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A numerical analysis solution of water table drawdown between tile drains.

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A NUMERICAL ANALYSIS SOLUTION OF WATER TABLE DRAWDOWN BETWEEN TILE DRAINS

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ABSTRACT

The description of the analysis of unconfined transient drainage problems between parallel tile drains is presented. A non-linear partial differential equation is used to predict the water table variation with respect to time. A computer programme which solves the Laplace's partial differential equation of flow by finite difference techniques is written for an IBM7094 and fully described. Theoretical water table profiles, which are obtained for different ratios of average aquifer depth to distance between drains, are compared to those found experimentally with a Hele-Shaw model by R.A. Sherry and those theoretically predicted from Dupuit-Forchheimer theory by R.E. Glover. The results indicate a much better agreement with the Hele-Shaw model than the Dupuit-Forchheimer approach.

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CHAPTER I

INTRODUCTION

A drainage problem in agricultural areas is caused by an excess of water on the surface of the soil or in the root zone beneath the surface of the soil. Much of the water will be removed naturally by surface runoff, deep seepage, evaporation and transpiration; however, if these processes are too slow to prevent crop damage, artificial means must be used to improve drainage.

The main purpose of drainage is to provide conditions in the soil which are suitable for the maximum growth of plants. This is achieved by stabilizing the fluctuations of the ground water table and removing gravitational water from the soil.

High water tables may slow plant growth in many ways. It is found that dissolved oxygen in waterlogged soils is completely absent. This results in the production of toxic organic compounds by the anaerobic decomposition of organic material. Since the rate of decomposition is slowed down, the nitrogen available to plants is limited. In addition, a fluctuating water table may cause plant roots to rot off. When the water table drops, there are no roots for the plant to use to extract moisture from the wet-root zone.

The best removal of water is most frequently obtained by a combination of surface and subsurface drainage systems. It is usually most economical to remove surface water from tiled fields by installing shallow surface ditches constructed to grade which do not interfere with cultivation. Subsurface drainage may be achieved by either tile drains or open drains. In laying out a drainage system, specific drainage requirements, such as rate of lowering the water table, depth of the water table below the surface, and the degree, length and frequency of fluctuations in the water table level must be determined.

Parallel tile drains which are designed to provide subsurface drainage for agricultural lands are the subject of this thesis. Parallel line systems are used on poorly drained soils having little slope and approximately uniform texture. Tile drainage removes excess water from the soil through a continuous line of tile laid at a specified depth and grade. Free water enters through the tile joints and flows out by gravity, so that the water table is lowered below the root zone of the plants.

For uniform drainage, drains are parallel and the same distance apart. This provides for a uniform water table midway between drains and allows each line to cover its maximum area.

One of the main problems in drainage design is the determination of the proper depth and spacing of drains and the subsequent behaviour of the water table. A mathematical analysis of the field problem is usually obtained only by simplifying assumptions. In most cases, the assumptions will not exactly correspond to the situation in the field. It is then necessary to use judgment in applying the theory. In some cases a theory will work quite well and is suitable for determining the depth and spacing of drains but in other cases a theory is useful only as a first approximation. However, an acceptable theoretical solution of water table response to parallel tile drains will contribute to the understanding of field experimentation findings. This solution could also serve as a basis for design practises if used with a sound judgment of local conditions.

This thesis consists of a comparison between theoretical and experimental values of water table level drawdown profiles for various values of spacing between parallel tile drains. Experimental values were obtained from experiments performed by Shery [1966] with a Hele-Shaw model and theoretical values were calculated by a finite difference solution of Laplace's equation and a partial differential equation which describes the movement of the water table. A comparison of water table drawdown profiles is also presented between this theoretical solution and one presented by Glover (Dumm, [1954]).

CHAPTER II

REVIEW OF LITERATURE

INTRODUCTION

In recent years, there has been a growing interest in the problem of drainage in agricultural lands. In particular, many experimental studies and theoretical analyses have been presented on fluid flow between parallel drains. A survey of the literature pertaining to this subject will illustrate the varied approaches of different research projects.

Most theoretical analyses to date have been based on Dupuit-Forchheimer, radial flow and potential theory assumptions. An evaluation and comparison of various methods of designing subsurface drainage systems has been presented by van Schilfgaarde, Kirkham and Frevert [1956]. Both steady and non-steady solutions were reviewed for problems of saturated flow and special attention was given to theories based upon either radial or horizontal flow assumptions as presented by Hooghoudt [1937, 1940], van Deemter [1949], Glover (Lee D. Dumm, [1954]), Ferris [1950] and Walker [1952]. Theoretically predicted water table levels and drain spacings were compared with experimental field data.

DUPUIT-FORCHHEIMER THEORY

Glover (Dumm, [1954]), using the Dupuit-Forchheimer assumptions, obtained an approximate equation, relating the spacing of tile drains to the rate of drop of the water table. The Dupuit-Forchheimer equation was linearized by using the average depth of the soil cross-section, rather than the variable depth. Since this method was based on the Dupuit-Forchheimer assumptions, neither the effect of convergence near the drains, nor the vertical component of flow was considered. The resulting rate of fall of the water table was over-estimated.

The formulas presented by Glover (Dumm, [1954]) were improved by Brooks [1961], who took the drawdown of the water table into account. The Dupuit-Forchheimer equations were not linearized since a solution was obtained in the form of a perturbation series. Therefore, Brooks' theory, unlike Glover's (Dumm, [1954]), was accurate when the distance from the drains to the impermeable barrier was small compared to the drawdown. The solution was compared with the one developed by Isherwood [1959] and Glover (Dumm, [1954]), and field data obtained by Klinge [1955]. A good comparison of water table levels was found when the ratio of drain spacing to depth was large.

A rational analysis relating the spacing and depth of drains to water table drawdown rate was proposed by van Schilfgaarde [1963]. An improved solution was obtained since the Dupuit-Forchheimer equation was not linearized and the thickness of the aquifer was considered a variable. In addition, convergence of flow near the drains was corrected for by the use of Hooghoudt's "equivalent depth" chart.

A discharge formula was combined with drain spacing computations by Dumm and Winger [1964] in order to develop area-discharge curves. These curves were used to predict whether a drainage system under irrigation was in dynamic equilibrium. Some limitations of the Dupuit-Forchheimer assumptions were overcome by using Hooghoudt's method of correcting for convergence. The analysis calculated the drain spacing required to keep the water table lower than a predetermined soil elevation in an area subjected to repeated irrigation.

Dumm [1964] altered the development which Glover (Dumm, [1954]) presented by using an initial water table which fit a fourth degree parabola. Moody's and Hooghoudt's methods of correcting for convergence, as the streamlines approach the drain, were applied. Data from experimental field investigations of water table recession rates in Canada and Australia were checked against the theoretical results obtained by the author's formulas.

Because of repeated criticism concerning the accuracy of quantities calculated from Dupuit-Forchheimer theory, van Schilfgaarde [1965] and Glover [1965] reviewed the underlying assumptions and accompanying restrictions of this theory. Van Schilfgaarde compared the time required for a specific drawdown as calculated by Glover (Dumm, [1954]), Tapp and Moody (Dumm, [1964]), van Schilfgaarde [1963] and Guyon [1964] and found a close agreement between Guyon's potential theory and van Schilfgaarde's non-linearized Dupuit-Forchheimer theory. Glover gave examples of favourable comparisons between field drainage results and answers calculated from Dupuit-Forchheimer theory. Both authors concluded that the use of Dupuit-Forchheimer assumptions for field problems was permissible if the limitations of the theory were recognized.

A relationship was developed by van Schilfgaarde [1965] that described the water table behaviour after any precipitation pattern. In calculating the water table response to precipitation, the probability distribution of the precipitation was taken into account.

Moody [1966] solved the Dupuit-Forchheimer partial differential equation in unconfined aquifers by numerical techniques which avoided linearizing the differential equation. In addition, he avoided the assumption that the ratio of drawdown distance to distance from the drains to the impermeable layer is small. Dimensionless curves of the maximum water table height, rate of discharge and volume of water removed between the drains were presented for varied spacings of parallel drains.

The depth from the drains to the impermeable barrier was varied from zero to infinity in the analysis.

POTENTIAL THEORY

Potential theory assumptions have led to the description of ground water flow by non-linear partial differential equations for transient conditions and by Laplace's equation for steady state conditions. Because of the difficulty in solving the non-linear equations, transient flow problems have been solved by either numerical techniques or by linearizing the governing equation.

An analysis was given by Kirkham [1949] for calculating the quantity of flow into parallel drains overlying an impermeable layer with ponded water at the soil surface. The formulas which were presented for the calculation of flow were functions of drain depth and spacing, depth to the impervious layer, drain diameter, thickness of the ponded water and soil permeability. The effect of these parameters on the quantity of flow was studied extensively by the derived formulas and the results were presented in graphical form for ease of interpretation.

Hammad [1962] investigated the seepage flow to parallel tile drains and a spacing formula based on the transient water table was obtained. Laplace's equation for two dimensional seepage flow was solved and relations between discharge, head and spacings of tiles were derived by complex function theory and conformal mapping; however, boundary conditions at the water table were approximated. The effect of evaporation from the soil surface on the drain spacing formula was introduced and unit discharge formulas for thin and thick pervious strata were derived.

An additional study by Hammad [1964] was made of the important quantities affecting the design of a tile drainage system in an arid region. Equations were presented which determine the following parameters: distribution of moisture in the capillary zone, drainage and irrigation requirements, allowable water table fluctuations, and the depth and spacing of drains. All formulas were based upon the physical properties of the soil, the salt tolerance of plants and the salinity of the irrigation water.

Kirkham [1964] made use of a theoretical model which inserted rigid thin impervious membranes along natural streamlines and replaced soil by gravel in the region above the level of the drain tube centers. This model justified the integration of the water table with respect to time in the subsequent theory which was developed. The presence of the membranes caused the water to move along unnatural streamlines causing theoretical water table levels which were too high; however, the author was able to conclude that his results represented the theoretical upper limit for the height of the water table. The equations were valid for recharge conditions and also accounted for the effects of convergence around the drains.

An approximate method of predicting water table positions, as a result of variable rainfalls, was obtained by Dagan [1964]. The drains were assumed to be on the water table and the soil was homogeneous and isotropic. Laplace's equation of flow with appropriate boundary conditions was solved for the potential. However, it was necessary to linearize the differential boundary condition describing the water table and introduce an approximation into the analysis.

Dagan [1965] used a method of analysis similar to the one presented in a previous article (Dagan, [1964]) to investigate the drainage of deep anisotropic soil layers. The solution of a drainage problem in anisotropic soil was transformed into one of isotropic soil by proper scale changing. The potential distribution and movement of the free surface caused by recharge was determined by conformal mapping.

NUMERICAL TECHNIQUES

A numerical method called the "relaxation method" was used by Luthin and Gaskell [1950] to solve Laplace's equation for drainage in both a uniform and a stratified soil overlying an impervious barrier. This method allowed the solution in layered soils with irregular boundaries. The quantity of water flowing into the drains compared favourably with the answers obtained by Kirkham [1949]. In both problems presented, water was ponded on the surface of the soil; therefore, steady state conditions prevailed.

Kirkham and Gaskell [1951] determined the positions of a transient water table by numerical techniques and compared the results to those obtained by Childs [1946] with an electric analog. An equation of the falling water table as a function of hydraulic head, slope, soil permeability and drainable porosity was derived by considering the geometry of two successive positions of the water table. Initially, Laplace's equation was solved with the appropriate boundary conditions; thereupon, the equation of the water table was applied and the new upper boundary of the free water zone was calculated. These steps were repeated and the water table position calculated from a series of steady states.

A digital computer was used by Isherwood [1959] to find the mid-point water table recession rate in a homogeneous, tile-drained soil overlying an impermeable barrier. The effect of tile-depth, spacing, depth to barrier, hydraulic conductivity and drainable pore space was studied. In the region above the barrier and below the water table, fluid flow was assumed to obey Laplace's equation. An analysis similar to that presented by Kirkham and Gaskell [1951] was used to obtain successive positions of the water table.

Taylor and Luthin [1963] used a numerical solution proposed by Luthin and Gaskell [1951] to solve Laplace's equation for ponded flow in a stratified soil. However, computation time was substantially shortened by the use of an electronic computer. The effect of residual and mesh size on the accuracy of answers was illustrated by a comparison of answers with the theoretical solutions found by Kirkham [1949]. Overrelaxation constants were shown to reduce computer running time and therefore reduce the cost of numerical analysis techniques with respect to drainage problems.

An iteration procedure which solved the problem of flow through unsaturated regions of soil as well as below the water table was derived by Sewell and van Schilfgaarde [1963]. The potential at each grid point was solved by numerical solution of Laplace's equation with hydraulic conductivity coefficients present. In the unsaturated zones of the drainage cross-section, the hydraulic conductivity was assumed to vary with the soil moisture tension. The potentials at the grid points were used to draw equipotentials and streamlines and these were used to calculate the quantity of flow from the cross-section into the drains.

EMPIRICAL ANALYSES

In contrast to many theoretical analyses which are based on questionable assumptions and result in formulas which are too complex to be used by design engineers, Luthin [1959] developed a drain spacing equation which was based on empirical assumptions. The author substantiates his assumptions with experimental data in a publication by Luthin and Worstell [1959].

Bouwer and van Schilfgaarde [1963] presented a simplified procedure for predicting the rate of fall of the water table at the midpoint in tile and ditch drained land. The derived equation, relating mid-point water table height and drainage rate was based on the transient assumption that the water table falls without a change of shape and a steady state assumption that the flux per unit area of the water table is uniform and independent of time. Hooghoudt's steady state formula relating drainage rate to hydraulic conductivity, drain spacing, height of the water table above the drains, drain radius, and the "equivalent depth" to the impermeable layer was used to integrate the author's equation and obtain the relation between the same quantities as the Hooghoudt equation for the steady case.

RADIAL FLOW THEORY

A method of analysis based on the analogy between radial flow into a well and the flow into a tile drain was proposed by Walker [1952]. His analysis estimated the most efficient depth and spacing of parallel drains in stratified soils with different porosities and permeabilities from permeability measurements.

EXPERIMENTAL STUDIES

Experimental studies have been performed in the field or by means of porous material tanks, Hele-Shaw models and electric analogues. Data received from these tests have been used to check the validity of theoretical analyses and to develop empirical relationships between such parameters as discharge, head and tile spacing.

POROUS MATERIAL MODELS

One advantage of porous material tanks is the ease of studying the effect of the capillary zone. A sand-filled tank 6X10X2 feet was used to study the distribution of hydraulic head and quantity of tile outflow during drainage by Luthin and Worstell [1957]. Experimental elevations of the water table were compared with those predicted by Childs [1947] and Kirkham and Gaskell [1951]. From the experiment the authors concluded that it was wrong to assume that the soil above the water table was drained to the same moisture content; moreover, the capillary layer was an important part of the area of flow and capillary conductivity varied with soil moisture tension. Since theoretical and experimental water table recession rates did not agree, it was recommended that Laplace's equation was valid below the water table but Richard's equation must be solved above the water table.

Ligion, Johnson and Kirkham [1962] made use of a column of initially saturated porous material to show that drainable porosity was nearly constant for large pore sizes but a function of soil moisture tension and time for smaller pore sizes. Equations were derived for the fluid surface location and rate of discharge when the capillary fringe was taken into account as the upper level of saturation, a constant drainable porosity was assumed and complete drainage occurred.

Kraijenhoff [1962] measured the transient water table level and outflow rates of parallel tile drains using a scaled model filled with screened sand and a spirit-water mixture. Theoretical discharge quantities calculated from Dupuit-Darcy assumptions agreed quite closely with experimental data; however, there was a discrepency in the water table elevations. An investigation of the effect of flow in the unsaturated capillary zone on the saturated zone was performed and it was concluded that the neglect of capillary effects above the phreatic surface could result in a serious error when calculating water table elevations.

The shape and position of a water table falling through a porous material composed of glassbeads was photographed by Grover and Kirkham [1964]. Various sizes of beads were used to simulate both homogeneous and stratified soils between tile drains. Experimental results were presented graphically and formulas derived which predicted water table heights and discharge under field conditions.

HELE-SHAW MODELS

An experimental study of the falling water table and the rate of outflow in an unconfined rectangular aquifer was carried out by Ibrahim and Brutsaert [1965] with the aid of a Hele-Shaw model. Experimental discharge, mid-point water table height and water table shape were compared to theoretical predictions proposed by Glover (Dumm, [1954]), Kirkham and Gaskell [1951], Boussinesq [1904] and Brooks [1961], and Kraijenhoff [1962]. Dimensionless graphs were prepared from the empirical results so that drawdown and discharge from horizontal unconfined aquifers could be predicted.

Shery [1966] investigated the effect of the ratio of average depth of saturated aquifer to the distance between the drains on transient water table elevations with a Hele-Shaw model. The drawdown profiles were compared to theoretical values predicted by Glover (Dumm, [1954]).

ELECTRIC ANALOGS

Electric analogs are useful for studying seepage problems because they are adaptable to a wide variety of flow conditions which cannot be analyzed by mathematics. Childs [1947] employed an electric Analog to study rising and falling water tables in soils with and without capillary fringes and with drains placed above and on an impermeable soil layer. The water table was found to be quite flat except near the drains; in addition, it was realized that the water table would fall at a slower rate if the drains were placed on the impermeable barrier. The effect of the capillary fringe on the movement of the water was reported.

Worstell and Luthin [1959] constructed a resistance network analog of carbon resistors which was used to solve the flow net for a three layered soil which was tile drained. The model was an improvement over previous models in that its cost was relatively low and it was quickly changed from one set of conditions to another.

Successive positions of a falling water table were determined with a modified form of the Kirkham-Gaskell [1951] equation by Brutsaert, Taylor and Luthin [1961]. The capillary fringe replaced the water table as the upper boundary of the region in which flow occurred and drainable porosity was considered variable. Piezometric head in the cross-section was evaluated by a resistance network. In addition, the vertical and horizontal component of the potential gradients at the upper capillary fringe boundary were plotted versus distance from the drain for the successive positions of the falling water table. There was close agreement between the authors' water table elevations and those obtained by Luthin and Worstell [1957]. However, Kirkham and Gaskell's original analysis showed a discrepency in shape and level of the water table because the capillary fringe was neglected.

A resistance network analog with some improved features in accuracy and ease of operation was described by Vimope, Tyra, Thiel and Taylor [1962]. A logarithmic expression, which took into account the curvature of the drain, was used to calculate the resistance around the drain.

FIELD INVESTIGATIONS

Field investigation of water table response between drains was found to be quite expensive; furthermore, uncontrollable parameters, which are not accounted for by theoretical analyses, are introduced into the results. In spite of this, experimental field studies have been performed and the resulting data used to check the validity and usefulness of drain spacing formulas.

A Humic-Gley tile drained soil was studied by Taylor and Goins [1957] and the relationship between measurable physical soil quantities and water removal was illustrated. Goins and Taylor [1959] also reported an experimental field study which evaluated the effect of two depths and two spacings of drains on tile flow rates and water table drawdown. Outflow hydrographs showed a sharp increase of drain outflow shortly after rain began and an exponential decline decline similar for all tile depths and spacing after rainfall ceased. A linear relationship between mid-point water table height and drain flow was reported for spacings of less than 100 feet.

An investigation of discharge and water table elevations for transient conditions in farm drainage systems was presented by Talsma and Haskew [1959] and experimental results were compared with analyses by Hooghoudt [1940], Glover (Dumm, [1954]) and Kirkham [1958]. The predictions of all theories seemed to agree with the experimental data in the regions where their underlying assumptions were valid.

Falling water table prediction equations presented by Luthin and Worstell [1959], van Schilfgaarde [1963], Hooghoudt (van Schilfgaarde, [1963]) and Toksoz and Kirkham (van Schilfgaarde, [1963]) were compared by Johnston, Letey and Pillsbury [1965] with field data collected from drainage systems in the San Joaquin

Valley of California. It was found that van Schilfgaarde's [1963] equation was quite accurate while Luthin and Worstell's [1959] equation underestimated the time for a specified water table drop and the other integrated steady state equations overestimated the time.

Hoffman and Schwab [1964] presented a method of predicting tile spacing which was based on drain outflow. Hydraulic conductivity of the stratified anisotropic soil was determined as a function of drain outflow and tile spacings were calculated by van Schilfgaarde's [1963] proposed equation.

CHAPTER III

SECTION A: DRAINAGE THEORY

POTENTIAL THEORY

The development of mathematical expressions, which quantitatively express the flow of fluids through porous media is based on the following premise:

> The net inward flux into any element of volume in the region of flow must equal the rate at which the fluid is accumulating within that volume.

In the case of unconfined flow, some simplifying assumptions are made. These assumptions may be expressed in the following manner:

1. The aquifer is homogeneous and isotropic.
2. Darcy's Law is valid.
3. All fluid flow takes place below the water table.
4. The drainable porosity is a constant and represents the total fraction of soil volume which is drained as the water table passes through any cross-section. All the water is drained out instantaneously at the moment the water table passes. Therefore, the presence and effect of a capillary zone is ignored.
5. The hydraulic conductivity (k) is considered to be constant.

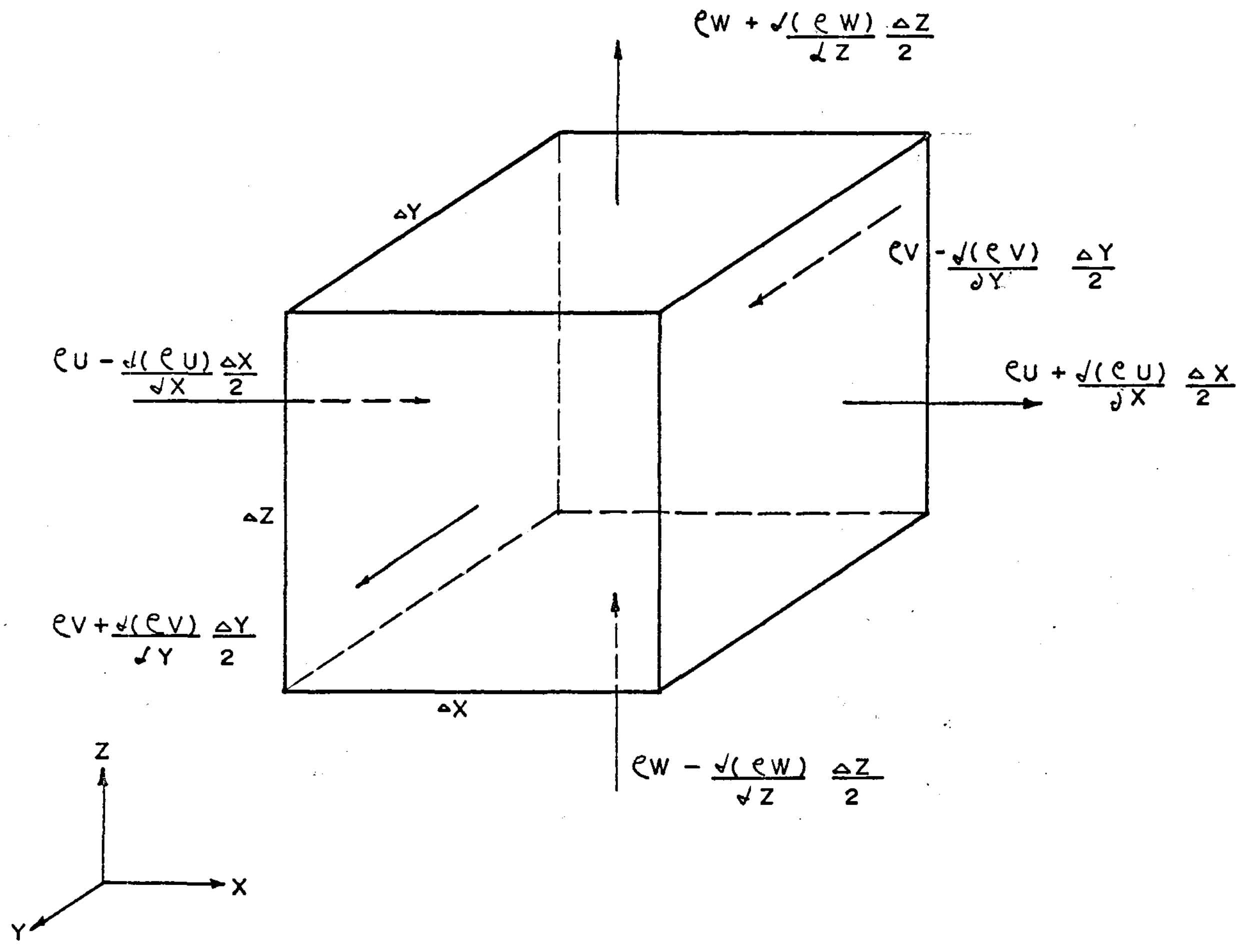

Fig. 1. Elemental Parallelepipedum of Porous Medium Completely Saturated (After De Wiest [1965])

The derivation of the differential equation of flow, which is given below, is essentially the same as presented by De Wiest [1965].

Water is assumed to flow through the faces of an elemental parallelepiped of volume $\Delta x \Delta y \Delta z$. The mass flow rate through an elemental area ΔA is expressed as $\rho V_n \Delta A$ where V_n is the normal component of the velocity to the area ΔA. The density of the fluid is considered to be a function of the space co-ordinates. The fluid velocity in the x, y and z directions is expressed by u, v and w respectively as shown in figure 1.

The contributions of the mass inflow rate are:

In the

x-direction $\left[\rho u - \frac{\partial(\rho u)}{\partial x} \frac{\Delta x}{2}\right] \Delta y \Delta z$

y-direction $\left[\rho v - \frac{\partial(\rho v)}{\partial y} \frac{\Delta y}{2}\right] \Delta x \Delta z$

z-direction $\left[\rho w - \frac{\partial(\rho w)}{\partial z} \frac{\Delta z}{2}\right] \Delta x \Delta y$

Similarly, the contributions of the mass outflow rate are:

In the

x-direction $\left[\rho u + \frac{\partial(\rho u)}{\partial x} \frac{\Delta x}{2}\right] \Delta y \Delta z$

y-direction $\left[\rho v + \frac{\partial(\rho v)}{\partial y} \frac{\Delta y}{2}\right] \Delta x \Delta z$

z-direction $\left[\rho w + \frac{\partial(\rho w)}{\partial z} \frac{\Delta z}{2}\right] \Delta x \Delta y$

The change of mass storage in time is equal to the mass inflow rate minus the mass outflow rate. Thus:

$$\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] \Delta x \Delta y \Delta z \qquad (1)$$

The mass storage may be expressed by the equation:

$$\Delta M = n\rho \Delta x \Delta y \Delta z$$

where n = porosity of the media.

In the above expression n, ρ, and Δz are considered variable; therefore, the rate of change of mass of water is

$$\frac{\partial(\Delta M)}{\partial t} = \left[n\Delta z \frac{\partial \rho}{\partial t} + \rho \Delta z \frac{\partial n}{\partial t} + \rho n \frac{\partial \Delta z}{\partial t}\right] \Delta x \Delta y \qquad (3)$$

If the three terms on the right hand side of equation (3) are expressed in terms of α_a, β and p

where α_a = compressibility of the aquifer

β = compressibility of the fluid

p = pore pressure

the following relation is obtained:

$$\frac{\partial M}{\partial t} = \rho (\alpha_a + n\beta) \frac{\partial p}{\partial t} \Delta x \Delta y \Delta z \qquad (4)$$

When equation (1) and equation (4) are equated, the resulting equation is

$$-\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] = \rho (\alpha_a + n\beta) \frac{\partial p}{\partial t}$$

If u, v and w are expressed in term of Darcy's law and low-angle flow with a small $\frac{\partial \phi}{\partial z}$ is assumed, the differential equation of unsteady unconfined flow results:

$$\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right] = \frac{\rho g (\alpha_a + n\beta)}{k} \frac{\partial \phi}{\partial t} \qquad (5)$$

where k = hydraulic conductivity of the porous media

$\phi = z + p/\gamma$

z = elevation above an arbitrary datum

γ = specific weight of the fluid.

In an unconfined aquifer the volume of water which is removed is due to the lowering of the water table with only a negligible amount due to the compressibility of the aquifer and fluid. Therefore, the compressibility is unimportant compared to the movement of the water table and the right side of equation (5) may be ignored.

The resulting differential equation for unsteady flow in an unconfined aquifer is

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

Since this equation does not contain a time variable, solutions of this equation for unsteady flow require a boundary condition which describes the moving water table as a function of time.

DUPUIT-FORCHHEIMER THEORY

Because of the difficulty in solving Laplace's equation for unsteady unconfined flow, an analysis based on Dupuit's simplifying assumptions has been developed. These conditions as summarized by Jacob [1950] are the following:

1. The aquifer rests on a horizontal impermeable layer.

2. The flow is horizontal at the water table and everywhere below.

3. The velocity of flow along the water table is proportional to the tangent of its angle of inclination instead of the sine.

4. The flow velocity is uniform from top to bottom of the aquifer.

In the derivation a vertical prism of the aquifer of cross section $\Delta x \Delta y$ and of variable height h is considered. The flow in the x direction through the elemental area $h\Delta y$ at x may be expressed as

$$Q\ (x) = -k \frac{\partial h}{\partial x} h\Delta y$$

and similarly in the y direction

$$Q\ (y) = -k \frac{\partial h}{\partial y} h\Delta x$$

The flow rate through the elemental area at $x + \Delta x$ may be expressed as

$$Q\ (x + \Delta x) = -k \frac{\partial h}{\partial x} h\Delta y + \Delta x \frac{\partial}{\partial x} \left(-k \frac{\partial h}{\partial x} h\Delta y \right)$$

The net inward flux through the two faces that are normal to the x direction is

$$\Delta x \Delta y\ k \frac{\partial}{\partial x} \left(h \frac{\partial h}{\partial x} \right) = \Delta x \Delta y\ k \frac{\partial}{\partial x} \left(\frac{1}{2} \frac{\partial h^2}{\partial x} \right) \qquad (6)$$

Similarly the net inward flux for the y direction is

$$k\ \Delta x \Delta y \frac{\partial}{\partial y} \left(\frac{1}{2} \frac{\partial h^2}{\partial y} \right) \qquad (7)$$

By the principle of continuity, the sum of these two terms must equal the rate of increase of storage which is

$$\Delta x \Delta y \quad S \frac{\partial h}{\partial t} \tag{8}$$

where

S = storage coefficient or the amount of water in storage released from a column of aquifer with unit cross section under a unit decline of head.

The net inward flux is found by adding equation (6) and equation (7). If this sum is equated to equation (8), the result is

$$\frac{k}{2}\left[\frac{\partial^2 (h^2)}{\partial x^2} + \frac{\partial^2 (h^2)}{\partial y^2}\right] = S \frac{\partial h}{\partial t}$$

This equation is non-linear and very difficult to solve.

GLOVER'S DRAINAGE FORMULA

The above equation may be simplified so that it becomes identical with those used in the theory of heat conduction in solids. It is assumed that flow through any cross section may be expressed by

$$q = kd \frac{\partial h}{\partial x}$$

where d = average depth of the aquifer both in space and in time.

The resulting equation for unsteady unconfined flow is

$$kd\left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right] = S \frac{\partial h}{\partial t} \tag{9}$$

Glover (Dumm, [1954]) has defined d by the value

$$d = D + \frac{H}{2}$$

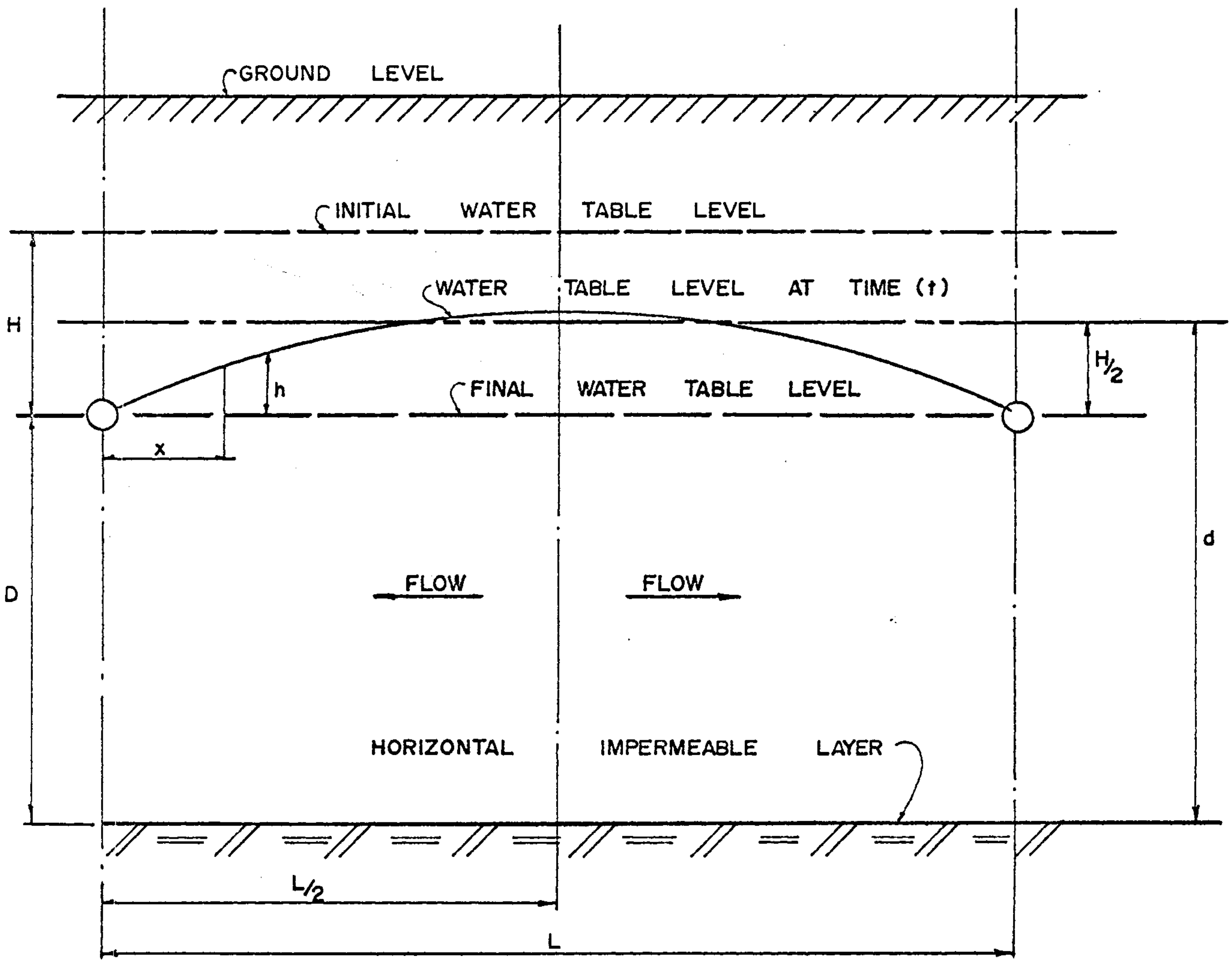

Fig. 2. Unsteady, Unconfined Flow Between Parallel Tile Drains (After Shery [1966])

where H is the original height of the water table above parallel drains D is the vertical distance from the impermeable layer to the drains as shown in figure 2.

Defining $\alpha = \frac{kd}{S}$

equation (9) becomes

$$\alpha \frac{\partial^2 h}{\partial x^2} = \frac{\partial h}{\partial t}$$

The solution of this equation for the boundary conditions of unsteady unconfined flow between two drains;

when $x = 0$	$h = 0$	for $t > 0$
when $x = L$	$h = 0$	for $t > 0$
when $t = 0$	$h = H$	for $0 < x < L$

is

$$h = H \frac{4}{\pi} \sum_{n = 1, 3, 5}^{n = \infty} \frac{e}{n}^{\frac{-\alpha n^2 \pi^2 t}{L^2}} \sin \frac{n\pi x}{L}$$

where L = the horizontal distance between two parallel drains.

BOUNDARIES

It is necessary to idealize the conditions on the boundaries of the system under investigation in order that they can be defined by mathematical expressions.

IMPERMEABLE BOUNDARIES

The component of groundwater velocity which crosses an impermeable boundary is zero; therefore,

$$v_N = - k \frac{\partial \phi}{\partial N} = 0 \qquad \text{or } \frac{\partial \phi}{\partial N} = 0$$

where N = direction of a line drawn normal to the bounding surface.

Hence, in two dimensional flow, an impermeable boundary is a streamline.

LINES OF SYMMETRY

The area of flow to be analyzed may be substantially reduced by recognizing symmetrical flow regions. The axis of symmetry is defined as a boundary which is a streamline and mathematically expressed as

$$\frac{\partial \phi}{\partial N} = 0$$

WATER TABLE EQUATION

If Laplace's equation is to be solved for unsteady unconfined flow, a boundary condition which describes the position of the water table as a function of time must be developed. At the water table, the piezometric head ϕ is always equal to the elevation head z since the pressure p is equal to zero. In addition to the assumptions made for the development of the potential theory, Boulton [1954] has stated these additional conditions in his derivation of the water table equation.

1. The water table is initially horizontal and replenishment of this free surface by rainfall is neglected.
2. A particle of fluid which is once on the water table never leaves it.

Since the pressure on the free-surface boundary is equal to zero, the equation of the surface is

$$\phi \;(x, z, t) - z = 0$$

Since a particle of fluid, which is once on the free surface, never leaves it

$$\frac{D}{Dt}\,(\phi - z) = 0 \qquad (10)$$

where D denotes differentiation following the motion of the particle.

If equation (10) is expanded, the result is

$$\frac{\partial \phi}{\partial x}\,\frac{dx}{dt} + \frac{\partial \phi}{\partial z}\,\frac{dz}{dt} + \frac{\partial \phi}{\partial t} - \frac{dz}{dt} = 0$$

Now $\frac{dx}{dt}$ and $\frac{dz}{dt}$ are the true velocities in the x and z direction respectively.

Thus

$$V_x = \frac{dx}{dt} = -\frac{k}{S}\,\frac{\partial \phi}{\partial x}$$

$$V_z = \frac{dz}{dt} = -\frac{k}{S}\,\frac{\partial \phi}{\partial z}$$

where S = specific yield or porosity minus the field capacity

porosity = volume of voids in a soil volume divided by the total soil volume

field capacity = soil moisture content after the removal of water by gravitation has stopped

Substituting gives

$$k\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2 - \left(\frac{\partial \phi}{\partial z}\right)\right] = S\,\frac{\partial \phi}{\partial t}$$

as the equation of the water table.

SECTION B: NUMERICAL ANALYSIS OF PARTIAL DIFFERENTIAL EQUATIONS

Finite difference methods make it possible to solve many problems involving partial differential equations which are difficult to solve analytically. The region of the independent variables under consideration is replaced by a set of grid points and approximate values for the desired solution at these points are determined. The values at the mesh points are required to satisfy difference equations which have been formed by replacing partial derivatives with partial difference quotients. The Laplace equation is used to describe flow through the isotropic homogeneous porous media in the region under consideration and the corresponding difference equation is developed.

A grid of uniformly spaced straight lines is drawn over the region under consideration so that the boundaries of the region fall along the sides of the mesh. Figure 3 represents part of the region in which flow is taking place with the notation $\phi\,(x)$ indicating the total head as a function of distance. The four points adjacent to (x_0, z_0) are situated in the interior of the flow region.

By Taylor's theorem the variation of piezometric head in the x-direction may be written as

$$\phi_1 = \phi_0 + \Delta x\left(\frac{\partial \phi}{\partial x}\right)_0 + \frac{(\Delta x)^2}{2!}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_0 + \frac{(\Delta x)^3}{3!}\left(\frac{\partial^3 \phi}{\partial x^3}\right)_0 + \dots \quad (11)$$

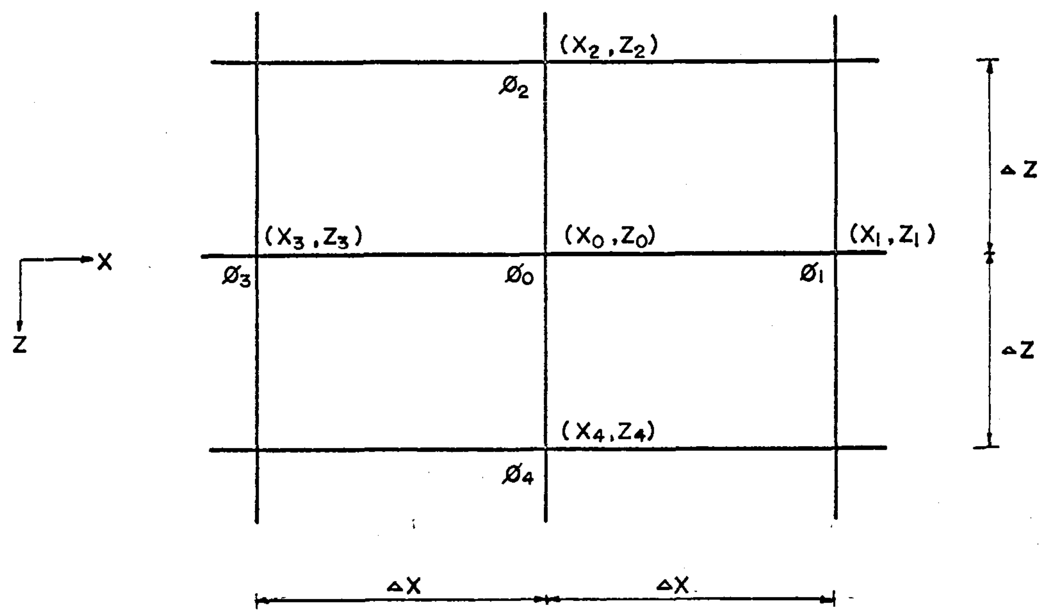

Fig. 3. Grid Spacing for Interior Flow Region

$$\phi_3 = \phi_0 - \Delta x\left(\frac{\partial \phi}{\partial x}\right)_0 + \frac{(\Delta x)^2}{2!}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_0 - \frac{(\Delta x)^3}{3!}\left(\frac{\partial^3 \phi}{\partial x^3}\right)_0 + \ldots \quad (12)$$

where the subscripts 0, 1, 3 indicate the function or derivatives at the points 0, 1, 3.

Adding equation (11) and (12) and ignoring terms containing derivatives higher than the second power, the second derivative of the piezometric head with respect to distance in the x-direction is given approximately by

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\phi_1 + \phi_3 - 2\,\phi_0}{(\Delta x)^2}$$

Similarly, the second derivative in the s direction may be expressed as

$$\frac{\partial^2 \phi}{\partial s^2} = \frac{\phi_2 + \phi_4 - 2\phi_0}{(\Delta s)^2}$$

Thus the finite difference equation representing Laplace's equation of flow may be written in the following manner:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial s^2} = \frac{1}{(\Delta x)^2}(\phi_1 + \phi_3 - 2\phi_0) + \frac{1}{(\Delta s)^2}(\phi_2 + \phi_4 - 2\phi_0) = 0 \quad (13)$$

Manipulation of equation (13) allows the value of ϕ_0 to be expressed as a function of the values at the four surrounding grid points.

$$\phi_0 = \frac{(\Delta s)^2}{2((\Delta x)^2 + (\Delta s)^2)}\left[\phi_1 + \phi_3\right] + \frac{(\Delta x)^2}{2((\Delta x)^2 + (\Delta s)^2)}\left[\phi_2 + \phi_4\right] \quad (14)$$

If a square mesh is used with $\Delta x = \Delta s$, equation (14) may be written as

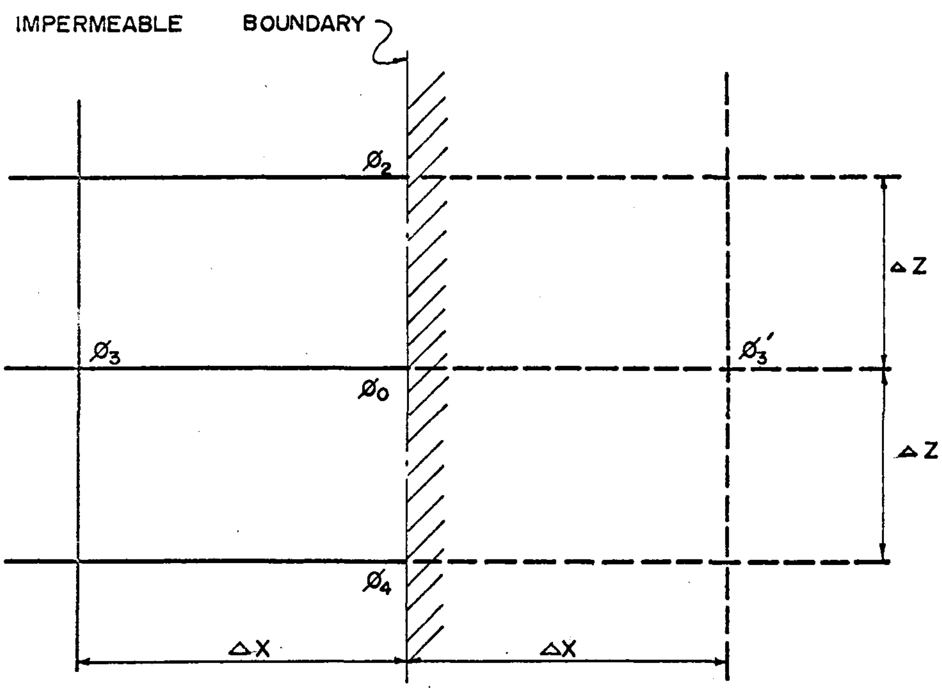

Fig. 4. Fictitious Grid Spacing at an Impermeable Boundary

$$\phi_o = \frac{1}{4}\left[\phi_1 + \phi_2 + \phi_3 + \phi_4\right] \qquad (15)$$

or

$$\phi_1 + \phi_2 + \phi_3 + \phi_4 - 4\phi_o = 0 \qquad (16)$$

Equation (15) may be used to calculate a better approximation of piezometric head at grid points until a final limiting head is obtained at each grid point.

NUMERICAL REPRESENTATION OF BOUNDARIES

LINES OF SYMMETRY AND IMPERMEABLE BOUNDARIES

Head distribution along boundaries which are impermeable or lines of symmetry may be calculated numerically by setting up a fictitious grid point ϕ_3' as shown in figure 4.

Since the potential gradient normal to the boundary is zero, the numerical value of ϕ_3' is assumed to equal ϕ_3 at all times. The head at ϕ_o, if $\Delta x = \Delta z$, is expressed as

$$\phi_o = \frac{1}{4}(\phi_3' + \phi_2 + \phi_3 + \phi_4)$$

or

$$\phi_o = \frac{1}{4}(\phi_2 + \phi_4 + 2\phi_3)$$

WATER TABLE

Grid points along the water table are numerically represented by the elevation of the water table above an arbitrarily chosen datum since the piezometric head equals the elevation head. The change in elevation of the water table with respect to time is predicted by applying the equation of the water table in the finite form;

$$\Delta \phi = \frac{k}{S} \Delta t \left[\left(\frac{\Delta \phi}{\Delta x} \right)^2 - \left(\frac{\Delta \phi}{\Delta z} \right) + \left(\frac{\Delta \phi}{\Delta z} \right)^2 \right]$$

in which $\frac{\Delta \phi}{\Delta x}$ and $\frac{\Delta \phi}{\Delta z}$ are the horizontal and vertical derivatives of the piezometric head at a grid point on the water table.

DRAINS

The piezometric head at the grid point representing the drain is also equal to its elevation above a datum since water in the drain is assumed to be flowing without any back pressure.

CURVED SURFACES

If the boundary represented by the water table cuts across grid lines, as shown in figure 5, instead of falling along the sides of the mesh, the finite difference equation, which approximates Laplace's equation, must be developed with coefficients that adequately describe the effect of the partial mesh lengths: R and H.

By Taylor's theorem, the variation of head in the x-direction, if terms of third order and higher are neglected, is given as

$$\phi_1 = \phi_0 + \Delta x\left(\frac{\partial \phi}{\partial x}\right)_0 + \frac{(\Delta x)^2}{2!}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_0 + \cdots \quad (17)$$

$$\phi_3 = \phi_0 - H\left(\frac{\partial \phi}{\partial x}\right)_0 + \frac{H^2}{2!}\left(\frac{\partial^2 \phi}{\partial x^2}\right)_0 + \cdots \quad (18)$$

Similarly, the variation of head in the z-direction may be written as:

$$\phi_2 = \phi_0 + R\left(\frac{\partial \phi}{\partial z}\right)_0 + \frac{R^2}{2!}\left(\frac{\partial^2 \phi}{\partial z^2}\right)_0 + \cdots \quad (19)$$

$$\phi_4 = \phi_0 - \Delta z\left(\frac{\partial \phi}{\partial z}\right)_0 + \frac{(\Delta z)^2}{2!}\left(\frac{\partial^2 \phi}{\partial z^2}\right)_0 + \cdots \quad (20)$$

Adding equation (17) and (18), after multiplying the respective equations by factors which will remove the $\left(\frac{\partial \phi}{\partial x}\right)$ term, gives the following expression for $\left(\frac{\partial^2 \phi}{\partial x^2}\right)$

$$\frac{\partial^2 \phi}{\partial x^2} = \phi_1\left[\frac{2}{\Delta x(\Delta x + H)}\right] + \phi_3\left[\frac{2}{H(\Delta x + H)}\right] - \phi_0\left[\frac{2}{H\Delta x}\right]$$

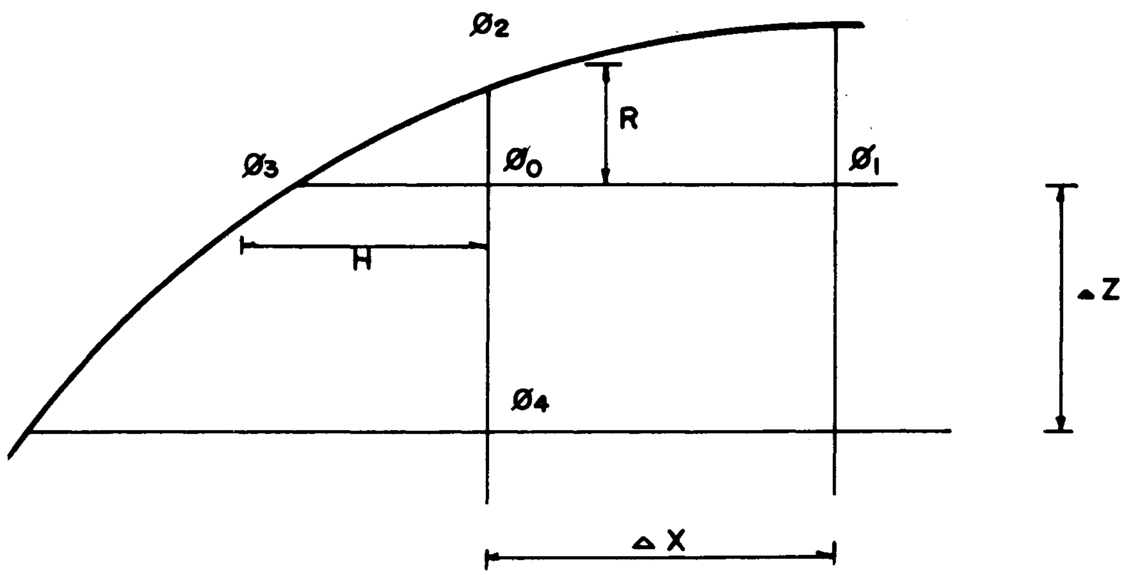

Fig. 5. Grid Spacing for Curved Boundary

Similarly the second derivative in the z-direction is

$$\frac{\partial^2 \phi}{\partial z^2} = \phi_2 \left[\frac{2}{R(R+\Delta z)}\right] + \phi_4 \left[\frac{2}{\Delta z \, (R+\Delta z)}\right] - \phi_o \left[\frac{2}{\Delta z R}\right]$$

The finite difference equation representing Laplace's equation becomes:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = \phi_1 \left[\frac{1}{\Delta x(\Delta x+H)}\right] + \phi_2 \left[\frac{1}{R(R+\Delta z)}\right] + \phi_3 \left[\frac{1}{H(\Delta x+H)}\right] + \phi_4 \left[\frac{1}{\Delta z(R+\Delta z)}\right] - \phi_o \left[\frac{R\Delta z+H\Delta x}{H\Delta x R\Delta z}\right] = 0 \qquad (21)$$

Manipulation of equation (21) allows the value of ϕ_o to be expressed as a function of the values at the four surrounding grid points

$$\phi_o' = \phi_1 \left[\frac{HR\Delta x}{(R\Delta z + H\Delta x)(\Delta x+H)}\right] + \phi_2 \left[\frac{H\Delta x\Delta z}{(R+\Delta z)(R\Delta z+H\Delta x)}\right] + \phi_3 \left[\frac{\Delta x R\Delta z}{(\Delta x+H)(R\Delta z+H\Delta x)}\right] + \phi_4 \left[\frac{HR\Delta x}{(R+\Delta z)(R\Delta z+H\Delta x)}\right]$$

RELAXATION METHOD

Equation (16) is only valid when ϕ_o and the surrounding grid points have the correct numerical value. For intermediate calculations equation (16) is written as

$$\phi_1 + \phi_2 + \phi_3 + \phi_4 - 4\phi_o = R_o \qquad (17)$$

where R_o = residual at point o which represents the numerical error at point o.

If the value of ϕ_o is altered by an amount $\Delta\phi_o$ the residual will be altered.

$$R_o + \Delta R_o = \phi_1 + \phi_2 + \phi_3 + \phi_4 - 4(\phi_o + \Delta\phi_o) \qquad (18)$$

By subtracting (17) and (18)

$$\Delta R_o = -4\Delta\phi_o$$

If the residual is made equal to zero or $R_o = -R_o$ the value of the function at o is changed by

$$\Delta\phi_o = \tfrac{1}{4}R_o$$

The new approximation of ϕ_o is obtained by the formula

$$\phi_{o(n+1)} = \phi_{o(n)} + \tfrac{1}{4}\left(\phi_1 + \phi_2 + \phi_3 + \phi_4 - 4\phi_{o(n)}\right) \qquad (22)$$

when n = value of ϕ_o after n iterations.

OVERRELAXATION FACTOR

Equation (22) may be written as;

$$\phi_{o(n+1)} = \phi_{o(n)} + w\left[\beta_1\phi_{1(n)} + \beta_2\phi_{2(n+1)} + \beta_3\phi_{3(n)} + \beta_4\phi_{4(n)} - \phi_{o(n)}\right] \qquad (23)$$

where β_1, β_2, β_3, and β_4 are coefficients which describe the effect of the grid spacing,

and w = overrelaxation constant

If w = 1 equation (23) reduces to equation (22) which is known as the Gauss-Seidel method of iteration. An indication of how rapidly the errors in numerical values are decreasing in a grid, when equation (22) is used is given by;

$$\frac{d_{(n)}}{d_{(n-1)}} = \frac{\left|\phi_{(n)} - \phi_{(n-1)}\right|}{\left|\phi_{(n-1)} - \phi_{(n-2)}\right|}$$

An approximate formula for the limiting value of $\frac{d_{(n)}}{d_{(n-1)}}$ in a rectulangular region is

$$\lambda = \frac{1}{4}\left[\cos\frac{\pi}{(N-1)} + \cos\frac{\pi}{(M-1)}\right]^2$$

where N = number of horizontal mesh lines

M = number of vertical mesh lines.

The best estimation of the overrelaxation factor as suggested by Todd [1962] is;

$$w_b = 1 + \frac{\lambda}{(1 + \sqrt{1-\lambda})^2}$$

where λ = limiting value of $\frac{d_{(n)}}{d_{(n-1)}}$ for the Gauss-Seidel method.

CHAPTER IV

MATHEMATICAL PROCEDURE

METHOD OF ANALYSIS

The shape and position of the water table is calculated by successive solutions of the finite difference equation defined by the flow equation;

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

with the boundary and initial conditions;

when $z = 0$	$\frac{\partial \phi}{\partial z} = 0$	for $t > 0$
when $x = \frac{L}{2}$	$\frac{\partial \phi}{\partial x} = 0$	for $t > 0$
when $x = 0$, $z = D$	$\phi = D$	for $t > 0$
when $x = 0$, $0 \leq z \leq D + H$, $z \neq D$	$\frac{\partial \phi}{\partial x} = 0$	for $t > 0$
when $z = D + H$	$\phi = D + H$	for $t = 0$

and subsequent application of the equation;

$$\frac{\partial \phi}{\partial t} = \frac{k}{S}\left[\left(\frac{\partial \phi}{\partial x}\right)^2 - \left(\frac{\partial \phi}{\partial z}\right) + \left(\frac{\partial \phi}{\partial z}\right)^2\right]$$

to the water table.

COMPUTER PROGRAMME

The problem, as stated above, was solved with the use of a digital computer. The programme was initially written in Fortran II language for an IBM 1620-40^{k} computer but was changed to Fortran IV and executed on an IBM 7094. This was necessary because of the excessive running time required on the IBM 1620. The length of running time required to obtain answers of pre-determined accuracy was considerably shortened by the use of an over-relaxation factor.

The programme is designed for the analysis of unconfined drainage problems in a closed system between parallel tile drains flowing with the absence of back pressure. Although initial water table profiles were always taken as horizontal, it is possible to calculate water table curves when the initial free surface is curved.

The necessary input data are:

(1) the initial height of the water table, position of drain and dimensions of flow profile under consideration

(2) the hydraulic conductivity and specific storage of the soil

(3) the horizontal and vertical mesh spacing and the number of vertical and horizontal mesh lines used to cover the flow profile

(4) the over-relaxation constant

(5) the time steps.

The organization of the programme is given in the flow chart shown in figure 6. A complete listing of the programme can be found in Appendix A. Notation used in the programme is presented in Appendix B.

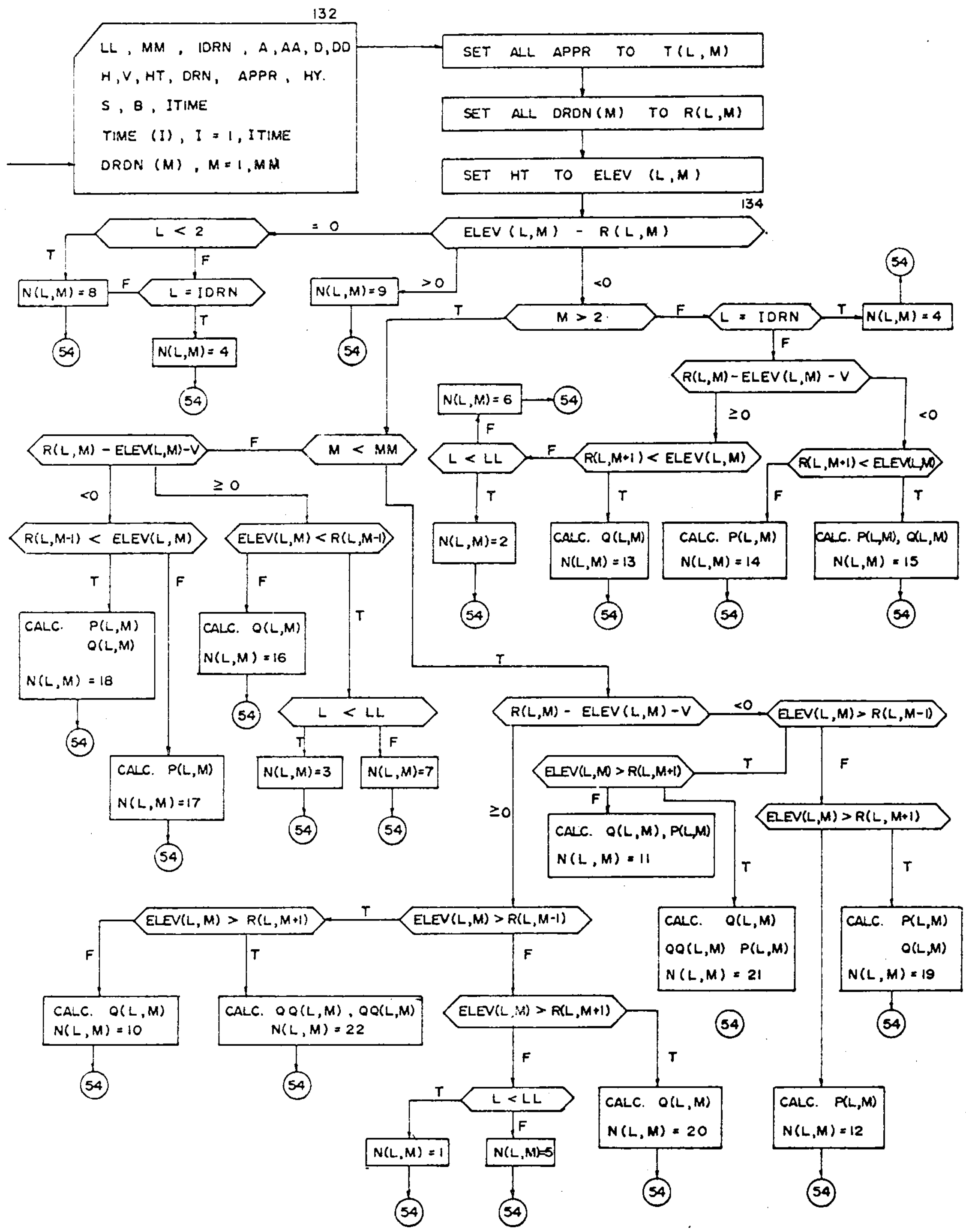

Fig. 6. Simplified Flow Chart

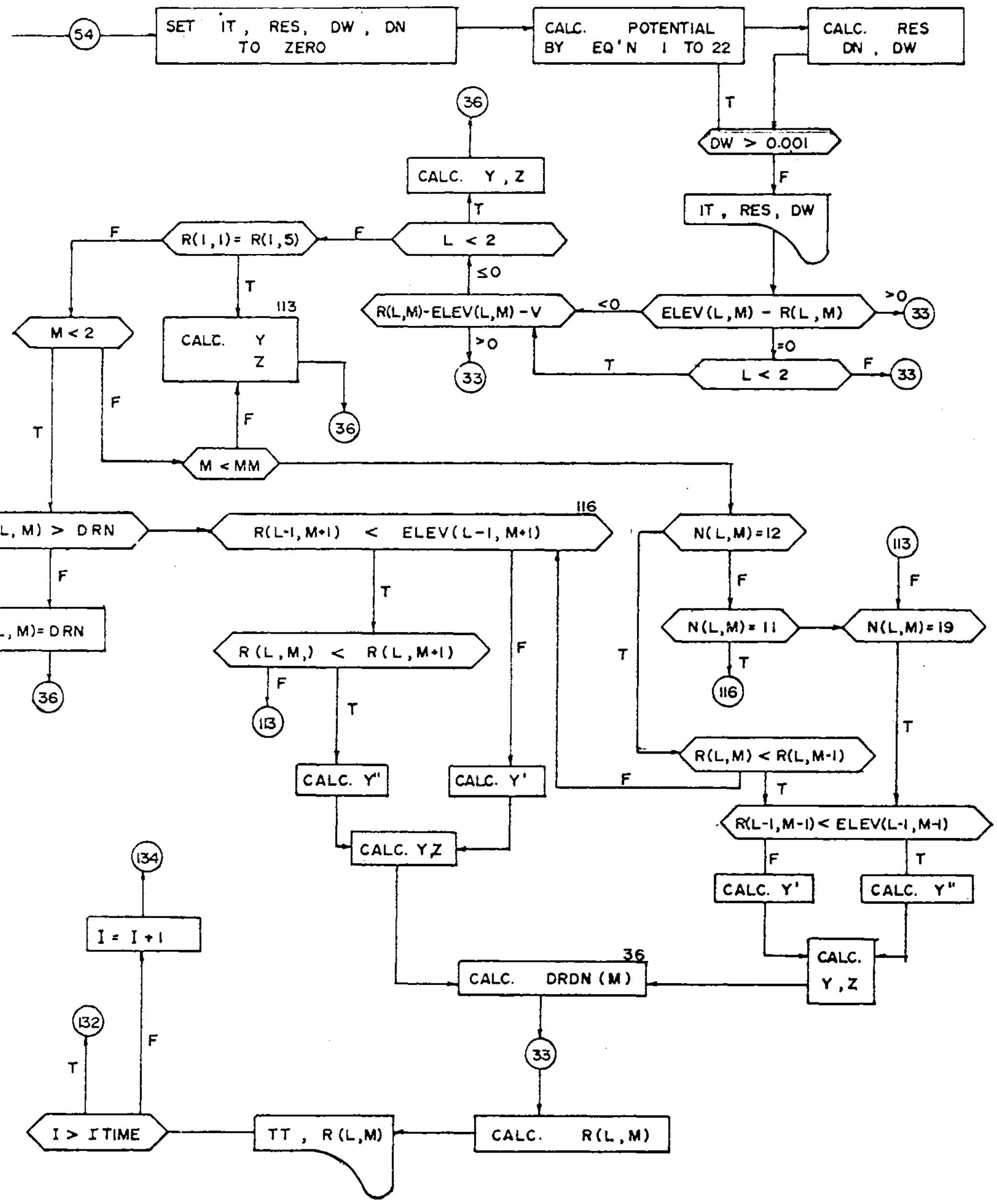
54
SET IT, RES, DW, DN TO ZERO
CALC. POTENTIAL BY EQ'N 1 TO 22
CALC. RES DN, DW
DW > 0.001
IT, RES, DW
36
CALC. Y, Z
L < 2
R(1,1)= R(1,5)
M < 2
113
CALC. Y Z
R(L,M)-ELEV(L,M) -V
ELEV(L,M) - R(L,M)
33
L < 2
M < MM
116
R(L, M) > DRN
R(L-1, M+1) < ELEV(L-1, M+1)
N(L,M)=12
N(L,M)= 11
N(L,M)= 19
R(L,M)= DRN
R(L,M,) < R(L,M+1)
R(L,M) < R(L,M-1)
CALC. Y"
CALC. Y'
CALC. Y,Z
R(L-1,M-1)< ELEV(L-1,M-1)
134
I = I + 1
CALC. DRDN (M)
132
I > ITIME
TT, R(L,M)
CALC. R(L,M)

Fig. 6. (cont'd)

CHAPTER V

RESULTS AND DISCUSSION

PRESENTATION OF DATA

A comparison of all corresponding free water surface levels as obtained experimentally by B. Shery [1966] and theoretically by numerical analysis is given in Table I. The surface level readings are tabulated for values of $\frac{d}{H}$ from 0.035 to 0.600 at various time intervals when $\frac{H}{L} = 0.25$. Both the theoretical and experimental profiles are plotted in Figures 7 to 21 with surface level readings in terms of $\frac{h}{H}$, and horizontal distances in terms of $\frac{x}{L}$ for various values of t.

The value of S will be 1.0 for all numerical calculations since the specific yield of the Hele-Shaw model used by Shery [1966] will always equal 1.0. In the "Presentation of Data" by Shery [1966], the values of $\sqrt{\frac{4\alpha t}{L^2}}$ where $\alpha = \frac{kd}{S}$ are given for the various times at which surface profile readings were made. Since the numerical values of d, S, t and L are known, the corresponding hydraulic conductivity, to be used in the numerical analysis solution, may be calculated.

Corresponding surface profiles predicted by Glover (Dumm, [1954]) and the numerical solution of the flow equation are tabulated in Table II for various values

TABLE I

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		11		45		101	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	3.80		0		0		0	
10	--	3.80	1.3949	2.60	0.8194	1.70	0.6256	1.30
20	--	--	2.2520	3.10	1.3228	2.10	0.9957	1.50
40	--	--	3.2452	3.60	2.2006	2.75	1.6534	2.10
70	--	--	3.7029	3.80	3.0820	3.35	2.4719	2.85
100	--	--	3.7865		3.5178	3.70	3.0634	3.15
130	--	--	3.7857		3.7026	3.80	3.4377	3.50
160	--	--	3.7868		3.7636		3.6158	3.60
190	--	--	3.7870		3.7800		3.7052	3.70
220	--	--	3.7871		3.7830		3.7307	3.80

$d = 15.3$ cm $\quad L = 440$ cm $\quad \frac{d}{L} = 0.035$

$K = 2.848$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	180		281		404			
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0			
10	0.5021	1.05	0.4073	0.85	0.3250	0.70		
20	0.7931	1.20	0.6427	1.00	0.5123	0.80		
40	1.2952	1.70	1.0784	1.40	0.8623	1.10		
70	2.0490	2.30	1.6904	1.95	1.3142	1.60		
100	2.6417	2.70	2.2118	2.30	1.7092	1.90		
130	3.1258	3.10	2.6045	2.55	1.9830	2.10		
160	3.4241	3.30	2.7964	2.85	2.1687	2.35		
190	3.5192	3.45	2.8599	3.05	2.2736	2.50		
220	3.5515	3.55	2.9305	3.10	2.3086	2.60		

$d = 15.3$ cm $\quad L = 440$ cm $\quad \frac{d}{L} = 0.035$

$K = 2.848$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		9		33		75	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	3.80		0		0		0	
10	--	3.80	1.5174	2.70	0.9254	1.90	0.7192	1.40
20	--	--	2.4279	3.20	1.4913	2.30	1.1167	1.70
40	--	--	3.3702	3.70	2.4214	2.95	1.8580	2.30
70	--	--	3.7322	3.80	3.4657	3.55	2.7627	3.05
100	--	--	3.7807	--	3.6481	3.70	3.3810	3.40
130	--	--	3.7856	--	3.7508	3.80	3.5992	3.60
160	--	--	3.7862	--	3.7825	--	3.7089	3.70
190	--	--	3.7863	--	3.7837	--	3.7386	3.75

$d = 15.3$ cm $L = 380$ cm $\frac{d}{L} = 0.043$

$K = 2.836$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	133		208		300		407	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	0.5804	1.20	0.4927	0.90	0.4447	0.70	0.1851	0.65
20	0.9169	1.35	0.7674	1.10	0.6964	0.90	0.2819	0.70
40	1.5213	1.85	1.2488	1.55	1.1395	1.30	0.4211	1.05
70	2.3013	2.55	1.9976	2.10	1.7327	1.75	0.4895	1.40
100	2.9003	2.95	2.5452	2.50	2.0844	2.10	0.3766	1.65
130	3.2962	3.25	2.9634	2.80	0	2.30	0	1.85
160	3.5072	3.45	3.2434	3.00	2.6816	2.50	0.4809	1.95
190	3.5620	3.50	3.3535	3.10	2.9190	2.55	0.7100	2.05

$d = 15.3$ cm $\quad L = 380$ cm $\quad \frac{d}{L} = 0.043$

$K = 2.836$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		7		30		67	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	6.00		0		0		0	
10	--	6.00	2.5302	4.50	1.4836	2.80	1.1084	2.10
20	--	--	4.0106	5.20	2.2963	3.40	1.7078	2.70
40	--	--	5.3347	5.70	3.5407	4.50	2.6446	3.50
70	--	--	5.8478	5.90	4.8264	5.40	3.8087	4.50
100	--	--	5.9610	6.00	5.5052	5.70	4.6767	5.10
130	--	--	5.9857	--	5.8083	5.90	5.2542	5.50
160	--	--	5.9909	--	5.9273	5.95	5.5978	5.75
190	--	--	5.9921	--	5.9688	6.00	5.7713	5.90
220	--	--	5.9924	--	5.9786	--	5.8232	5.95

$d = 24$ cm $L = 440$ cm $\frac{d}{L} = 0.055$

$K = 2.682$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	120		188		271		368	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	0.8814	1.70	0.7625	1.45	0.6168	1.20	0.4006	1.00
20	1.3059	2.10	1.1231	1.70	0.9063	1.45	0.5506	1.20
40	2.0143	2.85	1.7237	2.30	1.3779	1.90	0.6879	1.50
70	3.0151	3.70	2.5455	3.10	2.0109	2.55	1.4351	2.10
100	3.9985	4.30	3.2707	3.70	2.3979	3.05	1.9144	2.50
130	4.5288	4.90	3.8642	4.15	2.8950	3.50	2.2813	2.85
160	5.0080	5.20	4.3012	4.50	3.4183	3.80	2.5530	3.10
190	5.2339	5.20	4.5700	4.75	3.6812	4.00	2.7185	3.30
220	5.3192	5.55	4.6600	4.90	3.7626	4.10	2.7746	3.40

$d = 24$ cm $L = 440$ cm $\frac{d}{L} = 0.055$

$K = 2.682$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	481		609					
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0					
10	0.2837	0.80	0.2544	0.70				
20	0.4147	0.90	0.3696	0.75				
40	0.6374	1.25	0.5554	1.00				
70	0.9738	1.70	0.8086	1.30				
100	1.3147	1.90	1.0744	1.50				
130	1.6360	2.25	1.2133	1.75				
160	1.8988	2.45	1.3443	1.90				
190	2.0612	2.60	1.4247	2.05				
220	2.0924	2.70	1.4512	2.10				

$d = 24$ cm $L = 440$ cm $\frac{d}{L} = 0.055$

$K = 2.682$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		5		22		49	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	6.00		0		0		0	
10	--	6.00	2.8937	4.80	1.6785	3.10	1.2526	2.35
20	--	--	4.4530	5.40	2.6101	3.70	1.9324	2.90
40	--	--	5.55	5.85	3.9588	4.80	3.0080	3.85
70	--	--	5.9049	6.00	5.1701	5.60	4.2266	4.85
100	--	--	5.9737	--	5.7000	5.85	5.0505	5.30
130	--	--	5.9873	--	5.8960	5.90	5.5334	5.70
160	--	--	5.9899	--	5.9598	6.00	5.7653	5.85
190	--	--	5.9904	--	5.9741	--	5.8318	5.90

$d = 24$ cm $L = 380$ cm $\frac{d}{L} = 0.063$

$K = 2.745$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	88		137		197		268	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	1.0242	1.90	0.8959	1.60	0.7597	1.30	0.6499	1.10
20	1.5629	2.30	1.3239	1.90	1.1174	1.65	0.9558	1.35
40	2.4159	3.10	2.0286	2.60	1.7065	2.15	1.4573	1.75
70	3.5161	4.10	2.9665	3.50	2.5125	2.90	2.1475	2.35
100	4.3800	4.70	3.7679	4.00	3.2448	3.35	2.7404	2.75
130	4.9762	5.10	4.3395	4.50	3.7622	3.80	3.2137	3.10
160	5.3061	5.40	4.6915	4.75	4.0727	4.05	3.5186	3.35
190	5.4132	5.55	4.8103	4.90	4.1521	4.10	3.6196	3.40

$d = 24$ cm $L = 380$ cm $\frac{d}{L} = 0.063$

$K = 2.745$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	350							
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0							
10	0.5548	0.90						
20	0.8133	1.10						
40	1.2342	1.50						
70	1.8129	1.90						
100	2.3095	2.20						
130	2.7012	2.45						
160	3.2887	2.70						
190	3.0236	2.75						

$d = 24$ cm $L = 380$ cm $\frac{d}{L} = 0.063$

$K = 2.745$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		4		16		35	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	6.00		0		0		0	
10	--	6.00	3.1721	5.10	1.8770	3.50	1.4368	2.65
20	--	--	4.7606	5.55	2.9424	4.20	2.2173	3.25
40	--	--	5.6731	5.90	4.4137	5.10	3.4109	4.15
60	--	--	5.8878	6.00	5.2280	5.70	4.3257	4.95
80	--	--	5.8561	--	5.6405	5.90	4.9666	5.35
100	--	--	5.9784	--	5.8350	5.95	5.4021	5.55
130	--	--	5.9871	--	5.9441	6.00	5.7308	5.85
160	--	--	5.9885	--	5.9671	--	5.8213	5.90

d = 24 cm L = 320 cm $\frac{d}{L} = 0.075$

K = 2.701 cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	63		98		141		191	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	1.1347	2.15	0.9900	1.80	0.8611	1.50	0.7245	1.25
20	1.7469	2.70	1.4698	2.20	1.2694	1.85	1.0625	1.50
40	2.7027	3.50	2.2589	2.90	1.9339	2.45	1.6138	2.00
60	3.5501	4.25	2.9599	3.60	2.[illegible]266	3.05	2.1154	2.50
80	4.2571	4.75	3.5775	4.05	3.0551	3.45	2.5696	2.85
100	4.7830	5.05	4.0703	4.35	3.4787	3.75	2.9483	3.05
130	5.2590	5.40	4.5588	4.75	3.9122	4.00	3.3209	3.35
160	5.4086	5.50	4.7403	4.85	4.0680	4.15	3.4471	3.50

$d = 24$ cm $L = 320$ cm $\frac{d}{L} = 0.075$

$K = 2.701$ cm/sec.

TABLE 1 (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	250		316		390			
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0			
10	0.6311	1.05	0.5311	0.85	0.4696	0.70		
20	0.9252	1.20	0.7[illegible]5	1.00	0.6850	0.80		
40	1.3963	1.65	1.1792	1.30	0.9056	1.00		
60	1.8087	2.05	1.5483	1.65	1.3370	1.30		
80	2.1781	2.30	1.8763	1.80	1.6091	1.40		
100	2.5014	2.45	2.1396	1.90	1.8352	1.50		
130	2.8199	2.70	2.3973	2.10	2.0671	1.60		
160	2.9312	2.75	2.4870	2.20	2.1112	1.70		

d = 24 cm L = 320 cm $\frac{d}{L} = 0.075$

K = 2.701 cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		10		23		41	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	6.00		0		0		0	
10	--	6.00	2.3930	3.95	1.8291	2.95	1.4874	2.50
20	--	--	3.6620	4.70	2.7033	3.70	2.1624	3.00
40	--	--	5.0524	5.50	3.9645	4.70	3.1906	3.95
60	--	--	5.6114	5.90	4.8270	5.35	4.0167	4.75
80	--	--	5.8387	5.95	5.3521	5.65	4.6307	5.10
100	--	--	5.9300	6.00	5.6394	5.80	5.0237	5.35
120	--	--	5.9617	--	5.7612	5.85	5.2200	5.50
130	--	--	5.9651	--	5.7753	5.90	5.2450	5.50

$d = 24$ cm $L = 260$ cm $\frac{d}{L} = 0.092$

$K = 2.789$ cm/sec.

TABLE 1 (cont'd)

Explicit and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	64		92		125		163	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	1.2996	2.10	1.0777	1.75	0.8813	1.50	0.7129	1.25
20	1.8338	2.55	1.5147	2.15	1.2144	1.80	0.9761	1.50
40	2.6579	3.40	2.1877	2.90	1.7457	2.40	1.3812	1.95
60	3.3583	4.10	2.7389	3.50	2.1850	2.95	1.7223	2.40
80	3.9002	4.50	3.1717	3.85	2.5337	3.20	1.9857	2.65
100	4.2645	4.75	3.4929	4.10	2.7776	3.50	2.1892	2.80
120	4.4584	4.90	3.6568	4.20	2.8855	3.50	2.2864	2.85
130	4.4798	4.90	3.6750	4.20	2.8936	3.55	2.2985	2.90

$d = 24$ cm $L = 260$ cm $\frac{d}{L} = 0.092$

$K = 2.789$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	207							
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0							
10	0.5754	1.00						
20	0.7818	1.20						
40	1.1161	1.55						
60	1.3370	1.95						
80	1.5286	2.15						
100	1.6628	2.25						
120	1.7298	2.30						
130	1.7382	2.30						

d = 24 cm   L = 260 cm $\frac{d}{L} = 0.092$

K = 2.789 cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		6		14		24	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	6.00		0		0		0	
10	--	6.00	3.7372	4.70	2.1557	3.60	1.8289	2.90
20	--	--	4.8546	5.30	3.2660	4.40	2.6969	3.60
40	--	--	5.6318	5.85	4.6661	5.30	3.9369	4.65
60	--	--	5.8621	6.00	5.3697	5.75	4.7601	5.30
80	--	--	5.9380	--	5.6799	5.90	5.2151	5.50
100	--	--	5.9512	--	5.7657	5.90	5.3576	5.55

$d = 24$ cm $L = 200$ cm $\frac{d}{L} = 0.120$

$K = 2.769$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	38		55		74		97	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	1
10	1.5295	2.50	1.3522	2.15	1.1442	1.85	0.9277	1.50
20	2.2372	3.10	1.9083	2.65	1.6055	2.25	1.3008	1.90
40	3.2762	4.00	2.7503	3.50	2.2864	2.90	1.8813	2.45
60	4.0478	4.70	3.3830	4.10	2.8130	3.50	2.3091	2.90
80	4.5244	5.00	3.8069	4.35	3.1437	3.75	2.5814	3.10
100	4.6852	5.10	3.9031	4.45	3.2572	3.80	2.6755	3.15

d = 24 cm L = 200 cm $\frac{d}{L}$ = 0.120

K = 2.76 cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		7		16		30	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	12.00		0		0		0	
10	--	12.00	4.5013	9.30	3.2613	7.75	2.7121	6.20
20	--	--	7.6102	10.30	5.3414	9.25	4.2649	7.30
40	--	--	10.3285	11.30	8.1494	10.35	6.4617	8.90
70	--	--	11.3917	11.75	10.2391	11.20	8.6747	10.15
100	--	--	11.7354	11.90	11.1353	11.70	10.0260	11.00
130	--	--	11.8765	12.00	11.5515	11.85	10.8065	11.40
160	--	--	11.9344	--	11.7381	11.90	11.2017	11.65
190	--	--	11.9501	--	11.7913	11.95	11.3215	11.70

d = 48 cm L = 380 cm $\frac{d}{L}$ = 0.126

K = 2.6794 cm/sec

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	50		76		105		150	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	2.3000	5.25	1.9885	4.70	1.8846	4.05	1.5487	3.45
20	3.6079	6.20	3.0464	5.50	2.7017	4.80	2.1940	4.00
40	5.3214	7.70	4.4456	6.75	3.8162	5.90	3.0724	4.90
70	7.2300	9.05	6.0302	8.05	5.0968	7.05	4.0544	5.95
100	8.6468	10.10	7.2880	9.05	6.1356	8.10	4.8448	6.80
130	9.6132	10.65	8.2049	9.70	6.9232	8.50	5.4363	7.30
160	10.1623	11.00	8.7670	10.10	7.4106	9.10	5.8154	7.70
190	10.3407	11.10	8.9544	10.35	7.5736	9.50	5.9471	7.80

$d = 48$ cm $L = 380$ cm $\frac{d}{L} = 0.126$

$K = 2.6794$ cm/sec

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	201		250		300			
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0			
10	1.2525	2.90	1.0333	2.50	0.8526	2.15		
20	1.7744	3.35	1.4540	2.90	1.1940	2.50		
40	2.4331	4.10	1.9760	3.45	1.6188	2.90		
70	3.1796	4.90	2.5702	4.10	2.0856	3.50		
100	3.7781	5.65	3.0381	4.70	2.4665	3.95		
130	4.2183	6.10	3.3837	5.10	2.7476	4.30		
160	4.4937	6.40	3.5949	5.35	2.9202	4.50		
190	4.5859	6.50	3.6646	5.40	2.9781	4.55		

$d = 48$ cm $L = 380$ cm $\frac{d}{L} = 0.126$

$K = 2.6794$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		8		20		35	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	12.0		0		0		0	
10	--	12.0	4.2188	8.90	3.0728	6.60	2.5785	5.35
20	--	--	7.2544	9.90	4.9516	7.90	4.0314	6.50
40	--	--	10.0756	11.10	7.5558	9.55	6.0816	8.20
60	--	--	11.0369	11.50	9.1963	10.45	7.5907	9.25
80	--	--	11.4588	11.70	10.1844	10.95	8.7165	9.95
100	--	--	11.6774	11.90	10.7909	11.35	9.5259	10.55
130	--	--	11.8305	11.95	11.2631	11.60	10.2405	10.95
160	--	--	11.8717	12.00	11.3997	11.70	10.4658	11.10

d = 48 cm L = 320 cm $\frac{d}{L}$ = 0.150

K = 2.644 cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	60		80		100		140	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	2.1108	4.70	1.9920	4.20	1.8287	3.75	1.4612	3.10
20	3.2971	5.55	2.9100	5.00	2.6181	4.50	2.0638	3.70
40	4.8294	6.90	4.1686	6.15	3.6815	5.55	2.8784	4.50
60	6.0044	7.95	5.1495	7.10	4.4936	6.35	3.4881	5.20
80	6.9713	8.65	5.9465	7.75	5.1735	6.95	3.9955	5.70
100	7.7309	9.25	6.6104	8.35	5.7208	7.50	4.3866	6.15
130	8.4703	9.75	7.2589	8.85	6.2655	7.95	4.7859	6.55
160	8.7187	10.00	7.4774	9.00	6.4525	8.15	4.9215	6.65

$d = 48$ cm $L = 320$ cm $\frac{d}{L} = 0.150$

$K = 2.644$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	180		220					
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0					
10	1.1884	2.60	0.9744	2.25				
20	1.6802	3.10	1.3693	2.60				
40	2.2917	3.70	1.8569	3.10				
60	2.7565	4.30	2.2183	3.55				
80	3.1370	4.75	2.5163	3.95				
100	3.4404	5.05	2.7537	4.20				
130	3.7399	5.40	2.9888	4.50				
160	3.8395	5.50	3.0681	4.55				

$d = 48$ cm $L = 320$ cm $\frac{d}{L} = 0.150$

$K = 2.644$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		3		7		12	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	6.00		0		0		0	
10	--	6.00	4.4504	5.35	2.8486	4.50	2.3833	3.70
20	--	--	5.4094	5.75	4.3735	5.30	3.5440	4.50
30	--	--	5.7070	5.95	5.0749	5.60	4.3321	5.05
40	--	--	5.8346	6.00	5.4326	5.75	4.8423	5.35
50	--	--	5.8983	--	5.6223	5.90	5.1494	5.55
70	--	--	5.937	--	5.7472	5.95	5.3692	5.70

d = 24 cm

L = 140 cm

$\frac{d}{L} = 0.172$

K = 2.741 cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	19		27		36		48	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	1.7894	3.15	1.6278	2.75	1.0900	2.40	0.8799	2.05
20	2.6276	3.90	2.2468	3.35	1.2173	2.90	1.1572	2.50
30	3.3553	4.50	2.6924	3.90	1.8859	3.40	1.3841	2.90
40	3.8439	4.90	3.0017	4.25	2.3858	3.75	1.6021	3.10
50	3.9737	5.10	3.2045	4.55	2.6440	4.00	1.8008	3.40
70	3.5026	5.30	3.3018	4.75	2.8287	4.20	2.0011	3.50

$d = 24$ cm $L = 140$ cm $\frac{d}{L} = 0.172$

$K = 2.741$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	60		74					
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0					
10	0.7841	1.70	0.6420	1.45				
20	1.0208	2.10	0.8132	1.75				
30	1.2087	2.50	0.9280	2.05				
40	1.3566	2.65	1.0814	2.20				
50	1.4676	2.90	1.1724	2.40				
70	1.5604	3.05	1.2494	2.50				

$d = 24$ cm $L = 140$ cm $\frac{d}{L} = 0.172$

$K = 2.741$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		6		15		26	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	12.00		0		0		0	
10	--	12.00	6.2940	9.40	3.4442	7.10	3.1175	5.80
20	--	--	9.0710	10.40	5.6813	8.50	4.7050	7.10
40	--	--	10.8972	11.40	8.6848	10.15	6.9596	8.80
60	--	--	11.4507	11.70	10.0915	10.85	8.4786	9.80
80	--	--	11.6855	11.85	10.8000	11.30	9.4603	10.45
100	--	--	11.7980	11.95	11.1631	11.50	10.0595	10.85
120	--	--	11.8449	12.00	11.3200	11.55	10.3373	10.95
130	--	--	11.8503	--	11.3384	11.60	10.3698	11.00

$d = 48$ cm $L = 260$ cm $\frac{d}{L} = 0.185$

$K = 2.582$ cm/sec

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	35		50		70		81	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	2.9174	5.10	2.6238	4.75	2.2558	4.10	2.0696	3.85
20	4.2716	6.25	3.7868	5.65	3.2231	4.90	2.9379	4.50
40	6.1664	7.95	5.2972	7.05	4.4476	6.10	4.0168	5.65
60	7.5518	8.95	6.4107	8.00	5.2887	6.95	4.7889	6.40
80	8.5290	9.70	7.2539	8.07	6.0634	7.55	5.3696	7.00
100	9.1583	10.10	7.8206	9.15	6.3986	7.95	5.7579	7.40
120	9.4630	10.35	8.1017	9.35	6.6291	8.15	5.9884	7.50
130	9.4995	10.35	8.1362	9.35	6.6581	8.15	5.9900	7.50

d =48 cm L = 260 cm $\frac{d}{L}$ = 0.185

K =2.582 cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	113		142					
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0					
10	1.7041	3.10	1.4050	2.60				
20	2.3127	3.70	1.8917	3.05				
40	3.0944	4.50	2.4668	3.75				
60	3.6503	5.15	2.8678	4.25				
80	4.0576	5.60	3.1700	4.60				
100	4.3305	5.90	3.3763	4.90				
120	4.4693	6.05	3.4806	4.95				
130	4.4869	6.10	3.4922	5.00				

$d = 48$ cm $\quad L = 260$ cm $\quad \frac{d}{L} = 0.185$

$K = 2.582$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		7		15		22	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	12.00		0		0		0	
10	--	12.00	5.3270	9.10	3.3009	7.35	2.8403	6.50
20	--	--	8.4019	10.20	5.6183	8.60	4.7101	7.70
40	--	--	10.5501	11.25	8.4831	10.10	7.1462	9.30
60	--	--	11.2159	11.60	9.8054	10.75	8.5756	10.10
80	--	--	11.4726	11.75	10.3930	11.10	9.3085	10.50
100	--	--	11.5434	11.80	10.5638	11.25	9.5296	10.70

d = 48 cm L = 200 cm $\frac{d}{L} = 0.240$

K = 2.668 cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	31		40		50		70	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	2.8435	5.50	2.6103	4.85	2.3339	4.50	1.9228	3.70
20	4.1638	6.65	3.7741	5.90	3.3420	5.30	2.6372	4.45
40	6.0289	8.83	5.2785	7.40	4.5973	6.60	3.5312	5.40
60	7.2846	9.10	6.2997	8.30	5.4403	7.50	4.0866	6.10
80	7.9987	9.60	6.9213	8.75	5.9461	7.95	4.4726	6.50
100	8.2336	9.80	7.1281	8.90	6.1126	8.10	4.5919	6.60

$d = 48$ cm $L = 200$ cm $\frac{d}{L} = 0.240$

$K = 2.668$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	80							
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0							
10	1.7417	2.95						
20	2.3687	4.05						
40	3.1457	4.90						
60	3.6466	5.50						
80	4.1943	5.90						
100	4.0499	6.00						

d = 48 cm L = 200 cm $\frac{d}{L}$ = 0.240

K = 2.668 cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		2		4		6	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	6.00		1.0713		0		0	
10	--	6.00	4.9943	5.50	3.9063	4.95	3.0710	4.50
20	--	--	5.6166	5.80	5.0809	5.45	4.4890	5.10
30	--	--	5.7767	5.90	5.4312	5.75	4.9949	5.50
40	--	--	5.8144	5.95	5.5171	5.75	5.1289	5.50

$d = 24$ cm $L = 80$ cm $\frac{d}{L} = 0.300$

$K = 2.778$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	9		12		16		20	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	2.5297	4.10	2.2431	3.50	1.9578	3.10	1.7012	2.70
20	3.7336	4.75	3.2092	4.20	2.7098	3.75	2.3161	3.30
30	4.3311	5.15	3.7567	4.65	3.1380	4.15	2.6681	3.65
40	4.5025	5.20	3.9271	4.70	3.2821	4.25	2.7885	3.75

$d = 24$ cm $L = 80$ cm $\frac{d}{L} = 0.300$

$K = 2.778$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	24							
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0							
10	1.4944	2.35						
20	2.0308	2.85						
30	2.2839	3.20						
40	2.3773	3.30						

d = 24 cm L = 80 cm $\frac{d}{L}$ = 0.300

K = 2.778 cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		6		10		15	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	12.00		0.3522		0		0	
10	--	12.00	7.1607	9.40	4.1602	8.30	2.5439	7.00
20	--	--	9.3466	10.35	7.3751	9.35	5.2164	8.30
30	--	--	10.3340	10.90	8.8922	10.10	7.0273	9.10
40	--	--	10.8285	11.25	9.7019	10.60	8.1077	9.70
50	--	--	11.0832	11.40	10.1399	10.85	8.7371	10.05
70	--	--	11.2456	11.50	10.4342	10.95	9.1721	10.25

$d = 48$ cm $\quad L = 140$ cm $\quad \frac{d}{L} = 0.343$

$K = 2.615$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	20		25		33		41	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0		0	
10	2.9619	6.10	2.6573	5.35	2.4563	4.70	2.1869	4.20
20	4.4836	7.30	4.0383	6.50	3.4909	5.65	3.0308	5.00
30	5.8469	8.30	5.0822	7.45	4.2192	6.40	3.6060	5.60
40	6.8293	8.90	5.8585	8.10	4.8167	7.00	4.0115	6.05
50	7.4792	9.25	6.4469	8.50	5.2347	7.30	4.3364	6.40
70	7.9496	9.50	6.8909	8.70	5.5618	7.55	4.5853	6.55

$d = 48$ cm $L = 140$ cm $\frac{d}{L} = 0.343$

$K = 2.615$ cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	0		3		5		7	
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	12.00		6.7448		3.4805		0	
10	--	12.00	9.0588	10.25	7.4271	9.30	5.8463	8.50
20	--	--	10.3745	10.90	9.2633	10.20	8.0964	9.50
30	--	--	10.8638	11.25	9.9801	10.70	9.0315	10.05
40	--	--	10.9885	11.40	10.1782	10.85	9.2843	10.20

d = 48 cm   L = 80 cm $\frac{d}{L} = 0.600$

K = 2.778 cm/sec.

TABLE I (cont'd)

Laplace and Hele-Shaw Water Table Profiles for Different Values of Time

t (sec.)	9		11		14			
x (cm)	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.	h (cm) THEOR.	h (cm) EXP.
0	0		0		0			
10	4.1615	7.80	2.9030	7.10	2.4614	6.45		
20	6.8600	8.90	5.6516	8.30	4.3518	7.55		
30	7.9509	9.50	6.8751	8.90	5.5023	8.20		
40	8.2652	9.70	7.2247	9.10	5.8542	8.50		

$d = 48$ cm $\quad L = 80$ cm $\quad \frac{d}{L} = 0.600$

$K = 2.778$ cm/sec.

TABLE II

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	0		11		45		101	
$\sqrt{\frac{4\kappa t}{L^2}}$	0		0.10		0.20		0.30	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	3.80	3.80	0		0		0	
10	--	--	1.3949	0.8143	0.8194	0.3690	0.6256	0.3362
20	--	--	2.2520	1.5744	1.3229	0.9849	0.9957	0.6694
40	--	--	3.2452	2.8892	2.2006	1.8914	1.6534	1.3202
70	--	--	3.7029	3.8000	3.0820	2.8956	2.4719	2.1601
100	--	--	3.7765	3.8000	3.5178	3.4556	3.0634	2.8059
130	--	--	3.7957	3.8000	3.7026	3.6882	3.4377	3.2492
160	--	--	3.7868	3.8000	3.7636	3.7656	3.6158	3.5210
190	--	--	3.7870	3.8000	3.7800	3.7656	3.7052	3.6573
220	--	--	3.7871	3.8000	3.7830	3.8000	3.7307	3.6984

d = 15.3 cm   L = 440 cm $\frac{d}{L}$ = 0.035

K = 2.848 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	180		281		404			
$\sqrt{\frac{4\alpha t}{L^2}}$	0.40		0.50		0.60			
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0	0	0	0	0		
10	0.5021	0.2507	0.4073	0.1959	0.3250	0.1519		
20	0.7931	0.5020	0.6427	0.3933	0.5123	0.3048		
40	1.2952	1.0007	1.0784	0.7865	0.8623	0.6100		
70	2.0490	1.6813	1.6904	1.3328	1.3142	1.0357		
100	2.6417	2.2632	2.2118	1.8149	1.7092	1.4139		
130	3.1258	2.3859	2.6045	2.2116	1.9830	1.7276		
160	3.4241	3.0559	2.7964	2.5098	2.1687	1.9653		
190	3.5192	3.2501	2.8599	2.6899	2.2736	2.1099		
220	3.5515	3.3143	2.9305	2.7507	2.3086	2.1588		

d = 15.3 cm L = 440 cm $\frac{d}{L}$ = 0.035

K = 2.848 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	0		7		30		67	
$\sqrt{\frac{4\alpha t}{L^2}}$	0		0.10		0.20		0.30	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	6.00	6.00	0	0	0		0	0
10	--	--	2.5302	1.2734	1.4836	0.7577	1.1084	0.5081
20	--	--	4.0106	2.4628	2.2965	1.4921	1.7078	1.0128
40	--	--	5.3347	4.5237	3.5407	2.8757	2.6446	2.0017
70	--	--	5.8478	6.0000	4.8264	4.4416	3.8087	3.2919
100	--	--	5.9610	6.0000	5.5052	5.3600	4.6767	4.3052
130	--	--	5.9857	6.0000	5.8083	5.7775	5.2542	5.0216
160	--	--	5.9909	6.0000	5.9273	5.9316	5.5978	5.4834
190	--	--	5.9921	6.0000	5.9688	5.9870	5.7713	5.7131
220	--	--	5.9924	6.0000	5.9786	6.0000	5.8232	5.7859

d = 24 cm L = 440 cm $\frac{d}{L} = 0.055$

K = 2.682 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	120		188		271		368	
$\sqrt{\frac{4\alpha t}{L^2}}$	0.40		0.50		0.60		0.70	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0	0	0	0	0	0	0
10	0.8814	0.3774	0.7625	0.3235	0.6168	0.2203	0.4006	0.1602
20	1.3059	0.7563	1.1231	0.6494	0.9065	0.4423	0.5506	0.3217
40	2.0143	1.5089	1.7237	1.2993	1.3779	0.8854	0.6979	0.6440
70	3.0151	2.5416	2.5455	2.2040	2.0109	1.5038	1.4351	1.0938
100	3.9985	3.4329	3.2707	3.0049	2.3974	2.0538	1.9144	1.4938
130	4.5288	4.1467	3.8642	3.6667	2.8950	2.5106	2.2813	1.8261
160	5.0080	4.6685	4.3012	4.1660	3.4183	2.8572	2.5530	2.0782
190	5.2339	4.9767	4.5700	4.4686	3.6812	3.0681	2.7185	2.2316
220	5.3192	5.0793	4.6600	4.5708	3.7626	3.1395	2.7746	2.2835

d = 24 cm   L = 440 cm $\frac{d}{L} = 0.055$

K = 2.682 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	481		609					
$\sqrt{\frac{4\alpha t}{L^2}}$	0.80		0.90					
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0	0	0				
10	0.2837	0.1108	0.2544	0.0727				
20	0.4147	0.2221	0.3696	0.1459				
40	0.6374	0.4445	0.5554	0.2920				
70	0.9738	0.7549	0.8086	0.4960				
100	1.3147	1.0310	1.0744	0.6774				
130	1.6360	1.2604	1.2133	0.8281				
160	1.8988	1.4343	1.3443	0.9424				
190	2.0612	1.5402	1.4247	1.0120				
220	2.0924	1.5761	1.4512	1.0355				

d = 24 cm L = 440 cm $\frac{d}{L}$ = 0.055

K = 2.682 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	0		4		16		35	
$\sqrt{\frac{4\alpha t}{L^2}}$	0		0.10		0.20		0.30	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	6.00	6.00	0	0	0		0	0
10	--	--	3.1721	1.7335	1.8770	1.0815	1.4368	0.7407
20	--	--	4.7606	3.3143	2.9424	2.1102	2.2173	1.4638
40	--	--	5.6731	4.7347	4.4137	3.8665	3.4109	2.8136
60	--	--	5.8878	6.0000	5.2280	5.0171	4.3257	3.9077
80	--	--	5.9561	--	5.6405	5.6256	4.9666	4.7374
100	--	--	5.9784	--	5.8350	5.8679	5.4021	5.2898
130	--	--	5.9871	--	5.9441	5.9777	5.7308	5.7309
160	--	--	5.9885	--	5.9671	6.0000	5.8213	5.8534

d = 24 cm L = 320 cm $\frac{d}{L}$ = 0.075

K = 2.701 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	63		98		141		191	
$\sqrt{\frac{4\kappa t}{L^2}}$	0.40		0.50		0.60		0.70	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0	0		0		0	0
10	1.1347	0.5519	0.9900	0.4335	0.8611	0.3366	0.7245	0.2547
20	1.7469	1.0964	1.4698	0.8629	1.2694	0.6699	1.0625	0.5070
40	2.7027	2.1500	2.2589	1.7028	1.9339	1.3242	1.6138	1.0024
60	3.5501	3.0765	2.9599	2.4600	2.5266	1.9185	2.1154	1.4519
80	4.2571	3.8672	3.5775	3.1284	3.0551	2.4474	2.5696	1.8521
100	4.7830	4.4761	4.0703	3.6635	3.4787	2.8749	2.9483	2.1757
130	5.2590	5.0645	4.5588	4.2041	3.9122	3.3114	3.3209	2.5060
160	5.4086	5.2580	4.7403	4.3882	4.0680	3.4612	3.4471	2.6194

d = 24 cm

K = 2.701 cm/sec.

L = 320 cm

$\frac{d}{L} = 0.075$

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	250		316		390			
$\sqrt{\frac{4\alpha t}{L^2}}$	0.80		0.90		1.00			
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0	0	0	0	0		
10	0.6311	0.1825	0.5311	0.1260	0.4696	0.0832		
20	0.9252	0.3633	0.7775	0.2508	0.6850	0.1655		
40	1.3963	0.7183	1.1792	0.4958	0.9056	0.3273		
60	1.8087	1.0403	1.5483	0.7182	1.3370	0.4740		
80	2.1781	1.3272	1.8763	0.9162	1.6091	0.6047		
100	2.5014	1.5590	2.1396	1.0762	1.8352	0.7103		
130	2.8199	1.7954	2.3973	1.2396	2.0671	0.8181		
160	2.9312	1.8769	2.4870	1.2957	2.1112	0.8552		

d = 24 cm

L = 320 cm

$\frac{d}{L} = 0.075$

K = 2.701 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	0		6		14		24	
$\sqrt{\frac{4\alpha t}{L^2}}$	0		0.20		0.30		0.40	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	6.00	6.00	0	0	0	0	0	0
10	--	--	3.7372	1.6550	2.1557	1.1018	1.8289	0.8413
20	--	--	4.8546	3.1251	3.2660	2.1459	2.6969	1.6552
40	--	--	5.6318	5.0729	4.6661	3.8809	3.9369	3.1042
60	--	--	5.8621	5.7971	5.3697	5.0119	4.7601	4.1979
80	--	--	5.9380	5.9676	5.6799	5.5886	5.2151	4.8639
100	--	--	5.9512	6.0040	5.7657	5.7573	5.3576	5.0859

$d = 24$ cm $L = 200$ cm $\frac{d}{L} = 0.120$

$K = 2.769$ cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	38		55		74		97	
$\sqrt{\frac{4\alpha t}{L^2}}$	0.50		0.60		0.70		0.80	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0	0	0	0	0	0	0
10	1.5295	0.6466	1.3522	0.4868	1.1442	0.3566	0.9277	0.2449
20	2.2372	1.2764	1.9083	0.7616	1.6055	0.7044	1.3008	0.4837
40	3.2762	2.4220	2.7503	1.8284	2.2864	1.3397	1.8813	0.9199
60	4.0478	3.3241	3.3830	2.5158	2.8130	1.8439	2.3091	1.2662
80	4.5244	3.8985	3.8069	2.9567	3.1437	2.1676	2.5817	1.4884
100	4.6852	4.0956	3.9031	3.1086	3.2572	2.2792	2.6755	1.5651

d = 24 cm

K = 2.769 cm/sec

L = 200 cm

$\frac{d}{L} = 0.120$

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	0		8		20		35	
$\sqrt{\frac{4\alpha t}{L^2}}$	0		0.20		0.31		0.42	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	12.0	12.0	0	0	0	0	0	0
10	--	--	4.2188	2.0791	3.0728	1.3288	2.5785	0.9994
20	--	--	7.2544	4.0651	4.9516	2.6322	4.0314	1.9871
40	--	--	10.0756	7.4966	7.5558	5.1050	6.0816	3.9105
60	--	--	11.0369	9.8052	9.1963	7.1856	7.5907	5.6255
80	--	--	11.4588	11.1021	10.1849	8.8529	8.7165	7.1176
100	--	--	11.6774	11.6692	10.7909	10.0428	9.5259	8.2927
130	--	--	11.8305	11.9433	11.2631	11.0897	10.2405	9.4584
160	--	--	11.8717	12.0000	11.3991	11.4071	10.4656	9.8498

d = 48 cm   L = 320 cm $\frac{d}{L}$ = 0.150

K = 2.644 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	60		80		100		140	
$\sqrt{\frac{4\alpha t}{L^2}}$	0.55		0.63		0.70		0.83	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0	0	0	0	0	0	0
10	2.1108	0.7152	1.9920	0.5586	1.8287	0.4372	1.4612	0.2680
20	3.2971	1.4241	2.9100	1.1119	2.6181	0.8704	2.6638	0.5336
40	4.8294	2.8142	4.1686	2.1983	3.6815	1.7207	2.8784	1.0549
60	6.0044	4.6719	5.1495	3.1837	4.4936	2.4922	3.4881	1.5279
80	6.9713	5.1931	5.9465	4.0611	5.1735	3.1794	3.9955	1.9491
100	7.7309	6.0959	6.6104	4.7699	5.7208	3.7347	4.3866	2.2896
130	8.4703	7.0154	7.2589	5.4935	6.2655	4.3017	4.7859	2.6372
160	8.7187	7.3273	7.4774	5.7418	6.4525	4.4964	4.9215	2.9093

d = 48 cm   L = 320 cm $\frac{d}{L}$ = 0.150

K = 2.644 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	180		220					
$\sqrt{\frac{4\alpha t}{L^2}}$	0.94		1.04					
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0	0	0				
10	1.1884	0.1643	0.9744	0.1008				
20	1.6802	0.3271	1.3693	0.2006				
40	2.2907	0.6467	1.8569	0.3965				
60	2.7565	0.9367	2.2183	0.5743				
80	3.1370	1.1949	2.5163	0.7326				
100	3.4409	1.4037	2.7537	0.8606				
130	3.7399	1.6168	2.9888	0.9913				
160	3.8395	1.6899	3.0681	1.0361				

$d = 48$ cm $L = 320$ cm $\frac{d}{L} = 0.150$

$K = 2.644$ cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	0		7		15		22	
$\sqrt{\frac{4\alpha t}{L^2}}$	0		0.30		0.44		0.53	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	12.00	12.00	0	0	0	0	0	0
10	--	--	5.3270	2.3177	3.3009	1.5206	2.8403	1.1972
20	--	--	8.4019	3.9625	5.6183	2.9974	4.7101	2.3641
40	--	--	10.5501	7.8602	8.4831	5.6588	7.1462	4.4906
60	--	--	11.2159	10.0017	9.8054	7.7172	8.5756	6.1711
80	--	--	11.4726	11.2383	10.3930	9.0037	9.3085	7.2450
100	--	--	11.5434	11.6725	10.5638	9.4399	9.5296	7.6143

d = 48 cm

L = 200 cm

$\frac{d}{L}$ = 0.240

K = 2.668 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	31		40		50		70	
$\sqrt{\frac{4\alpha t}{L^2}}$	0.63		0.71		0.80		0.94	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0	0	0	0	0	0	
10	2.8435	0.8979	2.6103	0.6754	2.3339	0.4924	1.9228	0.2617
20	4.1638	1.7737	3.7741	1.3342	3.3420	0.9726	2.6372	0.5170
40	6.0289	2.7992	5.2785	2.5375	4.5973	1.8498	3.5312	0.9833
60	7.2846	4.6418	6.2997	3.4926	5.4403	2.5461	4.0866	1.3534
80	7.9987	5.4558	6.9213	4.1056	5.9461	2.9930	4.4726	1.5910
100	8.2336	5.7364	7.1281	4.3170	6.1126	3.1471	4.5919	1.6729

d = 48 cm

L = 200 cm

$\frac{d}{L}$ = 0.240

K = 2.668 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	80							
$\sqrt{\frac{4\alpha t}{L^2}}$	1.01							
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0						
10	1.7417	0.1908						
20	2.3687	0.3770						
40	3.1457	0.7170						
60	3.6466	0.9868						
80	4.1943	1.1601						
100	4.0499	1.2198						

d = 48 cm

L = 200 cm

$\frac{d}{L} = 0.240$

K = 2.668 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	0		2		4		6	
$\sqrt{\frac{4\alpha t}{L^2}}$	0		0.30		0.40		0.50	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	6.00	6.00	1.0713	0	0	0	0	0
10	--	--	4.9943	2.7583	3.9063	1.9959	3.0710	1.5868
20	--	--	5.6166	4.6746	5.0809	3.6250	4.4890	2.9220
30	--	--	5.7767	5.5908	5.4312	4.7031	4.9949	3.8056
40	--	--	5.8144	5.8285	5.5171	5.0008	5.1289	4.1126

d = 24 cm

L = 80 cm

$\frac{d}{L} = 0.300$

K = 2.778 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	9		12		16		20	
$\sqrt{\frac{4\alpha t}{L^2}}$	0.61		0.70		0.82		0.91	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0	0	0	0	0	0	0
10	2.5297	1.1595	2.2431	0.8517	1.9578	0.5643	1.7012	0.3740
20	3.7336	2.1418	3.2092	1.5736	2.7098	1.0426	2.3161	0.6911
30	4.3311	2.7981	3.7567	2.0564	3.1380	1.3625	2.6681	0.9032
40	4.5025	3.0278	3.9271	2.2254	3.2821	1.4745	2.7885	0.9774

d = 24 cm L = 80 cm $\frac{d}{L}$ = 0.300

K = 2.778 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	24							
$\sqrt{\frac{4\alpha t}{L^2}}$	1.00							
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0						
10	1.4944	0.2479						
20	2.0308	0.4581						
30	2.2839	0.5987						
40	2.3773	0.6479						

d = 24 cm L = 80 cm $\frac{d}{L} = 0.300$

K = 2.778 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	0		3		5		7	
$\sqrt{\frac{4\alpha t}{L^2}}$	0		0.50		0.64		0.76	
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	12.00	12.00	6.7448	0	3.4805	0	0	0
10	--	--	9.0588	3.1734	7.4271	2.0916	5.8463	1.3862
20	--	--	10.3745	5.8438	9.2633	3.8643	8.0964	2.5612
30	--	--	10.8638	7.6109	9.9801	5.0493	9.0315	3.3471
40	--	--	10.9885	8.2249	10.1782	5.4640	9.2843	3.6222

d = 48 cm L = 80 cm $\frac{d}{L} = 0.600$

K = 2.778 cm/sec.

TABLE II (cont'd)

Laplace and Glover's Water Table Profiles for Different Values of Time

t (sec.)	9		11		14			
$\sqrt{\frac{4\alpha t}{L^2}}$	0.87		0.96		1.08			
x (cm)	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER	h (cm) THEOR.	h (cm) GLOVER
0	0	0	0	0	0	0		
10	4.1615	0.9187	2.9030	0.6090	2.4614	0.3282		
20	6.8600	1.6975	5.6516	1.1252	4.3518	0.6064		
30	7.9509	2.2184	6.8751	1.4705	5.5023	0.7924		
40	8.2652	2.4007	7.2247	1.5913	5.8542	0.8576		

d = 48 cm

L = 80 cm

$\frac{d}{L}$ = 0.600

K = 2.778 cm/sec.

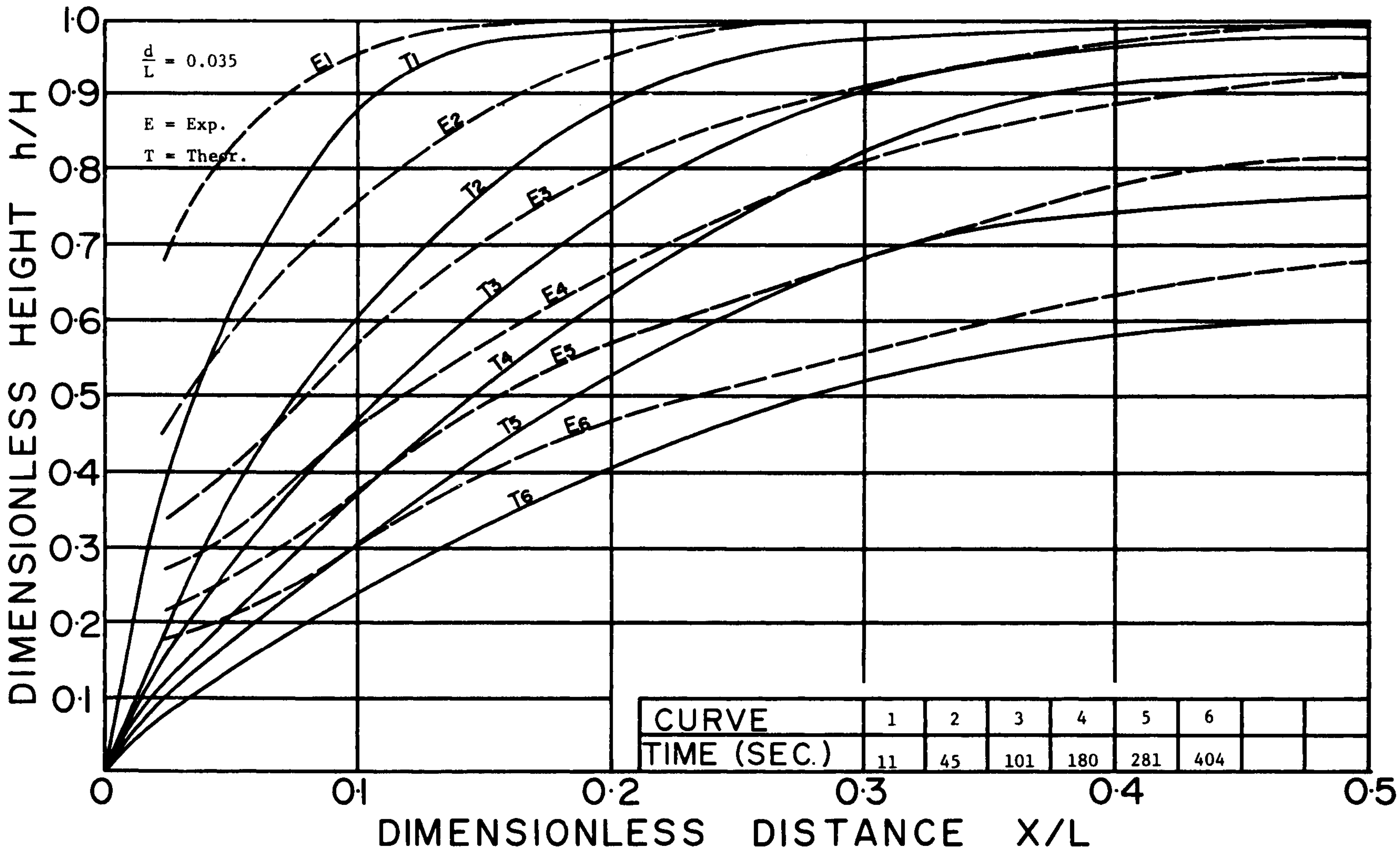

Fig. 7. Experimental and Theoretical Water Table Profiles

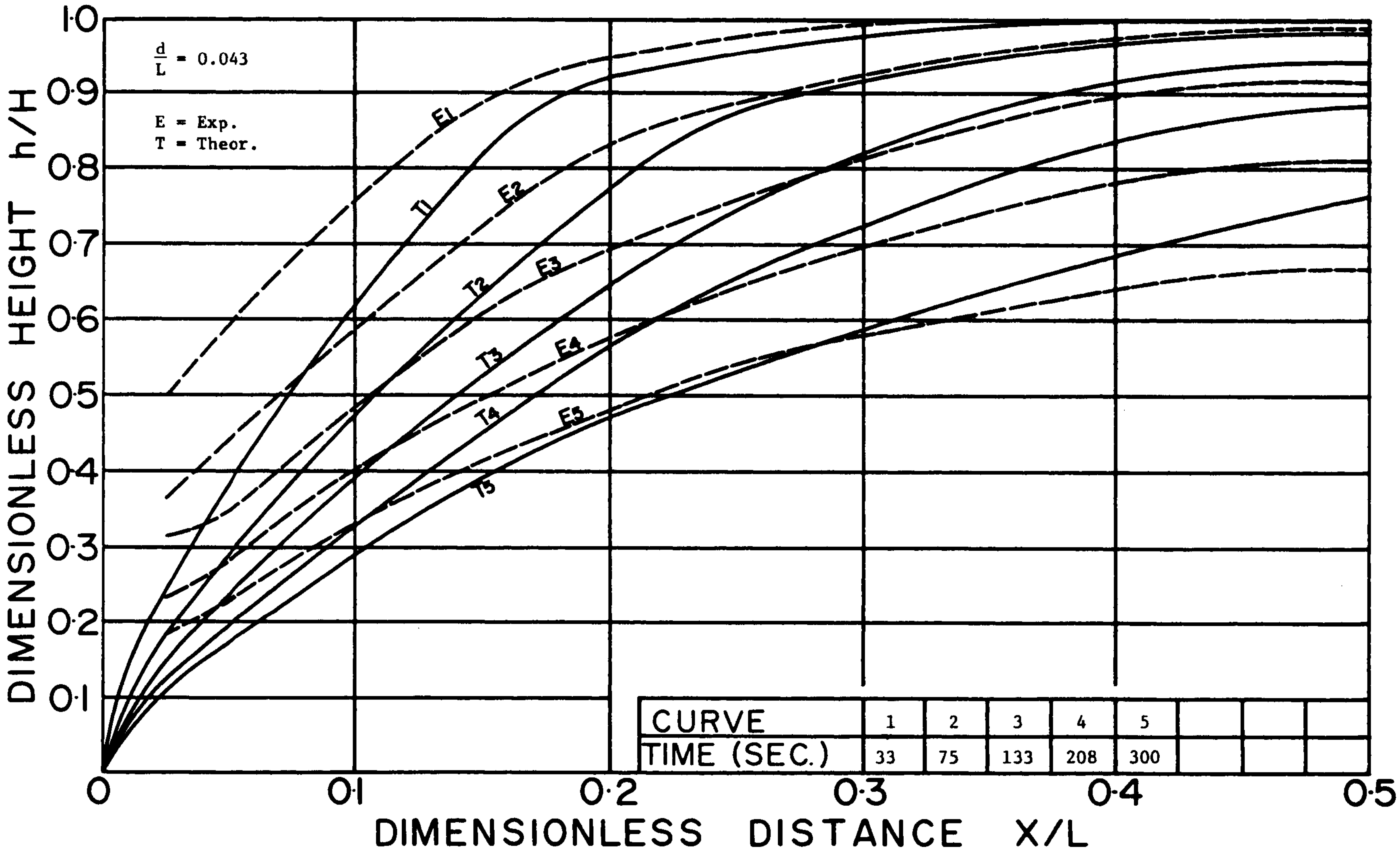

Fig. 8 Experimental and Theoretical Water Table Profiles

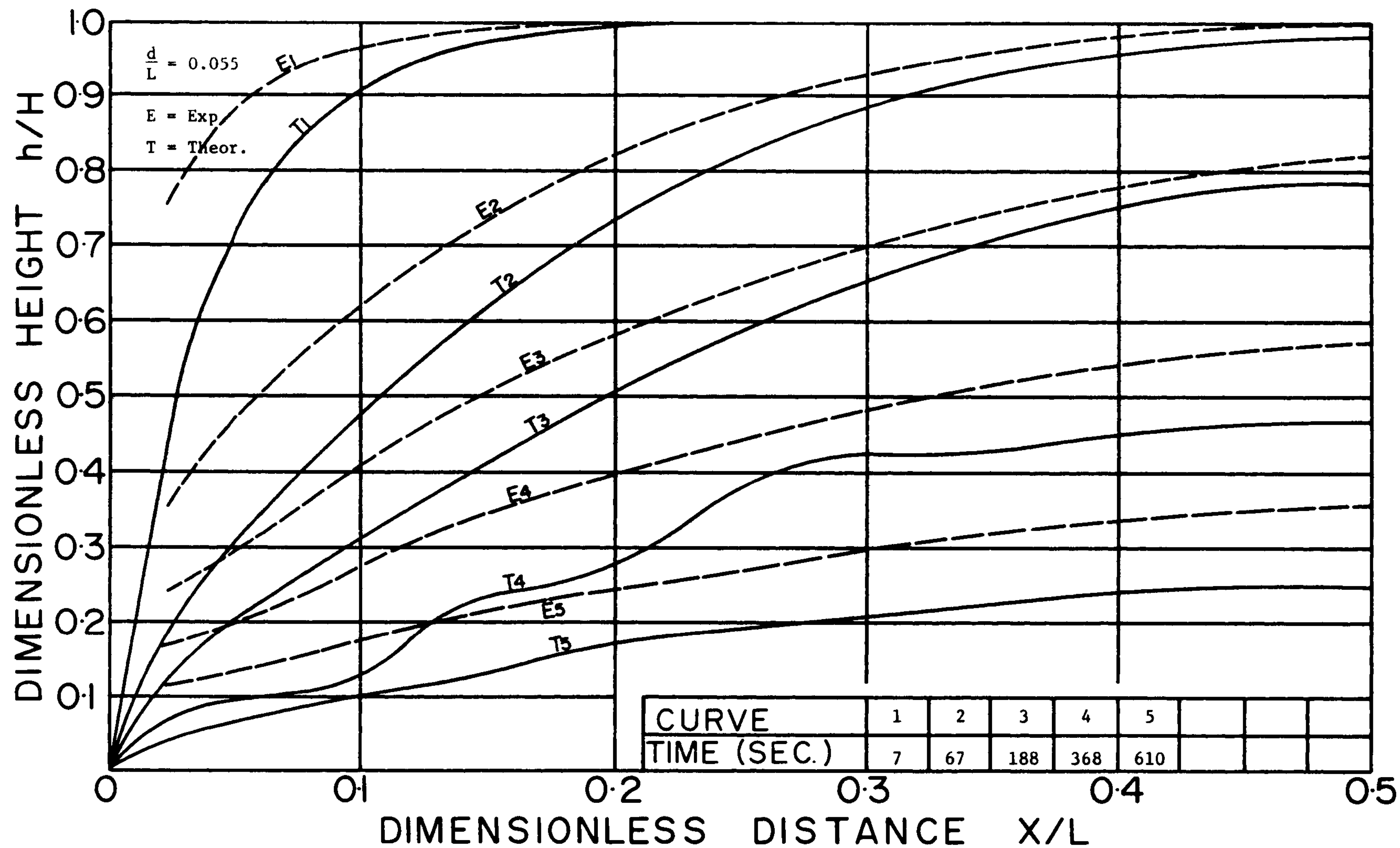

Fig. 9. Experimental and Theoretical Water Table Profiles

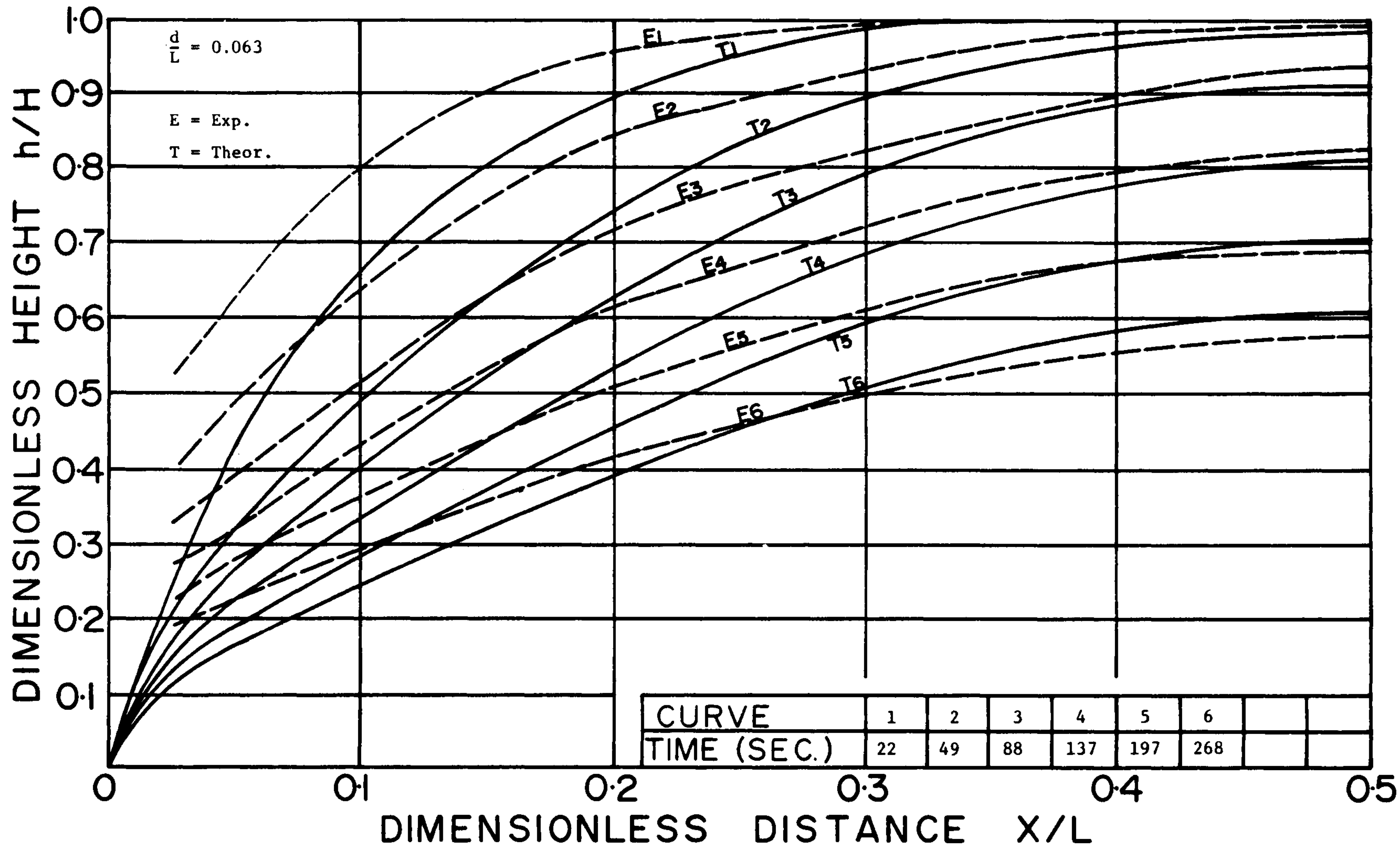

Fig. 10. Experimental and Theoretical Water Table Profiles

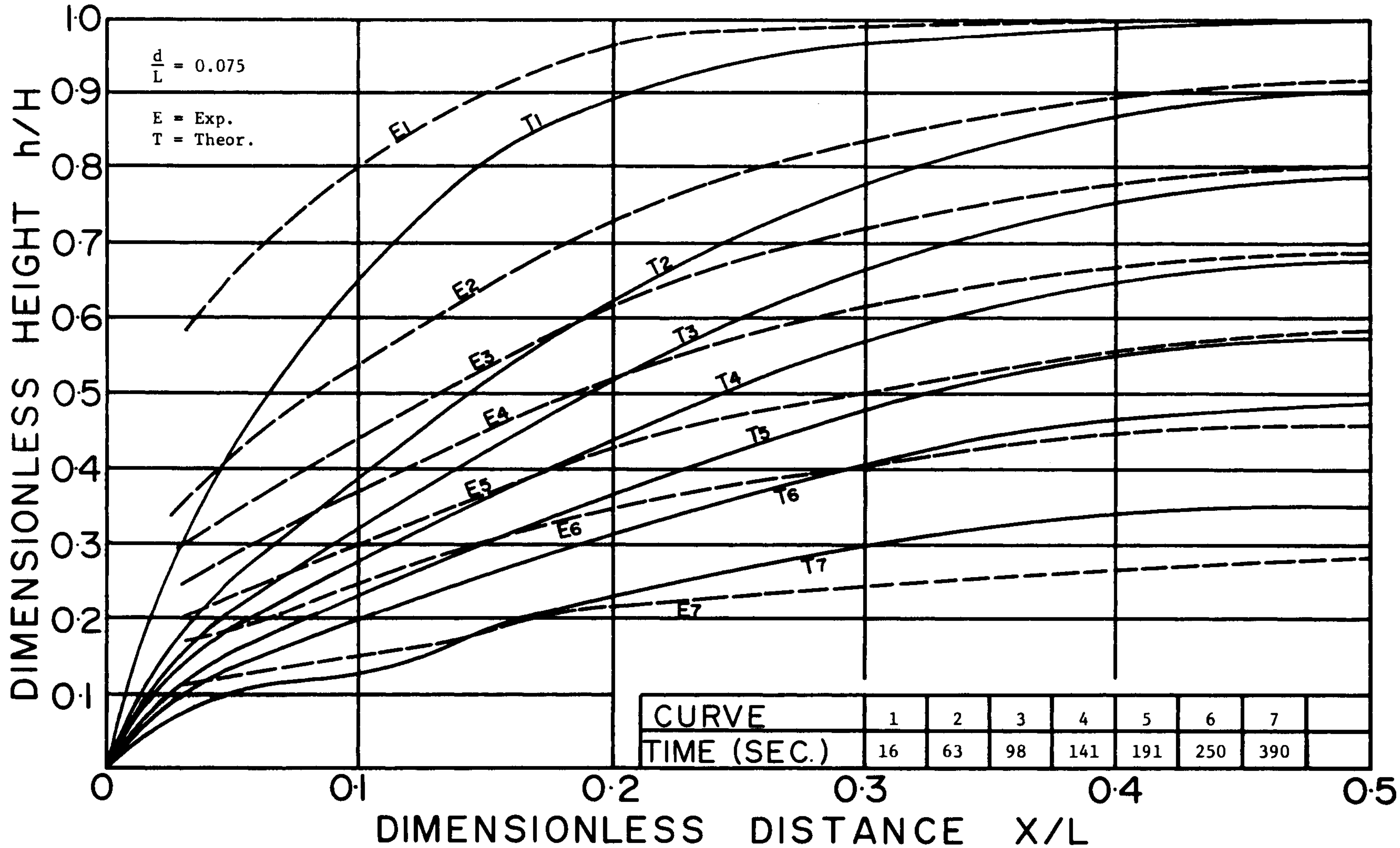

Fig. 11. Experimental and Theoretical Water Table Profiles

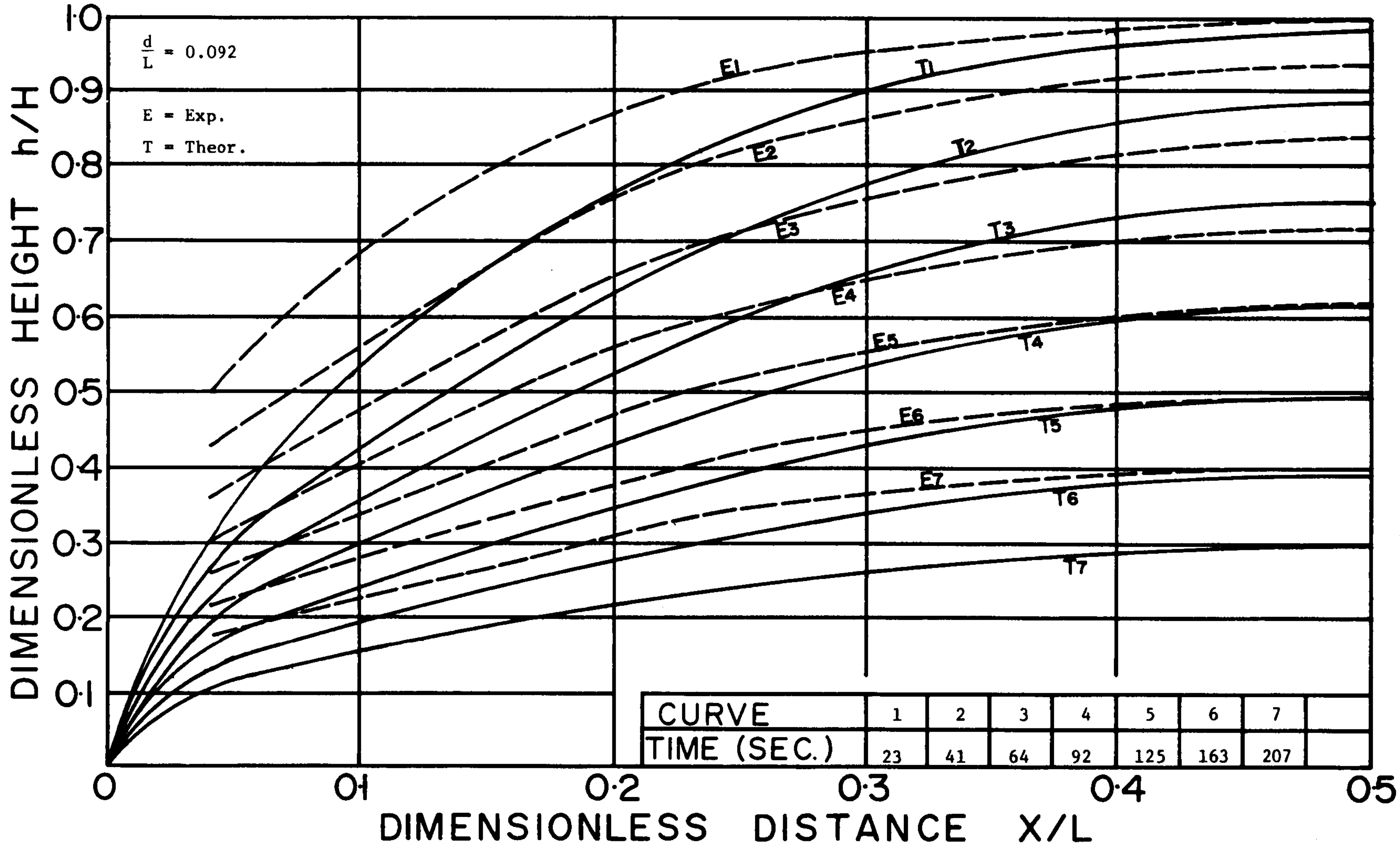

Fig. 12. Experimental and Theoretical WaterTable Profiles

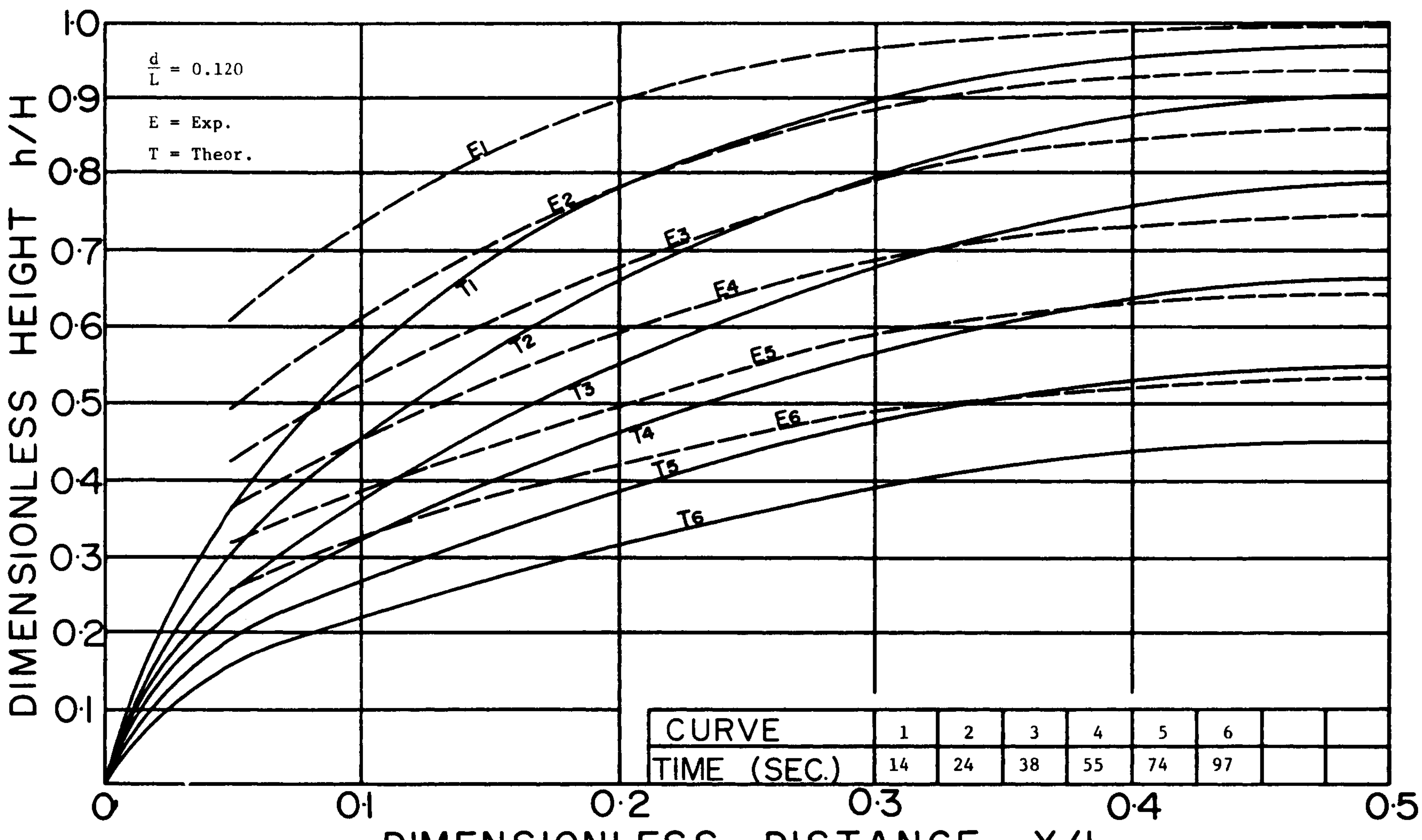

Fig. 13. Experimental and Theoretical Water Table Profiles

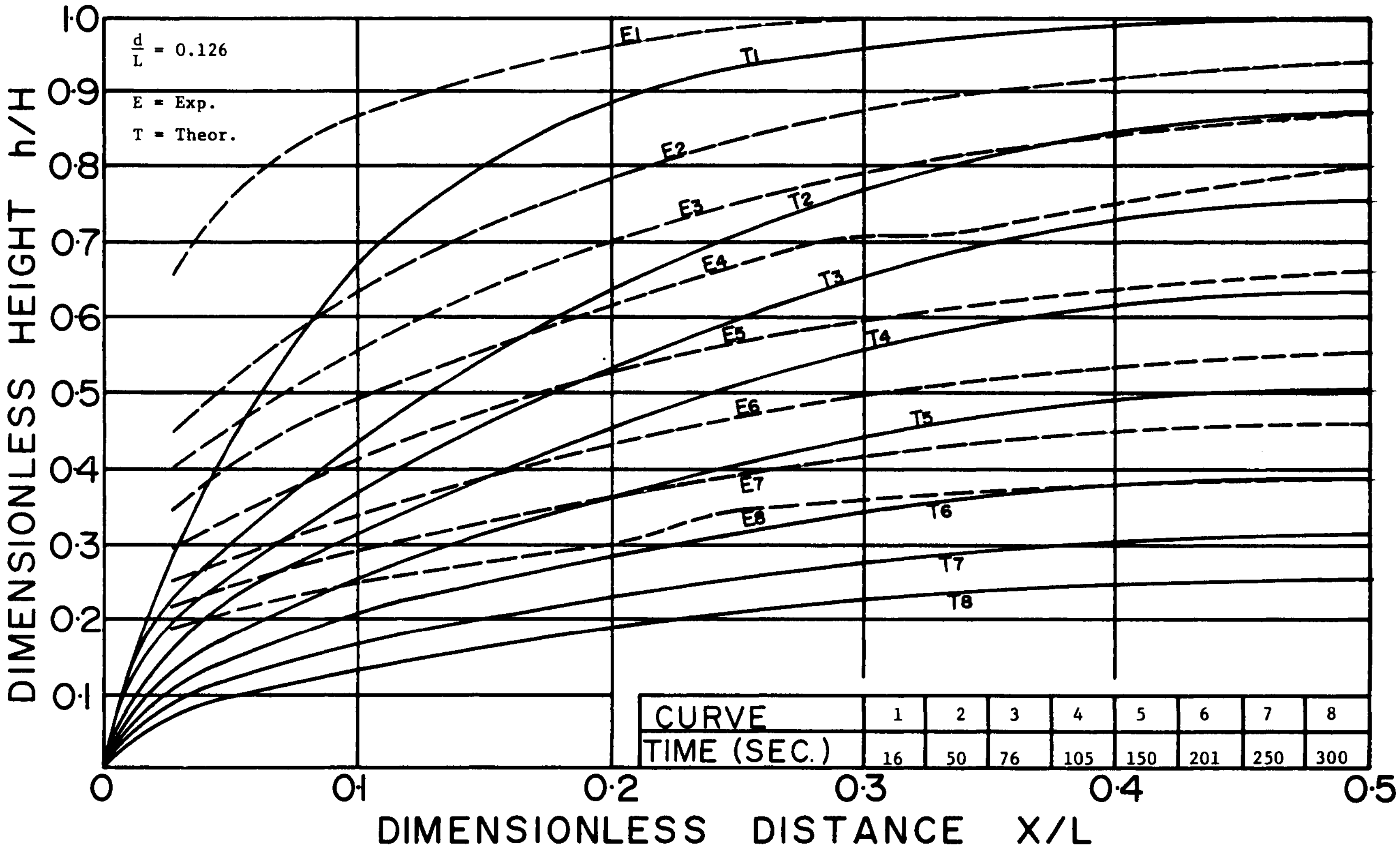

Fig. 14. Experimental and Theoretical Water Table Profiles

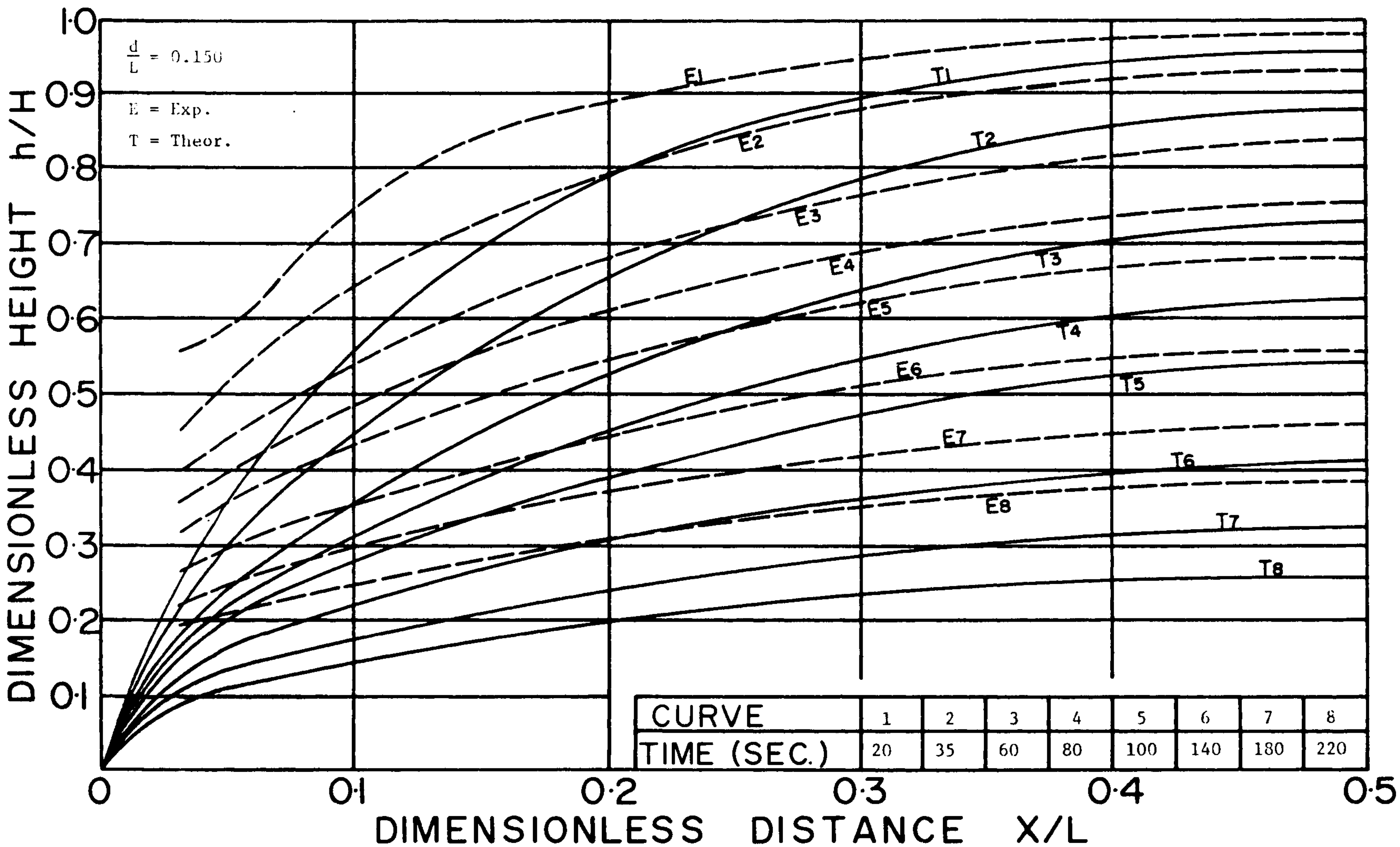

Fig. 15. Experimental and Theoretical Water Table Profiles

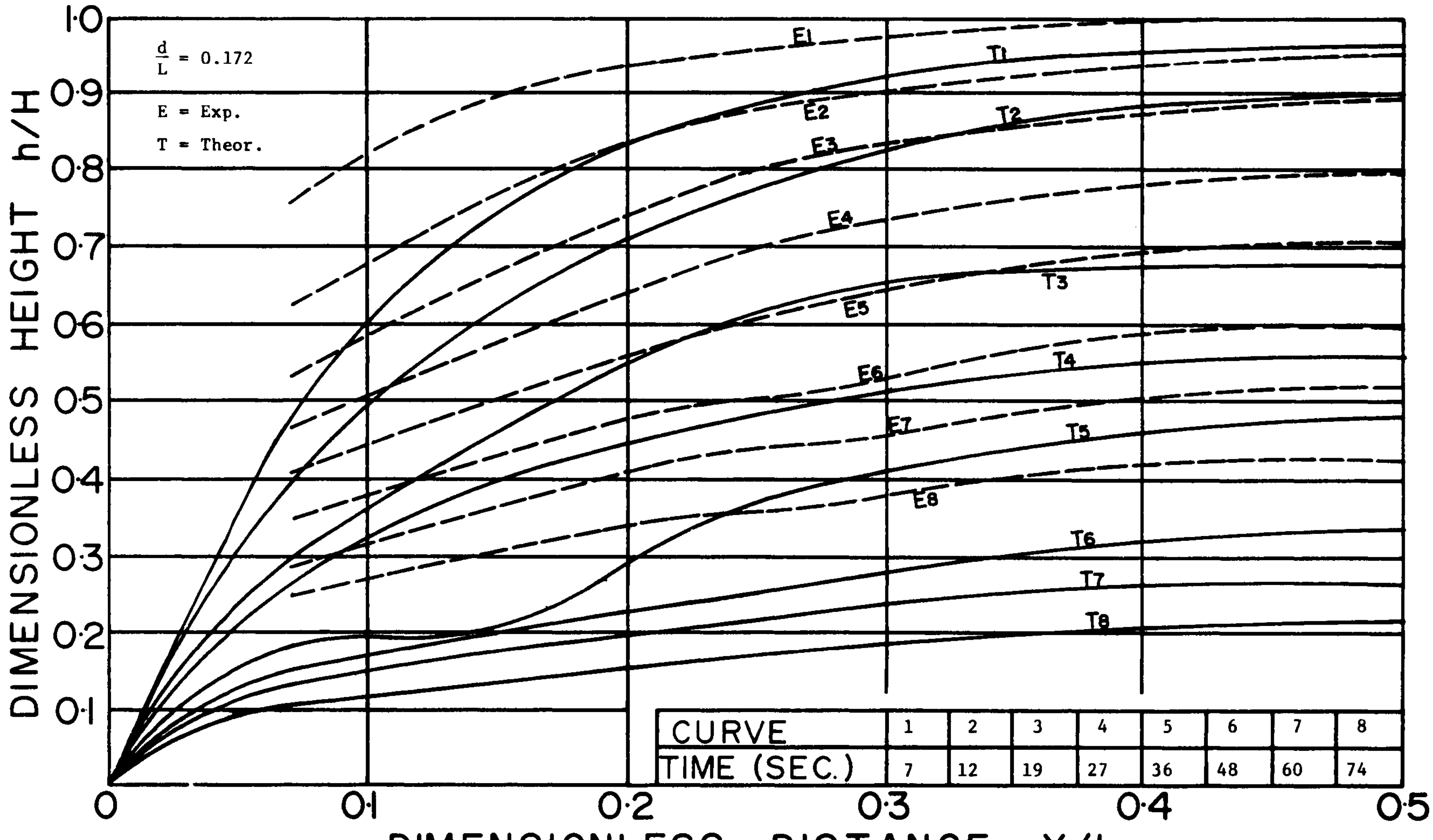

Fig. 16. Experimental and Theoretical Water Table Profiles

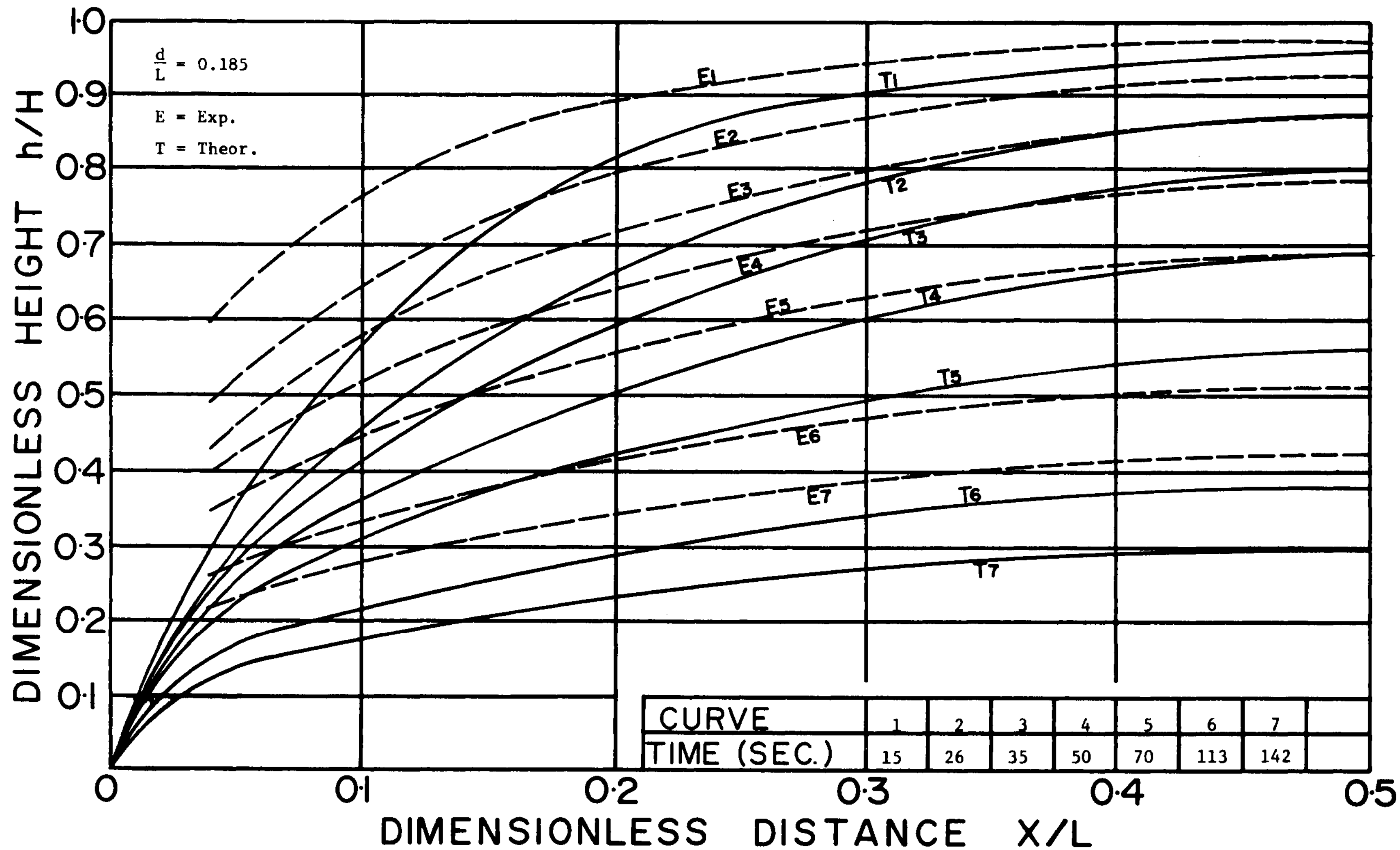

Fig. 17. Experimental and Theoretical Water Table Profiles

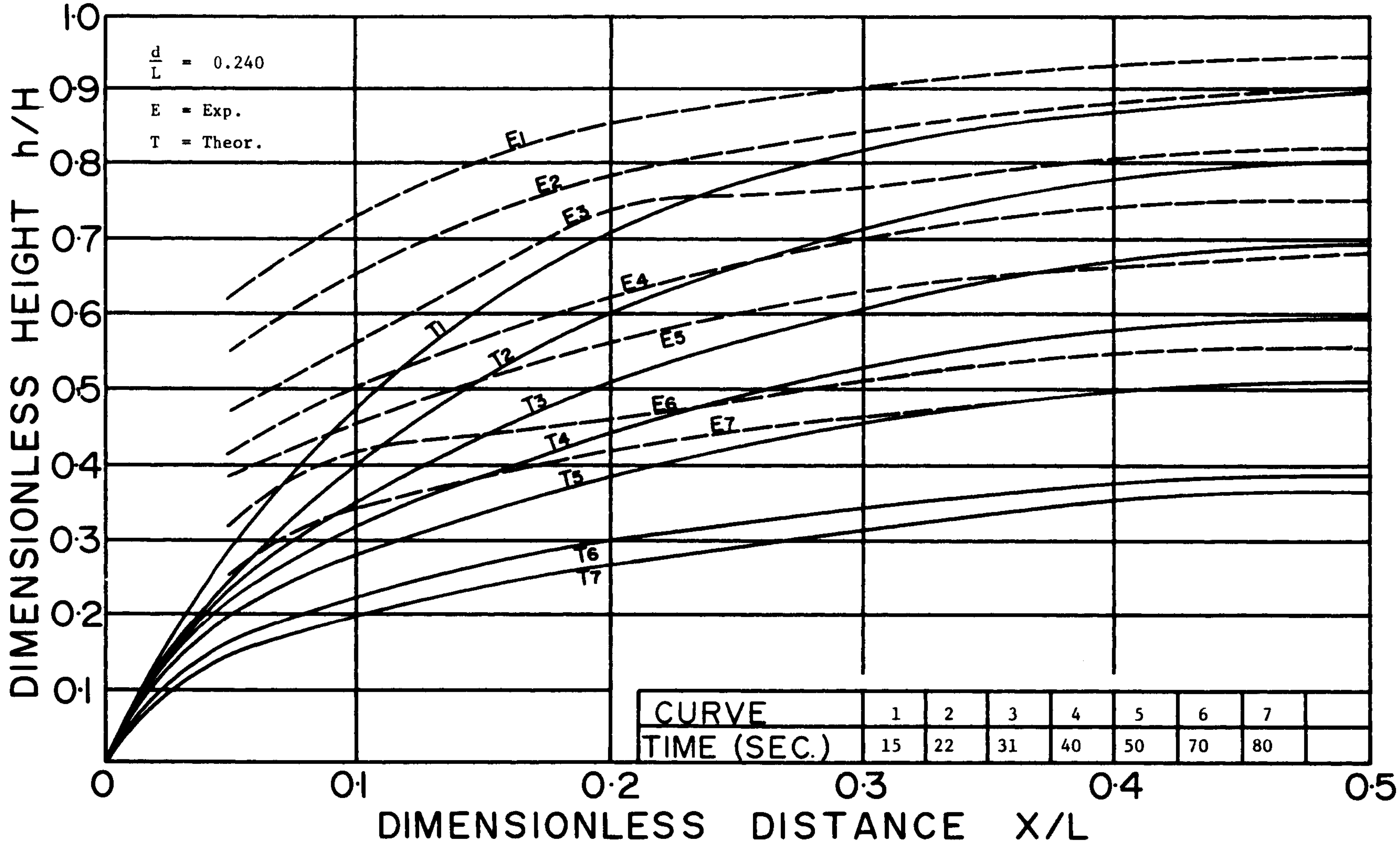

Fig. 18. Experimental and Theoretical Water Table Profiles

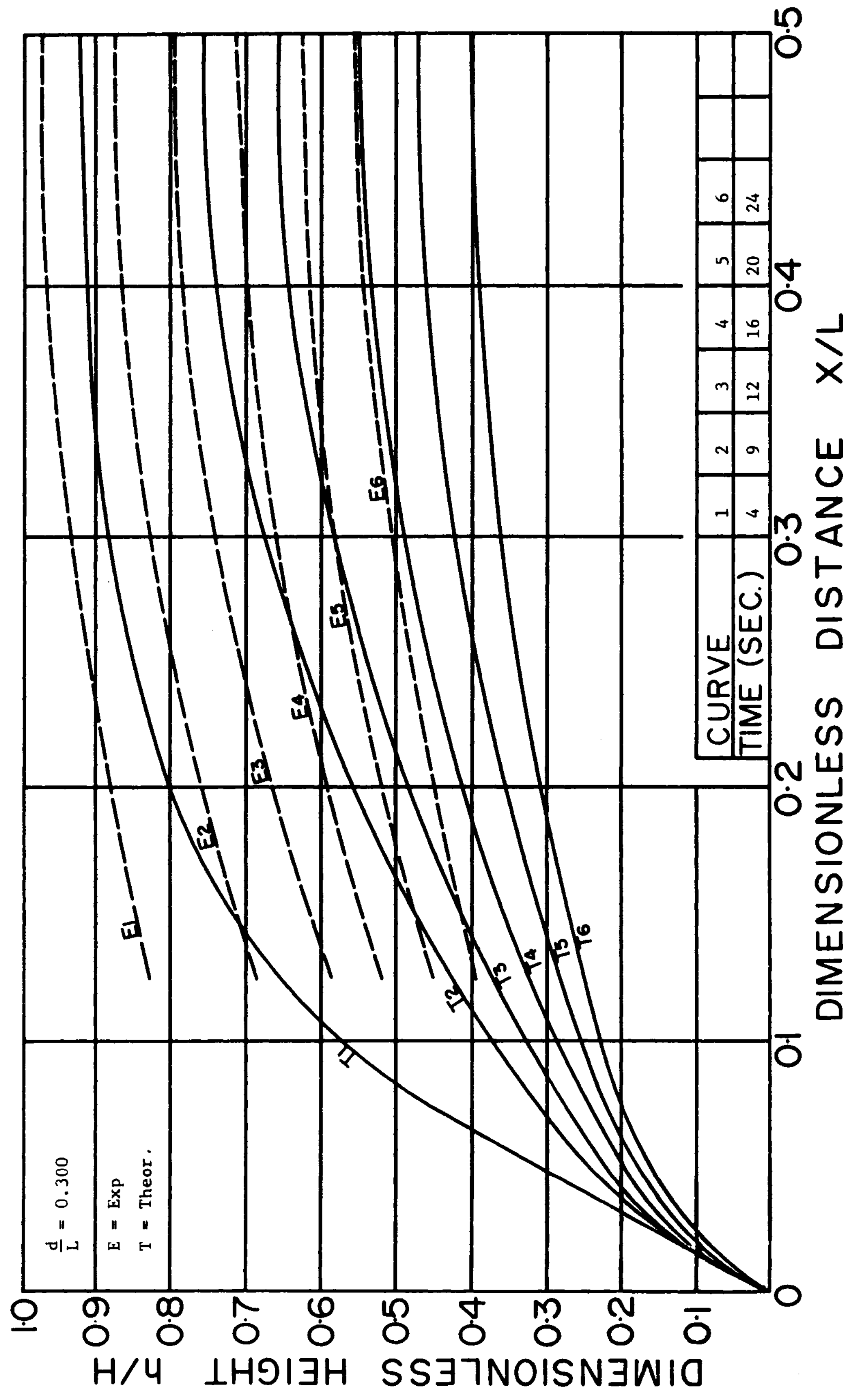

Fig. 19. Experimental and Theoretical Water Table Profiles

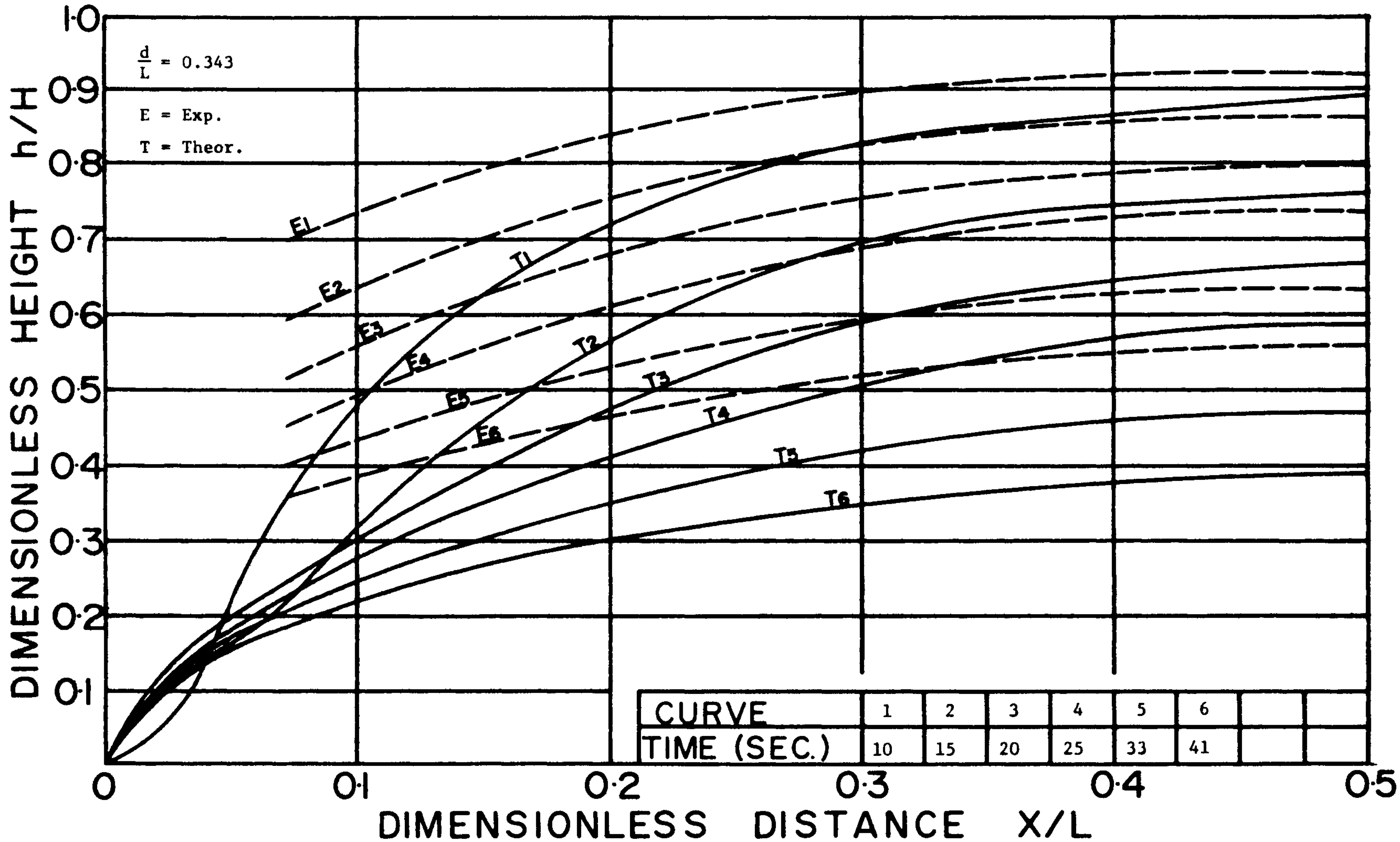

Fig. 20. Experimental and Theoretical Water Table Profiles

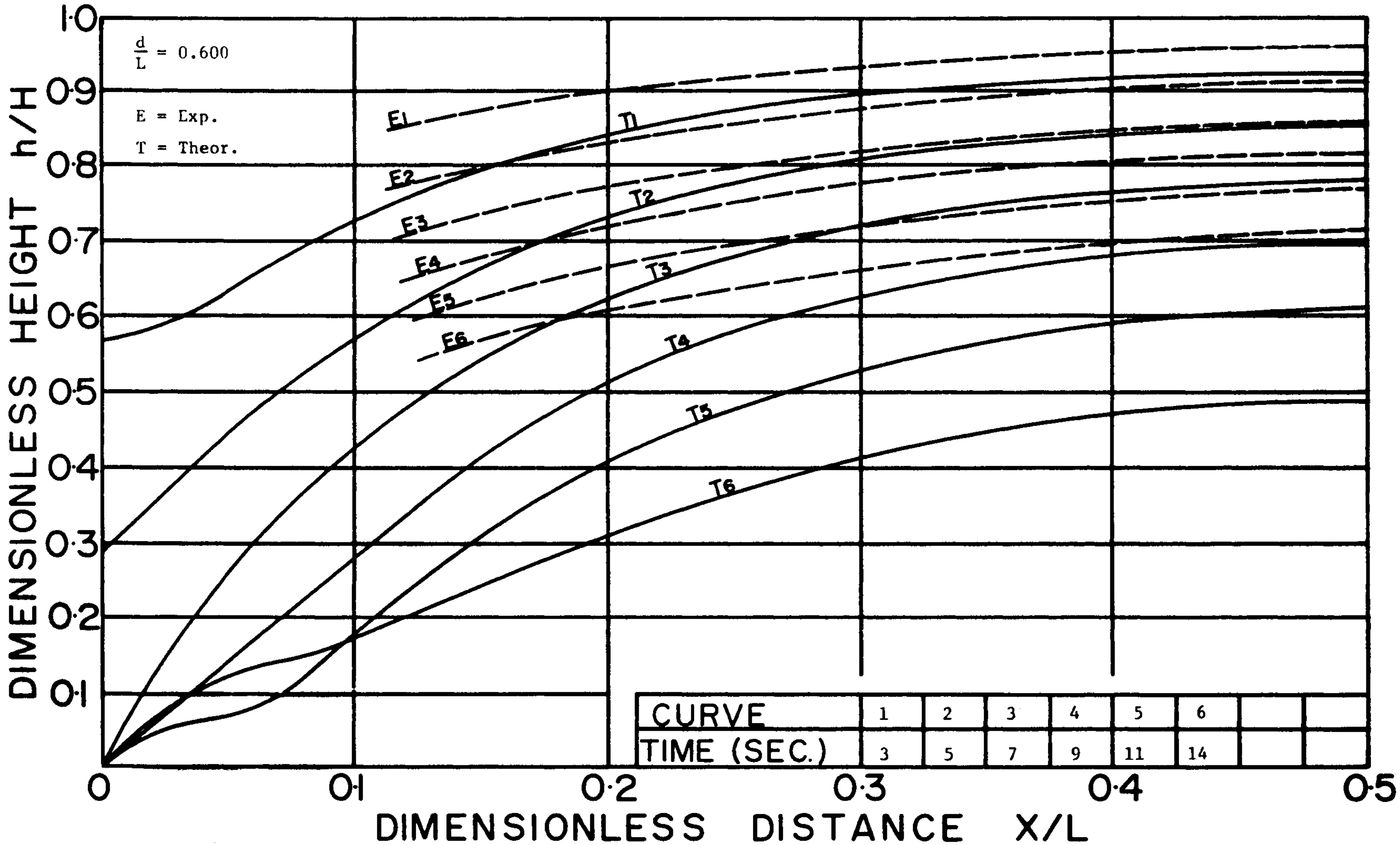

Fig. 21. Experimental and Theoretical Water Table Profiles

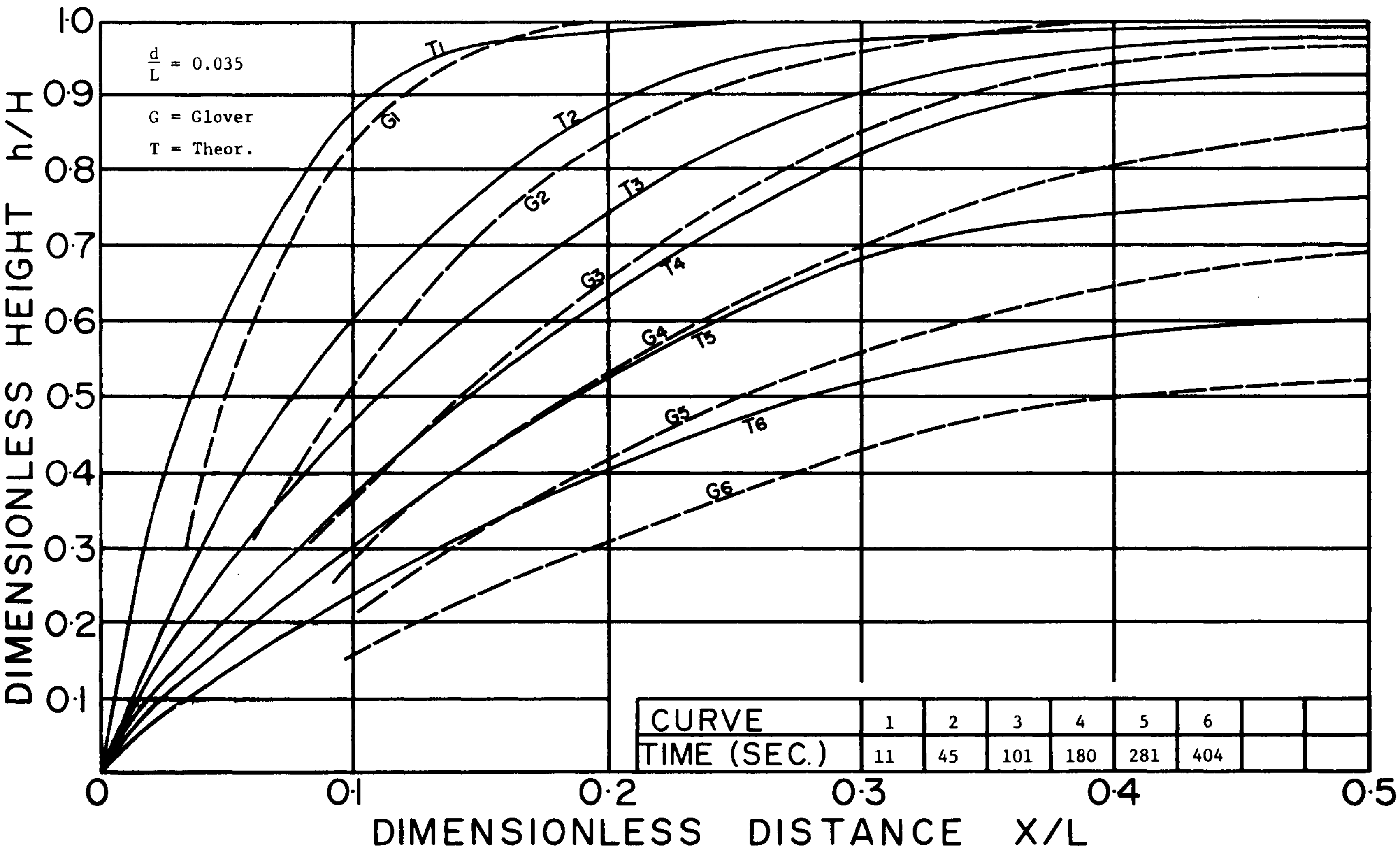

Fig. 22. Glover and Theoretical Water Table Profiles

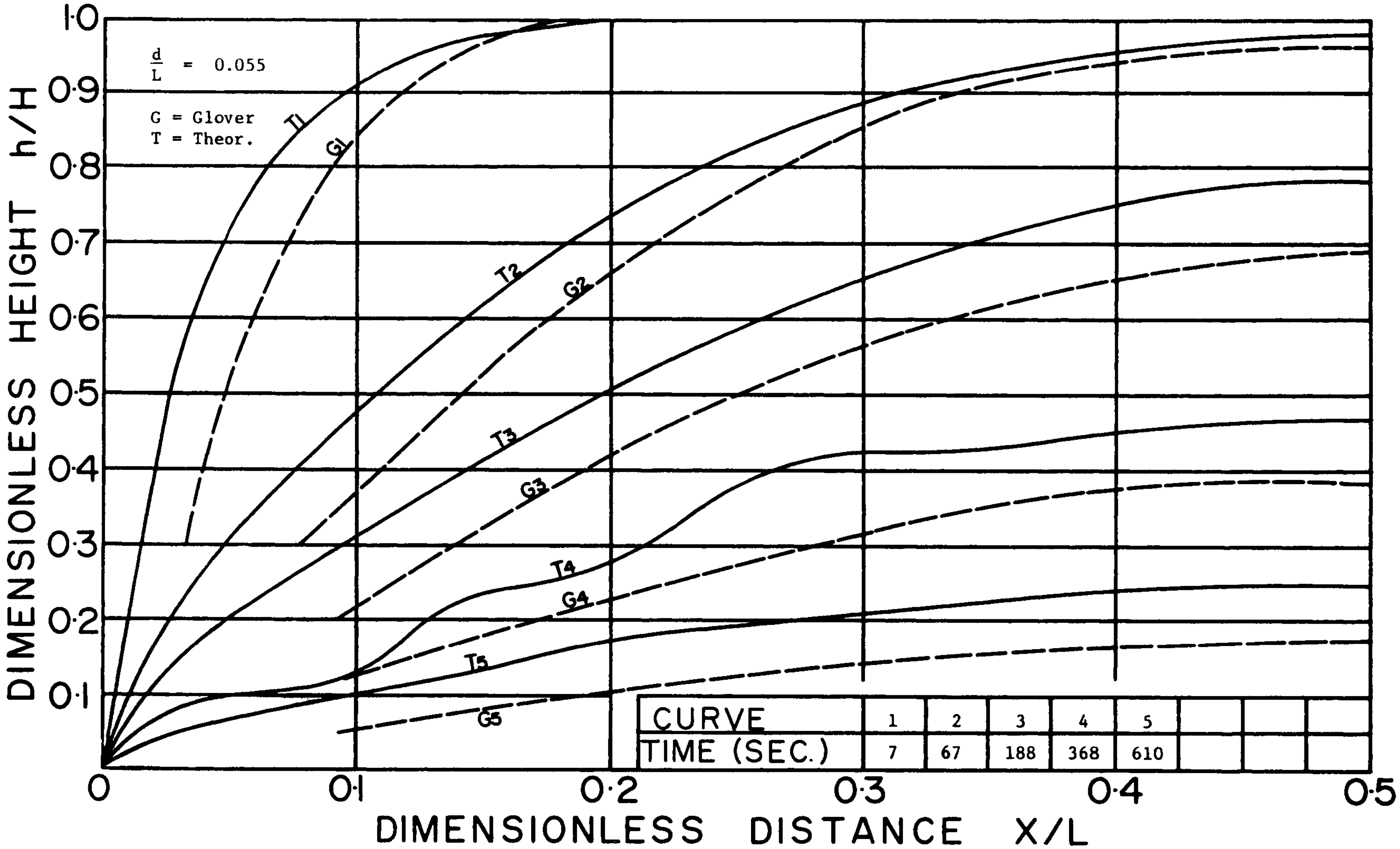

Fig. 23. Glover and Theoretical Water Table Profiles

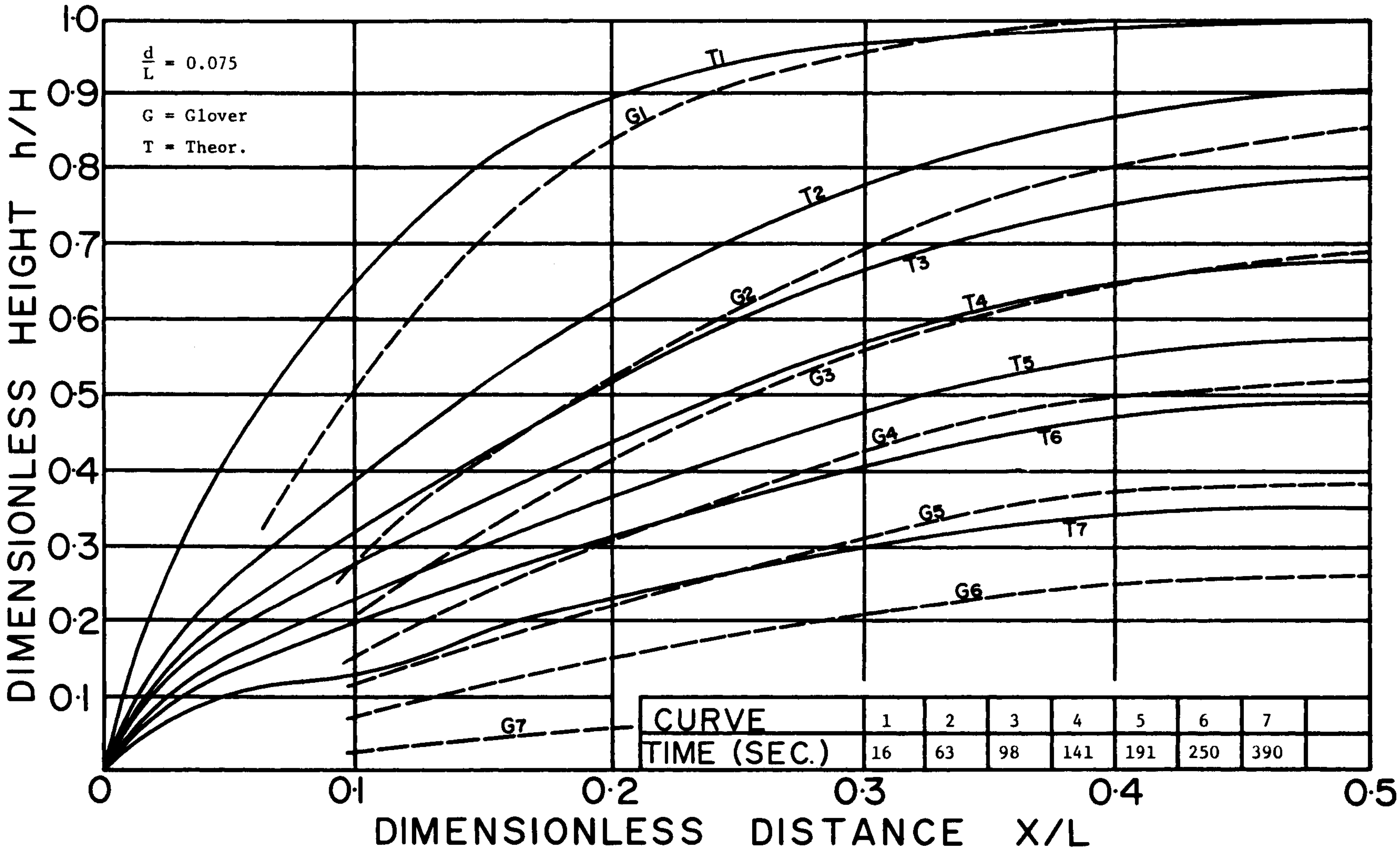

Fig. 24. Glover and Theroetical Water Table Profiles

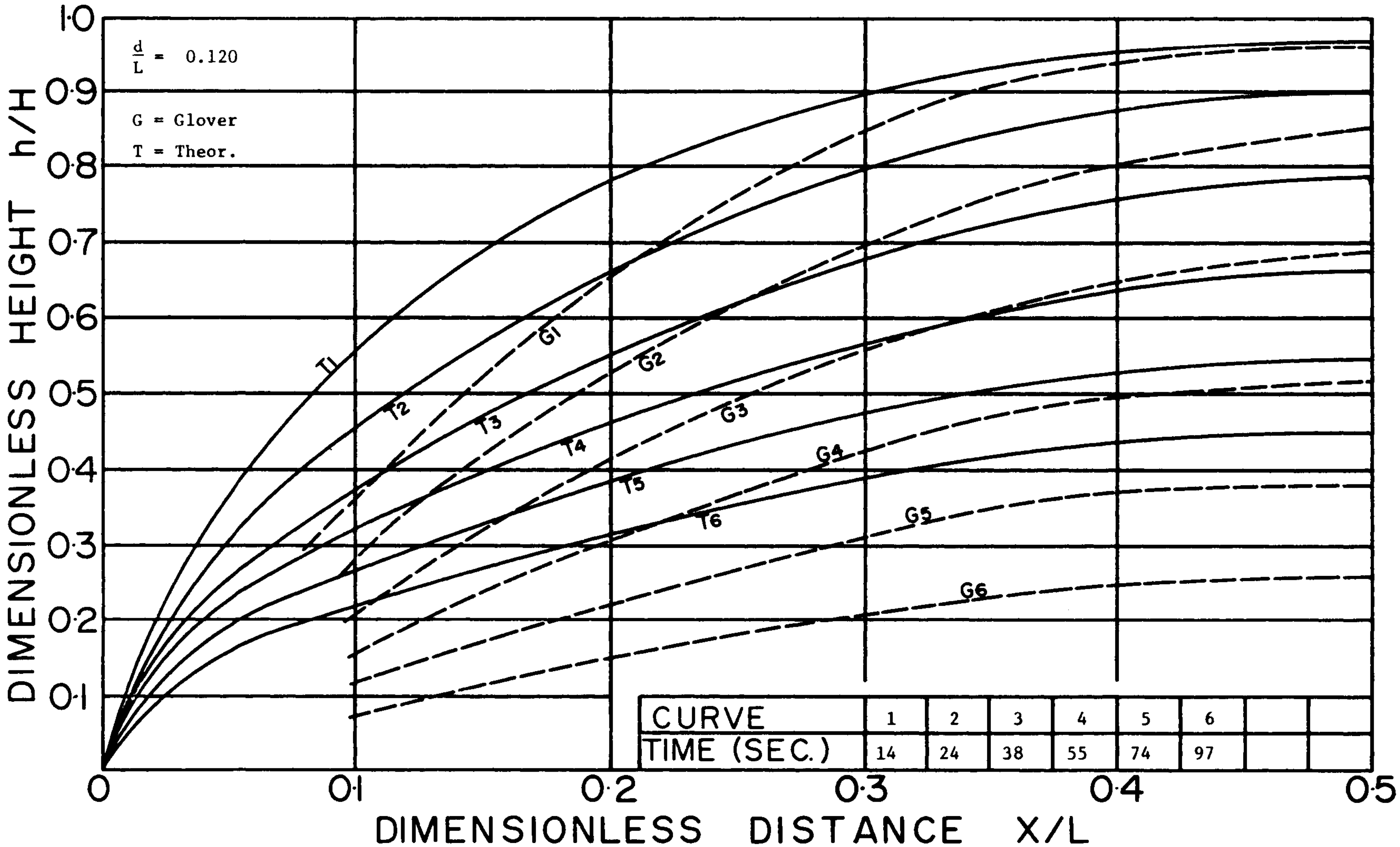

Fig. 25. Glover and Theoretical Water Table Profiles

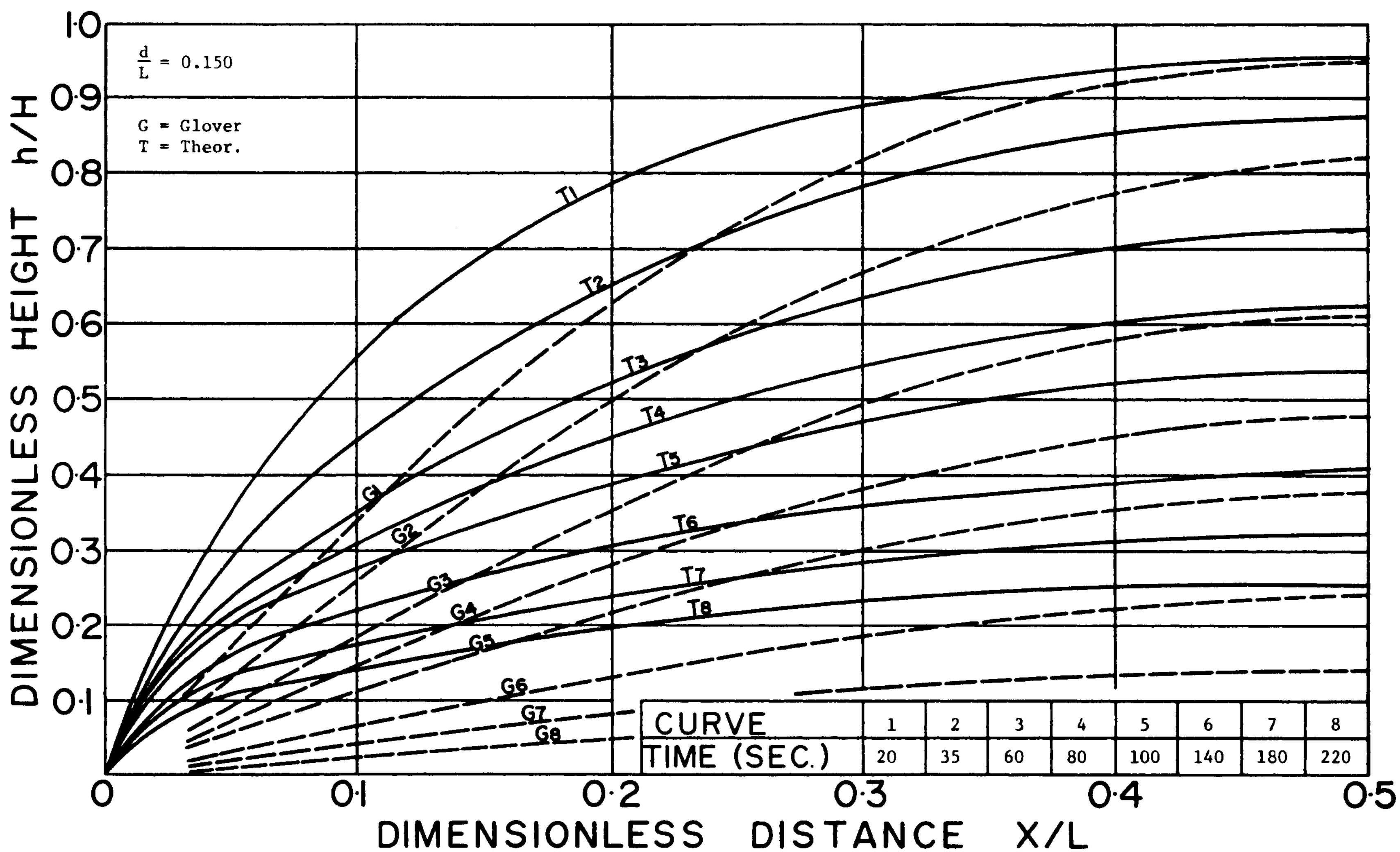

Fig. 26. Glover and Theoretical Water Table Profiles

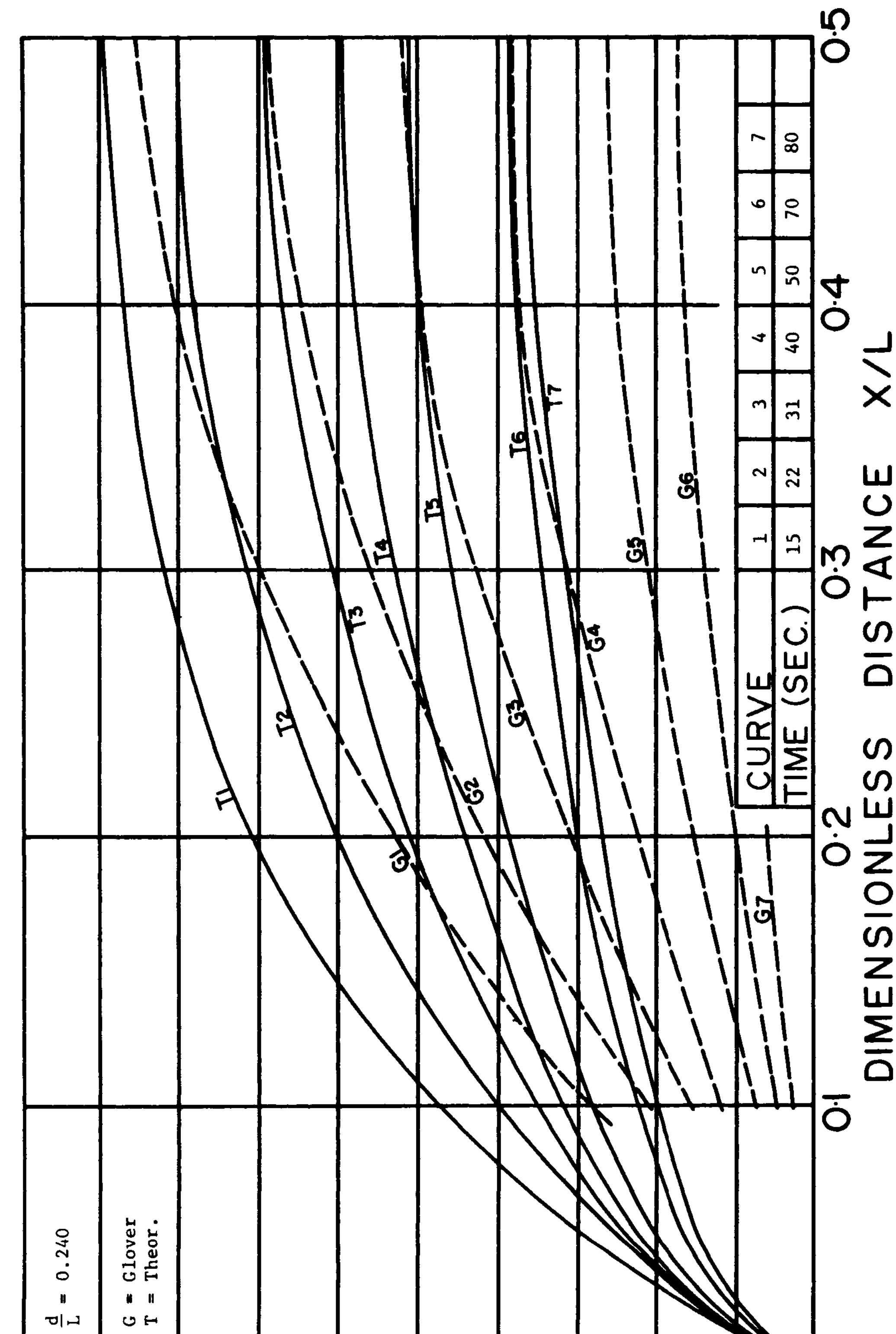

Fig. 27. Glover and Theoretical Water Table Profiles

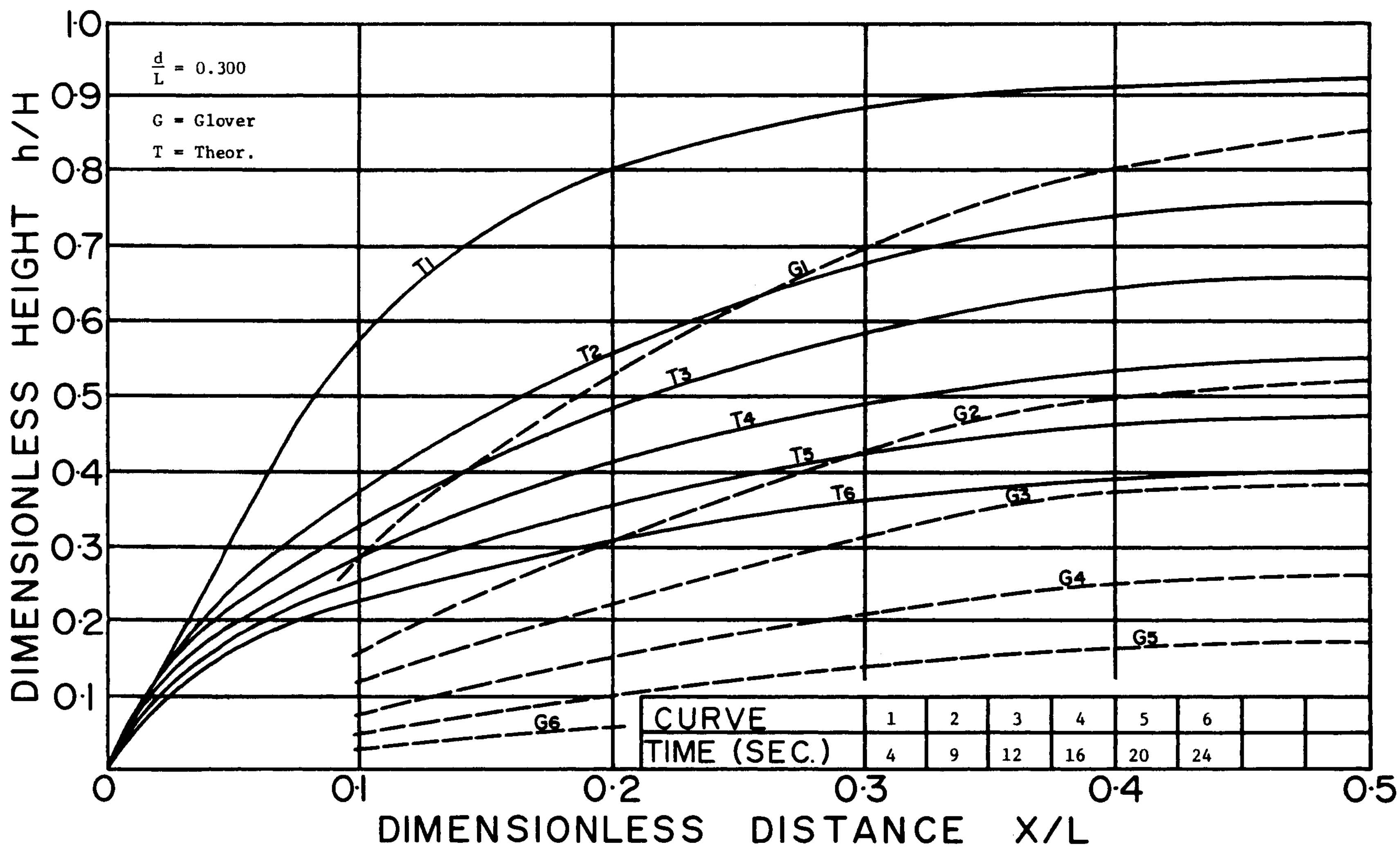

Fig. 28. Glover and Theoretical Water Table Profiles

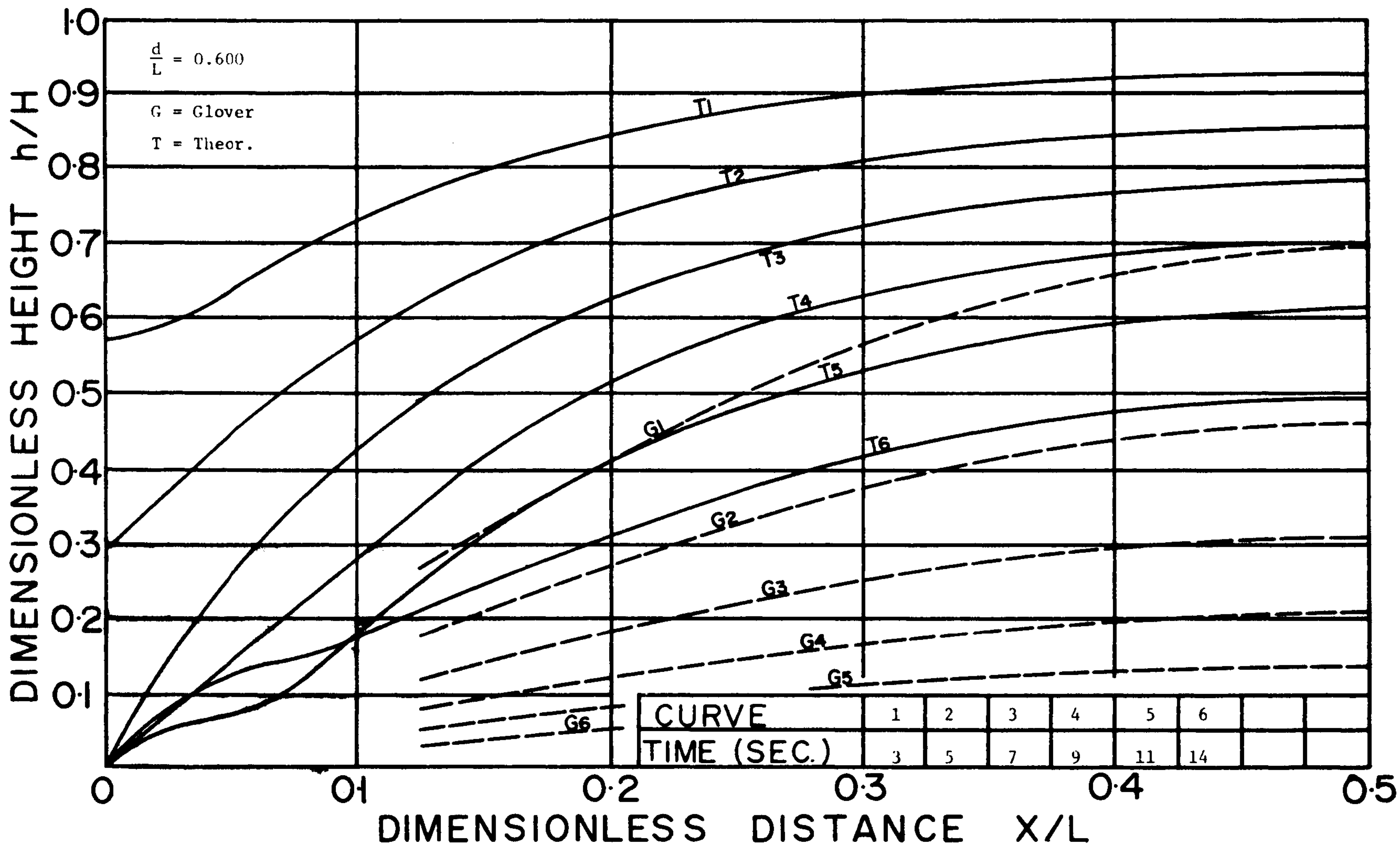

Fig. 29. Glover and Theoretical Water Table Profiles

of $\frac{d}{L}$ and $\frac{H}{d}$ = 0.25. Surface level readings are plotted in Figure22 to29 in the same manner as the data in Table I was plotted.

In Table I and II, the theoretical values refer to answers obtained by the finite difference solution of Laplace's flow equation.

WATER TABLE PROFILES

This thesis is a continuation of a project reported by Shery [1966]. In the initial work performed by Shery [1966], an experimental study, with the aid of a Hele-Shaw model, was completed and water table profiles were compared with those obtained by Glover (Dumm [1954]) for corresponding values of $\frac{d}{L}$ and a fixed ratio of $\frac{H}{d}$. It was found that experimental and predicted water table profiles were reasonably close for small values of $\frac{d}{L}$. However, when values of $\frac{d}{L}$ were larger than 0.1, the comparison steadily deteriorated until the predicted free surface elevations at $\frac{d}{L}$ = 0.600 did not resemble the experimental results. This discrepancy was partially due to the limitations of the Dupuit-Forchheimer assumptions on which Glover's theory was based and the boundary conditions imposed in the development of his theory.

In this study, a numerical solution based on potetntial theory was performed with the belief that predicted water table profiles would compare favourably with experimental results obtained by Shery [1966] because of the more realistic assumptions in the theory. A comparison of water table behaviour, as predicted by experimentation and Dupuit-Forchheimer and potential theories, is presented. Graphical presentation of water table profiles illustrates the accuracy of both Glover's formula and the numerical solution. In addition, the limitations imposed by the assumptions of two quite different drainage theories may be compared.

The solid lines (labelled T) in Figures 7 to 29 are the graphical representation of the numerical solution of Laplace's drainage formula. Drawdown of the initially horizontal water table occurs quite quickly near the drain resulting in steep free surface profiles for small drainage times. As time progresses, the influence of the drain on the water table is felt at increasing distances away from the drain. The value of $\frac{h}{H}$ will decrease at all values of $\frac{x}{L}$ as the drainage time increases; however, the gradient of the water table becomes less pronounced because recession rates tend to be greater farther from the drain. All values of $\frac{h}{H}$ will become zero for any value of $\frac{x}{L}$ when the drainage time goes to infinity.

In Figures 22 to 29, the broken lines represent the solution of Glover's equation (labelled G). For all values of time, the theoretical drawdown profiles, obtained by numerically solving Laplace's flow equation, lag behind those solved by Glover. This difference in elevation between curves increases as the value of $\frac{d}{L}$ becomes larger until there is little resemblance between corresponding water table profiles at a value of $\frac{d}{L} = 0.600$. The rate of fall of the water table is predicted with fairly close agreement for the smaller values of $\frac{d}{L}$ but even for these cases Glover's drainage formula underestimates the recession time. Shery [1966] also found a growing discrepancy in his comparison of experimental and Glover's curves as the ratio of $\frac{d}{L}$ increased. This illustrates that the assumption of horizontal flow lines in Glover's theory is reasonable only when the drain spacing is very large in comparison to the depth of the aquifer.

A comparison of Shery's experimental water tables with those obtained by Laplace's equation in Figures 7 to 21 gives much better results than obtained when Shery [1966] compared his results to Glover's predicted curves.

From the experimental curve, it is evident that some fluid was present directly over the drains for varying values of $\frac{d}{L}$ even after a considerable amount of time had elapsed. In the finite difference solution, the pressure head at the drain was set equal to zero since the drain was assumed to be only partially full. This condition is not as restrictive as the boundary conditions;

when $x = 0$ $h = 0$ for $t>0$

when $x = L$ $h = 0$ for $t>0$

which were imposed in Glover's solution but it did cause a rapid drawdown in the vicinity of the drain. However, the experimental drawdown profiles would suggest that the drains were flowing full and under pressure. This would tend to slow the rate of drawdown and flatten the water tables since the gradient near the drains would be smaller. It is possible that under field conditions tile drains do flow under pressure and free surface profiles would compare more favourably with experimental results presented by Shery [1966]. If a field study verified this condition, the boundary condition at the drains could be changed in the finite difference solution.

Despite the discrepency of drawdown rates in the vicinity of the drains, the corresponding curves compared favourably near the mid-point of the drains. The theory did tend to overestimate the rate of drawdown especially at larger values of the $\frac{d}{L}$ ratio. Some error could be attributed to experimental error in determining the viscosity of the fluid in the Hele-Shaw model and the plate spacing. In addition, the height of fluid standing above the drains was more pronounced for larger values of $\frac{d}{L}$.

The theoretical water table elevations at the mid-point between the drains are found to be more accurate than Glover's when compared with experimental results. Drawdown rates predicted from the finite difference analysis are far more conservative than those based on Dupuit-Forchheimer theory for all values of $\frac{d}{L}$.

SUMMARY AND CONCLUSIONS

A numerical analysis solution of Laplace's flow equation with a moving boundary is presented. Because of the assumptions in the theory, the predicted rate of water table drawdown compares favourably with those obtained experimentally by Shery [1966] over a wide range of $\frac{d}{L}$ values. The water table profiles lag behind those predicted by Glover (Dumm [1954]) at high values of $\frac{d}{L}$ as found by Shery [1966] in a comparison of his experimental water tables with Glover's.

The conclusions of this study may be summarized as follows;

1. Glover's solution overestimates the drawdown rate of the water table between parallel drains.
2. This overestimation increases as the spacing between tile drains decreases (keeping all other dimensions constant).
3. The finite difference solution presented is a definite improvement over Glover's drainage formula.

FURTHER STUDY

An analysis of the recession rates of the water table at the mid-point between drains is not presented. Because of the interest in the highest point on the water table for design purposes, a graphical representation of this recession rate for various values of $\frac{d}{L}$ would be very useful. In addition, water table response to recharge by rainfall or repeated irrigation could be studied by including a factor in the water table which represents this effect.

An investigation of water table profiles, when tile drains are flowing under pressure, can be performed by varying the piezometric head at the drain in the given computer programme.

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NOMENCLATURE

Units are expressed in the (F-L-T) system, where

(F) = Force

(L) = Length

(T) = Time

ΔA an element of cross sectional area of the flow region (L^2).

D the vertical distance between the drain and the impermeable layer (L).

d the average depth of the saturated aquifer equal to $D + \frac{h}{2}$ (L).

g the acceleration due to gravity $\frac{(L)}{(T^2)}$.

H the original height of the water table above the parallel drains before descending (L). In the finite difference equations, the horizontal distance from the curved water table to the closest grid point (L).

h the height of the water table above the tile drain at any point (L).

k the coefficient of permeability, permeability or hydraulic conductivity of a porous medium $\frac{(L)}{(T)}$.

L the horizontal spacing of the tile drains (L).

M In overrelaxation constant formula, the number of vertical mesh lines in the grid (dimensionless).

ΔM the change of mass storage in an element of volume $\Delta x \Delta y \Delta z$ $\frac{(FT^2)}{(L)}$.

N the direction of a line normal to a bounding surface. In overrelaxation constant formula, the number of vertical mesh lines in the grid (dimensionless).

n the porosity of the flow region (dimensionless).

p the hydrostatic pressure $\frac{(F)}{(L^2)}$.

Q the rate of flow of fluid through a porous medium $\frac{(L^3)}{(T)}$.

q the rate of flow of fluid through an element of unit width of aquifer $\frac{(L^2)}{(T)}$.

R In the finite difference equations, the vertical distance from the curved water table to the closest grid point (L).

R_0 In the finite difference equations, the residual at grid point 0, which represents the numerical error at point 0 (L).

S storage coefficient, specific yield, or the amount of water in storage released from a column of aquifer with unit cross section under a unit decline of head (dimensionless).

t the time (T).

u the fluid velocity in the x-direction in the potential theory development $\frac{(L)}{(T)}$.

V the fictitious fluid velocity determined by the Darcy equation $\frac{(L)}{(T)}$.

V_N the component of the velocity normal to a bounding surface $\frac{(L)}{(T)}$.

V_n the component of the fluid velocity normal to the element of area ΔA $\frac{(L)}{(T)}$.

V_x the true fluid velocity in the x-direction $\frac{(L)}{(T)}$.

V_z the true fluid velocity in the z direction $\frac{(L)}{(T)}$.

v the fluid velocity in the y-direction in the potential theory development $\frac{(L)}{(T)}$.

w — the fluid velocity in the z-direction in the potential theory development $\frac{(L)}{(T)}$.

x — the horizontal axis, parallel to the drainage profile

Δx — an element of distance in the x-direction (L). In the finite difference equations, the horizontal spacing of the vertical grid lines (L).

y — the horizontal axis, perpendicular to the drainage profile

Δy — an element of distance in the y-direction (L).

Z — the elevation head above an arbitrary datum (L).

z — the vertical axis, parallel to the drainage profile.

Δz — an element of distance in the z-direction (L). In the finite difference equation, the vertical spacing of the horizontal grid lines (L).

α — the numerical value of $\frac{kd}{S} \frac{(L^2)}{(T)}$.

γ — the specific weight of a fluid $\frac{(F)}{(L^3)}$.

ϕ — the piezometric head (L).

ρ — the density of a fluid $\frac{(FT^2)}{(L^4)}$.

α_{ε} — the compressibility of the aquifer $\frac{(L^2)}{(F)}$.

β_f — the compressibility of a fluid $\frac{(L^2)}{(F)}$.

β — a coefficient which describes the effect of the grid spacing (dimensionless).

ω — the overrelaxation constant (dimensionless).

ω_b — the best estimate of the overrelaxation constant (dimensionless).

λ — an approximation of the limiting value of $\frac{d(n)}{d(n-1)}$ in the Gauss-Seidel method (dimensionless).

APPENDIX A - COMPUTER PROGRAMME

```
132 READ(5,50) LL,MM,IDRN,A,AA,D,DD,H,V,HT
    READ(5,161) DRN,APPR,HY,S,B,ITIME
    DIMENSION T(30,30),N(30,30),R(30,30),QQ(30,30)
    DIMENSION ELEV(30,30),P(30,30),Q(30,30),TIME(900),DRDN(30)
    READ(5,130) (TIME(I),I=1,ITIME)
    READ(5,98) (DRDN(M),M=1,MM)
    I=1
    TT=0.0
    DO 25 L=1,LL
    DO 25 M=1,MM
 25 T(L,M)=APPR
    DO 99 M=1,MM
    DO 99 L=1,LL
 99 R(L,M)=DRDN(M)
 52 DO 51 L=1,LL
    DO 51 M=1,MM
    ELEV(L,M)=HT
    IF (M-MM)51,53,53
 53 HT=HT-V
 51 CONTINUE
134 DO 54 L=1,LL
    DO 54 M=1,MM
    IF (ELEV(L,M)-R(L,M)) 55,100,56
100 IF (L-2) 101,560,560
560 IF (L-IDRN) 101,79,101
101 N(L,M)=8
    GO TO 54
 56 N(L,M) =9
    T(L,M)=0.0
    GO TO 54
 55 IF (M-2) 57,58,58
 58 IF(M-MM) 59,60,60
 59 IF (R(L,M)-ELEV(L,M)-V) 62,61,61
 57 IF (L-IDRN) 80,79,80
 79 N(L,M)=4
    GO TO 54
 80 IF (R(L,M)-ELEV(L,M)-V) 63,64,64
 64 IF (R(L,M+1)-ELEV(L,M)) 78,77,77
 78 Q(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M+1))
    N(L,M)=13
    GO TO 54
 63 IF (R(L,M+1)-ELEV(L,M)) 81,82,82
 82 P(L,M)=R(L,M)-ELEV(L,M)
    N(L,M)=14
    GO TO 54
 81 P(L,M)=R(L,M)-ELEV(L,M)
    Q(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M+1))
    N(L,M)=15
    GO TO 54
 77 IF (L-LL) 65,66,66
 65 N(L,M)=2
    GO TO 54
```

```
 66 N(L,M)=6
    GO TO 54
 61 IF(ELEV(L,M)-R(L,M-1)) 90,90,905
905 IF (ELEV(L,M)-R(L,M+1)) 74,74,906
 90 IF (ELEV(L,M)-R(L,M+1)) 73,73,87
906 Q(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M+1))
    QQ(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M-1))
    N(L,M) = 22
    GO TO 54
 74 Q(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M-1))
    N(L,M)=10
    GO TO 54
 87 Q(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M+1))
    N(L,M)=20
    GO TO 54
 73 IF(L-LL) 67,68,68
 67 N(L,M)=1
    GO TO 54
 68 N(L,M)=5
    GO TO 54
 62 IF (ELEV(L,M)-R(L,M-1)) 75,75,901
 75 IF (ELEV(L,M)-R(L,M+1)) 88,88,89
901 IF (ELEV(L,M)-R(L,M+1)) 76,76,902
902 Q(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M+1))
    P(L,M) = R(L,M)-ELEV(L,M)
    QQ(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M-1))
    N(L,M) = 21
    GO TO 54
 76 Q(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M-1))
    P(L,M)=R(L,M)-ELEV(L,M)
    N(L,M) = 11
    GO TO 54
 88 P(L,M)=R(L,M)-ELEV(L,M)
    N(L,M)=12
    GO TO 54
 89 P(L,M)=R(L,M)-ELEV(L,M)
    Q(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M+1))
    N(L,M)=19
    GO TO 54
 60 IF (R(L,M)-ELEV(L,M)-V) 69,70,70
 70 IF (ELEV(L,M)-R(L,M-1)) 83,84,84
 84 Q(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M-1))
    N(L,M)=16
    GO TO 54
 83 IF (L-LL) 71,72,72
 71 N(L,M) =3
    GO TO 54
 72 N(L,M) = 7
    GO TO 54
 69 IF (R(L,M-1)-ELEV(L,M)) 85,86,86
 86 P(L,M)=R(L,M)-ELEV(L,M)
    N(L,M)=17
    GO TO 54
 85 P(L,M)=R(L,M)-ELEV(L,M)
```

```
      Q(L,M)=(R(L,M)-ELEV(L,M))*H/(R(L,M)-R(L,M-1))
      N(L,M)=18
   54 CONTINUE
      IT = 0
   26 RES = 0.0
      DW=0.0
      DN=0.0
      DO 23 L=1,LL
      DO 23 M=1,MM
      K=N(L,M)
      IF (K-10) 91,91,92
   91 GO TO (1,2,3,4,5,6,7,8,9,10),K
    1 X=T(L,M)+B*(AA/DD*(T(L+1,M)+T(L-1,M))+A/DD*(T(L,M+1)+T(L,M-1))-T(L
     1,M))
      GO TO 24
    2 X=T(L,M)+B*(A/D*T(L,M+1)+AA/DD*(T(L-1,M)+T(L+1,M))-T(L,M))
      GO TO 24
    3 X=T(L,M)+B*(A/D*T(L,M-1)+AA/DD*(T(L-1,M)+T(L+1,M))-T(L,M))
      GO TO 24
    4 X=DRN
      GO TO 24
    5 X=T(L,M)+B*(AA/D*T(L-1,M)+A/DD*(T(L,M-1)+T(L,M+1))-T(L,M))
      GO TO 24
    6 X=T(L,M)+B*(AA/D*T(L-1,M)+A/D*T(L,M+1)-T(L,M))
      GO TO 24
    7 X=T(L,M)+B*(AA/D*T(L-1,M)+A/D*T(L,M-1)-T(L,M))
      GO TO 24
    8 X=R(L,M)
      GO TO 24
    9 X=0.0
      GO TO 24
   10 X=T(L,M)+B*(V**2*Q(L,M)/((H+Q(L,M))*(H*Q(L,M)+V**2))*T(L,M+1)+(H*Q
     1(L,M))/(2.0*(Q(L,M)*H+V**2))*T(L-1,M)+(V**2*H)/((H+Q(L,M))*(H*Q(L,
     1M)+V**2))*ELEV(L,M)+(H*Q(L,M))/(2.0*(Q(L,M)*H+V**2))*T(L+1,M)-T(L,
     1M))
      GO TO 24
   92 J=K-10
      GO TO (11,12,13,14,15,16,17,18,19,20,21,22),J
   11 X=T(L,M)+B*(V*Q(L,M)*P(L,M)/((Q(L,M)+H)*(V*P(L,M)+Q(L,M)*H))*T(L,M
     1+1)+V*H*Q(L,M)/((V+P(L,M))*(Q(L,M)*H+V*P(L,M)))*R(L,M)+H*V*P(L,M)/
     1((Q(L,M)+H)*(V*P(L,M)+H*Q(L,M)))*ELEV(L,M)+H*P(L,M)*Q(L,M)/((V+P(L
     1,M))*(H*Q(L,M)+V*P(L,M)))*T(L+1,M)-T(L,M))
      GO TO 24
   12 X=T(L,M)+B*(V*P(L,M)/(2.0*(P(L,M)*V+H**2))*T(L,M+1)+V*H**2/((V+P(L
     1,M))*(H**2+V*P(L,M)))*R(L,M)+V*P(L,M)/(2.0*(P(L,M)*V+H**2))*T(L,M-
     11)+H**2*P(L,M)/((V+P(L,M))*(H**2+V*P(L,M)))*T(L+1,M)-T(L,M))
      GO TO 24
   13 X=T(L,M)+B*(V**2/(2.0*(V**2+Q(L,M)**2))*ELEV(L,M)+Q(L,M)**2/(2.0*(
     1V**2+Q(L,M)**2))*T(L-1,M)+V**2/(2.0*(V**2+Q(L,M)**2))*ELEV(L,M)+Q(
     1L,M)**2/(2.0*(V**2+Q(L,M)**2))*T(L+1,M)-T(L,M))
      GO TO 24
   14 X=T(L,M)+B*(V*P(L,M)/(2.0*(H**2+V*P(L,M)))*T(L,M+1)+V*H**2/((V+P(L
     1,M))*(H**2+P(L,M)*V))*R(L,M)+V*P(L,M)/(2.0*(H**2+V*P(L,M)))*T(L,M+
     11)+H**2*P(L,M)/((V+P(L,M))*(H**2+P(L,M)*V))*T(L+1,M)-T(L,M))
```

```
      GO TO 24
   15 X=T(L,M)+B*(V*P(L,M)/(2.0*(P(L,M)*V+Q(L,M)**2))*ELEV(L,M)+V*Q(L,M)
     1**2/((V+P(L,M))*(Q(L,M)**2+P(L,M)*V))*R(L,M)+V*P(L,M)/(2.0*(P(L,M)
     1*V+Q(L,M)**2))*ELEV(L,M)+P(L,M)*Q(L,M)**2/((V+P(L,M))*(Q(L,M)**2+V
     1*P(L,M)))*T(L+1,M)-T(L,M))
      GO TO 24
   16 X=T(L,M)+B*(V**2/(2.0*(V**2+Q(L,M)**2))*ELEV(L,M)+Q(L,M)**2/(2.0*(
     1V**2+Q(L,M)**2))*T(L-1,M)+V**2/(2.0*(V**2+Q(L,M)**2))*ELEV(L,M)+Q(
     1L,M)**2/(2.0*(V**2+Q(L,M)**2))*T(L+1,M)-T(L,M))
      GO TO 24
   17 X=T(L,M)+B*(V*P(L,M)/(2.0*(H**2+V*P(L,M)))*T(L,M-1)+H**2*V/((V+P(L
     1,M))*(H**2+P(L,M)*V))*R(L,M)+P(L,M)*V/(2.0*(H**2+V*P(L,M)))*T(L,M-
     11)+H**2*P(L,M)/((V+P(L,M))*(H**2+P(L,M)*V))*T(L+1,M)-T(L,M))
      GO TO 24
   18 X=T(L,M)+B*(V*P(L,M)/(2.0*(P(L,M)*V+Q(L,M)**2))*ELEV(L,M)+V*Q(L,M)
     1**2/((V+P(L,M))*(Q(L,M)**2+P(L,M)*V))*R(L,M)+V*P(L,M)/(2.0*(V*P(L,
     1M)+Q(L,M)**2))*ELEV(L,M)+P(L,M)*Q(L,M)**2/((V+P(L,M))*(Q(L,M)**2+V
     1*P(L,M)))*T(L+1,M)-T(L,M))
      GO TO 24
   19 X=T(L,M)+B*(H*V*P(L,M)/((Q(L,M)+H)*(H*Q(L,M)+V*P(L,M)))*ELEV(L,M)+
     1H*V*Q(L,M)/((P(L,M)+V)*(V*P(L,M)+H*Q(L,M)))*R(L,M)+V*P(L,M)*Q(L,M)
     1/((H+Q(L,M))*(H*Q(L,M)+V*P(L,M)))*T(L,M-1)+H*Q(L,M)*P(L,M)/((V+P(L
     1,M))*(V*P(L,M)+H*Q(L,M)))*T(L+1,M)-T(L,M))
      GO TO 24
   20 X=T(L,M)+B*(H*V**2/((H+Q(L,M))*(H*Q(L,M)+V**2))*ELEV(L,M)+H*Q(L,M)
     1/(2.0*(V**2+H*Q(L,M)))*T(L-1,M)+V**2*Q(L,M)/((H+Q(L,M))*(H*Q(L,M)+
     1V**2))*T(L,M-1)+H*Q(L,M)/(2.0*(V**2+Q(L,M)*H))*T(L+1,M)-T(L,M))
      GO TO 24
   21 X=T(L,M)+B*(V*P(L,M)*QQ(L,M)/((Q(L,M)+QQ(L,M))*(QQ(L,M)*Q(L,M)+V*P
     1(L,M)))*ELEV(L,M)+V*Q(L,M)*QQ(L,M)/((V+P(L,M))*(QQ(L,M)*Q(L,M)+V*P
     1(L,M)))*R(L,M)+V*P(L,M)*Q(L,M)/((Q(L,M)+QQ(L,M))*(QQ(L,M)*Q(L,M)+V
     1*P(L,M)))*ELEV(L,M)+Q(L,M)*P(L,M)*QQ(L,M)/((V+P(L,M))*(QQ(L,M)*Q(L
     1,M)+V*P(L,M)))*T(L+1,M)-T(L,M))
      GO TO 24
   22 X=T(L,M)+B*(V**2*QQ(L,M)/((QQ(L,M)+Q(L,M))*(V**2+QQ(L,M)*Q(L,M)))*
     1ELEV(L,M)+Q(L,M)*QQ(L,M)/(2.0*(QQ(L,M)*Q(L,M)+V**2))*T(L-1,M)+V**2
     1*Q(L,M)/((QQ(L,M)+Q(L,M))*(V**2+QQ(L,M)*Q(L,M)))*ELEV(L,M)+QQ(L,M)
     1*Q(L,M)/(2.0*(QQ(L,M)*Q(L,M)+V**2))*T(L+1,M)-T(L,M))
   24 RES = RES+ABS(T(L,M)-X)
      DN =ABS(T(L,M)-X)
      IF (DN-DW) 23,23,46
   46 DW=DN
   23 T(L,M) = X
      IT=IT+1
      IF (DW-0.001) 28,28,26
   28 CONTINUE
  604 WRITE(6,30) IT,RES,DW
   39 DO 33 M=1,MM
      DO 33 L=1,LL
      IF (ELEV(L,M)-R(L,M)) 105,915,33
  915 IF (L-2) 105,33,33
  105 IF(R(L,M)-ELEV(L,M)-V) 165,165,33
  165 IF (L-2) 166,107,107
  166 Y=0.0
```

```
      Z=((T(L+1,M)-T(L,M))/V)**2-(T(L+1,M)-T(L,M))/V
      GO TO 36
  107 IF (R(1,1)-R(1,5)) 401,113,401
  401 IF (M-2) 108,109,109
  109 IF (M-MM) 110,113,113
  108 IF (R(L,M)-DRN) 112,112,116
  112 R(L,M)=DRN
      GO TO 36
  113 Y=0.0
      Z=((T(L,M)-R(L,M))/(R(L,M)-ELEV(L,M)))**2-(T(L,M)-R(L,M))/(R(L,M)-
     1ELEV(L,M))
      GO TO 36
  110 IF (N(L,M)-12) 115,114,115
  114 IF (R(L,M)-R(L,M-1)) 117,116,116
  115 IF (N(L,M)-11) 911,116,911
  116 IF (R(L-1,M+1) - ELEV(L-1,M+1)) 910,204,204
  910 IF (R(L,M)-R(L,M+1)) 205,113,113
  911 IF (N(L,M)-19) 113,117,113
  204 Y=T(L,M+1)+(R(L,M)-ELEV(L,M))*(T(L-1,M+1)-T(L,M+1))/V
      GO TO 206
  205 Y=T(L,M+1)+(R(L,M)-ELEV(L,M))*(R(L,M+1)-T(L,M+1))/(R(L,M+1)-ELEV(L
     1,M+1))
  206 Y=((Y-R(L,M))/H)**2
      Z=((T(L,M)-R(L,M))/(R(L,M)-ELEV(L,M)))**2-(T(L,M)-R(L,M))/(R(L,M)-
     1ELEV(L,M))
      GO TO 36
  117 IF (R(L-1,M-1)-ELEV(L-1,M-1)) 207,208,208
  208 Y=T(L,M-1)+(R(L,M)-ELEV(L,M))*(T(L-1,M-1)-T(L,M-1))/V
      GO TO 209
  207 Y=T(L,M-1)+(R(L,M)-ELEV(L,M))*(R(L,M-1)-T(L,M-1))/(R(L,M-1)-ELEV(L
     1,M-1))
  209 Y=((R(L,M)-Y)/H)**2
      Z=((T(L,M)-R(L,M))/(R(L,M)-ELEV(L,M)))**2-(T(L,M)-R(L,M))/(R(L,M)-
     1ELEV(L,M))
   36 DRDN(M)=HY/S*(Y+Z)*TIME(I)
   33 CONTINUE
      DO 125 L=1,LL
      DO 125 M=1,MM
      R(L,M)=R(L,M)-DRDN(M)
      IF (R(L,M)-DRN)131,131,125
  131 R(L,M)=DRN
  125 CONTINUE
      TT=TT+TIME(I)
      WRITE(6,37) TT
      WRITE(6,38) ((R(L,M),M=1,5),L=1,1)
      IF (MM-10) 202,202,203
  202 WRITE(6,38) ((R(L,M),M=6,MM),L=1,1)
      GO TO 145
  203 WRITE(6,38) ((R(L,M),M=6,10),L=1,1)
      IF (MM-15)140,140,141
  140 WRITE(6,38) ((R(L,M),M=11,MM),L=1,1)
      GO TO 145
  141 WRITE(6,38) ((R(L,M),M=11,15),L=1,1)
      IF (MM-20)142,142,143
```

```
142 WRITE(6,38) ((R(L,M),M=16,MM),L=1,1)
    GO TO 145
143 WRITE(6,38) ((R(L,M),M=16,20),L=1,1)
    IF (MM-25) 300,300,301
300 WRITE(6,38) ((R(L,M),M=21,MM),L=1,1)
    GO TO 145
301 WRITE(6,38) ((R(L,M),M=21,25),L=1,1)
    WRITE(6,38) ((R(L,M),M=26,MM),L=1,1)
145 CONTINUE
    IF(I-ITIME) 133,133,135
133 I=I+1
    GO TO 134
135 CONTINUE
    GO TO 132
 30 FORMAT (5X,I5,E14.6,3X,E14.6)
 29 FORMAT (10F7.3)
161 FORMAT (5F9.4,I3)
 98 FORMAT (10F6.2)
210 FORMAT (2E16.8,2I3)
 37 FORMAT (6X,//30HHEIGHT OF WATER TABLE AT TIME ,F6.1,5H SEC.,/)
 50 FORMAT(3I3,7F9.4)
130 FORMAT(12F6.1)
 38 FORMAT (5E14.6)
    END
```

APPENDIX B - COMPUTER PROGRAMME NOMENCLATURE

APPR	the initial assumed potential at each grid point
A,AA D,DD	coefficients which describe the effect of the grid spacing
B	the over-relaxation constant
DW	the largest residual in the flow region after a complete iteration
DRN	the height of the drain above the impermeable boundary
H	the horizontal grid spacing
HY	the hydraulic conductivity
IDRN	the horizontal grid line representing the drain location
IT	the number of iterations
ITIME	the number of time steps
LL	the number of horizontal grid lines in the initial flow region
MM	the number of vertical grid lines in the initial flow region
RES	the residual at a grid point
S	the specific storage
TT	the elapsed drainage time
V	the vertical grid spacing
Y	the horizontal component of the water table equation
Z	the vertical components of the water table equation
DRDN(M)	the initial height of the water table above the impermeable boundary and the subsequent drawdown increments

ELEV(L,M)	the elevation of each grid point above the impermeable boundary
N(L,M)	classification of grid point location with respect to the water table and other boundaries
P(L,M)	the partial vertical mesh length from the water table to a grid point
Q(L,M)	the partial horizontal mesh length from the water table to a grid point
QQ(L,M)	the partial horizontal mesh lengths from the water table to a grid point
R(L,M)	the water table elevations above the impermeable boundary
T(L,M)	the value of the piezometric head at each grid point
TIME(I)	the drainage time steps

VITA AUCTORIS

1944	Ronald Bryan Wigle was born in Windsor, Ontario Canada, on November 9, 1944.
1949	In September, 1949, entered Kingsville Public School in Kingsville, Ontario.
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