1-1-1976

The calculation of the loss ratio of pipe-type cable systems using the finite element numerical technique.

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THE CALCULATION OF THE LOSS RATIO OF PIPE-TYPE CABLE SYSTEMS USING THE FINITE ELEMENT NUMERICAL TECHNIQUE

by

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A Thesis
submitted to the Faculty of Graduate Studies
through the Department of
Electrical Engineering in Partial Fulfillment
of the requirements for the Degree
of Master of Applied Science at
The University of Windsor

Windsor, Ontario, Canada

1976

11e-y 1529

We the undersigned hereby find this Thesis acceptable for partial fulfillment for the Degree of Master of Applied Science in Electrical Engineering.

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ABSTRACT

The ratio of effective A. C. resistance to D. C. resistance, the loss ratio, is calculated for a single isolated conductor and for three conductors in various arrangements in a magnetic steel pipe.

The finite element numerical method is used to solve the dual problem to the solution of Maxwell's field equations. A simple but effective technique is used to specify the boundary condition for the single conductor and an approximate condition, based on skin depth, is used for three conductors in the pipe.

A real and imaginary part functional is presented and shown to be equivalent to the Maxwell field equation, including an eddy current term. The calculations are performed using both a complex arithmetic and a real arithmetic formulation and the program listings appear in the Appendix, including a useful iterative method for solving matrix equations.

Results for the single conductor case agree well with analytic calculations and the results for the three conductor pipe arrangements compare reasonably to measured values.

ACKNOWLEDGEMENTS

I would like to express my sincere thanks to Prof. P. H. Alexander, my supervisor, for his guidance throughout this project and for his considerable patience and willingness to provide assistance when conceptual impasses arose. His resourceful recommendations illuminated both previous pitfalls and possible new avenues of thought and his reservoir of experience was invaluable. I am additionally grateful for his assistance in the preparation of this thesis.

I would also like to thank Dr. E. Kuffel, Dr. M. Shridhar, Dr. M. Raghuveer and Dr. J. Soltis for their interest and helpful suggestions.

Appreciation is also due to Dr. S. I. Tang and the Academic Programming consultants for their aid with programming difficulties and job control information.

This work was made possible through financial assistance from the National Research Council of Canada and I would like to state my sincere appreciation.

A final mention of thanks is due to General Cable Corporation for their co-operation in making available data and results from their recent measurements.

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CHAPTER 1

Introduction

1.1 Loss Ratio

One of the parameters of major importance in the selection of the correct cable size for a specific application is the resistance of the cable. The direct current resistance of the cable is easily calculated with a knowledge of only the conductor conductivity and the conductor geometry. However, under alternating current excitation, it has long been known that the resistance of a cable increases to a value greater than the d. c. value. This alternating current resistance has been termed the 'effective resistance' and the ratio of effective resistance to d. c. resistance is known as the 'loss ratio'.

Fundamentally, the increase in resistance is due to an unequal distribution of current over the conductor cross section caused by the magnetic fields of the alternating current. For d. c. excitation, the current is distributed evenly over the conductor cross-section, but under a. c. excitation, the current is concentrated in the outer regions of the conductor and the conductor cross-section is inefficiently used.

For proper cable size selection, a knowledge of the loss ratio is important because, since the d. c. resistance is easily calculated, a knowledge of the loss ratio implies a knowledge of effective, or alternating current, resistance.

Investigations have previously been undertaken to calculate the loss ratio for various cable systems. An analytic solution has been obtained for the single conductor case where axial and rotational symmetry exist. For more than one conductor, the rotational symmetry no longer exists, and an analytic solution ceases to be feasible. Efforts have been made by several investigators to derive semi-empirical expressions to describe the loss ratio for varying conductor sizes and geometries.

In this work, efforts were expended to formulate a numerical solution for the problem of finding the loss ratios of multi-conductor cable systems of varying geometries. For this reason, the electromagnetic equations describing the vector magnetic potential were solved approximately by using the finite element method, and the loss ratio was calculated from the resultant knowledge of vector magnetic potential values.

A specific type of cable system was selected for this work so that the results of the numerical calculations could be compared to measured values which were recently obtained in an industrial laboratory. The cable system in question is a three-conductor, three phase pipe-type cable. The three cables are compact, segmental, stranded conductor cables loosely enclosed in a quarter inch thick, ten inch diameter magnetic steel pipe. Figure 1 illustrates the compact segmental cables and shows the two most common pipe configurations.

1.2 Scope and Limitations

For this work, it is requisite that the loss ratio be computed for many different cable sizes and configurations within the pipe. In the finite element method, the cable and pipe geometry is modelled by a triangular grid, with large numbers of triangles in regions where field quantities are expected to exhibit a rapid variation with a change in position. Thus, it was essential to find some manner in which the large number of grid points could be easily and quickly specified for each cable configuration of interest.

However, limits exist as to the number of grid points that can be employed for the modelling of the cable system. Each grid point results in an additional equation that must be solved and as the grid points increase in number, the set of elements in the matrix increases accordingly. For matrices of size greater than 100 by 100, solution methods such as Gaussion elimination break down because of round-off errors. For this reason, the cable configurations must be modelled somewhat coarsely. This means that round cable

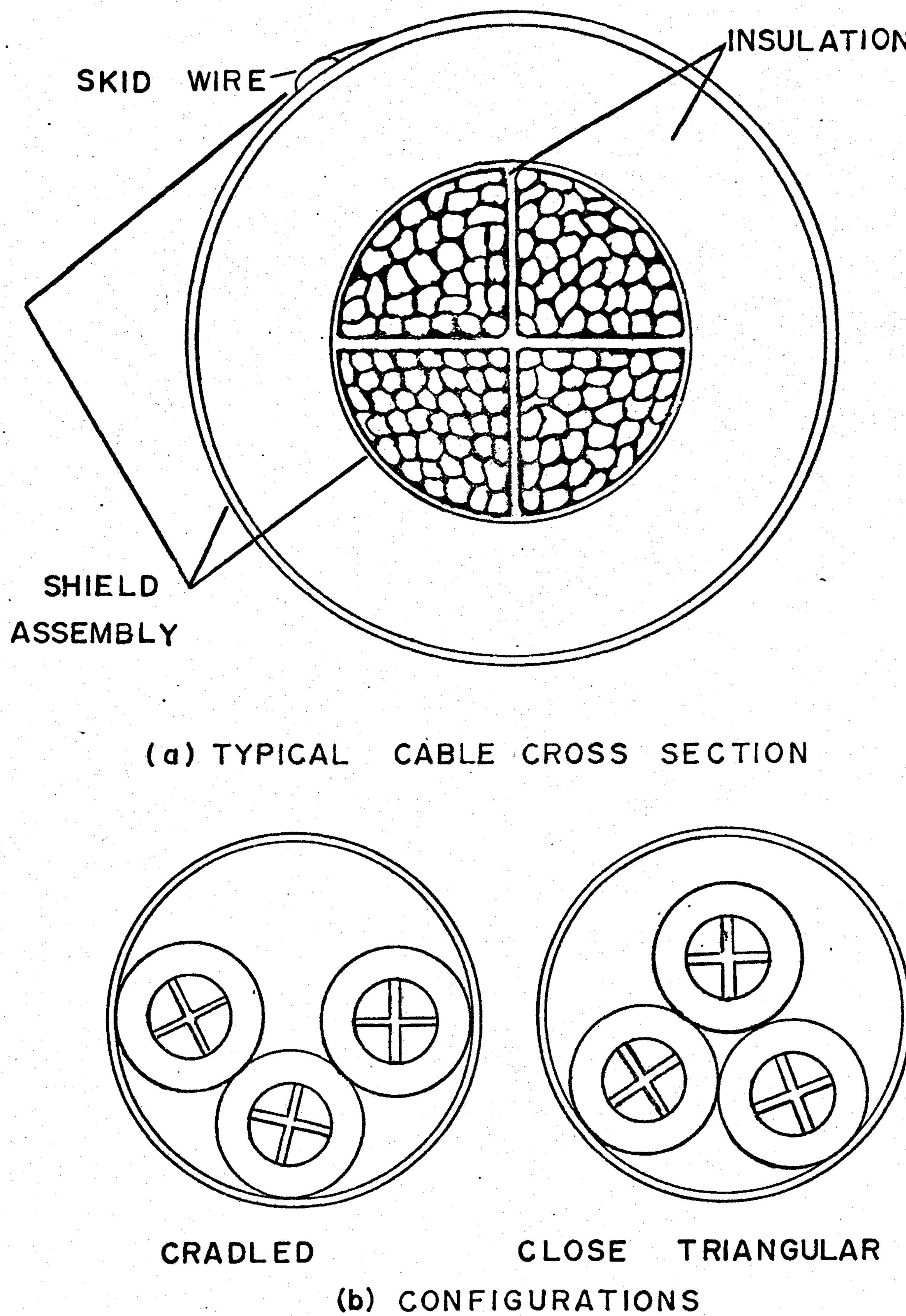


FIGURE I CABLE GEOMETRY

or pipe contours are portrayed by fewer straight line segments than may be desirable . For example octagonal figures are used to represent the round conductors. Also, it is impossible to model the stranding of the conductors without an enormous number of grid points, so the stranded conductor is modelled by a solid conductor. A stranding factor is used to modify the conductor conductivity and thus account for the stranding.

A limit also exists as to the amount of computer time that can reasonably be used for each loss ratio computation and this time constraint results in the exclusion of a number of techniques that might otherwise have been used to specify boundary conditions for the electromagnetic equations.

It is, therefore, the goal of this work to endeavour to achieve the best possible results for loss ratio calculations within the limits which exist for cable system modelling and computer time and storage requirements.

CHAPTER 2

Loss Ratio Theory

2.1 Component Losses

Several investigators have broken down the extra losses which arise under alternating current excitation into constituent parts which are attributed to different phenomena. It was hoped that a simple theory could be found to predict each of the component losses and thus obtain the effective resistance. The theory behind the various components is explained in an American Institute of Electrical Engineers Committee Report (1).

The cable power dissipation is classified into three main types: eddy losses, proximity losses and pipe losses. All three types may occur in each part of the cable. However, the power dissipated in some cable sections is minimal and the most important losses are described below:

The extra losses that occur in a single isolated conductor under a.c. excitation are termed the conductor skin effect losses. These are related to the non-uniform distribution of current which is caused by the electromagnetic fields due to the current in the conductor itself.

If two or more conductors are brought close together, the losses increase to a value greater than those due to skin effect. This increase is termed the conductor proximity effect and is the result of an intensification of the non-uniformity of the current distribution due to the magnetic fields of the neighbouring conductor.

The non-uniformity of current results in current flow mainly in the outer regions of the conductor, and the area of the conductor is inefficiently used, resulting in higher power dissipation than under d. c. excitation, where the current is evenly distributed. In small conductors, the current distribution is almost uniform, resulting in a small loss ratio. In very large conductors, it is thought that, under a. c. excitation, current flows in quasi-periodically positioned radial bands.

When conductors are placed in a magnetic steel pipe, the electromagnetic field is strengthened and distorted by the influence of the pipe material. This results in a further increase in the conductor losses and is known as the magnetic pipe effect.

These three conductor component losses, skin effect, proximity effect and pipe effect, contribute to the bulk of the extra a. c. losses for the cable system. However, in certain instances, other cable system component parts can contribute significantly. The shield assemblies of the three cables are periodically in contact with each other and form three phase circuits which allow the flow of current which is induced by the magnetic fields due to the conductor currents. This circulating current flow results in increased power dissipation. An additional loss contribution is termed the shield-assembly proximity effect and occurs when local eddy currents exist in the shield. However, this component is minor in modern cables.

The one remaining significant loss element is that occurring in the pipe itself. In addition to the extra power dissipation caused in other cable parts by the pipe, there exist eddy current losses in the pipe itself which are not negligible. Pipe hysteresis loss may also exist, but this is usually small when compared to eddy loss. The use of non-magnetic pipes would not significantly reduce dissipation in the pipe unless the material had high resistivity to suppress eddy currents, high permeability to reduce the depth of current penetration and a low hysteresis loss characteristic. However, a pipe of such a material would be prohibitively expensive.

2.2 Loss Calculation Methods

Several investigators have attempted to derive expressions for the loss ratio of a magnetic pipe-type cable system using analytical, semi-empirical and empirical means.

Perhaps the earliest investigator was Lord Kelvin, who derived an analytical solution for the problem of alternating current flow in an isolated cylindrical conductor.

The solution is in terms of Bessel functions of the first kind and a similar derivation is presented by Stevenson (2). The skin effect is calculated as a function of the frequency of the current, and the permeability, conductivity, and radius of the conductor.

The proximity effect was investigated in two papers by Arnold (3), (4). In one paper, the proximity effect due to a single phase line and its return is considered and a modified Bessel function solution is presented with the stranding of the cable treated by means of a factor which modifies the resistivity. In the second paper, Arnold's formulae are extended to include multicore cables and to account for effects due to the lead sheath and armouring. The paper handles conductor stranding by considering the cross-conductor conductivity to be about one-half the normal conductivity for concentric stranded conductors and about eighty per cent for compact segmental conductors. The equations in the paper are somewhat complicated and are empirical modifications of solutions developed for simpler geometries.

Arnold's papers did not really refer to cables in steel pipes but research into the resistance of cables in steel pipes was reported by Wiseman (5) and Meyerhoff and Eager (6). Wiseman reported on loss ratio measurements that were carried out on a number of cables, and he used a modified version of Arnold's equations to account for the magnetic pipe in calculations of the loss ratio. The Meyerhoff and Eager measurements were done on compact segmental cables. Some of the equations presented are again modifications of Arnold's equations, but some new derivations are also performed to calculate the pipe loss. An empirical factor was included in the equations to account for cable segmentation.

In an effort to consolidate the work of the foregoing and other researchers, and to standardize the equations for the loss ratio of segmental conductors in steel pipes, a report was written by the American Institute of Electrical Engineers (1). This report also investigates possible design variations for reducing the cable losses. Another paper by Neher and McGrath (7) investigates all the aspects of cable rating and includes a section on the calculation of A.C. resistance. Stranding factors for various cables are furnished and functions used in determining the losses are presented in tabular and graphical form. The paper by Neher and McGrath is the paper that is most usually quoted in recent literature for the calculation of loss ratio.

Attempts at the numerical solution of the loss ratio problem are not abundant in the literature, but Stoll (8) has used the finite difference method to calculate eddy currents and Silvester (9), (10), (11), (12) has developed a modal analysis to calculate the a.c. resistance of certain types of conductors. Silvester's papers use an entirely different method than the technique employed in this work. However, the analysis is never applied to pipe-type cables and it is difficult to determine whether or not only the skin effect loss component is treated. Additional study of these papers is probably warranted to see if the application to pipe-type cables is valid.

2.3 Factors Affecting Loss Ratio

In the literature, many factors which could affect the loss ratio are discussed, but various researchers are not always in agreement on the effect or the importance of the factors.

Conductor conductivity and radius, current frequency and cable geometry are factors on which almost everyone agrees. Increased conductivity, radius and frequency results in a higher loss ratio and the loss ratio is usually higher for a cradled configuration than for a triangular configuration in a pipe. Wiseman (5) does report one measurement where the opposite is true.

For other factors, agreement among researchers is not good. This is true for the effect of current and temperature. Some researchers, such as Arnold, state that the only consequences of increased current are those effects caused by the increase in temperature resulting from the increased loss. Other authors reason that, as the current is increased, more and more of the cross-sectional area is used up, thus decreasing the skin effect. A reduction of the loss ratio with an increase in current at a particular temperature is attributed to the improved current distribution at the higher current level.

With regard to the effect of temperature, it appears that an even greater difference of opinion exists. Wiseman (5) and the AIEE Committee (1) report that the difference between the a.c. and d.c. resistance does not change with temperature. As the heat level increases both the a.c. and d.c. resistance increase by the same amount and the loss ratio decreases. However, recent measurements indicate that temperature plays a much larger role and that heat cycling occurs. When a cable is heated to a high value (about 150°C) and then brought back to room level the loss ratio at room temperature is found to have increased to a value greater than that before heating. This holds true for various starting temperatures and the loss ratio at a given temperature is a function of the maximum conductor heating that occurs after installation. It is thought that the inter-strand contact resistance is permanently altered by the heating to high temperatures. Subsequent temperature cycling to the same high level does not significantly change the loss ratio and the overall change appears independent of the current magnitude at a specific temperature.

The decrease in transverse strand resistance is suspected of increasing the loss ratio by making it easier for the currents to travel in the outer portions of the cable and thus increasing the skin effect. Other researchers have also noticed the relationship between inter-strand resistance and the loss ratio. Arnold (4), who introduced a stranding factor to account for the decrease in resistance in a stranded cable, noted that the stranding factor depends on the surface condition of strands, the lay of strands, core impregnation and the tightness of insulation. Meyerhoff and Eager (6) noted that the stranding factor was influenced by cable treatment and the type of binder used. The AIEE Committee found that such uncontrollable factors as strand oxidation and handling have an effect and that even cables manufactured by the same company exhibited different characteristics. This Committee recommended that the a.c. resistance could be reduced by insulating strands to increase the transverse resistance and they found that coating the strands with varnish did reduce the loss ratio.

Other cable design parameters can also have an effect Wiseman (5) reported that the loss ratio was less for segmental than for concentric stranded cables.and Meyerhoff and Eager (6) noted that it was dependent on the number of segments that were insulated. They also stated that the construction of the cable shield affects the loss ratio, with high losses occurring in cables with low resistance shields and suggestions were presented for methods of increasing the shield assembly resistance.

The AIEE Committee examined a large number of possible loss ratio parameters and concluded that the loss ratio was not affected by the phase sequence of the current, by twisting the cables or by placing oil in the pipe (1). They did report that the effective resistance increased with the use of skid wires made of magnetic steel instead of the usual brass or copper, and that, although losses did not change with the use of aluminum or other non-magnetic pipe materials, they did become larger with increasing pipe size for a cradled configuration and decreased with increasing pipe size for a close triangular configuration.

In the present loss ratio analysis, the computer program models some of the parameters, but not all. The temperature is assumed to be constant at room temperature but variations could be implemented by changing conductor conductivities. The current is read into the program and both magnitude and phase can be adjusted to any value. Stranding variations can be implemented by altering the stranding factor. The shield assembly is not modelled but provision is made for the representation of conductor segmentation. Pipe material can be modified by changing the conductivity and permeability characteristics and the pipe can be made any size and thickness. The cables can be placed in any spatial geometry, either with or without the pipe and the conductors can be any size. Provision is also made for any number of conductors and the fineness of the finite element grid can be adjusted over a fairly wide range.

These features ensure that the programme is sufficiently flexible to be useful in studying an extensive variety of practical cable configurations.

CHAPTER 3

Numerical Methods

3.1 Available Methods

Two numerical methods are widely used for the solution of differential equations and accompanying boundary conditions. The oldest method is the finite difference method where the differential operator itself is approximated. A rectangular grid is drawn over the region of interest and the differential equation is approximated by writing difference equations between neighbouring grid points. A set of equations results which is usually solved by over-relaxation iterative techniques.

The disadvantage of this method is that, for programming ease, the grid must be uniform and rectangular over the entire region of interest. This means that for irregular or curved boundaries, the grid pattern must be very fine and the boundary must be approximated in a step-wise fashion. Advances in the finite difference technique have been developed to allow an irregular grid and to permit special equations to be written at the boundaries, but this results in significant programming difficulties.

Many of the problems inherent in the use of the finite difference method can be eliminated by the use of a different numerical procedure, the finite element method. Here, a regular grid is not required, but instead, a triangular grid is used with small triangles in regions of high interest and larger triangles elsewhere. With a judicious choice of the number and size of triangles, even complicated boundaries can be faithfully modelled, and this, basically, is the most important reason for the preference of the finite element method over the finite difference method.

3.2 The Finite Element Method

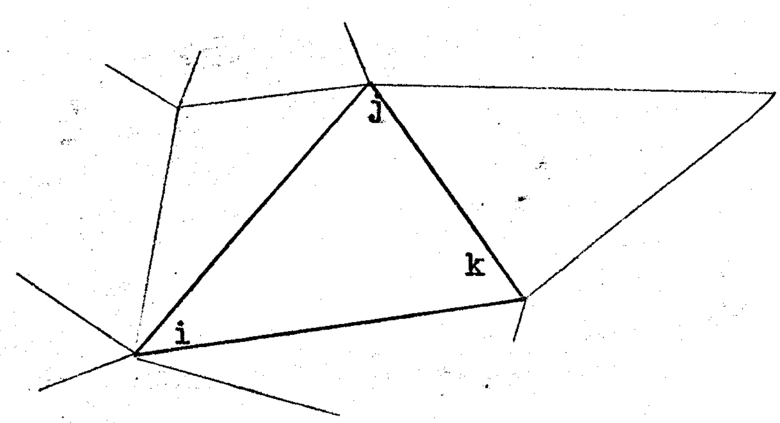
The finite element method has evolved from the work of a large number of people who were active in a variety of fields. Perhaps the most direct development of the method resulted from work on the stress analysis of trusses and beams. When a finite number of connection points did not exist in a structure under analysis, the structure was divided into sections with imaginary connection points and an approximate stress analysis was done using these points. Gradually, it was realized that this technique was related to the Rayleigh-Ritz variational technique and a mathematical basis for the finite element method was derived.

The application to electromagnetic and other fields was first demonstrated in a now famous paper by Zienkiewicz and Cheung in 1965 (13). In this paper, an energy functional was minimized jointly over each subregion in the area of interest

It was shown that the minimization of the functional was equivalent to the solution of Poisson's equation, and the finite element method was thus made available for applications in heat flow, electromagnetic phenomena and other continuous field problems.

The first step in attacking any finite element problem is to divide the domain into triangular sub-regions. The nodal points of the triangles are the variables of the problem and the field in each triangle is expressed in terms of nodal values. Usually, interpolation polynomials of order one or greater are used to characterize the field in the triangles and to ensure that the field is continuous between adjoining triangles. In this work, first order equations are used throughout, but there would be no substantial difficulty in increasing the order.

Consider a typical triangle.



The potential is assumed to be everywhere of the form

$$= A + Bx + Cy$$

Triangle i,j,k has nodal potentials Φ_i, Φ_j and Φ_k which must be determined to solve the problem. At each of the triangle nodes the potential must satisfy the general form

$$\begin{aligned}
\Phi_i &= A + Bx_i + Cy_i \\
\Phi_j &= A + Bx_j + Cy_j \\
\Phi_k &= A + Bx_k + Cy_k
\end{aligned}$$

The three variables A, B and C can be found by solving the three equations with the use of Cramer's rule. In Matrix form:

$$\begin{bmatrix} \Phi_i \\ \Phi_j \\ \Phi_k \end{bmatrix} = \begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$A = \frac{\begin{vmatrix} \Phi_i & x_i & y_i \\ \Phi_j & x_j & y_j \\ \Phi_k & x_k & y_k \end{vmatrix}}{\begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix}}$$

$$A = \frac{\Phi_i(x_j y_k - x_k y_j) + \Phi_j(x_k y_i - x_i y_k) + \Phi_i(x_i y_j - x_j y_i)}{x_j y_k - x_k y_j + x_k y_i - x_i y_k + x_i y_j - x_j y_i}$$

If
$$a_i = x_j y_k - x_k y_j$$
$$a_j = x_k y_i - x_i y_k$$
$$a_k = x_i y_j - x_j y_i$$

then
$$A = \frac{a_i \Phi_i + a_j \Phi_j + a_k \Phi_k}{2\Delta}$$

Similarly
$$B = \frac{\Phi_i (y_j - y_k) + \Phi_j (y_k - y_i) + \Phi_k (y_i - y_j)}{2\Delta}$$

$$C = \frac{\Phi_i (x_k - x_j) + \Phi_j (x_i - x_k) + \Phi_k (x_j - x_i)}{2\Delta}$$

where Δ=triangle area

By defining

$b_i = y_j - y_k$; $c_i = x_k - x_j$,

$b_j = y_k - y_i$, $c_j = x_i - x_k$,

$b_k = y_i - y_j$, and $c_k = x_j - x_i$,

one can write

$$\Phi(x,y) = \frac{(a_i + b_i x + c_i y)\,\Phi_i + (a_j + b_j x + c_j x)\,\Phi_j + (a_k + b_k x + c_k y)\,\Phi_k}{2\Delta}$$

Thus, once the nodal potentials Φ_i, Φ_j and Φ_k are known, the potential anywhere in the triangle is given in terms of the nodal potentials and the nodal positions.

If shape functions are defined as

$$N_i = \frac{a_i + b_i x + c_i y}{2\Delta}$$

then the potential for one element can be written in matrix form:

$$\Phi = \begin{bmatrix} N_i & N_j & N_k \end{bmatrix} \begin{bmatrix} \Phi_i \\ \Phi_j \\ \Phi_k \end{bmatrix}$$

It should also be noted that, since the potential is defined as a linear function of the nodal values along a triangle edge, the potential along a triangle edge is uniquely specified and continuity between adjoining triangles is assured.

Therefore, the entire finite element problem boils down to selecting the nodal potential values in such a manner that the defining equations and boundary conditions are satisfied. This is done by a transformation into a variational form. Instead of solving the field equation directly, an expression, known as a functional, is sought such that the minimization of this functional is equivalent to solving the field equation. It may not always be easy, or even possible, to find a functional for every field equation, but when it exists, its equivalence to the field equation can be shown and the finite element method can be used.

As an example consider the equation

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\mu J$$

The functional whose minimization is equivalent to this equation's solution is

$$X = \int \left\{ \left| \frac{\partial A}{\partial x} \right|^2 + \left| \frac{\partial A}{\partial y} \right|^2 - 2\,\mu J A \right\} dxdy$$

A full test for equivalence appears in Appendix 1, but a quick check for equivalence is obtained by applying Euler's criteria:

$$\frac{\partial F}{\partial w} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial w_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial w_y}\right) = 0$$

where $w_x = \frac{\partial w}{\partial x}$ and $w_y = \frac{\partial w}{\partial y}$, and where w is the field variable and F is the integrand of the functional.

For the above functional, $W \equiv A$, so

$$\frac{\partial F}{\partial w} = -2\,\mu J \qquad \frac{\partial F}{\partial w_x} = 2\,\frac{\partial A}{\partial x} \qquad \frac{\partial F}{\partial w_y} = 2\,\frac{\partial A}{\partial y}$$

Therefore Euler's equation gives

$$-2\,\mu J - \frac{\partial}{\partial x}\left(2\frac{\partial A}{\partial x}\right) - \frac{\partial}{\partial y}\left(2\,\frac{\partial A}{\partial y}\right) = 0 \quad \text{or}$$

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\,\mu J$$ which is the original field equation.

This, together with the derivation in Appendix 1, shows the equivalence between the solution of the field equation and the minimization of the functional. Thus, to solve the desired equation, it is necessary only to adjust the potential values until the functional is minimized. This is most easily done by differentiating the functional with respect to each of the nodal values and setting the resulting expressions to zero. This results in a set of N equations in N unknowns where N is the number of nodes. First the expression derived for the potential is placed in the functional and this is then differentiated with respect to nodal values.

$$\chi = \int \left\{ \left| \frac{\partial A}{\partial x} \right|^2 + \left| \frac{\partial A}{\partial y} \right|^2 \right\} dxdy - 2 \int \mu \, JA \, dxdy$$

From the expression for potential,

$$\frac{\partial A}{\partial x} = \left[\frac{b_i \quad b_j \quad b_k}{2\Delta} \right] \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix}$$

$$\frac{\partial A}{\partial y} = \left[\frac{c_i \quad c_j \quad c_k}{2\Delta} \right] \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix} .$$

Δ is the triangle area

$$\frac{\partial \chi}{\partial A_i} = \int \left\{ \left[b_i \frac{\left[b_i \quad b_j \quad b_k \right]}{2\Delta^2} + c_i \frac{\left[c_i \quad c_j \quad c_k \right]}{2\Delta^2} \right] \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix} \right\} dxdy - 2 \int \mu \, J \, N_i \, dxdy = 0$$

In Appendix 2, it is shown that $\int N_i \, dxdy$ equals $\frac{\Delta}{3}$.

Therefore, since $\int dxdy = \text{Area} = \Delta$

$$\frac{\partial \chi}{\partial A_i} = \left[b_i \frac{\begin{bmatrix} b_i & b_j & b_k \end{bmatrix}}{4\Delta} + c_i \frac{\begin{bmatrix} c_i & c_j & c_k \end{bmatrix}}{4\Delta} \right] \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix}$$

$$- \frac{J\mu\Delta}{3} = 0$$

By similarly differentiating with respect to the other nodes, three equations in three unknowns result, which can be put in matrix form

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix} = \begin{bmatrix} + \frac{\mu J\Delta}{3} \\ + \frac{\mu J\Delta}{3} \\ + \frac{\mu J\Delta}{3} \end{bmatrix}$$

where

$$S_{ij} = \frac{b_i b_j + c_i c_j}{4}$$

After this matrix equation is written for each element in the region, these sub-matrices are used to form the master matrix for the entire problem. Each entry in the sub-matrix is placed into the master matrix in a position given by the overall node numbering of the triangle vertices. For example, a triangle with nodes 7, 21, and 23 will have its value of S_{11} placed in row 7, column 7 of the master matrix and S_{13} will be placed in row 7, column 23. Since adjoining triangles share the same nodes, many positions in the master matrix will have contributions from more than one triangle. This provides the interaction needed to assure a continuity of solution.

Once the contributions from all the triangles have been added to the master matrix, it is only necessary to impose the existing boundary conditions before the equation is solved and the solution is obtained.

3.3 Features of the Finite Element Method

From the simple description of the finite element method described above, sophistications have been applied to make the method applicable to a greater variety of problems.

For greater accuracy, the use of higher order polynomial approximations to the variation within each triangle is recommended. Zienkiewicz (14) gives a good discussion of the derivation, use, and ramifications of higher order finite elements. In order to facilitate the solving of problems with curved boundaries, several persons have developed finite elements with curved sides. Silvester and Rafinejad (15) and Ergatoudis, Iron and Zienkiewicz (16) directly employed elements with curved sides whereas Richards and Wexler (17) espoused a technique where straight-sided elements were used but integration was carried out over the curved boundary.

In addition, several authors have written about the use of three-dimensional elements. Zienkiewicz, Bahrani and Arlett (18), (19) showed the use of tetrahedral elements as a base for building up larger eight-cornered element blocks and Ziekiewicz and Parekh (20) demonstrated the use of curved three-dimensional elements. The same paper also illustrated a possible treatment of transient problems by using finite elements in space and time. Finite elements of order one greater than the number of spatial dimensions are used and the extra element order is used to time-step the solution.

One other development that results in an even greater range of applications for the finite element technique is the use of an alternate procedure for deriving the finite element equations. This is useful for those instances where a functional cannot be found which is equivalent to the field equation which is to be solved. Several authors (14), (20), (21) have described the process wherein the weighted residual of the field equation is minimized. Usually, element shape functions are used as the weighting functions and a transformation is performed to decrease the order of the resulting system of equations. This technique greatly increases the number of problems that can be tackled. As a result, the finite element method can be used for a large variety of problems if a method for the treatment of boundary conditions can be specified.

Boundary conditions may be incorporated into the problem in either of two ways: an additional term may be added to the functional to satisfy the conditions or the final matrix may be modified after it has been formed.

Dirichelet conditions are usually applied by modifying the master matrix. For a node which is set at some potential value, the row values of the node are set to zero and the diagonal term is set to unity. The fixed potential value is then placed in the corresponding place in the source matrix. For example:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix}$$

To fix the potential value of the second node to a value of 23, the matrix becomes

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ 0 & 1 & 0 \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} J_1 \\ 23 \\ J_3 \end{bmatrix}$$

However, this leads to an asymmetric matrix. The matrix may be made symmetric again by the following technique.

$$\begin{bmatrix} S_{11} & 0 & S_{13} \\ 0 & 1 & 0 \\ S_{31} & 0 & S_{33} \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix} = \begin{bmatrix} J_1 - S_{12} \times 23 \\ J_2 \\ J_2 - 23 \times S_{32} \end{bmatrix}$$

If convenient, the row and column may be eliminated and the order of the matrix reduced.

Although this is the usual way of imposing Dirichelet conditions, Hazel and Wexler (22), present a term which may be added to the functional to account for the Dirichelet conditions directly. In the same paper, the authors illustrate how the mixed or Cauchy boundary condition is handled. The condition $\frac{\partial \phi}{\partial n} + \sigma(s)\phi(s) = h(s)$

is imposed by including a term of the form

$$\oint \left[\sigma(s)\phi^2 - 2h(s)\phi \right] ds \quad \text{in the functional.}$$

By letting $\sigma(s)$ be equal to zero, Neumann conditions can be applied. The homogeneous Neumann condition, $\frac{\partial \phi}{\partial n} = 0$, is special to the finite element method because it is a natural boundary condition for the method such that no added term in the functional is needed. In the absence of another applied boundary condition, the homogeneous Neumann condition is in effect. This can be seen by letting $\sigma(s)$ and h(s) go to zero in the above equations. The functional term vanishes and the differential equation becomes $\left.\frac{\partial \phi}{\partial n}\right|_s = 0$, which is the homogeneous Neumann condition.

3.4 Electromagnetic Equations

Before the correct functional can be found and the finite element method applied, the correct electromagnetic equations which describe the physical problem must be determined. For this work, it was decided to formulate the equations in terms of the vector magnetic potential and the derivation of the equations was found in a paper by Stoll (8).

The basic electromagnetic equations, in the abscence of displacement currents are:

$$\nabla \times \overline{H} = \overline{J}_{tot} = \overline{J}_s + \overline{J}_e$$

$$\nabla \cdot \overline{B} = 0$$

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

$$\nabla \cdot \overline{J}_e = 0$$

$$\overline{B} = \mu \overline{H}$$

$$\overline{J}_e = \sigma \overline{E}$$

Here $\overline{J}_s$ is the source current, $\overline{J}_e$ is the eddy current, and $\overline{E}$ is the partial field related to the eddy current. The vector magnetic potential, $\overline{A}$, can be defined such that

$$\nabla \times \overline{A} = \overline{B}$$

$$\nabla \cdot \overline{A} = 0$$

The first equation is consistent with $\nabla \cdot \overline{B} = 0$, since the divergence of the curl of any vector field is identically zero. The second equation, together with the equation $\nabla \cdot \overline{J}_e = 0$, indicates that both the vector magnetic potential and the eddy current must sum to zero over the conductor area. This is seen by integrating the point form of the divergence expression over the volume of interest

$$\int_{vol} \nabla \cdot \overline{A}\, dv = 0$$

Applying the divergence theorem to this integral gives

$$\oint_{s} \overline{A} \cdot d\overline{S} = 0$$

where $d\overline{S}$ is the area vector of the cable cross-section. This expression indicates that the average value of vector magnetic potential must be zero over the cable cross-section area.

To derive the second order differential equation consider:

$$\nabla \times \overline{E} = -\frac{\partial}{\partial t}\overline{B} = -\frac{\partial}{\partial t}\nabla \times \overline{A}$$

$$\overline{E} = -\frac{\partial}{\partial t}\overline{A} \qquad (+\nabla\phi \text{ in general})$$

Since

$$\sigma\overline{E} = \overline{J}_E,\ \nabla \times \overline{H} = \overline{J}_e + \overline{J}_s,\ \text{and } \overline{B} = \mu\overline{H}$$

$$\nabla \times \frac{1}{\mu}\overline{B} = -\sigma\frac{\partial\overline{A}}{\partial t} + J_s$$

$$\text{or } \nabla \times \frac{1}{\mu}\nabla \times \overline{A} = -\sigma\frac{\partial\overline{A}}{\partial t} + J_s$$

For a long conductor, it is assumed that there is no variation of $\overline{A}_z$ in the z-direction and that the component of vector magnetic potential in the z-direction is the only component that exists. Therefore,

$$\nabla \times \overline{A} = \frac{\partial A_z}{\partial y}\hat{a}_x - \frac{\partial A_z}{\partial x}\hat{a}_y$$

Using this result,

$$\nabla \times \frac{1}{\mu} \nabla \times \bar{A} = -\sigma \frac{\partial \bar{A}_z}{\partial t} + \bar{J}_s$$

$$= \nabla \times \frac{1}{\mu}\left(\frac{\partial \bar{A}_z}{\partial y} \hat{a}_x - \frac{\partial \bar{A}_z}{\partial x} \hat{a}_y\right)$$

$$= -\frac{\partial}{\partial y} \frac{1}{\mu} \frac{\partial \bar{A}_z}{\partial y} \hat{a}_z - \frac{\partial}{\partial x} \frac{1}{\mu} \frac{\partial \bar{A}_z}{\partial x} \hat{a}_z = -\sigma \frac{\partial \bar{A}_z}{\partial t} + \bar{J}_s$$

Therefore,

$$\frac{\partial}{\partial x} \frac{1}{\mu} \frac{\partial A}{\partial x} + \frac{\partial}{\partial y} \frac{1}{\mu} \frac{\partial A}{\partial y} = \sigma \frac{\partial A}{\partial t} - J_s$$

The equation is now a scalar equation since all the components are in the z-direction.

It remains only to specify boundary conditions before the differential equation can be solved. However, for this problem, there are no real boundary conditions that can be applied. The vector potential cannot be specified to be constant anywhere and it does not go to zero at infinity. The only equation that can really be used to implement a boundary specification of sorts is $\nabla \cdot \bar{A} = 0$. This can be used as a check. If the boundary conditions are specified in some manner, they can be adjusted until the divergence equation is satisfied. When the iterated boundary conditions allow the divergence equation to be satisfied, then the boundary condition is the correct one.

Other boundary conditions may be able to be specified in the presence of special materials, such as high permeability steels, or other techniques may be applied to simulate the boundary condition with the use of Green's functions or to calculate them by means of iterative techniques (23), (24), (25), (26), (27), (28).

3.5 Matrix Solution Techniques

The application of the finite element method to an electromagnetic field problem results in a large set of simultaneous equations which can be put in matrix form. The solution of the equations requires a major portion of the computer time used in the problem and, as a result, the efficiency of the solution technique is of considerable importance. Both the matrix entries and the solution vector are complex quantities and either complex computer operations must be used or two systems of real equations must be solved. In this work, both of these methods were used, with the complex arithmetic method being used predominantly.

The complex matrix equation in this work was solved by using a complex gaussion elimination subroutine called CSOLVE*. The subroutine uses a maximum pivot strategy and was used at various times in single and double precision form. CSOLVE was used in conjunction with another subroutine, CMPRUV, which was used to improve the values obtained from CSOLVE by adjusting the solution values iteratively. For each matrix row, the largest element was

* See Appendix 4

chosen and the corresponding solution vector entry was set to zero and recalculated by subtracting, from the right hand side, the vector product of the row and the solution vector and by then dividing the result by the previously found largest entry. The procedure was continued until successive solution vector changes were sufficiently small. However, the residual of the solution obtained using CSOLVE alone was invariably good enough so that no iterative adjustment was necessary, and the use of subprogram CMPRUV was dropped from later program runs.

When the real and imaginary part method of solution was used, a completely different concept was used in the solution subprogram.

In this procedure, called CONGRH, a system of equations, $Ax = \mathbf{b}$, is solved by an iterative search technique which hunts along conjugate directions and minimizes the distance between the search vector and the solution vector at each step. If round-off errors are insignificant, the search terminates at the solution after N iterations where N is the order of the system.

The program CONGRH is a modification of a program written by Dr. M. Shridhar and the theoretical basis can be found in a paper by F. S. Beckman (29). The advantages of this solution method are that both less computer storage area and less computer time are needed to solve the problem since no zero matrix entries are stored. For one problem, a comparison between the use of CONGRH and the use of SIMQ, an IBM library subroutine, showed that CONGRH was ten times faster and required only one-third the core area. The only difficulty with the method is that it does not always converge and it is often difficult to determine why convergence is not achieved for a specific problem. A listing of the program appears in Appendix 4.

3.6 Loss Ratio From Vector Magnetic Potential

The solution of the finite element method matrix system results in a set of vector magnetic potential values for various locations within the cable system. From these potentials, a value for the loss ratio can be obtained by using the finite element equations.

The power per unit length for a conductor of conductivity σ, is given by $\int_R \frac{|\overline{J}|^2}{\sigma}\, d[\text{Area}] = \int_R \frac{\overline{J}\cdot\overline{J}^*}{\sigma}\, d[\text{Area}]$

where $\overline{J}$ is the total current density.

$$\overline{J} = \overline{J}_s - j\,\omega\,\sigma\,\overline{A} \qquad \frac{\delta \overline{A}}{\delta t} = j\omega\overline{A} \text{ for a sinusoidal input}$$

J_s is the source current

$$P_{ac} = \frac{1}{\sigma} \int_R (\overline{J}_s - j\,\omega\,\sigma\,\overline{A}) \cdot (\overline{J}_s - j\,\omega\,\sigma\,\overline{A})^*\, d[\text{Area}]$$

$$= \frac{1}{\sigma} \int_R (\overline{J}_s\overline{J}_s^* - j\omega\sigma\,\overline{A}\overline{J}_s^* + j\omega\sigma\,\overline{A}^*\overline{J}_s + \sigma^2\omega^2\overline{A}\overline{A}^*)\, d[\text{Area}]$$

The loss ratio is given by the ratio of the a.c. power to the source current power

$$\text{Loss Ratio} = \frac{P_{ac}}{\frac{1}{\sigma}\int_R \bar{J}_s \cdot \bar{J}_s^* \, d\,\text{Area}} = \frac{P_{ac}}{P_s}$$

$$= 1 + \frac{\sigma\omega^2 \int \bar{A} \cdot \bar{A}^* d\,\text{Area}}{P_s} + j\omega \frac{\int \left\{ \bar{A}^* \bar{J}_s - \bar{J}_s^* \bar{A} \right\} d[\text{Area}]}{P_s}$$

The integrations in the loss ratio calculations are easily performed since the source current density is assumed constant and the vector potential in any triangle is expressed in polynomial form. The integration can be performed for each triangle in turn and the results summed to obtain the overall loss ratio. Since the integrands are polynomial functions, two methods exist for determining the exact result. The integration may be performed analytically, or Gaussion quadrature of a sufficient order to assure exactness may be used. Either method is equally easy, and the analytic method was used in this work.

CHAPTER 4

Single Conductor Results

4.1 Small Radius

The first calculations using the finite element method to compute a.c. resistance were done for a single, solid, small conductor with a radius of one centimeter. This conductor size was chosen so that the finite element solution could be compared with results that were available from analytic and finite difference calculations.

For this conductor size, the eddy current term in the field equation was neglected because a functional was available for this modified field equation in the paper by Zienkiewicz and Cheung (13). Without the eddy current term, $\sigma \mu \frac{\partial \bar{A}}{\partial t}$, the field equation is

$$\frac{\partial^2 \bar{A}}{\partial x^2} + \frac{\partial^2 \bar{A}}{\partial y^2} = - \mu \bar{J}$$

The corresponding functional to be minimized is

$$\chi = \int \left\{ \left| \frac{\partial \bar{A}}{\partial x} \right|^2 + \left| \frac{\partial \bar{A}}{\partial y} \right|^2 - 2 \mu \bar{J} \bar{A} \right\} dx\, dy$$

The boundary conditions used for the computations were those suggested in the paper by Stoll (8). Because of rotational symmetry in the problem, and because all boundary points are also symmetrically placed, the potential is the same on all boundary points. Stoll suggests that the boundary potential be fixed at some arbitrary value and that the potential in the conductor be calculated using this boundary condition. The average value of potential is then calculated over the conductor area and this average value is subtracted from each of the computed potential values and the loss ratio is calculated using these adjusted values.

With the average value of potential subtracted out, the average value of potential over the conductor area is zero. This assures that the eddy currents go and return in the same path and that the following equation is satisfied:

$$\nabla \cdot \bar{A} = 0$$

The results obtained, using this method of specifying the boundary condition, and using the modified equation, compared favourably with analytic and finite difference results.

TABLE 1

Comparison Between Loss Ratio Calculation Methods

ANALYTIC	FINITE DIFFERENCE	FINITE ELEMENT
1.0145	1.0142	1.0146

These results were obtained for an isolated, solid, round conductor with a conductivity of 5.80×10^7 mhos per meter and a radius of 1 centimeter. The triangular mesh for the finite element method consisted of 196 triangles with 97 nodes. This mesh comprised five conducting rings and one insulating ring divided into sixteen segments as shown in Figure 2.

Theoretically, the accuracy of the finite element method can be increased, so that the analytic result is reached, simply by making the triangular mesh sufficiently fine, but this accuracy is not needed. The closeness of the result obtained using the relatively coarse mesh is sufficient to lend confidence to the belief that the finite element method can be used to calculate the loss ratio to a useable accuracy.

4.2 Larger Conductor Radius

After the success of the calculations for the small radius conductor, various parameters were varied to determine their effect on the answer. The conductors are divided into triangles by specifying the number of radial rings and the number of segmental divisions and the grid fineness is adjusted by changing these variables. The rings may be specified to be either conducting or insulating, the position of the rings can be adjusted, and the outer boundary can be placed at any position. Also, the outer conductor radius is adjusted so that the cross-sectional area of the straight-sided conductor model is the same as the corresponding circular conductor. In addition, the cable can be

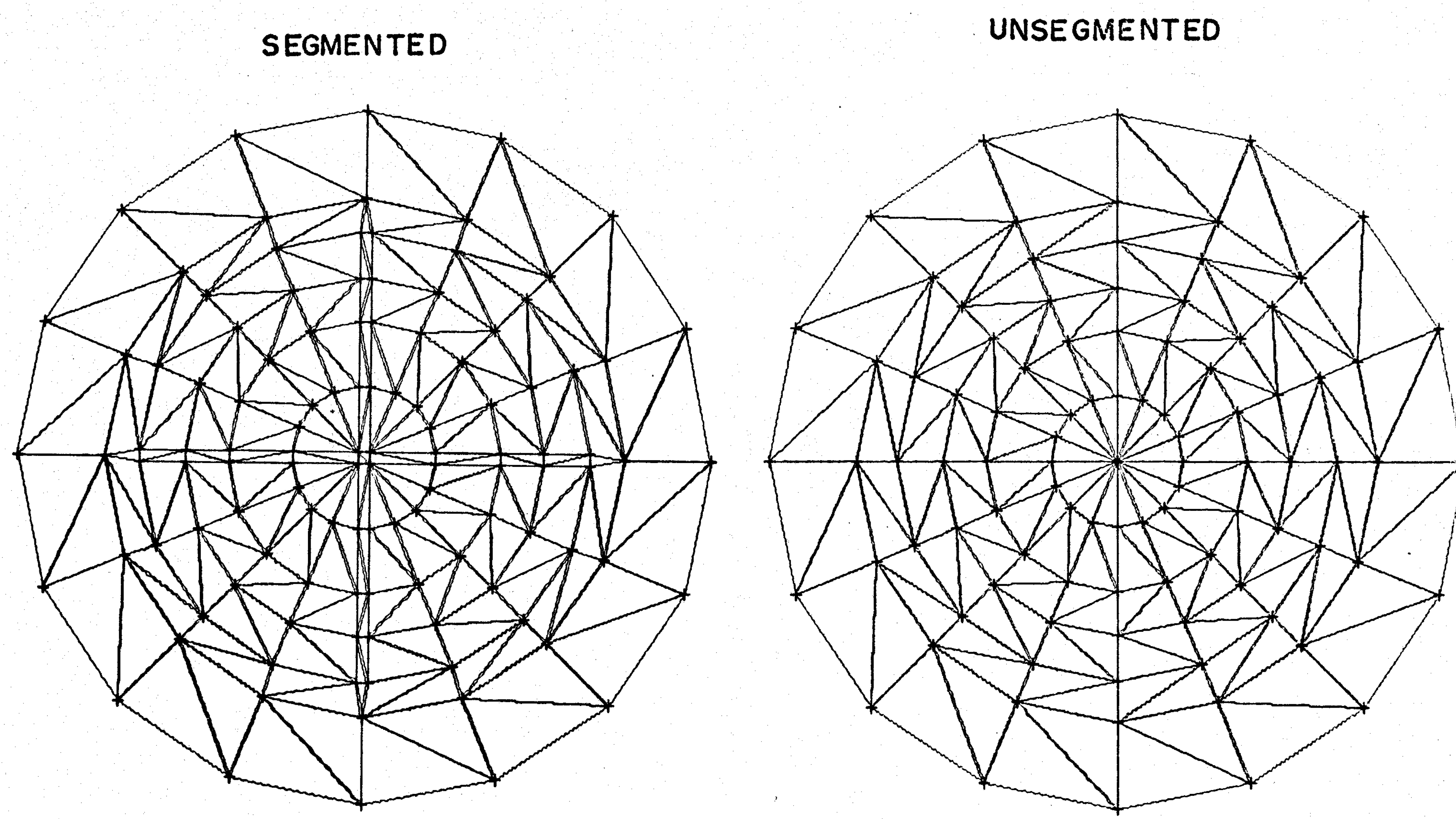


FIGURE 2 SINGLE CONDUCTOR ELEMENT PATTERNS

specified to be either a solid conductor or a segmental conductor by separating cable quarters by a strip of insulation. Results are presented in Table 2.

From the foregoing data, the effects of the changes in the various parameters can be seen. An increase in radius or conductivity increases the loss ratio. An increase in the number of triangles due to an increase in the number of segments results in increased accuracy. The number of insulating rings does not affect the answer at all, but a change in the position of the conducting rings (location of break-points) does change the answer slightly. Finally, it can be seen that the segmented cable has a lower loss ratio than the solid cable (1.56 compared to 1.59).

When these results were compared to the corresponding analytical results, it was found that results for small conductor cables had good correlation. However, as the conductor radius and conductivity increased, the numerical results deviated more and more from the analytic data.

For the analytic calculation of the loss ratio, formulae and tables are usually presented as a function of a variable mr, which is defined as

$$mr = r\sqrt{2\pi f \sigma \mu}$$

where r - conductor radius

f - source current frequency

μ - conductor permeability

σ - conductor conductivity

TABLE 2

Single Conductor Results (Without Eddy Term)

Radius	# Segments	Rings	Conductivity $x10^7$ ℧/m	Loss Ratio	Special
1 cm	16	5/6	3.54	1.014601	
1 cm	8	4/7	3.54	1.013538	
1 cm	16	5/6	3.54	1.01348	Unadjusted Radius
1 cm	8	4/5	3.54	1.013502	
1 cm	8	4/4	3.54	1.013502	
1 cm	8	4/4	3.54	1.013590	Different Break Points
1 cm	12	4/7	3.54	1.015406	Different Break Points
1 cm	12	4/5	3.54	1.014285	
1 cm	8	4/4	5.80	1.03625	
1 cm	12	4/5	5.80	1.03835	
.791 in	8	4/7	5.80	1.590352	
.791 in	12	4/5	5.80	1.622965	
.791 in	12	4/5	5.80	1.540116	Unadjusted Radius
.791 in	12	8/8	5.80	1.536107	Unadjusted Radius
.791 in	12	4/5	3.54	1.232067	
.791 in	12	4/6	3.54	1.232067	
.791 in	12	4/7	3.54	1.232067	
.791 in	8	4/7	5.80	1.55032	Diff. Segmented
.791 in	8	4/7	5.80	1.561762	Segmented
.791 in	8	4/7	5.80	1.55209	Unadj. Rad. & Segm.

Note: The listing N/M under rings means that N of the M rings are conducting

In an attempt to obtain a clearer view of what was happening as the conductor size increased, it was decided to plot both the analytically and numerically calculated loss ratios as a function of mr, as shown in Figure 3.

TABLE 3

Comparison of Numerical and Analytical Loss Ratio Data for Solid Circular Conductors

Radius	Conductivity	mr	Loss Ratio Range	Analytic L. R.
1 cm	3.54E7	1.295	1.0135 - 1.0154	1.0145
.791 in	5.80E7	3.33	1.53 - 1.62	1.43
1 cm	5.80E7	1.658	1.036 - 1.038	1.038
.791 in	3.54E7	2.602	1.23	1.20

From the table and the graph, the deviation of numerical loss ratio values from the values obtained analytically is clearly evident as the radius and conductivity increase. Upon evaluation, it was decided that this difference was due to the deletion of the eddy current term from the field equation and the functional, and it was therefore decided to formulate a functional to include the eddy term. The functional chosen was

$$\chi = \int \left\{ \left|\frac{\partial \bar{A}}{\partial x}\right|^2 + \left|\frac{\partial \bar{A}}{\partial y}\right|^2 - 2\mu \bar{J}\bar{A} + \sigma \mu \bar{A} \frac{\partial \bar{A}}{\partial t} \right\} dxdy$$

For the time harmonic case, the term $\frac{\partial \bar{A}}{\partial t}$ can be replaced by the expression $j\omega\bar{A}$, since $\bar{A} = \bar{A}_o e^{j\omega t}$ and $\frac{\partial \bar{A}}{\partial t} = j\omega\bar{A}_o e^{j\omega t}$.

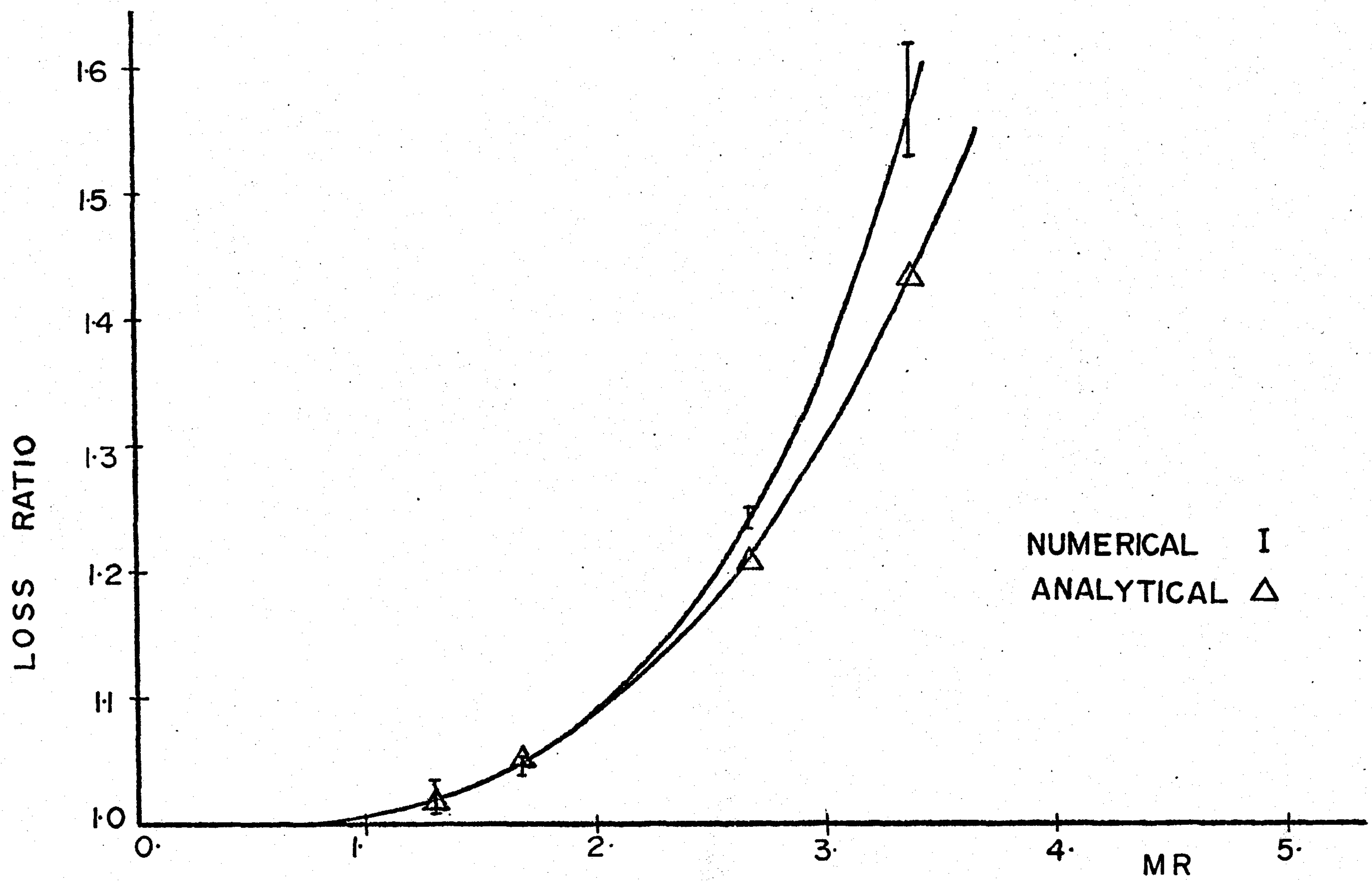


FIGURE 3· ANALYTICAL AND NUMERICAL LOSS RATIO (NO EDDY TERMS)

Since $e^{j\omega t}$ is common to all terms in the equation, it can be removed and the functional becomes

$$\chi = \int \left\{ \left| \frac{\partial \bar{A}_o}{\partial x} \right|^2 + \left| \frac{\partial \bar{A}_o}{\partial y} \right|^2 - 2\mu \bar{J}\bar{A}_o + j\omega\mu\sigma \bar{A}_o^2 \right\} dxdy$$

By applying Euler's equation, $\frac{\partial F}{\partial A} - \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial A_x}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F}{\partial A_y}\right) = 0,$

$$\frac{\partial F}{\partial A} = -2\mu\bar{J} + 2j\omega\mu\sigma\bar{A}, \quad \frac{\partial F}{\partial A_x} = 2\frac{\partial \bar{A}}{\partial x}, \quad \frac{\partial \bar{A}}{\partial A_y} = 2\frac{\partial \bar{A}}{\partial y}.$$

$$\therefore \; -2\mu\bar{J} + 2j\omega\mu\sigma\bar{A} - 2\frac{\partial^2 \bar{A}}{\partial x^2} - 2\frac{\partial^2 \bar{A}}{\partial y^2} = 0$$

This is the required field equation with eddy terms included.

The chosen functional must be minimized with respect to the nodal potentials and, since this has been done for the other terms already, this must be done only for the eddy term, $j\omega\sigma\mu \int \bar{A}_o^2 \, dxdy$.

By inserting the finite element expression for $\bar{A}$, this integration is easily evaluated for each triangle. The derivation appears in Appendix 3.

Using the new finite element equations, the loss ratio was again calculated for the large conductor. However, once again the results were not close to the known analytic values. After many possible reasons for the discrepancy were examined and rejected, it was decided that the method of specifying the boundary conditions was no longer valid for the extended functional. This was because, with the addition of the eddy current term, the vector magnetic potential values became complex numbers instead of real numbers as was the case previously. Therefore, although the potential values on the boundary were still the same everywhere on the boundary, the relative size of the real and imaginary parts of the boundary value could not be arbitrarily fixed. Since there was no way to determine what the complex boundary constant should be, it was decided to try an iterative search technique whose purpose was to alter the boundary constant with the goal of minimizing the average value of potential over the conductor area.

The first technique used was a simple one. The initial boundary value was set at zero, the matrix equation was solved, and the average potential value was determined. This average value was then subtracted from the initial boundary value and the result was used as the new boundary condition. The system was solved again and the process repeated until the average potential value became sufficiently small. This technique is a negative feedback approach and the results obtained were very good. The search pattern, when plotted, was shown to be an inward spiral and convergence was achieved in every case examined. This was the case no matter what initial boundary constant was selected and this can be seen in the plot showing the real part and the imaginary part of the boundary constant plotted in the

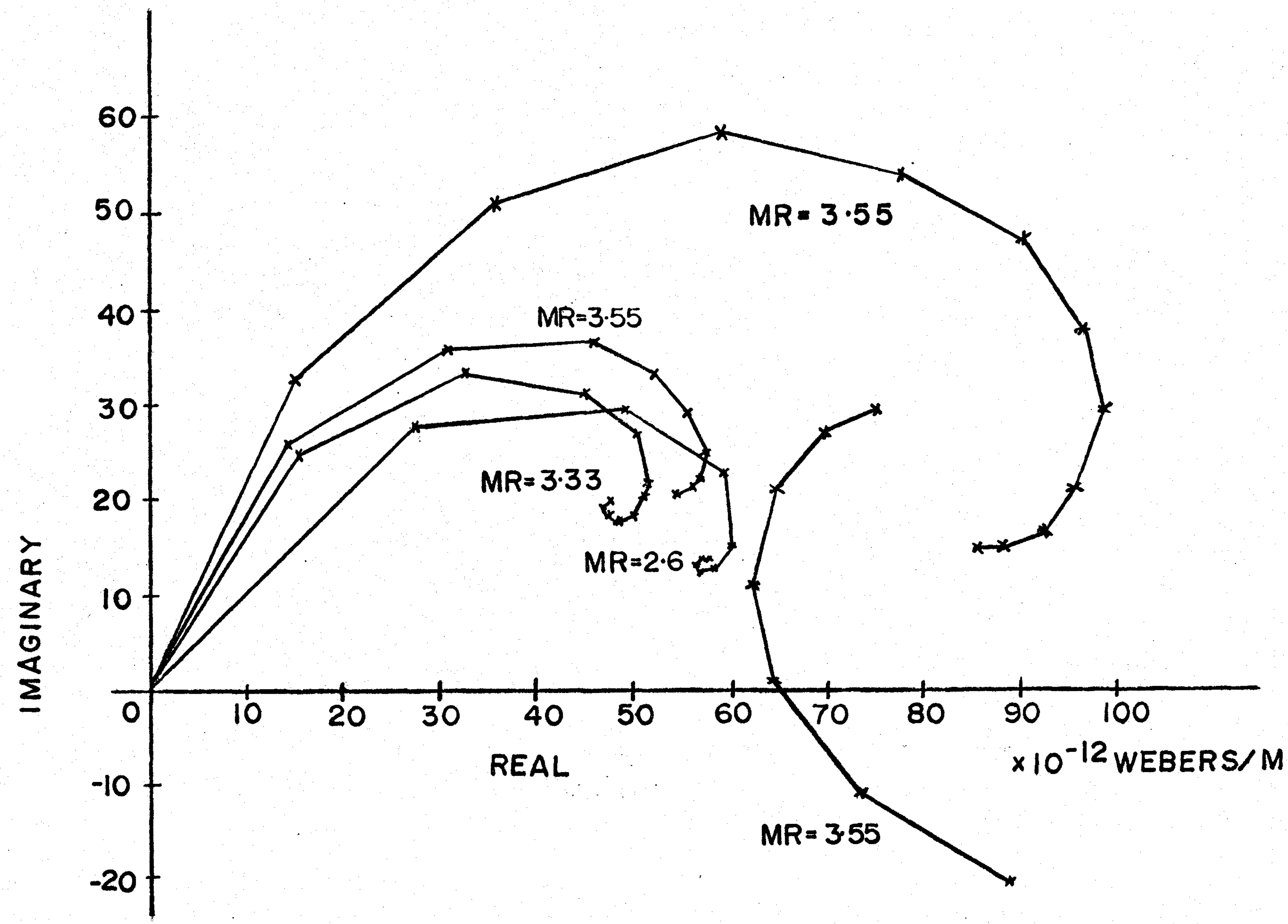


FIGURE 4 ITERATIVE SEARCH PATTERNS OF COMPLEX BOUNDARY VALUES

complex plane with the number of iterations for a particular conductor size as a parameter. The particular search pattern is not only a function of the parameter mr. It also depends on the grid pattern used to model the conductor and the position of the boundary.

From the plot, it can be seen that, usually, about ten iterations were required to achieve completion of the first curl of the spiral and so achieve a bound for the solution value. For a more exact answer, the number of iterations can be increased. However, with this technique it is necessary to solve the matrix of equations for each iterative step and this is very time consuming. This fact means that the number of iterations is limited in practice and prevents this technique from being used for unsymmetric problems where more than one boundary constant must be found.

However, for this symmetric problem results could be determined for various cable sizes within a reasonable amount of time.

The effect of changing the various parameters can be seen from the chart. The segmented cables can be seen to produce lower loss ratios than the solid conductors. However, the validity of the segmental results is questionable because the rotational symmetry that was assumed in the boundary specification technique disappears when the conductor is segmented, and the boundary specification is no longer correct.

TABLE 4

Single Conductor Loss Ratio Results With Eddy Term

Radius (in.)	# of Rings	# of Segm.	# of Iter.	Conduct. $\times 10^7$ ℧/m	CVA $\times 10^{-10}$ wb/m	Loss Ratio	Special
.791	4	8	10	3.54	.0016	1.1905	
.791	4	8	20	5.80	.0001	1.4217	
.842	4	8	10	5.80	.007	1.4938	
.791	4	8	12	5.80	.002	1.3993	Segmented
.751	4	8	8	5.51	.009	1.3128	Segmented
.791	4	8	8	5.51	.0116	1.3735	Segmented
.751	4	8	8	5.80	.0097	1.3409	Segmented
.842	5	8	6	5.80	.06	1.4456	Arbitrary Starting Point
.842	5	8	11	5.80	.038	1.5059	
.991	4	8	20	5.80	.001	1.75	
.791	4	8	10	5.80	.01	1.4319	Under-relaxation
.791	4	8	10	2.90	.0005	1.1048	
.393	4	8	10	3.54	.0000	1.013	

Note: CVA is the average potential value over the conductor

However, the unsegmented cable results should be correct , and they can be compared with corresponding analytic results.

TABLE 5

Comparison of Numerical and Analytical Loss Ratio Data for Solid Circular Conductors

Radius (in)	Conduct. ℧/m	mr	Num. Loss Ratio	Analytic L. R.
.791	3.54E7	2.60	1.19	1.20
.791	5.80E7	3.33	1.42	1.43
.842	5.80E7	3.55	1.50	1.51
.991	5.80E7	4.17	1.75	1.74
.791	2.90E7	2.36	1.13	1.14
.393	3.54E7	1.29	1.01	1.01

From this data and from the corresponding graph, it can be seen that a much better agreement exists between the analytical and numerical results with the eddy current term included.

However, in spite of the success of the calculation technique, as evidenced by these results, it was decided to investigate several possible ways to improve the efficiency of the method.

First, the basic technique was applied, but with the boundary changes either under or over-relaxed. This did not

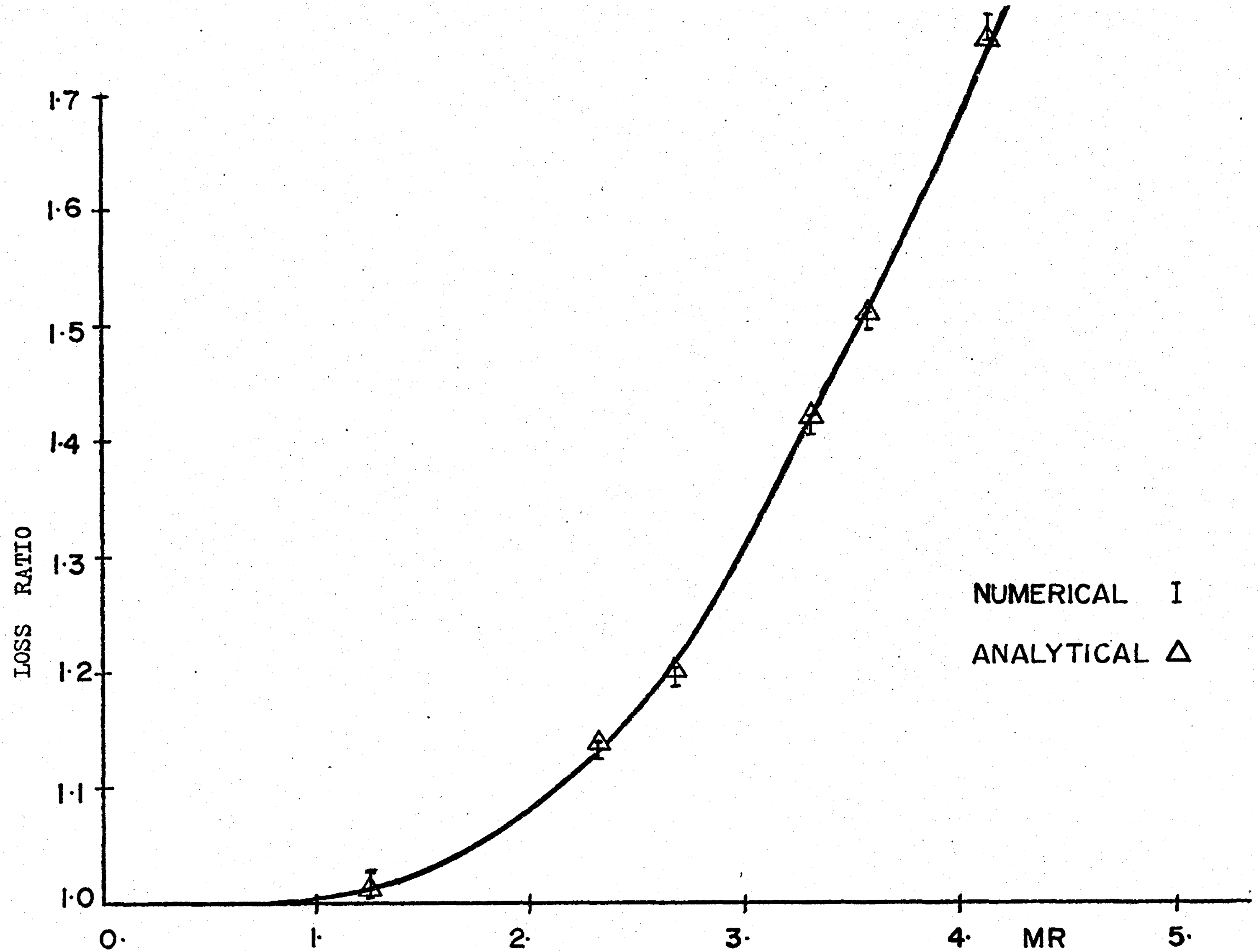


FIGURE 5 ANALYTICAL AND NUMERICAL LOSS RATIOS (WITH EDDY TERMS)

seem to have any effect on the result. Next, a different method of varying the boundary constant was tried. The program was reorganized so that a Hooke and Jeeves search could be used to vary the boundary constant while minimizing the average potential. Although this search method also converged to the right answer, it was found to be slower than the negative feedback approach.

Other program modifications were also implemented to vary the way in which the eddy current term was handled, since an independent confirmation of the correctness of the chosen functional was not available. As an alternate method, it was decided to treat the eddy current term as part of the source current term. Initially, the problem was solved without an eddy term. Then, using these results, a tentative value for the eddy term was calculated and added to the source current. This process was repeated in conjunction with a Hooke and Jeeves search to determine the boundary constant and convergence was quite rapid to the correct answer. A program listing appears in Appendix 4.

The source current approach was also tried with negative feedback boundary specification and convergence was also good here.

One final eddy term variation was tried in which a simplified method of approximating the eddy term was employed. The potential in a triangle was assumed to be the average of the three nodal potentials. This method gave fairly accurate results but has no firm theoretical basis.

Overall, the results of the calculations for the loss ratio of a single solid conductor were good and compared very well with the analytic results available.

CHAPTER 5

Three Conductor Results

5.1 Formulation Difficulties

The major goal of this work is the calculation of the loss ratio for three phase cables in a magnetic steel pipe. There are, however, many difficulties encountered in going from the single phase calculations to this new problem.

First, it was necessary to devise a computer program to formulate the element pattern for the extended problem. It was necessary that the three conductors and the pipe be modelled with sufficient flexibility to allow the positions of the conductors to be shifted as required to model various cable configurations. In addition it was necessary to be able to vary the grid fineness without forming greatly elongated triangles. Such a program has been written to automatically generate the finite elements. A listing and several computer-generated plots illustrating its capability appear in Appendices 4 and 6. Basically, the position and size of the conductors are first specified and triangles are formed within the conductor cross-section. Segmentation can be implemented if desired. Then, a rectangular grid, with variable grid size is placed over the region and the positions of grid points on the boundary are modified to conform to the circular contour of the pipe. In doing this, the best fit is made between the grid points and the conductor elements in order to obtain good triangles in the area around the conductors. Finally, triangles are formed in one or more circular layers to represent the pipe. Extensive use has proven this program to be adequate for forming the grid for almost any desired cable configuration.

As soon as this pattern generation program was operational, the finite element method was applied to the three phase problem. Initially, a zero potential boundary condition was assumed with no pipe present. The eddy term was not included and the result was quite accurate when applied to small diameter conductors.

However, for larger conductors, the inclusion of the eddy term is necessary. This complicates both the functional and the boundary conditions.

With the addition of the eddy term, the vector magnetic potential becomes complex and the terms in the functional are also complex. The squared magnitude of a complex number equals the number times its conjugate and this results in the addition of new variables to account for the modelling of the conjugate vector magnetic potential. In addition, it has not been possible to find a functional such that the Euler equation is satisfied for both the vector potential and its conjugate, without adding on a term that is equal to zero. However, it was suggested (30) that this problem might possibly be eliminated if all the equations were expressed in terms of real and imaginary parts and a functional was formulated to satisfy the necessary criteria. For the field equation

$$\frac{\partial^2 \bar{A}}{\partial x^2} + \frac{\partial^2 \bar{A}}{\partial y^2} = -\mu \bar{J} + j\omega\sigma\mu \bar{A},$$

the real and imaginary part equations are:

$$\frac{\partial^2 \bar{A}_R}{\partial x^2} + \frac{\partial^2 \bar{A}_R}{\partial y^2} = -\mu \bar{J}_R - \omega\sigma\mu \bar{A}_I \quad \text{and}$$

$$\frac{\partial^2 \bar{A}_I}{\partial x^2} + \frac{\partial^2 \bar{A}_I}{\partial y^2} = -\mu \bar{J}_I + \omega\sigma\mu \bar{A}_R$$

where $\bar{A}_R$ is the real part of the vector potential and $\bar{A}_I$ is the imaginary part.

The functional chosen to correspond to these equations is:

$$\chi = \int \Big\{ \left| \frac{\partial \bar{A}_R}{\partial x} \right|^2 + \left| \frac{\partial \bar{A}_R}{\partial y} \right|^2 - \left| \frac{\partial \bar{A}_I}{\partial x} \right|^2 - \left| \frac{\partial \bar{A}_I}{\partial y} \right|^2$$

$$-2\omega\sigma\mu \bar{A}_R\bar{A}_I + 2\mu \bar{J}_I\bar{A}_I - 2\mu \bar{J}_R\bar{A}_R \Big\} \, dxdy$$

Applying Euler's equation twice, gives

$$-2\mu\sigma\omega \bar{A}_I - 2\mu \bar{J}_R - 2\frac{\partial^2 \bar{A}_R}{\partial x^2} - 2\frac{\partial^2 \bar{A}_R}{\partial y^2} = 0 \quad \text{and}$$

$$-2\mu\sigma\omega \bar{A}_R + 2\mu \bar{J}_I + 2\frac{\partial^2 \bar{A}_I}{\partial x^2} + 2\frac{\partial^2 \bar{A}_I}{\partial y^2} = 0$$

These two equations correspond exactly to the real and imaginary part equations stated previously. A more detailed proof of the equivalence is presented in Appendix 1.

The use of the real and imaginary part functional results in the need for twice as many variables as with the complex formulation and the final solution matrix is four times the size. However, this problem may be reduced by using a solution technique which does not store zero elements. In this work, a conjugate gradient routine was used for this reason. This subprogram, CNGRH, is a search routine that solves a matrix equation, $A\,\mathbf{x} = B$, by minimizing the quadratic, $y = \frac{1}{2}\,\mathbf{x}^T A\,\mathbf{x} - B^T\mathbf{x}$. At the point of minimization, the gradient of the quadratic is zero:

$$\nabla y = A\,\mathbf{x} - B = 0 \qquad \therefore\ A\,\mathbf{x} = B$$

Thus, the values of x that minimize the quadratic are also those values that satisfy the matrix equation.

It was, however, also desired to continue using the complex formulation and this was done by examining the matrix which resulted from employing the real and imaginary part functional. When this functional was minimized, the matrix equation that resulted had the following form, for each triangular element:

$$\begin{bmatrix} S_{ii} & S_{ij} & S_{ik} & -E_{ii} & -E_{ij} & -E_{ik} \\ S_{ji} & S_{jj} & S_{jk} & -E_{ji} & \cdots & \\ S_{ki} & \cdots & & -E_{ki} & \vdots & \\ -E_{ii} & \vdots & & -S_{ii} & & \\ -E_{ji} & & & -S_{ji} & & \\ -E_{ki} & & & -S_{ki} & & \end{bmatrix} \begin{bmatrix} A_{Ri} \\ A_{Rj} \\ A_{Rk} \\ A_{Ii} \\ A_{Ij} \\ A_{Ik} \end{bmatrix} = \begin{bmatrix} J_R \frac{\Delta}{3} \\ J_R \frac{\Delta}{3} \\ J_R \frac{\Delta}{3} \\ -J_I \frac{\Delta}{3} \\ -J_I \frac{\Delta}{3} \\ -J_I \frac{\Delta}{3} \end{bmatrix}$$

Δ - area of triangle

The S_{ij} terms have been previously defined and the E_{ij} terms are the corresponding entries resulting from the minimization of the eddy current term. A_R, A_I; and J_R, and J_I are the real and imaginary parts of the vector potential and source current density respectively.

This matrix can be obtained by manipulating the complex number formulation of the problem. Consider the following matrix equation and then separate it into real and imaginary parts:

$$\left[S - jE\right] \left[A_R - jA_I\right] = k \left[J_R - jJ_I\right]$$

$$SA_R - jEA_R - EA_I - jSA_I = k\,(J_R - jJ_I)$$

$$\begin{bmatrix} S & -E \\ -E & -S \end{bmatrix} \begin{bmatrix} A_R \\ A_I \end{bmatrix} = k \begin{bmatrix} J_R \\ -J_I \end{bmatrix}$$

This matrix has the same form and the same entries as the real and imaginary part matrix. Thus, the correct matrix can be obtained using complex arithmetic if the source current is conjugated and the eddy current term is subtracted from the original S-matrix to form a complex S-matrix. The resulting answer must also be conjugated to get the correct value of vector potential.

From program test runs, it has been found that both formulations give the same answer. It has also been observed that the matrix equation used for the single conductor calculations is simply the complex conjugate of the modified complex equation described above.

However, from observations of both three phase and single phase calculations, it was noticed that the sign of the right hand side and the sign of the eddy current term do not affect the answer unless Dirichelet boundary conditions are specified. This is logical since the sign of the right hand side only determines the current direction and the eddy current term is in quadrature.

Thus, from these results, it can be concluded that the problem of the determination of the correct functional for this particular application has been resolved through the use of a functional in real and imaginary part form and by the complex formulation derived from this functional.

5.2 Boundary Conditions

The remaining major difficulty with the three phase calculations is the specification of a boundary condition. There is no real condition that can be specified since there is no equipotential surface and nothing can actually be said about the potential except that the average value over the conductor area should be zero. It is even very difficult to impose the far field boundary condition since the vector magnetic potential does not approach zero at infinity. Therefore some approximations were necessary in order to establish a useable condition.

For the first calculations with the cables in air, a Dirichelet condition was set at a distance far from the cables by treating the conductors as filamentary conductors and summing the contributions by superposition. However, this method is not accurate, especially for large conductors.

It is also not possible to set the boundary condition by using a search technique such as the one used in the single conductor calculations because there is no simple symmetry in the three phase case. This means that a multivariable search would be necessary with many cost function evaluations and the computer time used would be of the order of hours.

The possibility does exist, however, that a technique based on work already described in the literature may soon be developed to handle the problem. Silvester and Hsieh (25), Hseih (26), Csendes (28), and McDonald and Wexler (27) have all written papers on the use of an exterior element to specify boundary conditions for unbounded problems.

The methods basically use Green's functions to relate the field in the exterior regions to the potential on the finite element boundary and so derive a relationship between the exterior field energy and the boundary potentials. This is equivalent to specifying a boundary condition. The first three papers seem to be applicable only for electrostatic problems since parts of the derivations are based on assumptions valid only for these problems. However, the work described in the final paper is more general in scope and, in the future, may be able to be applied to this problem.

An initial investigation into these methods has been begun, but the application of this theory to the loss ratio calculations rests with future work, because of the complexity involved in applying exterior element methods to the time harmonic diffusion equation. It was therefore decided to concentrate on solutions for the case of three conductors enclosed in a steel pipe by using an approximate boundary condition. This approximate pipe boundary specification is based on the skin depth of the pipe material. The skin depth is that distance wherein the magnitude of the incident wave is reduced by a factor of 'e^{1}' and the approximate formula for skin depth is:

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

The particular pipe modelled had a thickness of 0.25 inches and a conductivity of 8.2×10^6 mhos per meter. The permeability varied throughout the pipe but the average relative permeability was 100. Therefore the skin depth was:

$$\delta = \sqrt{\frac{1}{60 \times \pi^2 \times 4 \times .82 \times 100}} = .00227 \text{ meters}$$

$$\delta = .0895 \text{ inches.}$$

The pipe is approximately three times the skin depth. This means that the flux density, $\overline{B}$, at the outer pipe boundary has been reduced in magnitude by a factor of e^3 or about 20. Since the flux density at the outer pipe boundary is only 1/20 of the flux density at the inner pipe wall, the flux density can be taken to be approximately zero at the outer wall of the pipe. Since the flux density is assumed to be zero, the curl of vector magnetic potential is zero at the outer pipe boundary.

$$\nabla \times \overline{A} = 0$$

Since it is assumed that only a z-component exists for vector potential, the curl expands to: (in cylindrical coordinates)

$$\frac{1}{r} \frac{\partial A}{\partial \phi} \hat{a}_r - \frac{\partial A}{\partial r} \hat{a}_\phi = 0$$

To satisfy the equation, both components must be zero. Therefore $\frac{\partial A}{\partial r} = 0$ and $\frac{\partial A}{\partial \phi} = 0$. The second condition indicates that the potential is the same everywhere on the boundary. This is an indication of the amount of approximation that is inherent in accepting that the flux density is zero when it is not really so. From intuition, it is apparent that the potential should not be the same everywhere on the boundary, especially for the cradled configuration. For this work, this second equation is ignored and only the first one is applied. However, it is reasonable that the use of this chosen condition involves about the same amount of approximation that was implied by the use of the unused equation, and that much of the deviation of answers obtained using this boundary condition, from answers obtained by measurement, may be due to the inexactness of the boundary specification.

The boundary constraint that is used is the homogenous Neumann condition $\frac{\partial \bar{A}}{\partial r} = 0$ The application of this condition is accomplished without effort in the finite element method since this condition is 'natural' in the method. This means that it is automatically satisfied, in the absence of any other boundary specification, simply by writing the basic element equations.

5.3 Permeability Specification

All that remained, before the in-pipe calculations could be performed, was to determine a way to specify the permeability variation throughout the pipe. Measurements on the variation of permeability with flux density were available for the particular pipe material of interest. To get them into a form useable in a computer program, the data was curve-fitted to a polynomial expression using a library least-squares curve fitting subroutine. Because of the large range of the data, it was broken into five parts and fitted by five polynomials. In this form, the value of relative permeability can be determined for any value of flux density and the flux density can be calculated from a knowledge of vector potential.

An iterative technique is needed to determine the permeability variation. First, a constant value of permeability is used for all the triangles which make up the pipe. Using this permeability, the matrix equation is written and solved to yield the vector magnetic potential. From the vector potential, the flux density is calculated and a new value of permeability, which is different for each triangle, is determined. The matrix equation is written and solved again with the new permeability variation and the process is repeated until the vector potential is sufficiently similar for successive iterations. In practice, only two iterations were necessary to obtain the correct permeability variation and it was even found that there was no great difference in the vector potential values when constant permeability values, as compared with varying permeability values, were used. The permeability of the pipe triangles did not affect the vector potential in the conductors very much. This can be seen in the program printouts in Appendix 4.

5.4 Results

The following tables list the results obtained for the loss ratio for the three-conductor configuration using the boundary approximations already described.

TABLE 6

Loss Ratio in Air Using Filamentary Superposition

Formulation	Special Notes	Radius	Loss Ratio
Real & Imag.	Used CONGRH	.833	2.02
Complex	Unconjugated	.833	2.03
Real & Imag.	Used SIMQ	.833	2.02
Real & Imag.	Using CONGRH	.393	1.83

The final entry in the table illustrates the change in loss ratio that results with a variation in the conductor radius.

These results are consistent and compare reasonably well with available measured values. In each case, the use of the stranding factor reduces the loss ratio considerably, but the use of segmentation, the thickness of the pipe and a change of conductor radius does not alter the loss ratio as appreciably as might be expected. This may be because the boundary specification has an overpowering effect on the potential distribution.

For each case with similar parameters, the loss ratio for the cradled configuration is consistently higher than that for the close triangular configuration, and this is as

TABLE 7

Pipe Loss Ratios Using Neumann Boundary Conditions

Configuration	Special Notes	Radius (inches)	Numerical Loss Ratio	Measured Loss Ratio
Centered Triang.	Constant Permeab.	.791	1.95	
Centered Triang.	L. P. R. & Im. Form.	.833	2.02	
Centered Triang.	T. P. SF.	.833	1.86	
Cradled	T. P.	.791	2.09	
Cradled	No Pipe	.791	1.98	
cradled	T. P. SF.	.791	1.89	
Cradled	T. P. SF.	.745	1.88	
Cradled	T. P. SF.	.707	1.86	
Cradled	T. P.	.707	1.95	
Cradled	Tn. P.	.745	1.89	
Cradled	Tn.P. S.F. Segmented.	.745	1.88	1.78-1.95
Close Triangle	T. P. SF.	.833	1.90	
Close Triangle	T. P. SF.	.745	1.87	
Close Triangle	Tn. P. SF.	.745	1.88	
Close Triangle	Tn. P. SF. Segmented	.745	1.87	1.49-1.70
Close Triangle	Unconjugated Form.	.745	1.87	

Note: L.P.-Large Pipe 15 inches in diameter
T.P.-Thick 1 inch pipe
Tn. P.-Thin .25 inch pipe

S.F.-Stranding Factor .435
Normal Pipe Diameter 10 inches

was anticipated. However, based on measured values, the difference is not as great as was expected and the loss ratio for the close triangular arrangement is much higher than that obtained by measured values. Once again, the lack of a greater difference between the cradled and triangular loss ratios is probably due to the fact that both have the same specified boundary condition. This condition probably forces the potential values of the two cases to be much more similar than they would be if the correct boundary potential distribution were known. Recent measurements on 2000 Kc mil. segmental cables (corresponding to a .745 inch corrected radius) indicate a range in loss ratio, at room temperature, from 1.78 to 1.95 for the cradled arrangement and from 1.49 to 1.70 for the close triangular configuration.

The range is due to the dependence of loss ratio on current, and heat cycling and corresponds to calculated loss ratio values of 1.88 for cradled and 1.87 for close triangular arrangements obtained in this work.

It should be noted that a computational adjustment was necessary to prevent all answers from being completely unreasonable. As noted previously the loss ratio can be specified as a ratio of the total current to the source current loss:

$$\text{Loss Ratio} = \frac{\int \left| \bar{J}_s - j\sigma\omega\bar{A} \right|^2 dxdy}{\int \left| \bar{J}_s \right|^2 dxdy}$$

$$= 1 + \frac{\int\left\{\omega^2 \sigma^2 \left|\bar{A}\right|^2 + j\omega\sigma \left[\bar{A}^*\bar{J}_s - \bar{A}\bar{J}_s^*\right]\right\} dxdy}{\int \left|\bar{J}_s\right|^2 dxdy}$$

It is the cross-term that causes the difficulty. In the single conductor case this term is found to be zero and thus is no problem. However, in the three conductor case, the real parts of the bracketed term cancel, leaving a term which, when included, adds to the remaining terms in such a way that they cancel out and the loss ratio becomes much less than unity. This is clearly an impossibility and for this reason it was necessary to ignore the contribution of this imaginary term.

In the single conductor calculations the imaginary term vanishes because the average value of vector potential is zero over the conductor. In the three conductor case, the average of the vector magnetic potential is also zero over the three conductors but this average is weighted by a different value of current in each conductor, and, although the currents also have a zero average value, the weighting is such that the integral is prevented from being zero.

Although this indicates what actually happens in the performance of the calculations, it does not explain the fundamental theoretical reasons for the difficulty involved in including the cross term. A reference in a paper by Stoll (8) also indicates that this cross term is ignored, but it is not clear why, or if this omission is valid in general or only in specific cases.

CHAPTER 6

Conclusions and Future Work

6.1 Single Conductor Results

The single conductor loss ratio results compare very favourably to analytical results and can provide very good accuracy if a sufficiently fine grid is used.

From the accuracy and consistency of the results, it can be concluded that a workable functional and an acceptable method of formulating the boundary conditions have been developed. The results also confirm the soundness of the basic finite element procedure and attest to the success of the solution routines.

In terms of specific results, the effect on the loss ratio due to a change in a number of parameters was observed. The finer the triangular grid, the greater the accuracy. However, the accuracy also greatly depends on the judicousness of the choice of the ring break points which determine where the finest portion of the grid is placed. However, the number of rings outside of the conductor and the position of the outer boundary does not affect the loss ratio. In terms of physical cable parameters, the loss ratio increases with an increase in radius and conductivity and decreases when the conductor is segmented. As expected, a change in radius results in a larger change in loss ratio than does a corresponding percentage variation in conductivity.

To obtain these results, a variety of techniques were employed. Overall, it appears that the simple negative feedback approach for specifying the boundary is computationally most efficient. The Hooke and Jeeves search also worked, but the added complexity was not necessary. However, the approach using the iterative source current variation was appealing since it gave a physical indication of how the distribution of the source current becomes modified to cause an increase in the loss ratio.

Overall, the single conductor analysis results were very successful and this was of some significance since these results provided the basis for the three conductor calculations.

6.2 Three Conductor Results

The results obtained in the three conductor loss ratio calculations are reasonable, but the accuracy is limited by the inexactness of the boundary condition specification.

The finite element pattern generation program appears to be able to satisfactorily model almost any imaginable pipe and conductor arrangement as long as the grid lines are judiciously located. Results also indicate that the use of the real and imaginary part functional and the derived modified complex formulation has solved the problem of finding the correct functional corresponding to the complete field equation.

With regard to the matrix equation solution routines, the complex gaussian elimination scheme has proven to be very stable and gives good results, even when the matrix order is greater than 150. However, for a high order matrix, a very great amount of core area is needed and much of this is wasted by storing zero entries. For this reason, the use of the conjugate gradient subroutine becomes significant. Large matrices can be solved quickly with comparatively little core storage, since zero entries are not retained. In other programs, similar to this work, matrices of order greater than 2200 have been solved in less than five minutes of CPU time using less than 300 k-bytes of memory. Using an elimination subroutine, more than 20,000 k-bytes of storage would have been required and the round-off errors probably would have rendered the solution useless. Thus, the use of the conjugate gradient solution routine opens up many more areas in which the finite element method can be employed. Much finer modelling, with many more grid points, can be used and, because of the rapid solution time, iterative techniques, such as those used for boundary specification, can be employed where they would otherwise not be feasible. There only remains to perform some work on the convergence of the method since convergence did not occur in every case tested and a satisfactory explanation of this failure is not yet available.

The results of the iterative permeability specification appear to be consistent with intuitive expectations. The permeability varies from triangle to triangle in the pipe, but is constant throughout any one triangle. It has been found, however, that the specification of permeability has not been as significant as was believed. Results obtained using a reasonably arbitrary constant permeability have differed from results using the variable permeability by an average of less than one per cent. It should be noted that the permeability measurements for the pipe material sample used were those obtained under direct current conditions and that pipe hysteresis was not modelled or treated in the program at all. Nonetheless, this would not account for the lack of dependence of the loss ratio on permeability of the pipe.

However, it may be that the manner in which the boundary is specified accounts for the lack of sensitivity. Normally, it might be expected that the permeability variation in the pipe would exert a considerable influence on the boundary conditions prevailing; but, if the boundary condition is fixed, as in this work, the permeability variation may not be able to control the potential distribution in the expected manner.

The fixedness of the boundary is also likely to be responsible for the closeness in results for the cradled and the close triangular arrangements. It is reasonable that as the cable configuration changes, the boundary conditions should change, but with the boundary specifications used in this work, this is not possible.

In spite of the boundary approximations used, the results are reasonable and compare favourably with other methods of calculation.

The loss ratio calculations for cables in air gave results that are higher than expected results but this is due to the inexact boundary values that were applied. For the cables in the pipe, the calculated loss ratio for the cradled configuration was close to measured values but the loss ratio for the close triangular arrangement was calculated to be higher than expected. The thickness of the pipe did not significantly affect the results, since the boundary condition was the same in both cases. The use of a stranding factor significantly decreased the loss ratio result but the application of segmentation did not have a great effect.

The effect of a change of a number of variables can be handled by the program. Loss ratio decreases with increasing conductor radius and conductivity and also changes with the position of the cables or the size of the pipe. No provision has been made for the variation in loss ratio with a change in temperature, but this could be implemented by changing the conductor conductivity. The magnitude and phase of the source current can be altered to any value but provision has not been made to account for the change in conductivity due to the heating caused by larger currents. Also the effect of heat cycling is not considered but could be handled by modifying the stranding factor. Some cable components, such as the shield assembly, are not modelled and the inclusion of these parts in the cable model is one of the things that could be done to improve the program in the future.

6.3 Summary of Results

Good accuracy was obtained for single isolated conductors of all sizes by using the finite element method and including an eddy current term in the functional. A valid boundary condition was specified by iterative techniques which minimized the average vector potential over the conductor cross-section. An alternate formulation procedure illustrated the modification of the source current distribution responsible for the increased losses.

For the three conductor and pipe calculation, an automatic pattern generation program was written which permits the modelling of any size conductor in various arrangements. Grid fineness is easily modified either in the conductors, the pipe or in the air, and the cables may be represented as either solid or segmented conductors.

A real and imaginary part functional was developed and implemented, both in a real and imaginary part and in a modified complex formulation. Preliminary evaluation of an alternate matrix solution routine was undertaken in an effort to reduce both computer storage and computation time. Approximate boundary conditions were applied to the three conductor case and work was initiated on the specification of a more accurate boundary constraint. Pipe permeability was expressed in a polynomial form and several loss calculations were made and compared to measured values.

6.4 Future Work

A number of areas exist in this work which are worthy of consideration for additional study in the future. The most immediate focus of effort should be directed toward the determination of a more accurate boundary condition.

Several avenues exist for the attacking of this problem. With the use of the conjugate gradient routine to decrease the matrix solution time, a modified search routine, similar to that used for the single conductor case, might be feasible. However, the most promising method of defining a boundary condition appear to be that involving the exterior element concept. The paper by McDonald and Wexler (27) provides a foundation from which it should eventually be possible to obtain a valid boundary condition specification which should improve the results significantly.

Other modifications to improve the computational accuracy include the use of an increased number of grid points to more accurately model the cable geometry. Also, an increase in the order of the defining equations for the electromagnetic vector potential would lead to greater accuracy. The best possible positioning of the triangles of the grid could also improve the results and this might best be accomplished by the use of an interactive graphics terminal. With this type of device, a program to generate the triangular subdivision of the region of interest would not be necessary since the grid would be directly and easily constructed by the user.

In addition to this work undertaken to achieve greater accuracy, a comprehensive study should be initiated in an attempt to more completely resolve the difficulty that arose with respect to the cross-term that had to be omitted in the loss ratio calculations.

If most of this work can be successfully accomplished, it seems certain that a method for confidently evaluating the loss ratio for a pipe-type cable will result.

APPENDIX 1

REAL AND IMAGINARY PART FUNCTIONAL VERIFICATION

The full field equation in terms of vector magnetic potential is given by

$$\frac{\partial^2 \overline{A}}{\partial x^2} + \frac{\partial^2 \overline{A}}{\partial y^2} = -\mu \overline{J} + j\omega\sigma\mu \overline{A}$$

Both $\overline{A}$ and $\overline{J}$ are complex quantities and the field equation can be separated into real and imaginary parts.

$$\frac{\partial^2}{\partial x^2}(\overline{A}_R + j\overline{A}_I) + \frac{\partial^2}{\partial y^2}(\overline{A}_R + j\overline{A}_I) = -\mu(\overline{J}_R + j\overline{J}_I) + j\omega\sigma\mu(\overline{A}_R + j\overline{A}_I)$$

$$\frac{\partial^2 \overline{A}_R}{\partial x^2} + \frac{\partial^2 \overline{A}_R}{\partial y^2} = -\mu \overline{J}_R - \omega\sigma\mu \overline{A}_I \quad \text{(Real)}$$

$$\frac{\partial^2 \overline{A}_I}{\partial y^2} + \frac{\partial^2 \overline{A}_I}{\partial x^2} = -\mu \overline{J}_I + \omega\sigma\mu \overline{A}_R \quad \text{(Imaginary)}$$

For the minimization of a functional to be equivalent to the basic field equation, it must be shown that the functional reduces to the real and imaginary equations written here. Let the proposed functional be:

$$\chi = \tfrac{1}{2} \iint \left[\left(\frac{\partial \overline{A}_R}{\partial x}\right)^2 + \left(\frac{\partial \overline{A}_R}{\partial y}\right)^2 - \left(\frac{\partial \overline{A}_I}{\partial x}\right)^2 - \left(\frac{\partial \overline{A}_I}{\partial y}\right)^2 \right] dxdy$$

$$+ \iint \left[-\omega\sigma\mu A_R A_I + \mu J_I A_I - \mu J_R A_R \right] dxdy$$

Now the functional must be minimized. Let the optimal (minimum) solution be A_R^* and A_I^* and let the solution be perturbed a small amount from the optimum:

$$A_R = A_R^* + \xi \eta_R(x,y) \qquad\qquad A_I = A_I^* + \xi \eta_I(x,y)$$

ξ is a small number and $\eta(x,y)$ is an arbitrary function. Substituting these values into the functional gives:

$$\chi = \tfrac{1}{2} \iint \Bigg[\left(\frac{\partial (A_R^* + \xi \eta_R(x,y))}{\partial x} \right)^2 + \left(\frac{\partial (A_R^* + \xi \eta_R(x,y))}{\partial y} \right)^2 - \left(\frac{\partial (A_I^* + \xi \eta_I(x,y))}{\partial x} \right)^2 - \left(\frac{\partial (A_I^* + \xi \eta_I(x,y))}{\partial y} \right)^2 \Bigg] dxdy$$

$$+ \iint \Big[-\sigma\mu\omega (A_R^* + \xi \eta_R(x,y))(A_I^* + \xi \eta_I(x,y)) + \mu J_I (A_I^* + \xi \eta_I(x,y)) - \mu J_R (A_R^* + \xi \eta_R(x,y)) \Big] dxdy$$

Now take a variation on the functional with respect to and set the functional to zero as ξ approaches zero

$$\underset{\xi \to 0}{\delta \chi} = \iint \Bigg[\frac{\partial A_R^*}{\partial x}\frac{\partial \eta_r}{\partial x} + \frac{\partial A_R^*}{\partial y}\frac{\partial \eta_r}{\partial y} - \frac{\partial A_I^*}{\partial x}\frac{\partial \eta_I}{\partial x} - \frac{\partial A_I^*}{\partial y}\frac{\partial \eta_I}{\partial y} - \sigma\mu\omega \eta_R A_I^* - \sigma\mu\omega \eta_I A_R^* + \mu J_I \eta_I - \mu J_R \eta_R \Bigg] dxdy = 0$$

Terms such as $\int \frac{\partial A_R^*}{\partial x}\frac{\partial \eta_R}{\partial x} d_x d_y$ may be integrated by parts

$$\int \frac{\partial A_R^*}{\partial x}\frac{\partial \eta_R}{\partial x}\, dxdy = \frac{\partial A_R^*}{\partial x}\eta_R \Bigg|_{x_1}^{x_2} - \int \frac{\partial^2 A_R^*}{\partial x^2}\eta_R \, dxdy$$

Since $\eta_R(x,y)$ and $\eta_I(x,y)$ are arbitrary, they can be chosen to be zero at the endpoints. Thus,

$$\int \frac{\partial A_R^*}{\partial x}\frac{\partial \eta_R}{\partial x}\, dxdy = - \int \frac{\partial^2 A_R^*}{\partial x^2}\eta_R \, dxdy$$

Therefore the functional can be rewritten as:

$$\delta X = \int\Big[-\frac{\partial^2 A_R^*}{\partial x^2}\eta_R - \frac{\partial^2 A_R^*}{\partial y^2}\eta_R + \frac{\partial^2 A_I^*}{\partial x^2}\eta_I + \frac{\partial^2 A_I^*}{\partial y^2}\eta_I$$

$$- \omega\sigma\mu\eta_R A_I^* - \omega\sigma\mu\eta_I A_R^* + \mu J_I\eta_I - \mu J_R\eta_R\Big] dxdy = 0$$

Rearranging and collecting like terms gives

$$\delta X = \int\Big[\eta_R\Big(\frac{-\partial^2 A_R^*}{\partial x^2} - \frac{\partial^2 A_R^*}{\partial y^2} - \sigma\omega\mu A_I^* - \mu J_R\Big)$$

$$+ \eta_I\Big(\frac{\partial^2 A_I^*}{\partial x^2} + \frac{\partial^2 A_I^*}{\partial y^2} - \sigma\omega\mu A_R^* + \mu J_I\Big)\Big] dxdy = 0$$

Since η_R and η_I are arbitrary, the two bracketed terms are zero. Thus, if the minimum of the functional is found, then

$$\frac{\partial^2 A_R}{\partial x^2} + \frac{\partial^2 A_R}{\partial y^2} = -\mu J_R - \omega\sigma u A_I$$

$$\frac{\partial^2 A_I}{\partial x^2} + \frac{\partial^2 A_I}{\partial y^2} = -\mu J_I + \omega\sigma u A_R$$

These equations correspond exactly to the real and imaginary parts of the field equation previously derived and this proves the equivalence between the minimized functional and the field equation.

APPENDIX 2

INTEGRAL EVALUATION

One integral occurs in finite element derivations a sufficient number of times so that its evaluation once and for all is warranted:

$$I = \int N_i \, dxdy$$

$$N_i = \frac{a_i + b_i x + c_i y}{2\Delta} \qquad \Delta \text{ is triangle area}$$

It is well known that for a triangle

$$\int x \, dxdy = \bar{x}\,\Delta \, dxdy \qquad \text{and} \qquad \int y \, dxdy = \bar{y}\,\Delta \, dxdy$$

where $\bar{x}$ and $\bar{y}$ are the centroid values.

$$I = \iint \left\{ \frac{a_i + b_i x + c_i y}{2\Delta} \right\} dxdy = \frac{a_i + b_i \bar{x} + c_i \bar{y}}{2}$$

Substituting for a_i, b_i and c_i

$$I = \frac{x_j y_k - x_k y_j + (y_j - y_k)\left\{\frac{x_i + x_j + x_k}{3}\right\} + (x_k - x_j)\left\{\frac{y_i + y_j + y_k}{3}\right\}}{2}$$

$$I = 1/6 \left[(x_j y_k - x_k y_j) + (y_j x_i - x_j y_i) + (x_k y_i - y_k x_i) \right]$$

$$= 1/3 \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix} \Big/ 2 = 1/3\,\Delta$$

APPENDIX 3

EVALUATION OF THE EDDY CURRENT TERM

The contribution of the eddy current term towards the total functional is given by the integral:

$$\chi_E = j\omega\sigma\mu \int \bar{A}^2 \; dxdy$$

$$\bar{A} = \begin{bmatrix} N_i & N_j & N_k \end{bmatrix} \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix}$$

$$N_i = \frac{a_i + b_i x + c_i y}{2\Delta}$$

To obtain the master matrix entries the functional must be differentiated with respect to each of the nodal potentials.

$$\frac{\partial \chi_E}{\partial A_i} = 2j\omega\sigma\mu \int N_i \begin{bmatrix} N_i & N_j & N_k \end{bmatrix} \begin{bmatrix} A_i \\ A_j \\ A_k \end{bmatrix} dxdy$$

This expression can be separated into three terms, each of which will be added to the appropriate place in the master matrix.

$$E_{ij} = \left[2j\omega\mu\sigma \int N_i N_j \; dxdy \right] A_j$$

The expression in brackets is added to the i^{th} row and the j^{th} column of the master matrix after the integration is performed.

$$E_{ij} = \frac{j2\omega\sigma\mu}{4\Delta^2} \int (a_i + b_i x + c_i y)(a_j + b_j x + c_j y)\, dxdy$$

$$= \frac{j2\omega\sigma\mu}{4\Delta^2} \int (a_i a_j + a_i b_j x + a_j b_i x + a_j c_i y + a_i c_j y + b_i b_j x^2$$

$$+ c_i c_j y^2 + b_i c_j xy + c_i b_j xy)\, dxdy$$

$$E_{ij} = \frac{j\omega\sigma\mu}{2\Delta} \Big[a_i a_j + \bar{x}(a_i b_j + a_j b_i) + \bar{y}(a_i c_j + a_j c_i)$$

$$+ (b_i c_j + c_i b_j)\left(\bar{x}\bar{y} + \frac{x_1 y_1 + x_2 y_2 + x_3 y_3}{12}\right)$$

$$+ b_i b_j \left(\bar{x}^2 + \frac{x_1^2 + x_2^2 + x_3^2}{12}\right) + c_i c_j \left(\bar{y}^2 + \frac{y_1^2 + y_2^2 + y_3^2}{12}\right)\Big]$$

$\bar{x}$ and $\bar{y}$ are triangle centroid co-ordinates

x_1, y_1 are nodal co-ordinates minus centroid co-ordinates

APPENDIX 4

Computer program listings

The first listing is for a single conductor with a negative feedback boundary specification.

```
$JOB          XXXXXXXXXX
      IMPLICIT COMPLEX*16(C),REAL*8(A-B,D-H,O-Z)
      DIMENSION XO(3),YO(3),RO(3)
      DIMENSION IFADDX(4),IFADDY(4),SIGMM(56),ISEG(8)
      DIMENSION RADIUS(16),X(33),Y(33),NODE(56,3),RADI(3)
      REAL*8 MU(56),MU1
      COMMON X,Y,NODE,NSEG,NRAD,SIGMM,BIGJ,MU
      DIMENSION PTNW(10)
      DATA IFADDX/1,0,-1,0/,IFADDY/0,1,0,-1/
      COMPLEX*16 BIGJ(56),BIGJ1,BIGJ2,VAL
      REAL*8 CDABS
      PI=3.14159265359D0
C      READ SEGMENTATION CONSTANTS. THICKNESS IS IN INCHES. OTHER
C      CONSTANTS ARE 4,6 AND 3 FOR SEGMENTATION. ELSE 0,0,0
      READ(5,830) THIK,NSEGC,IFSEG,KSEG
C      READ NUMBER OF CABLES, CENTRE OF REGION AND RADIUS OF REGION
      READ(5,700) NPHMAX,XORG,YORG,RO
      THIK2=THIK/2.D0
      NDNO=1
      NDINIT=NDNO
      JSTART=0
      J0=1
      NPHASE=0
      SIGMA=5.80D7
      DO 113L=1,NPHMAX
C      READ NUMBER OF RADIAL RINGS AND SEGMENTS
      READ(5,101) NRAD,NSEG
C      READ RADIAL BREAK POINTS AS PERCENTAGE OF RADIUS
      READ(5,102) (RADIUS(I),I=1,NRAD)
C      READ POSITION AND RADIUS
      READ (5,703) XO(L),YO(L),RO(L)
      WRITE(6,31) NRAD,NSEG
   31 FORMAT(1H1,10X,'CYLINDRICAL CONDUCTOR FINITE ELEMENT ANALYSIS'//
     15X,'NUMBER OF RADIAL SEGMENTS:',I5/5X,'NUMBER OF ANGULAR SEGMENTS:
     2',I5/)
C      READ NUMBER OF CONDUCTING RINGS,PERMEABILITY,AND CONDUCTOR
C      AND INSULATOR CURRENTS
      READ(5,22)NCON,MU1,BIGJ1,BIGJ2
   22 FORMAT(I2,F10.9,4G10.5)
      RADI(L)=RADIUS(NCON)*RO(L)
      ANEW=PI*(RADI(L)**2)
C      ADJUST CONDUCTOR BOUNDARY
      RADIUS(NCON)=DSQRT((PI*RADIUS(NCON)**2)/(NSEG*DCOS(PI/NSEG)*DSIN
     .(PI/NSEG)))
      WRITE(6,32) (RADIUS(I),I=1,NRAD)
   32 FORMAT(10X,'RADIAL BREAK-POINTS (INCHES)'/2X,(10G12.5))
      WRITE(6,33)NCON,MU1,BIGJ1,BIGJ2
   33 FORMAT(15X,'NUMBER OF RINGS WITHIN THE CONDUCTOR:',I5/5X,
     1'MAGNETIC PERMEABILITY:',G12.5/5X,'CURRENT DENSITY:'/10X,'WITHIN T
     .HE CONDUCTOR:',G12.5,G12.5/10X,'WITHIN THE INSULATION:',2G12.5/)
      NPHASE=NPHASE+1
      JMAX=NSEG
      IF(NSEGC.NE.0) GO TO 650
C
C      UNSEGMENTED CABLE
C
C (II) COMPUTE COORDINATES OF EACH NODE, X(N) AND Y(N)  IN METERS
C    REQUIRED INPUTS: NRAD, NSEG, RADIUS(I)
C    RESULTS: X(N), Y(N)
      ANGLE=2.D0*3.14159265359D0/NSEG
      N=2
      DO 6 I=1,NRAD
      THETA=0.D0
      DO 6 J=1,NSEG
      X(N)=RADIUS(I)*DCOS(THETA)*1.0D-2 *2.54010D0*RO(L)
      Y(N)=RADIUS(I)*DSIN(THETA)*1.0D-2 *2.54010D0*RO(L)
      N=N+1
    6 THETA=THETA+ANGLE
C     SET COORDINATES OF NODE 1
      X(1)=0.D0
      Y(1)=0.D0
C (III) DETERMINE NODE NUMBERS OF EACH ELEMENT, NODE(I,J)
C    REQUIRED INPUTS: NRAD, NSEG
C    RESULTS: NODE(I,J) (I=ELEMENT NUMBER; J=L,M,N (1,2,3) )
C  (1) DETERMINE NODE NUMBERS OF ELEMENTS IN 1ST (INNER) RING
      DO 1 J=1,NSEG
      SIGMM(J)=SIGMA
      NODE(J,1)=1
      NODE(J,2)=J+2
    1 NODE(J,3)=J+1
```

```
      NODE(NSEG,2)=2
      GO TO 651
C
C       SEGMENTED CABLE- INNER RING
C
  650 CONTINUE
      DO111J=1,4
      X(NDNO)=XO(L)+(-1)**((J+4)/2)*THIK2
      Y(NDNO)=YO(L)+(-1)**((J+3)/2)*THIK2
  111 NDNO=NDNO+1
      DO72 J=1,4
      ISEG(2*J-1)=NDNO-6+J
      IF(ISEG(2*J-1).LT.NDINIT) ISEG(2*J-1)=NDINIT+3
   72 ISEG(2*J)=NDNO-5+J
      DTHETA=PI/NSEG*2.D0
      THETA=DTHETA
      NPTS=NSEG/4
      DO 123 M=1,NCON
      ADD=RO(L)*RADIUS(M)
      DO 124 J=1,4
      X(NDNO)=X(ISEG(2*J-1))+ADD*IFADDX(J)
      Y(NDNO)=Y(ISEG(2*J-1))+ADD*IFADDY(J)
      X(NDNO+1)=X(ISEG(2*J))+ADD*IFADDX(J)
      Y(NDNO+1)=Y(ISEG(2*J))+ADD*IFADDY(J)
      NDNO=NDNO+2
      NPTSM=NPTS-1
      IF(NPTSM.LE.0) GO TO 123
      DO 125 K=1,NPTSM
      X(NDNO)=       RO(L)*DCOS(THETA)*RADIUS(M)+X(ISEG(2*J))
      Y(NDNO)=      +RO(L)*DSIN(THETA)*RADIUS(M)+Y(ISEG(2*J))
      THETA=THETA+DTHETA
  125 NDNO=NDNO+1
      THETA=THETA+DTHETA
  124 CONTINUE
  123 CONTINUE
      NIST=NCON+1
      IF(NIST.GT.NRAD) GO TO 782
      DO 375 J=NIST,NRAD
      THETA=0.D0
      DO 375 K=1,NSEG
      Y(NDNO)=YO(L)+RO(L)*RADIUS(J)*DSIN(THETA)
      X(NDNO)=XO(L)+RO(L)*RADIUS(J)*DCOS(THETA)
      THETA=THETA+DTHETA
  375 NDNO=NDNO+1
  782 CONTINUE
      NALL=NDNO-1
      DO 365 J=1,NALL
      X(J)=X(J)*2.5401D-2
  365 Y(J)=Y(J)*2.5401D-2
  830 FORMAT(F10.5,3(I2))
C FORM TRIANGLES FOR INNER RADIUS
      DO 83 J=1,2
      NODE(JO,1)=NDINIT
      NODE(JO,2)=J+NDINIT+1
      NODE(JO,3)=J+NDINIT
      SIGMM(JO)=0.D0
   83 JO=JO+1
      NDNOW=NDINIT+4
      DO 41J=1,NSEGC
      NODE(JO,1)=NDNOW
      NODE(JO,2)=NDINIT+J-2
      NODE(JO,3)=NDINIT+J-1
      SIGMM(JO)=0.D0
      IF(NODE(JO,2).LT.NDINIT) NODE(JO,2)=NDINIT+3
      JO=JO+1
      NUM=NPTS+1
      DO 51 K=1,NUM
      SIGMM(JO)=SIGMA
      IF(K.EQ.1) SIGMM(JO)=0.D0
      NODE(JO,1)=NDINIT+J-1
      NODE(JO,2)=NDNOW+1
      IF((K.EQ.NUM).AND.(J.EQ.NSEGC)) NODE(JO,2)=NDINIT+4
      NODE(JO,3)=NDNOW
      NDNOW=NDNOW+1
   51 JO=JO+1
   41 CONTINUE
  651 CONTINUE
C
C       FORM OTHER CONDUCTING RINGS
C
```

```
C  (2) DETERMINE NODE NUMBERS FOR RINGS 2, 4, ETC.
      IF(NSEGC.NE.0) NSE4=NSEG/NSEGC
      NTR=NSEG+NSEGC
      K1=((NSEG+NSEGC)*(2*NRAD-1)+IFSEG)*(NPHASE-1)+NSEG+NSEGC+1+IFSEG
      K2=1+NDINIT+KSEG
      DO 2 I=2,NRAD,2
       IF((I.GT.NCON).AND.(NSEGC.NE.0)) GO TO1370
      DO 3 J=1,NTR
      NODE(K1,1)=K2
      NODE(K1,2)=K2+NSEG+1+NSEGC
      NODE(K1,3)=K2+NSEG +NSEGC
      K1=K1+1
      NODE(K1,1)=K2
      K2=K2+1
      NODE(K1,2)=K2
      NODE(K1,3)=K2+NSEG+NSEGC
    3 K1=K1+1
      NODE(K1-1,2)=K2-NSEG-NSEGC
      NODE(K1-1,3)=K2
      NODE(K1-2,2)=K2
      K1=K1+2*NSEG+2*NSEGC
    2 K2=K2+NSEG+NSEGC
C  (3) DETERMINE NODE NUMBERS FOR RINGS 3, 5, ETC.
 1370 K1=3*NTR+IFSEG+JSTART+1
      K2=NSEG+1+NDINIT+NSEGC+KSEG
      DO4I=3,NRAD,2
       IF((I.GT.NCON).AND.(NSEGC.NE.0)) GO TO 370
      DO 5 J=1,NTR
      NODE(K1,1)=K2
      K2=K2+1
      NODE(K1,2)=K2
      NODE(K1,3)=K2+NSEG-1+NSEGC
      K1=K1+1
      NODE(K1,1)=K2
      NODE(K1,2)=K2+NSEG+NSEGC
      NODE(K1,3)=K2+NSEG-1+NSEGC
    5 K1=K1+1
      NODE(K1-1,1)=K2-NSEG-NSEGC
      NODE(K1-1,2)=K2
      NODE(K1-2,2)=K2-NSEG-NSEGC
      K1=K1+2*NSEG+2*NSEGC
    4 K2=K2+NSEG+NSEGC
      GO TO 65
  370 J0=K1
C FORM FIRST INSULATING RING FOR SEGMENTED CABLE
      K1=K2
      KODREV=NCON/2
      KODREV=NCON-2*KODREV
      IF(KODREV.EQ.0) K1=K1+NTR
      IF(KODREV.NE.0) J0=J0-2*NTR
      LST=K1-NTR
      DO 371 K=1,4
      NODE(J0,1)=LST
      NODE(J0,2)=LST+1
      NODE(J0,3)=K1
      J0=J0+1
      LST=LST+1
      DO 372 J=1,NSE4
      NODE(J0,1)=LST
      NODE(J0,2)=LST+1
      NODE(J0,3)=K1
      J0=J0+1
      NODE(J0,1)=K1
      NODE(J0,2)=LST+1
      NODE(J0,3)=K1+1
      J0=J0+1
      LST=LST+1
  372 K1=K1+1
  371 CONTINUE
C       FORM INSULATING LAYERS FOR SEGMENTED CABLE
      NODE(J0-2,2)=K1-NSEG-NSEGC-NSEG
      NODE(J0-1,2)=K1-NSEG-NSEGC-NSEG
      NODE(J0-1,3)=K1-NSEG
      NSTT=NCON+2
      DO 373 K=NSTT,NRAD
      DO 374 J=1,NSEG
      NODE(J0,1)=K1
      NODE(J0,2)=K1-NSEG
      NODE(J0,3)=K1+1
      J0=J0+1
```

```
      NODE(J0,1)=K1-NSEG
      NODE(J0,2)=K1+1-NSEG
      NODE(J0,3)=K1+1
      K1=K1+1
  374 J0=J0+1
      NODE(J0-1,2)=K1-2*NSEG
      NODE(J0-1,3)=K1-NSEG
      NODE(J0-2,3)=K1-NSEG
  373 CONTINUE
   65 NTR2=NTR*2
C        SPECIFY ELEMENT CONDUCTIVITIES
      JNOW=JSTART+1+IFSEG+NTR
      ICOUN=JNOW
      SIG=SIGMA
      DO 63 JJ=2,NRAD
      IF(((JJ-1).EQ.NCON).AND.(NSEGC.NE.0)) NTR2=NTR2-4
      IF(((JJ-2).EQ.NCON).AND.(NSEGC.NE.0)) NTR2=NTR2-4
      IF(JJ.GT.NCON) SIG=0.0D0
      DO 64 KK=1,NTR2
      JARG=JNOW+(JJ-2)*NTR2+KK-1
      IF(JJ.GT.NCON+0)JARG=JARG+NSEGC*NCON-NSEGC
      IF(JJ.GT.NCON+1) JARG=JARG+NSEGC*NCON+NSEGC-NSEGC
   64 SIGMM(JARG)=SIG
      IF((NSEGC.EQ.0).OR.(JJ.GT.NCON)) GO TO 63
      DO 62 LL=1,NSEGC
      SIGMM(ICOUN)=0.D0
      SIGMM(ICOUN +1)=0.D0
   62 ICOUN=ICOUN+NSE4 *2+2
   63 CONTINUE
C        ASSIGN CURRENTS AND PERMEABILITIES
      CALL MUANDJ(MU,BIGJ,NRAD,NSEG,NCON,MU1,BIGJ1,BIGJ2,SIGMM,IFSEG,
     .KSEG,NTR)
      NDINIT=(NTR*NRAD+1+KSEG)*NPHASE+1
      JSTART=JSTART+(NCON*2-1)*NTR+(NRAD-NCON)*NSEG*2+IFSEG
      IF(NRAD.GT.NCON) JSTART=JSTART+NSEGC
  113 CONTINUE
 102  FORMAT(16F5.3)
 101  FORMAT(2I2)
  703 FORMAT(3(F7.4))
  700 FORMAT(I2,3(F10.5))
      VAL=(0.D0,0.D0)
C         INITIALIZE VARIABLES AND CALL SEARCH ROUTINE
      AREAL=0.D0
      AIMAG=0.D0
      CALL RJHSER(AREAL,AIMAG,VAL)
      STOP
      END
```

```
      SUBROUTINE MUANDJ(MU,BIGJ,NRAD,NSEG,NCON,MU1,BIGJ1,BIGJ2,SIGMM,
     .IFSEG,KSEG,NTR)
      IMPLICIT COMPLEX*16(C),REAL*8(A-B,D-H,O-Z)
C  ASSIGNS VALUES OF CURRENT DENSITY (BIGJ) AND PERMEABILITU(MU)
C  TO EACH ELEMENT.
C  NCON=NUMBER OF RINGS IN CONDUCTOR
C  BIGJ1=CURRENT DENSITY IN CONDUCTOR
C  BIGJ2=CURRENT DENSITY IN INSULATION
C  MU1=MAGNETIC PERMEABILITY
      REAL*8 MU( 56),MU1,SIGMM( 56)
      COMPLEX*16 BIGJ( 56),BIGJ1,BIGJ2
      NLCONS=NCON*2*NTR-NTR+IFSEG
      NLMNTS=NLCONS+2*(NRAD-NCON)*NSEG
      DO 1 N=1,NLCONS
      MU(N)=MU1
      BIGJ(N)=BIGJ1
      IF(SIGMM(N).EQ.0.D0) BIGJ(N)=BIGJ2
    1 CONTINUE
      IF(NLCONS.EQ.NLMNTS) RETURN
      NLCONS=NLCONS+1
      DO 2 N=NLCONS,NLMNTS
      MU(N)=MU1
    2 BIGJ(N)=BIGJ2
      RETURN
      END
```

```
      SUBROUTINE RJHSER(AREAL,AIMAG,VAL)
      COMPLEX*16 VAL
      COMPLEX*16 DCONJG,DCMPLX
      REAL*8 AREAL,AIMAG
C
CTHIS IS A SIMPLE NEGATIVE FEEDBACK SEARCH ROUTINE
C
C        VAL IS THE AVERAGE POTENTIAL
C        AREAL IS THE REAL BOUNDARY VALUE
C        AIMAG IS THE IMAGINARY BOUNDARY VALUE
C
C        OLD BOUNDARY MINUS AVERAGE POTENTIAL EQUALS NEW BOUNDARY
C
      DO 1 J=1,20
      AREAL=AREAL-(DCONJG(VAL)+VAL)/2.D0
      AIMAG=AIMAG+(DCMPLX(0.D0,1.D0)*((VAL-DCONJG(VAL))/2.D0))
      WRITE(6,3) J
    3 FORMAT(' ','***** ITERATION NUMBER',2X,I2,' *****')
      CALL CVI(AREAL,AIMAG,VAL)
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CVI(AREAL,AIMAG,VAL)
      IMPLICIT REAL*8(A-B,D-H,O-Z),COMPLEX*16(C)
      COMMON X(33),Y(33),NODE(56,3),NSEG,NRAD,SIGMM(56),BIGJ(56),MU(56)
      COMMON/A3/ XP(50)
      DIMENSION YP(8),SP(8,8),TEMP(8,8),F1(8,8),VMUL(8),IECT(8),IEC2(8)
      DIMENSIONCSM( 650),R(3),Q(3),T(3),RADI(1)
      COMPLEX*16 BIGJ,S(3,3),RHS,CV(33),VAL,DCMPLX,ALPHA
      REAL*8 MU,CDABS
      COMPLEX*16 DP,DQ,DR,DS,CW(3),DCONJG,CRES(33)
      DATA CV/33*(0.D0,0.D0)/
C
C      THIS SUBROUTINE COMPUTES VECTOR POTENTIAL,CALCULATES LOSS
C      RATIO AND FINDS NEW AVERAGE VALUE
C
C      THIS TECHNIQE IS NOT VALID FOR SEGMENTED CABLES DUE TO A LACK
C      OF ROTATIONAL SYMMETRY
      SIGMA=5.80D7
      NPOT=1+(NRAD-1)*NSEG
      NTR=NSEG
      NPOT8=NPOT+8
      NLMNTS=NRAD*2*NSEG-NSEG
      NPOT1=NPOT+1
      NPOTSQ=NPOT*NPOT
      NSPOQ=NPOT*(NPOT-1)
      BETA=100.
      NMZ=NSEG+1
      ALPHA=DCMPLX(AREAL,AIMAG)
      ATOT=0.0
      NDTOT=NPOT
C      INITIALIZE MASTER MATRIX
      DO 7 I=1,NPOT
      DO 7 J=1,NPOT1
    7 CSM(I+NPOT*(J-1))=(0.D0,0.D0)
C
C      FORM MATRIX FOR EACH ELEMENT
      DO 100 N=1,NLMNTS
      XCENT=(X(NODE(N,1))+X(NODE(N,2))+X(NODE(N,3)))/3.D0
      YCENT=(Y(NODE(N,1))+Y(NODE(N,2))+Y(NODE(N,3)))/3.D0
      X1=X(NODE(N,1))-XCENT
      X2=X(NODE(N,2))-XCENT
      X3=X(NODE(N,3))-XCENT
      Y1=Y(NODE(N,1))-YCENT
      Y2=Y(NODE(N,2))-YCENT
      Y3=Y(NODE(N,3))-YCENT
      J=2
      K=3
C      CALCULATE FINITE ELEMENT CONSTANTS
      DO 8 I=1,3
      R(I)=(X(NODE(N,J))-X(NODE(N,K)))
      Q(I)=(Y(NODE(N,K))-Y(NODE(N,J)))
      T(I)=X(NODE(N,K))*Y(NODE(N,J))-X(NODE(N,J))*Y(NODE(N,K))
      J=J+1-J/3*3
    8 K=K+1-K/3*3
      AREA=  (X(NODE(N,1))*(Y(NODE(N,3))-Y(NODE(N,2)))
     .       + X(NODE(N,2))*(Y(NODE(N,1))-Y(NODE(N,3)))
     .       + X(NODE(N,3))*(Y(NODE(N,2))-Y(NODE(N,1))))*5.D-1
C      FORM MATRIX
      DO 9 I=1,3
      DO99 J=1,3
      SREAL=(Q(I)*Q(J)+R(I)*R(J))/(AREA*4.D0)
      S(I,J)=DCMPLX(SREAL,0.D0)
      IF(SIGMM(N).EQ.0.D0) GO TO 98
C      CALCULATE EDDY TERM
      EDCUR=     ( 60.D0*3.141592653D0*MU(N)*SIGMA*(T(I)*T(J)+XCENT*(Q(I)
     .*T(J)+T(I)*Q(J))+YCENT*(R(I)*T(J)+T(I)*R(J))+(Q(I)*R(J)+R(I)*Q(J))
     .*(XCENT*YCENT+(X1*Y1+X2*Y2+X3*Y3)/12.D0)+Q(I)*Q(J)*(XCENT**2+(
     .X1**2+X2**2+X3**2)/12.D0)+R(I)*R(J)*(YCENT**2+(Y1**2+Y2**2+Y3**2)
     ./12.D0)) )/(AREA*2.D0)
      S(I,J)=S(I,J)+DCMPLX(0.D0,EDCUR)
   98 CONTINUE
   99 CONTINUE
    9 CONTINUE
C      CALCULATE SOURCE TERM
      RHS=BIGJ(N)*AREA*MU(N)/3.D0
C
C      PLACE ELEMENT MATRIX INTO MASTER MATRIX
      DO 10 I=1,3
      IF(NODE(N,I).GT.NPOT)GO TO 10
      DO 11 J=1,3
      IF (NODE(N,J).GT.NPOT)GO TO211
```

```
      CSM(NODE(N,I)+(NODE(N,J)-1)*NPOT)=CSM(NODE(N,I)+(NODE(N,J)-1)*NPOT
     1)+S(I,J)
      GO TO 11
  211 CSM(NODE(N,I)+NPOTSQ)=CSM(NODE(N,I)+NPOTSQ)-S(I,J)*ALPHA
   11 CONTINUE
      CSM(NODE(N,I)+NPOTSQ)=CSM(NODE(N,I)+NPOTSQ)-RHS
   10 CONTINUE
  100 CONTINUE
C
C       SOLVE MASTER MATRIX
C
      CALL CSOLVE(CSM,CV,NPOT,CDET)
C
C       CALCULATE AVERAGE POTENTIAL
      VAL=(0.D0,0.D0)
      DO118 N=1,NLMNTS
      AREA=  (X(NODE(N,1))*(Y(NODE(N,3))-Y(NODE(N,2)))
     .      + X(NODE(N,2))*(Y(NODE(N,1))-Y(NODE(N,3)))
     .      + X(NODE(N,3))*(Y(NODE(N,2))-Y(NODE(N,1))))*5.D-1
      IF(SIGMM(N).NE.0.D0) ATOT=ATOT+AREA
      DO118 J=1,3
      IF(NODE(N,J).GT.NPOT) CV(NODE(N,J))=ALPHA
      IF(SIGMM(N).NE.0.D0)  VAL=VAL+CV(NODE(N,J))*AREA/3.D0
  118 CONTINUE
      VAL=VAL/ATOT
      VAB=CDABS(VAL)
      WRITE(6,800)
  800 FORMAT(' ',15X,'NEW BOUNDARY',15X,'AVERAGE POTENTIAL')
      WRITE(6,77) AREAL,AIMAG,VAL
   77 FORMAT(' ',10X,4G14.7)
      CVA=VAL
C
C       PRINT OUT RESULTS
      RNORM=0.D0
      VNORM=0.D0
      DO 14 I=1,NPOT
      CRES(I)=CSM(I+NPOTSQ)
      DO 13 J=1,NPOT
   13 CRES(I)=CRES(I)-CSM(I+(J-1)*NPOT)*CV(J)
      VNORM=VNORM+CDABS(CV(I)**2)
   14 RNORM=RNORM+CDABS(CRES(I)**2)
      RNORM=DSORT(RNORM)
      VNORM=DSQRT(VNORM)
      WRITE(6,216) VNORM,RNORM
  216 FORMAT(' ',2G14.7)
      WRITE(6,105) CVA
  105 FORMAT(//'    CVA=(',G14.7,',',G14.7,')',/
     1        ' NO.',12X,'RAW POTENTIAL',26X,'RESIDUAL',22X,'ADJUSTED P
     2OTENTIAL'/)
      DO 12 I=1,NPOT
      CVAD=CV(I)-CVA
   12 WRITE (6,103) I,CV(I),CRES(I),CVAD
  103 FORMAT(I4,':  ',3('(',G14.7,' +J ',G14.7,')     '))
      PI=3.1415926
C
C       CALCULATE LOSS RATIO
C
      DP=(0.D0,0.D0)
      DQ=(0.D0,0.D0)
      DR=(0.D0,0.D0)
      DS=(0.D0,0.D0)
      RADI(1)=.791D0
      NLCONS=NSEG*(2*NRAD-1)
      DO 400 N=1,NLCONS
      IF(SIGMM(N).EQ.0.D0) GO TO 400
      XCENT=(X(NODE(N,1))+X(NODE(N,2))+X (NODE(N,3)))/3.D0
      YCENT=(Y(NODE(N,1))+Y(NODE(N,2))+Y (NODE(N,3)))/3.D0
      X1=X(NODE(N,1))-XCENT
      X2=X(NODE(N,2))-XCENT
      X3=X(NODE(N,3))-XCENT
      Y1=Y(NODE(N,1))-YCENT
      Y2=Y(NODE(N,2))-YCENT
      Y3=Y(NODE(N,3))-YCENT
      CW(1)=CV(NODE(N,1))
      CW(2)=CV(NODE(N,2))
      CW(3)=CV(NODE(N,3))
      CTRA=(CW(1)-CVA)*(X(NODE(N,3))*Y(NODE(N,2))-X(NODE(N,2))*Y(NODE(N,
     .3)))+(CW(2)-CVA)*(X(NODE(N,1))*Y(NODE(N,3))-X(NODE(N,3))*Y(NODE(N,
     .1)))+(CW(3)-CVA)*(X(NODE(N,2))*Y(NODE(N,1))-X(NODE(N,1))*Y(NODE(N,
     .2)))
```

```
      CTRB=(CW(1)-CVA)*(Y(NODE(N,3))-Y(NODE(N,2)))+(CW(2)-CVA)*(Y(NODE(N
     *,1))-Y(NODE(N,3)))+(CW(3)-CVA)*(Y(NODE(N,2))-Y(NODE(N,1)))
      CTRC=(CW(1)-CVA)*(X(NODE(N,2))-X(NODE(N,3)))+(CW(2)-CVA)*(X(NODE(N
     *,3))-X(NODE(N,1)))+(CW(3)-CVA)*(X(NODE(N,1))-X(NODE(N,2)))
      AREA=    (X(NODE(N,1))*(Y(NODE(N,3))-Y(NODE(N,2)))
     C          +X(NODE(N,2))*(Y(NODE(N,1))-Y(NODE(N,3)))
     C          +X(NODE(N,3))*(Y(NODE(N,2))-Y(NODE(N,1))))*5.D-1
      VP=((CDABS(CTRB))**2*((X1**2+X2**2+X3**2)/12.D0+XCENT**2)+(CDABS(
     *CTRA))**2+(CDABS(CTRC))**2*(((Y1**2+Y2**2+Y3**2)/12.D0)+YCENT**2)
     *+      (CTRB*DCONJG(CTRC)+CTRC*DCONJG(CTRB))*(((X1*Y1+X2*Y2+X3*Y3)
     */12.D0)+XCENT*YCENT)+CTRB*DCONJG(CTRA)*XCENT+CTRA*DCONJG(CTRB)*
     *XCENT+CTRA*DCONJG(CTRC)*YCENT+CTRC*DCONJG(CTRA)*YCENT)/(AREA*4.D0)
C
C       DP IS THE EDDY CONTRIBUTION
C       DQ IS THE SOURCE CONTRIBUTION
C       DR AND DS ARE CROSS TERMS
C
      DP=DP+(60.D0*60.D0*SIGMA*VP*4.D0*PI*PI)
      DQ=DQ+AREA/SIGMA*BIGJ(N)*DCONJG(BIGJ(N))
      DR=DR+40.D0*PI*AREA*(CW(1)+CW(2)+CW(3)-3.D0*CVA)*DCONJG(BIGJ(N))
      DS=DS+40.D0*PI*AREA*BIGJ(N)*DCONJG(CW(1)+CW(2)+CW(3)-3.D0*CVA)
      IF(SIGMM(N).EQ.0.D0) GO TO 400
  400 CONTINUE
      CPTOT=DP+DQ+DCMPLX(0.D0,1.D0)*(DS-DR)
      PDC=DQ
      RACDC=(CPTOT+DCONJG(CPTOT))/(2.D0*PDC)
      WRITE(6,500) RACDC
      WRITE(6,801)
  801 FORMAT(' ',10X,'D.C. LOSS',30X,'CROSS TERMS',30X,'TOTAL LOSS')
      WRITE(6,503) DQ,DR,DS,CPTOT
  503 FORMAT(' ',8(G14.7))
      WRITE(6,802)
  802 FORMAT(' ',5X,'EDDY LOSS',20X,'D.C.')
      WRITE(6,501) DP,PDC
      WRITE(6,803)
  803 FORMAT(' ',5X,'AREA')
      WRITE(6,501) ATOT
  500 FORMAT(' ','RATIO OF AC TO DC RESISTANCE IS =',G14.7)
  501 FORMAT(' ',3(G14.7))
   15 CONTINUE
 9999 CONTINUE
      RETURN
      END
```

```
*** ITERATION NUMBER    1 *****
          NEW BOUNDARY                          AVERAGE POTENTIAL
       -0.0000000      0.0000000     -0.1263589D-10 0.2950464D-10
0.2283068D-09 0.0000000

 CVA=(-0.1263589D-10, 0.2950464D-10)
0.         RAW POTENTIAL                                RESIDUAL                          ADJUSTED POTENTIAL

  1: (-0.6655684D-11 +J  0.5824549D-10) (-0.0000000     +J -0.0000000    ) ( 0.5780210D-11 +J  0.2874085D-10)
  2: (-0.1152151D-10 +J  0.5414775D-10) (-0.0000000     +J -0.0000000    ) ( 0.1114386D-11 +J  0.2464311D-10)
  3: (-0.1152151D-10 +J  0.5414775D-10) (-0.0000000     +J -0.0000000    ) ( 0.1114386D-11 +J  0.2464311D-10)
  4: (-0.1152151D-10 +J  0.5414775D-10) (-0.0000000     +J -0.0000000    ) ( 0.1114386D-11 +J  0.2464311D-10)
  5: (-0.1152151D-10 +J  0.5414775D-10) (-0.0000000     +J -0.0000000    ) ( 0.1114386D-11 +J  0.2464311D-10)
  6: (-0.1152151D-10 +J  0.5414775D-10) (-0.0000000     +J -0.0000000    ) ( 0.1114386D-11 +J  0.2464311D-10)
  7: (-0.1152151D-10 +J  0.5414775D-10) (-0.0000000     +J -0.0000000    ) ( 0.1114386D-11 +J  0.2464311D-10)
  8: (-0.1152151D-10 +J  0.5414775D-10) (-0.0000000     +J -0.0000000    ) ( 0.1114386D-11 +J  0.2464311D-10)
  9: (-0.1152151D-10 +J  0.5414775D-10) (-0.0000000     +J -0.0000000    ) ( 0.1114386D-11 +J  0.2464311D-10)
 10: (-0.1804271D-10 +J  0.4139829D-10) (-0.0000000     +J -0.0000000    ) (-0.5406812D-11 +J  0.1189365D-10)
 11: (-0.1804271D-10 +J  0.4139829D-10) (-0.0000000     +J -0.0000000    ) (-0.5406812D-11 +J  0.1189365D-10)
 12: (-0.1804271D-10 +J  0.4139829D-10) (-0.0000000     +J -0.0000000    ) (-0.5406812D-11 +J  0.1189365D-10)
 13: (-0.1804271D-10 +J  0.4139829D-10) (-0.0000000     +J -0.0000000    ) (-0.5406812D-11 +J  0.1189365D-10)
 14: (-0.1804271D-10 +J  0.4139829D-10) (-0.0000000     +J -0.0000000    ) (-0.5406812D-11 +J  0.1189365D-10)
 15: (-0.1804271D-10 +J  0.4139829D-10) (-0.0000000     +J -0.0000000    ) (-0.5406812D-11 +J  0.1189365D-10)
 16: (-0.1804271D-10 +J  0.4139829D-10) (-0.0000000     +J -0.0000000    ) (-0.5406812D-11 +J  0.1189365D-10)
 17: (-0.1804271D-10 +J  0.4139829D-10) (-0.0000000     +J -0.0000000    ) (-0.5406812D-11 +J  0.1189365D-10)
 18: (-0.1789288D-10 +J  0.2571613D-10) (-0.0000000     +J -0.0000000    ) (-0.5256984D-11 +J -0.3788512D-11)
 19: (-0.1789288D-10 +J  0.2571613D-10) (-0.0000000     +J -0.0000000    ) (-0.5256984D-11 +J -0.3788512D-11)
 20: (-0.1789288D-10 +J  0.2571613D-10) (-0.0000000     +J -0.0000000    ) (-0.5256984D-11 +J -0.3788512D-11)
 21: (-0.1789288D-10 +J  0.2571613D-10) (-0.0000000     +J -0.0000000    ) (-0.5256984D-11 +J -0.3788512D-11)
 22: (-0.1789288D-10 +J  0.2571613D-10) (-0.0000000     +J -0.0000000    ) (-0.5256984D-11 +J -0.3788512D-11)
 23: (-0.1789288D-10 +J  0.2571613D-10) (-0.0000000     +J -0.0000000    ) (-0.5256984D-11 +J -0.3788512D-11)
 24: (-0.1789288D-10 +J  0.2571613D-10) (-0.0000000     +J -0.0000000    ) (-0.5256984D-11 +J -0.3788512D-11)
 25: (-0.1789288D-10 +J  0.2571613D-10) (-0.0000000     +J -0.0000000    ) (-0.5256984D-11 +J -0.3788512D-11)
RATIO OF AC TO DC RESISTANCE IS =  1.152600
        D.C. LOSS                                  CROSS TERMS                                      TOTAL LOSS
0.3432186D-10 0.0000000      -0.9454498D-26 0.1297518D-25-0.9454498D-26-0.1297518D-25 0.3955938D-10 0.0000000
      EDDY LOSS                         D.C.
0.5237522D-11 0.0000000      0.3432186D-10
       AREA
0.1990668D-02
**** ITERATION NUMBER    2 *****
          NEW BOUNDARY                          AVERAGE POTENTIAL
        0.1263589D-10-0.2950464D-10-0.1630367D-10 0.1554326D-10
0.2324385D-09 0.0000000

 CVA=(-0.1630367D-10, 0.1554326D-10)
0.         RAW POTENTIAL                                RESIDUAL                          ADJUSTED POTENTIAL

  1: (-0.1473526D-10 +J  0.6442280D-10) (-0.0000000     +J -0.0000000    ) ( 0.1568408D-11 +J  0.4887555D-10)
  2: (-0.2127900D-10 +J  0.5639235D-10) (-0.0000000     +J -0.0000000    ) (-0.4975331D-11 +J  0.4084910D-10)
  3: (-0.2127900D-10 +J  0.5639235D-10) (-0.0000000     +J -0.0000000    ) (-0.4975331D-11 +J  0.4084910D-10)
  4: (-0.2127900D-10 +J  0.5639235D-10) (-0.0000000     +J -0.0000000    ) (-0.4975331D-11 +J  0.4084910D-10)
  5: (-0.2127900D-10 +J  0.5639235D-10) (-0.0000000     +J -0.0000000    ) (-0.4975331D-11 +J  0.4084910D-10)
  6: (-0.2127900D-10 +J  0.5639235D-10) (-0.0000000     +J -0.0000000    ) (-0.4975331D-11 +J  0.4084910D-10)
  7: (-0.2127900D-10 +J  0.5639235D-10) (-0.0000000     +J -0.0000000    ) (-0.4975331D-11 +J  0.4084910D-10)
  8: (-0.2127900D-10 +J  0.5639235D-10) (-0.0000000     +J -0.0000000    ) (-0.4975331D-11 +J  0.4084910D-10)
  9: (-0.2127900D-10 +J  0.5639235D-10) (-0.0000000     +J -0.0000000    ) (-0.4975331D-11 +J  0.4084910D-10)
 10: (-0.2848469D-10 +J  0.3361605D-10) (-0.0000000     +J -0.0000000    ) (-0.1218102D-10 +J  0.1807279D-10)
 11: (-0.2848469D-10 +J  0.3361605D-10) (-0.0000000     +J -0.0000000    ) (-0.1218102D-10 +J  0.1807279D-10)
 12: (-0.2848469D-10 +J  0.3361605D-10) (-0.0000000     +J -0.0000000    ) (-0.1218102D-10 +J  0.1807279D-10)
 13: (-0.2848469D-10 +J  0.3361605D-10) (-0.0000000     +J -0.0000000    ) (-0.1218102D-10 +J  0.1807279D-10)
 14: (-0.2848469D-10 +J  0.3361605D-10) (-0.0000000     +J -0.0000000    ) (-0.1218102D-10 +J  0.1807279D-10)
 15: (-0.2848469D-10 +J  0.3361605D-10) (-0.0000000     +J -0.0000000    ) (-0.1218102D-10 +J  0.1807279D-10)
 16: (-0.2848469D-10 +J  0.3361605D-10) (-0.0000000     +J -0.0000000    ) (-0.1218102D-10 +J  0.1807279D-10)
 17: (-0.2848469D-10 +J  0.3361605D-10) (-0.0000000     +J -0.0000000    ) (-0.1218102D-10 +J  0.1807279D-10)
```

```
**** ITERATION NUMBER    3 *****
              NEW BOUNDARY                         AVERAGE POTENTIAL
        0.2893957D-10-0.4504790D-10-0.1481249D-10 0.5522931D-11
0.2323317D-09 0.0000000


 CVA=(-0.1481249D-10, 0.5522931D-11)
D.             RAW POTENTIAL                               RESIDUAL                        ADJUSTED POTENTIAL

  1: (-0.2152537D-10 +J  0.6623094D-10)  (-0.0000000     +J -0.0000000    )  (-0.6712884D-11 +J  0.6070801D-10)
  2: (-0.2819405D-10 +J  0.5514452D-10)  (-0.0000000     +J -0.0000000    )  (-0.1338156D-10 +J  0.4962159D-10)
  3: (-0.2819405D-10 +J  0.5514452D-10)  (-0.0000000     +J -0.0000000    )  (-0.1338156D-10 +J  0.4962159D-10)
  4: (-0.2819405D-10 +J  0.5514452D-10)  (-0.0000000     +J -0.0000000    )  (-0.1338156D-10 +J  0.4962159D-10)
  5: (-0.2819405D-10 +J  0.5514452D-10)  (-0.0000000     +J -0.0000000    )  (-0.1338156D-10 +J  0.4962159D-10)
  6: (-0.2819405D-10 +J  0.5514452D-10)  (-0.0000000     +J -0.0000000    )  (-0.1338156D-10 +J  0.4962159D-10)
  7: (-0.2819405D-10 +J  0.5514452D-10)  (-0.0000000     +J -0.0000000    )  (-0.1338156D-10 +J  0.4962159D-10)
  8: (-0.2819405D-10 +J  0.5514452D-10)  (-0.0000000     +J -0.0000000    )  (-0.1338156D-10 +J  0.4962159D-10)
  9: (-0.2819405D-10 +J  0.5514452D-10)  (-0.0000000     +J -0.0000000    )  (-0.1338156D-10 +J  0.4962159D-10)
 10: (-0.3307101D-10 +J  0.2571044D-10)  (-0.0000000     +J -0.0000000    )  (-0.1825853D-10 +J  0.2018751D-10)
 11: (-0.3307101D-10 +J  0.2571044D-10)  (-0.0000000     +J -0.0000000    )  (-0.1825853D-10 +J  0.2018751D-10)
 12: (-0.3307101D-10 +J  0.2571044D-10)  (-0.0000000     +J -0.0000000    )  (-0.1825853D-10 +J  0.2018751D-10)
 13: (-0.3307101D-10 +J  0.2571044D-10)  (-0.0000000     +J -0.0000000    )  (-0.1825853D-10 +J  0.2018751D-10)
 14: (-0.3307101D-10 +J  0.2571044D-10)  (-0.0000000     +J -0.0000000    )  (-0.1825853D-10 +J  0.2018751D-10)
 15: (-0.3307101D-10 +J  0.2571044D-10)  (-0.0000000     +J -0.0000000    )  (-0.1825853D-10 +J  0.2018751D-10)
 16: (-0.3307101D-10 +J  0.2571044D-10)  (-0.0000000     +J -0.0000000    )  (-0.1825853D-10 +J  0.2018751D-10)
 17: (-0.3307101D-10 +J  0.2571044D-10)  (-0.0000000     +J -0.0000000    )  (-0.1825853D-10 +J  0.2018751D-10)
 18: (-0.2285028D-10 +J -0.5323745D-11)  (-0.0000000     +J -0.0000000    )  (-0.8037797D-11 +J -0.1084668D-10)
 19: (-0.2285028D-10 +J -0.5323745D-11)  (-0.0000000     +J -0.0000000    )  (-0.8037797D-11 +J -0.1084668D-10)
 20: (-0.2285028D-10 +J -0.5323745D-11)  (-0.0000000     +J -0.0000000    )  (-0.8037797D-11 +J -0.1084668D-10)
 21: (-0.2285028D-10 +J -0.5323745D-11)  (-0.0000000     +J -0.0000000    )  (-0.8037797D-11 +J -0.1084668D-10)
 22: (-0.2285028D-10 +J -0.5323745D-11)  (-0.0000000     +J -0.0000000    )  (-0.8037797D-11 +J -0.1084668D-10)
 23: (-0.2285028D-10 +J -0.5323745D-11)  (-0.0000000     +J -0.0000000    )  (-0.8037797D-11 +J -0.1084668D-10)
 24: (-0.2285028D-10 +J -0.5323745D-11)  (-0.0000000     +J -0.0000000    )  (-0.8037797D-11 +J -0.1084668D-10)
 25: (-0.2285028D-10 +J -0.5323745D-11)  (-0.0000000     +J -0.0000000    )  (-0.8037797D-11 +J -0.1084668D-10)
ATIO OF AC TO DC RESISTANCE IS =   1.662378
          D.C. LOSS                              CROSS TERMS                                  TOTAL LOSS
0.3432186D-10 0.0000000     -0.1291208D-25 0.1611801D-25-0.1291208D-25-0.1611801D-25 0.5705590D-10 0.0000000
     EDDY LOSS                        D.C.
0.2273404D-10 0.0000000      0.3432186D-10
     AREA
0.1990668D-02
**** ITERATION NUMBER    4 *****
              NEW BOUNDARY                         AVERAGE POTENTIAL
        0.4375205D-10-0.5057083D-10-0.1108196D-10-0.5295144D-12
0.2273803D-09 0.0000000


 CVA=(-0.1108196D-10,-0.5295144D-12)
D.             RAW POTENTIAL                               RESIDUAL                        ADJUSTED POTENTIAL

  1: (-0.2640547D-10 +J  0.6552139D-10)  (-0.0000000     +J -0.0000000    )  (-0.1532351D-10 +J  0.6605091D-10)
  2: (-0.3231041D-10 +J  0.5242894D-10)  (-0.0000000     +J -0.0000000    )  (-0.2122845D-10 +J  0.5295846D-10)
  3: (-0.3231041D-10 +J  0.5242894D-10)  (-0.0000000     +J -0.0000000    )  (-0.2122845D-10 +J  0.5295846D-10)
  4: (-0.3231041D-10 +J  0.5242894D-10)  (-0.0000000     +J -0.0000000    )  (-0.2122845D-10 +J  0.5295846D-10)
  5: (-0.3231041D-10 +J  0.5242894D-10)  (-0.0000000     +J -0.0000000    )  (-0.2122845D-10 +J  0.5295846D-10)
  6: (-0.3231041D-10 +J  0.5242894D-10)  (-0.0000000     +J -0.0000000    )  (-0.2122845D-10 +J  0.5295846D-10)
  7: (-0.3231041D-10 +J  0.5242894D-10)  (-0.0000000     +J -0.0000000    )  (-0.2122845D-10 +J  0.5295846D-10)
  8: (-0.3231041D-10 +J  0.5242894D-10)  (-0.0000000     +J -0.0000000    )  (-0.2122845D-10 +J  0.5295846D-10)
  9: (-0.3231041D-10 +J  0.5242894D-10)  (-0.0000000     +J -0.0000000    )  (-0.2122845D-10 +J  0.5295846D-10)
 10: (-0.3384556D-10 +J  0.1934312D-10)  (-0.0000000     +J -0.0000000    )  (-0.2276360D-10 +J  0.1987264D-10)
 11: (-0.3384556D-10 +J  0.1934312D-10)  (-0.0000000     +J -0.0000000    )  (-0.2276360D-10 +J  0.1987264D-10)
 12: (-0.3384556D-10 +J  0.1934312D-10)  (-0.0000000     +J -0.0000000    )  (-0.2276360D-10 +J  0.1987264D-10)
 13: (-0.3384556D-10 +J  0.1934312D-10)  (-0.0000000     +J -0.0000000    )  (-0.2276360D-10 +J  0.1987264D-10)
 14: (-0.3384556D-10 +J  0.1934312D-10)  (-0.0000000     +J -0.0000000    )  (-0.2276360D-10 +J  0.1987264D-10)
 15: (-0.3384556D-10 +J  0.1934312D-10)  (-0.0000000     +J -0.0000000    )  (-0.2276360D-10 +J  0.1987264D-10)
```

```
18: (-0.4117651D-11 +J -0.1381109D-10)  (-0.0000000    +J -0.0000000   )  (-0.4063654D-11 +J -0.1382013D-10)
19: (-0.4117651D-11 +J -0.1381109D-10)  (-0.0000000    +J -0.0000000   )  (-0.4063654D-11 +J -0.1382013D-10)
20: (-0.4117651D-11 +J -0.1381109D-10)  (-0.0000000    +J -0.0000000   )  (-0.4063654D-11 +J -0.1382013D-10)
21: (-0.4117651D-11 +J -0.1381109D-10)  (-0.0000000    +J -0.0000000   )  (-0.4063654D-11 +J -0.1382013D-10)
22: (-0.4117651D-11 +J -0.1381109D-10)  (-0.0000000    +J -0.0000000   )  (-0.4063654D-11 +J -0.1382013D-10)
23: (-0.4117651D-11 +J -0.1381109D-10)  (-0.0000000    +J -0.0000000   )  (-0.4063654D-11 +J -0.1382013D-10)
24: (-0.4117651D-11 +J -0.1381109D-10)  (-0.0000000    +J -0.0000000   )  (-0.4063654D-11 +J -0.1382013D-10)
25: (-0.4117651D-11 +J -0.1381109D-10)  (-0.0000000    +J -0.0000000   )  (-0.4063654D-11 +J -0.1382013D-10)
RATIO OF AC TO DC RESISTANCE IS =  1.754154
        D.C. LOSS                              CROSS TERMS                                    TOTAL LOSS
0.3432186D-10 0.0000000     -0.1652190D-25 0.1939967D-25-0.1652190D-25-0.1939967D-25 0.6020582D-10 0.0000000
    EDDY LOSS                     D.C.
0.2588396D-10 0.0000000     0.3432186D-10
    AREA
0.1990668D-02
***** ITERATION NUMBER  20 *****
            NEW BOUNDARY                  AVERAGE POTENTIAL
        0.6241842D-10-0.3457114D-10-0.3733365D-13-0.9085800D-14
0.1917688D-09 0.0000000


 CVA=(-0.3733365D-13,-0.9085800D-14)
NO.           RAW POTENTIAL                          RESIDUAL                            ADJUSTED POTENTIAL




 1: (-0.2911355D-10 +J  0.5834628D-10)  (-0.0000000    +J -0.0000000   )  (-0.2907622D-10 +J  0.5835537D-10)
 2: (-0.3171371D-10 +J  0.4478304D-10)  (-0.0000000    +J -0.0000000   )  (-0.3167637D-10 +J  0.4479212D-10)
 3: (-0.3171371D-10 +J  0.4478304D-10)  (-0.0000000    +J -0.0000000   )  (-0.3167637D-10 +J  0.4479212D-10)
 4: (-0.3171371D-10 +J  0.4478304D-10)  (-0.0000000    +J -0.0000000   )  (-0.3167637D-10 +J  0.4479212D-10)
 5: (-0.3171371D-10 +J  0.4478304D-10)  (-0.0000000    +J -0.0000000   )  (-0.3167637D-10 +J  0.4479212D-10)
 6: (-0.3171371D-10 +J  0.4478304D-10)  (-0.0000000    +J -0.0000000   )  (-0.3167637D-10 +J  0.4479212D-10)
 7: (-0.3171371D-10 +J  0.4478304D-10)  (-0.0000000    +J -0.0000000   )  (-0.3167637D-10 +J  0.4479212D-10)
 8: (-0.3171371D-10 +J  0.4478304D-10)  (-0.0000000    +J -0.0000000   )  (-0.3167637D-10 +J  0.4479212D-10)
 9: (-0.3171371D-10 +J  0.4478304D-10)  (-0.0000000    +J -0.0000000   )  (-0.3167637D-10 +J  0.4479212D-10)
10: (-0.2576379D-10 +J  0.1349588D-10)  (-0.0000000    +J -0.0000000   )  (-0.2572646D-10 +J  0.1350497D-10)
11: (-0.2576379D-10 +J  0.1349588D-10)  (-0.0000000    +J -0.0000000   )  (-0.2572646D-10 +J  0.1350497D-10)
12: (-0.2576379D-10 +J  0.1349588D-10)  (-0.0000000    +J -0.0000000   )  (-0.2572646D-10 +J  0.1350497D-10)
13: (-0.2576379D-10 +J  0.1349588D-10)  (-0.0000000    +J -0.0000000   )  (-0.2572646D-10 +J  0.1350497D-10)
14: (-0.2576379D-10 +J  0.1349588D-10)  (-0.0000000    +J -0.0000000   )  (-0.2572646D-10 +J  0.1350497D-10)
15: (-0.2576379D-10 +J  0.1349588D-10)  (-0.0000000    +J -0.0000000   )  (-0.2572646D-10 +J  0.1350497D-10)
16: (-0.2576379D-10 +J  0.1349588D-10)  (-0.0000000    +J -0.0000000   )  (-0.2572646D-10 +J  0.1350497D-10)
17: (-0.2576379D-10 +J  0.1349588D-10)  (-0.0000000    +J -0.0000000   )  (-0.2572646D-10 +J  0.1350497D-10)
18: (-0.4097554D-11 +J -0.1383617D-10)  (-0.0000000    +J -0.0000000   )  (-0.4060220D-11 +J -0.1382708D-10)
19: (-0.4097554D-11 +J -0.1383617D-10)  (-0.0000000    +J -0.0000000   )  (-0.4060220D-11 +J -0.1382708D-10)
20: (-0.4097554D-11 +J -0.1383617D-10)  (-0.0000000    +J -0.0000000   )  (-0.4060220D-11 +J -0.1382708D-10)
21: (-0.4097554D-11 +J -0.1383617D-10)  (-0.0000000    +J -0.0000000   )  (-0.4060220D-11 +J -0.1382708D-10)
22: (-0.4097554D-11 +J -0.1383617D-10)  (-0.0000000    +J -0.0000000   )  (-0.4060220D-11 +J -0.1382708D-10)
23: (-0.4097554D-11 +J -0.1383617D-10)  (-0.0000000    +J -0.0000000   )  (-0.4060220D-11 +J -0.1382708D-10)
24: (-0.4097554D-11 +J -0.1383617D-10)  (-0.0000000    +J -0.0000000   )  (-0.4060220D-11 +J -0.1382708D-10)
25: (-0.4097554D-11 +J -0.1383617D-10)  (-0.0000000    +J -0.0000000   )  (-0.4060220D-11 +J -0.1382708D-10)
RATIO OF AC TO DC RESISTANCE IS =  1.754751
        D.C. LOSS                              CROSS TERMS                                    TOTAL LOSS
0.3432186D-10 0.0000000     -0.1794816D-25 0.1647142D-25-0.1794816D-25-0.1647142D-25 0.6022633D-10 0.0000000
    EDDY LOSS                     D.C.
0.2550447D-10 0.0000000     0.3432186D-10
    AREA
0.1990668D-02

CORE USAGE      OBJECT CODE=   30064 BYTES,ARRAY AREA=   1824 BYTES,TOTAL AREA AVAILABLE=   68544  BYTES

DIAGNOSTICS       NUMBER OF ERRORS=      0, NUMBER OF WARNINGS=      0, NUMBER OF EXTENSIONS=      0

COMPILE TIME=    3.15 SEC,EXECUTION TIME=   181.67 SEC,  WATFIV - JUL 1973 V1L4     14.12.17   WEDNESDAY   10 NOV 76
```

Single conductor with source current variation and a Hooke and Jeeves search.

Mainline is the same as that of the previous program except:

(1) Variable 'VAL' is omitted from statement 9

(2) Statements 241,242 and 243 are omitted

(3) Statement 244 becomes 'CALL HKNJV(VAL, PTNW,2)'

```
      SUBROUTINE HKNJV(VAL,PTNW,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION CORD(10),CTEST(10),PTNW(10)
      COMMON/IO/ IFWRIT
      DELTA=1.D-10*1.D-1
      READ(5,27)(CORD(J),J=1,N)
   27 FORMAT(10F8.4)
      DELTO=DELTA
      IFWRIT=1
      CALL FUNCT(CORD,N,VAL)
      IFWRIT=0
C TRY FOR A BETTER POINT BY TESTING ALL DIRECTIONS
      VALOLD=VAL
    5 CALL NEWDIR(N,DELTA,CORD,PTNW,VAL)
      IFWRIT=1
      CALL FUNCT(PTNW,N,VAL)
      IFWRIT=0
      WRITE(6,18) VAL,(PTNW(L),L=1,N)
C TEST FOR THE NUMBER OF SUCESSFUL CO-ORDIATES
   20 ITEST=0
      DO 2 J=1,N
      IF(CORD(J).NE.PTNW(J)) ITEST=ITEST+1
    2 CONTINUE
C IF NO SUCESSFUL SEARCHES DECREASE DELTA
      IF(ITEST.NE.0) GO TO 4
      DELTA=DELTA/2
      IF(DABS(DELTA/DELTO).LT.1.E-3) GO TO 7
      GO TO 5
C IF LESS THAN TWO NEW CO-ORDINATES DO A LOCAL SEARCH
    4 IF(ITEST.NE.1) GO TO 3
   17 DO 21 J=1,N
   21 CORD(J)=PTNW(J)
      GO TO 5
C IF MORE THAN ONE NEW POINT ACCELERATE
    3 DO 16 J=1,N
   16 CTEST(J)=2.*PTNW(J)-CORD(J)
      CALL FUNCT(CTEST,N,VNEW)
      IF(VNEW.GE.VAL) GO TO 17
      DO 19 J=1,N
   19 CORD(J)=PTNW(J)
      VAL=VNEW
      CALL NEWDIR(N,DELTA,CTEST,PTNW,VAL)
      IFWRIT=1
      CALL FUNCT(PTNW,N,VAL)
      IFWRIT=0
      GO TO 20
    7 WRITE(6,18) VAL,(PTNW(L),L=1,N)
   18 FORMAT(' ',8G14.7/8G14.7)
      RETURN
      END
```

```
      SUBROUTINE NEWDIR(N,DELTA,CORD,PTNW,VAL)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION CORD(10),PTNW(10)
      REAL*8 MAX
      DO 2 L=1,N
    2 PTNW(L)=CORD(L)
      MAX=VAL
      DO 1 J=1,N
      TNPT=CORD(J)
      PTNW(J)=CORD(J)+DELTA
      CALL FUNCT(PTNW,N,VAL)
      IF(VAL.GE.MAX) GO TO 3
      MAX=VAL
      TNPT=PTNW(J)
    3 PTNW(J)=CORD(J)-DELTA
      CALL FUNCT(PTNW,N,VAL)
      IF(VAL.GE.MAX) GO TO 1
      MAX=VAL
      TNPT=PTNW(J)
    1 PTNW(J)=TNPT
      VAL=MAX
      RETURN
      END
```

```
      SUBROUTINE FUNCT(Z,NQR,VAB)
      IMPLICIT REAL*8(A-B,D-H,O-Z),COMPLEX*16(C)
      COMMON X(33),Y(33),NODE(56,3),NSEG,NRAD,SIGMM(56),BIGJ(56),MU(56)
      DIMENSION CSM(650),R(3),Q(3),T(3),RADI(1)
      COMPLEX*16 BIGJ,S(3,3),RHS,CV(33),VAL,DCMPLX,ALPHA
      COMPLEX*16 SJ(3),CK(100)
      DIMENSION Z(10)
      REAL*8 MU,CDABS
      REAL*8 EDCUR(3,3)
      COMPLEX*16 DP,DQ,DR,DS,CW(3),DCONJG,CRES(33)
      COMMON/IO/ IFWRIT
      DATA CV/33*(0.D0,0.D0)/
      DATA VAL/(0.D0,0.D0)/
      AREAL=Z(1)
      AIMAG=Z(2)
      SIGMA=5.80D7
      NPOT=1+(NRAD-1)*NSEG
      NPOT8=NPOT+8
      NLMNTS=NRAD*2*NSEG-NSEG
      NPOT1=NPOT+1
      NPOTSQ=NPOT*NPOT
      ALPHA=DCMPLX(AREAL,AIMAG)
      ATOT=0.0
      DO 7 I=1,NPOT
      DO 7 J=1,NPOT1
    7 CSM(I+NPOT*(J-1))=(0.D0,0.D0)
      DO 422 J=1,NPOT
  422 CK(J)=CV(J)-VAL
      DO 421 J=NPOT1,NPOT8
  421 CK(J)=ALPHA
      DO 100 N=1,NLMNTS
      XCENT=(X(NODE(N,1))+X(NODE(N,2))+X(NODE(N,3)))/3.D0
      YCENT=(Y(NODE(N,1))+Y(NODE(N,2))+Y(NODE(N,3)))/3.D0
      X1=X(NODE(N,1))-XCENT
      X2=X(NODE(N,2))-XCENT
      X3=X(NODE(N,3))-XCENT
      Y1=Y(NODE(N,1))-YCENT
```

```
      DO 8 I=1,3
      R(I)=(X(NODE(N,J))-X(NODE(N,K)))
      Q(I)=(Y(NODE(N,K))-Y(NODE(N,J)))
      T(I)=X(NODE(N,K))*Y(NODE(N,J))-X(NODE(N,J))*Y(NODE(N,K))
      J=J+1-J/3*3
    8 K=K+1-K/3*3
      AREA=   (X(NODE(N,1))*(Y(NODE(N,3))-Y(NODE(N,2)))
     .       + X(NODE(N,2))*(Y(NODE(N,1))-Y(NODE(N,3)))
     .       + X(NODE(N,3))*(Y(NODE(N,2))-Y(NODE(N,1))))*5.D-1
      DO 9 I=1,3
      DO99 J=1,3
      SREAL=(Q(I)*Q(J)+R(I)*R(J))/(AREA*4.DO)
      S(I,J)=DCMPLX(SREAL,0.D0)
      IF(SIGMM(N).EQ.0.D0) GO TO 98
      EDCUR(I,J)=(60.D0*3.141592653D0*MU(N)*SIGMA*(T(I)*T(J)+XCENT*(Q(I)
     .*T(J)+T(I)*Q(J))+YCENT*(R(I)*T(J)+T(I)*R(J))+(Q(I)*R(J)+R(I)*Q(J))
     .*(XCENT*YCENT+(X1*Y1+X2*Y2+X3*Y3)/12.D0)+Q(I)*Q(J)*(XCENT**2+(
     .X1**2+X2**2+X3**2)/12.D0)+R(I)*R(J)*(YCENT**2+(Y1**2+Y2**2+Y3**2)
     ./12.D0)) )/(AREA*2.D0)
   98 CONTINUE
   99 CONTINUE
    9 CONTINUE
      RHS=BIGJ(N)*AREA*MU(N)/3.D0
      DO 10 I=1,3
      SJ(I)=(0.D0,0.D0)
      IF(NODE(N,I).GT.NPOT)GO TO 10
      DO 11 J=1,3
      IF(SIGMM(N).NE.0.D0) SJ(I)=SJ(I)-DCMPLX(0.D0,EDCUR(I,J))*CK(NODE
     .(N,J))
      IF (NODE(N,J).GT.NPOT)GO TO211
      CSM(NODE(N,I)+(NODE(N,J)-1)*NPOT)=CSM(NODE(N,I)+(NODE(N,J)-1)*NPOT
     1)+S(I,J)
      GO TO 11
  211 CSM(NODE(N,I)+NPOTSQ)=CSM(NODE(N,I)+NPOTSQ)-S(I,J)*ALPHA
   11 CONTINUE
      CSM(NODE(N,I)+NPOTSQ)=CSM(NODE(N,I)+NPOTSQ)+RHS-SJ(I)
   10 CONTINUE
  100 CONTINUE
      CALL CSOLVE(CSM,CV,NPOT,CDET)




      VAL=(0.D0,0.D0)
      DO 18 N=1,NLMNTS
      AREA=   (X(NODE(N,1))*(Y(NODE(N,3))-Y(NODE(N,2)))
     .       + X(NODE(N,2))*(Y(NODE(N,1))-Y(NODE(N,3)))
     .       + X(NODE(N,3))*(Y(NODE(N,2))-Y(NODE(N,1))))*5.D-1
      IF(SIGMM(N).NE.0.D0) ATOT=ATOT+AREA
      DO 18 J=1,3
      IF(NODE(N,J).GT.NPOT) CV(NODE(N,J))=ALPHA
      IF(SIGMM(N).NE.0.D0) VAL=VAL+CV(NODE(N,J))*AREA/3.D0
   18 CONTINUE
      VAL=VAL/ATOT
      VAB=CDABS(VAL)
      WRITE(6,77) AREAL,AIMAG,VAL
   77 FORMAT(' ',10X,4G14.7)
      CVA=VAL
      IF(IFWRIT.EQ.0) GO TO 9999
      RNORM=0.D0
      VNORM=0.D0
      DO 14 I=1,NPOT
      CRES(I)=CSM(I+NPOTSQ)
      DO 13 J=1,NPOT
   13 CRES(I)=CRES(I)-CSM(I+(J-1)*NPOT)*CV(J)
      VNORM=VNORM+CDABS(CV(I)**2)
   14 RNORM=RNORM+CDABS(CRES(I)**2)
      RNORM=DSQRT(RNORM)
      VNORM=DSQRT(VNORM)
      WRITE(6,105) CVA
  105 FORMAT(//'    CVA=(',G14.7,',',G14.7,')',/
     1           ' NO.',12X,'RAW POTENTIAL',26X,'RESIDUAL',22X,'ADJUSTED P
     2OTENTIAL'/)
      DO 12 I=1,NPOT
      CVAD=CV(I)-CVA
   12 WRITE (6,103) I,CV(I),CRES(I),CVAD
  103 FORMAT(I4,': ',3('(',G14.7,' +J ',G14.7,')    '))
      PI=3.1415926
      DP=(0.D0,0.D0)
      DQ=(0.D0,0.D0)
```

```
      IF(SIGMM(N).EQ.0.D0) GO TO 400
      XCENT=(X(NODE(N,1))+X(NODE(N,2))+X(NODE(N,3)))/3.D0
      YCENT=(Y(NODE(N,1))+Y(NODE(N,2))+Y(NODE(N,3)))/3.D0
      X1=X(NODE(N,1))-XCENT
      X2=X(NODE(N,2))-XCENT
      X3=X(NODE(N,3))-XCENT
      Y1=Y(NODE(N,1))-YCENT
      Y2=Y(NODE(N,2))-YCENT
      Y3=Y(NODE(N,3))-YCENT
      CW(1)=CV(NODE(N,1))
      CW(2)=CV(NODE(N,2))
      CW(3)=CV(NODE(N,3))
      CTRA=(CW(1)-CVA)*(X(NODE(N,3))*Y(NODE(N,2))-X(NODE(N,2))*Y(NODE(N,
     .3)))+(CW(2)-CVA)*(X(NODE(N,1))*Y(NODE(N,3))-X(NODE(N,3))*Y(NODE(N,
     .1)))+(CW(3)-CVA)*(X(NODE(N,2))*Y(NODE(N,1))-X(NODE(N,1))*Y(NODE(N,
     .2)))
      CTRB=(CW(1)-CVA)*(Y(NODE(N,3))-Y(NODE(N,2)))+(CW(2)-CVA)*(Y(NODE(N
     .,1))-Y(NODE(N,3)))+(CW(3)-CVA)*(Y(NODE(N,2))-Y(NODE(N,1)))
      CTRC=(CW(1)-CVA)*(X(NODE(N,2))-X(NODE(N,3)))+(CW(2)-CVA)*(X(NODE(N
     .,3))-X(NODE(N,1)))+(CW(3)-CVA)*(X(NODE(N,1))-X(NODE(N,2)))
      AREA=    (X(NODE(N,1))*(Y(NODE(N,3))-Y(NODE(N,2)))
C             +X(NODE(N,2))*(Y(NODE(N,1))-Y(NODE(N,3)))
C             +X(NODE(N,3))*(Y(NODE(N,2))-Y(NODE(N,1))))*5.D-1
      VP=((CDABS(CTRB))**2*((X1**2+X2**2+X3**2)/12.D0+XCENT**2)+(CDABS(
     .CTRA))**2+(CDABS(CTRC))**2*(((Y1**2+Y2**2+Y3**2)/12.D0)+YCENT**2)
     .+     (CTRB*DCONJG(CTRC)+CTRC*DCONJG(CTRB))*(((X1*Y1+X2*Y2+X3*Y3)
     ./12.D0)+XCENT*YCENT)+CTRB*DCONJG(CTRA)*XCENT+CTRA*DCONJG(CTRB)*
     .XCENT+CTRA*DCONJG(CTRC)*YCENT+CTRC*DCONJG(CTRA)*YCENT)/(AREA*4.D0)
      DP=DP+(60.D0*60.D0*SIGMA*VP*4.D0*PI*PI)
      DQ=DQ+AREA/SIGMA*BIGJ(N)*DCONJG(BIGJ(N))
      DR=DR+40.D0*PI*AREA*(CW(1)+CW(2)+CW(3)-3.D0*CVA)*DCONJG(BIGJ(N))
      DS=DS+40.D0*PI*AREA*BIGJ(N)*DCONJG(CW(1)+CW(2)+CW(3)-3.D0*CVA)
      IF(SIGMM(N).EQ.0.D0) GO TO 400
  400 CONTINUE
      CPTOT=DP+DQ+DCMPLX(0.D0,1.D0)*(DS-DR)
      PDC=           (PI*RADI(1)*RADI(1)*(2.5401D-2**2))/SIGMA
      RACDC=(CPTOT+DCONJG(CPTOT))/(2.D0*PDC)
      WRITE(6,500) RACDC



      WRITE(6,503) DQ,DR,DS,CPTOT
  503 FORMAT(' ',8(G14.7))
      WRITE(6,501) DP,PDC
      WRITE(6,501) ATOT
  500 FORMAT(' ','RATIO OF AC TO DC RESISTANCE IS =',G14.7)
  501 FORMAT(' ',3(G14.7))
   15 CONTINUE
 9999 CONTINUE
      RETURN
      END
```

Three conductors in a steel pipe
using homogeneous Neumann boundary
condition with a complex formulation

```
C       FEPCAB
C       FINITE ELEMENT PATTERN GENERATOR FOR A CABLE CONFIGURATION
C       *      *      *                                  ***
C       THIS PROGRAM GENERATES  THE FINITE ELEMENT PATTERN FOR A MULTI-PHASE
C       ENCASED CABLE CONFIGURATION. SEVERAL CURRENT-CARRYING CONDUCTORS CAN
C       BE LOCATED ANYWHERE WITHIN AN OUTER CASING. A STANDARD FINITE ELEMEN
C       PATTERN IS GENERATED WITHIN EACH CONDUCTOR, AND THE USER CAN SPECIFY
C       THE NUMBER AND ARRANGEMENT OF ADDITIONAL ELEMENTS FOR COMPLETING THE
C       PATTERN WITHIN THE  REMAINDER OF THE REGION.
C
        IMPLICIT REAL*8(A-H,O-Z)
        REAL XT(6),YT(6),A
        DIMENSION IFADDX(4), IFADDY(4),SIGMM(400),ISEG(8),RXR(3),NSRG(9)
        DATA IFADDX/1,0,-1,0/,IFADDY/0,1,0,-1/
        DIMENSION XO(3),YO(3),RO(3),RADI(3),JEND(3),RADIUS(10),X(200),
       .Y(200),NODE(400,3)
        DIMENSICN DIVH(15),NC(3),DIVV(15),NBF(3),XB(3),YB(3),NBOUN(3,25),
       .NNN(5),LL(5),NT(50),NSEGM(3),NODES(3),NST(15),LROW(25),ANG(25)
C        CONDUCTOR CONDUCTIVITY
        SIGMA=5.80D7
C        STRANDING FACTOR
        SIGMA=SIGMA*.435
C        PIPE CONDUCTIVITY
        SIGMA1=8.19D6
C        READ SEGMENTATICN THICKNESS IN INCHES, AND 3 CONSTANTS WHICH ARE
C        4,6, AND 3 FOR SEGMENTED CABLES AND 0,0,0 FOR UNSEGM. CABLES
        READ(5,830) THIK,NSEGC,IFSEG,KSEG
        THIK2=THIK/2.D0
C        READ NUMBER OF CONDUCTORS AND CENTRE AND INSIDE RADIUS OF PIPE
        READ(5,700) NPHMAX,XORG,YORG,R0
C        READ OUTSIDE PIPE RADIUS AND NUMBER OF PIPE SEGMENTS
        READ(5,902) ROUT,NSOUT
        WRITE(6,43) NSOUT,RO,ROUT
   43   FORMAT(' ',I3,'INSIDE RAD.',G14.7,10X,'OUTSIDE RAD.',G14.7)
        PI=3.14159265359D0
        JSTART=0
        J0=1
        NDNO=1
        NDINIT=NDNO
        NPHASE=0
        DO 1 L=1,NPHMAX
        NPHASE=NPHASE+1
C        READ NUMBER OF RADIAL DIVISIONS, SEGMENTS, AND CONDUCTING
C        RADIAL  DIVISIONS FOR EACH CONDUCTOR
        READ(5,701)NRAD,NSEGM(L),NCON
        NCM1=NCON-1
        NSEG=NSEGM(L)
        NTR=NSEG+NSEGC
        JMAX=NSEG
        DTHETA=2.*PI/NSEG
C        READ RADIAL BREAK-POINTS AS PERCENTAGE OF CONDUCTOR RADIUS
        READ(5,702)(RADIUS(K),K=1,NRAD)
C        READ POSITION AND RADIUS OF CONDUCTOR
        READ(5,703) XO(L),YO(L),RO(L)
        RADI(L)=RADIUS(NCON)*RO(L)
        XO=XO(L)
        YO=YO(L)
        RQ=RO(L)
        IF(NSEGC.NE.0) GC TO 650
C        CALCULATE NODES FOR UNSEGMENTED CONDUCTOR
C
C       CALCULATE POSITICN OF NODES FOR THIS CONDUCTOR
C
        JMAX2=J0+NSEG-1
        X(NDNO)=XO
        Y(NDNO)=YO
        NDNO=NDNO+1
        DO 25 I=1,NRAD
        THETA=0
        DO 25 J=1,NSEG
        X(NDNO)=RQ*RADIUS(I)*DCOS(THETA)+XO(L)
        Y(NDNO)=RQ*RADIUS(I)*DSIN(THETA)+YO(L)
        NDNO=NDNO+1
   25   THETA=THETA+DTHETA
        NODES(L)=NDNO-1
C
C       CALCULATE NODE ORDERING FOR EACH ELEMENT FOR THIS CONDUCTOR
C
```

```
C           INNERMOST CORE OF ELEMENTS
      DO 30 J=J0,JMAX2
      SIGMM(J)=SIGMA
      NODE(J,1)=NDINIT
      NODE(J,2)=J-J0+1+NDINIT+1
      IF(J.EQ.JMAX2) NODE(J,2)=NDINIT+1
   30 NODE(J,3)=J-J0+1+NDINIT
      GO TO 651
  650 CONTINUE
C CALCULATE NODES FOR SEGMENTED CONDUCTOR
C     INNER RING
      DO111 J=1,4
      X(NDNO)=X0(L)+(-1)**((J+4)/2)*THIK2
      Y(NDNO)=Y0(L)+(-1)**((J+3)/2)*THIK2
  111 NDNO=NDNO+1
      DO72 J=1,4
      ISEG(2*J-1)=NDNO-6+J
      IF(ISEG(2*J-1).LT.NDINIT) ISEG(2*J-1)=NDINIT+3
   72 ISEG(2*J)=NDNO-5+J
      DTHETA=PI/NSEG*2.D0
      THETA=DTHETA
      NPTS=NSEG/4
C      CALCULATE NODE POSITIONS FOR CONDUCTING RINGS
      DO 123 M=1,NCM1
      ADD=RO(L)*RADIUS(M)
      DO 124 J=1,4
      X(NDNO)=X(ISEG(2*J-1))+ADD*IFADDX(J)
      Y(NDNO)=Y(ISEG(2*J-1))+ADD*IFADDY(J)
      X(NDNO+1)=X(ISEG(2*J))+ADD*IFADDX(J)
      Y(NDNO+1)=Y(ISEG(2*J))+ADD*IFADDY(J)
      NDNO=NDNO+2
      NPTSM=NPTS-1
      DO 125 K=1,NPTSM
      X(NDNO)=        RO(L)*DCOS(THETA)*RADIUS(M)+X(ISEG(2*J))
      Y(NDNO)=       +RO(L)*DSIN(THETA)*RADIUS(M)+Y(ISEG(2*J))
      THETA=THETA+DTHETA
  125 NDNO=NDNO+1
      THETA=THETA+DTHETA
  124 CONTINUE
  123 CONTINUE
      THETA=0.D0
      J=NCON
C      CALCULATE NODE POSITIONS FOR OUTER RING
      DO 375 K=1,NSEG
      Y(NDNO)=Y0(L)+RO(L)*RADIUS(J)*DSIN(THETA)
      X(NDNO)=X0(L)+RO(L)*RADIUS(J)*DCOS(THETA)
      THETA=THETA+DTHETA
  375 NDNO=NDNO+1
  782 CONTINUE
      NODES(L)=NDNO-1
      NALL=NDNO-1
  830 FORMAT(F10.5,3(I2))
C FORM TRIANGLES FOR INNER RADIUS
      DO 83 J=1,2
      NODE(J0,1)=NDINIT
      NODE(J0,2)=J+NDINIT+1
      NODE(J0,3)=J+NDINIT
      SIGMM(J0)=0.D0
   83 J0=J0+1
      NDNOW=NDINIT+4
      DO 41 J=1,NSEGC
      NODE(J0,1)=NDNOW
      NODE(J0,2)=NDINIT+J-2
      NODE(J0,3)=NDINIT+J-1
      SIGMM(J0)=0.D0
      IF(NODE(J0,2).LT.NDINIT) NODE(J0,2)=NDINIT+3
      J0=J0+1
      NUM=NPTS+1
      DO 51 K=1,NUM
      SIGMM(J0)=SIGMA
      IF(K.EQ.1) SIGMM(J0)=0.D0
      NODE(J0,1)=NDINIT+J-1
      NODE(J0,2)=NDNOW+1
      IF((K.EQ.NUM).AND.(J.EQ.NSEGC)) NODE(J0,2)=NDINIT+4
      NODE(J0,3)=NDNOW
      NDNOW=NDNOW+1
   51 J0=J0+1
   41 CONTINUE
```

```
  651 CONTINUE
C
C              EVEN - NUMBERED RINGS (2,4,6,...)
C
      IF(NSEGC.NE.0) NSE4=NSEG/NSEGC
      NTF=NSEG+NSEGC
      K1=(NTR*(2*NCON-1)+IFSEG-NSEGC)*(NPHASE-1)+NTR+1+IFSEG
      K2=1+NDINIT+KSEG
      DO 40 I=2,NRAD,2
       IF((I.GT.NCM1).AND.(NSEGC.NE.0)) GO TO1370
      DO 35 J=1,NTR
      NODE(K1,1)=K2
      NODE(K1,2)=K2+NSEG+1+NSEGC
      NODE(K1,3)=K2+NSEG+NSEGC
      K1=K1+1
      NODE(K1,1)=K2
      K2=K2+1
      NODE(K1,2)=K2
      NODE(K1,3)=K2+NSEG+NSEGC
   35 K1=K1+1
      NODE(K1-1,2)=K2-NSEG-NSEGC
      NODE(K1-1,3)=K2
      NODE(K1-2,2)=K2
      K1=K1+2*NSEG+2*NSEGC
   40 K2=K2+NSEG+NSEGC
C
C              ODD - NUMBERED RINGS (3,5,7,...)
C
 1370 K1=3*NTR+IFSEG+JSTART+1
      K2=NSEG+1+NDINIT+NSEGC +KSEG
      DO 60 I=3,NRAD,2
       IF((I.GT.NCM1).AND.(NSEGC.NE.0)) GO TO 370
      DO 55 J=1,NTR
      NODE(K1,1)=K2
      K2=K2+1
      NODE(K1,2)=K2
      NODE(K1,3)=K2+NSEG-1 +NSEGC
      K1=K1+1
      NODE(K1,1)=K2
      NODE(K1,2)=K2+NSEG+NSEGC
      NODE(K1,3)=K2+NSEG-1+NSEGC
   55 K1=K1+1
      NODE(K1-1,1)=K2-NSEG-NSEGC
      NODE(K1-1,2)=K2
      NODE(K1-2,2)=K2-NSEG-NSEGC
      K1=K1+2*NSEG+2*NSEGC
   60 K2=K2+NSEG+NSEGC
      IF(NSEGC.NE.0) GC TO 370
      GO TO 65
  370 J0=K1
C
C      FORM TRIANGLES FOR OUTER RING
C
      K1=K2
      KODREV=NCON/2
      KODREV=NCON-2*KODREV
      IF(KODREV.NE.0) K1=K1+NTR
      IF(KODREV.EQ.0) J0=J0-2*NTR
      LST=K1-NTR
      DO 371 K=1,4
      NODE(J0,1)=LST
      NODE(J0,2)=LST+1
      NODE(J0,3)=K1
      J0=J0+1
      LST=LST+1
      DO 372 J=1,NSE4
      NODE(J0,1)=LST
      NODE(J0,2)=LST+1
      NODE(J0,3)=K1
      J0=J0+1
      NODE(J0,1)=K1
      NODE(J0,2)=LST+1
      NODE(J0,3)=K1+1
      J0=J0+1
      LST=LST+1
  372 K1=K1+1
  371 CONTINUE
      NODE(J0-2,2)=K1-NSEG-NSEGC-NSEG
```

```
      NODE(JO-1,2)=K1-NSEG-NSEGC-NSEG
      NODE(JO-1,3)=K1-NSEG
      NSTT=NCON+2
   65 NTR2=NTR*2
      JNOW=JSTART+1+IFSEG+NTR
      ICOUN=JNOW
      SIG=SIGMA
      DO 63 JJ=2,NRAD
      IF(((JJ-1).EQ.NCM1).AND.(NSEGC.NE.0)) NTR2=NTR2-4
      IF(((JJ-2).EQ.NCM1).AND.(NSEGC.NE.0)) NTR2=NTR2-4
      IF(JJ.GT.NCON) SIG=0.D0
      DO 64 KK=1,NTR2
C      COMPUTE CONDUCTIVITY
      JARG=JNOW+(JJ-2)*NTR2+KK-1
      IF(JJ.GT.NCM1+0) JARG=JARG+NSEGC*NCM1-NSEGC
      IF(JJ.GT.NCM1+1) JARG=JARG+NSEGC*NCM1+NSEGC-NSEGC
   64 SIGMM(JARG)=SIG
      IF((NSEGC.EQ.0).OR.(JJ.GT.NCON))GO TO 63
      DO 62 ML=1,NSEGC
      SIGMM(ICOUN)=0.D0
      IF(JJ.NE.NCON) GO TO 66
      ICOUN=ICOUN-1
      GO TO 62
   66 SIGMM(ICOUN+1)=0.D0
   62 ICOUN=ICOUN+NSE4*2+2
   63 CONTINUE
      JSTART=JSTART+NTR*(NCON*2-1)-NSEGC +IFSEG
      JEND(L)=JSTART
C
C      DRAW ALL ELEMENTS
C
      JO=JSTART+1
      NDINIT=(NTR*NCON+1+KSEG-NSEGC)*NPHASE+1
    1 CONTINUE
  700 FORMAT(I2,3(F5.3))
  701 FORMAT(3(I2))
  702 FORMAT(10(F5.3))
  703 FORMAT(3(F7.4))
C     SELECT GRID SIZE
C      READ NUMBER OF HORIZ. AND VERT. GRID LINES
      READ(5,701) NHORZ,NVERT
C      READ POSITIONS OF GRID LINES AS +/- FRACTIONS OF INNER PIPE RAD.
      READ(5,705)(DIVV(J),J=1,NVERT)
      READ(5,705)(DIVH(J),J=1,NHORZ)
C
C  ***NOTE*** A HORIZONTAL GRID LINE SHOULD NOT PASS EXACTLY THROUGH
C      ZERO. GRID LINES SHOULD BE PLACED SO THAT AT LEAST ONE LINE
C      SEPARATES CONDUCTORS
C
      WRITE(6,800)
  800 FORMAT(' ','VERTICAL AND HORIZONTAL GRID POSITIONS')
      WRITE(6,46)(DIVV(J),J=1,NVERT)
      WRITE(6,46)( DIVH(J),J=1,NHORZ)
   46 FORMAT(' ',12F10.5)
C     INITIALIZE BOUNDARY COUNTERS AND FLAGS
      DO 207 J=1,NPHASE
      NC(J)=1
  207 NBF(J)=1
      NTH=1
C     NUMBER THE NODES
      DO 202 J=1,NHORZ
      NTV=1
      IFLAG=0
      DO 203 K=1,NVERT
      X(NDNO)=XORG+RO*DIVV(K)
      Y(NDNO)=YORG+RO*DIVH(J)
C     CHECK IF NODE IS IN SHEATH CIRCLE
      IF(DSQRT((X(NDNO)-XORG)**2+(Y(NDNO)-YORG)**2).LE.RO) GO TO 999
C     CHECK IF NODE POSITION SHOULD BE MODIFIED
      IF(X(NDNO).GE.0) NTV=-1
      KK=K+NTV*1
      JJ=J+NTH*1
      XS=XORG+RO*DIVV(KK)
      YS=YORG+RO*DIVH(JJ)
      IF(DSQRT((X(NDNO)-XORG)**2+(YS-YORG)**2).LT.RO) GO TO 204
      IF(DSQRT((Y(NDNO)-YORG)**2+(XS-XORG)**2).LT.RO) GO TO 205
C     DO NOT ASSIGN A NUMBER TO THIS NODE
      GO TO 203
```

```
C     MODIFY VALUE OF X FOR THIS NODE
  205 X(NDNO)=DSQRT(R0**2-Y(NDNO)**2)
      IF(DIVV(K).LT.0) X(NDNO)=-X(NDNO)
      GO TO 999
C     MODIFY VALUE OF Y FOR NODE
  204 Y(NDNO)=DSQRT(R0**2-X(NDNO)**2)
      IF(DIVH(J).LT.0) Y(NDNO)=-Y(NDNO)
C TEST IF NODE FALLS WITHIN CONDUCTOR RADIUS
  999 DO 106 L=1,NPHASE
      IF(DSQRT((X(NDNO)-XO(L))**2+(Y(NDNO)-YO(L))**2).LE. RO(L)) GO TO
     . 203
  106 CONTINUE
C STORE FIRST NODE OF EACH GRID ROW
      IF(IFLAG.EQ.0) NST(J)=NDNO
      IFLAG=1
C TEST IF NODE IS A CONDUCTOR BOUNDARY NODE
      JP=J+1
      JM=J-1
      KM=K-1
      KP=K+1
      IF(K.NE.1)XB(1)=XORG+R0*DIVV(KM)
      IF(K.NE.NVERT)XB(2)=XORG+R0*DIVV(KP)
      IF(J.NE.1) YB(1)=YORG+R0*DIVH(JM)
      IF(J.NE.NHORZ) YB(2)=YORG+R0*DIVH(JP)
      XB(3)=X(NDNO)
      YB(3)=Y(NDNO)
      DO 117 L=1,NPHASE
      DO 108 JJ=1,3
      DO 108 KK=1,3
      IF(((JJ.EQ.1).AND.(JM.EQ.0)).OR.((JJ.EQ.3).AND.(JP.GT.NHORZ)).OR.
     .((KK.EQ.1).AND.(KM.EQ.0)).OR.((KK.EQ.3).AND.(KP.GT.NVERT))) GO TO
     . 108
      IF(DSQRT((XB(KK)-XO(L))**2+(YB(JJ)-YO(L))**2).LT.RO(L)*.9999D0)
     .GO TO 209
      IF(((JJ.EQ.3).OR.(KK.EQ.3)).AND.(DSQRT((XB(KK)-XO(L))**2+(YB(JJ)
     .-YO(L))**2).LE.RO(L))) GO TO 209
  108 CONTINUE
      GO TO 117
  209 NBOUN(L,NC(L))=NDNO
      NC(L)=NC(L)+1
      IF((X(NDNO).GT.XO(L)).AND.(Y(NDNO).LE.YO(L)+.0001).AND.(NBF(L).EQ.
     .1)) GO TO1211
      GO TO 117
 1211 NBF(L)=0
      LL(L)=NC(L)-1
      NNN(L)=NDNO
  117 CONTINUE
      NDNO=NDNO+1
  203 CONTINUE
      IF(J.NE.NHORZ) YNEXT=YORG+R0*DIVH(J+1)
      IF(YNEXT.LE.YORG)NTH=-1
  202 CONTINUE
      NFIN=NDNO
C ORDER THE BOUNDARY NODES
      DO 210 L=1,NPHASE
      NC(L)=NC(L)-1
      NCCL=NC(L)
      DO 231 J=1,NCCL
      IF(X(NBOUN(L,J)).NE.XO(L)) GO TO 232
      ANG(J)=PI/2.
      IF(Y(NBOUN(L,J)).LT.YO(L))  ANG(J)=-ANG(J)
      GO TO 233
  232 ANG(J)=DATAN((Y(NBOUN(L,J))-YO(L))/(X(NBOUN(L,J))-XO(L)))
  233 CONTINUE
      IF(X(NBOUN(L,J)).LT.XO(L)) ANG(J)=PI+ANG(J)
      IF((X(NBOUN(L,J)).GE.XO(L)).AND.(Y(NBOUN(L,J)).LT.YO(L))) ANG(J)=
     .2.*PI+ANG(J)
  231 CONTINUE
      CALL ORDER(NCCL,ANG,NT)
      DO 240 J=1,NCCL
  240 LROW(J)=NBOUN(L,J)
      DO 241 J=1,NCCL
      K=NCCL-J+1
  241 NBOUN(L,K)=LROW(NT(J))
  210 CONTINUE
C FORM TRIANGLES AROUND CONDUCTORS
      DO 301 L=1,NPHASE
      NTRIA=NC(L)+NSEGM(L)
```

```
      NBC=1
      NR=NODES(L)-NSEGM(L)+1
      NODES(L)=NR
      J01=J0
      DO 300 J=1,NTRIA
      NRP1=NR+1
      NBCP1=NBC+1
      IF(NRP1.GT.(NODES(L)+NSEGM(L)-1))NRP1=NODES(L)
      IF(NBCP1.GT.NC(L))NBCP1=1
C
C       CALCULATE TEST DISTANCES
C
      D1=DSQRT((X(NR)-X(NBOUN(L,NBCP1)))**2+(Y(NR)-Y(NBOUN(L,NBCP1)))**
     .2)
      D2=DSQRT((X(NRP1)-X(NBOUN(L,NBC)))**2+(Y(NRP1)-Y(NBOUN(L,NBC)))**
     .2)
      D3=DSQRT((X(NR)-X(NBOUN(L,NBC)))**2+(Y(NR)-Y(NBOUN(L,NBC)))**2)
      D4=DSQRT((X(NRP1)-X(NBOUN(L,NBCP1)))**2+(Y(NRP1)-Y(NBOUN(L,NBCP1))
     .)**2)
      D5=DSQRT((X(NR)-X(NRP1))**2+(Y(NR)-Y(NRP1))**2)
C
C       TEST FOR SHORTEST DISTANCES
C
      IF(D1.LE.D2) GO TO 302
      IF((D2/D3).GE.((D3+RO(L)*DTHETA*.8)/D3)) GO TO 302
  305 IF(D2/(D3+D5).GE..99) GO TO 306
      XTEST=X(NRP1)-.25D0*(X(NRP1)-X(NBOUN(L,NBC)))
      YTEST=Y(NRP1)-.25D0*(Y(NRP1)-Y(NBOUN(L,NBC)))
      RTEST=DSQRT((XTEST-XO(L))**2+(YTEST-YO(L))**2)
C
C       CHECK FOR IMPROPER PLACEMENT
      IF(RTEST.LE.RO(L)) GO TO 306
  303 NODE(J0,1)=NR
      NODE(J0,3)=NBOUN(L,NBC)
      NR=NR+1
      IF(NR.GT.NODES(L)+NSEGM(L)-1) NR=NODES(L)
      NODE(J0,2)=NR
      IF((J.EQ.NTRIA).AND.(NBC.NE.1))GO TO 710
      GO TO 300
  710 NBC=NBC+1
      IF(NBC.GT.NC(L)) NBC=1
      NODE(J0,2)=NBOUN(L,NBC)
      GO TO 300
  302 IF((D1/D4).GE.((D4+RO(L)*DTHETA*.8)/D4)) GO TO 305
  306 NODE(J0,1)=NR
      NODE(J0,3)=NBOUN(L,NBC)
      NBC=NBC+1
      IF(NBC.GT.NC(L)) NBC=1
      NODE(J0,2)=NBOUN(L,NBC)
  300 J0=J0+1
C     DRAW TRIANGLES
      JFIN=J01+NTRIA-1
      DO 301 J=J01,JFIN
      SIGMM(J)=0.0D0
  301 CONTINUE
  705 FORMAT(15(F4.2))
  902 FORMAT(F5.2,I3)
C FORM TRIANGLES INSIDE STEEL PIPE
      J01=J0
      NHMI=NHCRZ-1
      DO 400 J=1,NHMI
C       INITIALIZE FLAGS AND COUNTERS
      ITEST=(-1)**J
      K=J
      NCOWN=0
      NCCUN=0
      F1=1
      F2=1
      F3=1
      IF((Y(NST(J)).GE.YORG).AND.(Y(NST(J+1)).LT.YORG)) NSTAR=J+1
      IF(J.EQ.1)GO TO 410
      IF(J.EQ.NHMI) GO TO 411
  401 IF((NST(J+1)+NCCUN+1).GE.NFIN) GO TO 400
      F1=1
      IF(X(NST(J)+NCOWN+1).LE.X(NST(J)+NCOWN))F2=0
      IF(X(NST(J+1)+NCCUN+1).LE.X(NST(J+1)+NCCUN))GO TO 412
C       CHECK IF TRIANGLES ALREADY FORMED IN THIS REGION
      DO 402 L=1,NPHASE
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      IF((Y(NST(J+1)).LE.(YO(L)+RO(L))).AND.((X(NST(J+1)+NCOUN).LE.XO(L
     *)).AND.(X(NST(J+1)+NCOUN+1).GE.XO(L))).AND.(Y(NST(J+1)+NCOUN).GE.
     *(YO(L)-RO(L)))) GO TO 403
      IF((Y(NST(J)).GE.(YO(L)-RO(L))).AND.((X(NST(J)+NCOWN).LE.XO(L)).
     *AND.(X(NST(J)+NCOWN+1).GE.XO(L))).AND.(Y(NST(J)).LE.(YO(L)+RO(L))
     *)) GO TO 406
  402 CONTINUE
      F1=0
C      CHECK GRID DIRECTION
      IF(ITEST.EQ.-1)GO  TO 404
      GO TO 412
C      FORM FIRST TRIANGLE OF FIRST ROW
  410 NODE(J0,1)=NST(J)
      NODE(J0,2)=NST(J+1)+1
      NODE(J0,3)=NST(J+1)
      J0=J0+1
      NCOUN=1
      GO TO 401
C      FORM FIRST TRIANGLE OF LAST ROW
  411 NODE(J0,1)=NST(J)
      NODE(J0,2)=NST(J)+1
      NODE(J0,3)=NST(J+1)
      NCOWN=1
      J0=J0+1
      GO TO 401
C      ADJUST COUNTERS FOR GRID NEAR CONDUCTORS
  403 NCOUN=NCCUN+1
  407 NCOWN=NCOWN+1
      IF(X(NST(J)+NCOWN).GE.(XO(L)+RO(L))) GO TO 401
      IF(X(NST(J+1)+NCOUN).LT.(XO(L)+RO(L))) GO TO 403
      GO TO 407
  406 NCOWN=NCOWN+1
  408 NCOUN=NCOUN+1
      IF(X(NST(J+1)+NCOUN).GE.(XO(L)+RO(L)))GO TO 401
      IF(X(NST(J)+NCOWN).LT.(XO(L)+RO(L))) GO TO 406
      GO TO 408
C      FORM REGULAR TRIANGULAR GRID
  404 NODE(J0,1)=NST(J)+NCOWN
      NODE(J0,2)=NST(J+1)+NCOUN+1
      NODE(J0,3)=NST(J+1)+NCOUN
      IF(F3.EQ.1) LROW(J)=J0
      F3=0
      J0=J0+1
  405 IF(F2.EQ.0) GO TO 400
      NODE(J0,1)=NST(J)+NCOWN
      NODE(J0,2)=NST(J)+NCOWN+1
      NODE(J0,3)=NST(J+1)+NCOUN+1
      IF(F3.EQ.1) LROW(J)=J0
      F3=0
      J0=J0+1
      NCOWN=NCOWN+1
      NCOUN=NCOUN+1
      GO TO 401
C      FORM REGULAR TRIANGULAR GRID
  412 IF(F2.EQ.0) GO TO 400
      NODE(J0,1)=NST(J)+NCOWN
      NODE(J0,2)=NST(J)+NCOWN+1
      NODE(J0,3)=NST(J+1)+NCOUN
      J0=J0+1
      IF(F3.EQ.1) LROW(J)=J0
      F3=0
      IF(F1.EQ.1) GO TO 400
      NODE(J0,1)=NST(J)+NCOWN+1
      NODE(J0,2)=NST(J+1)+NCOUN+1
      NODE(J0,3)=NST(J+1)+NCOUN
      J0=J0+1
      NCOWN=NCOWN+1
      NCOUN=NCOUN+1
      IF(F3.EQ.1) LROW(J)=J0
      F3=0
      GO TO 401
  400 CONTINUE
      NODE(J),1)=NST(K)+NCOWN
      NODE(J0,2)=NST(K)+NCOWN+1
      NODE(J0,3)=NST(K+1)+NCOUN
C      SPECIFY CONDUCTIVITY
      DO 747 J=J01,J0
  747 SIGMM(J)=0.000
```

```
      J0=J0+1
      J01=J0
      NBEG=NDNC
      THETA=0.D0
      DTHETA=2.D0*PI/NSOUT
C      FORM INNER PIPE TRIANGLES
      DO 903 I=1,NSOUT
      X(NDNO)=ROUT*DCOS(THETA)
      Y(NDNO)=ROUT*DSIN(THETA)
      NDNO=NDNO+1
  903 THETA=THETA+DTHETA
      NM=1
      NN=2
      I=1
      J=NSTAR
C ORDER NODE NUMBERS OF INNER PIPE BOUNDARY
  900 NT(I)=NST(J)-1
      I=I+1
      J=J-1
      IF(J.NE.1) GO TO 900
  901 NT(I)=NST(2)-NN
      NN=NN+1
      I=I+1
      IF(NT(I-1).NE.NST(1)) GO TO 901
  904 NT(I)=NST(J+1)
      I=I+1
      J=J+1
      IF(J.NE.NHORZ) GO TO 904
  905 NT(I)=NST(NHORZ)+NM
      I=I+1
      NM=NM+1
      IF(NT(I-1).NE.NFIN-1) GO TO 905
  906 NT(I)=NST(J)-1
      I=I+1
      J=J-1
      IF(J.NE.NSTAR) GO TO 906
      NPTS=I-1
      NTPIA=NPTS+NSOUT
      NBC=1
      NR=NBEG
C      FORM PIPE TRIANGLES
      DO 910 J=1,NTRIA
      NRP1=NR+1
      NBCP1=NBC+1
      IF(NRP1.GT.(NBEG+NSOUT-1)) NRP1=NBEG
      IF(NBCP1.GT.NPTS) NBCP1=1
      D1=DSQRT((X(NR)-X(NT(NBCP1)))**2+(Y(NR)-Y(NT(NBCP1)))**2)
      D2=DSQRT((X(NRP1)-X(NT(NBC)))**2+(Y(NRP1)-Y(NT(NBC)))**2)
      IF(D1.LT.D2) GO TO 922
      NODE(J0,1)=NR
      NODE(J0,2)=NT(NBC)
      NR=NR+1
      IF(NR.GT.NBEG+NSOUT-1) NR=NBEG
      NODE(J0,3)=NR
      GO TO 910
  922 NODE(J0,1)=NR
      NODE(J0,2)=NT(NBC)
      NBC=NBC+1
      IF(NBC.GT.NPTS) NBC=1
      NODE(J0,3)=NT(NBC)
  910 J0=J0+1
C      READ NUMBER OF EXTRA RINGS, NUMBER OF RING SEGMENTS, AND RING
C      RADII IN INCHES
      READ(5,883) NXRNG,(NSRG(J),J=1,9     ),(RXR(J),J=1,NXRNG)
      WRITE(6,801)
  801 FORMAT(' ','EXTRA RING RADII')
      WRITE(6,44)(RXR(J),J=1,NXRNG)
   44 FORMAT(' ',8G14.7)
  883 FORMAT(10I2,5F10.5)
C      FORM EXTRA RINGS
      DO 881 J=1,NXRNG
      NNRG=NSRG(J)
      THETA=0.0
      DTHETA=2.D0*PI/NNRG
      NXBEG=NDNO
      DO 880 K=1,NNRG
      X(NDNO)=RXR(J)*DCOS(THETA)
      Y(NDNO)=RXR(J)*DSIN(THETA)
```

```
      NODE(J0,1)=NBEG+K-1
      NODE(J0,2)=NBEG+K
      NODE(J0,3)=NDNO+1
      J0=J0+1
      NODE(J0,1)=NDNO
      NODE(J0,2)=NBEG+K-1
      NODE(J0,3)=NDNO+1
      J0=J0+1
      NDNO=NDNO+1
  880 THETA=THETA+DTHETA
      NODE(J0-1,3)=NDNO-NNRG
      NODE(J0-2,2)=NBEG
      NBEG=NXBEG
  881 NODE(J0-2,3)=NDNO-NNRG
      NDTOT=NDNO-1
      JFIN=J0-1
      DO 748 J=J01,JFIN
  748 SIGMM(J)=SIGMA1
C       CONVERT INCHES TO METERS
      DO 277 I=1,NDTOT
      X(I)=X(I)*2.5401D-2
  277 Y(I)=Y(I)*2.5401D-2
C WRITE DATA ON TAPE
      REWIND2
      WRITE(2)NDTOT,JFIN,NSCUT,NPHMAX,NTRIA,(NODES(L),L=1,3),(NSEGM(L),
     .L=1,3),(JEND(L),L=1,3),(RADI(L),L=1,NPHMAX),(XO(L),L=1,3),(YO(L),
     .L=1,3)
      WRITE(2)((NODE(J,L),L=1,3),J=1,JFIN),(SIGMM(L),L=1,JFIN)
      WRITE(2)(X(I),I=1,NDTOT)
      WRITE(2)(Y(I),I=1,NDTOT)
      WRITE(2) NXRNG,(NSRG(J),J=1,NXRNG)
      ENDFILE2
      STOP
      END
```

21 MAIN DATE = 76304 05/43/38

```
*  ID,EBCDIC,SOURCE,LIST,NODECK,LOAD,NOMAP
*  NAME = MAIN     , LINECNT =       80
URCE STATEMENTS =       546,PROGRAM SIZE =     29342
IAGNOSTICS GENERATED
```

```
      SUBROUTINE ORDER(NCCL,ANG,NT)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ANG(25),NT(50)
C
C      THIS SUBROUTINE ORDERS GRID NODES WHICH BOUND CONDUCTORS
C
      DO 2 K=1,NCCL
      AMAX=-1.
      DO 1 J=1,NCCL
      IF(ANG(J).LT.AMAX) GO TO 1
      AMAX=ANG(J)
      NT(K)=J
    1 CONTINUE
      ANG(NT(K))=-2.
    2 CONTINUE
      RETURN
      END
```

```
 16INSIDE RAD.   5.120000                       OUTSIDE RAD.   5.300000
VERTICAL AND HORIZONTAL GRID POSITIONS
  -1.00000  -0.55000  -0.30000   0.0        0.30000   0.55000   1.00000
   1.00000   0.50000   0.05000  -0.30000  -0.60000  -0.75000  -1.00000
EXTRA RING RADII
  5.350000     5.400000
```

```
      IMPLICIT REAL*8(A-H,O-Z)
C (I) READ IN SPECIFICATIONS OF NODE PATTERN:
      DIMENSION RADIUS(16),X(200),Y(200),NODE(400,3),NODES(3),NSEGM(3),
     CRADI(3)    ,             JEND(3), Q(3),R(3),T(3),  NLCON(3),NCOD(3)
      REAL*8 DSQRT,DLOG,DSIN,DCOS,RPT,THETA,DTHETA,ATOT(4)
      REAL MU(400),MU1,MUO,VAL,XT(3),YT(3)
      DIMENSION XO(3),YO(3),SIGMM(400)
      DIMENSION NSRG(3)
      COMPLEX CV(200),CSM(40000),CRES(200),CVAD,CVOLD(200),CW(3),
     .BIGJ(400),BIGJ1,BIGJ2,RHS,CBOUN(32),CONJG,CDET,CT(3)
      COMPLEX*16 CTRA,CTRB,CTRC,C,DCONJG
      COMPLEX*16 DCMPLX,CPTOT,S(3,3),       DP,DQ,DR,DS,VP,CVA(4)
C       READ DATA FROM TAPES
      REWIND2
      READ(2) NDTOT,JFIN,NSOUT,NPHMAX,NTRIA,(NODES(L),L=1,3),(NSEGM(L),
     .L=1,3),(JEND(L),L=1,3) ,(RADI(L),L=1,3),(XO(L),L=1,3),(YO(L),L=1,3
     .)
      READ(2) ((NODE(J,L),L=1,3),J=1,JFIN),(SIGMM(J),J=1,JFIN)
      READ(2) (X(I),I=1,NDTOT)
      READ(2) (Y(I),I=1,NDTOT)
      READ(2) NXRNG,(NSRG(J),J=1,NXRNG)
      WRITE(6,444)NDTOT,JFIN,NSOUT,NPHMAX,NTRIA,(NODES(L),L=1,NPHMAX),
     .(NSEGM(L),L=1,NPHMAX),(JEND(L),L=1,3),(RADI(L),L=1,3)
  444 FORMAT(' ',14(I3),3(F7.4))
      PI=3.14159265359D0
      NBEG=1
      MUO=.000001257D0
C       SPECIFY BOUNDARY RADIUS
      RPT=15.D0*2.5401D-2
      DTHETA=2.D0*PI/NSOUT
      THETA=0.D0
      NUMOUT=NDTOT-NSOUT
C CALCULATE APPROXIMATE BOUNDARY POTENTIAL FOR USE IF DESIRED
      DO749 J=1,NSOUT
      CBOUN(J)=-MUO/(2.D0*PI)*(DLOG(DSQRT((RPT*DCOS(THETA))**2+(RPT*
     .DSIN(THETA)-.0587D0)**2))+DCMPLX(-.5D0,.866D0)*DLOG(DSQRT((RPT*
     .DCOS(THETA)-.0499D0)**2+(RPT*DSIN(THETA)+.0293D0)**2))+
     .DCMPLX(-.5D0,-.866D0)*DLOG (DSQRT((RPT*DCOS(THETA)+.0499D0)**2
     .+(RPT*DSIN(THETA)+.0293D0)**2)))
      CBOUN(J)=CBOUN(J)*800.
  749 THETA=THETA+DTHETA
      DO 777 L=1,NPHMAX
C       READ NUMBER OF COND. RINGS,PERMEABILITY,AND COND. AND INSULATION
C       CURRENT
      READ(5,22)NCON,MU1,BIGJ1,BIGJ2
      BIGJ1=BIGJ1*800./(PI*(RADI(L)*.025401)**2)
      NCOD(L)=NCON
   22 FORMAT(I2,F10.9,4G10.5)
      WRITE(6,33)NCON,MU1,BIGJ1,BIGJ2
   33 FORMAT(15X,'NUMBER OF RINGS WITHIN THE CONDUCTOR:',I5/5X,
     1'MAGNETIC PERMEABILITY:',G12.5/5X,'CURRENT DENSITY:'/10X,'WITHIN T
     2HE CONDUCTOR:',2(G12.5)/10X,'WITHIN THE INSULATION:',2(G12.5)/)
      WRITE(6,34) XO(L),YO(L)
   34 FORMAT(' ','X AT CENTRE IS',F8.4,'Y AT CENTRE IS',F8.4)
      NLCON(L)=(2*NCOD(L)-1)*NSEGM(L)
      IF(L.NE.1) NLCON(L)=NLCON(L)+JEND(L-1)
      IF(L.NE.1) NBEG=JEND(L-1)+1
      NFI1=NLCON(L)
      NFI2=JEND(L)
C       ASSIGN PERMEABILITIES
C (IV) GET VALUES OF PERMEABILITY AND CURRENT DENSITY FOR EACH ELEMENT
      CALL MUANDJ(MU,BIGJ,NBEG,NFI2,SIGMM,MU1,BIGJ1,BIGJ2)
  777 CONTINUE
      CVA(4)=(0.D0,0.D0)
      ATOT(4)=0.D0
      BIGJ2=(0.D0,0.D0)
      BIGJ1=(0.D0,0.D0)
      NBEG=JEND(NPHMAX)+1
      NFI1=JFIN-NTRIA-2*(NSRG(1)+NSRG(2))
      NFI2=JFIN
C       ASSIGN PIPE PERMEABILITY
      CALL MUANDJ(MU,BIGJ,NBEG,NFI2,SIGMM,MU1,BIGJ1,BIGJ2)
      NFI1=NFI1+1
C       INITIALIZE PIPE PERMEABILITY
      DO 440 J=NFI1,JFIN
  440 MU(J)=300.*MUO
C (V) SET SM AND RM TO ZERO. RM(I)=CSM(I+NPOTSQ).
C       IFPIPE IS ZERO IF NO PIPE IS PRESENT
```

```
      READ(5,22) IFPIPE
      JLAST=JEND(NPHMAX)
C      NPOT IS THE NUMBER OF UNFIXED NODES
      NDTST=0
      IF(IFPIPE.NE.0) NDTST=IFPIPE/IFPIPE
      NPOT=NDTOT+NSOUT*(NDTST-1)
      NPOT1=NPOT+1
      NPOTSQ=NPOT*NPOT
      WRITE(6,800)
  800 FORMAT(' ',' APPROXIMATE BOUNDARY POTENTIALS')
      DO 750 J=1,NSOUT
  750 WRITE(6,501) CBOUN(J)
  501 FORMAT(' ',2(G14.7))
      DO 15 JJJ=1,2
      DO 441 L=1,4
      CVA(L)=(0.D0,0.D0)
  441 ATOT(L)=0.D0
      DO 7 I=1,NPOT
      DO 7 J=1,NPOT1
    7 CSM(I+NPOT*(J-1))=(0.D0,0.D0)
C (VI) MAIN LOOP-CALCULATE MATRIX ENTRIES FOR EACH ELEMENT
C     AND PLUG THEM INTO MATRIX EQUATION (SM()A)=(RM)
C     REQUIRED INPUTS: NRAD,NSEG,NODE,X,Y,MU,BIGJ
C     RESULTS:CSM
      NLMNTS=JFIN
      DO 100 N=1,NLMNTS
C  (1) CALCULATE Q(I) AND R(I)
      XCENT=(X(NODE(N,1))+X(NODE(N,2))+X(NODE(N,3)))/3.D0
      YCENT=(Y(NODE(N,1))+Y(NODE(N,2))+Y(NODE(N,3)))/3.D0
      X1=X(NODE(N,1))-XCENT
      X2=X(NODE(N,2))-XCENT
      X3=X(NODE(N,3))-XCENT
      Y1=Y(NODE(N,1))-YCENT
      Y2=Y(NODE(N,2))-YCENT
      Y3=Y(NODE(N,3))-YCENT
      J=2
      K=3
      DO 8 I=1,3
      Q(I)=(Y(NODE(N,K))-Y(NODE(N,J)))
      R(I)=(X(NODE(N,J))-X(NODE(N,K)))
      T(I)=X(NODE(N,K))*Y(NODE(N,J))-X(NODE(N,J))*Y(NODE(N,K))
      J=J+1-J/3*3
    8 K=K+1-K/3*3
C  (2) CALCULATE ELEMENT MATRIX ENTRIES
      AREA=    (X(NODE(N,1))*(Y(NODE(N,3))-Y(NODE(N,2)))
     C          +X(NODE(N,2))*(Y(NODE(N,1))-Y(NODE(N,3)))
     C          +X(NODE(N,3))*(Y(NODE(N,2))-Y(NODE(N,1))))*5.D-1
      DO 9 I=1,3
      DO99 J=1,3
      SREAL=(Q(I)*Q(J)+R(I)*R(J))/(AREA*4.D0)
      S(I,J)=DCMPLX(SREAL,0.D0)
      IF(SIGMM(N).EQ.0.D0) GO TO 98
      EDCUR=(60.D0*PI*MU(N)*SIGMM(N)*(T(I)*T(J)+XCENT*(Q(I)
     .*T(J)+T(I)*Q(J))+YCENT*(R(I)*T(J)+T(I)*R(J))+(Q(I)*R(J)+R(I)*Q(J))
     .*(XCENT*YCENT+(X1*Y1+X2*Y2+X3*Y3)/12.D0)+Q(I)*Q(J)*(XCENT**2+(
     .X1**2+X2**2+X3**2)/12.D0)+R(I)*R(J)*(YCENT**2+(Y1**2+Y2**2+Y3**2)
     ./12.D0)) )/(AREA*2.D0)
      S(I,J)=S(I,J)-DCMPLX(0.D0,EDCUR)
   98 CONTINUE
      S(I,J)=S(I,J)/MU(N)
   99 CONTINUE
    9 CONTINUE
C  (3) CALCULATE RIGHT HAND SIDE OF ELEMENT MATRIX
      RHS= CONJG(BIGJ(N))*AREA/3.D0
C  (4) ADD ENTRIES INTO BIG MATRIX EQUATION: (SM)(A)=(RM)
      DO 10 I=1,3
      IF(NODE(N,I).GT.NPOT)GO TO 10
      DO 11 J=1,3
      CSM(NODE(N,I)+(NODE(N,J)-1)*NPOT)=CSM(NODE(N,I)+(NODE(N,J)-1)*NPOT
     1)+S(I,J)
   11 CONTINUE
      CSM(NODE(N,I)+NPOTSQ)=CSM(NODE(N,I)+NPOTSQ)+RHS
   10 CONTINUE
  100 CONTINUE
      CALL CSOLVE(CSM,CV,NPOT,CDET)
      WRITE(6,802)
  802 FORMAT(' ','VECTOR POTENTIAL VALUES')
C CORRECTION OF FUNCTIONAL
```

```
      DO 920 J=1,NPOT
      CV(J)=CONJG(CV(J))
  920 WRITE(6,505) J,CV(J)
      NZBU=NCON*NSEGM(1)+1
      NZBUN=3*NZBU
      NPOT1=NPOT+1
      NPOTSQ=NPOT*NPOT
C     THIS SECTION RE - EVALUATES SM THEN EVALUATES THE RESIDUAL
C       IF DESIRED CMPRUV IS USED HERE
      DO 200 N=1,NLMNTS
      AREA=    (X(NODE(N,1))*(Y(NODE(N,3))-Y(NODE(N,2)))
     C          +X(NODE(N,2))*(Y(NODE(N,1))-Y(NODE(N,3)))
     C          +X(NODE(N,3))*(Y(NODE(N,2))-Y(NODE(N,1))))*5.D-1
      IF(SIGMM(N).EQ.0.D0) GO TO 29
C
C       CALCULATE AVERAGE POTENTIAL
C
      L=(N-1)/JEND(1)+1
      IF(L.GT.4) L=4
      ATOT(L)=ATOT(L)+AREA
      DO 247 I=1,3
  247 CVA(L)=CVA(L)+CV(NODE(N,I))*AREA/3.
      GO TO 29
  950 CONTINUE
   29 CONTINUE
      IF(N.NE.JFIN) GO TO 402
      DO 403 J=1,4
      IF(ATOT(J).EQ.0.) GO TO 402
  403 CVA(J)=CVA(J)/ATOT(J)
      WRITE(6,44)(ATOT(L),L=1,4)
   44 FORMAT(' ',4F14.7)
  402 CONTINUE
      IF(N.LE.JFIN) GO TO 200
C
C       THIS SECTION IS USED TO RECONSTRUCT THE MASTER MATRIX IF
C       CMPRUV IS USED
      DO 18 I=1,3
      IF(NODE(N,I)   .GT. NPOT) CV(NODE(N,I))=CBOUN(NODE(N,I)-NUMOUT)
      R(I)=(X(NODE(N,J))-X(NODE(N,K)))
      Q(I)=(Y(NODE(N,K))-Y(NODE(N,J)))
      T(I)=X(NODE(N,K))*Y(NODE(N,J))-X(NODE(N,J))*Y(NODE(N,K))
   61 J=J+1-J/3*3
   18 K=K+1-K/3*3
      DO 19 I=1,3
      DO119 J=1,3
      SREAL=(Q(I)*Q(J)+R(I)*R(J))/(AREA*4.D0)
      S(I,J)=DCMPLX(SREAL,0.D0)
      IF(SIGMM(N).EQ.0.D0) GO TO 195
      EDCUR=(60.D0*PI*MU(N)*SIGMM(N)*(T(I)*T(J)+XCENT*(Q(I)
     .*T(J)+T(I)*Q(J))+YCENT*(R(I)*T(J)+T(I)*R(J))+(Q(I)*R(J)+R(I)*Q(J))
     .*(XCENT*YCENT+(X1*Y1+X2*Y2+X3*Y3)/12.D0)+Q(I)*Q(J)*(XCENT**2+(
     .X1**2+X2**2+X3**2)/12.D0)+R(I)*R(J)*(YCENT**2+(Y1**2+Y2**2+Y3**2)
     ./12.D0)) )/(AREA*2.D0)
      S(I,J)=S(I,J)-DCMPLX(0.D0,EDCUR)
  195 CONTINUE
  119 CONTINUE
   19 CONTINUE
      RHS=CONJG(BIGJ(N))*AREA*MU(N)/3.0
      DO 20 I=1,3
      IF(NODE(N,I).GT.NPOT)GO TO 20
      DO 21 J=1,3
      CSM(NODE(N,I)+(NODE(N,J)-1)*NPOT)=CSM(NODE(N,I)+(NODE(N,J)-1)*NPOT
     1)+S(I,J)
   21 CONTINUE
      CSM(NODE(N,I)+NPOTSQ)=CSM(NODE(N,I)+NPOTSQ)+RHS
   20 CONTINUE
  200 CONTINUE
C
C       PUT CALL TO SUBROUTIE CMPRUV HERE IF DESIRED
C
      DO 510 L=1,4
  510 WRITE(6,105) CVA(L)
 105 FORMAT(//'     CVA=(',G14.7,',',G14.7,')',/
     1      ' NO.',12X,'RAW POTENTIAL',26X,'RESIDUAL',22X,'ADJUSTED P
     2OTENTIAL'/)
      CVA(1)=CVA(1)*ATOT(1)+CVA(2)*ATOT(2)+CVA(3)*ATOT(3)+CVA(4)*ATOT(4)
      CVA(1)=CVA(1)/(ATOT(1)+ATOT(2)+ATOT(3)+ATOT(4))
      NEND=NPHMAX+1
```

```
      NBEG=1
      DQT=0.
      DPT=0.
C
C      CALCULATE LOSS RATIO
C
      DO 401 L=1,NEND
      DP=(0.D0,0.D0)
      DQ=(0.D0,0.D0)
      DR=(0.D0,0.D0)
      DS=(0.D0,0.D0)
      IF((IFPIPE.EQ.0).AND.(L.EQ.NEND)) GO TO 401
      IF(L.NE.1) NBEG=JEND(L-1)+1
      IF(L.EQ.NEND ) NBEG=JFIN-NTRIA+1-2*(NSRG(1)+NSRG(2))
      IF(L.NE.NEND) NFIN=JEND(L)
      IF(L.EQ.NEND) NFIN=JFIN
      IF(L.NE.NEND) WRITE(5,504) NEND,IFPIPE,JEND(L),NLCON(L),NFIN
  504 FORMAT(' ',I2,I2,I4,I4,I4)
      DO 400 N=NBEG,NFIN
      IF(SIGMM(N).EQ.0.D0) GO TO 400
      XCENT=(X(NODE(N,1))+X(NODE(N,2))+X(NODE(N,3)))/3.D0
      YCENT=(Y(NODE(N,1))+Y(NODE(N,2))+Y(NODE(N,3)))/3.D0
      X1=X(NODE(N,1))-XCENT
      X2=X(NODE(N,2))-XCENT
      X3=X(NODE(N,3))-XCENT
      Y1=Y(NODE(N,1))-YCENT
      Y2=Y(NODE(N,2))-YCENT
      Y3=Y(NODE(N,3))-YCENT
      CW(1)=CV(NODE(N,1))-CVA(1)
      CW(2)=CV(NODE(N,2))-CVA(1)
      CW(3)=CV(NODE(N,3))-CVA(1)
      CTRA=(CW(1)     )*(X(NODE(N,3))*Y(NODE(N,2))-X(NODE(N,2))*Y(NODE(N,
     .3)))+(CW(2)     )*(X(NODE(N,1))*Y(NODE(N,3))-X(NODE(N,3))*Y(NODE(N,
     .1)))+(CW(3)     )*(X(NODE(N,2))*Y(NODE(N,1))-X(NODE(N,1))*Y(NODE(N,
     .2)))
      CTRB=(CW(1)     )*(Y(NODE(N,3))-Y(NODE(N,2)))+(CW(2)     )*(Y(NODE(N
     .,1))-Y(NODE(N,3)))+(CW(3)     )*(Y(NODE(N,2))-Y(NODE(N,1)))
      CTRC=(CW(1)     )*(X(NODE(N,2))-X(NODE(N,3)))+(CW(2)     )*(X(NODE(N
     .,3))-X(NODE(N,1)))+(CW(3)     )*(X(NODE(N,1))-X(NODE(N,2)))
      AREA=    (X(NODE(N,1))*(Y(NODE(N,3))-Y(NODE(N,2)))
     C          +X(NODE(N,2))*(Y(NODE(N,1))-Y(NODE(N,3)))
     C          +X(NODE(N,3))*(Y(NODE(N,2))-Y(NODE(N,1))))*5.D-1
      VP=((CDABS(CTRB))**2*((X1**2+X2**2+X3**2)/12.D0+XCENT**2)+(CDABS(
     .CTRA))**2+(CDABS(CTRC))**2*(((Y1**2+Y2**2+Y3**2)/12.D0)+YCENT**2)
     .+      (CTRB*DCONJG(CTRC)+CTRC*DCONJG(CTRB))*(((X1*Y1+X2*Y2+X3*Y3)
     ./12.D0)+XCENT*YCENT)+CTRB*DCONJG(CTRA)*XCENT+CTRA*DCONJG(CTRB)*
     .XCENT+CTRA*DCONJG(CTRC)*YCENT+CTRC*DCONJG(CTRA)*YCENT)/(AREA*4.D0)
  505 FORMAT(' ',I4,2(G14.7))
C
C      DQ IS THE D.C. LOSS
C
      DQ=DQ+AREA/SIGMM(N)*BIGJ(N)*CONJG(BIGJ(N))
      DS=DS+40.D0*PI*AREA*BIGJ(N)* CONJG(CW(1)+CW(2)+CW(3))
      DR=DR+40.D0*PI*AREA*(CW(1)+CW(2)+CW(3))            * CONJG(BIGJ(N)
     .)
C
C      DP IS THE EDDY LOSS
      DP=DP+3600.D0*SIGMM(N)*VP*4.D0*PI*PI
  400 CONTINUE
      CPTOT=DP+DQ+DCMPLX(0.D0,1.D0)*(DS-DR)
      WRITE(5,803)
  803 FORMAT(' ',5X,'D.C. LOSS',25X,'IMAGINARY CROSS TERMS',30X,'TOTAL
     .LOSS')
      WRITE(6,503) DQ,DR,DS,CPTOT
      PDC=DQ
      DQT=DQT+DQ
      DPT=DPT+DP
      IF(L.NE.NEND) RACDC=(CPTOT+DCONJG(CPTOT))/(2.D0*PDC)
      IF(L.NE.NEND) RACDC=1.+DP/DQ
  503 FORMAT(' ',8(G14.7))
      IF(L.NE.NEND) WRITE(5,500) RACDC
      WRITE(6,804)
  804 FORMAT(' ','EDDY LOSS')
      WRITE(6,501) DP,PDC ,ATOT(L)
  401 CONTINUE
      RACDC=1.+DPT/DQT
      WRITE(6,500) RACDC
C
```

21 MAIN DATE = 76316 06/32/18

```
C       THIS SECTION ADJUSTS PERMEABILITY
C
      DO 16 J=NFI1,JFIN
      DO 17 K=1,3
      XT(K)=X(NODE(J,K))
      YT(K)=Y(NODE(J,K))
   17 CT(K)=CV(NODE(J,K))-CVA(4)
      CALL MUADJS(XT,YT,CT,VAL,J)
   16 MU(J)=.1257E-5*VAL
   15 CONTINUE
  500 FORMAT(' ','RATIO OF AC TO DC RESISTANCE IS =',G14.7)
 9999 CONTINUE
      STOP
      END
```

```
      SUBROUTINE CSOLVE(CA,CX,N,CDET)
      IMPLICIT COMPLEX(C)
      REAL CABS
      REAL*8 CDABS
      DIMENSION IPIV(200),JPIV(200),CA(1),CX(1)
      LOGICAL F1,F2
C      THIS SUBROUTINE SOLVES A SYSTEM OF COMPLEX SIMULTANEOUS
C      EQUATIONS BY GAUSSIAN ELIMINATION
C
C      WRITTEN BY P.H. ALEXANDER
C
C      USES MAXIMUM PIVOT STRATEGY
C
      F1=.FALSE.
      F2=.FALSE.
      NM1=N-1
      NP1=N+1
      DO 5 I=1,NP1
      IPIV(I)=I
    5 JPIV(I)=I
      CDET=(1.D0,0.D0)
      DO 100 I=1,N
      IP1=I+1
      CELMAX=CA((JPIV(I)-1)*N+IPIV(I))
      DO 25 II=I   ,N
      DO 20  J=I   ,N
      IF(CABS(CA((JPIV(J)-1)*N+IPIV(II))).LE.CABS(CELMAX)) GO TO 20
      ISAVE=JPIV(I)
      JPIV(I)=JPIV(J)
      JPIV(J)=ISAVE
      ISAVE=IPIV(I)
      IPIV(I)=IPIV(II)
      IPIV(II)=ISAVE
      CELMAX=CA((JPIV(I)-1)*N+IPIV(I  ))
   20 CONTINUE
   25 CONTINUE
      IF(F1 .OR. F2) GO TO 26
      CDET=CDET*CELMAX
      IF( CABS(CDET).LE.1.D-70) F1=.TRUE.
      IF( CABS(CDET) .GT. 1.D70) F2=.TRUE.
      IF(F1 .OR. F2) K=I
   26 CONTINUE
      DO 30 J=IP1,NP1
   30 CA((JPIV(J)-1)*N+IPIV(I))=CA((JPIV(J)-1)*N+IPIV(I))/CELMAX
      CA((JPIV(I)-1)*N+IPIV(I))=(1.D0,0.D0)
      DO 40 II=  1,N
      IF(II .EQ. I) GO TO 36
      DO 35 J=IP1,NP1
   35 CA((JPIV(J)-1)*N+IPIV(II))=CA((JPIV(J)-1)*N+IPIV(II))-CA((JPIV(I)-
     11)*N+IPIV(II))*CA((JPIV(J)-1)*N+IPIV(I))
      CA((JPIV(I)-1)*N+IPIV(II))=(0.D0,0.D0)
   36 CONTINUE
   40 CONTINUE
  100 CONTINUE
      DO 105 J=1,N
  105 CX(J)=CA(N*N+J)
      IF(F1) GO TO 110
      IF(F2) GO TO 109
      RETURN
  109 WRITE(6,108) K,K,N,N,CDET
  108 FORMAT(' :::: PROBLEM ILL-CONDITIONED'/'    DETERMINANT CALCULATED
     1TO THE (',I3,',',I3,') ELEMENT OF THIS ',I3,'X',I3,' SYSTEM=(',
     2G12.5,',',G12.5,').')
      RETURN
  110 WRITE(6,111) K,K,N,N,CDET
  111 FORMAT(' ::::: MATRIX PRACTICALLY SINGULAR.'/'    DETERMINANT CALCU
     1LATED TO (',I3,',',I3,') ELEMENT OF THIS ',I3,'X',I3,' SYSTEM=(',
     2G12.5,',',G12.5,').')
      RETURN
      END
```

```
      SUBROUTINE CMPRUV(CA,CX,N,ITMAX,CRES,CRIT,*)
      IMPLICIT COMPLEX*16 (C)
      DIMENSION CRES(33),CA(1122),CX(33),CXOLD(100)
      REAL*8 XNORM,RNORM,CRIT ,CMAX
      REAL*8 CDABS,DSQRT
C
C      THIS SUBROUTINE MAY BE USED TO ITERATIVELY ADJUST THE VALUES
C      OBTAINED BY CSOLVE
C
C      WRITTEN BY P.H. ALEXANDER
C
C      FOR EACH ROW, SOLUTION ENTRY  CORRESPONDING TO LARGEST ROW
C      ELEMENT IS ADJUSTED BY SUBTRACTING  RESIDUAL  OF OTHER ROW
C      ELEMENTS  FROM THE RIGHT HAND SIDE
C
      NSQ=N*N
      XNORM=0.D0
      RNORM=0.D0
      DO 2 I=1,N
      CRES(I)=CA(NSQ+I)
      DO 1  J=1,N
    1 CRES(I)=CRES(I)-CA((J-1)*N+I)*CX(J)
      XNORM=XNORM+(CDABS(CX(I)))**2
    2 RNORM=RNORM+(CDABS(CRES(I)))**2
      RNORM=DSQRT(RNORM)
      XNORM=DSQRT(XNORM)
      IF(RNORM/XNORM .LE. CRIT   ) GO TO 100
      ITER=1
    3 XNORM=0.D0
      DO 5 I=1,N
      CXOLD(I)=CX(I)
      CMAX=0.D0
      DO 9  J=1,N
      IF(CDABS(CA(I+(J-1)*N)) .LT. CMAX) GO TO 9
      CMAX=CDABS(CA(I+(J-1)*N))
      JMAX=J
    9 CONTINUE
      CX(JMAX)=CA(NSQ+I)
      DO 4  J=1,N
      IF(J .EQ. JMAX.OR.CDABS(CA(I+N*(J-1))).EQ.0.D0) GO TO 16
      CX(JMAX)=CX(JMAX)-CA(I+N*(J-1))*CX(J)
   16 CONTINUE
    4 CONTINUE
      CX(JMAX)=CX(JMAX)/CA(I+N*(JMAX-1))
      XNORM=XNORM+(CDABS(CX(I)))**2
    5 CONTINUE
      XNORM=DSQRT(XNORM)
      DO 6 I=1,N
      IF(CDABS((CX(I)-CXOLD(I))/XNORM).GT. CRIT) GO TO 7
    6 CONTINUE
      GO TO 10
    7 IF(ITER .GT. ITMAX) GO TO 200
      DO 8 I=1,N
    8 CXOLD(I)=CX(I)
      ITER=ITER+1
      GO TO 3
   10 RNORM=0.D0
      DO 12 I=1,N
      CRES(I)=CA(NSQ+I)
      DO 11  J=1,N
   11 CRES(I)=CRES(I)-CA(I+(J-1)*N)*CX(J)
   12 RNORM=RNORM+(CDABS(CRES(I)))**2
      RNORM=DSQRT(RNORM)
      WRITE(6,13)  XNORM,RNORM,ITER,CRIT
   13 FORMAT(5X,'CONVERGENCE WITH XNORM=',G12.5,' AND RNORM=',G12.5,' AF
     1TER ',I5,' ITERATIONS'//10X,'ERROR CRITERION=',G14.7)
      RETURN
  100 WRITE(6,101) XNORM,RNORM
  101 FORMAT(5X,'NO ITERATIONS NECESSARY. XNORM=',G12.5,';RNORM=',G12.5,
     1'.')
      RETURN
  200 WRITE(6,201) CRIT,ITMAX,XNORM,RNORM
  201 FORMAT(5X,'"CONVERGENCE" NOT ATTAINED TO WITHIN ',G12.5,' BETWEEN
     1SUCCESSIVE ITERATIONS FOR ALL SOLUTION ELEMENTS WITHIN',I5,' ITERA
     2TIONS'/10X,'XNORM=',G12.5,';RNORM=',G12.5,'.')
      RETURN
      END
```

```
      SUBROUTINE MUANDJ(MU,BIGJ,NBEG,NFI1,SIGMM,MU1,BIGJ1,BIGJ2)
C     ASSIGNS VALUES OF CURRENT DENSITY  (BIGJ) AND PERMEABILITU(MU)
C     TO EACH ELEMENT.
C     NCON=NUMBER OF RINGS IN CONDUCTOR
C     BIGJ1=CURRENT DENSITY IN CONDUCTOR
C     BIGJ2=CURRENT DENSITY IN INSULATION
C     MU1=MAGNETIC PERMEABILITY
      IMPLICIT COMPLEX*16(C),REAL*8(A-B,D-H,O-Z)
      REAL MU(400),MU1
      REAL*8 SIGMM(400)
      COMPLEX BIGJ(400),BIGJ1,BIGJ2
      DO 1 N=NBEG,NFI1
      MU(N)=MU1
      BIGJ(N)=BIGJ1
      IF(SIGMM(N).EQ.0.) BIGJ(N)=BIGJ2
    1 CONTINUE
    3 RETURN
      END
```



```
      SUBROUTINE MUADJS(XT,YT,CT,VAL,L)
      DIMENSION XT(3),YT(3),B(3),C(3)
      COMPLEX CT(3),CONJG,GRADX,GRADY
C      THIS SUBROUTINE CALCULATES FLUX DENSITY IN EACH TRIANGLE AND
C      COMPUTES CORRESPONDING ELEMENT PERMEABILITY
      J=2
      K=3
      DO 18 I=1,3
      B(I)=YT(K)-YT(J)
      C(I)=XT(J)-XT(K)
      J=J+1-J/3*3
   18 K=K+1-K/3*3
      AREA=XT(1)*B(1)+XT(2)*B(2)+XT(3)*B(3)
      GRADX=(0.0,0.0)
      GRADY=(0.0,0.0)
      DO 2 J=1,3
      GRADX=GRADX+B(J)*     (CT(J))/AREA
    2 GRADY=GRADY+C(J)*     (CT(J))/AREA
      TEMP=GRADX*CONJG(GRADX)+GRADY*CONJG(GRADY)
C      MAGNITUDE OF FLUX DENSITY
      ST=SQRT(TEMP)
      S=ST*1.E4
C      VAL IS RELATIVE PERMEABILITY
      IF(S.LT..44) VAL=60.
      IF((S.LT.5.).AND.(S.GE..44))VAL=64.7661-4.789063*S+3.070068*S**2-
     ..368408*S**3
      IF((S.LE.70.).AND.(S.GE.5.)) VAL=74.22656-1.496094*S+.2026367*S**2
     .-.003891945*S**3+.2253056E-4*S**4
      IF((S.LT.970.).AND.(S.GT.70.)) VAL=113.5195+.8176117*S-.2273917E-3
     .*S**2
      IF((S.LT.9420.).AND.(S.GT.970.)) VAL=22.56250+.7792969*S
     .-.9357929E-4*S**2+.2755769E-8*S**3
      IF((S.LE.15370.).AND.(S.GE.9420.)) VAL=3235.188-.2082367*S+
     . .8009374E-6*S**2
      WRITE(6,3) S,VAL,L
```

```
198378 16   3 36 29 65101   8   8   8 62124186 0.7450 0.7450 0.7450
                    NUMBER OF RINGS WITHIN THE CONDUCTOR:      3
   MAGNETIC PERMEABILITY: 0.12570E-05
   CURRENT DENSITY:
        WITHIN THE CONDUCTOR: 0.71109E 06 0.0
        WITHIN THE INSULATION: 0.0            0.0

X AT CENTRE IS  0.0    Y AT CENTRE IS  0.0200
                    NUMBER OF RINGS WITHIN THE CONDUCTOR:      3
   MAGNETIC PERMEABILITY: 0.12570E-05
   CURRENT DENSITY:
        WITHIN THE CONDUCTOR:-0.35555E 06-0.61581E 06
        WITHIN THE INSULATION: 0.0            0.0

X AT CENTRE IS -1.8000Y AT CENTRE IS -2.9000
                    NUMBER OF RINGS WITHIN THE CONDUCTOR:      3
   MAGNETIC PERMEABILITY: 0.12570E-05
   CURRENT DENSITY:
        WITHIN THE CONDUCTOR:-0.35355E 06 0.61581E 06
        WITHIN THE INSULATION: 0.0            0.0

X AT CENTRE IS  1.8000Y AT CENTRE IS -2.9000
 APPROXIMATE BOUNDARY POTENTIALS
-0.2764999E-05 0.3629262E-04
 0.1216281E-04 0.3153372E-04
 0.2609560E-04 0.2288664E-04
 0.3613133E-04 0.1195312E-04
 0.3980934E-04-0.3182817E-17
 0.3613133E-04-0.1195312E-04
 0.2609560E-04-0.2288664E-04
 0.1216281E-04-0.3153372E-04
-0.2764999E-05-0.3629262E-04
-0.1612074E-04-0.3554457E-04
-0.2616302E-04-0.2847021E-04
-0.3217344E-04-0.1583846E-04
-0.3414442E-04 0.1289341E-16
-0.3217344E-04 0.1583846E-04
-0.2616302E-04 0.2847021E-04
-0.1612074E-04 0.3554457E-04
:::: PROBLEM ILL-CONDITIONED
  DETERMINANT CALCULATED TO THE ( 11, 11) ELEMENT OF THIS 198X198 SYSTE
VECTOR POTENTIAL VALUES
   1 0.2407048E-04-0.7458791E-04
   2 0.2368317E-04-0.7433284E-04
   3 0.2389619E-04-0.7397970E-04
   4 0.2432622E-04-0.7423776E-04
   5 0.2628200E-04-0.7246452E-04
   6 0.2602152E-04-0.7304535E-04
   7 0.2440588E-04-0.7392271E-04
   8 0.2273034E-04-0.7336284E-04
   9 0.2241599E-04-0.7330622E-04
  10 0.2137230E-04-0.7202337E-04
  11 0.2128237E-04-0.7027391E-04
  12 0.2151199E-04-0.7009019E-04
  13 0.2254512E 04-0.6897577E-04
  14 0.2461579E-04-0.6891467E-04
  15 0.2519233E-04-0.6938528E-04
  16 0.2675102E-04-0.7084789E-04
  17 0.2751230E-04-0.6614157E-04
  18 0.2700747E-04-0.6612475E-04
  19 0.2388915E-04-0.6821498E-04
  20 0.2084345E-04-0.6817062E-04
  21 0.2037438E-04-0.6764481E-04
  22 0.1816150E-04-0.6558362E-04
  23 0.1764839E-04-0.6279668E-04
  24 0.1753168E-04-0.6215533E-04
  25 0.1861389E-04-0.5942491E-04
  26 0.2370267E-04-0.5910975E-04
  27 0.2403901E-04-0.5899303E-04
  28 0.2826660E-04-0.6158471E-04
  29 0.2631001E-04-0.5804948E-04
  30 0.2193202E-04-0.6061372E-04
  31 0.1797020E-04-0.6149348E-04
  32 0.1424872E-04-0.5859340E-04
  33 0.1291651E-04-0.5516413E-04
  34 0.1261235E-04-0.5074529E-04
  35 0.2039093E-04-0.4886874E-04
  36 0.2693890E-04-0.5089206E-04
  37-0.7476914E-04 0.1929761E-04
```

38	-0.7523845E-04	0.1972725E-04
39	-0.7504257E-04	0.2032220E-04
40	-0.7455921E-04	0.1991025E-04
41	-0.6944388E-04	0.1711946E-04
42	-0.6985418E-04	0.1549364E-04
43	-0.7142672E-04	0.1435765E-04
44	-0.7333656E-04	0.1473860E-04
45	-0.7400666E-04	0.1525477E-04
46	-0.7547106E-04	0.1794836E-04
47	-0.7541203E-04	0.2068648E-04
48	-0.7539049E-04	0.2134533E-04
49	-0.7432955E-04	0.2284389E-04
50	-0.7206404E-04	0.2218377E-04
51	-0.7169246E-04	0.2182391E-04
52	-0.6993537E-04	0.1976812E-04
53	-0.5995159E-04	0.1453049E-04
54	-0.5978483E-04	0.1392633E-04
55	-0.6279646E-04	0.8222132E-05
56	-0.6766524E-04	0.8501253E-05
57	-0.6773185E-04	0.8958385E-05
58	-0.7147019E-04	0.1440772E-04
59	-0.7248895E-04	0.1992418E-04
60	-0.7210222E-04	0.2041883E-04
61	-0.7104543E-04	0.2370978E-04
62	-0.6642351E-04	0.2298152E-04
63	-0.6562911E-04	0.2260957E-04
64	-0.6147704E-04	0.1938743E-04
65	-0.4951749E-04	0.1414150E-04
66	-0.5226415E-04	0.4714701E-05
67	-0.5933696E-04	0.4148834E-05
68	-0.6484447E-04	0.1156964E-04
69	-0.6755618E-04	0.1888952E-04
70	-0.6698121E-04	0.2366788E-04
71	-0.6007809E-04	0.2338176E-04
72	-0.5306852E-04	0.2010747E-04
73	0.5334687E-04	0.5393554E-04
74	0.5285910E-04	0.5422918E-04
75	0.5327554E-04	0.5464217E-04
76	0.5378513E-04	0.5432111E-04
77	0.5423851E-04	0.5146264E-04
78	0.5387558E-04	0.5120540E-04
79	0.5173139E-04	0.5012062E-04
80	0.4951200E-04	0.4993037E-04
81	0.4913686E-04	0.5033551E-04
82	0.4775934E-04	0.5199255E-04
83	0.4783721E-04	0.5414303E-04
84	0.4836428E-04	0.5476490E-04
85	0.5079911E-04	0.5592342E-04
86	0.5341214E-04	0.5512126E-04
87	0.5409584E-04	0.5488923E-04
88	0.5543637E-04	0.5336042E-04
89	0.5223922E-04	0.4656923E-04
90	0.5163036E-04	0.4598845E-04
91	0.4789376E-04	0.4356034E-04
92	0.4414628E-04	0.4235153E-04
93	0.4361582E-04	0.4220019E-04
94	0.4027164E-04	0.4518565E-04
95	0.3930638E-04	0.5083019E-04
96	0.3947807E-04	0.5098508E-04
97	0.4447839E-04	0.5424368E-04
98	0.5060367E-04	0.5329686E-04
99	0.5098140E-04	0.5272687E-04
100	0.5445413E-04	0.5042282E-04
101	0.4932580E-04	0.4089915E-04
102	0.4464058E-04	0.3661121E-04
103	0.4013321E-04	0.3333938E-04
104	0.3471221E-04	0.3616980E-04
105	0.3129945E-04	0.4472288E-04
106	0.3784662E-04	0.4961427E-04
107	0.4687543E-04	0.4923072E-04
108	0.5230936E-04	0.4677082E-04
109	-0.1462346E-04	-0.2183788E-04
110	-0.1196997E-04	-0.2580149E-04
111	-0.8771849E-05	-0.2918235E-04
112	-0.6721972E-05	-0.2943516E-04
113	-0.4611136E-05	-0.2810005E-04
114	-0.1775935E-04	-0.1375857E-04
115	-0.1273169E-04	-0.2269851E-04
116	-0.5456885E-05	-0.3150925E-04
117	0.2132503E-05	-0.3823248E-04
118	0.4650799E-05	-0.3479263E-04
119	0.4183550E-05	-0.2923246E-04
120	0.1686932E-05	-0.2622395E-04
121	-0.2363530E-04	-0.3917711E-05
122	-0.1805816E-04	-0.1896231E-04
123	-0.2964507E-05	-0.3752143E-04
124	0.2203304E-04	-0.3841337E-04
125	0.1959939E-04	-0.2158761E-04
126	0.1462506E-04	-0.1879156E-04
127	-0.3463859E-04	0.8225046E-05
128	-0.4141484E-04	0.2563507E-06
129	-0.3302365E-04	-0.1059231E-04
130	0.6475452E-05	-0.1525384E-04
131	0.3153246E-04	0.5643015E-05
132	0.3357172E-04	0.9763377E-05
133	0.2738426E-04	-0.4354095E-05
134	-0.4838681E-04	0.2025256E-04
135	-0.6231198E-04	0.1988714E-04
136	-0.5402936E-05	0.2392523E-04
137	0.4810444E-04	0.3571413E-04
138	0.4247740E-04	0.1827670E-04
139	-0.5531432E-04	0.2559336E-04
140	-0.5953910E-04	0.2558103E-04
141	-0.5174387E-04	0.2383014E-04
142	-0.4337487E-05	0.2978356E-04
143	0.4032782E-04	0.4636979E-04
144	0.5042447E-04	0.3749410E-04
145	0.4926078E-04	0.3076333E-04
146	-0.5491597E-04	0.2818188E-04
147	-0.3686704E-04	0.2908500E-04
148	-0.6037374E-06	0.3042404E-04
149	0.3553387E-04	0.3777754E-04
150	0.4955957E-04	0.3496725E-04
151	-0.9267646E-05	0.1547686E-05
152	-0.7501321E-05	0.1004523E-04
153	-0.3271622E-05	0.1431801E-04
154	-0.3643236E-05	0.9070785E-05
155	0.2087558E-06	0.1422188E-04
156	-0.5879824E-06	0.9080134E-05
157	0.5292138E-05	0.1190049E-04
158	0.8517592E-05	0.6855599E-05
159	0.1017453E-04	0.2800167E-05
160	0.1778976E-04	0.1171055E-05
161	0.2128544E-04	0.2974515E-05
162	0.2113612E-04	0.3758149E-06
163	0.4615948E-05	-0.1455712E-04
164	-0.8403776E-05	-0.1983540E-04
165	-0.9102843E-05	-0.1820257E-04
166	-0.1234516E-04	-0.8044894E-05
167	0.4800322E-06	0.3265036E-05
168	0.2604125E-05	0.2673829E-05
169	0.4535349E-05	0.1004025E-05
170	0.2312034E-05	0.3919477E-06
171	0.4398335E-05	0.2519183E-06
172	0.2286948E-05	-0.4370293E-06
173	0.3780609E-05	-0.1085123E-05
174	0.2134242E-05	-0.2882371E-05
175	0.7688312E-06	-0.2842894E-05
176	0.5696563E-06	-0.4791885E-05
177	0.5324947E-06	-0.5550169E-05
178	0.3207241E-06	-0.6413908E-05
179	-0.4963045E-05	-0.3012802E-05
180	-0.6123597E-05	0.2154337E-05
181	-0.5057363E-05	0.2800154E-05
182	-0.2783325E-05	0.4403277E-05
183	0.3007403E-05	-0.5394007E-06
184	0.2002490E-05	-0.1806507E-05
185	0.1095455E-05	-0.3460078E-05
186	-0.6643405E-06	-0.1884782E-05
187	0.5614820E-06	-0.2773812E-05
188	-0.1067371E-05	-0.1860122E-05
189	-0.4148467E-06	-0.2341287E-05
190	-0.2310239E-05	-0.1935020E-05
191	-0.2252465E-05	-0.2051664E-06
192	-0.3055810E-05	-0.4306954E-06
193	-0.3766266E-05	0.3603193E-08
194	-0.4142217E-05	-0.3153945E-06
195	-0.3438944E-05	0.3580869E-05
196	0.1280383E-05	0.5260323E-05
197	0.2483385E-05	0.3339934E-05

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 CVA=( 0.2198995D-04,-0.6425363D-04)
IO.            RAW POTENTIAL                          RESIDUAL                         ADJUSTED POTENTIAL


 CVA=(-0.6677971D-04, 0.1686861D-04)
 O.            RAW POTENTIAL                          RESIDUAL                         ADJUSTED POTENTIAL


 CVA=( 0.4709178D-04, 0.4823118D-04)
 O.            RAW POTENTIAL                          RESIDUAL                         ADJUSTED POTENTIAL


 CVA=(-0.1132692D-05,-0.4347533D-06)
 O.            RAW POTENTIAL                          RESIDUAL                         ADJUSTED POTENTIAL
410   62  40  62
     D.C. LOSS                         IMAGINARY CROSS TERMS                               TOTAL   LOSS
 18.66217      0.0           5.622109      -15.99032       5.622109        15.99032        2.250509      0.0
ATIO OF AC TO DC RESISTANCE IS =  1.834253
DDY LOSS
 15.56858      0.0
18.66217      0.9311668D-03
410  124 102 124
     D.C. LOSS                         IMAGINARY CROSS TERMS                               TOTAL   LOSS
 18.66134      0.8326673D-16  4.579556   -16.45072       4.579556        16.45072        1.602095    -0.3608225D-15
ATIO OF AC TO DC RESISTANCE IS =  1.848931
DDY LOSS
 15.84219      0.0
18.66134      0.9311668D-03
410  186 164 186
     D.C. LOSS                         IMAGINARY CROSS TERMS                               TOTAL   LOSS
 18.66134     -0.8326673D-16  4.524562   -16.33933       4.524562        16.33933        1.581742    -0.8326673D-16
ATIO OF AC TO DC RESISTANCE IS =  1.835902
DDY LOSS
 15.59966      0.0
18.66134      0.9311668D-03
     D.C. LOSS                         IMAGINARY CROSS TERMS                               TOTAL   LOSS
0.0            0.0           0.0           0.0             0.0             0.0             1.774709      0.0
DDY LOSS
 1.774709      0.0
.0            0.5584230D-02
ATIO OF AC TO DC RESISTANCE IS =  1.871395
TRESS=    94.74437         NEW MU=  188.9424       FOR ELEMENT 279
TRESS=    49.12943         NEW MU=  159.5690       FOR ELEMENT 280
TRESS=   170.0793          NEW MU=  246.0005       FOR ELEMENT 281
TRESS=    51.97505         NEW MU=  161.3383       FOR ELEMENT 282
TRESS=   199.7915          NEW MU=  267.7944       FOR ELEMENT 283
TRESS=    67.56274         NEW MU=  167.2925       FOR ELEMENT 284
TRESS=   145.2057          NEW MU=  227.4468       FOR ELEMENT 285
TRESS=    98.08626         NEW MU=  191.5282       FOR ELEMENT 286
TRESS=    98.18907         NEW MU=  191.6077       FOR ELEMENT 287
TRESS=   137.8684          NEW MU=  221.9201       FOR ELEMENT 288
TRESS=    63.37546         NEW MU=  166.0781       FOR ELEMENT 289
TRESS=   193.2135          NEW MU=  255.6940       FOR ELEMENT 290
TRESS=    46.17064         NEW MU=  156.4441       FOR ELEMENT 291
TRESS=   148.6552          NEW MU=  230.0367       FOR ELEMENT 292
TRESS=    44.55316         NEW MU=  154.3822       FOR ELEMENT 293
TRESS=   101.8010          NEW MU=  194.3966       FOR ELEMENT 294
TRESS=    64.54605         NEW MU=  166.3608       FOR ELEMENT 295
TRESS=   185.9495          NEW MU=  257.6912       FOR ELEMENT 296
TRESS=    94.89539         NEW MU=  189.0601       FOR ELEMENT 297
TRESS=   324.6111          NEW MU=  354.9641       FOR ELEMENT 298
TRESS=   150.5035          NEW MU=  231.4222       FOR ELEMENT 299
TRESS=   215.0803          NEW MU=  278.8525       FOR ELEMENT 300
TRESS=   405.4775          NEW MU=  407.6563       FOR ELEMENT 301
TRESS=   123.4052          NEW MU=  210.9541       FOR ELEMENT 302
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STRESS=   140.4834         NEW MU=   223.8926        FOR ELEMENT 310
STRESS=   235.1536         NEW MU=   328.1743        FOR ELEMENT 311
STRESS=   74.57811         NEW MU=   173.2307        FOR ELEMENT 312
STRESS=   142.0389         NEW MU=   225.1021        FOR ELEMENT 313
STRESS=   57.23013         NEW MU=   164.4690        FOR ELEMENT 314
STRESS=   100.5608         NEW MU=   193.4307        FOR ELEMENT 315
STRESS=   79.53416         NEW MU=   177.1091        FOR ELEMENT 316
STRESS=   123.9522         NEW MU=   211.3781        FOR ELEMENT 317
STRESS=   100.5112         NEW MU=   193.4014        FOR ELEMENT 318
STRESS=   84.34836         NEW MU=   180.8659        FOR ELEMENT 319
STRESS=   123.8822         NEW MU=   211.3173        FOR ELEMENT 320
STRESS=   117.2360         NEW MU=   206.2477        FOR ELEMENT 321
STRESS=   84.54810         NEW MU=   181.0215        FOR ELEMENT 322
STRESS=   79.64130         NEW MU=   177.1928        FOR ELEMENT 323
STRESS=   117.0873         NEW MU=   206.1340        FOR ELEMENT 324
STRESS=   105.0919         NEW MU=   196.9317        FOR ELEMENT 325
STRESS=   79.85304         NEW MU=   177.3583        FOR ELEMENT 326
STRESS=   93.38467         NEW MU=   187.8889        FOR ELEMENT 327
STRESS=   105.0271         NEW MU=   196.8826        FOR ELEMENT 328
STRESS=   88.03546         NEW MU=   183.7360        FOR ELEMENT 329
STRESS=   93.50203         NEW MU=   187.9798        FOR ELEMENT 330
STRESS=   146.5484         NEW MU=   228.4556        FOR ELEMENT 331
STRESS=   88.09987         NEW MU=   183.7860        FOR ELEMENT 332
STRESS=   180.2527         NEW MU=   253.5078        FOR ELEMENT 333
STRESS=   146.3043         NEW MU=   228.2723        FOR ELEMENT 334
STRESS=   175.7339         NEW MU=   250.1790        FOR ELEMENT 335
STRESS=   180.1308         NEW MU=   253.4182        FOR ELEMENT 336
STRESS=   120.5396         NEW MU=   208.7701        FOR ELEMENT 337
STRESS=   175.9227         NEW MU=   250.3182        FOR ELEMENT 338
STRESS=   177.7482         NEW MU=   251.6641        FOR ELEMENT 339
STRESS=   120.6075         NEW MU=   208.8219        FOR ELEMENT 340
STRESS=   171.6523         NEW MU=   247.1644        FOR ELEMENT 341
STRESS=   177.5069         NEW MU=   251.4863        FOR ELEMENT 342
STRESS=   125.7926         NEW MU=   212.7708        FOR ELEMENT 343
STRESS=   171.7845         NEW MU=   247.2622        FOR ELEMENT 344
STRESS=   79.30591         NEW MU=   176.9307        FOR ELEMENT 345
STRESS=   126.0546         NEW MU=   212.9700        FOR ELEMENT 346
STRESS=   36.28156         NEW MU=   139.8508        FOR ELEMENT 347
STRESS=   36.68597         NEW MU=   140.7101        FOR ELEMENT 348
STRESS=   45.24252         NEW MU=   155.2922        FOR ELEMENT 349
STRESS=   36.35707         NEW MU=   140.0121        FOR ELEMENT 350
STRESS=   30.00601         NEW MU=   124.8905        FOR ELEMENT 351
STRESS=   45.24185         NEW MU=   155.2912        FOR ELEMENT 352
STRESS=   39.28751         NEW MU=   145.8873        FOR ELEMENT 353
STRESS=   30.06844         NEW MU=   125.0609        FOR ELEMENT 354
STRESS=   29.17151         NEW MU=   122.7234        FOR ELEMENT 355
STRESS=   39.25539         NEW MU=   145.8270        FOR ELEMENT 356
STRESS=   35.20847         NEW MU=   137.5031        FOR ELEMENT 357
STRESS=   29.23659         NEW MU=   122.8943        FOR ELEMENT 358
STRESS=   36.43796         NEW MU=   140.1850        FOR ELEMENT 359
STRESS=   35.22697         NEW MU=   137.5443        FOR ELEMENT 360
STRESS=   32.15904         NEW MU=   130.3372        FOR ELEMENT 361
STRESS=   36.49442         NEW MU=   140.3049        FOR ELEMENT 362
STRESS=   45.58171         NEW MU=   155.7229        FOR ELEMENT 363
STRESS=   32.21439         NEW MU=   130.4732        FOR ELEMENT 364
STRESS=   56.40550         NEW MU=   164.1651        FOR ELEMENT 365
STRESS=   45.55772         NEW MU=   155.6929        FOR ELEMENT 366
STRESS=   60.68936         NEW MU=   165.4585        FOR ELEMENT 367
STRESS=   56.38394         NEW MU=   164.1567        FOR ELEMENT 368
STRESS=   54.26283         NEW MU=   163.2010        FOR ELEMENT 369
STRESS=   60.77393         NEW MU=   165.4792        FOR ELEMENT 370
STRESS=   64.43169         NEW MU=   166.3325        FOR ELEMENT 371
STRESS=   54.43770         NEW MU=   163.2902        FOR ELEMENT 372
STRESS=   60.65369         NEW MU=   165.4500        FOR ELEMENT 373
STRESS=   64.47359         NEW MU=   166.3425        FOR ELEMENT 374
STRESS=   52.71710         NEW MU=   162.3223        FOR ELEMENT 375
STRESS=   60.73087         NEW MU=   165.4690        FOR ELEMENT 376
STRESS=   36.56934         NEW MU=   140.4640        FOR ELEMENT 377
STRESS=   52.79764         NEW MU=   162.3724        FOR ELEMENT 378
:::: PROBLEM IL-CONDITIONED
  DETERMINANT CALCULATED TO THE ( 11, 11) ELEMENT OF THIS 198X198 SYSTE
VECTOR POTENTIAL VALUES
    1  0.2443051E-04-0.7484922E-04
    2  0.2405213E-04-0.7461202E-04
    3  0.2423923E-04-0.7426844E-04
    4  0.2466096E-04-0.7450864E-04
    5  0.2659608E-04-0.7260137E-04
    6  0.2635991E-04-0.7317525E-04
    7  0.2484824E-04-0.7403517E-04
    8  0.2324789E-04-0.7351355E-04
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9	0.2294224E-04	-0.7348000E-04
10	0.2190258E-04	-0.7228076E-04
11	0.2173809E-04	-0.7059568E-04
12	0.2193839E-04	-0.7042584E-04
13	0.2235918E-04	-0.6932694E-04
14	0.2486177E-04	-0.6921693E-04
15	0.2542823E-04	-0.6967185E-04
16	0.2699238E-04	-0.7105416E-04
17	0.2781028E-04	-0.6608429E-04
18	0.2732818E-04	-0.6605058E-04
19	0.2441386E-04	-0.6807278E-04
20	0.2153334E-04	-0.6809569E-04
21	0.2107446E-04	-0.6757760E-04
22	0.1890725E-04	-0.6569228E-04
23	0.1825698E-04	-0.6307854E-04
24	0.1811648E-04	-0.6243994E-04
25	0.1896707E-04	-0.5977746E-04
26	0.2389964E-04	-0.5938938E-04
27	0.2423119E-04	-0.5925143E-04
28	0.2844953E-04	-0.6168136E-04
29	0.2658207E-04	-0.5780648E-04
30	0.2247616E-04	-0.6021703E-04
31	0.1876013E-04	-0.6115036E-04
32	0.1514759E-04	-0.5847332E-04
33	0.1365870E-04	-0.5532583E-04
34	0.1303833E-04	-0.5063787E-04
35	0.2058300E-04	-0.4909880E-04
36	0.2709571E-04	-0.5089105E-04
37	-0.7503449E-04	0.1868667E-04
38	-0.7547782E-04	0.1909655E-04
39	-0.7528650E-04	0.1967674E-04
40	-0.7482916E-04	0.1923414E-04
41	-0.6977622E-04	0.1656040E-04
42	-0.7018668E-04	0.1595268E-04
43	-0.7170392E-04	0.1384387E-04
44	-0.7350495E-04	0.1417595E-04
45	-0.7415403E-04	0.1467273E-04
46	-0.7552192E-04	0.1728082E-04
47	-0.7542223E-04	0.1994308E-04
48	-0.7541230E-04	0.2058662E-04
49	-0.7441276E-04	0.2205503E-04
50	-0.7223988E-04	0.2142812E-04
51	-0.7190136E-04	0.2108798E-04
52	-0.7024517E-04	0.1912231E-04
53	-0.6025365E-04	0.1398417E-04
54	-0.6007355E-04	0.1339358E-04
55	-0.6298206E-04	0.7761235E-05
56	-0.6763615E-04	0.7972555E-05
57	-0.6766693E-04	0.8412399E-05
58	-0.7118998E-04	0.1372382E-04
59	-0.7211353E-04	0.1909457E-04
60	-0.7171884E-04	0.1958055E-04
61	-0.7080012E-04	0.2278830E-04
62	-0.6637999E-04	0.2207895E-04
63	-0.6560005E-04	0.2172442E-04
64	-0.6169718E-04	0.1865299E-04
65	-0.4972055E-04	0.1358254E-04
66	-0.5232151E-04	0.4267412E-05
67	-0.5908249E-04	0.3649163E-05
68	-0.6423799E-04	0.1093262E-04
69	-0.6676753E-04	0.1807397E-04
70	-0.6637117E-04	0.2272523E-04
71	-0.5974030E-04	0.2240085E-04
72	-0.5311336E-04	0.1929210E-04
73	0.5302667E-04	0.5427793E-04
74	0.5255351E-04	0.5455755E-04
75	0.5294487E-04	0.5495241E-04
76	0.5343914E-04	0.5464497E-04
77	0.5376653E-04	0.5180531E-04
78	0.5343313E-04	0.5157210E-04
79	0.5140928E-04	0.5054972E-04
80	0.4927910E-04	0.5033291E-04
81	0.4892115E-04	0.5072504E-04
82	0.4755934E-04	0.5230293E-04
83	0.4757379E-04	0.5436780E-04
84	0.4807820E-04	0.5497463E-04
85	0.5040815E-04	0.5607624E-04
86	0.5292337E-04	0.5527602E-04
87	0.5359133E-04	0.5505210E-04
88	0.5490209E-04	0.5361504E-04
89	0.5156861E-04	0.4693355E-04
90	0.5098287E-04	0.4626505E-04
91	0.4748162E-04	0.4402346E-04
92	0.4393222E-04	0.4277664E-04
93	0.4341021E-04	0.4260516E-04
94	0.4011510E-04	0.4543304E-04
95	0.3905874E-04	0.5090880E-04
96	0.3920773E-04	0.5104370E-04
97	0.4402752E-04	0.5414993E-04
98	0.4994623E-04	0.5317762E-04
99	0.5030703E-04	0.5261040E-04
100	0.5368194E-04	0.5048068E-04
101	0.4849212E-04	0.4099779E-04
102	0.4410636E-04	0.3703449E-04
103	0.3988302E-04	0.3373862E-04
104	0.3454571E-04	0.3634844E-04
105	0.3105048E-04	0.4466130E-04
106	0.3738633E-04	0.4928622E-04
107	0.4612388E-04	0.4980232E-04
108	0.5138021E-04	0.4656671E-04
109	-0.1174963E-04	-0.1885048E-04
110	-0.9550354E-05	-0.2223783E-04
111	-0.6795029E-05	-0.2522457E-04
112	-0.5261974E-05	-0.2524750E-04
113	-0.3592918E-05	-0.2394349E-04
114	-0.1418109E-04	-0.1199234E-04
115	-0.1008114E-04	-0.2082574E-04
116	-0.3447701E-05	-0.2958199E-04
117	0.3506665E-05	-0.3612273E-04
118	0.5495991E-05	-0.3221881E-04
119	0.4630295E-05	-0.2620999E-04
120	0.1634588E-05	-0.2250998E-04
121	-0.1968119E-04	-0.3398555E-05
122	-0.1584075E-04	-0.1846973E-04
123	-0.1607782E-05	-0.3725429E-04
124	0.2213313E-04	-0.3752168E-04
125	0.1956975E-04	-0.2008384E-04
126	0.1308228E-04	-0.1650497E-04
127	-0.3072103E-04	0.8029099E-05
128	-0.3970531E-04	0.1354327E-07
129	-0.3242900E-04	-0.1081243E-04
130	0.6570077E-05	-0.1545360E-04
131	0.3131537E-04	0.5985237E-05
132	0.3275272E-04	0.1037451E-04
133	0.2517056E-04	-0.3757546E-05
134	-0.4517997E-04	0.1960793E-04
135	-0.6099144E-04	0.1909187E-04
136	-0.5509934E-05	0.2335454E-04
137	0.4698463E-04	0.3547831E-04
138	0.4041029E-04	0.1756998E-04
139	-0.5285608E-04	0.2471686E-04
140	-0.5785342E-04	0.2466408E-04
141	-0.5129534E-04	0.2278053E-04
142	-0.4345179E-05	0.2872189E-04
143	0.3965749E-04	0.4561833E-04
144	0.4905291E-04	0.3650431E-04
145	0.4748936E-04	0.2960073E-04
146	-0.5283093E-04	0.2723758E-04
147	-0.3534340E-04	0.2752054E-04
148	-0.3032082E-06	0.2827052E-04
149	0.3467618E-04	0.3572096E-04
150	0.4807688E-04	0.3340235E-04
151	-0.6115885E-05	-0.2978557E-05
152	-0.7883436E-05	0.3845040E-05
153	-0.5881177E-05	0.7160637E-05
154	-0.6662892E-05	0.2594723E-05
155	-0.4363171E-05	0.7907391E-05
156	-0.5246294E-05	0.3921577E-05
157	-0.6490844E-06	0.8143371E-05
158	0.2238269E-05	0.5916213E-05
159	0.3974513E-05	0.5146345E-05
160	0.1044651E-04	0.6834745E-05
161	0.1282604E-04	0.1016694E-04
162	0.1422978E-04	0.8288664E-05
163	0.7547858E-05	-0.7744968E-05
164	0.2138265E-05	-0.1590262E-04
165	0.1382640E-05	-0.1576041E-04
166	-0.4467716E-05	-0.9655951E-05
167	-0.3408685E-05	0.2140952E-05
168	-0.7085554E-06	0.5104485E-05

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82-0.6907816E-05-0.2148639E-07
83-0.2255287E-05 0.3437653E-05
84 0.1123270E-05 0.5782084E-05
85 0.4546016E-05 0.3673013E-05
86 0.4988786E-07 0.2394399E-05
87 0.4707889E-05 0.3671160E-05
88 0.8809272E-06 0.2128425E-05
89 0.4753923E-05 0.1761605E-05
90 0.3548766E-05-0.1302837E-05
91 0.2662018E-05-0.1563421E-05
92 0.3704685E-05-0.4943848E-05
93 0.3725723E-05-0.5688313E-05
94 0.4138608E-05-0.6062377E-05
95-0.4154281E-05-0.5954216E-05
96-0.8376245E-05-0.5545003E-06
97-0.6652137E-05-0.1182942E-06
98-0.3502532E-05 0.2681416E-05
   0.0009312      0.0009312     0.0009312     0.0055842

CV A=( 0.2243061D-04,-0.6436060D-04)
.            RAW POTENTIAL                        RESIDUAL                        ADJUSTED POTENTIAL

CV A=(-0.6675212D-04, 0.1618062D-04)
.            RAW POTENTIAL                        RESIDUAL                        ADJUSTED POTENTIAL

CV A=( 0.4665377D-04, 0.4839661D-04)
.            RAW POTENTIAL                        RESIDUAL                        ADJUSTED POTENTIAL

CV A=(-0.1198196D-05,-0.1113739D-06)
.            RAW POTENTIAL                        RESIDUAL                        ADJUSTED POTENTIAL
10  62  40  62
   D.C. LOSS                         IMAGINARY CROSS TERMS                                  TOTAL   LOSS
18.66217     0.0           5.733850      -16.05336        5.733850        16.05336       2.297598     0.4440892D-15
TIO OF AC TO DC RESISTANCE IS =  1.843533
DY LOSS
15.74215     0.0
8.66217      0.9311668D-03
10 124 102 124
   D.C. LOSS                         IMAGINARY CROSS TERMS                                  TOTAL   LOSS
18.66134     0.8326673D-16  4.755439      -16.33920        4.755439        16.33920       1.711315     0.8326673D-16
TIO OF AC TO DC RESISTANCE IS =  1.842832
DY LOSS
15.72838     0.0
8.66134      0.9311668D-03
10 186 164 186
   D.C. LOSS                         IMAGINARY CROSS TERMS                                  TOTAL   LOSS
13.36134    -0.8326673D-16  4.582646      -16.24863        4.582646        16.24863       1.625987    -0.8326673D-16
TIO OF AC TO DC RESISTANCE IS =  1.828553
DY LOSS
15.46190     0.0
8.66134      0.9311668D-03
   D.C. LOSS                         IMAGINARY CROSS TERMS                                  TOTAL   LOSS
.0           0.0           0.0            0.0              0.0            0.0            1.712966     0.0
DY LOSS
1.712966     0.0
0            0.5584230D-02
TIO OF AC TO DC RESISTANCE IS =  1.868903
RESS=  71.83008        NEW MU=  171.0753        FOR ELEMENT 279
RESS=  37.33987        NEW MU=  142.0694        FOR ELEMENT 280
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STRESS= 127.4958	NEW MU= 214.0653	FOR ELEMENT 281
STRESS= 38.68434	NEW MU= 144.3552	FOR ELEMENT 282
STRESS= 145.2086	NEW MU= 227.4401	FOR ELEMENT 283
STRESS= 48.68719	NEW MU= 159.1536	FOR ELEMENT 284
STRESS= 105.8623	NEW MU= 197.5254	FOR ELEMENT 285
STRESS= 73.51547	NEW MU= 172.3976	FOR ELEMENT 286
STRESS= 73.62625	NEW MU= 172.4845	FOR ELEMENT 287
STRESS= 99.92375	NEW MU= 192.9478	FOR ELEMENT 288
STRESS= 45.32510	NEW MU= 155.3979	FOR ELEMENT 289
STRESS= 132.2921	NEW MU= 217.7034	FOR ELEMENT 290
STRESS= 33.62444	NEW MU= 133.8670	FOR ELEMENT 291
STRESS= 106.9982	NEW MU= 198.3991	FOR ELEMENT 292
STRESS= 31.92755	NEW MU= 129.7661	FOR ELEMENT 293
STRESS= 73.95512	NEW MU= 172.7424	FOR ELEMENT 294
STRESS= 47.29199	NEW MU= 157.7250	FOR ELEMENT 295
STRESS= 144.4592	NEW MU= 226.8857	FOR ELEMENT 296
STRESS= 75.22522	NEW MU= 173.7377	FOR ELEMENT 297
STRESS= 266.8318	NEW MU= 315.4939	FOR ELEMENT 298
STRESS= 125.7872	NEW MU= 212.7567	FOR ELEMENT 299
STRESS= 180.8804	NEW MU= 253.9697	FOR ELEMENT 300
STRESS= 345.9727	NEW MU= 369.1721	FOR ELEMENT 301
STRESS= 103.1828	NEW MU= 195.4620	FOR ELEMENT 302
STRESS= 193.7197	NEW MU= 263.3735	FOR ELEMENT 303
STRESS= 81.79846	NEW MU= 178.8774	FOR ELEMENT 304
STRESS= 82.08598	NEW MU= 179.1017	FOR ELEMENT 305
STRESS= 213.7988	NEW MU= 277.9294	FOR ELEMENT 306
STRESS= 111.1319	NEW MU= 201.5738	FOR ELEMENT 307
STRESS= 339.9319	NEW MU= 365.1753	FOR ELEMENT 308
STRESS= 170.5804	NEW MU= 246.3715	FOR ELEMENT 309
STRESS= 117.9784	NEW MU= 206.8150	FOR ELEMENT 310
STRESS= 236.8586	NEW MU= 294.4204	FOR ELEMENT 311
STRESS= 59.23871	NEW MU= 165.0884	FOR ELEMENT 312
STRESS= 110.3962	NEW MU= 201.0094	FOR ELEMENT 313
STRESS= 43.46880	NEW MU= 152.8562	FOR ELEMENT 314
STRESS= 58.51462	NEW MU= 164.8323	FOR ELEMENT 315
STRESS= 46.64305	NEW MU= 156.9985	FOR ELEMENT 316
STRESS= 70.32591	NEW MU= 169.8941	FOR ELEMENT 317
STRESS= 58.62373	NEW MU= 164.9084	FOR ELEMENT 318
STRESS= 44.73853	NEW MU= 154.6315	FOR ELEMENT 319
STRESS= 70.25758	NEW MU= 169.8405	FOR ELEMENT 320
STRESS= 65.05496	NEW MU= 166.4934	FOR ELEMENT 321
STRESS= 44.86511	NEW MU= 154.7997	FOR ELEMENT 322
STRESS= 42.03851	NEW MU= 150.6653	FOR ELEMENT 323
STRESS= 64.97025	NEW MU= 166.4707	FOR ELEMENT 324
STRESS= 54.39516	NEW MU= 163.2638	FOR ELEMENT 325
STRESS= 42.18492	NEW MU= 150.8997	FOR ELEMENT 326
STRESS= 46.64752	NEW MU= 157.0035	FOR ELEMENT 327
STRESS= 54.45804	NEW MU= 163.3006	FOR ELEMENT 328
STRESS= 44.60327	NEW MU= 154.4502	FOR ELEMENT 329
STRESS= 46.77179	NEW MU= 157.1458	FOR ELEMENT 330
STRESS= 89.91017	NEW MU= 185.1929	FOR ELEMENT 331
STRESS= 44.71756	NEW MU= 154.6035	FOR ELEMENT 332
STRESS= 122.2216	NEW MU= 210.0525	FOR ELEMENT 333
STRESS= 89.73624	NEW MU= 185.0578	FOR ELEMENT 334
STRESS= 115.3485	NEW MU= 204.8042	FOR ELEMENT 335
STRESS= 122.0784	NEW MU= 209.9433	FOR ELEMENT 336
STRESS= 73.25615	NEW MU= 172.1943	FOR ELEMENT 337
STRESS= 115.6422	NEW MU= 205.0290	FOR ELEMENT 338
STRESS= 122.0711	NEW MU= 209.9378	FOR ELEMENT 339
STRESS= 73.59059	NEW MU= 172.4565	FOR ELEMENT 340
STRESS= 115.6781	NEW MU= 205.0564	FOR ELEMENT 341
STRESS= 121.9252	NEW MU= 209.8266	FOR ELEMENT 342
STRESS= 78.08469	NEW MU= 175.9760	FOR ELEMENT 343
STRESS= 115.8076	NEW MU= 205.1555	FOR ELEMENT 344
STRESS= 46.31653	NEW MU= 156.6176	FOR ELEMENT 345
STRESS= 78.40227	NEW MU= 176.2243	FOR ELEMENT 346
STRESS= 14.62684	NEW MU= 84.54848	FOR ELEMENT 347
STRESS= 14.07843	NEW MU= 83.35202	FOR ELEMENT 348
STRESS= 18.53449	NEW MU= 93.98636	FOR ELEMENT 349
STRESS= 14.85061	NEW MU= 85.04735	FOR ELEMENT 350
STRESS= 9.907484	NEW MU= 75.72661	FOR ELEMENT 351
STRESS= 18.53824	NEW MU= 93.99654	FOR ELEMENT 352
STRESS= 15.77183	NEW MU= 87.16144	FOR ELEMENT 353
STRESS= 9.959862	NEW MU= 75.80193	FOR ELEMENT 354
STRESS= 9.771969	NEW MU= 75.53052	FOR ELEMENT 355
STRESS= 15.76579	NEW MU= 87.14726	FOR ELEMENT 356
STRESS= 12.95493	NEW MU= 81.02592	FOR ELEMENT 357
STRESS= 9.837185	NEW MU= 75.62442	FOR ELEMENT 358
STRESS= 12.55284	NEW MU= 80.23770	FOR ELEMENT 359
STRESS= 13.08832	NEW MU= 81.29277	FOR ELEMENT 360

Three conductors in air using a computed boundary condition and a real and imaginary formulation with a conjugate gradient solution routine.

```
      INTEGER NODE(312,3),NSEG(3)
      DIMENSION CVAR(4),CVAI(4)
      DIMENSION CVR(3),CVI(3),CVR(350),CVI(175),CBR(350)
      DIMENSION A(350)
      DIMENSION RADIUS(16),FDCUF(3,3),LOC(3000),NODES(3),NSEGM(3)
     .,T(3),VAL(3000),JEND(3),Q(3),R(3),S(3,3),NLCON(3),NCOD(3)
      REAL X(225),Y(225),RADI(3),SIGMM(350),XO(3),YO(3)
      REAL MU(350),MU1,ATOT(4),MU0
      COMPLEX BIGJ(350),BIGJ1,BIGJ2,CSIN,DP,DQ,DR,DS,CPTOT
      COMPLEX CTRA,CTRB,CTRC, CZZ ,CONJG,RHS,CBOUN(32),CTEMP
      COMPLEX CMPLX,CT(3)
      DIMENSION XT(3),YT(3)
C (I) READ IN SPECIFICATIONS OF NODE PATTERN:
      REWIND2
      READ(2) NDTOT,JFIN,NSOUT,NPHMAX,NTRIA,(NODES(L),L=1,3),(NSEGM(L),
     .L=1,3),(JEND(L),L=1,3) ,(RADI(L),L=1,3),(XO(L),L=1,3),(YO(L),L=1,3
     .)
      READ(2) ((NODE(J,L),L=1,3),J=1,JFIN),(SIGMM(J),J=1,JFIN)
      READ(2) (X(I),I=1,NDTOT)
      READ(2) (Y(I),I=1,NDTOT)
      READ(2) NXRNG,(NSEG(J),J=1,NXRNG)
      WRITE(6,444)NDTOT,JFIN,NSOUT,NPHMAX,NTRIA,(NODES(L),L=1,NPHMAX),
     .(NSEGM(L),L=1,NPHMAX),(JEND(L),L=1,3),(RADI(L),L=1,3)
  444 FORMAT(' ',14(I3),3(F7.4))
      NLMNTS=JFIN
      NBEG=1
      NUMOUT=NDTOT-NSOUT
      RPT=15.D0*2.5401D-2
      PI=3.14159265359
      MU0=.0000012570D0
      DTHETA=2.D0*PI/NSOUT
      THETA=0.D0
C       COMPUTE APPROXIMATE BOUNDARY VALUES
      DO749 J=1,NSOUT
      CBOUN(J)=-MU0/(2.D0*PI)*(ALOG( SQRT((RPT* COS(THETA))**2+(RPT*
     . SIN(THETA)-.0587   )**2))+ CMPLX(-.5   ,.866   )*ALOG( SQRT((RPT*
     . COS(THETA)-.0499   )**2+(RPT* SIN(THETA)+.0293   )**2))+
     . CMPLX(-.5   ,-.866   )*ALOG ( SQRT((RPT* COS(THETA)+.0499   )**2
     .+(RPT* SIN(THETA)+.0293   )**2)))
      CBOUN(J)=CBOUN(J)*PI*(RADI(1)*.025401)**2
      WRITE(6,247) J,CBOUN(J)
  749 THETA=THETA+DTHETA
      DO 777 L=1,NPHMAX
      ATOT(L)=0.
C       READ  NUMBER COND. RINGS, PERMEABILITY, AND COND. AND INSULATOR
C       CURRENT
      READ(5,22)NCON,MU1,BIGJ1,BIGJ2
      NCOD(L)=NCON
   22 FORMAT(I2,F10.9,4G10.5)
      WRITE(6,33)NCON,MU1,BIGJ1,BIGJ2
   33 FORMAT(15X,'NUMBER OF RINGS WITHIN THE CONDUCTOR:',I5/5X,
     1'MAGNETIC PERMEABILITY:',G12.5/5X,'CURRENT DENSITY:'/10X,'WITHIN T
     2HE CONDUCTOR:',2(G12.5)/10X,'WITHIN THE INSULATION:',2(G12.5)/)
      NLCON(L)=(2*NCOD(L)-1)*NSEGM(L)
      IF(L.NE.1) NLCON(L)=NLCON(L)+JEND(L-1)
      IF(L.NE.1) NBEG=JEND(L-1)+1
      NFI1=NLCON(L)
      NFI2=JEND(L)
C (IV) GET VALUES OF PERMEABILITY AND CURRENT DENSITY FOR EACH ELEMENT
      CALL MUANDJ(MU,BIGJ,NBEG,NFI1,NFI2,MU1,BIGJ1,BIGJ2)
  110 CONTINUE
  777 CONTINUE
C       SPECIFY PIPE CURRENT AND PERMEABILITY
      BIGJ2=(0.D0,0.D0)
      BIGJ1=(0.D0,0.D0)
      NBEG=JEND(NPHMAX)+1
      NFI1=JFIN-NTRIA
      NFI2=JFIN
      CALL MUANDJ(MU,BIGJ,NBEG,NFI1,NFI2,MU1,BIGJ1,BIGJ2)
C (V) SET SM AND RM TO ZERO. RM(I)=CSM(I+NPOTSQ).
C       IFPIPE IS 0 FOR NO PIPE. ELSE NONZERO INTEGER
      READ(5,355)IFPIPE
      NFI1=NFI1+1
      NOTST=0
      IF(IFPIPE.NE.0) NOTST=IFPIPE/IFPIPE
      NPOT=NDTOT+NSOUT*(NOTST-1)
      NPOTSQ=NPOT*NPOT
      NPO2=2*NPOT
```

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      NPOTS2=NPO2*NPO2
      N2PSQ=2*NPOTSQ
  955 FORMAT(I1)
      NSFAR=1
C      INITIALIZE LOCATION, VALUE AND SOLUTION ARRAY
      DO1001J=1,2200
      LOC(J)=0
      VAL(J)=0.
      IF(J.GT.300)GOTO1001
      CBR(J)=0.
 1001 CONTINUE
C
C      FORMULATE FINITE ELEMENT MATRIX ENTRIES
C
      DO 100 N=1,NLMNTS
      XCENT=(X(NODE(N,1))+X(NODE(N,2))+X(NODE(N,3)))/3.D0
      YCENT=(Y(NODE(N,1))+Y(NODE(N,2))+Y(NODE(N,3)))/3.D0
      X1=X(NODE(N,1))-XCENT
      X2=X(NODE(N,2))-XCENT
      X3=X(NODE(N,3))-XCENT
      Y1=Y(NODE(N,1))-YCENT
      Y2=Y(NODE(N,2))-YCENT
      Y3=Y(NODE(N,3))-YCENT
C  (1) CALCULATE Q(I) AND R(I)
      AREA=    (X(NODE(N,1))*(Y(NODE(N,3))-Y(NODE(N,2)))
     C         +X(NODE(N,2))*(Y(NODE(N,1))-Y(NODE(N,3)))
     C         +X(NODE(N,3))*(Y(NODE(N,2))-Y(NODE(N,1))))*5.D-1
      J=2
      K=3
      DO 8 I=1,3
      Q(I)=(Y(NODE(N,K))-Y(NODE(N,J)))
      R(I)=(X(NODE(N,J))-X(NODE(N,K)))
      T(I)=X(NODE(N,K))*Y(NODE(N,J))-X(NODE(N,J))*Y(NODE(N,K))
      J=J+1-J/3*3
    8 K=K+1-K/3*3
C  (2) CALCULATE ELEMENT MATRIX ENTRIES
C      COMPUTE CONDUCTOR AREAS
      A(N)=AREA
      IF(N.GT.JEND(NPHMAX))GOTO29
      DO950L=1,3
      IF(N.GT.NLCON(L).AND.N.LE.JEND(L))GOTO29
      IF(N.GT.NLCON(L))GOTO950
      ATOT(L)=ATOT(L)+AREA
      GOTO29
  950 CONTINUE
   29 CONTINUE
      DO 9 I=1,3
      DO 9 J=1,3
      EDCUR(I,J)=0.0
      S(I,J)=(Q(I)*Q(J)+R(I)*R(J))/(AREA*4.)
      S(I,J)=S(I,J)/MU(N)
      IF(SIGMM(N).EQ.0.) GO TO 9
      EDCUR(I,J)=(60.D0*PI*MU(N)*SIGMM(N)*(T(I)*T(J)+XCENT*(Q(I)
     .*T(J)+T(I)*Q(J))+YCENT*(R(I)*T(J)+T(I)*R(J))+(Q(I)*R(J)+R(I)*Q(J))
     .*(XCENT*YCENT+(X1*Y1+X2*Y2+X3*Y3)/12.D0)+Q(I)*Q(J)*(XCENT**2+(
     .X1**2+X2**2+X3**2)/12.D0)+R(I)*R(J)*(YCENT**2+(Y1**2+Y2**2+Y3**2)
     ./12.D0)) )/(AREA*2.D0)
      EDCUR(I,J)=EDCUR(I,J)/MU(N)
    9 CONTINUE
C  (3) CALCULATE RIGHT HAND SIDE OF ELEMENT MATRIX
      RHS=BIGJ(N)*AREA/3.D0*MU(N)
      RHS=RHS/MU(N)
C  (4) ADD ENTRIES INTO BIG MATRIX EQUATION:  (SM)(A)=(RM)
      IF(N.EQ.(JEND(NPHMAX)+1)) NSFIN=NSFAR-4
      IF(N.EQ.1) GO TO1857
      IF((SIGMM(N).NE.0.).AND.(SIGMM(N-1).EQ.0.))NSNOW=NSFAR
 1857 CONTINUE
      DO 10 I=1,3
      IF(NODE(N,I).GT.NPOT)GO TO 10
      DO 11 J=1,3
      IF(NODE(N,J).GT.NPOT) GO TO 211
C      CHECK FOR UNSTORED ENTRIES
      IF(NODE(N,J).GT.NPOT.OR.NODE(N,I).GT.NODE(N,J).OR.ABS(S(I,J)).
     1LT.1.E-30)GOTO11
C COMPUTE LOCATION WHERE NON-ZERO VALUE OCCURS
      LOCTMP=(NODE(N,I)-1)*NPO2+NODE(N,J)
      IF(N.LE.JEND(NPHMAX)) NSFIN=NSFAR-4
  957 NSSTR=NSFIN+4
```

```
      IF(I.EQ.J) EDCUR(I,J)=EDCUR(I,J)/2.
      IF(NSFIN.LT.1) GO TO 852
C      CHECK TO SEE IF ENTRY FOR THIS LOCATION ALREADY EXISTS
      DO 850 NN=1,NSFIN,4
      K=NN
      IF(LOC(NN).EQ.LOCTMP)GOTO851
  850 CONTINUE
      IF(N.LE.JEND(NPHMAX)) GO TO 852
      IF(SIGMM(N).NE.0.) GO TO 1855
      IF(NSSTR.EQ.NSFAR) GO TO 854
      NSFER=NSFAR-2
      DO 853 NN=NSSTR,NSFER,2
      K=NN
      IF(LOC(NN).EQ.LOCTMP) GO TO 851
  853 CONTINUE
      GO TO 854
 1855 IF(NSNOW.EQ.NSFAR) GO TO 852
      NSFEF=NSFAR-4
      DO 856 NN=NSNOW,NSFEF,4
      K=NN
      IF(LOC(NN).EQ.LOCTMP) GO TO 851
  856 CONTINUE
C
C      ADD NEW ENTRIES TO MASTER MATRIX -CONDUCTOR
C
  852 LOC(NSFAR)=LOCTMP
      VAL(NSFAR)=S(I,J)
      NSFAR=NSFAR+1
      LOC(NSFAR)=LOCTMP+N2PSQ+NPOT
      VAL(NSFAR)=-S(I,J)
      NSFAR=NSFAR+1
      LOC(NSFAR)=LOCTMP+NPOT
      VAL(NSFAR)=-EDCUR(I,J)
      NSFAR=NSFAR+1
      LOC(NSFAR)=(NODE(N,J)-1)*NP02+NODE(N,I)+NPOT
      VAL(NSFAR)=-EDCUR(I,J)
      NSFAR=NSFAR+1
      GO TO 11
C
C      ADD TO EXISTING MATRIX ENTRIES
C
  851 VAL(K)=VAL(K)+S(I,J)
      VAL(K+1)=VAL(K+1)-S(I,J)
      IF(SIGMM(N).EQ.0.) GO TO 11
      VAL(K+2)=VAL(K+2)-EDCUR(I,J)
      VAL(K+3)=VAL(K+3)-EDCUR(I,J)
      GOTO11
C
C      ADD NEW ENTRIES TO MASTER MATRIX -INSULATOR
C
  854 LOC(NSFAR)=LOCTMP
      VAL(NSFAR)=S(I,J)
      LOC(NSFAR+1)=LOCTMP+NPOT+N2PSQ
      VAL(NSFAR+1)=-S(I,J)
      NSFAR=NSFAR+2
      GO TO 11
C
C      INSERT BOUNDARY CONDITIONS INTO MATRIX
C
  211 CTEMP=CBCUN(NODE(N,J)-NUMCUT)
      CBR(NODE(N,I)+NPOT)=CBR(NODE(N,I)+NPOT)+S(I,J)*AIMAG(CTEMP)
      CBR(NODE(N,I))=CBR(NODE(N,I))-S(I,J)*REAL(CTEMP)
   11 CONTINUE
C
C      INSERT SOURCE ENTRIES
C
      CBR(NODE(N,I))=CBR(NODE(N,I))+REAL(RHS)
      CBR(NODE(N,I)+NPOT)=CBR(NODE(N,I)+NPOT)-AIMAG(RHS)
   10 CONTINUE
      IF(N.EQ.JEND(NPHMAX)) NSFIN=NSFAR-4
  100 CONTINUE
      NSFAR=NSFAR-1
      WRITE(6,503) (ATCT(L),L=1,3)
      IER=0
      TOL=1.D-7
      WRITE(6,450) NSFAR
  450 FORMAT(' ','THE NUMBER OF NON-ZERO ENTRIES IS',I6)
C
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```
C       CALL CONJUGATE GRADIENT SOLUTION ROUTINE
C
      CALL CHGRH(VAL,LCC,NSEAP,CVR,CBR,NPO2,TCL,IER)
      IF(IER.EQ.0)GOTO333
      WRITE(6,222)
  222 FORMAT('    CHECK MATRIX CSM FOR ERROR  ')
  333 CONTINUE
      L=1
      NBEG=1
C
C       CALCULATE AVERAGE POTENTIAL VALUES
C
      DO 246 LL=1,3
      L=LL
      CVAR(LL)=0.
      CVAI(LL)=0.
      IF(LL.NE.1) NBEG=JEND(LL-1)+1
      NFIN=NLCCN(LL)
      DO245N=NBEG,NFIN
      AREA=A(N)
      DO 242 I=1,3
      IF(NODE(N,I).LE.NPCT) GO TO 645
      CVR(NODE(N,I))=REAL(CBOUN(NODE(N,I)-NUMOUT))
      CVR(NODE(N,I)+NPCT)=AIMAG(CBOUN(NODE(N,I)-NUMOUT))
  645 CONTINUE
      CVAR(L)=CVAR(L)+CVR(NODE(N,I))*AREA/3.0
  242 CVAI(L)=CVAI(L)+CVR(NODE(N,I)+NPOT)*AREA/3.
  245 CONTINUE
      CVAR(L)=CVAR(L)/ATOT(L)
      CVAI(L)=CVAI(L)/ATOT(L)
      WRITE(6,247) L,CVAR(L),CVAI(L)
  246 CONTINUE
C
C       CALCULATE AVERAGE VALUE FOR PIPE
C
      ATOT(4)=0.0
      CVAR(4)=0.0
      CVAI(4)=0.
      IF(IFPIPE.EQ.0) GO TO 443
      DO 441 N=NFI1,JFIN
      ATOT(4)=ATOT(4)+A(N)
      DO 442 K=1,3
      CVAR(4)=CVAR(4)+A(N)*CVR(NODE(N,K))
  442 CVAI(4)=CVAI(4)+CVR(NODE(N,K)+NPOT)*A(N)
  441 CONTINUE
      CVAR(4)=CVAR(4)/ATOT(4)
      CVAI(4)=CVAI(4)/ATOT(4)
  443 LPI=4
      WRITE(6,247)LPI,CVAR(4),CVAI(4)
      WRITE(6,800)
  800 FORMAT(' ','VECTOR POTENTIAL VALUES')
      DO 212 I=1,NPOT
      CVI(I)=CVR(I+NPOT)
  212 WRITE(6,247) I,CVR(I),CVR(I+NPOT)
      L=1
  247 FORMAT(' ',I3,2G14.7)
      NEND=NPHMAX+1
      NBEG=1
C
C       COMPUTE OVERALL AVERAGE POTENTIAL
C
      CVAR(1)=CVAR(1)*ATOT(1)+CVAR(2)*ATOT(2)+CVAR(3)*ATOT(3)+CVAR(4)*
     *ATOT(4)
      CVAR(1)=CVAR(1)/(ATOT(1)+ATOT(2)+ATOT(3)+ATOT(4))
      CVAI(1)=CVAI(1)*ATOT(1)+CVAI(2)*ATOT(2)+CVAI(3)*ATOT(3)+CVAI(4)*
     *ATOT(4)
      CVAI(1)=CVAI(1)/(ATOT(1)+ATOT(2)+ATOT(3)+ATOT(4))
C
C       CALCULATE LOSS RATIO
C
      DO401J=1,NEND
      DP=(0.D0,0.D0)
      DQ=(0.D0,0.D0)
      DR=(0.D0,0.D0)
      DS=(0.D0,0.D0)
      IF((IFPIPE.EQ.0).AND.(J.EQ.NEND))GOTO401
      IF(J.NE.1)NBEG=JEND(J-1)+1
      IF(J.EQ.NEND)NBEG=JFIN-NTFIA+1
```

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      IF(J.NE.NEND)NFIN=NLCEN(J)
      IF(J.EQ.NEND)NFIN=JFIN
      DO400N=NBEG,NFIN
      AREA=A(N)
      XCENT=(X(NODE(N,1))+X(NODE(N,2))+X(NODE(N,3)))/3.D0
      YCENT=(Y(NODE(N,1))+Y(NODE(N,2))+Y(NODE(N,3)))/3.D0
      X1=X(NODE(N,1))-XCENT
      X2=X(NODE(N,2))-XCENT
      X3=X(NODE(N,3))-XCENT
      Y1=Y(NODE(N,1))-YCENT
      Y2=Y(NODE(N,2))-YCENT
      Y3=Y(NODE(N,3))-YCENT
      CWR(1)=CVR(NODE(N,1))-CVAR(L)
      CWI(1)=CVI(NODE(N,1))-CVAI(L)
      CWR(2)=CVR(NODE(N,2))-CVAR(L)
      CWI(2)=CVI(NODE(N,2))-CVAI(L)
      CWR(3)=CVR(NODE(N,3))-CVAR(L)
      CWI(3)=CVI(NODE(N,3))-CVAI(L)
      CWIT=CWI(1)+CWI(2)+CWI(3)
      CWRT=CWR(1)+CWR(2)+CWR(3)
      CTRA=(CMPLX(CWR(1),CWI(1)))*(X(NODE(N,3))*Y(NODE(N,2))-X(NODE(N,
     C2))*Y(NODE(N,3)))+(CMPLX(CWR(2),CWI(2)))*(X(NODE(N,1))*Y(NODE(N,
     C3))-X(NODE(N,3))*Y(NODE(N,1)))+(CMPLX(CWR(3),CWI(3)))*(X(NODE(N,
     C2))*Y(NODE(N,1))-X(NODE(N,1))*Y(NODE(N,2)))
      CTRB=(CMPLX(CWR(1),CWI(1)))*(Y(NODE(N,3))-Y(NODE(N,2)))+(CMPLX(C
     CWR(2),CWI(2)))*(Y(NODE(N,1))-Y(NODE(N,3)))+(CMPLX(CWR(3),CWI(
     C3)))*(Y(NODE(N,2))-Y(NODE(N,1)))
      CTRC=(CMPLX(CWR(1),CWI(1)))*(X(NODE(N,2))-X(NODE(N,3)))+(CMPLX(CW
     CR(2),CWI(2)))*(X(NODE(N,3))-X(NODE(N,1)))+(CMPLX(CWR(3),CWI(
     C3)))*(X(NODE(N,1))-X(NODE(N,2)))
      VP=((CABS(CTRB))**2*((X1**2+X2**2+X3**2)/12.+XCENT**2)+(CABS(
     CCTRA))**2+(CABS(CTRC))**2*(((Y1**2+Y2**2+Y3**2)/12.)+YCENT**2)
     C+(CTRB*CONJG(CTRC)+CTRC*CONJG(CTRB))*(((X1*Y1+X2*Y2+X3*Y3)
     C/12.)+XCENT*YCENT)+CTRB*CONJG(CTRA)*XCENT+CTRA*CONJG(CTRB)*
     CXCENT+CTRA*CONJG(CTRC)*YCENT+CTRC*CONJG(CTRA)*YCENT)/(AREA*4.)
      DQ=DQ+AREA/SIGMM(N)*BIGJ(N)*CONJG(BIGJ(N))
      DS=DS+40.*PI*AREA*BIGJ(N)*CMPLX(CWRT,-CWIT)
      DR=DR+40.*PI*AREA*CMPLX(CWRT,CWIT)*CONJG(BIGJ(N))
      DP=DP+3600.D0*SIGMM(N)*VP*4.D0*PI*PI
  400 CONTINUE
      CPTOT=DP+DQ+ CMPLX(0.00,1.00)*(DS-DR)
      WRITE(6,503) DQ,DR,DS,CPTOT
      PDC=            (PI*RADI(1)*RADI(1)*(2.5401D-2**2))/SIGMA
      PDC=DQ
      RACDC=(CPTOT+ CONJG(CPTOT))/(2.D0*PDC)
  503 FORMAT(' ',8(G14.7))
      RACDC=(DP+PDC)/PDC
      WRITE(6,500) RACDC
      WRITE(6,501) DP,PDC
  401 CONTINUE
  500 FORMAT(///' ','RATIO OF AC TO DC RESISTANCE IS ',G20.8///)
  501 FORMAT(' ',2G20.8)
      IF(IFPIPE.EQ.0) GO TO 9999
C
C      CALCULATE NEW PIPE PERMEABILITY
C
      DO 16 J=NFI1,JFIN
      DO 17 K=1,3
      XT(K)=X(NODE(J,K))
      YT(K)=Y(NODE(J,K))
   17 CT(K)=CMPLX(CVR(NODE(J,K)),CVI(NODE(J,K)))
      CALL MUADJS(XT,YT,CT,VAR,J)
   16 MU(J)=.1257E-5*VAR
   15 CONTINUE
 9999 CONTINUE
      STOP
      END
```

```
      SUBROUTINE CNGRH(A,L,NNZRO,X,B,N,EPS,IER)
      REAL*8 R,P,Y,ALPHA,BETA,SUM,SUM1,LAMBDA,DSQRT,DR,T,GAMMA
      REAL*8 RSQ,RTOT,XLSQ
      DIMENSION A(2200),L(2200),X(300),B(300),P(300),R(300),Y(300),T(300
     *),XLSQ(300)
      WRITE(6,2)NNZRO
    2 FORMAT(///' ',40X,I5///)
      DR=0.D0
      DO1J=1,N
C  INITIALIZE RESIDUAL  RO=B-AX
      R(J)=B(J)
      P(J)=0.D0
      DR=B(J)*B(J)+DR
      Y(J)=0.D0
      T(J)=0.D0
C INITIALIZE SOLUTION VECTOR
    1 X(J)=0.D0
      DO 12 JC=1,3
      DO 15 J=1,NNZRO
      KR=(L(J)-1)/N+1
      KC=L(J)-(KR-1)*N
C  INITIALIZE DIRECTION PO=A'RO
      P(KC)=P(KC)+A(J)*R(KR)
C  CALCULATE TO=A'RO
      T(KC)=T(KC)+A(J)*R(KR)
      IF(KR.EQ.KC) GO TO 15
      P(KR)=P(KR)+A(J)*R(KC)
      T(KR)=T(KR)+A(J)*R(KC)
   15 CONTINUE
      RSQ=0.D0
      DO 16 J=1,N
   16 RSQ=RSQ+R(J)*R(J)
      RTOT=1.D0/RSQ
      DO 17 J=1,N
   17 XLSQ(J)=X(J)/RSQ
      DO10JJ=1,N
      DO4J=1,NNZRO
      KR=(L(J)-1)/N+1
      KC=L(J)-(KR-1)*N
C  CALCULATE Y=AP
      Y(KR)=Y(KR)+A(J)*P(KC)
      IF(KR.EQ.KC)GOTO4
      Y(KC)=Y(KC)+A(J)*P(KR)
    4 CONTINUE
      SUM=0.D0
      LAMBDA=0.D0
      GAMMA=0.D0
C  GAMMA=(A'R)**2
C  LAMBDA=(AP)**2
      DO 6 J=1,N
      SUM=SUM+P(J)*Y(J)
      GAMMA=GAMMA+T(J)*T(J)
    6 LAMBDA=LAMBDA+Y(J)*Y(J)
C  ALPHA=(A'R)**2/(AP)**2
      ALPHA=GAMMA/LAMBDA
      DO 8 J=1,N
      T(J)=0.D0
      X(J)=X(J)+ALPHA*P(J)
C NEW RESIDUAL  R=R-ALPHA*AP
    8 R(J)=R(J)-ALPHA*Y(J)
      DO 7 J=1,NNZRO
      KR=(L(J)-1)/N+1
      KC=L(J)-(KR-1)*N
C  CALCULATE T=A'R
      T(KC)=T(KC)+A(J)*R(KR)
      IF(KR.EQ.KC) GO TO 7
      T(KR)=T(KR)+A(J)*R(KC)
    7 CONTINUE
      RSQ=0.D0
      SUM1=0.D0
C  CALCULATE  SUM1=(A'R)**2
      DO 14 J=1,N
      RSQ=RSQ+R(J)*R(J)
   14 SUM1=SUM1+T(J)*T(J)
      RTOT=1.D0/RSQ+RTOT
      BETA=SUM1/GAMMA
C  P=A'R+BETA*P  NEW DIRECTION
      DO9J=1,N
```

```
      XLSQ(J)=XLSQ(J)+X(J)/RSQ
      Y(J)=0.D0
    9 P(J)=T(J)+BETA*P(J)
      SUM1=SUM1/DR
      WRITE(6,13) SUM1,SUM
   13 FORMAT(' ',10X,G15.8,10X,G15.8)
      IF(SUM1.LT.EPS)GOTO11
   10 CONTINUE
C  RE-INITIALIZE
      DO3J=1,NNZRO
      KR=(L(J)-1)/N+1
      KC=L(J)-(KR-1)*N
C   Y=AX
      Y(KR)=Y(KR)+A(J)*X(KC)
      IF(KR.EQ.KC)GOTO3
      Y(KC)=Y(KC)+A(J)*X(KR)
    3 CONTINUE
      DO5J=1,N
C RESIDUAL   R=B-AX
      R(J)=B(J)-Y(J)
      XLSQ(J)=XLSQ(J)/FTCT
      WRITE(6,13) XLSQ(J),X(J)
      T(J)=0.D0
      P(J)=0.D0
    5 Y(J)=0.D0
   12 CONTINUE
      IF(SUM1.GT.EPS)IER=1
      DO 18 J=1,N
   18 X(J)=XLSQ(J)
   11 CONTINUE
      RETURN
      END
```

APPENDIX 5

FLUX DENSITY FROM VECTOR POTENTIAL

Once the nodal potentials have been determined by solving the matrix equation, the flux density is easily found. The potential in each triangle is given by

$$\bar{A}(x,y) = \left\{ \left(\frac{a_i + b_i x + c_i y}{2\Delta} \right) A_i + \left(\frac{a_j + b_j x + c_j y}{2\Delta} \right) A_j + \left(\frac{a_k + b_k x + c_k y}{2\Delta} \right) A_k \right\} \hat{a}_z$$

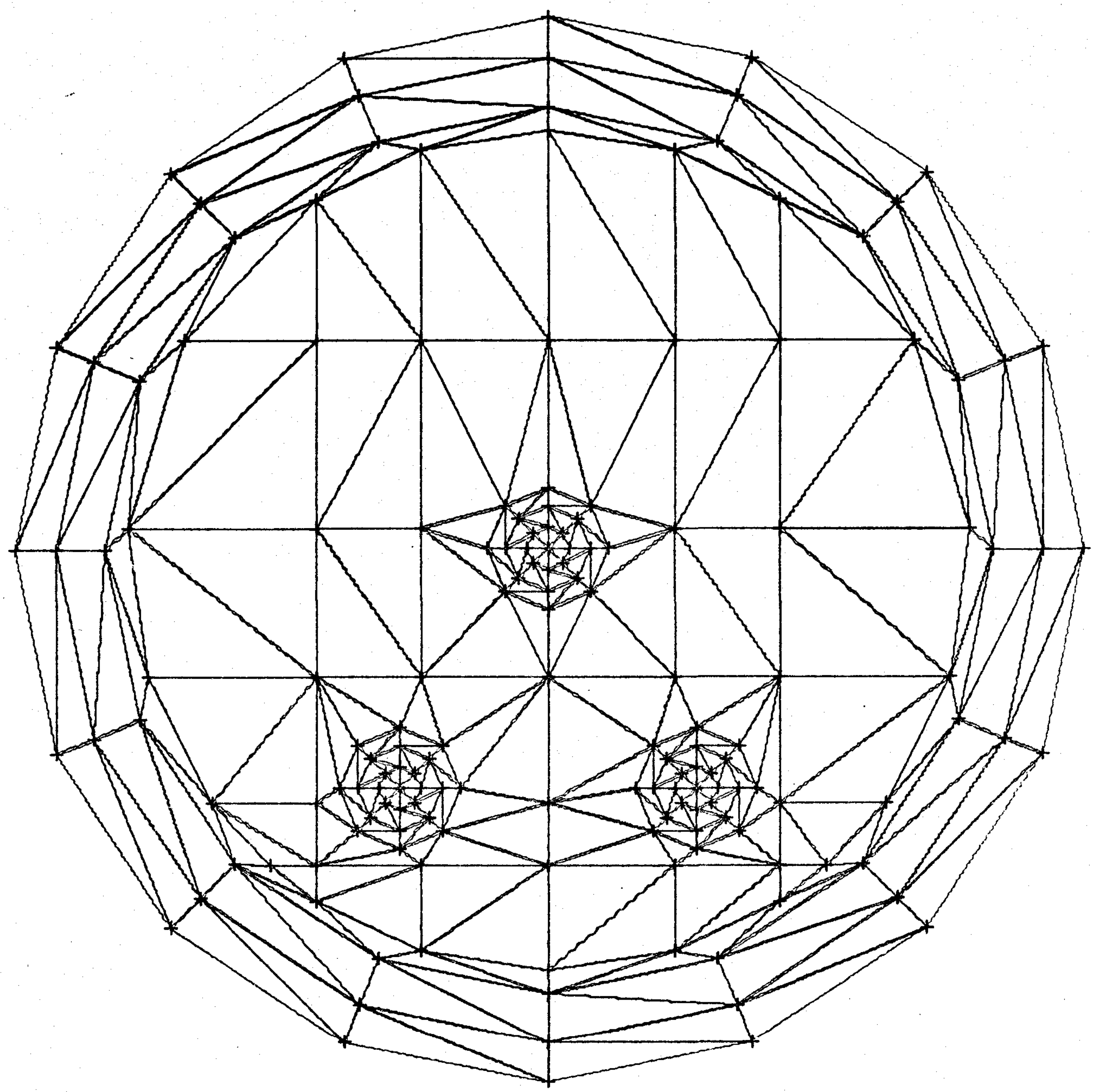
$$\bar{B} = \nabla \times \bar{A} = \frac{\partial A}{\partial y} \hat{a}_x - \frac{\partial A}{\partial x} \hat{a}_y$$

$$|\bar{B}| = \frac{1}{2\Delta} \sqrt{(b_i A_i + b_j A_j + b_k A_k)^2 + (c_i A_i + c_j A_j + c_k A_k)^2}$$

where the b_i's and c_i's have been previously defined.

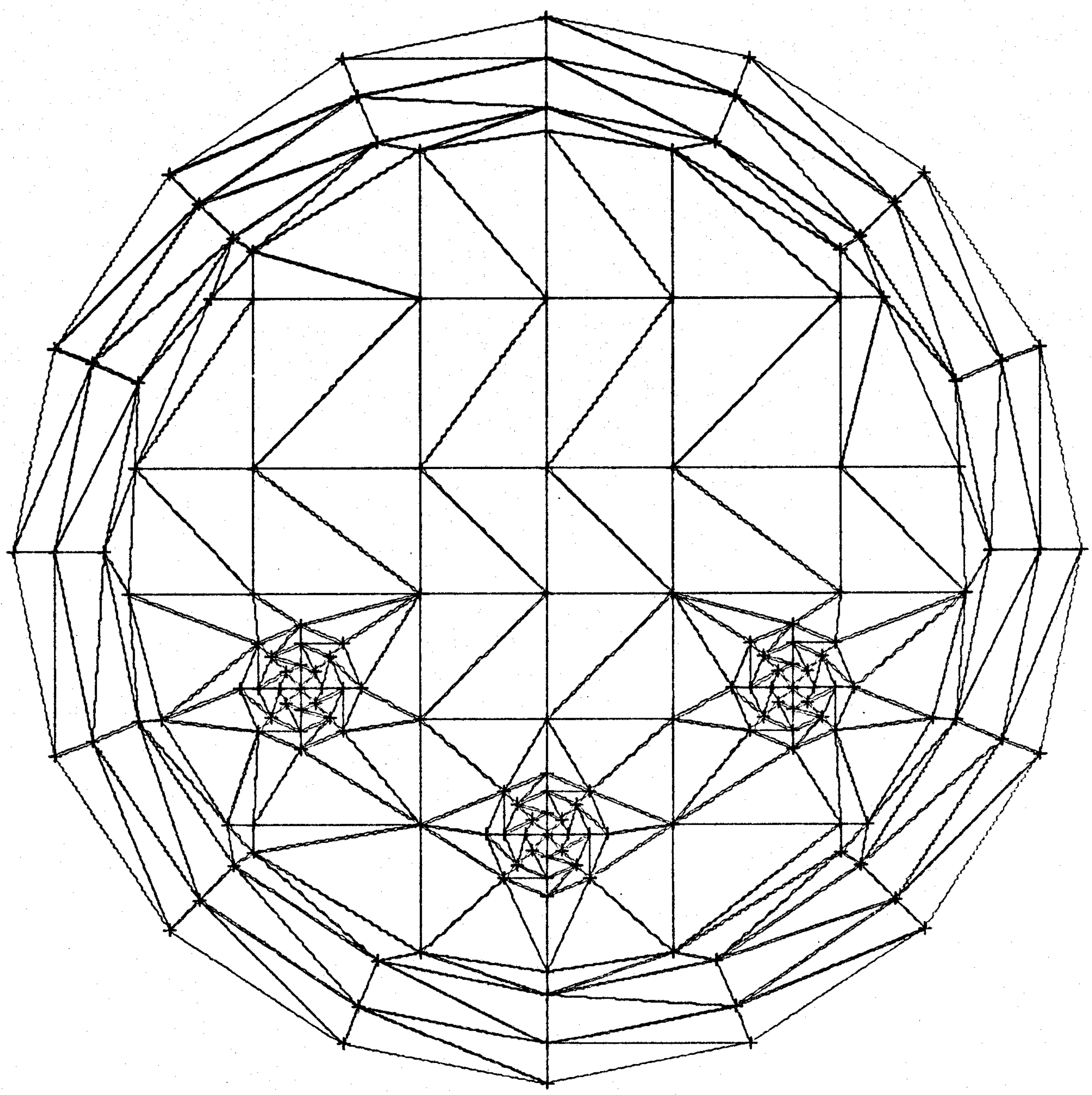
APPENDIX 6

Typical three cable finite element patterns.

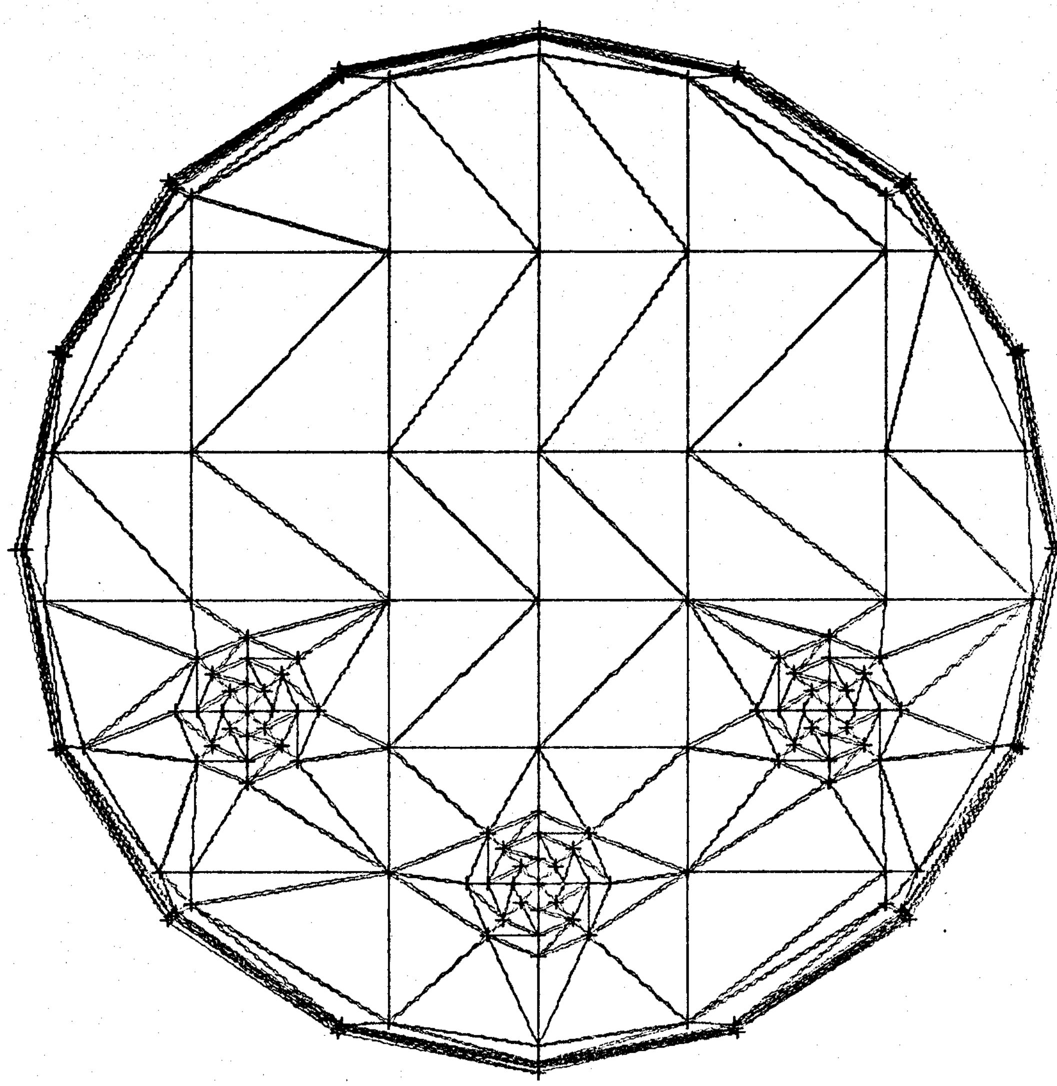


CLOSE TRIANGULAR CONFIGURATION

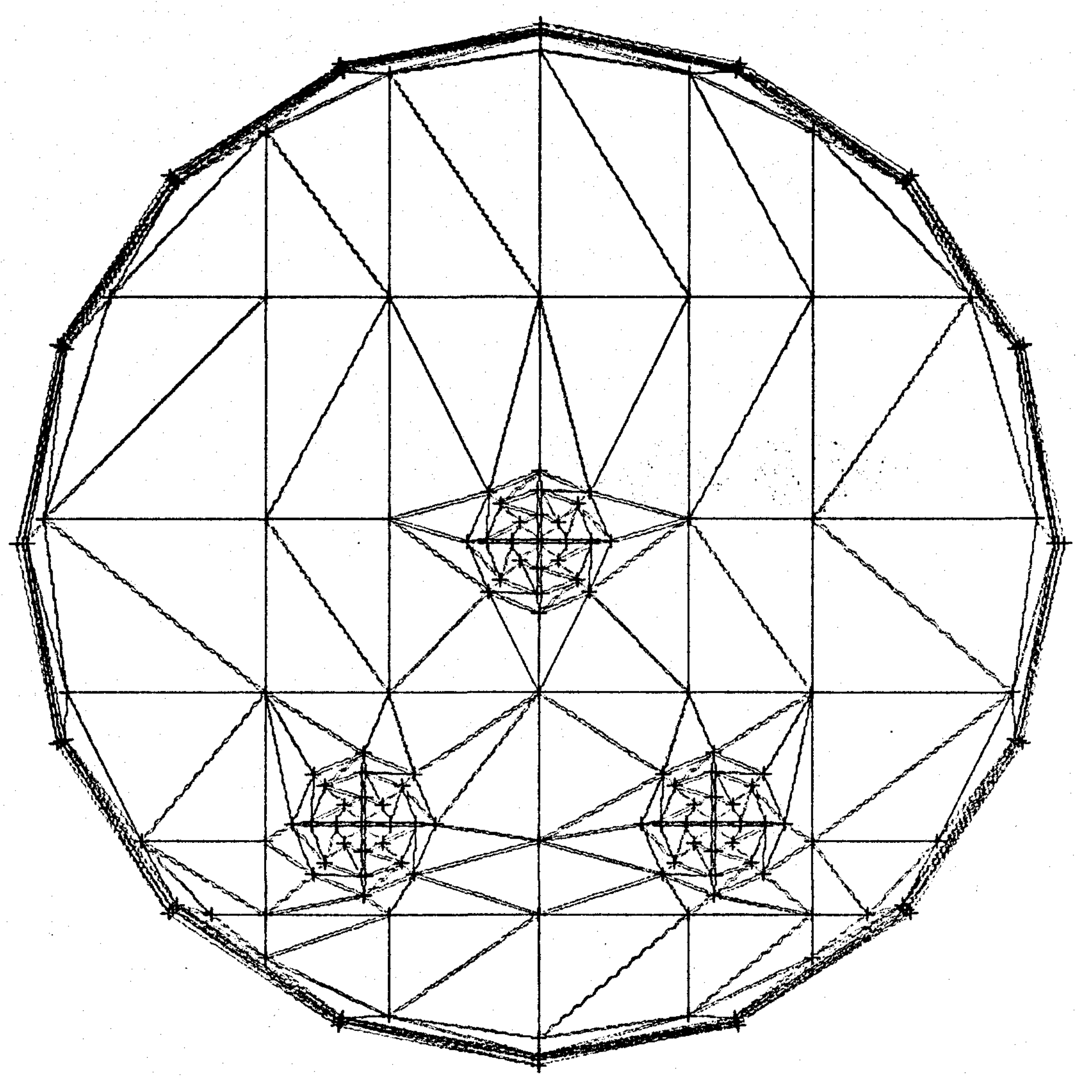
THICK PIPE



CRADLED CONFIGURATION IN A THICK PIPE



THIN PIPE - CRADLED



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