1-1-1964

The relationship between design practice and permissible stress intensities in structural steelwork.

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THE RELATIONSHIP BETWEEN DESIGN PRACTICE
AND
PERMISSIBLE STRESS INTENSITIES IN STRUCTURAL STEELWORK

A THESIS

Submitted to the Faculty of Graduate Studies through the Department of Civil Engineering in partial fulfillment of the Requirements for the Degree of Master of Applied Science at the University of Windsor.

by

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University of Windsor, 1964

Windsor, Ontario, Canada
1964.

ABG8860

APPROVED BY:

J. B. Kennedy

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ABSTRACT

In this thesis the relationships between the known properties of structural steel, the stress intensities permitted by design standards, and the design practices recognized by those standards, are discussed. An assessment is made of the effects of the simple or "conventional" method of designing building frames, with particular reference to secondary stresses, in order to determine the adequacy of the prescribed interval between the nominal strength of the material and the stress intensities permitted by current design standards. The conclusion is drawn, that this interval is adequate, but that in structures of a certain type the margin of excess capacity is small, and that there is a need for care and accuracy in the application of design standards.

ACKNOWLEDGEMENTS

The author wishes to express his appreciation of the assistance given him in assembling information by fellow members of the Canadian Standards Association Committees on the Specification for Structural Steel, G40 series, and Steel Structures for Buildings, S16. He also wishes to thank Dr. H.P. Herbich and Dr. J.B. Kennedy of the Department of Civil Engineering for their encouragement and advice.

"THE RELATIONSHIP BETWEEN DESIGN PRACTICE AND PERMISSIBLE STRESS INTENSITIES IN STRUCTURAL STEELWORK."

Table of Contents.

Table of Contents (contd.)

"THE RELATIONSHIP BETWEEN DESIGN PRACTICE AND PERMISSIBLE STRESS INTENSITIES IN STRUCTURAL STEELWORK."

SYNOPSIS.

The stress intensities permitted in the design of steel structures are determined by the appropriate design standards and specifications. Such permissible stresses are related to the properties of the material, and also take into account the effects of conventional design practices. This thesis examines the relationship between material properties, permissible stresses and design practices. Certain conclusions are presented as the result of the examination.

INTRODUCTION.

In the design of structural steelwork, the prime requirement is simple; the proportions of the members or components of the structure, and the properties of the material, must be such that the structure will remain stable and serviceable under the given conditions of loading. It follows that the designer must have sufficient information about the material, and must also be able to ascertain the nature and magnitude of the stresses which the given loading will cause in the various parts of the structure being designed. In the ordinary practice of the present day, he is not called on to decide the relationship to be observed between the properties of the material and the stress intensities in the structural members; that is usually laid down by a design specification or standard applicable to the type of structure under consideration. In similar fashion, the loading to be applied to the structure is commonly established by a code or standard.

Design specifications and tables of loads have not always been available. Engineers and architects of a few generations earlier frequently had to devise their own rules, and further, had to write their own material specifications. The formulation of rules and the writing of specifications are now usually delegated to technical committees and organizations. In effect, the individual designer and his client accept the decisions made by such organizations with regard to the properties of any particular material, and the maximum stress intensity which it may be permitted to carry.

It will be seen that responsibility for establishing the relationship between the material properties and the permissible stresses has not been eliminated; it has only been transferred from the individual designer to a group of designers. In like manner, the estimation of the magnitude and distribution of loads has been transferred, for the most part, to a group formed for the purpose. The individual is still responsible for the manner in which he uses these rules and this information, in selecting the form which the structure is to take, and the proportions of its members. The relationship between the prescribed permissible stresses and the actual stresses which will occur in the completed structure are his responsibility. The safety and serviceability of the finished work will depend on the competence of all concerned.

PROPERTIES OF MATERIAL.

Important Properties

The most important properties of structural steel, from the point of view of the designer and of the fabricator, are the tensile strength, the yield strength, ductility, weldability, and general fabricating properties. As a guarantee that the steel being supplied for any given project is suitable, the supplier is expected to make certain tests on representative samples, and to give to the purchaser certified copies of the test reports. Such test reports state the yield strength and tensile strength, provide information as to the ductility as determined by the elongation of a tensile specimen and by a bend test, and include information regarding the chemical composition of the steel which has a bearing on its weldability and some other qualities.

Significance of Yield Point.

It has been stated (1), that "the single, most important structural property of a mild steel is its yield point. Except where buckling enters the picture, allowable stresses in conventional design depend almost exclusively on this property." This is generally true to-day, but the yield point has not always been explicitly given in specifications for structural steel. It is not so long since specifications on the American continent gave a figure for the tensile strength, and stipulated that the yield point should bear a certain relationship to that figure. Current specifications for structural steel in European countries to-day include a grade "AO" for which only the tensile strength is given, and the French standards for the design of building steelwork provides for the use of this type of steel, on the basis of an assumed yield point.

Definition of Yield Point.

The importance of the yield point of steel renders it equally important to understand the meaning of the term. The tensile testing of structural steel commonly involves a state of the material in which the specimen continues to elongate without any increase in the applied load. If the load and elongation are being recorded graphically, the curve at this point doubles over quite suddenly, drops a little, sometimes wavers, and then levels off as the elongation continues. In the standard method of testing, there is reference to the "drop of the beam", or "halt in the gauge", as establishing the yield point. Sometimes a test fails to display these phenomena; it is then permitted to substitute for the yield point, that stress at which a permanent set of some stated amount, usually 0.2% , occurs. This may be referred to as the yield stress.

In the stress-strain curve for a tensile test, other characteristic points are noted. There is the proportional limit, or the point up to which the stress-strain relationship is expressed by a straight line. There is also the elastic limit, establishing the range through which the specimen will always return to its original length on removal of the load.Both of these points are relatively near to the yield point, and are sometimes treated as the same thing. In the Belgian standards (6), the term "apparent limit of elasticity" is used, and defined in such a manner as to make it exactly the same as the definition of the yield point above.

The designer's interest in the yield point for his chosen steel arises from his knowledge that continued loading of any part of the structure in excess of this level will cause a continued deformation at that point. Such unlimited deformation, if not foreseen and controlled in the design, would lead to a failure of the structure. In plastic design it is expected and assumed that certain parts of the structure will be stressed beyond the yield point, but this condition is so controlled that the structure remains stable, and in plastic design, as in elastic, a minimum value of the yield point of the material must be known.

Distinction between "Yield Point" and Yield Strength.

The property of the steel in which the designer is primarily interested is the stress at which elongation continues without any stress increase; this is the horizontal portion of the stress-strain curve. Moreover, the designer really wishes to know the value of this stress for the critical components of the structure, or at least for some pieces of steel which may be considered as truly representative of those components. It is, therefore, necessary to find the relationship between the required figure and that for the yield point, which is obtained from the report of the test made at the mill. There will be differences between these properties, and an attempt must be made to ascertain the nature of these differences.

George Winter (1) has identified three factors which affect the value of the yield point given in mill test reports, as an indication of the yield strength of any given piece of steel.

(a) The yield point determined in the mill test is the upper yield point, which is greater than the yield strength level at which continued elongation may occur.

(b) The value of the yield point depends to some extent on the speed at which the test is conducted.

(c) Test specimens for beam and channel sections are taken from the webs of the sections, and there may be an appreciable difference in this respect between the properties of the web and the flange. It is usually the properties of the flange which determine the behavious of the member as part of the structure.

Taking these three factors in order, Professor Winter surmises that each rarely exceeds 10%, and that the cumulative effect of all three would be from 5% to 30%. Unfortunately there is little experimental evidence on any of these phenomena. C.A.Edwards points out that the upper yield point can be raised by increasing the rate of loading (4), and gives upper and lower yield point figures for some tests on low-carbon steel with a difference of 0.76 to 1.88 ksi, with an average of 1.33 ksi. As a percentage of the upper yield, these differences range from 0.13 to 3.92. Tests reported by the Steel Structures Research Committee (Fig.1) show a stress drop at yield of from 6.29% to 12.1%. The rate of loading in the tests reported by Professor Edwards varied from 0.83 to 9.60 kips per square inch per hour. The Research Committee tests were said to have been at low speeds. For the purposes of this discussion, a figure of 5% will be assumed as the maximum difference between the upper yield point and the actual yield strength under a constant load.

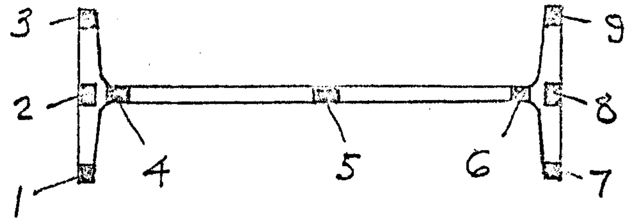
In the tests reported by Professor Edwards (4), the variation in the rate of loading did not produce any consistent variation in the value of the upper yield point. The Report of the Research Committee of the American Society for Testing Materials (Thirty-first Annual Meeting, Proc. 1928) shows that increasing the rate of strain from 0.005 to 0.2 per minute causes increases in the yield point of about 9%. In recognition of this effect, the standard rules for commercial tensile tests stipulate a maximum rate at which a test may be conducted; in Canada and the U.S.A. the speed of movement of the crosshead of the testing machine during approach to the yield point, must not exceed 1/16 inch per minute per inch of gauge length. With an 8" gauge length, this corresponds with a strain rate of 1/2" per minute, and since the part of the specimen under elongation will be about 10" long, the actual rate of strain will be approximately 0.05 in. per inch per minute, or 0.00083 per second.

Experiments by M.J.Majoine (2) have provided the data in Figures 2, 2a and 3. In tests at room temperatures, it appears that a reduction of the strain rate from 8.5×10^{-4} to 9.5×10^{-7} effects a reduction in the yield point of about 400 psi. Since the strain rate permitted in standard practice is 8.3×10^{-4}, it must be concluded that in commercial testing limited to this rate of strain, the effect of the rate of testing in raising the yield point may be assumed to be 1%. It may be noted here that George Dieter (3), in his study of slow elongation or creep, remarks that "It makes little difference in the results if the loading rate in a tension test is such that it requires two hours or two minutes to complete the test".

In the third of the factors cited above, there is an even greater scarcity of experimental data than in the first two. Figure 1 provides some comparisons between the properties of specimens taken from various parts of an I-beam, and shows that at the junction between the flange and the web, the yield point is lowest, while specimens from the middle of the web, or from the toes of the flanges, were higher. In the "wide-flange" shapes which form a large part of the structural steel used to-day, the thickness of the flanges, and therefore the variation between the edges and the centre of a flange, would not be so great, and the highest values of the yield point would be expected in the web. The variation is due to the difference in the rate of cooling, which is fastest in the web, and slower in the flange, being slowest in the middle of the flange. Metallurgists of the U.S. Steel Corporation estimate that on an average, the yield point in the web would be about 5 ksi higher than that in the flange. Metallurgists of the Algoma Steel Corporation made flange and web tests of a number of wide-flange sections, the results of which appear in Table 1. For the complete series of tests, the average difference between web and flange yield points is 4.58ksi, whichis fairly close to the figure estimated by the U.S.Steel metallurgists.

Steel Structures Research Committee: First Report.

Section:
24 x 7½ R.S.J.
(@ 90#/ft.)
Flge. thickness = 0.984 in.
Web thickness = 0.52 in.



Location of Test Specimens.

RESULTS OF TENSILE TESTS: (Stresses in pounds per sq. inch)

Specimen Location	Type of Specimen	Yield Stress	Ultimate Stress	Str.drop at Yield	Elongatn. %	"E" (psi)
1	Flat	38,100	71,000	-	19.25	28.6 x 10^6
1	Turned	40,300	72,800	11.8%	38.0	28.2
2	Flat	31,400	67,600	-	27.4	28.0
2	Turned	31,400	70,000	6.29	41.0	29.2
3	Flat	33,600	67,200	-	28.0	27.4
3	Turned	37,200	70,500	11.5	43.0	29.7
4	Flat	31,300	66,000	-	29.0	27.5
4	Turned	33,600	68,100	10.4	41.0	30.2
5	Flat	35,800	67,200	-	26.1	29.6
5	Turned	37,600	70,500	12.1	40.0	29.4
6	Flat	33,600	66,700	-	27.9	29.5
6	Turned	35,200	69,500	10.2	42.0	29.6
7	Flat	37,400	69,000	-	30.2	30.2
7	Turned	33,200	71,000	7.3	43.2	28.8
8	Flat	30,400	68,800	-	32.0	29.7
8	Turned	29,100	70,000	-	41.0	28.4
8	Turned normalized	37,700	72,100	10.7	39.0	29.6
8	Turned tempered	37,500	74,000	11.3	38.5	30.0
9	Flat	35,800	69,500	-	26.4	27.9
9	Turned	35,400	69,500	11.5	43.5	29.4

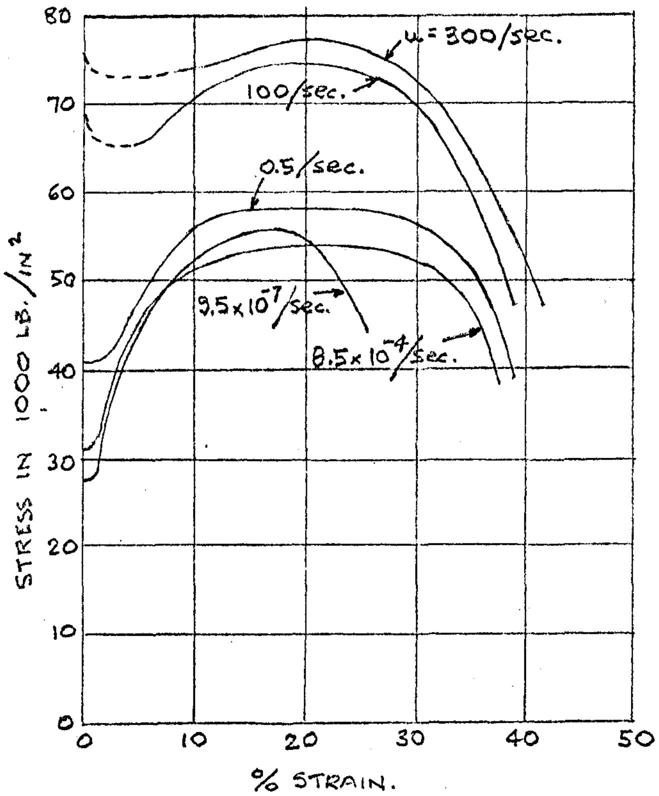
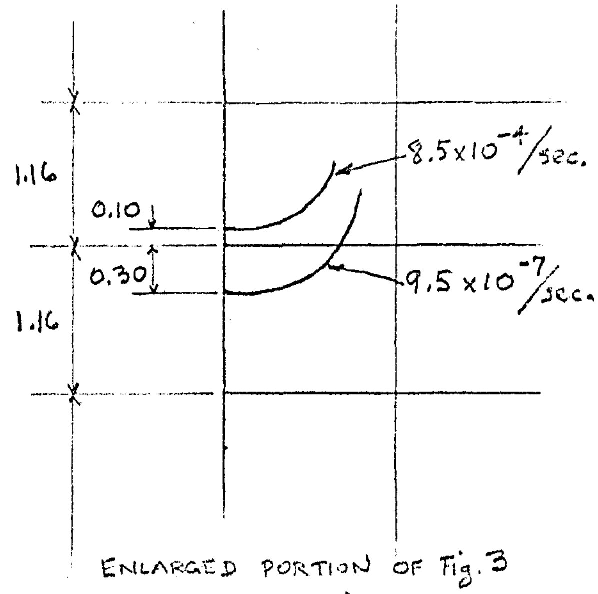


Fig. 2. STRESS–STRAIN CURVES OF MILD STEEL AT ROOM TEMPERATURES FOR VARIOUS RATES OF STRAIN.



ENLARGED PORTION OF Fig. 3 (SCALED IN CMS.)

$\frac{0.10}{1.16} = .086$ SAY .09

$\frac{0.30}{1.16} = .258$ SAY .26

Y.P. FOR 8.5×10^{-4} = 30.09 K/IN²

Y.P. FOR 9.5×10^{-7} = 29.74 K/IN²

Fig. 2a.

NOTE: YIELD-POINT STRESS IS TAKEN AT THE LOWER YIELD POINT.

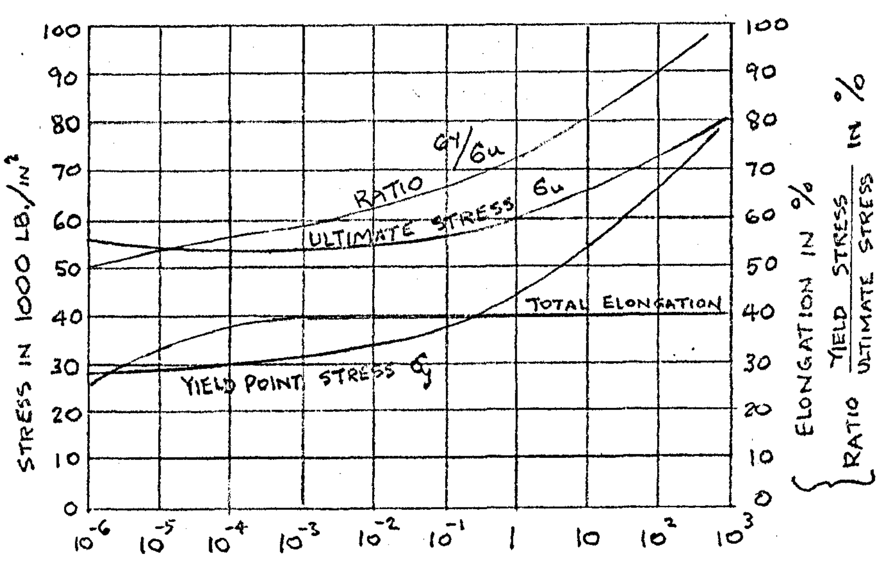


Fig. 3 INFLUENCE OF RATE OF STRAIN ON TENSILE PROPERTIES OF MILD STEEL AT ROOM TEMPERATURE.

EXTRACT FROM: M. J. MAJOINE; "INFLUENCE OF RATE OF STRAIN AND TEMPERATURE ON YIELD STRESSES OF MILD STEEL." ASME TRANSACTIONS, 1944.

TABLE 1.

COMPARISON OF MECHANICAL PROPERTIES OF WEBS AND FLANGES OF WIDE FLANGE BEAM SECTIONS: SPECIFICATION CSA G40.8 GRADE A.

TENSILE TEST DATA.

	Yield Strength psi.		Tensile Strength psi.	
	Web	Flange	Web	Flange
6"@15.5				
	44500	45600	65100	65100
	46100	44800	65100	67000
	48400	48100	68800	66800
	48000	45600	65900	65300
	49600	47500	69900	68900
	53600	45600	69900	66600
	51900	49100	70500	69100
	49100	47700	68400	67300
	52100	48300	71700	74500
Av.	49200	46900	68400	67900
6"@20.				
	44600	44800	65200	66800
	50100	45400	67700	66200
	52700	45500	76200	66900
	50200	41900	76200	65100
Av.	49400	44400	71300	66200
8"@31				
	54500	46800	74500	73200
	49100	48400	72400	73000
	49400	45900	71300	67600
	50100	46000	74900	69400
	52700	47900	73700	68700
	50100	45300	68800	69200
	47500	44300	70700	69000
	49000	44200	70800	69100
	47500	44200	71200	67400
	49900	45000	67800	67900
	44100	42600	66000	65100
	43000	41900	65600	65100
	48500	43900	68700	64700
	44500	45000	65300	66500
	47700	43800	69000	67600
	46300	43200	66700	66800
	49200	42900	69900	67300
	49800	46100	70200	67700
Av.	48490	44860	69860	68070

TABLE 1 (continued)

PROPERTIES OF WEBS AND FLANGES.

	Yield Strength psi.		Tensile Strength psi.	
	Web	Flange	Web	Flange
10" @ 21				
	53600	46400	71400	69600
	50600	45800	69200	67500
	53100	47600	74400	69300
	57400	47200	74800	69700
	50900	46500	70200	68600
	48700	44200	67800	67000
	48200	45800	69400	68000
	47600	46500	68000	68600
	51500	48000	73900	73700
	48000	46800	69700	71900
	51100	45700	70800	67300
	50300	46900	69700	68300
	51400	45400	70800	66300
	52200	45400	68300	66500
	50300	44800	70800	68000
	49000	45300	69400	68400
	49800	46200	68600	70700
	51200	45000	75400	67600
	50100	48000	70500	73500
	53100	47100	73700	70100
Av	50900	46230	70840	69030
10" @ 25				
	47900	42000	65400	62800
	46400	42600	65600	62400
	49000	44000	68700	66700
	49700	44300	71800	67000
	47700	42000	67400	64600
	46500	42100	64200	64600
	55400	46100	74500	69300
	49500	44300	70500	68400
	49400	45300	69500	68100
	50800	43400	70400	67100
Av.	49200	43600	68800	66100

TABLE 1 (continued)

PROPERTIES OF WEBS AND FLANGES.

	Yield Strength psi.		Tensile Strength psi.	
	Web	Flange	Web	Flange
12" @ 27				
	52100	46800	70000	67300
	55400	48900	73900	70900
	58700	48500	75700	71000
	58700	47600	74900	72100
	51100	47400	69300	69000
	50500	46400	69300	68400
	51500	47100	71200	67300
	52100	45800	72100	75800
	55700	44900	72000	62100
	54700	44500	67700	61800
	50900	46700	65300	64100
	50300	46900	66700	65800
	51500	46800	69500	67300
	51700	45700	70000	67000
	53000	47100	69200	67100
	52200	47300	70300	68200
	59500	49400	73300	69000
	56300	47100	72000	68200
Av.	53660	46940	70690	67910
14" @ 30				
	51900	46300	74300	74400
	52600	48500	74800	75300
	50500	45600	72400	72000
	53400		72100	
	51800		72100	
Av.	52000	46800	73100	73900
14" @ 34				
	47900	50900	68600	73600
	47800	45000	68700	66100
	50600	46300	69600	66700
	49900	48500	71000	69300
	53200	48200	72300	69900
	49000	46300	69600	67100
	50800	48000	75000	69300
Av.	49900	47600	70700	68900

(Information supplied by Algoma Steel Corporation.)

A comment on one feature of the web-flange relationship is necessary. Because the flange is usually the most highly-stressed portion of a beam or column section, it is important to the designer to know that the yield point of the flange will meet some minimum requirement. A difference of 8 ksi in the values for flange and web, to the detriment of the former, may therefore be regarded as serious. An examination of the figures in Table 1 will show, however, that where the greatest differences were found, the yield point in the web was much in excess of the minimum called for in the applicable specification. There is, therefore, no apparent probability that a beam will be produced with a yield point figure for the web just above the specified minimum, and that for the flange some 8 ksi lower than that. In the series of tests here reported, the steel was intended to have a minimum yield point of 44 ksi, and it will be seen that the flange yield point falls below this figure in 12 of the 89 tests, the lowest result being 41.9 ksi.

From the available information, it seems reasonable to assume that the yield point in beam flanges generally may be about 5 ksi lower than the yield point reported in the mill tests; this corresponds with a percentage of 12.5 .

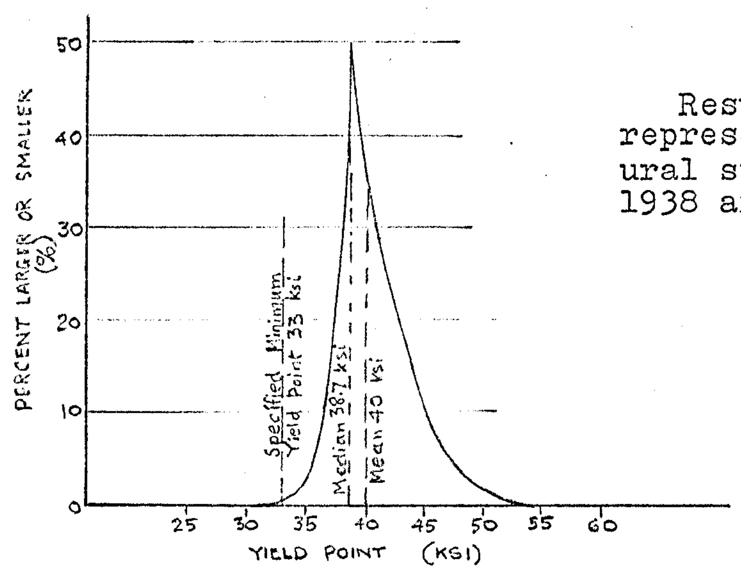
Rolling-Mill Practice.

The variations in properties of steel which have been considered so far are such as affect the relationship between the yield-point reported in the mill test result and the actual yield-point which would determine the usefulness of that particular piece of steel. There is another type of variation which affects the properties of structural steel,- the variation between different heats, between different parts of a heat, and even between different parts of a finished piece of steel.

The physical and chemical properties of the steel in an ingot are not uniform throughout the mass; during the cooling process, some of the constituents separate around the outside in the first stages of solidification, and any variations in the properties through the cross-section of the ingot are reflected in those of the billet and of the finished piece. The differences between rimmed and semi-killed and killed steel, which are quite important in certain applications of the end product, have their origin in the conditions under which the ingot is made. Mill practice is planned so as to minimize the effect of the variation in the ingot, but it is recognized that perfect uniformity is not attainable. There is also an inevitable difference between ingots and between heats. The specifications under which structural steel is produced, in Canadian and U.S. mills, require one tension test if the finished material from a heat is less than 30 tons, and two tests if the product exceeds that figure. Separate tests are required for finished sections when their thickness differs by 3/8" or more. When one test may represent 28 or 29 tons of a given section, it is obvious that a more extensive series of tests throughout the shipment would disclose considerable differences between the values obtained for the yield point and the tensile strength.

Whereas the routine mill tests include figures for the yield point and tensile strength of every heat of steel, ample information is available on these properties for the commonly-used steels. Figures 4 to 22 inclusive are graphical presentations of the relative frequencies of occurrence of the various strength levels in the test reports for several different grades of steel. In some of these graphs, the number of test reports available is small, so that no reliable statistical pattern can be established. Even in these cases, however, there are significant features, to which reference will be made below.

In the search for technical improvement and economic advantage, there is a continual process of development in the steel-making industry. Although the designer and the fabricator hear of some major changes, they do not necessarily know all the details, and may not appreciate the full significance of all the developments which take place. A case in point is the adoption by some mills of the practice of coiling plates after the last rolling pass; these are flattened and cut to length afterwards as required to meet customers' requirements. This practice generally applies to plates 3/8" or less in thickness. It might seem to have no bearing on our present discussion, - except for the fact that the coiled material cools more slowly in the inner portion, allowing more

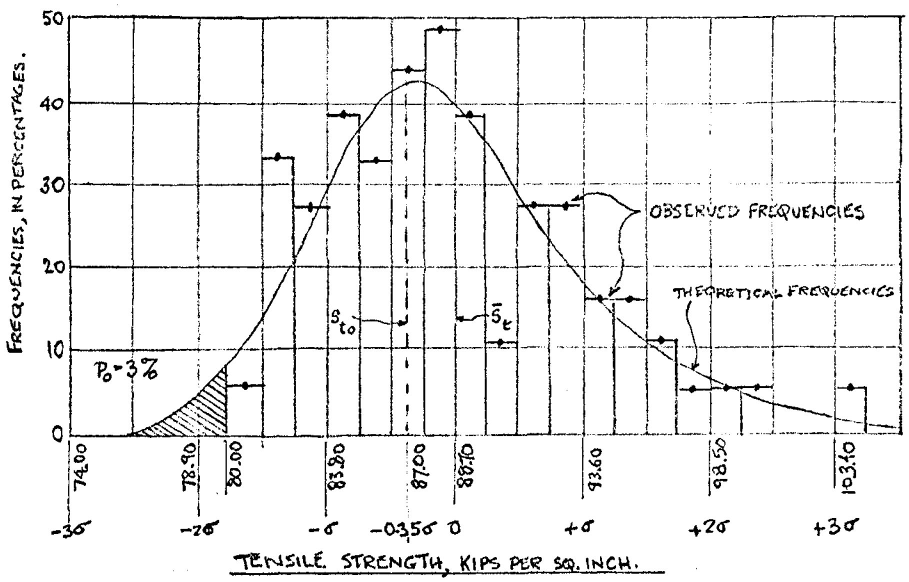


Results of 3,974 mill tests representing 33,000 tons of structural steel on nine projects, between 1938 and 1951.

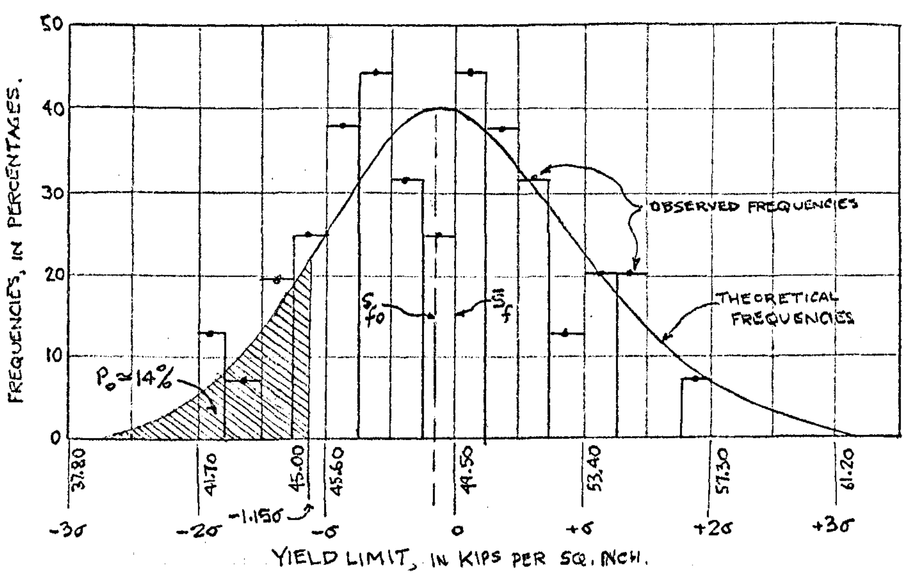
FIG. 4.- Probability Curve of Mill Tests for Steel (ASTM A-7)

From "Properties of Steel and Concrete and the Behavior of Structures", by George Winter. Trans. ASCE 1961

ACCEPTANCE TESTS, STRUCTURAL SHAPES OF SILICON STEEL, FOR THE TOWERS OF THE GOLDEN GATE BRIDGE.

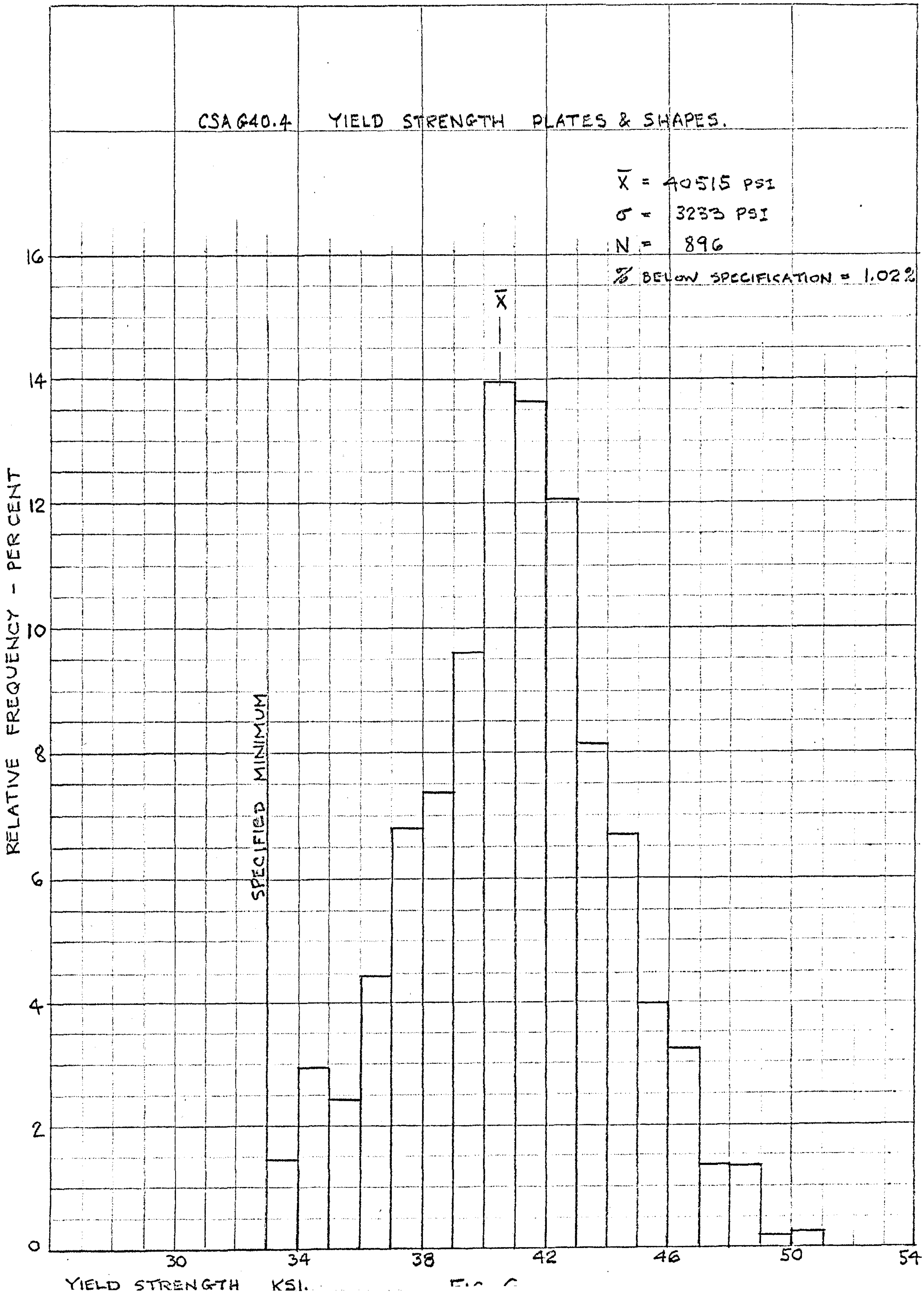


TENSILE STRENGTH: $\bar{S}_t = 88.70\,K/_{IN^2}$ $\sigma = \pm 4.9\,K/_{IN^2}$ $k = +0.72$



ELASTIC LIMIT: $\bar{S}_f = 49.5\,K/_{IN^2}$ $\sigma = \pm 3.9\,K/_{IN^2}$ $k = +0.27$

FIGURE 5, "THE SAFETY OF STRUCTURES", BY ALFRED M. FREUDENTHAL, TRANS. ASCE VOL. 112, 1947; p. 134.


CSA G40.4 YIELD STRENGTH PLATES & SHAPES.
X̄ = 40515 PSI
σ = 3233 PSI
N = 896
% BELOW SPECIFICATION = 1.02%
X̄
SPECIFIED MINIMUM
RELATIVE FREQUENCY - PERCENT
0
2
4
6
8
10
12
14
16
30
34
38
42
46
50
54
YIELD STRENGTH KSI.
DATE 1957

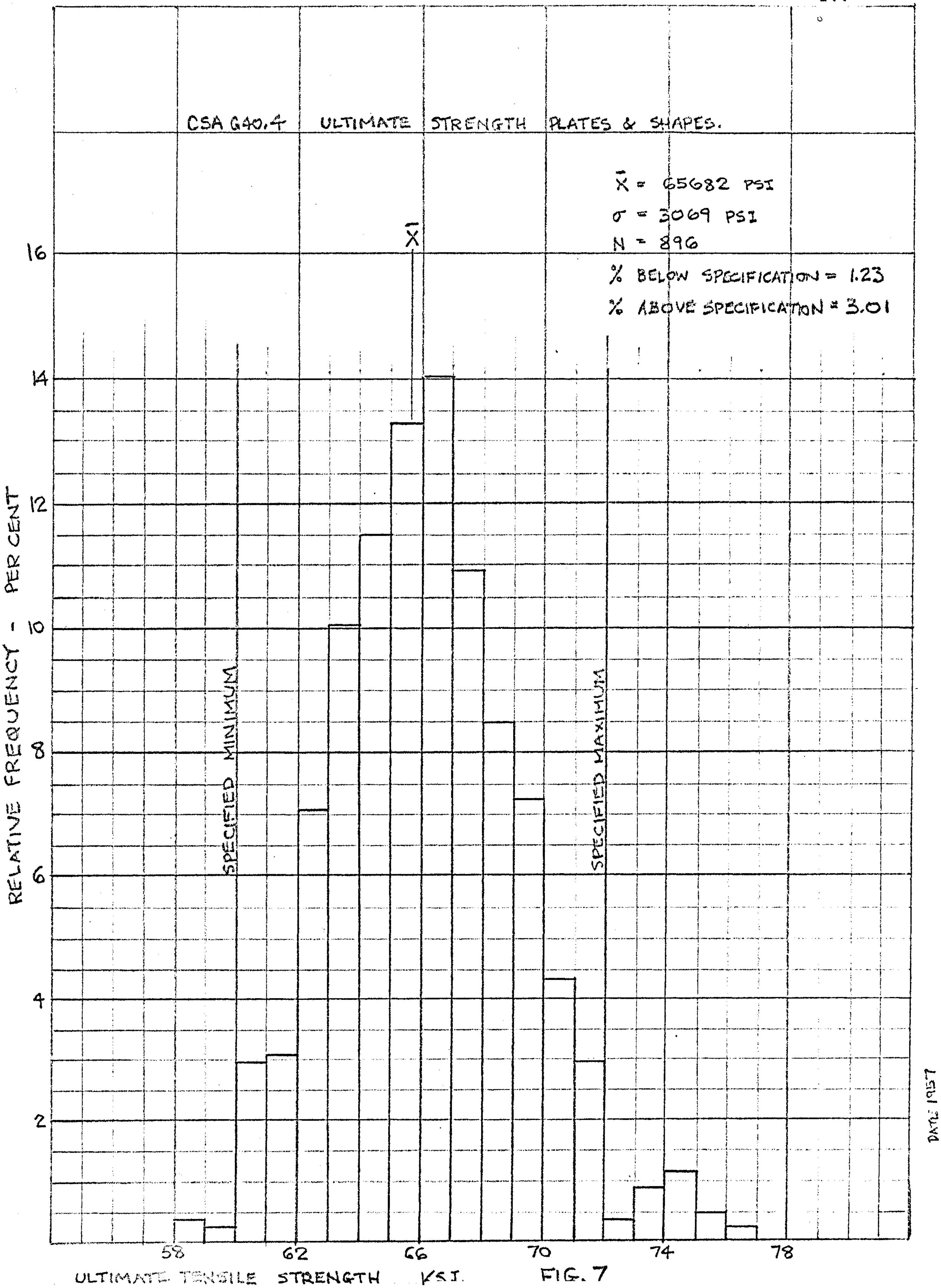

CSA G40.4 ULTIMATE STRENGTH PLATES & SHAPES.
$\bar{X}$ = 65682 PSI
σ = 3069 PSI
N = 896
% BELOW SPECIFICATION = 1.23
% ABOVE SPECIFICATION = 3.01
$\bar{X}$
SPECIFIED MINIMUM
SPECIFIED MAXIMUM
RELATIVE FREQUENCY - PER CENT
16
14
12
10
8
6
4
2
58
62
66
70
74
78
ULTIMATE TENSILE STRENGTH KSI.
DATE 1957

FIG. 7

ASTM A-7 STRUCTURAL PLATES & SHAPES: YIELD STRENGTH.

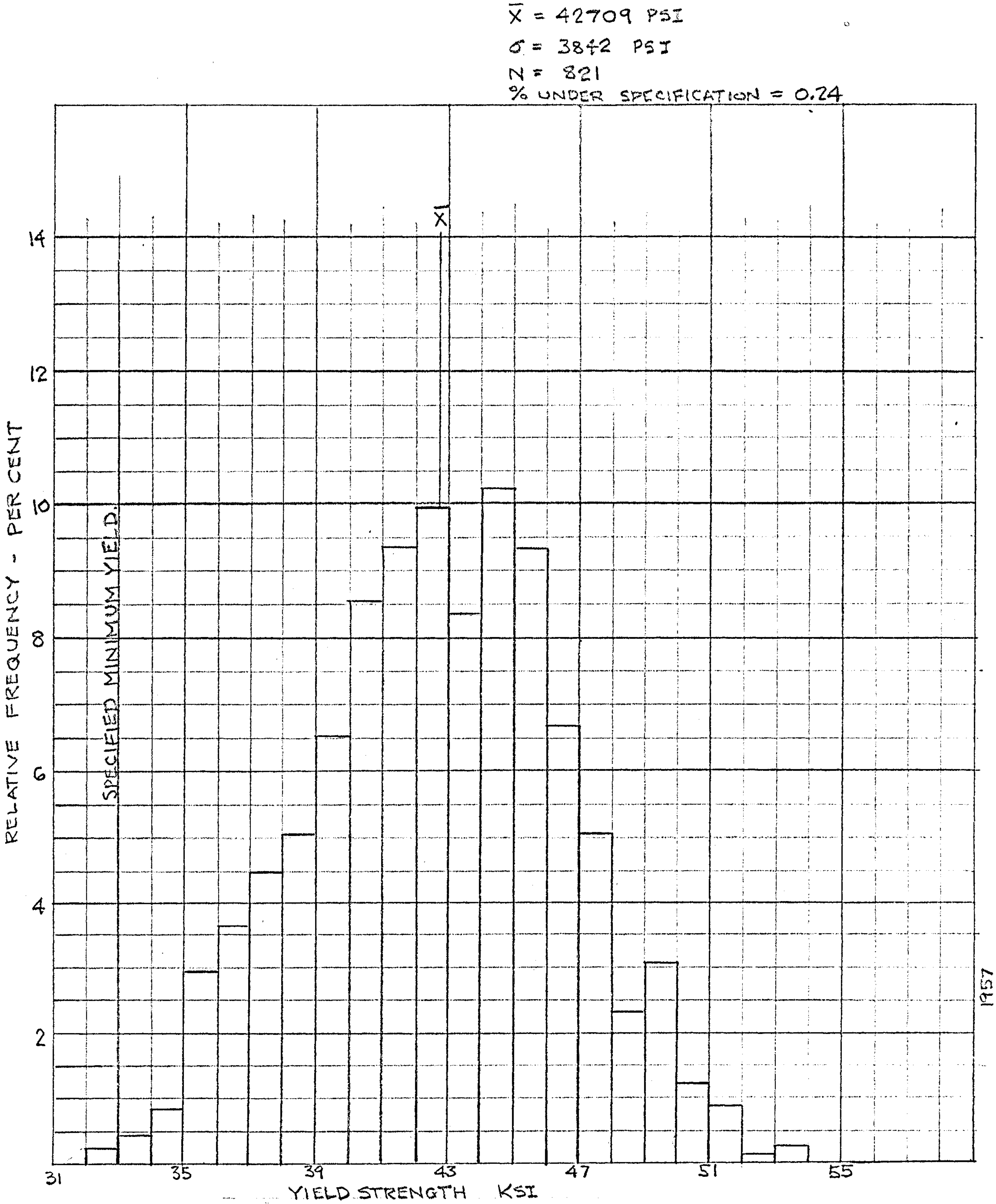


FIG. 8.

CSA G40.4 & ASTM A-7 YIELD STRENGTH

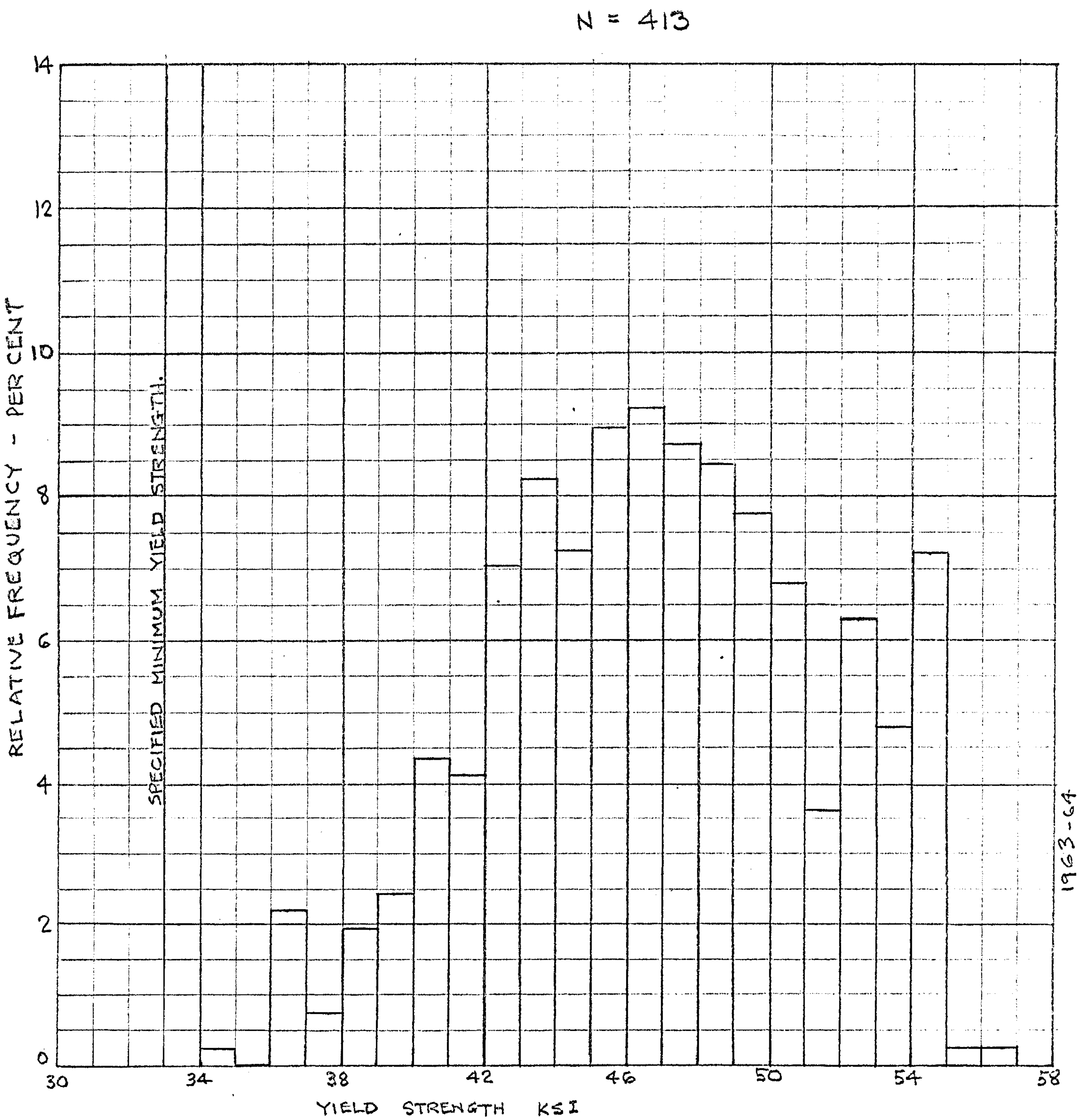
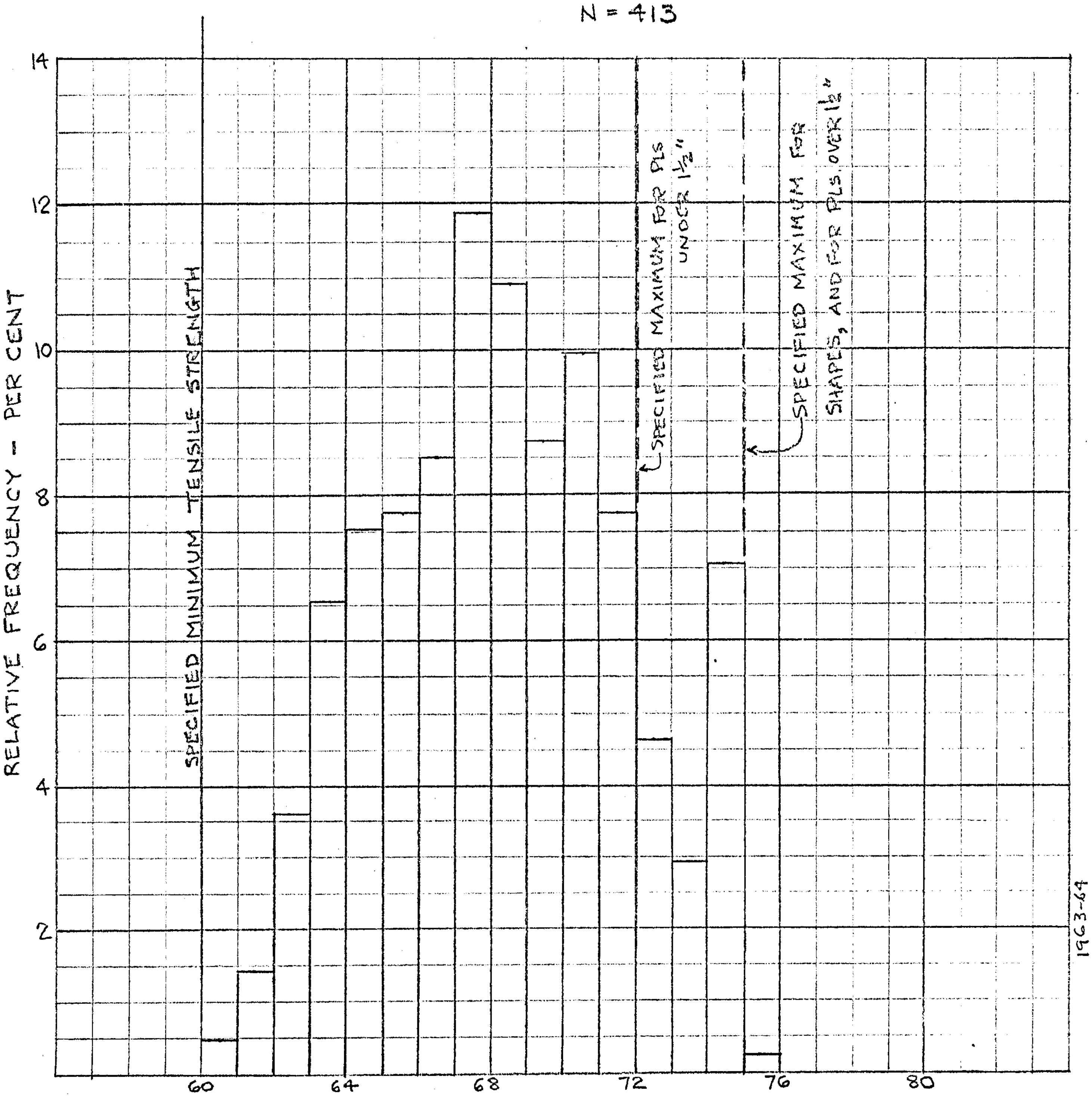


FIG. 9.

CSA G40.4 & ASTM A-7 TENSILE STRENGTH



TENSILE STRENGTH KSI.

FIG. 10

ASTM A-36 YIELD STRENGTH, PLATES & SHAPES.

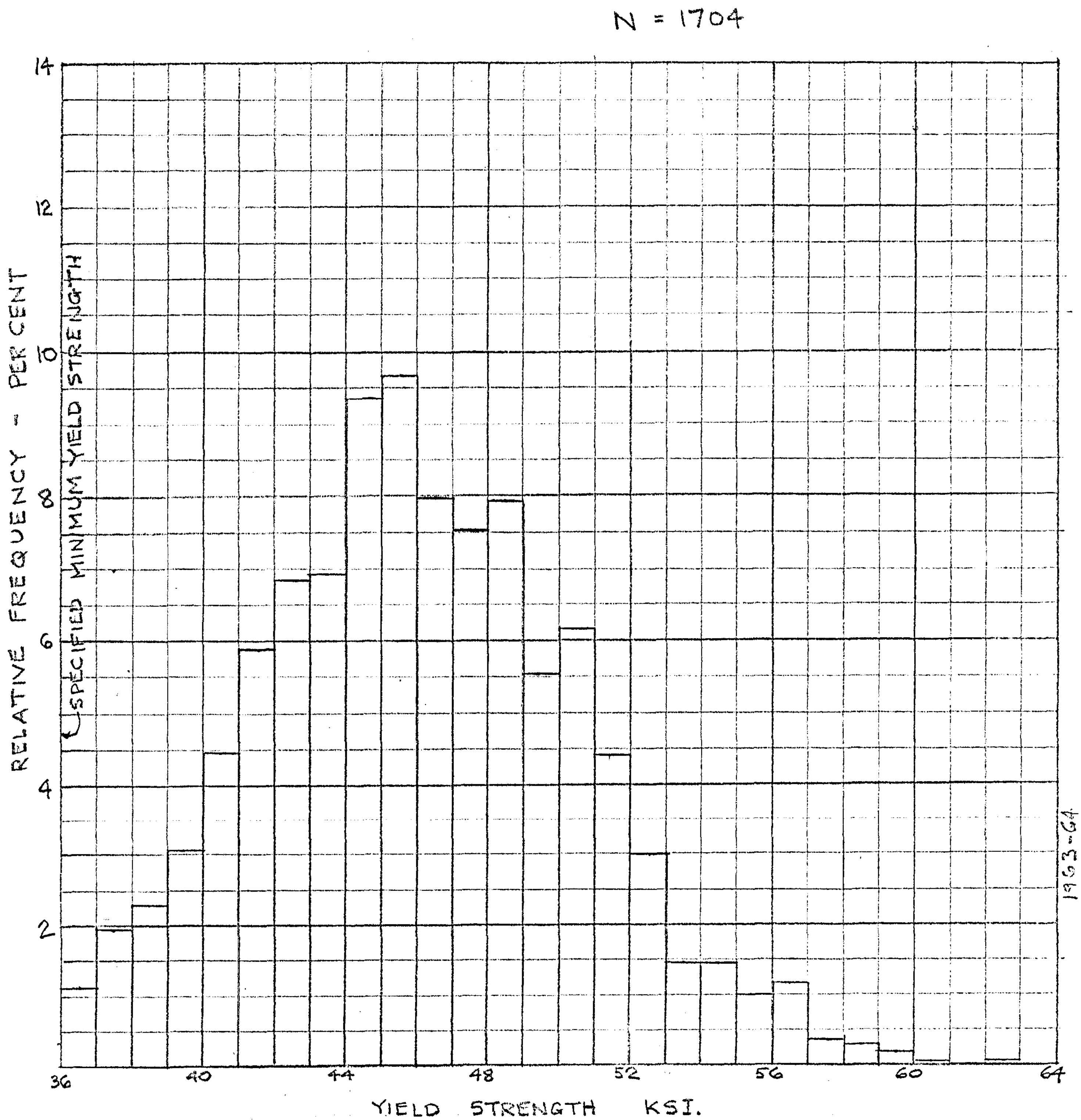
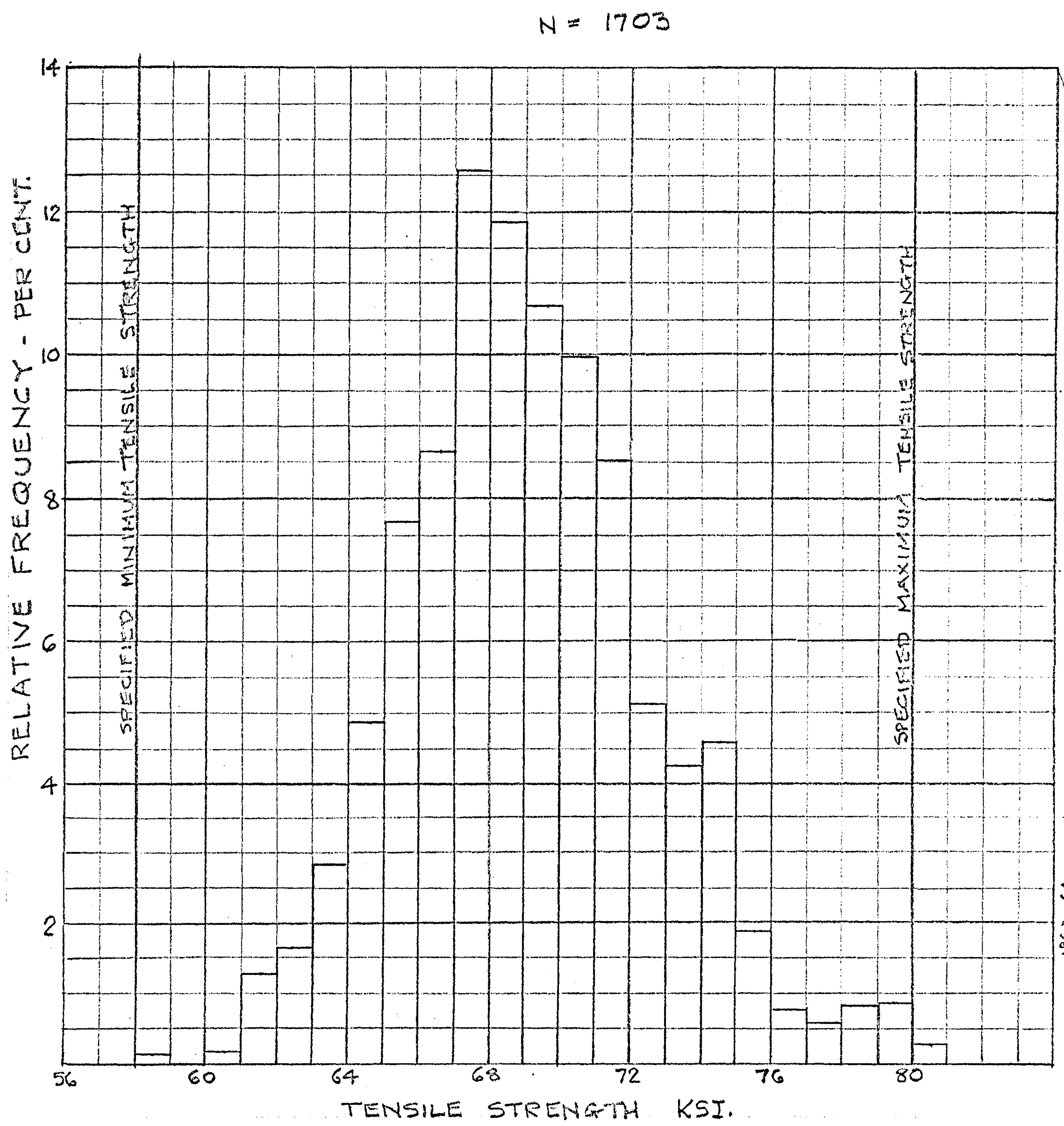


FIG. 11

ASTM A-36 TENSILE STRENGTH, PLATES & SHAPES.



SILICON STEEL, CSA G40.6-1959 AND ASTM A94-54.
YIELD STRENGTH; 54 TESTS.

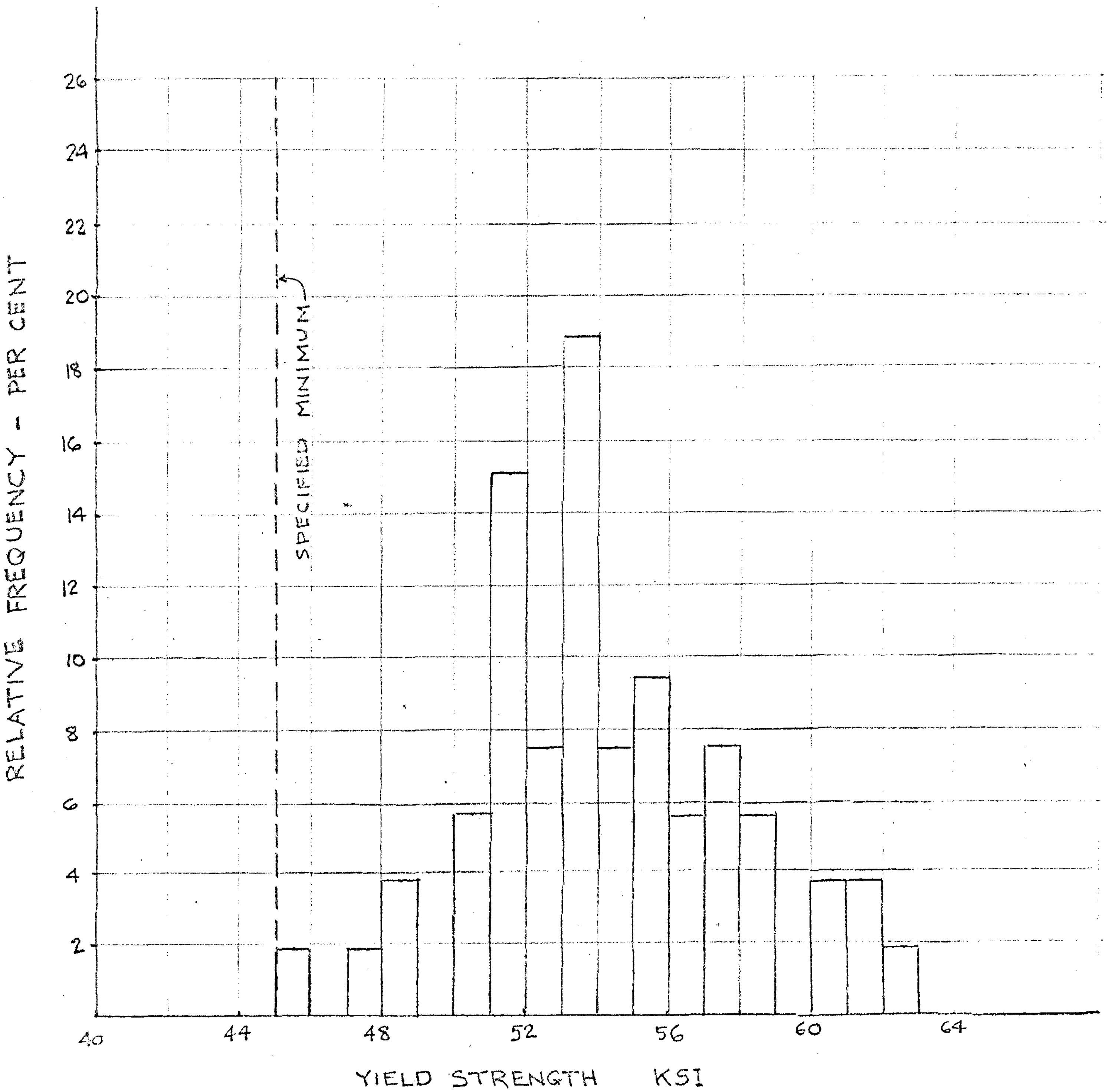


FIG. 13

SILICON STEEL, CSA G40.6-1959 AND ASTM A94-54.

TENSILE STRENGTH; 54 TESTS.

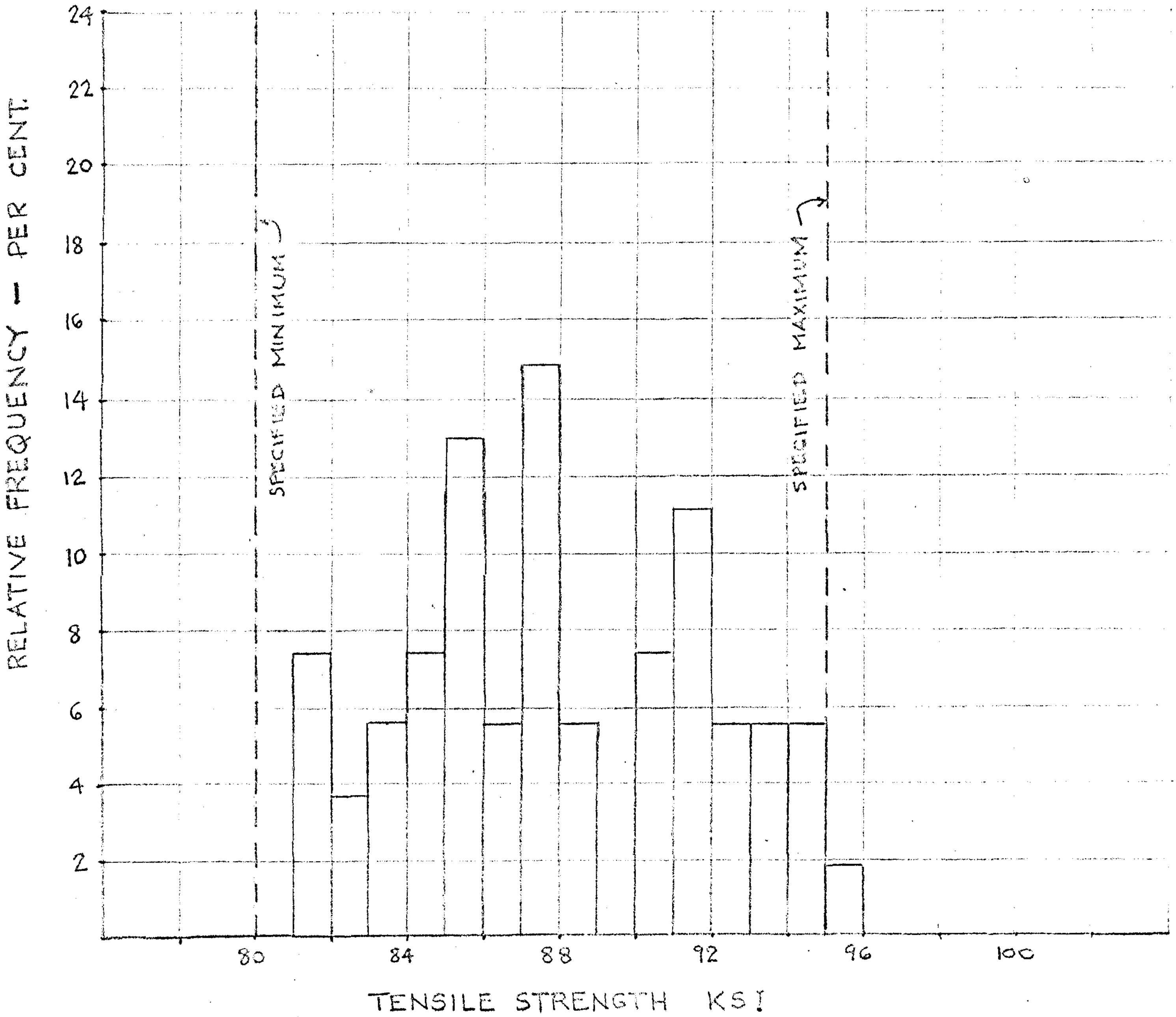


FIG. 14

C.S.A. G40.8 (UP TO AND INCL. 5/8" THICK)
YIELD STRENGTH 370 TESTS

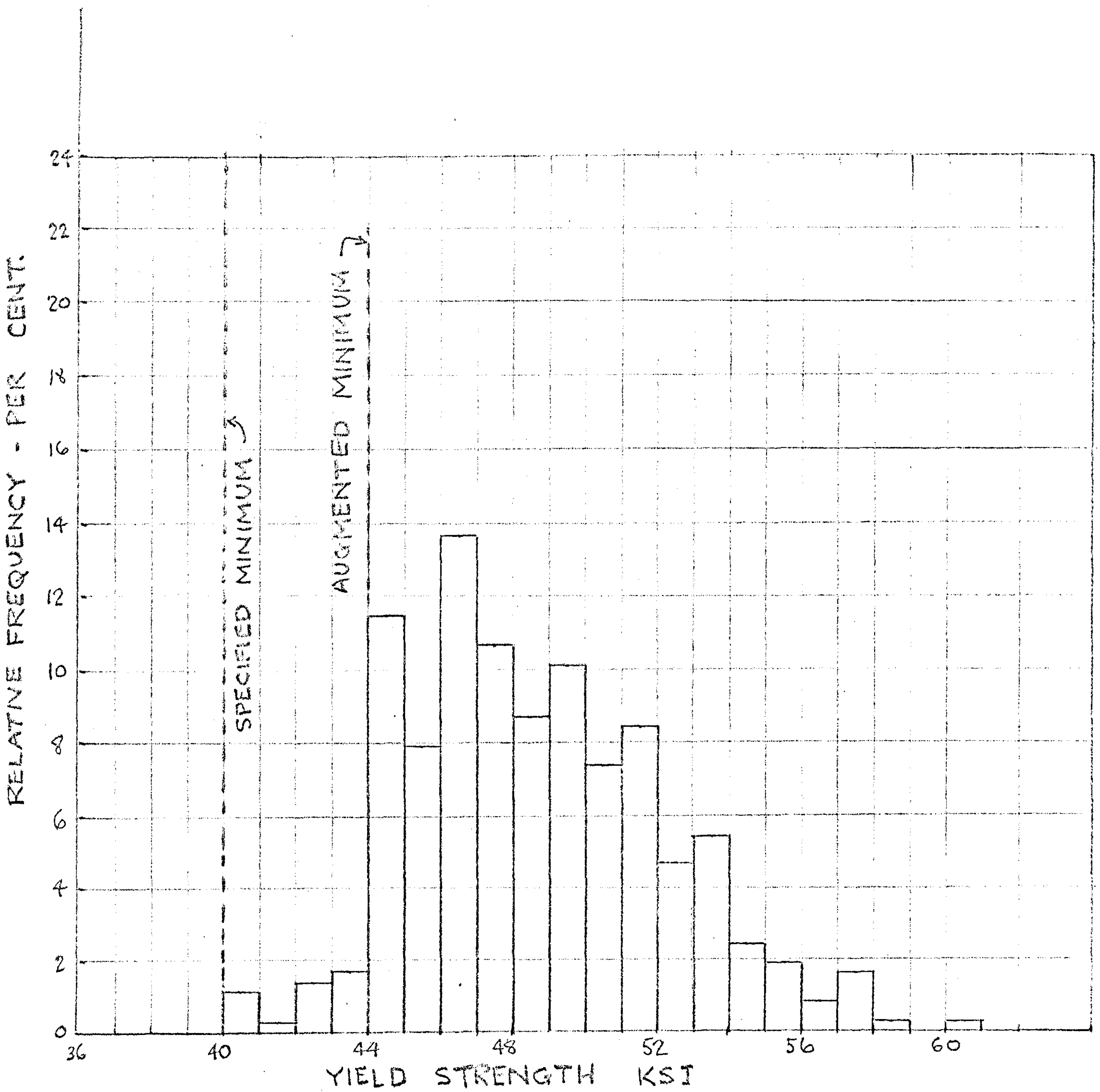


FIG. 15

C.S.A. G40.8 (UP TO AND INCL. 5/8" THICK)
TENSILE STRENGTH 370 TESTS

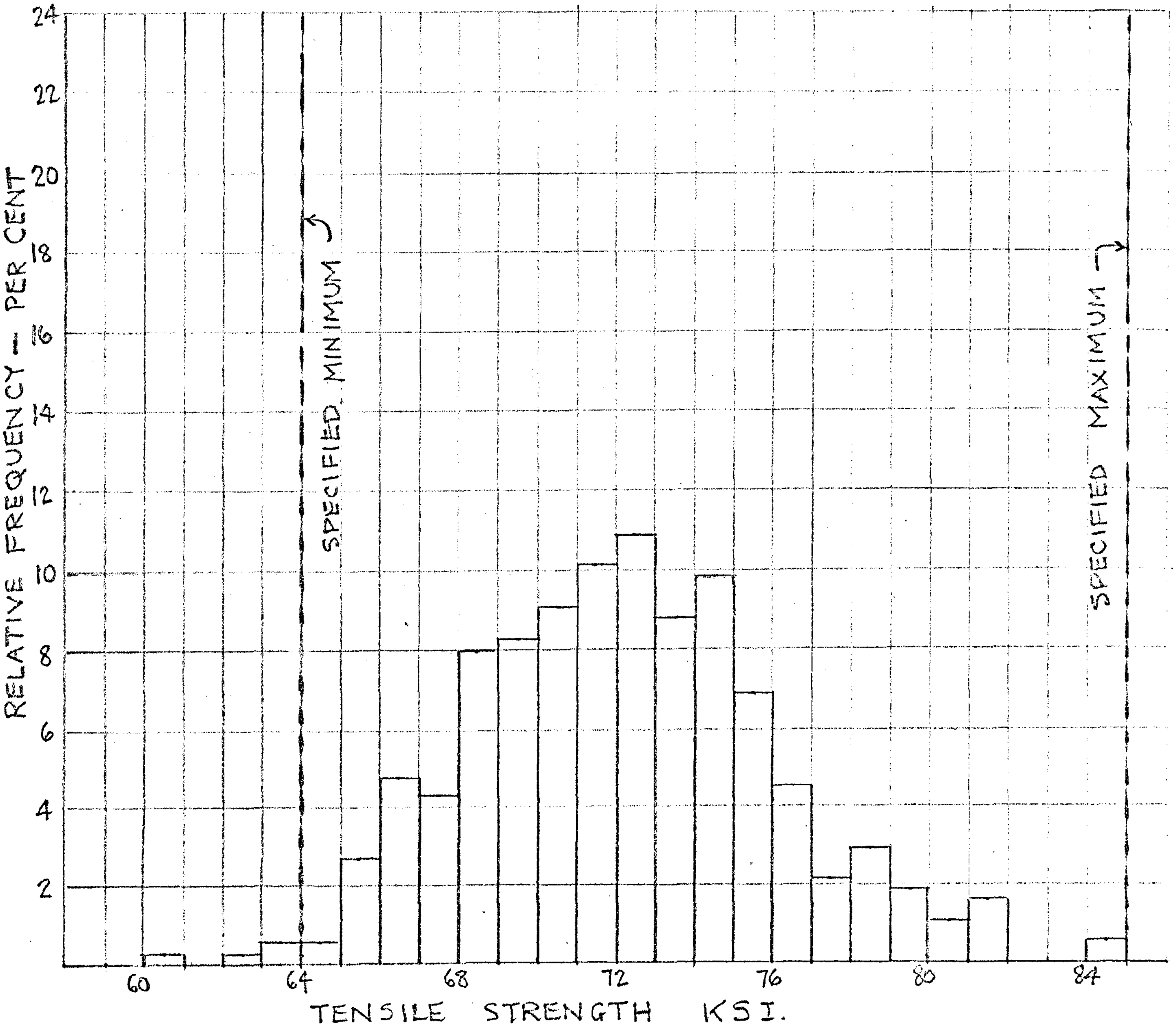


FIG. 16

ALGOMA "44"

YIELD STRENGTH; 34 TESTS.

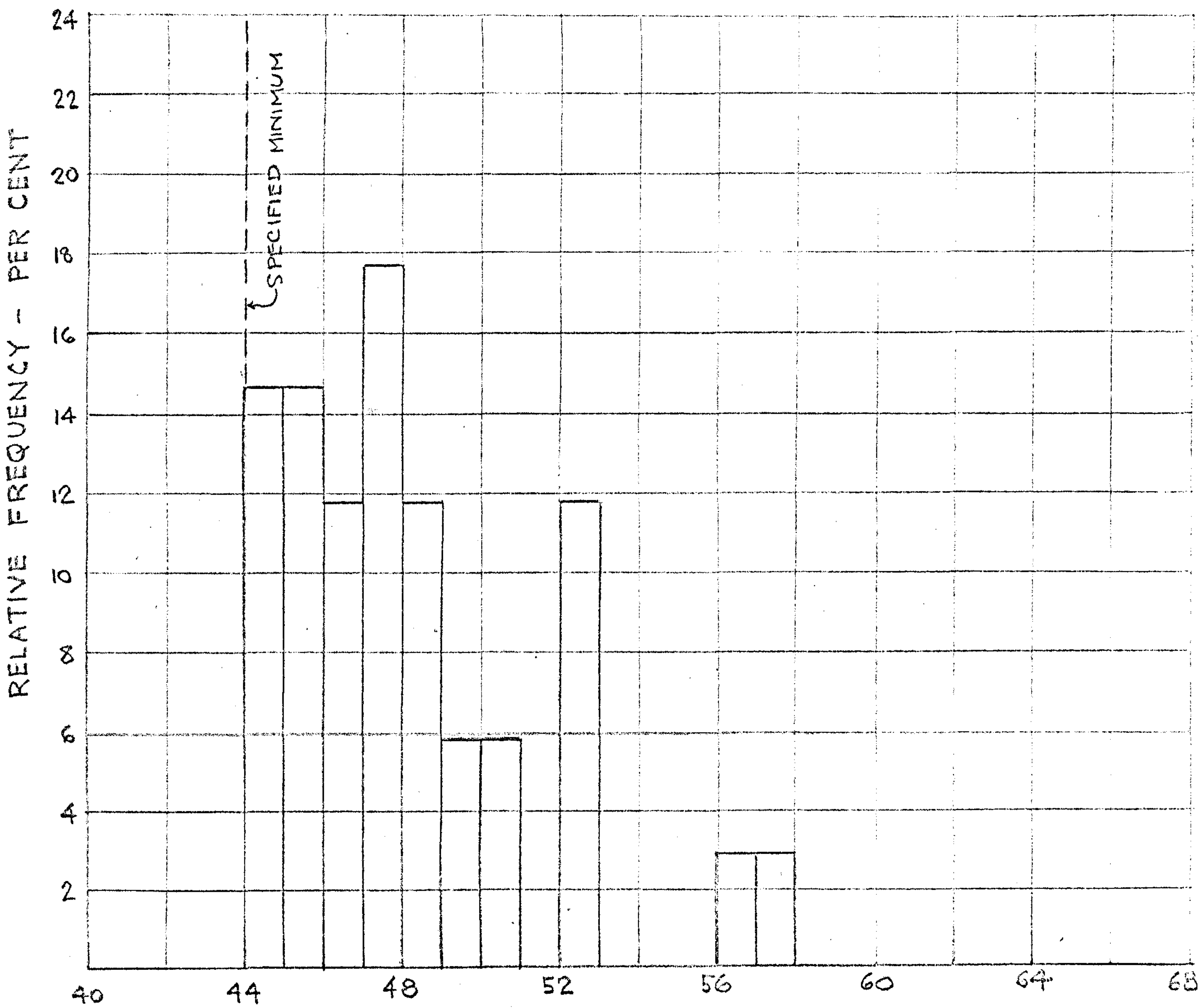


FIG. 17.

ALGOMA "44"

TENSILE STRENGTH; 34 TESTS.

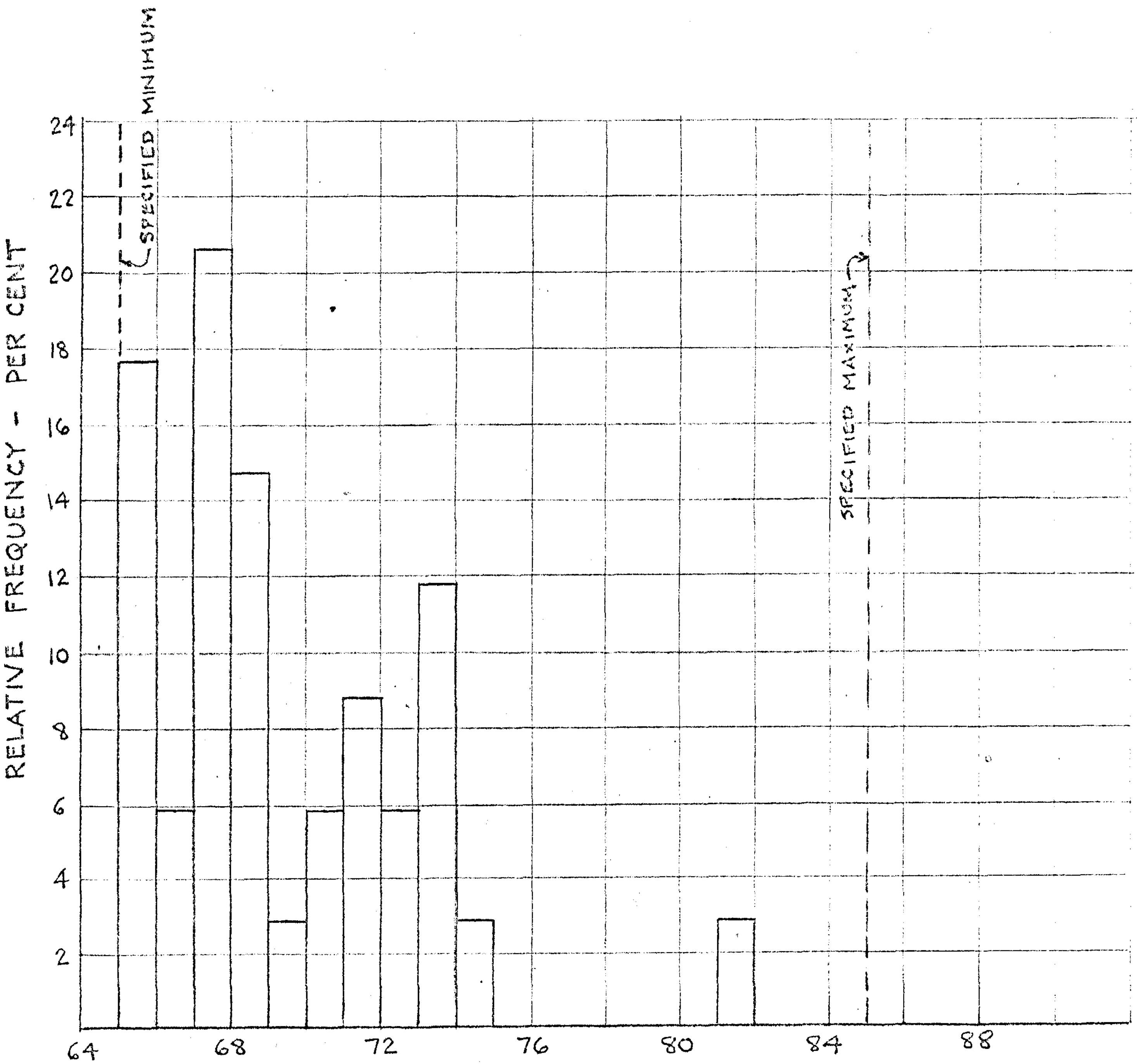


FIG. 18.

BETHLEHEM V45
U.S. STEEL EX-TEN 45
DOSCO W45

"45" TYPE STEELS
YIELD STRENGTH - 27 TESTS.

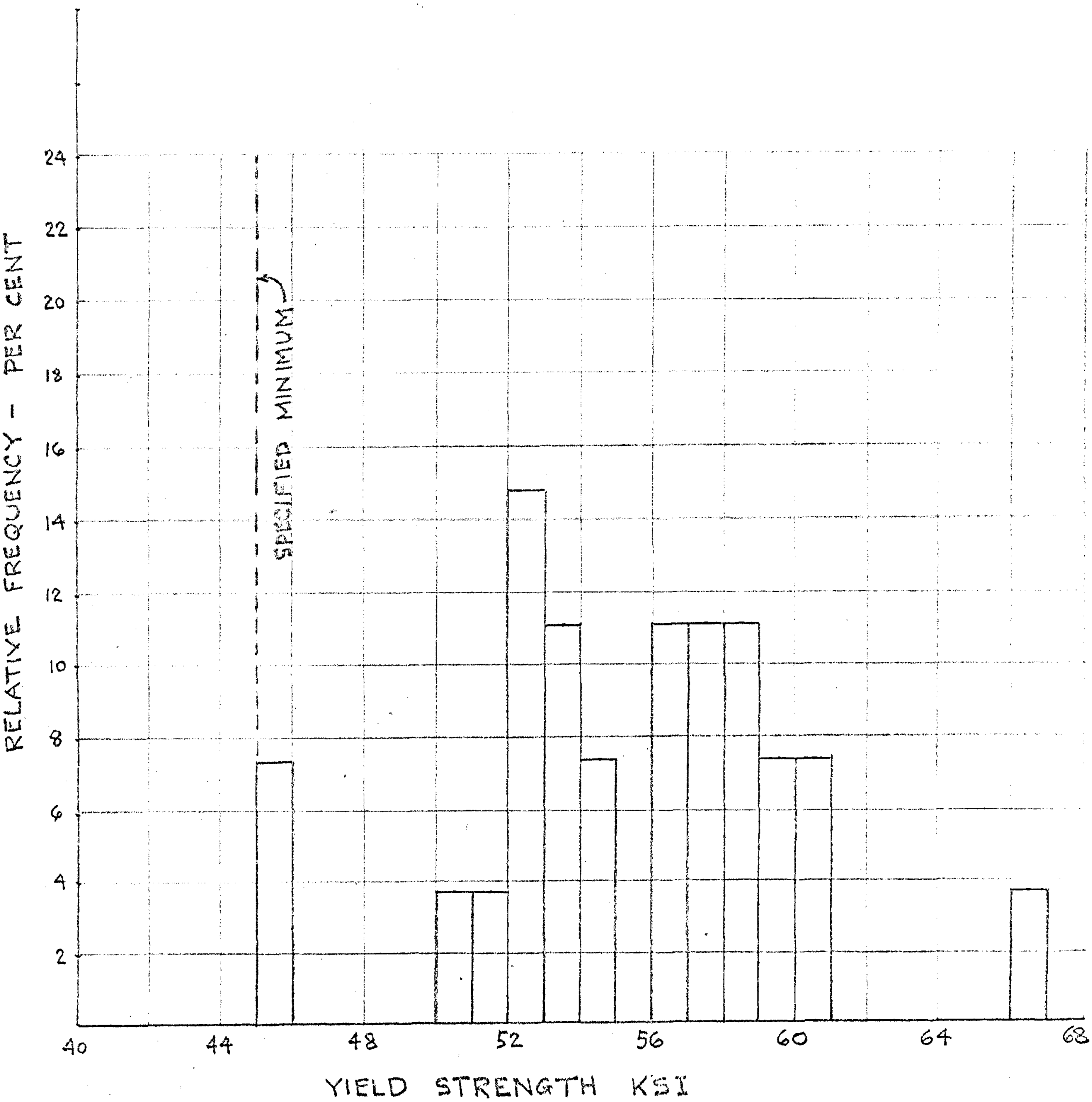


FIG. 19.

BETHLEHEM V45
U.S. STEEL EX-TEN 45
DOSCO W45
"45" TYPE STEELS
TENSILE STRENGTH - 27 TESTS.

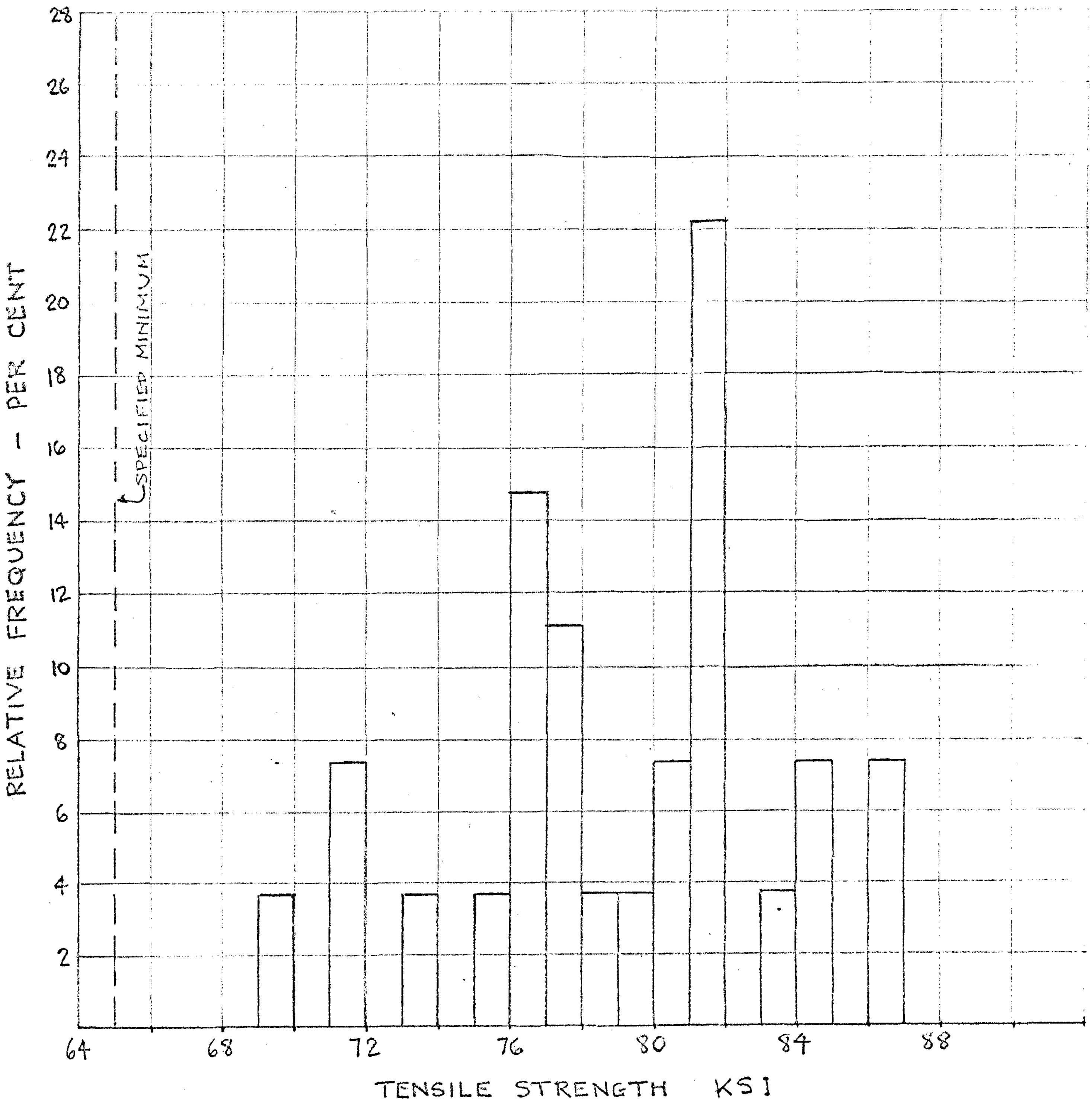


FIG. 20.

ALGOMA T60, DOSCO RB60, YIELD STRENGTH

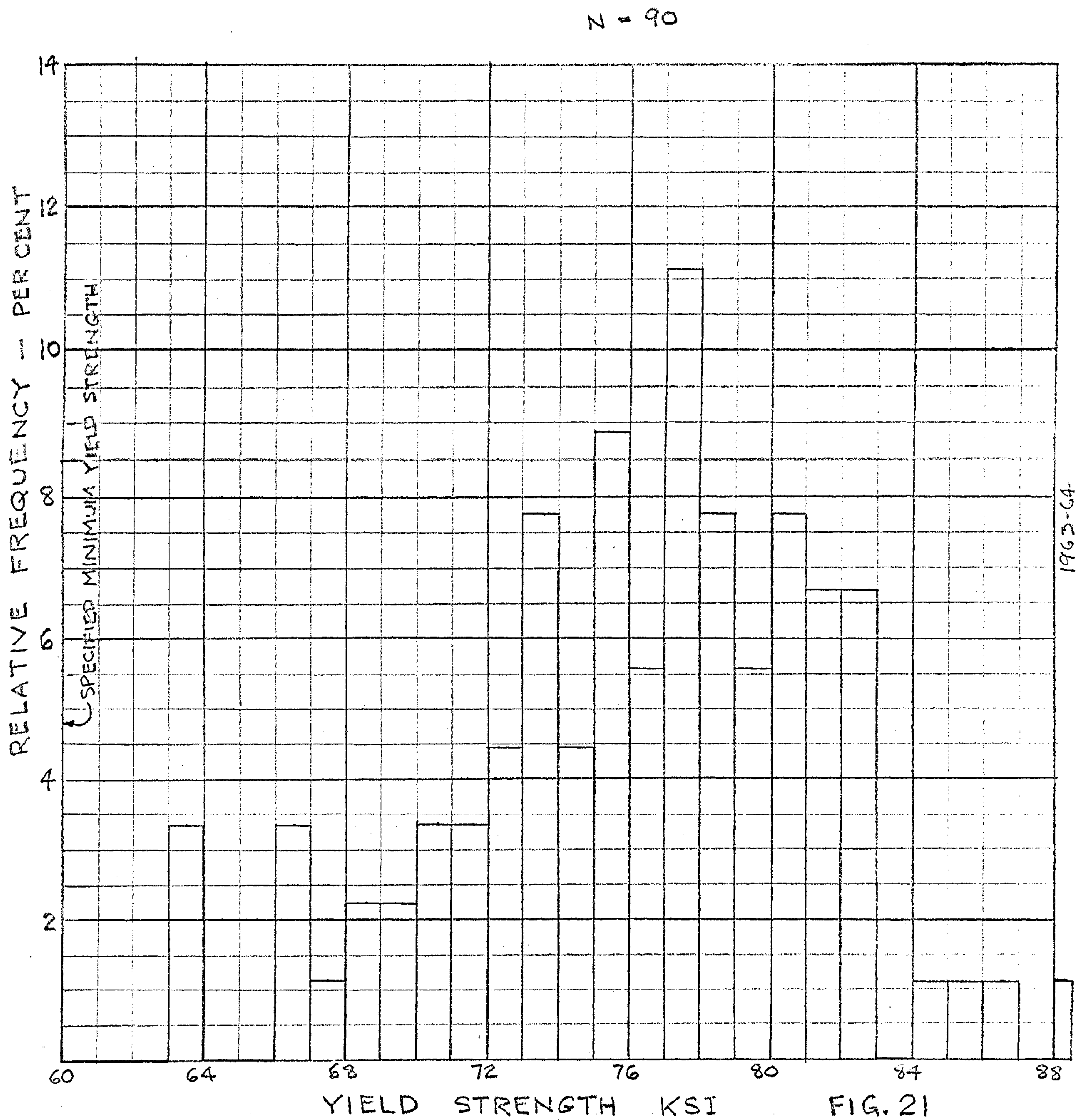


FIG. 21

ALGOMA T60, DOSCO RB60, TENSILE STRENGTH

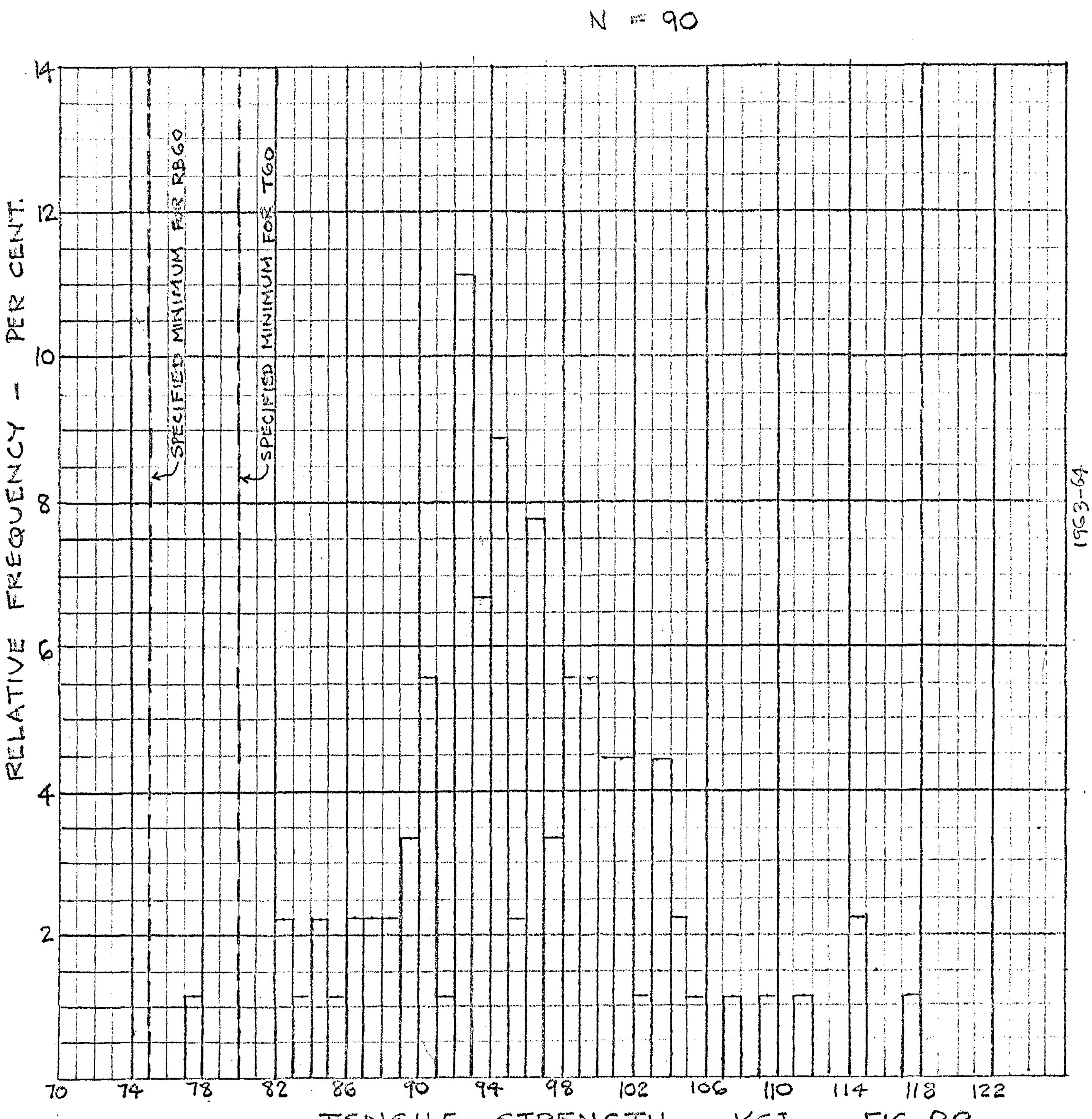


FIG. 22

time for grain growth, with a consequent lowering of the yield point. It should be noted that the material near the ends of the coil will cool more rapidly, and will have an appreciably higher yield point, and further, that the ends of the coil being more readily accessible, will be the sources of the coupons for the tension test performed at the mill. The difference in the yield point between specimens from the ends of the coil, and those from an intermediate part, might well be expected to exceed those noted between the web and flange of a wide-flange section. Information on this subject is not available. The ultimate effect of this feature may not be very serious, however, because in the majority of structures the lighter plates, which may be subject to the effects of coiling, will only appear in a secondary role, in gusset plates and similar details. Where plates begin to play a more important part in a structure, it will usually be found that they are the heavier sections which are not coiled at the mill.

The diagrams showing the yield points and tensile strengths for various types of steel, figures 4 to 22, illustrate the way in which the steel producer adjusts the factors at his disposal so that the finished product will have a yield point and a tensile strength on the safe side of the specified minimum. In other words, he "aims" at about 40 ksi in order to be reasonably sure of keeping above 33 ksi. The percentage of the product which fails to meet the requirements is very small. While this pattern is general, a possible exception must be noted; the results shown in figures 17 and 18 have an obvious tendency to "crowd" the specified minimum, and while the producer is meeting his obligations quite satisfactorily, it appears that his practice has been devised to permit a lower "aim" for this particular grade; the possibility exists that the production of this steel has been combined with a grade of lower strength, the latter furnishing an outlet for any material under the required level of the former.

The data in figures 4 to 22 refer, basically, to eight types of steel. Specifications G40.4 and A7 may be regarded as identical. It is instructive to compare figures 6 and 9, both of which refer to this type. Figure 6 dates from 1957, when this steel was the material used for 90% or more of structural work. Figure 9 is based on test reports of 1963 and 1964, after G40.4 had been largely replaced by a newer type, ASTM A36. The figures in figure 9 are indicative of a level generally higher than was found in the earlier survey, although the specification requirements had not changed. A change in mill operations is reflected; to simplify ingot production, practically all the heats are now being designed for the production of A36, and in cases where G40.4 or A7 is still being ordered, the material actually supplied is such as would be shipped out as A36.

The preceding comments on mill practice may serve to illustrate a point which has a bearing on the matter of design practice. In the first place, where the structural designer was confined to one type of steel as recently as ten years ago, he may now select his material from a long list, to which additions are made every year. For some of these new grades, standard specifications are available, while others are still identified only by the manufacturer's description. The development of new steels can affect the whole outlook regarding materials in several ways. The examples given above show that the margin between the specified minimum strength and the general average actually provided by the mills, may be either appreciably larger, or smaller, than was the case in earlier years. During an experimental period, a heat produced by a mill as the prototype of a new high-strength steel, could legitimately be used for the production of steel ordered to a "lower" specification. When a new specification has been published, the tendency may be for the steel-maker to err on the safe side in selecting the composition of the first heats. Either of these conditions lead to the production of steel which is well above the specified minimum. This condition will not continue, however, because growing familiarity will in time permit the metallurgist to effect economies by reducing the strength margin.

The objection may be raised, that so long as the designer, and the writer of a design specification, observe the properties set forth in the material specifications, they cannot be led into error by any changes in mill practice such as described above. There is an element of danger, however, in the tendency of some designers to follow some supposed safe precedent. The precedent may conceivably contain some approximation, some short-cut, or even some error, and yet be associated with a stable and satisfactory structure, owing to the live load being overestimated, and the actual yield point of the steel being substantially higher than the specified minimum. Application of the same method at a later date, using a more realistic figure for the live load, and taking advantage of a steel with a higher minimum yield point, may lead to the construction of something in which the margin between maximum stress intensity and the actual yield strength of the material disappears. In other words, an assumption which appears to be safe at one period may become unsafe owing to changes of circumstance.

The Elastic Modulus.

The modulus of elasticity of structural steel is not measured in the routine tests made by steel manufacturers. Text books and manuals usually assign a value between 29 and 30 million psi, with a preference by some for the value 29.6 million. It may be seen that in Figure 1, values of E appear which range from 27.5 to 30.2 million. In simple design, the value of E is not required unless it becomes necessary to calculate a deflection. In the design of indeterminate structures E forms part of the basic calculations. The elastic modulus, through its place in the Euler formula, influences the design rules for slender struts, but possible variations of its value in this respect are of less consequence than other factors in compression theory.

There is a design field in which the elastic modulus may be found at a lower level than in ordinary practice. In the testing of cold-formed sections, the initial slope of the stress-strain curve may correspond with a value of 25 to 27 million psi. This reduction appears to be the result of the cold-working which the metal has undergone. The effects of such variations have been examined in the work of the Column Research Council (9).

Tolerance on Sectional Dimensions.

Material specifications such as CSA G40.1 and ASTM A6 give tolerances for the main dimensions of the cross-section of a structural steel member. These tolerances are of importance when special accuracy of assembly is required, but are rarely of concern to the designer. There is a tolerance, however, which must be taken into account in the formulation of design rules; that is the plus or minus 2.5% allowed in the cross-sectional area and in the unit weight of a piece. This is the figure applicable to shapes; for plates, the minus tolerance is very small, most of the variation allowed being on the plus side. The tolerance in weight affects the calculation of dead loads, but although the change in weight is in proportion to the change in sectional area, the two effects do not necessarily cancel each other, because the added weight of some oversize components of a structure might increase the stress in other members which happened to be undersized.

DESIGN PRACTICE.

Definition.

The term "design practice"is intended to cover the principles and methods used in the adoption of certain basic assumptions regarding the nature of a structure and the applied loading, calculating the stresses in the various parts of the structure, selecting the sections to be used in its construction, and preparing any special instructions which may seem to be required for the guidance of the fabricator and erector.

Initial Decisions.

First, the general shape and function of the structure must be known; this information will include the outline, the main dimensions, the nature of the foundations or supports, and similar data.

Second, the magnitude and distribution of both live and dead loads must be determined, with due regard for such things as impact and vibration.

Third, the magnitude of the axial, bending, shear, and other stresses which may be permitted in the structural members must be decided.

Adoption of Loads.

In many cases, the live loads to be assumed in design calculations have already been established in a building code or some other applicable standard. The adoption of standardized loadings for various types of bridges and buildings is generally the result of much collective experience in that field, continually revised and amplified to keep pace with new developments. As an example of the revisions which are made in such bodies of information, one may cite the changes in the figures for snow loads, made in the National Building Code of Canada after some years of research, and the revision and amplification of the data on wind pressure in the same code.In general, such changes are intended to afford greater accuracy, and often permit the use of lower unit loads; hence it is in the interest of economical design to use the most up-to-date edition of such a code. At the same time, in the attempt to gain greater accuracy and economy, the rules are in some respects made a little more complicated, and it is the designer's duty to become familar with all phases of such rules. For example, a designer who takes full advantage of a reduction which may have been made in the snow loads for the region in which a structure is to be erected, must also pay attention to those provisions in the code which refer to the building up of heavier loads on canopies and sheltered roof areas.

The calculation of dead loads on a structure is not complicated, requiring only a knowledge of the unit weights of the various materials being used in its construction. The weight of the steel itself presents a problem, since the weight must be estimated before the sections have been selected. Obviously, this item should be checked after the design has been made, and if the original estimate should fall short of the design weights, a correction should be made, followed, if necessary, by a revision of the design.

Good design practice should include the preparation of reliable figures for the dead load and live load systems. This is not to say that such figures will represent exactly the actual loading on the structure. The unit weights of material are subject to some variation; the weight of concrete will depend on the type of aggregate used and other factors; the weight of the steel itself may vary by known percentages, and so forth. In the calculation of dead weights, only the novice is liable to be misled by the accuracy of his own arithmetic into believing that a similar accuracy will prevail in the finished structure. There may well be a difference of 5% between the calculated weight and the actual weight.

Even when information regarding live loads is provided by a building code or similar standard, the experienced designer is often aware of other loading which should be added. It is a common practice, born of experience, to design roof trusses for industrial buildings, for a certain amount of additional loading at the bottom chord, where conduits, conveyors, and other equipment is often supported. In office buildings, it is possible to find banks of weighty filing cabinets on floors originally planned as ordinary desk and working space. A case was reported not many years ago where a certain university found temporary storage for the contents of a library, during rebuilding operations, in a building designed for class-rooms or offices; the floors began to exhibit such obvious signs of distress that expert advice was sought, which led to the hasty removal of the overload. While the designer need not be expected to foresee such flagrant cases as this, he will usually be aware that overloads are possible, and may either make deliberate provision for such, or assume that some degree of overloading is provided for in the "safety factor" in the design specification he is using.

Determination of Permissible Stresses.

The individual designer rarely has to make the decisions as to the permissible stresses which will govern the selection of members for a structure. In buildings, bridges, and most other types of structure, these decisions have usually been made when the applicable design specification was written.

The function of design specifications and building codes is well expressed in the words of A.M.Freudenthal (10):

".. basic aspects of engineering science have a particular bearing on the preparation of standard specifications for structural design. The designer relies on the specifications for all relevant information about the fundamental assumptions and premises of his design. He cannot and should not be expected to ascertain, select and appraise, for each structure individually, the basic facts and conditions, such as load or permissible stresses, by which both the safety and economy of the structure will be determined. This is because only in exceptional cases will he command the wide knowledge and experience required for such selection and for the appraisal of all implications. With the increasing complexity of engineering problems he will have to rely increasingly on the organized collective experience of the profession and to substitute the considered objective judgment of the group for his individual opinion."

Further from the same source:

"Among the subjects of fundamental character covered by specifications for structural design are: (1) Design Load; (2) Resistance of structural members and shapes and their permissible stresses; and (3) Performance, selection, and quality control of structural materials. These are interdependent and must be coordinated. Thus, a specified permissible stress has real meaning only in conjunction with the specification of the design load and the method of structural analysis."

The relationship between material properties, applied loads, and design procedures, has received attention from a number of writers in recent years. Besides the paper (10) by Professor Freudenthal from which the above extract had been taken, there are three other papers (8), (11) and (12) by the same author, Professor Winter's paper (1) to which reference has been made, and others byC.B.Brown (13), Hsuan Loh Su (14), and Malcolm S. Gregory (15). These authors, and others, have pointed out that if engineering is to become more of a science and less of an art, data on the several aspects of material properties, applied loads, and distribution of stresses, must be collected and analyzed. It has also been pointed out that some of these factors should be the subject of statistical study, and in some phases of the subject, this approach has been used by Professor Freudenthal and others. In spite of all this, there seems to be difficulty in finding the numerical data without which the practicing engineer or the writerof specifications is at a loss. Yet the engineers who draft design specifications have to set down definite rules and definite permissible stresses, if their work is to be of any real value.

The extent to which the individual designer relies on design specifications is well expressed in the words of Professor Freudenthal, quoted above. This reliance must always be kept in mind by those who participate in the preparation of such specifications. In such work, decisions frequently have to be made as to the inclusion of some clause, or the degree to which some rule should be elaborated. It is necessary to foresee, as far as is possible, the assumptions which will be made, and the methods which will be used, by those who refer to the specification. The work may be primarily intended for the use of qualified designers, with sufficient technical background to enable them to understand the intent of the various clauses. The fact remains, however, that a design specification, particularly one for building frames, will be used not only by qualified engineers, but also by technicians with different degrees of experience, in architects' offices and in the fabricators' drafting rooms.

Misinterpretation of Specifications.

Questions as to the exact meaning of some clause in a specification may well be asked, because of some ambiguity in the wording, or some difference in the semantic backgrounds of the writer and the reader. Instances occur, unfortunately, when the reader forms his own conclusion without asking any questions, and derives from the written requirement a meaning which was never intended by the writer. This is the eventuality which gives most concern to the writer.

As a specific example of the manner in which a simple rule may be misinterpreted, it is of interest to turn to clause 11.1 of the Canadian Standards Association standard S16. This clause says that the effective length of beams and other flexural members should generally be taken as the distance centre-to-centre of supports. This is actually an approximation which has little to recommend it but some simplification of arithmetic. It has been, however, taken by some to mean that if the beam is designed as if supported at the axis of the supporting member, than the latter may be designed as if the reaction from the beam were actually delivered directly at the axis of the support. This is unfortunately not the case. Some provision may be made in truss connections for the development of a correcting moment which, in theory at least, will have the desired effect, but in the case of an ordinary beam connection, the moment developed, if any, will be additive to whatever moment is due to the eccentric connection of the beam. In spite of this, and of the final sentence in said clause 11.1, which reads:"The design of the supporting members shall, in any case, provide for any moment or eccentricity arising from the manner in which a beam, girder or truss is actually connected.", there are designers who disregard the moment caused by the connection of a beam to a column flange, and design the column as if the loading were purely axial.

SECONDARY STRESSES.

Definition of Secondary Stresses.

For the purposes of this discussion, secondary stresses will be defined as follows:

"The term secondary stresses is applied to those stresses which are omitted in those conventional methods of calculation that depend on the adoption of simplifying assumptions".

The above definition is taken from the Belgian standard for steelwork for buildings (16). An important feature of any design method lies in the simplifying assumptions which are made; something of the kind applies to any type of design, but it is in the so-called conventional method that these assumptions are most far-reaching. It is, therefore, with this particular aspect of design practice that those who formulate the rules for design must be chiefly concerned.

The Canadian, U.S. and British specifications for building steelwork recognize three types of design for such structures. These are the rigid, semi-rigid and conventional or simple methods.

In the rigid type of design, in which we may include the method of plastic design, the moments developed at the joints of a structure form an important part of the calculations, and joint details are assumed to be such that full rigidity is realized. In the semi-rigid type, the connections are not supposed to be perfectly rigid, but the effects of whatever rigidity they may possess are taken into account in the design calculations. In the conventional type, which is much simpler than either of the other two, the joints are supposed to act as hinges, and this "simplifying assumption" makes it easy for the designer to select the sections required for the structural members by reference to capacity tables in the design manuals.

From the foregoing, it will immediately be seen that in the conventional type of design, all bending stresses which occur at the joints are classified as secondary moments, and omitted from the calculations.

Because secondary stresses are omitted from conventional design calculations, designers sometimes assume that they are actually so small as to be negligible. The fact is, that such stresses are frequently of appreciable magnitude, and may be neglected from the calculations in the conventional method of design, only because some allowance has been made for them in the design rules.

In connection with conventional design, a striking inconsistency is to be found. Beams, trusses and columns are designed as if all the connections were frictionless hinges. At the same time, if the connections provided by the fabricator approached that condition, the result would be greeted with some dismay, by architect and builder alike. The building frame which is designed as "non-rigid", is expected to possess sufficient rigidity to remain stable when erected. On the completion of the building, there are usually walls and other elements which provide stability, but the occasion can and does arise when a steel frame may be left standing alone for some weeks, before walls and roof are added. As has been pointed out by a number of writers, the usual type of connection used in "non-rigid" frames actually does possess an appreciable degree of rigidity.

Increasing Importance of Secondary Stresses.

It has been pointed out, above, that changes in specifications and in building codes make it inadvisable to accept past experience as a guide without some critical scrutiny of the effects of the changes. This necessity is emphasized in the introductory paragraphs of a bulletin published in France (17), from which the following quotation is obtained:

"So long as metal structures were designed forlimits of unit stress corresponding with modest fractions of the elastic limit or the ultimate strength, it appeared to be unnecessary to be concerned about the secondary stresses which were likely to occur in current construction: it was thought that even if unusual cases might occur in which these secondary stresses might attain magnitudes comparable with the calculated stresses, structures would not on that account be involved in catastrophic failure."

"The situation has now changed. Permissible stresses have been progressively increased; they are now based on 16 kg/mm^2 (22,800 psi); if secondary stresses add much to that, the stability of structures could be compromised. The mild steel generally used fortunately lends itself readily, by so-called adaptive deformations, to the mitigation of local stress concentrations, but one must take into account, nowadays, steel of harder grades, of which the adaptive qualities are less satisfactory. Certain features of modern construction, such as the use of heavy sections and of welded connections, simultaneously reduce the ability of the structure to adapt itself, and increase what are termed secondary stresses. It becomes necessary, therefore, to make a study of these stresses, whether it be to take steps to reduce them, or to make allowance for their importance. (These are the respective approaches of the fabricator who prepares the details, and of the designer who, in the absence of a proper evaluation, tends to estimate them at excessive values.)"

Specification Rules on Secondary Stresses.

Since the definition of secondary stresses given above has been taken from the Belgian standards (16), it is fitting that the provisions of those standards be repeated here:
"Charpentes Métalliques; NBN 1"

"The calculation of secondary stresses is not required as a general rule. The adopted limits of safety take into account the additional stresses of this kind which may occur in a structure, in a job which has been correctly designed, and fabricated under normal conditions."

"It is a function of the designer to determine whether, in special cases, the secondary stresses may attain unusual importance. In such cases they lose the character of secondary stresses, and it becomes incumbent upon the project engineer to calculate the additional stresses and combine them with those obtained by the simplified method of analysis, or to increase the design loads, or to adopt a method of calculation which will give the actual stresses."

C.S.A. Standard S16, "Steel Structures for Buildings":

"18.1 Eccentric Connections. Members of a frame meeting at a joint shall, if practicable, be arranged so that their neutral axes intersect at a point. If not practicable, then the results of the eccentricity shall be provided for."

"18.2 Symmetry.

Members in general shall be of symmetrical sections. All connections and splices shall be, as nearly as practicable, symmetrical about the axes of the members connected thereby."

"18.3 Placement of Fasteners.

Rivets, bolts and welds at the ends of any member, which transmit stresses into that member, should have their centres of gravity on the gravity axis of the member; otherwise, provision shall be made for the effect of the resulting eccentricity. Pins may be placed to counteract the effect of bending due to dead load."

"18.4 Unrestrained Members.

Except as otherwise provided, all connections of beams, girders or trusses shall be designed as flexible, and may ordinarily be proportioned for the reaction shears only. If, however, the eccentricity of the connection is excessive, provision shall be made for the resulting moment."

"18.5 Restrained Members.

When beams, girders or trusses are subject both to reaction shear and end moment due to full or partial end restraint, or to continuous or cantilever construction, their connections shall conform to the requirements of Section 9 (combined stresses) or Section 33 (plastic design), whichever is applicable."

In the "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings" published by the American Institute of Steel Construction, the corresponding portion reads:

"Section 1.15 Connections.

"1.15.2 Eccentric Connections.

Axially stressed members meeting at a point shall have their gravity axes intersect at a point where practicable; if not, provision shall be made for bending stresses due to the eccentricity."

"1.15.3 Placement of Rivets, Bolts and Welds.

Except as hereinafter provided, the rivets, bolts or welds at the ends of any member transmitting axial stress into that member shall have their centers of gravity on the gravity axis of that member unless provision is made for the effect of the resulting eccentricity. Except in members subject to repeated variations in stress, as defined in Section 1.7, disposition of fillet welds to balance the forces about the neutral axis or axes for end connections of single angle, double angle, and similar type members is not required. Eccentricity between the gravity axes of such members and the gage lines for their riveted or bolted end connections may be neglected."

"1.15.4 Unrestrained Members.

Except as otherwise indicated by the designer, connections of beams, girders or trusses shall be designed as flexible, and may ordinarily be proportioned for the reaction shears only."

"Flexible beams connections shall permit the ends of the beam to rotate sufficiently to accommodate its deflection by providing for a horizontal displacement of the top flange determined as follows:

$e = 0.007d$ when the beam is designed for full uniform load and for live load deflection not exceeding 1/360 of span.

$e = \frac{f_b L}{3,600,000}$ when the beam is designed for full uniform load producing the unit stress f_b at mid-span.

where

e = the horizontal displacement of the end of the top flange, in the direction of the span, in inches,

f_b = the flexural unit stress in the beam at mid-span, in pounds per sq. inch.

d = the depth of the beam, in inches,

L = the span of the beam, in feet. "

"1.15.5 Restrained Members.

Fasteners or welds for end connections of beams, girders or trusses not conforming to the requirements of Sect. 1.15.4 shall be designed for the combined effect of end reaction shear and tensile or compressive stresses resulting from the moment induced by the rigidity of the connection when the member is fully loaded."

British Standard 449: 1959, "The Use of Structural Steel in Building" gives the following rules in Clause 43a:

"(i) Members of braced frames and trusses shall, where practicable, be disposed symmetrically about the resultant line of force, and the connections shall, where practicable, be arranged so that their centroid lies on the resultant of the forces they are intended to resist (see clause 48c)."

"(ii) In the case of bolted, riveted or welded trusses and braced frames, the strut members act under complex conditions and the effective length L shall be taken as between 0.7 and 1.0 times the distance between centres of intersections, depending on the degree of end restraint (but see also clause 30c)."

"Clause 48c: Members meeting at a joint.

For triangulated frames, designed on the assumption of pin jointed connections, members meeting at a joint should, where-ever practicable, have their centroidal axes meeting at a point; and wherever practicable the centre of resistance of a connection shall lie on the line of action of the load so as to avoid an eccentricity moment on the connections."

"However, where eccentricity of members or of connections is present, the members and the connections shall provide resistance to the induced bending moments."

"Where the design is based on non-intersecting members at a joint, all stresses arising from the eccentricity of the members shall be calculated and the stresses kept within the limits specified in the appropriate clauses of this British Standard."

"Clause 30c: Angles as Struts.

"(i) For single-angle discontinuous struts connected to gussets or to a section either by riveting or bolting by not less than two rivets or bolts in line along the angle at each end, or by their equivalent in welding, the eccentricity of the cnnection with respect to the centroid of the strut may be ignored and the strut designed as an axially -loaded member provided that the calculated average stress does not exceed the allowable stresses given in Table 17, in which L is taken as 0.85 times the length of the strut, centre-to-centre of intersections at each end, and r is the minimum radius of gyration."

"Single-angle struts with single-bolted or riveted connections shall be treated similarly, but the calculated stress shall not exceed 80 per cent of the values given in Table 17, and the full length L centre-to-centre of intersections shall be taken. In no case, however, shall the ratio of slenderness for such single angle struts exceed 180."

"(ii) For double angle discontinuous struts, back-to-back connected to both sides of a gusset or section by not less than two bolts or rivets in line along the angles at each end, or by the equivalent in welding, the load may be regarded as applied axially. The effective length L shall be taken as between 0.7 and 0.85 times the distance between intersections, depending on the degree of restraint, and the calculated average stress shall not exceed the values obtained from Table 17 for the ratio of slenderness based on the minimum radius of gyration about a rectangular axis of the strut. The angles shall be connected together in their length so as to satisfy the requirements of Clause 37."

"(iii) Double angle discontinuous struts back to back, connected to one side of a gusset or section by one or more bolts or rivets in each angle, or by the equivalent in welding, shall be designed as for single angles in accordance with c(i) and the angles shall be connected together in their length so as to satisfy the requirements of Clause 37."

"(iv) The provisions in this clause are not intended to apply to continuous angle struts such as those forming the rafters of trusses, the flanges of trussed girders, or the legs of towers, which shall be designed in accordance with Clause 26 and Table 17."

(Note.- Clause 26 deals with the effective length of compression flanges for beams and girders.)

With regard to the connections of beams to columns or stanchions, BS 449 gives the following rules:

"Clause 34a:

For the purpose of determining the eccentricity of beam reactions or similar loads on a stanchion or column, the load shall be assumed to be applied as given in Table 18 but at a distance not less than 2 in. from the centroid of the strut section, except in case (v) and where packing is used."

TABLE 18. ASSUMED ECCENTRICITY OF LOADS ON STANCHIONS AND SOLID COLUMNS.

Type of Connection.	Assumed point of Application.
(i) Stiffened Bracket	Mid-point of stiffened seating.
(ii) Unstiffened Bracket.	Outer face of vertical leg of bracket.
(iii) Cleats to web of beam.	Face of strut.
(iv) Cleats on tube.	Outside of tube.
(v) Cap: a) Beams.	Face of column shaft or stanchion section towards span of beam.
b) Roof truss bearings.	No eccentricity for simple bearings without connections capable of developing an appreciable moment.

"b. In effectively jointed and continuous stanchions the bending moments due to eccentricities of loading at any one floor or horizontal frame level may be taken as being:

(i) Ineffective at the floor or frame levels above and below that floor.

(ii) Divided equally between the stanchion lengths above and below that floor or frame level, provided that the moment of inertia of either stanchion section, divided by its actual length, does not exceed 1.5 times the corresponding value for the other length. In cases exceeding this ratio the bending moment shall be divided in proportion to the moments of inertia of the stanchion sections, divided by their respective lengths."

The corresponding French regulations, "Règles pour le Calcul et l'Exécution des Constructions Métalliques" contribute this:

"Art.3,402 Secondary Stresses.

"Riveted, bolted or welded assemblies shall be devised in such a manner that secondary stresses shall be kept to a minimum.

"In particular:

1) In trussed members, the centroids* of the chords and of web members shall intersect at a common point.
2) In main structural members, it is recommended that web members shall be arranged symmetrically with respect to the axial plane of the main member and shall be connected symmetrically.
3) If conditions (1) or (2) are not realized, it shall be necessary to take into account, in the calculations, the resulting increase in the stresses.

* It is assumed that the forces are transmitted along the centroidal axis of the member (the line at the centre of gravity)."

While the treatment of secondary stresses in the French specification is not very elaborate, the remark on the subject in the commentary, "Commentaires des Règles pour le Calcul etc. " (15) is of particular interest in the light of our present investigation:

"Secondary Stresses:

Once more it becomes necessary to call attention to secondary stresses, certain unfortunate occurrences to metal frames having been largely due to a lack of observation of the rules set forth in 3,402 1 and 2. "

Although it is not proposed to make a detailed study of the secondary stresses in bridges here, this is because the subject has been treated at some length in text books and specifications. The AASHO "Standard Specifications for Highway Bridges" and the AREA "Manual of Recommended Practice" (chapter 15) use identical language with regard to secondary stresses; this is found in Art. 1.6.7 of the AASHO publication, and Art. 30 of the AREA manual:

"The design and details shall be such that secondary stresses will be as small as practicable. Secondary stresses due to truss distortion usually need not be considered in any member the width of which, measured parallel to the plane of distortion, is less than 1/10 of its length. If the secondary stress exceeds 4000 psi for tension members and 3000 psi for compression members, the excess shall be treated as a primary stress."

In the AREA manual there is a commentary on the above:

"It is provided that if the secondary stress exceeds 4000 psi for tension members and 3000 psi for compression members, the excess shall be treated as a primary stress. The above stress of 4000 psi for tension members when added to the maximum allowable stresses for all of the forces named in Art.29 results in a maximum combined stress of 18,000 + 4,500 + 4,000 = 26,500 psi, which is considered to be the maximum permissible."

"This article also provides that secondary stresses due to truss distortion usually need not be considered in any member the width of which, measured parallel to the plane of distortion, is less than 1/10 of its length. Excepted from this general provision should be the effects of secondary truss members, such as floor-beam hangers and subverticals; these may produce excessive secondary stresses in the chord unless adjustment is made in the length of the verticals."

Secondary stresses in bridge trusses form the subject of a chapter in the text, "Modern Framed Structures", by Johnson, Bryan and Turneaure. In this field of design, they identify such stresses as arising from four causes;

(1) Bending stresses in the plane of the main truss due to rigidity of joints.
(2) Bending stresses in members of a transverse frame due to the deflection of floor beams.
(3) Stresses in a horizontal plane due to longitudinal deformation of chords, especially the stresses in floor beams and connections.
(4) Variation of axial stress in different elements of a member.

Of the foregoing, the bending stresses of type (1) are said to be, "sometimes of large amount and required careful consideration. It is generally possible and sufficient so to design a structure as to keep these stresses within reasonable limits and then to neglect them in the calculations, but in many special cases, and in large and important structures, they should be calculated."

The same text goes on to say:

"The secondary stresses are in general proportional to the primary stresses, and therefore, are conveniently expressed in percentages of primary stresses."

"Other things being equal or similar, the percentages of secondary stress are proportional to the distances from gravity axis to outer fibre in the plane of bending, and inversely proportional to the lengths of the members. When the members are symmetrical the secondary stresses are proportional to the ratios of widths to lengths"

"The more uniform the proportions of a truss, the less, in general, will be the secondary stresses. Sudden changes in length, width, or in moment of inertia, are likely to result in relatively large secondary stresses."

Typical distributions of secondary stresses in bridge trusses are shown diagrammatically in this text. For a riveted Pratt truss, with a 160-ft. span, trusses 31 feet deep and panels 26'8" long, the secondary bending stresses are shown as 10% in top chord and end posts; 20% in bottom chord; 10% in diagonals and 20% in verticals. In a riveted pony truss, span 105 feet, depth 10 feet, and panels 13' 1½" long, the stresses run 10% in end post; 20 and 30% in top chord; 30 and 40% in bottom chord; 10 and 20% in diagonals and 100 to 120% in the hangers. Other Pratt and Warren trusses are reported with somewhat greater values, reaching 30 to 40% and about 400 to 450 times the width/length ratio of the members.

Other points developed in the text are the adverse effect of subdividing the panels of the chords, and the possibility of serious secondary stresses in trestle bents due to axial deformation of the leg members, when certain a rrangements of the bracing are adopted.

The British Standard for "Steel Girder Bridges", No. 153, which appears to apply to truss as well as girder structures, makes fewer concessions to the convenience of the designer than the AREA and AASHO. The B.S. 153, Part 3B, after citing the way in which secondary stresses arise in triangulated structures, identify them as:

"(i) Stresses which are the result of eccentricity of connections and of off-joint loading generally (i.e. loads rolling direct on chords, self weight of member and wind loads on member),"

"(ii) Stresses which are the result of the elastic deformation of the structure and the rigidity of the joints."

B.S. 153 goes on to specify:

"b. Such structures shall be designed, fabricated and erected in such a manner as to minimize as far as possible secondary stresses.

"c. Secondary stresses which are the result of eccentricity of connections and of off-joint loading generally (a(i) above), shall be computed and combined with the co-existent axial stresses in accordance with clause 23, but secondary stresses due to the self weight and wind on the member shall be ignored in this case."

"d. Secondary stresses which are the result of the elastic defrmation of the structure and the rigidity of the joints (a(ii) above) need not be calculated for a truss in which the ratios of the widths of the members (in the plane of distortion) to their lengths between centres of intersections are not greater than 1/12 for chord members and 1/24 for web members, provided that the allowable working axial tensile stresses are reduced by 10% in all web members in which axial stresses due to live load and its dynamic effects are greater than those due to dead load."

"e. Where, for any combination of loading, all secondary stresses due to eccentricity of connections and off-joint loading generally, and to the elastic deformation of the structure combined with the rigidity of the joints, are computed and combined with the co-existent axial stresses in accordance with Clause 23, the appropriate allowable working stresses shall be increased by 20%."

"f. Structures may be prestressed to counteract the effects of secondary stresses, in which case it is permissible to ignore these stresses in the members relieved."

It is a general practice in the fabrication of bridge trusses, to adjust the lengths of the truss members by amounts equal to all, or some specified proportion, of the elastic deformation which those members are expected to develope. This adjustment is made in order to produce camber in the span. If at the same time, the gusset plates at the joints are laid out so that the angles provided between the truss members correspond with the original, uncambered lengths of members, then in theory at least, the members will come into the correct alignment after they have been stressed by dead load, or dead plus half live load, or full dead plus live load, as the case may be. In that case, the members are assumed to be assembled at the site with some initial bending in them, due to their being forced into alignment with the holes in the gussets, this initial bending being cancelled in whole or in part when the truss receives its elastic deformation under load. This practice, if applied correctly, would eliminate the bending stresses due to elastic deformation and to rigidity of joints. There is, of course, some question as to how accurately the members will assume the correct angles with relation to each other, during erection. As mentioned above, the members have to be forced into the required alignment, and even with connections reamed and matched in the proper manner, there could be some slight relaxtion at the joint. If, on the other hand, the geometry controlling the gusset details is that of the "cambered" truss, then the members will fit together at the site without any effort, but will be subject to the full effect of elastic deformation and joint rigidity when the loads are imposed.

The foregoing review of practice in bridge design is instructive in several respects, but chiefly, in this context, for information regarding the approximate magnitude of the allowance made for secondary stresses, in the selection of the permissible stresses in the specifications. In the AREA commentary, for example, we learn that while the basic stress is set at 18,000, this stress is selected so that allowance may be made for an additional 4,500psi for a certain combination of forces due to impact, centrifugal, lateral and longitudinal effects, and to this may be added yet another 4,000 psi for secondary stresses. These figures would apply to structural steel of the ASTM A7 grade, with a specified minimum yield strength of 33,000 psi.

The approach of the British Standard 153 is different, but produces much the same effect. In trusses with comparatively slender members, there is a proviso calling for a 10% reduction in the permissible stress for the more vulnerable web members; this 10% would be applied to a basic design stress of 20,000 psi, as against 18,000 for the same type of steel in the AREA manual. On the other hand, if the designer calculates and makes provision for the secondary stresses, the appropriate permissible stresses may be increased by 20%. This 20% would be 4,000 psi for this type of steel, which is the same figure as allowed by the AREA as the margin included for secondary stresses.

A comparison of the specifications for bridges with those for building frames, previously outlined, will disclose that the latter are less precise in the wording of their requirements where secondary stresses are concerned. They recognize their existence, request the designer to do what he can to minimize them, and more or less leave it at that. One reason for leaving the problem and its treatment in such general terms, is the far greater variety of conditions which can arise in building frames, as compared to bridges. It must be recognized that when a rule or requirement is stated in general terms, and is qualified by the phrase "when practicable", it is more easily ignored or misinterpreted by designers who should be warned and guided by it.

Structural failures provide effective but costly lessons for designers and specification writers; it is more desirable to foresee the causes than to learn them from their effects. The reference in the French commentary to "unfortunate occurrences" should serve as a warning to every designer, and a reminder that while the permissible stresses and the design methods contained in any design specification,make some provision for some factors which are omitted from calculations in the interests of simplification, there are limits to this process of omission.

Causes of Secondary Stresses.

The terms of various design specifications which have been quoted convey some information as to the way in which secondary stresses originate in a structure. This information may be limited by the intent and purpose of such specifications; for example, rules for bridge design need not envisage circumstances which would be expected to occur only in buildings.

The calculations in a structural design apply to a "model" of the structure, rather than to the structure itself. This model is the framework assumed by the designer as the basis for his work, supporting exactly those dead and live loads which he has estimated or adopted, and having connections which may be hinged, or rigid or semi-rigid, according to the design method which has been selected. The model will differ from the actual structure insofar as variations may arise in the loads, in the dimensions of the steel and in its properties, and also in the behaviour of the connections. Any of these differences between the model and the actual structure may cause secondary stresses. They may be classified as follows:

(a) Effect of Elastic Deformation: This is most readily seen in a truss designed on the common assumption that all connections are hinged. When loads are applied, the members of the truss suffer elastic deformation, becoming slightly longer or shorter, and thereby changing the geometrical properties of the truss, including the angles between the straight-line outline of the members. The angles between the members themselves are restrained from changing by whatever degree of rigidity the connections possess, and therefore the members are subject to some bending at the joints.

(b) Effect of Eccentric Connections: In members of a truss or triangulated frame, meeting at a joint, teh resultant of the forces in any group of those members should, by the rules of static stability, be exactly equal and opposite to the resultant of the forces in the remaining members at that joint. If the neutral axes of any group intersect at some point other than the intersection of the remainder, then the resultants in question may not be in equilibrium because they are not in alignment. The resulting eccentricity produces a moment at the joint.

Eccentricity may develope in another way: a beam connected to the flange of a column may be assumed to impart to the column a moment equal to its end reaction multiplied by the half-width of the column. This assumption is only exactly correct if the beam connection acts as a hinge. If, on the other hand, the connection resists to any degree the rotation of the end of the beam as it seeks to deflect, then this resistance will produce an additional moment, equivalent to the application of the reaction from the beam at some point further from the centre-line of the column.

(c) Effect of Inaccuracies in Fabrication: If the connection angles at the end of a beam are not assembled truly square with the beam, or if the connections at the ends of the top and bottom chords of a truss are not accurately squared, these connections will not be correctly aligned with the surfaces of the supporting members, and, being forced into contact when the rivets are driven or the bolts are tightened, will produce some moment at the joint. Such a moment might add to, or reduce, the moment arising from the partial rigidity inherent in the connections. A slight slanting of the end connections for a long beam may, in fact, be used as a device to minimize the negative moments which would otherwise be developed at the ends.

Generally, fabricators take considerable care to see that connections are accurately squared during assembly. In the elastic range, end moments could be modified appreciably by the small deviations from the correct angle, but the elastic range in a standard connection is small. When the connection begins to enter the plastic range, the deformation will render the effect of the initial deviation less important, and continued plastic deformation will become the controlling factor in the development of moment.

(d) Differential Settlement of Foundations: The behaviour of the foundations of a structure are primarily the responsibility of the designer of the substructure, who should select such dimensions and proportions for it, that unequal settlement will be avoided. If unusual conditions at a site threaten great difficulties in attaining this end, then the designer of the superstructure might well be requested to adopt an arrangement in which unequal settlement would cause minimum damage. In such a case, the moments expected at the joints would certainly have to be investigated, which would remove them from the class of "secondary" stress.

Reports of the Steel Structures Research Committee.

In 1929, the Steel Structures Research Committee, appointed by the British Government, began a study of current practice in the design and fabrication of steel structures, both bridges and buildings, with the intention of finding ways of applying modern theory where it would lead to more efficient and economical design. The work was mostly in connection with steel frames for buildings, and was the subject of three reports; a "First Report" in 1931, a "Second Report" in 1934 and a "Final Report" in 1936.

The experimental work of the Committee included strain measurements and stress analysis in several building frames at various stages of erection. The characteristic behaviour of various types of connections waseexamined.

The results of the experiments provided quantitative confirmation of the partial rigidity known to exist in conventional beam-to-column connections. They showed that the maximum moments in floorbeams were reduced thereby, while the columns supporting those beams were subjected to moments considerably greater than those assumed in the conventional design method.

Typical data from the Committee reports are presented in figures 23 to 26 and elsewhere. (Diagrams copied from the reports retain the original numbers for identification purposes.) Figures 23, 24 and 25 refer to the steel frame for a hotel building, six storeys high. The type of connection used in this was intended to make a substantial contribution to the rigidity of the frame; the detail appears in figure 23, with curves showing relative stiffness. Figures 24 and 25 show the bending moment diagrams constructed from strain measurements in columns and beams, to which the corresponding bending stresses have been added. The maximum bending stress found in a column under the applied loading was just less than 3 ksi.

Figure 26 shows a typical analysis by the Committee of stresses in the frame for an office building, with connections less rigid than those in the hotel. In this case, the maximum column bending stress is just under 2 ksi.

Certain features of the building construction practice in Britain differ from what is usually found in Canada and the U.S.A., rendering a direct application of some of the information limited. The "standard beam connection" on this continent consists of a pair of angles in the web of the beam, whereas structures such as those examined in London generally support beams on seats, stiffened where necessary, with angles attached to the top flanges for stability. Further, buildings such as those examined, when completed, have the main beams and the columns encased in concrete, the effect of which is taken into account in the design; this practice, which is rare on this continent, renders the beam-to-column connections much more rigid, and modifies the moments at the connections substantially.

It is of interest to note, that in the Supplement No.1 to

British Standard 449, which is the official result of the recommendations of the Steel Structures Research Committee, the calculation of the restraining moment at the end of a beam is determined according to the type of connection used, while on the other hand the calculation of the moments applied to the column is based on the fixed-end moments of the attached beams. This requirement is a consequence of the stiffening effect of the concrete casing.

Relative Rigidities of Connections.

In figures 27 to 39 inclusive are presented data on the properties of various beam connections, as determined by the Steel Structures Research Committee. Further information of the kind, derived from the work of J. Charles Rathbun (20), appears in figures 40 to 46 inclusive, while figure 47 contains the results obtained by Munse, Bell and Chesson (21).

The experimental results to which reference is made above are very useful, and provide a practical check upon any calculated properties of similar connections. Calculations become necessary when structures being analysed contain connections not exactly like any of those in the various research programmes.

FINAL REPORT OF THE STEEL STRUCTURES RESEARCH COMMITTEE.

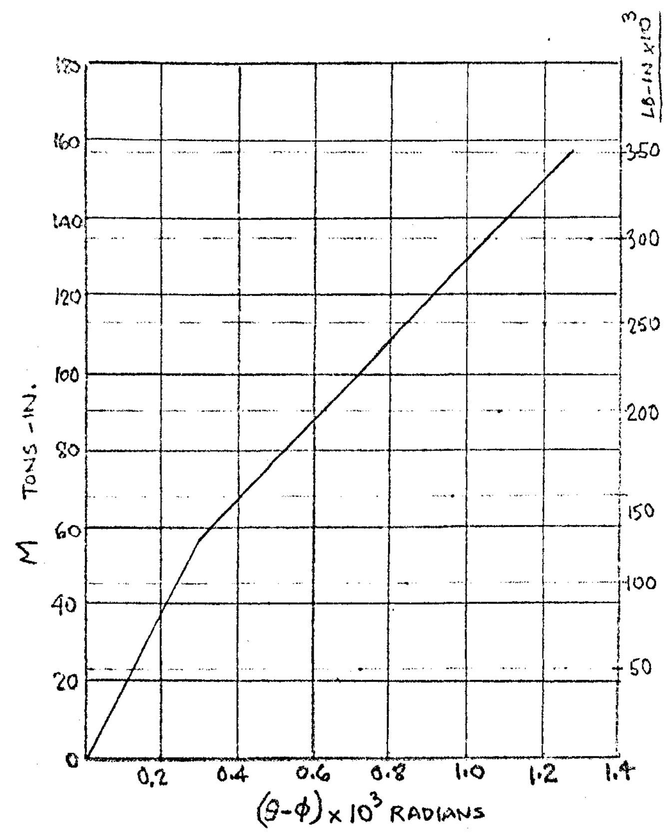


FIG. H12 p.23. CONNECTION OF H301.

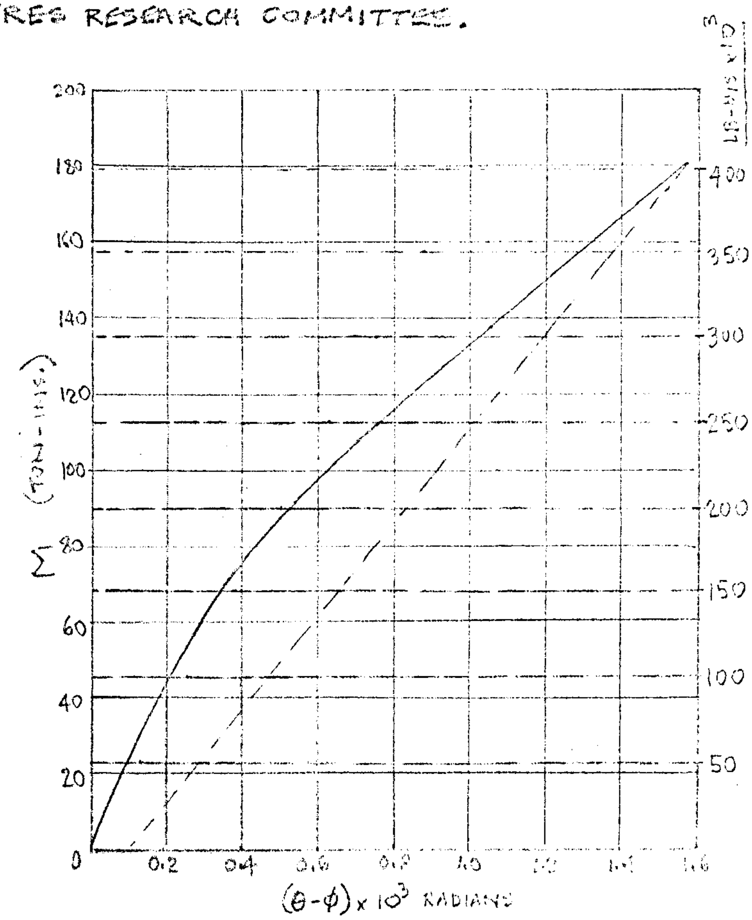


FIG. H22 p.39 CONNECTION OF BEAM E301.

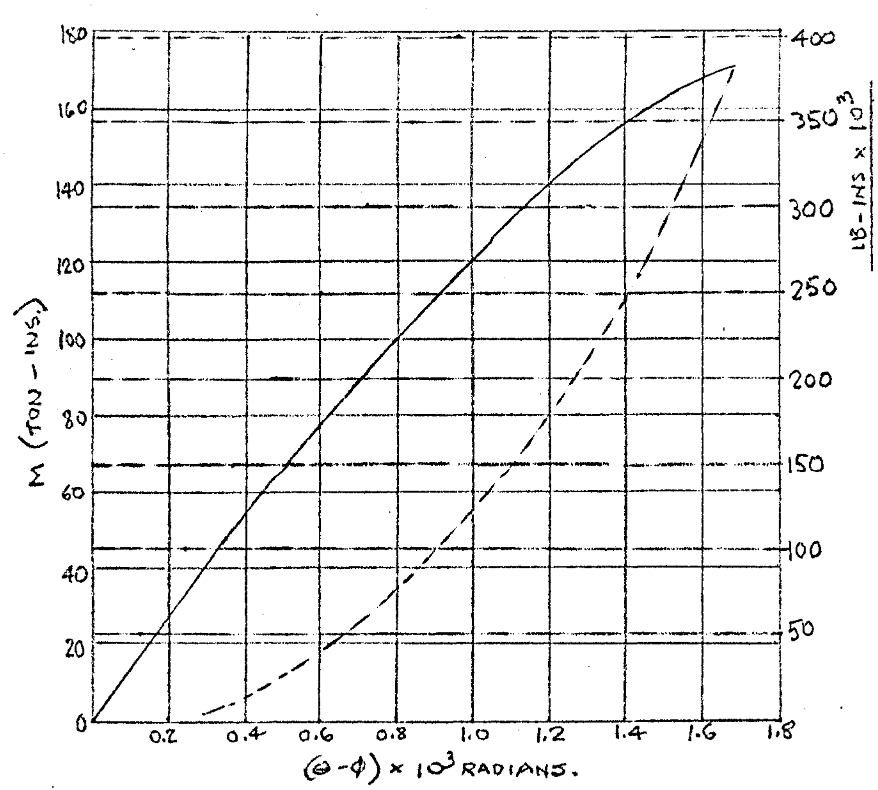
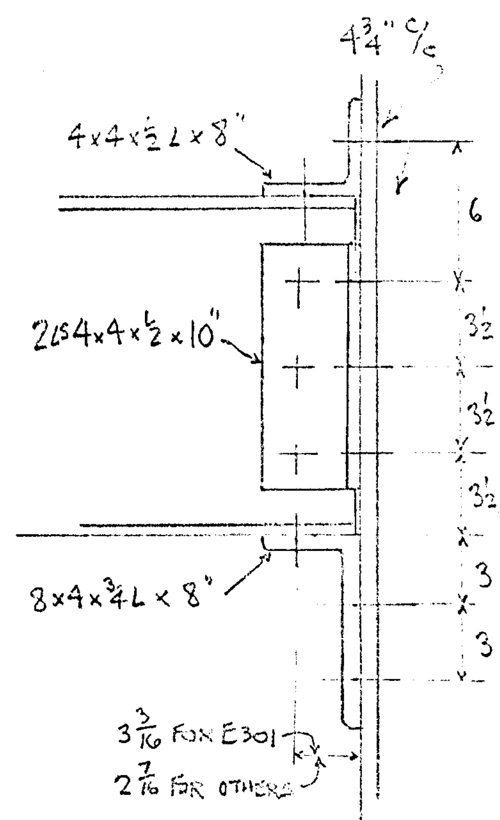


FIG. H18 p.33 CONNECTION OF BEAM F301.



BEAM CONNECTION DETAIL FOR E, F, H301 & OTHERS.

FIGURE 23.

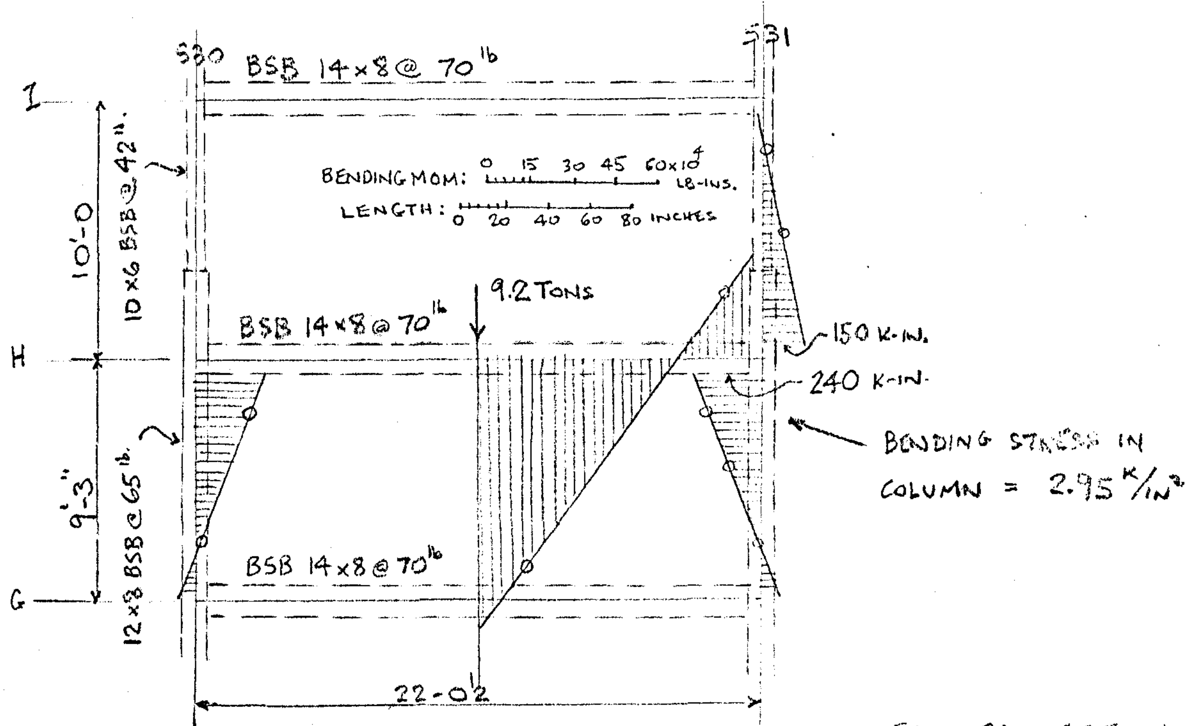


FIG. H10 p. 21

FOR CONNECTION DETAIL SEE FIG. 23.

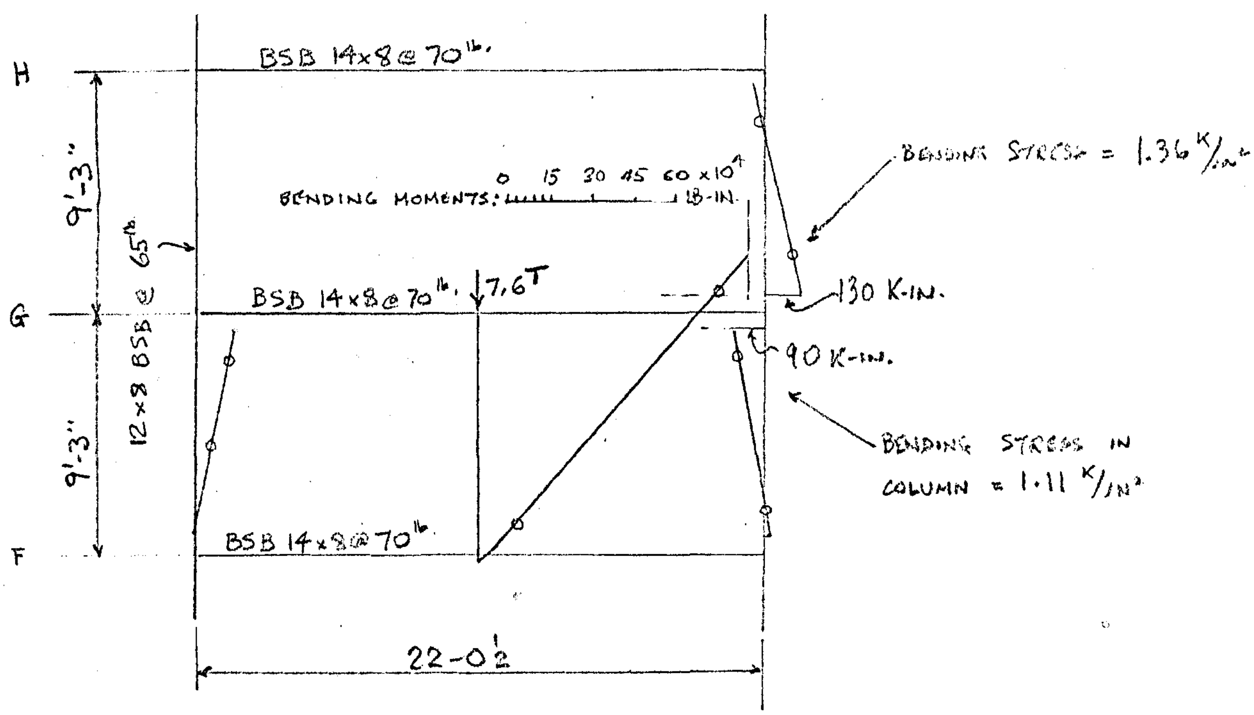


FIG. H13 p. 26

FIGURE 24.

FINAL REPORT OF THE STEEL STRUCTURES RESEARCH COMMITTEE.

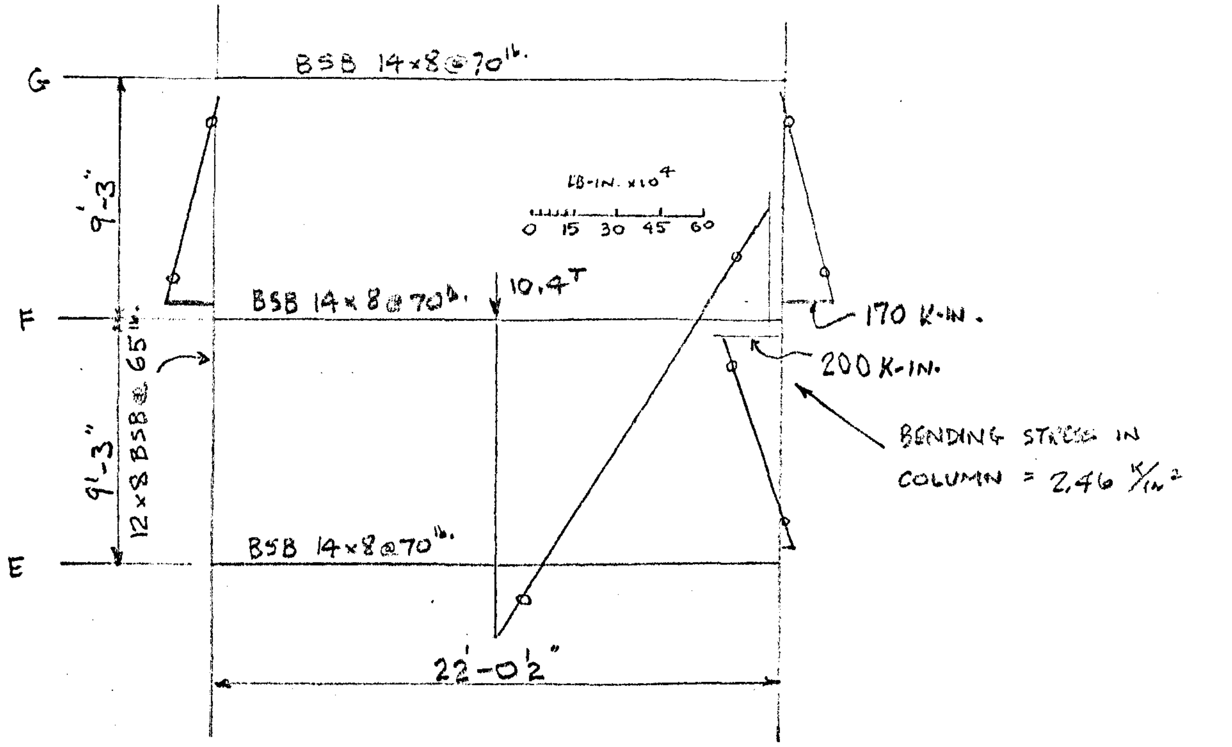


FIG. H 15 p.30

FOR CONNECTION DETAIL SEE FIG. 23.

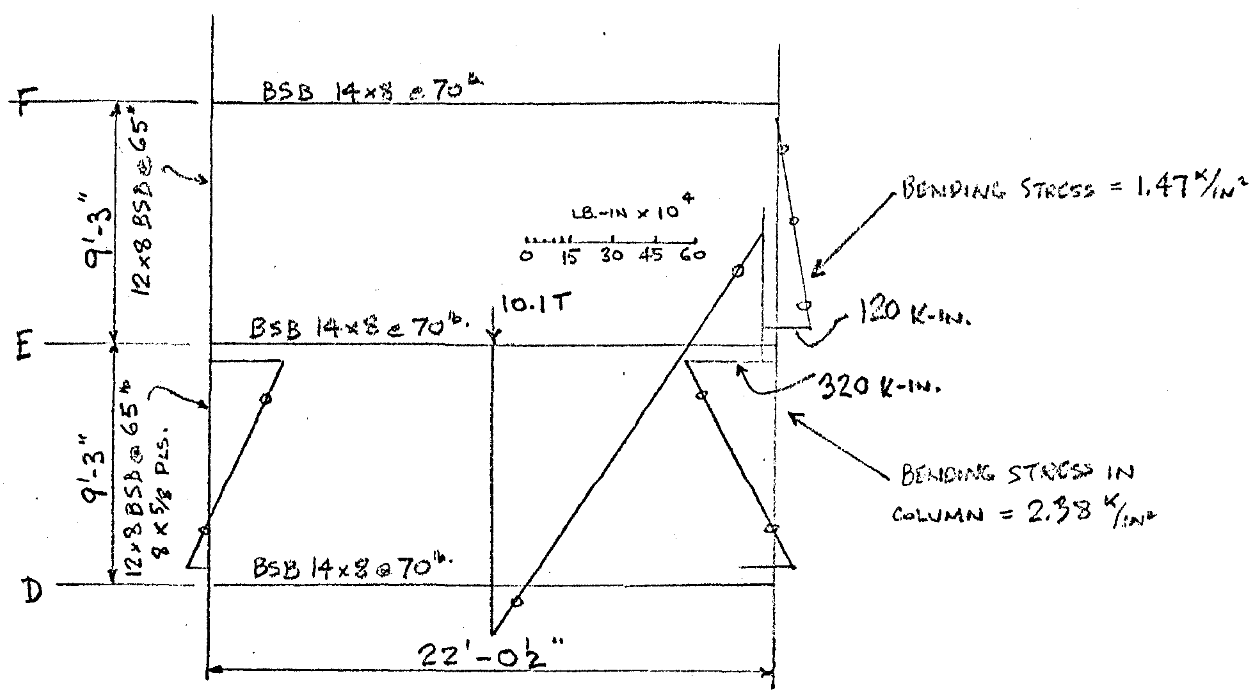


FIG. H 19 p.36

FIGURE 25

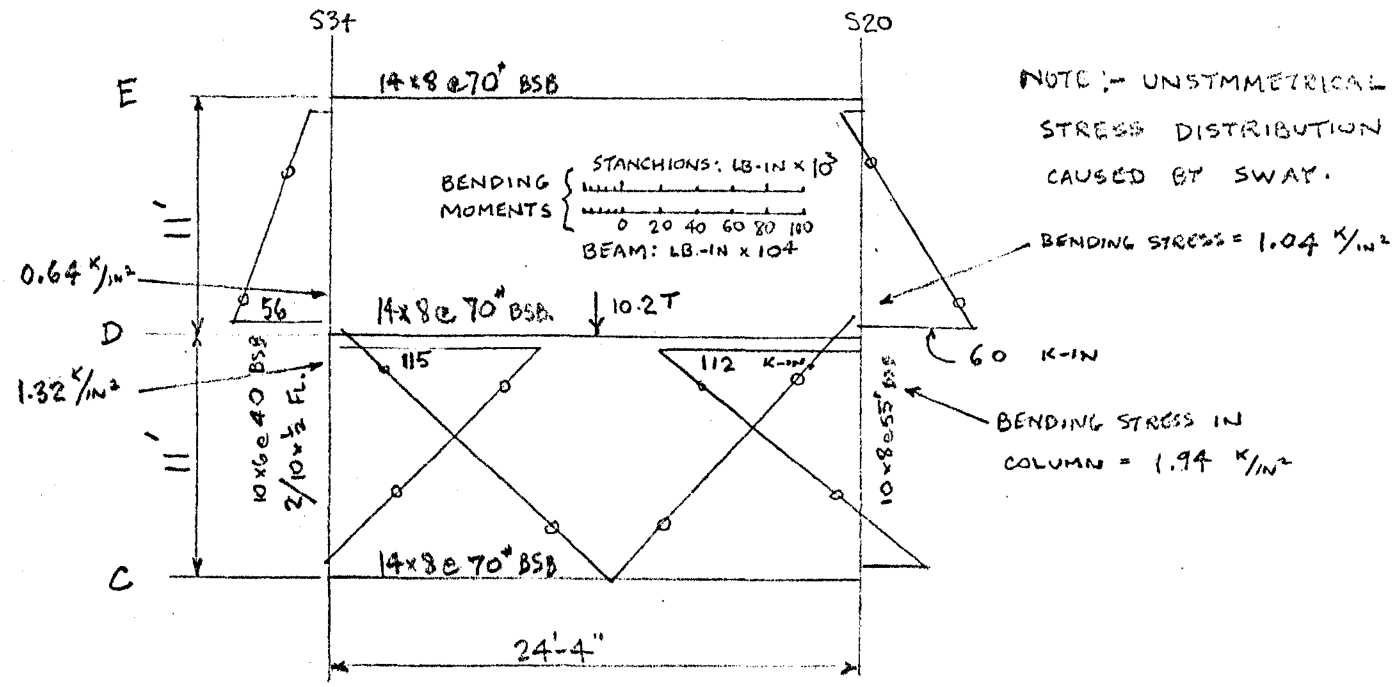
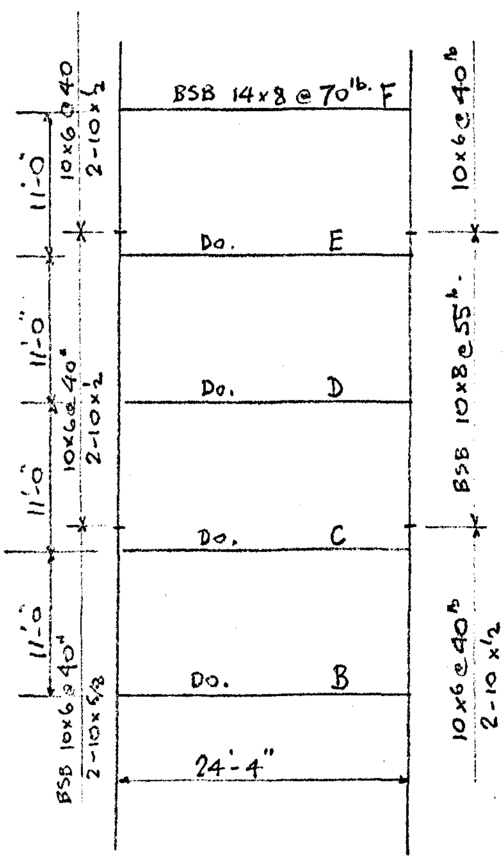
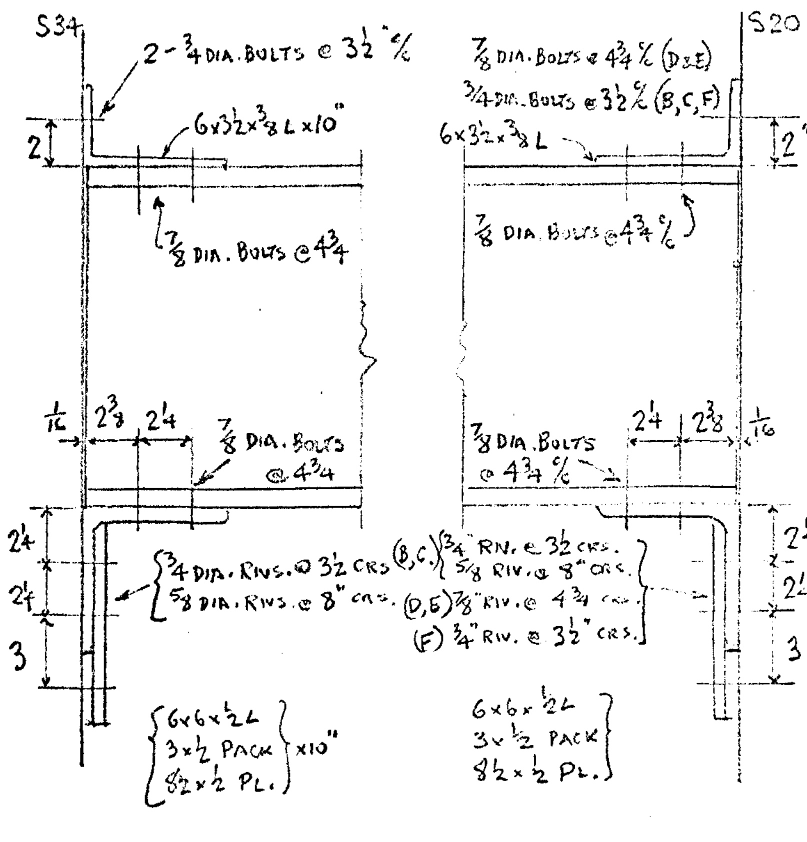


FIG. O10. p. 152.



SINGLE BAY FRAME.

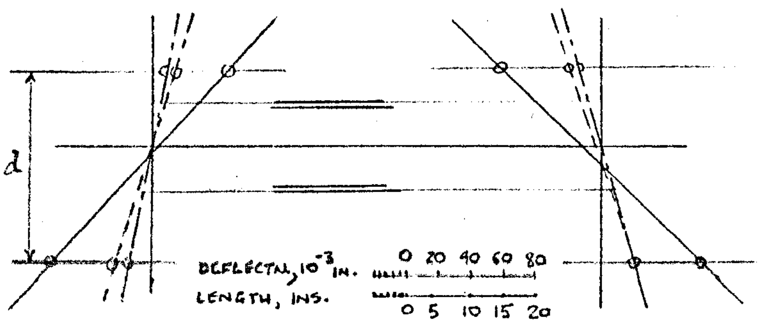
FIG. O3 (a) p. 142.



DETAILS OF CONNECTIONS.

FIG. O7. p. 147.

FIGURE 26.



APPLIED LOAD: { 3.2 TONS – – – – –
10.2 " ————
0.5 " –·–·–

FIGURE O 13, (p. 154)

DEFLECTIONS DUE TO CHANGE OF SLOPE BETWEEN STANCHION FACE AND BEAM END WITH THE APPLICATION OF CENTRAL CONCENTRATED LOADS TO BEAM No. 200 D.

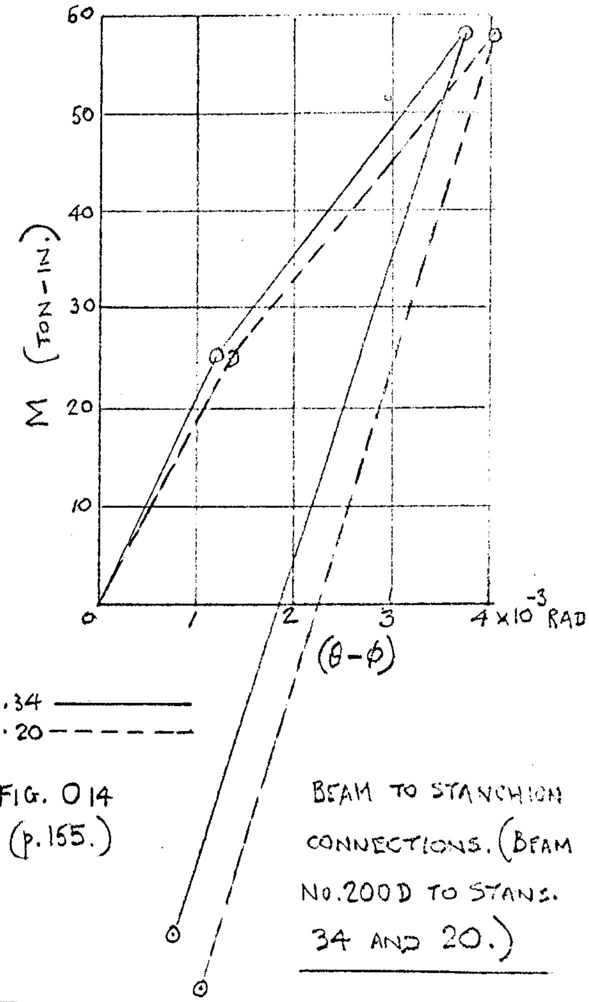


FIG. O 14 (p. 155.)

BEAM TO STANCHION CONNECTIONS. (BEAM No. 200 D TO STANS. 34 AND 20.)

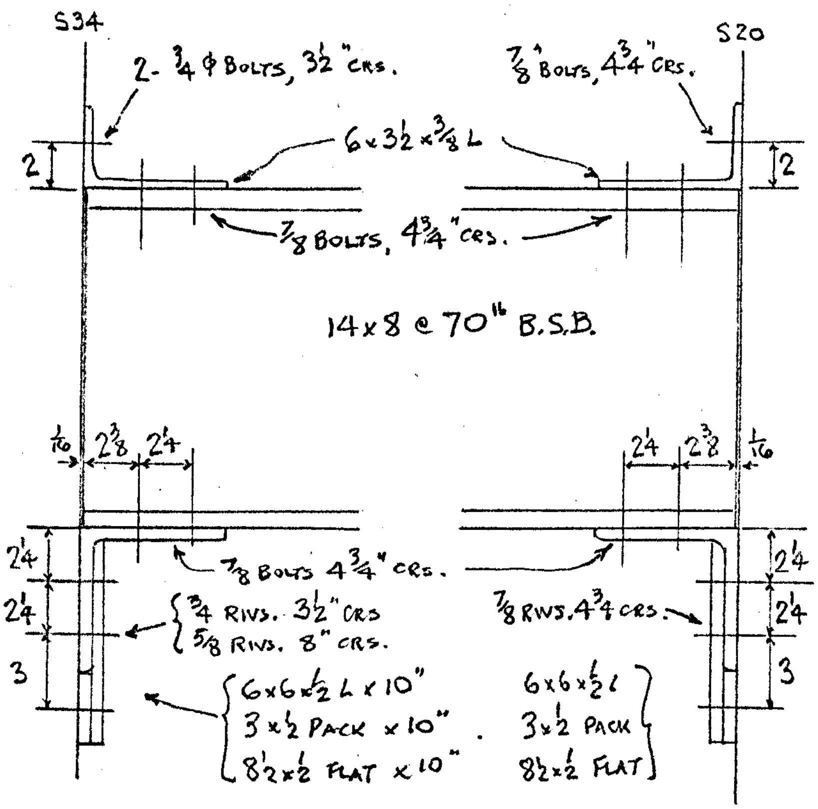
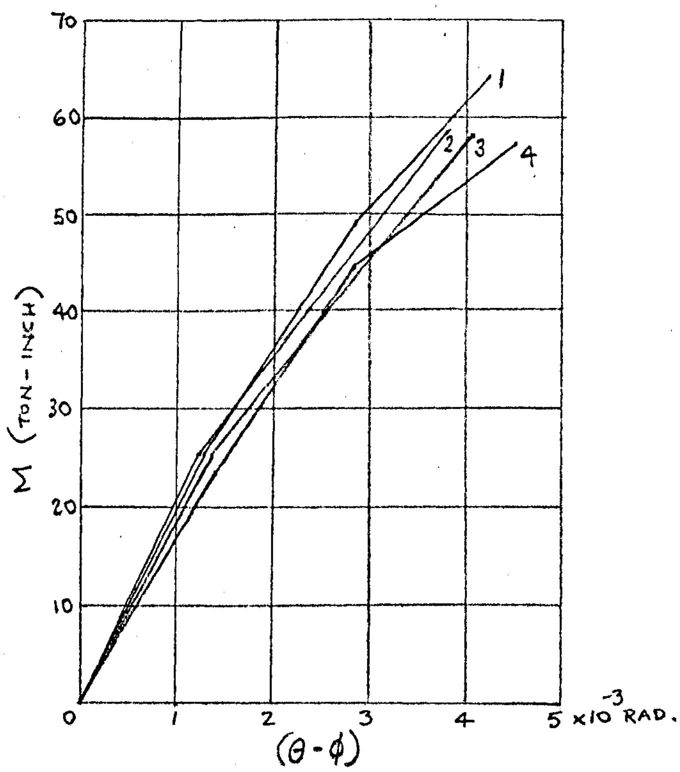


FIG. O 7 (p. 147) CONNECTIONS OF BEAM 200D

FIGURE 27.



1. BEAM 200E TO STAN. 34.
2. " 200D " " 34.
3 " 200D " " 20.
4. " 200E " " 20

FIG. O31 – COMPARISON OF $M/(\theta-\phi)$ CURVES FOR CONNECTIONS.
(p. 186)

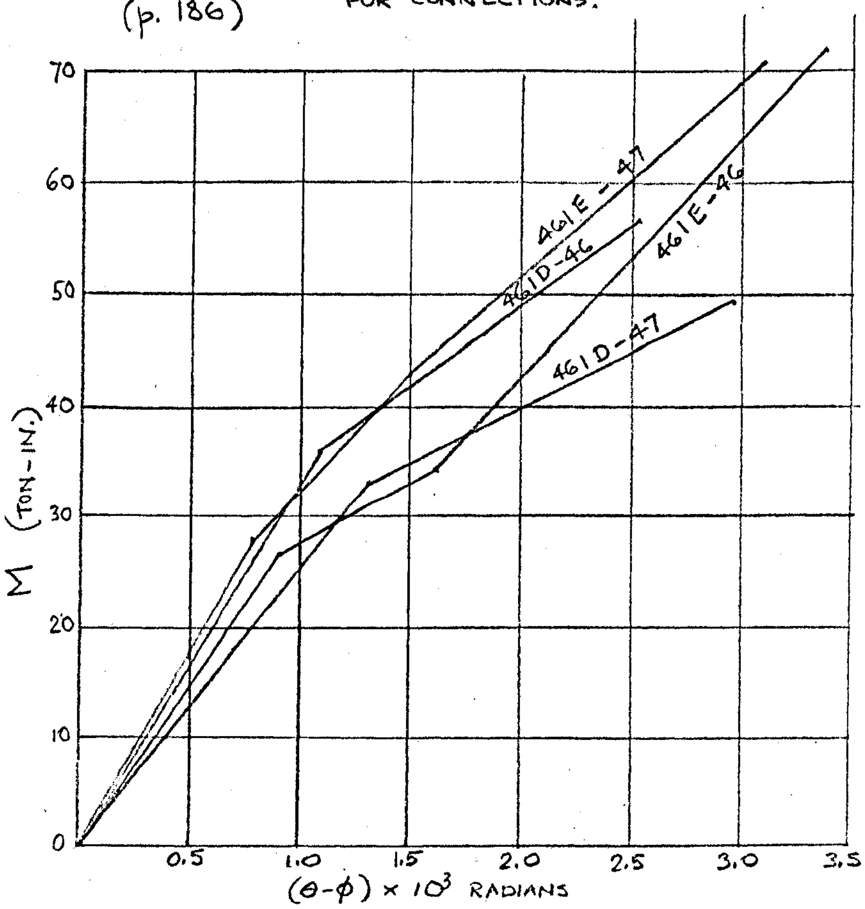


FIG. O53 (p. 226) BEAMS 461 D & E TO STANS. 46 & 47.

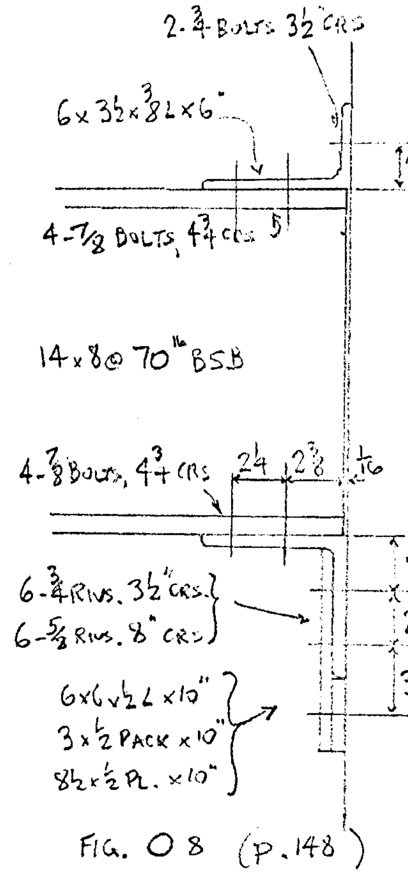
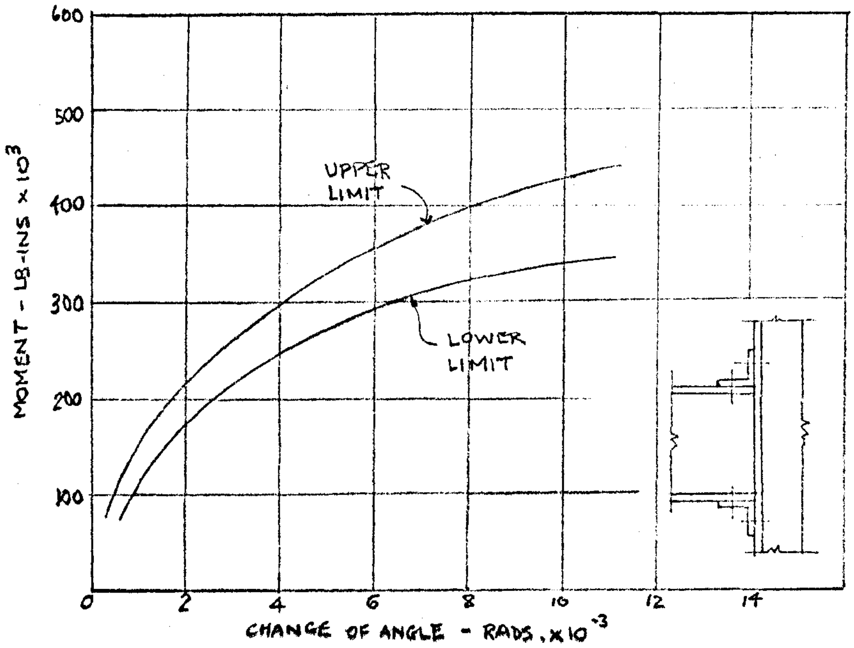


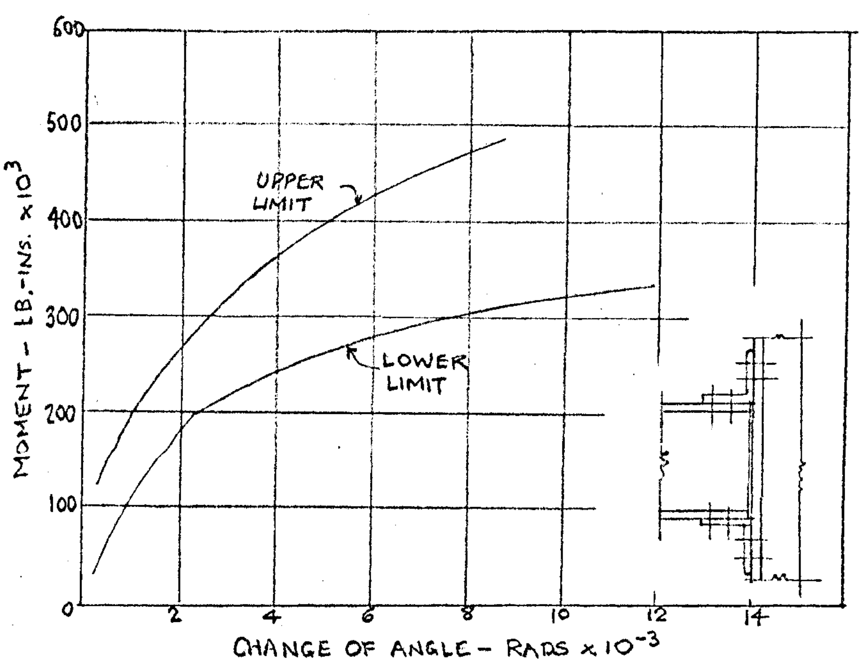
FIG. O8 (p. 148)
CONNECTIONS AT ENDS OF BEAMS 461 D & E.

FIG. 28.



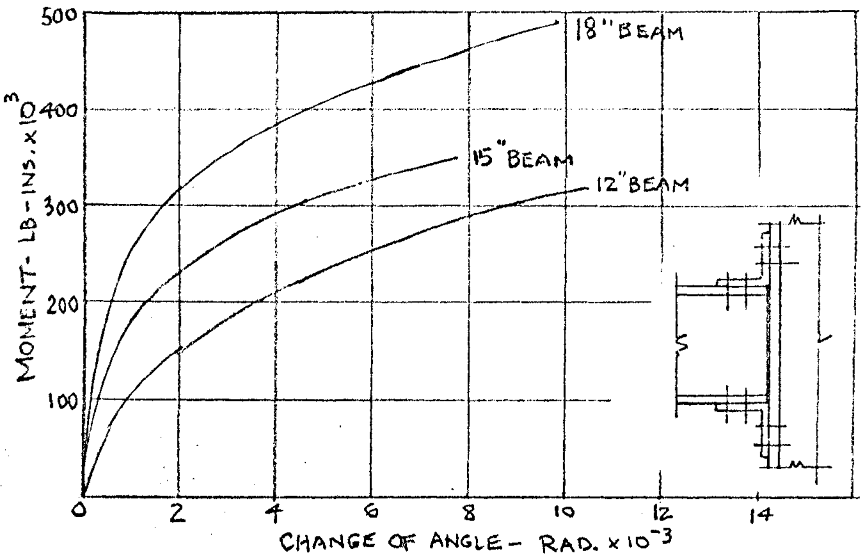
MOMENT - ROTATION CURVE.

CONNECTION ANGLES
$4 \times 4 \times \frac{1}{2}$ Ls
$\frac{3}{4}$" RIVETS.
12" RSJ @ 30 lbs.
RANGE FOR CONNECTIONS MADE BY SEVEN DIFFERENT FABRICATORS.
(REF. FIG. C2, p.282)



CONNECTION ANGLES
$6 \times 6 \times \frac{1}{2}$ Ls
$\frac{3}{4}$" RIVETS.
12" RSJ @ 30 lbs.
RANGE FOR CONNECTIONS MADE BY SEVEN FABRICATORS
(REF. FIG. C3, p.283)

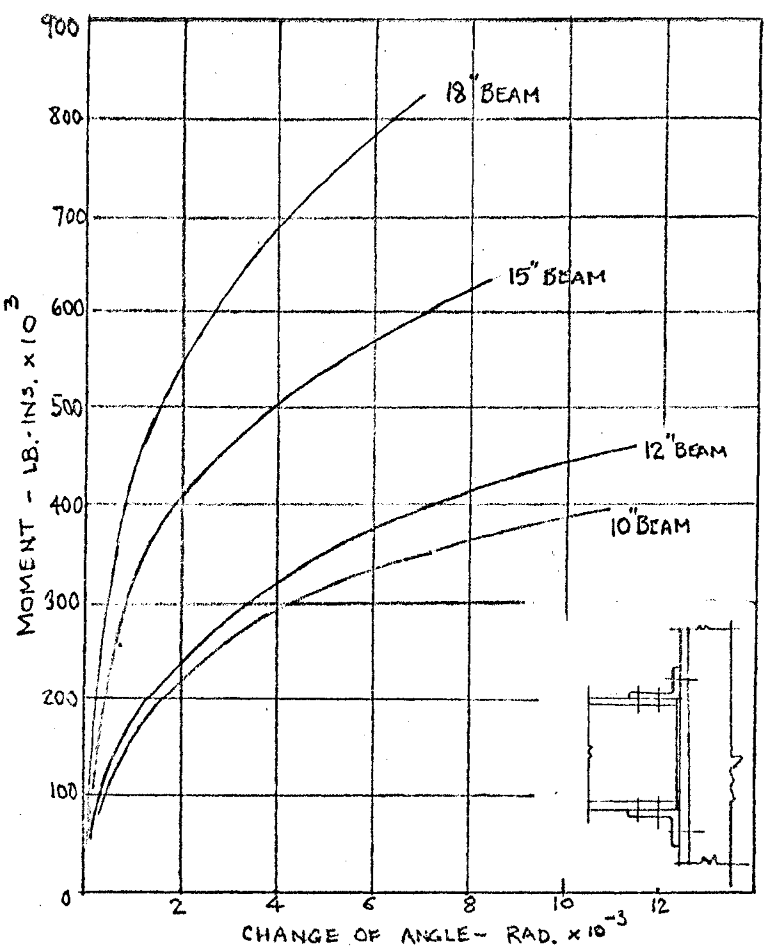
FIGURE 29.



MOMENT-ROTATION CURVES.

6 x 6 x 3/8 CONN. ANGLES.
HIGH-STRENGTH BOLTS.

(REF. FIG. C28, p. 308)

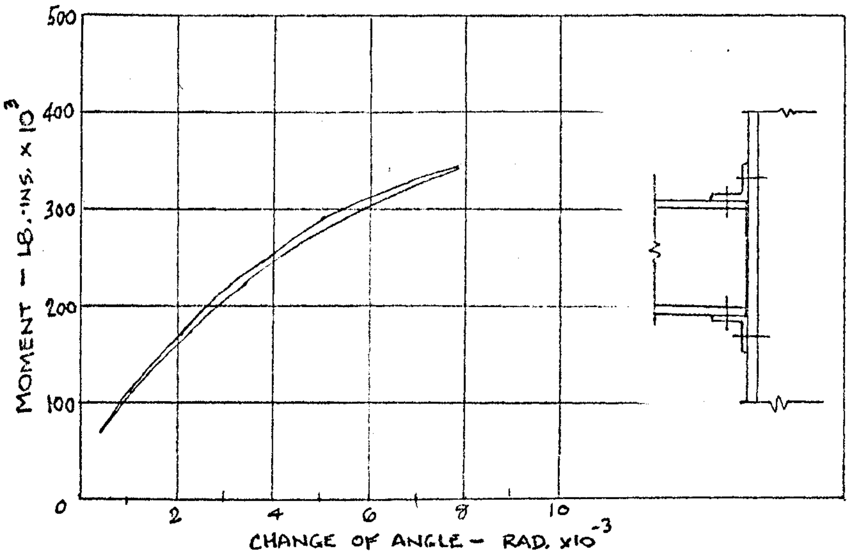


CONNECTION ANGLES 6 x 4 x 1/2
HIGH-STRENGTH BOLTS.

(REF. FIG. C29, p. 309)

FIGURE 30.

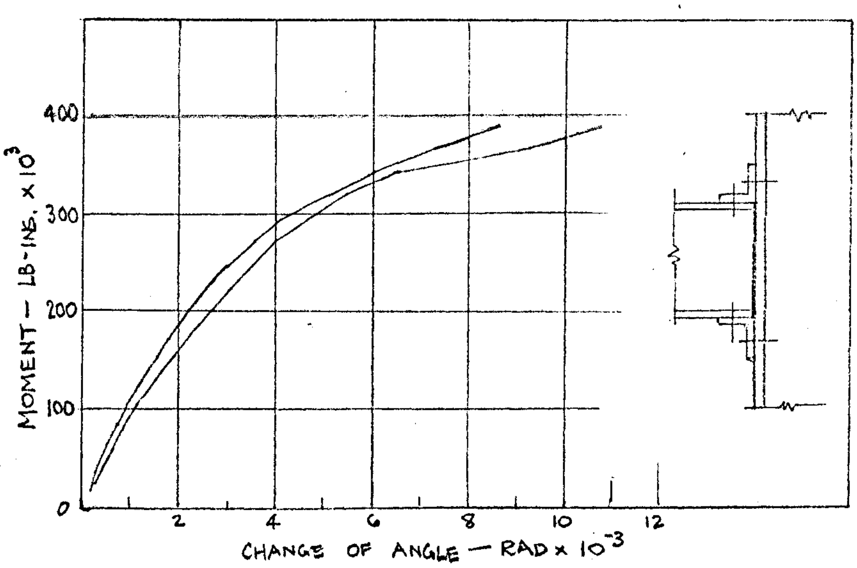
MOMENT - ROTATION CURVES.



BEAM — 12" x 5" RSJ @ 30#
COL. — 12" x 8" RSJ @ 65#

CONNECTIONS — 4" x 4" x ½" Ls x 5" LONG
¾" RIVETS

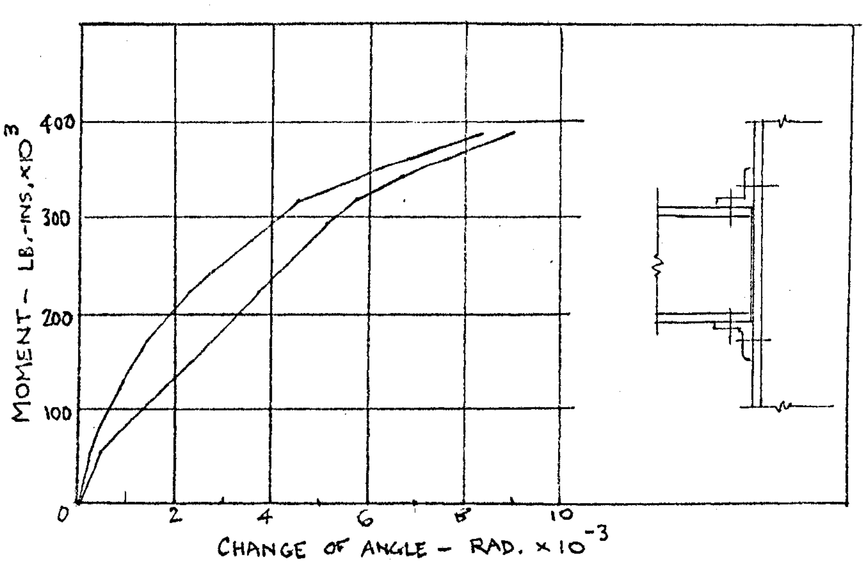
(REF. FIG. 30 p.75)



BEAM — 12" x 5" R.S.J. @ 30#
COL. — 12" x 8" R.S.J. @ 65#

CONNECTIONS — 4" x 4" x ¾" Ls x 5"
¾" RIVETS

(REF. FIG. 31 p.75)

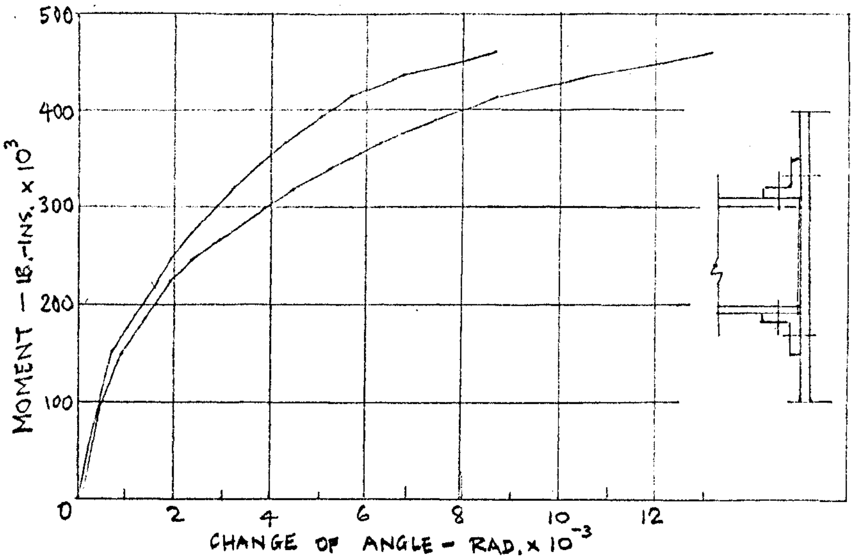


BEAM — 12" x 5" RSJ @ 30#
COL. — 12" x 8" RSJ @ 65#

CONNECTIONS — 4" x 4" x 1" Ls x 5"
¾" RIVETS

(REF. FIG. 32 p. 76)

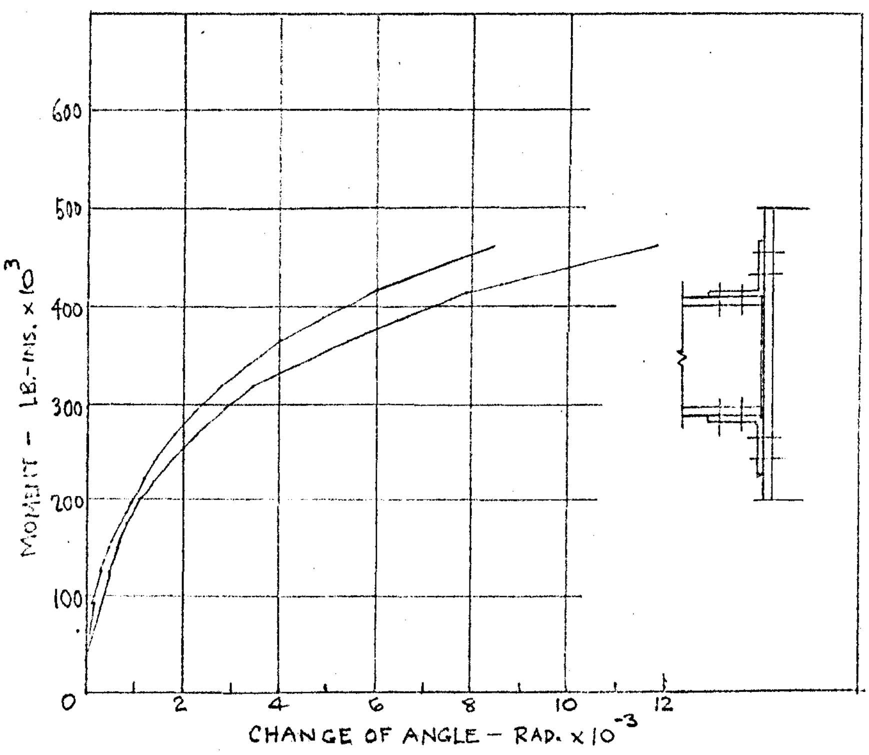
FIGURE 31



BEAM – 12"x5" RSJ @ 30#
COL. – 12"x8" RSJ @ 65#

CONNECTIONS – 4x4x1 Ls x 5"
7/8" RIVETS

(REF. FIG. 33 p.76)

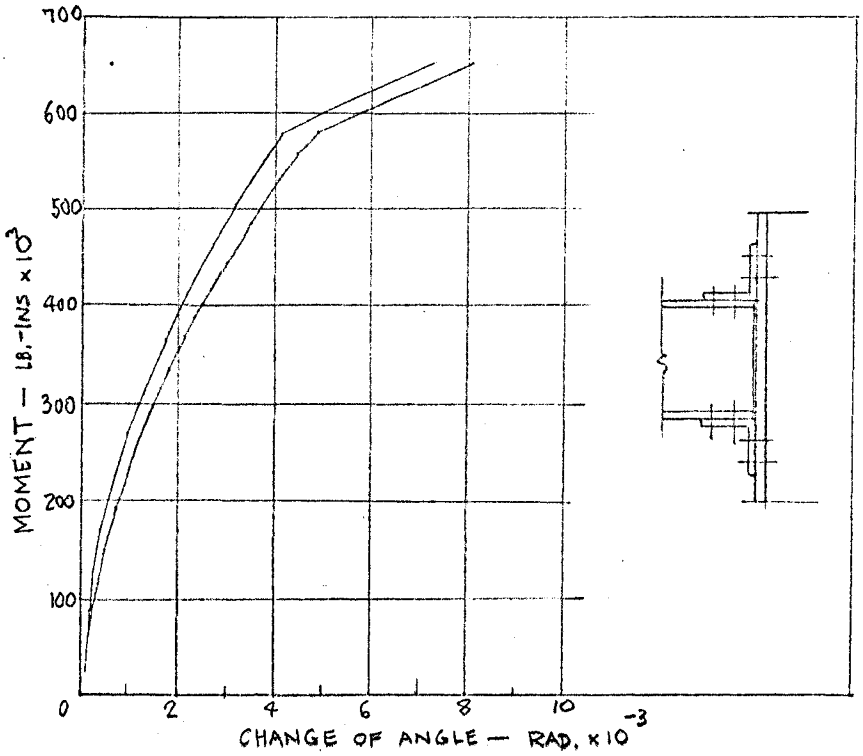


BEAM – 12"x5" RSJ @ 30#
COL. – 12"x8" RSJ @ 65#

CONNECTIONS – 6"x6"x½" Ls x 5"
3/4" RIVETS

(REF. FIG.34, p.77)

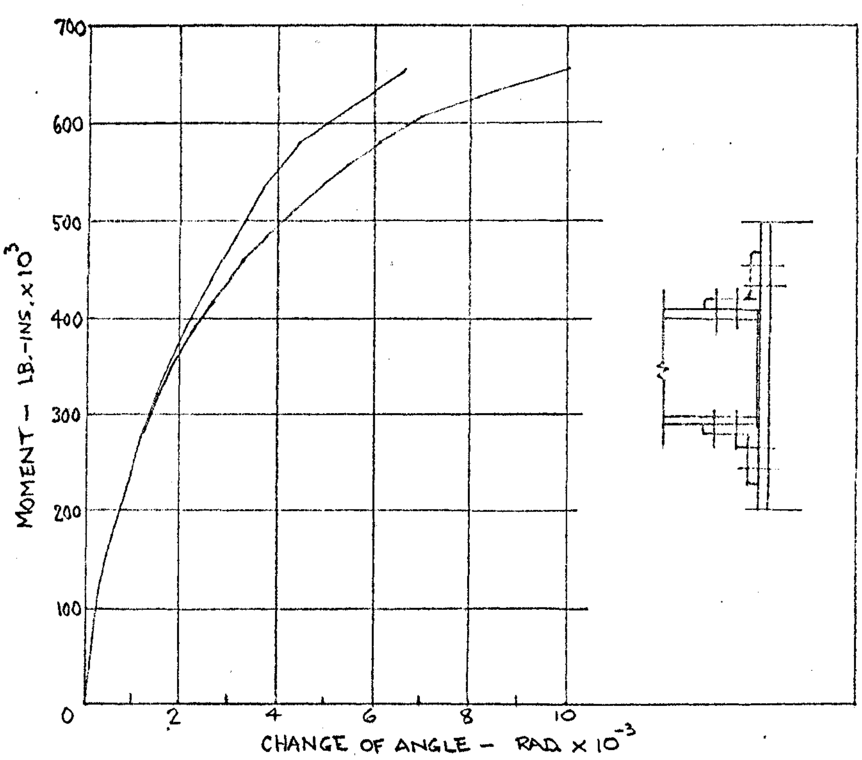
FIGURE 32.



BEAM - 12" x 5" RSJ @ 30#
COL. - 12" x 8" RSJ @ 65#

CONNECTIONS - 6" x 6" x ¾" ∠s x 5"
¾" RIVETS

(REF. FIG. 35, p. 77)

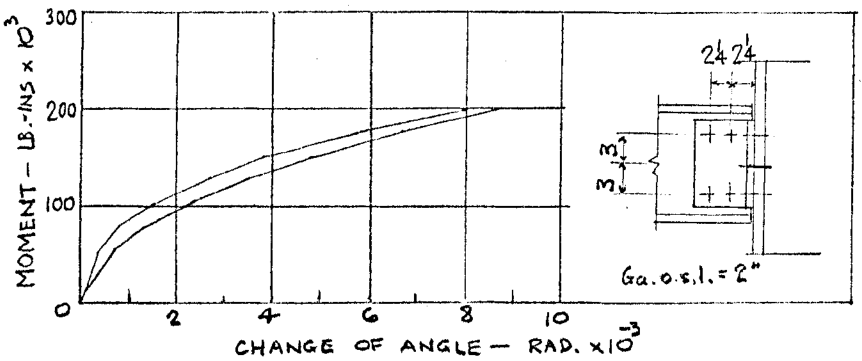


BEAM - 12" x 5" RSJ @ 30#
COL. - 12" x 8" RSJ @ 65#

CONNECTIONS - 6" x 6" x 1" ∠s x 5"
¾" RIVETS

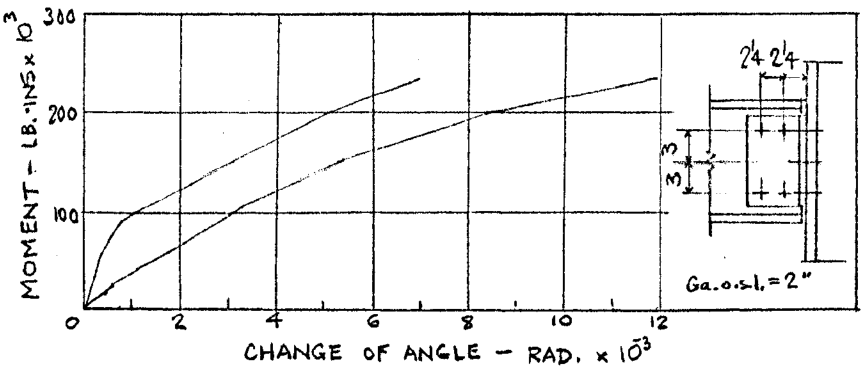
(REF. FIG. 36, p. 78)

FIGURE 33.



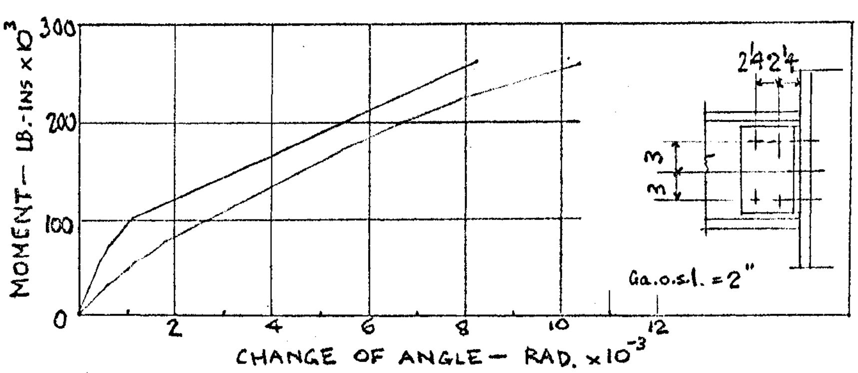
BEAM – 12"×5" RSJ @ 30#
COL. – 12"×8" RSJ @ 65#
CONNECTIONS – 2 Ls 6"×3½"×3/8"×9"
3/4" RIVETS

(REF. FIG. 37, p. 78)



BEAM – 12"×5" RSJ @ 30#
COL. – 12"×8" RSJ @ 65#
CONNECTIONS – 2 Ls 6"×3½"×½"×9"
3/4" RIVETS.

(REF. FIG. 38, p. 79)

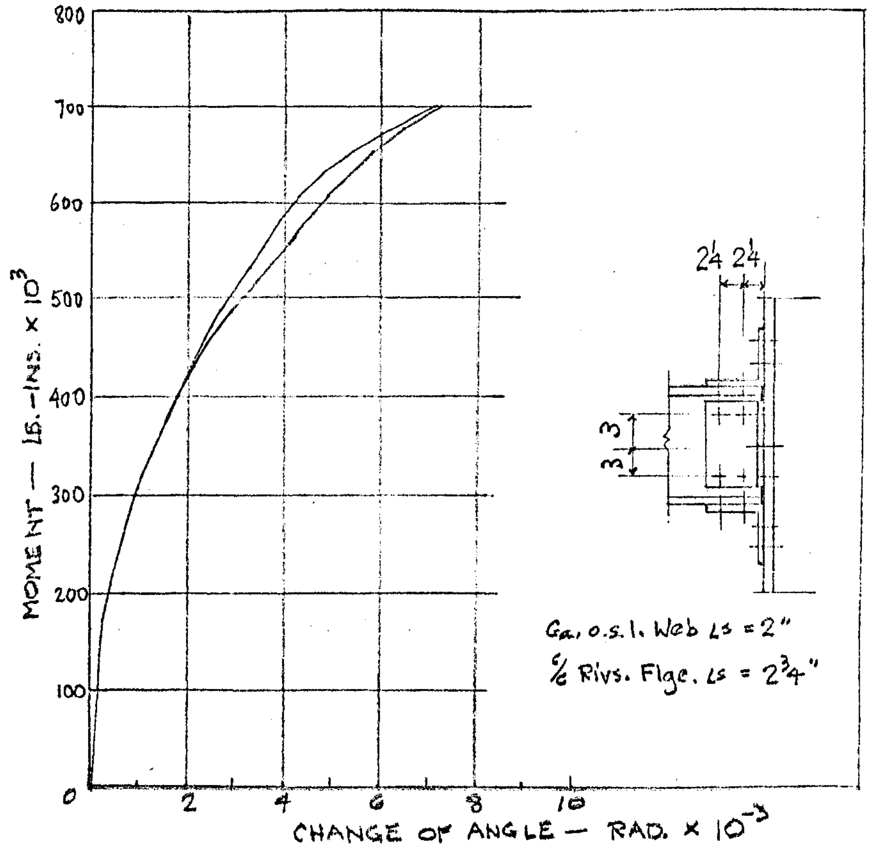


BEAM – 12"×5" RSJ @ 30#
COL. – 12"×8" RSJ @ 65#
CONNECTIONS – 2 Ls 6"×3½"×5/8"×9"
3/4" RIVETS.

(REF. FIG. 39, p. 79)

FIGURE 34.

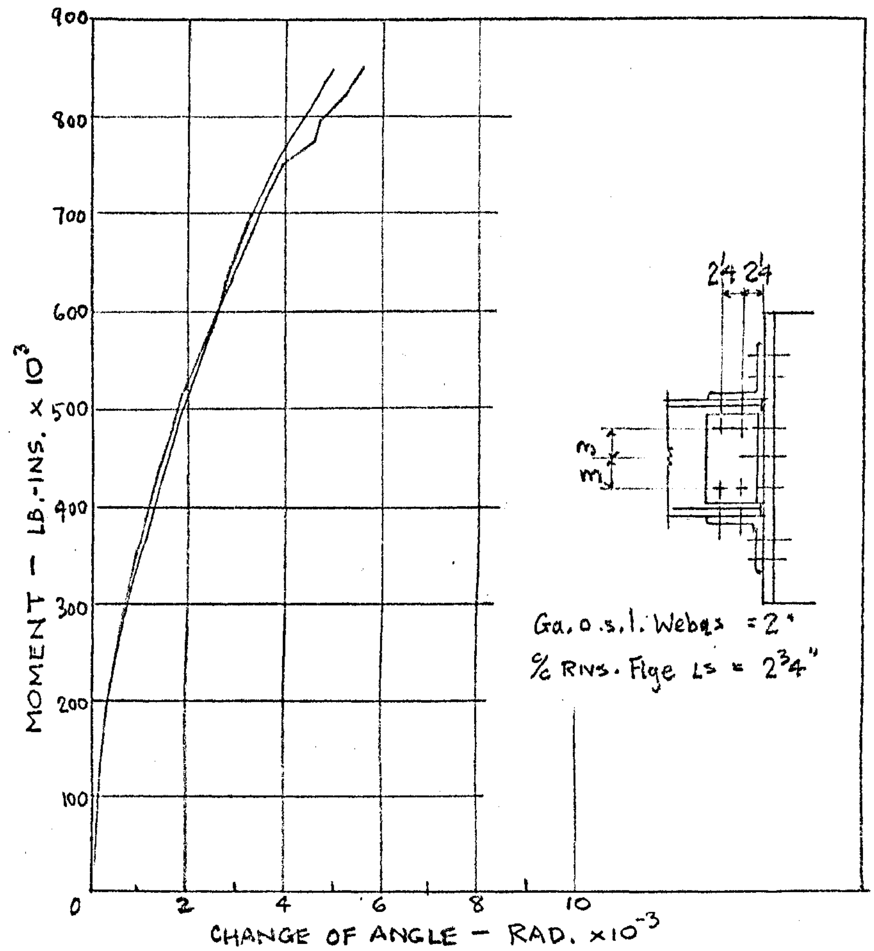
SECOND REPORT OF THE STEEL STRUCTURES RESEARCH COMMITTEE.



BEAM - 12" x 5" RSJ @ 30#
COL. - 12" x 8" RSJ @ 65#

CONNECTIONS - 2Ls 6" x 6" x ½" x 5"
2Ls 6" x 3½" x ⅜" x 9"
¾" RIVETS

(REF. FIG. 40, p. 80)

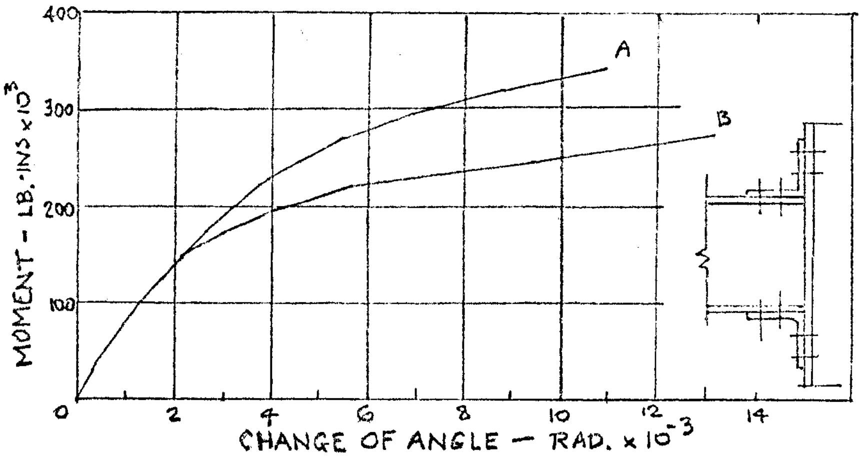


BEAM - 12" x 5" RSJ @ 30#
COL. - 12" x 8" RSJ @ 65#

CONNECTIONS - 2Ls 6" x 6" x ¾" x 5"
2Ls 6" x 3½" x ⅜" x 9"
¾" RIVETS

(REF. FIG. 41, p. 81)

FIG. 35

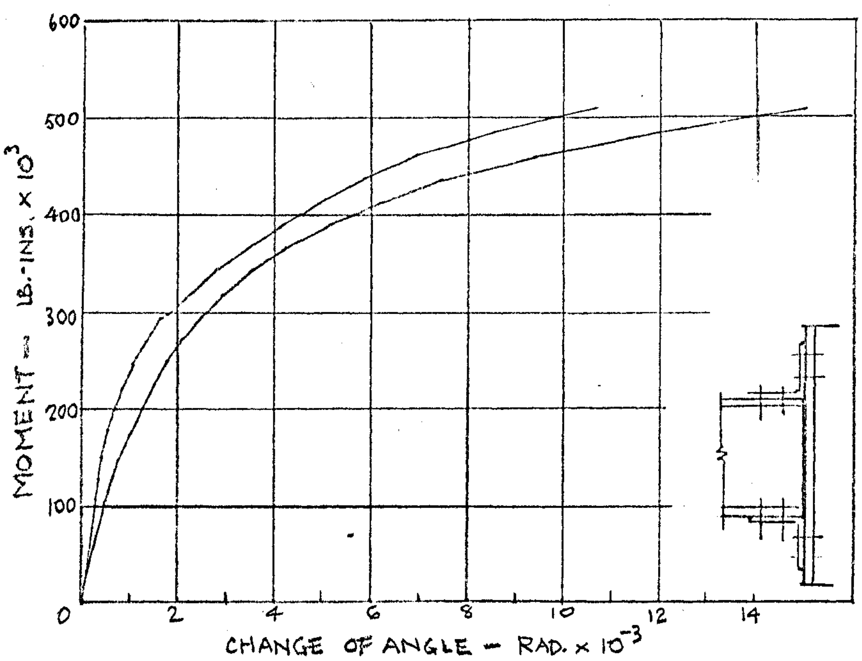


BEAM - 12" x 5" RSJ @ 30#
COL. - 12" x 8" RSJ @ 65#

CONNECTIONS - 6" x 6" x 3/8" Ls x 5"
3/4" H.T. BOLTS.

NOTE: TENSION L "A" WAS + 1/32 " AND TENSION L "B" WAS − 1/32 " IN THICKNESS.

(REF. FIG. 45, p. 85)

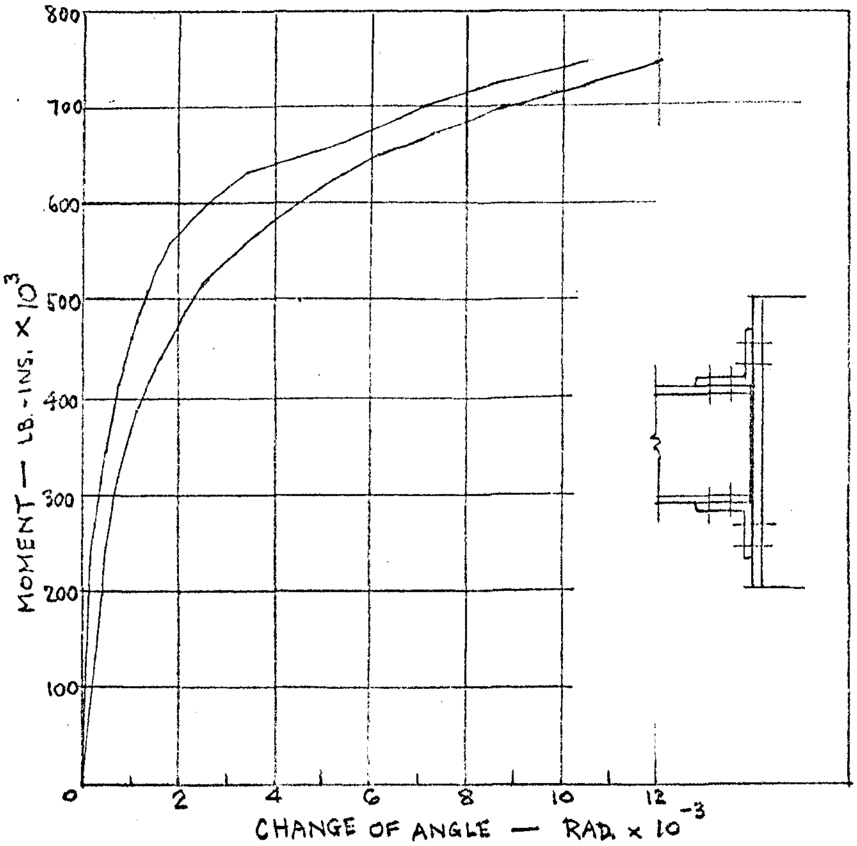


BEAM - 12" x 5" RSJ @ 30#
COL. - 12" x 8" RSJ @ 65#

CONNECTIONS - 6" x 6" x 1/2" L x 5"
3/4" H.T. BOLTS.

(REF. FIG. 46, p. 85)

FIGURE 36.

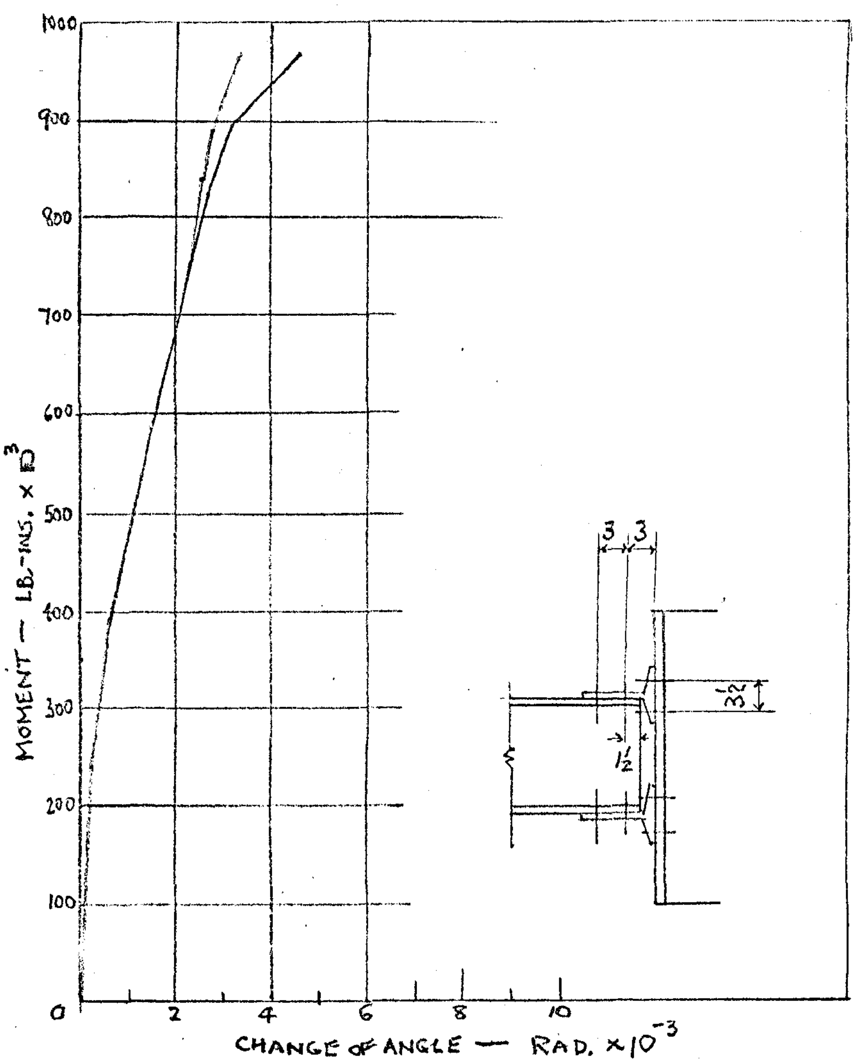


BEAM - 12"x5" RSJ @ 30#
COL. - 12"x8" RSJ @ 65#

CONNECTIONS - 6"x6"x3/4" Ls x 5"
3/4" H.T. BOLTS.

(REF. FIG. 47, p. 86)

FIGURE 37.

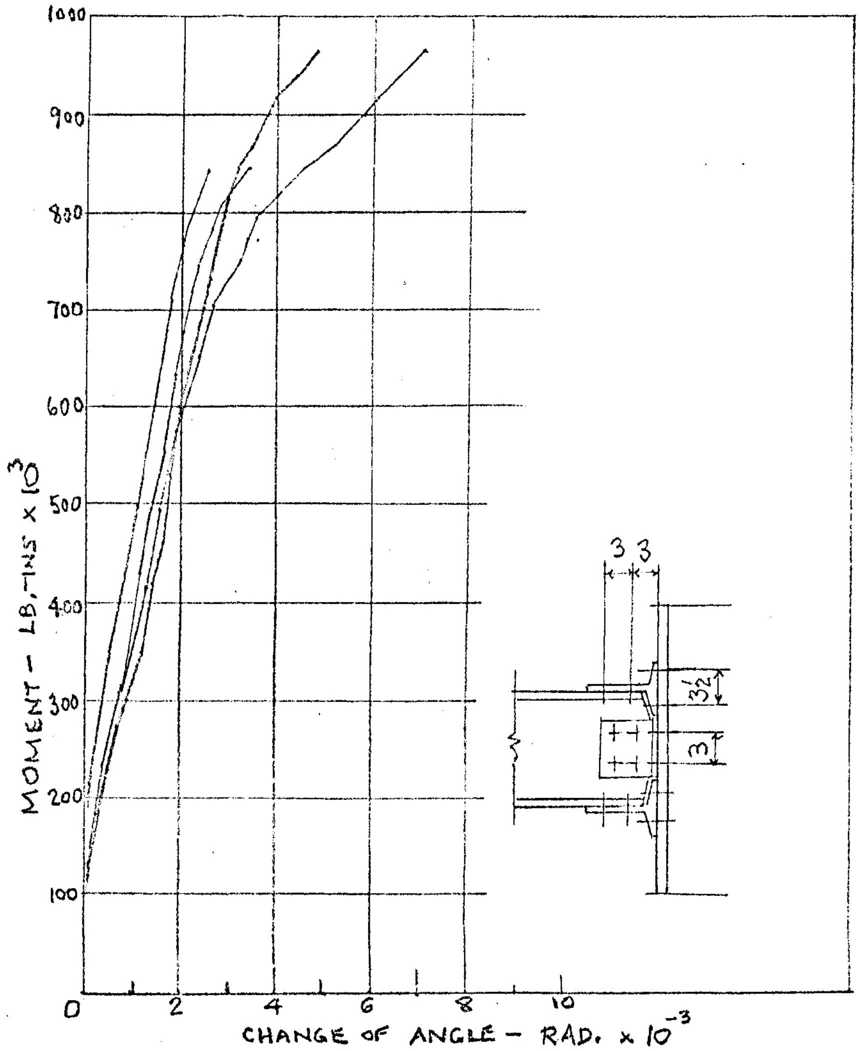


BEAM - 12" x 5" RSJ @ 30#
COL. - 12" x 8" RSJ @ 65#

CONNECTIONS: TEES CUT FROM 15" x 6" RSJ @ 45#, x 7 3/4" LG.
7/8" RIVETS 4 3/4" c/c

(REF. FIG. 42, p. 82)

FIGURE 38.



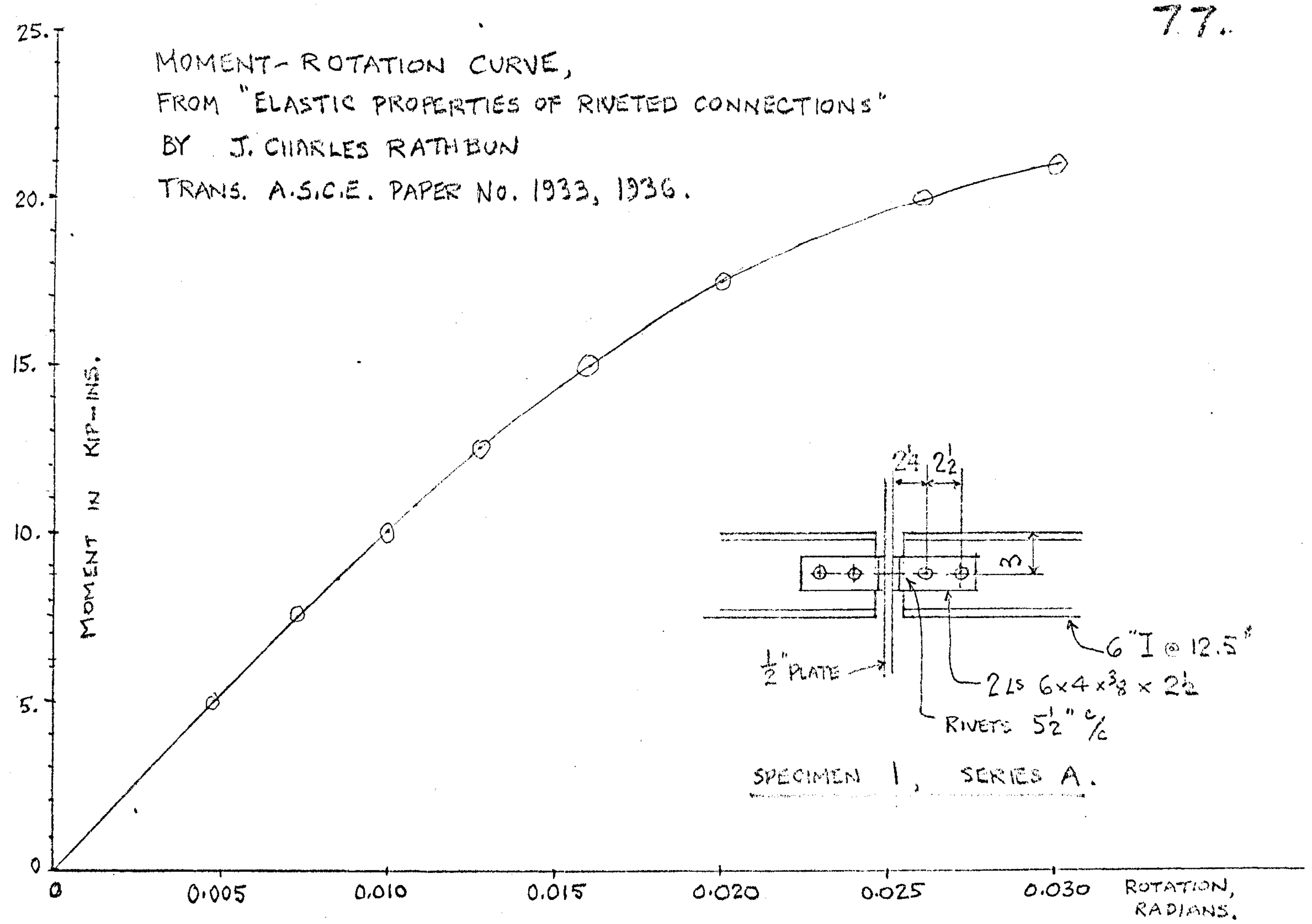
BEAM – 12"×5" RSJ @ 30#
COL. – 12"×8" RSJ @ 65#

FLANGE CONNECTIONS.–
TEES CUT FROM 15"×6" RSJ @ 45"
× 7¾" LG., ⅞" RIVS. @ 4¾" c/c

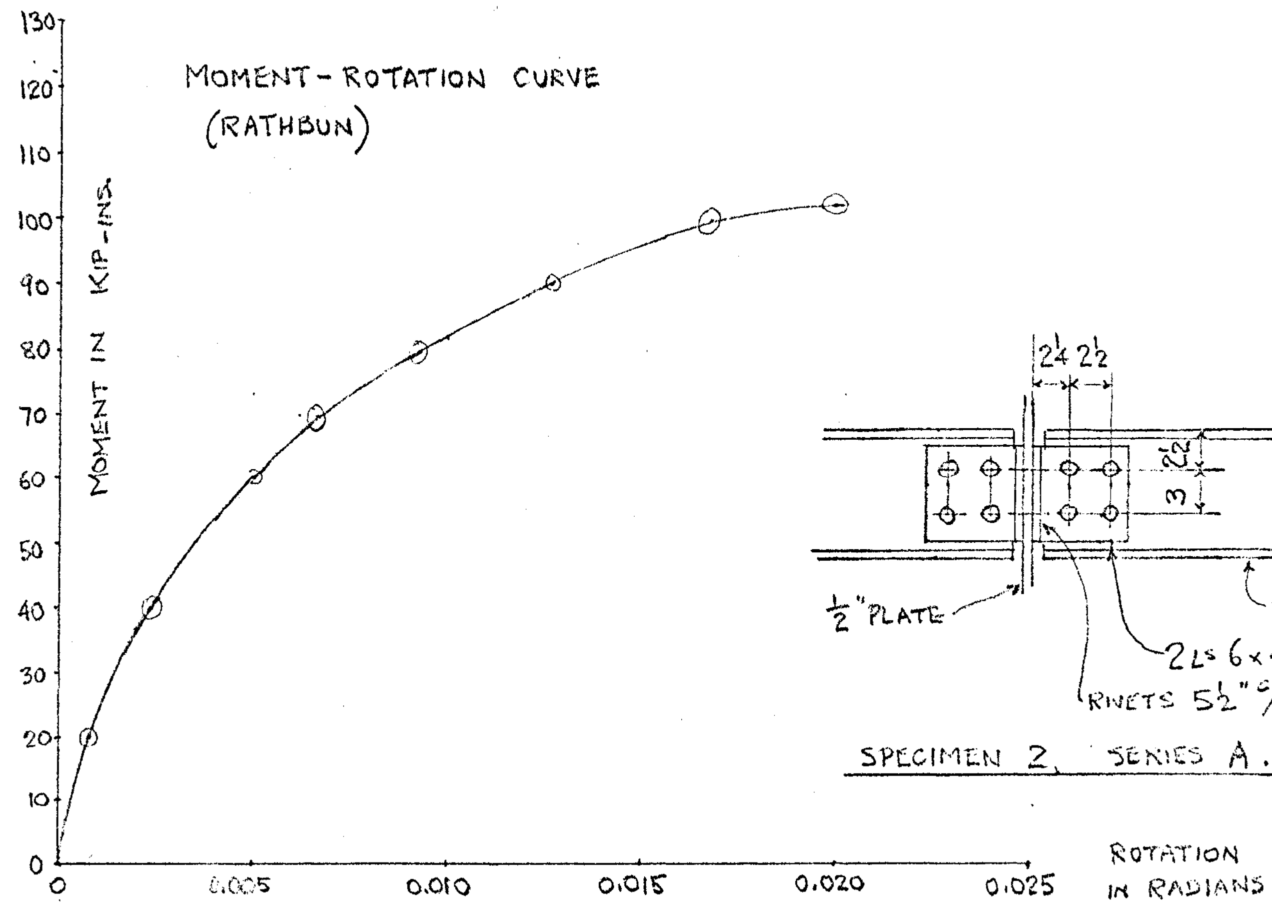
WEB CONNECTIONS.–
2Ls 6"×3½"×⅜"×6"
¾" RIVS. Ga. o.s.l. = 2"

(REF. FIG 43, p. 83)

FIGURE 39.

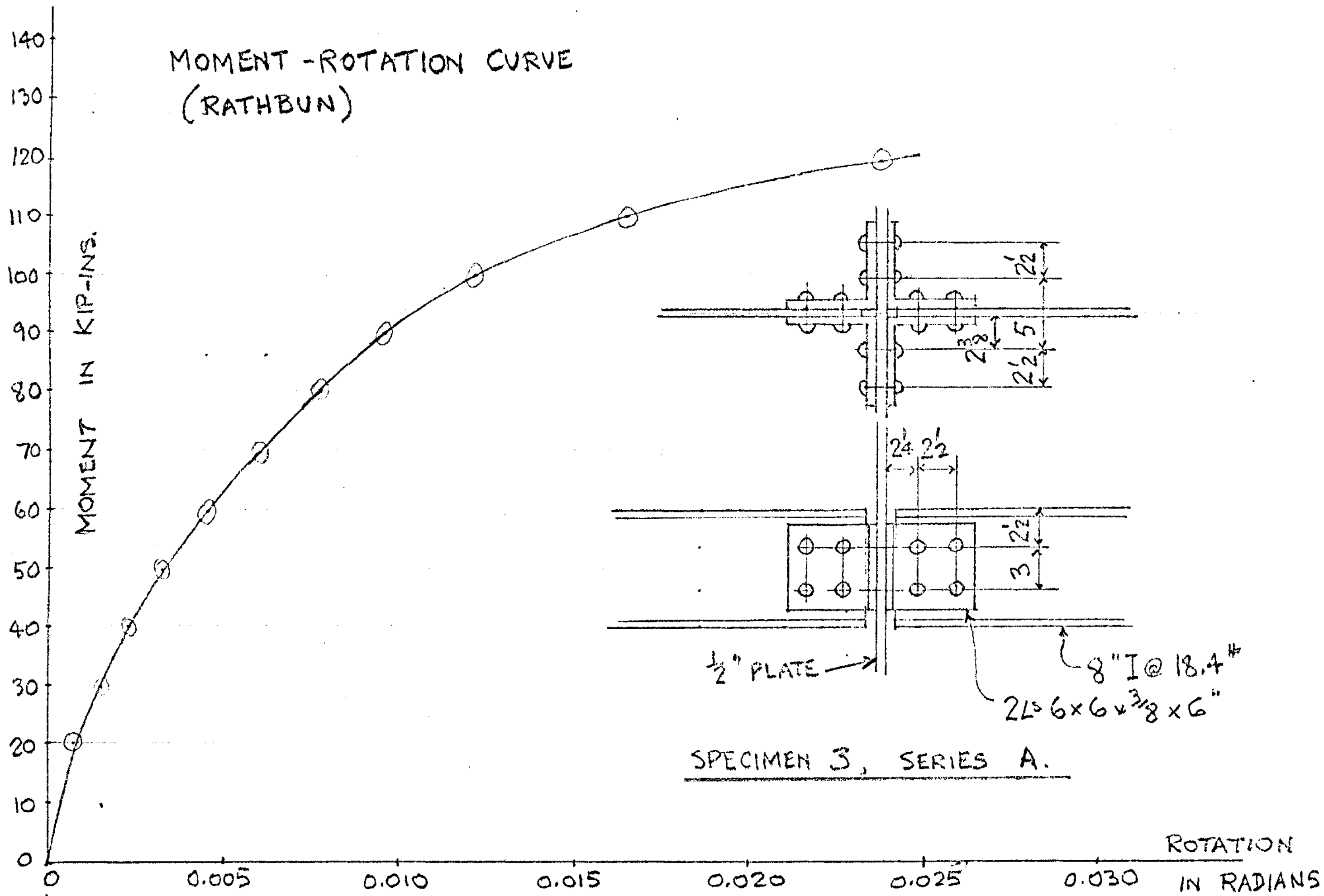


MOMENT-ROTATION CURVE,
FROM "ELASTIC PROPERTIES OF RIVETED CONNECTIONS"
BY J. CHARLES RATHBUN
TRANS. A.S.C.E. PAPER NO. 1933, 1936.
MOMENT IN KIP-INS.
ROTATION, RADIANS.
0
5.
10.
15.
20.
25.
0.005
0.010
0.015
0.020
0.025
0.030
2¼ 2½
3
½" PLATE
6" I @ 12.5#
2 Ls 6×4×⅜×2½
RIVETS 5½" c/c
SPECIMEN 1, SERIES A.



MOMENT-ROTATION CURVE
(RATHBUN)
MOMENT IN KIP-INS.
ROTATION IN RADIANS
0
10
20
30
40
50
60
70
80
90
100
110
120
130
0.005
0.010
0.015
0.020
0.025
2¼ 2½
2½
3
½" PLATE
8" I @ 18.4#
2 Ls 6×4×⅜×6"
RIVETS 5½" c/c
SPECIMEN 2, SERIES A.

FIG. 40


MOMENT - ROTATION CURVE
(RATHBUN)
MOMENT IN KIP-INS.
ROTATION IN RADIANS.
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
0.005
0.010
0.015
0.020
0.025
0.030
½" PLATE
8" I @ 18.4#
2Ls 6×6×⅜×6"
SPECIMEN 3, SERIES A.

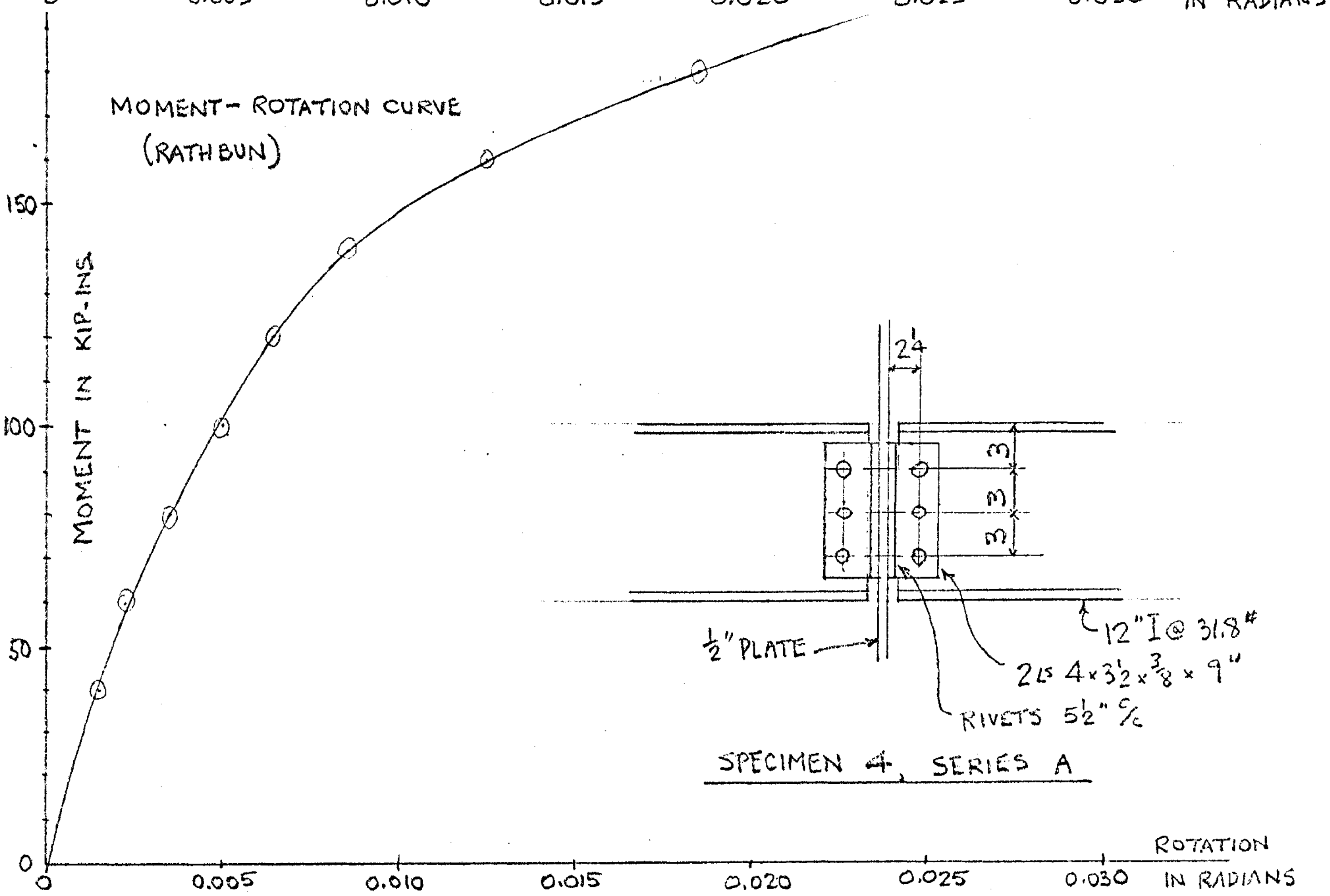
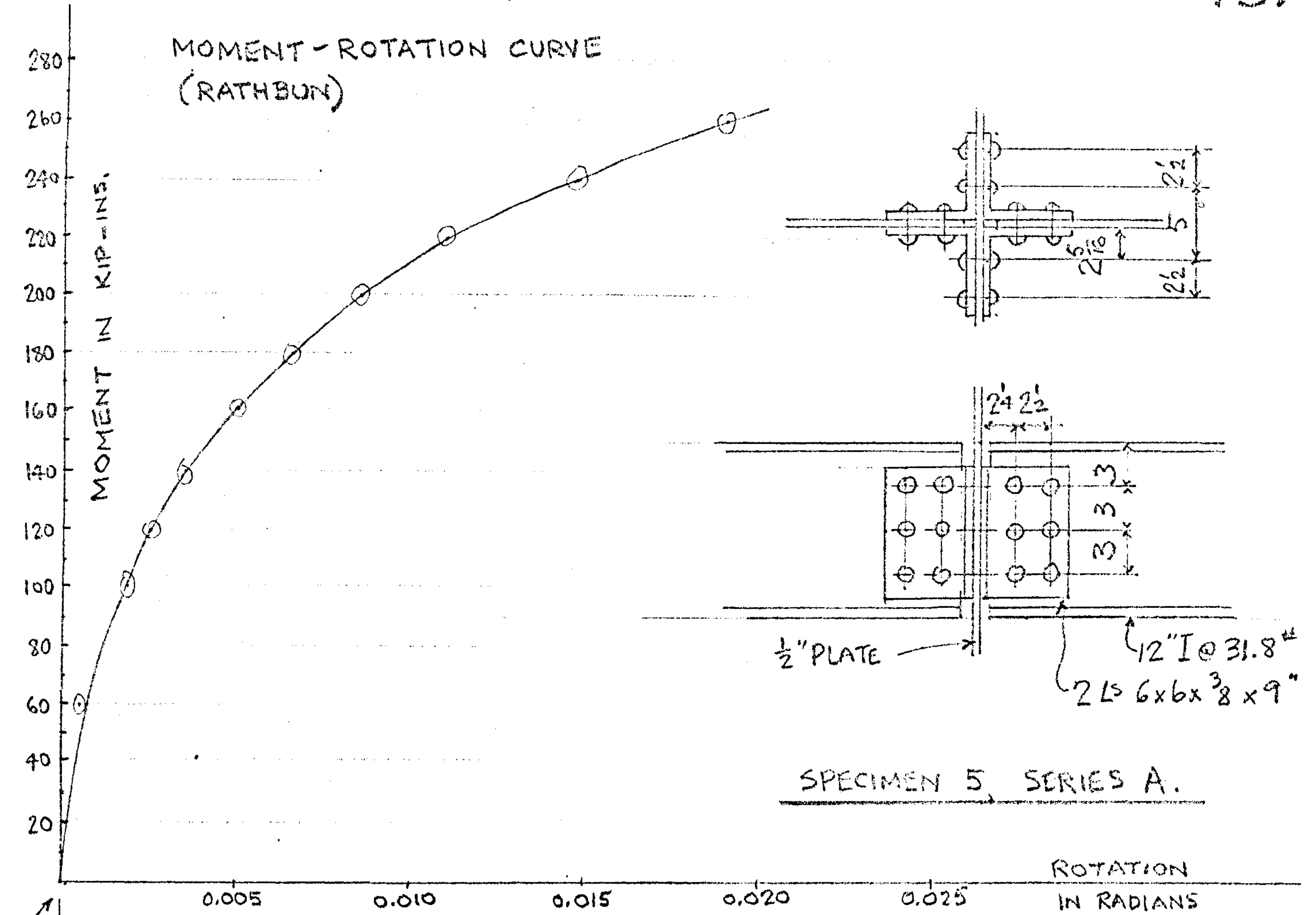

MOMENT - ROTATION CURVE
(RATHBUN)
MOMENT IN KIP-INS.
ROTATION IN RADIANS
0
50
100
150
0.005
0.010
0.015
0.020
0.025
0.030
½" PLATE
12" I @ 31.8#
2Ls 4×3½×⅜×9"
RIVETS 5½" ⅝
SPECIMEN 4, SERIES A

FIGURE 41


MOMENT-ROTATION CURVE
(RATHBUN)
MOMENT IN KIP-INS.
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0.005
0.010
0.015
0.020
0.025
ROTATION IN RADIANS
2½
5
2½
2 5/16
2¼ 2½
3
3
3
½" PLATE
12" I @ 31.8#
2 Ls 6x6x⅜x9"
SPECIMEN 5, SERIES A.

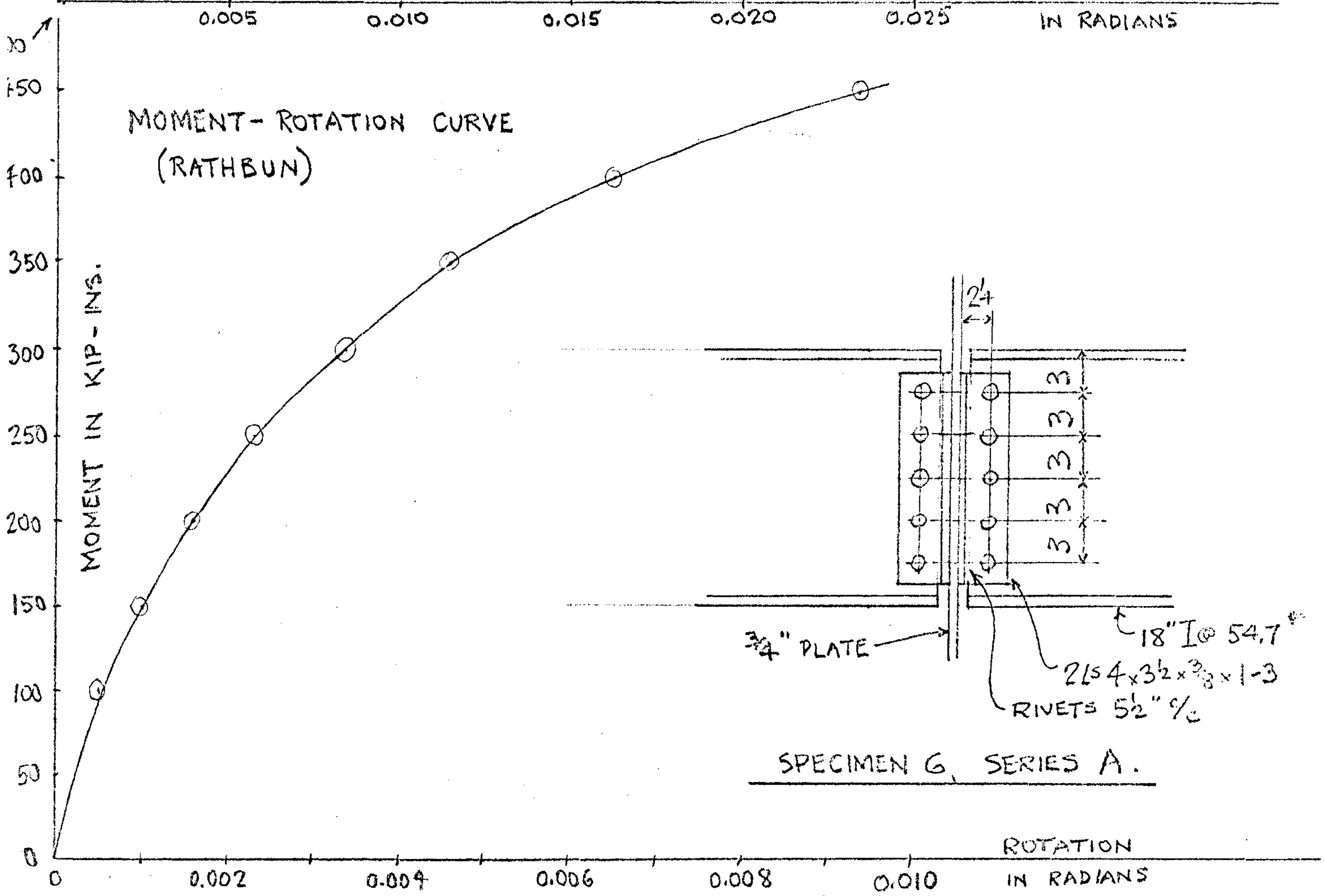

MOMENT-ROTATION CURVE
(RATHBUN)
MOMENT IN KIP-INS.
450
400
350
300
250
200
150
100
50
0
0
0.002
0.004
0.006
0.008
0.010
ROTATION IN RADIANS
2¼
3
3
3
3
3
¾" PLATE
18" I @ 54.7#
2Ls 4x3½x⅜x1-3
RIVETS 5½" c/c
SPECIMEN 6, SERIES A.

FIGURE 17

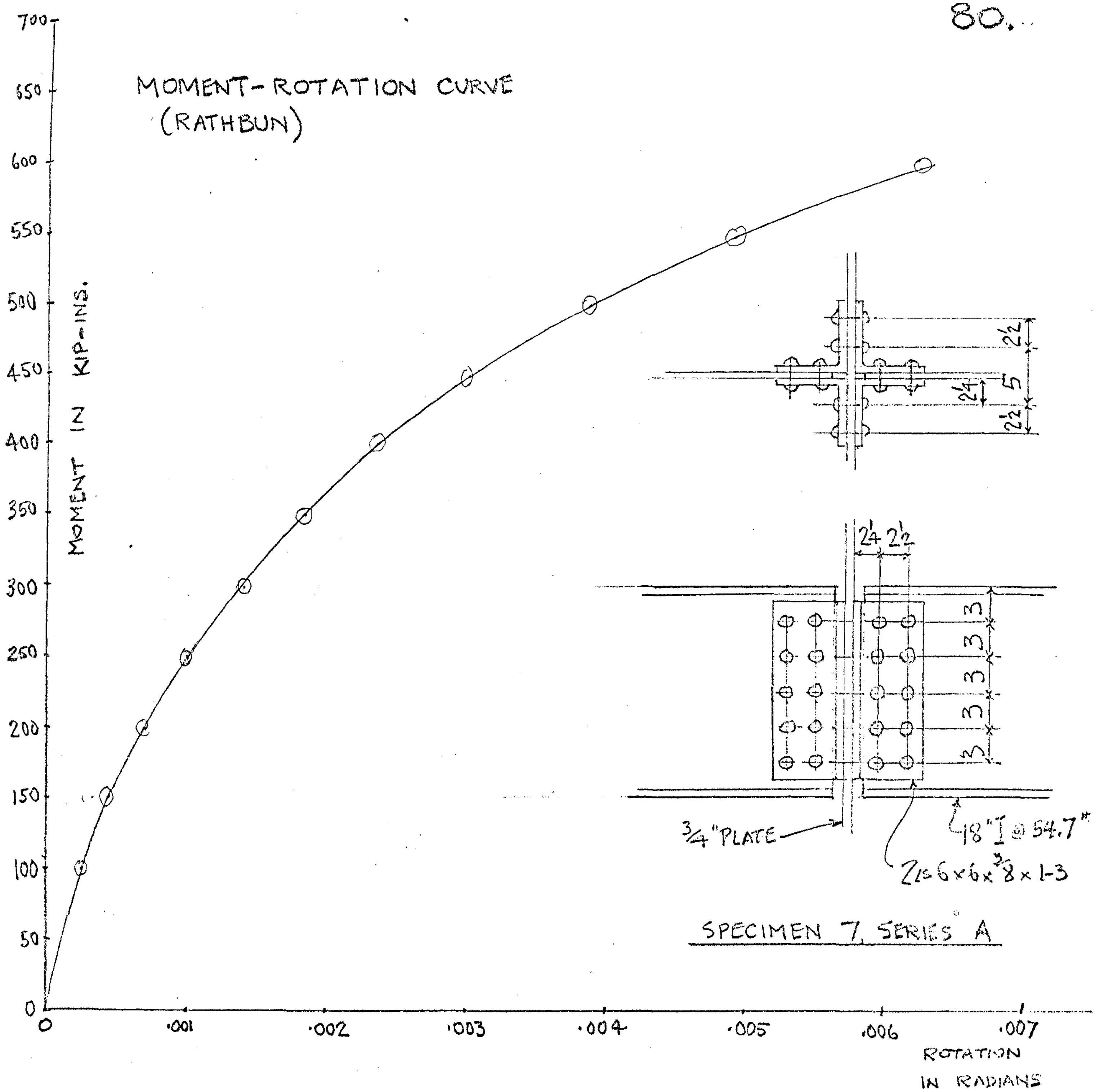
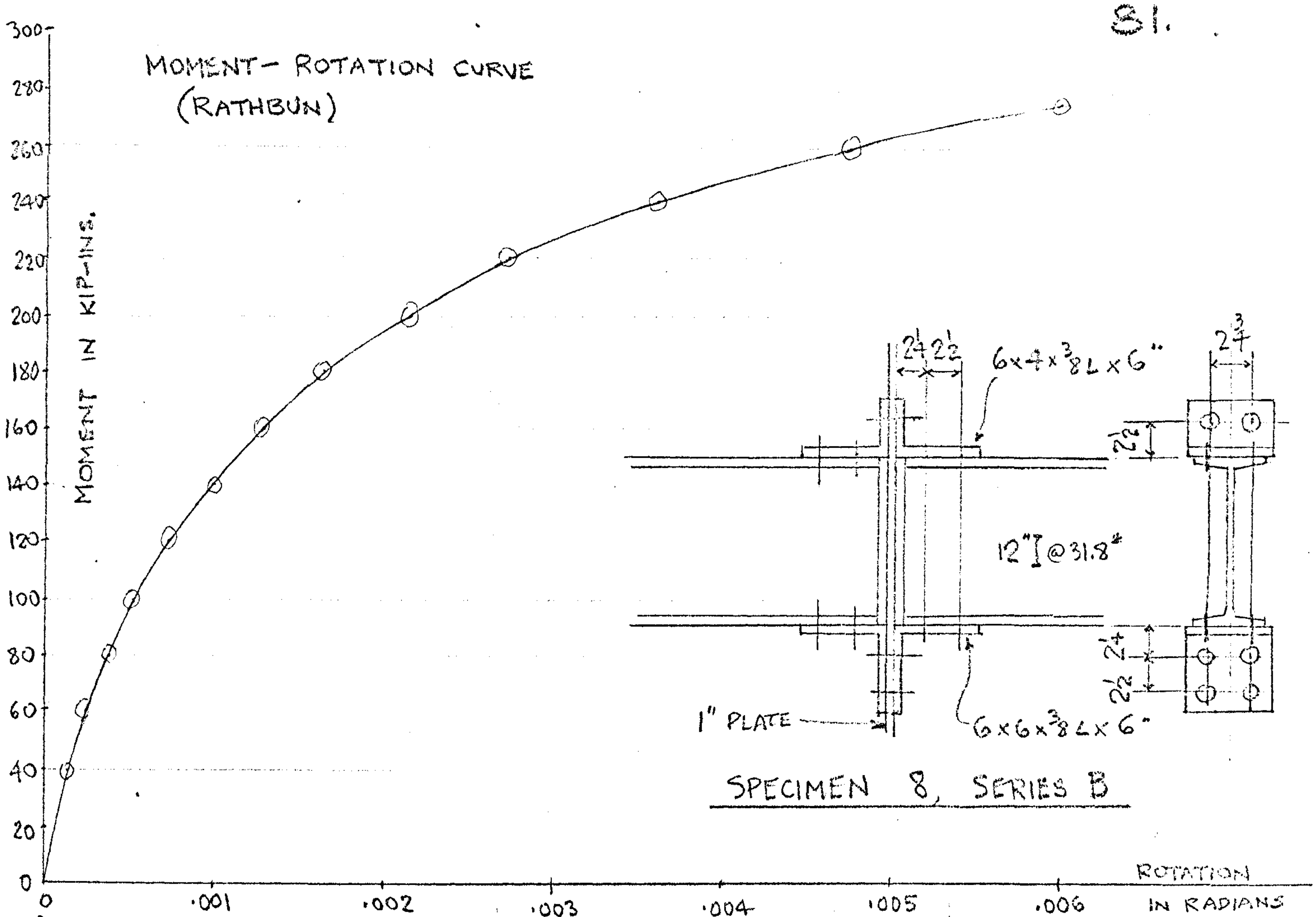

MOMENT-ROTATION CURVE
(RATHBUN)
MOMENT IN KIP-INS.
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
0
·001
·002
·003
·004
·005
·006
·007
ROTATION
IN RADIANS
2½
2½
2¼
5
2¼ 2½
3
3
3
3
3
¾"PLATE
18"I @ 54.7#
2∠s 6×6×⅜×1-3
SPECIMEN 7, SERIES A

FIGURE 43.


MOMENT- ROTATION CURVE
(RATHBUN)
MOMENT IN KIP-INS.
ROTATION IN RADIANS
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
·001
·002
·003
·004
·005
·006
6x4x3/8 L x 6"
12" I @ 31.8#
1" PLATE
6x6x3/8 L x 6"
SPECIMEN 8, SERIES B

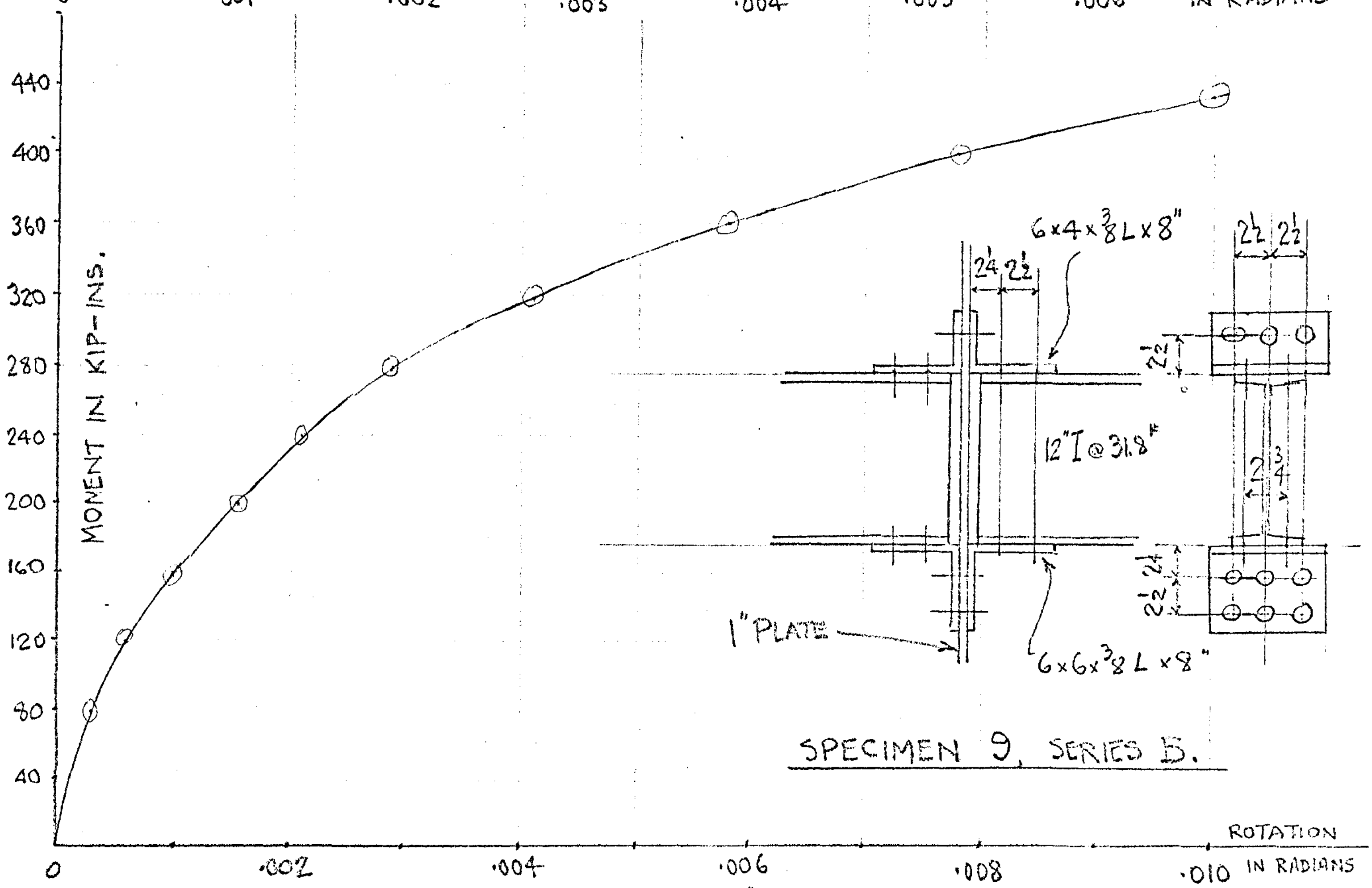

MOMENT IN KIP-INS.
ROTATION IN RADIANS
0
40
80
120
160
200
240
280
320
360
400
440
·002
·004
·006
·008
·010
6x4x3/8 L x 8"
12" I @ 31.8#
1" PLATE
6x6x3/8 L x 8"
SPECIMEN 9, SERIES B.

FIGURE 11

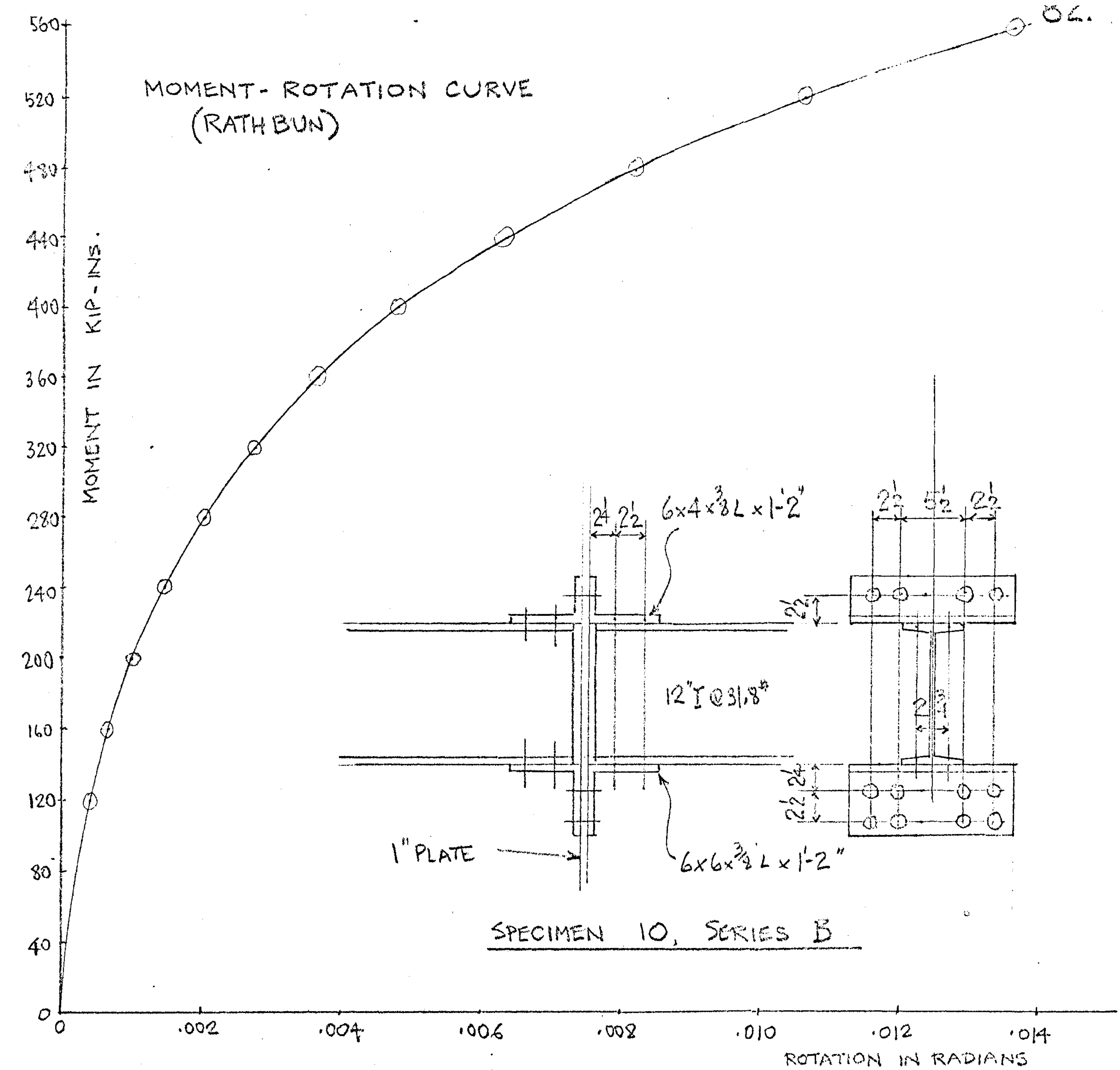
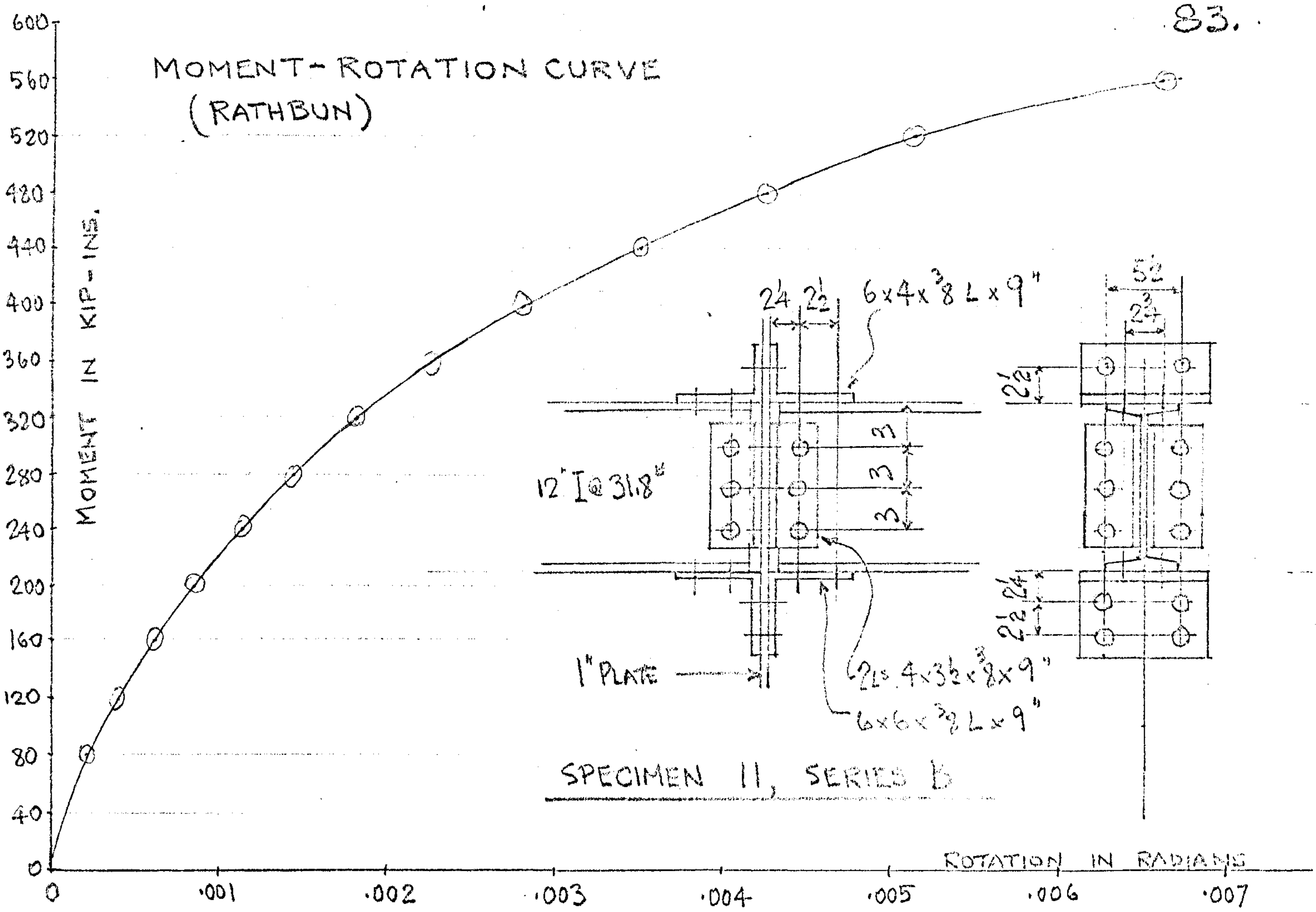

MOMENT-ROTATION CURVE
(RATHBUN)
MOMENT IN KIP-INS.
ROTATION IN RADIANS
0
40
80
120
160
200
240
280
320
360
400
440
480
520
560
0
.002
.004
.006
.008
.010
.012
.014
6x4x3/8 L x 1'-2"
12" I @ 31.8#
1" PLATE
6x6x3/8 L x 1'-2"
SPECIMEN 10, SERIES B

FIGURE 45


MOMENT-ROTATION CURVE
(RATHBUN)
MOMENT IN KIP-INS.
ROTATION IN RADIANS
12" I @ 31.8#
1" PLATE
6x4x3/8 L x 9"
2Ls 4x3½x3/8x9"
6x6x3/8 L x 9"
SPECIMEN 11, SERIES B

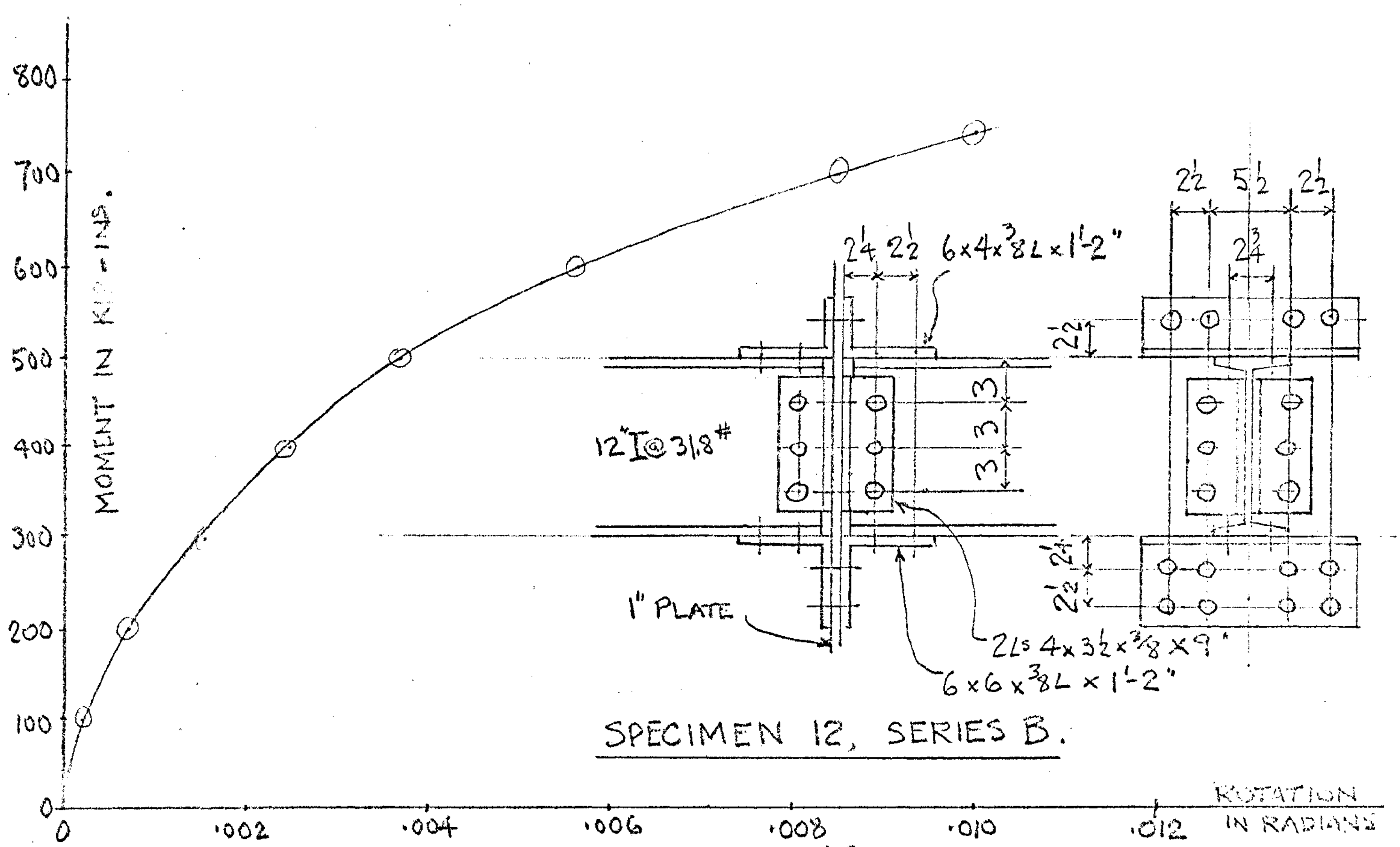
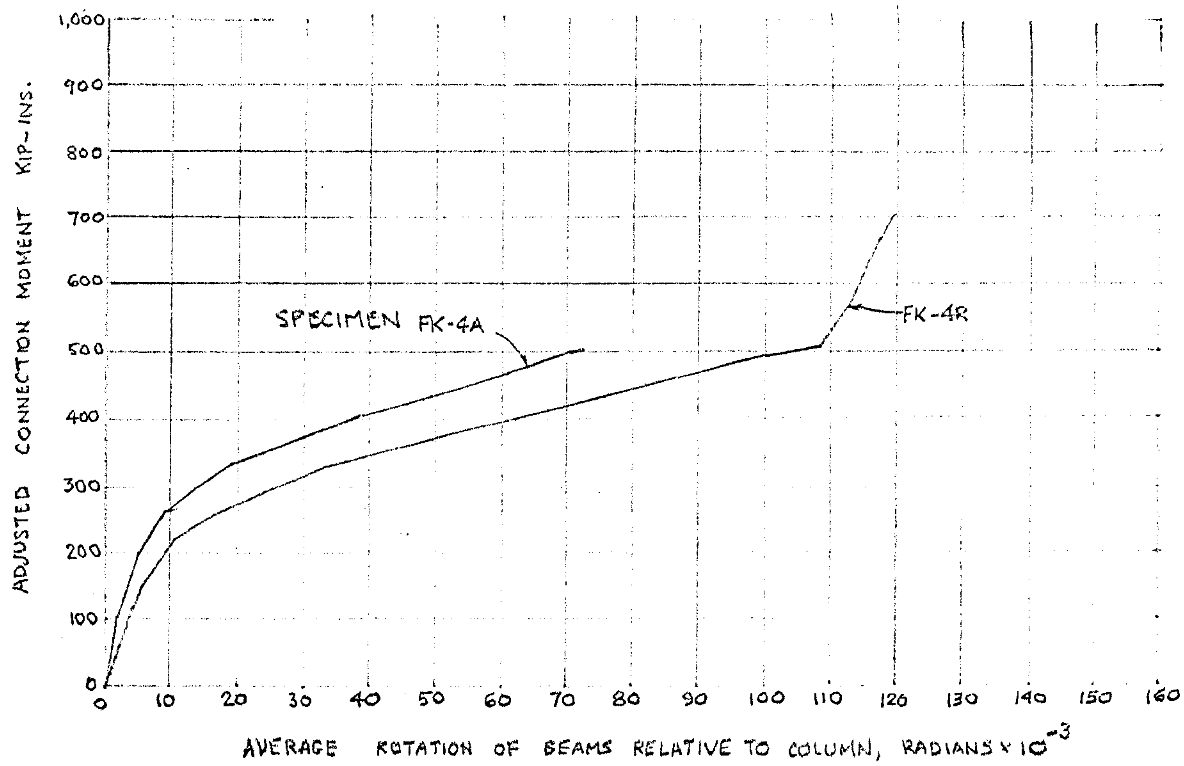

MOMENT IN KIP-INS.
ROTATION IN RADIANS
12" I @ 31.8#
1" PLATE
6x4x3/8 L x 1'-2"
2Ls 4x3½x3/8x9"
6x6x3/8 L x 1'-2"
SPECIMEN 12, SERIES B.

FIGURE 46.

TRANS. ASCE VOL. 126 (1961) "BEHAVIOR OF BEAM-TO-COLUMN CONNECTIONS."
BY MUNSE, BELL & CHESSON.

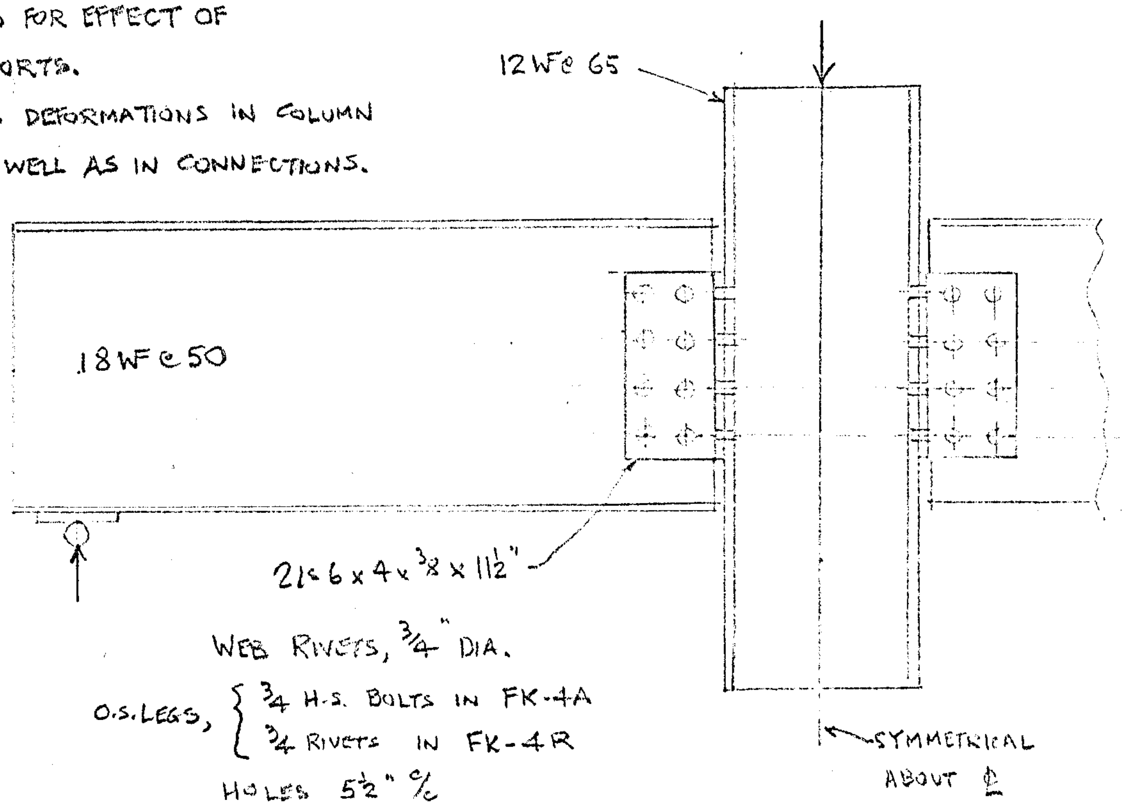


COMPARISON OF MOMENT-ROTATION CURVES FOR RIVETED AND BOLTED CONNECTIONS.

NOTE:—

MOMENT ADJUSTED FOR EFFECT OF FRICTION AT SUPPORTS.

ROTATION INCLUDES DEFORMATIONS IN COLUMN AND IN BEAMS AS WELL AS IN CONNECTIONS.



NOTE:- Y.P. OF ANGLE MATERIAL = 40.3 KSI (AV.)

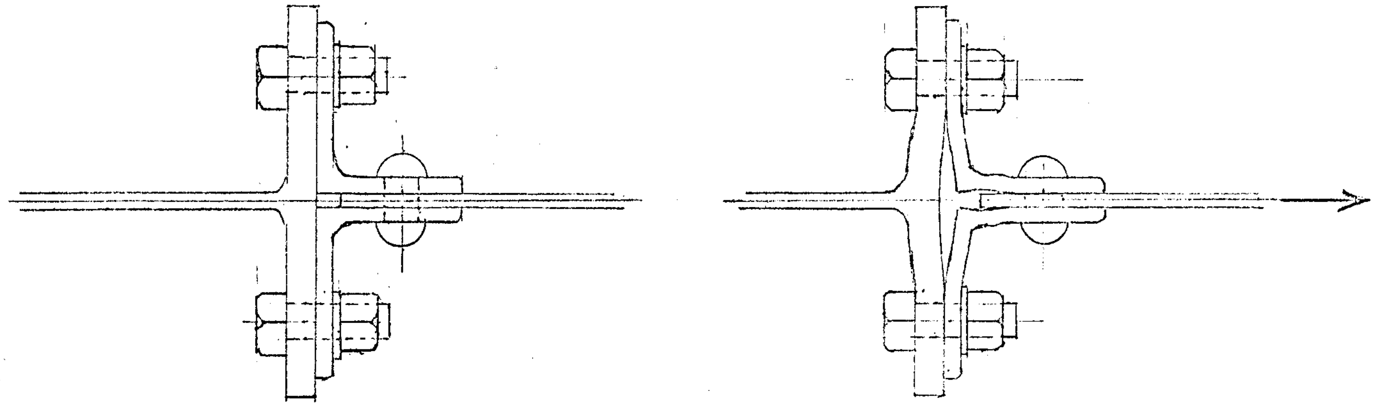
FIGURE 47.

Calculation of the Properties of Connections.

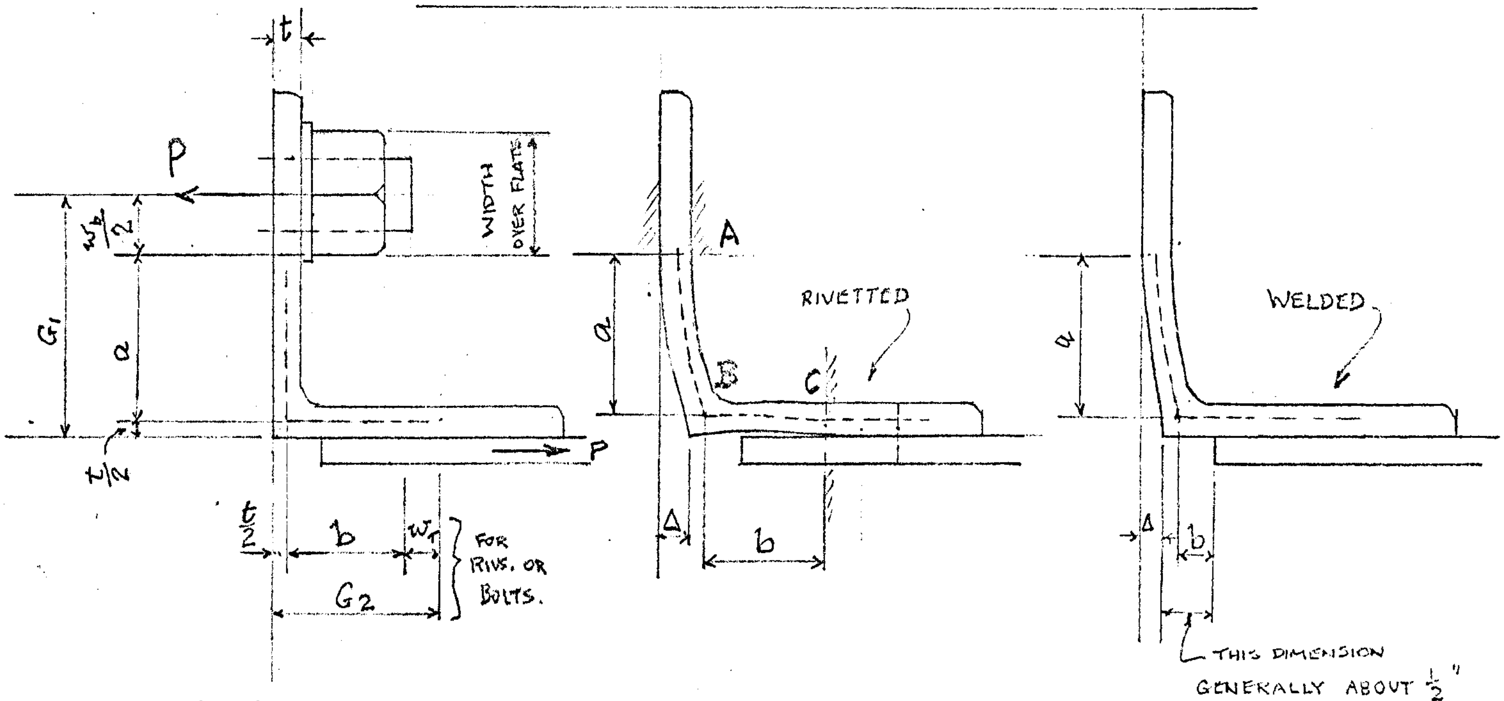
J.E.Lothers (22) has proposed a method of calculating the effect of angular deflection on a standard beam connection, and obtaining the relationship between the deflection and the resisting moment developed therein. A slightly modified form of this method has been adopted where required for the analysis of typical structures in the following pages. This modified method is set forth in figures 48, 49 and 50. It will be noted that an attempt has been made to include the deformation of the supporting member; this involves the adoption of some drastic approximations, in the face of the highly complicated problem of calculating the deformation of the flange or the web of a column section. Unfortunately Professor Lothers did not take this source of deformation into account, and the experiments of Professor Rathbun were conducted on pairs of connections attached on opposite sides of a solid plate. In figures 51 and 52 a connection identical with one tested by Professor Rathbun is subjected to analysis, the results of which are compared with the experimental curve.

In figure 53 a type of connection is analysed which differs materially from the "standard" beam connection. It consists of a plate or bar welded to the column, with the web of the connected beam bolted to the plate. It is evident that in this connection the plate is to all intents a part of the column, and that any rotation which takes place at the end of the beam will occur through slipping of the bolts. Research which has been carried out on joints with high-strength bolts in recent years furnish information on the resistance of such joints to slipping, and from such experimental data (19), the approximate moment which could be developed by such a connection has been calculated.

PROPERTIES OF CONNECTIONS:



DEFORMATIONS IN FLANGE CONNECTION.



USING SLOPE DEFLECTION METHOD;

$$M_{AB} = \frac{Pa(2a+b)}{4a+b}$$

$$M_{BA} = \frac{2Pa^2}{4a+b}$$

$$M_{CB} = \frac{Pa^2}{4a+b}$$

$$\Delta = \frac{Pa^3(a+b)}{3EI(4a+b)}$$

FOR WELDED CLIP ANGLES, IF $b_1 = 0$, THE EQUATIONS BECOME:

$$M_{AB} = \frac{Pa}{2}$$

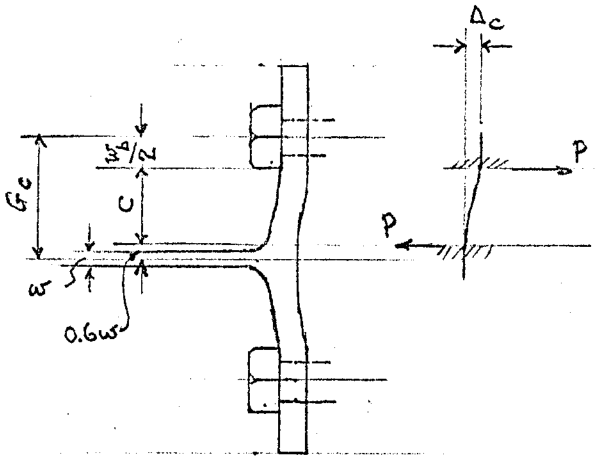
$$M_{BA} = \frac{Pa}{2}$$

$$\Delta = \frac{Pa^3}{12EI}$$

THESE EQUATIONS APPLY IN ELASTIC RANGE

NOTE:- THE ABOVE IS A MODIFICATION OF METHOD USED BY J.E. LOTHERS; TRANS. ASCE VOL. 116 (1951) p. 480.

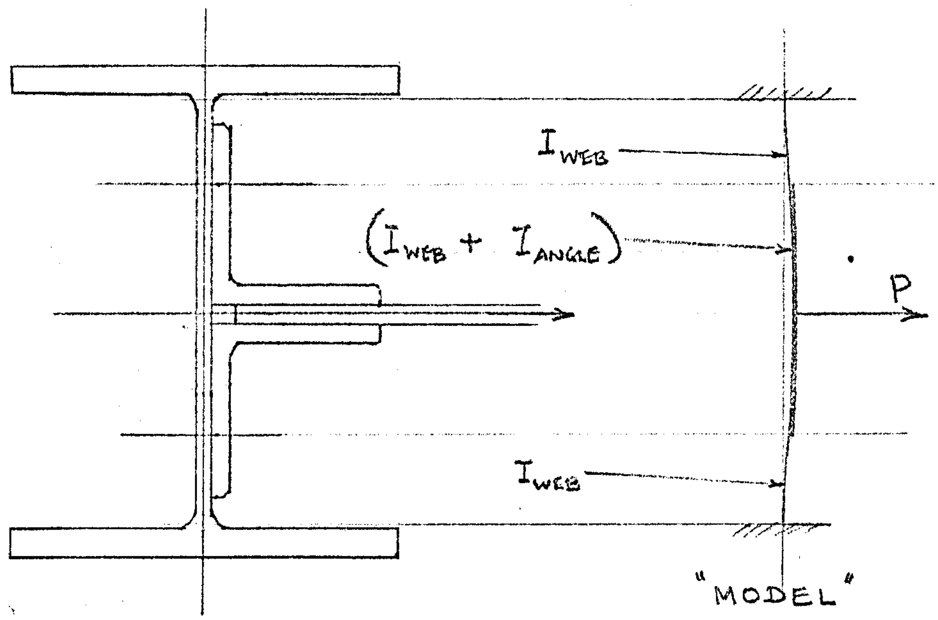
FIGURE 48.



DEFORMATION OF COLUMN FLANGE:

$$M_c = \frac{Pc}{2}$$

$$\Delta_c = \frac{Pc^3}{12EI_c}$$



CALCULATIONS SHOW THAT ALTHOUGH THE PULL ON THE CONNECTION TENDS TO SEPARATE THE ANGLES FROM THE WEB, THE MOMENTS TRANSMITTED TO THE WEB AT THE BOLTS, PLUS THE DIRECT PULL ON THE BOLTS, HAS THE EFFECT OF KEEPING ANGLES AND WEB IN CONTACT. RESULTANT APPROXIMATELY RESEMBLES THE MODEL SHOWN.

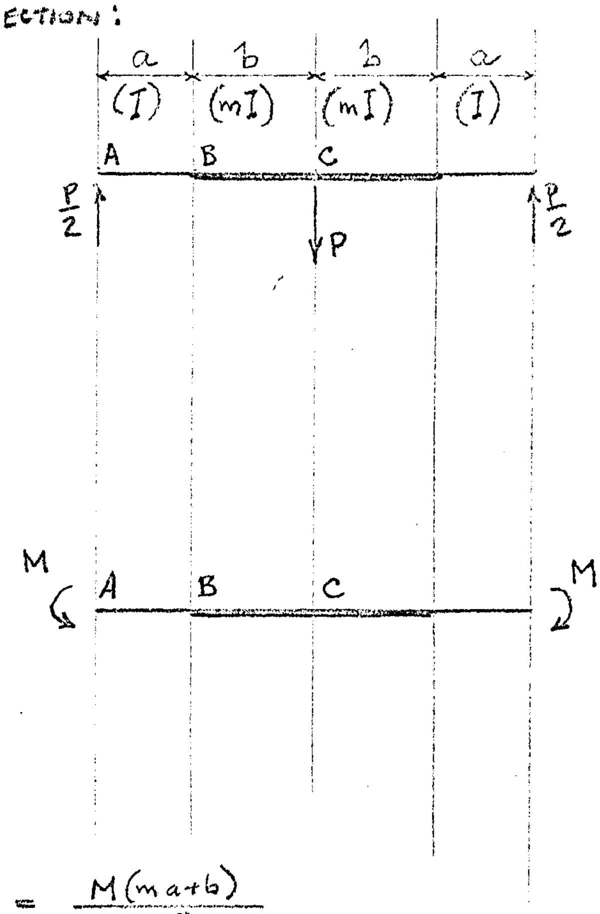
DEFORMATIONS IN WEB CONNECTION

WEB WILL BE RESTRAINED FROM ROTATION AT JUNCTURE WITH FLANGE ONLY IN PROPORTION TO THE RESISTANCE OFFERED AGAINST ROTATION OF THE FLANGE ITSELF. THIS WILL DEPEND ON THE WIDTH AND THICKNESS OF THE FLANGE, AND ALSO ON THE PRESENCE OR ABSENCE OF MEMBERS CONNECTED TO THE FLANGES IN THIS REGION.

NOTE THAT IF SIMILAR CONNECTIONS OCCUR ON BOTH SIDES OF THE WEB, THE LATTER WILL NOT BE DEFORMED, AND THE CONNECTION WILL BEHAVE IN A MANNER SIMILAR TO A FLANGE CONNECTION.

FIGURE 49

COMBINED DEFORMATION OF WEB AND CONNECTION:



CONDITION 1: EDGES OF WEB FREE TO ROTATE.

θ_A = SLOPE AT A

Δ_C = DEFLECTION AT C

$$\theta_A = \frac{Pa^2(m-1) + P(a+b)^2}{4mEI}$$

$$\Delta_c = \frac{Pa^3(m-1) + P(a+b)^3}{6mEI}$$

CONDITION 2: MOMENTS APPLIED AT EDGES OF WEB.

$$\theta_A = -\frac{M(ma+b)}{mEI}$$

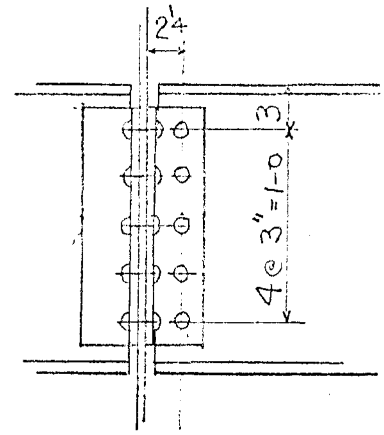
$$\Delta_c = -\frac{M(ma^2+2ab+b^2)}{2mEI}$$

RESULTANT: IF $\theta_A = 0$, $\dfrac{Pa^2(m-1)+P(a+b)^2}{4mEI} = \dfrac{M(ma+b)}{mEI}$

AND $\Delta_c = \dfrac{Pa^3(m-1)+P(a+b)^3}{6mEI} - \dfrac{M(ma^2+2ab+b^2)}{2mEI}$

FIGURE 50.

ANALYSIS OF A TYPICAL CONNECTION:



$2 L_S\ 4 \times 3\frac{1}{2} \times \frac{3}{8} \times 1\text{-}3$
RIVETS $5\frac{1}{2}''$ %
Ga. o.s.l. = $2\frac{9}{16}''$

TREAT CONNECTION ANGLES AS A SERIES OF 3" SECTIONS;

FOR ONE SUCH SECTION, $I = \frac{3 \times .375^3}{12} = .01318 \text{ IN}^4$

$S = \frac{3 \times .375^2}{6} = .0703 \text{ IN}^3$

FOR 4" LEG, $a = 2.5625 - .375 - .1875 = 2.000''$

FOR $3\frac{1}{2}''$ LEG, $b = 2.25 - .375 - .1875 = 1.6875''$

MOMENTS IN ANGLE, FOR TENSION P IN RIVET, (KIPS)

$M_A = \frac{Pa(2a+b)}{(4a+b)} = \frac{2(5.6875)P}{9.6875} = 1.175P$ K-INS.

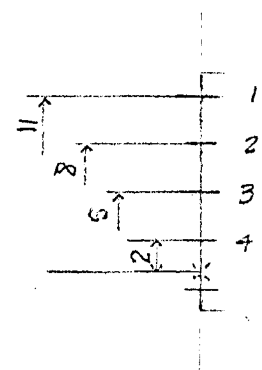
$M_B = \frac{8P}{9.6875} = 0.825P$ K-INS

DEFLECTION $\Delta = \frac{8(3.6875)P}{3EI(9.6875)} = \frac{1.016P}{EI}$

$= \frac{1.016P}{30{,}000 \times .01318} = 2.57 \times 10^{-3} P$ INS.

MAX. BENDING STRESS $= \frac{1.175P}{.0703} = 16.72P$ K/IN²

ASSUME CENTRE OF ROTATION AS SHOWN:



THEN FOR Δ DEFLECTION AT LEVEL 1,

ROTATION $= \frac{\Delta}{11} = 0.238 \times 10^{-3} P$ RADIANS

AT 1,	TENSION = 2P	MOMENT = 22P	
AT 2,	" $= 2P \times \frac{8}{11} = 1.455P$	" $= 1.455 \times 8P = 11.65P$	
AT 3,	" $= 2P \times \frac{5}{11} = 0.909P$	" $= .909 \times 5P = 4.55P$	
AT 4,	" $= 2P \times \frac{2}{11} = 0.364P$	" $= .364 \times 2P = 0.73P$	
	TOTAL MOMENT	$= 38.93P$ K-IN.	

ELASTIC CONDITION WILL PREVAIL UNTIL MAX BENDING STRESS = YIELD STRENGTH OF MATERIAL IN ANGLES. LET $f_y = 40$ KSI, THEN $16.72P = 40.0$ OR

MAX. VALUE $P = 40/16.72 = 2.39^k$

CORRESPONDING MOMENT $= 38.93 \times 2.39 = 93.2$ K-IN.

" ROTATION $= .238 \times 2.39 \times 10^{-3} = .570 \times 10^{-3}$ RAD.

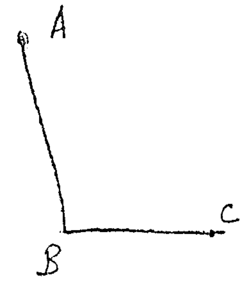
CONTINUED BENDING WILL TAKE PLACE WITH SEGMENT 1 IN PLASTIC CONDITION, UNTIL SEGMENTS 2, 3 AND 4 IN SUCCESSION REACH THAT CONDITION.

THIS SEQUENCE FOLLOWS:

FIGURE 51.

MAX. "PLASTIC" MOMENT FOR THE SECTION

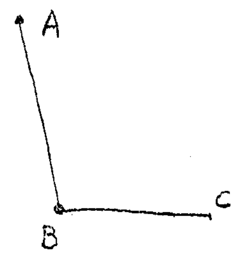
$$= \frac{f_y b d^2}{4}$$



1ST PLASTIC STAGE

WHEN ANGLE REACHES FULLY PLASTIC CONDITION (2ND. STAGE), $M_{AB} = M_{BA} = M_p$

OR $Pa = \frac{f_y b d^2}{2}$



2ND PLASTIC STAGE.

FOR THE GIVEN SECTION $\frac{bd^2}{4} = \frac{3 \times .1406}{4}$

$= .1055$

$\therefore P_p = \frac{40 \times .2110}{2.00} = 4.22^k$

STAGE 2.

1. TENSION	$= 8.44^k$	MOM. $= 8.44 \times 11 = 92.8$	ROTATION:
2. "	$= 4.78$	$4.78 \times 8 = 38.2$	$\frac{2.57 \times 2.39 \times 10^{-3}}{8}$
3. "	$= 4.78 \times \frac{5}{8} = 2.99$	$2.99 \times 5 = 15.0$	
4. "	$= 4.78 \times \frac{2}{8} = 1.20$	$1.20 \times 2 = 2.4$	$= \frac{6.15 \times 10^{-3}}{8}$

TOTAL MOMENT $= 148.4$ K-IN. $= 0.768 \times 10^{-3}$ RAD

STAGE 3.

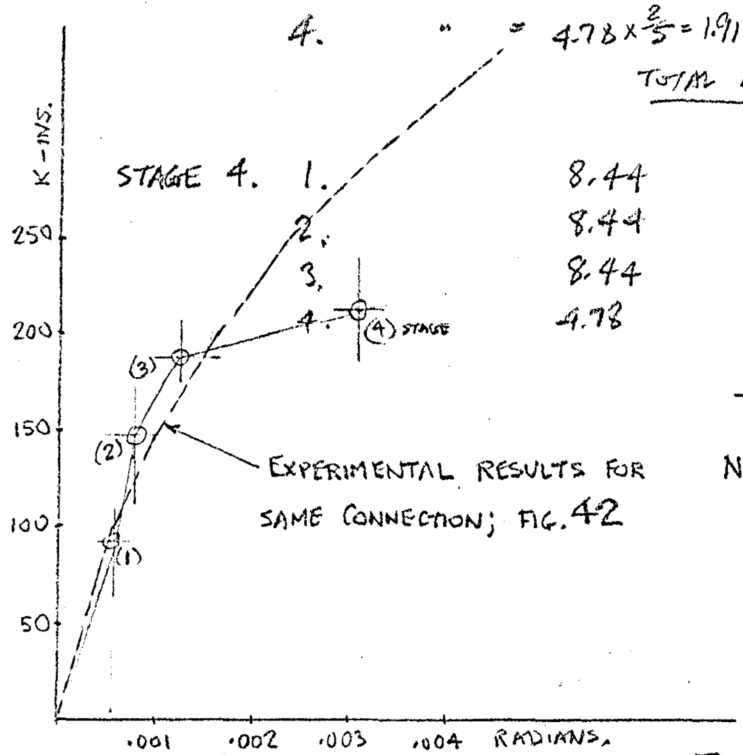
1. TENSION	$= 8.44$	MOM. $= 8.44 \times 11 = 92.8$	ROTATION:
2. "	$= 8.44$	$8.44 \times 8 = 67.5$	$\frac{6.15 \times 10^{-3}}{5}$
3. "	$= 4.78$	$4.78 \times 5 = 23.9$	
4. "	$= 4.78 \times \frac{2}{5} = 1.91$	$1.91 \times 2 = 3.8$	$= 1.23 \times 10^{-3}$ RAD

TOTAL MOMENT $= 188.0$ K-IN.

STAGE 4.

1.	8.44	$8.44 \times 11 = 92.8$	ROTATION:
2.	8.44	$8.44 \times 8 = 67.5$	$\frac{6.15 \times 10^{-3}}{2}$
3.	8.44	$8.44 \times 5 = 42.2$	
4.	4.78	$4.78 \times 2 = 9.6$	$= 3.075 \times 10^{-3}$ RAD

TOTAL MOMENT $= 212.1$ K-IN.



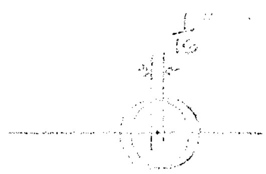
NOTE: MORE EXHAUSTIVE ANALYSIS OF THE CONNECTION WOULD PROBABLY DISCLOSE THAT AT STAGE 4 OR EARLIER, THE SEGMENT (1) WOULD BE SUBJECT TO STRAINING INTO THE STRAIN-HARDENING PART OF THE STRESS-STRAIN CURVE; HENCE THE CONTINUED RISE OF THE EXPERIMENTAL CURVE.

FIGURE 52.

PROPERTIES OF WELDED PLATE CONNECTION.

In this type of connection, before the bolts are tightened, there is sufficient play o bolts in the holes to permit some rotation of the end of the beam.

3/4" bolt in 13/16" hole allows relative movement of the connected parts = 1/16"



In the typical connection shown, the corresponding (maximum) rotation would be:

$$\theta_{max} = \frac{\frac{1}{16}''}{3\frac{3}{4}''} = \frac{1}{60} = 0.0167 \text{ RADIANS.}$$

After the bolts have been tightened, rotation of the beam relative to the column can occur only after maximum loading on a bolt has exceeded the "slip resistance" of the bolt. Experimental data for high strength bolts (19) give the following values:

7/8 dia. H.S. bolts, in connections with unpainted contact surfaces free from loose scale:

Average slip coefficient = 0.458

Specified clamping force for 3/4 dia. bolt = 28.4 K

∴ Slip resistance per 3/4" bolt in single-shear connection = 28.4 x 0.458 = 13.0 K

According to this, the connection shown would be able to develope, without slip, a moment equal to:

$$M = 2 \times 13.0 \times 3.75 + 2\left(13.0 \times \frac{1.25}{3.75}\right) \times 1.25 = 108 \text{ K-INS.}$$

Note: High strength bolts in friction-type connections are assumed to have a slip resistance equal to the shear value of a rivet of the same diameter. This is a "safe" value, which can be almost doubled in practice.
e.g. Double shear value of 7/8" rivet = 18.04 K
Slip value of 7/8" H.S. bolt in d.s. = 41.3 K (av) in tests.

FIGURE 53

Resistance to Rotation of Column Bases.

In any steel building frame, the magnitude of the moments which are produced at the joints by a given system of loads, will be affected by the resistance of the column bases to rotation. In the type of structure which is to be analysed, designed by the conventional method, column base details are of the simplest kind, not designed to provide any specific rigidity. A base plate, welded to the end of the column, and secured to the concrete by two, or more rarely four anchor bolts, comprises the usual base. The resistance of which it is capable is usually very small, and could well be neglected were it not for the fact that in some of the structures being studied the beam connections themselves have little resistance to rotation.

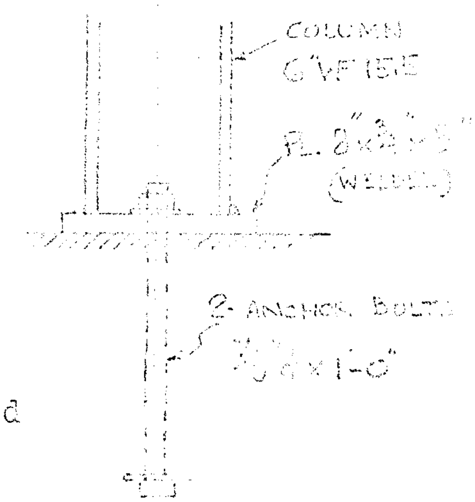
Methods used in calculating the properties of simple column bases are shown in figures 54, 55 and 56. The problem is highly indeterminate, but an approximate method has been adopted for each case, with certain aasumptions being made regarding the distribution of bending stress in a transverse direction, and the elongation of anchor bolts embedded in concrete. No claim is made that the results are accurate, but in each case it seems reasonable to suppose that the actual moment-resistance would be of the same order as the calculated figure. Fortunately, the behaviour of the base could vary considerably in its particular range without affecting seriously the distribution of moments in the upper part of the structure, which is where the important stresses are to be found.

It has been pointed out above, that secondary stresses would be caused by unequal settlement of the substructure of a building, but that this is of concern to the designer of the substructure rather than to the designer of the steel frame. In like manner, the possibility of the rotation of the concrete mass upon which a column is standing, is not primarily the concern of the desigher of the steelwork. If his design is to take official cognizance of some unavoidable weakness in the system of supports, then it will have to take into account the moments which will be likely to develope throughout the structure, and "secondary" stresses will cease to be such.

RIGIDITY OF SIMPLE COLUMN BASES.

The type of column base most commonly used on the structures being analysed is as shown. The base plate is attached by welding to the flanges.
Rotation of the column at the base will be due to -

a) Bending of the base plate
b) Elongation of the anchor bolts



Both of these factors are indeterminate, but approximate information may be obtained by making some simplifying assumptions.
It will be found in the analysis of typical structures that the moment at the column bases is not critical.

It will be assumed that under moments of the magnitude generally found, the concrete substructure resists rotation.

Let M = Moment at base (K-ins.)
$\therefore$ Uplift at r.h. weld = $\frac{M}{6}$ Kips.
$\therefore$ B.M. in plate = $\frac{M}{6}$ (2.35) = 0.39M K-ins.

For 8 x 3/4 plate, $\begin{cases} I = 0.281 \\ S = 0.75 \end{cases}$

$\therefore$ f_b in plate = $\frac{0.39M}{0.75}$ = 0.52M ksi

Deflection $\delta_p = \frac{.167M\ (2.35)^3}{3 \times 30{,}000 \times .281}$

$= 86 \times 10^{-6}$ M inches.

Rotation of Column = $\frac{86 \times 10^{-6} M}{6}$

$= 14.3 \times 10^{-6}$ M radians

Tension in each anchor bolt= $\frac{1}{2}(M/3)$
Assume 9" of bolt can extend; then
extension $\delta_b = \frac{.167M \times 9}{30{,}000 \times .60}$

$= 83 \times 10^{-6}$ M ins.

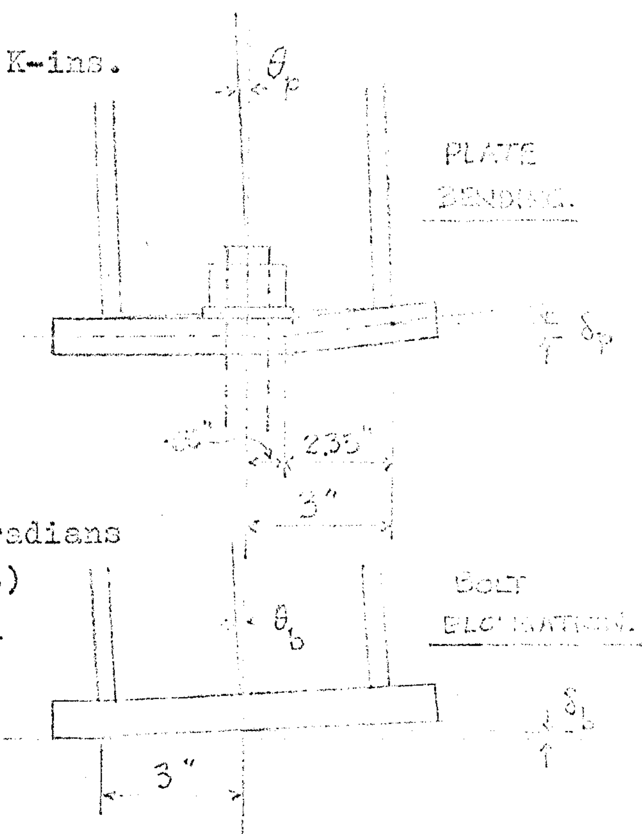


FIGURE 54.

Rotation of column due to elongation of anchor bolts

$$= \frac{\delta_b}{3} = \frac{83 \times 10^{-6} M}{3} = 27.8 \times 10^{-6} M \text{ Radians.}$$

Rotation of column due to both causes $= 14.3 \times 10^{-6} M + 27.8 \times 10^{-6} M$

$$= \underline{42.1 \times 10^{-6} M \text{ Radians}}$$

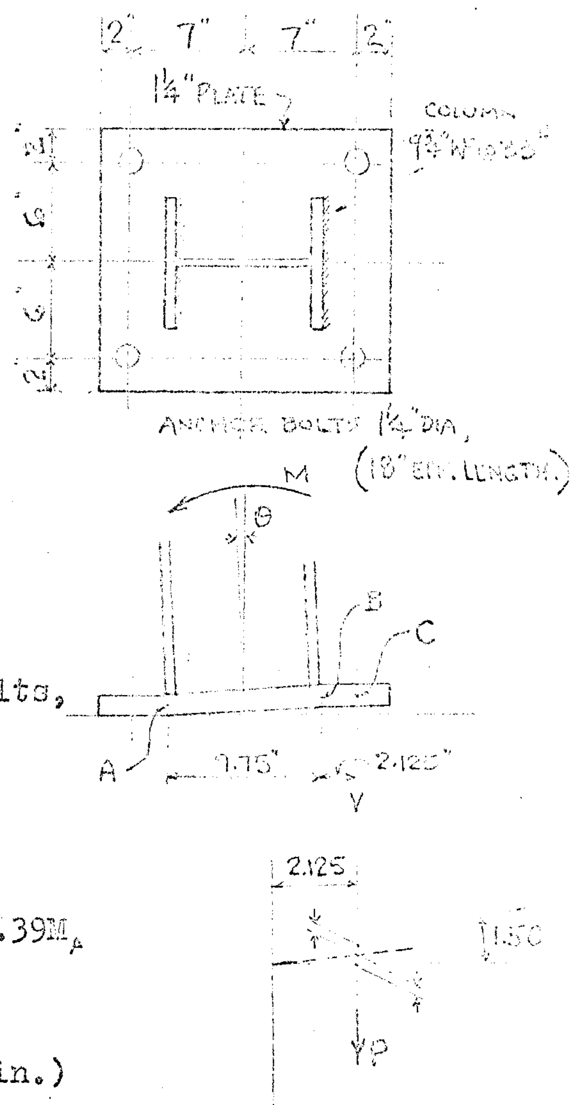
Stresses developed: f_b in plate = 0.52M K/sq.in.

f_a in bolts = .167M/.42 = 0.40M K/sq.in. (on nett area)

If f_y = 40 ksi, value of M at which plastic bending in plate begins = 40/0.52 = 77 K-ins.

Rotation/Moment ratio $= \gamma = \dfrac{42.1 \times 10^{-6} M}{M} = 42.1 \times 10^{-6} \left(\dfrac{\text{Radians}}{\text{K-ins.}}\right)$

Column Base with 4 Anchor Bolts.



If M_A = B.M. in plate at A, and upward displacement at B = δ, then

$\delta = \dfrac{M_A L^2}{3EI}$, but $\delta = L\theta$ $\therefore L\theta = \dfrac{M_A L^2}{3EI}$

$\therefore M_A = \dfrac{3EI\theta}{L} = \dfrac{EI\theta}{3.25} = 24{,}000\,\theta$

Slope of plate at B = $1.50\,\theta$

Deflection of plate between B and C due to tension in 2 anchor bolts,

$$\frac{(2.125)^3 P}{3EI} = \frac{3.2\,P}{EI}$$

$\therefore$ Resultant movement at C,

$$= 1.50\theta(2.125) + 9.75\theta - \frac{3.2P}{EI}$$

$$= 12.94\theta - \frac{3.2P}{EI} = \frac{42.0M_A}{EI} - \frac{3.2P}{EI}$$

But this movement at C = elongation in anchor bolts,

$$= \frac{18P}{2 \times 1.227E} = \frac{7.34P}{E}$$

Equating, $7.34\,PI = 42.0M_A - 3.2P$

where $I = \dfrac{16(1.25)^3}{12} = 2.60$,

$\therefore 22.28\,P = 42.0\,M_A$ or $P = 1.885\,M_A$

But total moment $M = M_A + 11.875P = 23.39M_A$

where $M_A = 24{,}000\,\theta$, $\therefore M = 56.16 \times 10^{4}\,\theta$

$\therefore \gamma = \theta/M = 1/56.16 \times 10^{4} = 1.78 \times 10^{-6}$

(Limit for M, f_b = 40 at B, M = 970 K-in.)

FIG. 55

Rigidity of Column Base about Y-Y Axis.

Assumption made in this case is that a strip of the base plate exerts beam-action between the lines of attachment to the column flanges. The bending of this strip, plus the elongation of the tensile anchor bolt, controls the rotation of the column.

Take strip of plate from column axis to right-hand edge (4.5")

$$I = \frac{4.5\ (.75)^3}{12} = 0.159$$

$$S = \frac{4.5\ (.75)^2}{6} = 0.422$$

Elastic deformation of plate at bolt

$$= \frac{P(8)^3}{48\ EI} = \frac{512P}{48 \times 30{,}000 \times .159}$$

$$= 0.00224P$$

Elongation of anchor bolt

$$= \frac{16\ P}{30{,}000 \times .44} = 0.00121P$$

$\therefore$ Total movement at bolt = (.00224 + .00121)P = 0.00345P

$\therefore$ Rotation = $\frac{0.00345P}{5.75}$ = 0.00060P Radians.

Moment = 5.75P K-ins.

$$\therefore\ \gamma = \frac{\theta}{M} = \frac{0.00060}{5.75} = 0.000104 = 104 \times 10^{-6} \text{ Radians/ K-ins.}$$

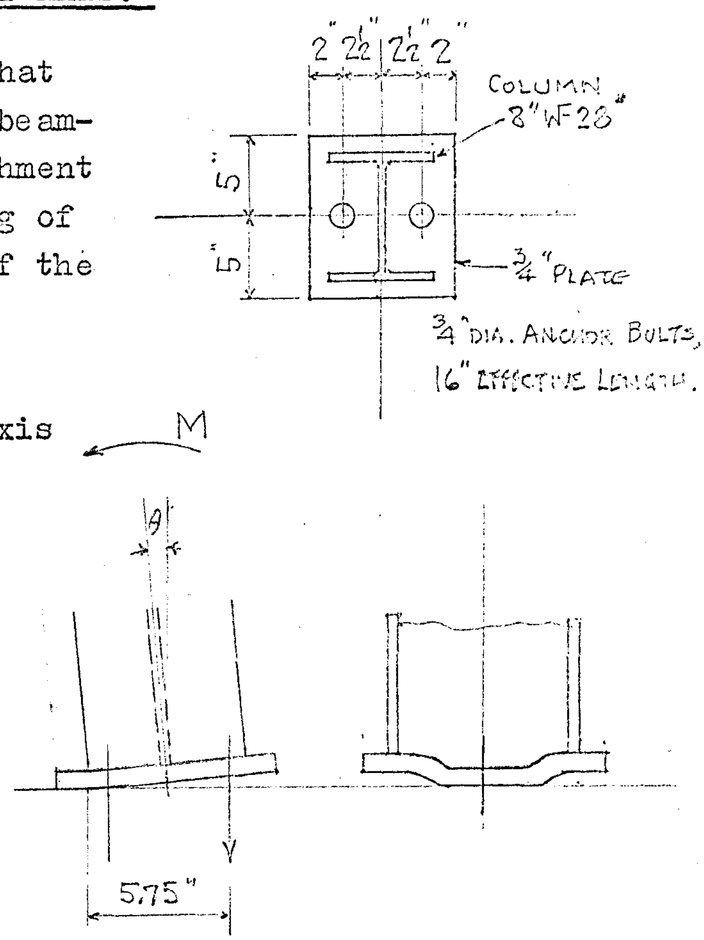


FIG. 56.

Analysis of Moments in Conventional Designs.

In order to determine the magnitude of the moments which may occur at the joints of steel structures designed according to the conventional method, in which such moments are assumed to be zero, a number of examples were selected from structures of this type which have been erected or are in course of erection.

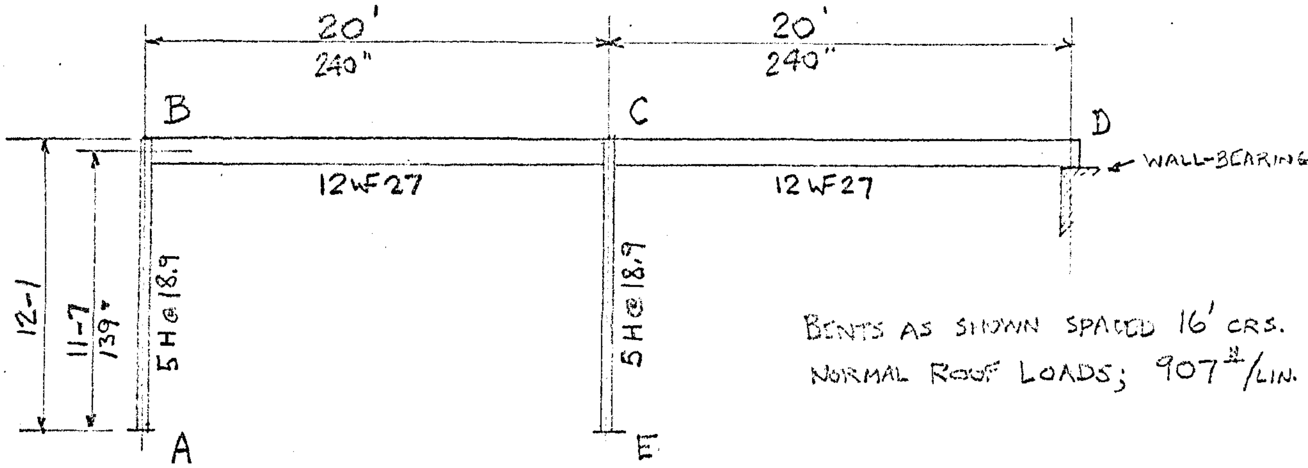
In structures designed on this "simple span" basis, there is actually no provision made in the design or detailing of the members and connections, for resistance to lateral forces such as wind loads. Lateral forces are supposed to be resisted by masonry construction or some other means, rather than by the steel frame. Since the intention here is to find what the condition is during the life of the structure, lateral forces are not included in the calculations, which are devoted to a study of the effects of the estimated vertical loading.

It must be admitted that an exact analysis of all the stresses in even a simple steel structure cannot be produced on a theoretical basis. For such a purpose, it would be necessary to obtain precise measurements of every piece of steel used, and also to know the yield point and elastic modulus pertaining to every part of the structure. At the same time, some form of approximate analysis must be made, if a true picture of the stress distribution is to be made available to the designer and to those who formulate the rules for design. In other words, the best practical use has to be made of the data on hand. The situation has been described by Professor Freudenthal in a paper already cited (10): "Much engineering knowledge is still descriptive, although its presentation is mathematical. ... The development toward an exact science by using mathematical abstractions and mathematical language, however, leads occasionally to the ascendancy (or domination) of the mathematical form over the real physical content - to an overvaluation of the mathematical exactness inherent in an expression of physical reality and, consequently, to a serious distortion of the perspective. To avoid the pitfalls of the mathematical approach to engineering problems, it is essential to realize and to check on its limitations."

The conventional method of design, even as regulated by the appropriate specifications, is not always consistent in the assumption that the ends of beams are hinged. Provision is made in CSA S16 (clause 12.4.2) and in the AISC specification (Section 1.5.1.4) for the design of fully-continuous and fixed-ended beams and girders, and in the actual design of some building frames there are properties implicit in the structural members selected and in the details prescribed, which are not explicitly set forth in the stress sheets. An example may be found in the portion of the frame from "Structure 790". In this bent, the column base is so embedded that it may be regarded as rigid, for reasons connected with the wind loading on the building. The truss, as detailed, is capable of developing considerable moments at the ends, particularly at the end attached to the column with the rigid base. Yet the stresses which are assigned to the members of this truss, on the design drawing, are simply those which would be caused by the vertical loads. Under these stresses, the truss members are considerably below their load-carrying capacity. This apparent over-designing of structural members will be discussed later.

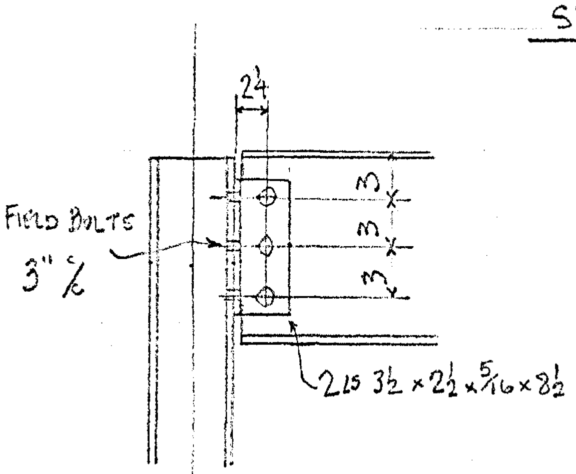
The analysis of typical structures which will be found in the following pages has been carried out on the basis of a treatment described in "The Analysis of Engineering Structures", by Pippard and Baker (1957). Generally, the intent has been to find the moments generated by the semi-rigid joints, or by the truss connections. The moment on columns due to the application of the beam reactions at some distance from the axes of the columns, is shown as a separate item where it has been introduced. This course has been adopted because the latter type of moment is here regarded as a primary rather than a secondary one, despite the fact that some designers fail to include it in their calculations.

Structures "42" and "90" represent a wide class of one-storey industrial buildings. As might be expected, the moments occurring at the tops of the interior columns are almost balanced, the only important column moment being the one at the exterior columns. A preliminary analysis was made, assuming the interior columns to remain vertical; it was found that this assumption does not always give results with the required degree of accuracy.



STRUCTURE "42"

BENTS AS SHOWN SPACED 16' CRS.
NORMAL ROOF LOADS; 907#/LIN. FOOT



BEAM CONNECTIONS ARE MODIFIED FORM OF SPECIMEN 4 SERIES A IN RATHBUN EXPERIMENT.

ASSUME $\gamma = \frac{.0035}{100} = 35 \times 10^{-6}$ RAD./K-IN.

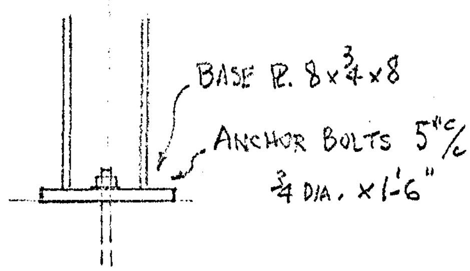
BASE DETAIL, $\gamma = 107.8 \times 10^{-6}$

COLUMN: $I = 23.8$ $S = 9.5$ $K = \frac{23.8}{139} = 0.171$

$2EK = 10,260$

BEAMS; $I = 204.1$ $S = 34.1$ $K = \frac{204.1}{240} = 0.852$

$2EK = 51,120$



SYMBOLS USED: θ = ANGULAR DEFLECTION OF COL.
ϕ = " " OF BEAM
COUNTERCLOCKWISE ROTATION +
MOMENT ACTING ON END OF MEMBER, + COUNTERCL.

AT COLUMN BASES; $\theta_A = -107.8 \times 10^{-6} M_{AB}$ $\quad \theta_E = -107.8 \times 10^{-6} M_{EC}$

AT BEAM CONNECTIONS; $(\theta_B - \phi_{BC}) = 35 \times 10^{-6} M_{BC}$

$$(\theta_C - \phi_{CB}) = 35 \times 10^{-6} M_{CB}$$

$$(\theta_C - \phi_{CD}) = 35 \times 10^{-6} M_{CD}$$

TOTAL DISTRIB. LOAD PER SPAN $= 907 \times 20 = 18.14^{k}$

$$F.E.M. = \frac{18.14 \times 240}{12} = 362.8 \text{ K-IN}$$

APPLYING SLOPE-DEFLECTION METHOD:

$M_{BC} = 51{,}120(2\phi_{BC} + \phi_{CB}) + 362.8$ ①

$M_{CB} = 51{,}120(\phi_{BC} + 2\phi_{CB}) - 362.8$ ②

$M_{CD} = 51{,}120(2\phi_{CD} + \phi_{DC}) + 362.8$ ③

$M_{DC} = 51{,}120(\phi_{CD} + 2\phi_{DC}) - 362.8$ ④

$M_{BA} = 10{,}260(2\theta_B + \theta_A)$ ⑤

$M_{AB} = 10{,}260(\theta_B + 2\theta_A)$ ⑥

$M_{CE} = 10{,}260(2\theta_C + \theta_E)$ ⑦

$M_{EC} = 10{,}260(\theta_C + 2\theta_E)$ ⑧

④ $M_{DC} = 0, \therefore 51{,}120(\phi_{CD} + 2\phi_{DC}) = 362.8$

$\therefore 51{,}120\,\phi_{DC} = 181.4 - \frac{51{,}120}{2}\phi_{CD}$

$\therefore 51{,}120(2\phi_{CD} + \phi_{DC}) = 181.4 + 51{,}120\phi_{CD}\left(2 - \frac{1}{2}\right) = 181.4 + 51{,}120\left(\frac{3\phi_{CD}}{2}\right)$

SUBSTITUTING IN ③

$M_{CD} = 181.4 + 51{,}120\left(\frac{3\phi_{CD}}{2}\right) + 362.8$

$= 76{,}680\,\phi_{CD} + 544.2$

ETC. ETC.

SOLUTION OF ABOVE EQUATIONS, INTRODUCING THE MOMENT-ANGLE RELATIONSHIPS ON THE PRECEDING PAGE, GIVES

$M_{BC} = +57.7$ K-IN. $M_{CB} = -142.7$ K-IN. $M_{CD} = +145.0$ K-IN.

$M_{BA} = -57.7$ $M_{CE} = -2.3$

$M_{AB} = -10.9$ $M_{EC} = -0.4$

NOTE THAT ABOVE SYSTEM DOES NOT INCLUDE THE MOMENTS IN THE COLUMNS DUE TO APPLICATION OF SHEAR REACTIONS TO COLUMN FLANGES. THIS WILL AFFECT OUTER COLUMN ONLY, THUS:

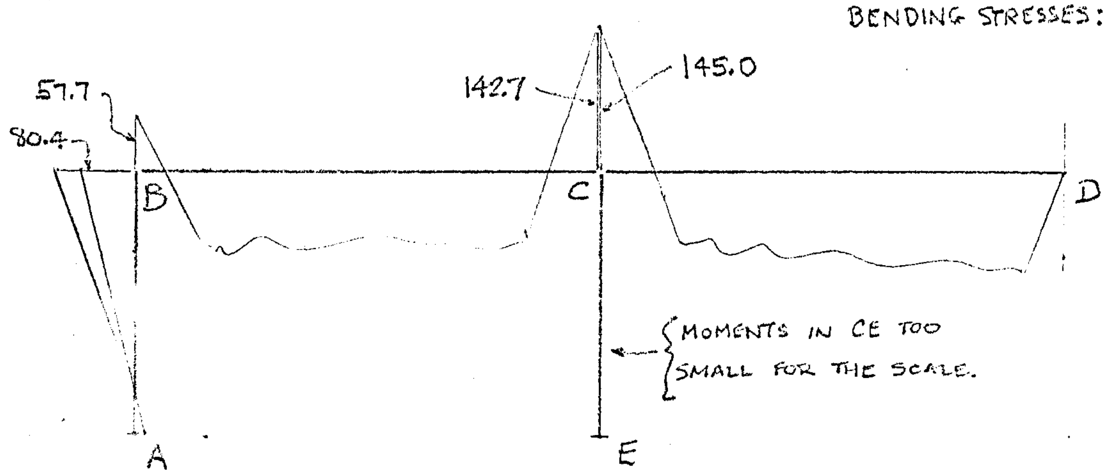
RESULTANT $M_{BA} = -57.7 - (9.07 \times 2.5) = -57.7 - 22.7 = -80.4$ K-IN.

BENDING STRESSES: $\frac{57.7}{9.5} = 6.07\ ^{K}/_{IN^2}$

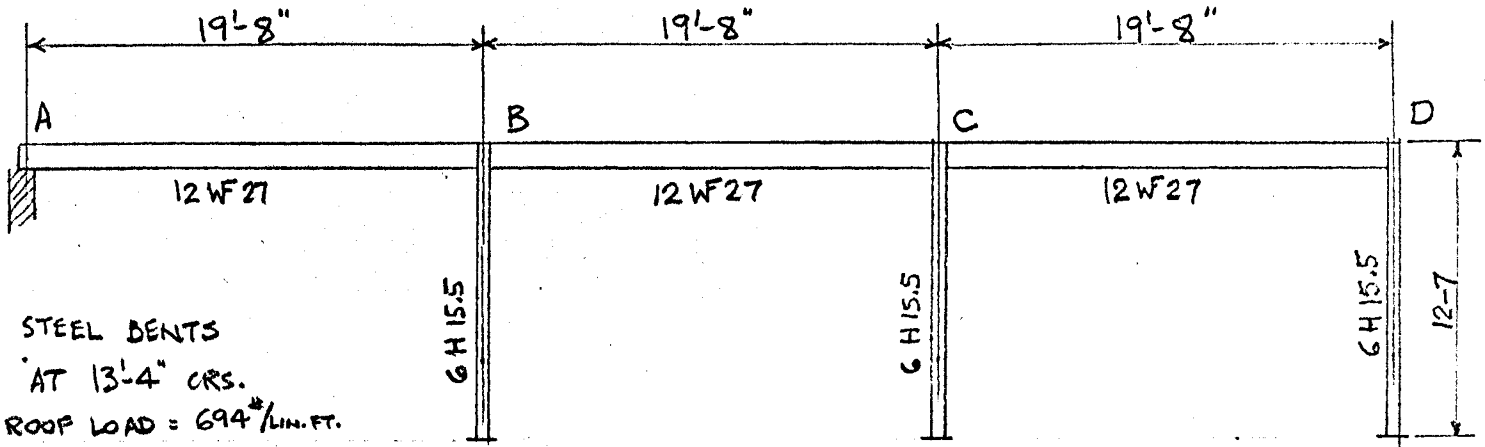
$\frac{80.4}{9.5} = 8.47\ ^{K}/_{IN^2}$

$\frac{11.8}{9.5} = 1.24\ ^{K}/_{IN^2}$

$\frac{145.0}{34.1} = 4.25\ ^{K}/_{IN^2}$



SECONDARY BENDING IN STRUCT. "42"



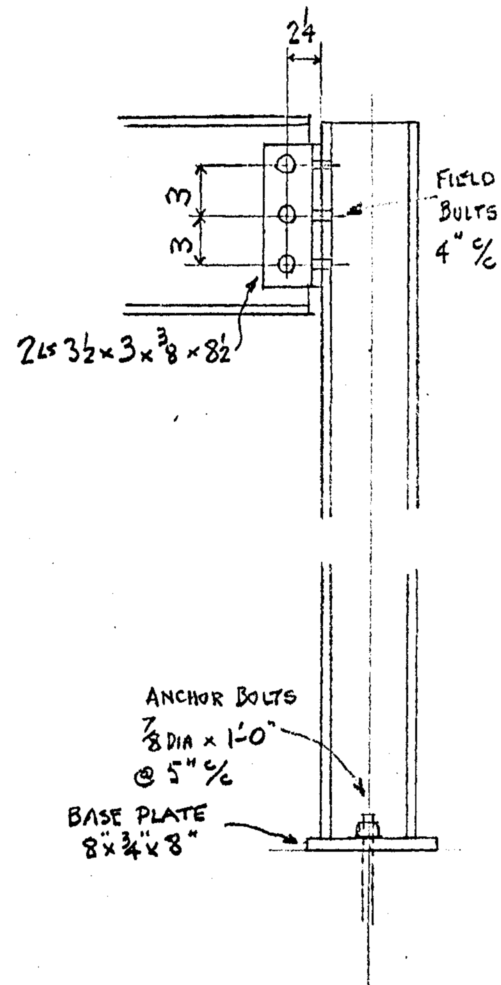
STRUCTURE "90"

FOR BEAM CONNECTIONS, $\gamma = \frac{.005}{100} = 50 \times 10^{-6}$

FOR COLUMN BASES, $\gamma = 42 \times 10^{-6}$

	I	S	L
BEAMS,	204.1	34.1	230"
COLUMNS,	30.3	10.1	145"

F.E.M. = 258 K-INS.



EQUATIONS:

$$(\theta_{BE} - \phi_{BA}) = 50 \times 10^{-6} M_{BA} \qquad \theta_{EB} = -42 \times 10^{-6} M_{EB}$$
$$(\theta_{BE} - \phi_{BC}) = 50 \times 10^{-6} M_{BC}$$
$$(\theta_{CF} - \phi_{CB}) = 50 \times 10^{-6} M_{CB} \qquad \theta_{FC} = -42 \times 10^{-6} M_{FC}$$
$$(\theta_{CF} - \phi_{CD}) = 50 \times 10^{-6} M_{CD}$$
$$(\theta_{DG} - \phi_{DC}) = 50 \times 10^{-6} M_{CD} \qquad \theta_{GD} = -42 \times 10^{-6} M_{GD}$$

$$M_{AB} = 53{,}200 (2\phi_{AB} + \phi_{BA}) + 258$$
$$M_{BA} = 53{,}200 (\phi_{AB} + 2\phi_{BA}) - 258$$
$$M_{BC} = 53{,}200 (2\phi_{BC} + \phi_{CB}) + 258$$
$$M_{CB} = 53{,}200 (\phi_{BC} + 2\phi_{CB}) - 258$$
$$M_{CD} = 53{,}200 (2\phi_{CD} + \phi_{DC}) + 258$$
$$M_{DC} = 53{,}200 (\phi_{CD} + 2\phi_{DC}) - 258 \qquad M_A = 0$$

$$M_{BE} = 12{,}600 (2\theta_{BE} + \theta_{EB}) \qquad M_{BA} + M_{BC} + M_{BE} = 0$$
$$M_{EB} = 12{,}600 (\theta_{BE} + 2\theta_{EB})$$
$$M_{CF} = 12{,}600 (2\theta_{CF} + \theta_{FC}) \qquad M_{CB} + M_{CD} + M_{CF} = 0$$
$$M_{FC} = 12{,}600 (\theta_{CF} + 2\theta_{FC})$$
$$M_{DG} = 12{,}600 (2\theta_{DG} + \theta_{GD}) \qquad M_{DC} + M_{DG} = 0$$
$$M_{GD} = 12{,}600 (\theta_{DG} + 2\theta_{GD})$$

SOLUTION OF THE SYSTEM OF EQUATIONS GIVES:

$M_{AB} = 0$	$M_{BE} = 2.9$ K-INS
$M_{BA} = -75.4$ K-INS.	$M_{EB} = 0.8$
$M_{BC} = 72.5$	$M_{CF} = -1.3$
$M_{CB} = -71.2$	$M_{FC} = -0.4$
$M_{CD} = 72.5$	$M_{DG} = 40.7$
$D_{DC} = -40.7$	$M_{GD} = 11.3$

SHEAR AT D = 6.55^K;

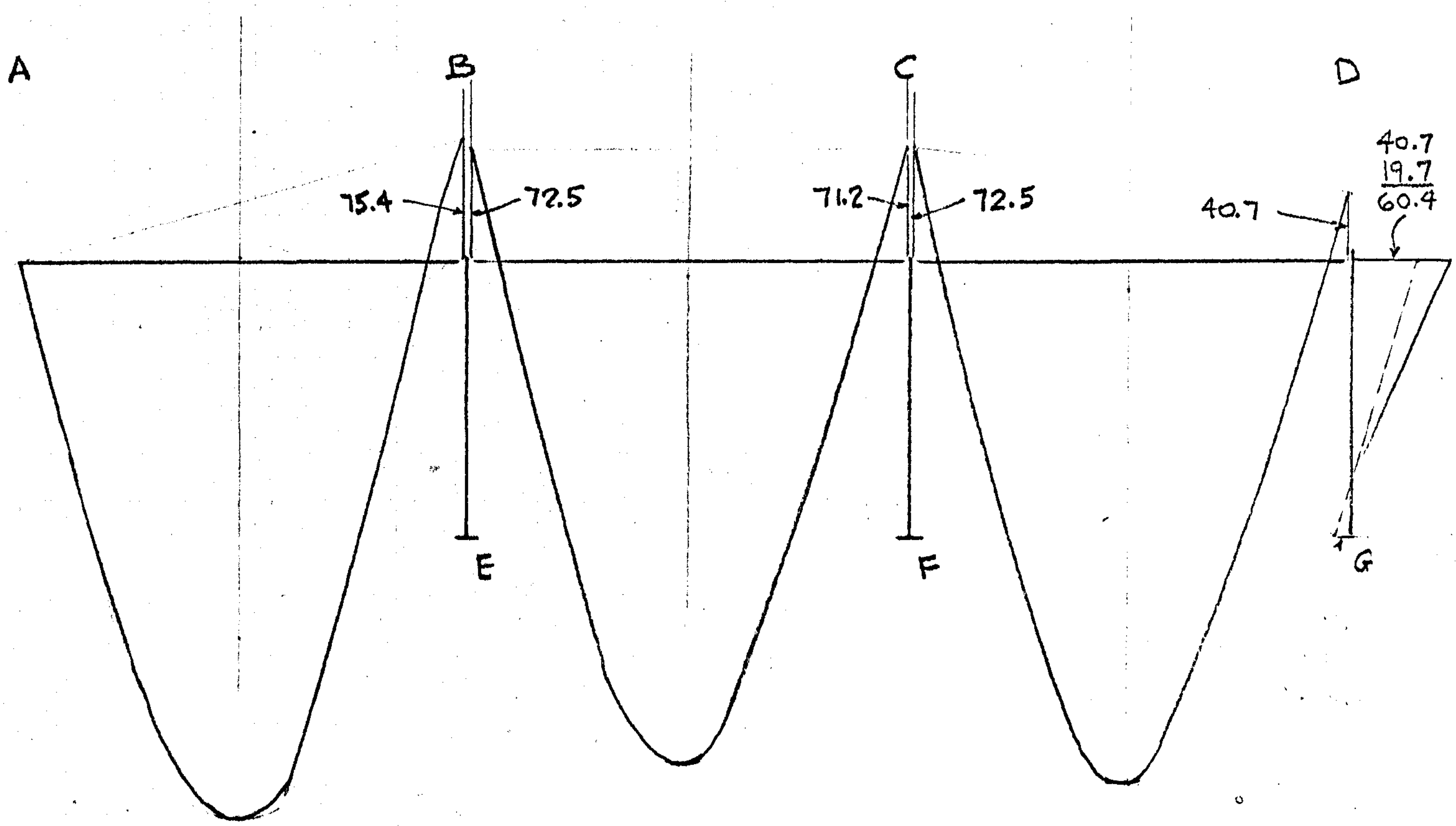
REACTION MOM. AT D = $6.55 \times 3 = 19.7$ K-IN.

SIGN CONVENTION: + INDICATES A COUNTERCLOCKWISE MOMENT APPLIED TO A MEMBER, M_{AB} BEING APPLIED TO MEMBER "AB" AT THE END "A".

ϕ_{AB} = ROTATION OF END A OF MEMBER "AB";

θ_{EB} = ROTATION OF END E OF COLUMN "EB";

COUNTERCLOCKWISE ROTATIONS ARE +



SECONDARY BENDING IN STRUCTURE "90"

BENDING STRESSES: $\frac{60.4}{10.1} = 6.0\ ^K/_{IN^2}$ $\quad \frac{40.7}{10.1} = 4.0\ ^K/_{IN^2}$ IN COL. DG.

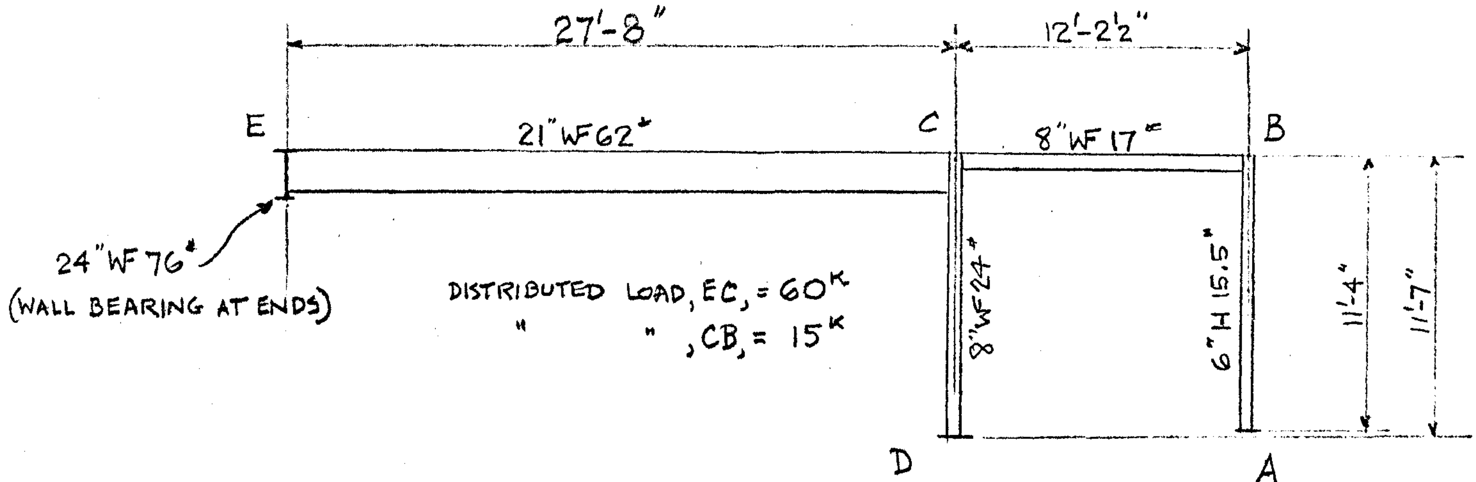
$\frac{75.4}{34.1} = 2.2\ ^K/_{IN^2}$ MAX. IN BEAMS AT SUPPORTS.

In structures "42" and "90", the maximum bending stresses at the tops of the columns, due to the stiffness of the connections alone, are 6 ksi and 4 ksi respectively. The addition of the reactions moments would increase these figures to about 8.5 ksi and 6 ksi respectively.

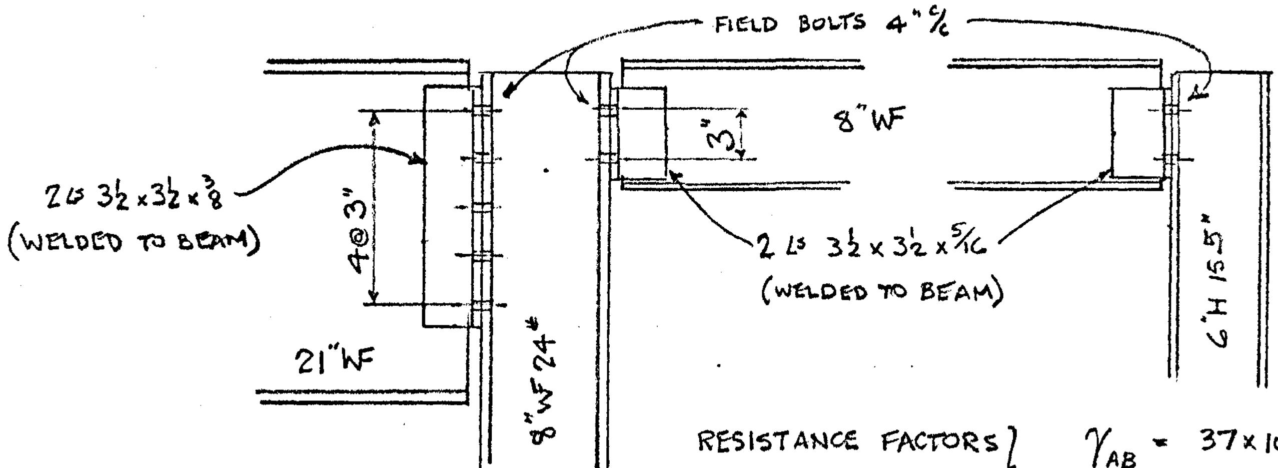
Structure "48" is unsymmetrical, so that the interior column is not even approximately balanced as regards the moments applied to it. The resultant moment is over 220 k-i, causing a bending stress at the top of the column of about 10.5 ksi. This is one of the most severe concentrations found in the series of structures investigated. When the reaction moment is added, a stress of almost 15 ksi is attained. Even more severe stresses could be produced in this column if the span EC were loaded and the span CB not loaded, but in a flat roof this extreme condition is not expected.

In buildings of more than one storey, it becomes more important to provide in the steel frame, some resistance to wind loads. Yet it is possible to find instances in which such frames appear to have been designed on the "simple-span" basis. Structure "96" is an example of this, where the masonry is evidently expected to provide all necessary lateral resistance. Local bending stresses in the columns due to connection stiffness are a little under 8 ksi, and the addition of the reaction moments increase this to about 9 ksi.

The connections in structures "42", "90" and "48" are standard connection angles, rivetted or welded to the beams, and attached to the columns by means of high strength bolts. In structure "99" the connections consist of plates welded to the columns, with the webs of the beams field bolted to these plates. The method of calculating the moments developed by this type of connection has been presented in figure 53. Structure "99" also differs in having different loading in the exterior bay. The maximum secondary bending stress, at the top of the exterior column, is 7.75 ksi, which is increased to 13.72 ksi on the addition of the reaction moment.



STRUCTURE "48"



PROPERTIES OF SECTIONS

	I	S
21WF62	1326.8	126.4
8WF17	56.4	14.1
8WF24	82.5	20.8
6H 15.5	30.3	10.1

RESISTANCE FACTORS OF CONNECTIONS

$\gamma_{AB} = 37 \times 10^{-6}$ RAD./K-IN.

$\gamma_{DC} = 29 \times 10^{-6}$ DO.

$\gamma_{BC} = \gamma_{CB} = 50 \times 10^{-6}$ DO.

$\gamma_{CE} = 6.67 \times 10^{-6}$ DO.

$\gamma_{EC} = \infty$ {TORSIONAL RESISTANCE OF WALL-BEARING BEAM NEGLIGIBLE.

CALCULATED SECONDARY BENDING: (K-INS.)

$M_{CE} = -350.1$ $\quad M_{CB} = 129.5$ $\quad M_{BC} = -45.5$

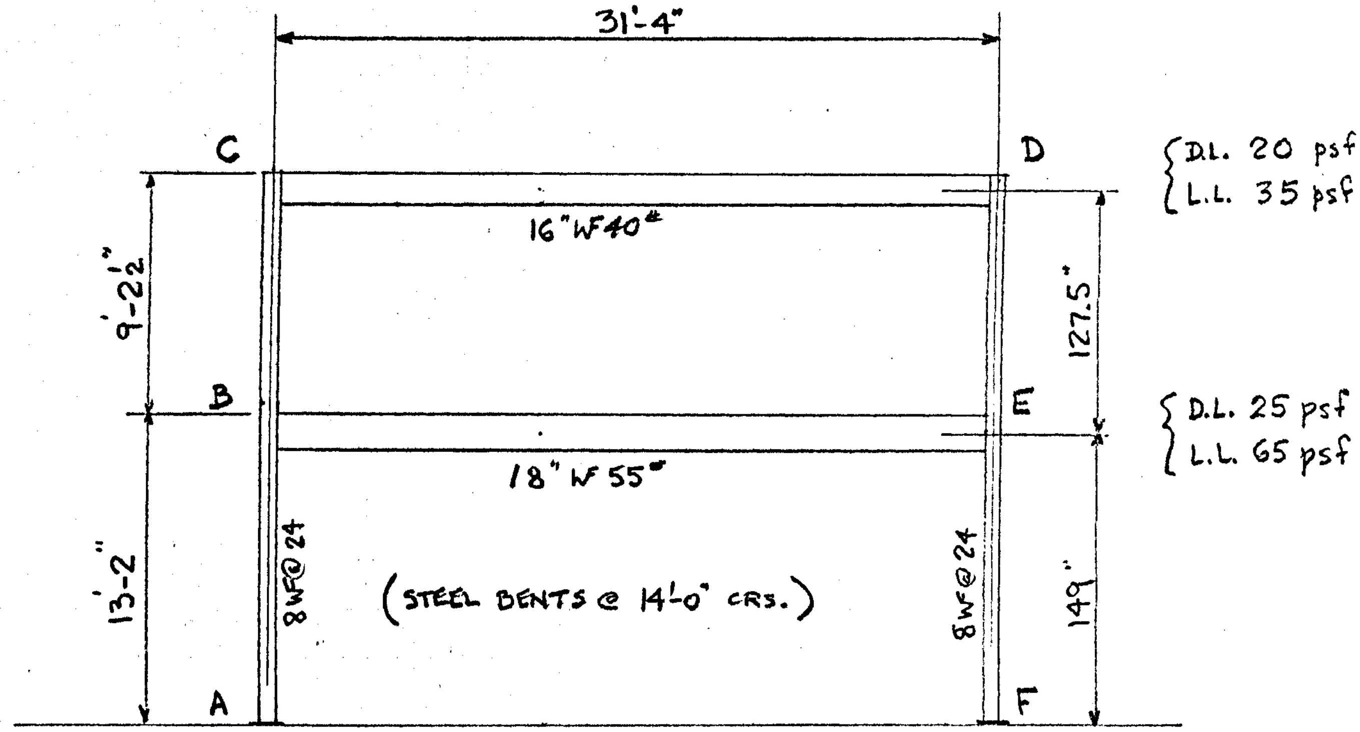
$M_{CD} = 220.6$ $\quad M_{BA} = 45.5$

$M_{DC} = 41.3$ $\quad M_{AB} = 12.9$

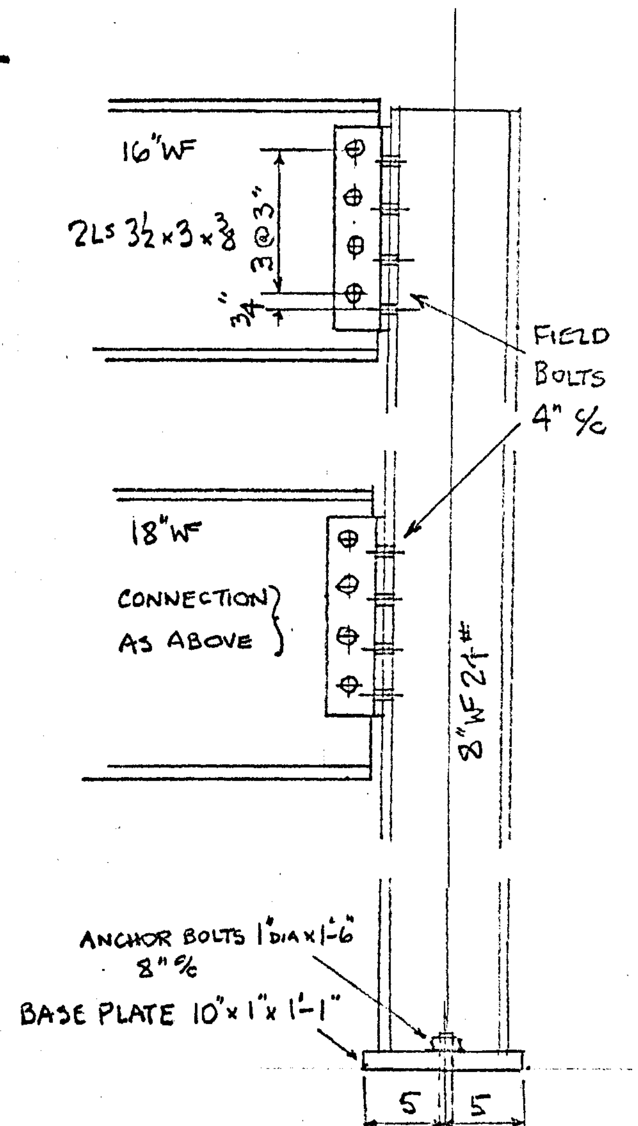
BENDING STRESSES AT TOPS OF COLUMNS:

COL. CD; $220.6/20.8 = 10.6$ K/IN² (SECONDARY)
$(30-7.5)\times 4 = 90$ K-IN; $90/20.8 = 4.3$ K/IN² (REACTION) } 14.9 K/IN² (RESULTANT)

COL. BA: $45.5/10.1 = 4.5$ K/IN² (SECONDARY)
$7.5 \times 3 = 22.5$ K-IN; $22.5/10.1 = 2.2$ K/IN² (REACTION) } 6.7 K/IN² (RESULTANT)



STRUCTURE "96"



ESTIMATED LOADS.

ROOF:	MAX. DISTRIBUTED LOAD ON BM.	= 24.1K
	MIN. " " "	= 8.8
FLOOR:	MAX. " " "	= 39.4
	MIN. " " "	= 11.0

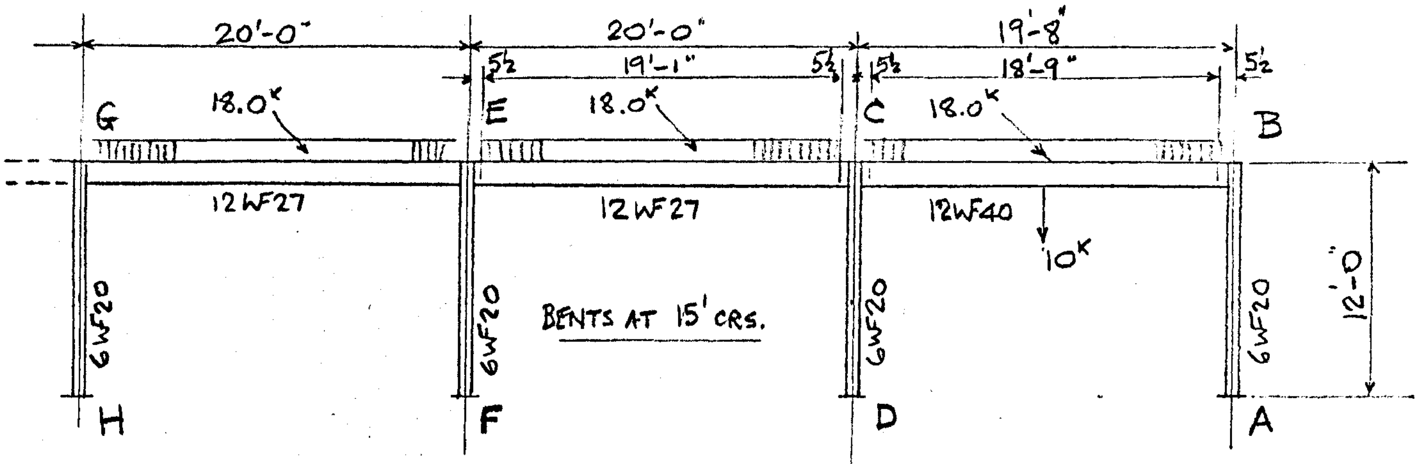
PROPERTIES OF CONNECTIONS:

COL. BASES, $\gamma = 29.14 \times 10^{-6}$

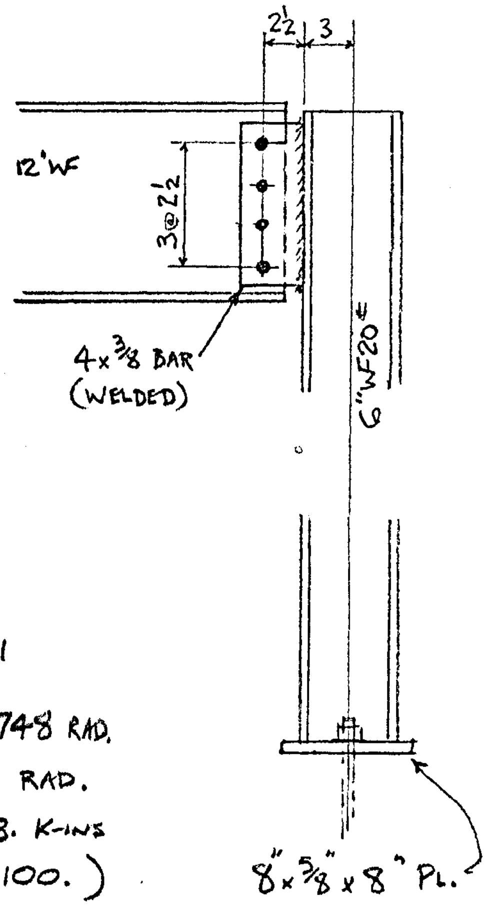
BEAM CONNS., $\gamma = 18 \times 10^{-6}$

LOADING CONDITION ANALYSED, MINIMUM ROOF LOAD AND MAXIMUM FLOOR LOAD:

		REACTION MOMENT.	RESULTANT MOMENT	f_b
M_{CD}	= + 100.3			
M_{CB}	= − 100.3	− 17.6	= −117.9	
M_{BC}	= − 162.7	+ 8.8	= −153.9	7.4 K/in²
M_{BE}	= + 270.0			
M_{BA}	= − 107.3	− 78.8	= −186.1	9.0 K/in²
M_{AB}	= − 21.9	+ 39.9	= − 61.8	



STRUCTURE "99"



THE TYPE OF BEAM CONNECTION USED IN THIS STRUCTURE HAS BEEN ANALYZED ON p.

SLOPE AT END OF SIMPLY-SUPPORTED BEAM

$= \dfrac{WL^2}{24EI}$ FOR DISTRIBUTED LOAD = W

$= \dfrac{WL^2}{16EI}$ FOR CONCENTRATED LOAD W AT MID-SPAN.

FOR SPANS GE, EC, THIS GIVES:

$$\theta = \frac{18.0 \times 229^2}{24 \times 30000 \times 204.1} = .00642 \text{ RAD.}$$

FOR SPAN CB, $\theta = \dfrac{18.0 \times 225^2}{24 \times 30000 \times 310.1} + \dfrac{10 \times 225^2}{16 \times 30000 \times 310.1}$

$$= .00408 + .00340 = .00748 \text{ RAD.}$$

PLAY IN HOLES PERMITS ROTATION $= \dfrac{.0625}{3.75} = .01667$ RAD.

SLIP RESISTANCE OF TIGHTENED H.S. BOLTS = 108. K-INS (SAY 100.)

F.E.M. FOR SPANS GE, EC: $= \dfrac{18.0 \times 229}{12} = 344$ K-IN.

F.E.M. FOR SPAN CB. $= \dfrac{18.0 \times 225}{12} + \dfrac{10 \times 225}{8} = 619$ K-IN.

∴ IN ALL SPANS, THE MOMENTS AT THE CONNECTIONS EXCEEDS THE SLIP-RESISTANCE OF THE BOLTS, AND THE MAXIMUM MOMENT DEVELOPED WILL IN EACH CASE BE DETERMINED BY THE LATTER.

THE ACTUAL MOMENT APPLIED AT THE TOP OF A COLUMN WILL BE THE SUM OF THE SLIP-RESISTANCE MOMENT, PLUS THE MOMENT DUE TO THE APPLICATION OF THE BEAM REACTIONS AT $5\frac{1}{2}$" FROM THE COLUMN C.L.

17

THESE CONSIDERATIONS LEAD TO THE FOLLOWING CONDITION:

$M_{BC} = -100. - (14 \times 5.5) = -177.$ K-IN. $\qquad M_{BA} = +177.$

$M_{CB} = +100. + (14 \times 5.5) = +177$

$M_{CE} = -100 - (9 \times 5.5) = -149.5$

$\left.\right\} \therefore M_{CD} = -27.5$

$M_{EC} = +149.5$

$M_{EG} = -149.5$

$\left.\right\} \therefore M_{EF} = 0$

RIGIDITY OF BASE :– $\gamma = 90 \times 10^{-6}$

	I	S	L	K
6WF20:	38.8	12.9	138	.281

$\therefore 2EK = 16{,}860.$

$\theta_D = -90 \times 10^{-6} M_{DC}$

$\theta_A = -90 \times 10^{-6} M_{AB}$

$M_{AB} = 16{,}860(2\theta_A + \theta_B)$

$M_{BA} = 16{,}860(\theta_A + 2\theta_B)$

$M_{CD} = 16{,}860(2\theta_C + \theta_D)$

$M_{DC} = 16{,}860(\theta_C + 2\theta_D)$

ALSO, $M_{BA} = +177.$

$M_{CD} = -27.5$

SOLUTION OF THESE EQUATIONS GIVES:

$M_{AB} = +27.0$ K-INS.

$M_{DC} = -4.2$ K-INS.

BENDING STRESSES IN COLUMNS:

AT B, $\frac{177.}{12.9} = 13.72$ K/IN.² / AT A, $\frac{27.}{12.9} = 2.09$ K/IN².

AT C, $\frac{27.5}{12.9} = 2.13$ K/IN². / AT D, $\frac{4.2}{12.9} = 0.38$ K/IN².

ABOVE STRESSES ARE DUE TO REACTION MOMENT PLUS ACTUAL SECONDARY BENDING.

SECONDARY BENDING STRESSES ONLY: $\left\{\begin{array}{l} M_{BA} = 100 \text{ K-IN} \\ M_{CD} = 0 \end{array}\right.$ $\quad f_b = 7.75$ K/IN²

Roof trusses are a common feature of industrial buildings, and are liable to secondary stresses from several causes. The clauses quoted from such specifications as B.S. 153 have already indicated some of these causes. When the truss is considered as a flexural member simply supported at the ends, the secondary stresses in it will be those due to the geometry changes arising from the elastic deformation of its members, and those due to eccentricity in the connections. When the truss is considered as part of a structure, moments may be developed at its connections to the supports, and these moments, not included in the design calculations, will affect the stresses in the truss members.

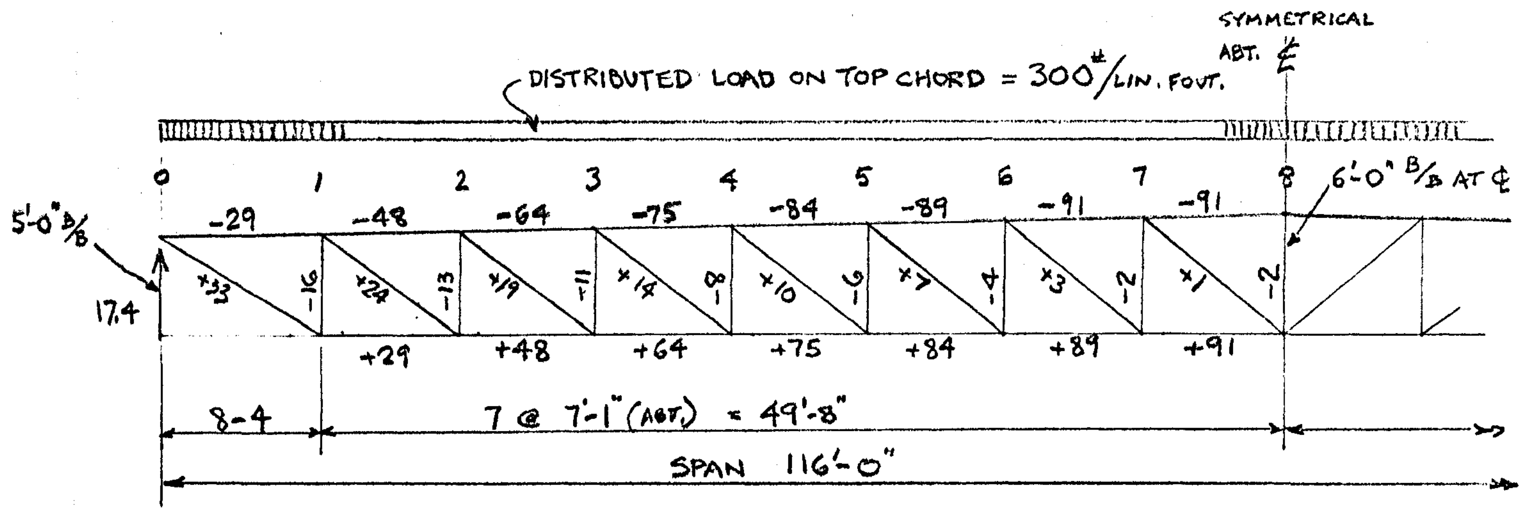
The subject of secondary stresses in trusses has been set forth in a very convenient manner in a bulletin published in France (17), to which reference has already been made. Using the moment-distribution method described here and elsewhere, several trusses have been treated as simply-supported structures, and the secondary stresses calculated.

The truss used in structure "76" is actually a "long-span" joist. It differs from many trusses in that the load is not delivered at specific points by purlins, but is distributed along the top chord. The nature of the supports is such that no end moments of any consequence could be developed. The truss was assembled by welding, with gravity axes of the members concurrent at the joints, rendering moments due to eccentricity non-existent. The secondary stresses in this truss are low, and in most cases the higher stresses are of opposite sign to the primary stress in the member, so that they cancel out. The bending in the top chord due to the distributed load, is treated as a primary stress in the design.

Trusses selected from structures "75" and "49" are of rivetted construction. According to the general practice, the geometrical outline of these trusses was made to depend on the conventional gauge lines in the members, so that the gauge lines intersected at a common point at any joint. Since the gauge line in an angle section rarely coincides with the position of the gravity axis, and may be almost an inch away from it, this practice produces eccentricity in most of the joints.

SECONDARY STRESSES IN TRUSSES.

STRUCTURE "76" TRUSS A1 — WELDED TRUSS ACTING AS "LONG SPAN JOIST."



DETAILS ARE SO ARRANGED THAT GRAVITY AXES OF CONNECTING MEMBERS ARE CONCURRENT AT PANEL POINTS.

FORCES IN MEMBERS ARE TAKEN FROM TRUSS DESIGN.

MEMBER	SECTION	MEMBER	SECTION	MEMBER	SECTION
U_0U_1	S.T. 8"@18	L_1L_2	S.T. 6"@ 13.5	U_0L_1	$2 ╥ 3 \times 2 \times \frac{1}{4}$
U_1U_2	"	L_2L_3	"	U_1L_2	$2 ╥ 2\frac{1}{2} \times 2 \times \frac{3}{16}$
U_2U_3	"	L_3L_4	"	U_2L_3	$2 ╥ 2 \times 2 \times \frac{3}{16}$
U_3U_4	"	L_4L_5	S.T. 8" @ 18	U_3L_4	$2 ╥ 1\frac{3}{4} \times 1\frac{3}{4} \times \frac{3}{16}$
U_4U_5	S.T. 8"@ 20	L_5L_6	"	U_4L_5	"
U_5U_6	"	L_6L_7	"	U_5L_6	"
U_6U_7	"	L_7L_8	"	U_6L_7	"
U_7U_8	"	U_4L_4	$2 ╥ 2\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{16}$	U_7L_8	"
U_1L_1	$2 ╥ 3 \times 2\frac{1}{2} \times \frac{5}{16}$	U_5L_5	$2 ╥ 2 \times 2 \times \frac{3}{16}$		
U_2L_2	$2 ╥ 2\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{16}$	U_6L_6	$2 ╥ 2\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{16}$ *		
U_3L_3	"	U_7L_7	$2 ╥ 2 \times 2 \times \frac{3}{16}$		
		U_8L_8	$2 ╥ 2\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{16}$ *		

* MEMBER SIZES INCREASED TO PROVIDE CONNECTIONS FOR BRACING.

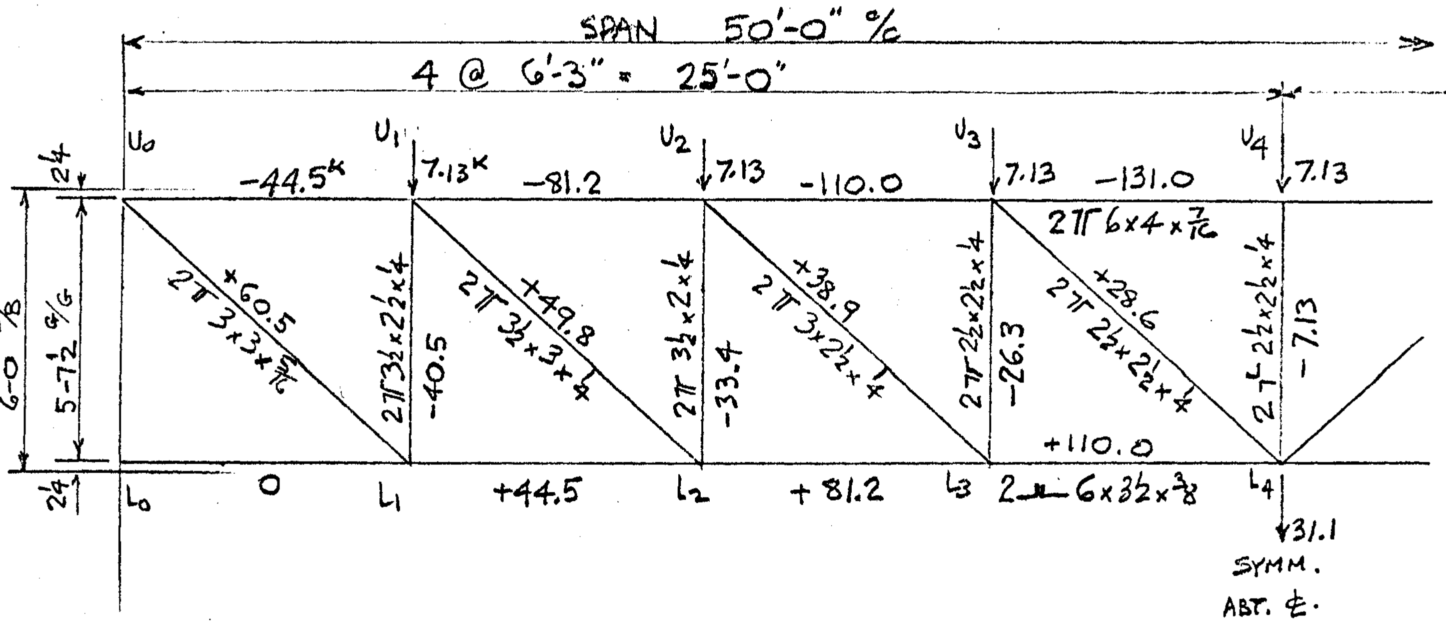
SECONDARY STRESSES DUE TO ECCENTRICITIES AT THE JOINTS WILL NOT OCCUR IN THIS TRUSS. ELASTIC DEFORMATION OF MEMBERS WILL CAUSE SOME BENDING STRESSES. BENDING IN THE TOP CHORD DUE TO DISTRIBUTED LOAD WAS TREATED AS A PRIMARY STRESS IN THE DESIGN.

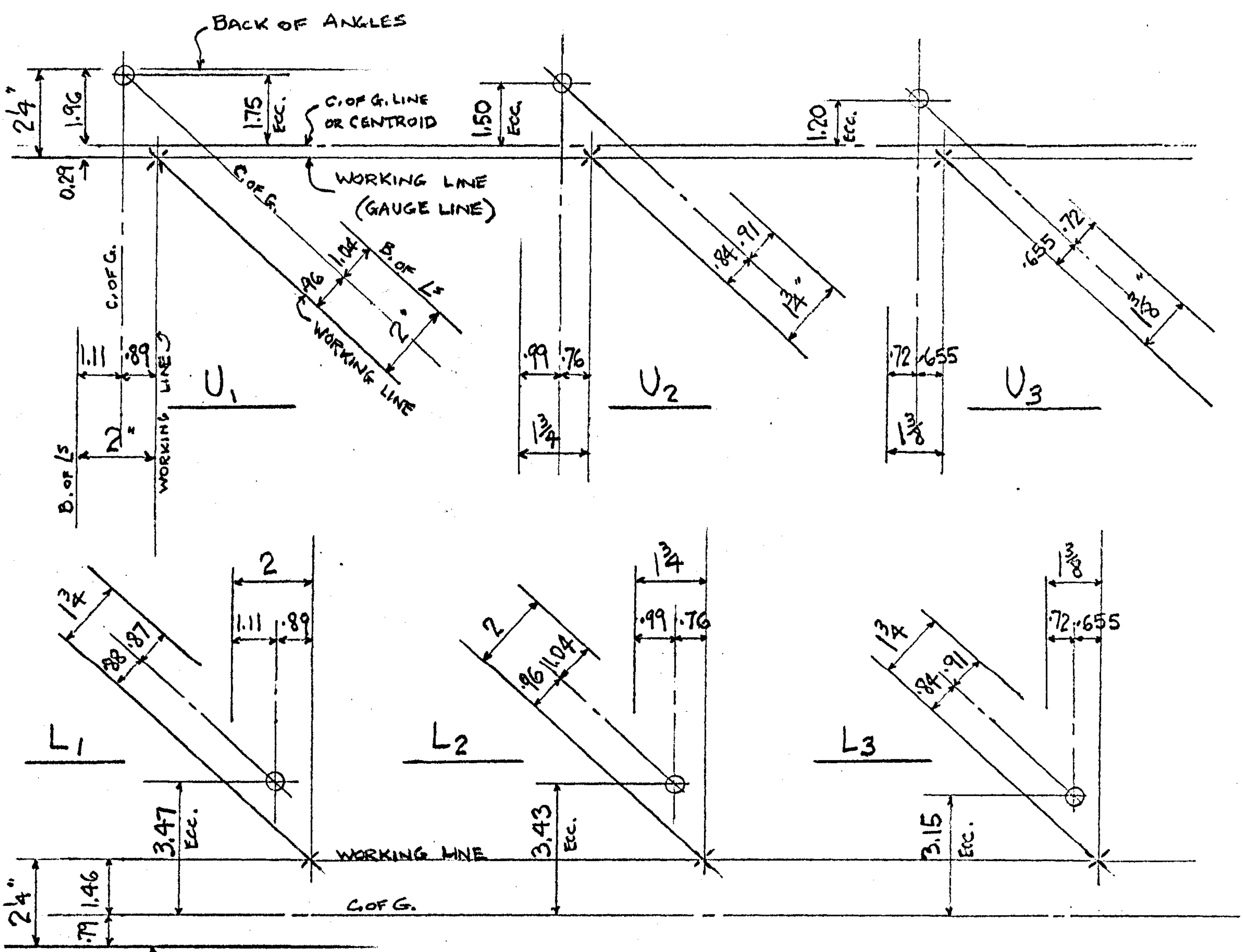
The method used for the calculation of secondary bending due to truss deformation, is that based on the principles developed by Cross, described in detail in several texts (17). The procedure may be summarized as follows:

1. Increase or decrease of length of each truss member under the given loading is calculated.
2. A Williot diagram is drawn, showing the relative displacement of all joints in the truss.
3. From the diagram, the relative displacement of the points at the ends of each member, measured at right angles to the member, are measured.
4. From data on the stiffness of each member, and the relative displacement of the ends, the bending moments at the ends are found.
5. The unbalanced moments at the joints are distributed through the truss, by means of the cross method.

LOCATION OF MOMENT	MOMENT (K-INS)	BENDING STRESS (K/IN^2)		LOCATION OF MOMENT	MOMENT (K-INS)	BENDING STRESS (K/IN^2)		LOCATION OF MOMENT	MOMENT (K-INS)	BENDING STRESS (K/IN^2)	
		HEEL	TOE			HEEL	TOE			HEEL	TOE
U_0U_1	+1	+.06	-.19	L_1L_2	+4	-.43	+1.67	U_0L_1	-1	-.45	+.91
U_1U_0	+4	-.25	+.78	L_2L_1	+5	+.53	-2.09	L_1U_0	+1	-.45	+.91
U_1U_2	+1	+.06	-.19	L_2L_3	-5	+.53	-2.09	U_1L_2	+1	+.76	-1.73
U_2U_1	+10	-.62	+1.96	L_3L_2	+3	+.32	-1.26	L_2U_1	+1	-.76	+1.73
U_2U_3	-10	-.62	+1.96	L_3L_4	-4	+.43	-1.67	U_2L_3	+1	+1.05	-2.63
U_3U_2	+16	-.99	+3.15	L_4L_3	+7	+.74	-2.92	L_3U_2	+1	-1.05	+2.63
U_3U_4	-16	-.99	+3.15	L_4L_5	-6	+.37	-1.12	U_3L_4	+1	+1.41	-3.57
U_4U_3	+5	-.31	+.93	L_5L_4	+14	+.87	-2.61	L_4U_3	0	0	0
U_4U_5	-5	-.28	+.93	L_5L_6	-15	+.93	-2.79	U_4L_5	+1	+1.41	-3.57
U_5U_4	+19	-1.04	+3.53	L_6L_5	+14	+.87	-2.61	L_5U_4	+1	-1.41	+3.57
U_5U_6	-18	-.99	+3.35	L_6L_7	-15	+.93	-2.79	U_5L_6	-1	-1.41	+3.57
U_6U_5	+14	-.77	+2.62	L_7L_6	+14	+.87	-2.61	L_6U_5	+1	-1.41	+3.57
U_6U_7	-15	-.82	+2.79	L_7L_8	-15	+.93	-2.79	U_6L_7	+1	+1.41	-3.57
U_7U_6	+17	-.93	+3.16	L_8L_7	+13	+.82	-2.42	L_7U_6	0	0	0
U_7U_8	-16	-.88	+2.98	U_3L_3	+1	-.63	+1.67	U_7L_8	-1	-1.41	+3.57
U_8U_7	+13	-.71	+2.42	L_3U_3	0	0	0	L_8U_7	0	0	0
U_1L_1	-6	+2.0	-4.28	U_4L_4	0	0	0	U_6L_6	+1	-.63	+1.67
L_1U_1	-4	-1.33	+2.86	L_4U_4	-1	-.63	+1.67	L_6U_6	0	0	0
U_2L_2	+1	-.63	+1.67	U_5L_5	-1	+1.05	-2.63	U_7L_7	0	0	0
L_2U_2	+1	+.63	-1.67	L_5U_5	-1	-1.05	+2.63	L_7U_7	0	0	0

STRUCTURE "75" TRUSS TYPE "T2"



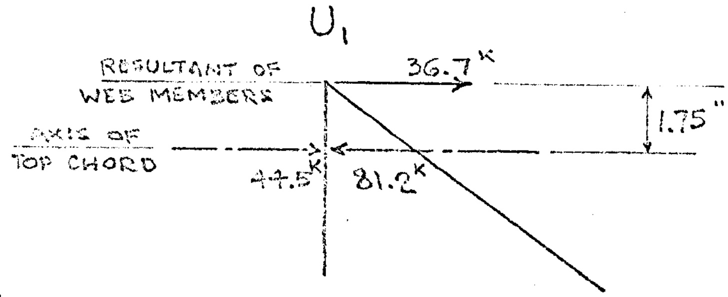


ECCENTRICITIES AT JOINTS.

Moments arising from Eccentricities in Connections.

Typical: At U_1 horizontal components equal to 36.7K are separated by a moment arm of 1.75" and produce a moment acting on the joint

= 36.7 x 1.75 = 64.2 K-ins.



This will cause a clockwise moment to act on the end of each member at the joint. The total moment will be distributed between these members in proportion to their relative stiffnesses. There will be some carry-over of moments to other parts of the truss, the whole being finally balanced with the respective resultant moments at the joints where eccentricity occurs.

Joint	U_1	U_2	U_3	L_1	L_2	L_3
Resultant	36.7K	28.8	21.0	44.5	36.7	28.8
Ecc'tricity	1.75"	1.50	1.20	3.47	3.43	3.15
Moment	64.2	43.2	25.2	154.3	126.0	90.6

Moments at joints due to elastic deformation are calculated by the method outlined on p.109. The two sets of moments, before distribution, are as below. (K-in units.)

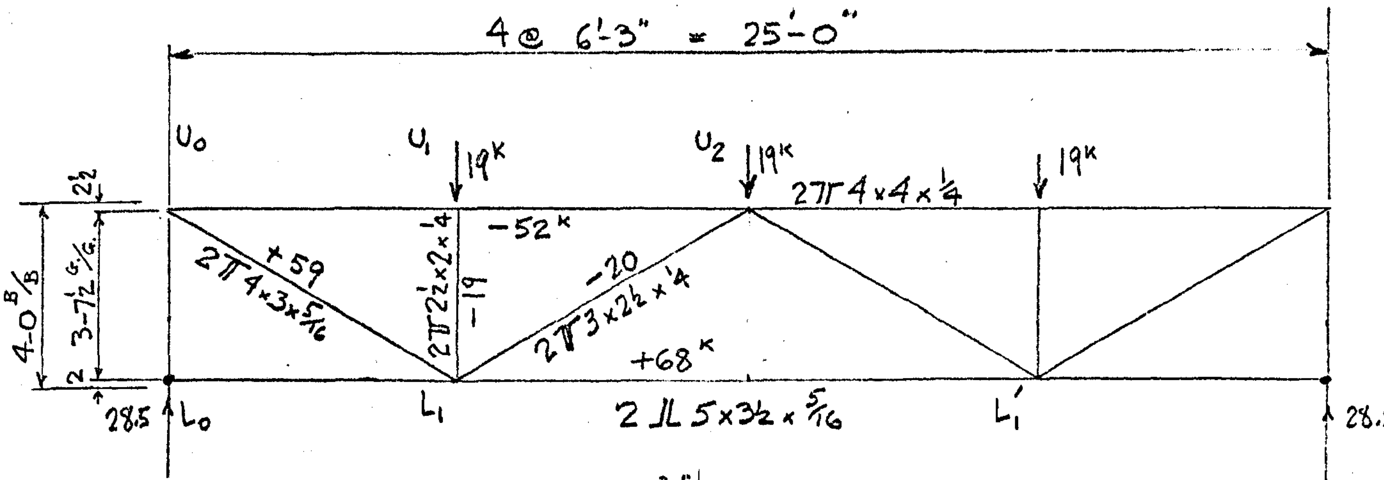
MOM. LOCATION.	DUE TO ECC.	DUE TO DEFRMN.	MOM. LOCATION	DUE TO ECC.	DUE TO DEFRMN.
U_0U_1	0	+394	U_1U_0	-30	+394
U_1U_2	-27	+277	U_2U_1	-20	+277
U_2U_3	-20	+210	U_3U_2	-12	+210
U_3U_4	-12	+101	U_4U_3	0	+101
L_0L_1	0	+76	L_1L_0	+44	+76
L_1L_2	+53	+59	L_2L_1	+13	+59
L_2L_3	+13	+46	L_3L_2	+11	+46
L_3L_4	+11	+25	L_4L_3	0	+25
U_1L_1	-4	+32	L_1U_1	+36	+32
U_2L_2	-2	+15	L_2U_2	+5	+15
U_3L_3	-1	+5	L_3U_3	+3	+5
U_0L_1	0	+25	L_1U_0	+21	+25
U_1L_2	-3	+27	L_2U_1	+6	+27
U_2L_3	-1	+12	L_3U_2	+4	+12
U_3L_4	0	+3	L_4U_3	0	+3

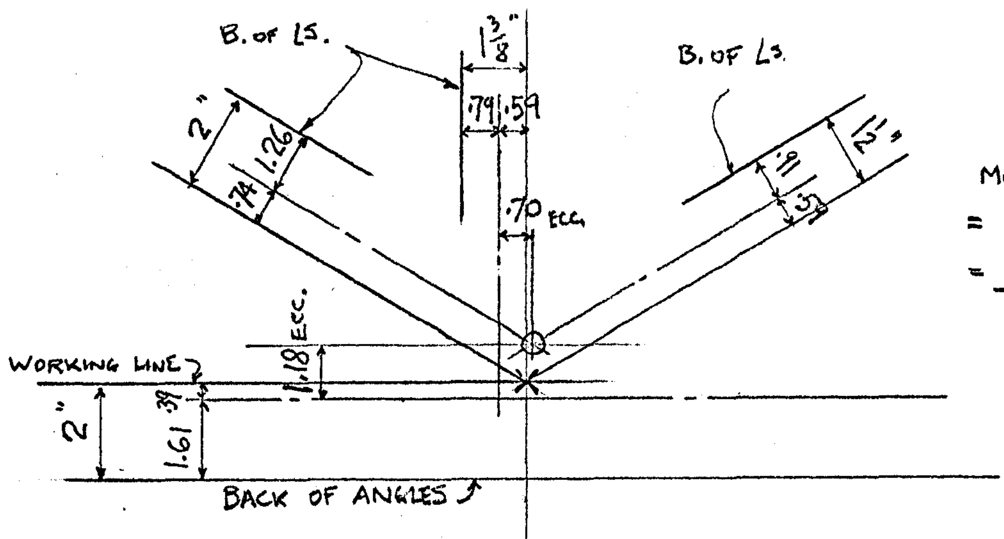
The following table gives the moments resulting from the distribution of the two series of moments, and the bending stresses at ends of members due to their combined effect.

LOCATION OF MOMENT	MOMENTS (DISTRIBUTED)			SECTION MODULUS		BENDING STRESS	
	ECC.	DEFMN.	TOTAL	HEEL	TOE	HEEL	TOE
U_0U_1	-10	+5	-5	15.8	7.7	-0.3	+0.7
U_1U_0	-35	+25	-10	"	"	+0.6	-1.3
U_1U_2	-40	-9	-49	"	"	-3.1	+6.4
U_2U_1	-27	+14	-13	"	"	+0.8	-1.7
U_2U_3	-16	-7	-23	"	"	-1.5	+3.0
U_3U_2	-17	+18	+1	"	"	-0.1	+0.1
U_3U_4	-9	-12	-21	"	"	-1.3	+2.7
U_4U_3	-4	+45	+41	"	"	-2.6	+5.3
L_0L_1	0	0	0	8.5	2.5	0	0
L_1L_0	+44	+33	+77	"	"	+9.1	-30.8
L_1L_2	+55	-9	+46	"	"	-5.4	+18.4
L_2L_1	+29	0	+29	"	"	+3.4	-11.6
L_2L_3	+8	+3	+11	"	"	-1.3	+4.4
L_3L_2	+12	+6	+18	"	"	+2.2	-7.2
L_3L_4	+11	0	+11	"	"	-1.3	+4.4
L_4L_3	+5	+13	+18	"	"	+2.2	-7.2
U_1L_1	+13	-14	-1	3.2	1.5	+0.3	-0.7
L_1U_1	+34	-18	+16	"	"	+5.0	-10.7
U_2L_2	-1	-5	-6	2.2	1.1	+2.7	-5.5
L_2U_2	0	-4	-4	"	"	-1.8	+3.6
U_3L_3	+1	-4	-3	1.9	0.78	+1.6	-3.9
L_3U_3	+2	-5	-3	"	"	-1.6	+3.9
U_0L_1	+10	-5	+5	3.5	1.4	+1.4	-3.6
L_1U_0	+21	-6	+15	"	"	-4.3	+10.7
U_1L_2	-2	-2	-4	3.7	1.6	-1.1	+2.5
L_2U_1	0	+1	+1	"	"	-0.3	+0.6
U_2L_3	+1	-2	-1	2.5	1.1	-0.4	+0.9
L_3U_2	+4	-1	+3	"	"	-1.2	+2.7
U_3L_4	0	-2	-2	1.9	0.78	-1.1	+2.6
L_4U_3	0	+1	+1	"	"	-0.5	+1.3

Moments are in Kip-ins.; Counterclockwise +
Stresses are in K/ sq. in.; Tension +

STRUCTURE "49" TRUSS A82





MOMENT DUE TO ECC:

$= 68^K \times 1.18'' + 19^K \times .70$

$= \underline{93.5 \text{ K-IN.}}$

ECCENTRICITY AT JOINT L1

AT U_1 MOMENT $U_1L_1 = 19 \times .59 = 11.2$ K-IN.

MOMENTS DUE TO ELASTIC DEFORMATION FOUND BY APPLICATION OF WILLIOT DIAGRAM.

SUMMARY OF SECONDARY MOMENTS AS GIVEN BELOW:

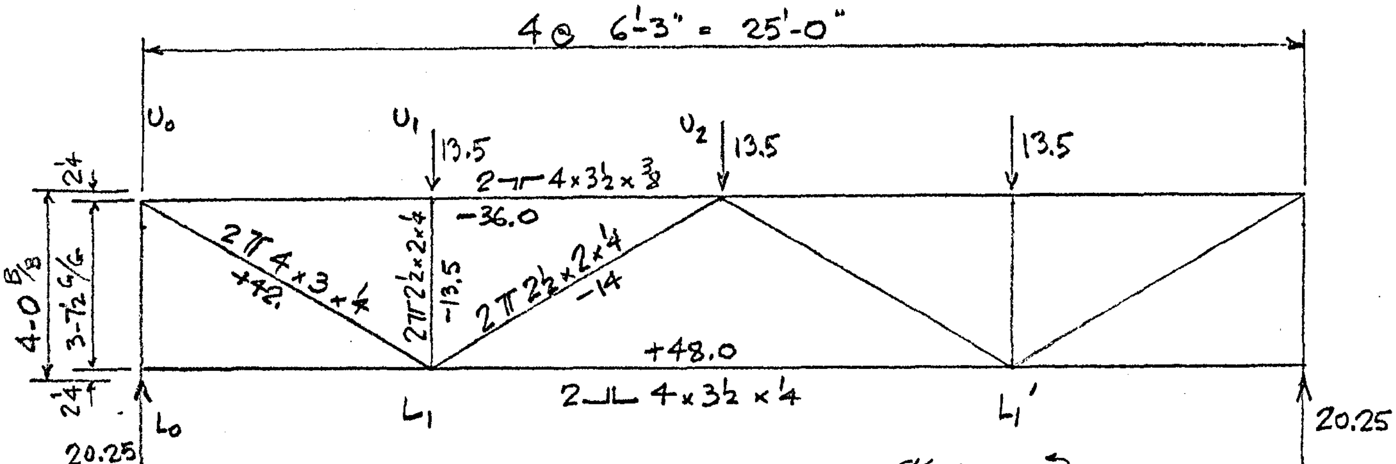
LOCATION OF MOMENT	DUE TO ECCENTRICITY	DUE TO DEFORMATION
U_0U_1	0	+61
U_1U_2	−4.2	+16
L_0L_1	0	+126
L_1L_1'	+19.8	0
U_0L_1	0	+69
U_1L_1	−2	+13
L_1U_2	+7.5	+8

LOCATION OF MOMENT	DUE TO ECCENTRICITY	DUE TO DEFORMATION
U_1U_0	−5	+61
U_2U_1	0	+16
L_1L_0	+34.2	+126
$L_1'L_1$	−19.8	0
L_1U_0	+23.7	+69
L_1U_1	+8.3	+13
U_2L_1	0	+8

Distribution of the unbalanced moments gives the following condition.

LOCATION OF MOMENT	MOMENT (K-INS.)	SECTION MODULUS		BENDING STRESS	
		HEEL	TOE	HEEL	TOE
U_0U_1	-7	5.6	2.1	-1.2	+3.3
U_1U_0	+3	5.6	2.1	-0.5	+1.4
U_1U_2	-11	5.6	2.1	-2.0	+5.5
U_2U_1	+2	5.6	2.1	-0.4	+1.0
L_0L_1	0	8.3	3.9	0	0
L_1L_0	+52	8.3	3.9	+6.3	-13.3
L_1L_1'	-2	8.3	3.9	+0.2	-0.5
$L_1'L_1$	+2	8.3	3.9	+0.2	-0.5
U_0L_1	+7	5.4	2.5	+1.3	-2.8
L_1U_0	+34	5.4	2.5	-6.3	+13.6
U_1L_1	-3	1.64	0.76	+1.8	-4.0
L_1U_1	+3	1.64	0.76	+1.8	-3.9
L_1U_2	+7	2.5	1.1	+2.8	-6.3
U_2L_1	+7	2.5	1.1	-2.8	+6.3

STRUCTURE "49" TRUSS B82



MOMENT AT L_1 DUE TO ECCENTRICITY = 86.0 K-IN.

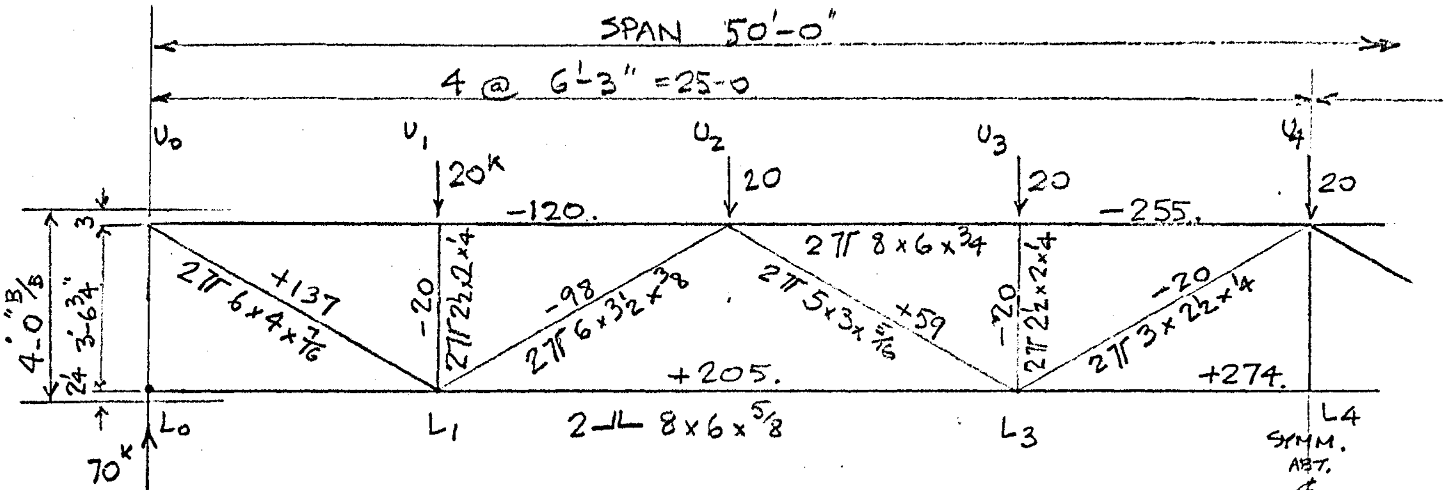
FOR THIS TRUSS, THE MOMENTS DUE TO ECCENTRICITY AND DUE TO DEFORMATION WERE TREATED SEPARATELY FOR PURPOSES OF COMPARISON.

LOCATION OF MOMENT	DUE TO ECCENTR'Y	DUE TO DEFORMATN.	COMBINED MOMENT	SECTION MODULUS		BENDING STRESS	
				HEEL	TOE	HEEL	TOE
U_0U_1	−12	+1	−11	6.3	2.3	−1.7	+4.8
U_1U_0	−10	+6	−4	"	"	+0.6	−1.7
U_1U_2	−5	−3	−8	"	"	−1.3	+3.5
U_2U_1	−3	+6	+3	"	"	−0.5	+1.3
L_0L_1	0	0	0	4.6	1.6	0	0
L_1L_0	+25	+3	+28	"	"	+6.1	−17.5
L_1L_1'	+8	−3	+5	"	"	−1.1	+3.1
$L_1'L_1$	−8	+3	−5	"	"	−1.1	+3.1
U_0L_1	+12	−1	+11	4.4	2.0	+2.5	−5.5
L_1U_0	+37	+4	+41	4.4	2.0	−9.3	+20.5
L_1U_2	+9	0	+9	1.64	.76	+5.5	−11.9
U_2L_1	+5	+2	+7	1.64	.76	−4.3	+9.2
U_1L_1	+7	−4	+3	1.64	.76	−1.8	+4.0
L_1U_1	+17	−4	+13	1.64	.76	+7.9	−17.1

MOMENTS IN KIP-INS. + INDICATES COUNTERCLOCKWISE MOMENT APPLIED TO MEMBER.

STRESSES: TENSION + COMPRESSION −

STRUCTURE "49" TRUSS A84



MOMENTS DUE TO ECCENTRICITY:

AT U_2 = 32. K-INS   AT L_1 = 150 K-INS   AT L_3 = 101 K-INS

MOMENTS DUE TO ECCENTRICITY AND TO ELASTIC DEFORMATION, AND BENDING STRESSES CAUSED, ARE TABULATED BELOW:

LOCATION OF MOMENT	MOMENT (K-INS.)	SECTION MODULUS		BENDING STRESS		LOCATION OF MOMENT	MOMENT (K-INS)	SECTION MODULUS		BENDING STRESS	
		HEEL	TOE	HEEL	TOE			HEEL	TOE	HEEL	TOE
U_0U_1	−70	49.5	23.3	−1.4	+3.0	U_1U_0	+161	49.5	23.3	−3.3	+6.9
U_1U_2	−158	"	"	−3.2	+6.8	U_2U_1	−63	"	"	+1.3	−2.7
U_2U_3	−122	"	"	−2.5	+5.2	U_3U_2	+95	"	"	−1.9	+4.1
U_3U_4	−94	"	"	−1.9	+4.0	U_4U_3	+138	"	"	−2.8	+5.9
L_0L_1	0	34.7	11.8	0	0	L_1L_0	−36	34.7	11.8	−1.0	+3.1
L_1L_3	−31	"	"	+0.9	−2.6	L_3L_1	+42	"	"	+1.2	−3.6
L_3L_4	−1	"	"	0	−0.1	L_4L_3	+86	"	"	+2.5	−7.3
U_0L_1	+70	15.8	7.7	+4.4	−9.1	L_1U_0	+123	15.8	7.7	−7.8	+16.0
L_1U_2	+95	12.6	6.5	+7.5	−14.6	U_2L_1	+127	12.6	6.5	−10.1	+19.5
U_2L_3	+24	7.5	3.8	+3.2	−6.3	L_3U_2	+56	7.5	3.8	−7.5	+14.8
L_3U_4	+3	2.5	1.1	+1.2	−2.7	U_4L_3	+7	2.5	1.1	−2.8	+6.4
U_1L_1	−2	1.64	.76	+1.2	−2.6	L_1U_1	−1	1.64	.76	−0.6	+1.3
U_3L_3	−1	1.64	.76	+0.6	−1.3	L_3U_3	0	1.64	.76	0	0

MOMENTS IN K-INS. COUNTERCLOCKWISE MOMENT APPLIED TO MEMBER +

STRESSES IN K/IN.² TENSION + COMPRESSION −

The analysis of the foregoing trusses showed that in a rivetted truss, the secondary stresses due to eccentricities at the joints are usually more important than those due to the elastic deformation of the members. The local bending caused by the latter tends to be distributed through the truss. The local moment caused by eccentricity, however, cannot be dissipated in such a manner; it is, to all intents, an external moment applied to the joint, and must be resisted by the resultant of the moments developed by the members meeting there. Only in truss A84 of structure "49" was this not true, owing to certain peculiarities in that particular truss. In the larger angle sections used, the dimension between the first gauge line and the centroid was quite small, so that the actual eccentricities were also small. At the same time, the low value of the depth-span ratio and the large loads applied enhanced the stresses caused by deformation of the members.

In "simple-span" design, trusses usually have a zero-stress panel of chord at each end. This member is supposed to be free from axial loading. However, it is connected in some way to the supporting steel, and even if this connection is not designed for any particular load, it will still provide sufficient resistance against vertical movement of the chord, and will therefore contribute to the rotation-resistance of the chord at the nearest panel point. In the analysis of the bending in the truss members, the chord so connected has been treated as a member with a hinge at the support, but continuous at the panel point.

An assumption sometimes made in the discussion of secondary stresses, is to the effect that at any connection where an appreciable moment tends to develope, there will be some slippage in the rivetted joint, which will largely relieve the local stresses. This is a convenient assumption, but it is difficult to justify when the results of some experiments on rivet capacities are considered. On page 118 some of the rivetted connections in the selected trusses have been checked for the probability of rivet slip, and no connection could be found in which the calculated capacity of the rivet group to develope moment without slipping did not exceed substantially the applied moment.

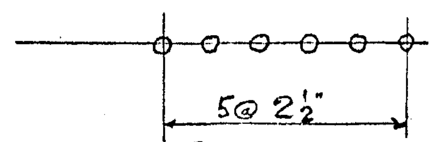
RESISTANCE OF RIVETTED CONNECTIONS TO SLIP.
DATA FROM PAPER BY BENDIGO, HANSEN AND RUMPF (19) FOLLOWS:

RIVET SHEAR STRESS AT SLIP = 24.8 K/IN² (AV.)

TAKE VALUE OF S.S. ¾" RIVET = 10.9^{K} S.S. ⅞" RIVET = 14.9^{K}
D.S. ¾" RIVET = 21.8^{K} D.S. ⅞" RIVET = 29.8^{K}

(P. 110, 111, 112) STRUCTURE "75" TRUSS T2.

LARGE BENDING STRESSES: L_1U_0 15 K-IN.



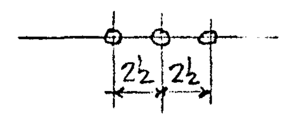
¾" RIVS. D.S.

"SLIP CAPACITY" OF GROUP:

$$2 \times 21.8 \times 6.25 = 272$$
$$2 \times \frac{21.8 \times 3.75}{6.25} \times 3.75 = 97$$
$$2 \times \frac{21.8 \times 1.25}{6.25} \times 1.25 = 7$$

376 K-IN.

THE SMALLEST CONNECTION IN THIS TRUSS — HAS SLIP CAPACITY OF

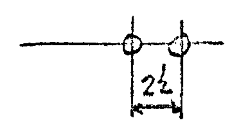


$$2 \times 21.8 \times 2.5 = 109.0 \text{ K-IN.}$$

CONCLUSION — SLIP IN THE JOINTS OF THIS TRUSS VERY IMPROBABLE.

(P. 113, 114) STRUCTURE "49" TRUSS A82.

¾ RIV. D.S.



SMALLEST CONNECTION IN THIS TRUSS IS GOOD FOR

$$2 \times 21.8 \times 1.25 = 54.5 \text{ K-IN.}$$

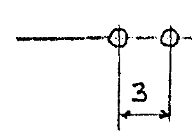
MAXIMUM MOMENT (OCCURRING IN A CONTINUOUS CHORD) = 52 K-IN.
" " (" IN A 6-RIVET CONNECTION) = 34 K-IN.

CONCLUSION — SLIP IN THE JOINTS OF THIS TRUSS VERY IMPROBABLE.

(P. 115) STRUCTURE "49" TRUSS B82. SAME ANALYSIS AS A82 ABOVE.

(P. 116) STRUCTURE "49" TRUSS A84

⅞" RIVS., D.S.



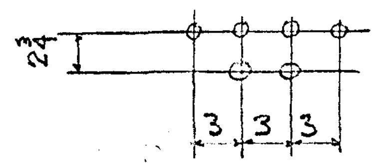
SMALLEST CONNECTION IN THIS TRUSS HAS SLIP CAPACITY.— $29.8 \times 3 = 89.4$ K-IN.

SUCH CONNECTIONS REQUIRED TO CARRY MAX. MOM. = 7. K-IN.

LARGER CONNECTIONS: U_2L_1 MOMENT = 127 K-IN.

$$2 \times 29.8 \times 4.5 = 268$$
$$2 \times 29.8 \times \frac{1.5}{4.5} \times 1.5 = 30$$
$$2 \times 29.8 \times \frac{1.7}{4.5} \times 1.7 = 38$$

336 K-IN.



⅞" RIVS, D.S.

BY INSPECTION OF OTHER CONNECTIONS IN THIS TRUSS, IT APPEARS THAT RIVET SLIP IS NOT PROBABLE.

Trusses, like beams, may be designed on the assumption that the ends are freely supported, and yet may develope end moments. Designers sometimes try to minimize such moments, by such devices as providing slotted holes in the connection at the end of the "null"chord member. Cambering of a truss could theoretically be so contrived that an initial moment would be produced at the ends of a truss equal and opposite to the end moments expected to develope under the vertical loading. Camber is rarely provided in exactly this way, however; while the lengths of truss members may be so modified that some negative deflection is introduced in the line of the chords, at least part of the inequality which should be made in the lengths of the chords, is restored in the end panels, with the result that the end moments are only reduced, but not eliminated. The reason for this compromise lies in the difficulty of making the field connections in a fully-cambered truss; the erector is not prepared to generate the necessary positive end-moment.

In order to obtain information regarding the secondary stresses in a truss-column system, portions of two structures, "05" and "790" have been selected for analysis. The first of these consists of three bays, the columns at either end being without loading on the far side. The system is symmetrical with regard to truss loading and design, but unsymmetrical in that the exterior columns are not alike. The trusses are of welded construction, with negligible eccentricity at the joints, and the connections at the ends of the top chord, where the zero-stress members are, are kept to minimum proportions in order to minimize the end moments in the trusses.

In structure "05", the calculation of the moment-rotation factor at the ends of the trusses presents some special problems. The magnitude of the negative moment which could be developed is limited to some extent by the detail of the top chord connection, which can develope only a small tensile reaction in the plastic condition. The moment is augmented, however, by the rather deep welded connection at the bottom chord level, which in itself possesses considerable moment capacity. The combined effect is represented graphically on page 126.

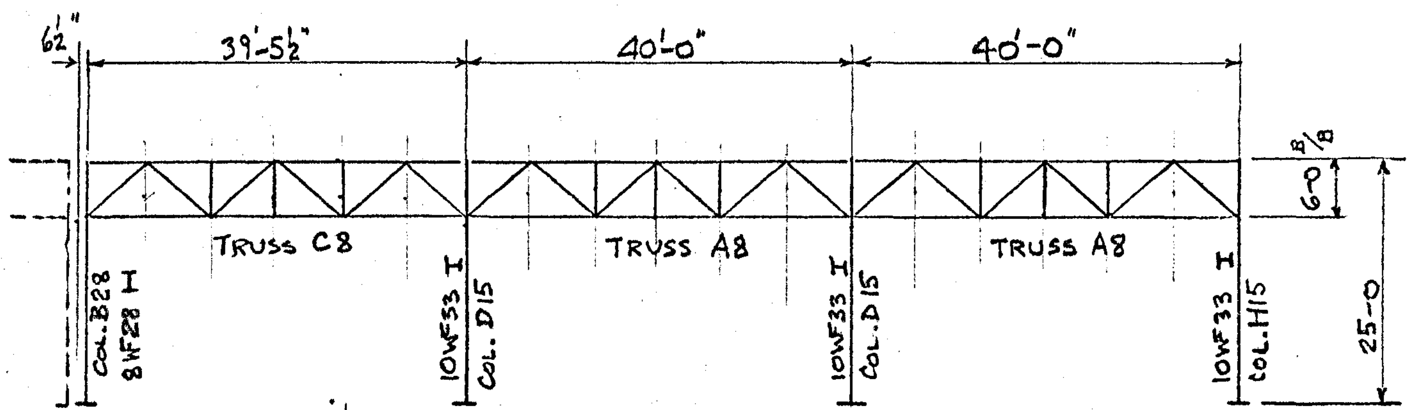
The distribution of the moments at the columns resembles that already noted in beam spans. The interior columns receive very little bending, while that in the exterior columns is greater, resulting in bending stresses at the tops of the outer columns (left and right) equal to about 2.5 and 3.6 ksi respectively. In the trusses, the end moments have the effect of reducing the stresses in most of the members from the calculated simple-span stresses; hence the effect of the secondary bending is to some extent offset by a reduction in the axial forces in the members. An exception to this is found in the "null" panel of the top chord.

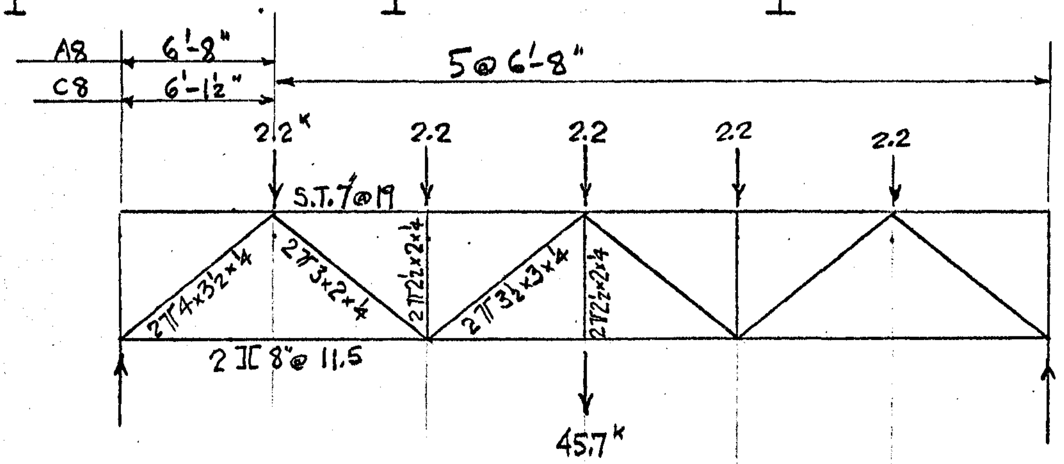
The truss-column system from structure "790" differs from the foregoing in several respects. The column at the right-hand side is unloaded on the outer flange, owing to the presence of an expansion joint in the building. The column at the left carries a truss similar in design and loading to that shown, and may be regarded as carrying a balanced load. This has justified the truss detailer in adopting a form of connection which, by itself, would impose a large moment on the column. The connection at the right, in contrast, is arranged to produce a vertical resultant at the centre-line of the column; this requires a much more substantial detail. In this truss, the "null" chord panels are at the ends of the bottom chord, which means that they are in compression when end moments are developed. While the connections are not large, they are still capable of transmitting a substantial compression reaction to the column.

The top of the left-hand column is assumed to remain vertical, due to the balanced loading. The base of the right-hand column, which is designed for rigidity, and is embedded in two feet of concrete, is assumed to be fixed.

Other significant features of the connection details in structure "790" will be developed later.

STRUCTURE "05"; THREE-BAY BENT.

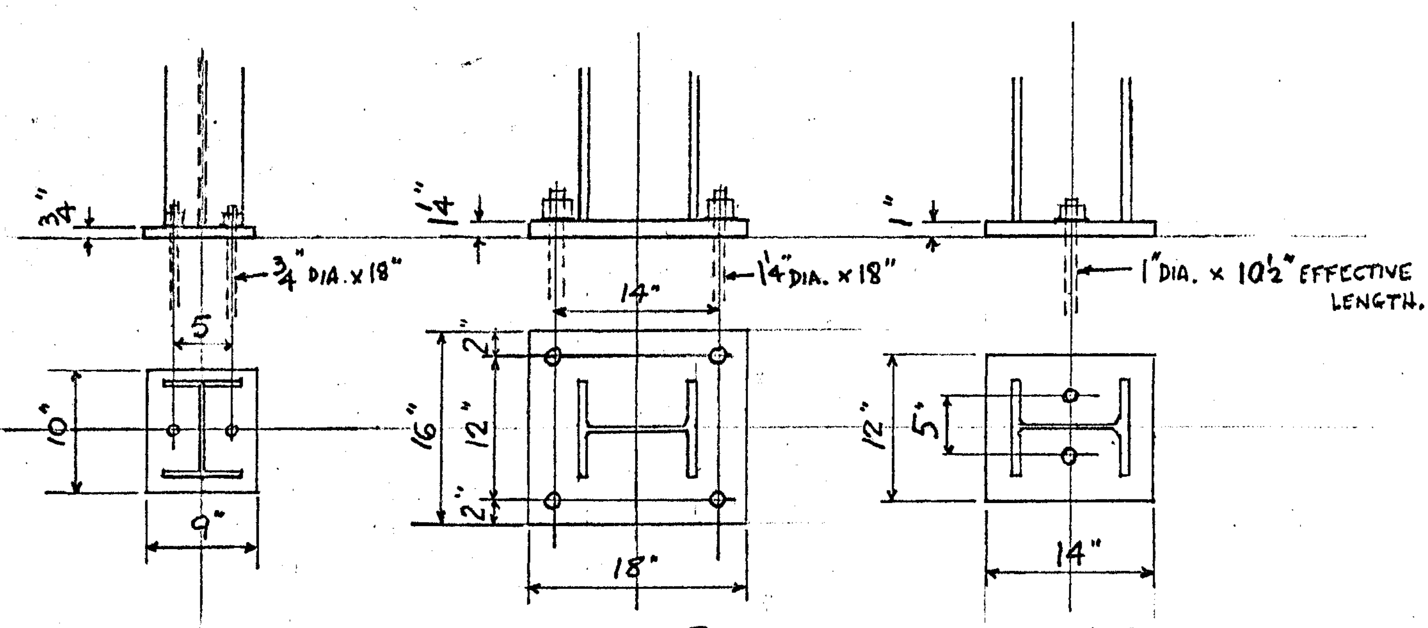




DESIGN DATA FOR TRUSSES A8, C8.

ALL SHOP CONNECTIONS IN TRUSSES WELDED; FIELD CONNECTIONS TO COLUMNS MADE WITH ¾ DIA. HIGH STRENGTH BOLTS.

WELDED JOINTS DESIGNED TO ELIMINATE ECCENTRICITY.

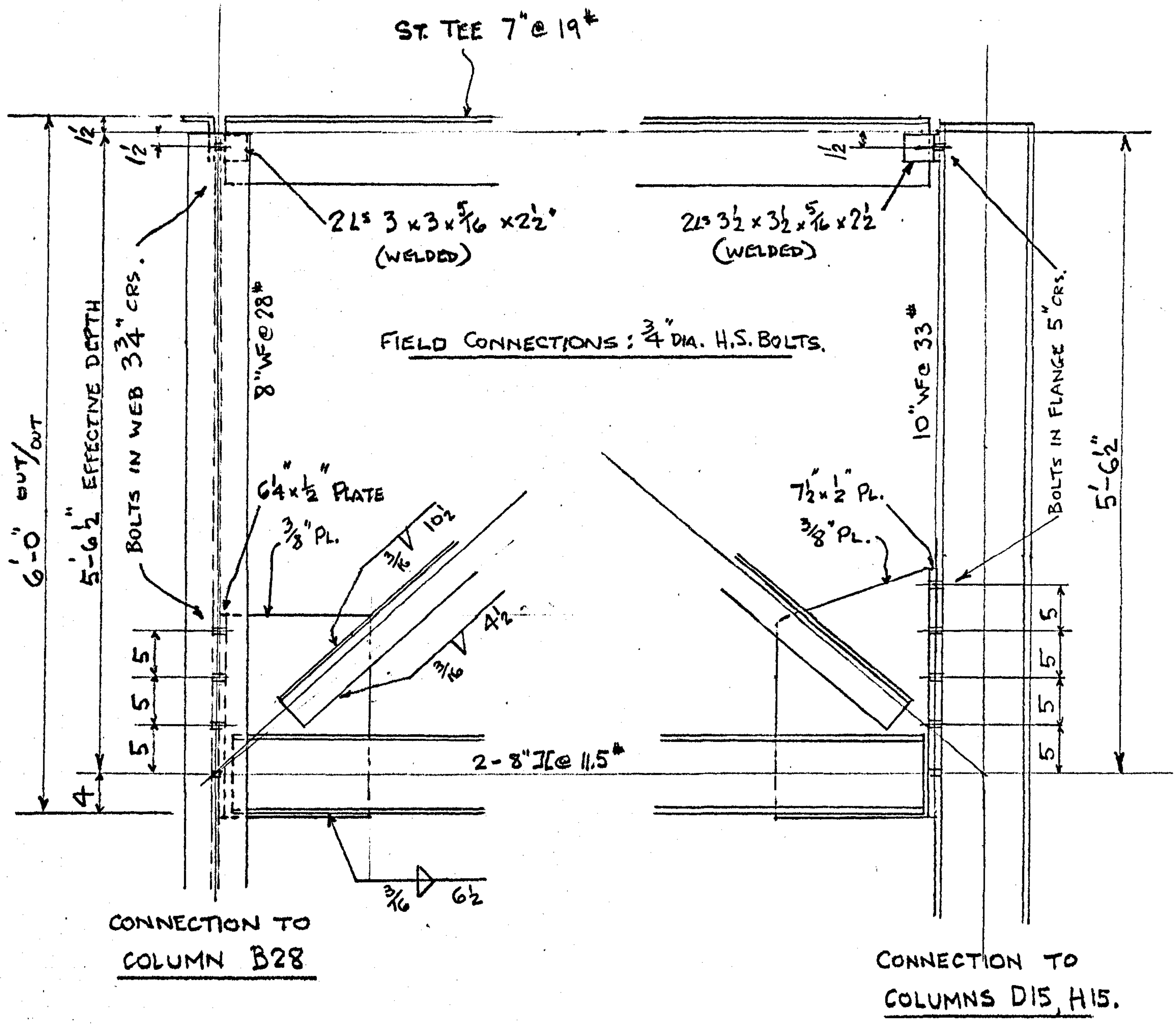


COL. B28

COLS. D15

COL. H15

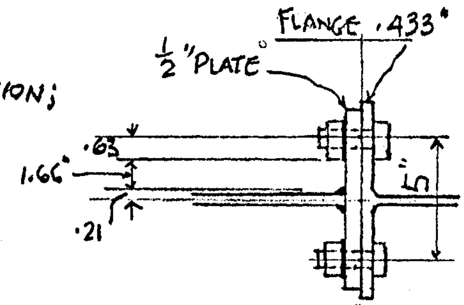
COLUMN BASE DETAILS.



DETAILS OF TRUSS-TO-COLUMN CONNECTIONS.

TOP CHORD CONNECTION TO COLUMN IS A "MINIMUM" CONNECTION. CALCULATION OF BENDING STRESSES IN THE CONNECTION ANGLES GIVES THE APPROXIMATE TENSILE CAPACITY OF THE CONNECTION AS 5^K, CORRESPONDING WITH DEVELOPMENT OF A PLASTIC BENDING CONDITION.

ANY ROTATION OF THE TRUSS RELATIVE TO THE COLUMN WILL BE RESISTED BY DIRECT HORIZONTAL REACTIONS AT THE CHORDS. IT WILL ALSO BE RESISTED BY THE AMOUNT OF RIGIDITY INHERENT IN THE BOTTOM CHORD CONNECTION. THE LATTER IS DETERMINED BY THE BENDING OF THE PLATE ELEMENT WHICH IS BOLTED TO THE COLUMN, AND BY ANY BENDING WHICH MAY OCCUR IN THE COLUMN FLANGE OR WEB, AS THE CASE MAY BE.



CONNECTION TO 10WF33" FLANGE.

FOR TENSION $= \frac{P}{2}$ PER 1" TRANSVERSE STRIP OF CONNECTION;

BENDING MOMENT IN ½" PL. $= \frac{P}{2}\left(\frac{1.66}{2}\right) = .415P$ K-INS.

" " " FLGE. $= \frac{P}{2}\left(\frac{1.66}{2}\right) = .415P$

FOR 1" STRIP OF PLATE, $I = .0104\ in^4 \quad S = .0417\ in^3$

" " OF FLANGE, $I = .00676 \quad S = .0313$

BENDING STRESS IN PLATE $= \frac{.415P}{.0417} = 10.0P\ ^K/_{in^2}$

" " IN FLANGE $= \frac{.415P}{.0313} = 13.3P$

DEFLECTION IN PLATE $= \frac{.5P \times .83^3 \times 2}{90{,}000 \times .0104} = .000612P$

" " FLANGE $= \frac{.5P \times .83^3 \times 2}{90{,}000 \times .00676} = .000941P$

IF f_y FOR MATERIAL = 40 KSI, ELASTIC LIMIT FOR FLANGE WOULD OCCUR WHEN

$13.3P = 40$ OR $P = 3.0^K$

ELASTIC LIMIT FOR PLATE WOULD BE REACHED WHEN $P = \frac{40}{10} = 4.0^K$

CORRESPONDING DEFLECTION WOULD BE: PLATE, $.000612 \times 4.0 = .002448"$

FLANGE, $.000941 \times 3.0 = .002823"$

THE DEFORMATION IN THE COLUMN FLANGE WILL BE MODIFIED TO SOME EXTENT BY THE CONTINUITY OF THE FLANGE. IT WILL BE ASSUMED, FOR THE PURPOSES OF THIS CALCULATION, THAT THE FLANGE AND THE PLATE WILL RESIST THE TENSION EQUALLY, AND THAT BOTH WILL REACH THE LIMIT OF ELASTIC BEHAVIOUR WHEN $P = 4.0^K$ AND THE COMBINED DEFLECTION = .005"

IF THE CENTRE OF ROTATION OF THE CONNECTION BE TAKEN AT THE LOWEST PAIR OF BOLTS, THE EFFECTIVE DEPTH WOULD BE 21.25", GIVING

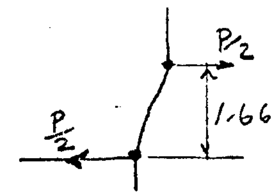
MOMENT $= \frac{PD^2}{3} = \frac{1}{3}(4.0)(21.25)^2 = 602$ K-IN.

ROTATION $= \frac{.005}{21.25} = 235 \times 10^{-6}$ RAD

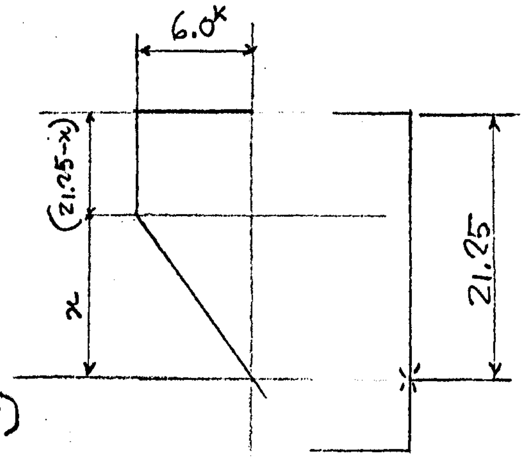
MAX. MOMENT DEVELOPED AT PLASTIC HINGES

$$= \frac{f_y b d^2}{4} = \frac{40 \times 1 \times .25}{4} = 2.5 \text{ K-in.}$$

$\therefore$ MAXIMUM VALUE OF P DEVELOPED $= 2\left(\frac{2.5}{.83}\right) = 6.03^K$



THE PLASTIC CONDITION WILL FIRST OCCUR AT THE TOP OF THE CONNECTION, WHERE MAXIMUM TENSION IS, AND WILL PROGRESS DOWNWARDS.



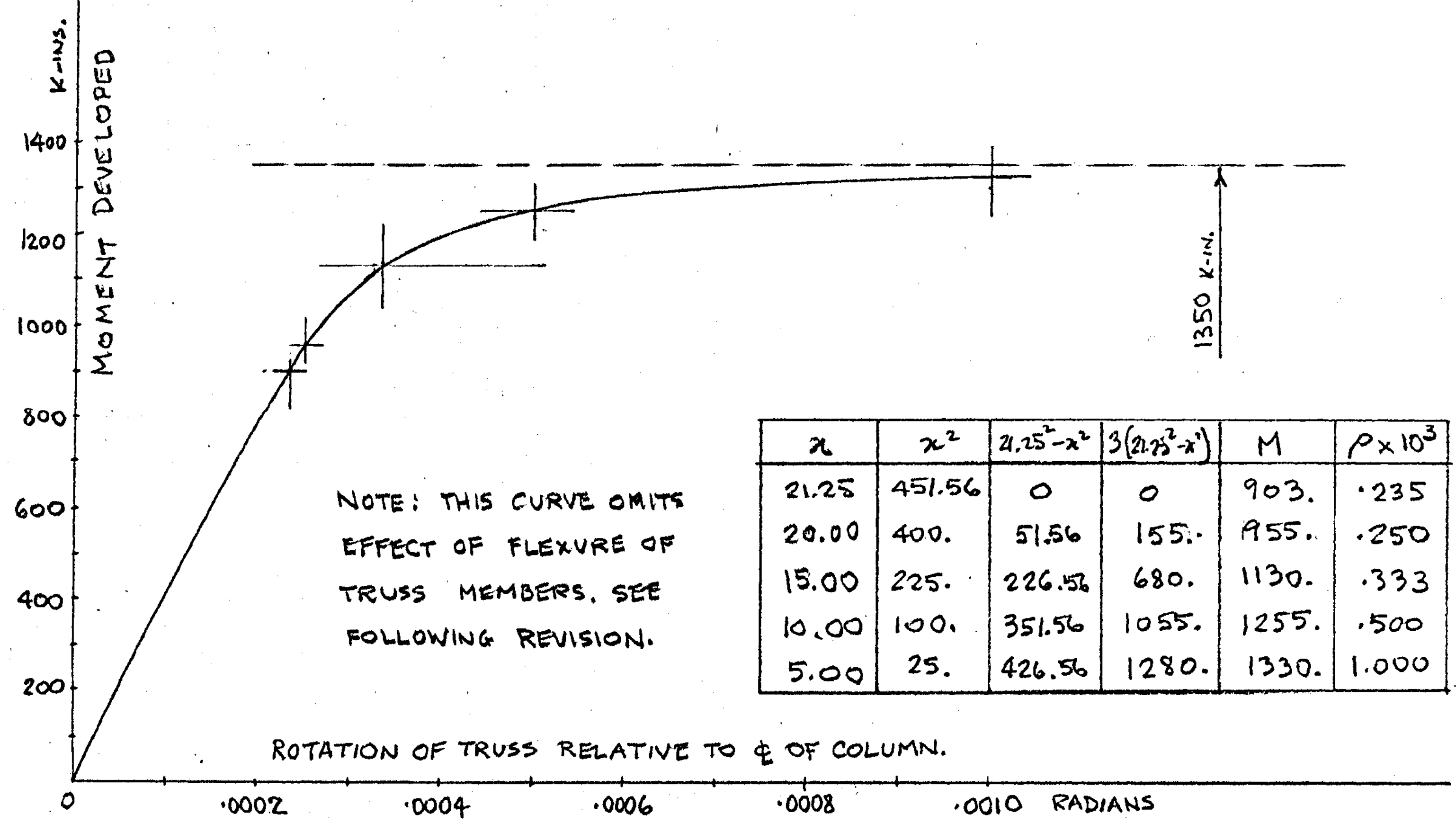
THE TOTAL MOMENT DEVELOPED IN THE CONNECTION WILL BE:

$$M = \frac{6 \times x^2}{3} + \frac{6(21.25-x)(21.25+x)}{2}$$

$$= 2x^2 + 3(21.25^2 - x^2)$$

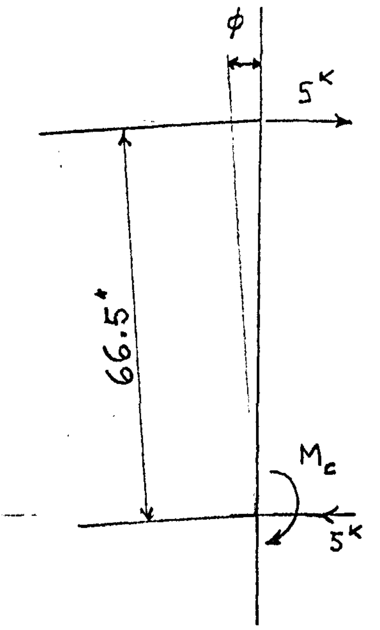
IF THE DEFLECTION AT $x = .005''$ (APPROXIMATELY)
THEN THE ROTATION $= \frac{.005}{x}$

THE CURVE BELOW SHOWS THE MOMENT-ROTATION RELATIONSHIP FOR THIS CONNECTION.



x	x^2	21.25^2-x^2	$3(21.25^2-x^2)$	M	$\rho \times 10^3$
21.25	451.56	0	0	903.	.235
20.00	400.	51.56	155.	955.	.250
15.00	225.	226.56	680.	1130.	.333
10.00	100.	351.56	1055.	1255.	.500
5.00	25.	426.56	1280.	1330.	1.000

THE MAXIMUM MOMENT WHICH CAN BE APPLIED TO THE END OF A TRUSS AT ONE OF THE CONNECTIONS TO COLUMN FLANGES, WILL BE :—



DUE TO HORIZONTAL REACTIONS.– $5^K \times 66.5'' = 333$ K-IN.

DUE TO RIGIDITY OF B.C. CONNECTION, M_c = $\underline{1350}$ K-IN.

TOTAL MOMENT CAPACITY = 1683 K-IN.

(SAY 1680)

THE ABOVE MOMENT CAN ONLY BE REALIZED IF THE CONNECTED TRUSS MEMBERS CAN DEVELOPE THE AXIAL LOADS AND BENDING MOMENTS SHOWN.

THE END DIAGONAL, $2\,\text{⊤⊤}\ 4 \times 3\frac{1}{2} \times \frac{1}{4}$ $\left\{\begin{array}{l} S_{HEEL} = 5.0 \\ S_{TOE} = 2.1 \end{array}\right\} I = 5.8$

THE BOTTOM CHORD, $2\,][\ 8 @ 11.5^{\#}$ $\{S = 16.2\}\ I = 64.6$

RELATIVE STIFFNESS OF MEMBERS $\left\{\begin{array}{ll} \text{DIAGONAL, } K = \frac{5.8}{90} = .0645 & (.13) \\ \text{CHORD, } K = \frac{64.6}{150} = \underline{.4310} & (.87) \\ \qquad\qquad\qquad .4955 & \end{array}\right.$

MOMENT IN DIAGONAL = $.13\,M_c$ BENDING STRESS (HEEL) = $+\frac{.13M_c}{5.0}$ (TOE) = $-\frac{.13M_c}{2.1}$

" " CHORD = $.87\,M_c$ " " (T. & B.) = $\pm\frac{.87M_c}{16.2}$

THE BENDING STRESSES MAY BE WRITTEN, $+.026M_c$ $-.062M_c$ $\pm .054M_c$ RESPECT'LY.

AXIAL STRESS IN DIAGONAL = $-\frac{44.4}{3.62} = -12.25\ ^K/_{IN^2}$

" " IN CHORD = $+\frac{34.1-5.}{6.72} = +4.33\ ^K/_{IN^2}$

ELASTIC LIMIT WILL BE ATTAINED IN DIAGONAL IF $\left\{\begin{array}{l} -12.25 + .026M_c = -40. \\ -12.25 - .062M_c = -40. \end{array}\right.$

OR M_c = $\underline{448 \text{ K-IN.}}$ (MINIMUM SOLUTION)

IN CHORD, $4.33 + .054M_c = 40.$, OR M_c = $\underline{660 \text{ K-IN.}}$

FROM THE ABOVE IT APPEARS THAT FOR ANY VALUE OF M_c GREATER THAN 448 K-IN., THE END DIAGONAL WILL EXPERIENCE COMBINED STRESSES AT THE ANGLE TOES IN EXCESS OF 40 KSI, THE ASSUMED YIELD POINT. VALUES OF M_c GREATER THAN THIS WOULD BE ACCOMPANIED BY PROGRESSIVE PLASTIC DEFORMATION IN THE DIAGONAL.

BOLT STRESSES,— WITH TENSION AT THE TOP OF THE BRACKET AT ITS MAXIMUM VALUE OF 6^K/IN., THE TOP TWO BOLTS WOULD BE EXPECTED TO CARRY $\frac{6 \times 3.75}{2} = 11.25^K$ EA. SINCE THE SPECIFIED INITIAL TENSION IN A 3/4" H.S. BOLT = 28.4^K, IT IS EVIDENT THAT THERE WILL BE NO (ADDITIONAL) BOLT ELONGATION.

THE PRECEDING CALCULATION SUGGESTS THAT THE MAXIMUM END MOMENT WHICH CAN BE MAINTAINED AT THE END OF THESE TRUSSES IS APPROXIMATELY:

DUE TO CHORD REACTION — 330

DUE TO CONNECTION AT B.C. — $\underline{470}$ = M_c

$\underline{800}$ K-INS. = M

THE VALUE OF M_c CORRESPONDS WITH A ROTATION IN THE CONNECTION PROPER = 183×10^{-6} RAD.

THE SIMULTANEOUS ROTATION OF THE ENDS OF THE TRUSS MEMBERS = $7{,}900 \times 10^{-6}$ RAD.

THE COMBINED EFFECT IS TO ALLOW THE END OF THE TRUSS TO ROTATE:

$$(7900 + 183) \times 10^{-6} = 8083 \times 10^{-6} \text{ RAD.}$$

AN APPROXIMATE CALCULATION OF THE SIMPLE-SPAN TRUSS GIVES A VALUE FOR THE END ROTATION, UNDER THE GIVEN LOADING, = 1835×10^{-6} RAD. THEREFORE THE DIAGONAL AND BOTTOM CHORD WILL NOT BE AS SEVERELY STRESSED AS ABOVE.

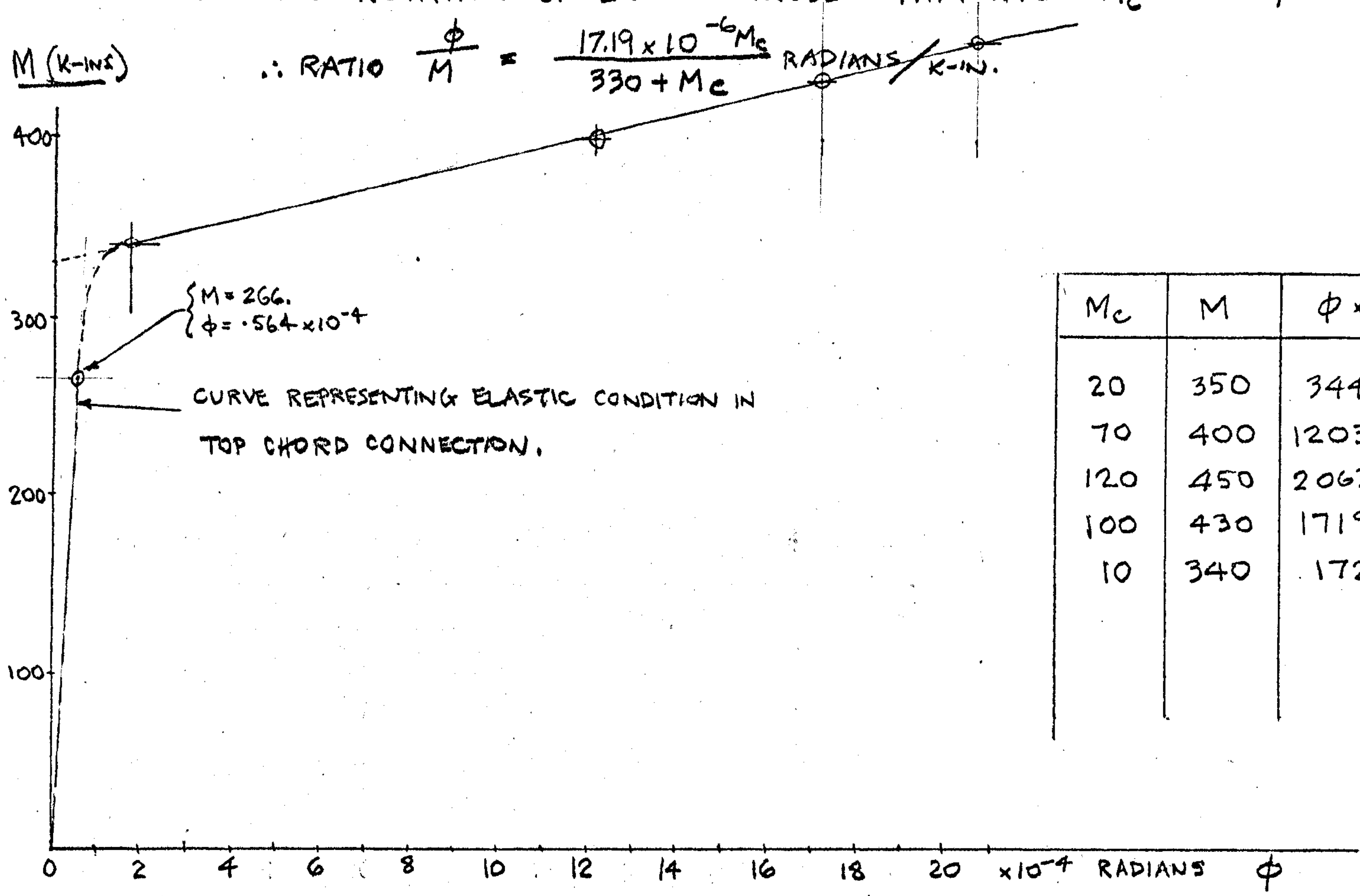
WE HAVE, TOTAL MOMENT AT END OF TRUSS = $330 + M_c$ K-IN.

ANGLE OF ROTATION IN CONNECTION = $0.390 \times 10^{-6} M_c$

" " " IN TRUSS MEMBERS = $16.8 \times 10^{-6} M_c$

∴ TOTAL ROTATION OF END OF TRUSS = $17.19 \times 10^{-6} M_c$ RAD = ϕ

$$\therefore \text{RATIO } \frac{\phi}{M} = \frac{17.19 \times 10^{-6} M_c}{330 + M_c} \text{ RADIANS/K-IN.}$$



M_c	M	$\phi \times 10^{6}$
20	350	344
70	400	1203
120	450	2063
100	430	1719
10	340	172

IN THE MIDDLE SPAN OF THE THREE-BAY BENT (p.A52) THE ROTATIONAL TENDENCIES WILL BE APPROXIMATELY BALANCED AT THE COLUMNS BY THE OUTER SPANS. THIS PROVIDES A CONVENIENT PLACE FOR THE CALCULATION OF THE END CONDITIONS.

I OF TRUSS $= 13{,}580 \text{ IN}^4$

$K = \frac{I}{L} = \frac{13{,}580}{470} = 28.89 \qquad 2EK = 60{,}000 \times 28.89 = 1{,}734{,}000$

"FIXED-END" MOMENT $= 3182$ K-INS.

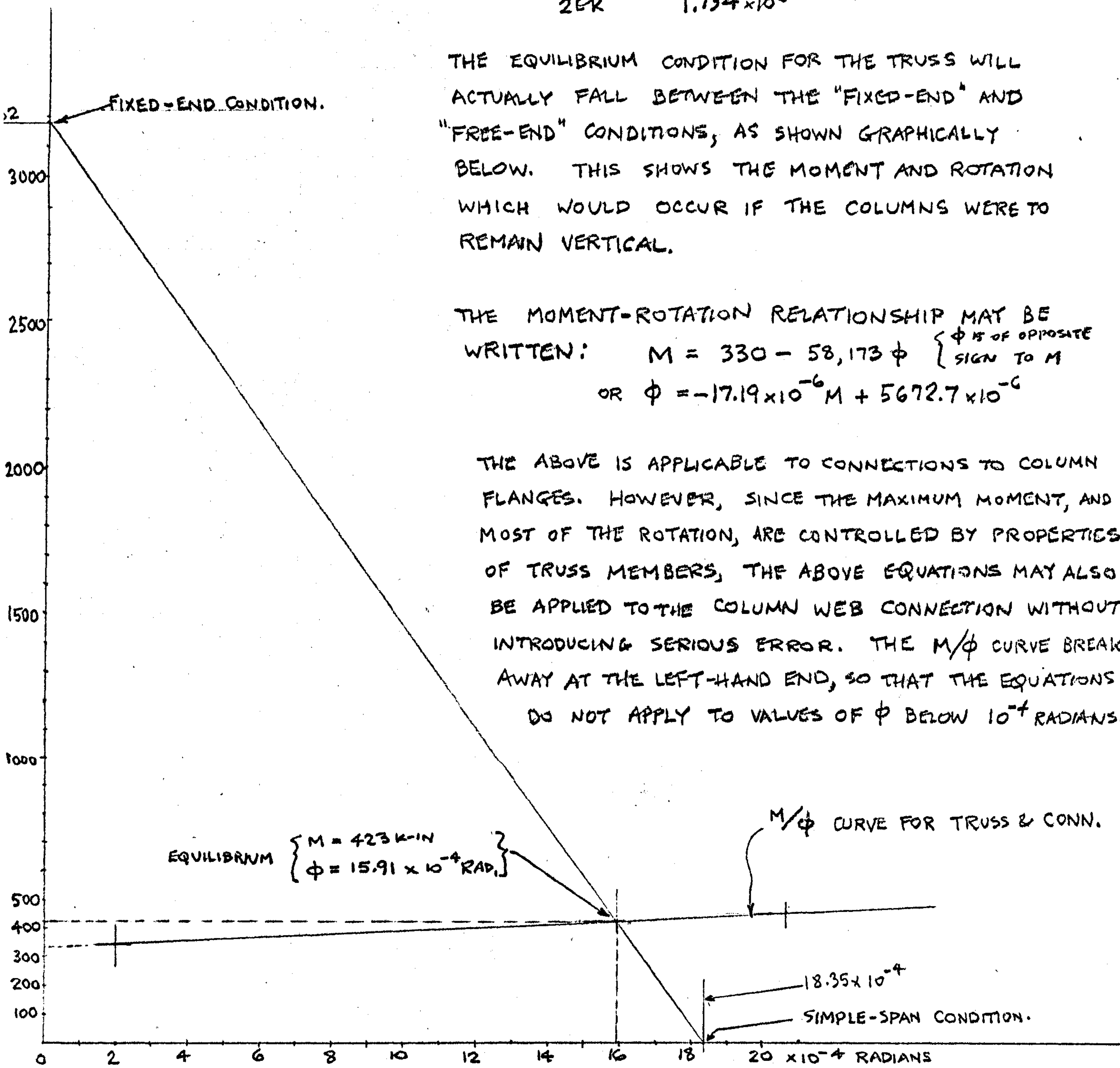
"FREE-END" ROTATION $= \frac{F.E.M}{2EK} = \frac{3182}{1.734 \times 10^6} = 1835. \times 10^{-6}$

THE EQUILIBRIUM CONDITION FOR THE TRUSS WILL ACTUALLY FALL BETWEEN THE "FIXED-END" AND "FREE-END" CONDITIONS, AS SHOWN GRAPHICALLY BELOW. THIS SHOWS THE MOMENT AND ROTATION WHICH WOULD OCCUR IF THE COLUMNS WERE TO REMAIN VERTICAL.

THE MOMENT-ROTATION RELATIONSHIP MAY BE WRITTEN: $M = 330 - 58{,}173\,\phi$ $\left\{ \begin{array}{l} \phi \text{ IS OF OPPOSITE} \\ \text{SIGN TO } M \end{array} \right.$

OR $\phi = -17.19 \times 10^{-6} M + 5672.7 \times 10^{-6}$

THE ABOVE IS APPLICABLE TO CONNECTIONS TO COLUMN FLANGES. HOWEVER, SINCE THE MAXIMUM MOMENT, AND MOST OF THE ROTATION, ARE CONTROLLED BY PROPERTIES OF TRUSS MEMBERS, THE ABOVE EQUATIONS MAY ALSO BE APPLIED TO THE COLUMN WEB CONNECTION WITHOUT INTRODUCING SERIOUS ERROR. THE M/ϕ CURVE BREAKS AWAY AT THE LEFT-HAND END, SO THAT THE EQUATIONS DO NOT APPLY TO VALUES OF ϕ BELOW 10^{-4} RADIANS.



FOR LOW VALUES OF ϕ, M_C BECOMES NEGLIGIBLE, AND TOP CHORD CONNECTION REMAINS ELASTIC. USING THE METHOD OF CALCULATION DESCRIBED ON p. 86, WE OBTAIN THESE EQUATIONS:

CONNECTION TO FLANGE:

$2\frac{1}{2} \times \frac{5}{16}$ SECTION; $I = .00637$ $S = .0408$ (FOR BOTH)

(2P = TENSION IN CHORD.)

$a = 1.52$

$b = 0.50$

$M_{AB} = .82P$

$M_{BA} = .70P$

$\Delta = .001875P$

$$\text{ROTATION} = \frac{1875 \times 10^{-6} P}{66.5}$$

$$= 28.2 \times 10^{-6} P \text{ RAD.}$$

$$\text{MAX. } f_b = \frac{.82P}{.0408} = 20P \text{ K/IN}^2$$

FOR $f_b = 40.0$, $P_e = 2.0^K$

$M_e = 2 \times 2.0 \times 66.5 = 266.$ K-IN.

$\phi_e = 56.4 \times 10^{-6}$ RADIANS

$$\frac{\phi_e}{M_e} = 0.212 \times 10^{-6}$$

CONNECTION TO WEB:

$a = 0.84$

$b = 0.50$

$M_{AB} = .475P$

$M_{BA} = .366P$

$\Delta = .00036P$

$$\text{ROTATION} = \frac{360 \times 10^{-6} P}{66.5}$$

$$= 5.4 \times 10^{-6} P \text{ RAD.}$$

$$\text{MAX. } f_b = \frac{.475P}{.0408} = 11.6P \text{ K/IN}^2$$

$P_e = 3.45^K$

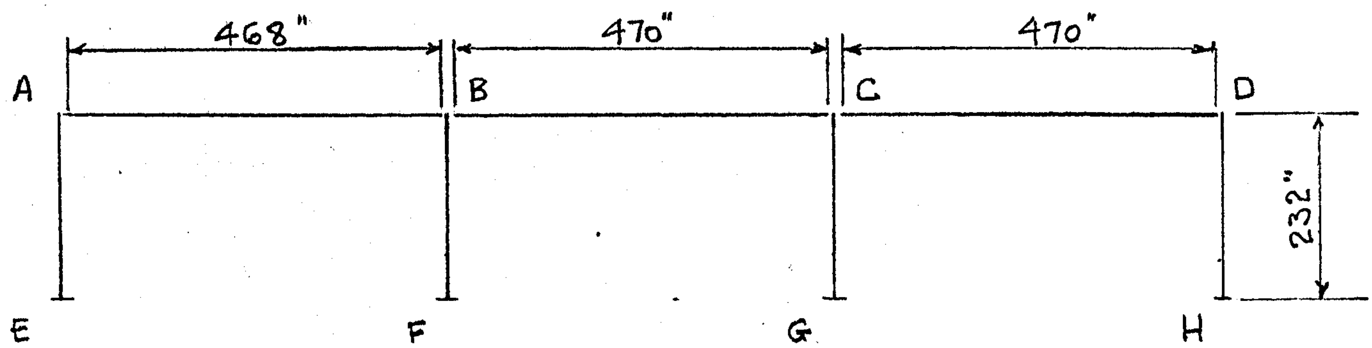
$M_e = 2 \times 3.45 \times 66.5 = 459.$ K-IN.

$\phi_e = 18.6 \times 10^{-6}$ RADIANS.

$$\frac{\phi_e}{M_e} = 0.0405 \times 10^{-6}$$

DEFORMATION OF FLANGE AND WEB, RESPECTIVELY, HAS BEEN OMITTED FROM ABOVE CALCULATION. IN BOTH CASES SUCH DEFORMATION WILL BE GREATLY REDUCED BY THE DISTRIBUTION OF THE LOAD OVER A SECTION OF COLUMN MUCH WIDER THAN THE SHORT CONNECTION ANGLES; IN ADDITION, THE WEB IS REINFORCED BY THE PRESENCE OF A SHELF-ANGLE ON THE OTHER SIDE.

IT WILL BE NOTED THAT THE MAXIMUM CHORD-REACTION MOMENT AT THE WEB-CONNECTED END WILL BE ABOUT $8 \times 66.5 = 530$ IN PLACE OF 330 K-IN. AT THE FLANGE-CONNECTED END.



$K_{AB} = \frac{13580}{468} = 29.02$ $\qquad 2EK = 1.741 \times 10^6$

NOTATION: SAME AS GIVEN ON p. 98.

$K_{BC} = K_{CD} = \frac{13580}{470} = 28.89$ $\qquad 2EK = 1.734 \times 10^6$

$K_{AE} = \frac{21.6}{232} = .093$ $\qquad 2EK = 5,580.$

$K_{BF} = K_{CG} = K_{DH} = \frac{170.9}{232} = .737$ $\qquad 2EK = 44,220.$

FIRST APPROXIMATION – IN BAY CD, ASSUME COLUMN CG REMAINS VERTICAL; ALSO ASSUME THAT M_{CD} WILL APPROACH THE MAXIMUM (p. 127) AND THAT MOMENT M_{DC} WILL REMAIN WITHIN THE ELASTIC RANGE (p. 128)

$$(\theta_{CG} - \phi_{CD}) = 17.19 \times 10^{-6} M_{CD} - 5672.7 \times 10^{-6} \quad (\text{IN WHICH } \theta_{CG} \text{ ASSUMED} = 0)$$

FOR COLUMN BASE AT H, USING THE METHOD SHOWN OF p. 93,

$$\theta_{HD} = -16.75 \times 10^{-6} M_{HD}$$

AT D, $(\theta_{DH} - \phi_{DC}) = 0.212 \times 10^{-6} M_{DC}$

F.E.M. FOR TRUSS = 3182 K-IN.

$$M_{CD} = 1.734 \times 10^6 (2\phi_{CD} + \phi_{DC}) + 3182$$
$$M_{DC} = 1.734 \times 10^6 (\phi_{CD} + 2\phi_{DC}) - 3182$$

$$M_{DH} = 44,220 (2\theta_{DH} + \theta_{HD})$$
$$M_{HD} = 44,220 (\theta_{DH} + 2\theta_{HD})$$

FROM THESE EQUATIONS WE OBTAIN:

$M_{CD} = 425.8$ K-INS.
$M_{DC} = -126.2$ K-INS.
$M_{DH} = +126.2$ K-INS.
$M_{HD} = +29.9$ K-INS.

$\phi_{CD} = -1647 \times 10^{-6}$ RAD.
$\phi_{DC} = +1705 \times 10^{-6}$
$\theta_{DH} = +1678 \times 10^{-6}$
$\theta_{HD} = -501 \times 10^{-6}$

A COMPLETE ANALYSIS OF THE BENT PRODUCED THE FOLLOWING RESULTS:

$M_{AB} = +16.8$ K-INS.	$M_{AE} = -16.8$	$M_{EA} = -4.5$
$M_{BA} = -425.5$	$M_{BF} = +1.8$	$M_{FB} = +0.8$
$M_{BC} = +423.7$		
$M_{CB} = -423.5$	$M_{CG} = -1.4$	$M_{GC} = -0.6$
$M_{CD} = +424.9$		
$M_{DC} = -126.3$	$M_{DH} = +126.3$	$M_{HD} = +29.9$

IT WILL BE SEEN THAT IN SPITE OF SOME LACK OF SYMMETRY IN THE COLUMN ARRANGEMENT, THE TWO INTERIOR COLUMNS RECEIVE VERY LITTLE MOMENT, AND THAT THE ANALYSIS OF BAY CD ON THE ASSUMPTION THAT THE COLUMN AT C REMAINS VERTICAL IS VERY CLOSE TO THE MORE ELABORATE ANALYSIS AS FAR AS THE MOMENT VALUES ARE CONCERNED.

THE CORRESPONDING BENDING STRESSES IN THE COLUMNS ARE:

COL. AE,	8"WF28	$S_y = 6.6$	$f_b = 2.55\ ^K/_{IN^2}$ (TOP)	$0.68\ ^K/_{IN^2}$ (BOTTOM)
COL. BF,	10"WF33	$S_x = 35.0$	0.05	0.02
COL. CG,	"	"	0.04	0.02
COL. DH,	"	"	3.61	0.85

BENDING STRESSES IN THE TRUSS MEMBERS WILL BE GOVERNED BY:

1) ELASTIC DEFORMATION OF TRUSS UNDER LOAD, SUPERIMPOSING ON THE EFFECTS OF THE SIMPLE SPAN LOADING, THE AXIAL FORCES DUE TO HORIZONTAL CHORD REACTIONS AT THE COLUMNS.

2) DISTRIBUTION THROUGH THE TRUSS OF THE BENDING CAUSED IN THE ENDS OF THE BOTTOM CHORD AND IN THE END DIAGONAL, BY THE RIGIDITY OF THE BOTTOM CHORD CONNECTIONS.

3) BENDING DUE TO ECCENTRICITY AT JOINTS WILL BE NIL, SINCE IN THE DETAILS OF THE WELDED TRUSSES, THE CENTROIDS OF THE MEMBERS HAVE BEEN MADE CONCURRENT AT THE JOINTS.

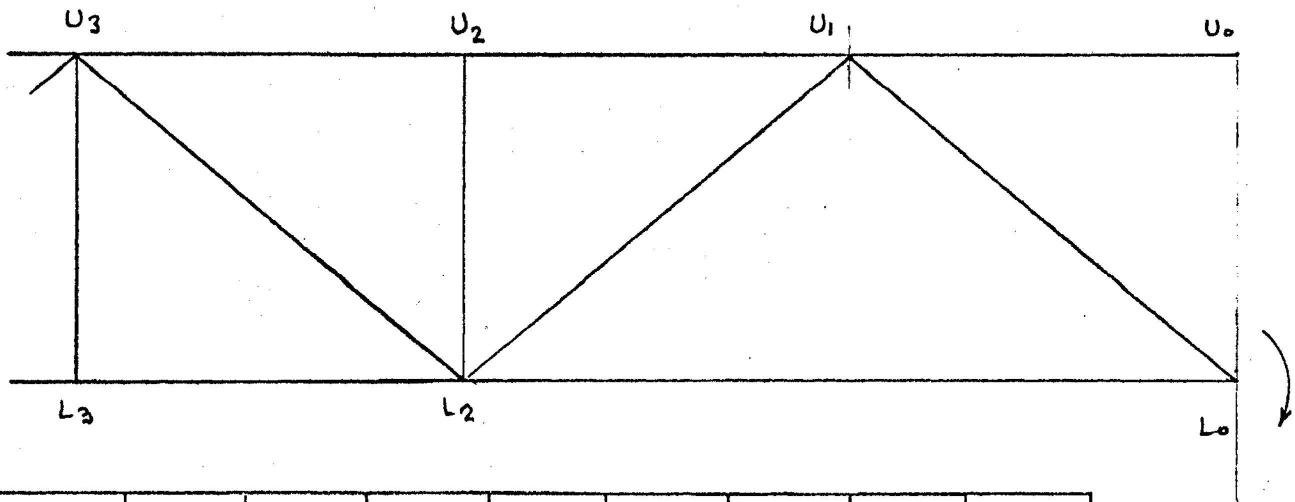
EFFECT OF END MOMENTS ON TRUSSES.

AT THE INTERIOR COLUMNS, TAKE MOMENT = 424 K-INS. FROM PREVIOUS ANALYSIS (p. 126, 127), THE VALUE OF THE MOMENT AT THE BOTTOM CHORD CONNECTION, $M_c = 424 - 330 = 94$ K-IN. THIS WILL BE DIVIDED BETWEEN THE BOTTOM CHORD AND THE END DIAGONAL THUS:

DIAGONAL: $94 \times .13 = 12.2$ K-IN.

CHORD : $94 \times .87 = 81.8$ K-IN.

REFERENCE (17) SUPPLIES A METHOD OF DISTRIBUTING EXTERNALLY-APPLIED MOMENTS SUCH AS THESE, EXTENDING TO THE ADJOINING PANELS ONLY;



MEMBER	L_0L_2	L_2L_3	L_0U_1	U_1L_2	U_1U_0	U_1U_2	L_2U_3	L_2U_2
I_{xx}	64.6	64.6	5.8	2.2	23.5	23.5	3.8	1.3
EFFECT. L	150	80	90	90	70	80	90	56
$K = I/L$	.431	.808	.065	.024	.336	.294	.042	.023

$M_{L0L2} = -81.8$

$M_{L0U1} = -12.2$

AT U_1 TAKING K' FOR $U_1U_0 = .336 \times \frac{3}{4} = .252$, THE RELATIVE STIFFNESSES OF THE MEMBERS AT THE JOINT ARE: U_1U_2 .463; U_1U_0 .397; U_1L_2 .038; U_1L_0 .102.

AT L_2 THE RELATIVE STIFFNESSES ARE: L_2L_0 .325; L_2L_3 .608; L_2U_3 .032; L_2U_1 .018; L_2U_2 .017.

FORMULA FOR $M_{L2L0} = \frac{M_{L0L2}}{2} \cdot \frac{1 - r_{BA}}{1 - \frac{r_{BA}}{4}} = -\frac{81.8}{2} \cdot \frac{1 - .325}{1 - .081} = -30.0$

$M_{U1L0} = -\frac{12.2}{2} \cdot \frac{1 - .102}{1 - .026} = -5.8$

$$M_{L2L3} = +\frac{81.8}{2}\cdot\frac{.608}{1-.081} = +27.1 \qquad M_{U1U0} = +\frac{12.2}{2}\cdot\frac{.397}{1-.026} = +2.5$$

$$M_{L2U1} = \quad \cdot\frac{.018}{\quad} = +\ 0.8 \qquad M_{U1U2} = \quad \cdot\frac{.463}{\quad} = +2.9$$

$$M_{L2U2} = \quad \cdot\frac{.017}{\quad} = +\ 0.8 \qquad M_{U1L2} = \quad \cdot\frac{.038}{\quad} = \underline{+0.3}$$

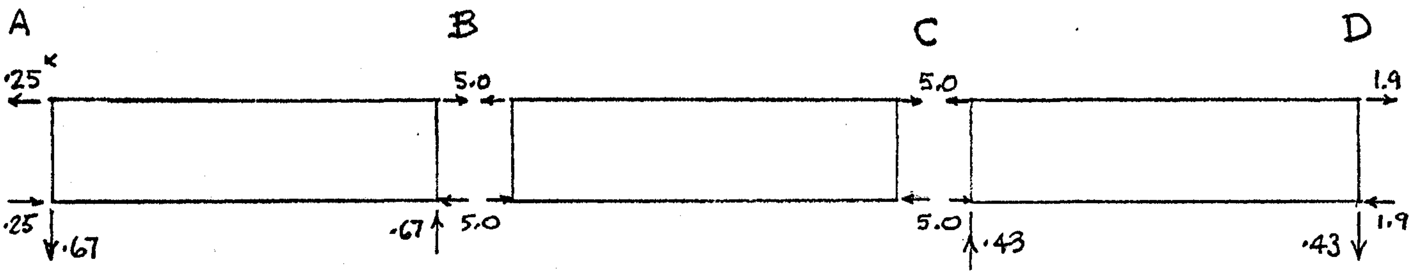
$$M_{L2U3} = \quad \cdot\frac{.032}{\quad} = \underline{+\ 1.4} \qquad \underline{+5.7}$$

$$\underline{+30.1}$$

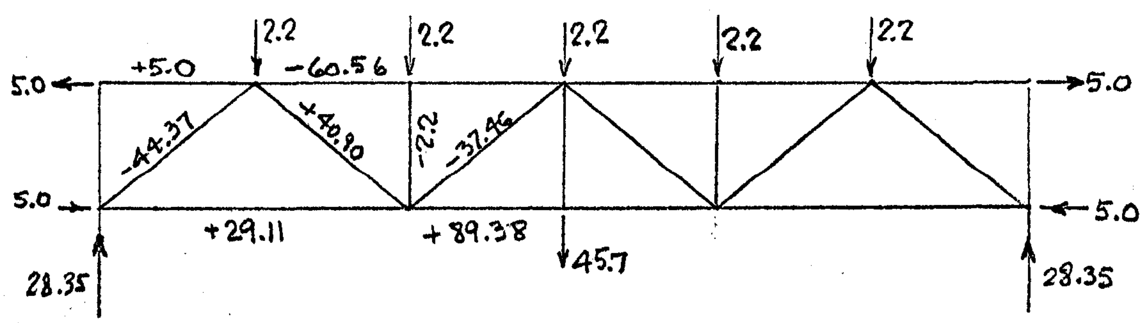
THE STRESSES TRANSMITTED BETWEEN U_1 AND L_2 ARE SMALL, AND ARE NOT RE-DISTRIBUTED.

AT THE EXTERIOR COLUMNS, THE END MOMENTS ARE ALMOST ENTIRELY DUE TO THE HORIZONTAL CHORD REACTIONS. THE MOMENT DUE TO THE RIGIDITY OF THE BOTTOM CHORD BECOMES SMALL ENOUGH TO BE NEGLECTED.

THE ARRANGEMENT OF THE HORIZONTAL REACTIONS IS AS SHOWN:



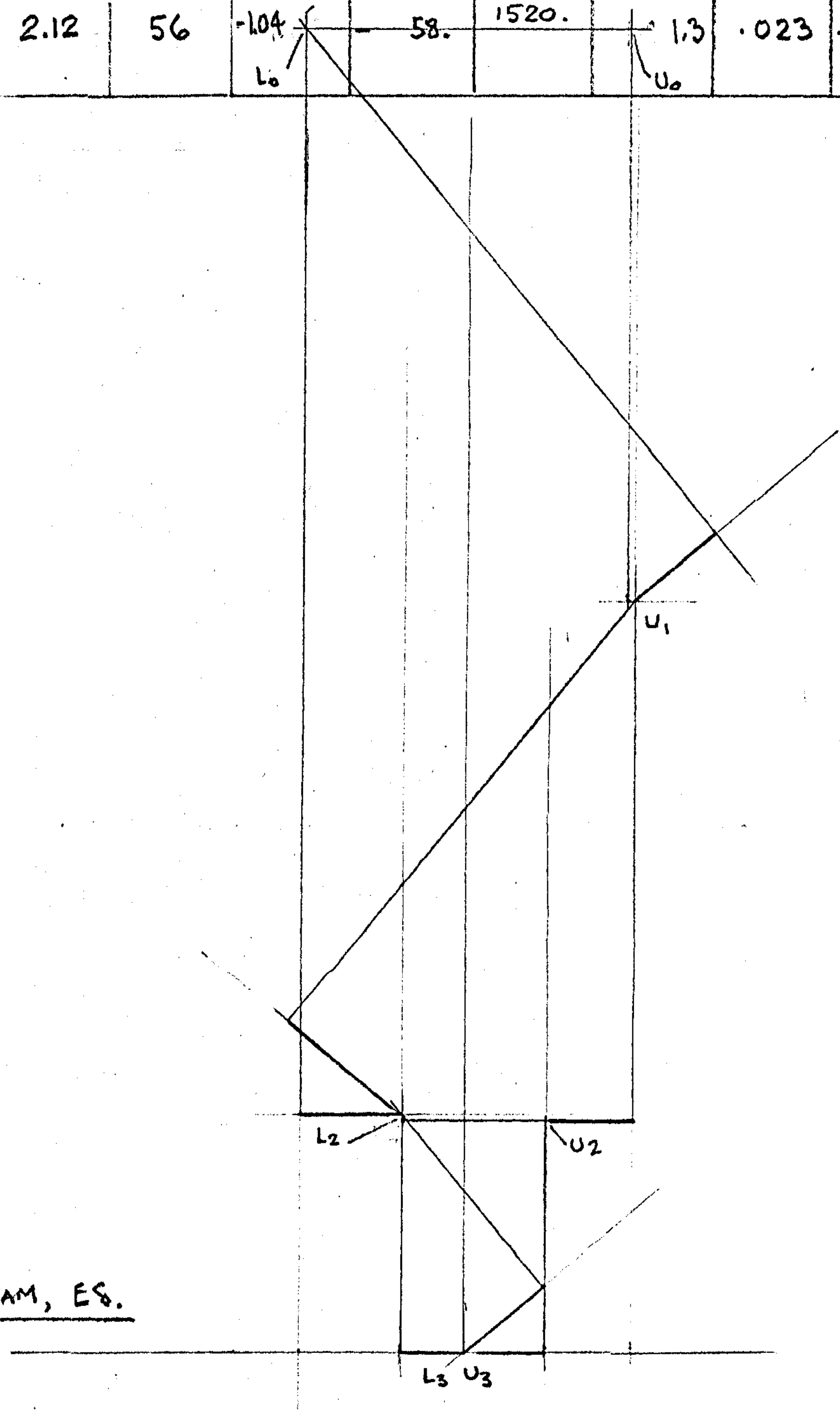
ELASTIC DEFORMATION OF TRUSS, SPAN BC; BELOW:



VERTICAL LOADING PLUS HORIZONTAL REACTIONS; FORCES IN TRUSS.

MOMENTS DUE TO ELASTIC DEFORMATION — UNDISTRIBUTED.

MEMBER	AXIAL FORCE (KIPS)	MEMBER AREA A (IN²)	EFFECTIVE LENGTH L (INS.)	P/A	Eδ = PL/A	Eδ₁ FROM W. DIAGRAM	I_{xx} (IN⁴)	K = I/L	Eδ₁/L = Eρ	6K	M = −6EKρ
U_0U_1	+5.0	5.59	70	+.89	+62	5900.	23.5	(.252)	−83.3	1.512	+126.
U_1U_2	−60.6	5.59	80	−10.85	−868.	5390.	23.5	.294	−67.4	1.764	+119.
U_2U_3	−60.6	5.59	80	−10.85	−868.	2400.	23.5	.294	−30.0	1.764	+53.
L_0L_2	+29.1	6.72	150	+4.34	+651.	11210.	64.6	.431	−74.8	2.586	+194.
L_2L_3	+89.4	6.72	80	+13.30	+1067.	2460.	64.6	.808	−30.8	4.848	+145.
L_0U_1	−44.4	3.62	90	−12.28	−1105.	6780.	5.8	.065	−75.3	.390	+30.
U_1L_2	+40.9	2.38	90	+17.18	+1546.	5600.	2.2	.024	−62.2	.144	+9.
L_2U_3	−37.5	3.12	90	−12.01	−1081.	2320.	3.8	.042	−25.8	.252	+7.
U_2L_2	−2.2	2.12	56	−1.04	−58.	1520.	1.3	.023	−27.2	.138	+4.

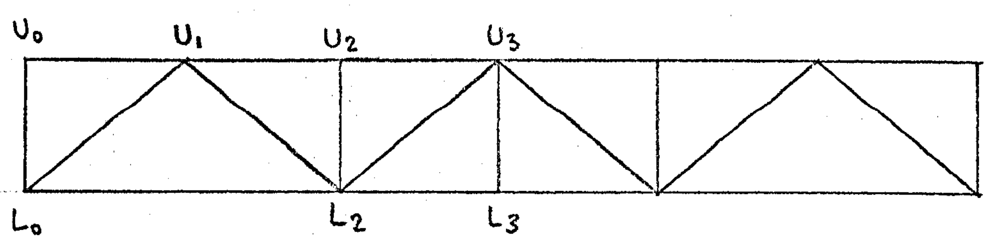


WILLIOT DIAGRAM, Eδ.

SCALE 20

Secondary moments and stresses in truss members (L.H. half)

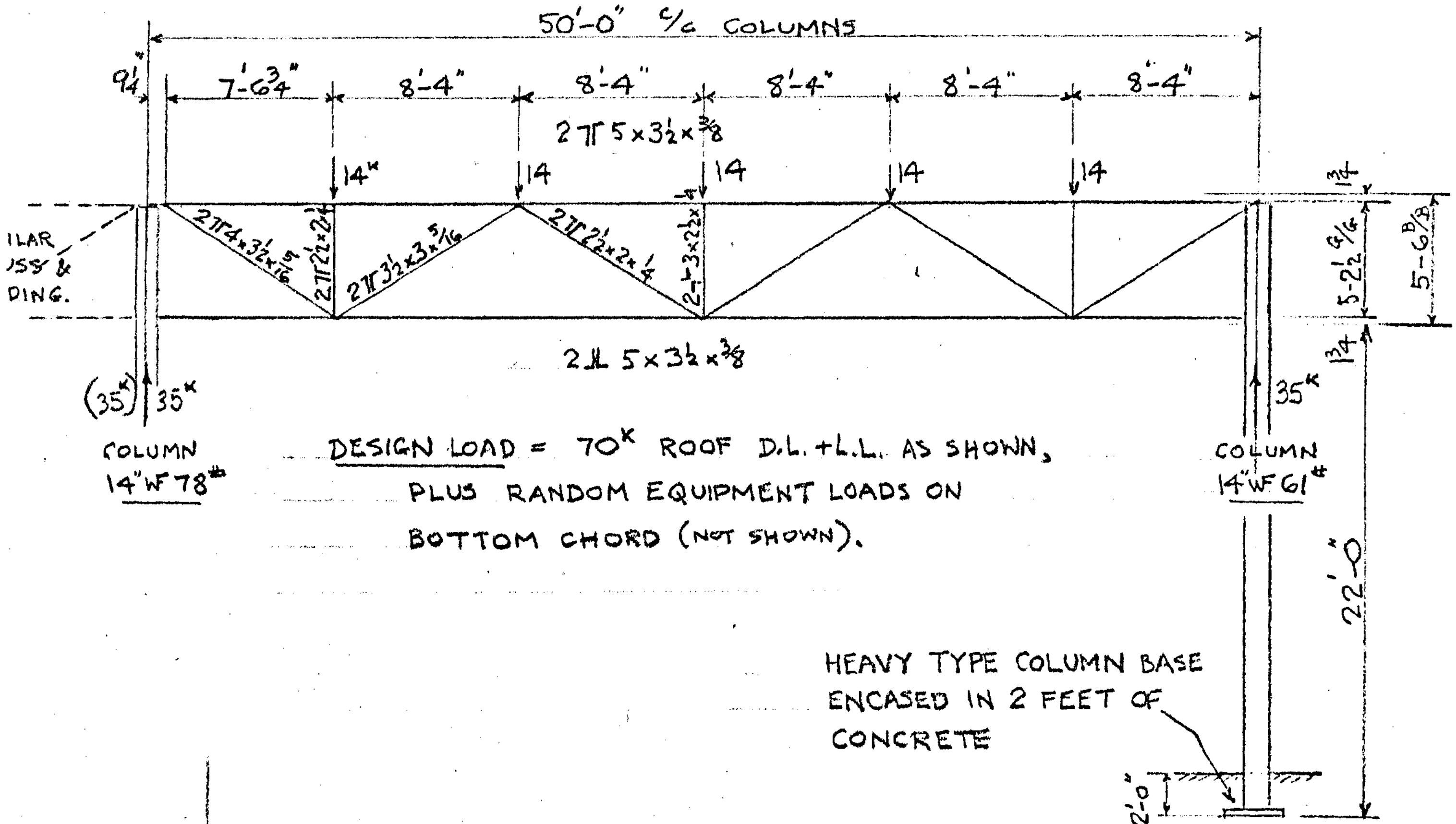
LOCATION OF MOMENT	MOMENTS			SECTION MODULUS		BENDING STRESS	
	DUE TO DEFORMN.	DUE TO END CONN.	COMBINED	HEEL	TOE	HEEL	TOE
U_0U_1	0	-	0	15.05	4.27	0	0
U_1U_0	+31	-3	+28	"	"	-1.9	+6.6
U_1U_2	-21	-3	-24	"	"	-1.6	+5.6
U_2U_1	+6	-	+6	"	"	-0.4	+1.4
U_2U_3	-4	-	-4	"	"	-0.3	+0.9
U_3U_2	+25	-	+25	"	"	-1.7	+5.9
L_0L_2	+5	+82	+87	16.2	16.2	-5.4	+5.4
L_2L_0	+30	+30	+60	"	"	+3.7	-3.7
L_2L_3	-25	-27	-52	"	"	+3.2	-3.2
L_3L_2	+60	-	+60	"	"	+3.7	-3.7
L_0U_1	-5	+12	+7	5.0	2.1	+1.4	-3.3
U_1L_0	-7	+6	-1	"	"	+0.2	-0.5
U_1L_2	-3	0	-3	2.2	1.1	-1.4	+2.7
L_2U_1	-1	-1	-2	"	"	+0.9	-1.8
L_2U_3	-2	-1	-3	3.7	1.6	-0.8	+1.9
U_3L_2	+3	-	+3	"	"	-0.8	+1.9
U_2L_2	-2	-	-2	1.6	0.76	+1.2	-2.6
L_2U_2	-2	-1	-3	"	"	-1.9	+3.9

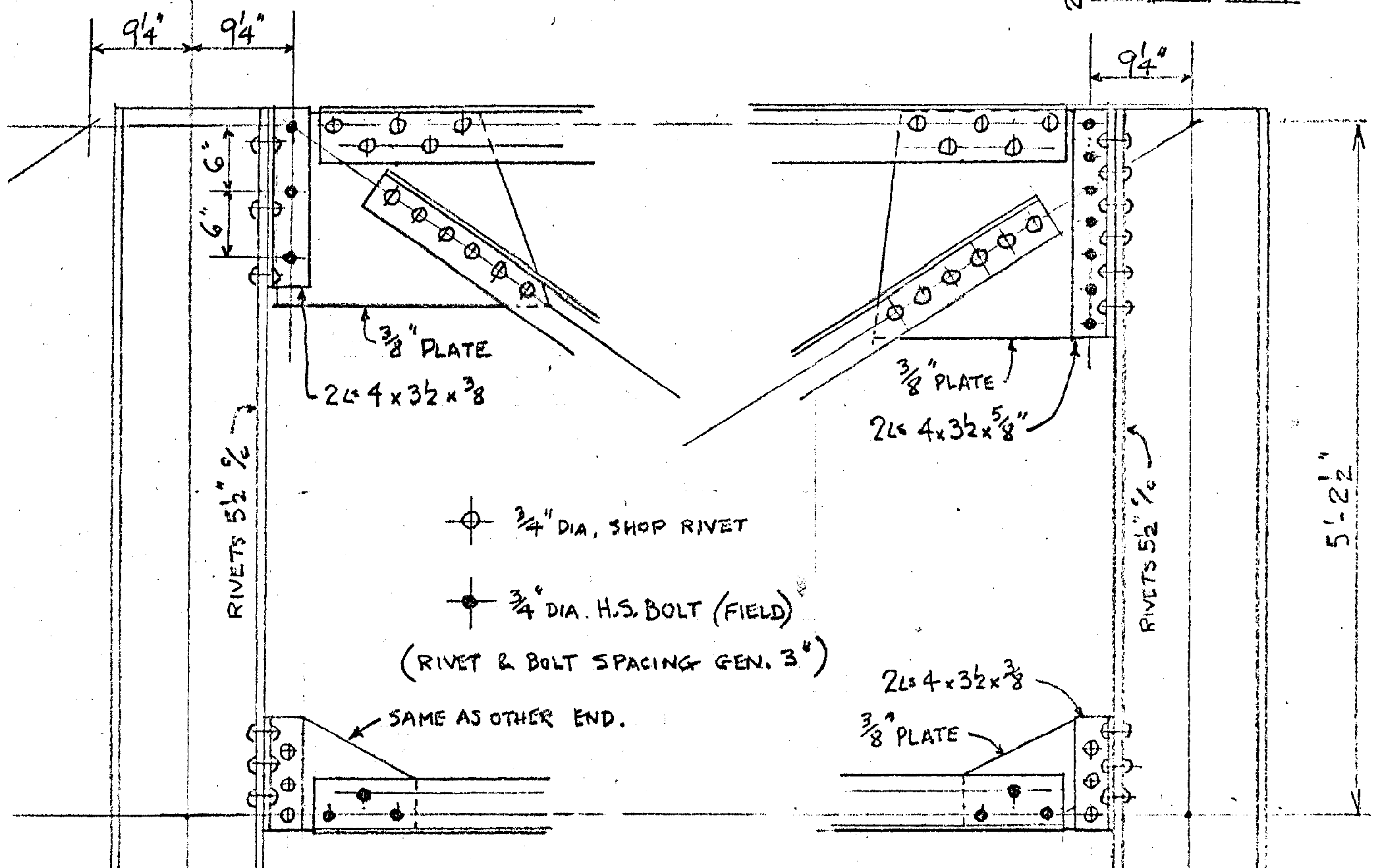


From the sketch of the connection details used in structure "790", it will be seen that the development of moments at the ends of the truss will depend on the degree to which the field bolted joints will resist slippage. The condition is somewhat analagous to the connection examined in figure 53. The behaviour of the arrangement will have to be considered in two stages; before and after the tightening of the bolts. In the "before" stage, while the bolts will not actually be loose, they will not provide very effective resistance to the movement of the ends of the truss when it deflects. Assuming that normal erection practice has prevailed, and that the final tightening of the high-strength bolts to the specified minimum tension, is not performed until all the steel in the immediate area has been erected, it will follow that any movement produced by the dead weight of the steel itself, will not be resisted unless the bolts are forced into bearing. In this case, it was assumed that 5 k of the total load of 14 k at each panel point would be due to the weight of steel alone. The relative horizontal displacement of the ends of the chords, under this load, has been measured from a Williot diagram at 0.075". Since this is much less than the play in the holes, which would permit a relative displacement of 0.125", it appears that the deformation of the truss under this portion of the load will not cause any appreciable end moments.

In the second stage, the bolts are assumed to be correctly tightened, and therefore capable of developing the resistance to slip indicated by the results of research (19). This condition is discussed in the following pages.

STRUCTURE "790", TRUSS T1.





TRUSS CONNECTIONS.

3/4" dia. H.S.Bolt in double shear = 25 K slip resistance.

Three-bolt groups at left top and right and left bottom connections would provide, each, 3 x 25 = 75 K slip resistance.
Seven bolts at top right: 7 x 25 = 175 K slip resistance.
These figures correspond with a maximum moment = 75 x 62.5 = 4680 K-in
In the bottom chord connections, however, a lower limit is imposed by the group of three shop rivets, to which may be assigned the following value, based on experiment (19): 3 x 22 = 66 K max.

giving a maximum moment = 66 x 62.5 = 4130 K-ins.

These resisting-moments will not be developed unless the components of the various connections can transmit the corresponding forces.

The bottom chord connections are quite adequate to transmit 66 K in compression.

Investigation of the connection at the left-hand end of the top chord follows:

Using method and symbols of pages 86,87:

$a = 2"$ $b = 1\text{-}7/16"$ For 1" length of 3/8" angle:

$I = .00439 \text{ in}^4$

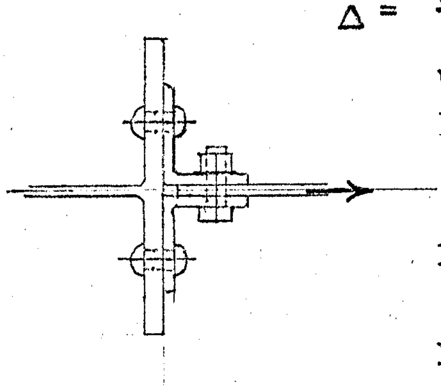
$S = .0234 \text{ in}^3$

$Z = .0352 \text{ in}^3$

$M_{riv.} = 1.46 \text{ P}$

$M_{heel} = 1.08 \text{ P}$

$\Delta = \dfrac{1.23 \text{ P}}{EI} = .00935 \text{ P}$



where P = pull per 1" per angle.

Bending stresses = $\dfrac{1.46P}{.0234} = 62.4 \text{ P}$ K/sq.in.

and $\dfrac{1.08P}{.0234} = 46.2 \text{ P}$ K/sq.in.

Plastic condition begins at rivet when

$62.4 \text{ P} = 40$ or $\text{P} = 0.64 \text{ K}$

For fully plastic condition, $M_{riv.} = M_{heel} = .0352 \times 40$
$= 1.41$ K-ins.

Giving maximum value of $\text{P} = \dfrac{2 \times 1.41}{2} = 1.41 \text{ K}$

If this value were developed uniformly through the full length (16.5") of the angles, the resulting tensile capacity would be

$1.41 \times 2 \times 16.5 = 46.5 \text{ K}$

If the connection detail produces more tension in the upper portion, partly owing to the location of the chord, and partly owing to a tendency for the system to rotate with the deflection of the truss, this maximum tension will not be realized. However, rotation will tend to be about the more resistant bottom chord connection, with the result that the plastic condition in the top connection will develope progressively from top to bottom of these angles.

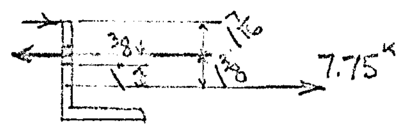
This calculation leads to the conclusion that at the left-hand end of the truss, the maximum moment which could be developed is

$$46.5 \times 62.5 = 2900 \text{ K-ins.}$$

Checking the total tension against the tensile capacity of the rivets in the column flange: 46.5/6 = 7.75 K per rivet, giving a tensile stress of 7.74/0.44 = 17.65 K/ sq.in. This is less than the tensile strength of a rivet. It should be noted, however, that these rivets are subjected to a prying action, in resisting the moment developed in the angles. This could produce a rivet tension

$$= \frac{7.75 \times 2.82}{1.44} = 15.18 \text{ K per rivet,}$$

or 15.18/ 0.44 = 34.5 K/sq.in.



Since the material specification for rivet steel requires a minimum yield point of 28 ksi and a tensile strength of 52 to 62 ksi, it appears that the development of the full plastic strength of the connection angles might be accompanied by deformation of the rivets.

<u>Connection at Right-hand End of Top Chord.</u>

1" section of 5/8" thick angle: $I = .0203$

$S = .0651$

$Z = .0976$

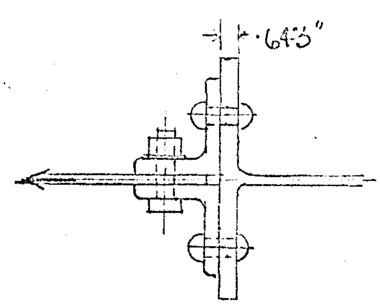
a = 1-7/8"

b = 1-3/8"

$M_{riv.} = 1.08\ P$

$M_{heel} = 0.793\ P$

$$\Delta = \frac{.804P}{EI} = .00132\ P$$



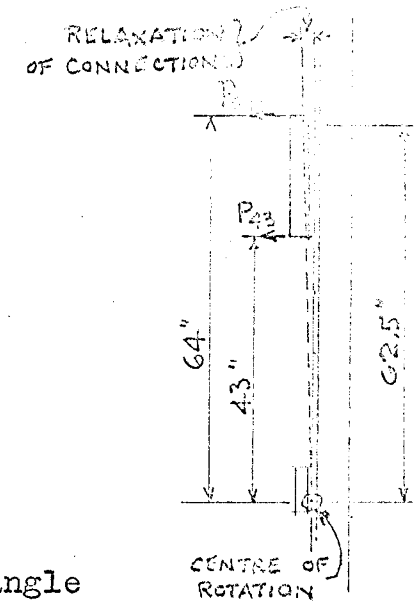
1" section of column flange: $I_c = .0222$

$$\therefore \Delta_c = Pc^3/\ 12EI_c$$

$$= \frac{P\ (2.5)^3}{12 \times 30{,}000 \times .0222}$$

$$= .00196\ P$$

Rotation of end of truss relative to axis of column will tend to be about a centre at the bottom chord, owing to lack of flexibility in that connection. In this case, elastic deformation and accompanying bending stresses in top chord connection angles will be in proportion to the distance from this centre.



If P is the tensile pull per inch of angle, one could write P_{64} (at top of conn.) = 64p

and P_{43} (at bott. ") = 43p

giving a resultant for the whole connection

$$= \frac{P_{64} + P_{43}}{2} \times 21p = \frac{107 \times 21p}{2} = 1122p \text{ K per angle}$$
$$= 2244p \text{ K total.}$$

As the maximum compression capacity of the bottm chord connection is 66K (page 137), the same magnitude will apply to the tension at the top;

$$\therefore 2244p = 66, \text{ or } p = .0294$$

This makes P_{64} = 64 x .0294 = 1.88 K per inch of angle.

From data on p.86, maximum bending in angles will be

$$1.08\ P_{64} = 2.03 \text{ K-in.}$$

and maximum bending stress = 2.03/ .0651 = 31.2 K/sq.in.

This indicates that stresses in the top chord connection remain in the elastic range.

Max. shear per bolt = 2 x 1.88 x 3 = 11.3 K (double shear, adequate)

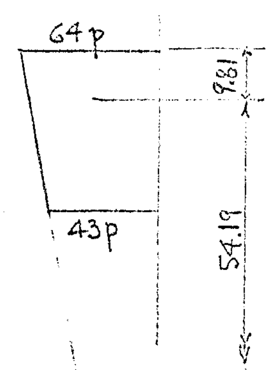
Max. tension per rivet = 1.88 x 4(abt.) = 7.5 K, causing

f_t in rivet = 7.5/ 0.44 = 17 K/sq. in. (elastic range)

Approximate moment arm at end of truss = 54.2"

Making maximum end moment = 66 x 54.2

= 3577 K-ins.



Deformation at top of connection (P = 1.88)

= (.00132 + .00196) 1.88 = .00617"

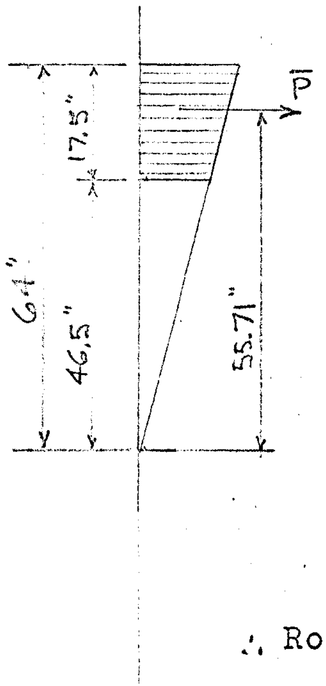
Angle of rotation = .00617/64 = .00009635 rad.

$\therefore \gamma$ for connection = $(96.35 \times 10^{-6})/ 3577$

= $.027 \times 10^{-6}$ radians/ K-in.

This is quite a rigid connection.

In the elastic range, the condition at the left-hand end of the truss produces the following:



If P_h = tension per inch per angle at distance h from centre of rotation at bottom chord,

$\overline{P}$ = total tension developed in connection

$= (P_{64} + P_{46.5}) \times 17.5$

Put $P_{64} = 64p$ and $P_{46.5} = 46.5p$,

then $\overline{P} = 110.5p \times 17.5 = 1933.75p$

Centre of tensile force is at (64 - 8.29) inches above centre of rotation,

$\therefore$ Moment $= 55.71\overline{P} = 107{,}729p$ K-ins.

$P_{64} = 64p$, referring to properties of connection, p.137

maximum bending stress $= 62.4P = 3993.6p$

and deflection $\Delta = .00935p = .598p$ inches.

$$\therefore \text{Rotation} = \frac{.598p}{55.71} = .01073p \text{ radians.}$$

$$\therefore \gamma = \frac{.01073p}{107{,}729p} = 0.0996 \times 10^{-6} \text{ radians/ K-ins.}$$

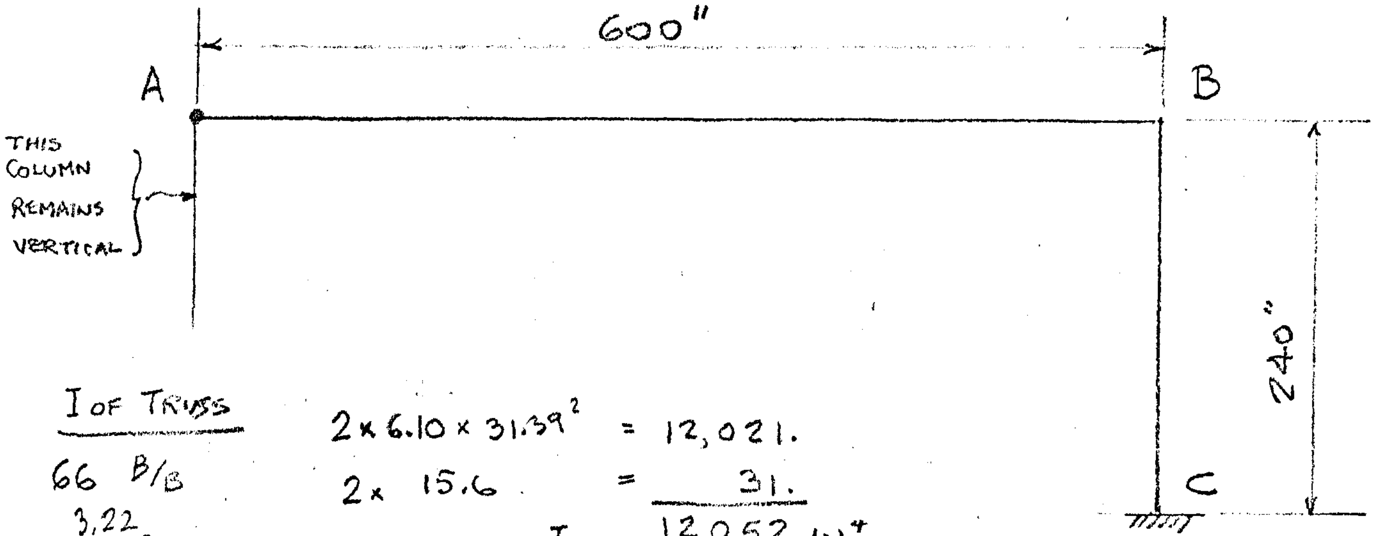
(say 0.100×10^{-6})

Limit of elastic range is attained when maximum stress = yield point; then if

$$3993.6p = 40.0,$$
$$p = .0100$$

This corresponds with $M = 107.729 \times 0.0100 = 1077.3$ K-ins.

(It will be found that the end moment developed at thia end of the truss is well below this figure. Hence the value of γ calculated above may legitimately be used.)



I OF TRUSS

66 B/B

$2)\overline{6.278}$ = 3.22 C/CENTROIDS

31.39"

$2 \times 6.10 \times 31.39^2 = 12{,}021.$

$2 \times 15.6 = 31.$

$I = 12{,}052 \text{ IN}^4$

K FOR TRUSS $= \dfrac{12{,}052}{600} = 20.09 \quad 2EK = 1{,}205{,}200$

K FOR COLUMN $= \dfrac{641.5}{240} = 2.68 \quad 2EK = 160{,}375$

F.E.M. FOR TRUSS (FOR 45^K LOAD ADDED AFTER BOLTS ARE TIGHTENED)

$$= \frac{45 \times 600}{12} = 2250 \text{ K-IN.}$$

TREATING A AS A FIXED END, B AS A RIGID JOINT, C AS FIXED:

$$\left.\begin{aligned}
M_{AB} &= 1.2052 \times 10^6 \left(2\phi_{AB} + \phi_{BA}\right) + 2250 \\
M_{BA} &= 1.2052 \times 10^6 \left(\phi_{AB} + 2\phi_{BA}\right) - 2250 \\
M_{BC} &= .1604 \times 10^6 \left(2\phi_{BC} + \phi_{CB}\right) \\
M_{CB} &= .1604 \times 10^6 \left(\phi_{BC} + 2\phi_{CB}\right)
\end{aligned}\right\}$$

SOLUTION GIVES:

$M_{AB} = +1256$ K-INS.

$M_{BA} = -M_{BC} = -265$ K-INS.

$M_{CB} = +133$ K-INS.

THE CONDITIONS AT THE JOINTS ARE MORE ACCURATELY REPRESENTED BY THE EQUATIONS:

$\phi_{AB} = -.100 \times 10^{-6} M_{AB}$

$(\theta_{BC} - \phi_{BA}) = .027 \times 10^{-6} M_{BA}$

UNDER THESE CONDITIONS, THE FOLLOWING MOMENTS ARE FOUND: $M_{AB} = +273$. K-INS. $M_{BC} = +301.$

$M_{BA} = -301. \quad M_{CB} = +150.$

f_b IN COLUMN AT B $= \frac{301}{92.2} = 3.26 \text{ K/in}^2$

IN THIS EXAMPLE, IT IS INSTRUCTIVE TO NOTE THAT WHERE THE COLUMN CAN DEFLECT, THERE IS LITTLE RELAXATION IN THE CONNECTION, BUT THAT AT THE RESTRAINED COLUMN THE FLEXIBILITY OF THE CONNECTION IS IMPORTANT.

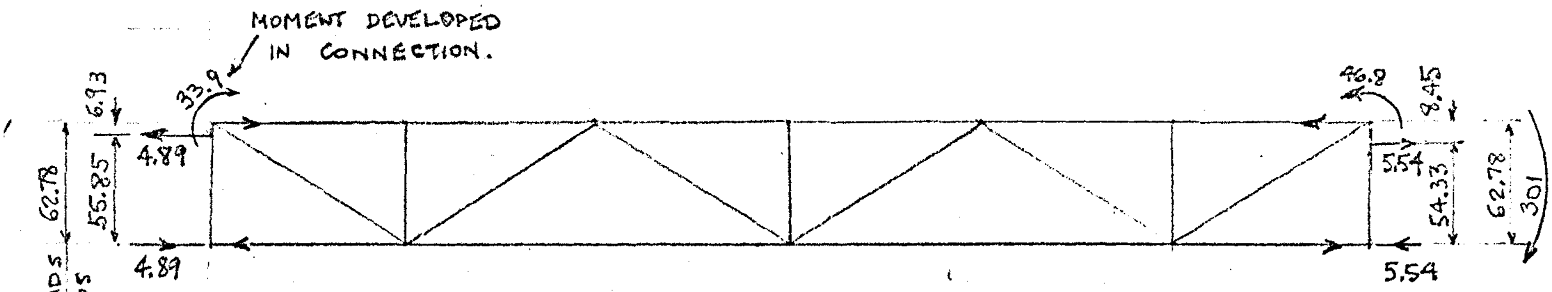
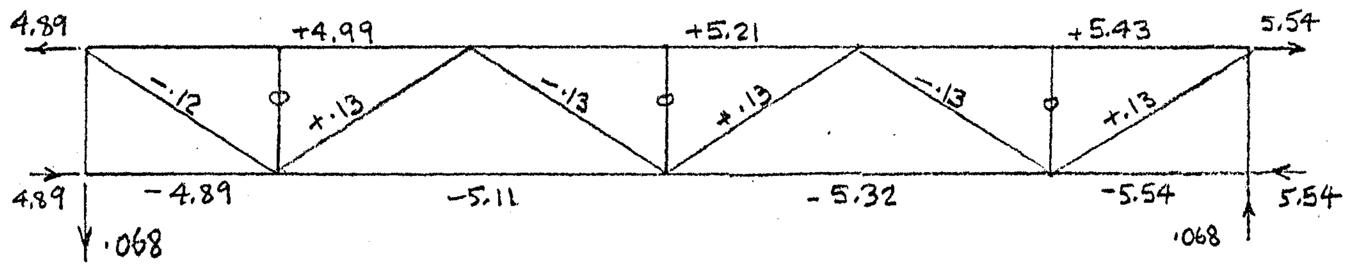
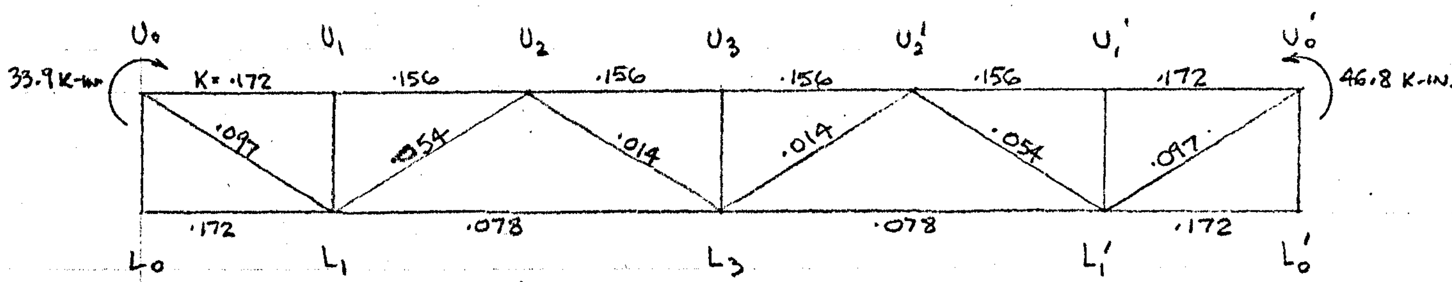


DIAGRAM OF END MOMENT APPLIED TO TRUSS, HORIZONTAL REACTIONS & "LOCAL" MOMENTS.



TRUSS FORCES DUE TO END REACTIONS.



EFFECT OF "LOCAL" MOMENTS IN T.C. CONNECTIONS.

USING THE METHOD OF DISTRIBUTION OF I.R.A.M. (17): (MOMENTS IN KIP-INS.)

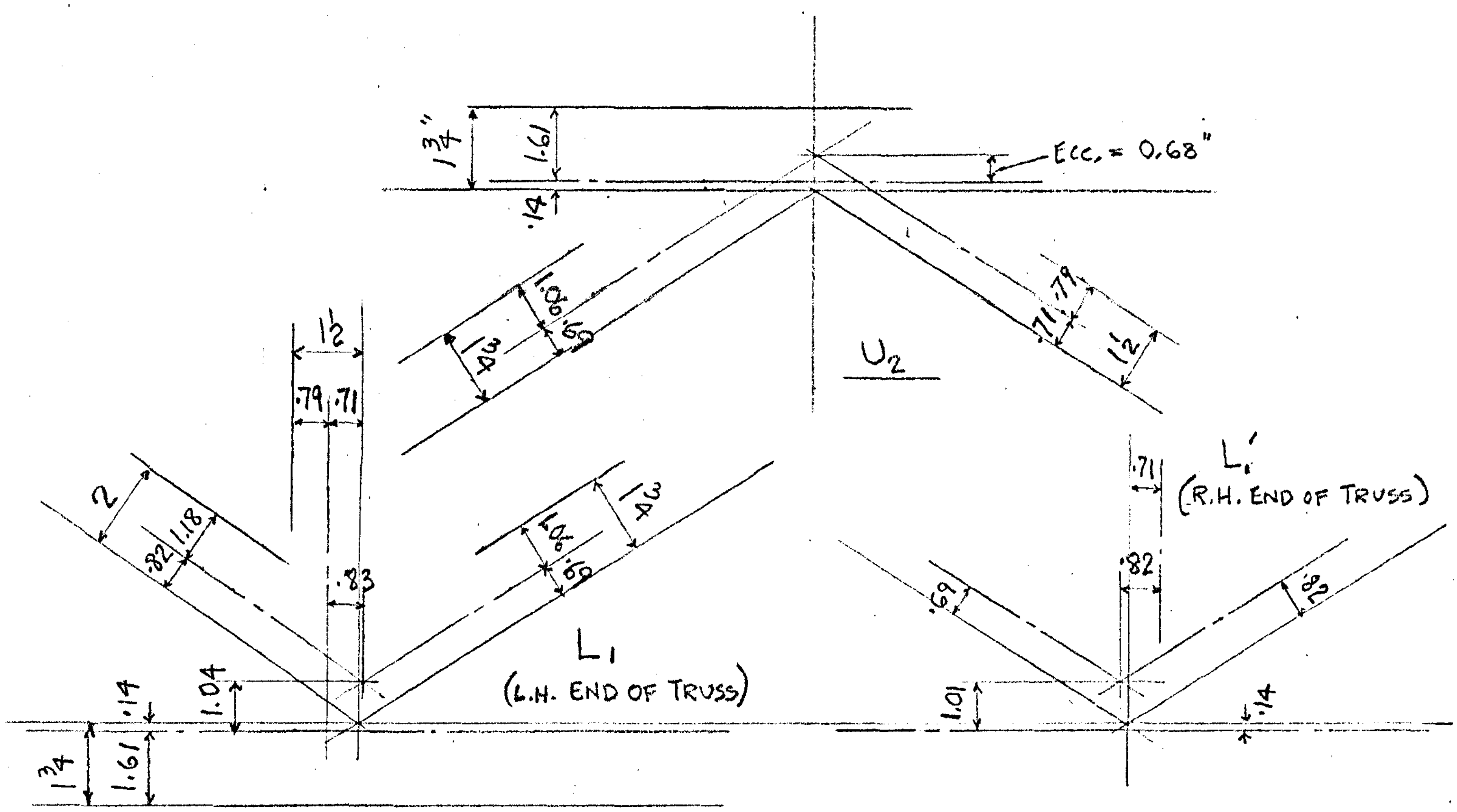
LOCATION	MOMENT	LOCATION	MOMENT	LOCATION	MOMENT
U_0U_1	−21.7	L_1L_3	+1.2	$L_1'U_1'$	−0.6
U_1U_0	−6.3	U_1L_1	+0.9	$L_1'U_2'$	−1.1
U_1U_2	+5.5	$U_0'U_1'$	+30.0	$L_1'L_3$	−1.6
U_0L_1	−12.2	$U_1'U_0'$	+8.8	$U_1'L_1'$	−1.3
L_1U_0	−5.0	$U_1'U_2'$	−7.5		
L_1L_0	+2.6	$U_0'L_1'$	+16.8		
L_1U_1	+0.4	$L_1'U_0'$	+6.9		
L_1U_2	+0.8	$L_1'L_0'$	−3.6		

Axial loads and deformations in truss members.

MEMBER	AREA A IN^2	EFFECTIVE LENGTH L IN.	FORCE DUE TO VERT. LOADING P_V	FORCE DUE TO HOR. REACTIONS P_H	TOTAL FORCE $P = P_V + P_H$	LONGITUDINAL DEFMN. × E $= E\delta = PL/A$
U_oU_1	6.10	87	-56.0	+5.0	-51.0	- 727
U_1U_2	"	100	-56.0	+5.0	-51.0	- 836
U_2U_3	"	100	-100.8	+5.2	-95.6	-1568
U_3U_2'	"	100	-100.8	+5.2	-95.6	-1568
$U_2'U_1'$	"	100	-56.0	+5.4	-50.6	- 828
$U_1'U_o'$	"	87	-56.0	+5.4	-50.6	- 721
L_oL_1	6.10	87	0	-4.9	- 4.9	- 70
L_1L_3	"	200	+89.6	-5.1	+84.5	+2770
L_3L_1'	"	200	+89.6	-5.3	+84.3	+2765
$L_1'L_o'$	"	87	0	-5.5	- 5.5	- 90
U_oL_1	4.50	77	+66.1	-0.1	+66.0	+1129
L_1U_2	3.86	87	-39.6	+0.1	-39.5	- 890
U_2L_3	2.12	96	+13.2	-0.1	+13.1	+ 593
L_3U_2'	2.12	96	+13.2	+0.1	+13.3	+ 602
$U_2'L_1'$	3.86	87	-39.6	-0.1	-39.7	- 894
$L_1'U_o'$	4.50	77	+66.1	+0.1	+66.2	+1132
U_1L_1	2.12	54	-14.0	0	-14.0	- 357
U_3L_3	2.62	54	-14.0	0	-14.0	- 289
$U_1'L_1'$	2.12	54	-14.0	0	-14.0	- 357

The vertical loading used above is the full roof design load of 14 K per panel point. The horizontal reactions and forces caused thereby are as shown on p. 142

ECCENTRICITIES AT TRUSS JOINTS.



MOMENTS DUE TO ECCENTRICITY:

AT U_2: $(95.6-51.0)\cdot 68 = 30$ K-IN.

AT L_1: $(84.5+4.9) \times 1.04 = 93$ K-IN.

AT L_1': $(84.3+5.5) \times 1.01 = 91$ K-IN.

IN VERTICALS:

U_1L_1 $14.0 \times .71 = 10.$

L_1U_1 $14.0 \times .83 = 12.$

$U_1'L_1'$ $14 \times .71 = 10$

$L_1'U_1'$ $14 \times .82 = 12$

MOMENTS ARE CALCULATED IN ACCORDANCE WITH FORCES IN TRUSS MEMBERS AS TABULATED UNDER "P" ON p. 143.

Moments developed in truss members due to elastic deformation, to eccentricity in connections, and to top chord connections.

MEMBER	L IN.	I IN^4	$K = \frac{I}{L}$	PERP. DISPLMNT. $E\delta_1$	$E\rho = E\delta_1/L$	$M_D = -6EK\rho$	M_D DISTRIB.	M_E DISTRIB.	$M_D + M_E = M_1$	MAX. M_2 DUE TO LOCAL MOM.	$M = M_1 + M_2$
U_0U_1	87	15.6	.179	11800	-135.7	145.8	-8	-8	-16	-30	-46
U_1U_0	87	"	.179	11800	-135.7	145.8	+15	-4	+11	- 9	+ 2
U_1U_2	100	"	.156	8040	- 80.4	75.1	-14	-7	-21	+ 8	-13
U_2U_1	"	"	"	8040	- 80.4	75.1	-2	-16	-18	-	-18
U_2U_3	"	"	"	3930	- 39.3	36.8	-7	-15	-22	-	-22
U_3U_2	"	"	"	3930	- 39.3	36.8	+14	-7	+ 7	-	+ 7
L_0L_1	87	15.6	.179	11420	-131.2	141.0	0	0	0	-	0
L_1L_0	87	"	"	"	"	"	0	+39	+39	+ 4	+43
L_1L_3	200	"	.078	11680	- 58.4	27.0	-13	+21	+ 8	+ 2	+10
L_3L_1	200	"	"	"	"	"	+6	+10	+16	-	+16
U_0L_1	77	7.1	.092	12820	-166.7	92.0	+8	+8	+16	-17	- 1
L_1U_0	"	"	"	"	"	"	+15	+26	+41	- 7	+34
L_1U_2	87	4.7	.054	9400	-108.0	35.0	+1	+13	+14	-	+14
U_2L_1	"	"	"	"	"	"	+9	+2	+11	-	+11
U_2L_3	96	1.3	.014	3900	- 40.6	3.4	0	-1	- 1	-	- 1
L_3U_2	"	"	"	"	"	"	+2	0	+ 2	-	+ 2
U_1L_1	54	1.3	.019	5180	- 96.0	10.9	-1	+1	0	+ 1	+ 1
L_1U_1	"	"	"	"	"	"	-3	+5	+ 2	+ 1	+ 3

Note: The slight lack of symmetry due to the horizontal reactions does not appear in the Williot diagram from which values of $E\delta_1$ were obtained. The moments due to distribution of the local moments are actually those calculated for the right-hand end of the truss.

Secondary Bending Stresses in Truss Members.

MOMENT LOCATION	MOMENT	SECTION MODULUS		BENDING STRESS	
		HEEL	TOE	HEEL	TOE
$U_0 U_1$	-46	9.7	4.6	-4.7	+10.0
$U_1 U_0$	+ 2	"	"	-0.2	+ 0.4
$U_1 U_2$	-13	"	"	-1.3	+ 1.9
$U_2 U_1$	-18	"	"	+1.9	- 3.9
$U_2 U_3$	-22	"	"	-2.3	+ 4.8
$U_3 U_2$	+ 7	"	"	-0.7	+ 1.5
$L_0 L_1$	0	9.7	4.6	0	0
$L_1 L_0$	+43	"	"	+4.4	- 9.3
$L_1 L_3$	+10	"	"	-1.0	+ 2.2
$L_3 L_1$	+16	"	"	+1.7	- 3.5
$U_0 L_1$	- 1	6.0	2.5	-0.2	+ 0.4
$L_1 U_0$	+34	"	"	-5.7	+13.6
$L_1 U_2$	+14	4.4	1.9	+3.2	- 7.4
$U_2 L_1$	+11	"	"	-2.5	+ 5.8
$U_2 L_3$	- 1	1.6	0.76	-0.6	+ 1.3
$L_3 U_2$	+ 2	"	"	-1.2	+ 2.6
$U_1 L_1$	+ 1	"	"	-0.6	+ 1.3
$L_1 U_1$	+ 3	"	"	+1.9	- 4.0

Moments are given in Kip-inches.
Stresses are given in K/ sq. in.
Sign Convention: + Moments act counterclockwise on members.
Tensile stress + Compressive stress -

Effect of Secondary Stresses in Beams.

Where secondary stresses are found in beams, they generally take the form of reversed or negative bending stresses at the ends of the span. These have the effect of reducing the positive bending moment in the mid-span region to some value less than what would be found in a true simple span. Since in the conventional design method the section of the beam is selected on the basis of the maximum simple-span moment, the secondary bending leaves the beam with bending stresses lower than those for which it was designed.

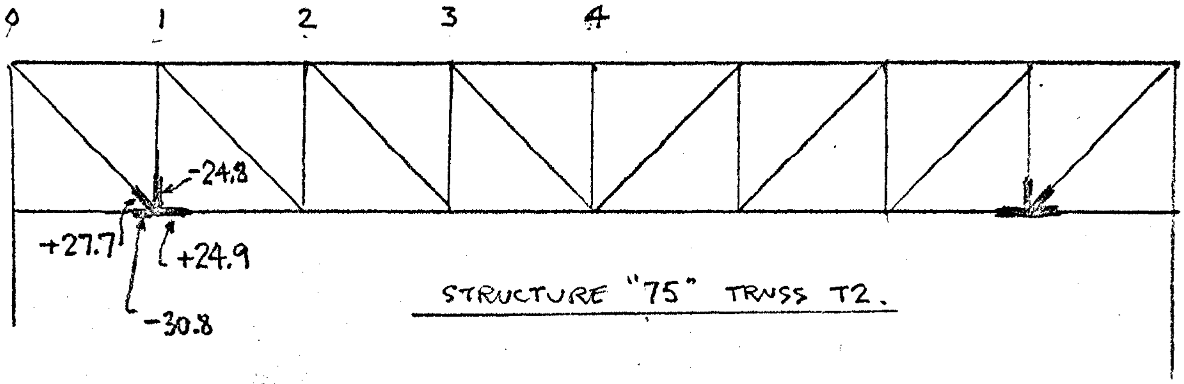
Effect of Secondary Stresses in Trusses.

The internal secondary stresses in trusses produce bending stresses in truss members in the region of the connections, and these stresses will be superimposed on those due to the axial loads. On the other hand, the effect of end moments at the supports will, like those in a beam span, reduce the maximum axial forces in the truss, and thereby cause some reduction in the internal secondary stresses also.

In figure 57 four of the trusses which have been analyzed as simple-span structures, are shown, with regions of high stress singled out. Figure 58 is an enlarged detail of one of these regions, showing both maximum and minimum stresses in the members. Some of the maxima appear to be very high, when the specified permissible stress is compared with them. However, whereas the axial stresses are approximately constant from one end of a member to the other, the bending stresses decrease quite rapidly away from the ends.

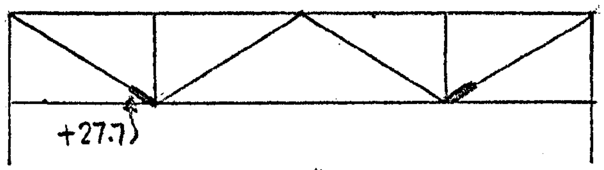
Research by Parcel and Murer (23) on the effect of secondary stress on ultimate strength led them to the conclusion that ultimate strength would only be affected adversely in a certain compression condition. To quote some of their conclusions: "As a result of the re-adjustment in stress-strain relations, it may be inferred:

(a) That any tension members tends, as the load increases, to a condition of uniformity of stress over the cross-section, except for a local over-strain in a short section near each end."



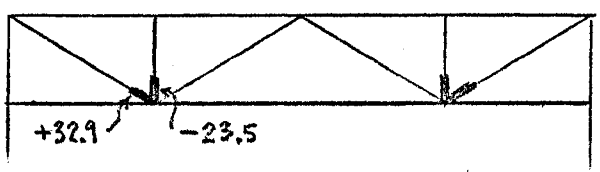
STRUCTURE "75" TRUSS T2.

SEE p. 110 FOR DETAILS.



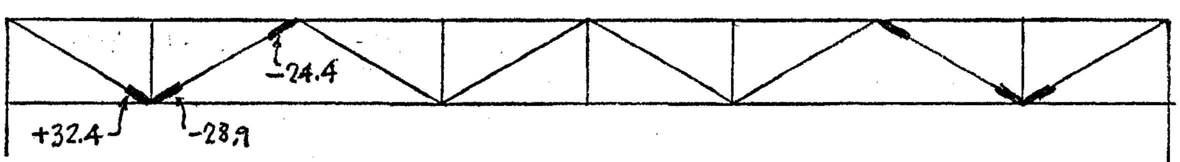
p. 113

STRUCTURE "49" TRUSS A82



p. 115

STRUCTURE "49" TRUSS B82

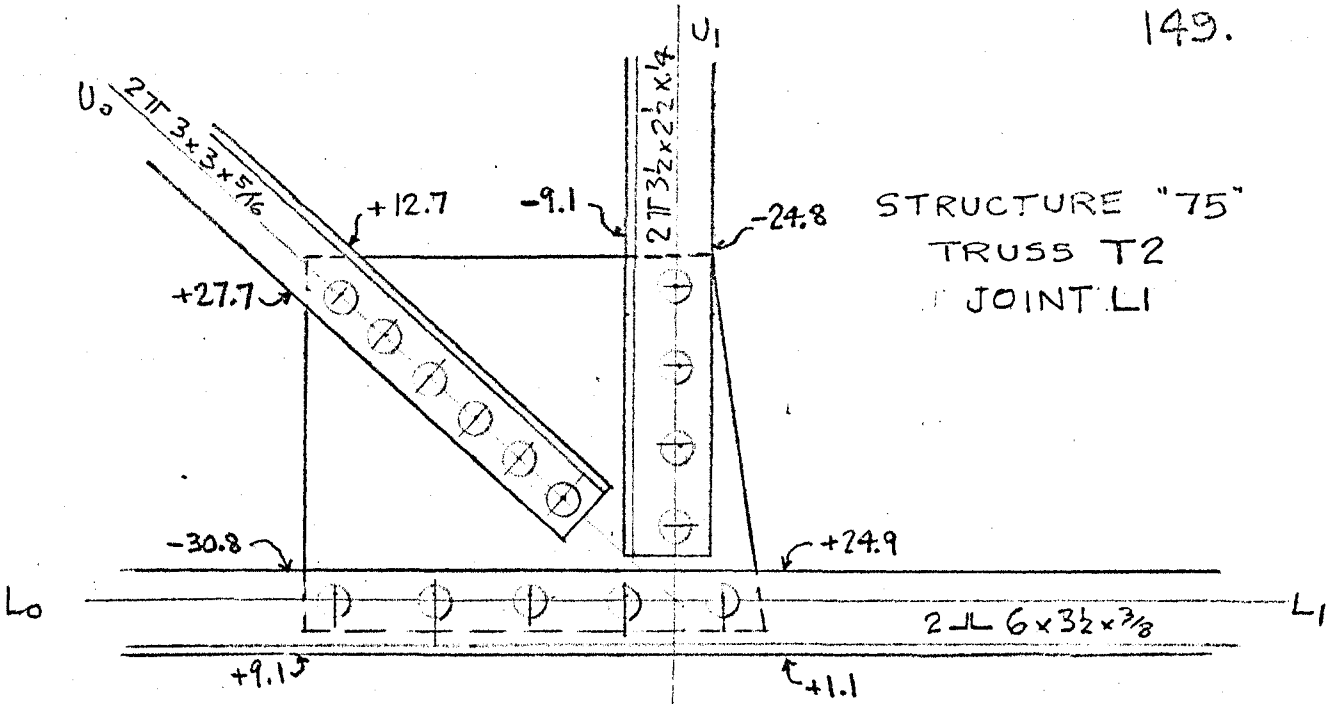


p. 116

STRUCTURE "49" TRUSS A84

DIAGRAMS SHOWING LOCATION OF OVER-STRESSED PARTS OF TRUSSES.
FIGURES GIVEN REPRESENT MAXIMUM COMBINED STRESS AT END OF BAR, IN K/IN^2.

FIGURE 57.



MEMBER:	L_1L_0	L_1U_0	L_1U_1	L_1L_2	
AXIAL FORCE:	0	+60.5	-40.5	+44.5	(KIPS)
AXIAL STRESS:	0	+17.0	-14.1	+6.5	(K/IN^2 ON GROSS AREA)
BENDING STRESS:					
HEEL	+9.1	-4.3	+5.0	-5.4	SEE P. 112
TOE	-30.8	+10.7	-10.7	+18.4	
RESULTANT:					
HEEL	+9.1	+12.7	-9.1	+1.1	
TOE	-30.8	+27.7	-24.8	+24.9	

FIGURE 58.

"(b) Compression members bent in single curvature may be seriously affected as regards L/r failure if the transverse deflections due to secondary bending become large. This can only occur, however, in the case of large secondary moments and flexible members, a combination that is not ordinarily realizable. It was noted previously that for values of L/r less than or equal to 70, the effect of secondary action in inducing L/r failure is small - well under that corresponding to a pin-ended column with normal 'equivalent eccentricities' at the ends. In such a case the rigid joint action which gives rise to secondary stress acts as a brake on the long column deflection before the point of ultimate column strength is reached."

"(c) For a column bent in double curvature, the secondary action, by forcing the curvature into two 'waves', may actually have a beneficial effect as regards L/r failure."

In a paper on the basis of design in B.S. 153 (24), Kerensky, Flint and Brown make the following statement:
"Since secondary stresses do not materially affect the ultimate strength of the structure, no additional margin of safety is provided on their account and all the above factors may in fact be further reduced by about 20% when the unavoidable secondary stresses develop."

The above findings on the effect of secondary stresses are reassuring to the designer. However, it must be remembered that in discussing the basis of B.S.153, the British writers have in mind the conditions prevailing in bridge design. Others who have contributed to the subject, such as Johnson, Bryan and Turneaure, have approached it from the same viewpoint. It is necessary, therefore, to question whether their conclusions can be applied, without reservation, to building frames. The eccentricity at connections, which can contribute a major portion of the secondary stresses in building trusses, is generally avoided in bridge details. A comparison of the width-length ratios of some of the truss members herein analyzed with the empirical rule given by the AREA or in BS 153, will indicate that it is possible, in an ordinary roof-truss, for a slender member to be affected by appreciable secondary bending. This can, in most cases, be traced to the effect of a large eccentricity in one of the joints.

In tension members, an analysis of the type conducted by Parcel and Murer confirms their conclusion that secondary bending well in excess of that found in these roof trusses, would cause plastic yielding in the area of the highest stress, with a redistribution of stress over the cross-section. In compression members, those in the web, which generally appear more critical, usually carry moments of opposite or double curvature, the advantages of which are mentioned by Parcel and Murer. It must still be observed that the effect of large eccentric moments on web members can hardly be viewed with complacency.

Effect of Secondary Stresses in Columns.

In the structures examined above, secondary bending stresses in columns have been found, ranging from 2 to 10 ksi. The addition of such stresses to the reaction moment stresses and the ordinary axial stresses could produce high concentrations in column flanges. Such would be the condition in the region of the beam or truss connection, where failure would have to take place as the result of yielding. In the mid-length region, where failure by buckling could occur, the effective moments would be less.

In the structures analyzed, omitting the column with the fixed base, since this detail is not usual in conventional design, it appears that the moment at the column base ranges from about 0.18 to 0.29 times the moment at the top. If the Massonnet formula is used to determine the effective moment at mid-length:

$$M_e = \sqrt{0.3(M_1^2 + M_2^2) - 0.4(M_1 M_2)}$$

then the effective moment will be 0.46 to 0.49 times the moment at the top of the column. Taking structure "90" as typical, the following would result:

TOP OF COLUMN: AXIAL STRESS $f_a = \frac{6.55}{4.62} = 1.42$ KSI; BENDING STRESS $f_b = \frac{60.4}{10.1} = 6.0$ KSI $\}$ RESULTANT = 7.42 KSI.

MID-LENGTH; FOR $L/r = \frac{143}{1.45} = 99$, $F_a = 14.2$ KSI. $M_e = 60.4 \times .47 = 28$ K-IN

$$f_b = \frac{28}{10.1} = 2.8 \qquad \frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{1.42}{14.2} + \frac{2.8}{22.0} = .10 + .13 = \underline{.23}$$

The column just taken as an example is evidently understressed, even when the secondary stresses are included in the calculation. In this type of structure, such a condition is not uncommon. Several causes may be distinguished. First, a certain minimum slenderness ratio must be maintained. Second, it is not practical to call for a column section which is too small to permit the correct type of beam connection to be used. Third, in some applications, columns are subject not so much to overload as to possible collision by moving equipment in the building. The experienced designer, knowing that only a slight increase in the weight of the section will provide a column with greatly increased capacity, will usually take advantage of the fact. Columns may be over-designed for these or for other reasons, but such a practice cannot be assumed as a constant factor, to be relied upon in the formulation of design rules. For that purpose, it must be assumed that column design will be such as to take full advantage of the unit stresses permitted by the design specification.

For a stocky compression member, the permissible stress would be equal to the basic design stress; 60% of the specified minimum yield point, in the CSA specification. This would be the limiting value for the combined effect of axial load plus reaction bending. In the same specification, the permissible axial stress for a member with L/r equal to 100, is about 14.5 ksi for all steels, the value being governed by the value of E rather than the yield point. If the calculated axial and bending stress (not secondary) at mid-length of the slender column should form the maximum combination permitted by the application of the interaction formula, then the addition of secondary bending stress would in effect reduce the margin between the maximum stress in the beam and that determined by the Euler crippling curve. If the bending stress at mid-length happened to be 2.5 ksi, for a steel with a basic design stress of 22 ksi, then the increase in the bending stress factor would be 0.113. The effect would be the same as an increase of 1.65 ksi in the axial stress.

COMBINED EFFECTS OF MATERIAL AND STRESS VARIATIONS.

In the discussion of material properties, it has been shown that the following conditions affect the manner in which the information supplied by the manufacturers on the strength of structural sections, must be interpreted:

(1) Difference between reported (upper) yield point and actual (plastic) yield strength: approximately 5%.

(2) Difference between yield point as determined in commercial tests, and as determined under very slow application of load: approximately 1%.

(3) Amount by which yield point in web of beam or channel section (reported) may exceed yield point in flange: about 12.5%

(4) Amount by which the sectional area, and corresponding geometrical properties, of a rolled section, may differ from the theoretical figure: 2.5%.

By combining the above factors, (1), (2) and (4) give a resultant factor:

$$1.05 \times 1.01 \times 1.025 = 1.087$$

For beam and channel shapes, which are affected by (3) above, the resultant of all factors is:

$$1.087 \times 1.125 = 1.222$$

From the above, the actual effective yield strength of any plate or angle section would have a minimum value of 1/1.087 (or 0.920) times the reported yield point, while for beam and channel shapes the ratio would be 1/1.222 (0.818).

In CSA S16, the currently accepted ratio between the guaranteed minimum yield point as given in the material specification and in the mill test reports, and the permissible basic design stress, is 1.65/1. The relationship arrived at above between this yield point and the effective minimum yield strength would reduce this factor as follows:

(a) for plate and angles: $1.65 \times 0.920 = 1.519$

(b) for beams and channels: $1.65 \times 0.818 = 1.350$

The next factor to be considered is that of possible overload on the structure, which is taken at 5% of the total loading. From this, the actual stress intensities in the structure may exceed those appearing in the design calculations by 5%. At any point in the system where the full permissible load occurs in the design, the actual stress intensity might reach 1.05 times that figure. This reduces the factors given above, thus:

(a) for plates and angles: 1.519/ 1.05 = 1.447

(b) for beams and channels: 1.350/ 1.05 = 1.286

Further, it becomes necessary to assess the effects of secondary stresses in the class pf structure most liable to be affected by them. It is obviously impractical to seek a formula which will express accurately all the possible conditions governing the magnitude of secondary stresses in building frames. The trusses which have been analyzed herein display local bending stresses ranging as high as 20 ksi in an "active" member, and over 30 ksi in a "null" member. These extremes occur in trusses analyzed as freely-supported structures, and would be reduced if the system analyzed were made to include the supports; but the possibility must be admitted of similar conditions arising in trusses actually free from end moments. The opinion has been advanced (23,24), that secondary stresses do not affect the ultimate strength of structures, but some doubt exists as to the universal application of this principle.

In compression chords of trusses, secondary stresses appear to range up to about 11% of the allowable axial stress, and plastic yielding in such members must be regarded as more serious than in the web members. In columns, while secondary stresses appear for the most part to lie in the range up to 5 ksi, at least one case has been found with a stress of 10 ksi. These are on the order of 23% and 46% of the permissible basic stress. The permissible axial stress in a column will usually be governed by the slenderness ratio, and will be less than the axial stress which would be permitted at the point of support. For a value of L/r equal to 50, the axial stress at mid length would be about 4 ksi less than that permitted at the support. This leaves, in effect, a margin of some 4 ksi at the top of the column where the highest bending stress occurs, so that the actual excess due to the secondary bending there will be, say, 10 ksi less 4 ksi, or 6 ksi. Altenratively, if conditions at mid-length are examined, the secondary stress will be reduced to, s ay, 0.47 of the value at the top (see page 151), or 2.3 to 4.7 ksi. Thus the secondary stress increment at the top works out at about 27%, and at mid-length 26% of the permitted axial stress.

Allowing for variations in the properties of steel, tolerance on sectional dimensions, and possible overloading, it has already been esimated that the ratio of yield strength to actual stress intensity will be 1.447 for plates and angles and 1.286 for beams and channels. This must now be modified to take into account the effects of secondary stresses. In trusses generally, the members are composed of pairs of angles. Where other sections are used, such as tees cut from beams, it will be found that the truss is of welded construction, and will be subject to relatively small secondary stresses. Assuming the 11% factor found for chords, and applying it to the factor already found for plates and angles, the resultant becomes:

$$1.447 / 1.11 = 1.304$$

Beams and channels, when used as flexural members, will generally be assisted by secondary bending. Beam or wide-flange shapes, however, used as columns, and having a possible stress increment of 26%, would be subject to a resultant factor of

$$1.286 / 1.26 = 1.021$$

The second of these factors suggests that the ratio between yield point and permissible stress at present prescribed for building frames may in certain cases provide adequate, but not excessive, allowance for safety.

CONCLUSIONS.

A steel structure could be designed so as to be exactly adequate for its prescribed function, without excess strength, only if the following conditions were realized: the mechanical properties and exact dimensions of each piece of steel to be used should be known, all the combinations of loading which would be applied to the structure should be known, and analytical facilities should be available for the calculation of all axial, bending, shear and torsional stresses throughout the members and their connections.

Since the conditions described above are not attainable in practice, it becomes necessary to examine the extent and the accuracy of the information which is available, and to estimate the effects of the methods which the designer may employ in using that information. From this examination and this estimate a conclusion must be drawn as to the safety and serviceability of structures as actually designed.

The first conclusion now presented is, that the information at present available is not sufficiently complete to permit an exact scientific analysis of all the factors which relate the properties of material to the adoption of permissible stresses and to design practices. It is true that research has been conducted over a considerable period, while manufacturing processes have been improved and refined, and design methds have become more sophisticated, but the large body of data which would be required for a complete statistical study of the problem does not appear to be available. This is, in fact, partly due to recent advances, in which a statistical pattern has not yet become established. In consequence of this situation, design practice must still be governed to some extent by what Professor Freudenthal has called "the organized collective experience of the profession".

The second conclusion is, that within the limits described, the present relationship prescribed between material properties and design stresses, in current design specifications, are adequate, but in some structural patterns do not provide an excess of strength such as might be surmised from a casual comparison of the figures. It should be noted that some structural items have a much more favourable factor of safety than others, when designed according to the conventional method, and that those methods of design which take into account some properties of structures which are ignored in the conventional method, provide an appreciably greater margin between working stress and the point of failure. Theoretically, a design method which took into account the so-called secondary stresses, could be allowed to extract more capacity from the material.

The third conclusion is, that standard specifications and other guides to design practice should be sufficiently explicit to guard against the adoption or perpetuation of any design practice whichis technically unsound, or which may lead to inaccuracies of serious proportions. An example of the first is the assumption that loads applied to columns may always be regarded as concentric with the axis of the column, no matter how they are actually transmitted. An example of the second is the assumption that common structural connections never produce an appreciable moment, or that the effects of eccentricity in connections may always be neglected.

The fourth conclusion is, that the precepts derived from past experience, while invaluable, cannot safely be used without due regard for the effects of changing circumstances.

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1926 In March 1926, entered the employment of Dorman Long and Company at their offices in London, England.

1929 In June 1929, arrived in Canada and entered employment in the drawing office of The Canadian Bridge Company in Windsor (Walkerville) Ontario. Became Chief Draftsman in 1941, and Chief Design Engineer in 1958.

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