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# Switching strategies for optimal control of third order systems using logic.

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# SWITCHING STRATEGIES FOR OPTIMAL CONTROL OF THIRD ORDER SYSTEMS USING LOGIC

BY

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# ABSTRACT

A design for time optimal third order bang bang control systems is proposed. It is known that a time optimum performance can be obtained by properly timing the relay operations: two switchings being required for optimal response of third order systems. By observing the movement of the state point in two dimensional projections of phase space, it is shown that optimal performance can be achieved if the first switching is delayed after the state point has crossed the optimum trajectories in both planes. The coordinate axis and the optimum boundary divide the phase space into sections described by logic variables, from which controllers have been developed for a third order system with two integrators and one time constant, and also for a system with one integrator and two time constants.

The designs of a delay circuit and function generator are also presented. The function generator determines the projections of the optimum trajectories, while the delay circuit delays the first switching operation.

In the second phase of this work, the optimal strategy developed earlier is simulated on a digital computer as a controller. Projections of optimal curves are obtained by storing sections of actual system trajectories in the form of tables in the computer memory. Optimal switching instants are determined by shifting and comparing the stored trajectory with the actual trajectory. Due to the in-

sensitivity of the required matrix transformation to the system parameters over a workable range, and that the switching curves are obtained by constantly storing and updating the system trajectories, the control remains practically optimum even when there is a change in parameters. The proposed method needs only the measurement of error in the system, derivatives of error are formed by using the difference equations, a technique ideally suited for digital formulation of state variables. Thus any direct measurements of the derivatives of error are avoided. Digital simulation results are presented to establish the usefulness of the control strategy.

## ACKNOWLEDGMENTS

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CHAPTER I

# INTRODUCTION

The time optimum performance of a control system is obtained when the system operates in the minimum time mode, the driving force applied is the maximum and the algebraic sign of the driving force is properly controlled. For ideal performance in which the position error is reduced to zero with no overshoot in minimum time, a bang bang or a relay controlled system is the best choice. The optimum response is achieved when the relay systems are used in conjunction with properly selected non-linear networks called the controllers. Maximum plant driving force must be used at all times except when the error is zero. Such a performance is possible with relays that have no dead zone, but do have a neutral position so that the contact is open when no signal is applied to the relay. Positive force is used to accelerate the output, but in order to prevent overshoot negative force is used to decelerate the output. Thus a control intelligence of some type is required to reverse the relay before the error reaches zero. The control elements determine only the instants of time at which the relay switches, otherwise the system is left alone to follow its own equations of motion without being affected by the error and its derivatives. Various switching criterions have been established in the literature for switching of relays and are described in the next section.

## 1.1 SURVEY OF LITERATURE

The earlier studies of contactor servo mechanisms were mainly centered on methods of improving system stability. In 1950 McDonald [1] suggested the application of phase plane analysis to optimize performance. He showed that for a second order system the switching criterion could be expressed as a non linear relationship between the error and the error rate. Optimum switching boundary was represented as a trajectory on the phase plane. A similar idea was also contained in a paper submitted by Hopkin[2] in 1951. He assumed the force applied to the load to be saturated and the resultant error signal included an intentionally non linear function of the output velocity. Similar ideas were discussed by Uttley and Hammond[3]. Later, in 1953, Neiswander and McNeal[4] extended Hopkins proposal to complex second order systems by considering output velocity as the independant variable with consequent optimization by non linearization of the rate feedback.

In 1954 Bognor and Kazda[5] extended the phase plane criteria for second order contactor servos to a phase space criteria for higher order contactor servos. The system error and its derivatives formed the principal coordinate phase space and were reduced to zero in minimum time. Starting at any point in phase space cooresponding to the initial conditions, the representative point moved in a series of discontinuous trajectories, determined by the sign of the controlled variable, eventually reaching the origin. The analysis presupposed a discontinuous forcing function, fixed in magnitude and reversible in sign, supplied to a dynamic linear system. The control, though non linear in a general sense can be more precisely described as piece wise linear. The sequence of sign reversals represented the switching criterion, the number of sign reversals being one less than the order of the system for optimum performance. Bognor and Kazda's switching criterion was based on the assumption that the error and its (n-1) order derivatives remain continuous through the switching transient. This assumption holds if uninterrupted circuits are available during switching. In 1955 Chang[6] suggested the method of open circuit switching i.e. keeping the contactor open during switching transient. Because of the large momentary counter voltage at the instant of open circuit switching, the value of higher order derivatives drops to zero in negligible time. This required removal of stored energy at the switching boundary by using shorting capacitors and inductors when the relay reversed. This method improved the overall response and the computer which formed the counter part of the controller was considerably simplified.

The concept of phase space techniques as applied to the design of higher order optimal servo systems was further extended by Kuba and Kazda [7,8] and Hopkin and Iwama[9]. As before, non linear circuits were intentionally introduced into the servo system to obtain optimum performance. Because of the complexity of the switching criteria for minimum response time, the control circuitory needed to accomplish the switching program reaches vast proportions for higher order systems. Kuba and Kazda proposed a method which determined precisely the quantitative nature of the non linearity to be inserted in the control system so that its response would approach as closely as desired to the response of the optimum contactor servo of the same order. Since the method of calculating the phase space trajectories from analytic expressions for E, $\dot{E}$ and $\ddot{E}$ is exceedingly tedious, Hopkin and Iwama obtained the switching instants (intersection between critical boundaries and the system trajectories) by the technique of reversing time axis by the use of an analog computer. This implied writing the differential equation by replacing t with -t and using the initial condition as the final steady state, $E = \dot{E} = \ddot{E} = 0$. This work was later extended to random inputs[10].

In practical systems the problem underlying optimization of higher order systems is lack of proper function generators to simulate the switching boundaries. Doll and Stout[11] describe the generation of the switching curves for a particular third order system. They used an electro optical two variable function generator for use in an analog computer investigation. In 1961, Garret[12] proposed a scheme for a third order system using positive-negative feedback. The control system operated in positive feedback for the first part of the transient response period and then switched into negative feedback for the later part of the response.

The optimum switching condition for step inputs was a linear function of the position error and its first and second time derivative. Flügge Lotz[13] presented analytical studies and analog simulation of third order contactor servos based on generalized linear switching functions. A still more generalized analytical approach was published in 1964 by Choudhary and Choudhary[14].

Several other schemes have been proposed during the past decade for optimization of relay controllers. However, their practical applications are limited to second order systems. Mitsumaki[15] gave a new method which replaced the switching curve with a switching line and nearly optimum response could be obtained by applying a simply calculated corrective action. Brown [16] tried to impose self optimization by adopting a pre-defined switching function. Mills[17] proposed a system which used a linear network in the forward path. This system, though simpler in construction, gave a poor response. Pandya[18] designed a bang bang controller using a three phase induction motor.

Until recently most of the studies concerning the relay controllers were analytic in nature with system simulation on analog computers. This put a restriction on the system to have fixed parameters, as for any variation in parameters new optimum trajectories had to be calculated requiring modification in the function generators. With the rapid growth of digital computer applications, some authors have tried introducing digital computers or logic circuits as a part of the controller. Smith[19], Elgerd and Scheiber[20] have developed logic circuit controllers for second order systems using switching circuits. In order to consider the effect of change in system parameters, some authors have used digital computer controllers. However, the use of digital computers has been restricted to off line computation of pre calculated trajectories or to using iterative techniques to find the switch points[21]. Sutton and Tomlinson[22] simulated a Ward Leonard system as a second order control system on an analog computer while the control was effected by a digital computer. Lee[23] synthesized time optimal adaptive control by combining off line memorization with simple on line calculations to determine the control signal. The approximation was obtained by linearizing the projection of the adaptive switching hyper surface in a subspace, and finally storing the parameters of the hyper planes in memory. The on line calculations corresponded to a function generator consisting of the memory containing the parameters and a linear interpolator. A threshold logic device was trained to obtain the desired control signal. The hybrid scheme of off-line memorization and on-line calculation was based on the concept of switching hyper surface. The generation of these surfaces is by no means simple even for a third order system. Pearson[24] presented an algorithm and simulation studies for adaptive control of linear systems. He used a modified gradient procedure for adjusting the parameters without the knowledge of the plant dynamics.

## 1.2 SCOPE OF PRESENT WORK

In general a number of authors have presented analysis of optimum control for higher order systems. As such, a number of rigorous mathematical theories based on the variational calculus approach leading to Pontryagin's maximum principle and Bellman's dynamic programming are now available. These can be applied to nth order systems resulting in n-dimensional switching hyper surfaces and curves. However, it is difficult to apply these to find switching instants even for third order systems as the complexity of computation increases with the order of the system. For third order systems the locations of torque reversal, which are lines in the phase plane for second order systems, become surfaces in three dimensional space. In case of still higher order systems these torque reversal surfaces are difficult to visualize. Also the generation of these surfaces is no easy problem. Moreover the surfaces cannot always be obtained in an explicit form, with the result that it is difficult to obtain unique switching instants. Because of the difficulty and expense of generating optimum reversal curves and because of the uncertainty of maintaining optimum performance in the face of noise and other inputs of arbitrary form, the practical applications of optimum relay control theory have been severely limited.

In view of the above discussion the study of contactor servo mechanisms is an interesting field and the problem of developing simple and cheap controllers for third order systems is worth considering. It was the purpose of this investigation to develop a simple logical design of optimum bang bang controllers for third order systems. In this work phase space analysis is used to design the controller. Since there are only two forcing functions +F and -F, every point in the phase space has only two trajectories passing through it, the switching boundary divides the phase space into these two regions and the position of any state point can thus be determined with respect to these regions. The function generator simulates the projections of the optimal switching boundary. Truth tables are developed defining locations of each state point in terms of the three coordinates of the phase space and the function generator variable. Based on these values, a logic circuit controller is designed which gives commands to the relay depending on the position of the state point. A delay circuit is used to delay the first switching interval to achieve the minimum time response.

Due to the nature of the method used it was considered worthwhile to examine the possibility of making the system adaptive by storing the system optimum trajectories in the memory of a digital computer. If this is done, then only the measurement of error is necessary as the derivatives of error may be formed by using difference equations. Having obtained the optimum trajectories generated from the actual system, the switching instants are determined from the stored and actual trajectories.

# CHAPTER II

## DEVELOPMENT OF LOGIC CONTROLLER: System with Two Integrators and One Time Constant

A conventional relay control system can be represented in the form of a block diagram as shown in figure 2.1. The relay is assumed to be an ideal one, with no dead zone, hysterisis or delay time. Without the controller, this will cause a positive driving force or accelerating torque to be applied to the system, and a negative error will result in a negative driving force. Zero error will completely remove force from the system. For optimum response, since error and all of its derivatives are to be simultaneously reduced to zero, it is required to consider the effect of error and its derivatives in determining the algebraic sign of the driving force. This sets a condition on the system that the output response depends on the forward or reverse torque applied.

### 2.1 PHASE SPACE ANALYSIS

For a third order system described by the transfer function

$$G(s) = \frac{K}{S^2(s+1)} \tag{2.1}$$

the normalized differential equation can be written as:

$$\frac{d^3E}{dt^3} + \frac{d^2E}{dt^2} = \pm F \tag{2.2}$$

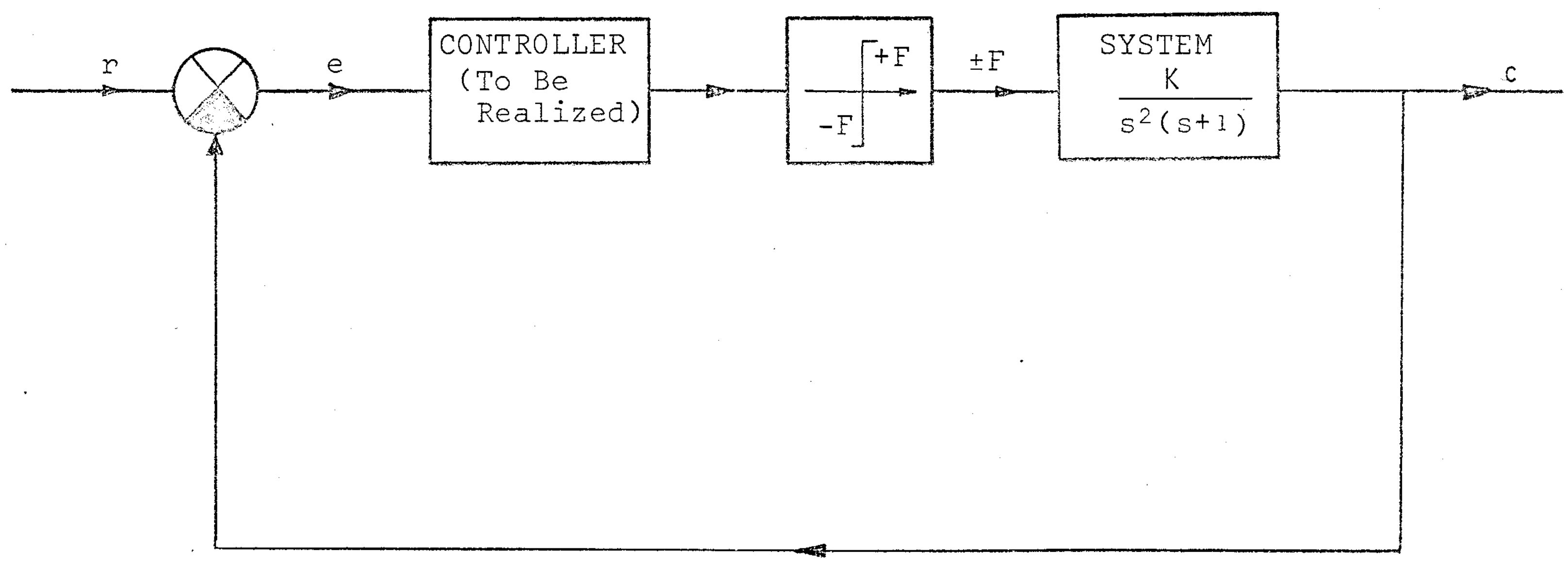



FIG. 2.1 BLOCK DIAGRAM OF THE SYSTEM

Replacing the driving force by $\delta$, where

$$\delta = +1 \quad \text{for} \quad F < 0$$
$$\delta = -1 \quad \text{for} \quad F > 0 \qquad (2.3)$$

If the state variables are defined as

$$E = X1 \qquad (2.4)$$

$$\dot{E} = X2 \qquad (2.5)$$

$$\ddot{E} = X3 \qquad (2.6)$$

The vector differential equations for the system become

$$\begin{bmatrix} \dot{X}1 & (t) \\ \dot{X}2 & (t) \\ \dot{X}3 & (t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} X1 & (t) \\ X2 & (t) \\ X3 & (t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \delta \qquad (2.7)$$

Since two of the eigenvalues are equal, the transformation matrix is of the form

$$P = \begin{bmatrix} 1 & 0 & 1 \\ \lambda_1 & 1 & \lambda_3 \\ \lambda_1^2 & 2\lambda_1 & \lambda_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2.8)$$

Where $\lambda_1 = \lambda_2 = 0$, $\lambda_3 = -1$ are the eigenvalues of the system. The transformed coordinates are given by

$$Y = P^{-1}X$$

$$\begin{bmatrix} \dot{Y}1 \\ \dot{Y}2 \\ \dot{Y}3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} Y1 \\ Y2 \\ Y3 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \delta \qquad (2.9)$$

The equation (2.9) may be written as

$$\frac{dY1}{dt} = Y2 - \delta \qquad (2.10)$$

$$\frac{dY2}{dt} = \delta \qquad (2.11)$$

$$\frac{dY3}{dt} = -Y3 + \delta \qquad (2.12)$$

From the equations (2.10) and (2.11), after eliminating the variable t, we get

$$\frac{dY1}{Y2-\delta} = \frac{dY2}{\delta}$$

Since $\delta = \pm 1$ (it can be transferred to the numerator) and the above equation can be written as

$$dY1 = \delta\ (Y2 - \delta)\ dY2$$

Integrating both sides

$$2Y1 - \delta(Y2-\delta)^2 = C1 \qquad (2.13)$$

From the equations (2.11) and (2.12) after eliminating the variable t

$$\frac{dY2}{\delta} = \frac{dY3}{-Y3+\delta}$$

Dividing both sides of the equation by $\delta$, we obtain

$$dY2 = \frac{dY3}{1-\delta Y3} \qquad (\text{Since } \delta^2 = 1)$$

Integrating both sides

$$Y2 = -\frac{1}{\delta}\ \ell n\ (1-\delta Y3) + C2$$

or

$$Y2 + \delta \ell n(1-\delta Y3) = C2 \tag{2.14}$$

The equations of optimal trajectories are hence given by

$$(2\ Y1 + \delta) - \delta(Y2 - \delta)^2 = 0 \tag{2.15}$$

$$Y2 + \delta\ \ell n(1-\delta\ Y3) = 0 \tag{2.16}$$

Furthermore we should note that

X1 = X2 = X3 = 0 implies that Y1 = Y2 = Y3 = 0 as it is a fixed end point minimum time optimal bang bang problem. The total time required to move from one state point to another is given by the equation

$$Y2 = \delta t + \alpha_2 \quad (\alpha_2 \text{ is initial value of } Y2) \tag{2.17}$$

If $\Delta$ is defined as the increment then

$$\Delta\ Y2 = \delta(\Delta t) \tag{2.18}$$

which means that the change in the coordinate Y2 in the right direction gives the time taken from one point to the other.

## 2.2 DEVELOPMENT OF SWITCHING CRITERION FOR OPTIMUM RESPONSE. [25]

It is well established that for optimum response, the system must reach the origin in the minimum possible time and this depends on the initial state of the system. Furthermore the switch curve is the locus of all states which can be forced to the origin by the control (+F, -F) without any switching. The system therefore normally follows an initial trajectory, then a trajectory along the switching surface which intersects the switch curves. Thus two switching sequences are possible i.e. (+F, -F, +F) or (-F, +F, -F).

In order to define the logic variables to describe the position of any state point with respect to the coordinate axes and the optimal trajectories, the whole phase space is divided into ten sections. Six of these sections are six octants of the three dimensional phase space and the other four sections are formed by dividing the other two octants by the optimal trajectories. These sections are shown in figures 2.2(a) and 2.2(b). It can be seen that the force is positive in sections 3,5,7,9, and 10, and is negative in sections 1,2,4,6, and 8. Sections 4 and 7 also include the optimal trajectories.

The function generator variable is defined as R, which designates the position of the state point with respect to the optimal trajectories. In the Y1 - Y2 plane R is positive, if the state point lies on or above the optimal trajectories and negative if it lies below. Similarly in the Y2 - Y3 plane R is positive if the point lies to the right of the optimal trajectories and negative if it lies to the left. The notation of any variable X as X(+) and X(-) [20] is used to define the value of X as positive or negative respectively with regard to the magnitude. Thus the criterion may be denoted as:

The force F is positive if

Y1(-), Y2(-), Y3(-), R(+) Section (3)

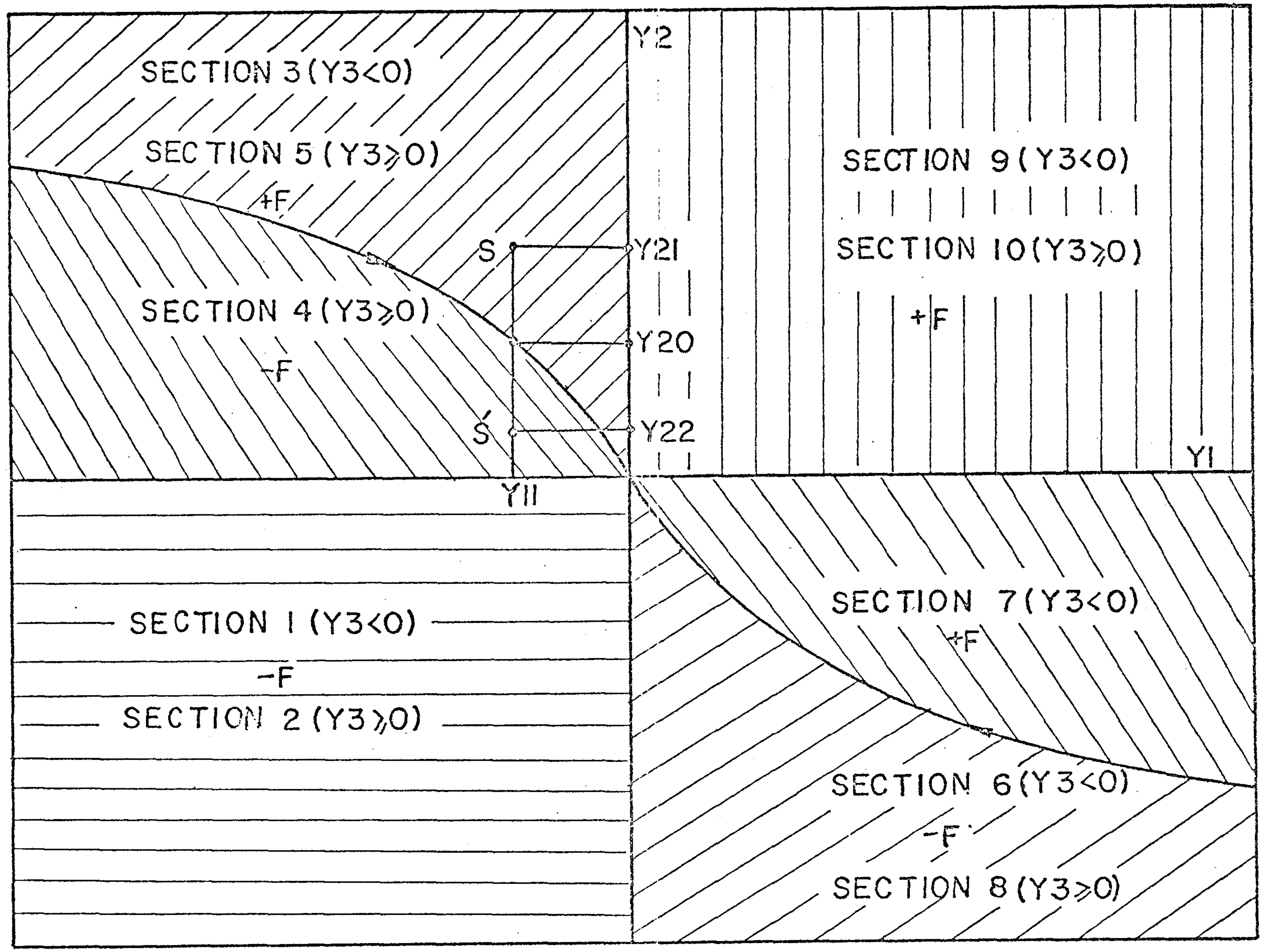



FIG 2.2(a) SECTIONS DESCRIBING LOGIC VARIABLES IN Y1-Y2 PLANE

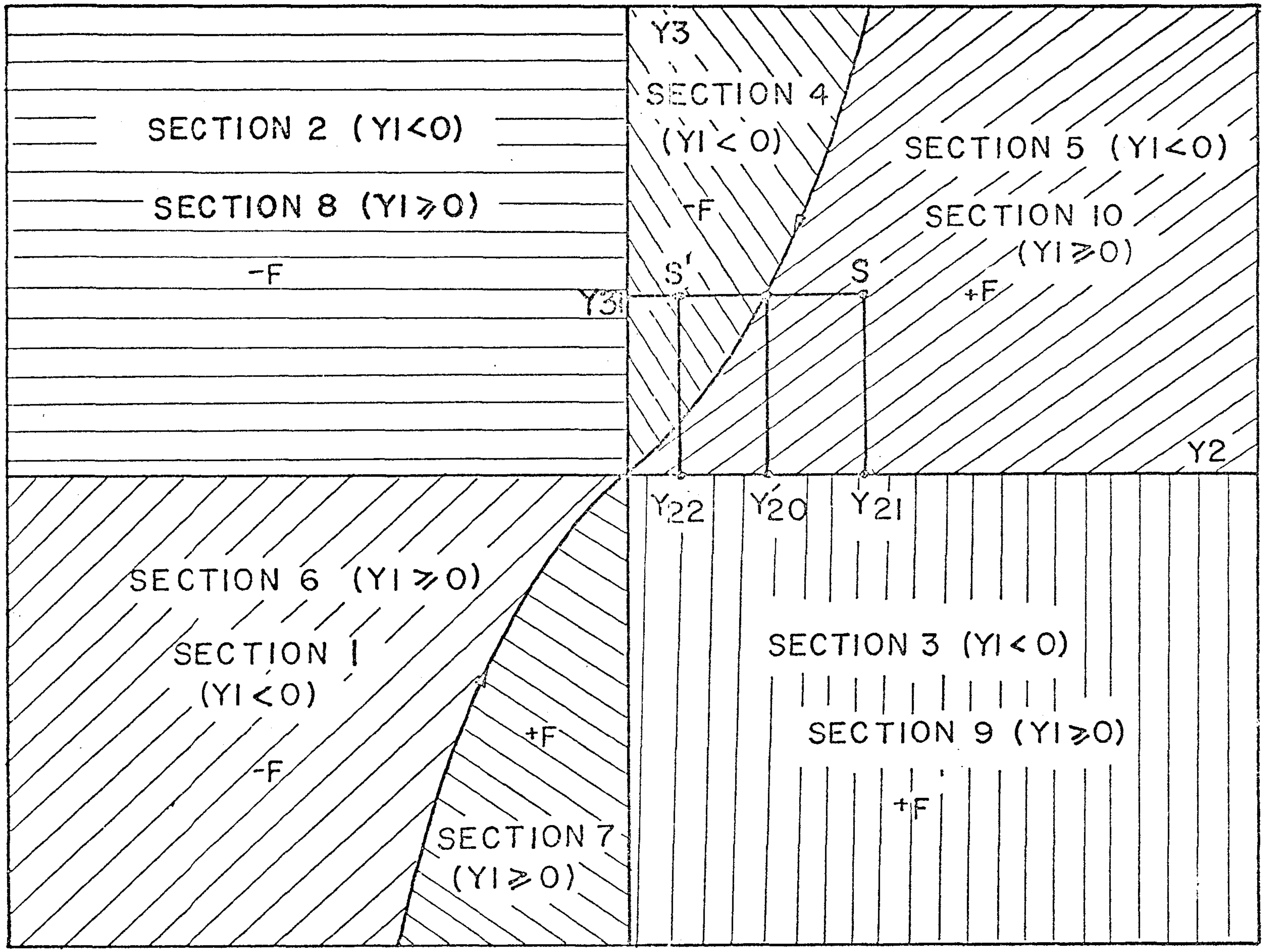



FIG 2.2(b) SECTIONS DESCRIBING LOGIC VARIABLES IN Y2-Y3 PLANE

| | | | | |
|---|---|---|---|---|
| Y1(-), | Y2(+), | Y3(+), | R(+) | Section (5) |
| Y1(+), | Y2(-), | Y3(-), | R(+) | Section (7) |
| Y1(+), | Y2(+), | Y3(-), | R(+) | Section (9) |
| Y1(+), | Y2(+), | Y3(+), | R(+) | Section (10) |

The force F is negative if

| | | | | |
|---|---|---|---|---|
| Y1(-), | Y2(-), | Y3(-), | R(-) | Section (1) |
| Y1(-), | Y2(-), | Y3(+), | R(-) | Section (2) |
| Y1(-), | Y2(+), | Y3(+), | R(-) | Section (4) |
| Y1(+), | Y2(-), | Y3(-), | R(-) | Section (6) |
| Y1(+), | Y2(-), | Y3(+), | R(-) | Section (8) |

The force F is zero if

Y1 = Y2 = Y3 = 0

## 2.3 FUNCTION GENERATOR

The value of R is determined from the comparison of the value of Y2 corresponding to the state point with the value Y2 which is greater in magnitude of the two values of Y2 corresponding to the points of intersection of the optimal trajectories and the vertical lines passing through the state point in the Y1 - Y2 and Y2 - Y3 planes. For example, in figures 2.2(a) and 2.2(b) consider the projections of the state point S having the coordinates Y11, Y21, Y31. In the Y1 - Y2 plane let the point of intersection of the vertical line passing through S with the optimal trajectory be Y11, Y20 and in the Y2 - Y3 plane let the intersection of the horizontal line, through S with the optimal trajectory be Y2́0, Y31. If |Y21| is greater than or equal to the greater of $|Y20|$ and $|Y'20|$ , R is positive, otherwise it is negative. Thus the state point S has the variable R as positive and $S'$ as negative.

The optimum trajectories from the equations (2.15) and (2.16) can be simulated using analog function generators and comparators. The simulation equations are:

$$YP = \sqrt{2|Y1|+1} - 1 \quad \text{from (2.15)} \tag{2.19}$$

$$YQ = \ln(1+|Y3|) \quad \text{from (2.16)} \tag{2.20}$$

$$YR = YP - YQ \tag{2.21}$$

$$\begin{aligned} YS &= YP \ (\text{IF } YR \geqslant 0) \\ YS &= YQ \ (\text{IF } YR < 0) \end{aligned} \tag{2.22}$$

$$W = -YS\ \text{SGN}(Y1) \tag{2.23}$$

$$\begin{aligned} R &= 1 \ (\text{IF } Y2 \geqslant W) \\ R &= 0 \ (\text{IF } Y2 < W) \end{aligned} \tag{2.24}$$

The corresponding function generator block diagram is given in figure 2.3.

## 2.4 DETERMINATION OF SWITCHING INSTANTS.

In the previous sections a criterion has been developed which locates any state point in the phase space. The function generator keeps track of the system states and determines when the system enters the +F region from the -F region or vice versa. This requires that the state point must change regions in both the projection planes though it is most likely that the change occurs in one plane prior to the other, as shown in figures 2.4(a) and 2.4(b). At first it might be expected that the relay should switch as the state point changes region in both the planes in order to reverse the force. But switching at this instant causes the system to follow the projection of the positive optimal trajectory in either the Y1 - Y2 plane or the Y2 - Y3 plane depending upon whether the change occurs in the Y2 - Y3 plane first or in the Y1 - Y2 plane first. Thus before the second switching for optimal control could be applied, the system reaches the origin of either of the two planes, bringing only two of the coordinates to zero. In this way the second switching becomes ineffective. Since transformed coordinates are used, all the coordinates must be simultaneously zero for the desired response.

This can be achieved by delaying the first switching instant after the state point has changed regions. The procedure for evaluating the delay is given in the next section and works out to be 0.11 units of time. Because it has been found that if the coordinate Y2 is changed by an amount 0.11 from the instant the system changes regions, the system follows a trajectory which intersects the negative optimum trajectory. Using equation (2.18) this gives a delay of 0.11 units of Y2(time) from the instant the state point enters the new region. The second switching occurs accurately at the optimum boundary giving the desired response. If the second switching is also delayed for some reason a third switching will bring the system to rest as shown in figures 2.4(a) and 2.4(b). If such unwanted switchings occur a number of times, the relay might chatter

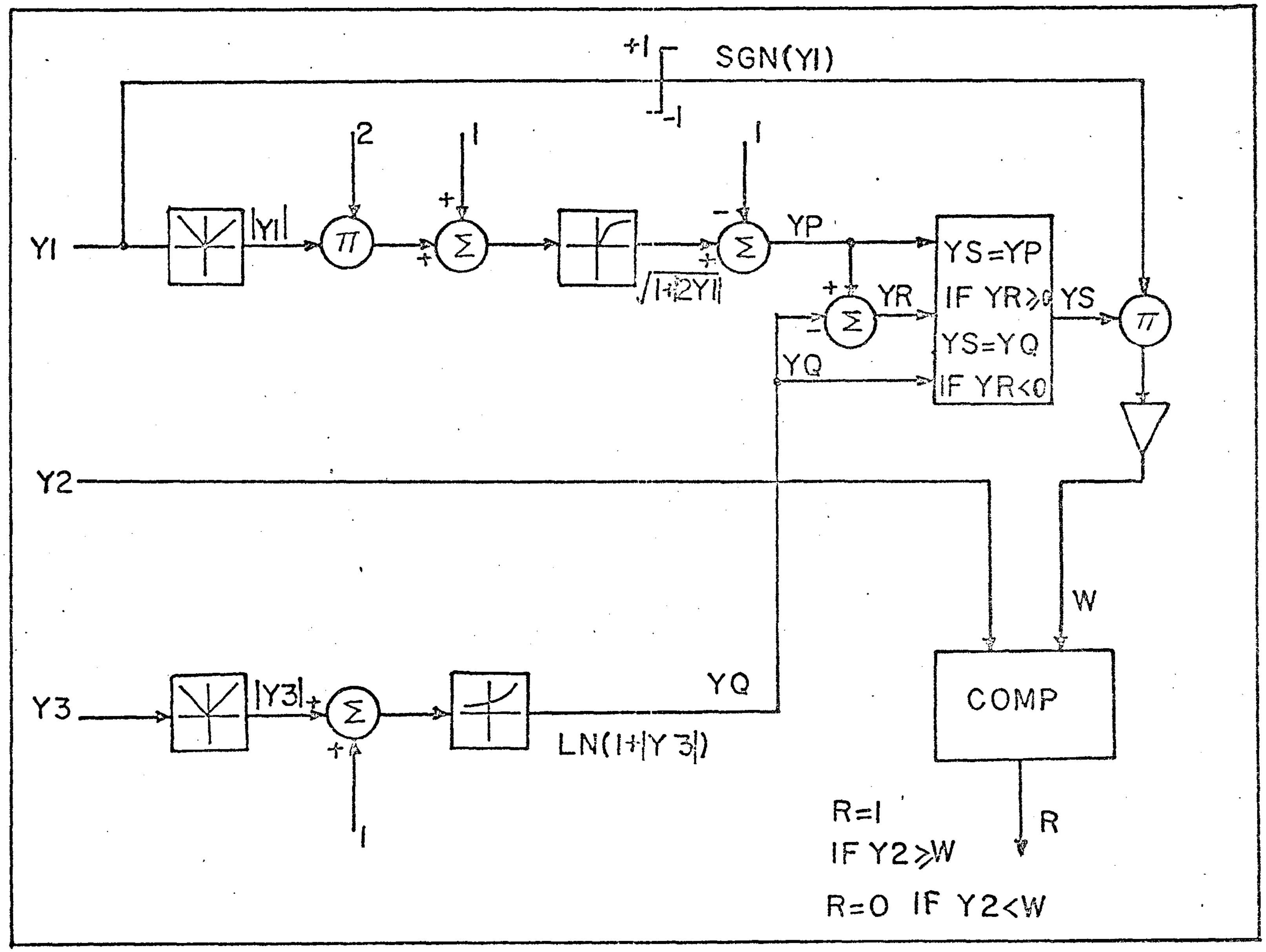



FIG 2.3 FUNCTION GENERATOR

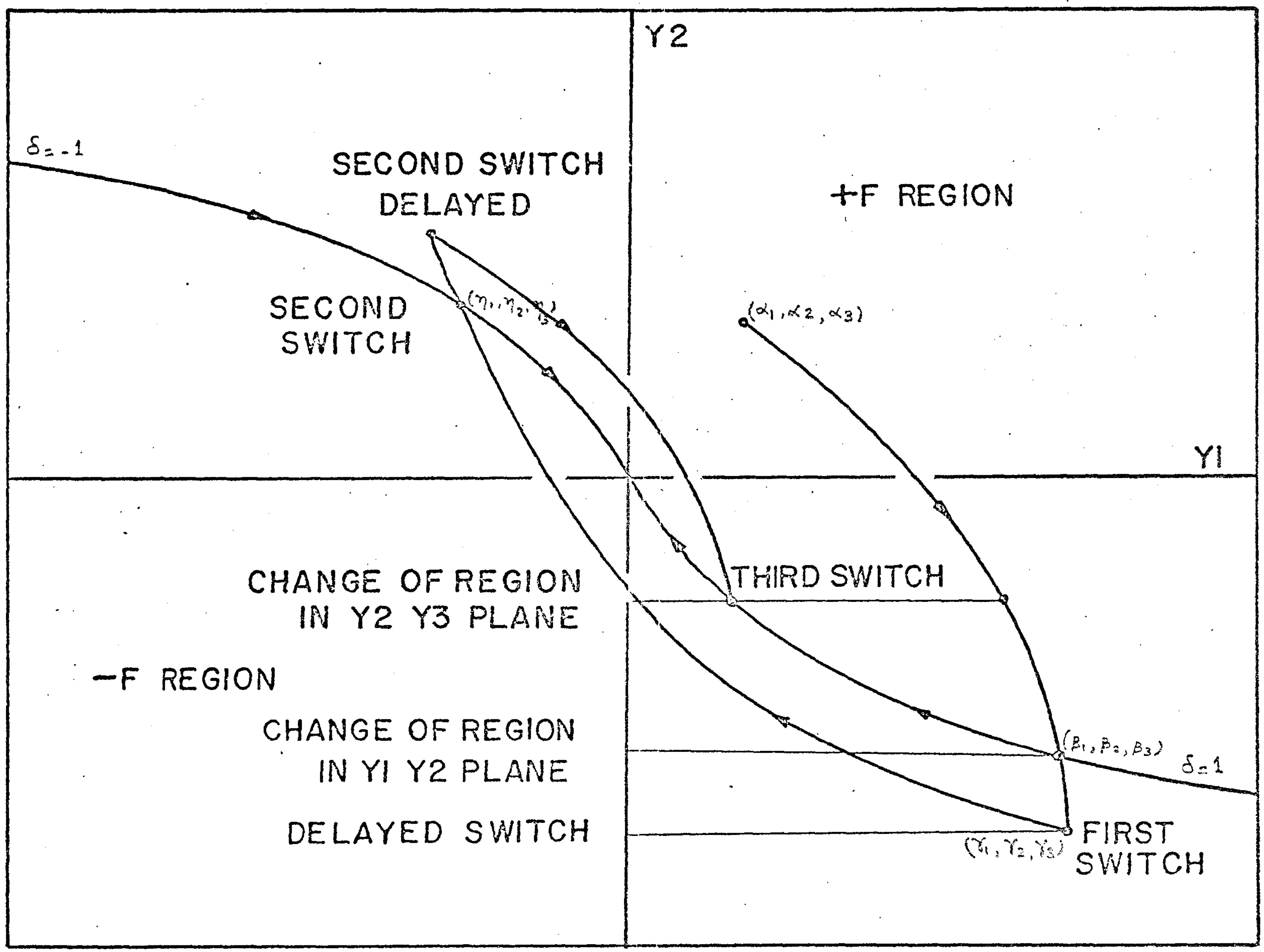



FIG 2.4(a) SWITCHING CRITERION IN Y1–Y2 PLANE

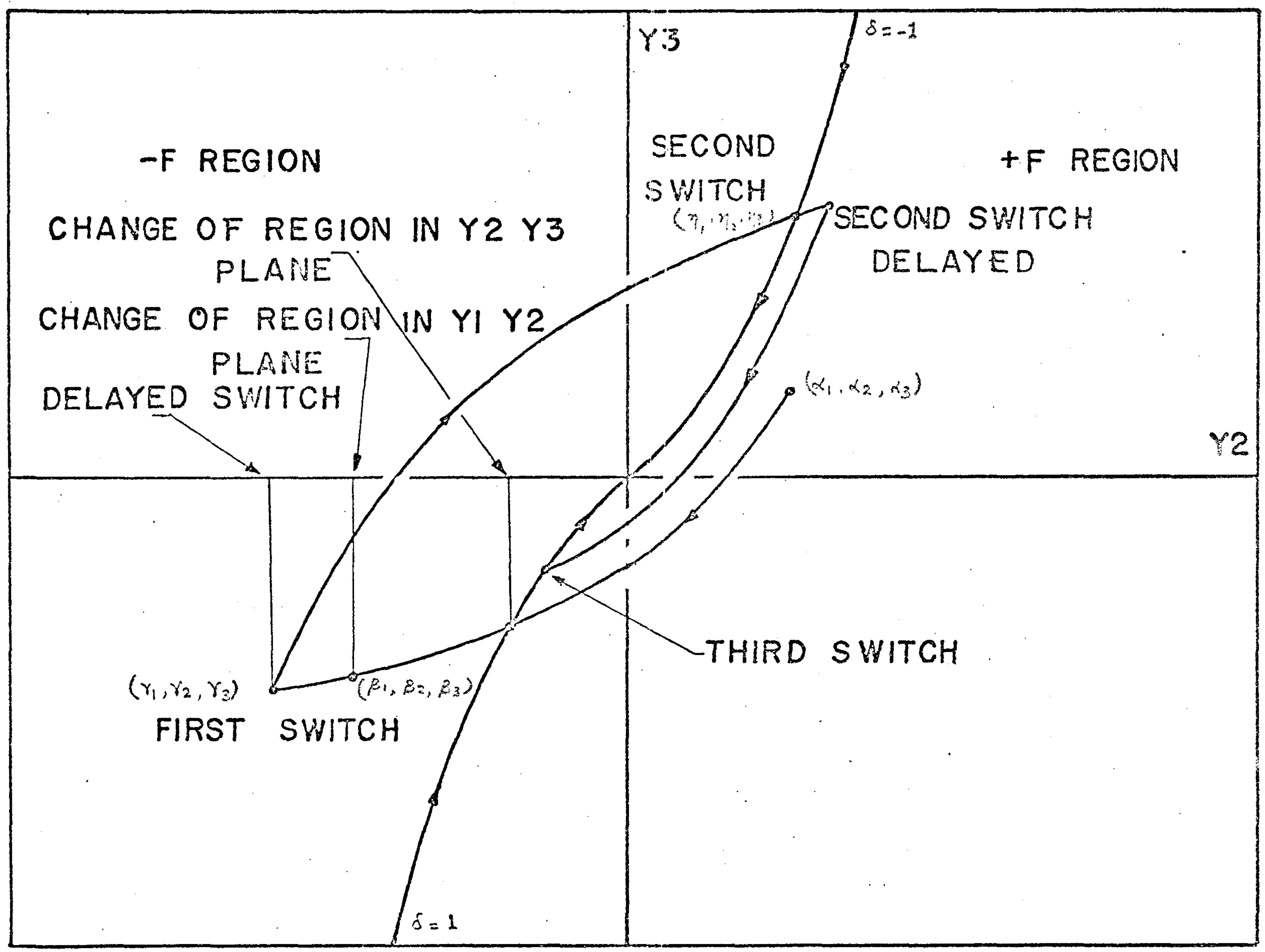



FIG 2.4(b) SWITCHING CRITERION IN Y2-Y3 PLANE

about the origin and a small dead zone in the relay would avoid this chatter.

## 2.5 DETERMINATION OF SWITCHING DELAY TIME

The trajectories of motion are given by the equations (2.13) and (2.14). If $(\alpha_1, \alpha_2, \alpha_3)$ is the initial state of the system, the constants C1 and C2 are given as:

$$C1 = 2\alpha_1 - \delta_1(\alpha_2 - \delta_1)^2 \qquad (2.25)$$

$$C2 = \alpha_2 + \delta_1 \ln(1 - \delta_1\alpha_3) \qquad (2.26)$$

Let $(\beta_1, \beta_2, \beta_3)$ be the state when the system just changes regions in both the planes. This point lies on the initial trajectory and the positive optimal trajectory in the plane where the change occurs later.

Hence

$$2\beta_1 - \delta_1(\beta_2 - \delta_1)^2 = C1 \qquad (2.27)$$

$$\beta_2 + \delta_1 \ln(1 - \delta_1\beta_3) = C2 \qquad (2.28)$$

and

$$(2\beta_1 + \delta_2) - \delta_2(\beta_2 - \delta_2)^2 = 0 \qquad (2.29)$$

(When change occurs in Y2 - Y3 plane first)

$$\beta_2 + \delta_2 \ln(1 - \delta_2\beta_3) = 0 \qquad (2.30)$$

(When change occurs in Y1 - Y2 plane first)

If $(\gamma_1, \gamma_2, \gamma_3)$ is the state when the first switching occurs after a delay $\Delta$, we obtain from the equation (2.17)

$$\gamma_2 = \delta_1\Delta + \beta_2 \qquad (2.31)$$

$$2\gamma_1 - \delta_1(\gamma_2 - \delta_1)^2 = C1 \qquad (2.32)$$

and

$$\gamma_2 + \delta_1 \ln(1 - \delta_1\gamma_3) = C2 \qquad (2.33)$$

Since the state point moves along the middle segment of the phase trajectory after the first switching, the constants

C1 and C2 have to be redefined as

$$C'1 = 2\gamma_1 - \delta 2(\gamma_2 - \delta_2)^2 \quad (2.34)$$

$$C'2 = \gamma_2 + \delta_2 \ell n(_1 - \delta_2 \gamma_3) \quad (2.35)$$

Let $(n_1, n_2, n_3)$ be the state when the second switching occurs and the point moves along the final switch curve. Then

$$2n_1 - \delta_2(n_2 - \delta_2)^2 = C'1 \quad (2.36)$$

$$n_2 + \delta_2 \ell n(1 - \delta_2 n_3) = C'2 \quad (2.37)$$

$$(2n_1 + \delta_3) - \delta_3(n_2 - \delta_3)^2 = 0 \quad (2.38)$$

$$n_2 + \delta_3 \ell n(1 - \delta_3 n_3) = 0 \quad (2.39)$$

$\delta_1$, $\delta_2$, $\delta_3$ are the values of $\delta$ along the initial, middle and final trajectories respectively. They have the same sign for the initial and final segments of the trajectory and reverse sign for the middle segment. The time delay $\Delta$ can be determined by solving the simultaneous equations (2.27) to (2.39). Since some of these equations are not linear, these equations do not have a unique solution. However by plotting trajectories obtained from equations (2.13), (2.14) and (2.27) to (2.39) for a number of initial conditions (figures 2.5(a), 2.5(b)) the value of $\Delta$ determined from the intersections is found to be very nearly equal to 0.11 as shown by the results of Table 1. Furthermore the delay circuit requires flip-flops which will cause the delay $\Delta$ only once in the first switching. The flip-flops have to be reset every time the system initially starts. Such a delay circuit is shown in figure 2.6.

TABLE I

THE DELAY TIME

| S. NO. | INITIAL CONDITIONS | | | TIME DELAY |
|---|---|---|---|---|
| | Y1 | Y2 | Y3 | (Change in Y2) |
| I | -0.5 | 1.0 | 0 | 0.1077 |
| II | 0.5 | 0 | 0 | 0.1076 |
| III | 1.0 | 0 | 0 | 0.1144 |
| IV | 0.5 | 0.5 | -0.5 | 0.1091 |
| V | 0.25 | 0 | 0 | 0.1075 |

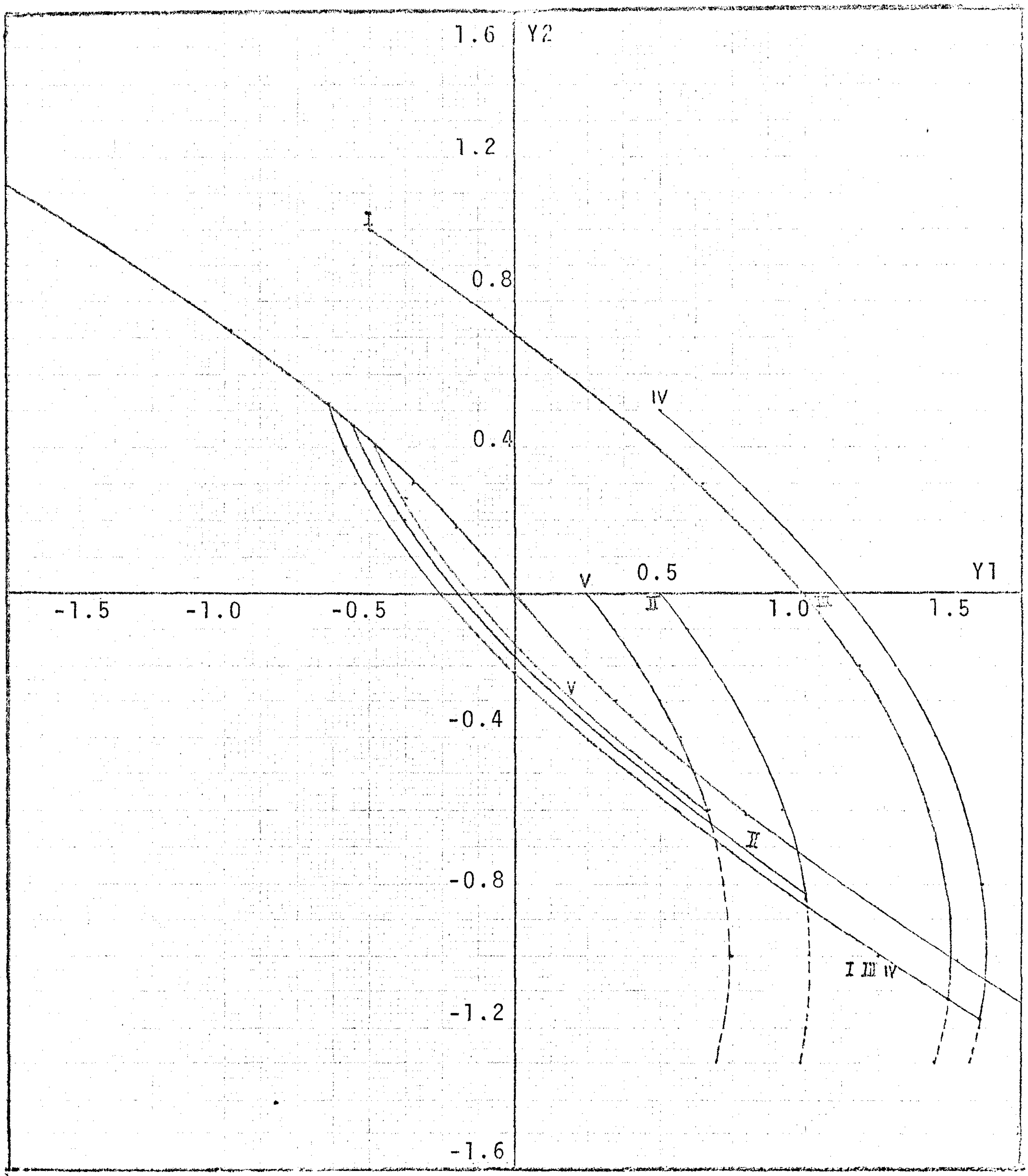



FIG. 2.5(a)

Evaluation of Delay Time in Y1 - Y2 Plane

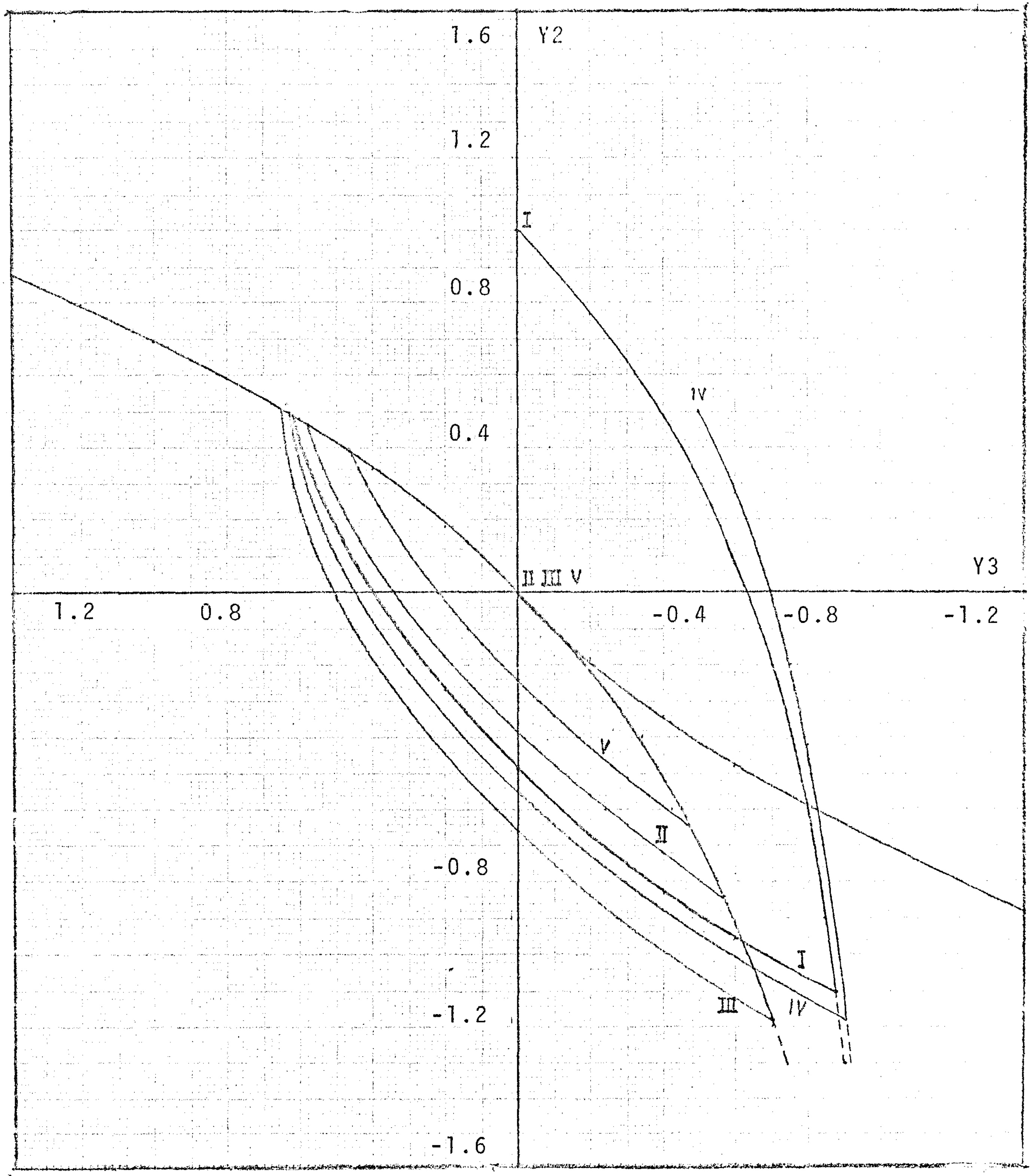



FIG. 2.5(b)

Evaluation of Delay Time in Y2 - Y3 Plane

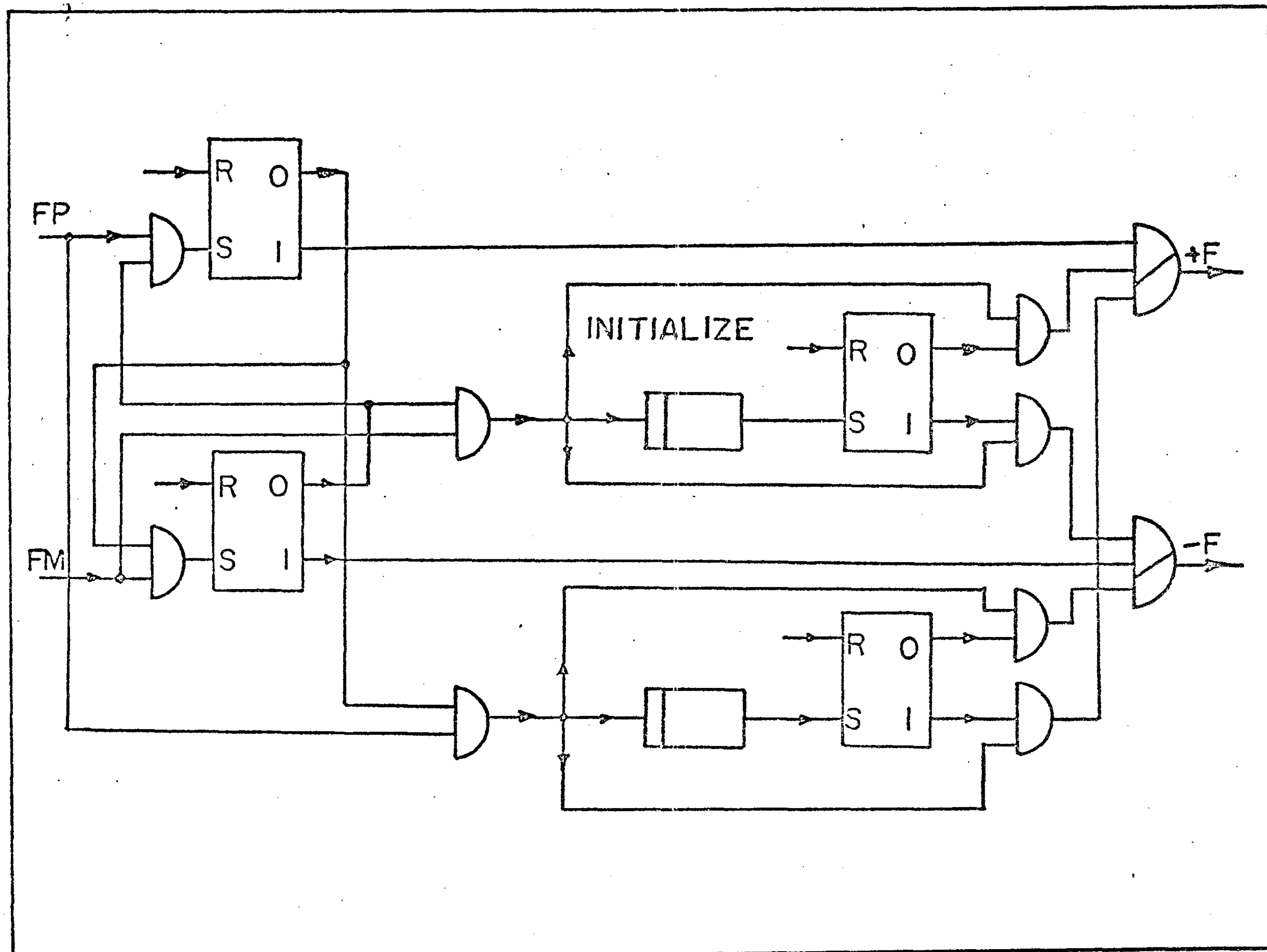



FIG 2.6 DELAY CIRCUIT BLOCK DIAGRAM

## 2.6 LOGIC CONTROLLER DESIGN

The switching logic equations for ±F are determined from the variables defined earlier for the different sections of figures 2.2(a) and 2.2(b). All the variables are considered having only two states as 1 and 0. The relationship between the state of any logic variable A and its continuous time variable is defined as follows.

$$A = 1 \quad \text{if} \quad A > 0 \tag{2.40}$$

$$A = 0 \quad \text{if} \quad A \leqslant 0$$

The complement of A is defined as

$$\overline{A} = 1 \quad \text{if} \quad -A > 0$$

$$\overline{A} = 0 \quad \text{if} \quad -A \leqslant 0 \tag{2.41}$$

Using the above notation, the sections described in terms of the logic variables are tabulated in the truth tables, Table 2a and Table 2b. Based on these tables the logic equations are

$$FP = R(\overline{Y1}\ Y2\ \overline{Y3} + \overline{Y1}\ Y2\ Y3 + Y1\ \overline{Y2}\ Y3 + Y1\ Y2\ \overline{Y3} + Y1\ Y2\ Y3) \tag{2.42}$$

$$FM = \overline{R}(\overline{Y1}\ \overline{Y2}\ \overline{Y3} + \overline{Y1}\ \overline{Y2}\ Y3 + \overline{Y1}\ Y2\ Y3 + Y1\ \overline{Y2}\ \overline{Y3} + Y1\ \overline{Y2}\ Y3) \tag{2.43}$$

Which can be simplified, using map techniques, to the form

$$FP = R(Y2 + Y1\ \overline{Y3}) \tag{2.44}$$

$$FM = \overline{R}(\overline{Y2} + \overline{Y1}\ Y3) \tag{2.45}$$

Based on these equations, the logic circuits required for the controller is given in figure 2.7 and the complete block diagram of the controlled system in figure 2.8.

TABLE 2a

TRUTH TABLE FOR POSITIVE FORCE

| $Y_1$ | $Y_2$ | $Y_3$ | R | FP |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

TABLE 2b

TRUTH TABLE FOR NEGATIVE FORCE

| $Y_1$ | $Y_2$ | $Y_3$ | $\overline{R}$ | FM |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

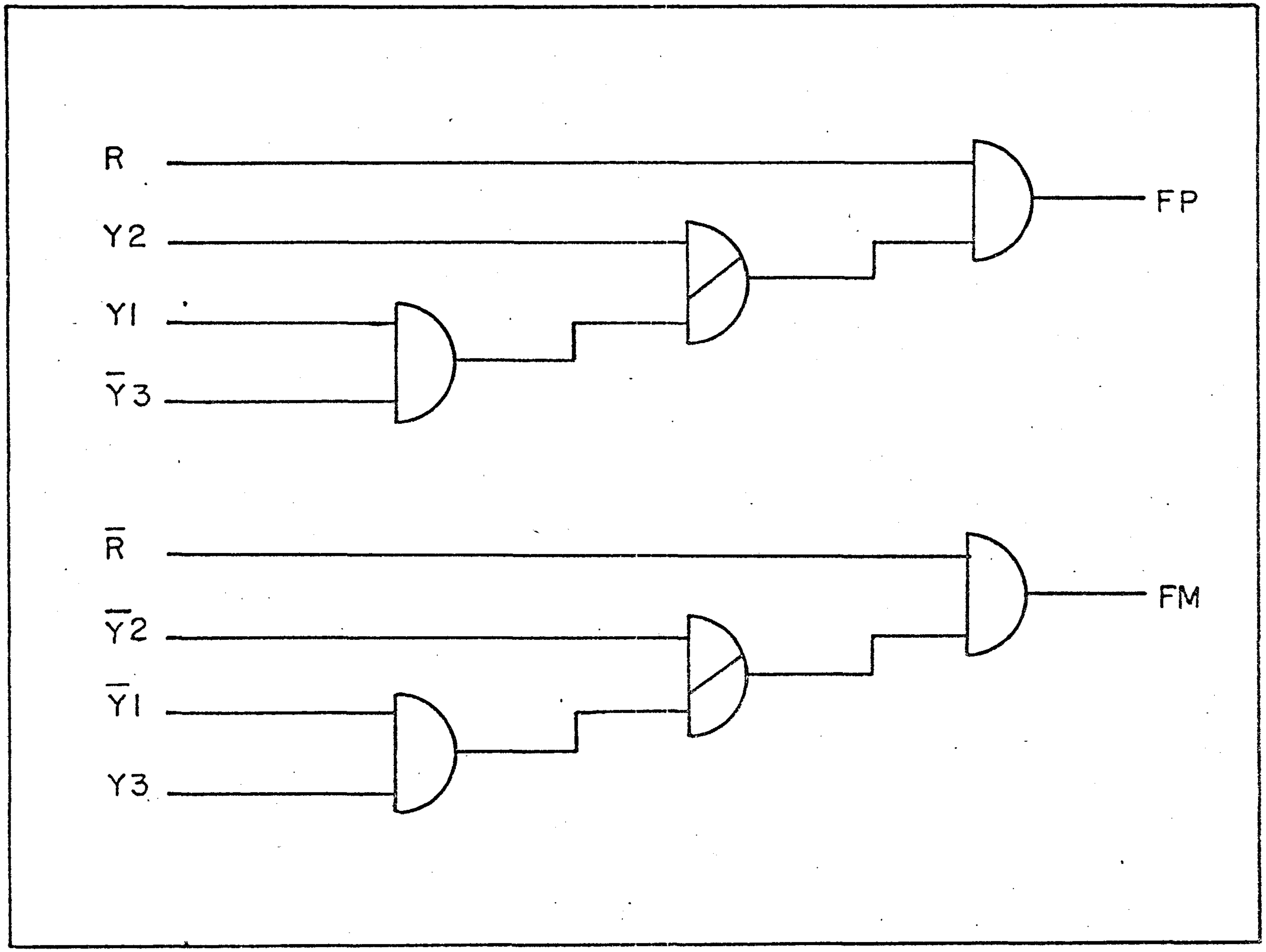



FIG 2.7 LOGIC CIRCUIT BLOCK DIAGRAM

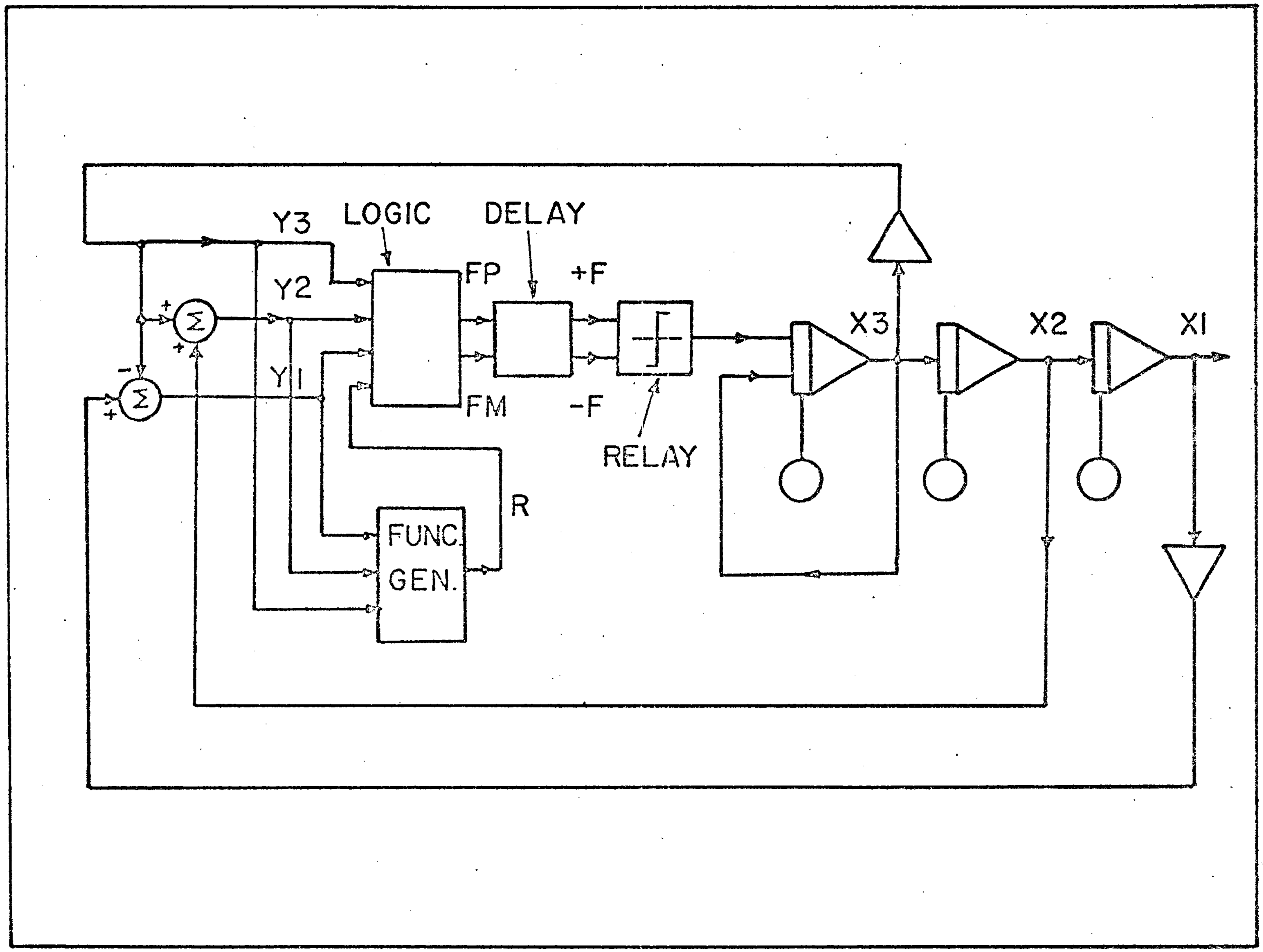



FIG 2.8 BLOCK DIAGRAM OF THE OPTIMUM SYSTEM

## 2.7 SIMULATION RESULTS

The controlled system of figure 2.8 with the detailed circuit was simulated on a digital computer (IBM 1620). The flow diagram is given in appendix in figure 2.9. The results from the plotter are shown in Figures 2.10(a) to 2.10(e) for positive initial conditions and 2.11(a) to 2.11(e) for negative initial conditions. Figures 2.10(a), 2.10(b), 2.11(a) and 2.11(b) show the phase plane projections of the trajectories starting from three different initial conditions. It is obvious that the controller gives the desired response. Figures 2.10(c) to 2.10(e) and 2.11(c) to 2.11(e) show the variations of E, dE/dt and $d^2E/dt^2$ with time.

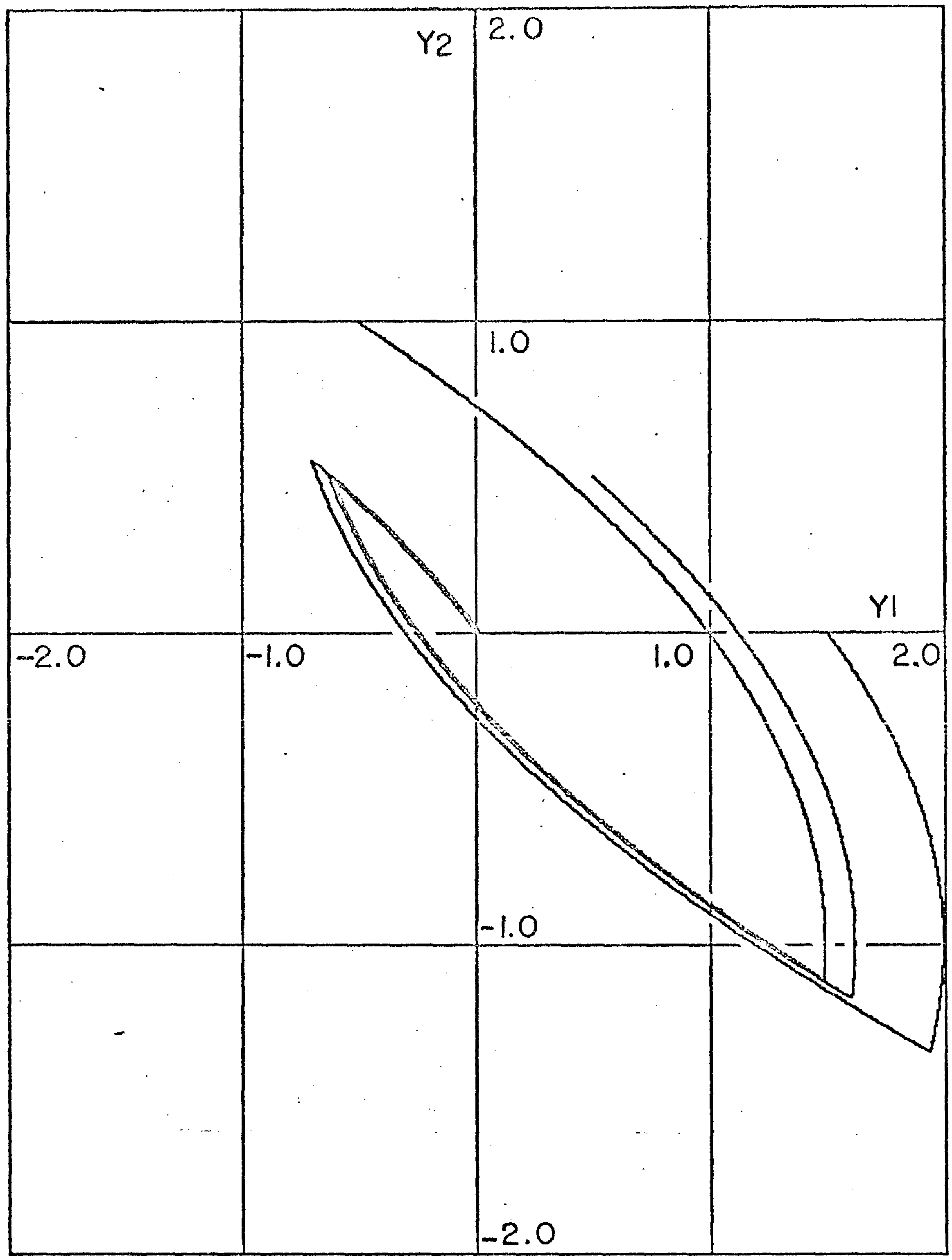



FIG 2.10(a) YI-Y2 PROJECTIONS OF THE SYSTEM TRAJECTORIES

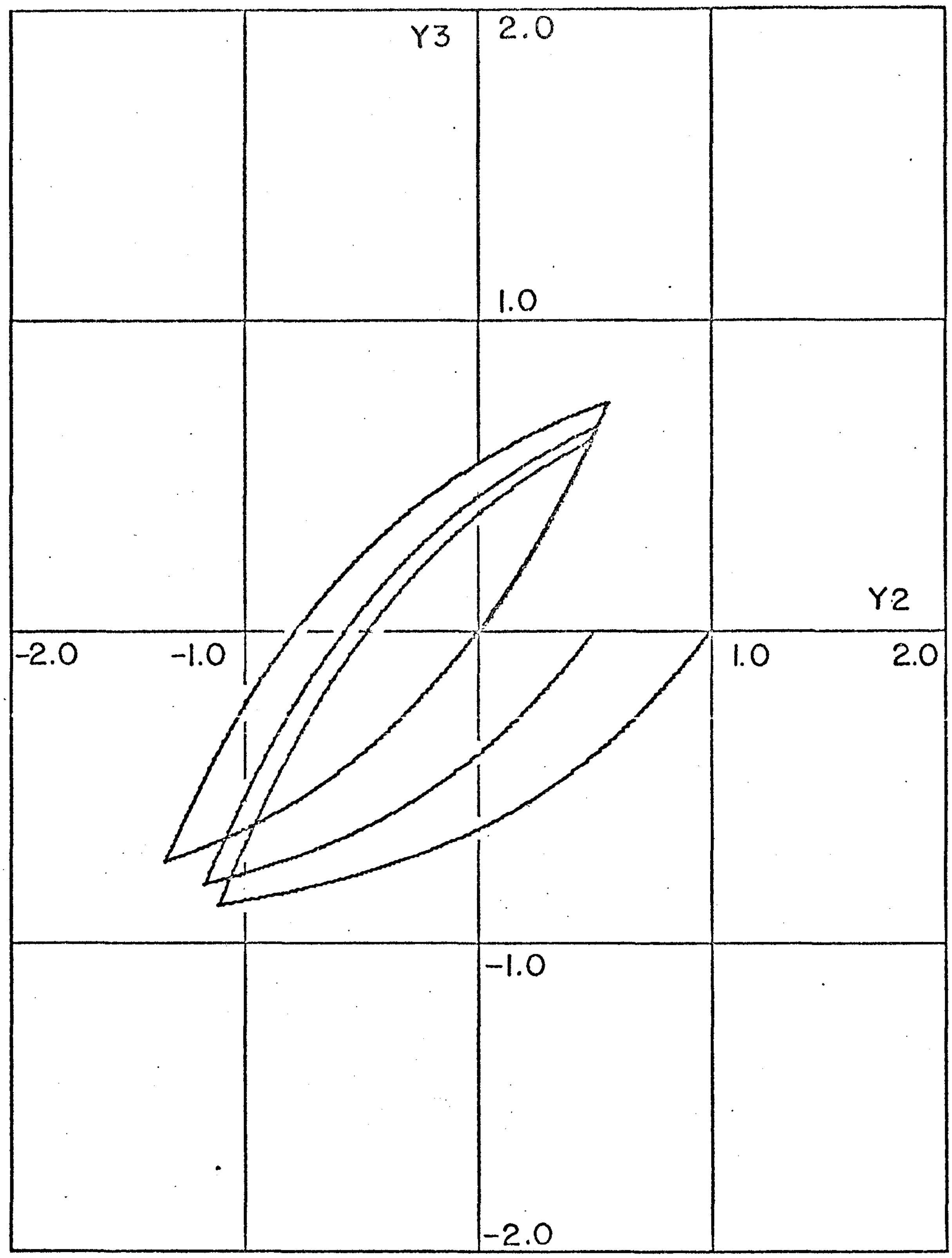



FIG 2.10(b) Y2–Y3 PROJECTIONS OF THE SYSTEM TRAJECTORIES

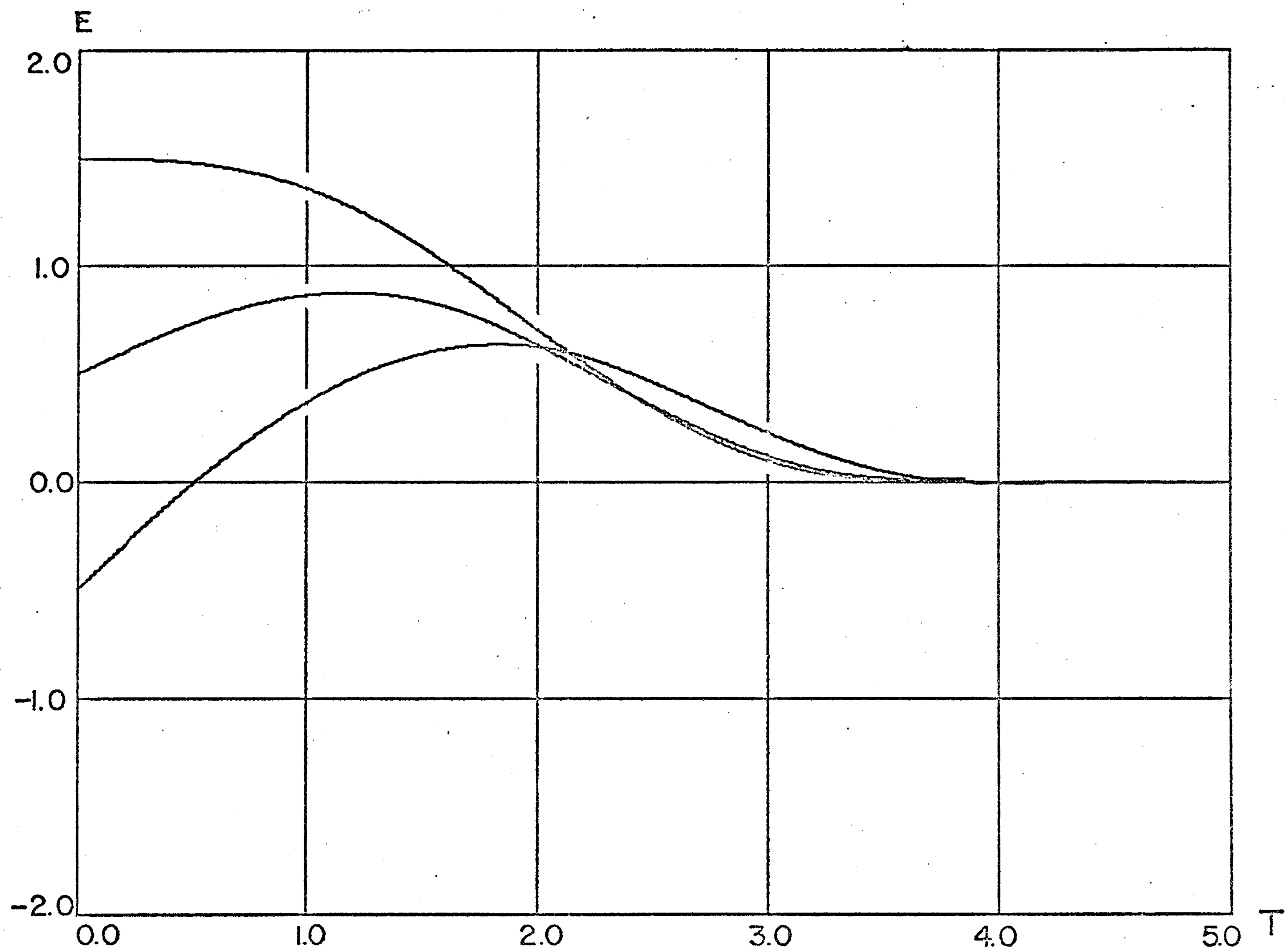



FIG 2.10(c) SYSTEMS TIME RESPONSE (ERROR V/S TIME)

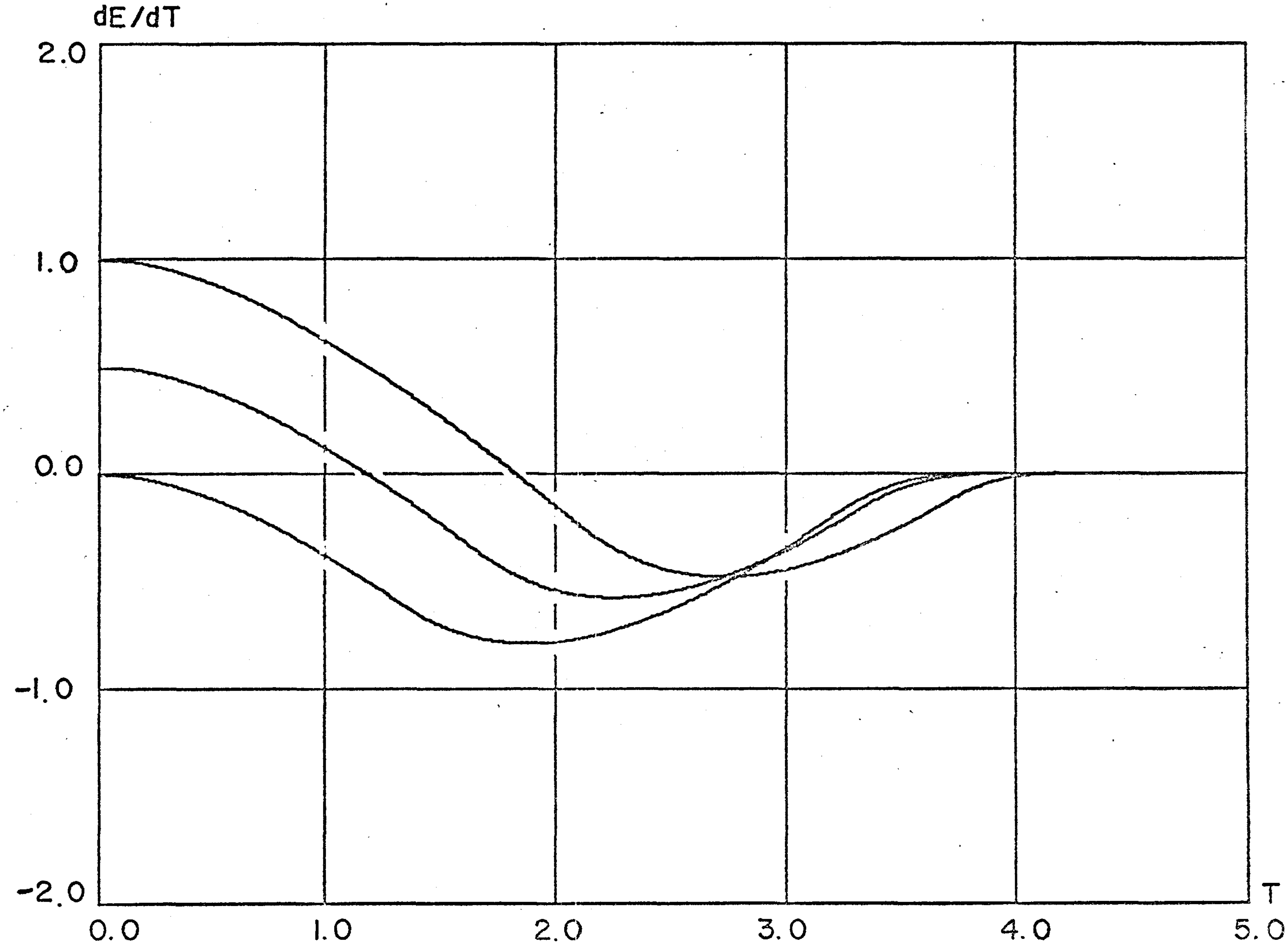



FIG 2.1(d) SYSTEMS TIME RESPONSE (RATE V/S TIME)

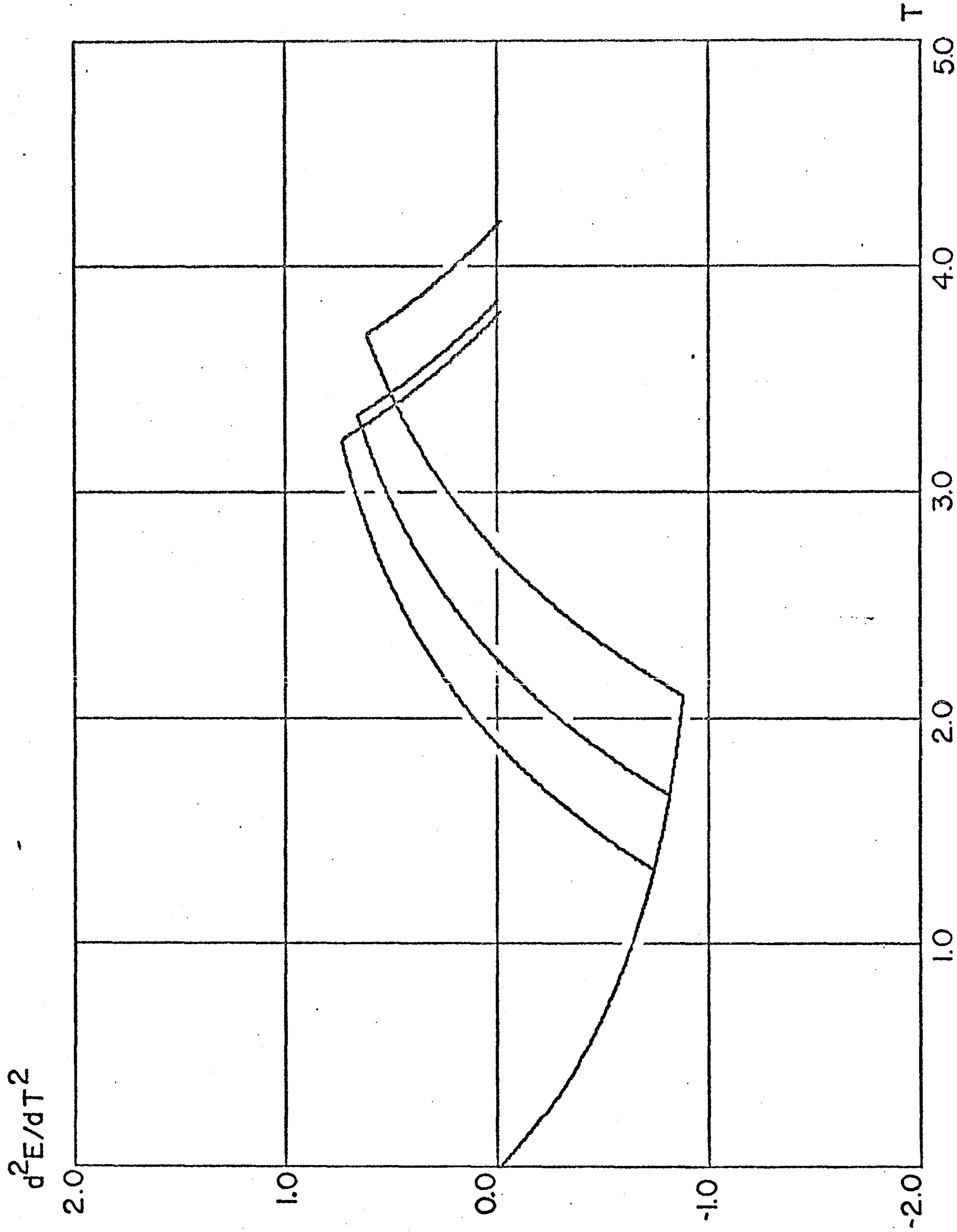



FIG 2.10(e) SYSTEMS TIME RESPONSE (ACCELERATION V/S TIME)

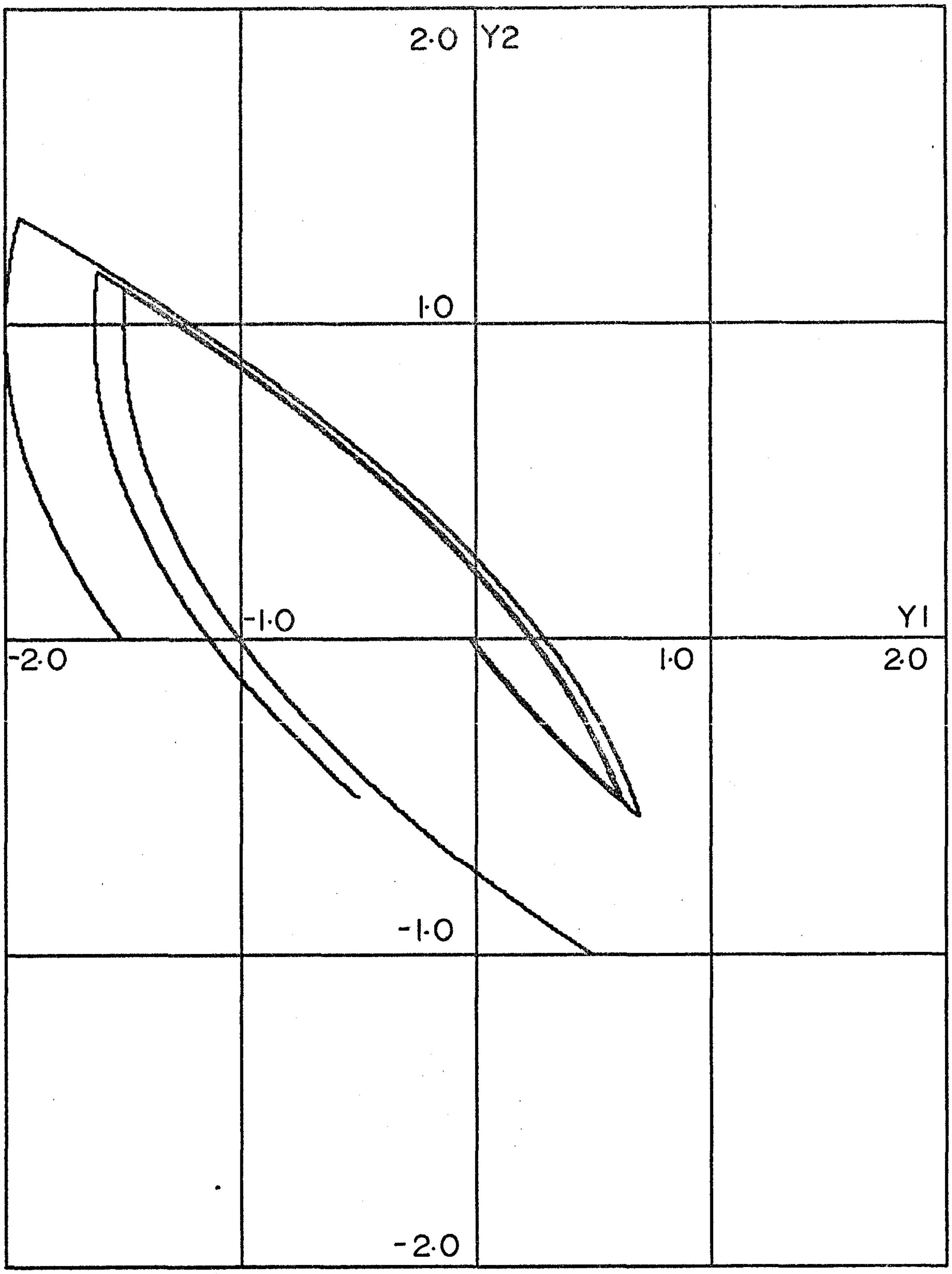



FIG 2.11(a)

YI-Y2 PROJECTIONS OF SYSTEM TRAJECTORIES

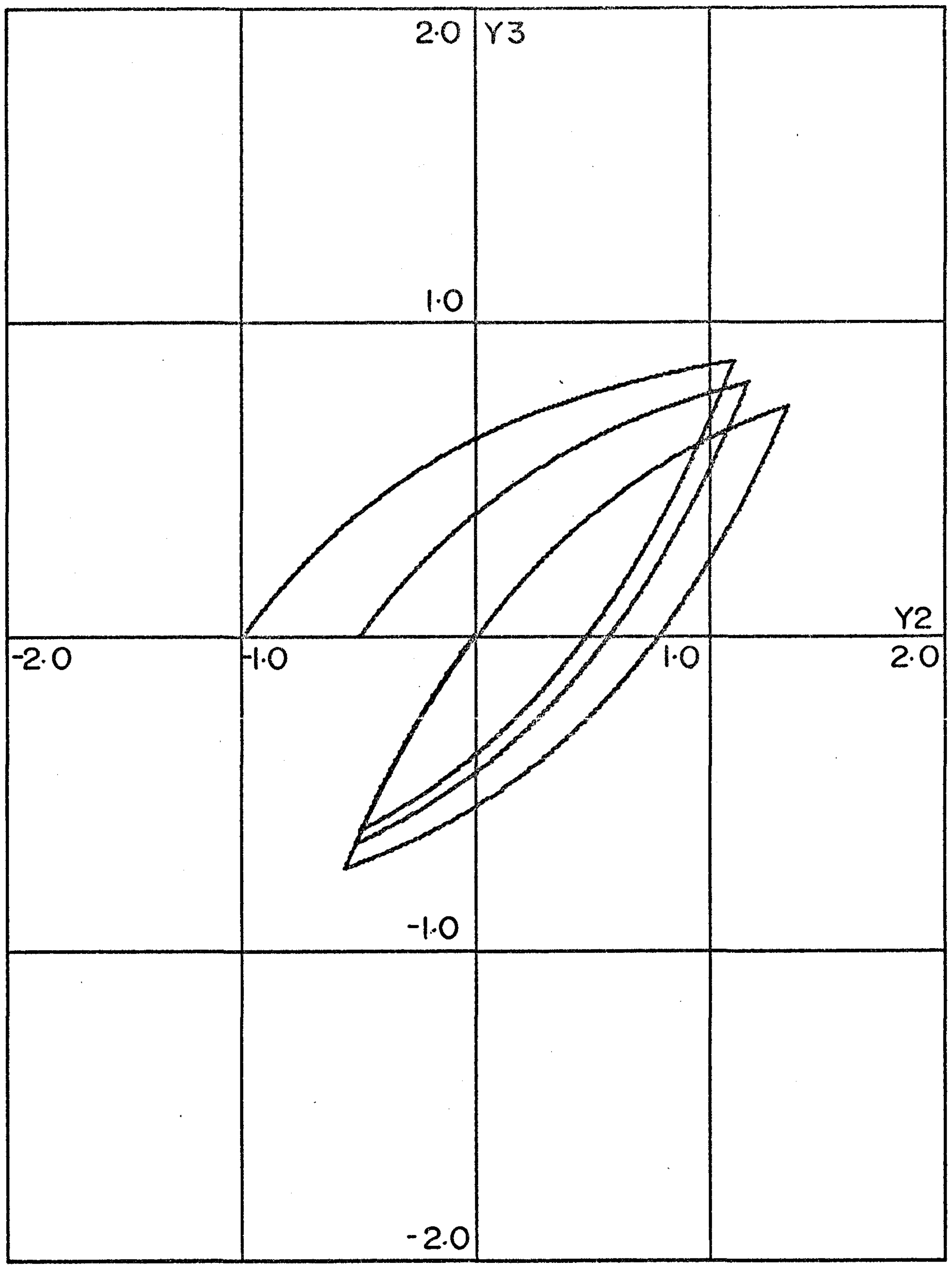



FIG 2.11(b)

Y2-Y3 PROJECTIONS OF SYSTEM TRAJECTORIES

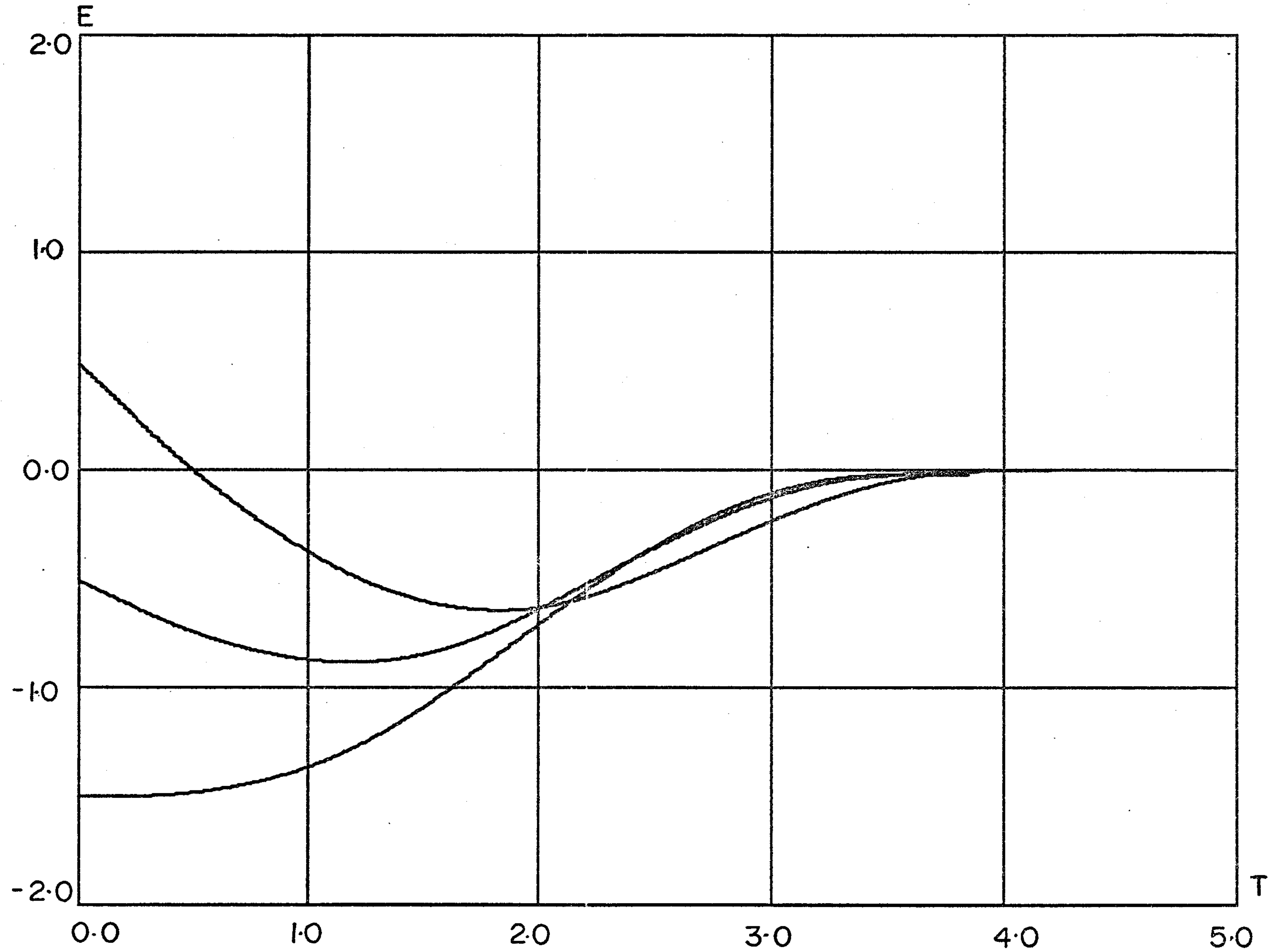



FIG 2.11(c) SYSTEMS TIME RESPONSE (ERROR V/S TIME)

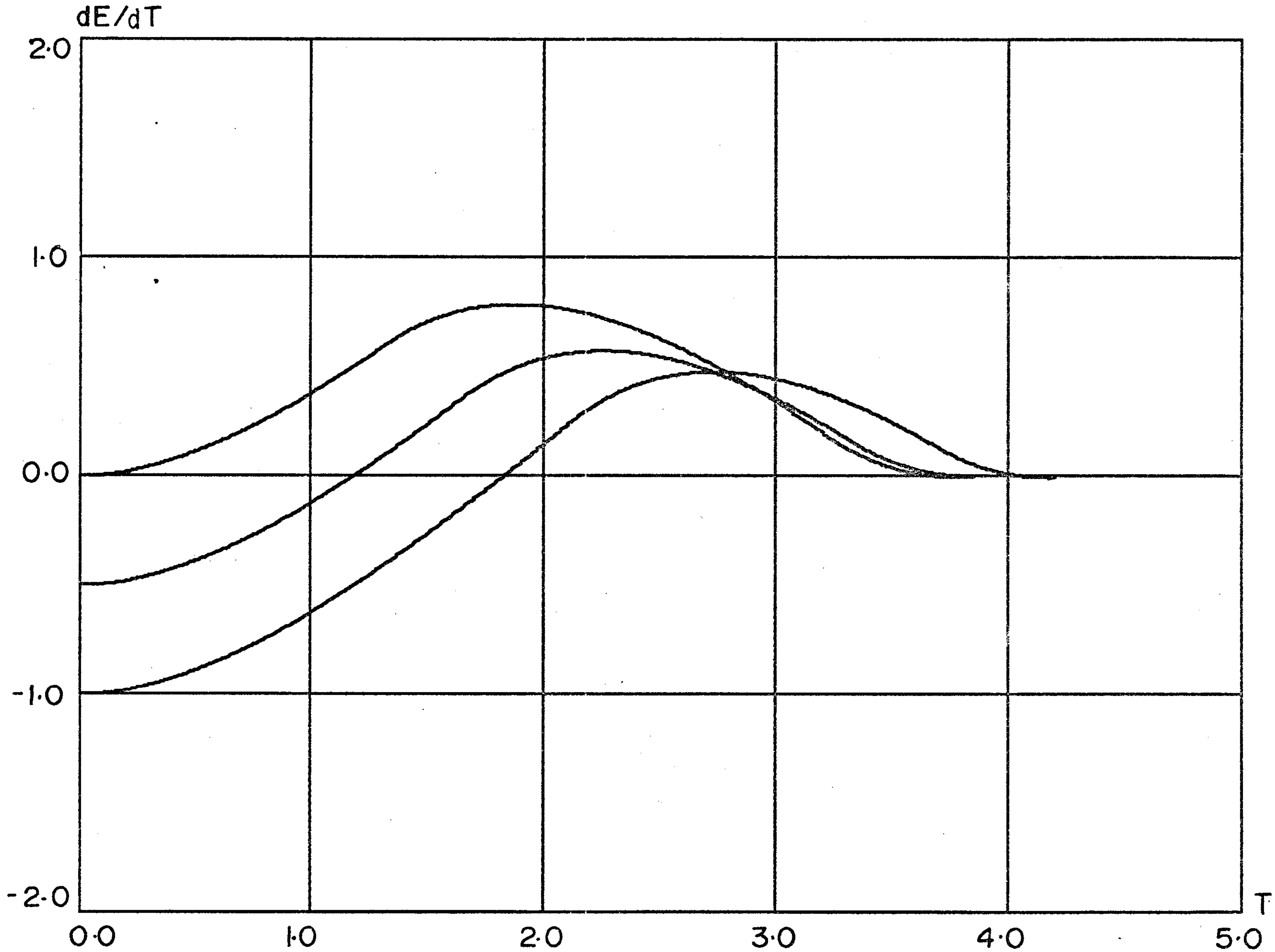



FIG 2.11(d) SYSTEMS TIME RESPONSE ( RATE V/S TIME )

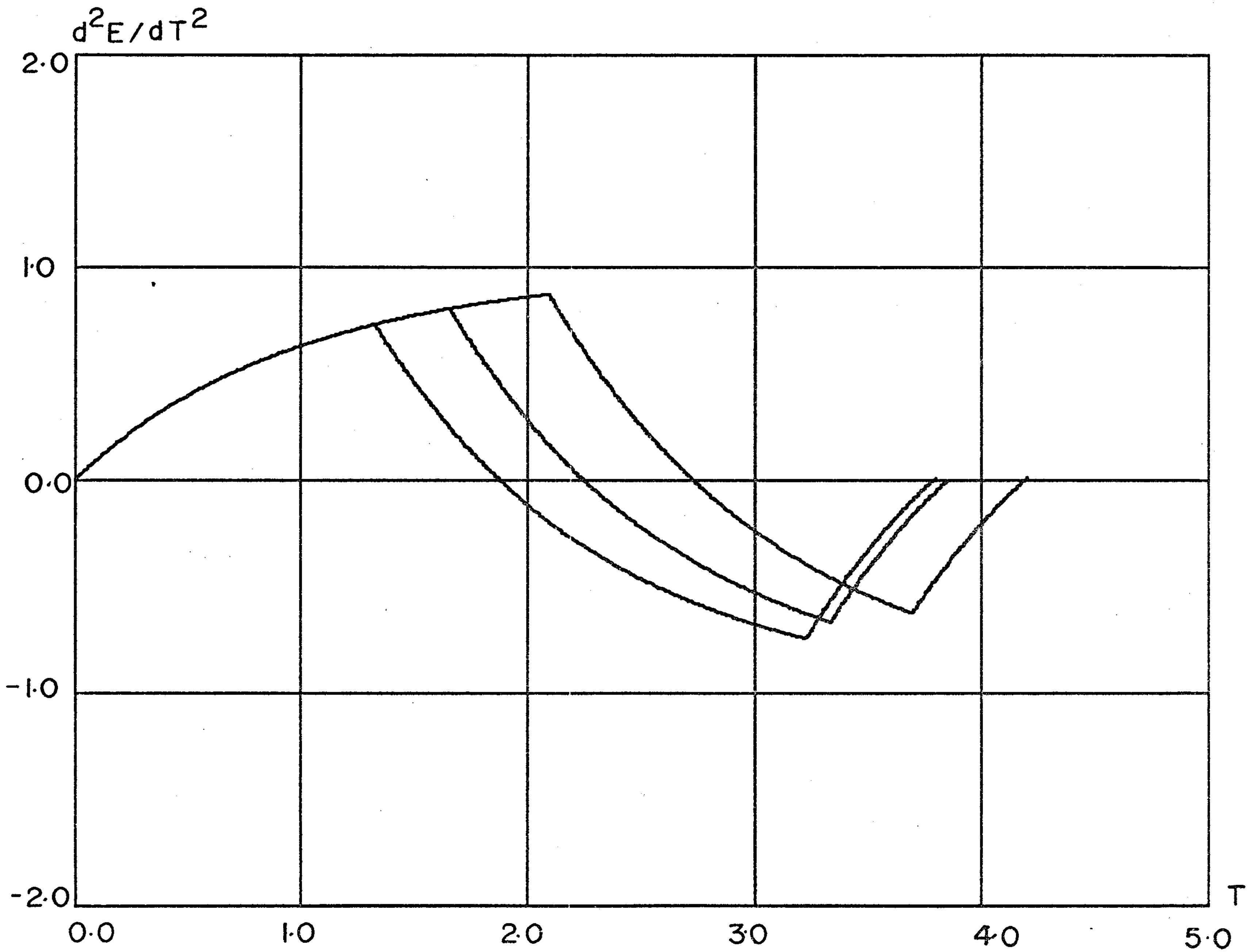



FIG 2.11(e) SYSTEMS TIME RESPONSE (ACCELERATION V/S TIME)

## CHAPTER III

### DEVELOPMENT OF LOGIC CONTROLLER:

### System with One Integrator and Two Time Constants

### 3.1 PHASE SPACE ANALYSIS

For a third order system described by the transfer function

$$G(s) = \frac{K}{s(s+1)\ (s+2)} \qquad (3.1)$$

The normalized vector differential equations have also been shown to reduce to

$$\begin{bmatrix} \dot{X}1 & (t) \\ \dot{X}2 & (t) \\ \dot{X}3 & (t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} X1 & (t) \\ X2 & (t) \\ X3 & (t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \delta \qquad (3.2)$$

and in this case, the similarity transformation matrix is

$$P = \begin{bmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 1 & 4 \end{bmatrix} \qquad (3.3)$$

Where $\lambda_1$, $\lambda_2$, $\lambda_3$ are the eigenvalues.

The transformed differential equation becomes

$$\begin{bmatrix} \dot{Y}1 \\ \dot{Y}2 \\ \dot{Y}3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} Y1 \\ Y2 \\ Y3 \end{bmatrix} + \begin{bmatrix} 1/2 \\ -1 \\ 1/2 \end{bmatrix} \delta \qquad (3.4)$$

Following the previous method the solution of these equations gives the projections of the phase trajectories in the Y1 - Y2 and Y2 and Y1 - Y3 planes of the Y space.

$$2Y1 + \delta \ell n(1+\delta Y2) = C1 \tag{3.5}$$

$$4Y1 + \delta \ell n(1-4\delta Y3) = C2 \tag{3.6}$$

For step displacement inputs C1 and C2 are determined from the initial conditions. The equations of the optimal trajectories are:

$$2Y1 + \delta \ell n(1+\delta Y2) = 0 \tag{3.7}$$

$$4Y1 + \delta \ell n(1-4\delta Y3) = 0 \tag{3.8}$$

The total time required is given by the relation

$$Y1 = \delta t + \alpha_1 \tag{3.9}$$

The increment $\Delta$ is given as

$$\Delta Y1 = \delta(\Delta t) \tag{3.10}$$

i.e. the change in the coordinate Y1 in the right direction gives the time taken from one point to the other.

## 3.2 SWITCHING CRITERION FOR OPTIMUM RESPONSE.

Again, the phase space is divided into ten sections similarly to those described in Chapter II, and is shown in figures 3.1(a) and 3.1(b). The force is positive in sections 4, 6, 8, 9 and 10 and negative in sections 1, 2, 3, 5 and 7. Sections 4 and 7 also include the optimum trajectories. The function generator variable R is positive if the state point lies to the left of the optimum trajectories in both the Y1 - Y2 and Y1 - Y3 planes. Thus the criterion is:

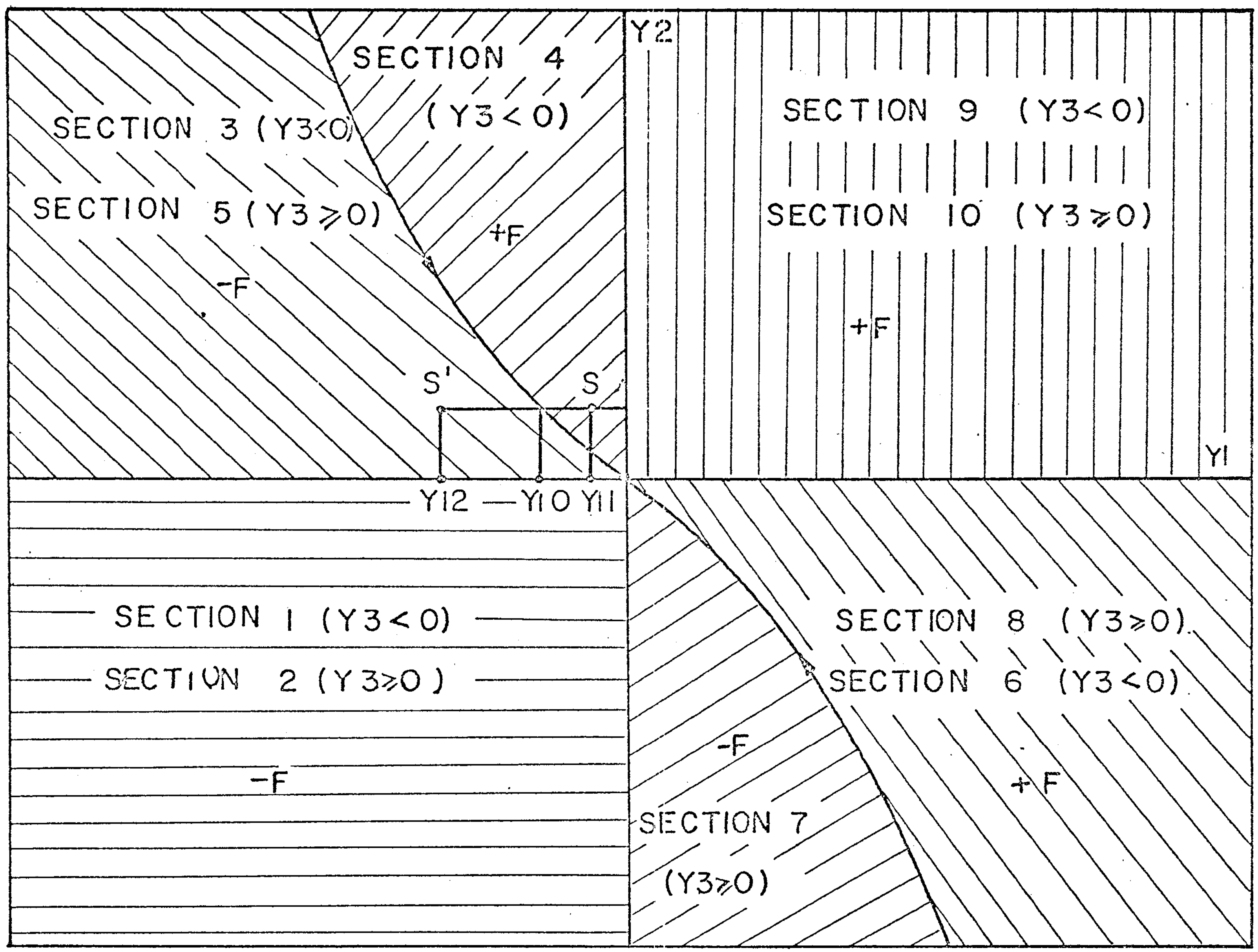



FIG 3.1(a) SECTIONS DESCRIBING LOGIC VARIABLES IN Y1-Y2 PLANE

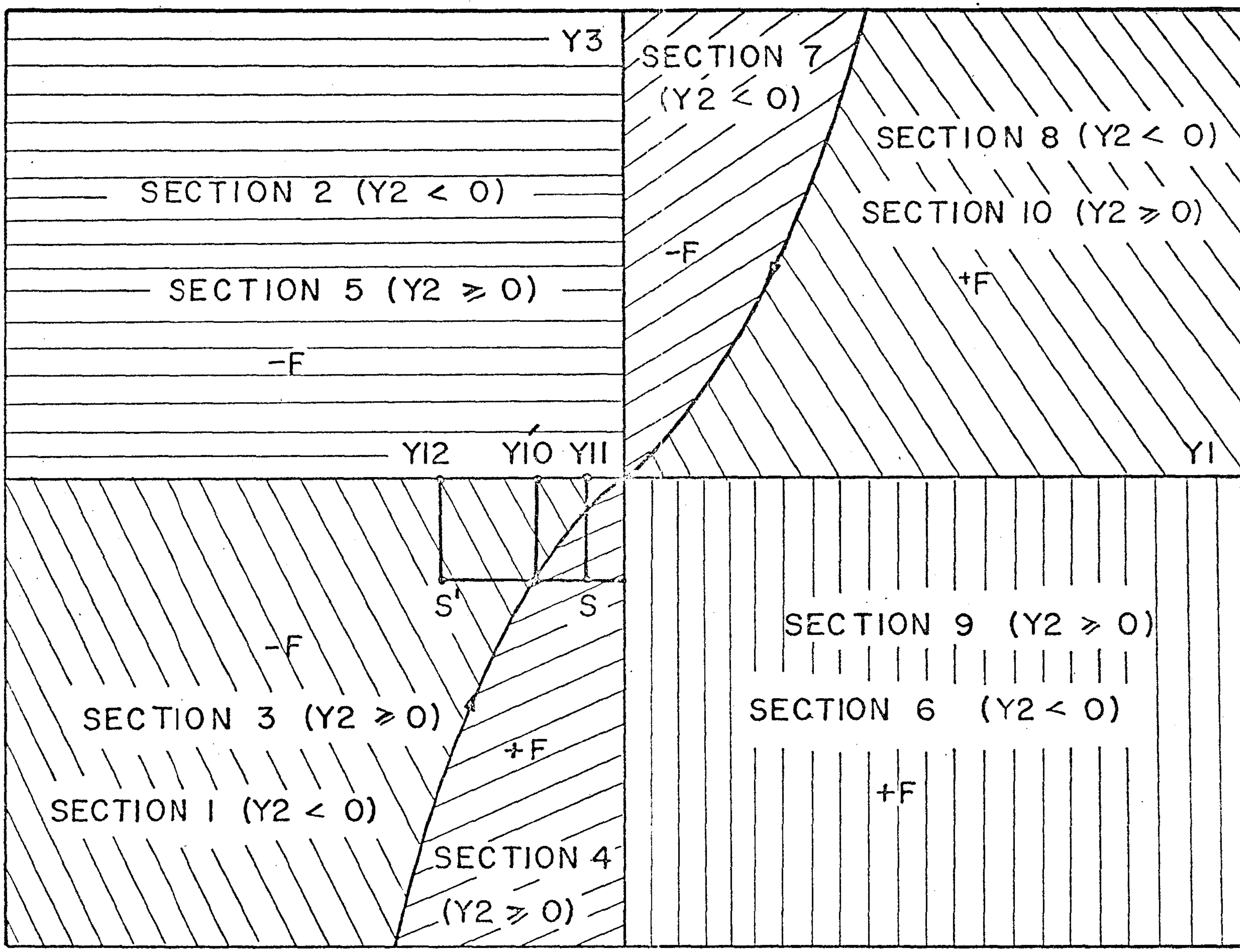



FIG 3.1(b) SECTIONS DESCRIBING LOGIC VARIABLES IN Y1–Y3 PLANE

The force F is positive if

| | |
|---|---|
| Y1(-), Y2(+), Y3(-), R(+) | Section (4) |
| Y1(+), Y2(-), Y3(-), R(+) | Section (6) |
| Y1(+), Y2(-), Y3(+), R(+) | Section (8) |
| Y1(+), Y2(+), Y3(-), R(+) | Section (9) |
| Y1(+), Y2(+), Y3(+), R(+) | Section (10) |

Driving force F is negative if

| | |
|---|---|
| Y1(-), Y2(-), Y3(-), R(-) | Section (1) |
| Y1(-), Y2(-), Y3(+), R(-) | Section (2) |
| Y1(-), Y2(+), Y3(-), R(-) | Section (3) |
| Y1(-), Y2(+), Y3(+), R(-) | Section (5) |
| Y1(+), Y2(-), Y3(+), R(-) | Section (7) |

F is zero if

$$Y1 = Y2 = Y3 = 0$$

## 3.3 FUNCTION GENERATOR

The value of R may be obtained as before by comparing the value of Y1 of the state point with the greater of the two values of Y1 corresponding to the points of intersection of the optimum trajectories and the horizontal line passing through the state point. If the state point has more positive Y1 then R is positive, otherwise it is negative. In figures 3.1(a) and 3.1(b) the point S has positive R, whereas $S'$ has negative R.

The optimal trajectories simulation equations are

$YP = \ell n(1+|Y2|)\ /2.0$ from (3.7) (3.11)

$YQ = \ell n(1+4|Y3|)\ /4.0$ from (3.8) (3.12)

$W = YS\ SGN(Y3)$ (3.13)

R is determined using equations (2.21, (2.22) and (2.24). The function generator block diagram is given in figure 3.2.

## 3.4 DETERMINATION OF SWITCHING INSTANTS

In the case of the present system the desired response can be achieved by similarly delaying the first switching after the state point has changed region in both the planes as shown in figures 3.3(a) and 3.3(b). It has been found in this case also that if the first switching is delayed by 0.11 units of Y1(time), the optimum response is achieved. The delay time equations are derived in the next section and the delay circuit of figure 2.6 can also be used for this system.

## 3.5 DETERMINATION OF DELAY TIME

Using the trajectory equations (3.5) and (3.6), and as described in section 2.5 if ($\alpha_1$, $\alpha_2$, $\alpha_3$), ($\beta_1$, $\beta_2$, $\beta_3$), ($\gamma_1$, $\gamma_2$, $\gamma_3$ ) and ($\eta_1$, $\eta_2$, $\eta_3$) represent the same states, we get

$$C1 = 2\alpha_1 + \delta_1 \ell n(1+\delta_1\alpha_2) \qquad (3.14)$$

$$C2 = 4\alpha_1 + \delta_1 \ell n(1-\delta_1\alpha_3) \qquad (3.15)$$

$$2\beta_1 + \delta_1 \ell n(1+\delta_1\beta_2) = C1 \qquad (3.16)$$

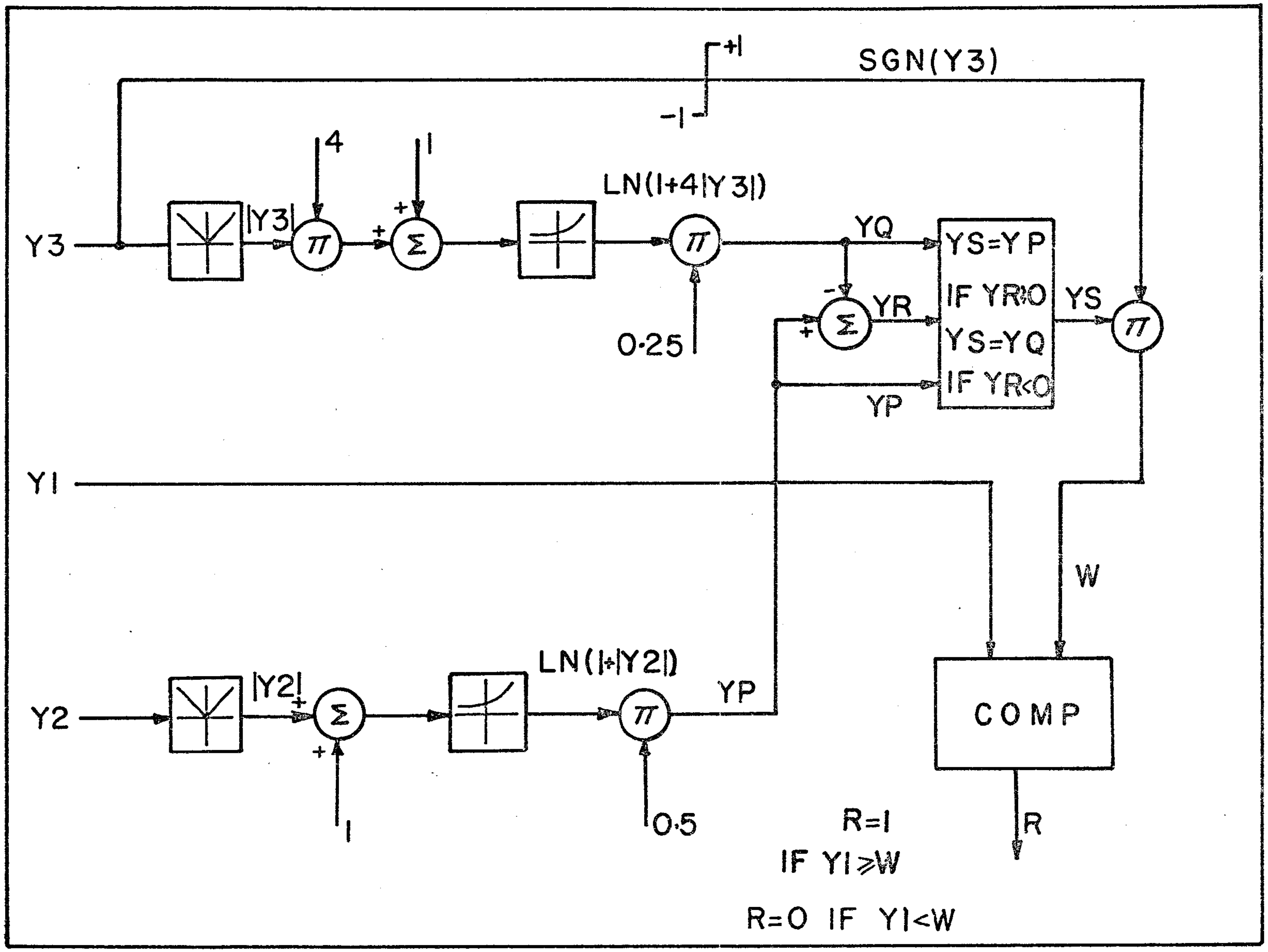



FIG 3.2 FUNCTION GENERATOR

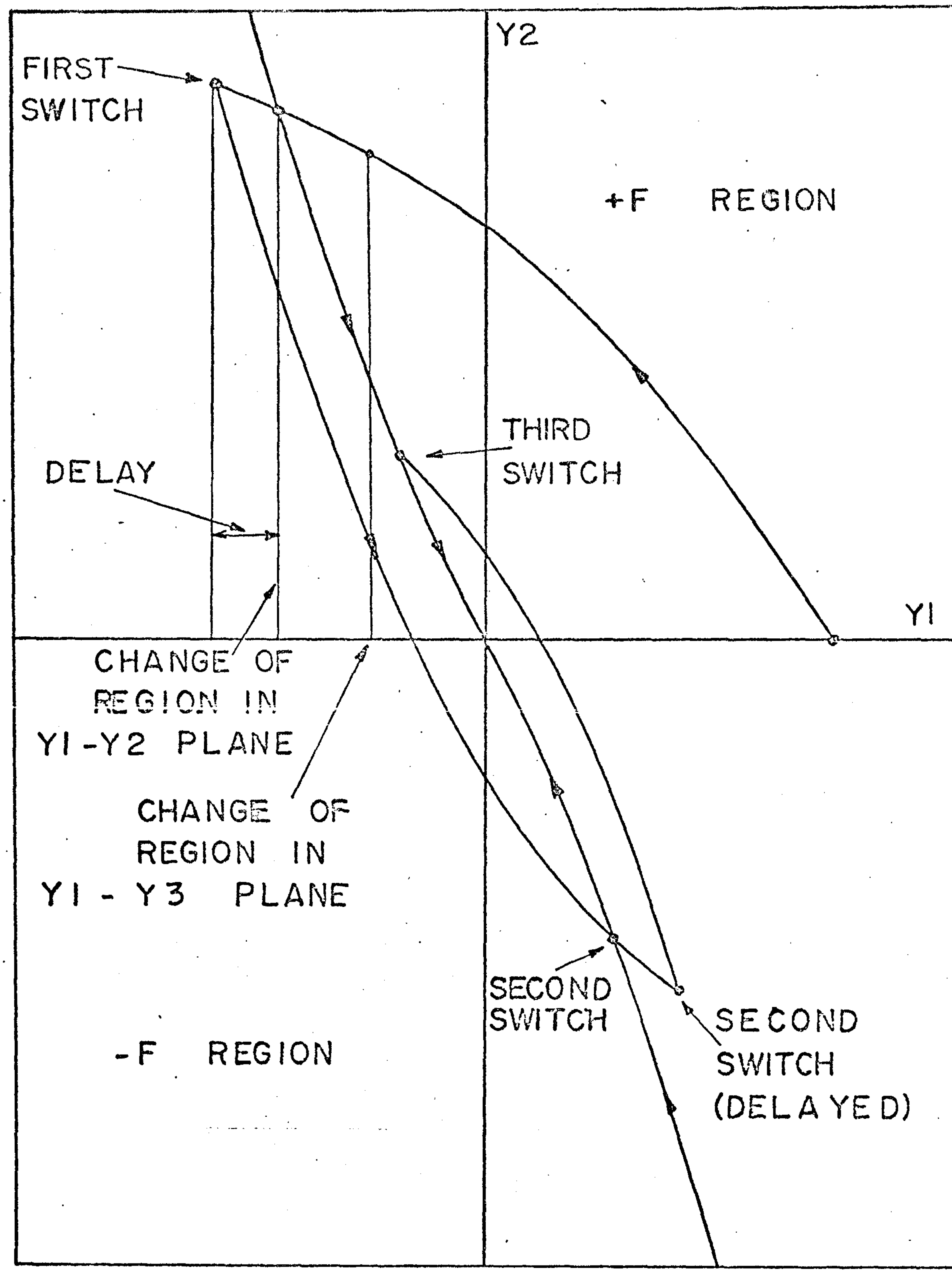



FIG 3.3(a) SWITCHING CRITERION IN Y1—Y2 PLANE

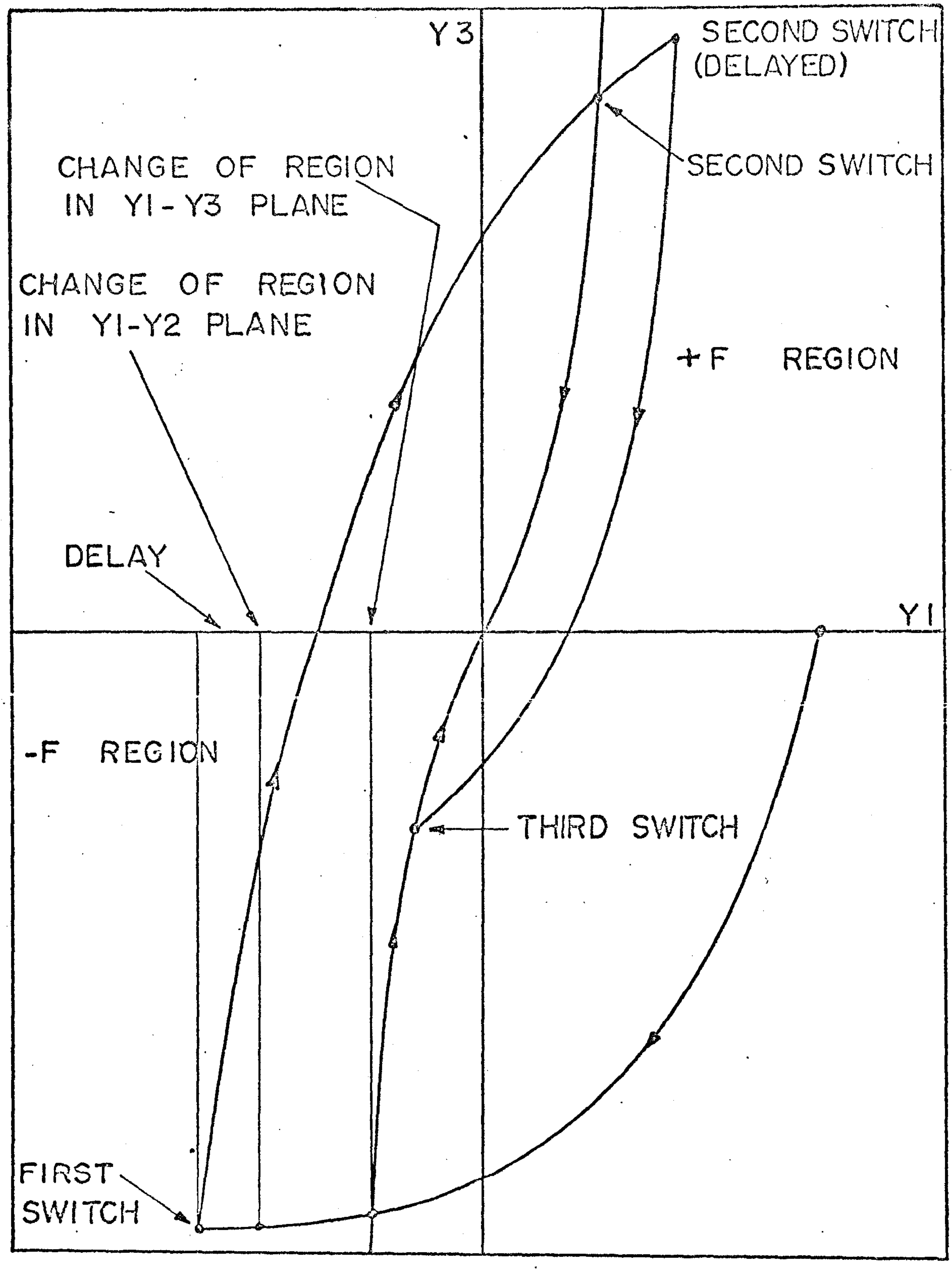



FIG 3.3(b) SWITCHING CRITERION IN Y1-Y3 PLANE

$$4\beta_1 + \delta_1 \ell n(1-4\delta_1\beta_3) = C2 \quad (3.17)$$

$$2\beta_1 + \delta_2 \ell n(1+\delta_2\beta_2) = 0 \quad (3.18)$$

(if change occurs in Y1 - Y3 plane first)

$$4\beta_1 + \delta_2 \ell n(1-4\delta_2\beta_3) = 0 \quad (3.19)$$

(if change occurs in Y1 - Y2 plane first)

From equation (3.9)

$$\gamma_1 = \delta_1 \Delta + \beta_1 \quad (3.20)$$

Also

$$2\gamma_1 + \delta_1 \ell n(1 + \delta_1\gamma_2) = C'1 \quad (3.21)$$

$$4\gamma_1 + \delta_1 \ell n(1 - 4\delta_1\gamma_3) = C'2 \quad (3.22)$$

$$C'1 = 2\gamma_1 + \delta_2 \ell n(1 + \delta_2\gamma_2) \quad (3.23)$$

$$C'2 = 4\gamma_1 + \delta_2 \ell n(1 - 4\delta_2\gamma_3) \quad (3.24)$$

$$2\eta_1 + \delta_2 \ell n(1+\delta_2\eta_2) = C'1 \quad (3.25)$$

$$4\eta_1 + \delta_2 \ell n(1-4\delta_2\eta_3) = C'2 \quad (3.26)$$

$$2\eta_1 + \delta_3 \ell n(1+\delta_3\eta_2) = 0 \quad (3.27)$$

$$4\eta_1 + \delta_3 \ell n(1-4\delta_3\eta_3) = 0 \quad (3.28)$$

By a similar procedure to that used in section 2.5, the time delay which satisfies equations (3.16) to (3.28) is also found to be 0.11 units of time.

## 3.6 LOGIC CONTROLLER DESIGN

The sections of figures 3.1(a) and 3.1(b) are again described in terms of logic variables and are tabulated in truth tables resulting in the logic equations for the controller.

$$FP = R(Y1 + Y2\ \overline{Y3}) \quad (3.29)$$

$$FM = \overline{R}(\overline{Y1} + \overline{Y2}\ Y3) \quad (3.30)$$

The logic circuit required for the controller is given in figure 3.4 and the complete block diagram of the controller in figure 3.5.

## 3.7 SIMULATION RESULTS

The phase plane projections of the trajectories for positive initial errors is given in figures 3.6(a) and 3.6(b), and for negative errors in figures 3.7(a) and 3.7(b). The time response is shown in figures 3.6(c) to 3.6(e) and 3.7(c) to 3.7(e). The flow chart is given in appendix (figure 3.8).

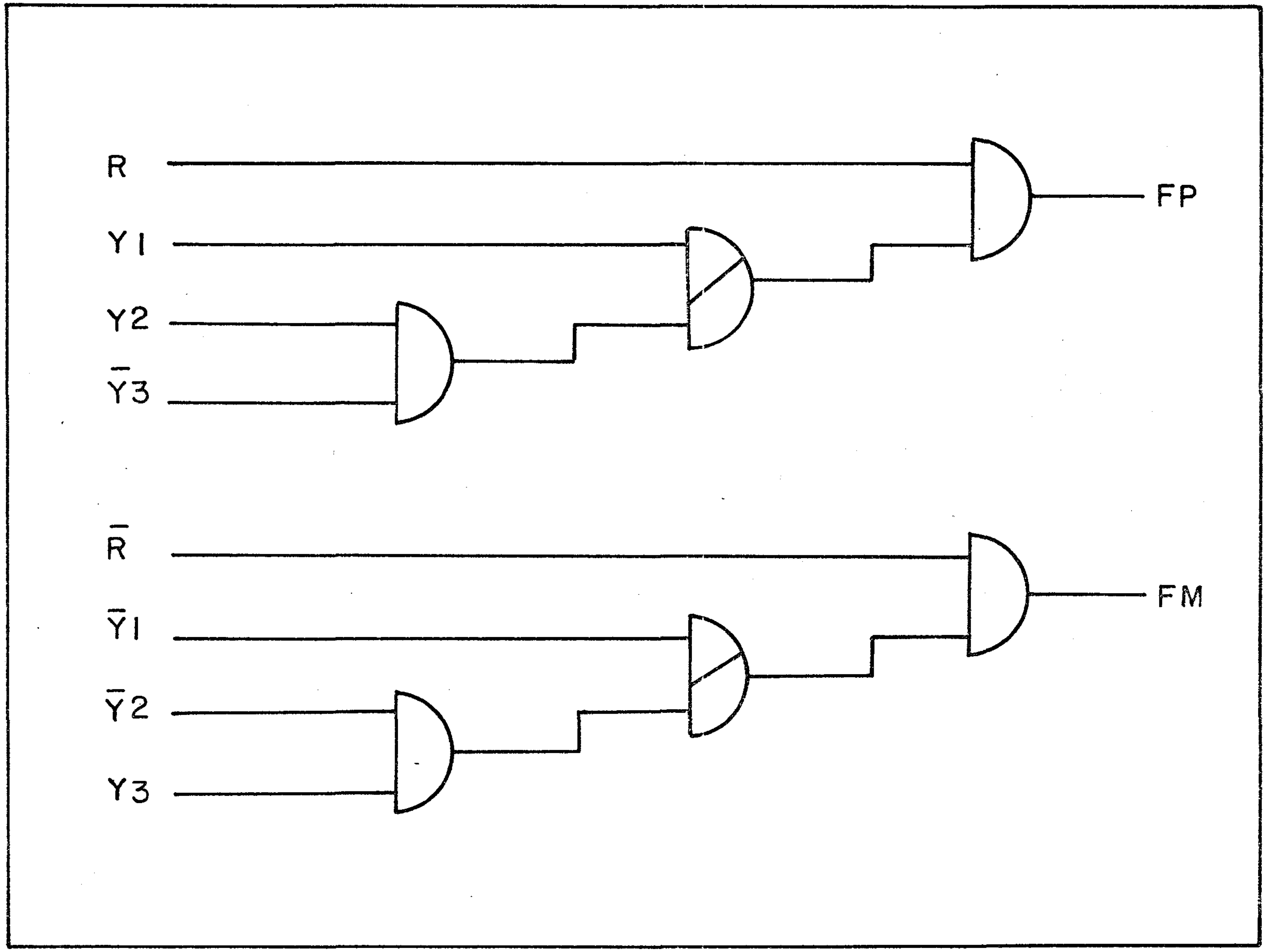



FIG 3.4 LOGIC CIRCUIT BLOCK DIAGRAM

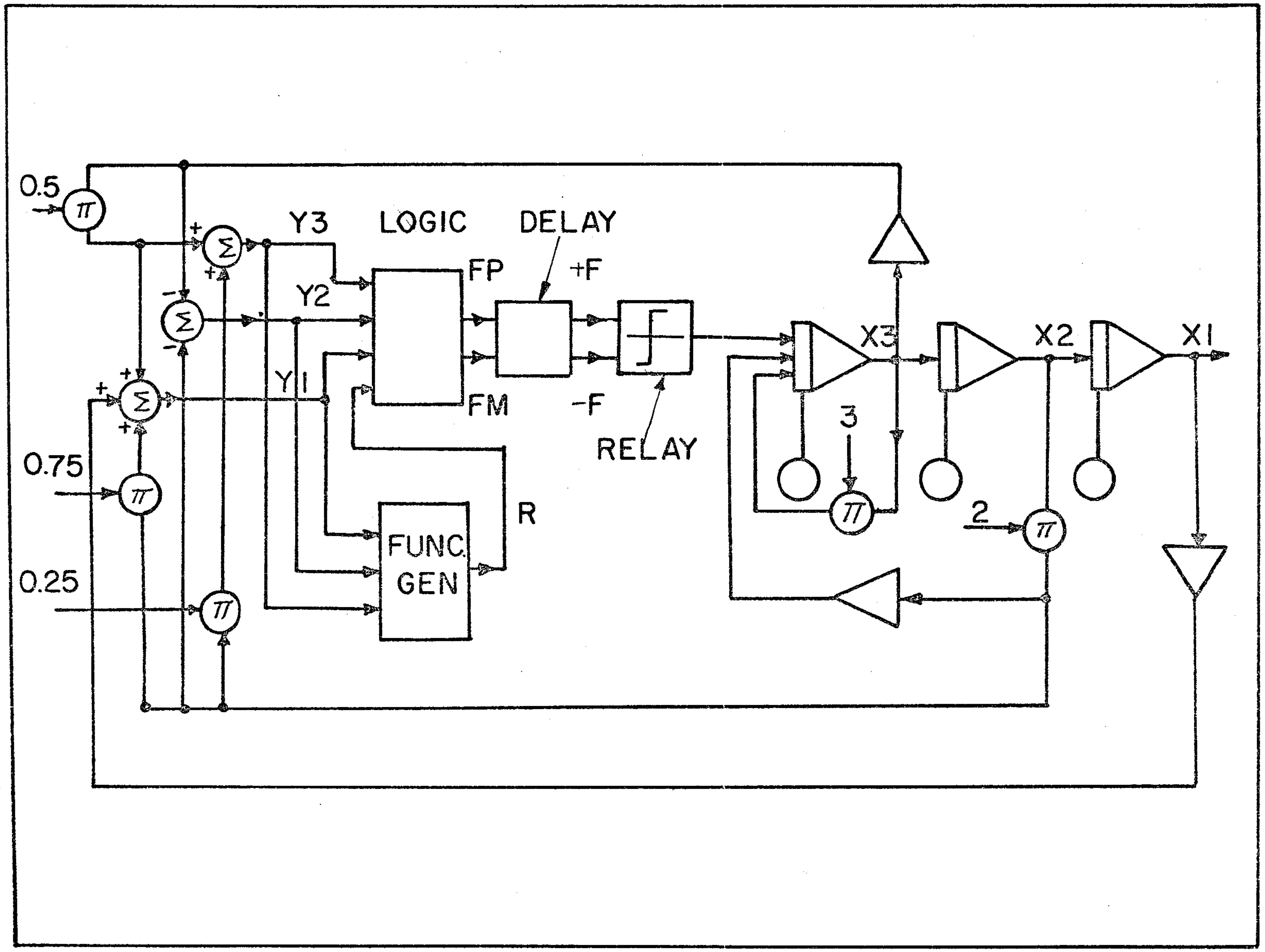



FIG 3.5 BLOCK DIAGRAM OF THE OPTIMUM SYSTEM

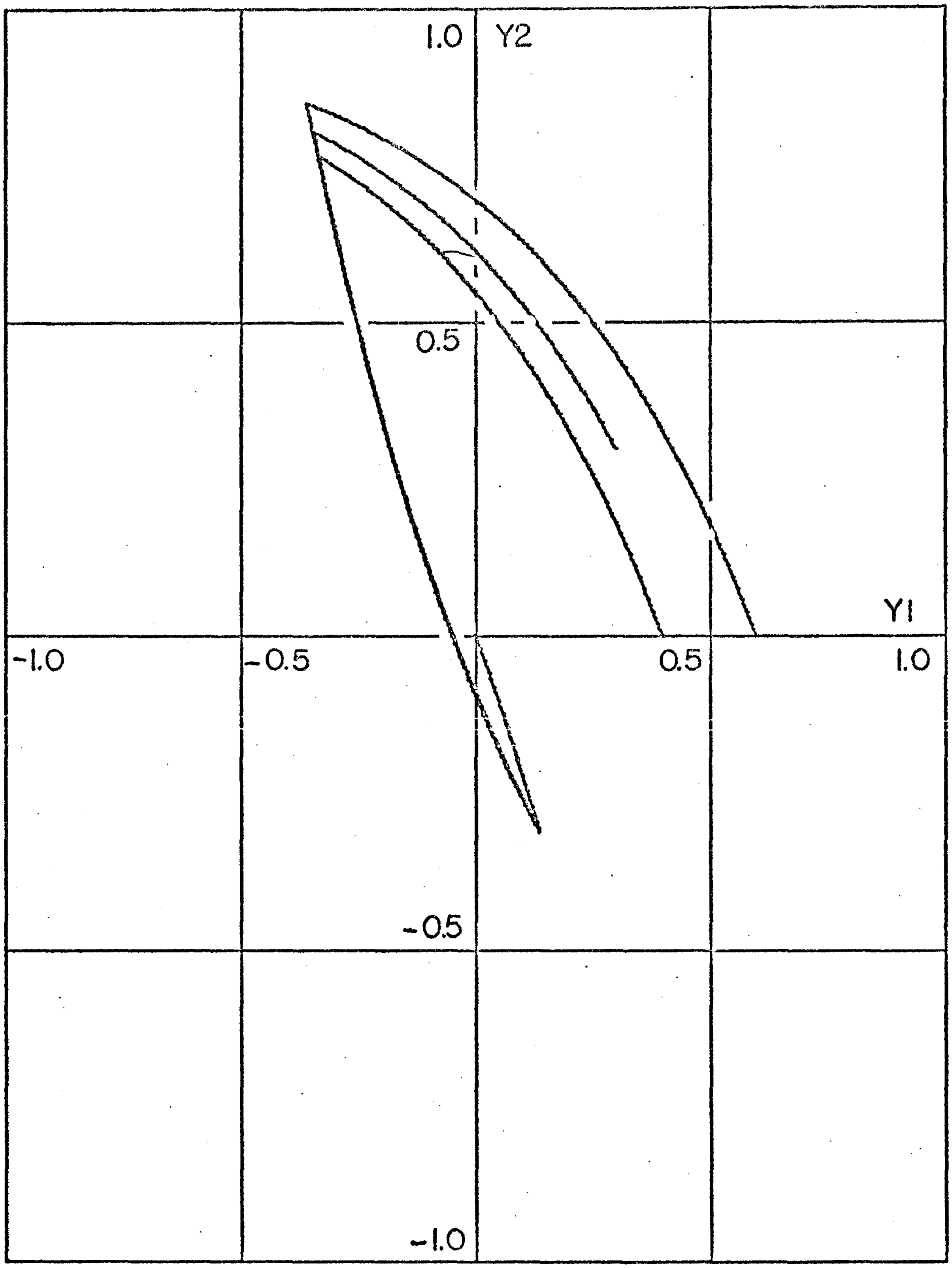



FIG 3.6(a) YI-Y2 PROJECTIONS OF THE SYSTEM TRAJECTORIES

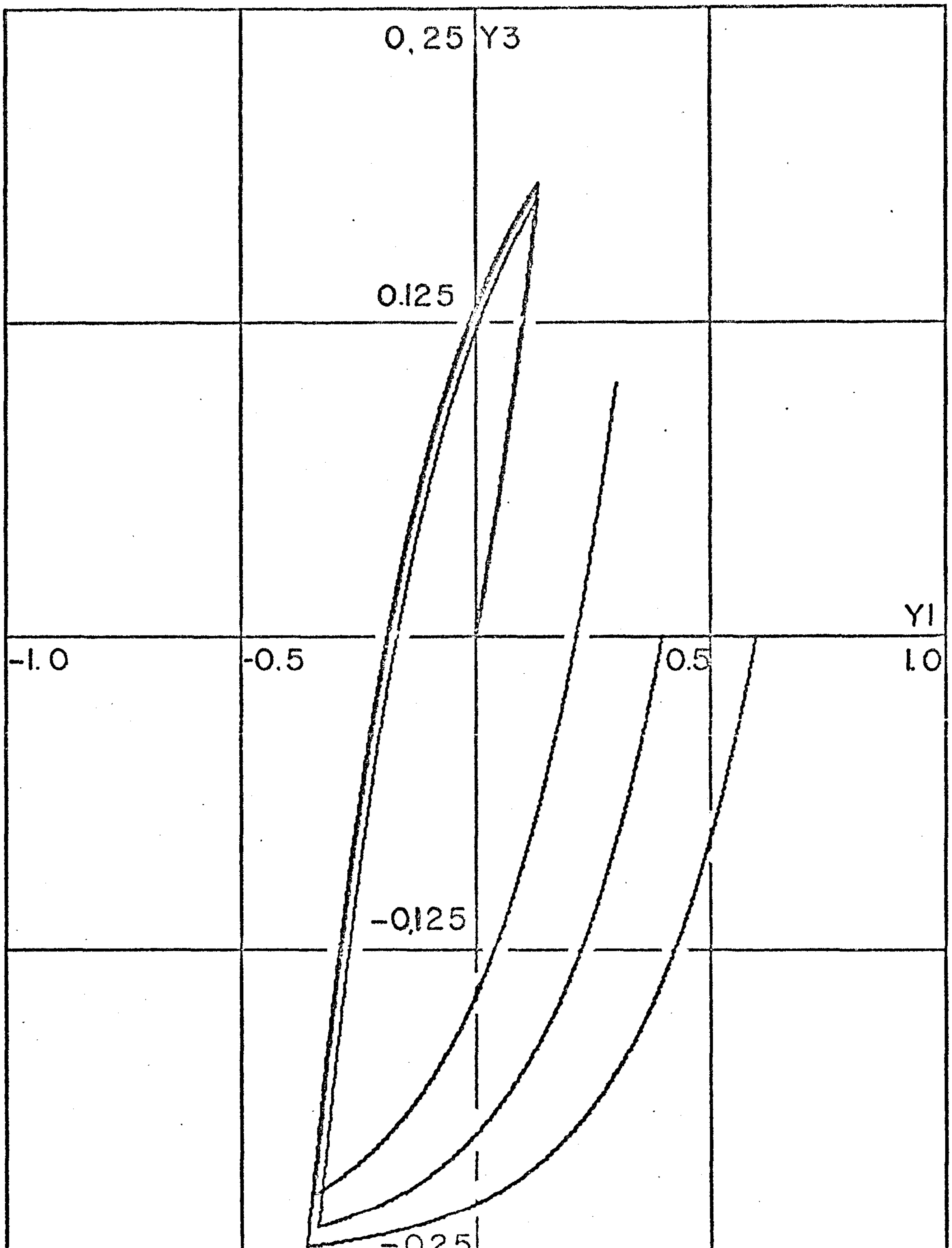



FIG 3.6(b) Y1–Y3 PROJECTIONS OF THE SYSTEM TRAJECTORIES

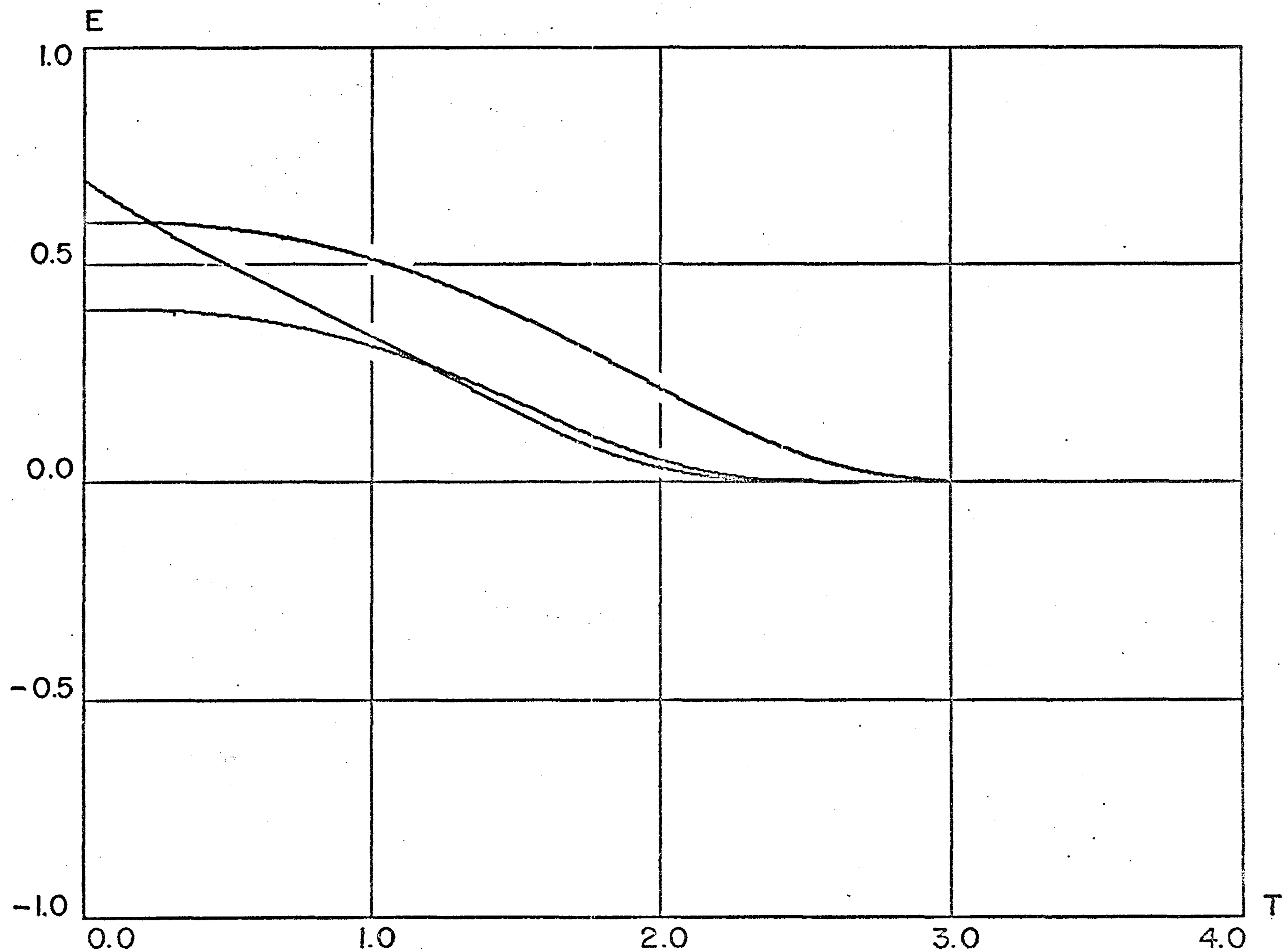



FIG 3.6(c) SYSTEMS TIME RESPONSE (ERROR V/S TIME)

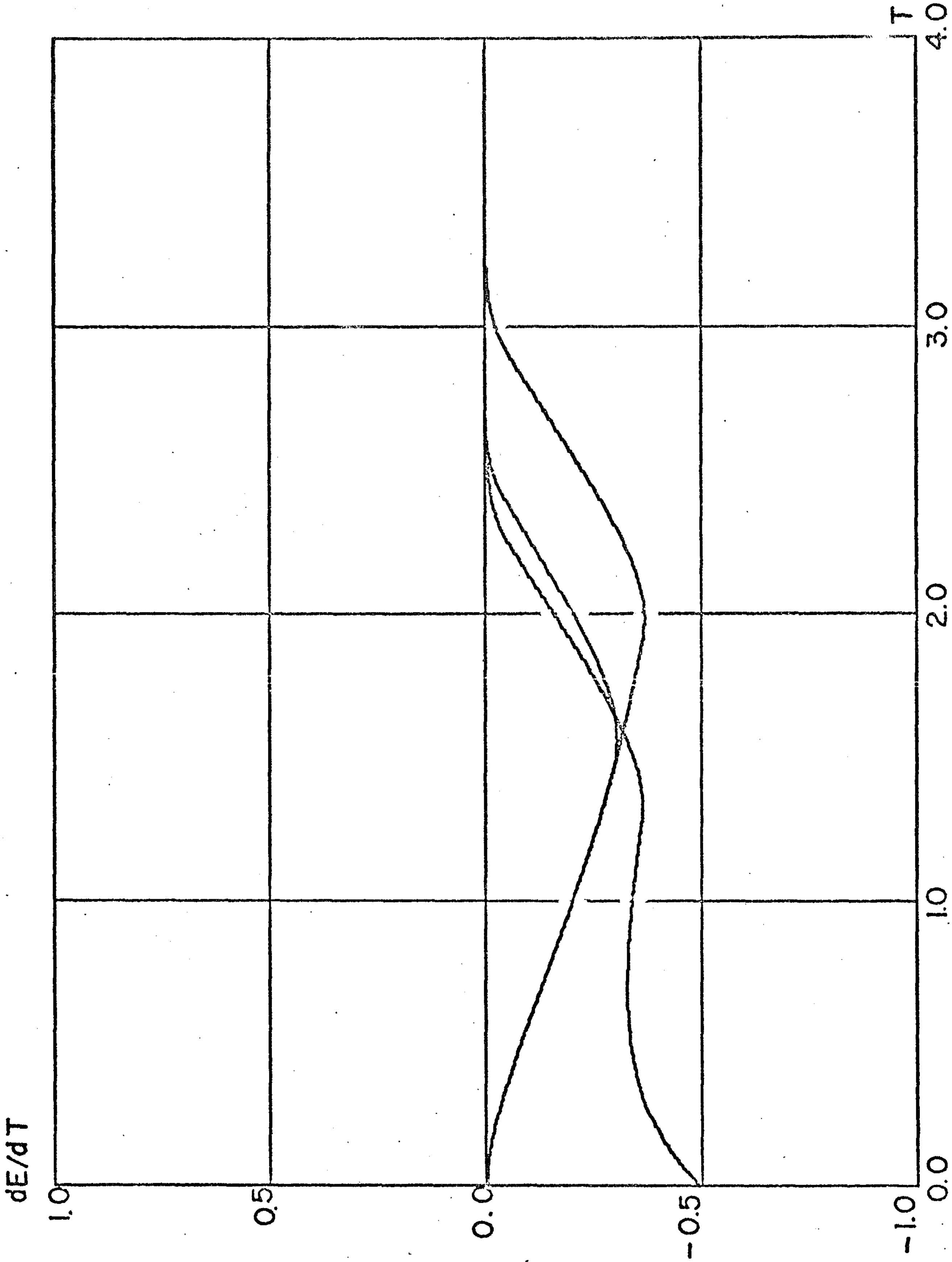



FIG 3.6(d) SYSTEMS TIME RESPONSE (RATE V/S TIME)

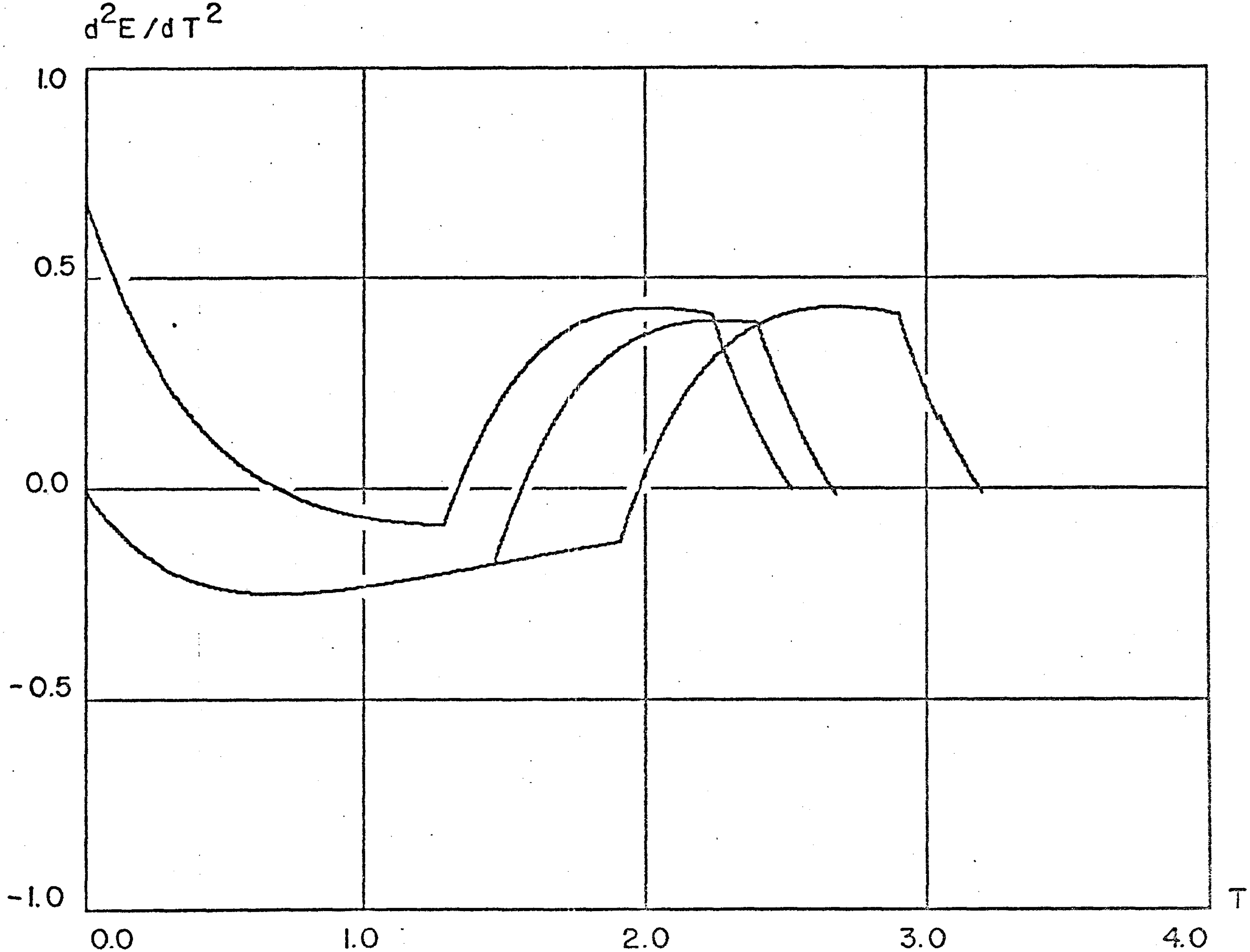



FIG 3.6(e) SYSTEMS TIME RESPONSE (ACCELERATION V/S TIME)

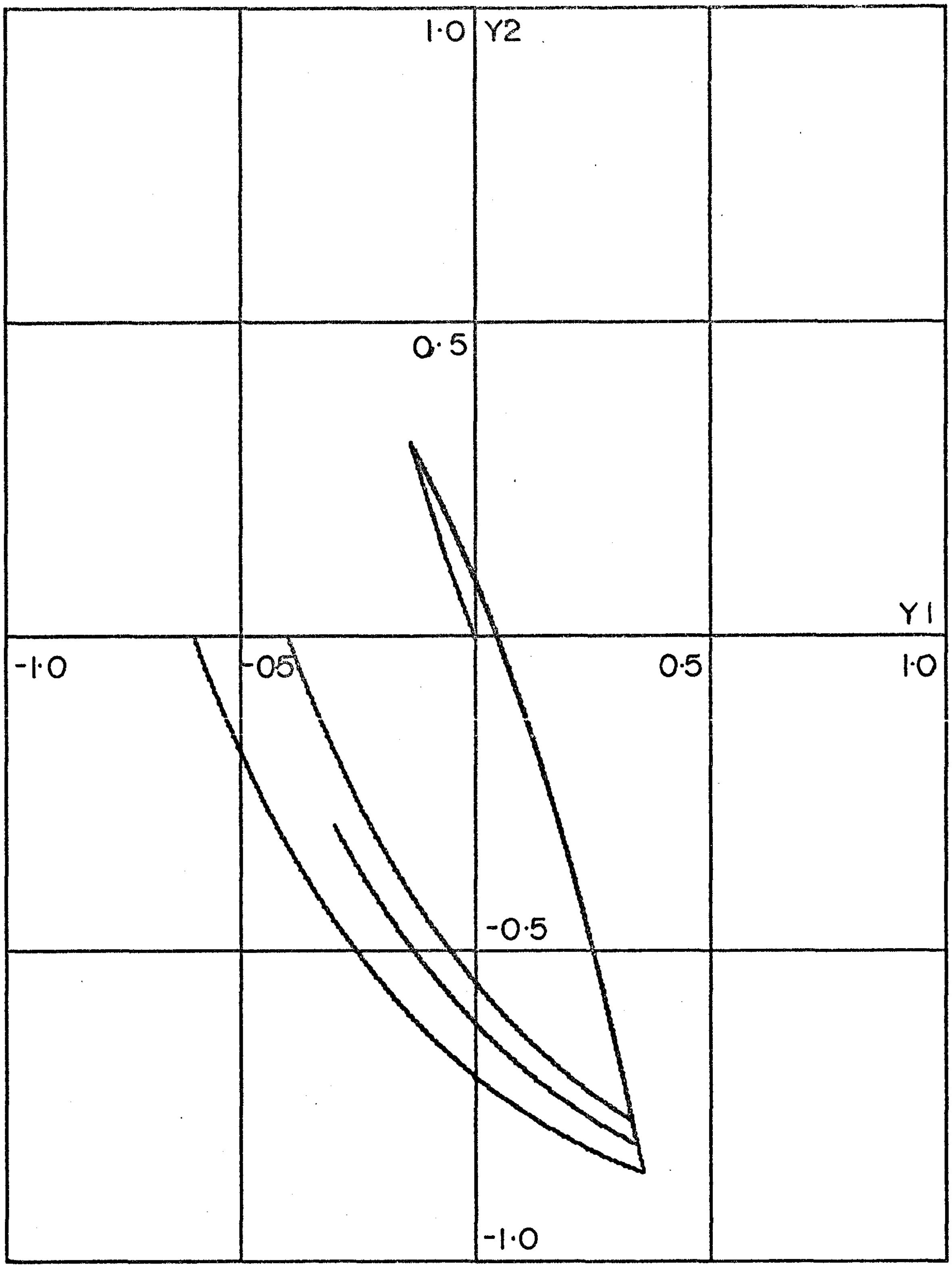



FIG 3.7(a)

YI-Y2 PROJECTIONS OF SYSTEM TRAJECTORIES

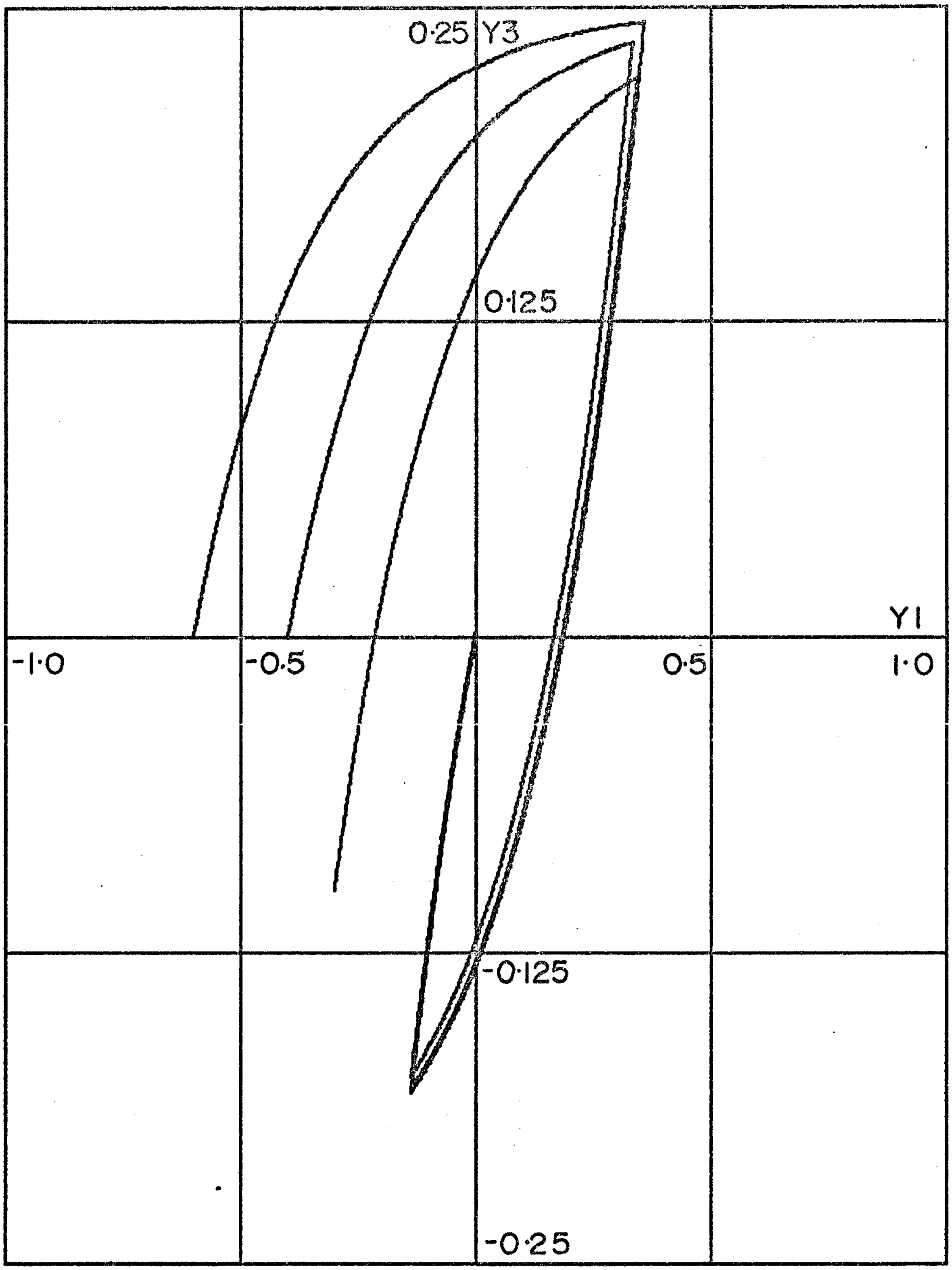



FIG 3.7(b)

YI-Y3 PROJECTIONS OF SYSTEM TRAJECTORIES

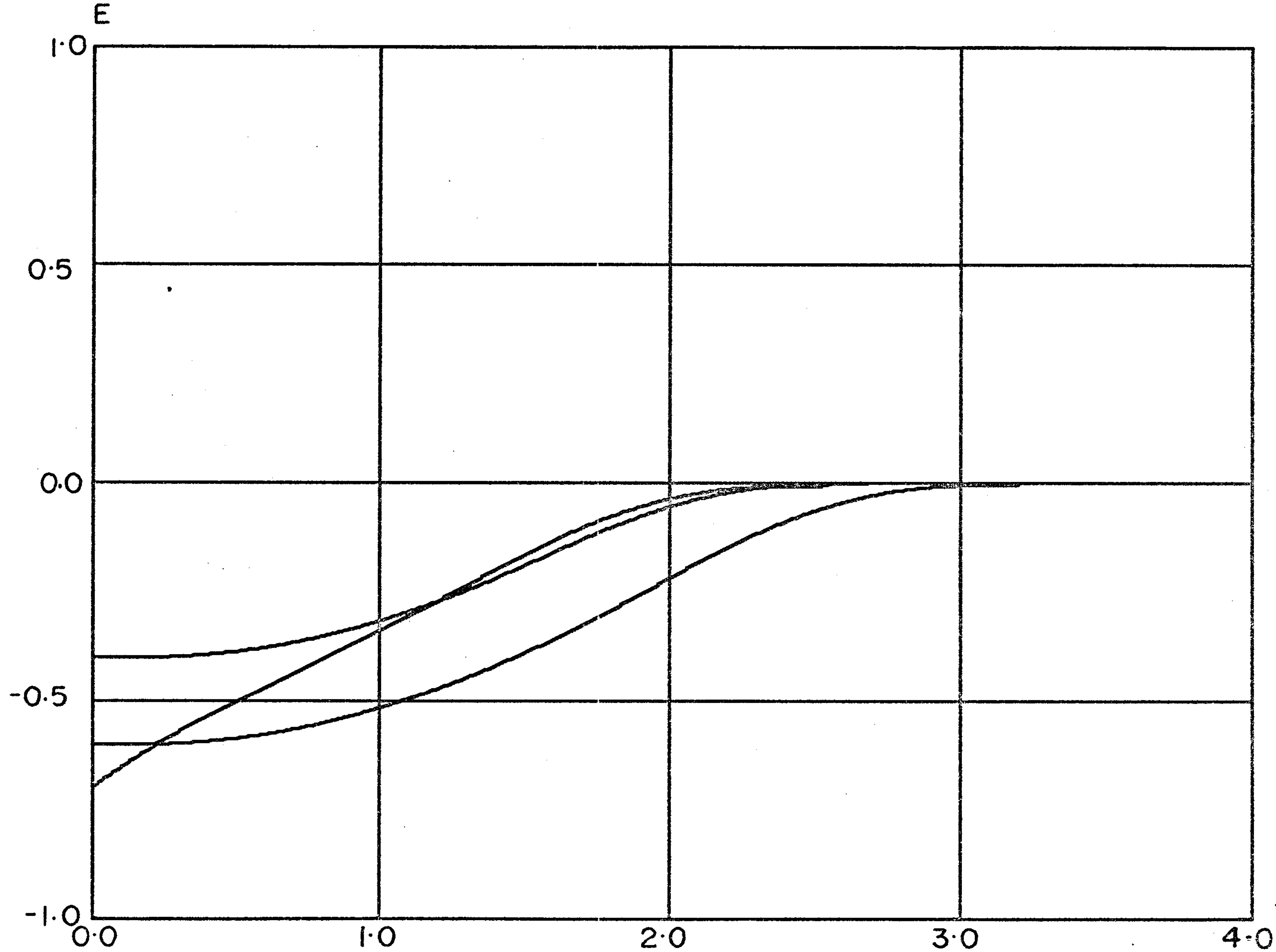



FIG 3.7(c) SYSTEMS TIME RESPONSE (ERROR V/S TIME)

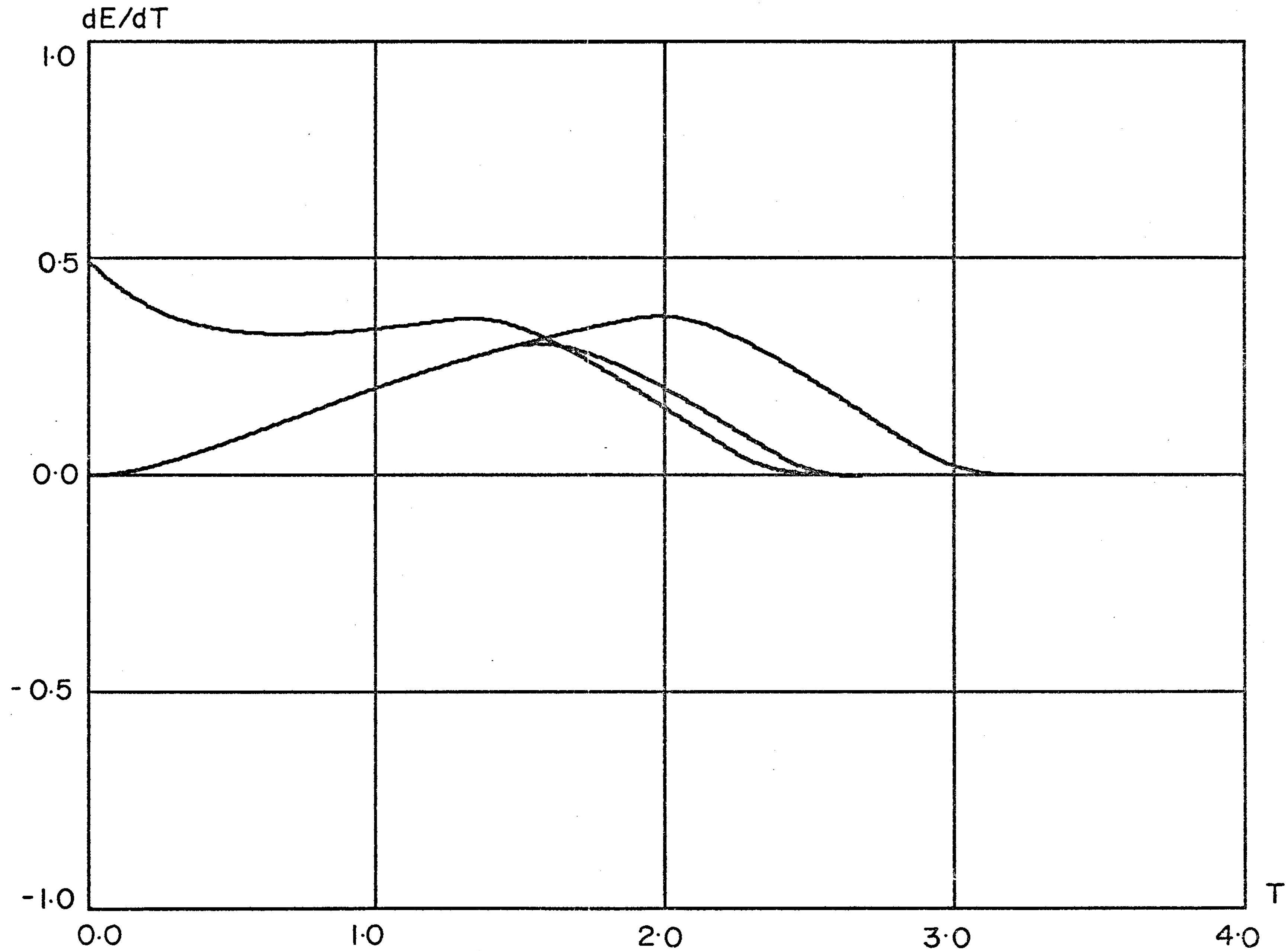



FIG 3.7(d) SYSTEMS TIME RESPONSE (RATE V/S TIME)

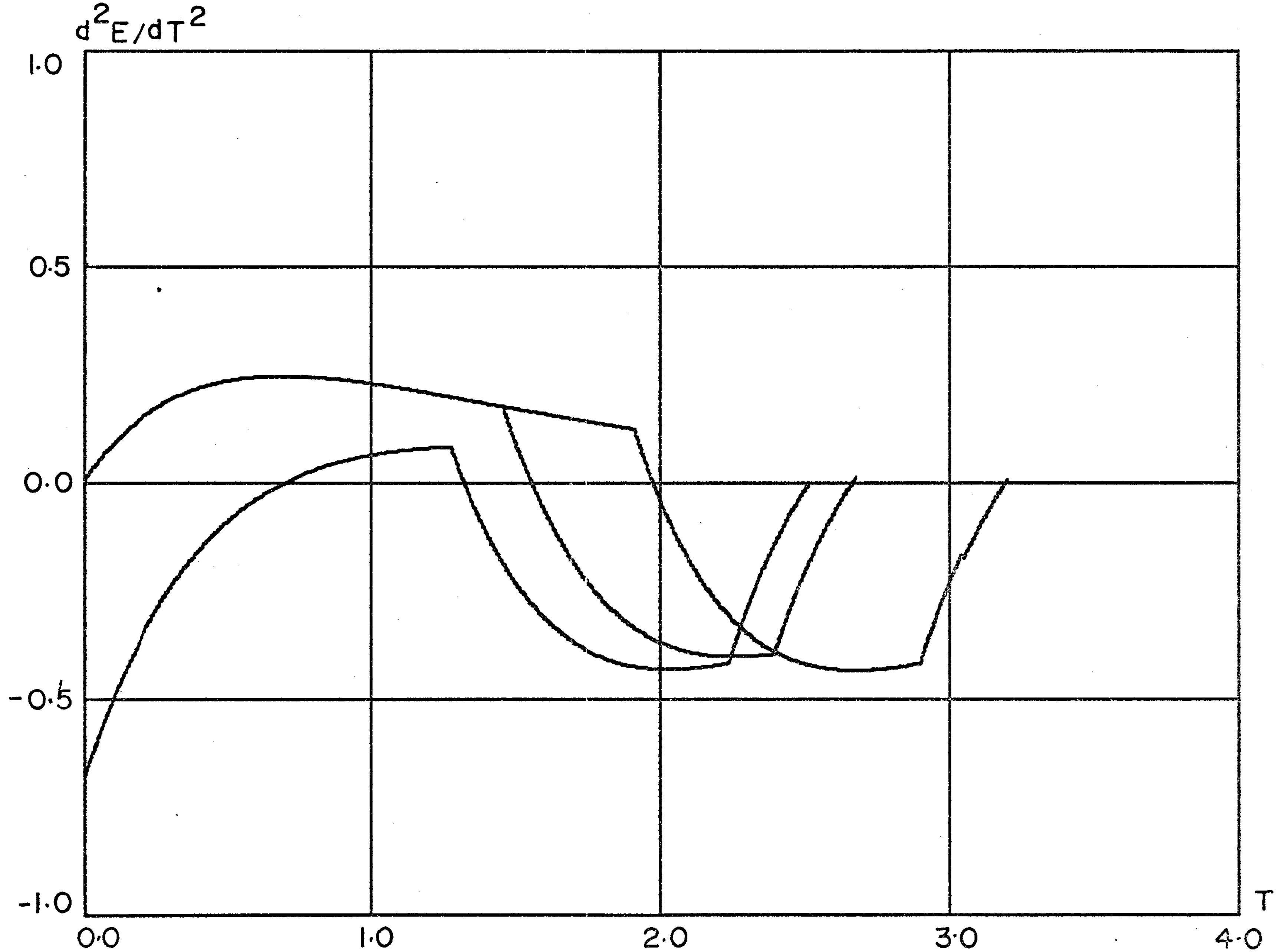



FIG 3.7(e) SYSTEMS TIME RESPONSE (ACCELERATION V/S TIME)

CHAPTER IV

# SWITCHING STRATEGIES FOR OPTIMAL CONTROL:
## Systems with Two Integrators and One Time Constant

In chapter II, a switching strategy was developed to obtain time optimal control of a system with two integrators and one time constant. It was assumed that the system parameters are time invariant, though small changes in the process to be controlled due to aging of the components or changes in load can be expected. In some cases the parameters of the controlled system transfer function are found to vary over extremely wide ranges with adverse effect on the system performance. In such cases however, the controller should possess the ability to self adjust or adapt its basic mode of operation to compensate for the parameter variations.[26,27] This chapter is concerned with automatic redesign of the controller characteristics to account for the changes in the system parameters. The parameter identification problems are not considered; the effort here is limited to an investigation of the parameteric sensitivity of the optimal strategy developed earlier.

### 4.1 SYSTEM EQUATIONS

To study the effect of parameter variations, the system of chapter II is considered in the general form of equation

$$\frac{d^3E}{dt^3} + a\frac{d^2E}{dt^2} = \delta \qquad (4.1)$$

where "a" is a parameter.

By properly selecting the coordinate transformation it is observed that the system trajectories become a set of parallel curves in the projection planes [28]. This characteristic of the system trajectories forms the basis for generating optimal trajectories in the computer (acting as a controller) memory without any mathematical manipulations. Hence the transformation materix is of the form

$$P = \begin{bmatrix} 1 & 0 & 1/a^2 \\ 0 & 1 & -1/a \\ 0 & 0 & 1 \end{bmatrix} \qquad (4.2)$$

Using the state equations (2.4) to (2.6) the projections of the phase trajectories in the Y1 - Y2 and Y2 - Y3 planes of Y space are given by

$$2\,a^3\,Y1 - \delta(a^2Y2 - \delta)^2 = C1 \qquad (4.3)$$

$$a^2\,Y2 + \delta\ell n(1 - a\delta Y3) = C2 \qquad (4.4)$$

The equations of the optimal trajectories are

$$(2\,a^3Y1 + \delta) - \delta(a^2Y2 - \delta)^2 = 0 \qquad (4.5)$$

$$a^2\,Y2 + \delta\ell n(1 - a\delta Y3) = 0 \qquad (4.6)$$

## 4.2 EVALUATION OF DERIVATIVES OF ERROR [29]

It is not always convenient to measure the derivatives of error directly by digital means. These derivatives can be easily determined by using simple difference equations. The error rate is given by[30]

$$\dot{E} = \frac{E_2 - E_1}{t_2 - t_1} = \frac{X1_2 - X1_1}{t_2 - t_1} \qquad (4.7)$$

Where

$X1_2 = E_2 =$ Present error sampled at time $t_2$

$X1_1 = E_1 =$ Previous error sampled at time $t_1$

If samples of errors are taken at a constant interval of time, then

$$t_2 - t_1 = \Delta t \tag{4.8}$$

and

$$\dot{E} = \frac{E_2 - E_1}{\Delta t} \tag{4.9}$$

Similarly error acceleration may be approximated as

$$\ddot{E} = \frac{E_2 - 2E_1 + E_o}{(\Delta t)^2} = \frac{X1_2 - 2X1_1 + X1_o}{(\Delta t)^2} \tag{4.10}$$

where $E_o$ is the error recorded prior to $E_1$. This method offers the obvious advantage of requiring only an error transducer to supply sufficient information to effect control. This is very significant as error derivative measurement devices are often very complex and expensive.

## 4.3 GENERATION OF OPTIMUM CURVES AND THE SWITCHING STRATEGY.

When the transformed coordinates are used, the projections of the system trajectories in the transformed space become parallel. As shown in figures 4.1(a) and 4.1(b), the optimum trajectories are the actual system trajectories shifted along the Y1 axis in the Y1 - Y2 plane and shifted along the Y2 axis in the Y2 - Y3 plane. Hence any point on the optimum trajectory bears the following relationship with the points on the system trajectories.

In the Y1 - Y2 plane

$$Y1\ opt = Y1\ act - EH1 \tag{4.11}$$

In the Y2 - Y3 plane

$$Y2\ opt = Y2\ act - EH2 \tag{4.12}$$

Where EH1 and EH2 are the curve offsets from the origin and Y1 act and Y2 act are the actual system response states.

The projections of optimum curves obtained from the system trajectories may be stored in the form of tables in the computer memory as Y2 a function of Y1 and Y3 i.e. $Y2_n\ (Y1_n)$ in the Y1 - Y2 plane and as $Y2_m\ (Y3_m)$ in Y2 - Y3 plane. Since the other half of the optimal curves in the opposite quadrants are merely mirror images, comparison is made between the absolute values of the actual state and the corresponding absolute values from optimum curves obtained by storing and shifting as described before. The memory tables may be compared as

$$Y1_n = |Y1_2| \qquad (4.13)$$

$$Y3_m = |Y3_2| \qquad (4.14)$$

$Y1_2$ and $Y3_2$ are the present values of the system state

$$YD1 = |Y2_n \quad (Y1_n - |EH1|)| \qquad (4.15)$$

$$YD2 = |Y2_m \quad (Y3_m)| - |EH2| \qquad (4.16)$$

The subscripts n and m indicate any point in general on the optimum (stored) curves. YD1 is the optimum value of Y2 from the stored curve in Y1 - Y2 plane and YD2 is the optimum value of Y2 from the stored curve in Y2 - Y3 plane.

$$YC = YD1 \quad \text{IF} \quad YD1 \geq YD2 \qquad (4.17)$$

$$YC = YD2 \quad \text{IF} \quad YD1 < YD2$$

The optimum value YC of Y2 enables the controller to be certain that the system has changed region in both the planes. For optimum switching the condition is therefore

$$|Y2_2| \geq YC \qquad (4.18)$$

## 4.4 THE SWITCHING SEQUENCE

From the strategy developed in the previous section, optimum switching can be easily effected by a simple table fill-in and comparison technique. However, since switching should occur in appropriate quadrants, the switching sequence should be properly maintained. This is easily accomplished by pre-setting various points along the trajectory and properly assigning the controller operation to be performed at each of these points. The points Pi (i = 1,2,3,.....,11) shown in figures 4.1(a) and 4.1(b) have the following operations associated with them.

$P_1$, Start comparing actual response with blank table.

$P_2$, Start to fill in tables (Y1 v/s Y2) and (Y2 v/s Y3) after switching on sgn(Y1)

$P_3$, Stop filling in table (Y2 v/s Y3). Store EH2

$P_4$, Stop filling in table (Y1 v/s Y2). Store EH1
Start comparing tables with actual response.

$P_5$, System ready for switching, start delay in switching.

$P_6$, Switch and reverse force on relay.

$P_7$, Start comparing tables with actual response on sgn (Y2)

$P_8$, Switch optimally

$P_9$, Unwanted delayed switching

$P_{10}$, Start comparing tables with actual response on sgn (Y2)

$P_{11}$, Switch optimally.

The compare cycle starts on each change of sgn (Y2). The tables fill in starts on change of sgn (Y1). To avoid undesirable responses the tables should be filled in only after the first switching following initiation of the response. This may be conveniently implemented by setting an index K and not filling in the tables if K>1. The index K is increased by unity after each switching, and is varied throughout the controller operation. The various values of K corresponding to the Pi on the figures 4.1(a) and 4.1(b) are given in Table 3.

TABLE 3

RELATIONSHIP BETWEEN Pi & K

| Position of the state point | Index K |
|---|---|
| IC $\longrightarrow$ $P_2$ | 0 |
| $P_2$ | 1 |
| $P_2$ $\longrightarrow$ $P_4$ | 1 |
| $P_4$ | 2 |
| $P_4$ $\longrightarrow$ $P_6$ | 2 |
| $P_6$ | 3 |
| $P_6$ $\longrightarrow$ $P_8$ | 3 |
| $P_8$ | 4 |
| $P_8$ $\longrightarrow$ origin | 4 |
| In case the system has delayed switching at $P_9$ | |
| $P_9$ (instead of $P_8$) | 4 |
| $P_9$ $\longrightarrow$ $P_{11}$ | 4 |
| $P_{11}$ | 5 |
| $P_{11}$ $\longrightarrow$ origin | 5 |

4.5 EFFECT OF CHANGE IN PARAMETER "a"

If any change in parameter occurs, the optimum trajectory is modified automatically. This can be explained by considering the fact that the memory will be refilled

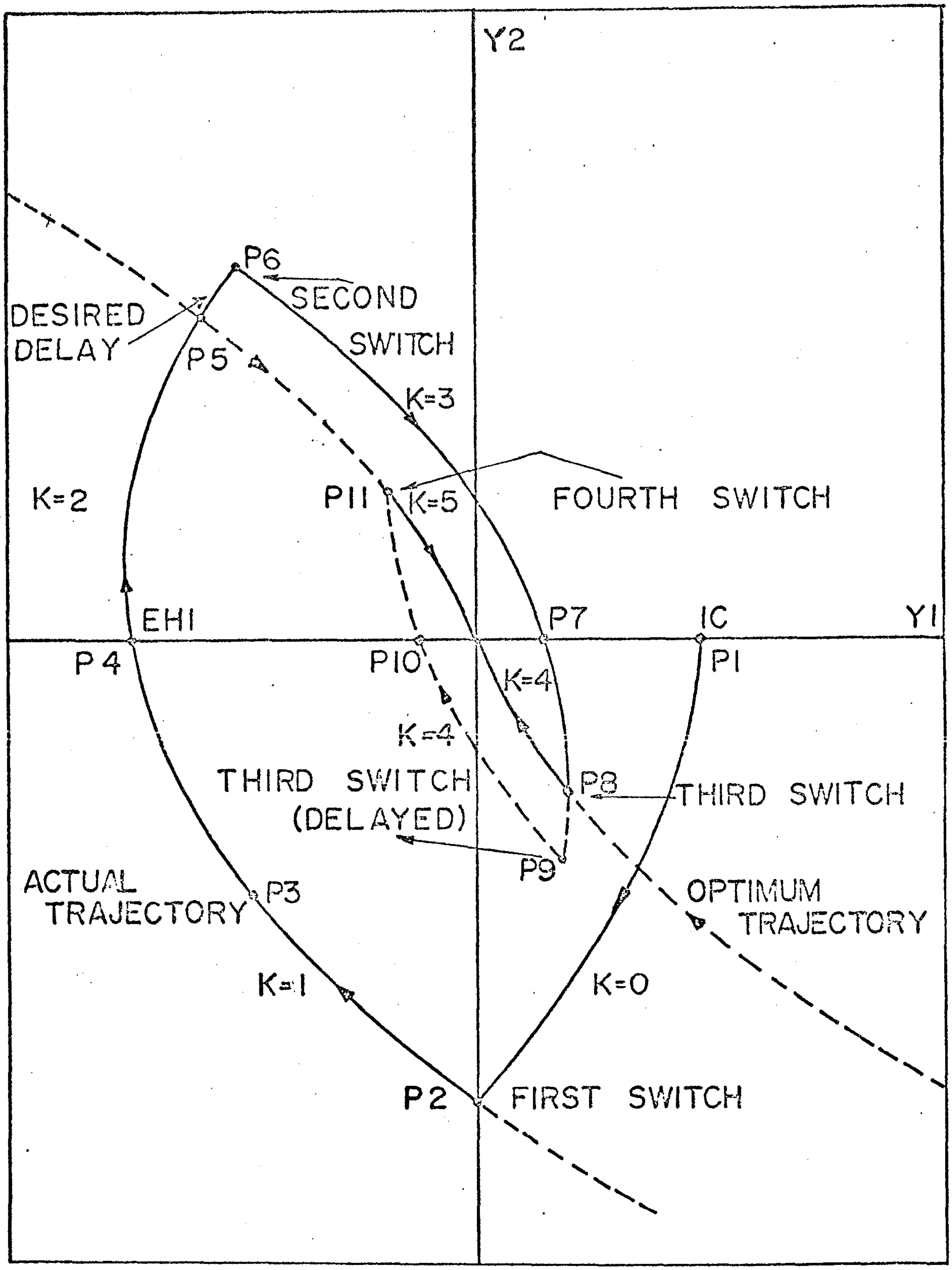



Fig 4.1(a) GENERATION OF OPTIMUM CURVES AND SWITCHING SEQUENCE IN Y1-Y2 PLANE.

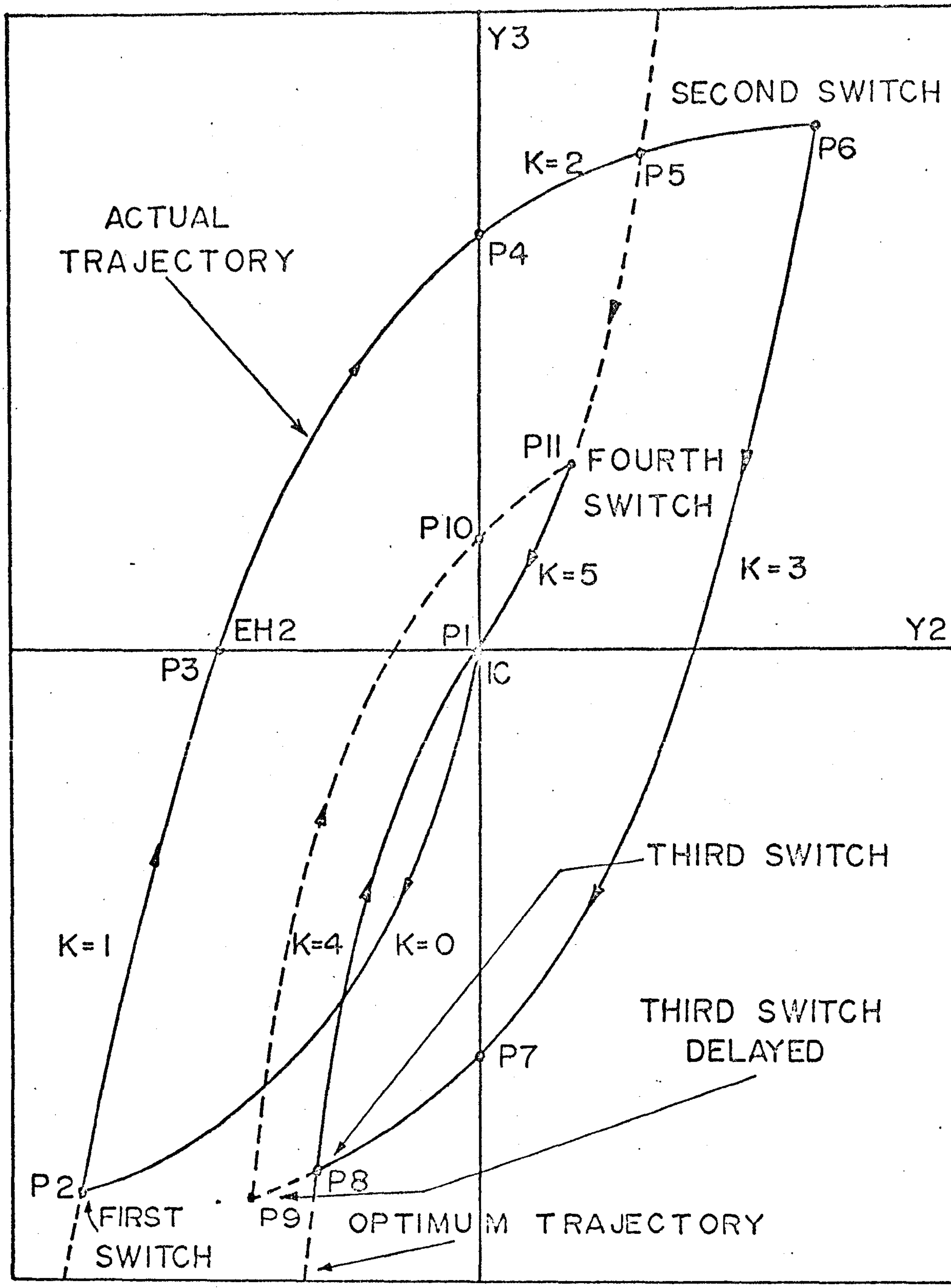



FIG 4.1(b) GENERATION OF OPTIMUM CURVES AND SWITCHING SEQUENCE IN Y2-Y3 PLANE

each time K=1, and the system always has a new reference generated by itself to compare with for the optimum switching. Changes in system parameter will yield new trajectories, but the system will be able to discern these changes because of the new switching tables. The first new response should exhibit one overshoot before switching optimally, and the next response should be optimum.

Because transformed coordinates are used, the transformation requires the knowledge of the change in the parameter "a", and this leads to dynamic identification problems which are not within the scope of this dissertation. However, if the parameters change gradually and within certain small limits, the method can be effective for obtaining nearly optimal control if the transformation is essentially insensitive to "a".

The sensitivity of the transformation to the value of "a" is now considered. Using the equation (4.2) the transformation equations are

$$Y1 = X1 - \frac{X3}{a^2} \tag{4.19}$$

$$Y2 = X2 + \frac{X3}{a} \tag{4.20}$$

$$Y3 = X3 \tag{4.21}$$

From equations (4.19) to (4.21) one can observe that only the variables Y1 and Y2 involve the parameter "a". Also from equation (4.6) it is evident that the maximum value of X3 decreases as "a" increases.

For "a" between 1.0 and 5.0, the values of $X3/_a$ and $X3/_{a^2}$ are shown in Table 4. The change in $X3/_a$ and $X3/_{a^2}$ values is quite large for a change in "a" from 1.0 to 2.0. It seems unlikely, an approximate controller setting would be sufficiently accurate for this range of system parameter values. However, for "a" values greater than 2.0, a fixed approximate controller setting may lead to performance that is nearly optimal. For example, if the controller parameter is set at 2.5, the values $X3/_{2.5}$ and $X3/_{6.25}$ seem close enough to true values to be used as the controller setting included in Table 4. If this approximation is used, then the final switching does not pass through the origin of the coordinate systems. However, as this is only an approximation, it is to be expected that the performance will not now be absolutely optimal.

## 4.6 SIMULATION RESULTS

To study the effect of parameter variation on the control strategy, simulation runs were carried out on a digital computer using the flow charts given in the appendix in figure 4.2. The controller parameter was set at 2.5. Figures 4.3(a) to 4.3(c) show the plotter result for various values of parameter "a". For each parameter the first switching is effected at the change in sgn(Y1) due to the absence of a stored trajectory.

Of particular interest is the behaviour of the control

TABLE 4

EFFECT OF PARAMETER VARIATION ON THE TRANSFORMATION MATRIX

| Parameter "a" | Max X3 | X3/a | $X3/a^2$ | X3/2.5 | X3/6.25 |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | | |
| 1.25 | 0.8 | 0.64 | 0.51 | | |
| 1.5 | 0.67 | 0.44 | 0.29 | | |
| 1.75 | 0.57 | 0.32 | 0.19 | | |
| 2.0 | 0.50 | 0.25 | 0.125 | 0.2 | 0.08 |
| 2.5 | 0.40 | 0.16 | 0.06 | 0.16 | 0.06 |
| 3.0 | 0.33 | 0.11 | 0.03 | 0.13 | 0.05 |
| 4.0 | 0.25 | 0.06 | 0.01 | 0.10 | 0.04 |
| 5.0 | 0.20 | 0.04 | 0.008 | 0.08 | 0.03 |

strategy when the system parameter changes from one value to the other. In figure 4.4 the responses are given for the situation when the system parameter changes from 2 to 3. Table 5 shows the final values for the runs when the trajectory just enters an arbitrary target volume defined by $E = \pm 0.07$, $\dot{E} = \pm 0.13$, $\ddot{E} = \pm 0.09$. Run 1, which is not included in figure 4.4 because of scale, is the same as the run corresponding to a=2 in figure 4.3(a). Run 2 uses the same parameter value (a=2), but uses the previously stored trajectory values in order to arrive at the steady state in a time shorter than that used for run 1. The response for run 2 is nearly optimal. Run 3, for which the system parameter is now changed to 3, uses the stored trajectory (a=2) for the first switch, after which the system generates a new table of trajectory values. This causes an overshoot resulting in suboptimal performance. The performance is improved in run 4 by using the newly stored trajectory values for the control. The total time for run 4 is not necessarily optimal, but it is best achievable with the strategy outlined. The performance is suboptimal due to approximations introduced in derivatives of error formulation and the fixed setting of the controller variable.

Figures 4.5 to 4.7 show the results for similar parameter variations when the system parameter changes from 3 to 2, 3 to 6 and 5 to 6 respectively.

TABLE 5

EFFECT OF PARAMETER VARIATION ON SYSTEMS RESPONSE

Controller Parameter "a" = 2.5

| SYSTEM PARAMETER | | RUN | FINAL | | | TOTAL NORMALIZED TIME |
|---|---|---|---|---|---|---|
| Change | "a" | | E | $\dot{E}$ | $\ddot{E}$ | |
| 2 to 3 | 2 | 1 | -0.02 | 0.08 | 0.09 | 15.35 |
| | | 2 | 0.01 | -0.07 | -0.09 | 7.11 |
| | 3 | 3 | 0.02 | -0.04 | -0.09 | 7.85 |
| | | 4 | -0.07 | 0.13 | -0.08 | 6.07 |
| 3 to 2 | 3 | 1 | 0.07 | -0.12 | 0.01 | 14.54 |
| | | 2 | -0.06 | 0.09 | 0.09 | 5.71 |
| | 2 | 3 | -0.04 | 0.13 | -0.005 | 6.52 |
| | | 4 | 0.04 | 0.02 | 0.09 | 4.70 |
| 3 to 6 | 3 | 1 | 0.07 | -0.12 | 0.01 | 14.54 |
| | | 2 | -0.06 | 0.09 | 0.09 | 5.71 |
| | 6 | 3 | -0.04 | -0.02 | -0.09 | 10.74 |
| | | 4 | 0.02 | 0.02 | 0.09 | 7.22 |
| 5 to 6 | 5 | 1 | -0.07 | -0.02 | -0.001 | 16.68 |
| | | 2 | 0.07 | 0.02 | 0.001 | 6.64 |
| | 6 | 3 | -0.06 | -0.08 | 0.01 | 11.38 |
| | | 4 | 0.06 | 0.07 | -0.01 | 7.80 |

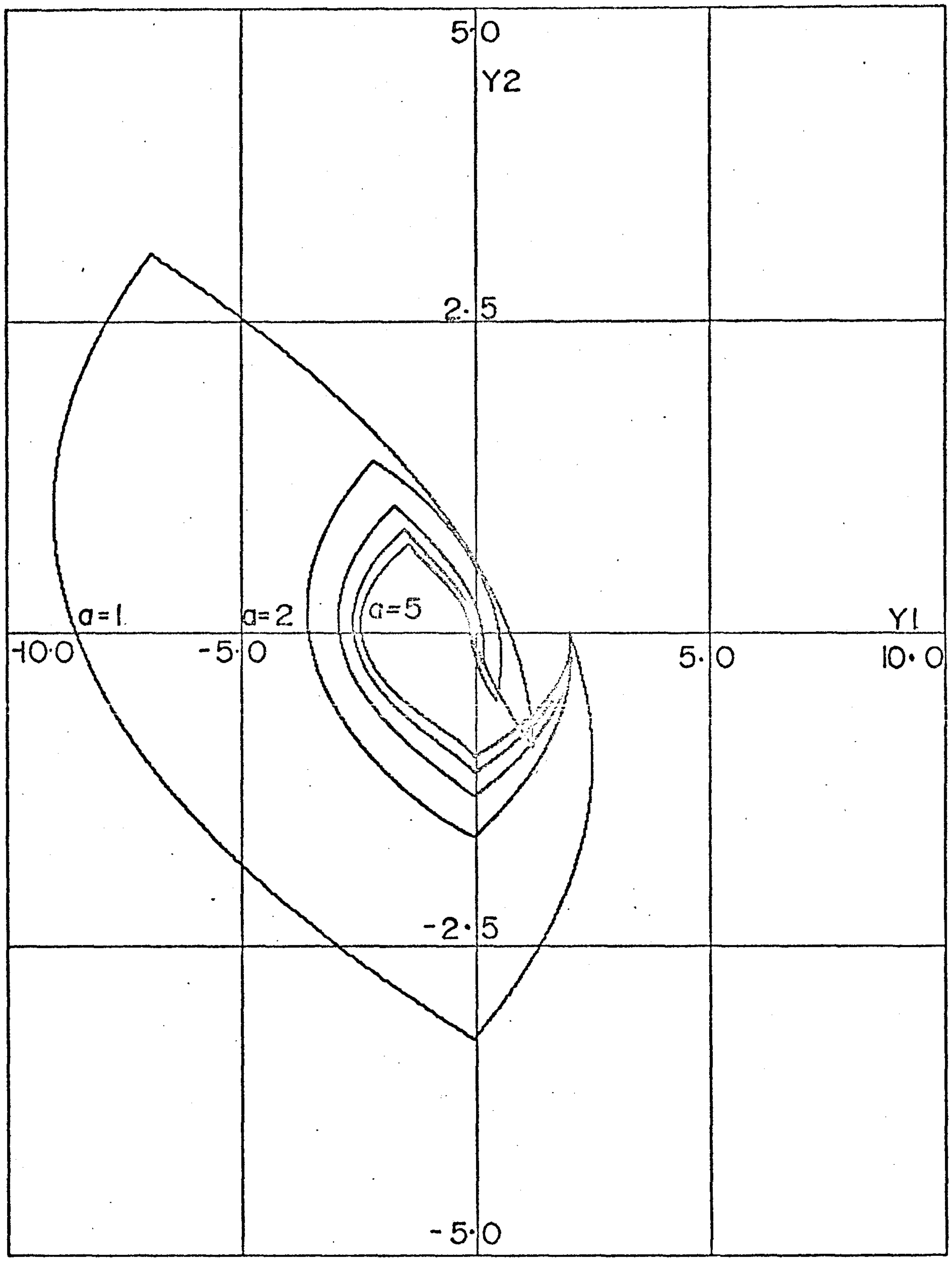



FIG 4.3 (a)

YI- Y2 PROJECTIONS OF SYSTEM TRAJECTORIES

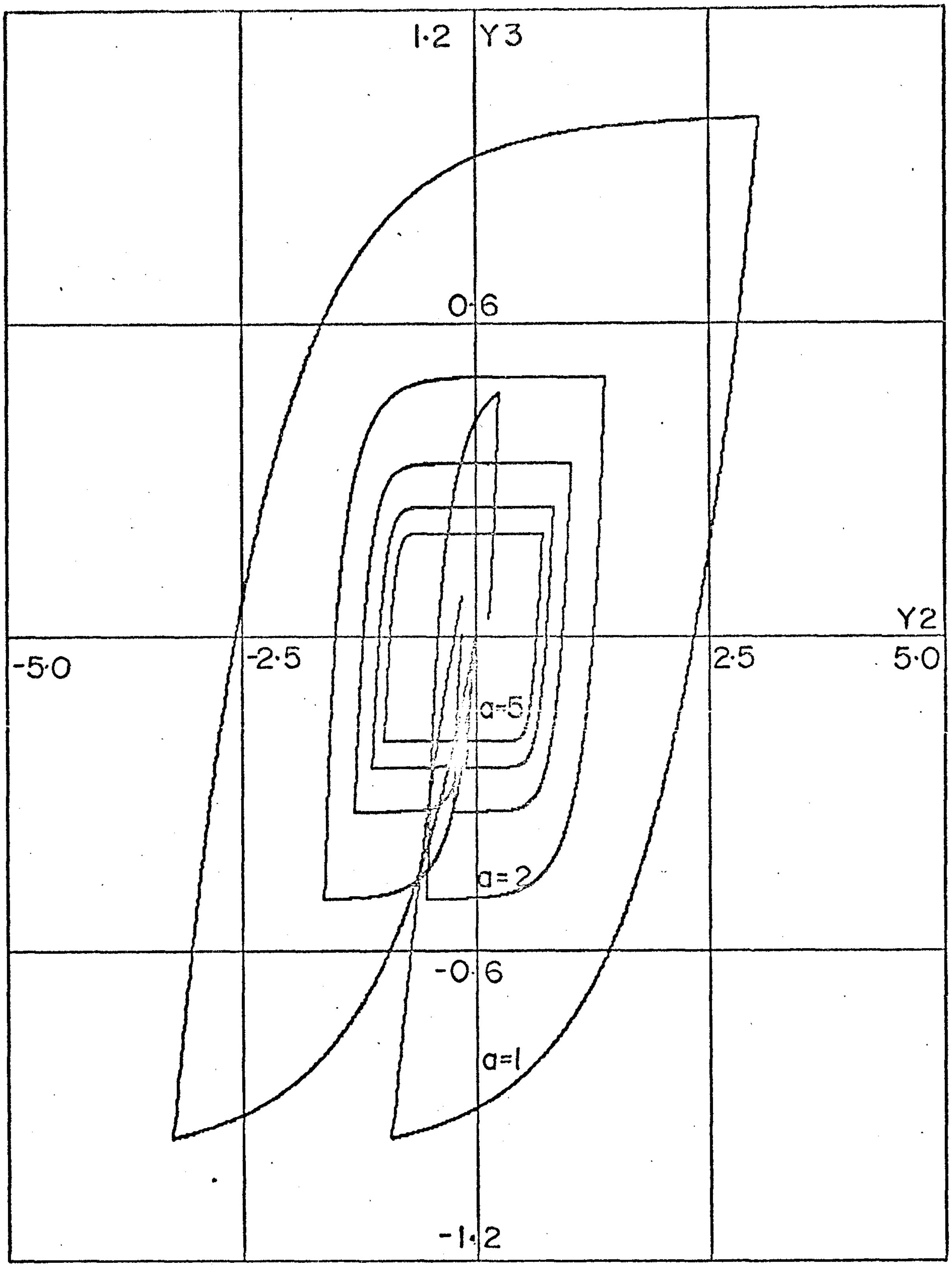



FIG 4.3(b)

Y2-Y3 PROJECTIONS OF SYSTEM TRAJECTORIES

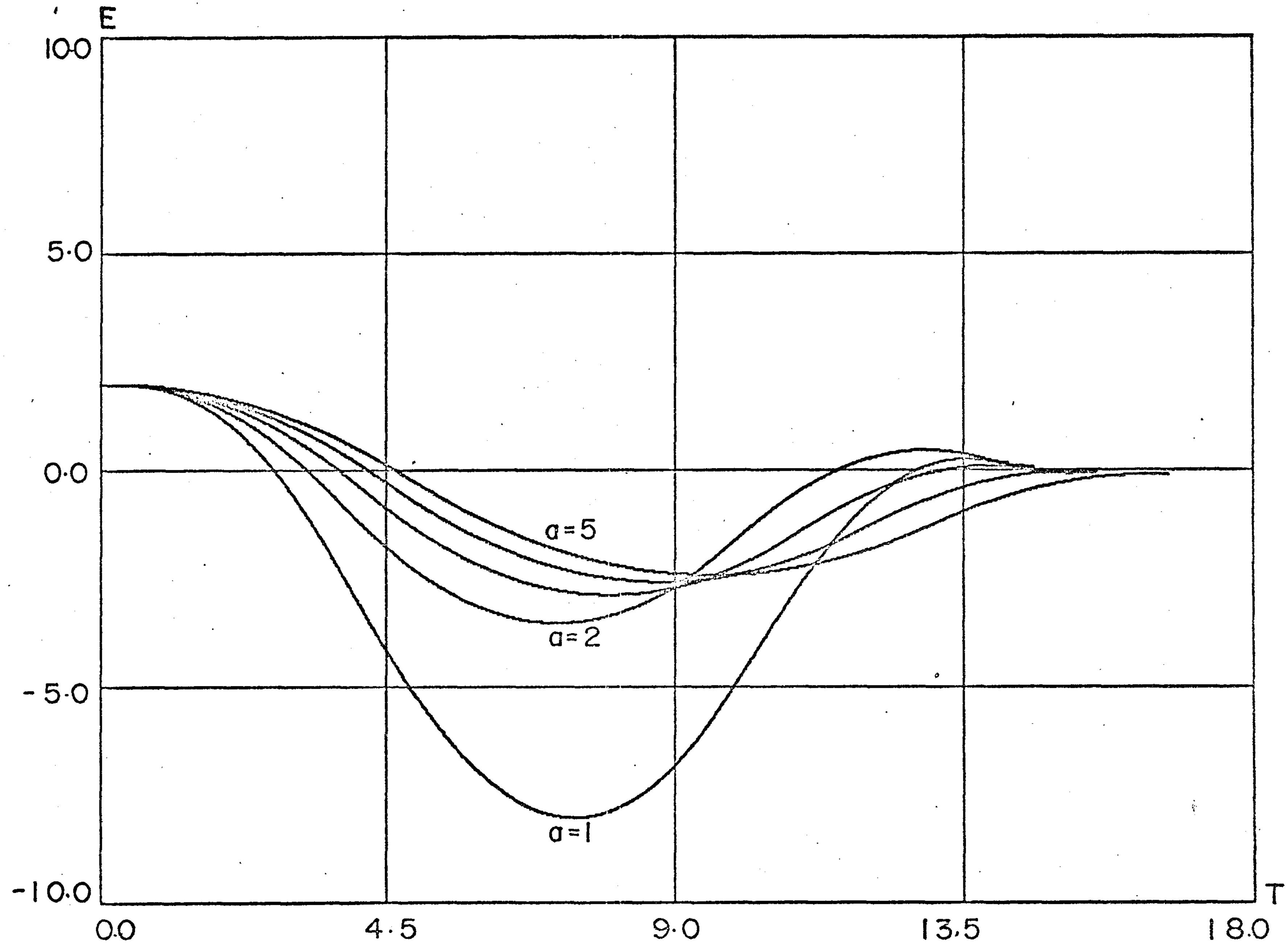



FIG 4.3(c) SYSTEMS TIME RESPONSE

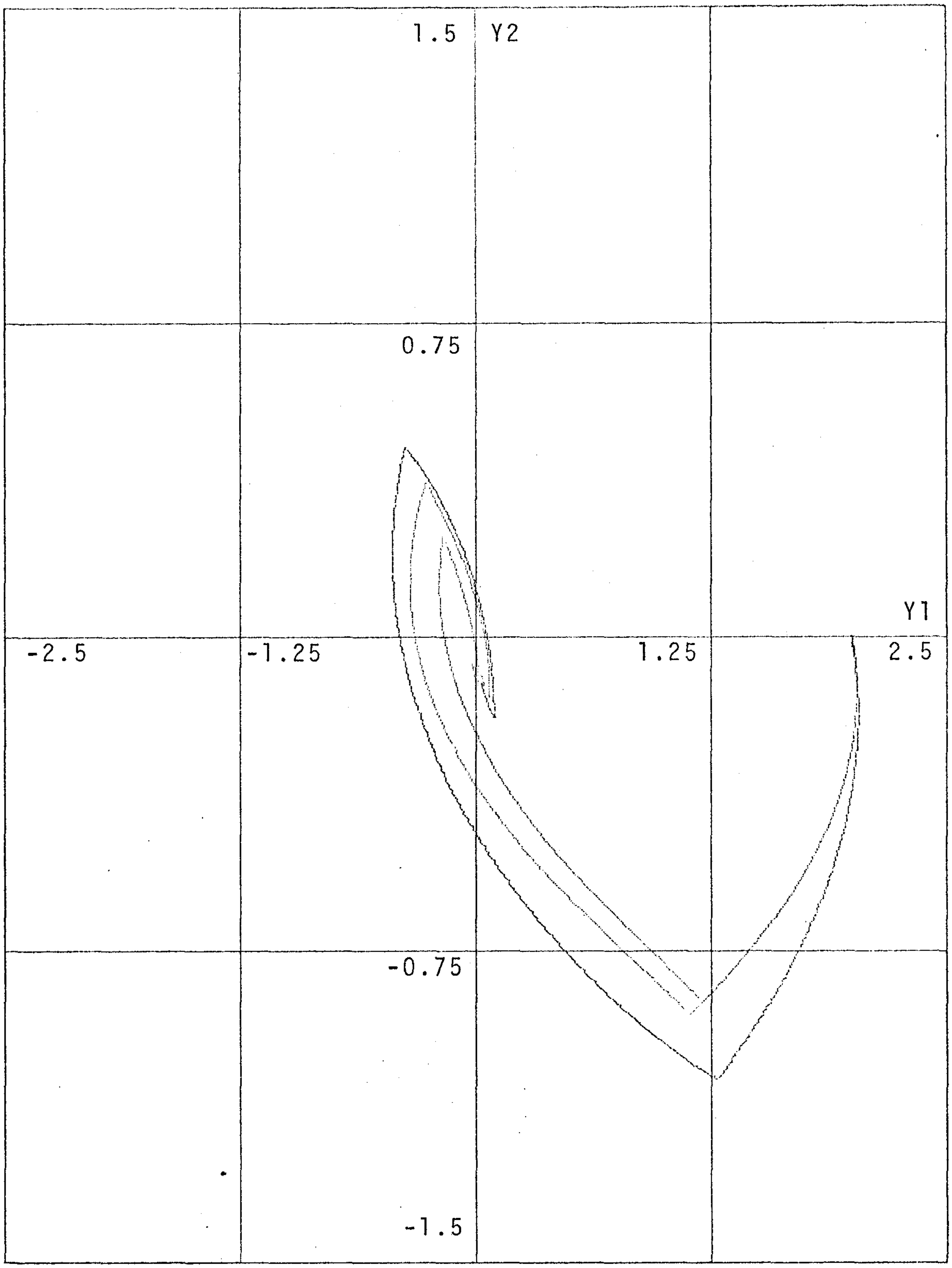



FIG. 4.4

SYSTEM TRAJECTORIES WITH PARAMETER

CHANGE 2 to 3

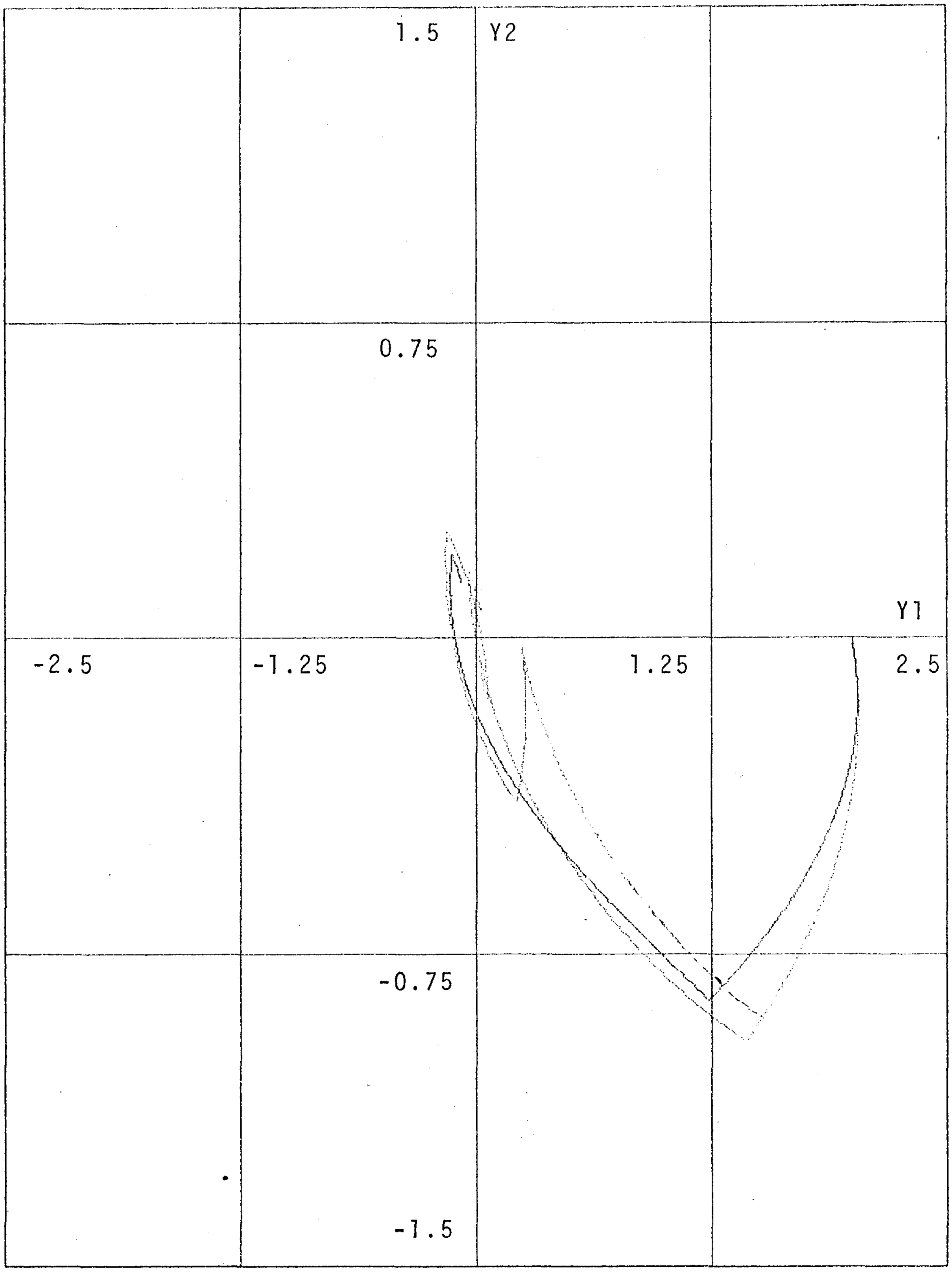



FIG. 4.5

SYSTEM TRAJECTORIES WITH PARAMETER
CHANGE 3 to 2

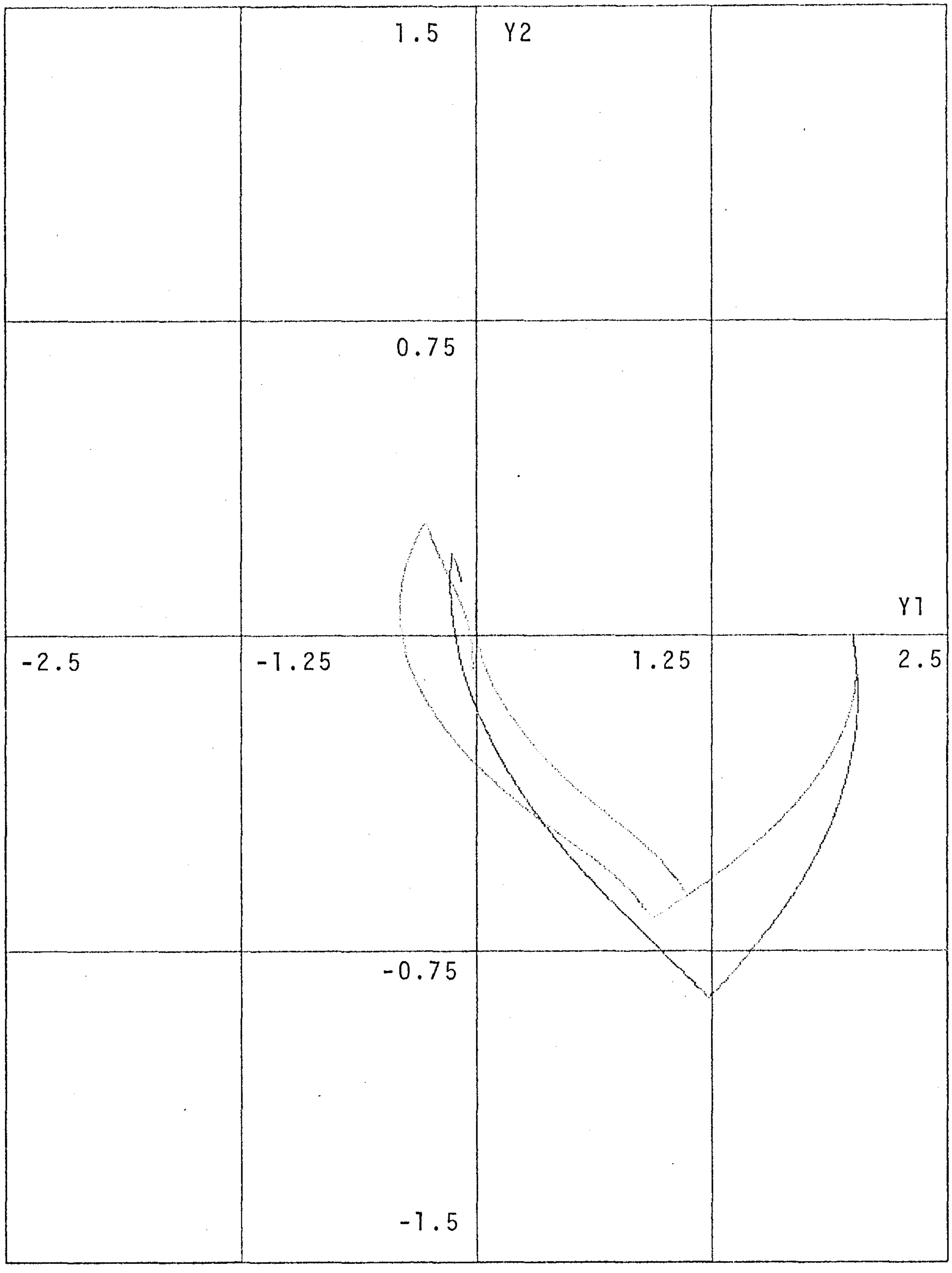



FIG. 4.6

SYSTEM TRAJECTORIES WITH PARAMETER

CHANGE 3 to 6

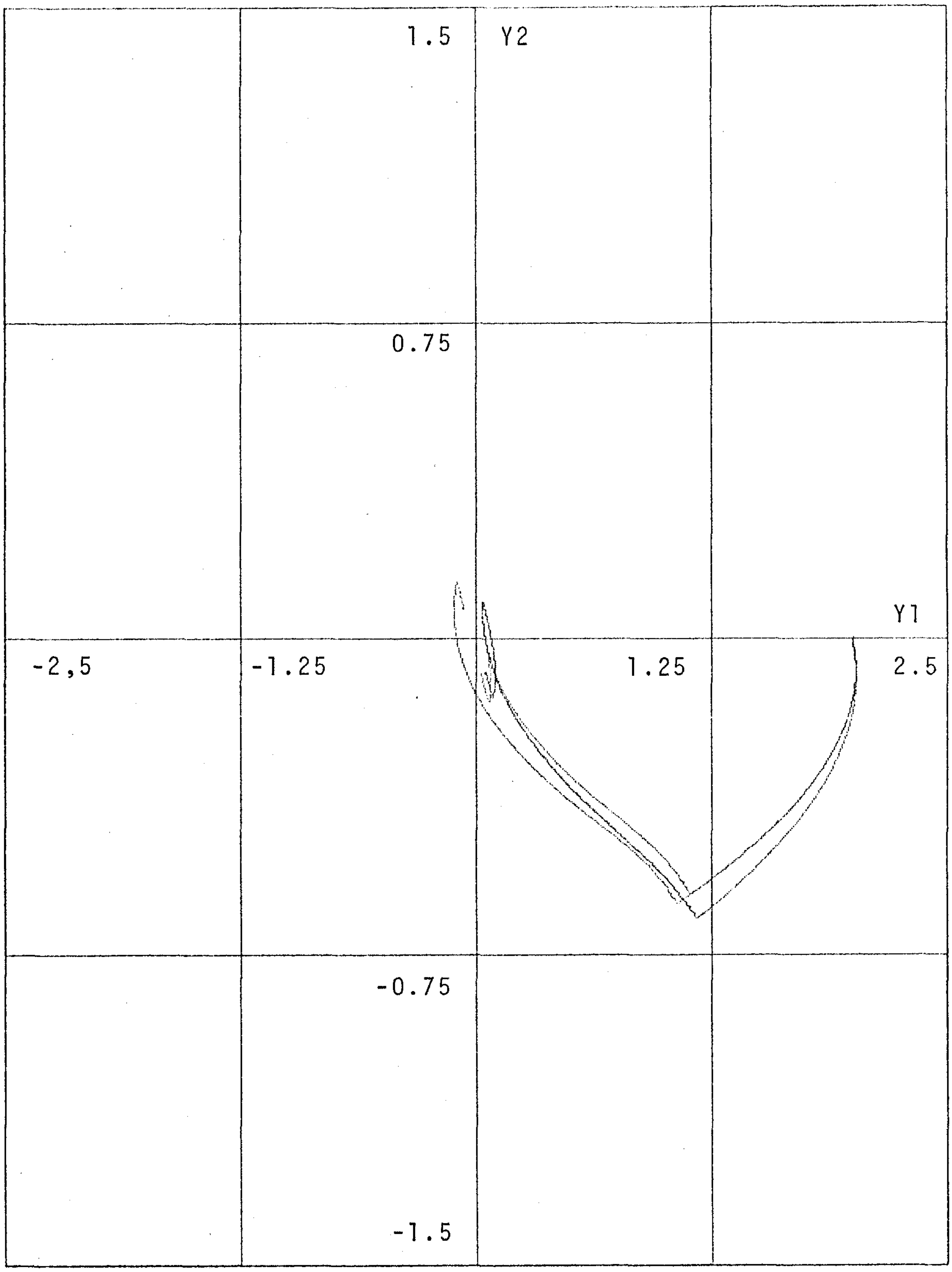



FIG. 4.7

SYSTEM TRAJECTORIES WITH PARAMETER

CHANGE 5 to 6

CHAPTER V

# SWITCHING STRATEGIES FOR OPTIMAL CONTROL: Systems with One Integrator and Two Time Constants

## 5.1 SYSTEM EQUATIONS

The differential equation describing a general system of the type considered in Chapter III, can be written as

$$\frac{d^3E}{dt^3} + (1+a)\frac{d^2E}{dt^2} + a\frac{dE}{dt} = \delta \qquad (5.1)$$

Where "a" is the parameter.

By following a procedure similar to that used in the previous chapter the transformation matrix is

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -a \\ 0 & 1 & a^2 \end{bmatrix} \qquad (5.2)$$

and the transformed coordinates are related by the equation

$$\begin{bmatrix} Y1 \\ Y2 \\ Y3 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1+a}{a} & \frac{1}{a} \\ 0 & \frac{a}{1-a} & \frac{1}{(1-a)} \\ 0 & \frac{1}{a(a-1)} & \frac{1}{a(a-1)} \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} \qquad (5.3)$$

The phase trajectories in the Y1 - Y2 and Y1 - Y3 planes are obtained as

$$a\,Y1 + \delta \ln[1+(a-1)\,\delta\,Y2] = C1 \qquad (5.4)$$

$$a^2 Y1 + \delta \ln[1-a^2(a-1)\,\delta Y3] = C2 \qquad (5.5)$$

and the equations of the optimal trajectories are

$$a\,Y1 + \delta \ell n[1+(a-1)\,\delta\,Y2] = 0 \qquad (5.6)$$

$$a^2 Y1 + \delta \ell n[1-a_2(a-_1)\,\delta\,Y3] = 0 \qquad (5.7)$$

The derivatives of error can be formed using equations (4.7) to (4.10).

## 5.2 GENERATION OF OPTIMUM CURVES AND THE SWITCHING STRATEGY[31]

As seen in figures 5.1(a) and 5.1(b), the optimum switch curves are the actual system trajectories shifted along Y1 axis. Hence any point on the optimal trajectory bears the following relationship with thepoints on the system trajectories.

In the Y1 - Y2 plane

$$Y1opt = Y1act - EH1 \qquad (5.8)$$

In the Y1 - Y3 plane

$$Y1opt = Y1act - EH2 \qquad (5.9)$$

Where EH1 and EH2 are the curve offsets from the origin and Y1act is actual system state.

The projections of the optimum curves may be stored as Y1 a function of Y2 and Y3 i.e. Y1n (Y2n) in the Y1 - Y2 plane and Y1m (Y3m) in the Y1 - Y3 plane. Since the other half of the optimal curves are mirror images comparison is made between the absolute values as

$$Y2n = |Y2_2| \qquad (5.10)$$

$$Y3m = |Y3_2| \qquad (5.11)$$

$Y2_2$ and $Y3_2$ are the present system states

$$YD1 = |Y1n\ (Y2w)| - |EH1| \qquad (5.12)$$

$$YD2 = |Y1m\ (Y3m)| - |EH2| \qquad (5.13)$$

The subscripts n and m indicate any point in general on the stored curves. YD1 is the optimum value of Y1 from the stored curve for the Y1 - Y2 plane and YD2 is the optimum value of Y1 in the Y1 - Y3 plane.

$$YC = YD1 \quad \text{IF} \quad YD1 \geq YD2 \qquad (5.14)$$

$$YC = YD2 \quad \text{IF} \quad YD1 < YD2 \qquad (5.15)$$

The value of YC enables the controller to be certain that the system has changed region in both the planes.

For optimum switching the condition is therefore

$$|Y1_2| \geq YC \qquad (5.15)$$

## 5.3 THE SWITCHING SEQUENCE

To obtain proper switching sequence various points are set along the trajectory and proper controller operations are assigned to each of these points. The points Pi (i = 1,2,3,4,5,6,7,8,9,10,11,12 and 13) in figures 5.1(a) and 5.1(b) have the following operations associated with them

$P_1$, Start comparing actual response with blank table

$P_2$, Start checking for $|Y2_2 - Y2_1| < \varepsilon$ after change of sgn(Y1)

$Y2_2$ and $Y2_1$ are the present and past values of the state Y2.

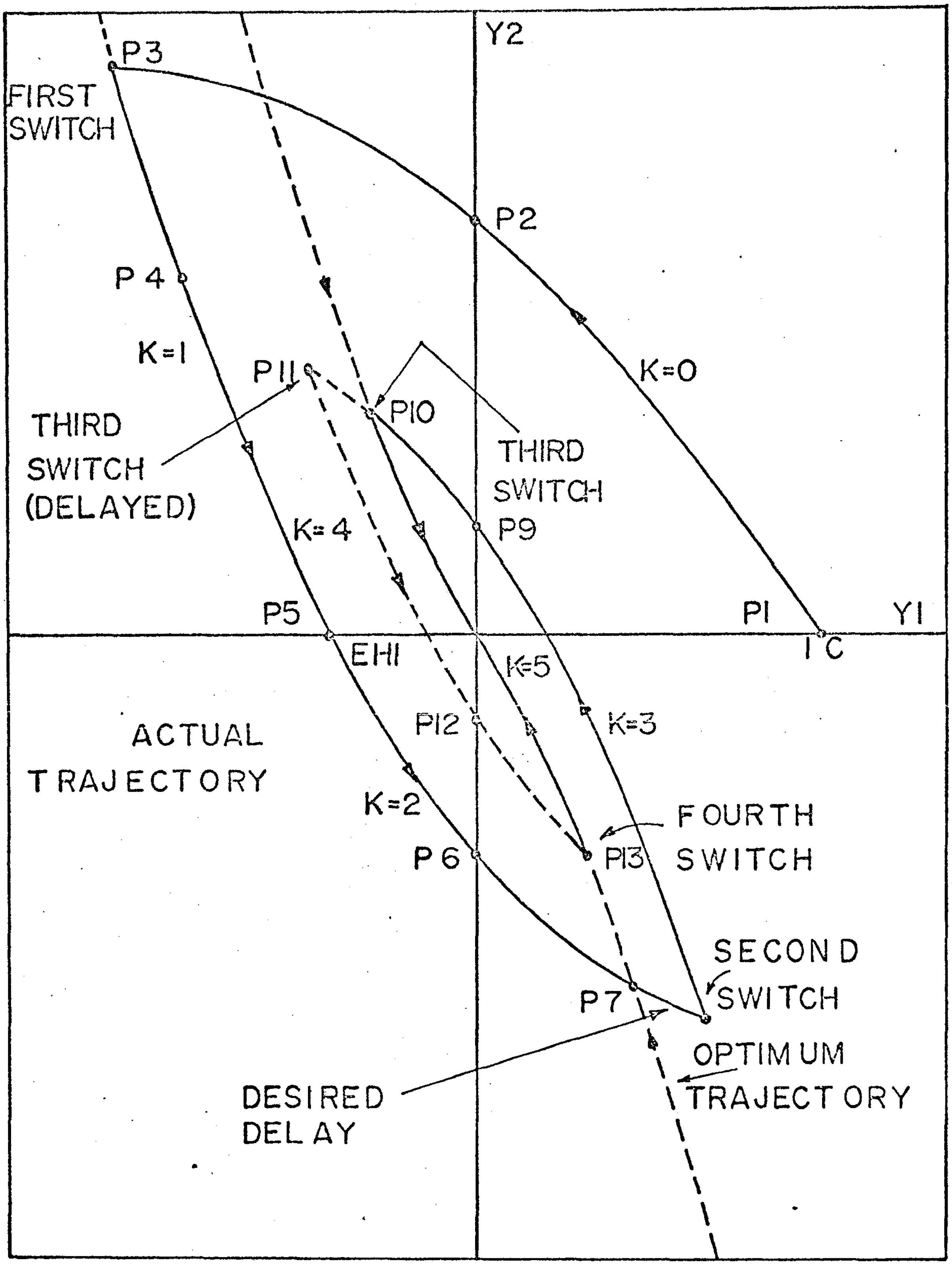



Fig 5.1(a) GENERATION OF OPTIMUM CURVES AND SWITCHING SEQUENCE IN Y1-Y2 PLANE.

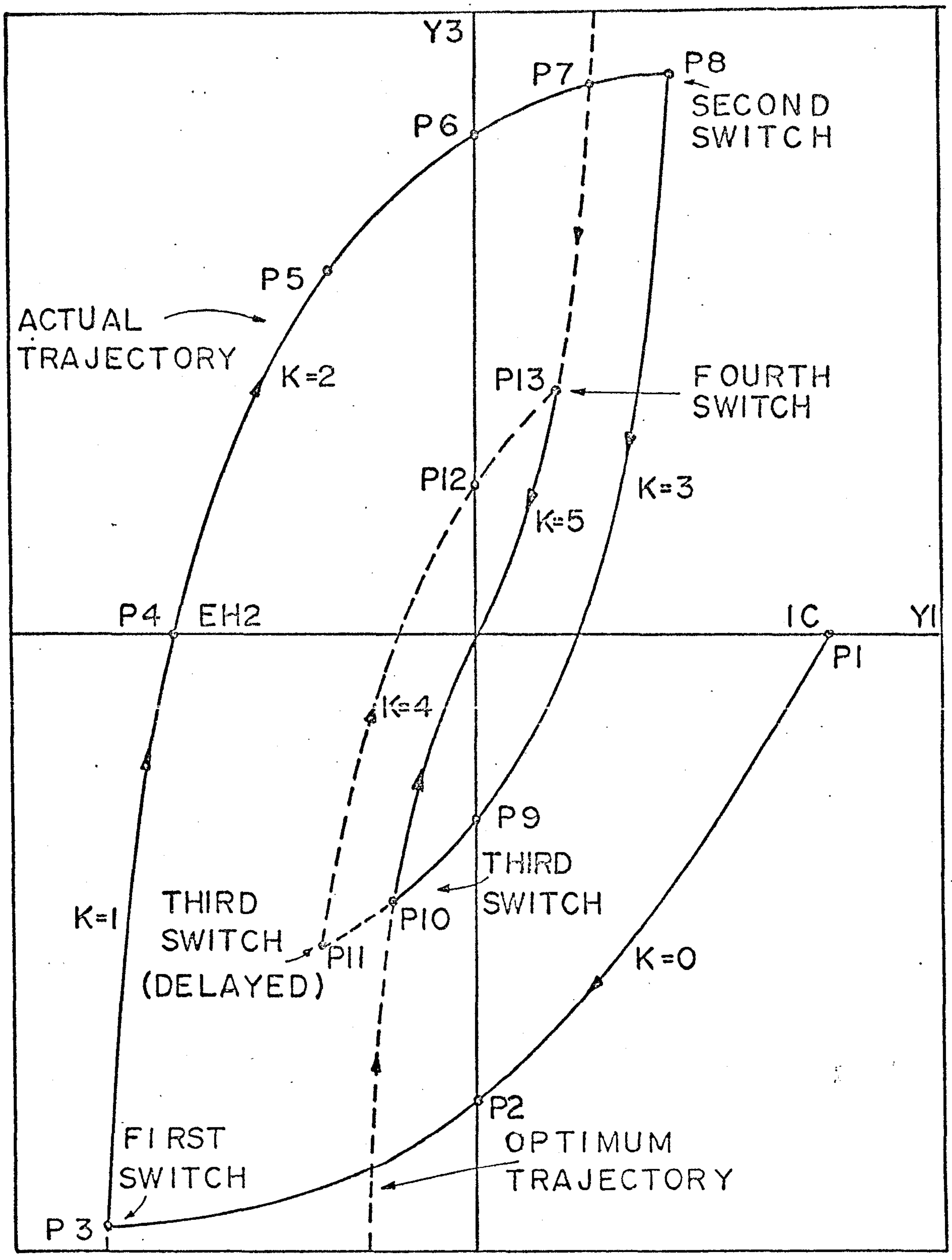



Fig 5.1(b) GENERATION OF OPTIMUM CURVES AND SWITCHING SEQUENCE IN Y1-Y3 PLANE

The function checks if the system has reached the stage when there is practically no change in the value of Y2 i.e. the difference between the present and past values of Y2 is less than a small quantity $\varepsilon$ necessary due to digital nature of the data.

$P_3$, Switch the relay and start filling in tables (Y1 v/s Y2) and (Y1 v/s Y3)

$P_4$, Stop filling in table (Y1 v/s Y3). Store the offset EH2

$P_5$, Stop filling in table (Y1 v/s Y2). Store EH1

$P_6$, Start comparing tables with actual response after change of sgn(Y1)

$P_7$, System ready for switching. Start delay in switching.

$P_8$, Switch and reverse force on relay.

$P_9$, Start comparing tables with actual response.

$P_{10}$, Switch optimally

$P_{11}$, Unwanted delayed switching.

$P_{12}$, Start comparing tables with actual response on change of sgn(Y1)

$P_{13}$, Switch optimally.

The compare cycle starts on each change of sgn(Y1). As described earlier in chapter IV the index K is set to avoid undesirable responses. The various values of K corresponding to the points Pi on the figures 5.1(a) and 5.1(b) are given in table 6.

5.4 EFFECT OF CHANGE IN PARAMETER "a"

The coordinate transformation equations are given in

TABLE 6

RELATION BETWEEN Pi AND K

| Position of the state point | Index K |
|---|---|
| IC $\longrightarrow$ $P_3$ | 0 |
| $P_3$ | 1 |
| $P_3 \longrightarrow P_5$ | 1 |
| $P_5$ | 2 |
| $P_5 \longrightarrow P_8$ | 2 |
| $P_8$ | 3 |
| $P_8 \longrightarrow P_{10}$ | 3 |
| $P_{10}$ | 4 |
| $P_{10} \longrightarrow$ origin | 4 |
| In case system has delayed switching at $P_{11}$ | |
| $P_{11}$(Instead of $P_{10}$) | 4 |
| $P_{11} \longrightarrow P_{13}$ | 4 |
| $P_{13}$ | 5 |
| $P_{13} \longrightarrow$ origin | 5 |

equation (5.3) and it can be seen that the change in parameter "a" involves the change in the state variables X2 and X3. By using the same procedure as used in section 4.5 the knowledge of "a" is found to be required for a<3 to get the proper transformed coordinates. For a>3 the approximate equations are:

$$Y1 = X1 + 1.29X2 + 0.285X3 \tag{5.16}$$

$$Y2 = -(1.40X2 + 0.4X3) \tag{5.17}$$

$$Y3 = -0.114(X2 + X3) \tag{5.18}$$

## 5.5 SIMULATION RESULTS

The logic flow diagram is given in figure 5.2 (see appendix). The effect of change in parameter "a" on the systems time response with controller parameter set at 3.5 is given figures 5.3(a) to 5.3(c).

The figures 5.4 to 5.7 describe the control strategy for a different set of parameter changes. The various runs 1 to 4 are the same as explained in the previous chapter. The target volume is defined by $E = \pm 0.02$, $\dot{E} = \pm 0.06$ and $\ddot{E} = \pm 0.01$. The total normalized times for these runs and the final steady state values are given in Table 7.

TABLE 7

EFFECT OF PARAMETER VARIATION ON SYSTEMS RESPONSE

Controller Parameter "a" = 3.5

| SYSTEM PARAMETER | | RUN | FINAL | | | TOTAL NORMALIZED TIME |
|---|---|---|---|---|---|---|
| Change | "a" | | E | $\dot{E}$ | $\ddot{E}$ | |
| 3 to 4 | 3 | 1 | 0.01 | 0.05 | -0.01 | 7.07 |
| | | 2 | -0.01 | 0.05 | 0.02 | 3.59 |
| | 4 | 3 | 0.008 | 0.02 | 0.01 | 3.78 |
| | | 4 | -0.01 | 0.04 | 0.01 | 4.01 |
| 4 to 3 | 4 | 1 | 0.01 | -0.03 | -0.01 | 6.73 |
| | | 2 | -0.01 | 0.03 | 0.02 | 3.94 |
| | 3 | 3 | 0.01 | -0.04 | -0.01 | 3.82 |
| | | 4 | -0.01 | 0.05 | 0.02 | 3.66 |
| 3 to 6 | 3 | 1 | 0.01 | 0.05 | -0.01 | 7.07 |
| | | 2 | -0.01 | 0.05 | 0.02 | 3.59 |
| | 6 | 3 | 0.005 | 0.01 | 0.01 | 4.72 |
| | | 4 | 0.01 | 0.02 | 0.01 | 4.86 |
| 6 to 7 | 6 | 1 | 0.008 | -0.01 | -0.01 | 11.64 |
| | | 2 | -0.008 | 0.01 | 0.01 | 4.80 |
| | 7 | 3 | 0.01 | -0.04 | 0.01 | 4.80 |
| | | 4 | -0.009 | 0.01 | 0.01 | 5.32 |

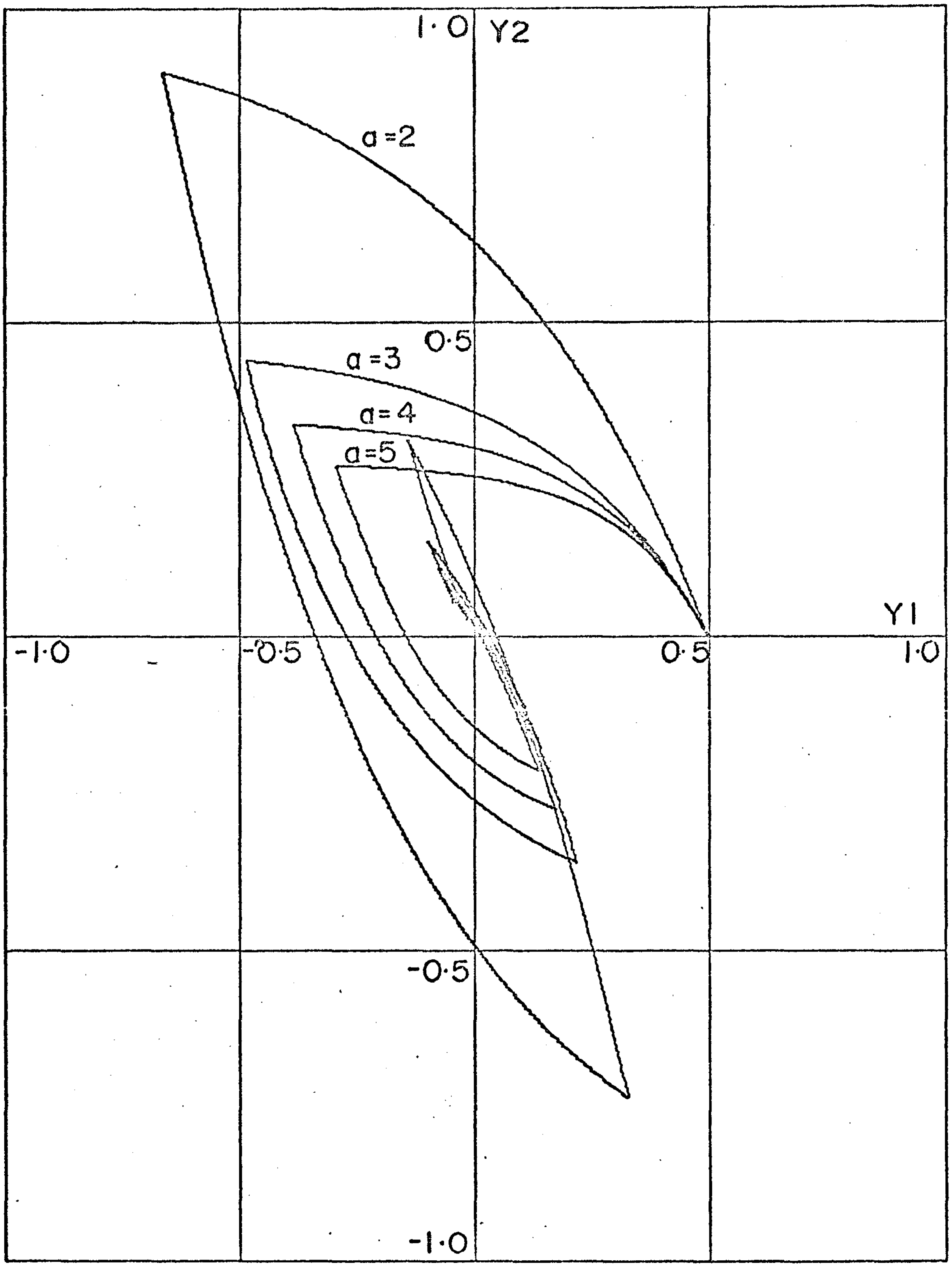



Fig 5.3(a)

Y1-Y2 PROJECTIONS OF SYSTEM TRAJECTORIES

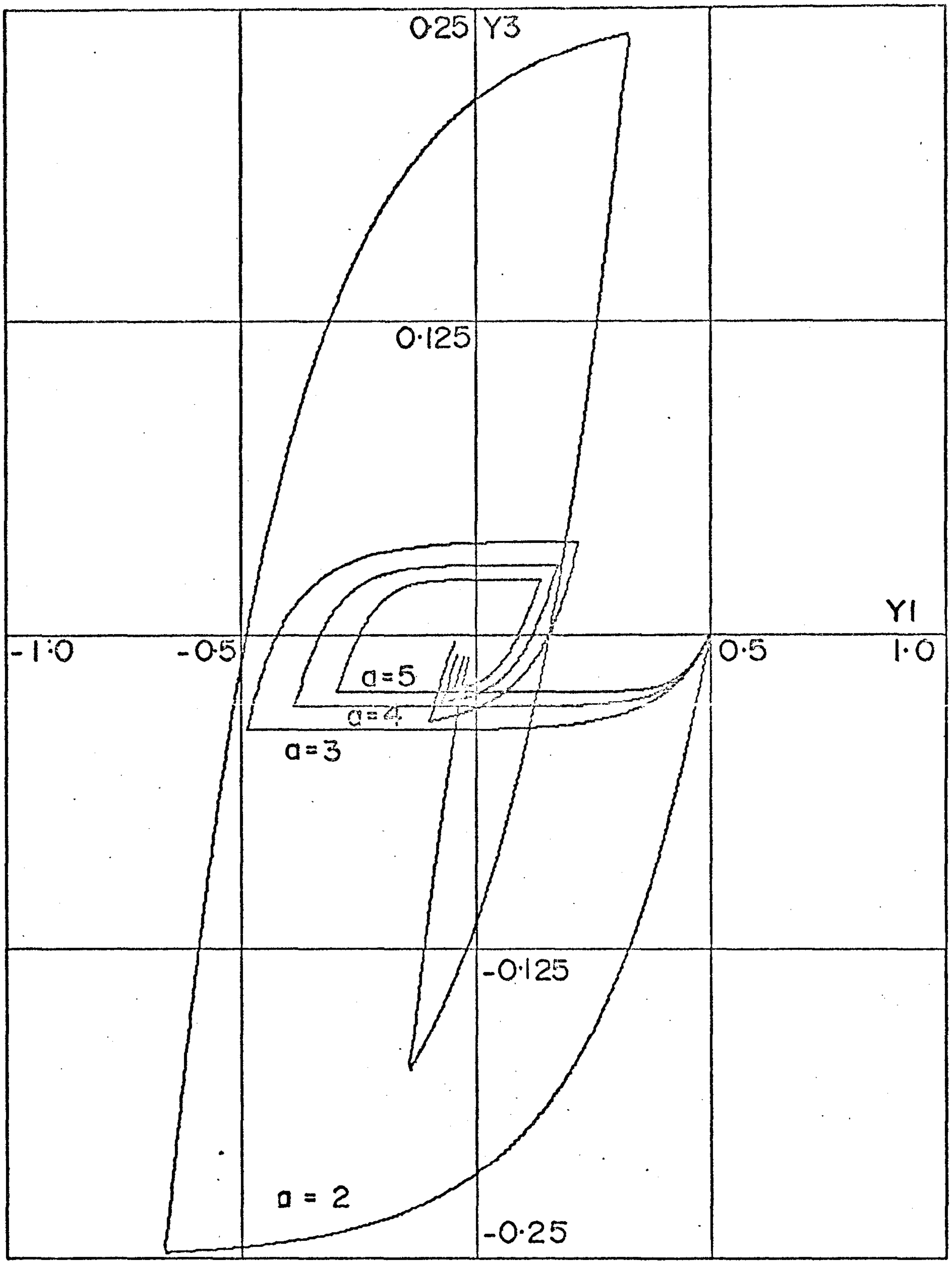



Fig 5.3(b)

YI-Y3 PROJECTIONS OF SYSTEM TRAJECTORIES

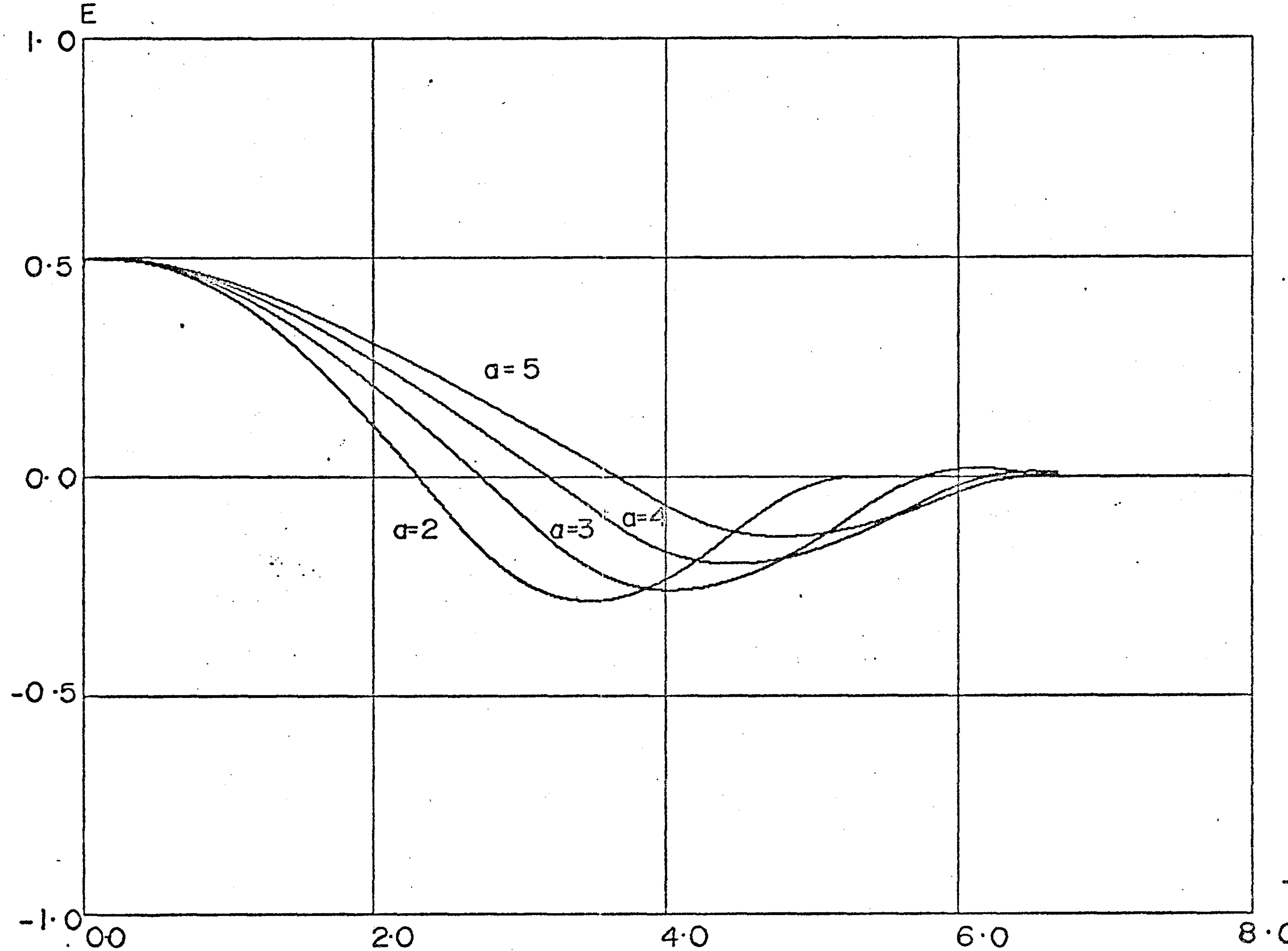



Fig 5.3(c) SYSTEMS TIME RESPONSE

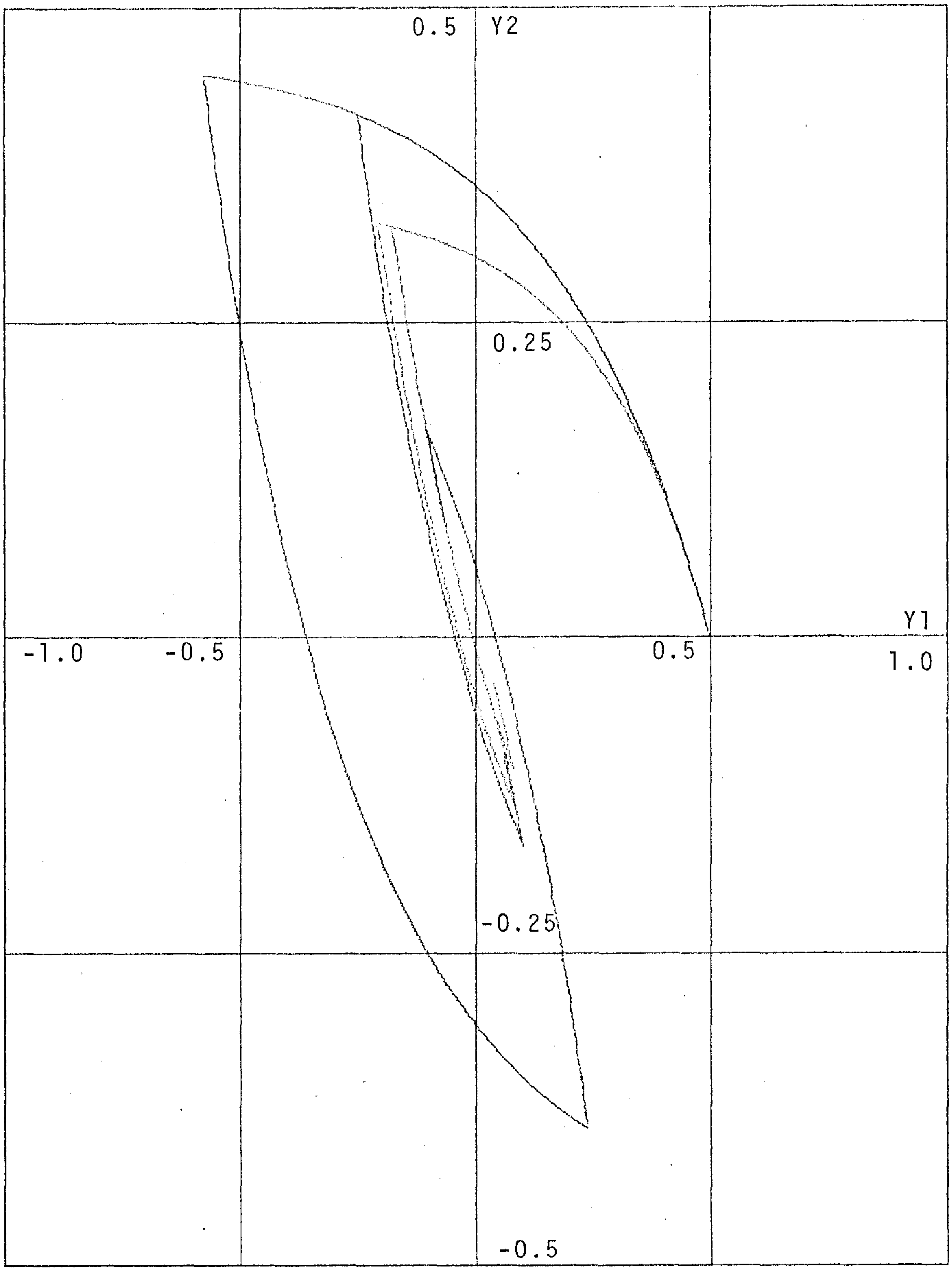



FIG. 5.4

SYSTEM TRAJECTORIES WITH PARAMETER

CHANGE 3 to 4

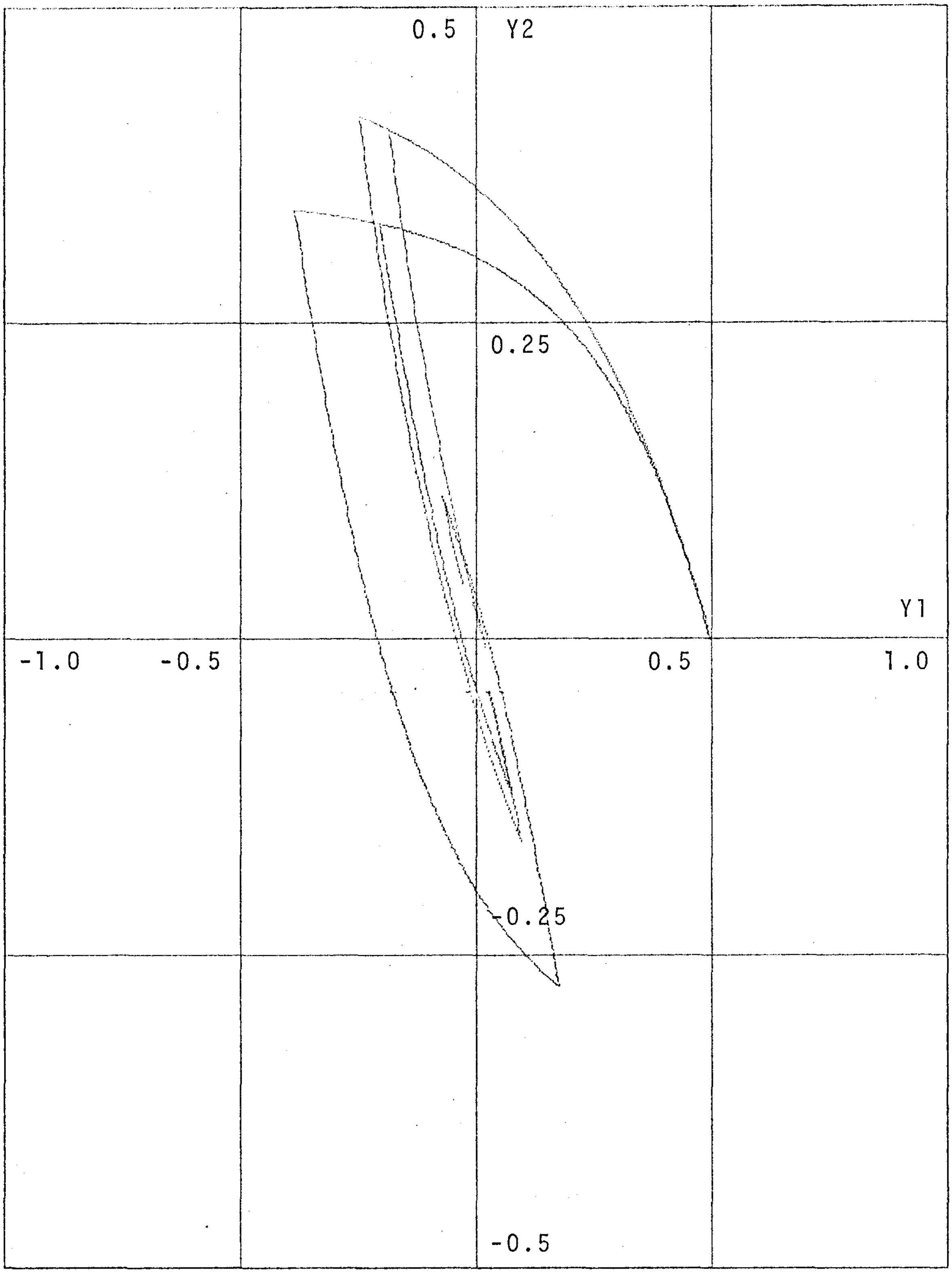



FIG. 5.5

SYSTEM TRAJECTORIES WITH PARAMETER

CHANGE 4 to 3

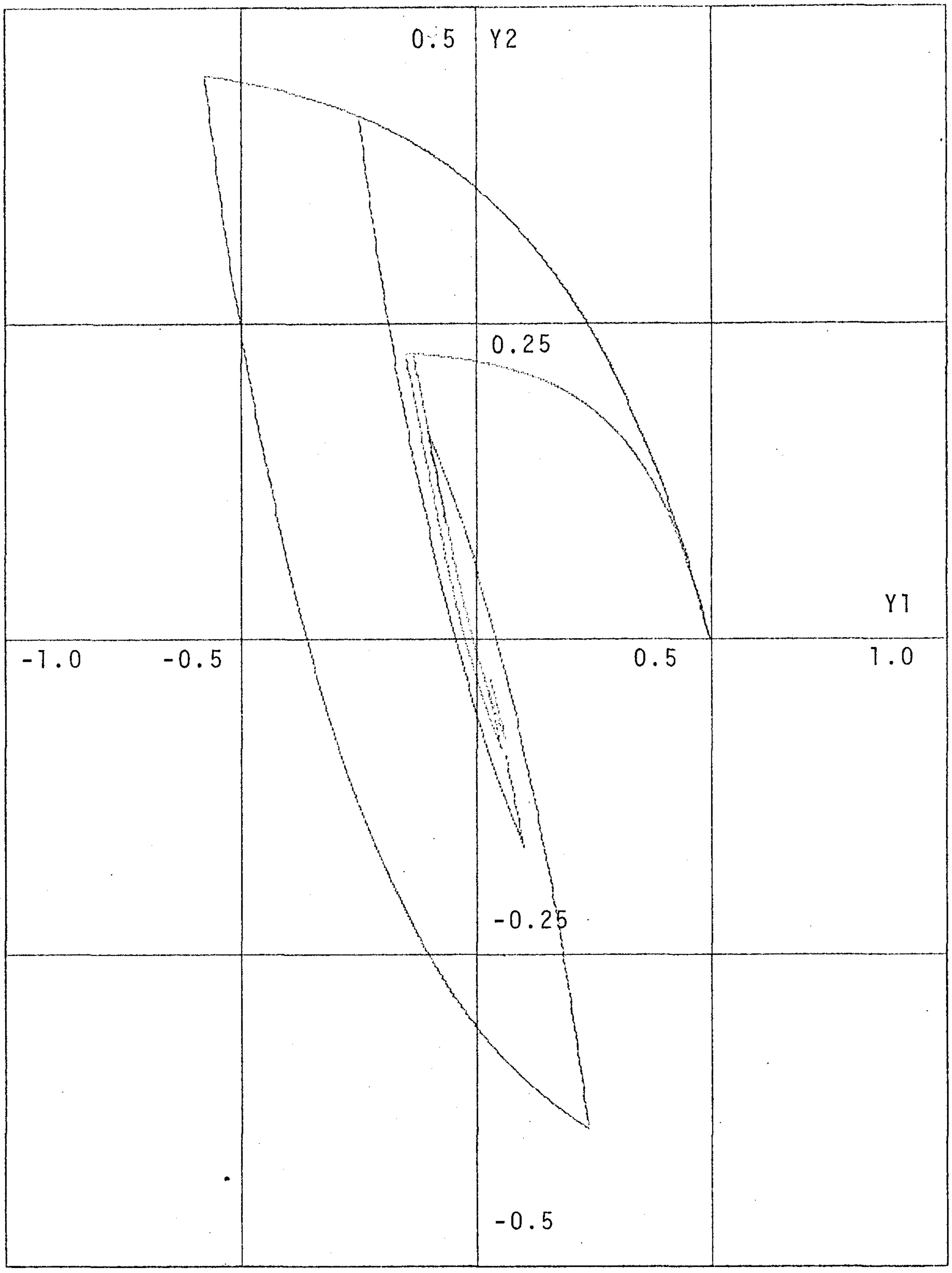



FIG. 5.6

SYSTEM TRAJECTORIES WITH PARAMETER

CHANGE 3 to 6

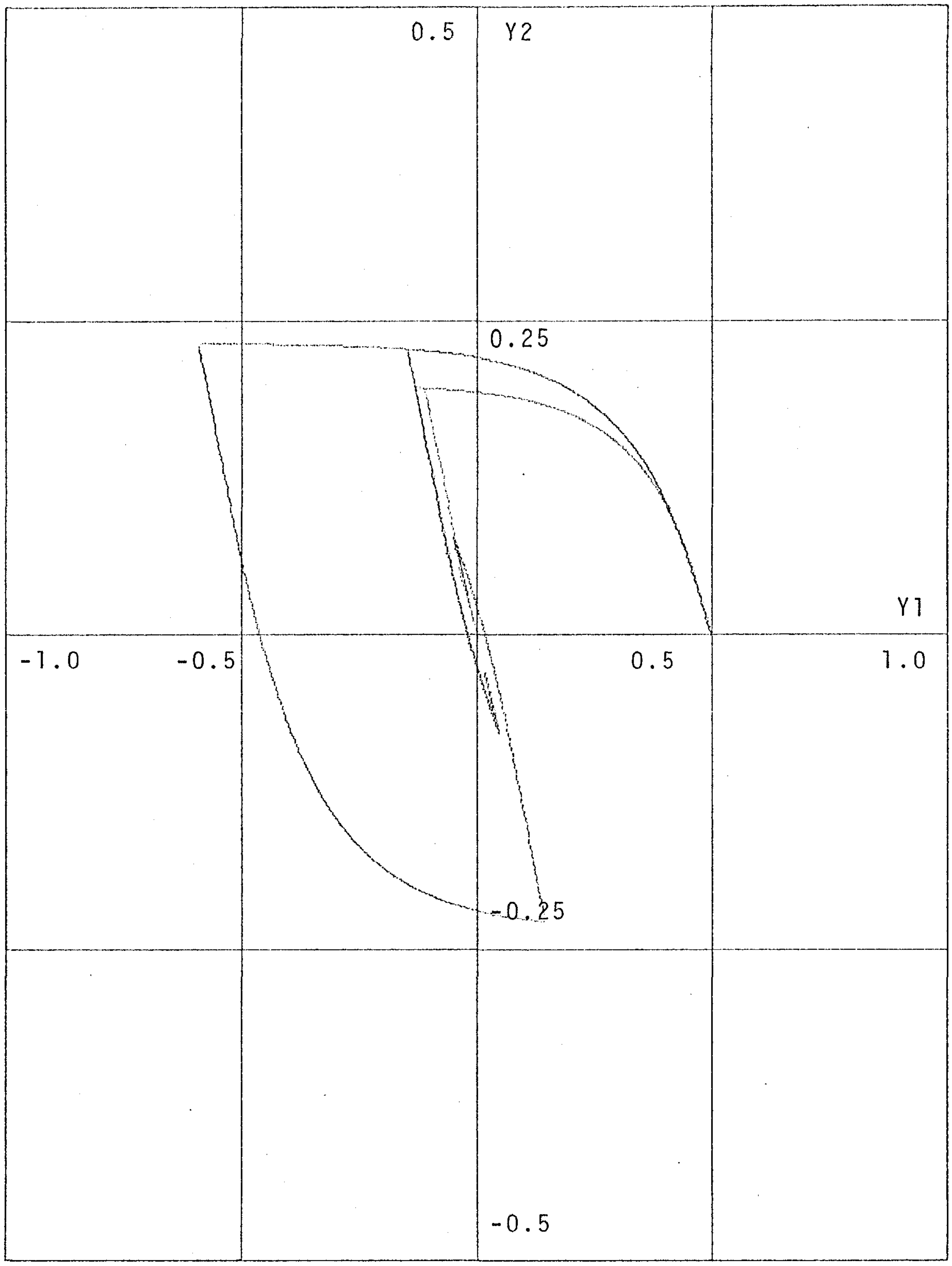



FIG. 5.7

SYSTEM TRAJECTORIES WITH PARAMETER

CHANGE 6 to 7

## CHAPTER VI

## CONCLUSIONS

On the basis of the studies made for the two third order systems, it can be said that if the first switching is delayed by 0.11 units of time after the system changes regions, optimum response will be achieved. If, due to some reasons, the second switching fails to bring the system to rest, the system will still be in the vicinity of the origin and another switching will bring the system to steady state, the time required for subsequent switchings being very small in comparison to the optimal response time of the system. One more important fact to note is that the logic circuitry required for the controller for each of the two systems is the same, except that the inputs are different. Thus with the pre-knowledge of the design equations the same controller can be used for different third order systems, only the function generators need be changed. The technique of developing time optimal controllers using logic is quite simple and requires no elaborate calculations and moreover, it is expected that such controllers will be cheaper in cost.

When attempting to generate optimum trajectories in a computer without mathematical manipulations, it is noted that initially no optimum trajectory exists in the computer memory. The switching criterions are therefore modified to switch first time on the change of Sgn(Y1) for the first system and on $|Y2_2 - Y2_1| < \varepsilon$ for the later. Thus an extra switching is required to initially generate the optimum curve in the computer memory. In this case the first response is sub-optimal. However, if there is no change in parameters and the system starts again from the same or a new initial condition, the system will switch optimally and the control is optimum. If any change in parameter occurs, the optimum trajectories are modified automatically as the controller has a new reference generated by monitoring the system error. This first new response exhibits an overshoot but the following response is more near optimum. Thus a quasi-adaptive optimal of the system is realized. The technique of self storing optimum trajectories is a simple direct method and requires no tedious mathematical manipulations. Moreover, The method has been shown by simulation to be easily implementable on a digital computer by using a suitable approximation and a small target volume. In a practical situation the size of this target volume will depend on the most critical performance characteristics of the system.

Further work in this area will be restricted until some suitable methods of dynamic parameter estimation can be developed which, coupled with the strategy developed in this work will lead to an adaptive optimal performance.

# APPENDIX

# LOGIC FLOW CHARTS

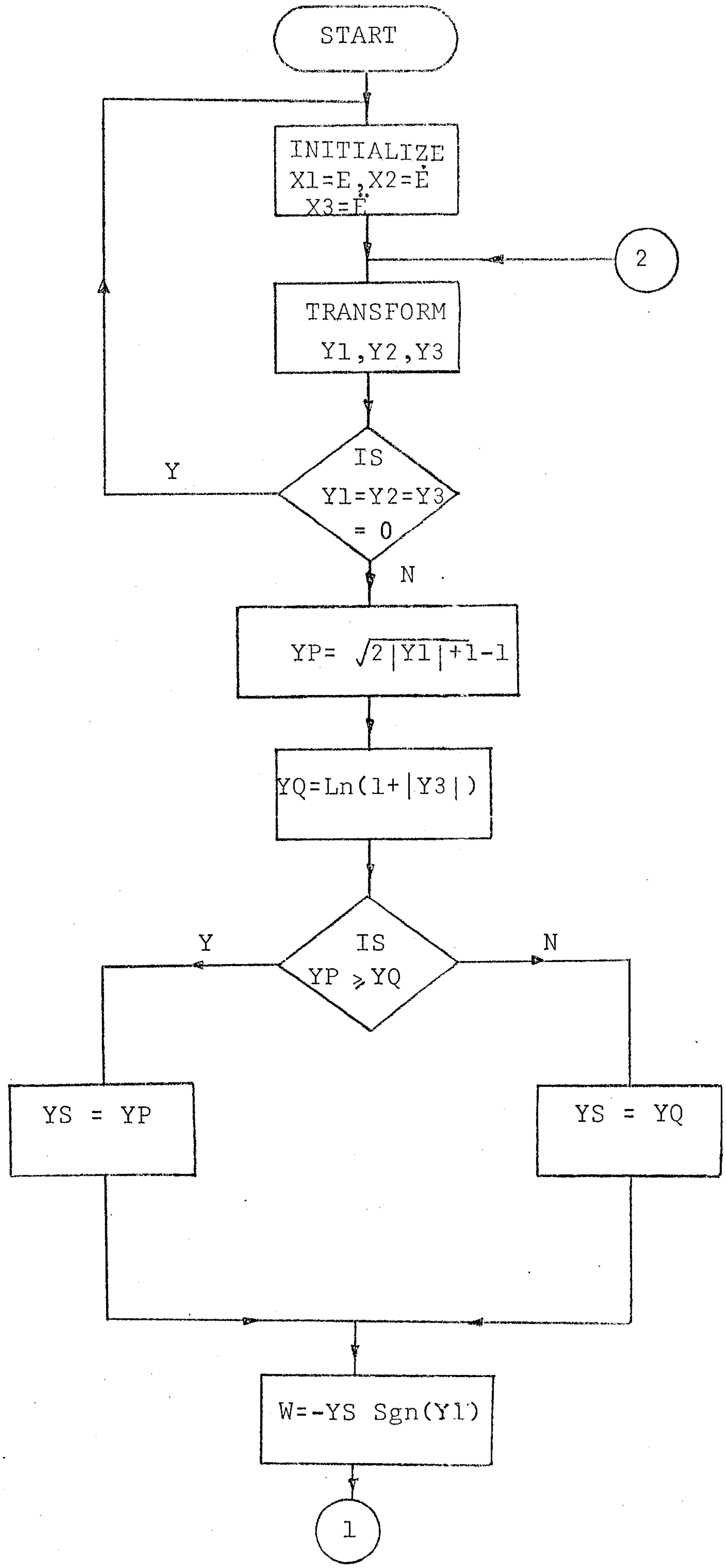
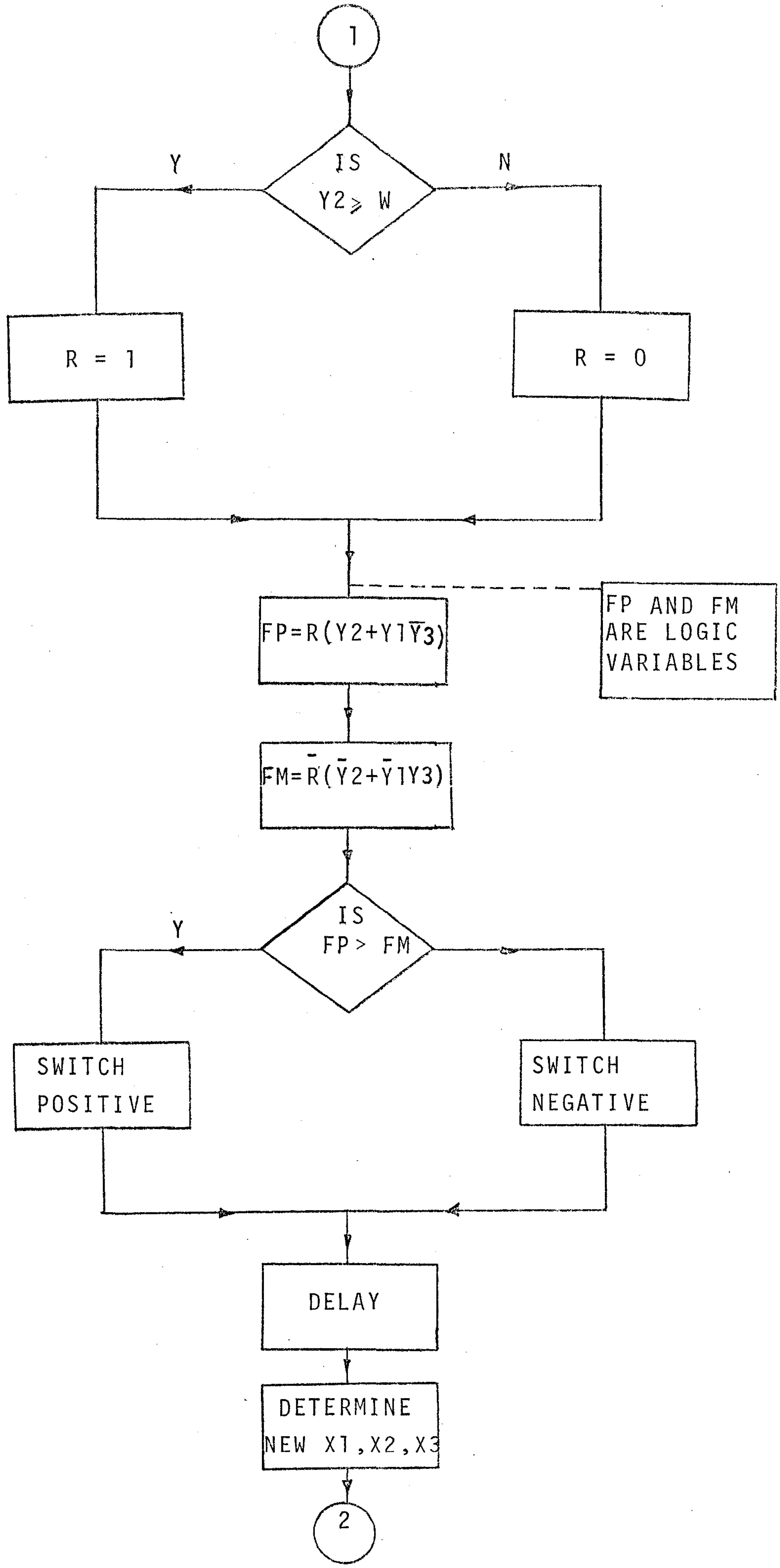



FIGURE 2.9 LOGIC FLOW CHART


1
IS
Y2 ≥ W
Y
N
R = 1
R = 0
FP=R(Y2+Y1$\overline{Y3}$)
FP AND FM ARE LOGIC VARIABLES
FM=$\overline{R}$($\overline{Y2}$+$\overline{Y1}$Y3)
IS
FP > FM
Y
SWITCH POSITIVE
SWITCH NEGATIVE
DELAY
DETERMINE NEW X1,X2,X3
2

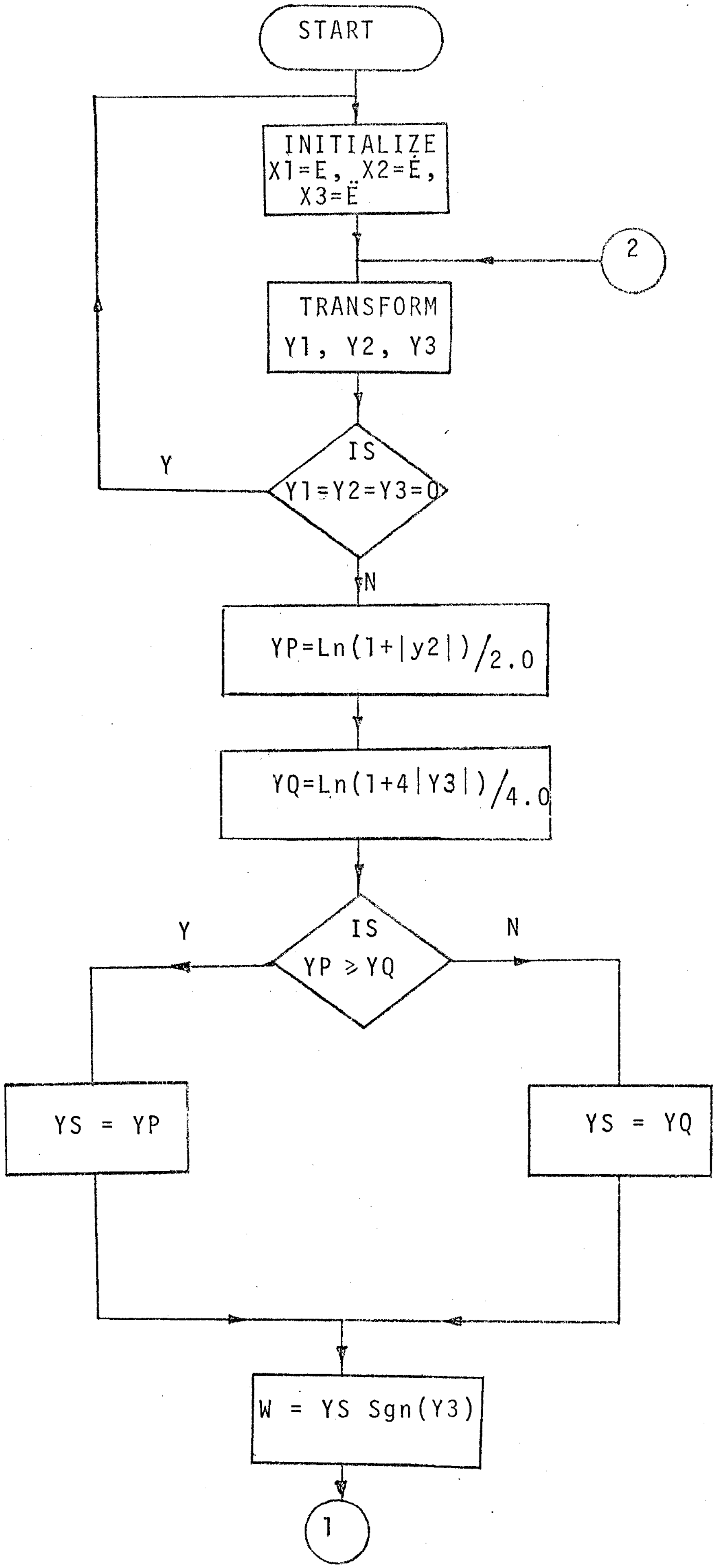
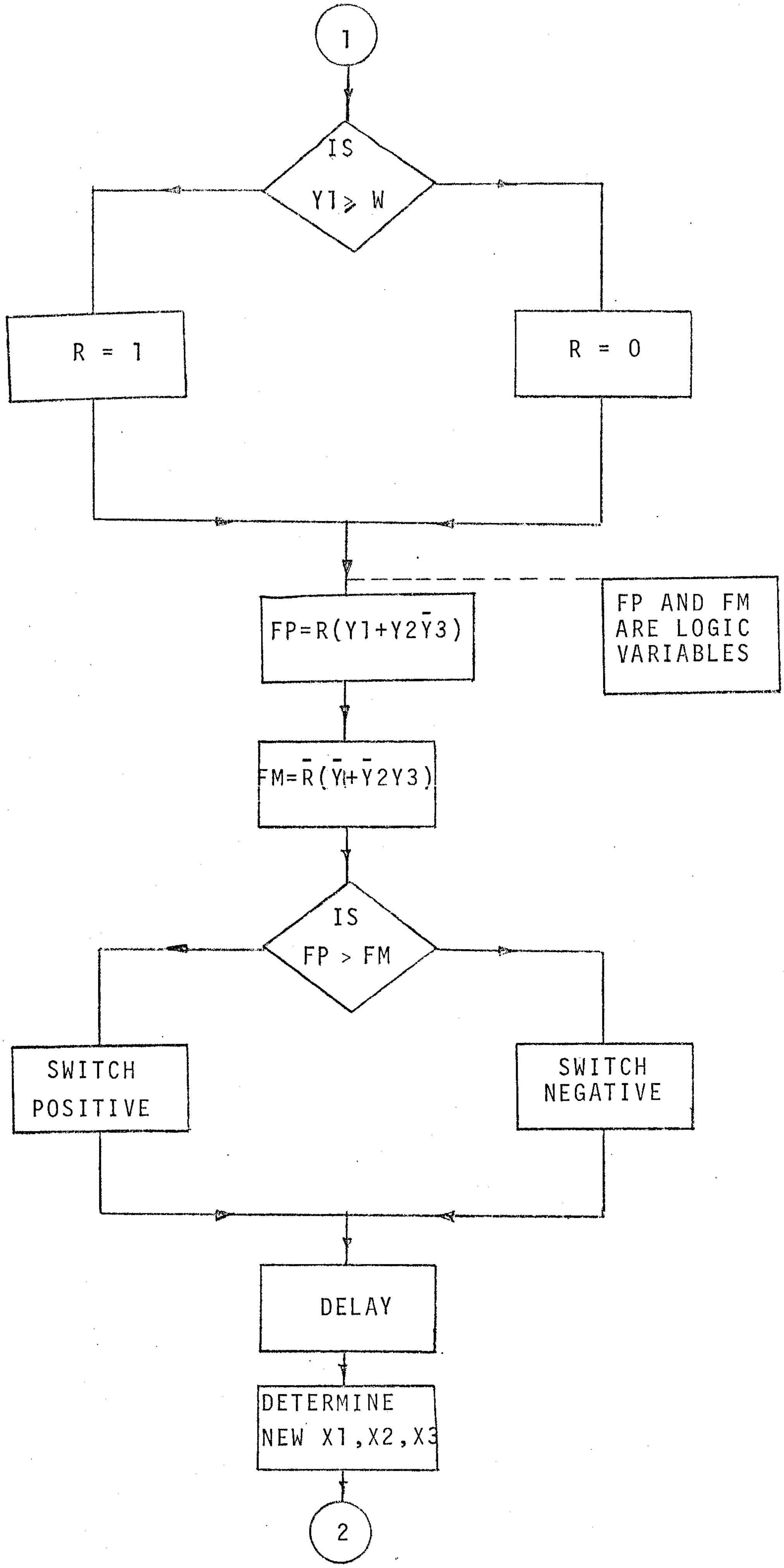



FIGURE 3.8 LOGIC FLOW CHART


1
IS
Y1 ≥ W
R = 1
R = 0
$FP=R(Y1+Y2\bar{Y}3)$
FP AND FM
ARE LOGIC
VARIABLES
$FM=\bar{R}(\bar{Y}1+\bar{Y}2Y3)$
IS
FP > FM
SWITCH
POSITIVE
SWITCH
NEGATIVE
DELAY
DETERMINE
NEW X1,X2,X3
2

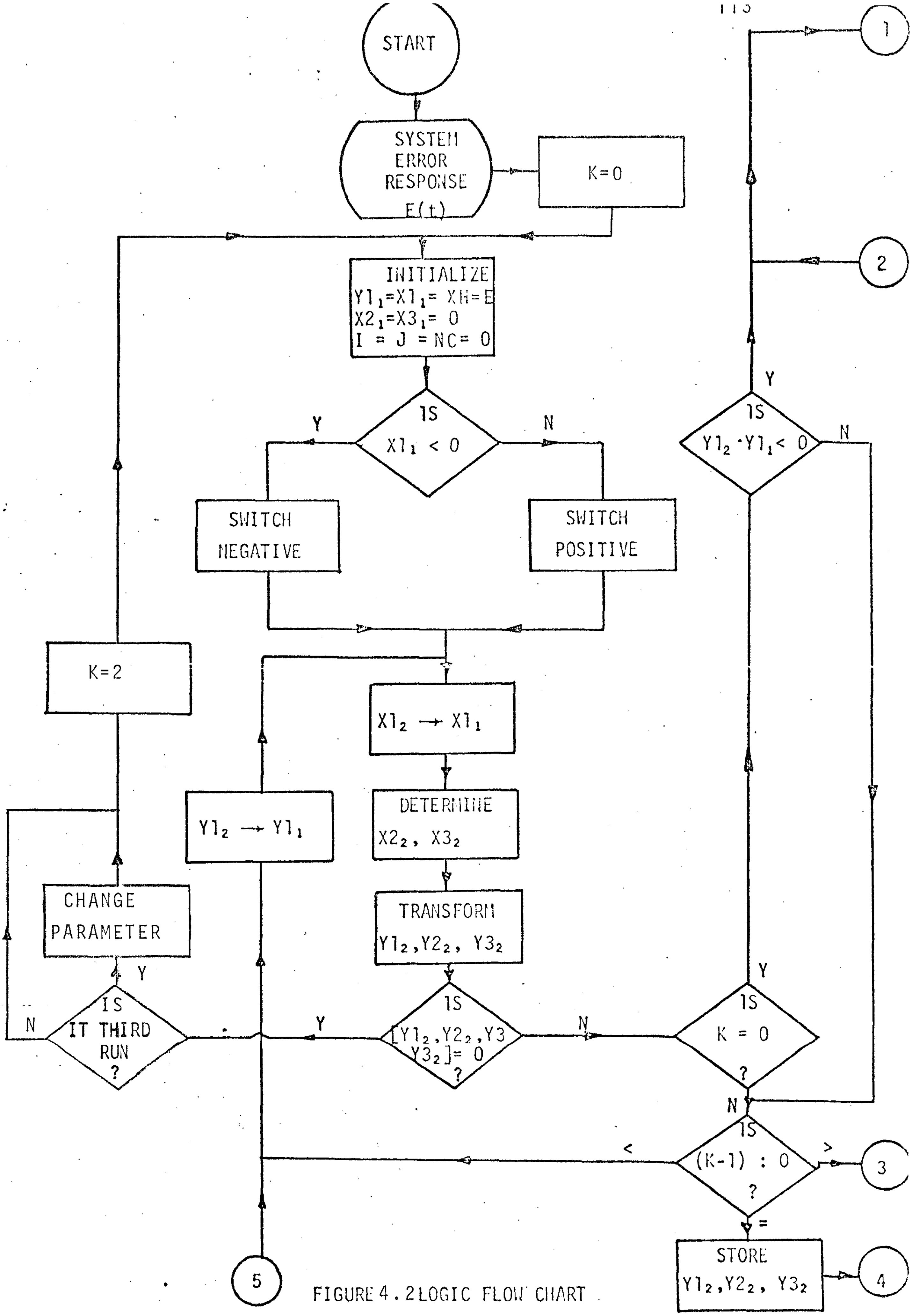
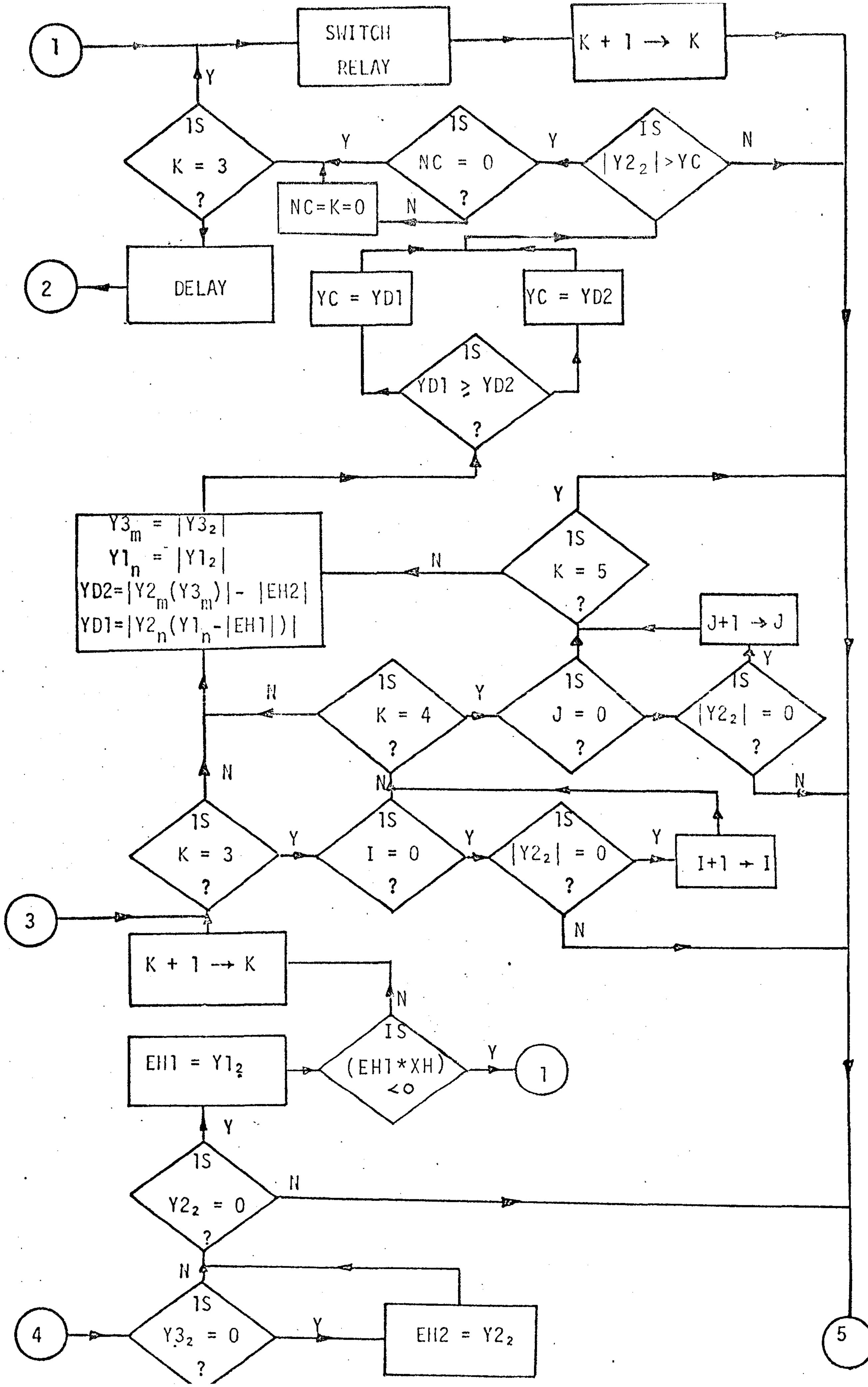



FIGURE 4.2 LOGIC FLOW CHART


1
SWITCH RELAY
K + 1 → K
IS K = 3 ?
IS NC = 0 ?
IS |Y2₂|>YC
NC=K=0
DELAY
2
YC = YD1
YC = YD2
IS YD1 ≥ YD2 ?
Y3m = |Y3₂|
Y1n = |Y1₂|
YD2=|Y2m(Y3m)|- |EH2|
YD1=|Y2n(Y1n-|EH1|)|
IS K = 5 ?
J+1 → J
IS K = 4 ?
IS J = 0 ?
IS |Y2₂| = 0 ?
IS K = 3 ?
IS I = 0 ?
IS |Y2₂| = 0 ?
I+1 → I
3
K + 1 → K
EH1 = Y1₂
IS (EH1*XH) <0
1
IS Y2₂ = 0 ?
4
IS Y3₂ = 0 ?
EH2 = Y2₂
5
Y
N

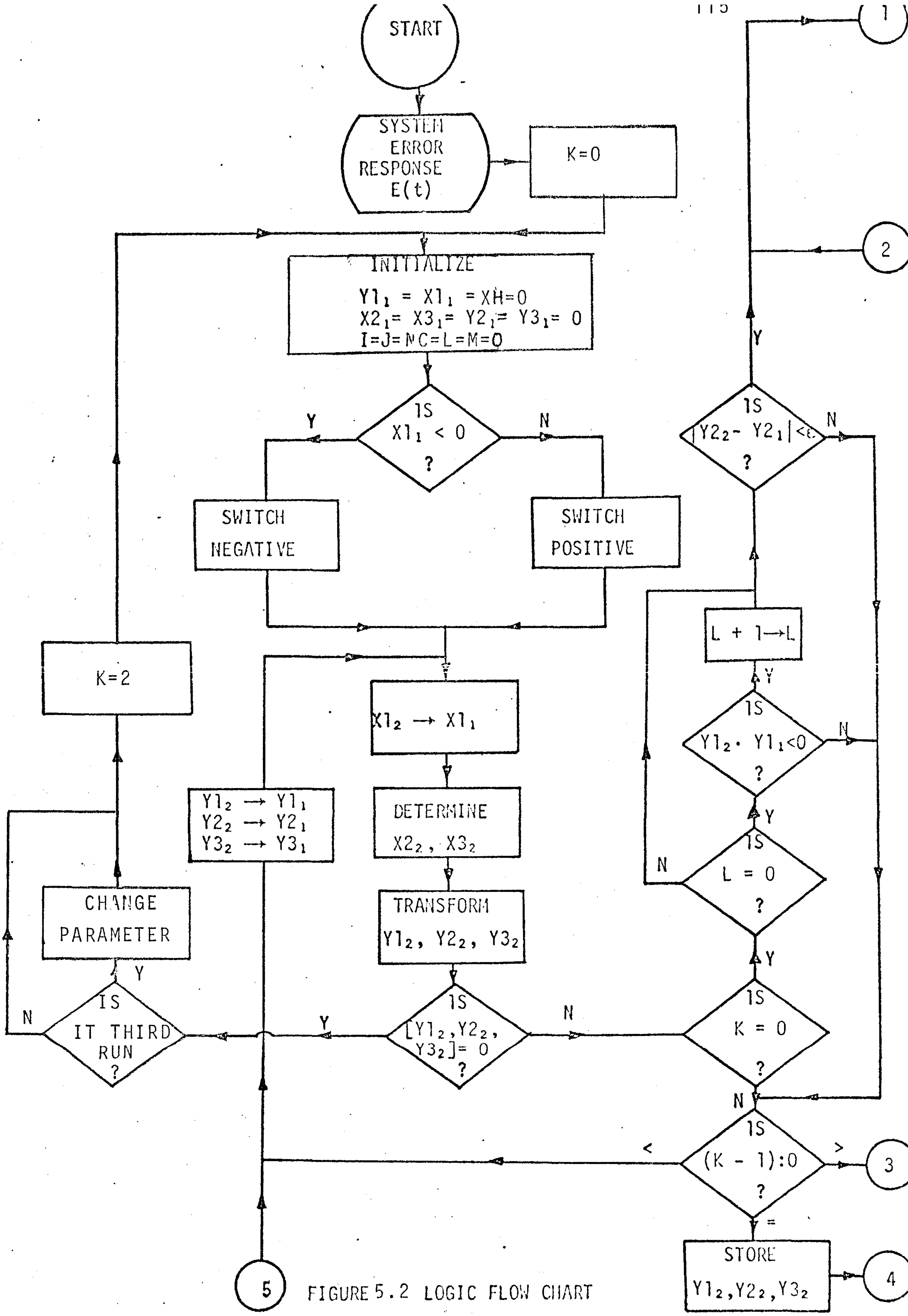
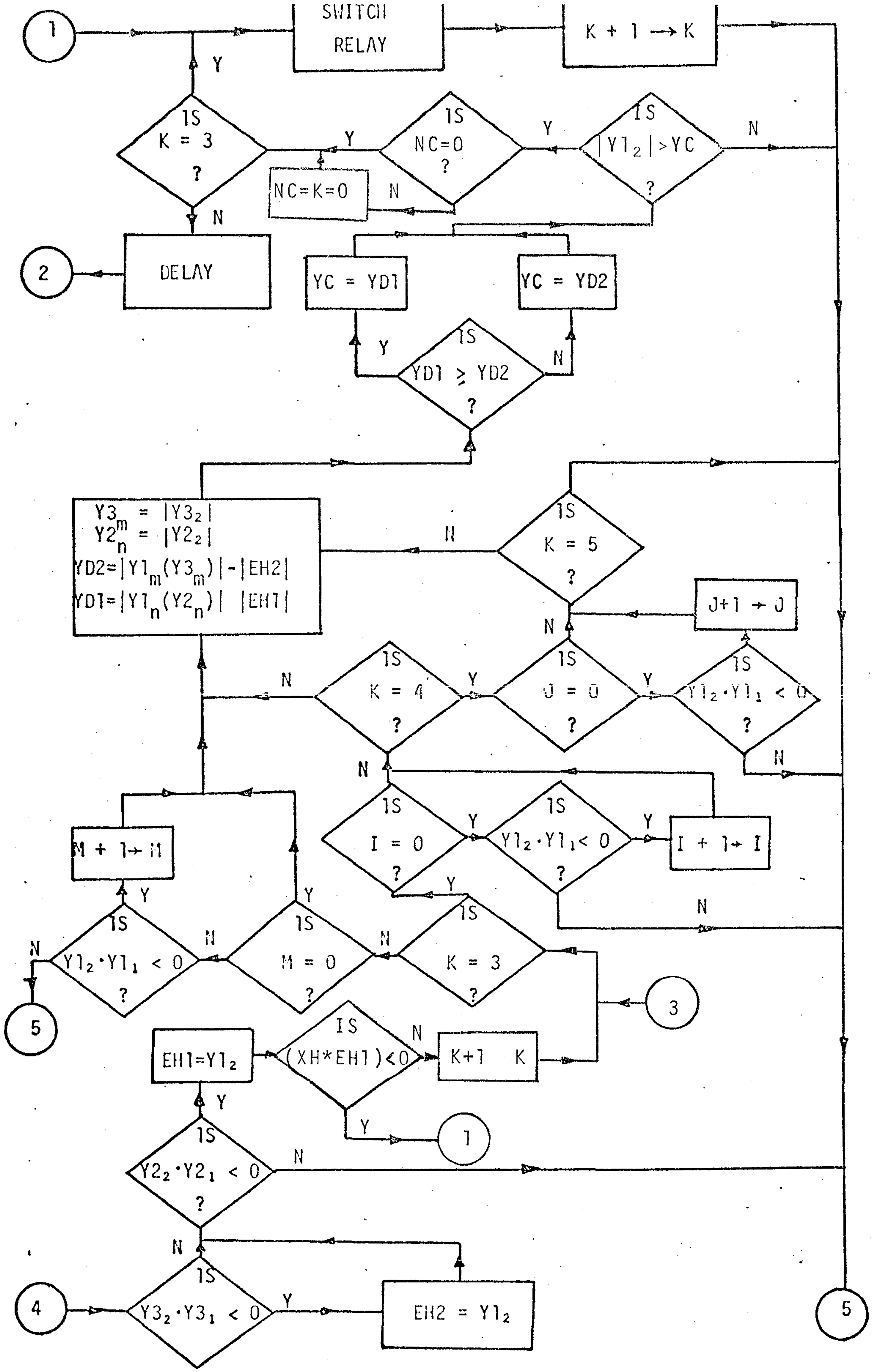



FIGURE 5.2 LOGIC FLOW CHART


1
SWITCH RELAY
K + 1 → K
IS K = 3 ?
IS NC=0 ?
IS |Y1₂|>YC ?
NC=K=0
DELAY
2
YC = YD1
YC = YD2
IS YD1 ≥ YD2 ?
Y3m = |Y3₂|
Y2n = |Y2₂|
YD2=|Y1m(Y3m)|-|EH2|
YD1=|Y1n(Y2n)| |EH1|
IS K = 5 ?
J+1 → J
IS K = 4 ?
IS J = 0 ?
IS Y1₂·Y1₁ < 0 ?
IS I = 0 ?
IS Y1₂·Y1₁< 0 ?
I + 1→ I
M + 1→ M
IS Y1₂·Y1₁ < 0 ?
IS M = 0 ?
IS K = 3 ?
3
5
EH1=Y1₂
IS (XH*EH1)<0
K+1 K
1
IS Y2₂·Y2₁ < 0 ?
IS Y3₂·Y3₁ < 0
EH2 = Y1₂
4
5
Y
N

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| | |
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| 1970 | Candidate for the degree of Doctor of Philosophy in Electrical Engineering at the University of Windsor. |

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