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Analysis of clamped skewed plates subjected to lateral uniform loading.

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ANALYSIS OF CLAMPED SKEWED PLATES SUBJECTED TO LATERAL UNIFORM LOADING

A THESIS

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by

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ABSTRACT

The Rayleigh-Ritz method is used to determine the deflections and moments of clamped skewed plates. A deflection configuration of the skewed plate is assumed in the form:

$$w = (1 - \alpha^2)^2 (\gamma^2 - \beta^2)^2 (A_o - A_1 \alpha\beta)$$

where A_o and A_1 are undetermined parameters defining the shape of the deflection surface.

By minimizing the total energy expression for the bending of plates, the parameters A_o and A_1 are evaluated and a deflection equation for the plate established. The deflection equation is then differentiated accordingly to obtain moments and stresses at various points of the plate.

An experiment was performed on the bending of a clamped skewed plate with a skew sides ratio of 1.23 and a skew angle of 55 degrees. The results of this experimental investigation are compared with those obtained by the Rayleigh-Ritz procedure.

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CHAPTER I

INTRODUCTION

Skewed plates and slabs are often required as component parts for large scale structures, such as bridges and building floor systems. The bending behaviour of such plates is also of great interest to the aircraft industry, where such plates are found frequently as parts of swept-back wings and fins of subsonic and supersonic aircrafts.

Many diverse and indirect methods are now available for the analysis of clamped rectangular plates subjected to uniform normal loadings. Some of these methods are cited by Timoshenko and Woinowsky-Krieger (1) who employ a double series which operates with two interdependant systems of infinite linear simultaneous equations. However, no general solutions are yet available for the skewed plate under similar boundary conditions. This is perhaps due to the fact that the analysis of a parallelogram plate is more complicated by its absence of orthogonal relationships.

The work embodied in this thesis comprises:

i) a theoretical analysis on the bending of clamped skewed plates subjected to a uniformly distributed lateral load;

and ii) an experimental investigation of the bending behaviour of a clamped skewed plate with a skew sides ratio of 1.23 and a skew angle of 35 degrees.

With reference to the theoretical work, the Rayleigh-Ritz method (2) is used to determine the deflections and moments of clamped skewed plates. Essentially, this method is one of the more important energy variational methods based on the well-known principle that when a system is in a position of stable equilibrium, its total energy is a minimum. Following the Rayleigh-Ritz procedure, a deflection configuration containing undetermined parameters was first assumed satisfying not only the boundary conditions of the plate but giving also polar symmetry ——— a condition quite evident in a uniformly loaded skewed plate. The assumed deflection function is next inserted into the expression for the complementary energy and the required integration carried out, the limits of the integration being extended to cover the entire surface of the plate. The resulting energy expression is then a function of the undetermined parameters. By minimizing this energy expression and solving the simultaneous equations thus obtained, the parameters are evaluated. Finally, the deflection equation with its determined parameters is differentiated to yield the required moments and hence stresses.

The experimental investigation consisted of the loading of a 1/4" thick aluminum plate rigidly clamped on all four edges. Meta-film strain rosette gauges were installed and moments and stresses were calculated from the strain measurements recorded for the different intensities of loading. Dial indicators were also installed to record the lateral deflection at selected points on the skewed plate.

To facilitate the computation of deflections, moments and the principal moments at specified points on the skewed plate, programmes using Fortran language were written and are included in Appendix A.

CHAPTER II

REVIEW OF LITERATURE

In 1941, Vernon P. Jensen (3) investigated slabs with various skew angles, boundary conditions and loading conditions. Finite difference equations were developed for a general system of skew co-ordinates. By means of these equations, he computed the principal moments for uniformly loaded slabs simply supported on two or all four edges.

In 1953, Eric Reissner (4) showed that the bending stiffness for a skewed plate is less than that of an unskewed plate and that pure bending of the skewed plate is always associated with a twisting deformation, the relative magnitude of which depends upon the angle of skew.

In the period between 1950 - 1960, a great deal of work was done on skewed plates and skewed structures. The outstanding researchers in this period are L.S.D. Morley (5), I. Mirsky (6), and P.D. Jones (7).

In 1953, F.M. Dorman (8) used the energy approach to investigate the bending behaviour of a clamped parallelogram plate but the function he assumed has the restrictive character of satisfying not only the polar symmetry but also quadrant symmetry ——— a condition which is non-existent in a skewed plate.

One of the more recent contributions to the solution of skewed plates and slabs is the one by Kennedy and M.W. Huggins (9) who presented an analytical solution for skewed stiffened plates under a uniformly distributed load. Stresses near the corners of skewed stiffened plates were also investigated by Kennedy and Martens (10) who have observed experimentally that critical stresses often occur in obtuse corners of such skewed plates.

CHAPTER III

THEORETICAL ANALYSIS

(a) The Plate Energy Equation in Oblique Dimensionless Co-ordinates

In investigating the bending behaviour of skew plates, it is often advantageous to adopt a co-ordinate system parallel to the edges of the plate, namely the oblique co-ordinates $\bar{\alpha}$ and $\bar{\beta}$ shown in Fig. 1.

By the transformation

$$\bar{x} = \bar{\alpha} \cos \theta \tag{3-1}$$

$$\bar{y} = \bar{\beta} + \bar{\alpha} \sin \theta \tag{3-2}$$

in which θ is the skew angle, the following relationships between the rectangular and the oblique co-ordinate systems hold:

$$\frac{\partial \bar{w}}{\partial \bar{x}} = \frac{\partial \bar{w}}{\partial \bar{\alpha}} \sec \theta - \frac{\partial \bar{w}}{\partial \bar{\beta}} \tan \theta \tag{3-3}$$

$$\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} = \frac{\partial^2 \bar{w}}{\partial \bar{\alpha}^2} \sec^2 \theta - 2 \frac{\partial^2 \bar{w}}{\partial \bar{\alpha} \partial \bar{\beta}} \sec \theta \tan \theta + \frac{\partial^2 \bar{w}}{\partial \bar{\beta}^2} \tan^2 \theta \tag{3-4}$$

Also,

$$\frac{\partial \bar{w}}{\partial \bar{y}} = \frac{\partial \bar{w}}{\partial \bar{\beta}} \tag{3-5}$$

$$\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} = \frac{\partial^2 \bar{w}}{\partial \bar{\beta}^2} \tag{3-6}$$

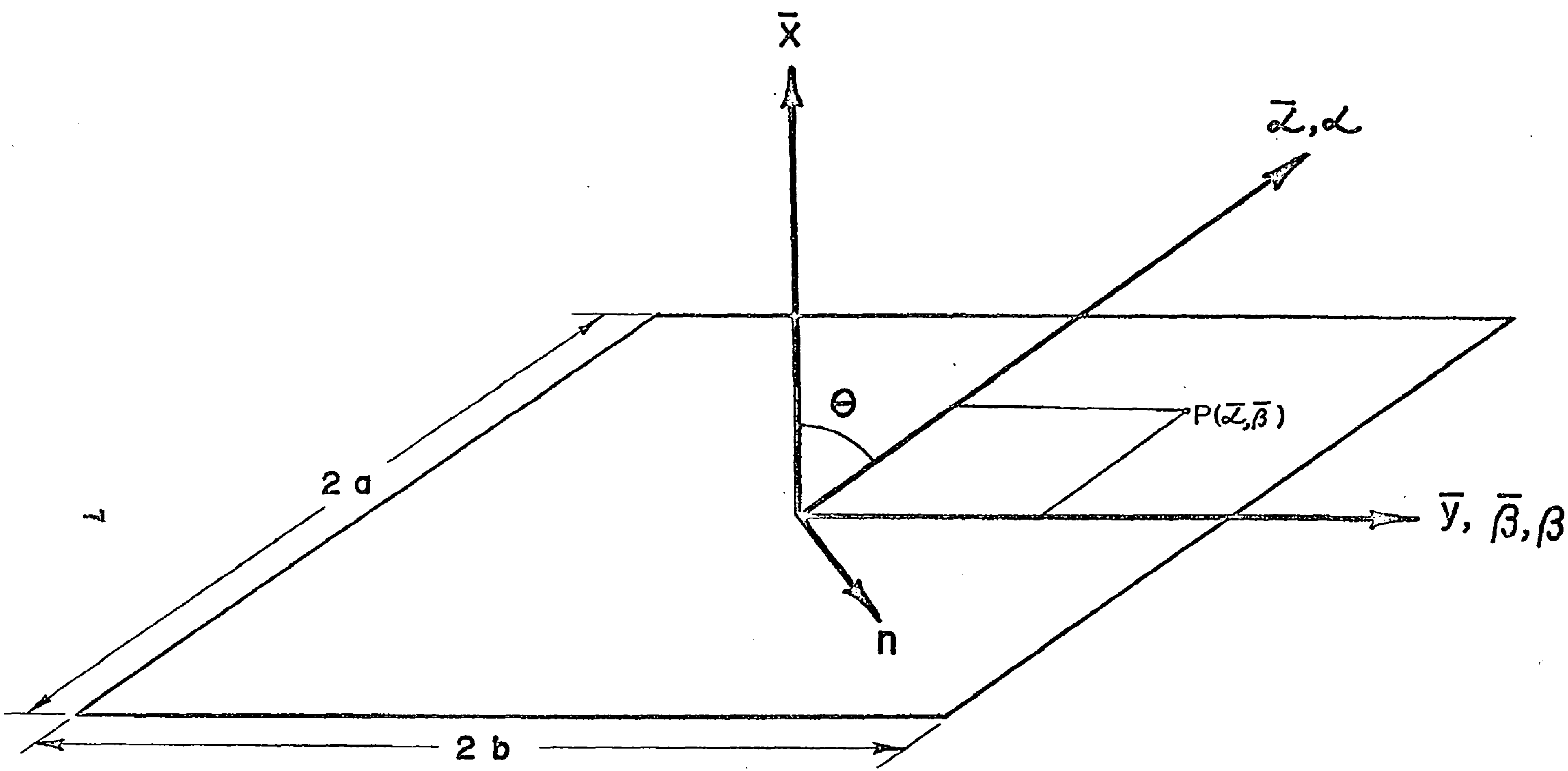

FIG. 1 THE RECTANGULAR AND OBLIQUE CO-ORDINATE SYSTEMS

From the classical small deflection theory of thin elastic plates, the total energy of bending consists of two parts: the strain energy due to bending and the potential energy of the load distributed over the plate. The expression for the strain energy of bending in rectangular co-ordinates can be written as,

$$J = \frac{1}{2} D \iint \left[\left(\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} \right)^2 - 2(1-\nu) \left[\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} - \left(\frac{\partial^2 \bar{w}}{\partial \bar{x} \partial \bar{y}} \right)^2 \right] \right] d\bar{x}\, d\bar{y} \qquad (3\text{-}7)$$

For a clamped skewed plate, the displacement $\bar{w}$ is zero around the boundary and Eq. (3-7) takes a simpler form since

$$\iint \left[\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} - \left(\frac{\partial^2 \bar{w}}{\partial \bar{x} \partial \bar{y}} \right)^2 \right] d\bar{x}\, d\bar{y} = 0 \qquad (3\text{-}8)$$

For a plate subjected to normal uniform load of intensity p, the potential energy of the total load is

$$- \iint \bar{w} p \; d\bar{x} \; d\bar{y}$$

Hence, the total energy of the system in rectangular co-ordinates is

$$I = \iint \left[\frac{D}{2} \left(\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} \right)^2 - \bar{w} p \right] d\bar{x} \; d\bar{y} \qquad (3\text{-}9)$$

Using the transformation relations Eqs. (3-1) and (3-2), Eq. (3-9) becomes, in oblique co-ordinates,

$$I = \cos\theta \iint \left\{ \frac{D}{2} \sec^4\theta \left[\frac{\partial^2 \bar{w}}{\partial \bar{\alpha}^2} - 2 \frac{\partial^2 \bar{w}}{\partial \bar{\alpha} \partial \bar{\beta}} \sin\theta + \frac{\partial^2 \bar{w}}{\partial \bar{\beta}^2} \right]^2 \right.$$

$$\left. - \bar{w}\, p \right\} d\bar{\alpha}\, d\bar{\beta} \qquad (3\text{-}10)$$

Now, by putting

$$\bar{\alpha} = a\alpha \qquad (3\text{-}11)$$

$$\bar{\beta} = a\beta \qquad (3\text{-}12)$$

$$q = \frac{pa^4}{Dh} \qquad (3\text{-}13)$$

$$\bar{w} = wh \qquad (3\text{-}14)$$

where, α and β are dimensionless oblique co-ordinates, 2a and 2b are the oblique dimensions of the plate (Fig. 1), h is the thickness of the plate and q the dimensionless load, the total energy expression can be further simplified into the following dimensionless form:

$$I = \frac{Dh^2}{a^2} \cos\theta \iint \left[\frac{(\nabla^2 w)^2}{2} - wq \right] d\alpha\, d\beta \qquad (3\text{-}15)$$

where

$$\nabla^2 w = \sec^2\theta \left(\frac{\partial^2 w}{\partial \alpha^2} - 2 \sin\theta \frac{\partial^2 w}{\partial \alpha \partial \beta} + \frac{\partial^2 w}{\partial \beta^2} \right)$$

(b) The Rayleigh-Ritz Procedure

To apply the Rayleigh-Ritz procedure, a deflection configuration for a clamped skewed plate is assumed as:

$$w = (1 - \alpha^2)^2 \ (\gamma^2 - \beta^2)^2 \ (A_o - A_1 \alpha\beta) \qquad (3\text{-}16)$$

where A_o and A_1 are undetermined parameters.

The boundary conditions for the clamped plate are:

$$w = 0 \text{ for } \alpha = \pm 1, \text{ and } \beta = \pm \gamma$$

and

$$\frac{\partial w}{\partial n} = 0 \text{ for } \beta = \pm \gamma, \text{ and } \frac{\partial w}{\partial \alpha} = 0 \text{ for } \alpha = \pm 1$$

where n is in a direction normal to the edge of the plate (Fig. 1).

From Eq. (3-16), it can be readily shown that all these boundary conditions are satisfied. It may be worth noting also that the deflection function satisfies polar symmetry, i.e., $w(\alpha,\beta) = w(-\alpha,-\beta)$ ——— a necessary condition for a uniformly loaded skewed plate.

In order to substitute into the plate energy expression (3-15), the assumed deflection function w is differentiated with respect to the dimensionless co-ordinates, α and β.

Thus:

$$\frac{\partial^2 w}{\partial \alpha^2} = (\gamma^2 - \beta^2)^2 \ (-4A_o + 12 A_1 \alpha\beta - 20 A_1 \alpha^3\beta + 12 A_o \alpha^2); \qquad (3\text{-}17)$$

$$\frac{\partial^2 w}{\partial \beta^2} = (1 - \alpha^2)^2 \, (12\, A_1 \, \gamma^2 \, \alpha\beta - 20\, A_1 \, \alpha\beta^3$$

$$- 4\, A_o \, \gamma^2 + 12\, A_o \, \beta^2 \,) \qquad (3\text{-}18)$$

and,

$$\frac{\partial^2 w}{\partial\alpha\partial\beta} = 16\, A_o \, (\gamma^2 \alpha\beta - \gamma^2 \alpha^3\beta - \alpha\beta^3 + \alpha^3\beta^3 \,)$$

$$+ A_1 \, (6\, \gamma^4\alpha^2 - 5\, \gamma^4\alpha^4 - \gamma^4 - 36\, \gamma^2 \, \alpha^2 \, \beta^2$$

$$+ 30\, \gamma^2 \, \alpha^4 \, \beta^2 + 6\, \gamma^2 \, \beta^2 + 30\, \alpha^2 \, \beta^4 - 25\, \alpha^4 \, \beta^4 - 5\, \beta^4) \qquad (3\text{-}19)$$

hence,

$$\nabla^2 w = \sec^2 \theta \, (\frac{\partial^2 w}{\partial\alpha^2} - 2 \sin\theta \, \frac{\partial^2 w}{\partial\alpha\partial\beta} + \frac{\partial^2 w}{\partial\beta^2} \,)$$

$$= \sec^2 \theta \, \Big\{ A_o \, \Big[-4\gamma^4 - 4\gamma^2 + (12\, \gamma^4 + 8\, \gamma^2) \, \alpha^2$$

$$+ (12 + 8\, \gamma^2) \, \beta^2 - \alpha^2\beta^2 \, (24\, \gamma^2 + 24\,) - 4\, \beta^4 + 12\, \alpha^2\beta^4$$

$$- 4\, \gamma^2 \, \alpha^4 + 12\, \alpha^4 \, \beta^2 + 32 \sin\theta \, \gamma^2 \, \alpha^3\beta + 32 \sin\theta \, \alpha\beta^3$$

$$- 32 \sin\theta \, \gamma^2 \, \alpha\beta - 32 \sin\theta \, \alpha^3\beta^3 \Big] + A_1 \, \Big[2 \sin\theta \, \gamma^4 + \alpha\beta \, (12\, \gamma^2$$

$$+ 12\, \gamma^4) - \alpha^3\beta \, (20\, \gamma^4 + 24\, \gamma^2) - \alpha\beta^5 \, (24\, \gamma^2 + 20) + \alpha^3\beta^3 \, (40\, \gamma^2$$

$$+ 40) + 12\, \alpha\beta^5 - 20\, \alpha^3 \, \beta^5 + 12\, \gamma^2 \, \alpha^5 \, \beta - 20\, \alpha^5 \, \beta^3 + 10 \sin\theta \, \gamma^4\alpha^4$$

$$+ 72 \sin\theta \, \gamma^2 \, \alpha^2 \, \beta^2 + 50 \sin\theta \, \alpha^4 \, \beta^4 + 10 \sin\theta \, \beta^4 - 12 \sin\theta \, \gamma^4\alpha^2$$

$$- 60 \sin\theta \, \gamma^2 \, \alpha^4 \, \beta^2 - 12 \sin\theta \, \gamma^2 \, \beta^2 - 60 \sin\theta \, \alpha^2 \, \beta^4 \Big] \Big\} \qquad (3\text{-}20)$$

Now, the total energy integral, Eq. (3-15), can be re-written as,

$$I = \frac{Dh^2}{a^2} \cos\theta \left\{ \int_{-\gamma}^{+\gamma} \int_{-1}^{+1} \frac{(\nabla^2 w)^2}{2} \, d\alpha \, d\beta - \int_{-\gamma}^{+\gamma} \int_{-1}^{+1} wq \, d\alpha \, d\beta \right\} \quad (3\text{-}21)$$

where the limits of the integration extends over the entire surface of the plate as shown.

Substituting the form of $\nabla^2 w$, Eq. (3-20), into the energy expression, Eq. (3-21), and after some tedious algebraic manipulations and integrations, the plate energy equation reduces to:

$$I = \frac{Dh^2}{a^2} \cos\theta \left[\left\{ \frac{\sec^4\theta}{2} \left[\left[20.8050793\, \gamma^5 + (11.8886168 + 23.7772337 \sin^2\theta)\, \gamma^7 + 20.8050793\, \gamma^9 \right] A_o^2 + 2 \left[A_o A_1 \right] \left[-5.9443083 \sin\theta\, \gamma^7 - 5.9443083 \sin\theta\, \gamma^9 \right] + \left[1.3509791\, \gamma^7 + (1.3209574 + 2.0419148 \sin^2\theta)\, \gamma^9 + 1.3509791\, \gamma^{11} \right] A_1^2 \right] \right\} - A_o q \left[\frac{256}{225} \gamma^5 \right] \right] \quad (3\text{-}22)$$

Following the Rayleigh-Ritz procedure, the total energy expression is now minimized with respect to each of the undetermined parameters, i.e.,

$$\frac{\partial I}{\partial A_o} = 0; \qquad \text{and} \qquad \frac{\partial I}{\partial A_1} = 0$$

and after partial differentiation, two linear simultaneous equations are obtained, viz.,

$$\Big[20.8050793\,\gamma^5 + (11.8866166 + 23.7732337\,\sin^2\theta)\,\gamma^7$$

$$+ 20.8050793\,\gamma^9\Big] A_o - \Big[5.9443003\,\gamma^7 \sin\theta$$

$$+ 5.9443003\,\gamma^9 \sin\theta\Big] A_1 = q\cos^4\theta\left[\frac{256}{225}\right]\gamma^9 \qquad (3\text{-}23)$$

and

$$\Big[-5.9443003\,\gamma^7 \sin\theta - 5.9443003\,\gamma^9 \sin\theta\Big] A_o$$

$$+ \Big[1.3509791\,\gamma^7 + (1.3209574 + 2.6419148\,\sin^2\theta)\,\gamma^9$$

$$+ 1.3509791\,\gamma^{11}\Big] A_1 = 0 \qquad (3\text{-}24)$$

Solving these equations simultaneously, the parameters A_o and A_1 are found to be:

$$A_o = \frac{q\cos^4\theta\,\left[1.5371140 + 1.5029560\,\gamma^2 + 3.0059119\,\gamma^2\sin^2\theta + 1.5371140\,\gamma^4\right]}{\psi}$$

and

$$A_1 = \frac{q\cos^4\theta\,\left[6.7033018\,\sin\theta + 6.7033018\,\gamma^2\sin\theta\right]}{\psi}$$

where

$$\psi = 28.1072273 + 43.5430963\,\gamma^2 + 51.7529915\,\gamma^2 \sin^2\theta$$

$$+ 71.9188109\,\gamma^4 - 7.6521758\,\gamma^4 \sin^2\theta$$

$$+ 62.0174256\,\gamma^4 \sin^4\theta + 43.5430963\,\gamma^6$$

$$+ 51.7529915\,\gamma^6 \sin^2\theta + 28.072273\,\gamma^6$$

These parameters are then inserted into the expression for the deflection Eq. (3-16), from which deflections at any point within the plate boundaries can be determined.

(c) Moment and Displacement Relationships

From the transformation equations (3-1) and (3-2), and the dimensionless ratios, equations (3-11) and (3-12), rectangular moments can be expressed in terms of deflections in oblique dimensionless coordinates, α and β. Thus:

$$M_{\bar{x}} = -D \left(\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + \nu \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} \right)$$

$$= -D \frac{h}{a^2} \left(\frac{\partial^2 w}{\partial \alpha^2} \sec^2 \theta - 2 \frac{\partial^2 w}{\partial \alpha \partial \beta} \sec \theta \tan \theta \right.$$

$$\left. + \frac{\partial^2 w}{\partial \beta^2} \tan^2 \theta + \nu \frac{\partial^2 w}{\partial \beta^2} \right) \qquad (3\text{-}25)$$

$$M_{\bar{y}} = -D \left(\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \nu \frac{\partial^2 \bar{w}}{\partial \bar{x}^2} \right)$$

$$= -D \frac{h}{a^2} \left(\frac{\partial^2 w}{\partial \beta^2} + \nu \left(\frac{\partial^2 w}{\partial \alpha^2} \sec^2 \theta - 2 \frac{\partial^2 w}{\partial \alpha \partial \beta} \sec \theta \tan \theta \right.\right.$$

$$\left.\left. + \frac{\partial^2 w}{\partial \beta^2} \tan^2 \theta \right) \right) \qquad (3\text{-}26)$$

$$M_{\bar{x}\bar{y}} = D \left(\frac{1 - \nu}{a^2} \right) h \left(\frac{\partial^2 w}{\partial \alpha \partial \beta} \sec \theta - \frac{\partial^2 w}{\partial \beta^2} \tan \theta \right) \qquad (3\text{-}27)$$

From the rectangular moments $M_{\bar{x}}$, $M_{\bar{y}}$ and the twisting moments $M_{\bar{x}\bar{y}}$, the principal moments can be calculated in the usual manner:

$$M_{\substack{max \\ min}} = \frac{1}{2} (M_{\bar{x}} + M_{\bar{y}}) \pm \frac{1}{2} \sqrt{(M_{\bar{x}} - M_{\bar{y}})^2 + 4 M_{\bar{x}\bar{y}}^2} \qquad (3\text{-}28)$$

The direction in which the maximum principal stress occurs is

$$\phi = \frac{1}{2} \tan^{-1} \frac{2 M_{\bar{x}\bar{y}}}{(M_{\bar{x}} - M_{\bar{y}})} \qquad (3\text{-}29)$$

where ϕ is the angle measured clockwise from the $\bar{x}$ direction.

Once these moments for the plate have been ascertained, the stresses are then readily obtainable by multiplying the corresponding moments by $\frac{6}{h^2}$, i.e.,

$$\sigma_{\bar{x}} = \frac{6 M_{\bar{x}}}{h^2} \qquad (3\text{-}30)$$

$$\sigma_{\bar{y}} = \frac{6 M_{\bar{y}}}{h^2} \qquad (3\text{-}31)$$

$$\sigma_{max} = \frac{6 M_{max}}{h^2} \qquad (3\text{-}32)$$

and

$$\sigma_{min} = \frac{6 M_{min}}{h^2} \qquad (3\text{-}33)$$

(d) Sample Calculations

As a sample calculation, deflections and moments were worked out at specified points for a skewed plate with a skew sides ratio of 1.25 and a skew angle of 55 degrees and these results are tabulated in Table I. Also, to demonstrate the adaptability of the digital computer to this type of problems, deflections, moments, and principal moments for the same skew plate were computed by the IBM 1620 and the results are included in Appendix B.

TABLE I

Theoretical deflections and Moments for a Clamped Skewed Plate having a skew sides ratio of 1.23 and a skew angle of 55 degrees. Plate is subjected to a uniform load of 1.3 lb./in^2.

Location	Deflection (w) in.	Max. Moment lb.in./in.	Min. Moment lb.in./in.
$\bar{\alpha} = 0$, $\bar{\beta} = 0$	.056	22.329	11.436
$\bar{\alpha} = \frac{a}{2}$, $\bar{\beta} = 0$	.032	15.861	6.157
$\bar{\alpha} = 0$, $\bar{\beta} = \frac{b}{2}$	.032	18.368	6.869
$\bar{\alpha} = \frac{a}{2}$, $\bar{\beta} = \frac{b}{2}$	.010	8.434	0.348

For D = 14,648.4375 lb.in; ν = 0.333; and E = 10,000,000 p.s.i.

CHAPTER IV

EXPERIMENTAL STUDY

(a) Materials and Apparatus

A 1/4" thick aluminum alloy (12) 6061-T6 plate was cut to the dimensions shown in Fig. II. With the dimensions shown, the plate has a skew sides ratio of 1.23 and a skew angle of 55 degrees. A total of 32 meta-foil strain rosette gauges were installed on the plate, seven on the top surface and twenty-nine on the bottom surface. All these rosette gauges are of the 3-gauge 45^{o} rectangular type, having a gage factor of 2.05 and a resistance of 120 ohms. Terminal strips (type T-50) were used to connect the lead wires to the gauge tabs. The wire leads are twelve feet long, made of No. 26 stranded copper wire and with vinyl insulation. To provide support for the plate, standard steel channels twelve inches deep and weighing 20.7 pounds per foot were cut to the required skew. It was thought that there might be a possibility that these channels would twist as the clamped plate was being loaded. As a safety measure, 1/4 inch thick vertical stiffeners were welded to the channels and were spaced approximately six inches apart. The plate and channel assembly was then seated on steel angles attached by 1 inch bolts to heavy steel posts. To assimulate the built-in edge condition, the aluminum plate was sandwiched between the flange of the channels and a 1 inch thick cold-rolled steel cover plate, 3 inches wide. The plate was cut in such a way that it provided a clamping edge of three inches while the flange width of the channel was also three inches. A wooden box having the same dimensions and skew as the plate was made. The box was 6 1/2 feet high and was heavily reinforced with 2 inch by 4 inch planks and steel straps. A waterproof sheet made of polyethylene material was placed inside the wooden box. The strains were measured by means of two standard S-R 4 strain indicator units, two switch and balancing units (model C-10T and C-10LTC) and a digital strain indicator which permitted the strains to be read off directly in micro-inch per inch. Seven dial indicators were used to measure the deflection of the plate. To clarify the description of the aforementioned apparatus and the general set-up of the experiment, photographs were taken and are included on pages 34 to 37

(b) Procedure and Results

The 1/4 inch thick aluminum plate was first sawed to the required dimensions and skew, leaving three inches all around for clamping purposes. The plate was then cleaned with acetone and the locations for both the top and bottom surface gauges laid with reference to the rectangular axes, $\bar{x}$ and $\bar{y}$. The co-ordinates of all the gauges are recorded in Table II while their alignment and locations are shown in Fig. 2 and 3. The installation of these rosette gauges followed a set procedure. The spot where the rosette was to be placed was first wiped clean with acetone and then sanded with a metal conditioner using silicon carbide paper. The exact location of the gauge was then marked with a ballpen. The same spot was again cleansed in turn first with a metal conditioner and then with a neutralizer. With the rosette and the terminal strip properly lined, Eastman 910 cement was applied to cement the assembly onto the plate. The gauge and terminal were then left to dry for one minute during which time pressure was applied to the gauge by means of the thumb. The insulation of both ends of the lead wires was next stripped and the bare copper strands twisted. Each lead wire consisted of three copper strands, one strand was soldered to one tab of the terminal strip while the other two strands were twisted together and soldered to the other tab of the terminal. The tabs of the terminal strip were then connected in turn with the tabs of the gauges by means of thin copper jumper wires. To provide waterproof and mechanical protection, besides the usual highly insulating gauge coating, a special gauge coat made of a two-component rubber-like resin was also applied to cover the entire gauge and part of the lead wires. This gauge coating is greenish in colour and photograph A on page 34 shows a clear picture of some of the gauges completely installed on the bottom surface of the plate. All 32 rosettes were thus installed and the gauge lead wires soldered in position.

The supporting structure for the plate is shown in photograph A. The plate was placed between the flange of the channel and the 1 inch steel cover plate and all four edges were then clamped by special structural erection clamps spaced six inches apart; allowance was made here to have clamps placed closer together near the corners of the plate. (See photograph B on page 35)

To allow a direct reading of the deflections, seven dial indicators were installed in locations previously marked on the bottom of the plate. These are numbered from 1 to 7 and their locations in terms of the rectangular co-ordinates are tabulated in Table II and are also shown in Fig. 4. The wooden skew tank was then placed to rest on the cover plates and, inside the tank a waterproof sheet of polyethylene material was carefully laid so that water can be used as a loading medium.

Next, gauge lead wires from 20 rosette gauges were soldered to twelve 5-channel receptacles especially provided for the three switch and balance units. Unit strains in micro-inch per inch could be read off directly from a screen of the digital strain indicator. The remaining 12 rosettes were hooked to the terminals of the two switch boxes which were in turn connected to a conventional S-R 4 strain indicator. Unit strains were read off from this indicator in the usual manner. These instruments are clearly depicted in photograph C on page 36.

Before the plate was loaded, all the gauges and dial indicators were "zeroed in". After the plate was loaded, strain and dial indicator readings were taken and recorded. Such readings were obtained for every one foot increment in height of water. Readings were also taken and recorded in a similar manner while the load was taken off the plate. The average of the loading and unloading readings was computed and the resulting strains tabulated in Table III.

Three strain values e_a, e_b, and e_c (See Fig. 2) were obtained from each rosette gauge. From these values the principal stresses and principal moments were then computed by using either the Mohr Circle or the following formulas:

$$\sigma_{max} = \frac{E}{2} \left(\frac{e_a + e_c}{1 - \nu} + \frac{1}{1 + \nu} \sqrt{2 (e_a - e_b)^2 + 2 (e_b - e_c)^2} \right) \quad (4\text{-}1)$$

$$\sigma_{min} = \frac{E}{2} \left(\frac{e_a + e_c}{1 - \nu} - \frac{1}{1 + \nu} \sqrt{2 (e_a - e_b)^2 + 2 (e_b - e_c)^2} \right) \quad (4\text{-}2)$$

$$\phi' = \frac{1}{2} \tan^{-1} \left(\frac{2e_b - e_a - e_c}{(e_a - e_c)} \right) - 45^\circ \quad (4\text{-}3)$$

where ϕ' gives the direction in which the maximum stress occurs, this angle being measured counter-clockwise from the $\bar{y}$ axis.

$$M_{max} = \frac{h^2}{6} \sigma_{max} \qquad (4\text{-}4)$$

$$M_{min} = \frac{h^2}{6} \sigma_{min} \qquad (4\text{-}5)$$

To facilitate the computation, a programme in Fortran was written to give the rectangular moments, principal moments, principal stresses and the direction of these principal stresses. As a sample calculation, the digital computer IBM 1620 was used to calculate the principal moments and principal stresses for each rosette and the results are tabulated in Table IV. The programme used for the computation is included in Appendix A.

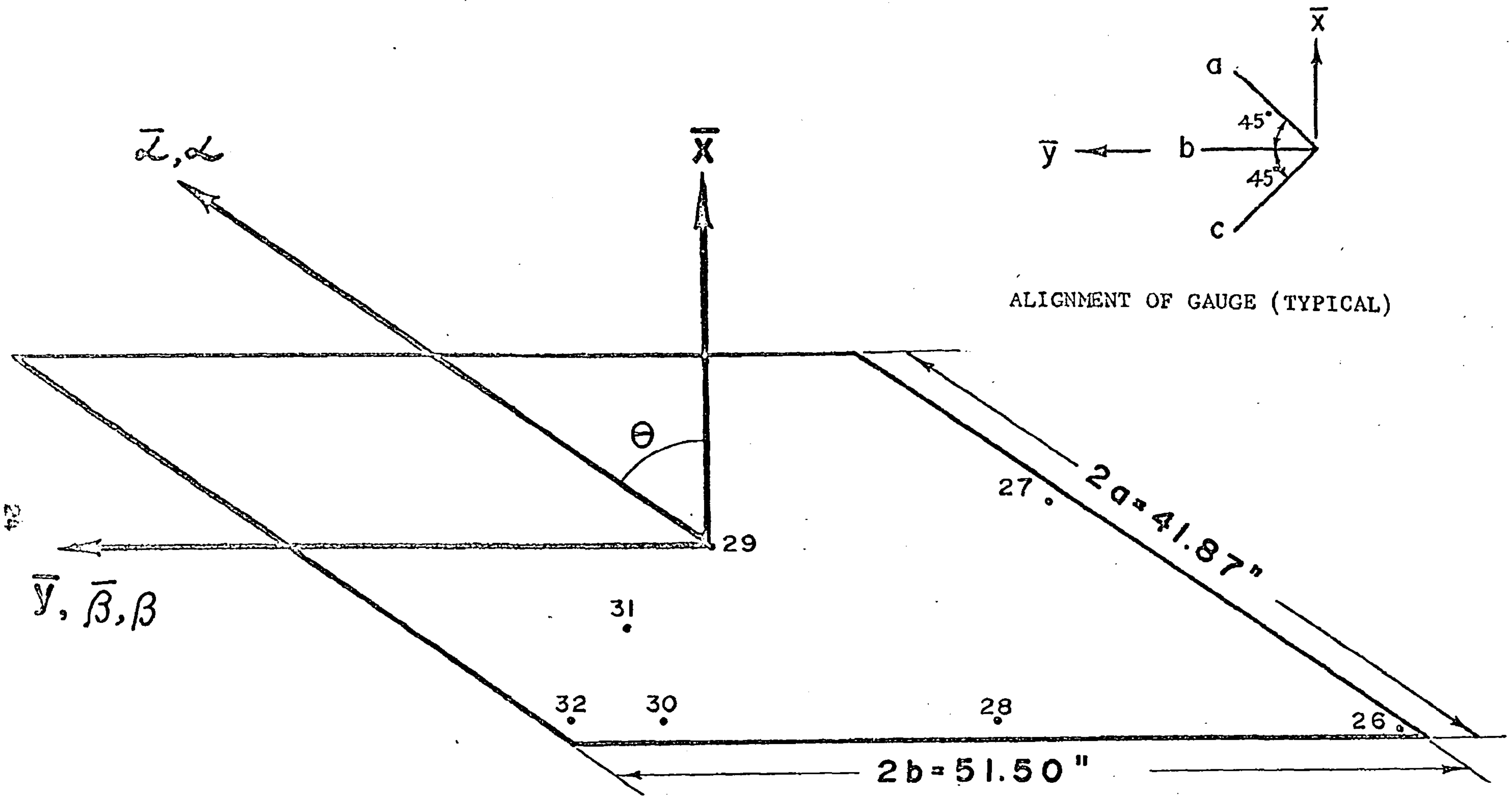

FIG. 2 LOCATION OF ROSETTE GAUGES ON TOP SURFACE AND GAUGE ALIGNMENT

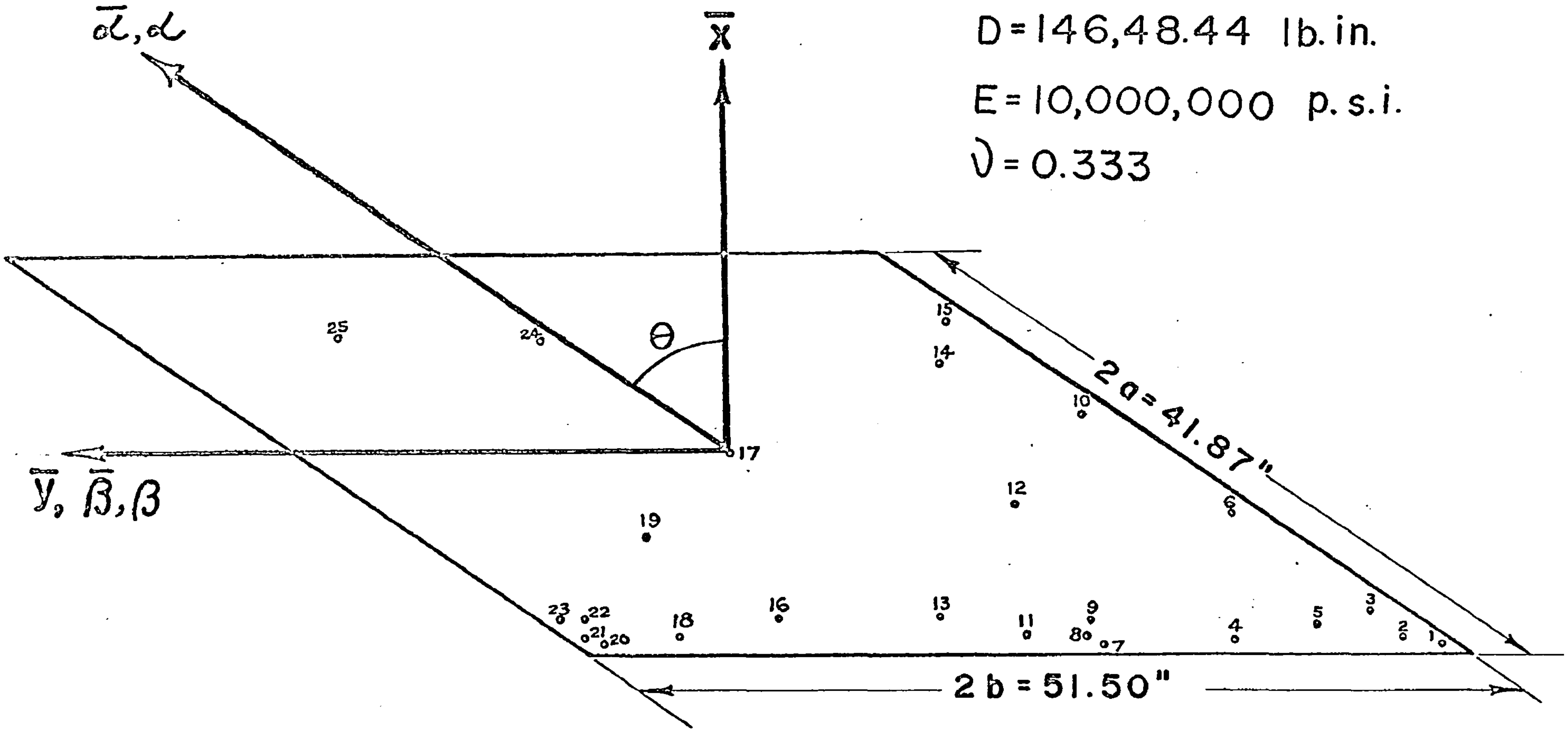

FIG. 3 DIMENSIONS OF SKEW PLATE AND LOCATION OF ROSETTE GAUGES ON BOTTOM SURFACE

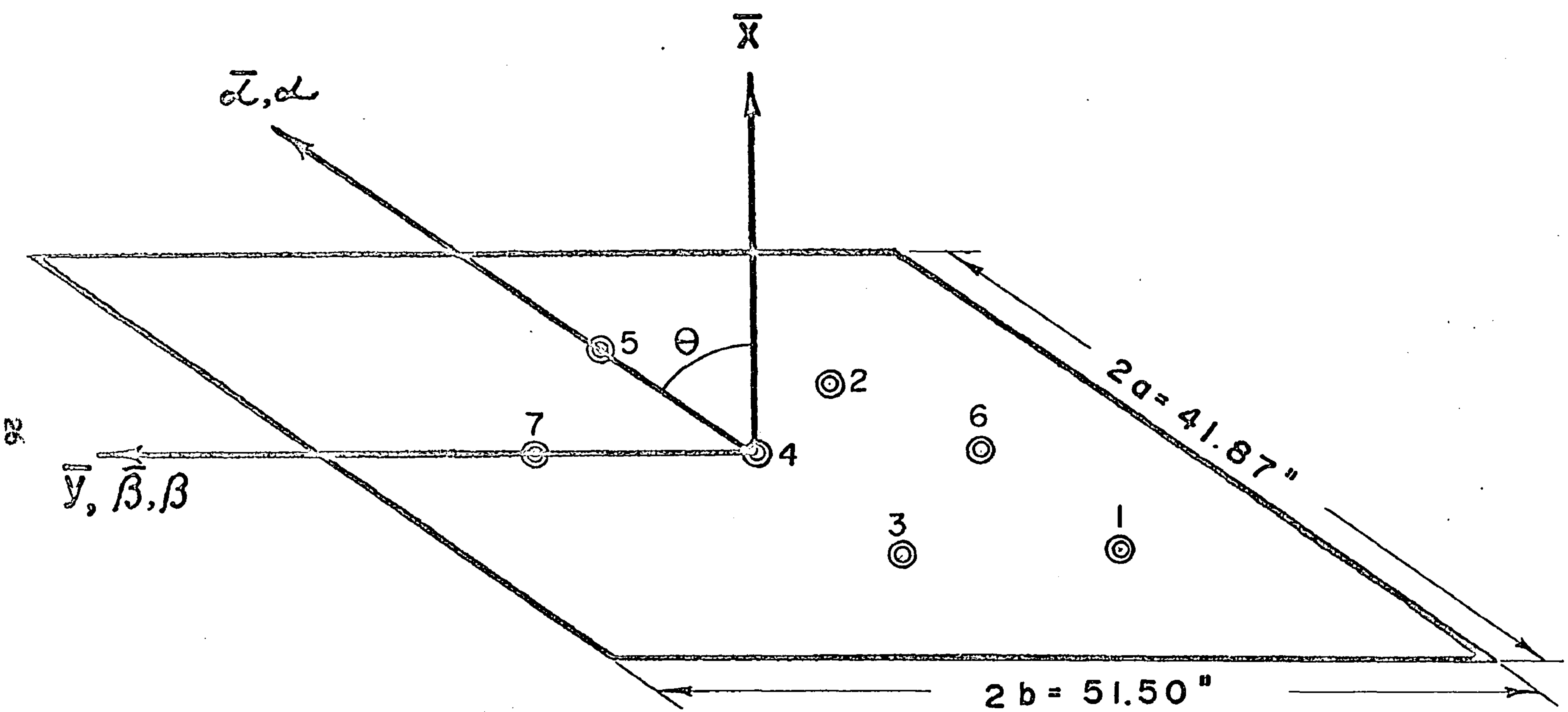

FIG. 4 LOCATION OF DIAL INDICATORS

TABLE II

Co-ordinates of Strain Rosette Gauges and Dial Indicators.

Gauge No.	Co-ordinates		Gauge No.	Co-ordinates	
	$\bar{x}$ (in)	$\bar{y}$ (in.)		$\bar{x}$ (in)	$\bar{y}$ (in)
1	-11.[illegible]	-41.4	21	-11.0	+[illegible].5
2	-11.0	-3[illegible]	22	-10.0	+[illegible].7
3	-[illegible]	-37.0	23	-10.0	+10.1
4	-11.0	-2[illegible]	24	+7.0	+11.
5	-10.0	-33.[illegible]	25	+7.0	-22.
[illegible]	-2 .	-[illegible].4	2[illegible]	-11.	-41.4
7	-11.5	-21.5	27	+2.4	-20.[illegible]
[illegible]	-11.0	-20.[illegible]	2[illegible]	-11.0	-1[illegible]
[illegible]	-10.0	-20.4	2[illegible]	0.0	0.0
10	+2.4	-20.	30	-11.0	+3.1
11	-11.0	-1[illegible]	31	-5.0	+5.[illegible]
12	-3.0	-1[illegible].5	32	-11.0	+[illegible].5
13	-10.0	-11.[illegible]	Dial 1	-[illegible].0	-21.4
14	+5.3	-11.[illegible]	Dial 2	+4.0	-4.3
[illegible]	+[illegible] 1	-12.4	Dial 3	-[illegible].0	-[illegible]
[illegible]	-10.0	-2.3	Dial 4	0.0	0.0
[illegible]	0.0	0.0	Dial 5	+[illegible].0	+[illegible]
[illegible]	-11.0	+3.1	Dial [illegible]	0.0	-12.[illegible]
[illegible]	-5.0	+5.1	Dial [illegible]	0.0	+12.[illegible]
20	-11.5	+[illegible]			

TABLE III

Strain Gauge Readings (Microinch per inch) and Dial Indicator Readings (inches) for different intensities of loading

Gauge No	Uniform Load in feet of water					
	1	2	3	4	5	5 1/2
1a	+2	+4	0	-1	[illegible]	[illegible]
1b	-1	-1	-3	[illegible]	[illegible]	[illegible]
1c	+1	[illegible]	0	-3	-1	0
2a	+3	+4	+4	[illegible]	+1	-1
2b	[illegible]	-6	-7	-8	-11	-10
2c	-1	-3	-1	+3	-4	-4
3a	+3	+3	[illegible]	[illegible]	[illegible]	[illegible]
3b	-4	-7	[illegible]	[illegible]	-14	[illegible]
3c	+2	-4	-4	-15	-13	+3
4a	+10	+4	+15	[illegible]	[illegible]	+1
4b	[illegible]	-8	-8	+15	+10	-35
4c	-13	-22	[illegible]	-38	-45	-52
5a	[illegible]	[illegible]	+1	+22	+25	[illegible]
5b	-11	-1	[illegible]	-14	-18	-25
[illegible]	-7	-13	[illegible]	[illegible]	-18	[illegible]
[illegible]a	[illegible]	-12	-2	-2	[illegible]	-40
[illegible]b	-2	-2	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]c	0	0	0	[illegible]	[illegible]	6
[illegible]a	[illegible]	[illegible]	-7	-2	[illegible]	-50
[illegible]b	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	+07

TABLE III (Cont'd.)

Strain Gauge Readings (Micro-inch per inch) and Dial Indicator Readings (inches) for Different Intensities of Loading

Gauge or Dial Number	Uniform Load in feet of Water					
	1	2	3	4	5	5 1/2
7 c	-28	-44	-65	-81	-99	-107
8 a	- 6	-13	-20	-25	-32	- 37
8 b	- 7	- 8	- 4	- 4	- 4	- 3
8 c	-28	-47	-61	-79	-94	-103
9 a	+ 4	+ 6	+ 8	+11	+14	+ 14
9 b	- 8	- 8	- 4	- 4	- 4	- 3
9 c	-26	-41	-54	-70	-83	- 88
10 a	-26	-55	-81	-110	-139	-150
10 b	-26	-40	-54	-71	-84	- 90
10 c	+ 1	+ 3	+ 7	+ 9	+ 8	+ 9
11 a	-15	-35	-56	-72	-90	-100
11 b	-17	-30	-40	-50	-59	- 65
11 c	-43	-78	-108	-139	-167	-182
12 a	+28	+60	+91	+121	+153	+169
12 b	-12	- 9	- 3	+ 2	+ 6	+ 6
12 c	- 3	+ 6	+18	+26	+33	+ 39
13 a	- 6	-13	-20	-26	-33	- 40
13 b	- 8	- 2	+ 5	+ 7	+ 9	+ 12
13 c	-33	-52	-69	-85	-101	-108
14 a	-12	-18	-20	-20	-14	- 14
14 b	-11	-13	-14	-17	-20	- 19

TABLE III (Cont'd.)

Strain Gauge Readings (Micro-inch per inch) and Dial Indicator Readings (inches) for Different Intensities of Loading

Gauge or Dial Number	Uniform Load in feet of Water					
	1	2	3	4	5	5 1/2
14 c	[illegible]	+ 7	+ 13	-11	+16	+16
15 a	-55	-111	-160	-210	-255	-266
15 b	-21	-41	-61	-82	-102	-113
15 c	- 9	-20	-28	-37	-50	-50
16 a	-17	-32	-43	-54	-63	-70
16 b	- 9	-11	-12	-16	-20	-23
16 c	-24	-42	-56	-75	-94	-103
17 a	+37	+79	+121	+159	+197	+214
17 b	+ 4	+20	+49	+69	+92	+99
17 c	+30	+70	+101	+130	+166	+168
18 a	-33	-57	-77	-95	-111	-122
18 b	+ 4	+12	+20	+23	+24	+26
18 c	- 6	-11	-14	-23	-35	-42
19 a	+18	+44	+73	+100	+128	+140
19 b	+ 9	+21	+42	+70	+93	+134
19 c	+19	+45	+69	+90	+106	+116
20 a	-44	-76	-100	-124	-143	-154
20 b	+ 5	+18	+32	+44	+56	+58
20 c	+10	+17	+23	+26	+28	+29
21 a	-53	-93	-125	-153	-176	-190
21 b	0	+ 7	+16	+23	+31	+37

TABLE III (Cont'd.)

Strain Gauge Readings (Micro-inch per inch) and Dial Indicator Readings (inches) for Different Intensities of Loading

Gauge or Dial Number	Uniform Load in feet of Water					
	1	2	3	4	5	5 1/2
21 c	+ 7	+ 9	+10	+ 8	+ 5	+ 4
22 a	-45	-78	-100	-121	-137	-146
22 b	+ 3	+ 9	+ 19	+26	+33	+37
22 c	- 2	- 6	-11	-19	-28	-31
23 a	-65	-123	-173	-210	-242	-264
23 b	-23	-41	-61	-79	-100	-107
23 c	-25	-38	-59	-83	-107	-108
24 a	+24	+49	+75	+98	+120	+131
24 b	- 8	+ 3	+11	+18	+24	+28
24 c	-11	-12	- 9	- 8	- 7	- 4
25 a	+16	+37	+56	+69	+88	+97
25 b	-13	-14	-15	-21	-27	-23
25 c	-14	-16	-18	-21	-27	-21
26 a	+ 5	+ 4	+ 6	+ 7	+ 6	+ 7
26 b	+ 4	+ 1	+ 1	+ 1	- 3	+ 1
26 c	+ 8	+ 8	+ 8	+12	+12	+18
27 a	0	- 4	- 9	-12	-17	-16
27 b	+16	+37	+55	+74	+91	+98
27 c	+35	+65	+96	+127	+157	+272
28 a	+27	+63	+100	+131	+160	+177
28 b	+ 5	+13	+24	+31	+37	+42

TABLE III (Cont.d.)

Strain Gauge Readings (Micro-inch per inch) and Dial Indicator Readings (inches) for Different Intensities of Loading

Gauge or Dial Number	Uniform Load in feet of Water					
	1	2	3	4	5	5 1/2
28 c	+19	+33	+48	+63	+78	+84
29 a	-59	-69	-95	-119	-144	-152
29 b	-27	-38	-44	-51	-55	-54
29 c	-44	-76	-100	-122	-138	-148
30 a	- 2	- 2	- 1	- 1	+ 3	+ 6
30 b	-18	-30	-42	-54	-63	-68
30 c	+25	+53	+80	+106	+136	+147
31 a	-34	-60	-83	-107	-130	-139
31 b	-31	-49	-64	-78	-93	-98
31 c	-23	-40	-52	-61	-68	-72
32 a	-10	-21	-35	-52	-67	-72
32 b	-11	-21	-30	-41	-54	-72
32 c	+38	-84	-133	+178	+224	+240
Dial 1	+ 3	+ 7	+11	+14	+18	+20
Dial 2	+16	+31	+45	+600	+72	+78
Dial 3	+13	+22	+34	+44	+53	+58
Dial 4	+21	+40	+57	+75	+89	+98
Dial 5	+13	+23	+35	+45	+54	+62
Dial 6	+13	+24	+36	+47	+57	+65
Dial 7	+13	+25	+36	+47	+57	+63

TABLE IV

Computation of Principal Moments and Principal Stresses for Rosette Gauges. Plate is Subjected to a Uniform Load of 1.3 lb./in^2.

Gauge Number	Principal Moment		Principal Stresses		ϕ (radians)
	Max	Min	Max	Min	
1	0.354	-0.198	34.017	-19.017	-.071
2	0.926	-0.458	88.950	-43.950	-1.427
3	1.030	-0.717	98.852	-68.852	-1.339
4	0.470	-3.284	45.156	-315.156	-0.764
5	2.103	-1.322	201.102	-126902	-1.178
6	0.514	-3.795	49.329	-364.329	-0.195
7	-2.365	-8.885	-227.060	-852.939	-0.384
8	-3.058	-9.599	-293.528	-922.471	-0.256
9	-0.753	-6.434	-72.304	-617.695	-0.510
10	-2.096	-9.466	-201.225	-908.774	-0.601
11	-8.953	-16.671	-859.527	-1600.472	-0.277
12	13.836	3.195	1328.299	306.700	-1.288
13	-2.638	-11.268	-253.265	-1081.735	-0.230
14	0.981	-2.075	94.182	-199.182	-0.502
15	-8.922	-20.452	-856.572	-1963.426	-1.017
16	-4.761	-10.707	-457.056	-1027.943	-0.086
17	22.250	12.437	2136.008	1193.989	-1.491
18	-1.431	-12.788	-137.393	-1227.606	-1.347

Moments are in lb. in./in. and stresses are in lb./in^2.

TABLE IV (Cont'd.)

Computation of Principal Moments and

Principal Stresses for Rosette Gauges. Plate is Subjected to a Uniform Load of 1.3 lb./in^2.

Gauge Number	Principal Moment		Principal Stresses		ϕ'
	Max	Min	Max	Min	
19	13.365	8.823	1283.016	846.983	-1.536
20	1.293	-13.324	124.161	-1279.161	-1.212
21	-1.188	-16.781	-114.057	-1610.942	-1.199
22	-1.892	-15.451	-181.661	-1483.337	-1.302
23	-11.937	-24.313	-1145.934	-2334.063	-1.169
24	8.860	1.452	850.598	139.402	-1.027
25	6.894	-0.957	661.870	-91.870	-1.157
26	1.569	0.619	150.621	59.379	-0.826
27	10.996	2.598	1055.585	249.414	-0.893
28	15.965	7.159	1532.669	687.329	-1.331
29	-11.050	-19.419	-1060.811	-1864.187	-0.233
30	13.281	-0.938	1275.062	-90.062	-0.230
31	-9.336	-11.758	-893.323	-1131.676	-0.896
32	-7.428	-18.831	-713.116	-1806.882	-0.368

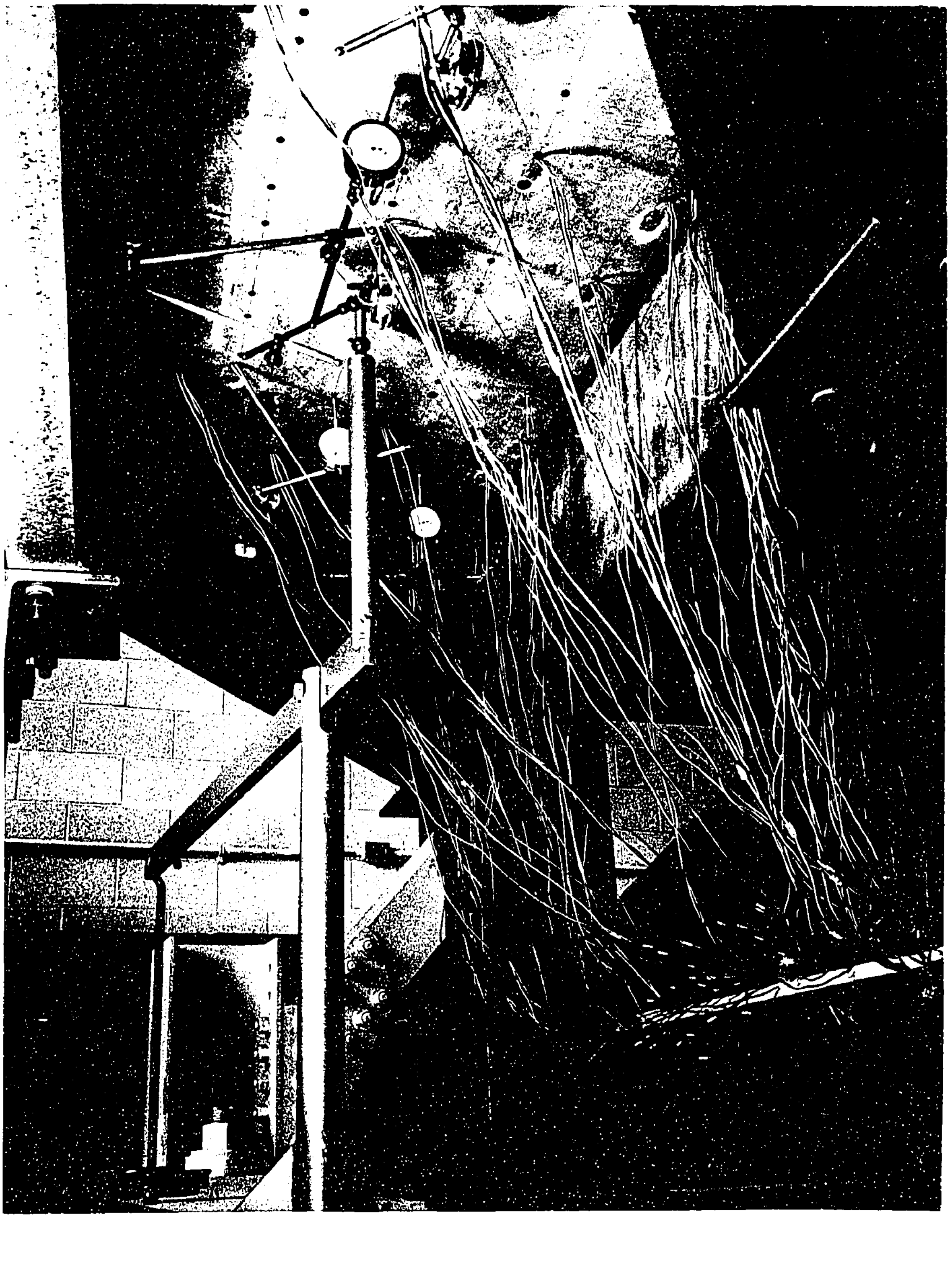

PHOTO. A PHOTOGRAPH SHOWING PART OF THE SUPPORTING STRUCTURE AND COMPLETED GAUGE AND DIAL INDICATOR INSTALLATIONS (BOTTOM OF PLATE)

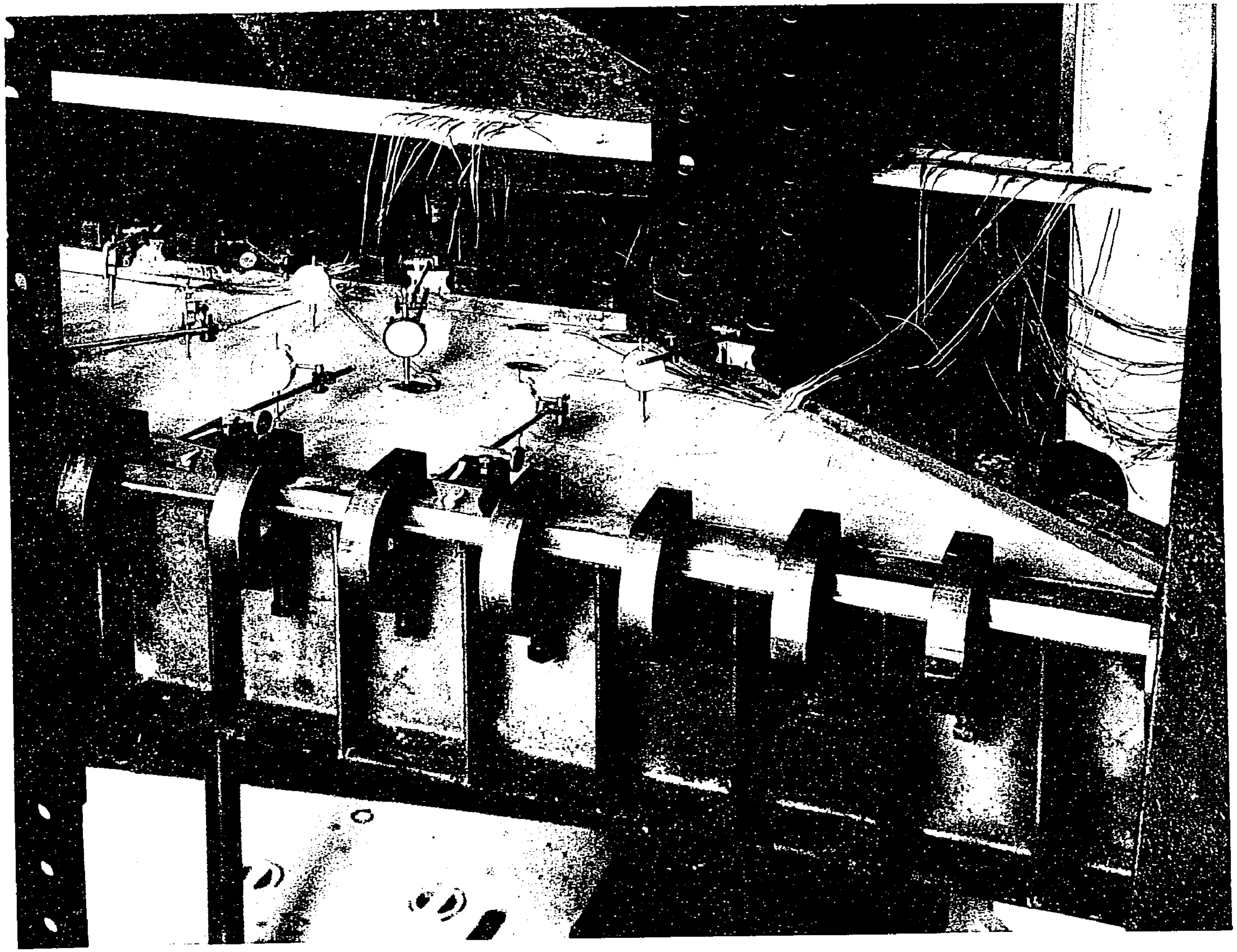

PHOTO. B PHOTOGRAPH SHOWING SUPPORTING CHANNELS AND CLAMPED EDGES

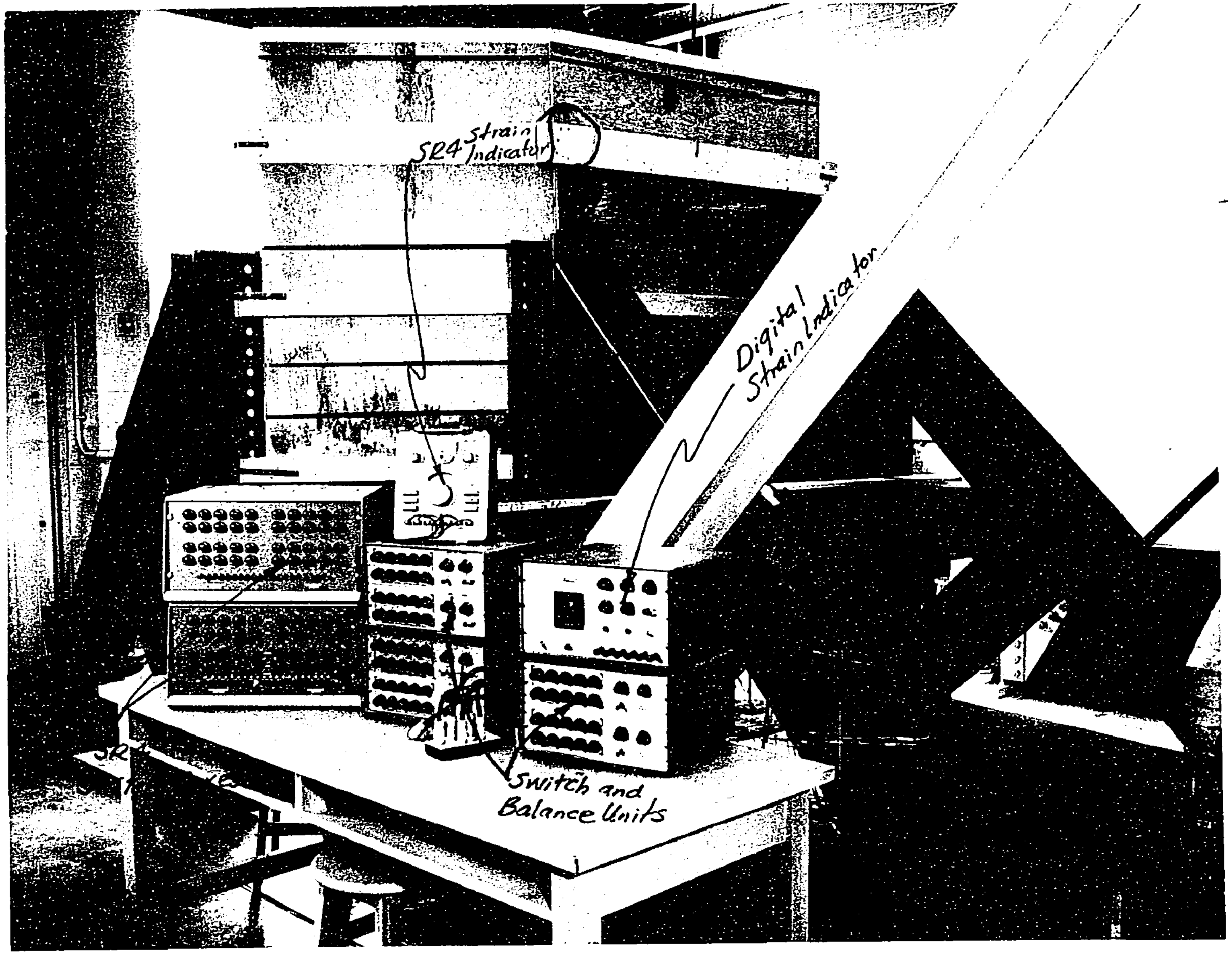

PHOTO. C PHOTOGRAPH SHOWING STRAIN INDICATORS AND WOODEN SKEW BOX IN POSITION ON TOP OF PLATE

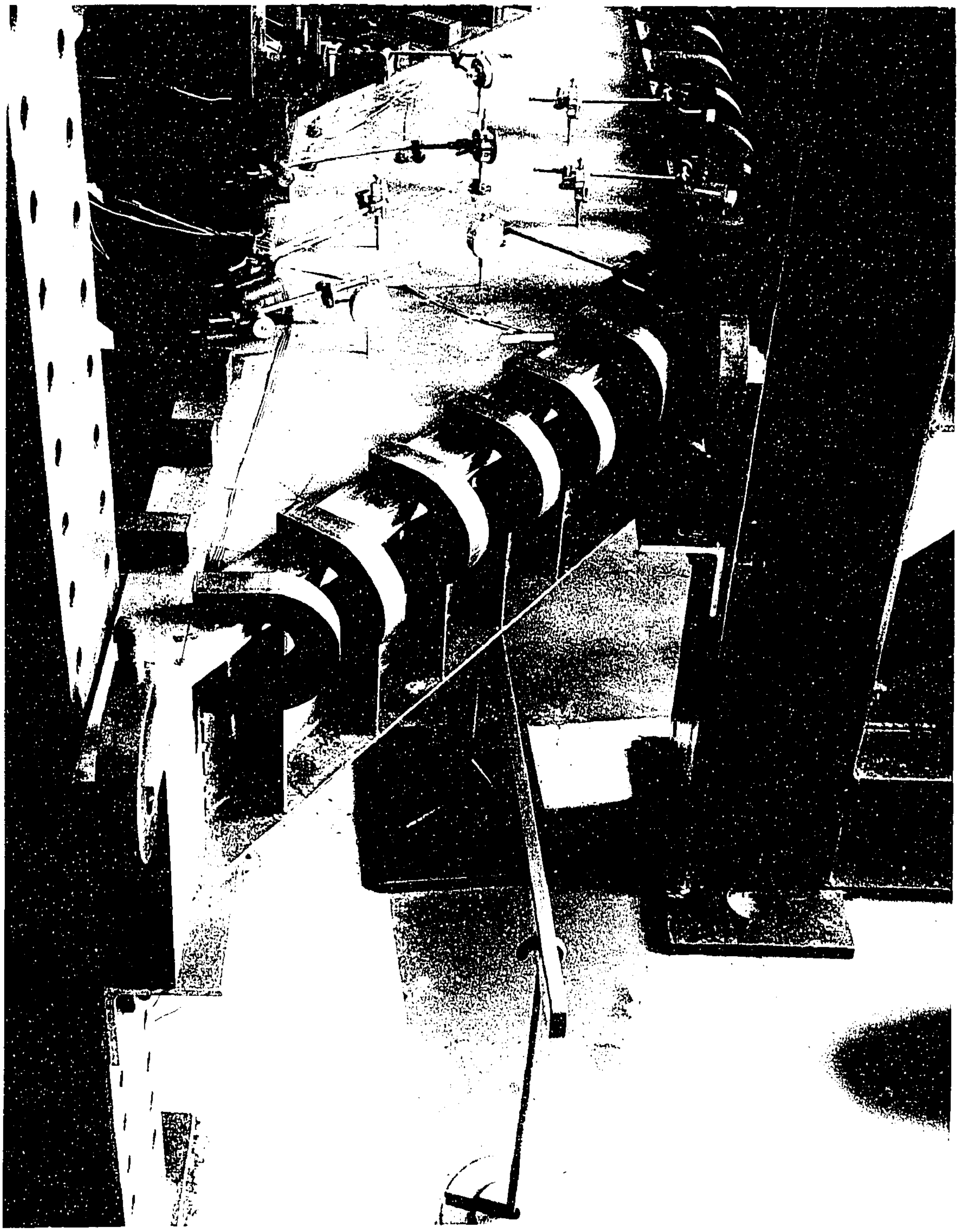

PHOTO. D PHOTOGRAPH SHOWING THE CLAMPED EDGES AND THE STRUCTURAL STEEL COLUMNS

CHAPTER V

COMPARISON OF RESULTS AND OBSERVATIONS

The deflections of the clamped skewed plate recorded at points where the dial indicators were located are compared with the deflections at the same points derived from the theoretical analysis. The comparison is tabulated in Table V.

A comparison of the theoretical and experimental principal stresses and moments is also made at two locations of the skewed plate, namely, at points where gauges 11 and gauge 17 (centre gauge) are located. These results are tabulated in Table VI.

By way of comparison, it is perhaps interesting to compare Timoshenko's results on clamped uniformly loaded rectangular plates with those derived from the theoretical analysis. In this particular case, the angle of skew in the deflection function is set to zero and the deflection calculated for different sides ratios of the rectangular plate. Table VII shows a comparison of the centre deflections, centre moments and the maximum edge moments.

As can be seen from Table V, the deflections of the particular skew plate investigated check very well with those from the theoretical analysis. Also the deflections for rectangular plates compare very closely with those given by Timoshenko and Krieger(1) for clamped rectangular plates. (See Table VII)

Centre deflections for clamped skewed plates having different skew sides ratios and skew angles compare quite well with those obtained by Kennedy. (See Table VIII)

Moments and stresses, on the other hand, do not compare so closely with either the experimental results or with other investigators (Tables VI and VII) The maximum or critical moments have been found to occur at the midpoint of the longitudinal sides. These moments are compared with Timoshenko's solution on clamped rectangular plates, and the deviation of the moment values is in the range of about 10 per cent. Comparisons of principal moments and stresses are also made with Kennedy's work (11) on clamped skewed plates and it is found that all principal moments and stresses compare quite favourably in the vicinity of the central portion of the plate but no close agreement could be observed for moments and stresses close to the built-in edges.

A comparison with Kennedy's results for the deflections and principal moments at the centre of skew plates with different skew sides ratios and skew angles is tabulated in Table VIII.

To conclude, it may be interesting to note that while based on the theoretical analysis, the deflections, moments and stresses are all proportional linearly with the intensity of loading. The experimental results, however, tend to deviate from this strict linear relationship, the deviation becomes increasingly evident for high intensities of loadings. This is understandable in the light that in the theoretical treatment of clamped skewed plates embodied in this thesis, only the bending effect on curvature is taken into account, whereas the tension in the middle surface of the plate effecting curvature, is ignored. This effect is presumably negligible when the lateral deflection $\bar{w}$ is small. [illegible] as the deflection gets larger, the membrane effect becomes more and more prominent until for large values of $\bar{w}$, the membrane effect becomes predominant and the bending stiffness is then negligible. It is quite evident from the experimental results in Table III that, for the particular skew plate under investigation, the effect of stretching of the middle plane of the plate becomes quite appreciable for loadings of four feet of water and higher.

TABLE V

Comparison of Deflections of a Clamped Skewed Plate (γ=1.23; θ=55^{o}) for Different Intensities of Loading Theoretical Studies Vs. Experimental Observations

Load in ft. of Water	Gauge 1		Gauge 2		Gauge 3		Gauge 4		Gauge 5		Gauge 6		Gauge 7	
	Experi	Theor	Experi.	Theor.	Experi.	Theor.	Experi.	Theor.	Experi.	Theor.	Experi.	Theor.	Experi.	Theor.
1'	.003"	.003"	.016"	.013"	.013"	.011"	.021"	.019"	.013"	.011"	.013"	.011"	.013"	.011"
2'	.007"	.007"	.031"	.026"	.023"	.021"	.040"	.037"	.023"	.021"	.024"	.021"	.025"	.021"
3'	.011"	.010"	.045"	.039"	.034"	.032"	.057"	.056"	.035"	.032"	.036"	.032"	.036"	.032"
4'	.014"	.014"	.060"	.052"	.044"	.042"	.075"	.075"	.045"	.042"	.047"	.042"	.047"	.042"
5'	.018"	.0173"	.072"	.065"	.053"	.053"	.089"	.093"	.054"	.053"	.057"	.053"	.057"	.053"
5 $\frac{1}{2}$'	.020"	.0191"	.078"	.071"	.058"	.058"	.098"	.102"	.062"	.058"	.063"	.058"	.063"	.058"

For Location of Dial Gauges See Fig. 4

TABLE VI

Comparison of Maximum Moment and Maximum Stress for a Clamped Skewed Plate (γ = 1.23; θ = 55°) for Two Different Intensities of Loading Theoretical Studies Vs. Experimental Results

Uniform Load in Ft. of Water	Gauge No.	Max. Moment (lb. in./ in.)		Max. Stress lbs./ in.^{2})	
		Experi.	Theor.	Experi.	Theor.
1	*17	7.55	7.44	725	714
	11	-3.09	-2.79	-296	-268
3	* 17	22.25	22.33	+2136	2144
	11	-8.95	-8.39	-859	-806

*Centre Gauge

ν = 0.3 D = 14,648.4375 lb. in

E = 10,000,000 p.s.i.

TABLE VII

Comparison of Centre Deflection, Centre Moments and Maximum Edge Moments for Clamped Rectangular Plates

b/a	Centre Deflection $\bar{\alpha} = 0, \beta = 0$		Max. Edge Moment $\bar{\alpha} = a, \bar{\beta} = 0$		Centre Moments $M_{\bar{x}}$ at $\bar{\alpha} = 0, \bar{\beta} = 0$		Centre Moments $M_{\bar{y}}$ at $\bar{\alpha} = 0, \bar{\beta} = 0$	
	From Theor.Anal.	Timo.	From Theor.Anal.	Timo.	From Theor.Anal.	Timo.	From Theor.Anal.	Timo.
1.0	.021 $\frac{pa^4}{D}$	.020 $\frac{pa^4}{D}$	-.170 pa^2	-.205 pa^2	.110 pa^2	.092 pa^2	.110 pa^2	.092 pa^2
1.1	.025 "	.024 "	-.203 "	-.232 "	.126 "	.106 "	.114 "	.092 "
1.2	.029 "	.028 "	-.233 "	-.256 "	.140 "	.120 "	.115 "	.091 "
1.3	.032 "	.031 "	-.259 "	-.275 "	.152 "	.131 "	.115 "	.089 "
1.4	.035 "	.033 "	-.282 "	-.290 "	.162 "	.140 "	.114 "	.085 "
1.5	.038 "	.035 "	-.301 "	-.303 "	.170 "	.147 "	.112 "	.081 "
1.6	.040 "	.037 "	-.318 "	-.312 "	.177 "	.152 "	.109 "	.077 "
1.7	.042 "	.038 "	-.332 "	-.320 "	.183 "	.157 "	.107 "	.073 "
1.8	.043 "	.039 "	-.344 "	-.325 "	.187 "	.160 "	.104 "	.070 "
1.9	.044 "	.040 "	-.354 "	-.329 "	.191 "	.163 "	.102 "	.066 "
2.0	.045 "	.041 "	-.362 "	.332 "	.195 "	.165 "	.099 "	.063 "

$\nu = 0.3$

TABLE VIII

Comparison of Centre Deflections and Centre Principal Moments for Clamped Skewed Plates with Different Skew Sides Ratios and Skew Angles

b/a	θ	Centre Deflection		Centre Max. Moment		Centre Min. Moment	
		Theor.	Kennedy	Theor.	Kennedy	Theor.	Kennedy
1.0	75°	.00009 $\frac{pa^4}{D}$	.00009 $\frac{pa^4}{D}$	.0061 pa^2	.0061 pa^2	.0021 pa^2	.0020 pa^2
1.25	15°	.0271 "	.0258 "	.1337 "	.1153 "	.1031 "	.0827 "
1.25	30°	.0178 "	.0170 "	.1011 "	.0925 "	.0710 "	.0596 "
1.25	60°	.0019 "	.0018 "	.0294 "	.0290 "	.0129 "	.0115 "
1.5	15°	.0331 "	.0309 "	.1557 "	.1345 "	.1002 "	.0741 "
1.5	60°	.0024 "	.0022 "	.0342 "	.0329 "	.0139 "	.0122 "
1.5	75°	.00017 "	.00015 "	.0088 "	.0085 "	.0029 "	.0027 "
1.6	0°	.0430 "	.0366 "	.1880 "	.1533 "	.1047 "	.0636 "
2.0	0°	.0454 "	.0379 "	.1951 "	.1566 "	.0998 "	.0576 "
2.0	60°	.0028 "	.0026 "	.0410 "	.0374 "	.0149 "	.0122 "

$\nu = 0.3$

CHAPTER VI

CONCLUSIONS

The Rayleigh-Ritz technique was employed to establish a deflection function for clamped uniformly loaded skewed plates. From this function, deflections, rectangular moments and principal moments were computed at arbitrary points of plates having different skew sides ratios and skew angles. These results were then compared with those observed from an experiment on a skewed plate with a skew sides ratio of 1.23 and a skew angle of 55 degrees. Also, the same results were used to compare with those obtained by Kennedy, Timoshenko and other investigators. These comparisons seem to indicate that the deflection function established for the clamped uniformly loaded skewed plate represents, to a reasonably high degree of accuracy, the true deflections of skew plates having the same boundary conditions.

Satisfactory agreements were also obtained when the principal moments and principal stresses in the central portion of the plate were compared with those observed from the experiment. However, as the edge of the plate is approached, the variation of moments and stresses is steep and it is in these areas that incongruencies exist in the values of the principal moments and stresses.

To gain confidence in the application of the theoretical analysis of clamped skewed plates embodied in this thesis another similar experiment will be conducted in the near future on a uniformly loaded clamped skewed plate with a skew sides ratio of 1.12 and a skew angle of 30°. The results of such an experiment will again be compared with those obtained from the [illegible]tical analysis.

APPENDIX A

Fortran Programme for the Theoretical Analysis of Uniformly Loaded Skewed Plates

```
C       ANALYSIS OF UNIFORMLY LOADED SKEWED PLATES
  100 READ 10,G,DEGRE,ANU
   10 FORMAT(3F10.5)
      THETA=DEGRE*0.017453292
      A1=(COSF(THETA))**4
      A2=(256./225.)*G**5
      A3=A1*A2
      A4=20.8050793*G**5+11.8886168*G**7+23.7772337*G**7
     1*(SINF(THETA))**2+20.8050793*G**9
      A001=A3/A4
  144 A2=1.5371140+1.5029560*G*G+3.0059119*G*G
     1*(SINF(THETA))**2+1.5371140*G**4
      A3=A1*A2
      A4=28.1072273+43.5438963*G**2+51.7529915*G**2*(SINF(THETA)**2)
      A5=71.9188109*G**4-7.8521768*G**4*(SINF(THETA)**2)
     1+62.8174256*G**4*(SINF(THETA)**4)
      A6=43.5438963*G**6+51.7529915*G**6
     1*(SINF(THETA)**2)+28.1072273*G**8
      A7=A4+A5+A6
      A002=A3/A7
      A2=6.7633018*(SINF(THETA))+6.7633018*G*G*SINF(THETA)
      A3=A1*A2
      A4=28.1072273+43.5438963*G**2+51.7529915*G**2*(SINF(THETA)**2)
      A5=71.9188109*G**4-7.8521768*G**4*(SINF(THETA)**2)
     1+62.8174256*G**4*(SINF(THETA)**4)
      A6=43.5438963*G**6+51.7529915*G**6
     1*(SINF(THETA)**2)+28.1072273*G**8
      A7=A4+A5+A6
      A022=A3/A7
      PUNCH 198
  198 FORMAT(24H  INPUT DATA AND RESULTS)
      PUNCH 199,G,THETA,ANU,A001,A002,A022
  199 FORMAT(3H G=F10.5,7H THETA=F10.5,5H ANU=F10.5/6H A001=F20.8,
     16H A002=F15.8/6H A022=F15.8)
  888 DIMENSION TBETA(3),TALPA(3)
      READ 101,(TBETA(I),I=1,3),(TALPA(J),J=1,3)
  101 FORMAT(6F10.5)
      DO 99 J=1,3
      ALPA=TALPA(J)
      DO 99 I=1,3
      BETA=TBETA(I)
```

```
      SEC=1./COSF(THETA)
      S=SINF(THETA)
      C=COSF(THETA)
      SEC3=SEC**3
      P1=(G**2-BETA**2)**2
      P2=12.*A001*ALPA**2-4.*A001
      P3=P1*P2
      P4=32.*S*A001
      P5=G**2*(ALPA**3)*BETA+ALPA*(BETA**3)
     1-G**2*ALPA*BETA-(ALPA**3)*(BETA**3)
      P6=P4*P5
      P7=S**2
      P8=C**2
      P9=(1.-ALPA**2)**2
      P10=12.*A001*BETA**2-4.*A001*G**2
      P11=P9*P10
      AMAL1=-SEC3*(P3+P6+(P7+ANU*P8)*P11)
      AMBE1=-SEC3*(P11+P6+(P7+ANU*P8)*P3)
      AMAB1=-SEC3*S*(P3+P6+P11)-SEC*(1.-ANU)*(-16.)*A001*(-P5)
      DEF1=P9*P1*A001
      P20=-4.*A002+12.*A022*ALPA*BETA
     1-20.*A022*(ALPA**3)*BETA+12.*A002*ALPA**2
      WAA=P1*P20
      P21=12.*A022*G**2*ALPA*BETA-20.*A022*ALPA*BETA**3-4.*A002*G**2
     1+12.*A002*BETA**2
      WBB=P9*P21
      P22=16.*A002*(G**2*ALPA*BETA-G**2*(ALPA**3)*BETA
     1-ALPA*BETA**3+(ALPA**3)*(BETA**3))
      P23=A022*(6.*(G**4)*ALPA**2-5.*G**4*ALPA**4-G**4-36.*G**2*ALPA**2
     1*BETA**2+30.*G**2*ALPA**4*BETA**2+6.*G**2*BETA**2+30.*ALPA**2
     2*BETA**4-25.*ALPA**4*BETA**4-5.*BETA**4)
      WAB=P22+P23
      AMAL2=-SEC3*(WAA-2.*S*WAB+(S**2+ANU*C**2)*WBB)
      AMBE2=-SEC3*(WBB-2.*S*WAB+(S**2+ANU*C**2)*WAA)
      AMAB2=-SEC3*S*(WAA-2.*S*WAB+WBB)+SEC*(1.-ANU)*WAB
      DEF2=P9*P1*(A002-A022*ALPA*BETA)
      P24=16.*A001*(G**2*ALPA*BETA-G**2*(ALPA**3)*BETA
     1-ALPA*(BETA**3)+(ALPA**3)*(BETA**3))
      TAN=S/C
      AMX1=-((SEC**2*P3)-2.*SEC*TAN*P24+TAN**2*P11+ANU*P11)
      AMY1=-(P11+ANU*(P3*(SEC**2)-2.*SEC*TAN*P24+P11*(TAN**2)))
      AMXY1=(1.-ANU)*(P24*SEC-P11*TAN)
      AMX2=-(SEC**2*WAA-2.*SEC*TAN*WAB+TAN**2*WBB+ANU*WBB)
      AMY2=-(WBB+ANU*(SEC**2*WAA-2.*SEC*TAN*WAB+TAN**2*(WBB)))
      AMXY2=(1.-ANU)*(WAB*SEC-WBB*TAN)
```

```
      PMMAX=(AMX2+AMY2)/2.+SQRTF(((AMX2-AMY2)/2.)**2+(AMXY2)**2)
      PMMIN=(AMX2+AMY2)/2.-SQRTF(((AMX2-AMY2)/2.)**2+(AMXY2)**2)
      PUNCH 150,AMAL2,AMBE2,AMAB2,AMX2,AMY2,AMXY2
   99 PUNCH 151,PMMAX,PMMIN,DEF2,ALPA,BETA
  150 FORMAT(7H AMAL2=E16.8,7H AMBE2=E16.8/7H AMAB2=E16.8/
     16H AMX2=E16.8,6H AMY2=E16.8/7H AMXY2=E16.8)
  151 FORMAT(7H PMMAX=E16.8,7H PMMIN=E16.8/
     26H DEF2=E16.8/6H ALPA=F6.3,6H BETA=F6.3///)
      GO TO 100
      END
      1.23        55.       0.3333
       0.0       0.615      1.23         0.0         0.5         1.0
```

Fortran Programme for the Experimental Studies of a Uniformly Loaded Skewed Plate

```
C     EXPERIMENTAL STUDIES OF A UNIFORMLY LOADED SKEWED PLATE
      READ 9,E,ANU,T
  100 READ 10,SA,SB,SC
    9 FORMAT(3F10.8)
   10 FORMAT(3F10.8)
      T1=(SA+SC)/(1.-ANU)
      T2=(1./(1.+ANU))*SQRTF(2.*(SA-SB)**2
     1+2.*(SB-SC)**2)
      SIGMX=(E/2.)*(T1+T2)
      SIGMN=(E/2.)*(T1-T2)
      DIMXM=0.5*ATANF((2.*SB-SA-SC)/(SA-SC))-45.*.017453292
      T4=T*T/6.
      PMMAX=T4*SIGMX
      PMMIN=T4*SIGMN
      PUNCH 11,SA,SB,SC,SIGMX,SIGMN,DIMXM,PMMAX,PMMIN
   11 FORMAT(4H SA=E16.8,4H SB=E16.8,4H SC=E16.8/
     17H SIGMX=E16.8,7H SIGMN=E16.8,7H DIMXM=E16.8/
     27H PMMAX=E16.8,7H PMMIN=E16.8///)
      GO TO 100
      END
 10000000.  0.333333      0.25
        0.          -3.        1.
        4.          -7.       -1.
        6.          -9.       -4.
```

00 00 0

APPENDIX B

Computer Solution for Deflections, Moments, and Principal Moments for a Uniformly Loaded Skewed Plate with a Skew Sides Ratio of 1.23 and a skew Angle of 55°

```
 INPUT DATA AND RESULTS
G=    1.23000 THETA=      .95993 ANU=      .33330
A001=                .00111395 A002=        .00143657
A022=          .00192663
AMAL2=     .67379836E-01 AMBE2=    .62196962E-01
AMAB2=     .58346108E-01
AMX2=     .38647488E-01 AMY2=    .20609022E-01
AMXY2=     .31517782E-02
PMMAX=     .39182325E-01 PMMIN=    .20074185E-01
DEF2=    .32881273E-02
ALPA= 0.000 BETA= 0.000


AMAL2=     .55376465E-01 AMBE2=    .49303281E-01
AMAB2=     .48392214E-01
AMX2=     .31762636E-01 AMY2=    .12518439E-01
AMXY2=     .30304716E-02
PMMAX=     .32228575E-01 PMMIN=    .12052499E-01
DEF2=    .18495715E-02
ALPA= 0.000 BETA=  .615


AMAL2=   -.71931256E-01 AMBE2=   -.92141318E-01
AMAB2=   -.75477746E-01
AMX2=   -.41258079E-01 AMY2=   -.29206948E-01
AMXY2=   -.16555108E-01
PMMAX=   -.17614936E-01 PMMIN=   -.52850090E-01
DEF2=    .00000000E-99
ALPA= 0.000 BETA= 1.230

                                                          00 00 00

AMAL2=     .44834382E-01 AMBE2=    .46696486E-01
AMAB2=     .42343381E-01
AMX2=     .25715946E-01 AMY2=    .12918020E-01
AMXY2=     .56172069E-02
PMMAX=     .27831660E-01 PMMIN=    .10802306E-01
DEF2=    .18495715E-02
ALPA=   .500 BETA= 0.000
```

```
AMAL2=    .22838868E-01 AMBE2=    .24214036E-01
AMAB2=    .23323437E-01
AMX2=    .13099839E-01 AMY2=    .23157578E-02
AMXY2=    .46149327E-02
PMMAX=    .14805100E-01 PMMIN=    .61049510E-03
DEF2=    .61133134E-03
ALPA=  .500 BETA=  .615


AMAL2=  -.70889517E-02 AMBE2=  -.90806883E-02
AMAB2=  -.74384642E-02
AMX2=  -.40660560E-02 AMY2=  -.28783959E-02
AMXY2=  -.16315349E-02
PMMAX=  -.17359828E-02 PMMIN=  -.52084690E-02
DEF2=    .00000000E-99
ALPA=  .500 BETA= 1.230


AMAL2=  -.13940060E+00 AMBE2=  -.10882480E+00
AMAB2=  -.11419028E+00
AMX2=  -.79956904E-01 AMY2=  -.26649636E-01
AMXY2=  -.89577538E-09
PMMAX=  -.26649637E-01 PMMIN=  -.79956903E-01
DEF2=    .00000000E-99
ALPA= 1.000 BETA= 0.000

                                                          00 00 0

AMAL2=  -.13738170E-01 AMBE2=  -.10724869E-01
AMAB2=  -.11253648E-01
AMX2=  -.78798907E-02 AMY2=  -.26263675E-02
AMXY2=    .14556350E-08
PMMAX=  -.26263675E-02 PMMIN=  -.78798905E-02
DEF2=    .00000000E-99
ALPA= 1.000 BETA=  .615


AMAL2=    .00000000E-99 AMBE2=    .00000000E-99
AMAB2=    .00000000E-99
AMX2=  -.00000000E-99 AMY2=  -.00000000E-99
AMXY2=    .00000000E-99
PMMAX=    .00000000E-99 PMMIN=    .00000000E-99
DEF2=    .00000000E-99
ALPA= 1.000 BETA= 1.230
```

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NOMENCLATURE

$\bar{x}$ $\bar{y}$	rectangular cartesian co-ordinates
$\bar{\alpha}$ $\bar{\beta}$	oblique co-ordinates
α β	dimensionless oblique co-ordinates
$\bar{w}$	lateral deflection of plate
2a	plate's transverse dimension (along $\bar{\alpha}$ axis)
2b	plate's longitudinal dimension (along $\bar{\beta}$ axis)
h	thickness of plate
θ	angle of skew
p	intensity of uniformly distributed load
γ	ratio of the longitudinal and transverse dimensions (ratio of skew sides)
ν	Poisson's ratio
D	flexural rigidity of the plate
J	strain energy integral for bending

I	total energy integral
$M_{\bar{x}}$	bending moment per unit length of plate perpendicula to the $\bar{x}$ axis
$M_{\bar{y}}$	bending moment per unit length of plate perpendicula to the $\bar{y}$ axis
$M_{\bar{x}\bar{y}}$	twisting moment per unit length of plate perpendicula to the $\bar{x}$ axis
$\sigma_{\bar{x}}$	normal component of stress parallel to the $\bar{x}$ axis
$\sigma_{\bar{y}}$	normal component of stress parallel to the $\bar{y}$ axis
σ_{max}	maximum principal stress
σ_{min}	minimum principal stress
A_o, A_1	parameter in the assumed deflection function
M_{max}	principal maximum moment
M_{min}	principal minimum moment
ϕ	angle measured clockwise from the $\bar{x}$ axis, giving the direction in which the maximum principal moment occurs
e_a, e_b, e_c	recorded unit strain in the three legs of a rosette gauge

VITA AUCTORIS

1938 Simon Shung Fun Ng was born in Canton, China, on March 6, 1938.

1945 In September, 1945, he entered Pui Ying School, Canton, China, where he obtained his elementary education.

1950 In September, 1950, he enrolled at Wah Yan College, Hong Kong, where he obtained his secondary education.

1957 In September, 1957, he enrolled in first year Science at the University of McGill, Montreal, Canada.

1958 In September, 1958, he entered the University of British Columbia, Vancoaver, Canada to study civil engineering.

1962 In May, 1962, he was graduated from the University of British Columbia with a Bachelor of Applied Science Degree. In June, he was employed as junior engineer in the hydro-electric design division of the International Power and Engineering Consultants, Ltd., Vancouver, Uritish Coluabia.

1963 In September, 1963 he enrolled at the University of Windsor in order to obtain the degree of Master of Applied Science in Civil Engineering.