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Structural synthesis of bonded prestressed concrete beams.

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STRUCTURAL SYNTHESIS OF BONDED PRESTRESSED CONCRETE BEAMS

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ABSTRACT

A structural synthesis capability to minimize the cost of bonded pretensioned prestressed concrete beams of various kinds, subjected to different loading conditions including all the live loading possibilities which may act is developed. The analysis is based on the Canadian Prestressed Concrete Institute code using the working load principal. The synthesis approach uses the penalty function method of Fiacco and McCormick which converts the constrained minimization problem to a sequence of unconstrained minimization problems, and enables the use of the Fletcher and Powell unconstrained minimization method. The design variables are the independent cross sectional dimensions, the prestressing stress, the area of prestressing steel in each row and their distances from the bottom fiber of the beam. Numerical results are presented which demonstrate the capability of the method and some properties of the design space.

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SYMBOLS

A_c - The net cross sectional area of concrete.

A_e - The transformed cross sectional area of concrete.

A_g - The gross cross sectional area.

A_s - The total area of prestress steel

$A_{s(i)}$ - The area of prestress steel in the i^{th} row, i = 1, 2,...,m where m = number of rows.

b - The number of behaviour constraints

B_f - The bottom flange width.

B_w - The web thickness.

C_c - The cost of unit volume of concrete including cost of placing and transportation.

C_r - The forming costs of the vertical, horizontal and sloping parts of the perimeter per unit area; respectively (r = 1,2,3)

C.g.s. - The center of gravity of the prestress steel

$C(\bar{V})$ - The objective function (the cost function)

d - The number of live loading conditions.

D_{efa} - The allowable deflection value of the beam.

$D_{ef}^{(x)}$ - The maximum deflection of the beam for the x^{th} critical loading combination, x = 1,2,...5

$e_s^{(i)}$ - The distance between the i^{th} steel row and the center of gravity of the transformed section.

E_c - The modulus of elasticity of concrete.

E_s - The modulus of elasticity of steel.

$f_{c_a}^{+}$ - The allowable tensile stress of concrete.

$f_{c_a}^{-}$ - The allowable compressive stress of concrete.

f_{c_b} - The normal stress of concrete at bottom fiber.

f_{ct} - The normal stress of concrete at top fiber.

$f_{cp}^{(i)}$ - The stress of concrete at the level of the i^{th} steel row due to the prestressing force.

$f_{cs}^{(i)}$ - The stress of concrete at the level of the i^{th} steel row.

f_{cy} - The stress of concrete at distance Y from the center of gravity of the transformed section.

F_c - The concrete stress

$f_{p(bed)}^{(i)}$ - The initial stress of the prestressing steel in the i^{th} row.

$f_{p(bed)}$ - The initial prestressing stress.

f_{ps} - The maximum principal tensile stress of concrete.

$f_{ps}^{(x)}$ - The maximum principal tensile stress of concrete due to the x^{th} critical loading combination.

f_{psa} - The allowable principal tensile stress of concrete.

f_s - The tensile stress of the prestress steel.

$f_s^{(i)}$ - The stress induced in the prestress steel in the i^{th} row due to bond between concrete and steel.

f_{s1} - The loss in the prestressing steel due to creep and shrinkage under girder weight and the prestressing force.

f_{s2} - The loss in the prestressing stress due to creep under superimposed dead load.

f_{sa} - The allowable tensile stress of the prestressing steel.

$f_{sg}^{(t)}$ - The stress induced in the prestressing steel due to girder weight.

$f_{sn}^{(i)}$ - The net initial prestressing stress of steel in the i^{th} row.

$f_{sn}^{(t)}$ - The net total prestressing stress.

$f_{sR}^{(i)}$ - The reduction in the initial prestressing stress of steel in the i^{th} row.

$f_{ssL}^{(t)}$ - The stress induced in the prestressing due to superimposed dead load.

$f(\vec{V})^*$ - The function value of a quadratic function at the minimum.

$F(\vec{V})$ - The Fiacco and McCormick function.

$F(\vec{V})^*$ - The minimum value of the Fiacco and McCormick function.

$g_j(\vec{V})$ - The j^{th} constraint function

H - The total depth of the beam.

$H_{s(i)}$ - The height of the i^{th} prestress steel row measured from the bottom of the beam.

H_w - The web depth

$H(\vec{V})$ - The matrix of the second partial derivatives of the $F(\vec{V})$ function.

I_e - The moment of inertia of the transformed section.

L_k - The lower limit of the constraint on the k^{th} variable.

M - The bending moment

n - The modular ratio

P_r - The length of the vertical, horizontal and sloping parts of the perimeter, respectively ($r = 1,2,3$).

$P(\vec{V})$	- The summation of the constraint functions
q	- The total number of side and behaviour constraints
R	- The constant multiplier of the Fiacco and McCormick function
s	- The number of side constraints.
$\vec{S}_k$	- Move direction
T_b	- The bottom flange thickness.
T_f	- The top flange width
T_{sb}	- The depth of the sloping portion of the bottom flange
T_{st}	- The depth of the sloping portion of the top flange.
T_t	- The top flange thickness
U_d	- The upper limit constraint on the total depth.
U_k	- The upper limit constraint on the k^{th} variable.
$\vec{V}$	- The vector of the design variables.
$\vec{V}^*$	- The solution vector
V_k	- The k^{th} design variable
Y	- The distance between the center of gravity of the transformed section and the point at which the stress is obtained.

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CHAPTER I

INTRODUCTION

Structural Synthesis has been defined as (Ref. 1) "the rational, directed evolution of a structural system, which, in terms of a defined objective, efficiently performs a set of specified functional purposes". The main problem of a structural designer is to select the "best" design for a certain structure which satisfies both structural and economical requirements especially if this structure is going to be constructed in large numbers. The selection of the best design has been left to the experience of the designer. There is no other means of choice between the different acceptable design possibilities except by doing a number of trials and much computational work. In the end the selected design may not be the optimum one. However, the present "Synthesis" capability can be viewed as a mathematical programming approach which can provide a useful tool to find the best or the optimum design automatically or at least provide a good means of choice between the different designs.

The structural synthesis concept, or in other words the automated optimum design of a structural system is based on the following considerations:

1. A set of "design variables": these are the quantities which are allowed to vary independently during the synthesis procedure. The goal of the structural synthesis concept is to

select these variables such that an acceptable optimum design is obtained.

2. An "objective function": this is a function of the design variables and provides a basis of choice between alternative acceptable designs; the most common objective functions used for evaluating merit in structural problems are the minimum cost or the minimum weight. By minimizing this function an optimum design can be obtained.
3. A set of "constraints": these represent the limits between the acceptable and unacceptable designs. They are namely, "behaviour" constraints and "side" constraints. The behaviour constraints are limits on the different kinds of stresses (i.e. normal, shear,..etc.) and on deflections. The side constraints are basically constraints on the design variables. None of these constraints must be violated during the synthesis procedure in order to reach to an acceptable optimum design.
4. A structural "analysis" capability: this is a well defined means of predicting the different structural behaviour (i.e. stresses, deflection,...etc.) during the synthesis procedure.
5. A powerful optimization method: this is a mathematical procedure which optimizes the objective function in order to obtain the optimum design.

The synthesis scheme can be represented graphically by a Cartesian space having a dimension equal to the number of design variables; this space is called the "design variable space". It can be divided into two portions by means of the "composite constraint surface" which is the collection of the "behaviour constraint surface" and the "side constraint surface". This composite surface represents the limit between the acceptable and unacceptable portions of the design variable space. The coordinates of any point in this space represent a design of the structural system. Any point can be identified as one of the following four types:

1. Free and acceptable.
2. Bound and acceptable.
3. Free and unacceptable.
4. Bound and unacceptable.

A graphical representation of the design variable space for a case of two design variables, V_1, and V_2, is given in Fig. 1. The optimum design in this case is a constrained optimum design (i.e. the optimum point is a bound acceptable point), but in many other cases the optimum design is an unconstrained design as shown in Fig. 2. In general there is no means of indicating that the optimum design is an absolute optimum or "Global" optimum. In the majority of practical cases an optimum design is only a "local" or "relative" optimum. Such a relative optimum is characterized by two or more designs, each one having no acceptable designs of better or equal merit within some finite neighbourhood about it. The relative minima concept is shown in Fig. 3. There is a way to build some confidence that a certain local optimum design is the global design; if the results obtained from various synthesis paths from widely separated initial designs converge toward the same particular design, then this design is probably a global optimum. But if they converge to different points the optimum design among them all can be considered as an absolute optimum as shown in Fig. 3.

The structural synthesis concept has been successfully applied to different structural problems. Schmit (Ref. 1) has treated the case of a planar statically indeterminate three bar truss under the influence of several distinct load conditions. A minimum weight design was obtained using a method of constrained steepest descent as the means of optimization. It is easily shown that the optimum design was not the fully stressed design. Rozvany (Ref. 2) applied the principal of reversed deformation method to synthesize a prestressed plate of minimum tendon volume and prestressed beam grids (grillage) of minimum weight. Goble and DeSantis (Ref. 3) synthesized a continuous, composite welded plate girder subject to the standard specifications of the American Association of State Highway Officials of a given span length and concrete deck dimensions. The optimization was based on minimum girder cost, using a smoothing technique from dynamic programming to determine the optimum number of flange splices and the material types. Goble and LaPay (Ref. 4) synthesized prestressed concrete simple beams of given span length, used in building structures where their flanges provide the structural surface, such that the optimum design obtained covers a large area for the least cost. The constrained steepest descent optimization method was used as the minimization procedure. The design variables were the independent cross sectional dimensions including the slopes of the top and bottom flanges, the area of transverse mild steel reinforcing, the area of prestress steel and the prestress stress. The center of gravity of the prestress steel was kept fixed. The analysis was based on the ACI code, using the ultimate load principle. The different loading conditions were uniformly distributed dead, live and superimposed dead loads, the prestress force and the losses in prestress force due to creep and shrinkage.

The aim of this work is to develop a structural synthesis capability to minimize the cost of bonded pretensioned prestressed concrete beams of various shapes. The different loading conditions are girder weight, superimposed dead load, all the live loading possibilities which may occur, prestress force and losses in prestress force due to creep and shrinkage. The analysis is based on the CPCI (Ref. 5) code using the working load principle; the normal stresses, principal stresses and deflections are obtained under critical loading combinations. The design variables are a set of independent cross sectional dimensions, the prestress stress, the area of prestress steel in the various rows and their distances from the bottom fiber of the beam.

The problem considered is a constrained minimization problem; it is converted to a sequence of unconstrained minimization problems using the "Penalty" function method of Fiacco and McCormick (Ref. 6) in order to be able to use one of the powerful unconstrained minimization methods. In this work the method of Fletcher and Powell (Ref.7) which is considered the most powerful unconstrained minimization method is used. A number of designs were obtained for a given simple beam; by varying the constraint requirements the synthesis capability converged to completely different optimum design. The relative minima concept was studied in some problems by starting from widely separated initial designs.

CHAPTER II

ANALYSIS OF BONDED PRESTRESSED CONCRETE BEAMS

2.1 Introduction

A well defined analysis must be used in predicting the different structural behaviours (i.e. stresses, deflection,... etc.). The analysis must possess the capability of analyzing beams having rather general cross sectional shapes (i.e. Rectangular, T, I...etc.) due to the synthesis scheme which may prescribe certain cross sectional dimensions to be zero (Fig. 4).

The analysis here is based on the CPCI (Ref. 5) code using the working load principal. The different loading conditions acting on the beam are girder weight, which varies during the synthesis procedure due to the changes in the cross sectional dimensions, superimposed dead load, all the live loading possibilities which may act, prestressing force and losses in prestressing force due to creep and shrinkage. In the analysis of a bonded prestressed concrete beam there are five critical loading combinations under which the different kinds of stresses (i.e. normal stresses, principal stresses, ...etc.) and the maximum deflection of the beam are determined. The normal stresses of the concrete are determined at the top and the bottom fibers of the cross section where the critical values of normal stresses of concrete occur. The stresses of the prestressing steel are determined at each prestressing steel row. In case that the section is not fully prestressed, mild steel may be used to resist the tensile stresses at the top or the bottom fibers. The prestressing steel can replace the mild steel if it is not fully stressed and if it is located at the same place where tensile stress occurs. In case that the shearing force acting on the section is large the maximum principal stress must be determined. It is pointed out that shear reinforcement may be used in case that the value of the maximum principal tensile stresses exceed the tensile stress that can be resisted by concrete.

2.2 Normal Stresses

2.2.1 Due to girder weight, superimposed dead and live loads.

The determination of normal stresses of concrete for each loading condition is based on the equation

$$f_c = \pm \frac{M.Y}{I_e} \qquad (2.1)$$

where

f_c = The concrete stress

M = The bending moment due to each loading condition

I_e = The moment of inertia of transformed section

Y = The distance between the center of gravity of transformed section and the point at which stress is obtained

Due to bond between concrete and the prestressing steel, stress is induced in the prestressing steel which is obtained by using the relation

$$f_s^{(i)} = n\, f_{cs}^{(i)} \qquad (2.2)$$

where

$f_s^{(i)}$ = The stress induced in the prestressing steel in the $i^{\underline{th}}$ row; i = 1,2,..., m where m = number of rows

$f_{cs}^{(i)}$ = The stress in concrete at the level of the $i^{\underline{th}}$ row

n = The modular ratio = $\frac{E_s}{E_c}$

2.2.2 Due to prestressing force

An initial prestressing force is applied on the concrete section at each prestressing steel row. The compressive stress in the concrete caused by these forces reduces the initial prestressing force in each row due to the bond action between the concrete and the prestressing steel. The reduction of the initial stress in the $i^{\underline{th}}$ row is given by

$$f_{sR}^{(i)} = n\, f_{cP}^{(i)} \qquad (2.3)$$

where

$f_{sR}^{(i)}$ - The reduction in the initial prestressing stress of the $i^{\underline{th}}$ row

$f_{cp}^{(i)}$ - The compressive stress of concrete at the i^{th} row level due to the total prestressing force

Let

$$\delta_i = \frac{f_{sR}^{(i)}}{f_{p(bed)}^{(i)}}$$

where

$f_{p(bed)}^{(i)}$ - The initial stress of the prestressing steel in the i^{th} row.

The net initial prestressing stress of the steel in the i^{th} row is given by

$$f_{sn}^{(i)} = (1 - \delta_i)\, f_{p(bed)}^{(i)} \tag{2.5}$$

The normal stress on the concrete due to the prestressing force only is

$$F_c = \frac{P_n}{A_e} \pm \frac{Y \sum_{i=1}^{m} P_n^{(i)} . e_s^{(i)}}{I_e} \tag{2.6}$$

where

P_n - The total net initial prestressing force where

$$P_n = \sum_{i=1}^{m} P_n^{(i)}$$

$P_n^{(i)}$ - The net initial prestressing force of the i^{th} row.

$e_s^{(i)}$ - The distance between the i^{th} row and the center of gravity of the transformed section.

A_e - The transformed cross sectional area (i.e. $A_e = A_g + (n - 1) A_s$ where A_s is the total area of the prestressing steel and A_g is the gross cross sectional area.

2.2.3 <u>Loss in the prestressing force due to creep and shrinkage.</u>

The loss in the prestressing force due to creep and shrinkage under the girder weight and the prestressing force has a maximum and a minimum value depending upon whether the upper or the lower limit of the creep factor ϕ and the shrinkage strain ε_s is used (Ref. 8). A suitable approximate method of obtaining the creep and shrinkage losses is to calculate these losses as if the prestressing steel is located at its center of gravity. The losses are obtained by using the following equation (Ref. 8):

$$f_{sl} = (1 - e^{-\gamma\phi}) \left[-f_{sn}^{(t)} + \frac{1-\gamma}{\gamma} \cdot f_{sg}^{(t)} + \frac{(1-\gamma)\, E_s\, \varepsilon_s}{\gamma \cdot \phi} \right] \tag{2.7}$$

where

f_{sl} - The loss in the prestressing steel stress due to creep and shrinkage under the girder weight and the prestressing force.

$f_{sn}^{(t)}$ - The net total prestressing stress

$f_{sg}^{(t)}$ - The stress induced in the prestressing steel due to girder weight

E_s - The modulus of elasticity of steel

$$\gamma = \sum_{i=1}^{m} f_{sR}^{(i)} / f_{P(bed)}; \text{ where } f_{P(bed)} \text{ is the total}$$

initial prestressing stress.

The loss in the prestressing stress due to superimposed dead load is obtained using equation 2.7:

$$f_{s2} = (1 - e^{-\gamma \phi_{sL}}) \left[\frac{(1 - \gamma) \cdot f_{ssL}^{(t)}}{\gamma} \right]$$

where

f_{s2} - The loss in prestressing stress due to creep under superimposed dead load

$f_{ssL}^{(t)}$ - The stress induced in the prestressing steel due to superimposed dead load.

ϕ_{sL} - The creep factor due to superimposed dead load.

2.3 Shear stress

The shear stress is obtained by using the following equation

$$q_s = \frac{V.Q}{I_e.t} \qquad (2.8)$$

where

q_s - The shear stress

Q - The static moment of the area above the section at which the shear stress is obtained, about the center of gravity of the transformed section.

V - The shearing force on the section.

t - The width of the section at which the shear is obtained.

2.4 Principal stress

The maximum principal tensile stress is obtained using the equation

$$f_{ps} = f_{cy}/2 + \sqrt{(f_{cy}/2)^2 + q_{sy}^2} .$$

where

f_{ps} - The maximum principal tensile stress.

f_{cy} - The stress of concrete at distance "y" from the center of gravity of the transformed section where the maximum tensile stress occur.

q_{sy} - The shear stress at the same distance "y".

In this work, for rectangular beams and beams with only small flange widths the principal tensile stress was obtained by dividing the cross section into 10 strips and calculating the normal and the shear stresses at each strip. The highest value among the principal tensile stresses obtained at each strip was considered to be the maximum principal tensile stress. For a beam having appreciable flange widths the maximum principal tensile stress was considered to be located at the bottom of the flange.

2.5 Stresses under critical loading combinations

The concrete and steel stresses previously obtained are combined together under five loading combinations which represent the critical loading conditions to which the beam will be subjected. These critical loading combinations are:

1. Girder weight + prestressing force (at transfer)
2. Fully loaded + minimum creep and shrinkage
3. Dead load + prestressing force + minimum creep and shrinkage
4. Fully loaded + maximum creep and shrinkage + creep due to superimposed dead load
5. Dead load + prestressing force + maximum creep and shrinkage + creep due to superimposed dead load

where

Fully loaded = prestressing force + girder weight + superimposed dead load + live load.

Dead load = Girder weight + superimposed dead load.

2.6 Deflections

Maximum deflection of a particular beam due to the prestressing force, girder weight, superimposed dead load, various live loading conditions and due to creep and shrinkage are obtained using standard elastic methods. In order to determine the critical deflections the combinations considered are the same as for determining critical stresses.

CHAPTER III

SYNTHESIS

3.1 Introduction

Prestressed concrete beams of various kinds and subjected to different loading combinations including all the live loading conditions which may act are synthesized in such a way that the cost of any cross section is minimized, and all the constraints on the design variables and on the behaviour of the structure are satisfied. The design variables in this work are a set of independent cross sectional dimensions of concrete, the prestressing stress, the area of prestressing steel in each row and their distances from the bottom fiber of the beam. A function of these design variables and of the costs of the different materials and labour is formed which is called the objective function; this function reflects the cost of a prestressed concrete cross-section per unit length of the beam. The goal of this work is to minimize this function in order to reach a local or global minimum. The minimization of the cost of a prestressed concrete cross section is a constrained minimization problem. In order to be able to use one of the successful unconstrained minimization methods, this constrained minimization problem must be converted to an unconstrained minimization problem by adding a "Penalty function" to the objective function. The penalty function has two factors; one factor is the constraint functions and the other is a constant multiplier. The work of this function is to hold the design away from the constraint boundaries and allows the design to reach these constraint boundaries in the limit as the value of the constant multiplier approaches zero. It is necessary that we start with an acceptable initial design; that is, due to the influence of the penalty function none of the constraints will be violated and the design will remain within the acceptable portion of the design variable space during the synthesis procedure. The unconstrained function which is formed is called the "Fiacco-McCormick" function (Ref. 6). The Fletcher and Powell unconstrained minimization method (Ref. 7) is used to minimize this function. This method is based on obtaining the gradient of the function, which is obtained in an exact way in this work using the ordinary partial derivatives of the function with respect to each design variable.

3.2 Design Variables

The design variables are those quantities which are allowed to vary independently during the synthesis procedure. The goal of this work is to select these design variables such that the constraints are not violated and the cost of the cross section is minimized. The design variables here are basically the independent cross sectional dimensions of the concrete, the prestressing stress, the area of prestressing steel in each row and their distances from the bottom fiber of the beam. They are shown in Fig. 5.

The vector of the design variables is

$$\vec{V} = \left\{H_w, T_t, T_{st}, T_{sb}, T_b, B_w, T_f, B_f, f_{p(bed)}, A_{s(1)}, A_{s(2)}, \ldots A_{s(m)}, H_{s(1)}, H_{s(2)}, \ldots H_{s(m)}\right\}$$

where

H_w - The web depth

T_t - The top flange thickness

T_{st} - The depth of the sloping portion of the top flange

T_{sb} - The depth of the sloping portion of the bottom flange

T_b - The bottom flange thickness

B_w - The web thickness

T_f - The top flange width

B_f - The bottom flange width

$f_{p(bed)}$ - The prestressing stress

$A_{s(i)}$ - The area of prestress steel in row i, $i = 1,2...,m$ where m = number of rows

$H_{s(i)}$ - The height of each prestress steel row measured from the bottom of the beam

3.3 Preassigned Parameters

The preassigned parameters are those quantities which remain fixed during the synthesis procedure. They are the span length and the number of prestress steel rows m.

In certain cases some of the design variables may be given a constant value in which case they then become preassigned parameters.

3.4 Objective function

The objective function is a function of the design variables which is used as a basis for choice between alternate acceptable designs. The goal of the synthesis procedure is to minimize this function in order to obtain a local or global minimum. The objective function in this work is a cost function which reflects the cost of a prestressed concrete section per unit length of the beam in terms of the design variables. The cost expression is taken as follows:

$$C(\vec{V}) = C_c A_c + C_{ps} A_s + \sum_{r=1}^{3} C_r P_r \qquad (3.1)$$

where

$\{\vec{V}\}$ - The vector of the design variables

A_c - The net cross sectional area of concrete

C_c - The cost of unit volume of concrete including cost of placing and transportation

A_s - The area of prestress steel

C_{ps} - The cost of prestress steel per pound including cost of pulling and placing of steel.

P_r - The lengths of the vertical, horizontal and sloping parts of the perimeter, respectively ($r = 1,2,3$).

C_r - The forming costs of the vertical, horizontal and sloping parts of the perimeter per unit area, respectively ($r = 1,2,3$).

3.5 Constraints

The optimum or the minimum cost design must be an acceptable design. The limits of acceptability are defined by side and behaviour constraints. Side constraints are basically constraints on the design variables. Constraints on the stresses and deflection of the structure are called behaviour constraints. In order to satisfy the acceptability condition, the optimum design must not violate any of these constraints. Most of these constraints are taken from CPCI (Ref. 5).

3.5.1 Side constraints

Side constraints are limits on the range of the design variables. These limits are prescribed in such a way as to satisfy the condition that the resulting cross section of the beam be a practical shape when the synthesis scheme is carried out (i.e. rectangular, T_{ee}, I,...etc.), as shown in Fig. 4. These limits can be controlled or additional side constraints can be added in order to reach to an optimum design of a particular desired shape as will be seen later on in this chapter and in Chapter IV.

In general, all the side constraints for the prestressed concrete beam can be represented as

$$U_k \geq V_k \geq L_k$$

where

V_k - The $k^{\underline{th}}$ design variable

L_k - The lower limit of the constraint on the K^{th} variable

U_k - The upper limit on the k^{th} design variable

For example

$$V_1 = H_w \geqslant L_1$$

$$V_2 = T_t \geqslant L_2$$

$$\vdots$$

$$V_n = H_{s(m)} \geqslant L_n$$

where n is the total number of design variables ($n = 9 + 2m$)

By choosing suitable values for the lower limits L_k ($k = 1,2,...,n$) such that all the constraint requirements are satisfied, various practical cross sectional shapes are obtained, as shown in Fig. 4.

In order to get an optimum section of a particular shape we need to add additional side constraints which are mainly upper limits on one or more of the design variables, or by controlling the lower limit values of some of the side constraints or setting limits on both of them. For example, in case that the beam is required to be of a limited depth, the following side constraints can be added:

$$H = H_w + T_t + T_{st} + T_b + T_{sb} \leqslant U_d$$

where

H - The total depth of the beam

U_d - The upper limit on the total depth

In case that the cross section of the beam is required to be of a particular shape (i.e. rectangular, T, i,...etc.) and the initial design is chosen the same shape as the required one, an optimum design of the desired shape can be obtained; for example, for a rectangular shape, the following side constraints can be added

$$T_f = 0$$

$$B_f = 0$$

By adding these constraint, the top and bottom flange widths T_f and B_f will remain fixed at the zero value during the synthesis procedure (i.e. they become preassigned parameters) which ensures that the optimum design obtained will have the same shape as in the initial design (i.e. rectangular). A similar procedure can be used in case that any other shape is required. In case that the optimum design is desired to be of a certain shape which differs from the initial shape (e.g. the initial design has a rectangular shape and the optimum design is desired to be of an I or T shape), it may be possible to obtain an optimum design having the required shape by controlling the upper limits on the top and bottom flange widths T_f and B_f respectively. For most of these cases, in order to obtain the desired shape the values of the design variables of the initial design and the variation of these values during the minimization procedure may have to be changed. By adding or altering the side constraints during the minimization procedure or controlling the existing side constraints, the desired shape may be obtained. Some of these cases will be discussed in more detail in Chapter IV.

3.5.2 Behaviour Constraints

The behaviour constraints are limits on the normal stresses, principal stresses and deflections obtained due to the five critical loading combinations previously mentioned in Chapter II. All these stresses and deflections must be kept within the allowable limits specified by the CPCI code (Ref.5) during the synthesis procedure.

The behaviour constraints on the normal stresses for each load combination are

$$f_{ca}^{-} \leq f_{ct} \leq f_{ca}^{+}$$

$$f_{ca}^{-} \leq f_{cb} \leq f_{ca}^{+}$$

$$f_{s} \leq f_{sa}$$

where

f_{ct} - The normal stress of concrete at the top fiber

f_{cb} - The normal stress of concrete at the bottom fiber

f_{s} - The tensile stress of the prestressing steel

f_{ca}^{+} - The allowable tensile stress of concrete (a positive quantity)

f_{ca}^{-} - The allowable compressive stress of concrete (a negative quantity)

f_{sa} - The allowable tensile stress of the prestressing steel

It should be noted that the allowable stresses may be different at different loading combinations.

The behaviour constraints on the principal tensile stresses and maximum deflections are as follows

$$f_{ps}^{(x)} \leqslant f_{psa}$$

$$D_{ef}^{(x)} \leqslant D_{efa}$$

where

$f_{ps}^{(x)}$ - The maximum principal tensile stress of concrete = 1,2,...,5 where the subscript x indicates the x^{th} critical loading combination.

f_{psa} - The allowable tensile stress that can be resisted by concrete alone.

$D_{ef}^{(x)}$ - The maximum deflection of the beam for the x^{th} load combination.

D_{efa} - The allowable deflection value of the beam.

3.6 Fiacco and McCormick function

The minimization of a prestressed concrete cross sectional cost is an inequality constrained minimization problem. It can be converted to a sequence of unconstrained minimization problems in order to be able to use one of the successful unconstrained minimization methods such as the Fletcher and Powell method (Ref. 7). This is done by adding what is called a "Penalty Function" to the objective function to form the Fiacco and McCormick function. The procedure is as follows (Ref. 6):

$$F(\vec{V}) = C(\vec{V}) + R \cdot P(\vec{V}) \tag{3.2}$$

where

$R \cdot P(\vec{V}) = R \cdot \sum_{j=1,2}^{q} 1/g_j(\vec{V})$ is called the penalty function

$C(\vec{V})$ - The objective function (Eq. 3.1)

$g_j(\vec{V})$ - The $j^{\underline{th}}$ constraint function; $j=1,2,\ldots q$ where q is the total number of side and behaviour constraints.

R - An arbitrary constant greater than zero which represents the relative weight of the $P(\vec{V})$ function in the $F(\vec{V})$ function.

All the constraint functions must be of the form

$$g_j(\vec{V}) \geqslant 0$$

For example, the constraint

$$U_k \geqslant V_k \geqslant L_k$$

can be written as the two constraints

$$V_k - L_k \geqslant 0$$

$$U_k - V_k \geqslant 0$$

In the case of various live loading conditions, the cross section obtained must be optimum for all live loading conditions; none of the behaviour constraints due to any loading conditions must be violated. Therefore, all the behaviour constraints for every loading condition must be added to the penalty function. The total number of constraints in this case will be

$$q = s + d.b \tag{3.3}$$

where

s - The number of side constraints

b - The number of behaviour constraints

d - The number of live loading conditions

For each value of the multiplier R a minimum value of the $F(\overline{V})$ function is sought. The value of R is reduced after each minimization process; theoretically in the limit the value of R reaches zero as the $F(\overline{V})$ function reaches its optimum value. Recommendations on the initial value of the multiplier R for a particular problem and on the rate of its reduction during the synthesis procedure is given in Chapter IV.

The initial design must be within the allowable portion of the design space (i.e. to be an acceptable design). The design will always remain within this acceptable portion during the synthesis procedure due to the influence of the penalty function which holds the design away from the constraint boundaries. In case that the design reaches the constraint boundaries where one of the constraints becomes zero, the function will be of an infinite value due to the influence of the penalty function; in such a case, a constrained minimum can be obtained as will be seen in Chapter IV, and thus none of the constraints is violated during the synthesis procedure. Practically, there is a possibility that a design point in the unacceptable region may be reached; this will be discussed later in the one dimensional minimization procedure in Chapter IV.

3.7 The Gradient

The Fletcher-Powell method (Ref. 7) requires the gradient of the $F(\vec{V})$ function. The gradient in this work is obtained in an exact way using the ordinary partial derivatives of the $F(\vec{V})$ function with respect to each design variables. It is sometimes difficult to differentiate this function (especially the penalty function portion) but on the other hand an "exact" gradient value is obtained. The finite difference method can be used in obtaining the gradient; however, experimentation indicated that the accuracy of the resulting gradient depended greatly on the increment used in the finite difference scheme. It was therefore felt that it was worth the extra effort to obtain exact values of the gradient.

CHAPTER IV

MINIMIZATION METHOD AND NUMERICAL RESULTS

4.1 Introduction

A computer program using the Fletcher-Powell unconstrained minimization method (Ref. 7) has been applied to several specific cases (eight in all). All these cases are studied under the same loading conditions, and they all represent a cross section at mid span of a simply supported beam. The first design is for a beam which is desired to be of a rectangular shape, starting with an initial design having the same shape as desired (i.e. rectangular shape). The second and third cases study the relative minima concept by starting from widely separated initial designs for the same side constraint set. Starting with different initial designs of a rectangular shape the second, third and fourth cases show the influence of the different side constraints and initial design on the optimum design which is desired to be of a T shape. The fifth design is for an I section of a limited cross sectional depth. The sixth design is for a wide flange I section of a limited depth. The seventh case is the same as in the sixth case but only one row of prestressing steel is allowed. The last case is for a limited depth I section, starting with an initial design of a rectangular shape. These specific cases shed some light on the following matters:

1. The influence of different additional side constraints on the optimum design.
2. The presence of relative minima in the design space.
3. The best design is not always the fully stressed design.
4. Operational characteristics of the synthesis technique for this work. This includes the choice of the initial value of the constant multiplier "R", the rate of its reduction and the effect of the different initial designs on choosing the initial value of the multiplier R.

4.2 Fletcher and Powell method

Unconstrained minimization methods can now be applied to minimize the $F(\vec{V})$ function for any value of the multiplier R. As R reaches zero in the limit, a local minimum of the cost function is obtained.

The method which is used in this work is the Fletcher and Powell method (Ref. 7) which is a "second order" gradient method. The logic behind this method is that the first partial derivatives of a function with respect to its independent variables vanish at its minimum. For example a Taylor series expansion about the minimum of a quadratic function $f(\vec{V})$ is

$$f(\vec{V}) = f(\vec{V}^*) + \frac{1}{2}(\vec{V} - \vec{V}^*)^T \cdot H(\vec{V})^* \cdot (\vec{V} - \vec{V}^*) \qquad (4.1)$$

where

$\vec{V}$ - The vector of variables

$\vec{V}^*$ - The solution vector

$f(\vec{V}^*)$ - The function value at the minimum

$H(\vec{V}^*)$ - The matrix of second partial derivatives of the $f(\vec{V}^*)$ function with respect to its variables. This is a symmetric positive definite matrix given by

$$H(\vec{V}^*) = \begin{bmatrix} \frac{\partial^2 f}{\partial v_1^{*2}} & \frac{\partial^2 f}{\partial v_1^* \partial v_2^*} & \cdots & \frac{\partial^2 f}{\partial v_1^* \partial v_n^*} \\ \vdots & \frac{\partial^2 f}{\partial v_2^{*2}} & & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{\partial^2 f}{\partial v_n^* \partial v_1^*} & \cdots & \cdots & \frac{\partial^2 f}{\partial v_n^{*2}} \end{bmatrix}$$

The gradient of the $f(\vec{V})$ function is

$$\nabla f(\vec{V}) = H(\vec{V}^*) \cdot (\vec{V} - \vec{V}^*) \tag{4.2}$$

From this relation we can get the vector of the variables at the minimum (i.e. the solution vector) as

$$\vec{V}^* = \vec{V} - H(\vec{V}^*)^{-1} \cdot \nabla f(\vec{V}) \tag{4.3}$$

where

$H(\vec{V}^*)^{-1}$ is the inverse of the $H(\vec{V}^*)$ matrix.

This equation allows $\vec{V}^*$ to be calculated in one step if $H(\vec{V}^*)^{-1}$ is available. However for a general function the elements of this matrix are not known. In order to approach $H(\vec{V}^*)^{-1}$, a method of successive linear searches in H-conjugate directions+ is used. In this method the $H(\vec{V}^*)^{-1}$ matrix is replaced by a positive definite matrix; in this work this matrix is taken as the identidy matrix during the first search iteration. A new H matrix is generated after each search iteration takes place. The minimum value of the function $F(\vec{V}^*)$ for a particular value of the multiplier R is obtained as the $H(\vec{V}^*)^{-1}$ matrix is approached. The total number of iterations required to approach $H(\vec{V}^*)^{-1}$ for any general function is not known; but, it has been shown that when applied to a quadratic function the minimum will be found in at most n iterations where n is the number of independent variables (Ref. 7).

The Fletcher-Powell method begins from an initial approximation, $\vec{V}_o$, to the minimum of $F(\vec{V})$. The initial direction of travel in the n-dimensional space is taken as the negative gradient direction, $\vec{S}_o = - \nabla F(\vec{V}_o)$. Subsequently, the method proceeds by generating direction of descent $\vec{S}_K (K = 1,2,\ldots)$ and choosing the step length $\alpha_K \geq 0$ such that $F(\vec{V}_K + \alpha_K \vec{S}_K)$ is a minimum along the direction $\vec{S}_K$ at α_K^*. The new approximation to

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+ A set of direction vector $\vec{s}_o, \vec{s}_1, \ldots, \vec{s}_{n-1}$ are said to be conjugate if $\vec{s}_i^T H \vec{s}_j = 0,\ i \neq j$

the minimum is achieved at $\vec{V}_{K+1} = \vec{V}_K + \alpha^*_K \vec{S}_K$ and subsequent directions are generated from the relation

$$\vec{S}_{K+1} = -H(\vec{V}_{K+1}) \cdot \nabla F(\vec{V}_{K+1}) \tag{4.4}$$

where

$\nabla F(\vec{V}_{K+1})$ - The gradient of the function at the current point $\vec{V}_{K+1}$

$H(\vec{V}_{K+1})$ - The generated H matrix

In order to find the step length α^*_K at which the function $F(\vec{V}_{K+1})$ reaches its minimum, a one dimensional interpolative minimization method is used which will be discussed in the next section. The updated $H(\vec{V}_{K+1})$ matrix after this iteration takes place is given by

$$H(\vec{V}_{K+1}) = H(\vec{V}_K) + A_K + B_K \tag{4.5}$$

where

$$A_K = \frac{\vec{\sigma}_K \cdot \vec{\sigma}_K^T}{\vec{\sigma}_K^T \cdot \vec{y}_K}$$

$$B_K = -\frac{H(\vec{V}_K) \cdot \vec{y}_K \cdot \vec{y}_K^T \cdot H(\vec{V}_K)}{\vec{y}_K^T \cdot H(\vec{V}_K) \cdot \vec{y}_K}$$

and

$$\vec{\sigma}_K = \alpha^*_K \cdot \vec{S}_K$$

$$\vec{y}_K = \nabla F(\vec{V}_{K+1}) - \nabla F(\vec{V}_K)$$

We notice that the current direction of search is not the steepest descent direction (i.e. the $-\nabla F(\bar{V}_K)$ direction which is perpendicular to the lines of equal function value in the design space). As a result of this the Fletcher-Powell method overcomes the difficulty of moving in a "Zig-Zag" fashion which converges very slowly for any general function (Fig. 6). Sometimes the updated H matrix becomes a non-positive definite matrix, in this case it is replaced with the identity matrix.

4.3 One Dimensional minimization

A one dimensional interpolative minimization method is used to find the required step length α_K^* which minimizes the function $F(\bar{V}_K + \alpha_K \bar{S}_K)$ of one variable α_K while searching along the direction $\bar{S}_K$. Starting from a current point $\bar{V}_K$ along this direction a step $\alpha_K = h$ is taken; if convergence tests indicate that the minimum has not been reached or passed, the step length α_K is doubled (i.e. $\alpha_K = h, 2h, 4h, \ldots$) and another search is made using the point represented by the vector $\bar{V}_K + \alpha_K \bar{S}_K$ as a new starting point. When tests indicate that the minimum has been passed, a cubic polynomial is fitted between the last starting point and the final point in order to obtain the minimum of the function.

The initial step length h is obtained by using the following equation:

$$h = 2(\text{est.} - F(\bar{V}_K))/(\bar{S}_K^T \cdot \nabla F(\bar{V}_K)) \leq 1 \qquad (4.6)$$

where

est. - The estimated minimum value of the function $F(\vec{V}_K + \alpha_K \vec{S}_K)$

The term $\vec{S}_K^T \cdot \nabla F(\vec{V}_K)$ represents the slope of $F(\vec{V}_K + \alpha_K \vec{S}_K)$ at $\alpha_K = 0$ i.e. $\left[\frac{d}{d\alpha_K} F(\vec{V}_K + \alpha_K \vec{S}_K) \alpha_{K=0} \right]$ which must be a negative quantity in order to ensure that this function is initially decreasing along $\vec{S}_K$ (i.e. the method will converge).

We must keep in mind that sometimes one or more of the constraints are violated while searching along the direction $\vec{S}_K$ (i.e. the design arrives to the unacceptable portion of the design space). In this case we must return to the last acceptable point in the design space and reduce α_K. In this work α_K was halved in such a case.

The convergence tests are based on obtaining the value of the function $F(\vec{V}_K + \alpha_K \vec{S}_K)$ and its derivatives with respect to the step length α_K at the different points. In case that the derivative of the function remains negative and the value of the function has been decreased after taking any step length α_K, more steps will be needed to pass the minimum. But if the derivative of the function becomes positive or if the function starts to increase this will be an indication that the minimum has been passed. The following cubic interpolative formula is used to estimate the value of α_K^* (Ref. 7).

$$\alpha_e = b - \frac{g'(b) + w - z}{g'(b) - g'(a) + 2w} \cdot (b - a) \qquad (4.7)$$

where

α_e - The distance to the minimum (i.e. α_K^*)

a & b - Are the points which bracket the minimum.

$$g'(b) = \nabla F(\vec{V}_b)^T \cdot \vec{S}_K$$

$$g'(a) = \nabla F(\vec{V}_a)^T \cdot \vec{S}_K$$

$$z = 3 \frac{(F(a) - F(b))}{(b - a)} + g'(a) + g'(b)$$

$$w = (z^2 - g'(a) \cdot g'(b))^{1/2}$$

If the value of the function at α_e is less than that at points a and b, then α_e is accepted as the estimate of α_K^*. Otherwise, the interpolation is repeated over the interval (a, α_e) or (α_e, b) according to whether the sign of the derivative of the function at α_e is positive or negative respectively.

4.4 Numerical Results

A computer program has been applied to eight specific designs. All these designs represent a cross section at mid span of a simply supported beam of 40 feet span length, and they all are studied under the same loading conditions, the values of which are given in Table 1.1. There are two distinct live loading conditions; one is a uniformly distributed load over the whole span length, and the other is a concentrated load at mid span, their values are given in Table 1.1 and Fig. 7. It is emphasized that the two live loading conditions act independently and not concurrently. The girder weight varies during the synthesis procedure due to the changes in the cross sectional dimensions. The initial prestressing stress is taken as 135 k.s.i. for all the problems. The different values of creep factors and shrinkage strains are given in Table 1.2. The various allowable stresses of concrete and steel and the allowable deflection of the beam are given in Table 1.4. The different material costs are given in Table 1.3. The average computer time per iteration is given in Table 1.3.

Design No. 1:

This design is for a beam which is required to be of a rectangular shape. The initial design has the same shape as required (Fig. 8 and Table 2.2). In order to achieve this condition additional side constraints on the top and bottom flange widths, T_f and B_f, respectively are added (Table 2.3) which keep their values as zero during the synthesis procedure (i.e. they become preassigned parameters). The optimum design obtained has a rectangular shape as desired (Fig. 9 and Table 2.2).

The initial design can be considered as a relatively good design. As a result of that and due to the additional side constraints on T_f and B_f the initial design was very close to the constraint boundaries; thus a small initial value of R was used in order to achieve a good balance between the cost and the penalty functions, $C(\vec{V})$ and $P(\vec{V})$ respectively, such that the cost is decreased after successive iterations. This value of R was taken to be equal to 0.0001 (Table 2.1). A total of 33 iterations were needed to reduce the cost of the initial design from 10.323 \$/ft to 8.197 \$/ft at the optimum using a rate of reduction of R equals to 10 (i.e. R = 10^{-5}, 10^{-6},.... etc.) (Table 2.1). Actually, most of the reduction of the cost was during the first iterations which reduced the cost to 8.407 \$/ft. Practically at this limit it can be considered that the optimum design is obtained since no appreciable reduction in the cost took place during the last two computer runnings; however, the last two computer runs were made in order to assure that the $F(\vec{V})$ function was actually minimized. The optimum design can be seen to be a constrained optimum since the stress of concrete at the top fiber due to the first loading combination is equal to 0.419 k.s.i. (Table 2.5) which is very close to 0.42 k.s.i., the allowable tensile stress of concrete (Table 1.3). The constrained optimum is also due to the fact that the difference between the height of successive prestressing steel rows is nearly equal to one inch which is the lower limit on the distance between the different prestressing steel rows (Table 2.2). The lower limits on the different design variables are given in Table 2.2

<u>Design No. 2:</u>

This design is for a beam which is required to be of a T shape starting with an initial design of a rectangular shape

(Fig. 10 and Table 3.2). The additional side constraint is on the bottom flange width B_f which keeps its value as zero during the synthesis procedure (Table 3.3). The initial design can be considered as a relatively good design. Due to this good initial design and to the additional side constraint on B_f, the initial design is very close to the constraint boundaries. As a result of that the desired T shape was not achieved since the synthesis scheme has no chance to work and achieve the required T shape. (Fig. 11 and Table 3.2).

A total of 43 iterations were needed to reduce the cost of the initial design from 10.348 $/ft to 7.454 $/ft, starting with an initial value of $R = 10^{-3}$ which was found to be a suitable value to achieve a good balance. Actually most of the reduction of the cost was during the first computer running which caused the cost to reduce to 8.2283 $/ft after 9 iterations only. At this limit it can be assumed that the optimum design was obtained and all the other 34 iterations were made only to improve the results. (Table 3.1). The optimum design obtained is a constrained optimum since the stress at the top fiber of concrete due to the first loading combination and the stress of concrete at bottom fiber due to the fourth loading combination are equal to 0.416 k.s.i. and 0.419 k.s.i. respectively (Table 3.5 and 3.6) which is very close to 0.42 k.s.i. the allowable tensile stress of concrete (Table 1.3). Also, the height of the fourth steel row is equal to 5.4981 inches (Table 3.2) which is very close to 5.5 inches, its upper limit (Table 3.3).

Design No. 3:

The aim of this design is to obtain an optimum design of a T shape, starting with an initial design of a rectangular shape as in the second design. In order to achieve the required T shape a significantly different initial design from that in the second design was chosen (i.e. the initial design was not as close to the constraint boundaries) as shown in Fig. 12 and Table 4.2. Using the same additional side constraints as in the second design, the optimum design obtained has a rectangular shape which does not satisfy the T-beam requirement (Fig. 13 and Table 4.2). But by changing the constraint on the upper limit on the height of the fourth steel row $H_{s(4)}$ to 5.5 inches an optimum design of the desired T shape was obtained (see Design No. 4). An initial value of R equal to 0.1 was found to be suitable to achieve a good balance in the $F(\vec{V})$ function. A total of 280 iterations were needed to reduce the cost of the design from 17.781 \$/ft to 6.713 \$/ft, but most of the reduction was during the first computer running after which the cost was reduced to 8.536 \$/ft; the rest of the computer runnings were made only to improve the design obtained. The optimum design obtained is a constrained optimum as the stress of concrete at bottom fiber due to the fourth loading combinations is equal to 0.417 k.s.i. (Table 4.6) which is very close to the upper limit of 0.42 k.s.i. (Table 1.3). The different lower limits on the design variables are given in Table 4.2.

Design No. 4:

The aim of this design is the same as for the third design. Starting with the same initial design and the same side constraint on the bottom flange width B_f as in the third design, the upper limit on the height of the fourth steel row was changed to 5.5 inches. The optimum design obtained is a T shape as desired (Fig. 14). The values of the different design variables of the optimum design are given in Table 5.2. The initial value of R was taken to be 0.1. The total number of iterations required to reach the optimum design was 140 (Table 5.1), after which the cost of the design was reduced from a value of 17.781 \$/ft to 7.609 \$/ft. But actually the major reduction of the cost was during the first and second runnings of the computer program after which the cost reached to 11.34 \$/ft and 8.268 \$/ft respectively (Table 5.1). It can easily be seen from Table 5.1 that the optimum design obtained is a constrained optimum since the value of the $F(\vec{V})$ is not close to the value of the cost function. The constrained optimum can also be seen from Table 5.6 where the value of the concrete stress at bottom fiber due to the fourth loading combination is equal to 0.416 k.s.i. which is very close to 0.42 k.s.i. the allowable tensile stress of concrete. The different lower limit values on the design variables are given in Table 5.2.

Design No. 5:

This design is for a beam which is required to be of an I section having a limited total depth H less or equal to 25 inches where

$$H = H_w + T_t + T_{st} + T_{sb} + T_b$$

The initial design was chosen to be of an I shape having a total depth H less than 25 inches in order to satisfy the additional side constraint on the total depth, H (Fig. 15 and Table 6.2). Due to the synthesis scheme none of the constraints are allowed to be violated and thus the total depth will be kept below the 25 inches as desired. The optimum design of the limited depth I shape is shown in Fig. 16. The values of the different design variables of the optimum design are given in Table 6.2. The initial design is not close to any of the constraint boundaries and therefore the initial value of R was taken as 0.1 to achieve a good balance between the cost and the constraint functions, $C(\vec{V})$ and $P(\vec{V})$ respectively (Table 6.1). The cost was reduced from 11.452 \$/ft to 6.319 \$/ft after 200 iterations, using a rate of reduction of R equals to 10 (i.e. R = 0.1, 0.01,....etc.). Actually, the major reduction of the cost was during the first running of the computer program in which the cost was reduced to 7.779 \$/ft after 60 iterations, (Table 6.1). The optimum design obtained is a constrained optimum since the stresses of concrete at the top fiber due to the fourth loading combination is equal to -2.647 k.s.i. (Table 6.5); this is very close to the upper limit on the allowable compressive stress of concrete which is equal to -2.7 k.s.i. (Table 1.3). The lower limits on the different design variables are given in Table 6.2 and the upper limit on the height of the fourth steel row is given in Table 6.3.

It was found by analyzing the optimum design obtained using the conditions of loading at the support that the allowable normal stresses were not satisfied. By using the same optimum design but increasing the height of different prestressing steel rows to a position at which all the normal stresses are satisfied, a minimization process was made in which all the design variables except the height of the different prestressing steel rows were kept fixed (i.e. they became preassigned parameters). The position of the steel were the only design variables left in the problem and finally the center of gravity of the steel coincided with the center of gravity of the transformed section as shown in Fig. 17. Obviously, the cost of the optimum design is identical to that at midspan since the objective function does not depend on the height of the prestressing steel rows. The only reduction was in the value of the $F(\vec{V})$ function which was reduced from 18.466 to 14.91 (i.e. all the minimization procedure was converted to minimize the $F(\vec{V})$ function).

Design No. 6:

This design is for a beam which is desired to have a wide flange I-section of limited depth and flange width. In order to achieve this design the initial design was chosen as an I-section of a total depth less than 25 inches, which is the upper limit on the total depth. The top and bottom flange widths were chosen equal to 8 inches which is greater than the minimum desired flange widths of 6 inches (Fig. 15). Additional side constraints were added on the top and bottom flange widths. (Tables 7.2 and 7.3). Due to the synthesis scheme none of the constraints can be violated which ensures that the optimum design will have a wide flange I shape. The optimum design obtained is shown in Fig.18 which satisfies all the requirements. The values of the different design variables of the optimum design are given in Table 7.2 An initial value of R to achieve a good balance in the $F(\vec{V})$ function was found to be 0.1 (Table 7.1). The cost was reduced from 11.452 $/ft (the same initial design as of the fifth design) to 8.022 $/ft after 54 iterations, using a rate of reduction of R equals to 10. The optimum design obtained is a constrained optimum since the top and bottom flange widths are 6.097 inches and 6.080 inches respectively which are very close to the lower limit specified (Table 7.2). Tables 4.4 to 4.8 reveal that the optimum design is not close to any of the behaviour constraint boundaries. The lower limits on the different design variables are given in Table 7.2.

Design No. 7:

This design is for a beam which is desired to have an optimum design of a wide flange I section of a depth less or equal to 25 inches, flange width greater or equal to 6 inches and having only one row of prestressing steel at a certain location. To achieve this design the initial design shown in Fig. 19 was chosen. Additional side constraints were added on the total depth, the top and bottom flange widths and on the height of the prestressing steel row. (Table 8.2 and 8.3). The optimum design obtained has the same properties as desired (Fig. 20). The value of the different design variables and the lower limit on each variable is given in Table 8.2. The initial design is not close to any constraint boundaries: an initial value of R equals to 10^{-3} was found to be suitable for this design. The cost was reduced from 11.452 $/ft to 9.359 $/ft after 21 iterations, using a rate of reduction of R equals to 10. It was found that further running of the computer program was not practical since the reduction in the cost during the last program running was not significant (Table 8.1). Again the optimum design obtained was not close to the constraint boundaries, since the design variables are not close to the upper or the lower limits and the stresses and deflections are not close to the allowable values. (Tables 8.2 to 8.7).

Design No. 8:

This design is for a beam which is desired to be of an I shape, starting with an initial design of a rectangular shape.

In order to achieve this design an initial design which is very far from the constraint boundaries was chosen (Fig. 21) and additional side constraints which are upper limits on the total depth of the beam and on the height of the fourth prestressing steel row were added; their values are given in Table 9.3. The optimum design obtained has an I shape as desired (Fig. 22). The values of the different design variables and the lower limit on each design variable is given in Table 9.2. The suitable initial value of R which gives a good balance in the $F(\vec{V})$ function was found to be 0.1. A total number of 163 iterations were needed to reduce the cost from 24.316 $/ft to 6.310 $/ft. The major reduction of the cost was during the first running of the computer program after which the cost was reduced to 7.799 $/ft after 63 iterations. Practically in order to save on computer time the optimum design can be assumed at this stage and the other two runnings of the program were only made to improve the design obtained (Table 9.1). The optimum design obtained was a constrained optimum since it is close to the behaviour constraint boundaries as given in Table 9.5 where the stress of concrete at the top fiber due to the fourth loading combination for case 1 of live loading is equal to -2.642 k.s.i. which is very close to -2.7 k.s.i., the allowable compressive stress of concrete.

4.5 Discussion of Results

It has been found from the different designs studied that the major reduction in the cost for a particular design occurs during the first running of the computer program as long as a suitable value of R is used. As a result of that the method can be considered as an effective method as it can save much of the computer time and give a relatively good design from only one trial.

The relative minima concept has been studied in the second and the third designs as the initial design in both cases are widely different from one another and both of them have the same set of constraints. The optimum design obtained have significantly different cross sectional dimensions and different costs.

In the case that an optimum design is required to have a certain shape and the initial design has the required shape (Designs 1, 5, 6 and 7), suitable additional side constraints must be added in order to achieve the desired shape. Starting with an initial design which satisfies these additional side constraints an optimum design having the desired shape will be obtained since none of the constraints will be violated during the minimization procedure due to the synthesis scheme.

In the case that the initial design has a different shape from the desired one (Designs 2, 3, 4 and 8) the choice of the initial design is critical and various initial designs may have to be tried. Generally it appears that in such a case it is best to choose the initial design such that it lies far away from the constraint boundaries. Also, it may be necessary to add some additional side constraints or change the values of the existing constraints in order to obtain an optimum design of the desired shape.

The effect of choosing an initial design having a relatively high cost (Designs 3, 4 and 8), is only in increasing the number of iterations required to obtain the optimum design. But since most of the cost for a particular design is reduced during the first running of the computer program this higher number of iteration is not of a great importance. The optimum design obtained may have a smaller cost than that of a good initial design, (Designs 2 and 3); this is due to the relative minima concept.

Practically speaking, the optimum designs obtained for a section at mid span, can be considered as an optimum design for the whole beam. This can be done by checking the stresses at the other critical sections; for a case of a simple beam the other critical section is at the support where the principal tensile stress is critical and the moment due to different loading conditions are zero. If the normal stresses are found to be unsafe, it is then required to change the position of the different prestressing steel rows in order to satisfy all the normal stress requirements. As shown in design 5 the cost remains constant since the cost function does not depend upon the height of the prestressing steel rows. This will cause the prestressing steel wires to have a curved shape along the entire length of the beam. Also, in case that the principal tensile stresses are not safe, the web width, B_w must be increased in order to satisfy the principal stresses.

4.6 Operational Characteristics

Choosing the initial value of the multiplier R differs from one problem to another, and depends mainly on the properties of the initial design. Due to the fact that R represents the weight of the constraint function $P(\vec{V})$ in the $F(\vec{V})$ function, a suitable value of R must be chosen in order to achieve a good balance which ensures the reduction of the cost function $C(\vec{V})$ after successive iterations. In the case that the initial design is close to the constraint boundaries the $P(\vec{V})$ function will have a large value which requires a smaller value of R than that of a design which is far from the constraint boundaries in order to achieve a good balance. Obviously, it is of no use to increase the value of R after the minimum is obtained for a particular value of R; if R is increased, the problem will be converted to minimize the constraint function $P(\vec{V})$ and the cost function $C(\vec{V})$ will remain without any reduction or it may start to increase since the $P(\vec{V})$ function has a large influence. From that a suitable initial value of R is taken as the value which causes the cost function $C(\vec{V})$ to decrease which is our goal (i.e. a good balance is achieved).

A suitable initial value of R for most of the cases studied is 0.1, and a rate of reduction of the value of R equals to 10 (i.e. R = 0.1, 0.01,...etc.) was found to be efficient for most of the cases. In case that the value of R is reduced too quickly, or when the initial value of R is too small, the synthesis method may encounter one of the constraints and thus the moves in the design space are found to move along the constraint boundaries; this was found to make the synthesis method inefficient.

4.7 Convergence Criteria

From experience it was found that most of the cost is reduced during the initial running of the computer program. In general the additional iterations for the program are made only to improve the resulting optimum design. If it is found that no appreciable reduction in the cost occurs after two successive R values, for practical purposes it can be assumed that convergence has taken place.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusion

An efficient structural synthesis capability to minimize the cost of bonded pretensioned prestressed concrete beams of various kinds, subjected to different loading conditions, including all the live loading possibilities which may act has been developed. This efficiency is due to a combination of several factors:

1. The use of the penalty function of Fiacco and McCormick causes the successive designs obtained during the synthesis procedure to stay away from the constraint boundaries and therefore ensures that the designs obtained remain in the acceptable region of the design space. Another advantage of using the penalty function is that the problem is converted to a sequence of unconstrained minimizations which enables the use of the Fletcher-Powell method which is considered as the most powerful method for finding the minimum of unconstrained general function.
2. The gradient of the Fiacco-McCormick function is obtained in an exact way by using the partial derivatives of that function with respect to each design variable. This ensures that the resulting gradient has high accuracy which is of a great importance as the Fletcher and Powell method is intimately related to gradient calculations.

Using the computer program on different designs, some general conclusions can be drawn from them.

1. Relative minima is present in the design space. (Design 2 and 3).
2. An optimum design of a desired shape can be obtain without difficulty if the initial design has the same shape as the desired one (Designs 1, 5, 6 and 7).
3. In the case that the initial design has a different shape from the desired one an optimum design having the desired shape may be obtained by a suitable choice of the initial design. In such a case it is recommended that the initial design be chosen far away from the constraint boundaries. It may also be required to add side constraints on some design variables or change the values of the existing side constraints (Designs 2, 3, 4 and 8).
4. Some designs of desired properties (e.g. limited depth, wide flange, limited number of prestressing steel rows, ...etc.) can be obtained without difficulty (Designs 5, 6 and 7).
5. Most of the optimum designs are constrained.
6. The major reduction of the cost is during the first iteration of the computer program. As a result, this method can be considered an efficient method since good design can be obtained after only one trial.
7. Starting with a relatively costly design does not significantly affect the minimum cost obtained. Due to

the relative minima concept the optimum design obtained may have a less cost than that of an initially good design. But the high cost initial design may need higher computer time to reach the minimum. (Design 3, 4 and 8). This is not of a significant effect since the major reduction in the cost is during the first iteration of the computer program.

8. The initial value of R varies from one problem to another, and for the same problem the closer the initial design to the constraint boundaries the smaller the initial value of R and vice versa. A good initial value of R is that value which causes a reduction of the cost function after each iteration.

9. In practice the optimum design obtained at the critical moment section can be considered as an optimum design for the entire beam. In case that the allowable normal stresses are not satisfied at other critical sections, changing the position of the different prestressing steel rows is required in order to satisfy the allowable normal stresses; this is accomplished without changing the other cross sectional dimensions and the cost therefore remains constant. If the principal tensile stresses are not satisfied, the web width, B_w must be increased.

5.2 Recommendations

1. Further studies on the initial value of the multiplier R and the rate of its reduction should be made.
2. The cost function in this work could be modified by considering the cost of the normal mild steel used in resisting the tensile stresses.
3. A synthesis capability should be developed in the same way as in this work but using the ultimate load principle in the analysis portion of the synthesis scheme. The method could be extended to post-tensioned concrete beams.
4. Further studies should be made in order to develop a synthesis capability to synthesize more complicated structures (e.g. frames, folded plates,....etc.).

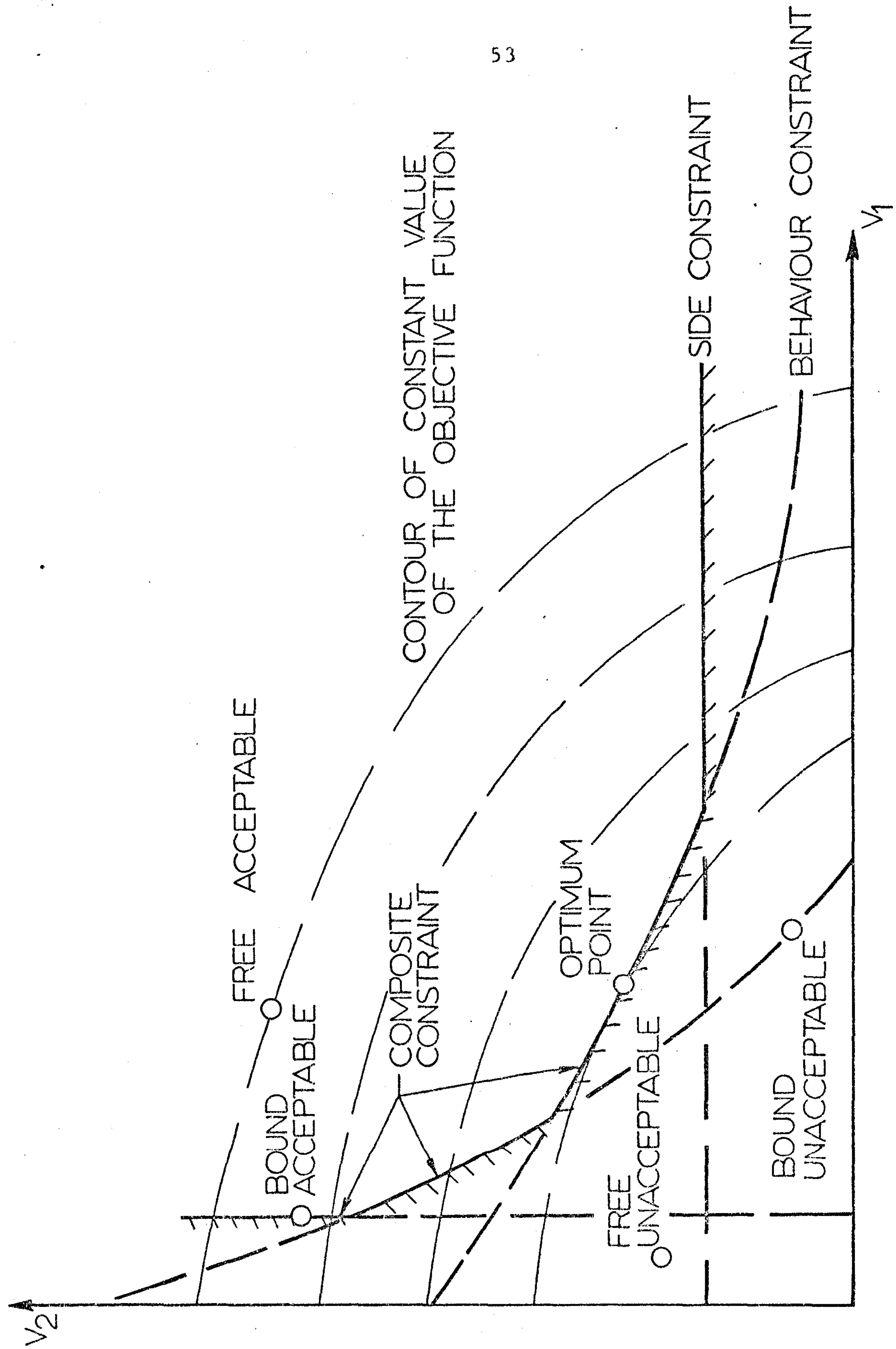


Fig. 1 DESIGN SPACE NOMENCLATURE

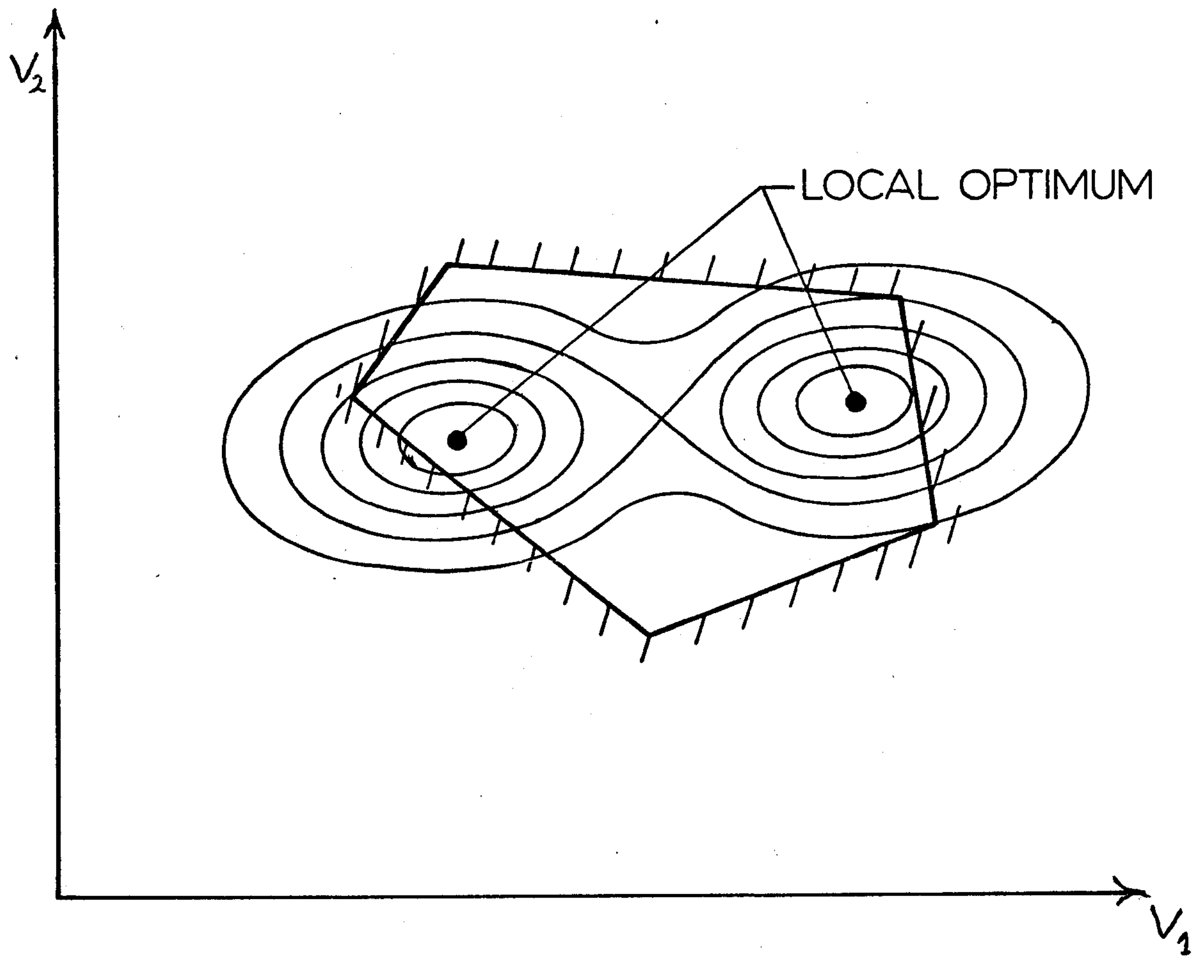


Fig. 2 UNCONSTRAINED LOCAL OPTIMA

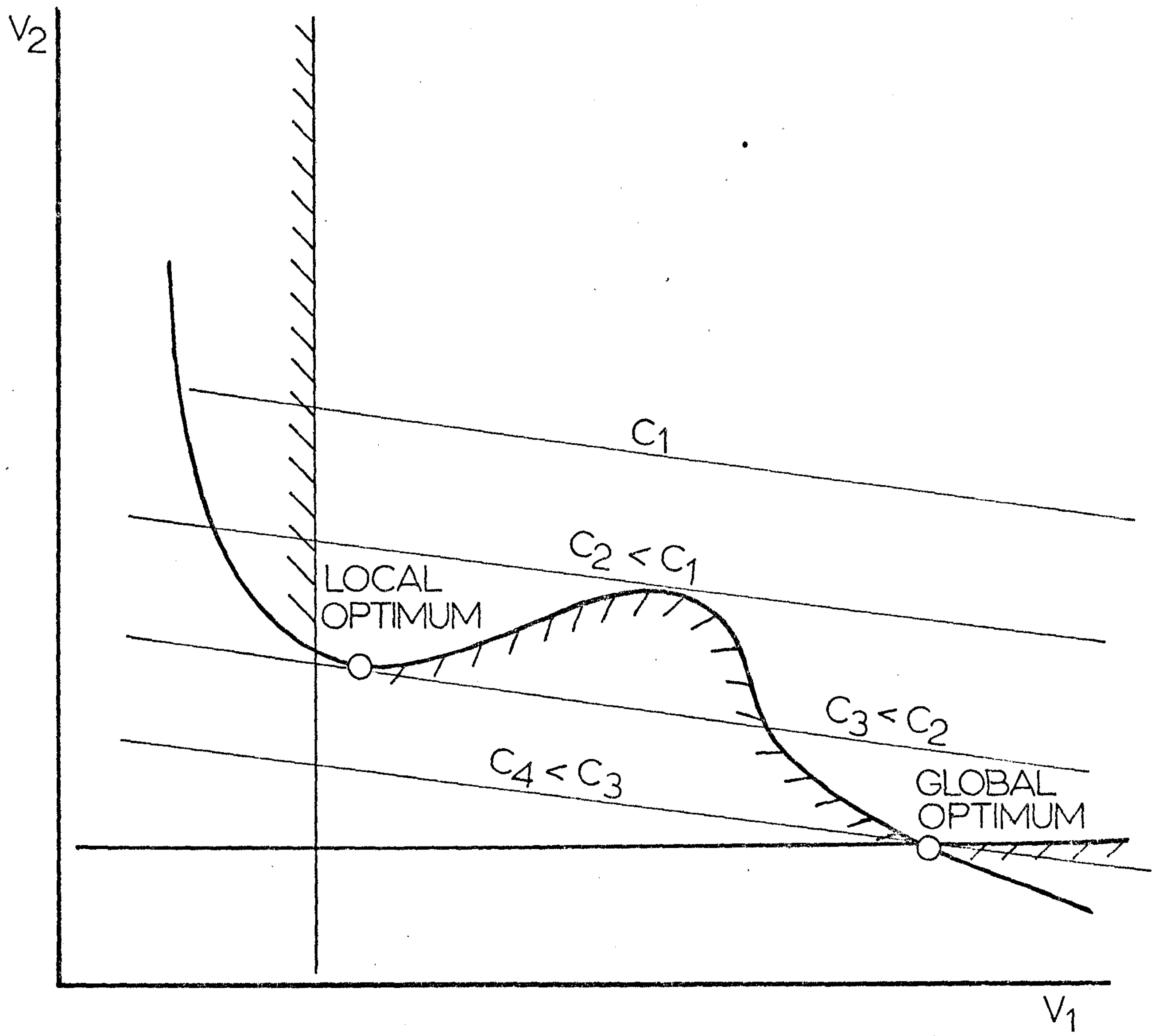


Fig. 3 RELATIVE MINIMA

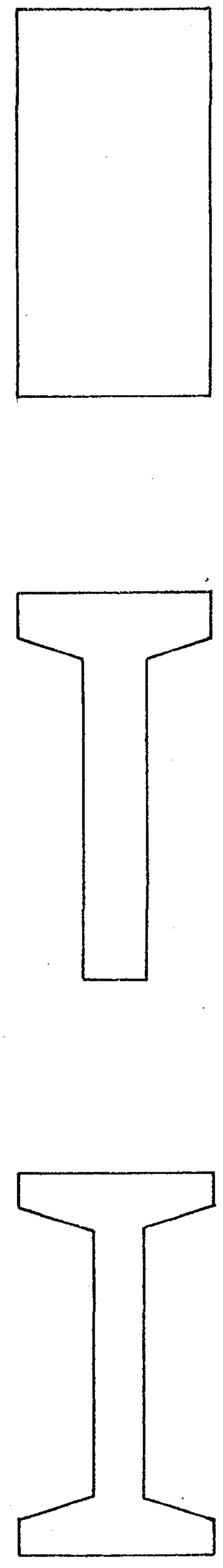


Fig. 4 PRACTICAL SHAPES

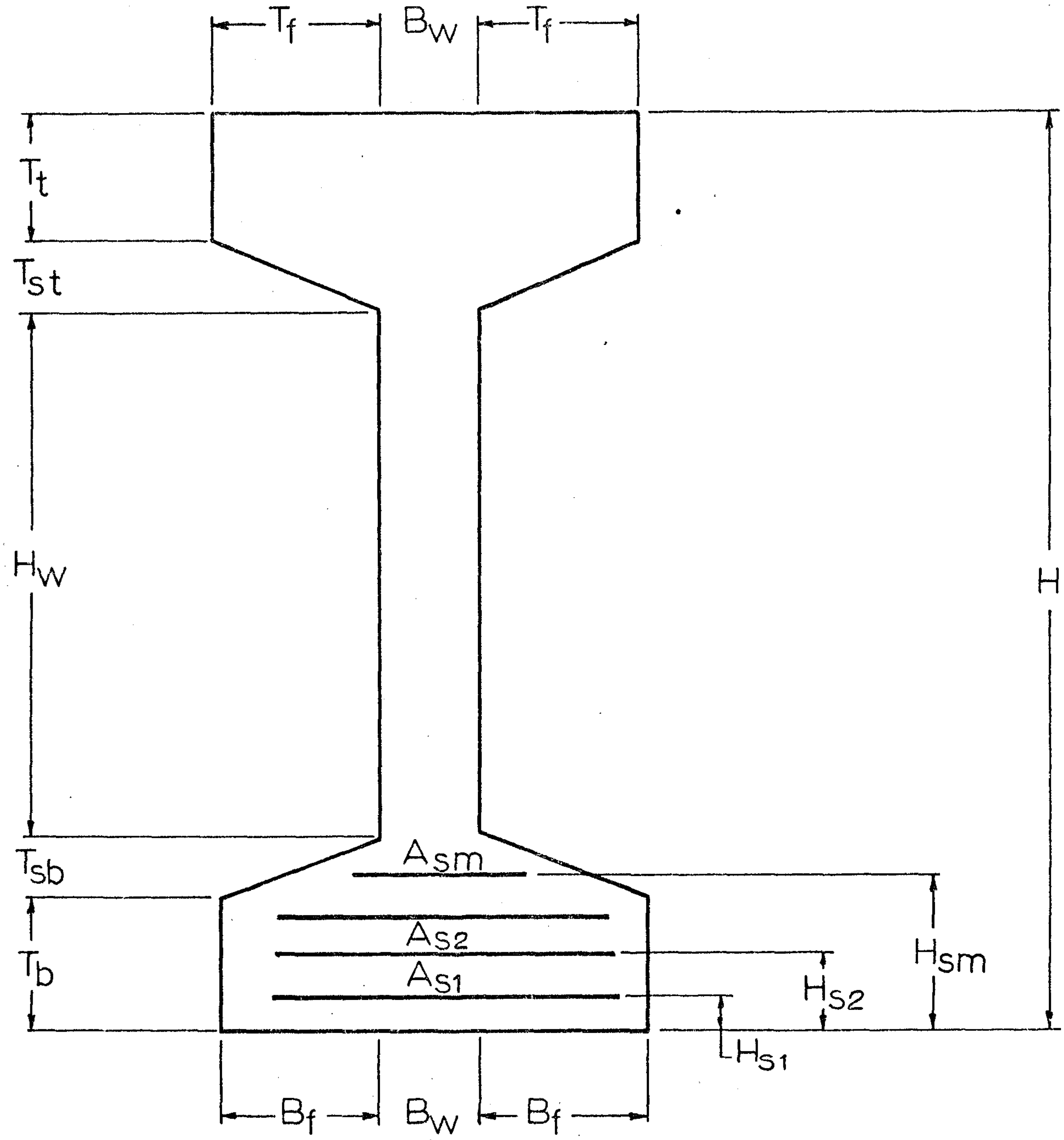


Fig. 5 DESIGN VARIABLES

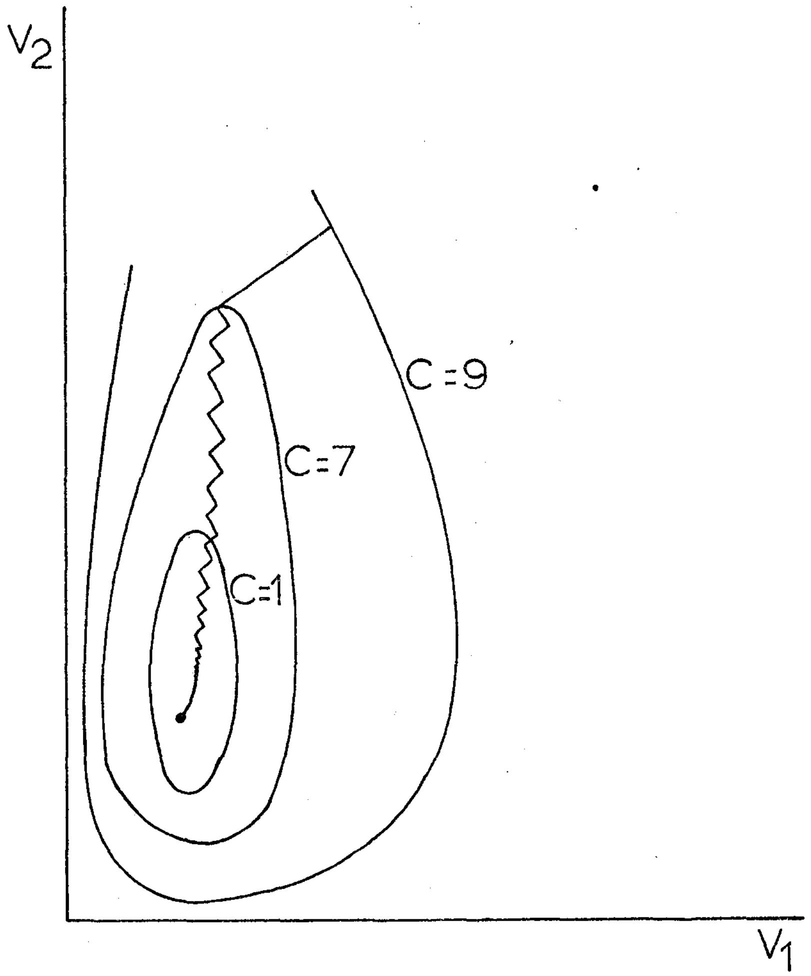
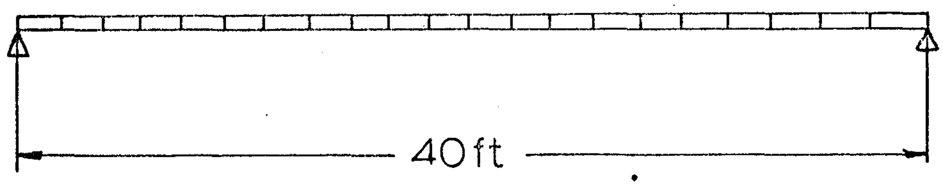
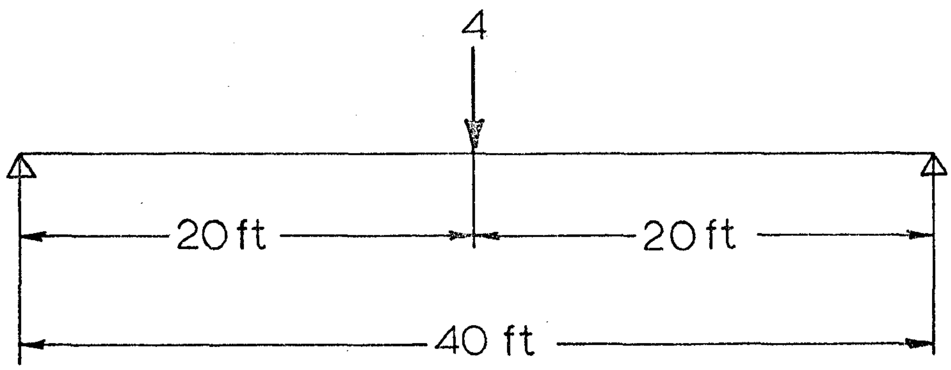


Fig. 6 STEEPEST DESCENT ZIG-ZAG PATTERN

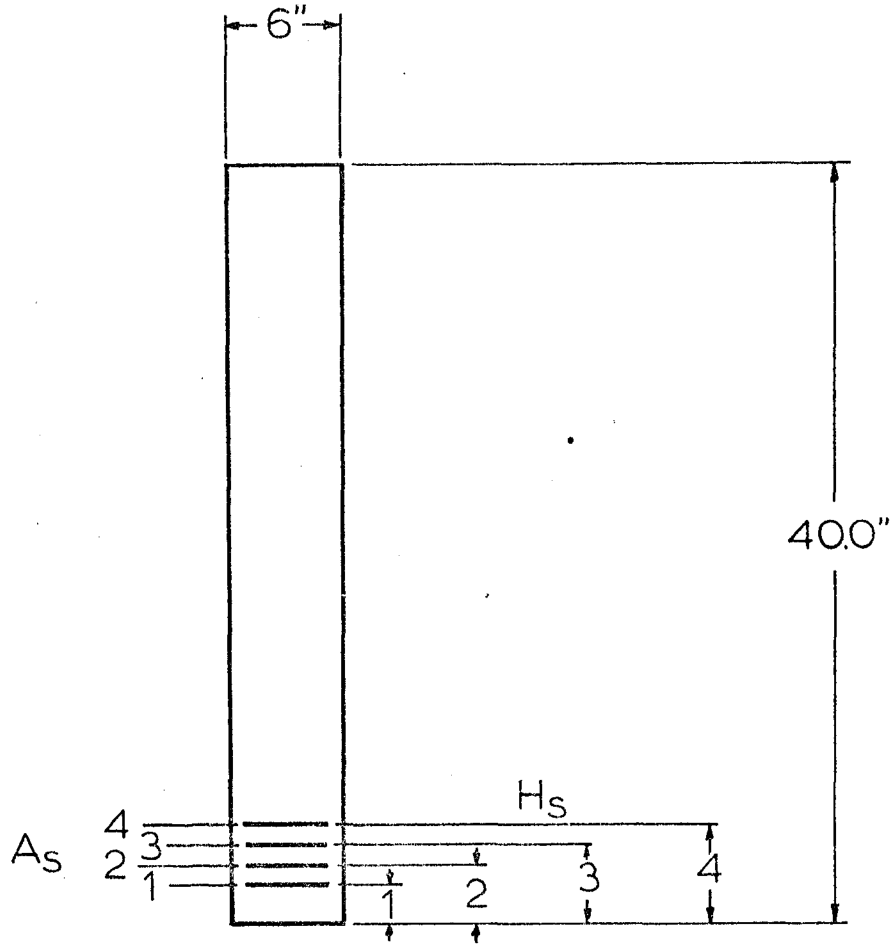


CONDITION NO. 1



CONDITION NO. 2

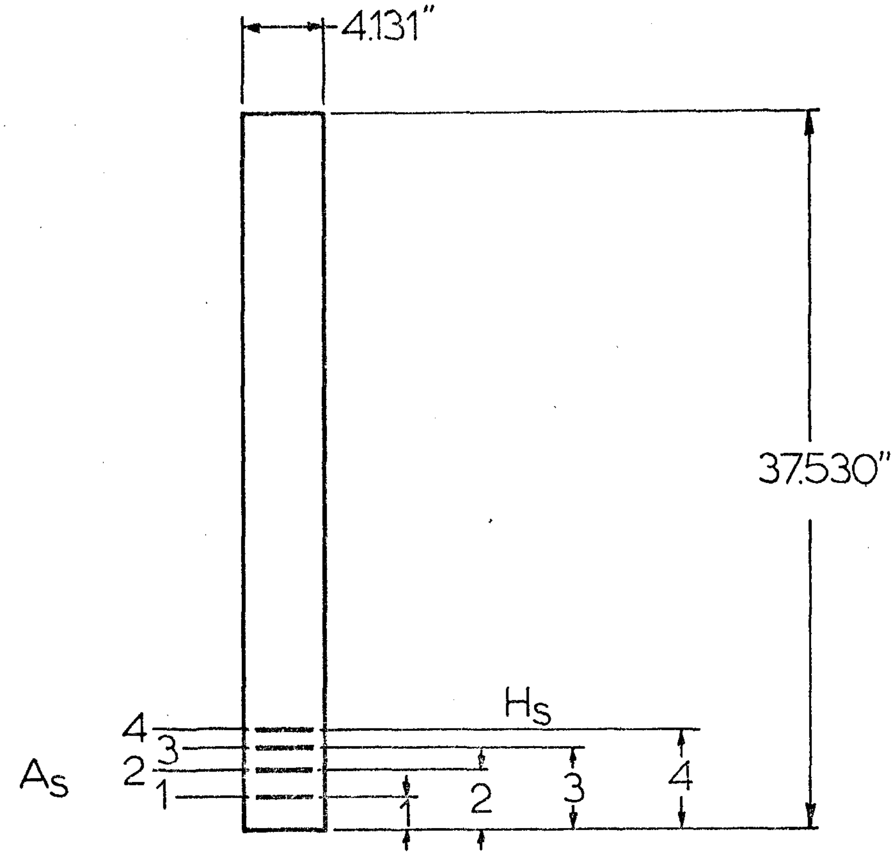
Fig. 7 LIVE LOADING CONDITIONS



A_s (1)	0.3 sq. in.
(2)	0.1 sq. in
(3)	0.1 sq. in.
(4)	0.1 sq. in.

H_s (1)	2.0 in.
(2)	3.1 in.
(3)	4.2 in.
(4)	5.3 in.

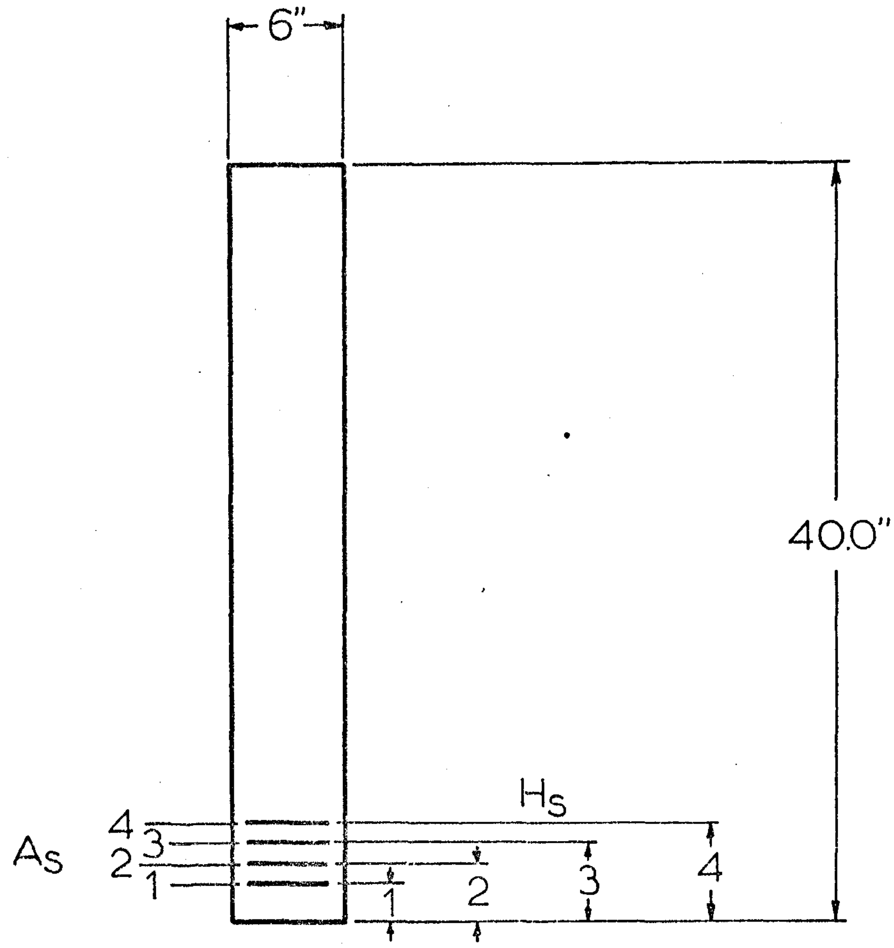
Fig. 8 INITIAL SECTION FOR DESIGN 1



A_S	(1)	0.29467 sq. in.
	(2)	0.12725 sq. in.
	(3)	0.12352 sq. in.
	(4)	0.11883 sq. in.

H_S	(1)	1.961 in.
	(2)	3.0994 in.
	(3)	4.1864 in.
	(4)	5.1898 in.

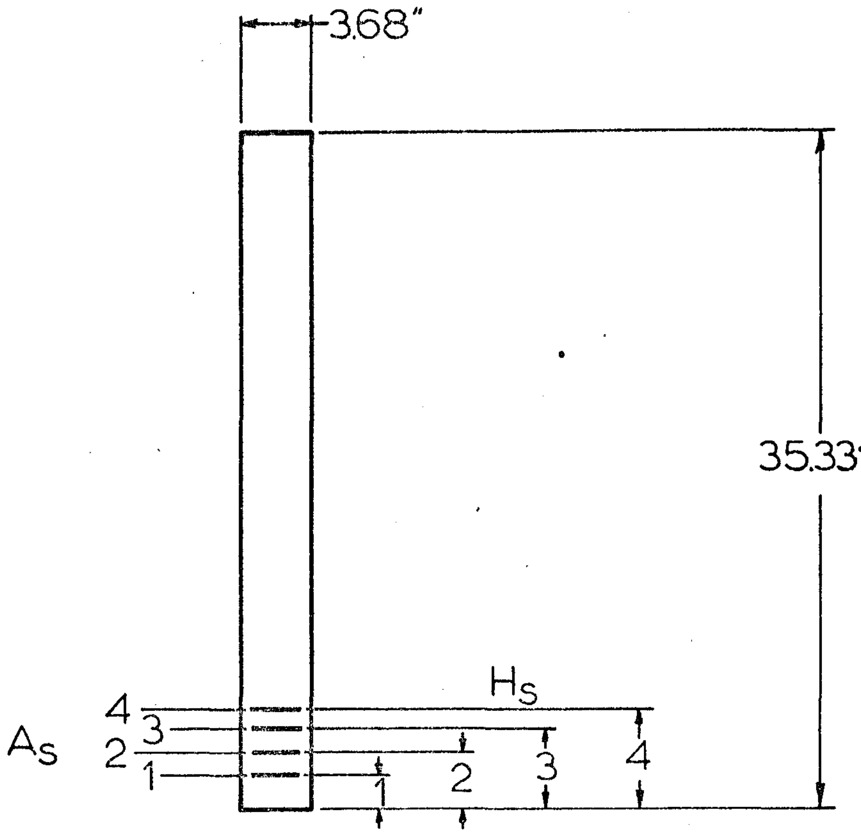
Fig. 9 OPTIMUM SECTION FOR DESIGN 1



A_s (1)	0.3 sq. in.
(2)	0.2 sq. in.
(3)	0.2 sq. in.
(4)	0.1 sq. in.

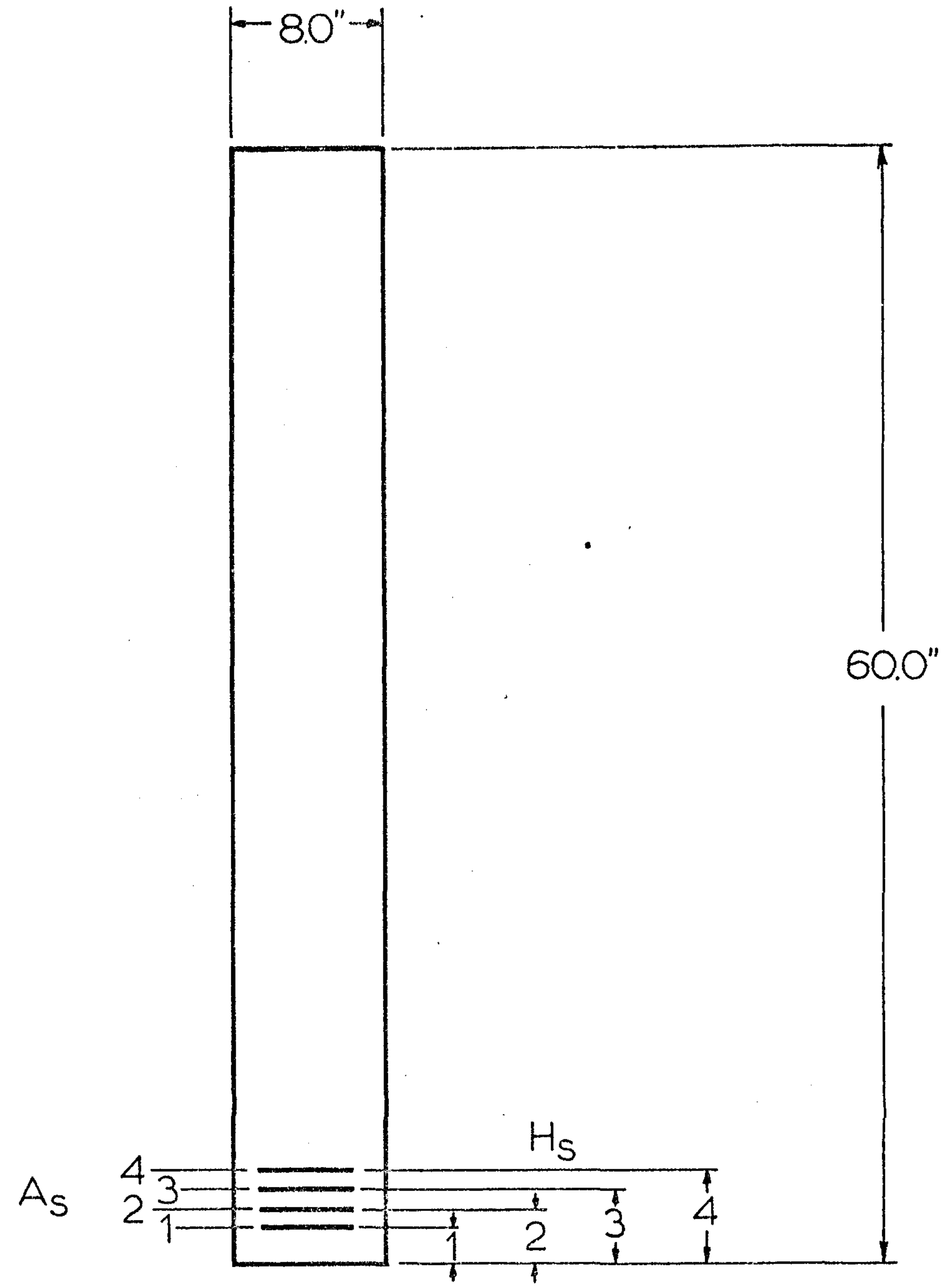
H_s (1)	2.0 in.
(2)	3.1 in.
(3)	4.2 in.
(4)	5.3 in.

Fig. 10 INITIAL SECTION FOR DESIGN 2



A_S (1)	0.000773 sq. in.		H_S (1)	1.6781 in.
(2)	0.001 sq. in.		(2)	3.0769 in.
(3)	0.0016122 sq. in.		(3)	4.308 in.
(4)	0.8178 sq. in.		(4)	5.4982 in.

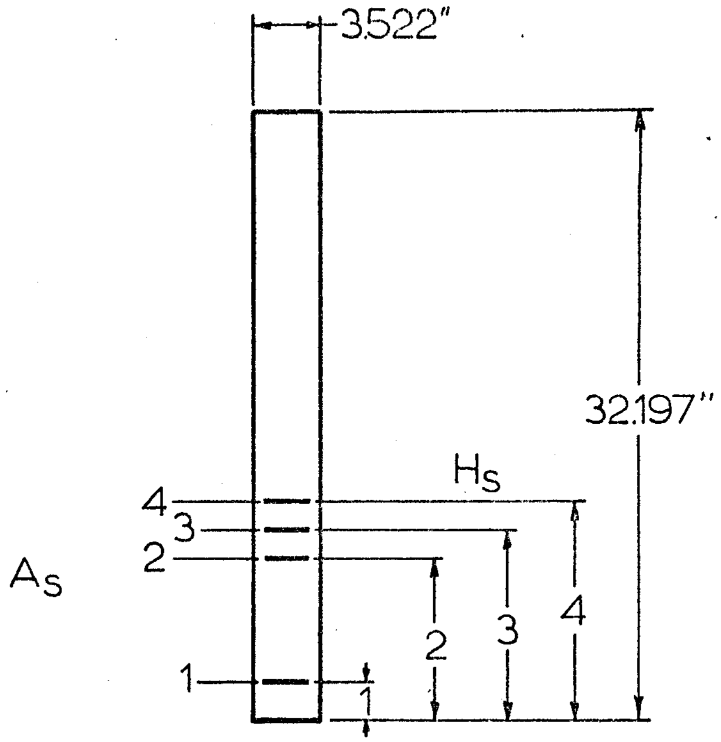
Fig. 11 OPTIMUM SECTION FOR DESIGN 2



A_S (1)	0.42 sq. in.
(2)	0.30 sq. in.
(3)	0.20 sq. in.
(4)	0.20 sq. in.

H_S (1)	2.0 in.
(2)	3.1 in.
(3)	4.2 in.
(4)	5.3 in.

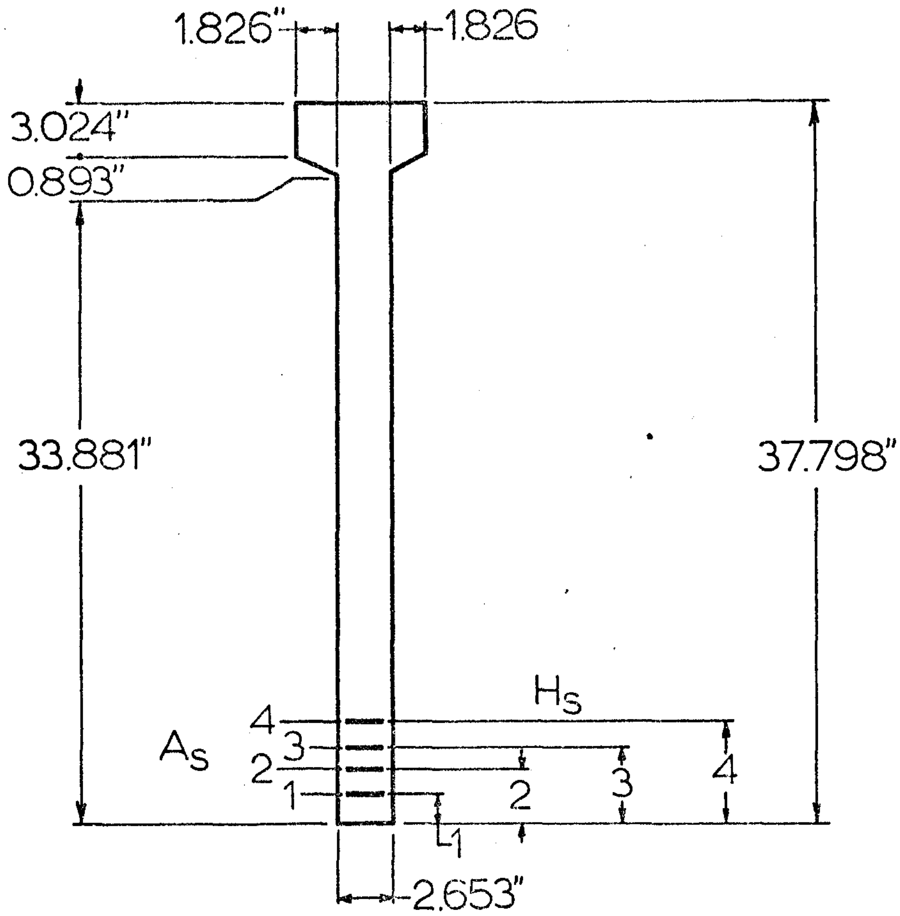
Fig. 12 INITIAL SECTION FOR DESIGNS 3 AND 4



A_s (1)	0.46382 sq. in.
(2)	0.1825 sq. in.
(3)	0.24745 sq. in.
(4)	0.33412 sq. in.

H_s (1)	1.9638 in.
(2)	8.88 in.
(3)	10.383 in.
(4)	11.798 in.

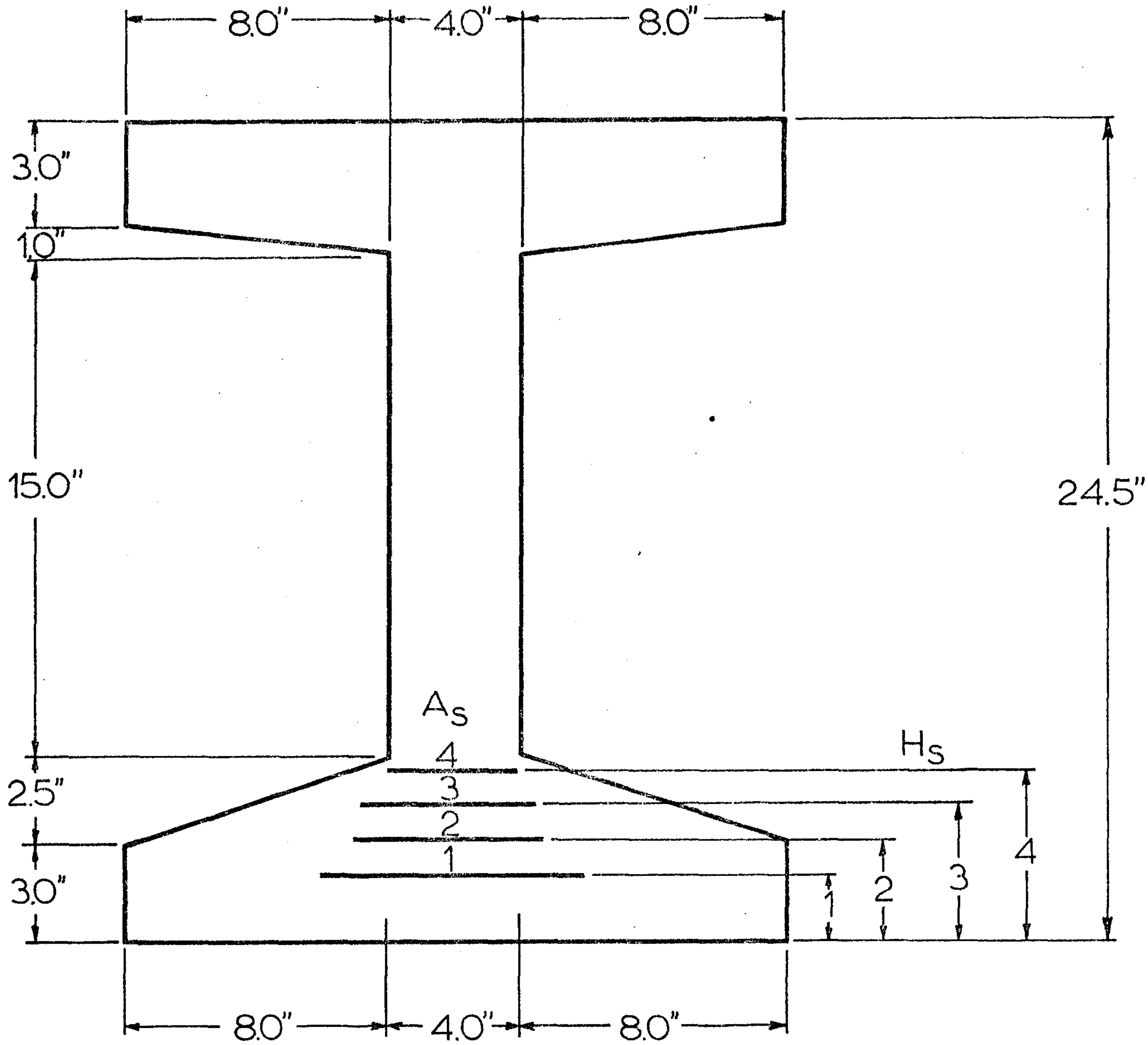
Fig. 13 OPTIMUM SECTION FOR DESIGN 3



A_S	
(1)	0.008 sq. in.
(2)	0.01 sq. in.
(3)	0.019 sq. in.
(4)	0.64842 sq. in.

H_S	
(1)	1.7273 in.
(2)	2.9544 in.
(3)	4.1998 in.
(4)	5.4285 in.

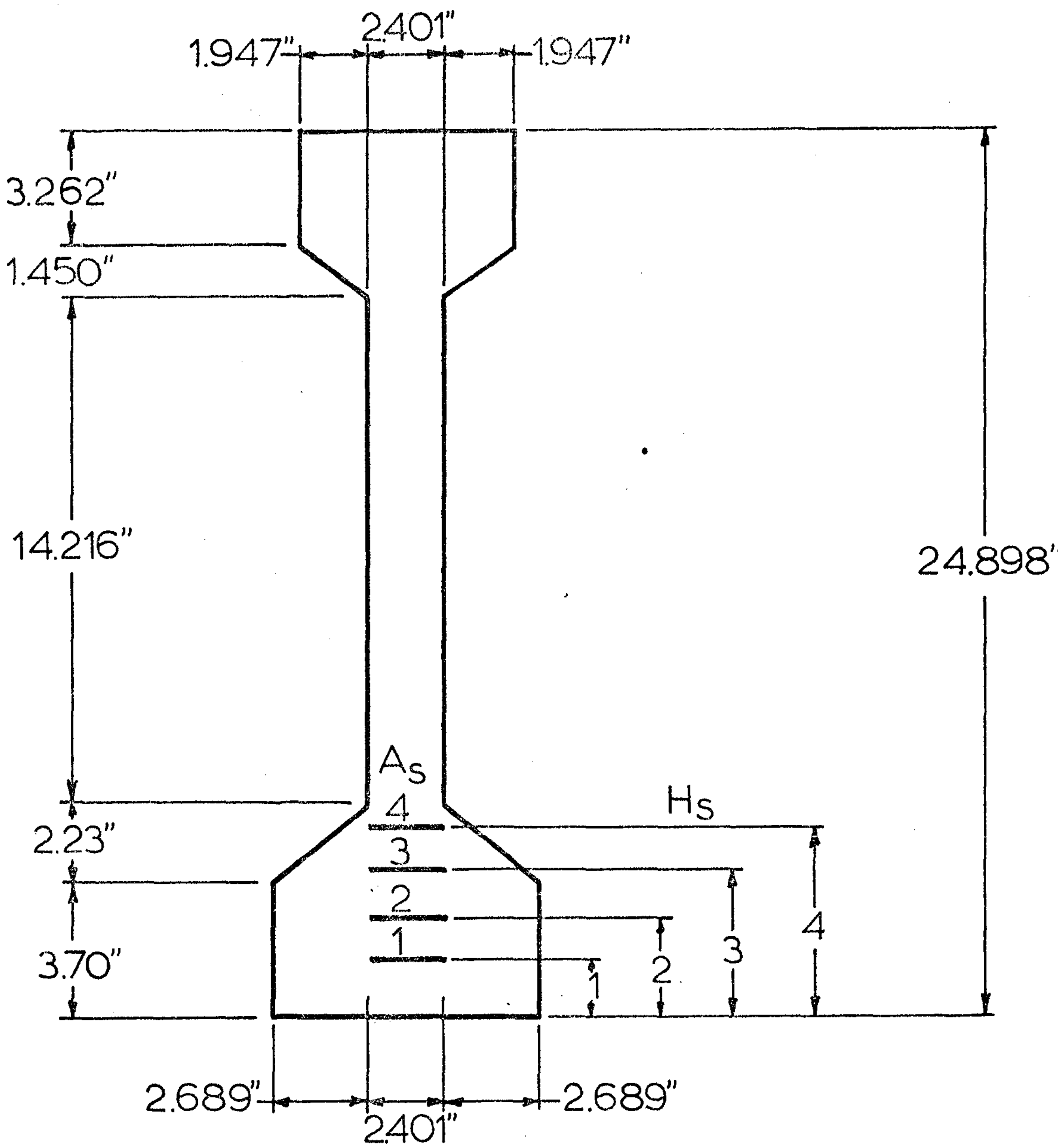
Fig. 14 OPTIMUM SECTION FOR DESIGN 4



As (1)	0.4 sq. in.
(2)	0.3 sq. in.
(3)	0.3 sq. in.
(4)	0.2 sq. in.

Hs (1)	2.0 in.
(2)	3.1 in.
(3)	4.2 in.
(4)	5.3 in.

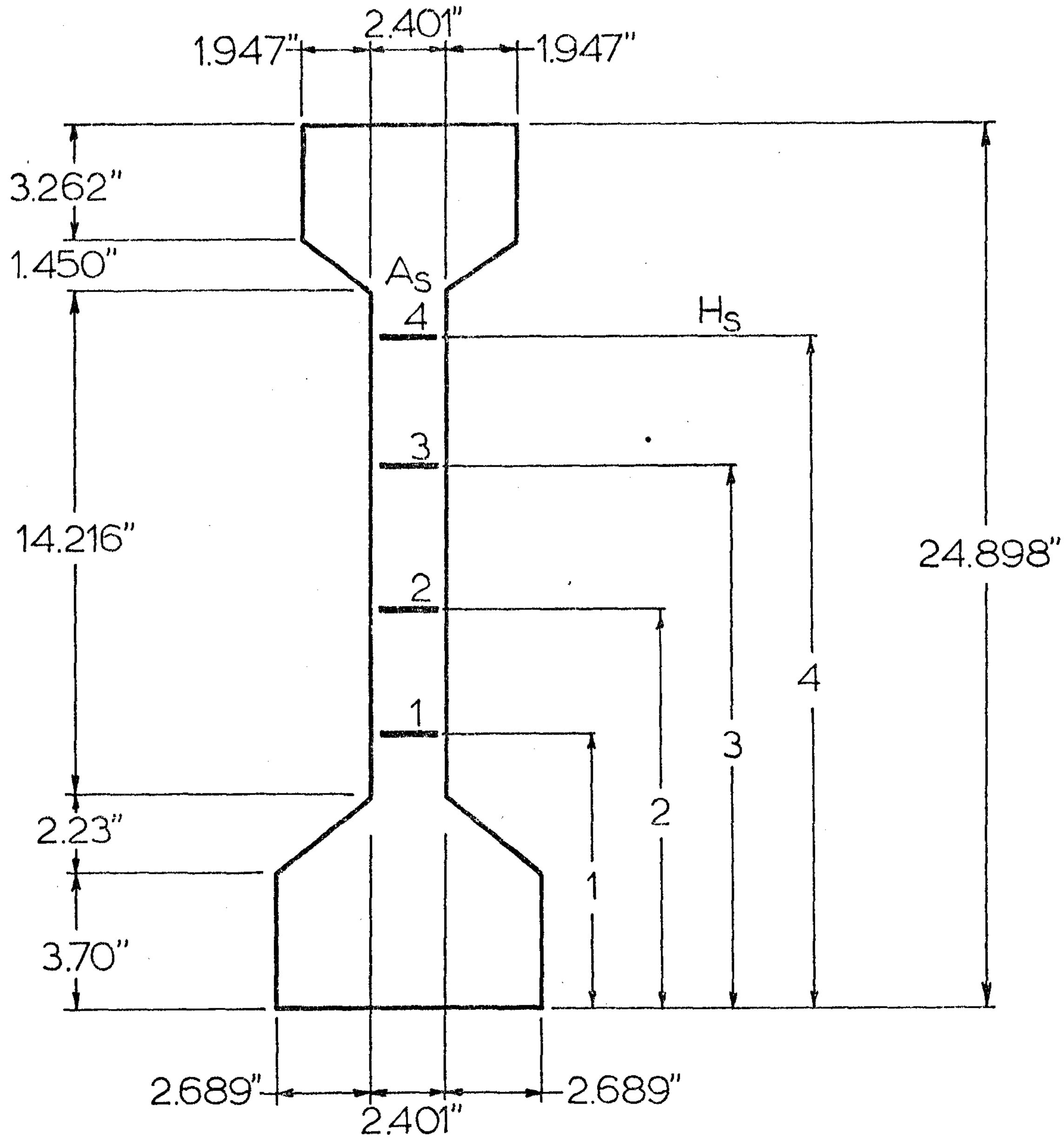
Fig. 15 INITIAL SECTION FOR DESIGNS 5 AND 6



As		
As	(1)	0.55519 sq. in.
	(2)	0.22581 sq. in.
	(3)	0.19098 sq. in.
	(4)	0.17498 sq. in.

Hs		
Hs	(1)	1.702 in.
	(2)	2.984 in.
	(3)	4.2739 in.
	(4)	5.5957 in.

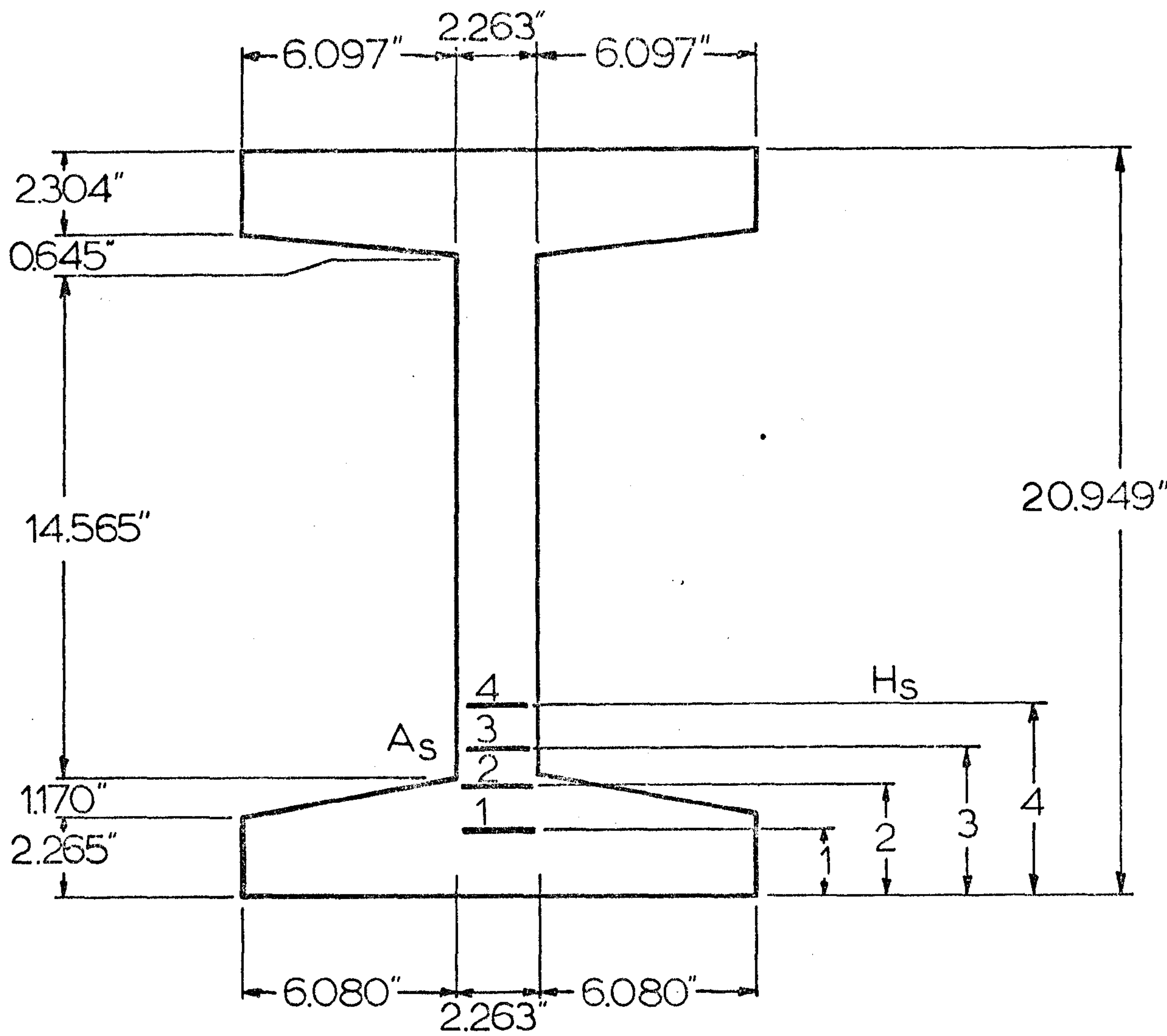
Fig. 16 OPTIMUM SECTION FOR DESIGN 5



A_S (1)	0.55519 sq. in.
(2)	0.22581 sq. in.
(3)	0.19098 sq. in.
(4)	0.17498 sq. in.

H_S (1)	7.8874 in.
(2)	11.449 in.
(3)	15.241 in.
(4)	18.886 in.

Fig. 17 OPTIMUM SECTION AT SUPPORT FOR DESIGN 5



A_S (1)	0.45178 sq. in.
(2)	0.3486 sq. in.
(3)	0.2946 sq. in.
(4)	0.26051 sq. in.

H_S (1)	1.7737 in.
(2)	2.9817 in.
(3)	4.1521 in.
(4)	5.3082 in

Fig. 18 OPTIMUM SECTION FOR DESIGN 6

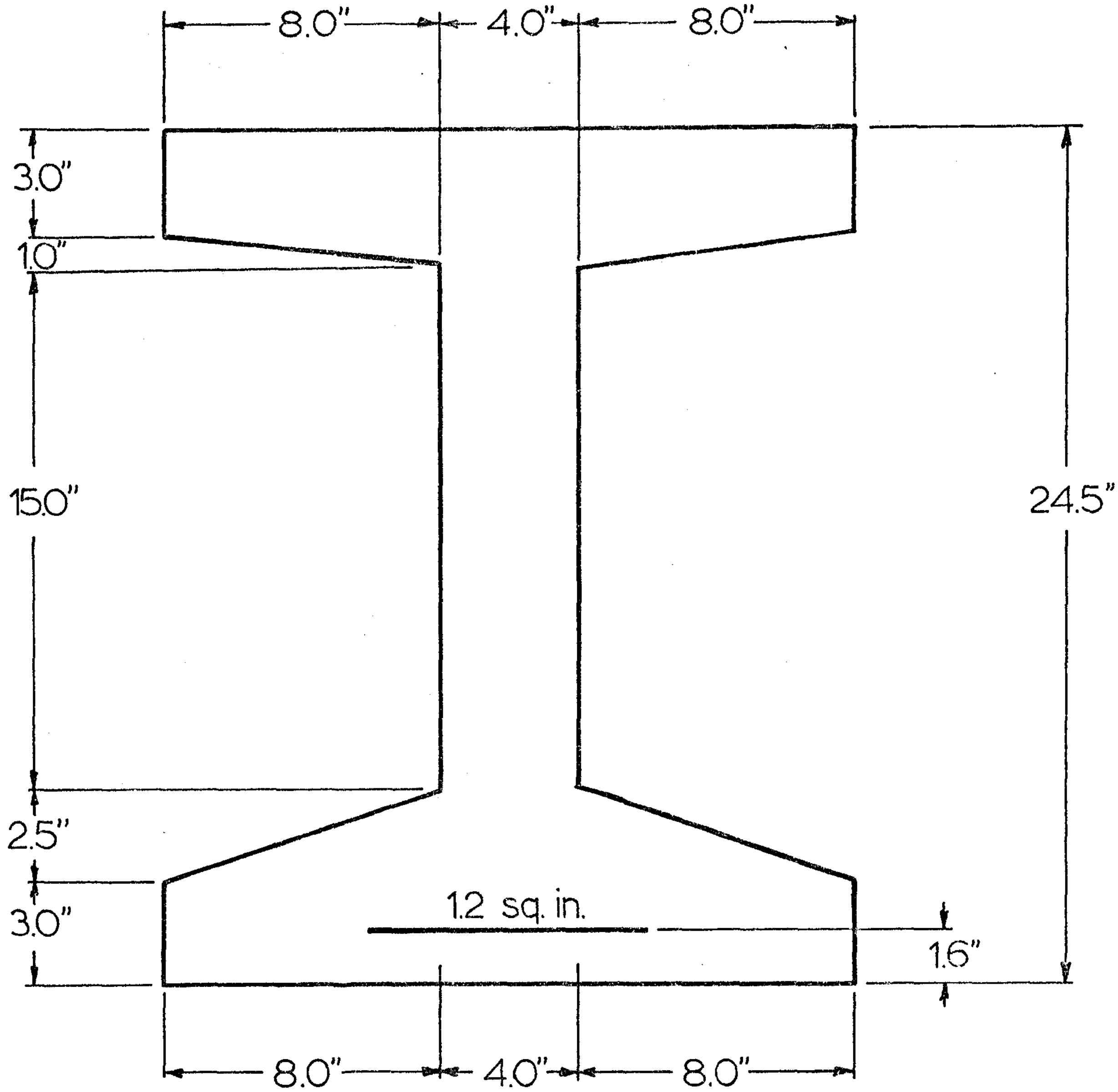


Fig. 19 INITIAL SECTION FOR DESIGN 7

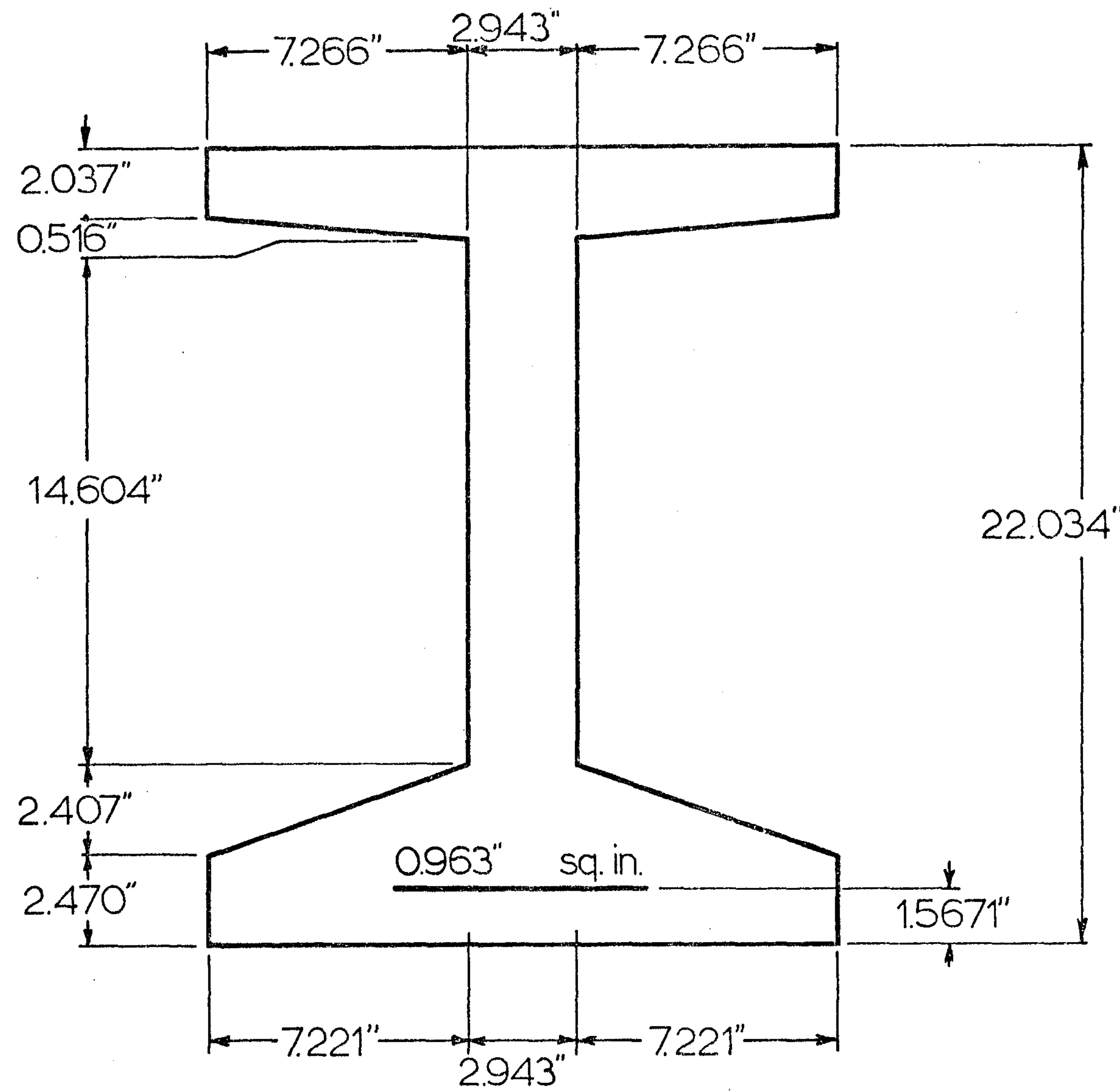
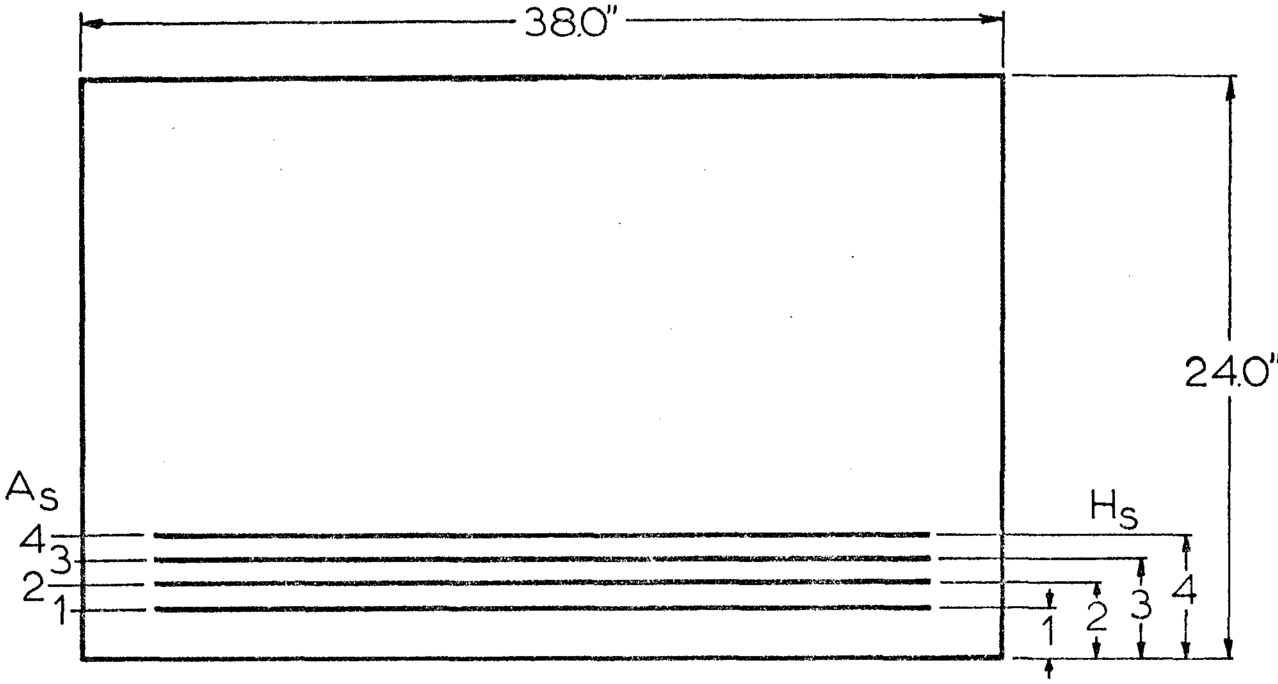


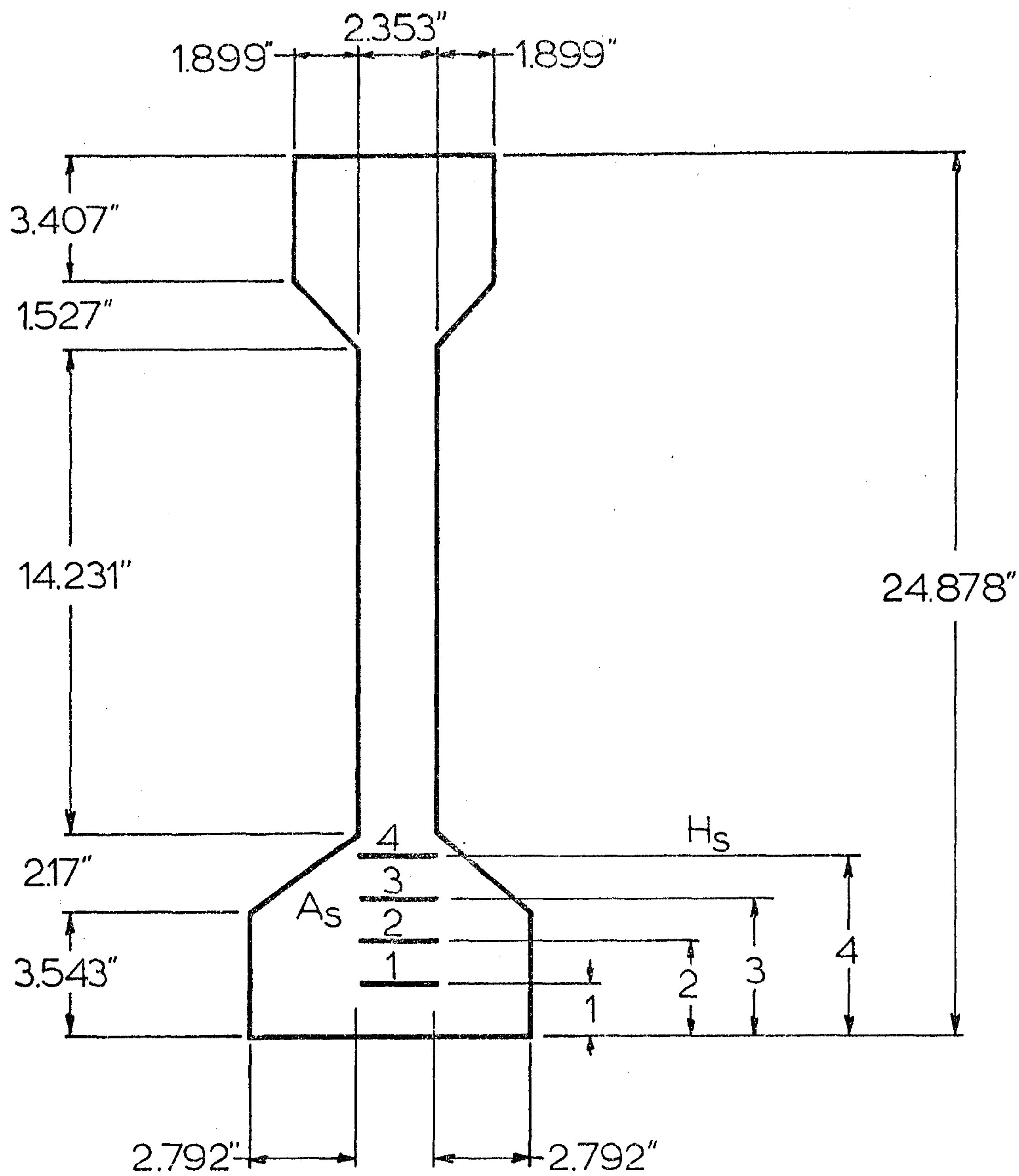
Fig. 20 OPTIMUM SECTION FOR DESIGN 7



A_s	(1)	1.0 sq. in.
	(2)	0.75 sq. in.
	(3)	0.50 sq. in.
	(4)	0.50 sq. in.

H_s	(1)	2.0 in.
	(2)	3.1 in.
	(3)	4.2 in.
	(4)	5.3 in

Fig. 21 INITIAL SECTION FOR DESIGN 8



A_S (1)	0.51593 sq. in.
(2)	0.2644 sq. in.
(3)	0.1958 sq. in.
(4)	0.15996 sq. in.

H_S (1)	1.6795 in.
(2)	2.8664 in.
(3)	4.0694 in.
(4)	5.2758 in.

Fig. 22 OPTIMUM SECTION FOR DESIGN 8

TABLE 1.1 Loads

Span Length in.	Live Loading Conditions		Superimposed Dead Load Kip/ft
	1 Kip/ft	2 Kips	
480.0	0.33	4.0	0.3

TABLE 1.2 Creep Factors and Shrinkage Strains

Maximum Creep Factor ϕ_{max}	Minimum Creep Factor ϕ_{min}	Creep Factor Due to Superimposed Dead Load ϕ_{SL}	Maximum Shrinkage Strain ε_{max}	Minimum Shrinkage Strain ε_{min}	Modular Ratio n
3.6	0.97	2.1	-2.5×10^{-4}	-0.7×10^{-4}	6

TABLE 1.3 Cost of Different Materials

Cost of Prestress Steel ¢/lb.	Cost of concrete $/cu.yard	Cost of Forming and Labour			Average Computer Time per Iteration seconds
		Vertical Portions ¢/sq.ft.	Horizontal Portions ¢/sq.ft.	Inclined Portions ¢/sq.ft.	
50	75	75	50	100	7.2

TABLE 1.4 Allowable Stresses and Deflection

Allowable Stresses of Concrete (k.s.i.)				Allowable Stress of Steel (k.s.i.)	Allowable Principal Tensile Stress (k.s.i.)	Allowable Deflection in.
At Transfer		Permanent				
Tensile	Compressive	Tensile	Compressive			
0.42	2.94	0.42	2.70	135.0	0.31	2.4

Initial Design	R	Shape	Cost $/ft	Function Value $F(\vec{V})$
	10^{-4}		10.323	18.340

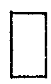
R	Number of Iterations	Shape	Minimum Cost $/ft	Function Value $F(\vec{V})$
10^{-4}	10		8.407	16.534
10^{-5}	16		8.328	9.156
10^{-6}	7		8.197	8.286
Total number of Iterations	33			

TABLE 2.1 Function values for Design 1

TABLE 2.2 Design Variables for Design 1

Design Variables	H_w in.	T_t in.	T_{st} in.	T_{sb} in.	T_b in.	B_w in.	T_f in.	B_f in.	$f_{p(bed)}$ k.s.i.
Initial	30.0	3.0	1.0	2.0	4.0	6.0	0.0	0.0	135.0
Final	29.474	2.478	0.386	1.549	3.641	4.131	0.0	0.0	135.0
Lower Limit	14.0	2.0	0.0	0.0	2.0	2.0	0.0	0.0	---

Design Variables	$A_{s(1)}$ sq.in.	$A_{s(2)}$ sq.in.	$A_{s(3)}$ sq.in.	$A_{s(4)}$ sq.in.	$H_{s(1)}$ in.	$H_{s(2)}$ in.	$H_{s(3)}$ in.	$H_{s(4)}$ in.
Initial	0.3	0.1	0.1	0.1	2.0	3.1	4.2	5.3
Final	0.2947	0.1273	0.1235	0.1188	1.961	3.099	4.186	5.19
Lower Limit	0.0	0.0	0.0	0.0	1.5	$H_{s(n+1)} - H_{s(n)} \geqslant 1.0$		

TABLE 2.3 Upper Limit Constraints for Design 1

Design Variables	$H_{s(4)}$ in.	T_f in.	B_f in.
Upper Limit	$T_b + T_{sb}$	0.0	0.0

TABLE 2.4 Average Steel Stresses for Design 1

Normal Stresses			Average Steel Stress at c.g.s. (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	130.71	111.82	109.41	116.76	114.35
		Final	126.87	108.36	104.54	106.79	102.97
	2	Initial	130.71	110.87	109.41	115.81	114.35
		Final	126.87	106.85	104.54	105.28	102.97

TABLE 2.5 Concrete Stresses at Top Fiber for Design 1.

Normal Stresses			Stress of Concrete at Top Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	0.128	-0.892	-0.405	-0.873	-0.386
		Final	0.419*	-1.267	-0.471	-1.277	-0.481
	2	Initial	0.128	-0.700	-0.405	-0.681	-0.386
		Final	0.419*	-0.953	-0.471	-0.963	-0.481

TABLE 2.6 Concrete Stresses at Bottom Fiber for Design 1.

Normal Stresses			Stress of Concrete at Bottom Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	-0.785	0.334	-0.142	0.290	-0.186
		Final	-1.518	0.340	-0.428	0.364	-0.404
	2	Initial	-0.785	0.146	-0.142	0.102	-0.186
		Final	-1.518	0.037	-0.428	0.061	-0.404

TABLE 2.7 Maximum Deflections for Design 1

			Maximum Deflection of the Beam in.				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	0.024	0.257	0.141	0.256	0.140
		Final	0.001	0.403	0.203	0.408	0.208
	2	Initial	0.024	0.197	0.141	0.196	0.140
		Final	0.001	0.300	0.203	0.305	0.208

Initial Design	R	Shape	Cost $/ft	Function Value $F(\vec{V})$
	10^{-3}	□	10.348	70.478

R	Number of Iterations	Shape	Minimum Cost $/ft	Function Value $F(\vec{V})$
10^{-3}	9	□	8.228	48.603
10^{-4}	12	□	7.727	11.813
10^{-5}	10	□	7.523	7.959
10^{-6}	12	□	7.454	7.504
Total number of Iterations	43			

TABLE 3.1 Function values for Design 2

TABLE 3.2 Design Variables for Design 2

Design Variables	H_w in.	T_t in.	T_{st} in.	T_{sb} in.	T_b in.	B_w in.	T_f in.	B_f in.	$f_{p(bed)}$ k.s.i.
Initial	30.0	3.0	1.0	2.0	4.0	6.0	0.0	0.0	135.0
Final	29.124	2.159	0.030	0.888	3.129	3.680	0.228	0.0	135.01
Lower Limit	14.0	2.0	0.0	0.0	2.0	2.0	0.0	0.0	--

Design Variables	$A_{s(1)}$ sq.in.	$A_{s(2)}$ sq.in.	$A_{s(3)}$ sq.in.	$A_{s(4)}$ sq.in.	$H_{s(1)}$ sq.in.	$H_{s(2)}$ sq.in.	$H_{s(3)}$ sq.in.	$H_{s(4)}$ sq.in.
Initial	0.3	0.2	0.2	0.1	2.0	3.1	4.2	5.3
Final	0.0007	0.001	0.0016	0.8178	1.678	3.076	4.308	5.498
Lower Limit	0.0	0.0	0.0	0.0	1.5	$H_{s(n+1)} - H_{s(n)} \geq 1.0$		

Table 3.3 Upper Limit Constraints for Design 2.

Design Variables	$H_{s(4)}$ in.	B_f in.
Upper Limit	5.5	0.0

TABLE 3.4 Average Steel Stresses for Design 2

Normal Stresses			Average Steel stress at c.g.s. (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	128.82	108.65	106.29	109.71	107.34
		Final	125.15	105.82	101.89	101.31	97.370
	2	Initial	128.82	107.72	106.29	108.77	107.34
		Final	125.15	104.27	101.89	99.755	97.370

TABLE 3.5 Concrete Stresses at Top Fiber for Design 2

Normal Stresses			Stress of Concrete at Top Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	0.279	-0.771	-0.287	-0.766	-0.281
		Final	0.416*	-1.643	-0.658	-1.673	-0.688
	2	Initial	0.279	-0.580	-0.287	-0.575	-0.281
		Final	0.416*	-1.255	-0.658	-1.285	-0.688

TABLE 3.6 Concrete Stresses at Bottom Fiber for Design 2

Nornal Stresses			Stress of Concrete at Bottom Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	-1.145	0.047	-0.424	0.035	-0.436
		Final	-2.022	0.331	-0.626	0.419*	-0.537
	2	Initial	-1.145	-0.138	-0.424	-0.150	-0.436
		Final	-2.022	-0.045	-0.626	0.042	-0.537

TABLE 3.7 Maximum Deflections for Design 2.

Maximum Deflection of the Beam in.							
Loading Combination			1	2	3	4	5
Live Loading condition	1	Initial	0.026	0.257	0.142	0.258	0.143
		Final	0.010	0.536	0.272	0.543	0.279
	2	Initial	0.026	0.198	0.142	0.199	0.143
		Final	0.010	0.400	0.272	0.407	0.279

Initial Design	R	Shape	Cost $/ft	Function Value $F(\vec{V})$
	0.1	□	17.781	21141

R	Number of Iterations	Shape	Minimum cost $/ft	Function Value $F(\vec{V})$
0.1	70	□	8.536	4015.6
0.01	70	□	7.109	408.08
0.001	70	□	6.780	46.943
0.0001	70	□	6.713	10.776
Total number of Iterations	280			

TABLE 4.1 Function values for Design 3

TABLE 4.2 Design Variables for Design 3

Design Variables	H_w in.	T_t in.	T_{st} in.	T_{sb} in.	T_b in.	B_w in.	T_f in.	B_f in.	$f_{p(bed)}$ k.s.i.
Initial	46.0	4.0	3.0	3.0	4.0	8.0	0.0	0.0	135.0
Final	15.185	2.674	0.219	2.371	11.747	3.522	0.155	0.0	131.14
Lower Limit	14.0	2.0	0.0	0.0	2.0	2.0	0.0	0.0	--

Design Variables	$A_{s(1)}$ sq.in.	$A_{s(2)}$ sq.in.	$A_{s(3)}$ sq.in.	$A_{s(4)}$ sq.in.	$H_{s(1)}$ in.	$H_{s(2)}$ in.	$H_{s(3)}$ in.	$H_{s(4)}$ in.
Initial	0.42	0.30	0.20	0.20	2.0	3.1	4.2	5.3
Final	0.4638	0.1825	0.2475	0.3341	1.964	8.88	10.383	11.798
Lower Limit	0.0	0.0	0.0	0.0	1.5	$H_{s(n+1)} - H_{s(n)} \geq 1.0$		

TABLE 4.3 Upper Limit Constraints for Design 3.

Design Variable	$H_{s(4)}$ in.	B_f in.
Upper Limit	$T_b + T_{sb}$	0.0

TABLE 4.4 Average Steel Stresses for Design 3

Normal Stresses			Average Steel stress at c.g.s. (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	130.14	107.75	106.91	110.34	109.49
		Final	118.03	96.804	93.023	87.437	83.657
	2	Initial	130.14	107.42	106.91	110.0	109.49
		Final	118.03	95.314	93.023	85.948	83.657

TABLE 4.5 Concrete Stresses at Top Fiber for Design 3

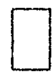
Normal Stresses			Stress of Concrete at Top Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	0.260	-0.143	0.018	-0.133	0.028
		Final	0.322	-2.220	-0.986	-2.281	-1.048
	2	Initial	0.260	-0.079	0.018	-0.069	0.028
		Final	0.322	-1.734	-0.986	-1.795	-1.048

TABLE 4.6 Concrete Stresses at Bottom Fiber for Design 3.

Normal Stresses			Stress of Concrete at Bottom Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	-0.871	-0.360	-0.519	-0.383	-0.542
		Final	-2.928	0.147	-1.035	0.417	-0.765
	2	Initial	-0.871	-0.423	-0.519	-0.445	-0.542
		Final	-2.928	-0.318	-1.035	-0.048	-0.765

TABLE 4.7 Maximum Deflections for Design 3

Live Loading Condition		Maximum Deflection of the Beam in.					
Loading Combination			1	2	3	4	5
	1	Initial	0.016	0.07	0.044	0.069	0.044
	1	Final	0.029	0.592	0.307	0.601	0.317
	2	Initial	0.016	0.056	0.044	0.056	0.044
	2	Final	0.029	0.445	0.307	0.455	0.317

Initial Design	R	Shape	Cost $/ft	Function Value $F(\vec{V})$
	0.1		17.781	5863.5

R	Number of Iterations	Shape	Minimum cost $/ft	Function Value $F(\vec{V})$
1×10^{-1}	43	T	11.344	4022
1×10^{-2}	69	T	8.268	409.9
1×10^{-3}	9	T	7.743	48.13
1×10^{-4}	19	T	7.609	11.695
Total number of Iterations	140			

TABLE 5.1 Function values for Design 4

TABLE 5.2 Design Variables for Design 4

Design Variables	H_w in.	T_t in.	T_{st} in.	T_{sb} in.	T_b in.	B_w in.	T_f in.	B_f in.	$f_{p(bed)}$ k.s.i.
Initial	46.0	4.0	3.0	3.0	4.0	8.0	0.0	0.0	135.0
Final	31.674	3.024	0.893	0.069	2.137	2.652	1.826	0.0	134.58
Lower Limit	14.0	2.0	0.0	0.0	2.0	2.0	0.0	0.0	--

Design Variables	$A_{s(1)}$ sq.in.	$A_{s(2)}$ sq.in.	$A_{s(3)}$ sq.in.	$A_{s(4)}$ sq.in.	$H_{s(1)}$ sq.in.	$H_{s(2)}$ in.	$H_{s(3)}$ in.	$H_{s(4)}$ in.
Initial	0.42	0.30	0.20	0.20	2.0	3.1	4.2	5.3
Final	0.0086	0.0107	0.0196	0.6484	1.727	2.954	4.199	5.428
Lower Limit	0.0	0.0	0.0	0.0	1.5	$H_{s(n+1)} - H_{s(n)} \geq 1.0$		

TABLE 5.3 Upper Limit Constraints for Design 4

Design Variables	$H_{s(4)}$ in.	B_f in.
Upper Limit	5.5	0.0

TABLE 5.4 Average Steel Stresses for Design 4

Normal Stresses			Average Steel Stress at c.g.s. (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	130.14	107.75	106.91	110.34	109.49
		Final	123.57	104.63	100.17	98.633	94.180
	2	Initial	130.14	107.42	106.91	110.0	109.49
		Final	123.57	102.87	100.17	96.879	94.180

TABLE 5.5 Concrete Stresses at Top Fiber for Design 4

Normal Stresses			Stress of Concrete at Top Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	0.260	-0.143	0.0186	-0.133	0.028
		Final	0.409	-1.391	-0.532	-1.426	-0.567
	2	Initial	0.260	-0.079	0.018	-0.069	0.028
		Final	0.409	-1.053	-0.532	-1.088	-0.567

TABLE 5.6 Concrete Stresses at Bottom Fiber for Design 4

Normal Stresses			Stress of Concrete at Bottom Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	-0.871	-0.360	-0.519	-0.383	-0.542
		Final	-2.202	+0.290	-0.713	0.416*	-0.587
	2	Initial	-0.871	-0.423	-0.519	-0.445	-0.542
		Final	-2.202	-0.104	-0.713	0.020	-0.587

TABLE 5.7 Maximum Deflections for Design 4

			Maximum Deflection of the Beam in.				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	0.016	0.07	0.044	0.069	0.044
		Final	0.031	0.612	0.352	0.684	0.361
	2	Initial	0.016	0.056	0.044	0.056	0.044
		Final	0.031	0.542	0.352	0.571	0.361

Inital Design	R	Shape	Cost $/ft	Function Value $F(\vec{V})$
	0.1	I	11.452	22.791

R	Number of Iterations	Shape	Minimum cost $/ft	Function Value $F(\vec{V})$
0.1	60	I	7.779	16.345
0.01	70	I	6.686	7.897
0.001	70	I	6.319	6.530
Total number of Iterations	200			

TABLE 6.1 Function values for Design 5

TABLE 6.2 Design Variables for Design 5

Design Variables	H_w in.	T_t in.	T_{st} in.	T_{sb} in.	T_b in.	B_w in.	T_f in.	B_f in.	$f_{p(bed)}$ k.s.i.
Initial	15.0	3.0	1.0	2.5	3.0	4.0	8.0	8.0	135.0
Final	14.216	3.262	1.45	2.229	3.7	2.4	1.947	2.689	135.3
Lower Limit	14.0	2.0	0.0	0.0	2.0	2.0	0.0	0.0	--

Design Variables	$A_{s(1)}$ sq.in.	$A_{s(2)}$ sq.in.	$A_{s(3)}$ sq.in.	$A_{s(4)}$ sq.in.	$H_{s(1)}$ in.	$H_{s(2)}$ in.	$H_{s(3)}$ in.	$H_{s(4)}$ in.
Initial	0.40	0.30	0.30	0.20	2.0	3.1	4.2	5.3
Final	0.5552	0.22581	0.1910	0.175	1.72	2.984	4.273	5.595
Lower Limit	0.0	0.0	0.0	0.0	1.5	$H_{s(n+1)} - H_{s(n)} \geq 1.0$		

TABLE 6.3 Upper Limit Constraints for Design 5

Design Variables	H in.	$H_{s(4)}$ in.
Upper Limit	25	$T_b + T_{sb}$

TABLE 6.4 Average Steel Stresses for Design 5

Normal Stresses			Average Steel Stress at c.g.s. (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	128.67	107.84	105.73	108.36	106.25
		Final	120.42	100.48	95.54	89.364	84.424
	2	Initial	128.67	107.01	105.73	107.53	106.25
		Final	120.42	98.531	95.54	87.418	84.424

TABLE 6.5 Concrete Stresses at Top Fiber for Design 5

Normal Stresses			Stress of Concrete at Top Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	-0.169	-1.267	-0.714	-1.266	-0.713
		Final	0.327	-2.574	-1.155	-2.647	-1.228
	2	Initial	-0.169	-1.049	-0.714	-1.048	-0.713
		Final	0.327	-2.015	-1.155	-2.088	-1.228

TABLE 6.6 Concrete Stresses at Bottom Fiber for Design 5

Nornal Stresses			Stress of Concrete at Bottom Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	-1.195	0.052	-0.443	0.046	-0.449
		Final	-2.869	0.085	-1.044	0.387	-0.742
	2	Initial	-1.195	-0.142	-0.443	-0.149	-0.449
		Final	-2.869	-0.359	-1.044	-0.057	-0.742

TABLE 6.7 Maximum Deflections for Design 5

Live Loading Condition		Maximum Deflection of the Beam in. Loading Combination 1	2	3	4	5
1	Initial	0.089	0.492	0.287	0.493	0.288
1	Final	0.038	1.007	0.515	1.024	0.532
2	Initial	0.089	0.386	0.287	0.387	0.288
2	Final	0.038	0.753	0.515	0.770	0.532

Initial Design	R	Shape	Cost $/ft	Function Value $F(\vec{V})$
	0.1	I	11.452	22.891

R	Number of Iterations	Shape	Minimum Cost $/ft	Function Value $F(\vec{V})$
0.1	9	I	9.193	19.664
0.01	45	I	8.022	9.527
Total number of Iterations	54			

TABLE 7.1 Function values for Design 6

TABLE 7.2 Design Variables for Design 6

Design Variables	H_w in.	T_t in.	T_{st} in.	T_{sb} in.	T_b in.	B_w in.	T_{f5} in.	B_f in.	$f_{p(bed)}$ k.s.i.
Initial	15.0	3.0	1.0	2.5	3.0	4.0	8.0	8.0	135.0
Final	14.565	2.304	0.644	1.17	2.265	2.263	6.097	6.08	135.0
Lower Limit	14.0	2.0	0.0	0.0	2.0	2.0	6.0	6.0	--

Design Variables	$A_{s(1)}$ sq.in.	$A_{s(2)}$ sq.in.	$A_{s(3)}$ sq.in.	$A_{s(4)}$ sq.in.	$H_{s(1)}$ in.	$H_{s(2)}$ in.	$H_{s(3)}$ in.	$H_{s(4)}$ in.
Initial	0.4	0.3	0.3	0.2	2.0	3.1	4.2	5.3
Final	0.4518	0.3486	0.2946	0.2605	1.773	2.981	4.152	5.308
Lower Limit	0.0	0.0	0.0	0.0	1.5	$H_{s(n+1)} - H_{s(n)} \geq 1.0$		

TABLE 7.3 Upper Limit Constraints for Design 6

Design Variables	H in.	$H_{s(4)}$ in.
Upper Limit	25.0	5.5

TABLE 7.4 Average Steel Stresses for Design 6

Normal Stresses			Average Steel Stress at c.g.s. (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	128.67	107.84	105.73	108.36	106.25
		Final	121.07	100.04	95.903	89.664	85.528
	2	Initial	128.67	107.01	105.73	107.53	106.25
		Final	121.07	98.410	95.903	88.035	85.528

TABLE 7.5 Concrete Stresses at Top Fiber for Design 6

Normal Stresses			Stress of Concrete at Top Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	-0.169	-1.267	-0.714	-1.266	-0.713
		Final	-0.169	-2.421	-1.272	-2.442	-1.294
	2	Initial	-0.169	-1.049	-0.714	-1.048	-0.713
		Final	-0.169	-1.968	-1.272	-1.990	-1.294

TABLE 7.6 Concrete Stresses at Bottom Fiber for Design 6

Normal Stresses			Stress of Concrete at Bottom Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	-1.195	0.052	-0.443	0.046	-0.449
		Final	-2.720	-0.004	-1.035	0.263	-0.766
	2	Initial	-1.195	-0.142	-0.443	-0.149	-0.449
		Final	-2.720	-0.410	-1.035	-0.142	-0.766

TABLE 7.7 Maximum Deflections for Design 6

			Maximum Deflection of the Beam in.				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	0.089	0.492	0.287	0.493	0.288
		Final	0.079	1.057	0.558	1.070	0.570
	2	Initial	0.089	0.386	0.287	0.387	0.288
		Final	0.079	0.800	0.558	0.813	0.570

Initial Design	R	Shape	Cost $/ft	Function Value $F(\vec{V})$
	1×10^{-3}	I	11.452	199.06

R	Number of Iterations	Shape	Minimum Cost $/ft	Function Value $F(\vec{V})$
1×10^{-3}	17	I	9.424	129.61
1×10^{-4}	4	I	9.359	21.394
Total number of Iterations	21			

TABLE 8.1 Function values for Design 7

TABLE 8.2 Design Variables for Design 7

Design Variables	H_w in.	T_t in.	T_{st} in.	T_{sb} in.	T_b in.	B_w in.	T_f in.	B_f in.	$f_{p(bed)}$ k.s.i.
Initial	15.0	3.0	1.0	2.5	3.0	4.0	8.0	8.0	135.0
Final	14.604	2.037	0.516	2.406	2.47	2.942	7.266	7.221	135.0
Lower Limit	14.0	2.0	0.0	0.0	2.0	2.0	6.0	6.0	--

Design Variables	A_s sq.in.	H_s in.
Initial	1.2	1.6
Final	0.9635	1.5671
Lower Limit	0.0	1.5

TABLE 8.3 Upper Limit Constraints for Design 7

Design Variables	H in.	$H_{s(4)}$ in.
Upper Limit	25.0	1.7

TABLE 8.4 Average Steel Stresses for Design 7.

Normal Stresses			Average Steel Stress at c.g.s. (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	127.37	106.65	104.12	105.08	102.55
		Final	126.82	107.98	104.3	106.19	102.51
	2	Initial	127.37	105.65	104.12	104.09	102.55
		Final	126.82	106.53	104.30	104.74	102.51

TABLE 8.5 Concrete Stresses at Top Fiber for Design 7

Normal Stresses			Stress of Concrete at Top Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	0.020	-1.109	-0.560	-1.114	-0.565
		Final	-0.035	-1.884	-0.959	-1.889	-0.965
	2	Initial	0.020	-0.893	-0.560	-0.898	-0.565
		Final	-0.035	-1.52	-0.959	-1.525	-0.965

TABLE 8.6 Concrete Stresses at Bottom Fiber for Design 7

Normal Stresses			Stress of Concrete at Bottom Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	-1.362	-0.081	-0.570	-0.060	-0.549
		Final	-1.465	0.307	-0.424	0.332	-0.398
	2	Initial	-1.362	-0.274	-0.570	-0.253	-0.549
		Final	-1.465	0.018	-0.424	0.0447	-0.398

TABLE 8.7 Maximum Deflections for Design 7

			Maximum Deflection of the Beam in.				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	0.078	0.479	0.276	0.482	0.278
		Final	0.079	0.787	0.426	0.792	0.431
	2	Initial	0.078	0.374	0.276	0.377	0.278
		Final	0.079	0.601	0.426	0.606	0.431

Initial Design	R	Shape	Cost $/ft	Function Value $F(\vec{V})$
	0.1	▭	24.316	34.521

R	Number of Iterations	Shape	Minimum Cost $/ft	Function Value $F(\vec{V})$
0.1	63	I	7.799	18.535
0.01	44	I	6.647	7.991
0.001	56	I	6.310	6.535
Total number of Iterations	163			

TABLE 9.1 Function values for Design 8

TABLE 9.2 Design Variables for Design 8

Design Variables	H_w in.	T_t in.	T_{st} in.	T_{sb} in.	T_b in.	B_w in.	T_f in.	B_f in.	$f_{p(bed)}$ k.s.i.
Initial	15.0	3.0	1.0	1.0	4.0	38.0	0.0	0.0	135.0
Final	14.231	3.407	1.527	2.169	3.543	2.353	1.899	2.792	135.0
Lower Limit	14.0	2.0	0.0	0.0	2.0	2.0	0.0	0.0	--

Design Variables	$A_{s(1)}$ sq.in.	$A_{s(2)}$ sq.in.	$A_{s(3)}$ sq.in.	$A_{s(4)}$ sq.in.	$H_{s(1)}$ in.	$H_{s(2)}$ in.	$H_{s(3)}$ in.	$H_{s(4)}$ in.
Initial	1.0	0.75	0.5	0.5	2.0	3.1	4.2	5.3
Final	0.5159	0.264	0.1958	0.16	1.679	2.866	4.069	5.275
Lower Limit	0.0	0.0	0.0	0.0	1.5	$H_{s(n+1)} - H_{s(n)} \geqslant 1.0$		

TABLE 9.3 Upper Limit Constraints for Design 8

Design Variables	H in.	H_s in.
Upper Limit	25.0	5.5

TABLE 9.4 Average Steel Stresses for Design 8

Normal Stresses			Average Steel Stress at c.g.s. (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	131.63	110.63	109.73	116.59	115.70
		Final	120.04	100.20	95.195	89.048	84.043
	2	Initial	131.63	110.27	109.73	116.24	115.70
		Final	120.04	98.228	95.195	87.076	84.043

TABLE 9.5 Concrete Stresses at Top Fiber for Design 8

Normal Stresses			Stress of Concrete at Top Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	-0.164	-0.645	-0.435	-0.625	-0.414
		Final	0.341	-2.568	-1.147	-2.642	-1.221
	2	Initial	-0.164	-0.562	-0.435	-0.542	-0.414
		Final	0.341	-2.008	-1.147	-2.082	-1.221

TABLE 9.6 Concrete Stresses at Bottom Fiber for Design 8

Nornal Stresses			Stress of Concrete at Bottom Fiber (k.s.i.)				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	-0.625	-0.016	-0.222	-0.073	-0.279
		Final	-2.869	+0.085	-1.043	0.389	-0.739
	2	Initial	-0.625	-0.097	-0.222	-0.154	-0.279
		Final	-2.869	-0.359	-1.043	-0.055	-0.739

TABLE 9.7 Maximum Deflections for Design 8

			Maximum Deflection of the Beam in.				
Loading Combination			1	2	3	4	5
Live Loading Condition	1	Initial	0.2205	0.386	0.301	0.385	0.300
		Final	0.035	1.004	0.512	1.022	0.530
	2	Initial	0.220	0.342	0.301	0.342	0.300
		Final	0.035	0.751	0.512	0.768	0.530

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