7-17-1965

The position of maximum velocity in annular flow.

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THE POSITION OF MAXIMUM VELOCITY IN ANNULAR FLOW

A THESIS

Submitted to the Faculty of Graduate Studies through the Department of Mechanical Engineering in Partial Fulfillment of the Requirements for the Degree of Master of Applied Science at the University of Windsor

by

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1965

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ABSTRACT

Turbulent flow of air through eleven annuli was studied. Diameter ratios ranged from .2 to .7. The Reynolds numbers ranged from 10,000 to 26,000.

Of prime interest was the position of maximum velocity. The effect of equivalent diameter, diameter ratio and Reynolds number was investigated by holding two of the former variables constant while varying the third. An iteration process was devised to obtain the position of maximum velocity, to the nearest .001 of an inch. The velocity distributions for the portion between the inner wall and the region of maximum velocity and the portion between the outer wall and the region of maximum velocity were solved for their point of intersection using this iteration process. Since it was difficult to obtain a completely concentric annulus, the effects of eccentricity were also studied. At the same time as the present investigation, R. Goel, graduate student in the Mechanical Engineering department at the University of Windsor, was conducting studies on settling lengths for turbulent flow in annuli. His results have indicated that fully developed turbulent flow does exist in all the annuli used for this investigation.

The results of this study show the position of maximum velocity to be independent of equivalent diameter and Reynolds number. The position of maximum velocity is closer to the inner wall than for laminar flow. As the diameter ratio decreased from .7 the position of maximum velocity moved closer to the inner tube, until, at a diameter

ratio of .2 the position was approximated by $r_m = \sqrt{r_1 r_2}$. The greatest eccentricity encountered in this work was of the order of $\pm$ 3%, and was shown to have no significant effect on the position of maximum velocity.

ACKNOWLEDGEMENTS

The author is grateful for the supervision and assistance provided by Dr. A. A. Nicol and Professor R. T. Griffiths. He also extends his appreciation to Mr. R. Myers, Laboratory Technician, for his co-operation.

This investigation was supported financially by a National Research Council Grant.

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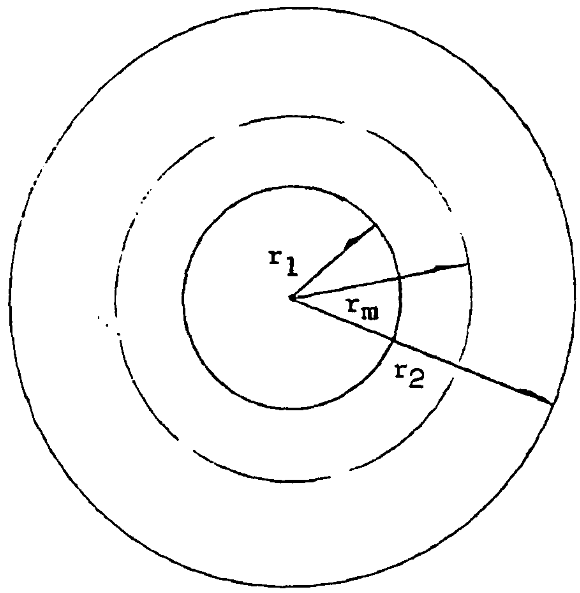
NOTATION



r_1 - outer radius of the inner tube (inner radius of the annulus)

r_2 - inner radius of the outer tube (outer radius of the annulus)

r_m - the radius of the point of maximum velocity

D_1 - outer diameter of the inner tube

D_2 - inner diameter of the outer tube

D_2-D_1 - equivalent diameter

V - velocity at radius r

r - radial location

V_{av} - the average velocity for the cross section

V_m - the maximum velocity for the cross section

Re - Reynolds number based on equivalent diameter

N_1 - the power for a velocity distribution between the inner wall and the position of maximum velocity

N_2 - the power for a velocity distribution between the position of maximum velocity and the outer wall

τ_1 - shear stress at inner wall

τ_2 - shear stress at outer wall

re_1 - equivalent radius for the portion of the annulus between the position of maximum velocity and the inner wall

re_2 - equivalent radius for the portion of the annulus between the position of maximum velocity and the outer wall

ν - kinematic viscosity

ρ - density

g_c - gravitational constant

e - the amount the gap between the two tubes differs from $(r_2 - r_1)$

Per cent Eccentricity - $\left[\frac{e}{(r_2 - r_1)}\right] \times 100$

y - distance from either wall of the annulus to the point of velocity measurement

y_m - distance from either wall of the annulus to the point of maximum velocity

f - Fanning's friction factor

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

It is important in many heat transfer and fluid mechanics applications to know the point of maximum velocity in annular flow. Nuclear reactors, heat exchangers, and turbo-machinery are typical applications. If the position of maximum velocity is known then the annulus can be divided about this surface of no shear and the effects of the inner and outer surfaces seperated. No theory exists which predicts this point for turbulent flow and only a limited amount of conflicting experimental data is available.

Some previous investigators have indicated that the position of maximum velocity could be correlated using the expression derived for laminar flow but considerable deviation from this curve exists in certain cases. The prime objective of this experimental work was to locate the position of maximum velocity under different geometrical and flow conditions in annuli. Eleven annuli, with diameter ratios from .202 to .698, were tested in the Reynolds numbers range from 11,000 to 80,000. A method was devised to obtain one specific value for the radius of maximum velocity rather than determine its location directly from the velocity profile. Possible variation of the position of maximum velocity with equivalent diameter, diameter ratio and Reynolds number was studied.

CHAPTER 2

LITERATURE SURVEY

For laminar flow in an annulus the position of maximum velocity r_m is obtained from

$$r_m = \sqrt{\frac{r_2^2 - r_1^2}{2 \ln (r_2/r_1)}}$$

This equation results from integration of the Navier Stokes equation employing the appropriate boundary conditions. Experimental values of r_m have agreed with those obtained from the theory for fully viscous flow.

Established correlations for turbulent flow in an annulus however are non-existent. Since turbulent flow is most common in practice numerous investigations have resulted.

Rothfus (1) investigated the flow of air through annuli of diameter ratios .162 and .650 with an outside diameter of approximately 3 inches. The Reynolds numbers were varied from 1,000 to 21,000 (laminar to turbulent region). He used smooth brass pipes supported by rods placed at 120 degree intervals around a cross-section located every 3.5 feet along the annulus. The rods had diameters from .036 to .094 inches with a threaded portion at the pipe wall. To ensure that the flow was fully developed a 20 foot calming length was allowed.

In laminar flow, measured values of r_m agreed with r_m calculated from theory. The onset of turbulence did not occur at the same Reynolds numbers for the inner and outer portions of the annuli. When the diameter ratio decreased the Reynolds number range for transition from laminar to turbulent flow increased. Rothfus suggests using the empirical equation

$$\frac{V}{V_m} = 1.06 \left(\frac{y}{y_m}\right)^{.136} - .06 \left(\frac{y}{y_m}\right)$$

for the entire annular cross-section. In turbulent flow the position of maximum velocity coincided with the laminar value.

Knudsen and Katz (2) probed an annulus of diameter ratio .278 at Reynolds numbers 9190, 35700 and 69900. Their results indicated that the position of maximum velocity for turbulent flow was approximately the same as its laminar counterpart. Non-dimensional logarithmic plots of velocity versus position indicated that the following velocity distribution equations could be used for the inner and outer portions of the annulus respectively:

$$\frac{V}{V_m} = \left(\frac{r - r_1}{r_m - r_1}\right)^{.102}$$

$$\frac{V}{V_m} = \left(\frac{r_2 - r}{r_2 - r_m}\right)^{.142}$$

These equations were valid for $Re \geqslant 10{,}000$. They measured the ratio V_{av}/V_m (average velocity to maximum velocity) which gave an average value of .876, with a range of, $\pm$ 1.8 per cent.

Using annuli of diameter ratios .392, .405 and .568 Owen (3) measured point velocities of water flowing at Reynolds numbers of 4,000 to 700,000. The position of maximum velocity was located approximately .4 of the distance between the inner and outer walls. Near the inner wall the velocity was higher than close to the outer wall. The shape of the velocity distribution for a given annulus was dependent on the Reynolds number. Owen presented his data on log plots of velocity versus distance from the wall for the inner and outer portions of the annulus. The velocity distribution could be represented by $V = a(r - r_1)^{N_1}$ and $V = b(r_2 - r)^{N_2}$ for the inner and outer sections respectively. The slope N appeared less at the core wall. As the diameter ratio decreased N decreased.

Prengle and Rothfus (4) injected dye filaments in water flowing at Reynolds numbers of 200 to 2,400 through annuli of diameter ratios .040, .055, .079, .217 and .558. For these visual studies they used a lucite outer tube with various inners mounted under tension.

Their studies showed the first deviation from viscous behaviour occurred at the position of maximum velocity near a Reynolds number of 1,225. From 1,225 to 2,100 Reynolds number the portion near the inner wall was turbulent while the rest was still viscous. During this same range of Reynolds numbers r_m decreases possibly to equalize the shear stress at the walls.

Rothfus, Monrad, Sikchi and Heideger (5) attacked the problem from another direction. They spring mounted the inner tube in order to measure the drag on this tube directly from the spring displacement during test. Annuli of diameter ratios .337 and .562 were employed. Pressure drop data measured at the outer wall gave the combined drag on the walls. From the two drag measurements it was possible to determine the shear stress on the inner wall (τ_1) and the outer wall (τ_2).

$$\frac{\tau_1}{\tau_2} = \frac{r_2}{r_1} \frac{(r_m^2 - r_1^2)}{(r_2^2 - r_m^2)}$$

From the above equation the radius of maximum velocity was calculated since the other quantities were known or measured.

The change in r_m coincided with the previous results of Prengle and Rothfus in the visual studies. At Reynolds numbers greater than 10,000 r_m approximated its laminar value.

Walker (6) worked with water in an annulus of diameter ratio .331 in the viscous, transitional and lower turbulent ranges. The position of maximum velocity shifts toward the inner wall then toward the outer wall then back to the laminar value during the transitional range. The ratio V_{av}/V_m (average velocity to maximum velocity) remained at .660 to a Reynolds number of approximately 1,000 then rapidly increased to .680 and continued constant to about 3,000. From 3,000 to 5,500 the ratio increased rapidly to .810. Near 5,500 the ratio increased gradually to .845 at approximately 13,000 - the upper limit for the investigation.

Croop (7) carried out an investigation similar to that of Walker. He used water flowing through three annuli of diameter ratios of .062, .197 and .500.

His results agreed with those of Walker with respect to the variation of r_m and V_{av}/V_m with Reynolds numbers. He noted also that the magnitude of the shift of r_m depended on the diameter ratio. When D_1/D_2 increased V_{av}/V_m was shown to increase.

Nicol and Medwell (8) plotted the results of past investigators and found that they seldom fell on the curve

$$r_m = \sqrt{\frac{r_2^2 - r_1^2}{2 \ln (r_2/r_1)}}$$

for turbulent flow. They observed that the previous curve and the curve

$r_m = \sqrt{r_1 r_2}$ enclosed most of the points. The latter equation is valid when $\tau_1 = \tau_2$ which occurs as r_1 and r_2 become large or r_1/r_2 approaches unity. They suggested that the position of maximum velocity might be affected by wall curvature since a log plot of the velocity distribution produces different powers for each wall.

Brighton (9) used aluminum annuli with diameter ratios of .0625, .125, .375 and .562. He probed the air flowing in the annuli at Reynolds numbers of 46,000 to 327,000. To obtain a specific location for r_m two pitot tubes (No. 23 hypodermic tubing) were positioned together to obtain the location where $dV/dr = 0$ or $r = r_m$.

Results showed r_m less than its laminar value but as D_1/D_2 increased r_m approached its laminar value. The ratio τ_1/τ_2 (shear stress - inner to shear stress - outer) is less than in laminar flow since r_m is less for a given D_1/D_2.

Using air flowing at Reynolds numbers from 12,000 to 100,000 in annuli with diameter ratios of .344, .531 and .781 Okiishi (10) measured velocity distributions. His values for r_m approximated the laminar values. The equation

$$\frac{V}{V_m} = 1.06 \left(\frac{y}{y_m}\right)^{.136} - .06 \left(\frac{y}{y_m}\right)$$

suggested by Rothfus in his Ph.D. thesis, appeared dependent on Reynolds numbers. Okiishi suggests more data on the position of maximum velocity would be helpful since it may not fall at its laminar value.

CHAPTER 3

THEORY

To isolate the effects of each wall on fluid flow within the annulus the cross-section is divided at the point of zero shear stress which is coincident with the point of maximum velocity. This procedure has been employed by past investigators. It is assumed that the portions of the annulus between the position of maximum velocity and the inner and outer walls may be replaced by equivalent pipes based on the equivalent radius concept. Also it is assumed that the Blasius friction factor equation $f = 0.079 \, (Re)^{-.25}$ may be used for both these pipes. The Blasius equation is considered valid for flow in smooth pipes at $Re < 100{,}000$. The power $-.25$ in Blasius' equation is related to the $1/7$ of the power velocity distribution law. Since the powers of the velocity distributions for the inner and outer portions of the annulus will differ from $1/7$ some error may be induced by using the Blasius equation with $-.25$.

The equivalent diameters for the inner and outer portions are respectively:

$$De_1 = 4 \, re_1 = \frac{2 \, (r_m^2 - r_1^2)}{r_1}$$

$$De_2 = 4 \, re_2 = \frac{2 \, (r_2^2 - r_m^2)}{r_2}$$

The corresponding Reynolds numbers and friction factors are as follows:

$$Re_1 = \frac{2 \, (r_m^2 - r_1^2) \, V_{av1}}{r_1 \, \nu}$$

$$Re_2 = \frac{2\,(r_2^2 - r_m^2)\,V_{av2}}{r_2\,\nu}$$

$$f_1 = .079\,(Re_1)^{-.25}$$

$$f_2 = .079\,(Re_2)^{-.25}$$

By equating forces acting on a differential length of annular fluid the following equation may be derived:

$$\frac{\tau_1}{\tau_2} = \frac{r_2\,(r_m^2 - r_1^2)}{r_1\,(r_2^2 - r_m^2)}$$

Since $$\tau = \frac{f\,\rho\,V_{av}^2}{2\,g_c}$$

Therefore $$\frac{\tau_1}{\tau_2} = \left[\frac{r_1\,(r_2^2 - r_m^2)}{r_2\,(r_m^2 - r_1^2)}\right]^{.25} \left(\frac{V_{av1}}{V_{av2}}\right)^{1.75}$$

$$\frac{r_2\,(r_m^2 - r_1^2)}{r_1\,(r_2^2 - r_m^2)} = \left[\frac{r_1\,(r_2^2 - r_m^2)}{r_2\,(r_m^2 - r_1^2)}\right]^{.25} \left(\frac{V_{av1}}{V_{av2}}\right)^{1.75}$$

$$\left(\frac{V_{av1}}{V_{av2}}\right) = \left[\frac{r_2\,(r_m^2 - r_1^2)}{r_1\,(r_2^2 - r_m^2)}\right]^{1.25}$$

Schlichting (16) gives the equation

$$\frac{V_{av}}{V_m} = \frac{2}{(N+2)(N+1)}$$

based on the power law velocity distribution

$$\frac{V}{V_m} = \left(\frac{y}{R}\right)^N$$

for turbulent flow in smooth pipes.

y - distance from pipe wall

R - radius of the pipe

It can be shown for the two portions of the annulus that the corresponding velocity distributions are:

$$\frac{V}{V_m} = \left\{\frac{Y_1}{re_1}\right\}^{N_1}$$

$$\frac{V}{V_m} = \left\{\frac{Y_2}{re_2}\right\}^{N_2}$$

where

$$Y_1 = \left(\frac{r_m + r_1}{r_1}\right) y_1$$

$$Y_2 = \left(\frac{r_2 + r_m}{r_2}\right) y_2$$

$$y_1 = r - r_1$$

$$y_2 = r_2 - r$$

Therefore

$$\frac{V_{av1}}{V_m} = \frac{2}{(N_1 + 2)(N_1 + 1)}$$

$$\frac{V_{av2}}{V_m} = \frac{2}{(N_2 + 2)(N_2 + 1)}$$

i.e.

$$\left\{\frac{r_2 (r_m^2 - r_1^2)}{r_1 (r_2^2 - r_m^2)}\right\}^{1.25} = \left\{\frac{(N_2 + 2)(N_2 + 1)}{(N_1 + 2)(N_1 + 1)}\right\}^{1.75}$$

This equation gives the radius of maximum velocity when the powers for the velocity distributions for the inner and outer portions of the annulus are known. If $N_1 = N_2$ then the equation will reduce to $r_m = \sqrt{r_1 r_2}$.

CHAPTER 4

EXPERIMENTAL EQUIPMENT

Using the suction from a centrifugal blower air was drawn through the annuli. The blower was belt driven at constant speed by a 3 H.P. motor and the assembly was supported by a baseplate on the floor (See figure 18). The annuli were made from three six-foot lengths of tubing joined to give a full eighteen feet. A table was constructed, using "handi angle", of dimensions 2 feet, 3 feet and 20 feet long. Plywood .75 inches thick was bolted on top of this frame to give a reference surface. Rectangular plywood supports, .75 inches thick were mounted across the table, at approximately two foot intervals from end to end of the table top. To hold the supports perpendicular to the table angle brackets were installed between the supports and the table top. On a centreline six inches above the table, holes of the same diameters as the outside of the annuli were cut in the supports. When viewed along the length of the table the holes in the supports were in line. Each support was sawed along the centreline of the holes leaving semi-circular holes in the two pieces. The two portions of each support were hinged together at one end and clamped together at the other end. In other words the annuli were supported both horizontally and vertically along their eighteen foot length at two foot intervals. The top of the supports could then be raised to permit the annuli to be placed on the bottom portion of the supports. The tops were then turned down and clamped rigidly (See figure 19).

Six foot lengths of cast and extruded Acrylic tubing were connected to form the inner and outer tubes of the annuli. The inner tubes were held together with aluminum tubing pushed inside the adjoining ends of the tubes. At either end of the eighteen foot long inner tubes hemispherically shaped wooden plugs were used to seal the ends so that no air leakage could take place. The outer tubes of the annuli were joined with external aluminum collars which slipped over the ends of adjoining tubes. To assure a tight fit all joints were faced on a lathe.

The inner tubes were supported in a concentric manner inside the outer tubes by two pins placed at locations approximately every two feet along the annuli. These pins passed through holes drilled in the inner and outer tubes. The pins at each location were inserted .50 inches apart and at right angles to one another. The pins were cut from .040 inch drill rod and all the holes in the tubes were drilled with a .040 inch diameter drill. The pins were a tight fit in the drilled holes and therefore were not anchored in any manner. Drill jigs were machined to fit each inner and outer tube. They were then slipped over the outside of these tubes, and held in position on the tube by means of setscrews. The jigs were of sufficient thickness to guide the drill during its operation. Four holes in the jig permitted drilling of the holes for two pins. Careful use of the jigs was necessary to ensure that the inner tubes would be centrally located in the outer tubes when assembled (See figure 20). To one end of each annulus a three foot length of hose was clamped with inside diameter equal to the outside diameter of the outer tube.

On the suction side of the blower a butterfly valve was provided to control the rate of flow. Also on the suction side, a three foot length of 2.75 inch inside diameter hose was connected to the blower. To affect a suitable connection between this hose and the various diameter hoses, 1.25 to 2.75 inches from the annulus, a cone shaped transition piece was fabricated. The small end of the cone faced upstream toward the annulus. The cone was smooth inside with the sides sloping at seven degrees or less. On the outside of this cone nine circular steps were machined to present a clamping surface for the hoses from the various annuli. When a large annulus was tested the small diameter of the cone would be restrictive so the cone seperated at two points. This essentially provided three cones which could be joined together as the annuli tested became smaller (See figure 21).

The actual pressure measurements were obtained with a pitot tube, wall tap and micromanometer. At approximately sixteen and a half feet from the open end of the annuli small slots were provided in the outer tubes to allow entry of the pitot tube. Diametrically opposite the pitot tube slots .040 inch diameter holes were drilled in the outer tubes to be used for static pressure readings. One inch sections of .250 inch outer diameter acrylic rod were fastened to the outer tubes around the .040 inch diameter holes. As each annulus was tested tygon tubing was used to connect between the acrylic rod (wall tap) and the micromanometer. The pitot tube was made from No. 23 hypodermic tubing with .0252 inch outer diameter and .0126 inch inner diameter. A two inch Starret micrometer head was mounted to a housing which contained the pitot tube assembly under spring tension. When the micrometer handle was turned the pitot tube assembly followed allowing accurate positioning to .001 of an inch. Since the pitot tube had to be located six inches above the table in a horizontal position, a heavy base to support the pitot tube - micrometer housing was provided. A threaded shaft between the housing and the base could be rotated to provide vertical adjustment (See figure 22). Tygon tubing of .125 inch inside diameter connected the pitot tube assembly to the micromanometer. A Meriam 10 inch model A - 750 micromanometer was used to obtain readings shown in this investigation.

CHAPTER 5

EXPERIMENTAL PROCEDURE

The various annuli tested were selected on the basis of the variables to be studied. To study the effect of changing the inner and outer tube diameters five annuli were used having various equivalent diameters but approximately the same diameter ratios. The effect of changing the diameter ratio was investigated with five annuli with approximately the same equivalent diameter but having diameter ratios ranging from .2 to .7. Some annuli used for the first study also satisfied the imposed conditions for the second study. The Reynolds numbers were held at about 13,000. One annulus was selected from the mid range of the equivalent diameters and diameter ratios being used. The Reynolds numbers were varied in this annulus from 10,000 to the limit of the blower. Restrictions placed on the above annuli were satisfied as closely as possible within the range of available tubes.

ECCENTRICITY

The annulus previously employed was also tested with the inner tube held slightly eccentric with respect to the outer tube. Since it was very difficult to mount the inner tube in a concentric manner within the outer tube this test would indicate the effect of slight eccentricity on the position of maximum velocity. The flow was controlled to give a Reynolds number of approximately 13,000.

VELOCITY DISTRIBUTIONS

The velocity distributions between the region of maximum velocity and the inner or outer walls of the annulus could be represented as power laws. When velocity versus distance from the wall was plotted on logarithmic graph paper a straight line resulted. The slope of the line represented the power (N) in the equation [velocity = constant x (distance from wall)^N]. From the slopes the powers for the two equations representing the velocity distribution in the inner and outer portions of the annulus were obtained. The powers were used to locate a unique value for the position of maximum velocity. This led to interest in the powers. A large outer tube was selected to be tested with four different inner tubes of various outer diameters. This would indicate the effect of changing the inner tube, on the powers at the inner and outer walls. Again the Reynolds numbers were approximately 13,000.

Air temperature and barometric pressure were measured using a thermometer and a barometer located in the laboratory. Since the Reynolds number, temperature, barometric pressure and equivalent diameter are known the average velocity may be calculated. Knudsen and Katz (2) measured the ratio of average to maximum velocity approximately equal to .876. Using this the maximum velocity may be determined. Knowing the maximum velocity the maximum pressure is calculated. The pitot tube was located at .4 of the distance from the inner wall to the outer wall as suggested by Owen (3) and the butterfly valve adjusted to give the maximum pressure corresponding to the Reynolds number desired.

The actual average velocity in each case was obtained from pressure readings taken at twenty points representing twenty equal area sections of the annulus cross section. A computer program was written with inputs of inner diameter and outer diameter of the annulus, and outputs of positions representing the twenty equal area sections. Additional points were provided between the approximate location of the position of maximum velocity and the inner tube since the positions representing equal area were widely spaced in this region. This gave thirty-two locations on a radius between the inner and outer walls of the annulus for pressure readings.

The outer diameter of the inner tubes could be measured with a micrometer at the test sections prior to the assembly of the annuli. The inside diameters of the outer tubes were measured indirectly. The inside diameters of the tubes were measured at both ends to .001 of an inch. Based on a linear change of diameter, if present, between the two points measured the diameter at the test section was approximated.

The pressure difference between that sensed by the pitot and wall tap was measured at the micromanometer to .001 of an inch of water. Ambient air temperature could be recorded to the nearest 0.1°F. Barometric pressure was obtained to the nearest .01 inches of mercury.

TABLE OF ANNULI TEST CONDITIONS AND NOTATION:

Annulus Test No.	r_1 in.	r_2 in.	r_1/r_2	$D_2 - D_1$ in.	Re.	$V_{av.}$ ft./sec.	Eccentricity in.
1	.626	1.241	.504	1.230	12,767	21.063	gap ± .000
2	.435	1.005	.432	1.140	13,908	24,138	gap + .012
3	.317	.751	.422	.868	14,253	32.305	gap + .001
4	.381	.751	.507	.740	13,139	36.303	gap ± .000
5	.250	.505	.495	.510	13,280	53.057	gap - .002
6	.867	1.241	.698	.748	12,658	34.306	gap - .003
7	.625	1.005	.621	.760	12,949	34.001	gap - .003
8	.136	.505	.269	.738	12,609	35.135	gap - .002
9 (a)	.381	.751	.507	.740	11,129	30.644	gap ± .000
9 (b)	.381	.751	.507	.740	13,139	36.303	gap ± .000
9 (c)	.381	.751	.507	.740	17,493	48.426	gap ± .000
9 (d)	.381	.751	.507	.740	22,440	62.191	gap ± .000
9 (e)	.381	.751	.507	.740	25,770	70.980	gap ± .000
10 (a)	.381	.751	.507	.740	13,309	36.608	gap ± .000
10 (b)	.381	.751	.507	.740	13,511	37.064	gap + .015
10 (c)	.381	.751	.507	.740	13,476	37.041	gap + .009
10 (d)	.381	.751	.507	.740	13,006	35.779	gap - .009
10 (e)	.381	.751	.507	.740	13,013	35.798	gap - .015
11	.251	1.241	.202	1.980	17,782	18.318	gap ± .000
12	.381	1.241	.307	1.720	17,433	20.694	gap ± .000
13	.435	1.241	.350	1.612	16,968	21.546	gap ± .000
14	.626	1.241	.504	1.230	17,289	28.773	gap ± .000

CHAPTER 6

DATA PROCESSING

All experimental data was reduced using an IBM 1620 computer. Considering a representative pitot tube traverse, the inputs to the computer were inner diameter, outer diameter, barometric pressure, air temperature, thirty-two radial positions and thirty-two pressure readings. The computer printed the following: diameter ratio, equivalent diameter, average velocity, Reynolds number, velocities, radii (corrected for effective centre displacement), distance from the inner wall (corrected), distance from the outer wall (corrected), power (with correlation coefficient) for the inner wall velocity distribution equation, power (with correlation coefficient) for the outer wall velocity distribution equation, the position of maximum velocity, the ratios of velocities to maximum velocity, and the ratios of (the radii (corrected) minus the inner radius) divided by (the outer radius minus the inner radius).

Using the statistical method of linear regression the computer located the line of best fit for log (velocity) = log a + N log (distance from the wall). The power for the velocity distribution equation is "N". Correlation coefficients indicated how well the data followed the line selected.

The powers for the inner and outer walls were used in an iteration process to find where the two velocity distributions intersect or the point of maximum velocity. This process determined the radius of maximum velocity to the nearest .001 of an inch.

CHAPTER 7

RESULTS

The results of this investigation are shown on figures one to seventeen. Figures 1, 3, 6, 9 and 10 present the basic velocity data. The other figures show more detailed information on the positions of maximum velocity, the ratios of average velocity to maximum velocity, the velocity distributions and the effects of eccentricity.

Figure 2 shows that the positions of maximum velocity do not vary with equivalent diameters if the diameter ratios and Reynolds numbers are held constant. If all five annuli had diameter ratios equal to .5 then they would have approximated the horizontal line shown. Test numbers 2 and 3 deviate for this reason. When corrections were applied to points 2 and 3, as indicated by the darkened points, deviations from the line were not greater than $^{+\,2.26}_{-\,1.69}$ per cent.

Figure 4 demonstrates a definite relation between the positions of maximum velocity and diameter ratio. Equivalent diameters and Reynolds numbers were held constant during these tests. This trend is similar to that found by past investigators. As the diameter ratios decrease the positions of maximum velocity move toward the inner tube.

Figure 5 presents the positions of maximum velocity for all the diameter ratios tested regardless of equivalent diameters or Reynolds numbers. The dark circles indicate the positions of maximum velocity measured by Brighton (9). The curve $r_m = \sqrt{r_1 r_2}$ suggested by Nicol and Medwell (8) is shown with the curve

$$r_m = \sqrt{\frac{r_2^2 - r_1^2}{2 \ln (r_2/r_1)}}$$

The positions of maximum velocity show a trend from their laminar locations at diameter ratios of about .6 to the locations specified by $r_m = \sqrt{r_1 r_2}$ at diameter ratios of about .2. The points from Brighton's research (9) agree with this trend from diameter ratios of .2 to .7 (the scope of this investigation). At diameter ratios of less than .2 a continuation of this trend may be in question. Brighton's results for diameter ratios less than .2 disagree with possible continuation of the trend. The fact that the points shown in figure 4 do not deviate from the trend shown in figure 5 when plotted with other experimental values, having various equivalent diameters, further supports the results of figure 2.

Figure 7 indicates that variation in Reynolds numbers does not affect the positions of maximum velocity when diameter ratios and equivalent diameters are constant. The Reynolds numbers ranged from 11,000 to 26,000. Rothfus and co-workers (1), (4), (15), (6) and (7) found the positions of maximum velocity to be constant at their laminar values for Reynolds numbers greater than 10,000. The above results indicate that the positions are constant but closer to the inner tubes than in the case of laminar flow.

The upper curve in figure 8 is for an annulus with a diameter ratio .507 and an equivalent diameter of .740 inches. An annulus of diameter ratio approximately .2 and an equivalent diameter of approximately 2 inches was tested to obtain the lower curve. The latter annulus was used in preliminary tests. The pins were made from .093 inch diameter rod. The sections which blocked the flow were filed to give an aerodynamic profile. Since the amount of eccentricity was not checked accurately these results do not have the same degree of accuracy as the others contained in this investigation. They were included to show that as the diameter ratio is decreased the value of average velocity to maximum velocity also decreases. Croop (7) found this to be the case at lower Reynolds number values. Both Walker (6) and Croop (7) have indicated that at about 11,000 to 13,000 Reynolds numbers (the upper limit of their work) the ordinate is still increasing slightly. This occurs in figure 8 but from 15,000 to 20,000 upwards the curve is almost flat. Nikuradse's work (15) with turbulent flow in pipes indicated a similar flattening of the curve above Reynolds numbers of 10,000. Knudsen and Katz (2) measured average velocity to maximum velocity equal to .876 $\pm$ 1.8 per cent. The results shown in figure 8 fall within this range. The results of this investigation show this ratio equal to .878 $\pm$ 1.7 per cent.

Figure 11 demonstrates the lack of correlation between the powers of the velocity distributions [velocity = constant x (distance from wall)^N] for the inner and outer walls and the equivalent diameters. Similarly figure 12 shows no trend between the powers and the diameter ratios. The powers were determined by the computer. All the correlation coefficients were greater than .990 - illustrating the "goodness of fit".

Figure 13 indicates that for Reynolds numbers greater than 13,000 there is essentially no change in powers as the Reynolds numbers are increased to 26,000. Since the range of Reynolds numbers tested was small no change was anticipated. The data of Nikuradse (15) for turbulent flow in pipes indicated a change in powers from .143 to .100 for a change in Reynolds numbers from 4,000 to 3,240,000. The reason for the large change in powers at Reynolds number 11,129 is not fully understood. Possibly fully developed turbulent flow does not exist across the entire cross-section. Above 13,000 the flow is fully developed.

Figure 14 gives the effect of eccentricity on the powers for the velocity distributions. Since all the annuli listed were within the limits + 2.1 per cent and - 0.8 per cent eccentricity the errors in the powers measured should be no greater than the order of magnitude of ± 1.5 per cent.

It is felt that the powers should depend on the inner diameter, outer diameter and the equivalent diameter. Possibly the effect of each, works in such a manner as to hide any trend between the powers and one of the variables. The outer diameter was held constant at 2.482 inches and the inner tube size varied. Figure 15 presents the results from this study. The upper curve shows the change of the power for the velocity distribution between the point of maximum velocity and the outer tube as the inner diameter is increased. Up to an inner diameter of .870 inches the powers decrease. The predominant effect in this region may be the curvature of the inner tube. As the diameter is increased further possibly a containment or gap effect predominates and the powers increase. The lower curve refers to the powers for the portion of the annuli between the point of maximum velocity and the inner tube. To an inner diameter of .782 inches the inner tube curvatures appear to increase the powers. Above .782 inches a containment effect seems predominant. This causes a decrease in the powers. Since the powers vary only slightly with inner diameter no definite conclusion concerning the curvature and containment effects can be made without further study. For all the annuli tested the powers for the outer walls have an average value of .159 with a range of ± 18.9 per cent. The powers for the inner wall have an average value of .152 with a range of

$^{+\,19.7}_{-\,12.5}$ per cent including 95 per cent of the annuli tested. In most cases the power at the inner wall is less. Owen (3) working with fewer annuli concluded that the power at the inner wall was always less.

Figure 16 shows what error results in the value of the position of maximum velocity when slight eccentricity occurs. Since the limits of eccentricity for the annuli tested were between + 2.1 per cent and - 0.8 per cent the curve shows that the position of maximum velocity would have an error magnitude of $^{+\,1.58}_{-\,0.00}$ per cent.

Figure 17 shows the positions of maximum velocity for laminar flow and as predicted by $r_m = \sqrt{r_1 r_2}$. The two broken lines enclosing the curve $r_m = \sqrt{r_1 r_2}$ result from calculations using the relation

$$\left\{\frac{r_2\,(r_m^2 - r_1^2)}{r_1\,(r_2^2 - r_m^2)}\right\}^{1.25} = \left\{\frac{(N_2 + 2)\,(N_2 + 1)}{(N_1 + 2)\,(N_1 + 1)}\right\}^{1.75}$$

with limiting experimental values N_1 and N_2. The upper and lower broken lines represent respectively, $N_1 = 1/7 = .143$, $N_2 = 1/5.5 = .182$, $N_1 = 1/6 = .167$ and $N_2 = 1/6.7 = .150$. These two curves enclose the experimental values (see fig. 5) for the position of maximum velocity. The positions of maximum velocity were calculated from the correlation mentioned previously using experimental values of N_1 and N_2 for annuli numbers 11, 12, 13 and 14. Since the calculated values of r_m deviated from the measured values of r_m by less than $^{+\,2.19}_{-\,1.07}$ per cent the assumptions employed in the development of the correlation produce negligible error. The points 11, 12, 13 and 14 on the graph indicate both the experimental and calculated values for the position of maximum velocity.

CHAPTER 8

CONCLUSIONS

(1) The position of maximum velocity is independent of equivalent diameter when the diameter ratio and Reynolds number are held constant.

(2) The position of maximum velocity is closer to the inner wall than for laminar flow. At diameter ratios of .7 the positions of maximum velocity are approximated by their laminar values. As the diameter ratios are decreased the positions of maximum velocity move closer to the inner walls until at diameter ratios of .2 the positions of maximum velocity are approximated by $r_m = \sqrt{r_1 r_2}$.

(3) The position of maximum velocity was not affected by Reynolds numbers in the range 11,000 to 26,000.

(4) The ratios of average velocity to maximum velocity were equal to .878 $\pm$ 1.7 per cent.

(5) There appears to be no predictable trend at this stage between the powers for the velocity distributions and either diameter ratios or equivalent diameters.

(6) The powers are not altered by changing the Reynolds number in the range 12,000 to 26,000.

(7) For eccentricities of $\pm$ 3.0 per cent the power for the inner wall will increase above its true value by not more than the order of magnitude of 4.0 per cent.

(8) The power for the inner wall is on the average less than the

power for the outer wall.

(9) For eccentricities of $\pm$ 3.0 per cent the position of maximum velocity will move toward the outer tube by not more than 1.56 per cent.

(10) When N_1 and N_2 are known the relation

$$\left\{\frac{r_2\,(r_m^2 - r_1^2)}{r_1\,(r_2^2 - r_m^2)}\right\}^{1.25} = \left\{\frac{(N_2 + 2)\,(N_2 + 1)}{(N_1 + 2)\,(N_1 + 1)}\right\}^{1.75}$$

may be used to determine the radius of maximum velocity.

Direct measurement of the drag on the inner and outer wall of the annulus in a manner similar to that employed by Rothfus and co-workers (5) would give the shear stress at either wall. From

$$\frac{\tau_1}{\tau_2} = \frac{r_2\,(r_m^2 - r_1^2)}{r_1\,(r_2^2 - r_m^2)}$$

the value for the radius of maximum velocity may be determined. This type of study might substantiate the trend between the radius of maximum velocity and diameter ratio found in the present investigation. Correlations between powers and annuli geometry will be necessary if the expression

$$\left\{\frac{r_2\,(r_m^2 - r_1^2)}{r_1\,(r_2^2 - r_m^2)}\right\}^{1.25} = \left\{\frac{(N_2 + 2)\,(N_2 + 1)}{(N_1 + 2)\,(N_1 + 1)}\right\}^{1.75}$$

is to be useful in practice.

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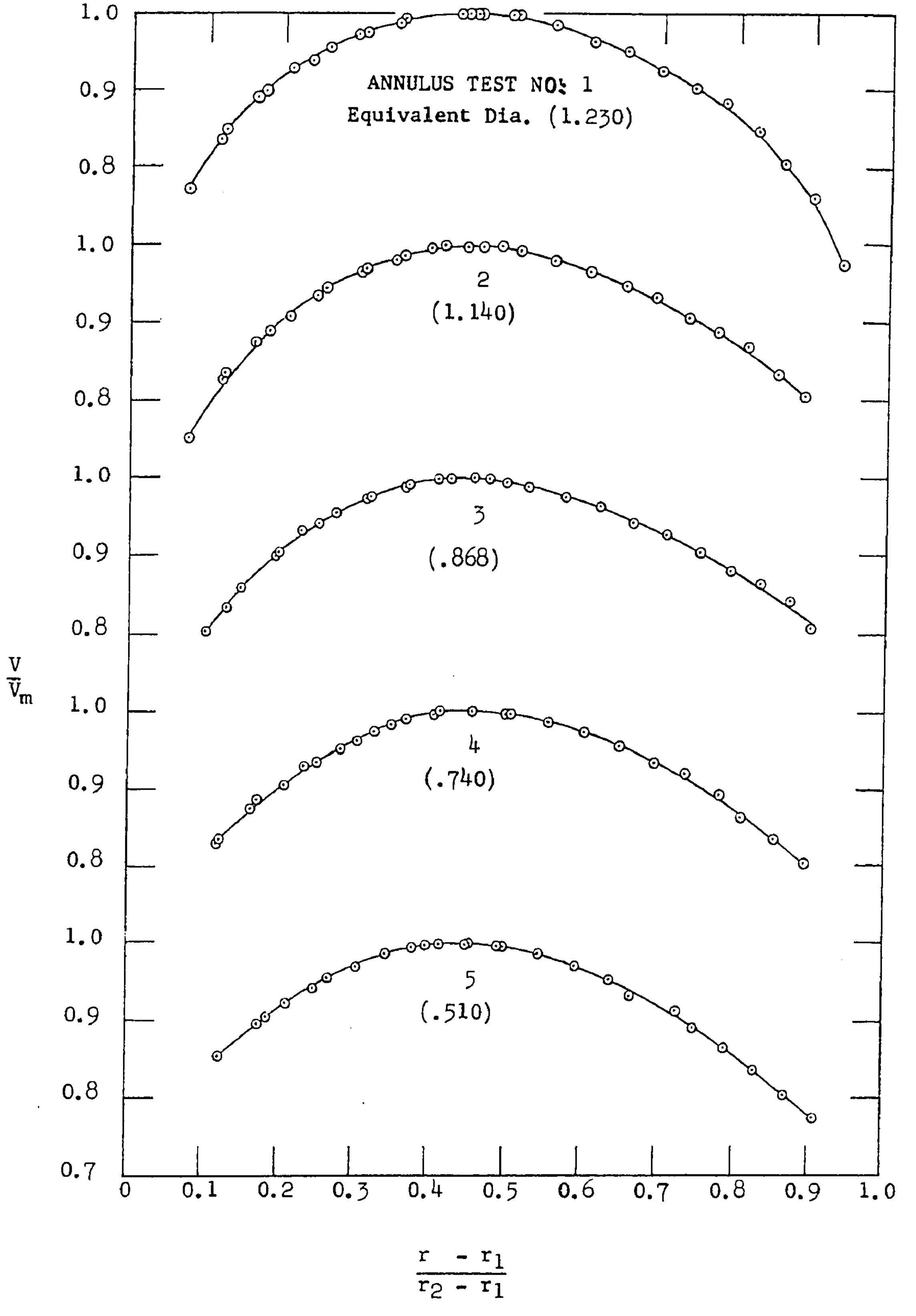

Fig. 1 EFFECT OF EQUIVALENT DIAMETER ON VELOCITY PROFILES (Diameter Ratio and Reynolds Number Constant)

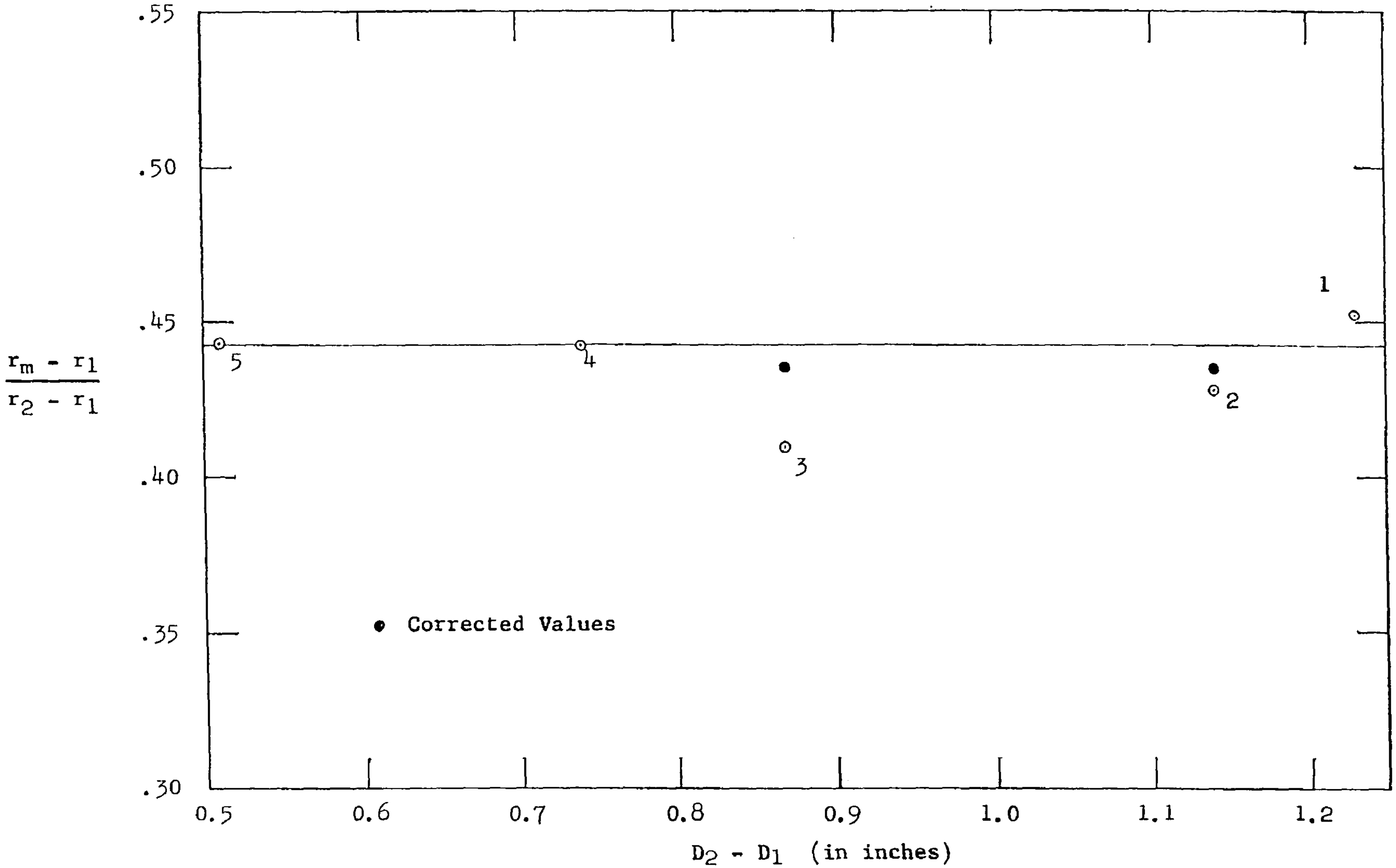

Fig. 2 POSITION OF MAXIMUM VELOCITY VERSUS EQUIVALENT DIAMETER
(Diameter Ratio and Reynolds Number Constant)

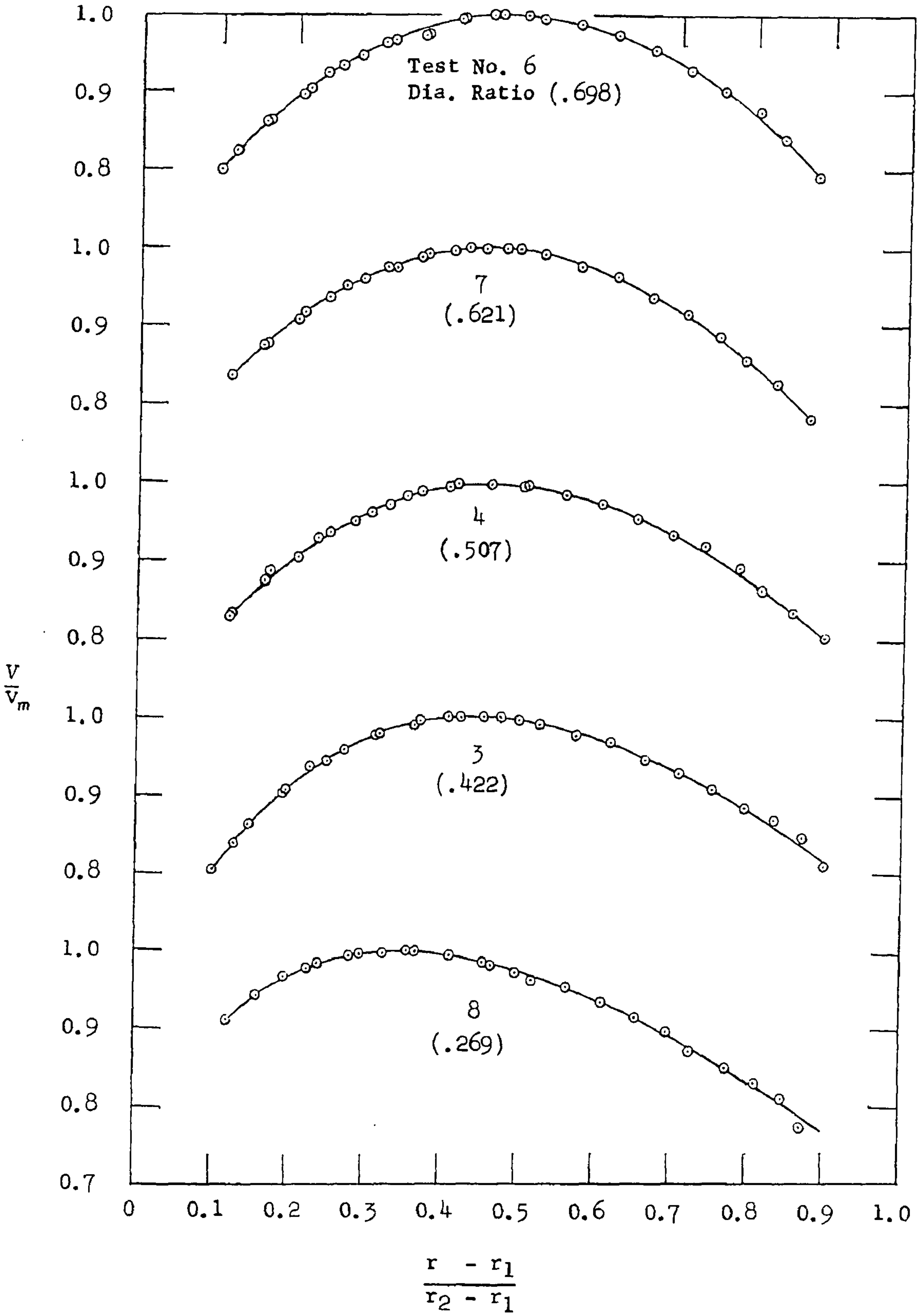

Fig. 3 EFFECT OF DIAMETER RATIO ON VELOCITY PROFILES
(Equivalent Diameter and Reynolds Number Constant)

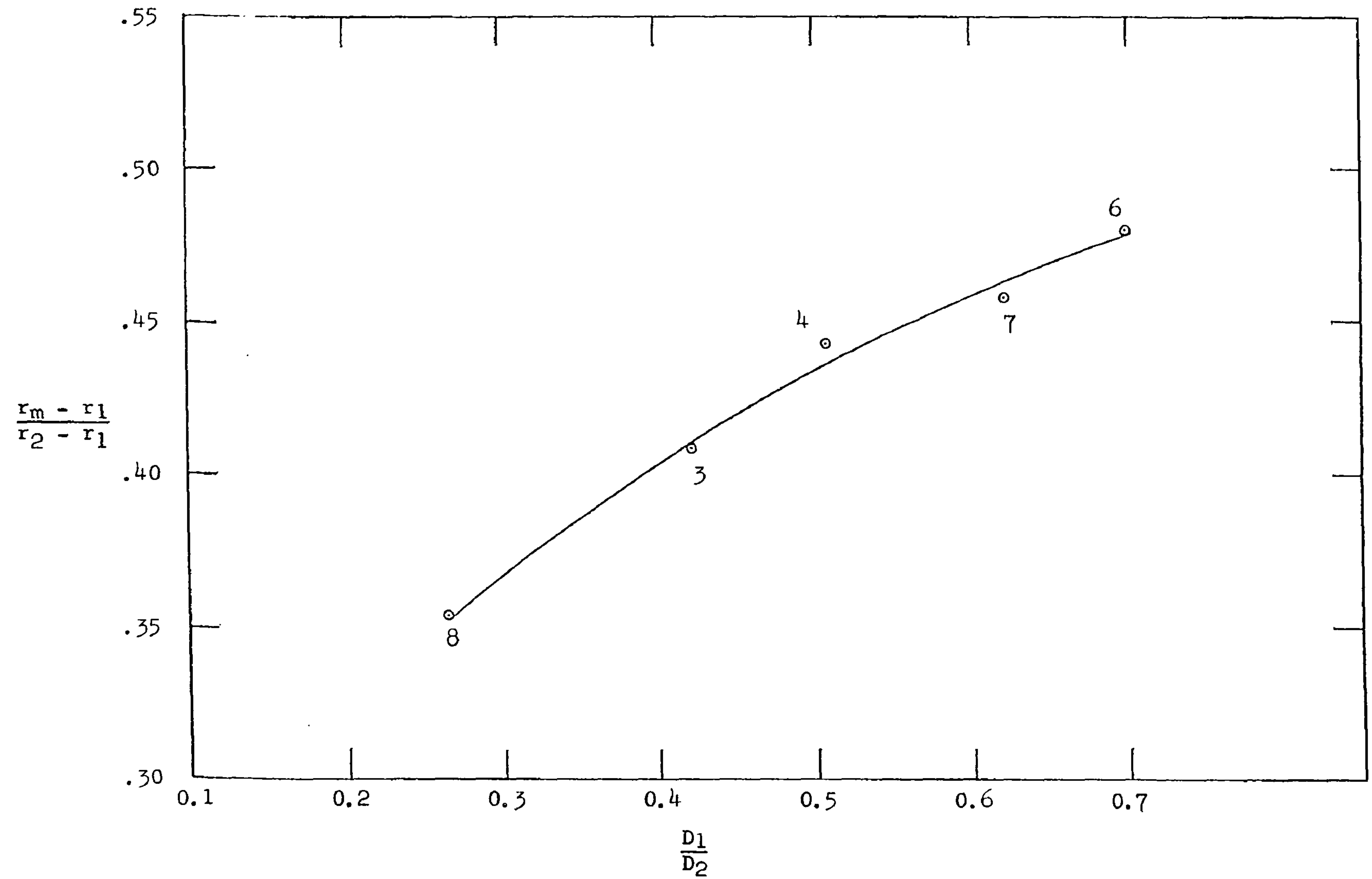

Fig. 4 POSITION OF MAXIMUM VELOCITY VERSUS DIAMETER RATIO (Equivalent Diameter and Reynolds Number Constant)

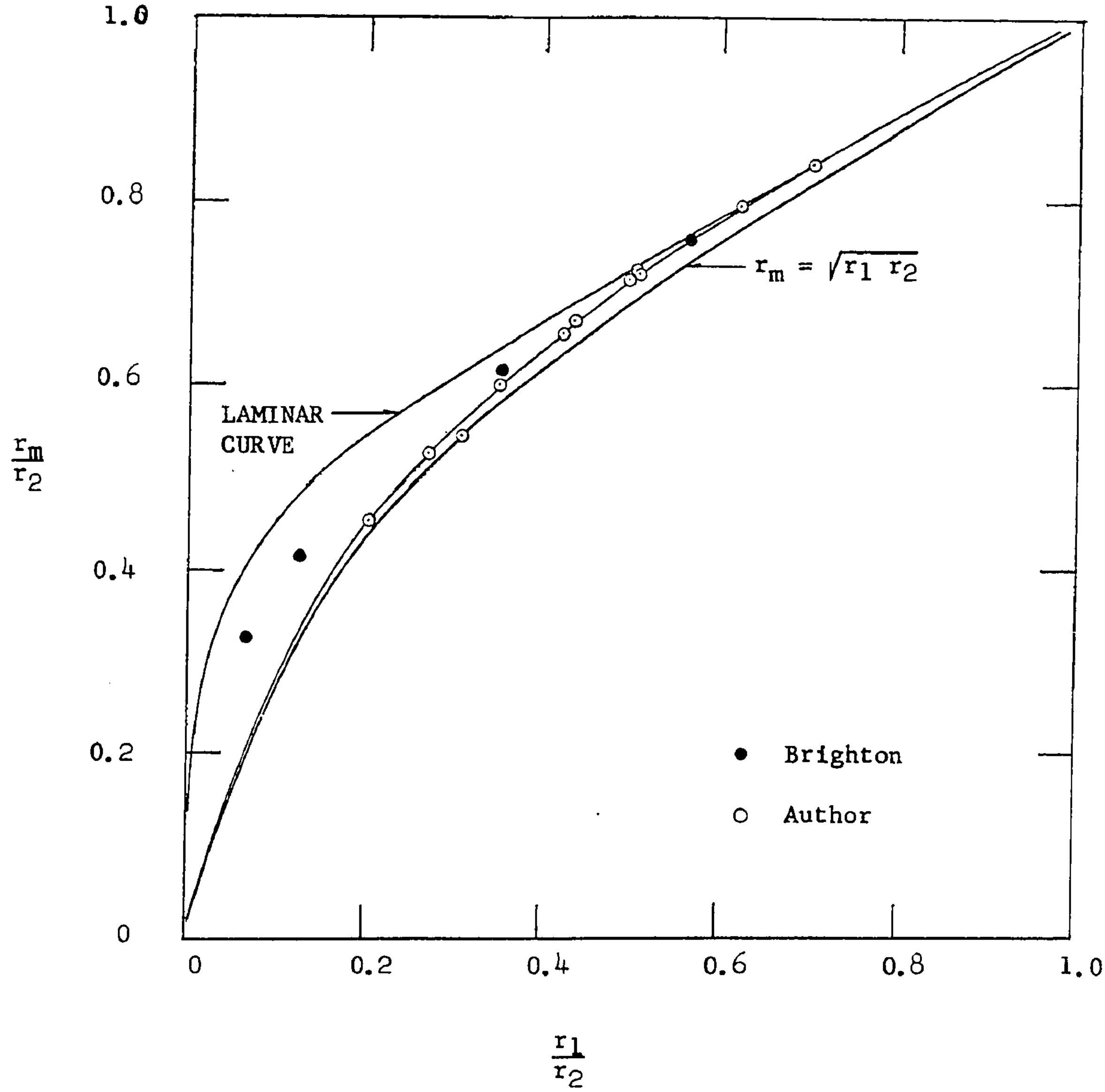

Fig. 5 POSITION OF MAXIMUM VELOCITY VERSUS DIAMETER RATIO

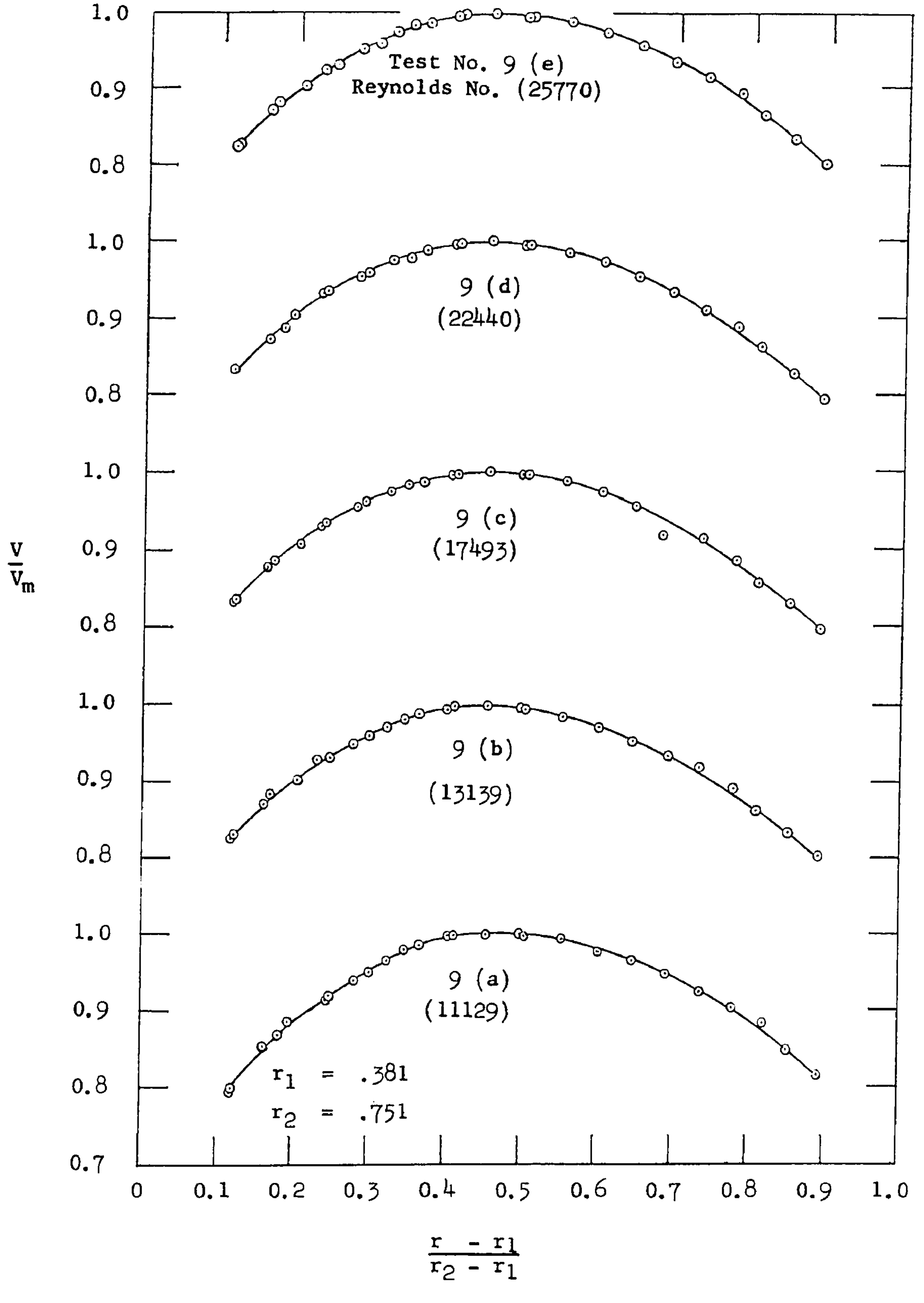

Fig. 6 EFFECT OF REYNOLDS NUMBER ON VELOCITY PROFILES
(Equivalent Diameter and Diameter Ratio Constant)

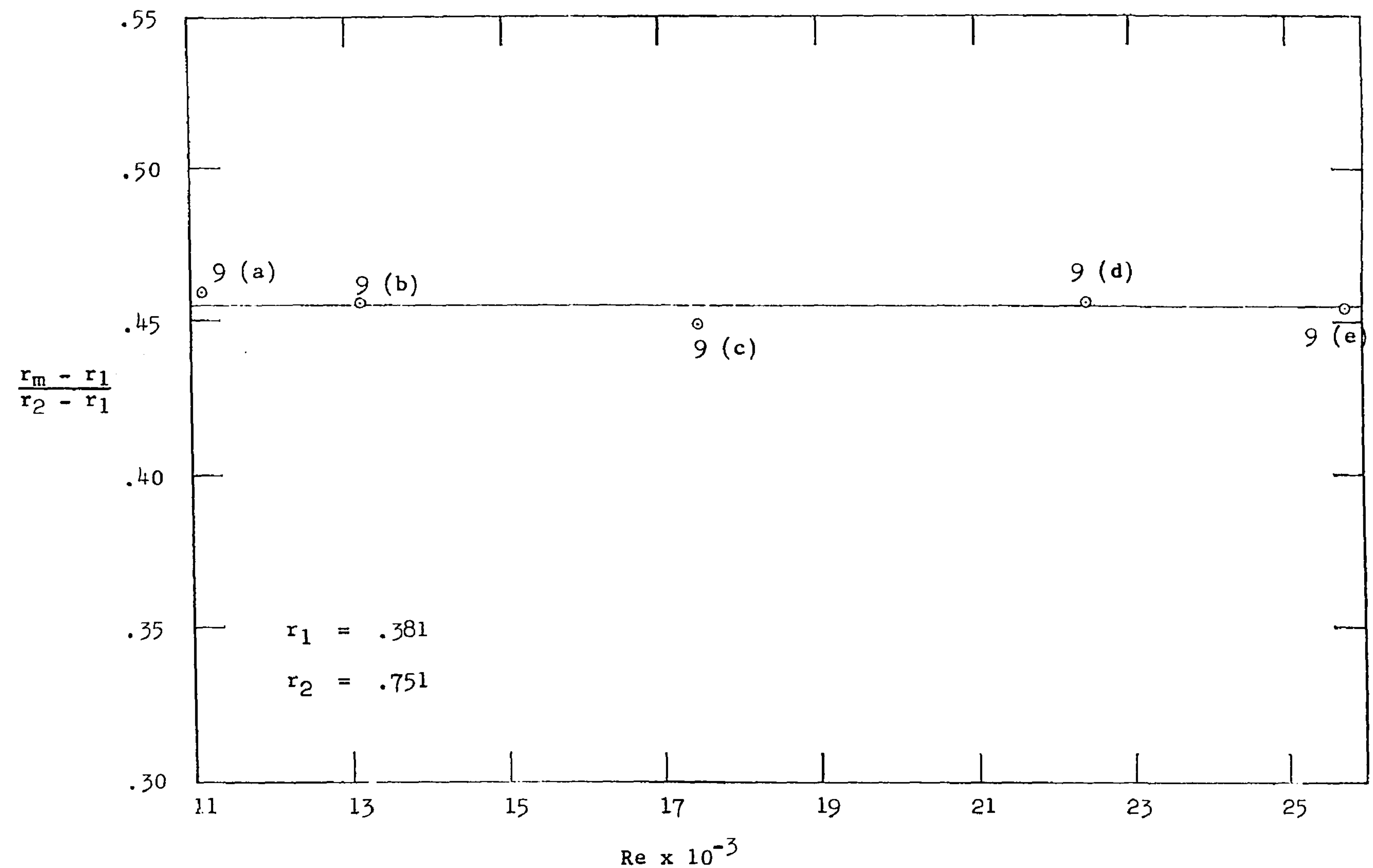

Fig. 7 POSITION OF MAXIMUM VELOCITY VERSUS REYNOLDS NUMBER (Equivalent Diameter and Diameter Ratio Constant)

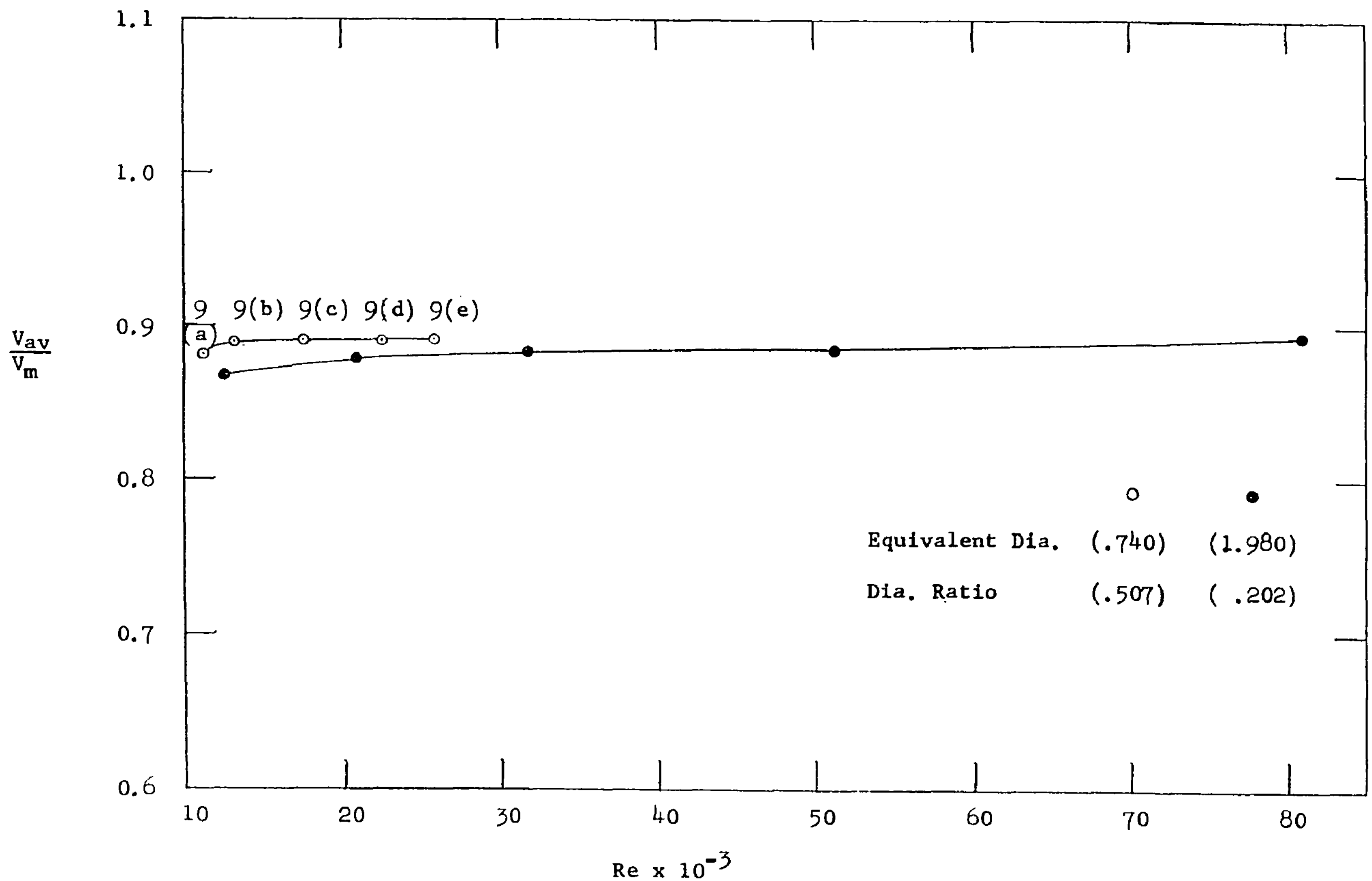

Fig. 8 RATIO OF AVERAGE VELOCITY TO MAXIMUM VELOCITY VERSUS REYNOLDS NUMBER

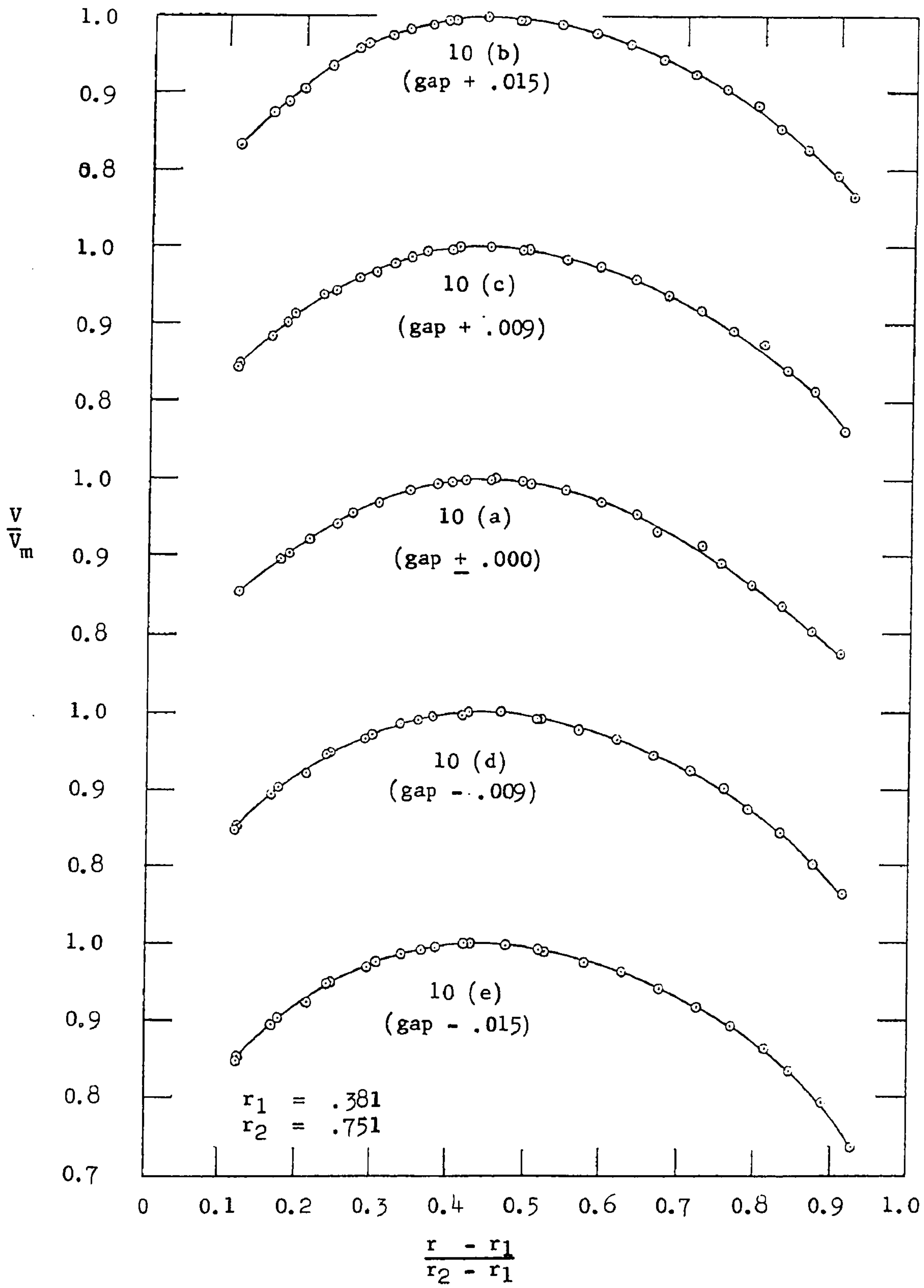

Fig. 9 EFFECT OF ECCENTRICITY ON VELOCITY PROFILES (Equivalent Diameter, Diameter Ratio and Reynolds Number Constant)

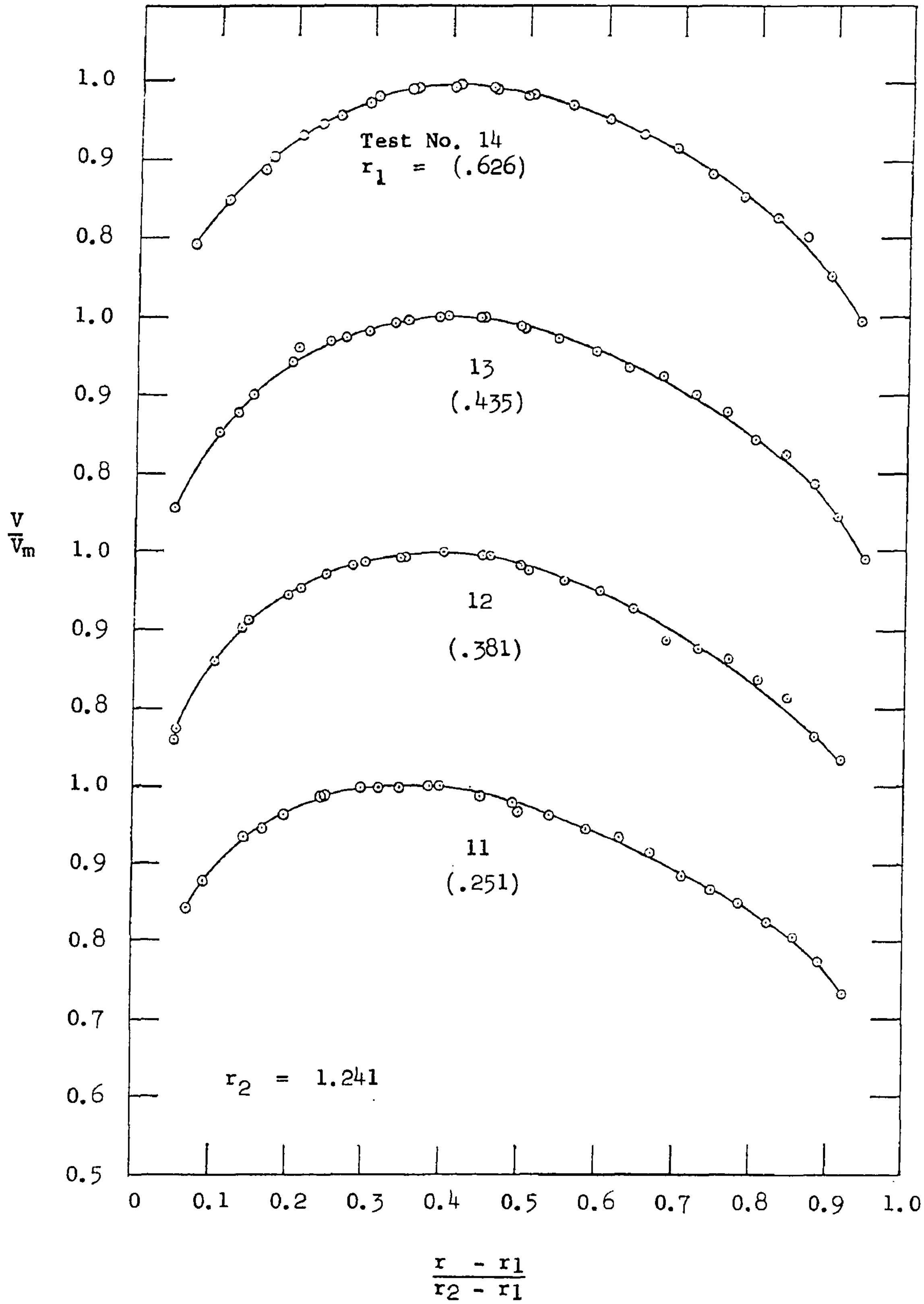

Fig. 10 VELOCITY PROFILES FOR POWER LAW DATA
(Outer Diameter and Reynolds Number Constant)

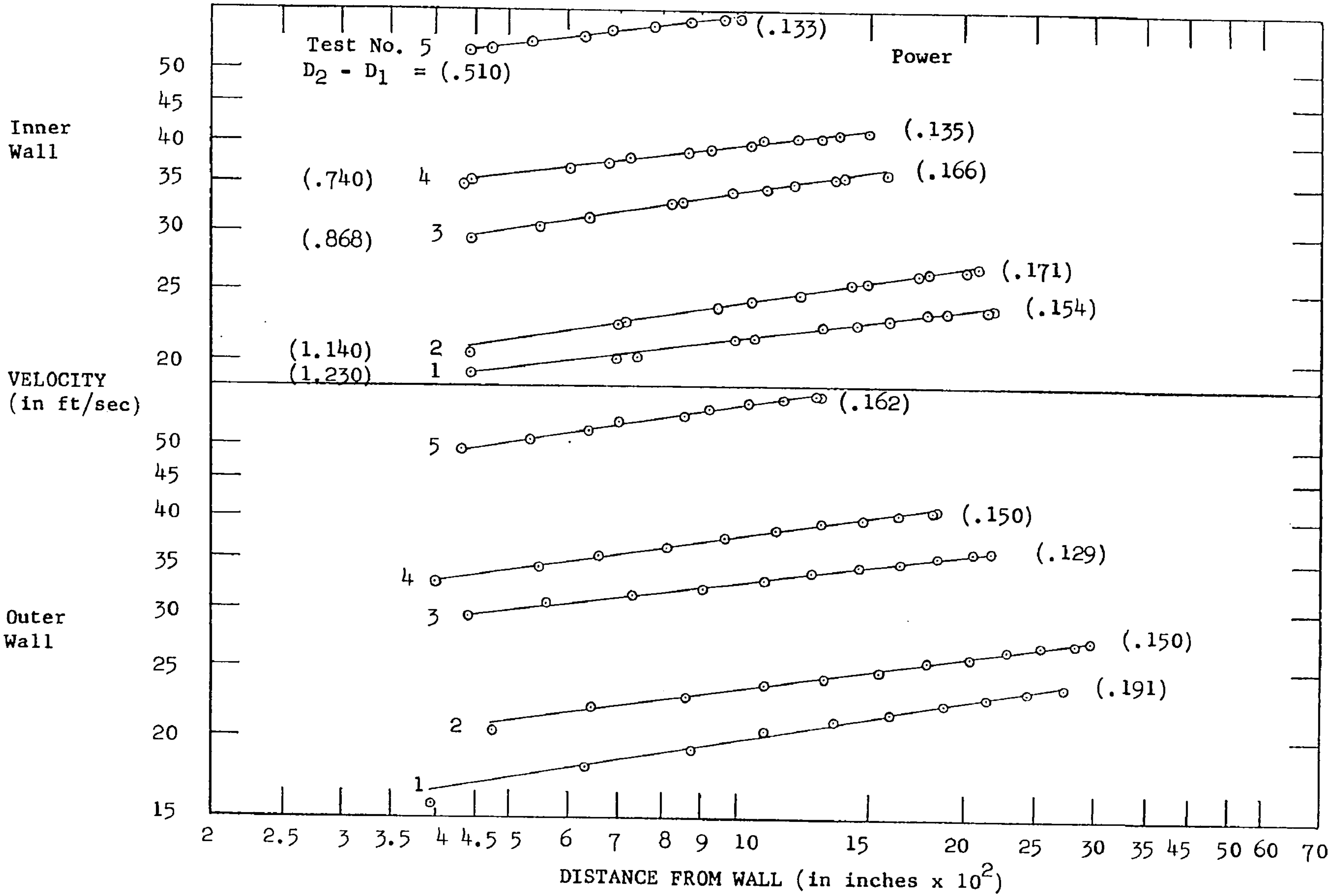

Fig. 11 EFFECT OF EQUIVALENT DIAMETER ON VELOCITY DISTRIBUTIONS
(Diameter Ratio and Reynolds Number Constant)

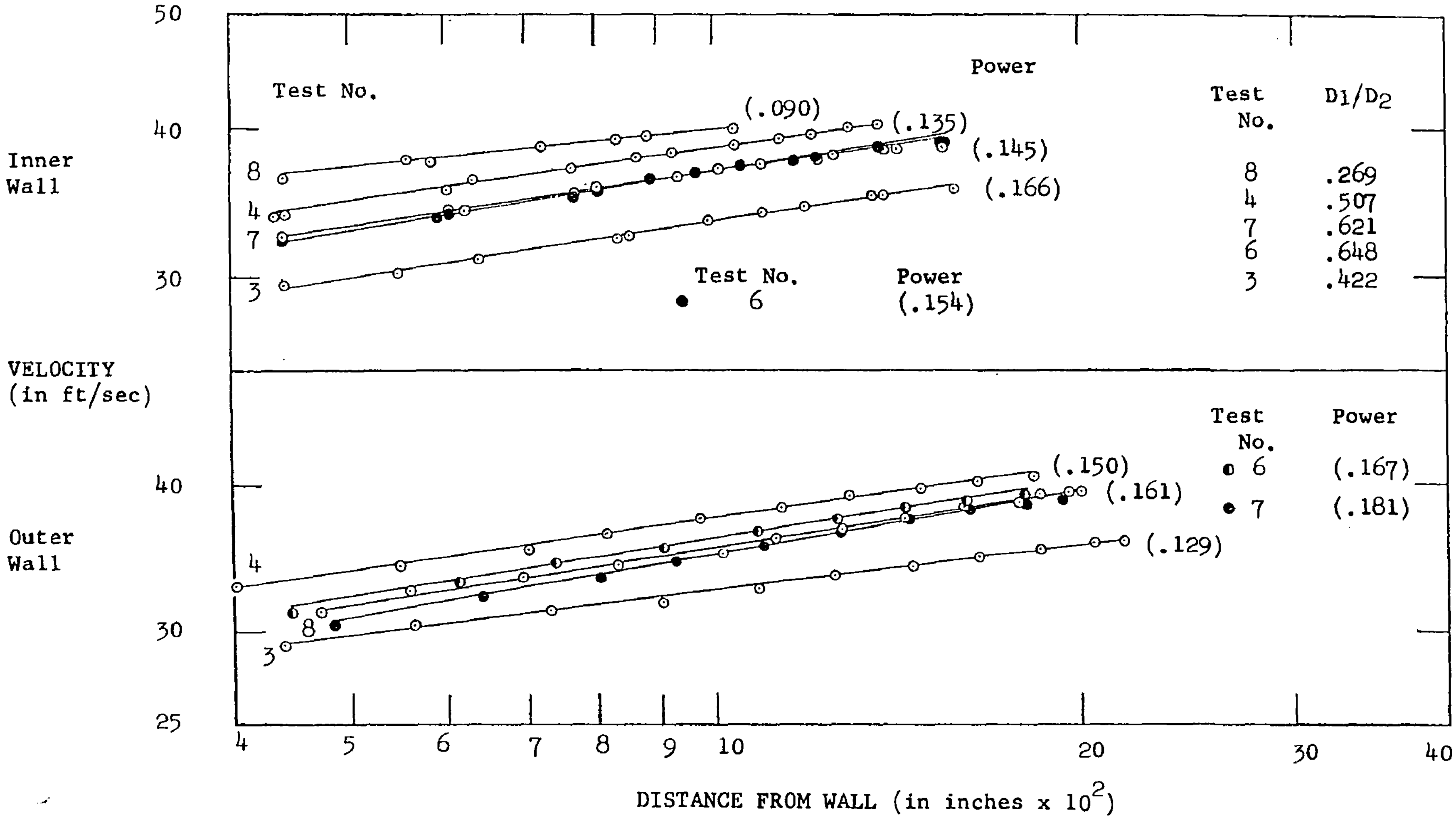

Fig. 12 EFFECT OF DIAMETER RATIO ON VELOCITY DISTRIBUTIONS
(Equivalent Diameter and Reynolds Number Constant)

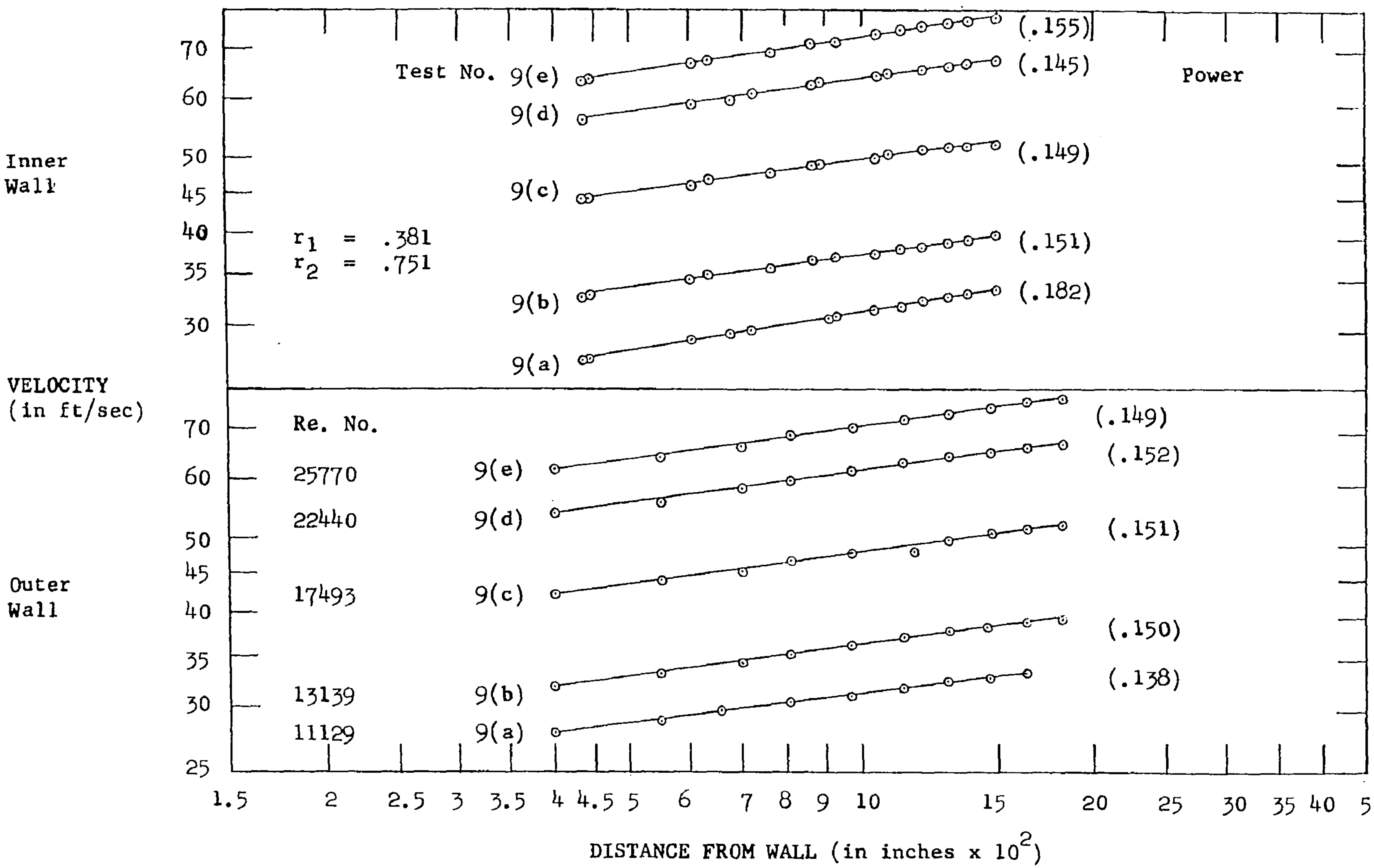

Fig. 13 EFFECT OF REYNOLDS NUMBER ON VELOCITY DISTRIBUTIONS
(Equivalent Diameter and Diameter Ratio Constant)

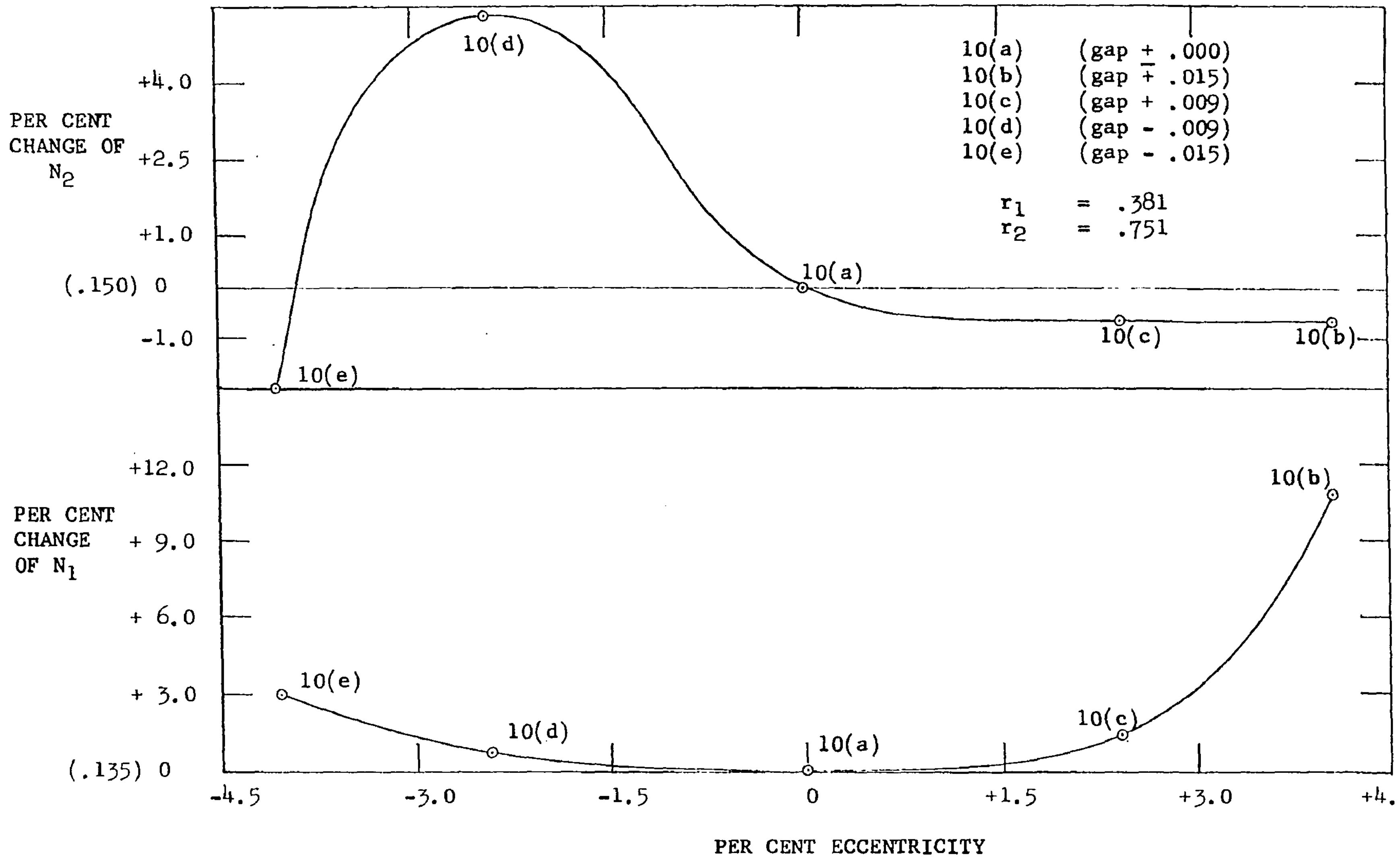

Fig. 14 PER CENT CHANGE IN POWERS VERSUS PER CENT ECCENTRICITY
(Equivalent Diameter, Diameter Ratio and Reynolds Number Constant)

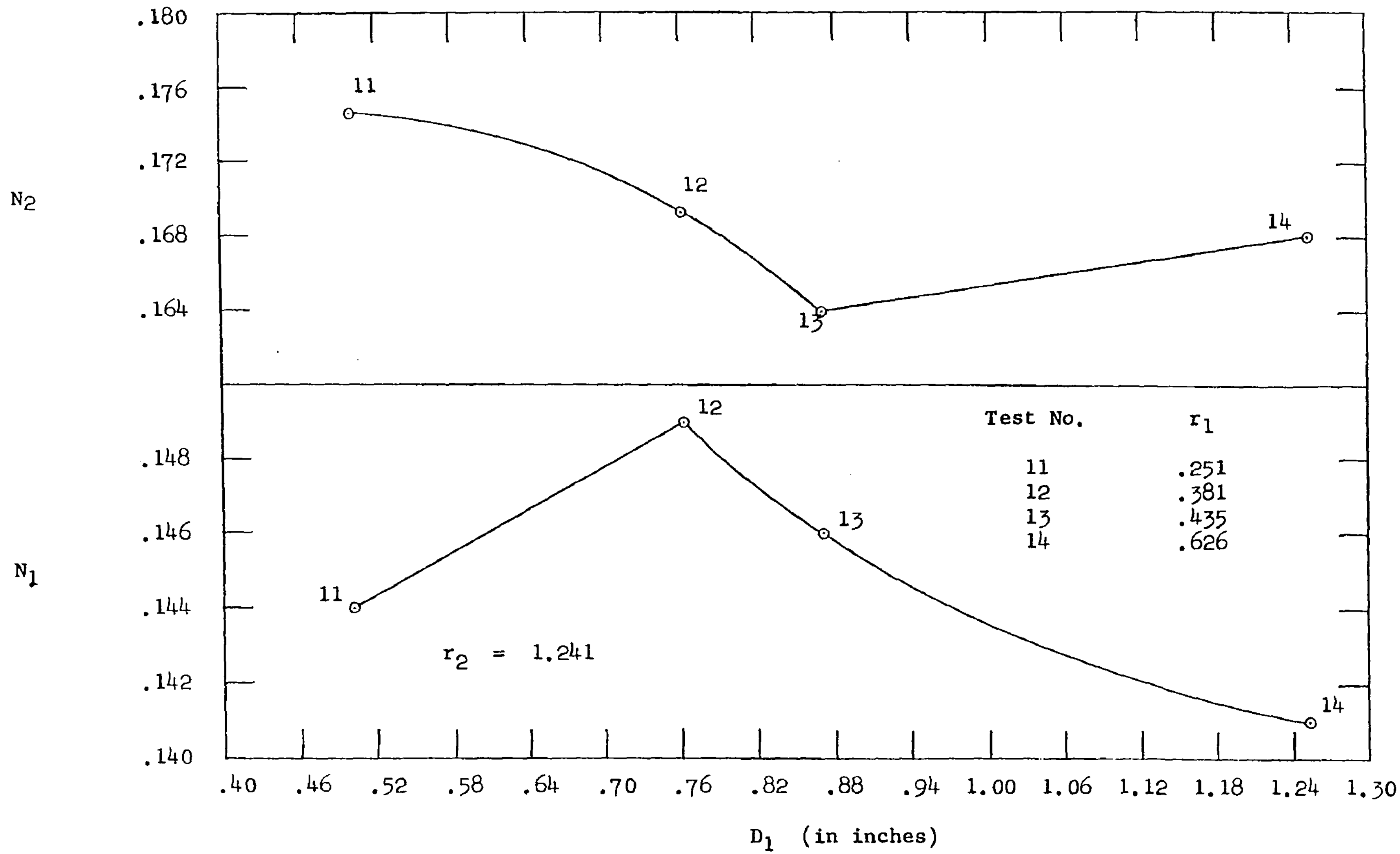

Fig. 15 POWERS VERSUS INNER DIAMETERS (Outer Diameter and Reynolds Number Constant)

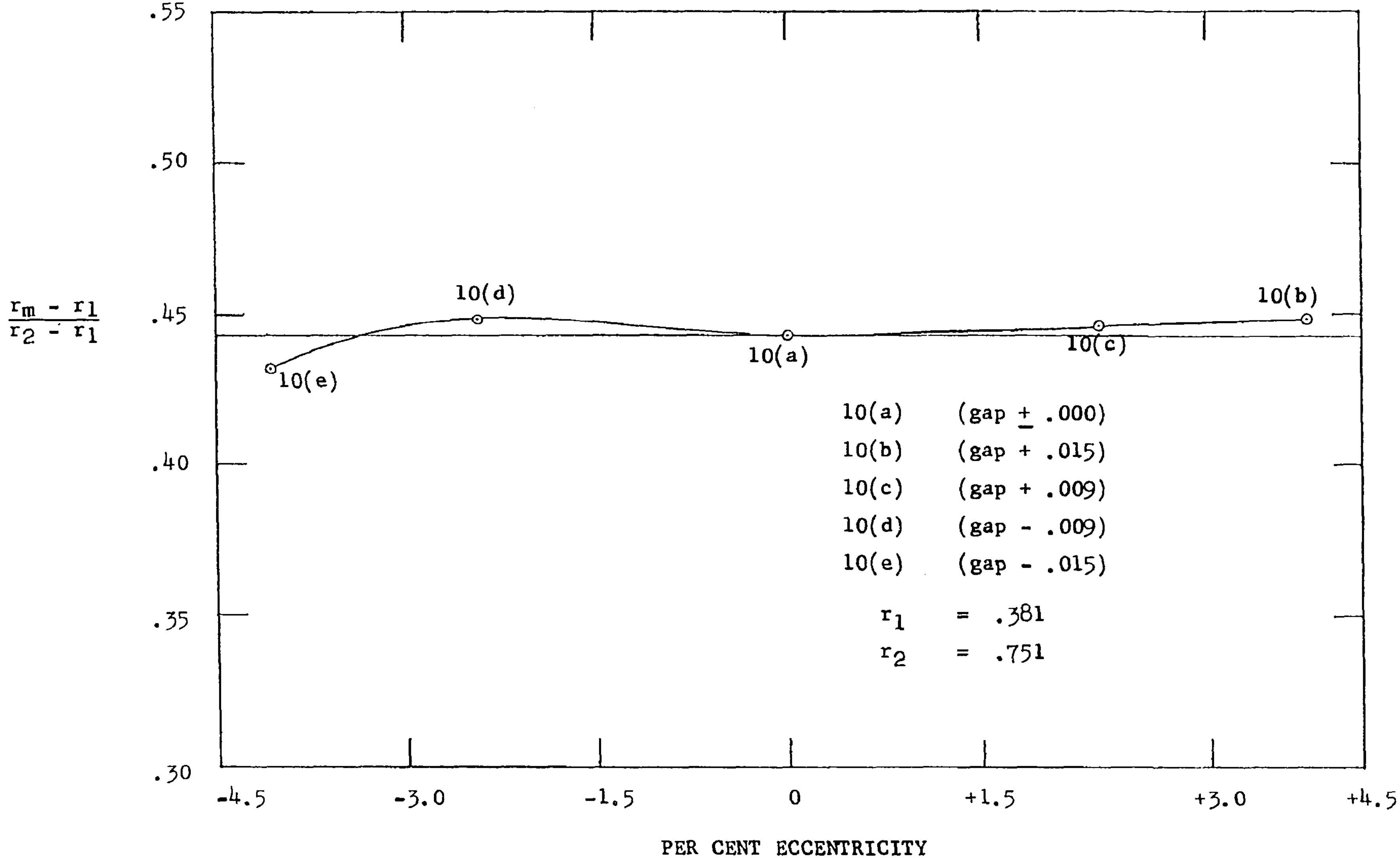

Fig. 16 EFFECT OF ECCENTRICITY ON THE POSITION OF MAXIMUM VELOCITY
(Equivalent Diameter, Diameter Ratio and Reynolds Number Constant)

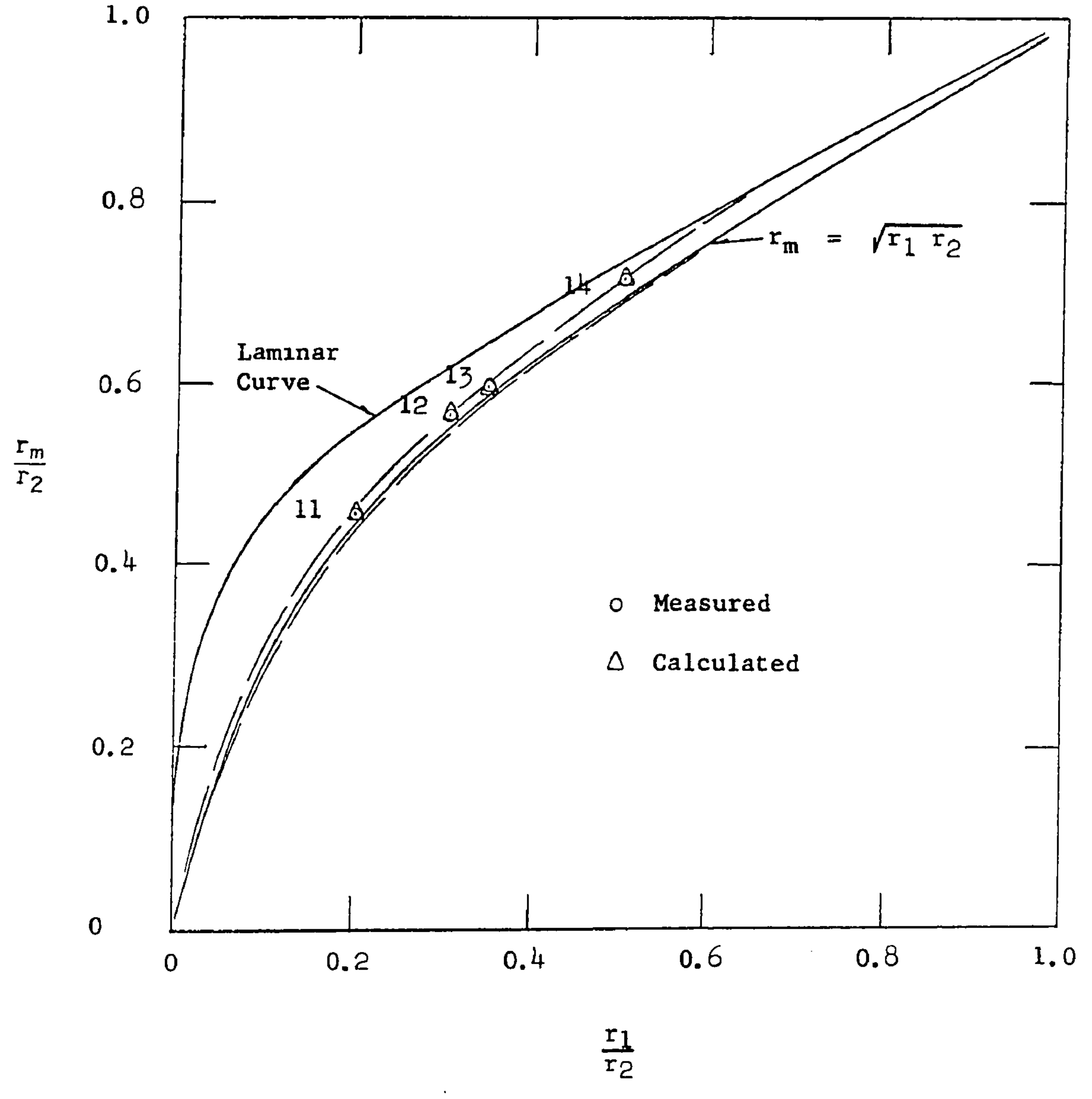

Fig. 17 RELATION FOR THE POSITION OF MAXIMUM VELOCITY

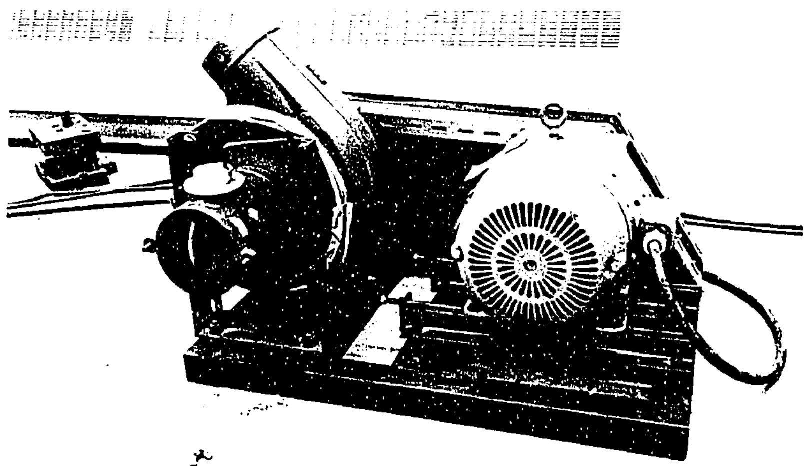

Fig. 18 BLOWER AND MOTOR ARRANGEMENT

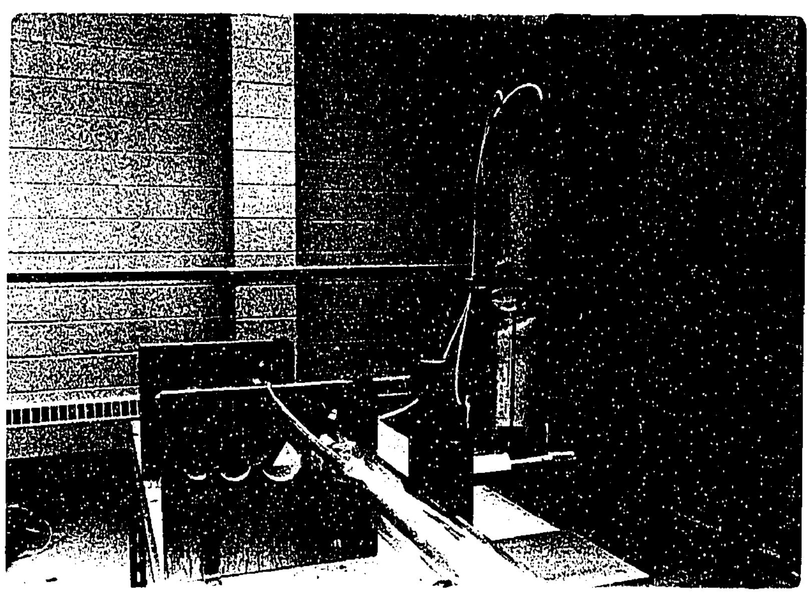

Fig. 19 ANNULI SUPPORT AND MANOMETER SYSTEM

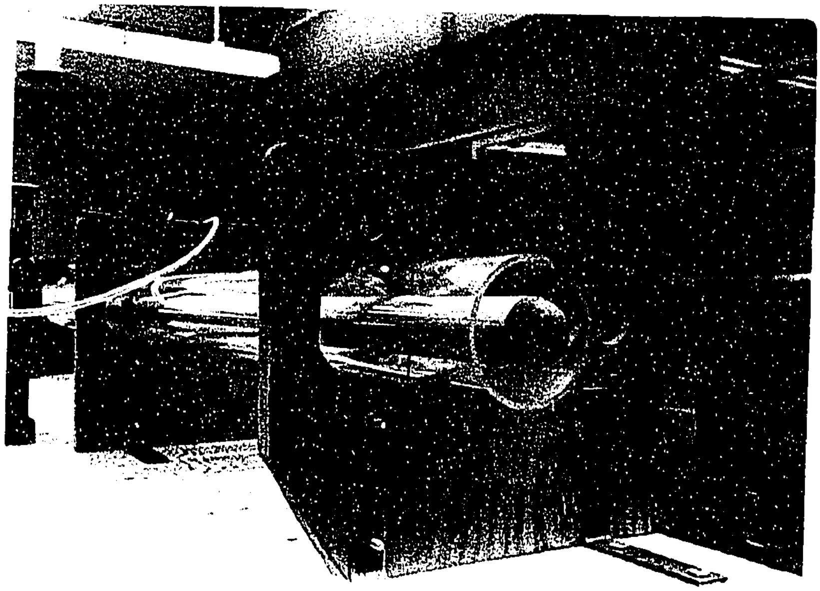

Fig. 20 PIN SUPPORTS

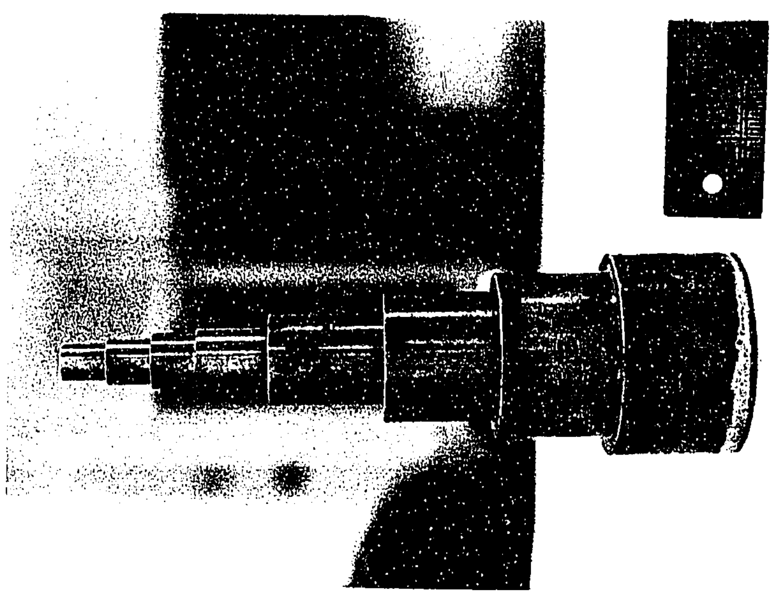

Fig. 21 TRANSITION PIECE

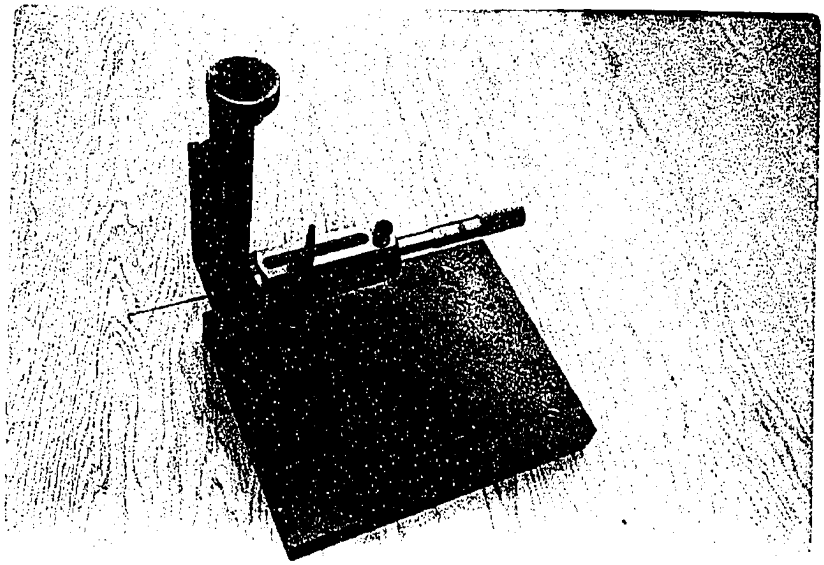

Fig. 22 PITOT TUBE - MICROMETER ASSEMBLY

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