)$$

where F_1 and F_2 are given by equations (64) and (63).

Open-Circuit Voltage:

The open-phase voltage is $e_b - e_c$ which equals, by equations (9), $-\sqrt{2}\, e_\beta$, and we know that $e_\beta = -\frac{d\phi_\beta}{dt}$.

The β-axis flux linkage for the present case is, by equation (24)

$$\phi_\beta = B \sin 2\theta\, i_\alpha + M_{af}\, i_f \sin\theta \qquad (73)$$

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Substituting equations (50) and (70 a) -- for i_α, i_f -- into above equation (73), and rearranging a few terms, there results,

$$\phi_\beta = \frac{1}{2} \frac{X_d' - X_q}{\omega} \sin 2\theta \, i_\alpha + \frac{\sqrt{\frac{3}{2}}\, E_f F_1 \sin \theta}{\omega}$$

$$= -\frac{1}{2} \frac{X_d' - X_q}{\omega} \sin 2\theta \left\{ \frac{\sqrt{6}\, E_f F_1}{A' + \sqrt{A'B'}} [\cos\theta + b \cos 3\theta + b^2 \cos 5\theta + \ldots.] - \frac{\sqrt{6}\, E_f \cos\theta_o}{\sqrt{A'B'}} F_2 [0.5 + b \cos 2\theta + b^2 \cos 4\theta + \ldots.] \right\}$$

$$+ \frac{\sqrt{3/2}\, E_f F_1 \sin \theta}{\omega}$$

$$= -\frac{\sqrt{3/2}\, E_f F_1 (X_d' - X_q)}{2\omega (A' + \sqrt{A'B'})} [(1-b)\sin\theta + (1-b^2)\sin 3\theta + b(1-b^2) \sin 5\theta + \ldots.]$$

$$+ \frac{\sqrt{3/2}\, E_f F_2 \cos\theta_o (X_d' - X_q)}{2\omega \sqrt{A'B'}} (1-b^2) [\sin 2\theta + b \sin 4\theta + b^2 \sin 6\theta + \ldots]$$

$$+ \frac{\sqrt{3/2}\, E_f F_1 \sin\theta}{\omega} \qquad (74)$$

Simplifying, we get

$$\phi_\beta = \frac{\sqrt{3/2}\, E_f F_1 (1+b)}{\omega} [\sin\theta + b \sin 3\theta + b^2 \sin 5\theta + \ldots.]$$

$$- \frac{2b \sqrt{3/2}\, E_f F_2 \cos\theta_o}{\omega} [\sin 2\theta + b \sin 4\theta + b^2 \sin 6\theta + \ldots.] \qquad (75)$$

By differentiation we have the open-phase voltage:

$$e_b - e_c = -\sqrt{2}\, e_\beta = \sqrt{2} \frac{d\phi_\beta}{dt}$$

$$= -\frac{\sqrt{3}\, E_f (X_d - X_d')(1+b)}{\omega\tau_d' (X_d + X_o' + X_2)} e^{-t/\tau_d'}$$

$$[\sin\theta + b \sin 3\theta + b^2 \sin 5\theta + \ldots.]$$

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$$+ \frac{2b\sqrt{3}\, E_f \cos\theta_o}{\omega\, \tau_a}\, e^{-t/\tau_a} . [\sin 2\theta + b \sin 4\theta + b^2 \sin 6\theta + \ldots]$$

$$+ \sqrt{3}\, E_f\, F_1\, (1 + b)\ [\cos\theta + 3b \cos 3\theta + 5b^2 \cos 5\theta + \ldots]$$

$$- 4b\sqrt{3}\, E_f\, F_2 \cos\theta_o\ [\cos 2\theta + 2b \cos 4\theta + 3b^2 \cos 6\theta + \ldots] \quad (76)$$

From equations (71) and (70), the sustained armature and field currents are:

$$i_a = - \frac{3\, E_f}{X_d + X_o' + X_2} [\cos\theta + b \cos 3\theta + b^2 \cos 5\theta + \ldots] \quad (77)$$

$$i_f = i_{fo} + \frac{\sqrt{3/2}\, M_{af}}{L_{ff}} \frac{E_f\,(1+b)}{X_d + X_o' + X_2} [\cos 2\theta + b \cos 4\theta + b^2 \cos 6\theta + \ldots] \quad (78)$$

Also, from equation (76), the sustained open-phase voltage is:

$$e_b - e_c = \sqrt{3}\, E_f \frac{2\, X_2 + X_o'}{X_d + X_o' + X_2} \left[\cos\theta + 3b \cos 3\theta + 5b^2 \sin 5\theta + \ldots\right] \quad (79)$$

We see that the armature current and voltage contain a fundamental frequency component and odd harmonics, while the field current contains even harmonics.

As the absolute value of b is less than unity, each succeeding harmonic is less than the preceding one, and when b is small the higher harmonics may be neglected.

The equations (77), (79) above, show that the fundamental frequency components of armature current and voltages are the same as those calculated by the method of symmetrical components. The quantity X_2 is defined as the negative sequence reactance for the line-to-neutral case, where

$$X_2 = \sqrt{A'B'} - 0.5\, X_o' = \sqrt{\left(X_d' + \frac{X_o'}{2}\right)\left(X_q + \frac{X_o'}{2}\right)} - 0.5\, X_o'.$$

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The general torque equation, developed in Chapter I, for the present case (line-to-neutral short circuit case) reduces to

$$T_{\ell-n} = \frac{K3P}{4} \phi_\beta \, i_\alpha \tag{80}$$

Short-Circuit Torque:

Substituting the values of ϕ_β and i_α as given by equations (74) and (52) into above equation (80), gives the following expression for the short-circuit torque:

$$T_{\ell-n} = \frac{K3P}{4}\left(\frac{1}{2}\,\frac{X_d' - X_q}{\omega}\sin 2\theta \, i_\alpha + \frac{\sqrt{3/2}\,E_f F_1 \sin\theta}{\omega}\right) i_\alpha$$

$$= -\frac{K3P}{4\omega}\left\{\frac{3 E_f^2 F_1 (F_1 \cos\theta - F_2 \cos\theta_o)\sin\theta}{(A' + B') + (A' - B')\cos 2\theta} + \frac{X_q - X_d'}{2}\,\frac{[\sqrt{6}\,E_f(F_1 \cos\theta - F_2 \cos\theta_o)]^2 \sin 2\theta}{[(A' B') + (A' - B')\cos 2\theta]^2}\right\} \tag{81}$$

With the aid of trigonometric expansions plus the expansion formulas given in Appendix A, the above equation (81) is resolved into Fourier series to yield,

$$T_{\ell-n} = -\frac{K3P\; 3 E_f^2}{4\omega\,(A' + \sqrt{A'B'})}\Big\{F_1F_2 \cos\theta_o\,[\sin\theta + 3b \sin 3\theta + 5b^2 \sin 5\theta + \ldots]$$

$$+ \left[F_1^2 \frac{\sqrt{A'B'}}{A' + A'B'} - F_2^2 \frac{A' - \sqrt{A'B'}}{\sqrt{A'B'}}\cos^2\theta_o\right]\cdot [\sin 2\theta + 2b \sin 4\theta + 3b^2 \sin 6\theta + \ldots]\Big\}$$

or

$$T_{\ell-n} = -\frac{K3P.\, 3 E_f^2}{4\omega(X_d' + X_o' + X_2)}\Big\{F_1F_2 \cos\theta_o [\sin\theta + 3b \sin 3\theta + 5b^2 \sin 5\theta + ..]$$

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$$+\left[F_1^2\frac{\left(X_2 + 0.5X_o'\right)}{X_d' + X_o' + X_2} - F_2^2\,\frac{\left(X_d' - X_2\right)}{X_2 + 0.5\,X_o'}\,\cos^2\theta_o\right]\cdot$$

$$\left.\cdot\left[\sin 2\theta + 2b \sin 4\theta + 3b^2 \sin 6\theta + \ldots\right]\right\} \qquad (82)$$

Summary of Results:

The phase currents have been found, by substituting equation (69) into equation (27), of the preceding chapter, and are given by equations (26) and (71). The open-phase voltage is given by equation (76), after differentiating equation (73) and employing the substituted values of equations (50) and (70 a). Substituting equations (74) and (52) into equation (80) gave the equation (81) for short-circuit torque. The sustained currents and voltages have been found (by letting time t approach infinity), and are given by equations (77, 78, 79).

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CHAPTER IV

CONCLUSIONS

In this dissertation, we have accomplished the following. We have derived differential equations that can be used to solve any number of problems of synchronous machines under unbalanced and transient conditions, and have developed new and complete expressions for armature and field currents of an alternator for a line-to-neutral fault. Most important, we have developed a new method for an approximate solution of differential equations with coefficients containing periodic functions and a small parameter. Lastly, but by no means least of all, we have paved the way for a complete and comprehensive investigation of our unbalanced system by considering the electro-magnetic action and inter-action involved.

In developing our performance equations, the matrix-tensor technique utilized proved to be a remarkably powerful and excellent tool for the study of electrical systems such as ours. It exhibited a clear-cut interpretation and understanding of the underlying physical phenomena in question. The transformation tensors proved to be labour-saving devices, as they were found to evolve quite naturally and painlessly, and required no great amount of mathematical knowledge.

Our developed method for an approximate solution to 'differential equations with coefficients containing periodic functions and a small parameter' is different from that given by any other author, and reveals more accurate answers to the case under investigation. The mathematical analysis, using our method of successive approximations plus Fourier series expansions, seems to be an effective, fool-proof and direct method to obtain the correct answer to the problem. There is no reason to doubt that the proposed method for solution to our problem is useful not only to machine analysis, but to other problems involved in other dynamic

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systems in which equations of a similar type are involved.

Finally, our investigation leads us to an encounter with electro-magnetics. Firstly, as a result of a current-carrying coil (i.e. the phase a armature coil with its sustained current) in a magnetic field (due to the sustained current in the field coil), a force or more precisely, a torque is produced on the armature coil. Secondly, with both current-carrying coils (i.e. armature and field) in close proximity, a force develops between them. Thirdly, the current flowing in the phase a armature coil causes a voltage to be induced in the open-phase armature coils. Hence, an extensive study of such electro-magnetic actions and inter-actions completes a thorough investigation of our system in question.

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APPENDIX A

The Mathematical Expressions necessary for the Series expansion are

$$\frac{\sin\theta}{x + y + (x - y)\cos 2\theta}$$

$$= \frac{1}{y + \sqrt{xy}} [\sin\theta + b \sin 3\theta + b^2 \sin 5\theta + b^3 \sin 7\theta + \ldots] \quad \text{(A-1)}$$

$$\text{where } b = \frac{\sqrt{y} - \sqrt{x}}{\sqrt{y} + \sqrt{x}} = \frac{\sqrt{xy} - x}{\sqrt{xy} + x} \quad \text{(A-2)}$$

$$\frac{\cos\theta}{x + y + (x - y)\cos 2\theta}$$

$$= \frac{1}{x + \sqrt{xy}} [\cos\theta + b \cos 3\theta + b^2 \cos 5\theta + \ldots.] \quad \text{(A-3)}$$

$$\frac{1}{x + y + (x - y)\cos 2\theta}$$

$$= \frac{1}{\sqrt{xy}} [0.5 + b \cos 2\theta + b^2 \cos 4\theta + \ldots.] \quad \text{(A-4)}$$

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APPENDIX B

LIST OF SYMBOLS

a, b, c axes	-	3-phase axes
d, q, o axes	-	direct, quadrature, zero axes respectively
α, β, o axes	-	orthogonal moving reference axes
i_α, i_β, i_c	-	instantaneous phase currents
i_d, i_q, i_o	-	direct, quadrature and zero components of current
e_a, e_b, e_c (v_a, v_b, v_c)	-	open phase voltages
e_d, e_q, e_o	-	direct, quadrature and zero axes components of armature voltage
e_α, e_β	-	moving axes components of voltage
ϕ_d, ϕ_q, ϕ_o	-	direct, quadrature, zero axes flux linkages
ϕ_α, ϕ_β	-	α-axis, β-axis flux linkages
$\theta(t)$	-	angular displacement of α and β axes from d and q axes, respectively
ω	-	synchronous speed, $2\pi f$
R_a	-	armature resistance per phase
R_f	-	field resistance
$X_o = \omega L_o$	-	zero sequence reactance
$X_d = \omega L_d$	-	synchronous reactance, direct axis
$X_q = \omega L_q$	-	synchronous reactance, quadrature axis
$X_{af} = \omega M_{af}$	-	mutual reactance between direct axis armature circuit and mainfield circuit

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$X_{ff} = \omega L_{ff}$	-	field reactance
T	-	torque, 3-phase
τ_a	-	armature time constant
τ_d'	-	field time constant
R_g	-	resistance from neutral to ground
L_g	-	inductance from neutral to ground
P	-	number of poles
p	-	$\frac{d}{dt}$

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