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An experimental study of the decay of anisotropic turbulence.

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AN EXPERIMENTAL STUDY OF

THE DECAY OF

ANISOTROPIC TURBULENCE

A Thesis

Submitted to the Faculty of Graduate Studies Through
the Department of Mechanical Engineering in Partial
Fulfillment of the Requirements for the Degree
of Master of Applied Science at the
University of Windsor

by

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ABSTRACT

Strongly anisotropic and approximately axisym-
metric homogeneous turbulence was obtained in a parall-
el flow section by passing nearly isotropic grid tur-
bulence through a 5 3/4 :1 axisymmetric contraction.
Measurement of the three components of turbulence in the
parallel flow section indicated strong tendency of aniso-
tropic turbulence towards isotropy in the initial period
of decay and the decay rate of the total turbulent energy
was found to follow nearly the same power law as isotropic
turbulent flow. Measurement of one dimensional energy spectra
for the three components was also carried out at two different
stations in the parallel flow section. Decay rate and
energy spectra in the nearly isotropic turbulence obtained
with the grid inserted at the starting of the parallel flow
section were also measured mainly for comparison purposes.

In both of the above studies a perforated steel
sheet grid was used for generating the turbulence but on
comparison with other investigations the characteristics
of turbulence were found to be nearly the same as obtain-
ed by the commonly used biplane grids.

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The wind tunnel used had considerably high free stream turbulence which was undesirable in the study of anisotropic turbulence because of low grid turbulence intensity obtainable after the contraction.

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NOMENCLATURE

Symbol	Description	Units
a	exponent of velocity in hot wire response equation	(dimensionless)
c	contraction ratio of flow section	(dimensionless)
E	instantaneous output voltage of hot wire	(volt)
$\bar{E}$	d.c. voltage across hot wire corresponding to $\bar{U}$	(volt)
$\bar{E}_\ell$	d.c. voltage across hot wire when linearizer is used in circuit	(volt)
$\bar{E}_o$	d.c. voltage across hot wire when it is not exposed to flow	(volt)
E_1, E_2, E_3	one dimensional spectrum functions of turbulence kinetic energy corresponding to u', v' and w' respectively	(L^3/t^2)
e	output fluctuating voltage of hot wire	(volt)
e'	r.m.s. value of fluctuating voltage	(volt)
f	coefficient of spatial longitudenal velocity correlation	(dimensionless)
f'	r.m.s. value of any turbulent velocity component in general, u', v' or $\sqrt{q^2/3}$	(L/t)
g	gravitational acceleration	(L/t^2)

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g_o	propotionality constant in Newton's law	$(M/F \; L/t^2)$
I	electric heating current through hot wire	(ampere)
k_t	coefficient of spatial triple velocity correlation	(dimensionless)
k	constant appearing in empirical effective velocity expression for slanting hot wire	(dimensionless)
k_1	one dimensional wave number in the direction of mean flow	$(1/L)$
Lx	spatial integral scale corresponding to the direction of mean flow	(L)
M	mesh length	(L)
Nu	Nusselt number	(dimensionless)
n	frequency	$(1/t)$
n	exponent in decay law	(dimensionless)
P	static pressure	(F/L^2)
Pr	Prandtl number	(dimensionless)
p	turbulent fluctuation of static pressure	(F/L^2)
q	$\overline{u_i u_i}$ = twice mean energy of turbulence	(L/t^2)
R_w	electric resistance of hot wire at operating conditions	(ohm)
R_g	electric resistance of hot wire at gas temperature	(ohm)

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Re	Reynolds number	(dimensionless)
Re_M	Reynolds number based on mesh length $= \bar{U}M/\nu$	(dimensionless)
Re_λ	turbulence Reynolds number $= u'\lambda/\nu$	(dimensionless)
r	coordinate in radial direction, $r = 0$ corresponds to pipe centre	(L)
r_o	pipe radius	(L)
r_1	distance between two points	(L)
S	sensitivity of hot wire	(volt t/L)
S_s	sensitivity of slanting hot wire	(volt t/L)
S_n	sensitivity of normal hot wire	(volt t/L)
t	time	(t)
t_o	time lapse between actual starting point and virtual origin	(t)
U	instantaneous total velocity	(L/t)
$\bar{U}$	time average of the velocity of flow	(L/t)
U_n	component of velocity of flow normal to the hot wire	(L/t)
U_t	component of velocity of flow tangential to the hot wire	(L/t)
U_{eff}	effective instantaneous velocity for slanting hot wire	(L/t)
U^*	shear stress velocity; $U^{*2} = -\nu \left(\dfrac{du}{dr}\right)_{r=a}$	(L/t)

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u,v,w instanteneous values of velocity

fluctuations in x, y and z directions,

respectively in cartesian coordinates

and in x, r and θ directions, respectively

in cylinderical coordinates   (L/t)

u'v'w' root mean square values of u, v and w

respectively.   (L/t)

w.r.t. refers to 'with respect to'

x,r,θ cylindrical coordinates   (L,L, dimensionless)

x,y,z rectangular cartesian coordinates   (L)

x_0 distance along x-axis between

physical and virtual origin   (L)

τ_0 wall shear stress in pipe flow (F/L^2)

θ azimuthal coordinate   (dimensionless)

λ spatial micro scale of turbulence   (L)

ν fluid kinematic viscosity (L^2/t)

ρ fluid mass density (M/L^3)

σ grid solidity   (dimensionless)

Q angle which the slanting wire

makes with the mean velocity of

flow   (dimensionless)

parallel flow section

refers to the flow section in which

mean flow stream lines are in one

direction.

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Subscripts

     e   indicates condition at the exit of contraction

     p   indicates precontraction condition

     ℓ indicates use of linearizer in hot wire circuit

     M   denotes mesh length

     n   normal direction

     t   tangential direction

     λ denotes micro scale of turbulence

x ,y , y , z , z indicate different positions of hot wire

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1. INTRODUCTION

1.1 Definitions

Turbulent flow is a certain type of irregular flow, in which the velocities vary in a random way with the distance and time at all points in the field. Recently Lin and Reid (22) stated, "It is often desirable to limit the term turbulence only to such random fluctuating motions of a fluid having statistical properties which are definite and experimentally reproducible."

Among conceivable turbulent flows the simplest is the isotropic homogeneous turbulence. The term isotropic means that all statistical functions are invariant to rotation of axes and to reflection with respect to the coordinate planes. The term homogeneous turbulence is used for the random motion where statistical properties of the field are independent of translation of the coordinate system. In brief, in isotropic turbulent flow field $\overline{u^2} = \overline{v^2} = \overline{w^2}$ at any point irrespective of the orientation of the coordinate system; and $\overline{uv} = \overline{vw} = \overline{wu} = 0$.

It has been found that if a uniform stream is passed through a regular array of holes in a rigid sheet or a grid of bars held at right angles to the stream, the turbulence thus generated is nearly isotropic and statistically homo-

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geneous in any plane parallel to the grid.

An isotropic homogeneous turbulence field would be statistically non stationary in time. The reason is the absence of Reynolds stresses necessary for the continuous generation of turbulent energy to compensate for the dissipation of this energy. The rate of turbulent energy decay has been one of the principal objects of theoretical and experimental works on homogeneous isotropic turbulence since the pioneering paper by Taylor (4). Though considerable research has been carried out in this field, it has to be admitted that no sound and experimentally confirmed hypothesis exists which can be used to determine the exact law of decay of isotropic turbulence.

1.2 Present Study

1.2.1 Type of Field: The kinds of turbulent flow which are encountered in nature or in the engineering applications are usually anisotropic and non homogeneous. The extreme complexity of such a field can be well imagined by appreciating the difficulties encountered even in the study of isotropic turbulence which is considered to be the most simple. Moreover the study of such a field would preclude comparison of the results obtained, with the existing experimental and theoretical information. Since there is extensive research information for the case of the ideal situation of isotropic turbulence it was decided to consider a type second in simplicity, that is anisotropic axisymmetric homogeneous field under a steady mean flow. This type of turbulence has sym-

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metry about a given direction. The main flow is assumed to be in this direction and uniform. If U is the mean velocity then $\overline{w^2} = \overline{v^2} \neq \overline{u^2}$ corresponds to this type of flow.

The perforated sheet grid used for generating turbulence was not the common biplane grid used by other workers for the study of grid turbulence. Therefore it was felt necessary to study the nearly isotropic turbulence obtained by perforated sheet grid in order to compare the present results with those obtained by other investigators using regular biplane grids.

1.2.2 <u>Realization of the Field</u>: For obtaining nearly isotropic turbulence a regular periodic grid was inserted at the beginning of the constant area working section of the wind tunnel. For obtaining anisotropic axisymmetric turbulence a variety of ways could be employed. In order to achieve simplicity in measurement and reproducibility, only two methods are considered.

The first method is to take nearly isotropic flow downstream of a periodic grid and pass it through an extremely fine mesh gauze of moderately high resistance. The passage of turbulence through the gauze has the effect of attenuating the longitudenal component of the turbulence more than the transverse components and the turbulence emerges anisotropic and axisymmetric. Experimental results reported by Townsend (26) indicate that an appreciable degree of anisotropy may be achieved only when the attenuation of turbulence intensities by the screen is very high. This means that the anisotropic turbulence obtained with this arrangement would consist of very low turbulence in-

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tensities.

The second method of obtaining anisotropic turbul-
ence is to pass isotropic turbulence through a distorted
passage and then allow it to recover in a parallel flow
section. The turbulence is acted on by the mean velocity
gradient and is made strongly anisotropic. If the distort-
ed passage is an axisymmetric contraction, the emerging
turbulence is also axisymmerric. Calculations made by
Ribner and Tucker and independently by Batchelor (32) show
that

$$\frac{\overline{u^2}_e}{\overline{u^2}_p} \simeq (3\overline{c}^{-2}/4)\ (\log 4c^3 -1) \qquad \text{for } c \gg 1 \qquad (1.1)$$

$$\frac{\overline{v^2}_e}{\overline{v^2}_p} \simeq 3c/4 \qquad \text{for } c \gg 1 \qquad (1.2)$$

where subscripts p and e correspond to enterance and exit,
respectively, of the contraction.

High degree of anisotropy and appreciable intensity
and homogenity of turbulence were the main requirements in
the present study. Keeping this in mind the use of the sec-
ond method was preferable to the first and hence the second
method was used.

1.2.3 Experimental Program: The present experiments were
undertaken to study the decay of anisotropic turbulence in
steady mean flow, to find the rate of decay of turbulent en-
ergy and also any possible tendency of turbulence towards
isotropy. For this purpose an existing large wind tunnel,
which is described later in section 4.1 was used. A per-

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forated grid was located in front of the wind tunnel
contraction which produced anisotropic turbulence in the
test section by the second method discussed in the prec-
eding section. The three fluctuating components of the
anisotropic turbulence were measured at points along the
centre line of the constant area test section. In addit-
ion the energy spectra for u', v' and w' were measured
at two different stations downstream of the grid. For
comparison, the same measurements were also carried out
in the case of nearly isotropic grid turbulence when the
grid was located after the contraction at the beginning
of the constant area test section.

Anisotropic turbulence was studied at two different
mean velocities of 39 ft/sec and 53 ft/sec and isotropic
turbulence was studied at the mean velocities of 27 ft/sec
and 36 ft/sec.

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2. LITERATURE SURVEY

2.1 Isotropic Turbulence

The concept of isotropic turbulence was first given by G.I. Taylor in 1935 (4). Prior to this time, as pointed out by Batchelor (4), theories of turbulence were based on analogies with the kinetic theory of gases which assumes discontinuous collisions between discrete entities. Taylor described measurements which showed that the turbulence generated by passing a uniform stream through a regular array of rods was approximately homogeneous and isotropic. Later measurements were reported by Dryden, Schubauer, Mock and Skramstad (10). Isotropy of the grid turbulence was also confirmed by the experimental study by McPhail (23).

A systematic theoretical approach to the dynamical problems was used by von Karman (4) who introduced, for the purposes of simplification, the assumption of self preservation of the shape of the velocity product functions during decay. In 1938 von Karman and Howarth (19, p.160) arrived at the following differential equation as a result of a theoretical analysis involving double and triple velocity correlations.

$$\frac{\partial}{\partial t}(u'^2 f) - u'^3 \frac{1}{r_1^4} \frac{\partial}{\partial r_1}(r_1^4 \, k_t) = 2 \nu u'^2 \frac{1}{r_1^4} \frac{\partial}{\partial r_1}(r_1^4 \, \frac{\partial f}{\partial r_1})$$

This equation played an important role in later studies of

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the dynamic behaviour of isotropic turbulence.

In 1941 Kolmogoroff (10) suggested the decay law, $\frac{1}{q^2} = D(t-t_o)^{10/7}$, where D is a constant. One of his main assumptions was the constancy of the Loitzianskii integral. The same law was derived by Comte-Bellot and Corrsin (10) using spectra method in 1966.

In 1943 Dryden (13) carried out a survey of the available experimental data on isotropic grid turbulence. He analysed the results obtained by Hall, by von Karman and by himself and his associates. He concluded that $\left(\frac{\overline{U}}{u'}\right)^2$ varies during decay as some power of t lying between 1 and 2.

Batchelor (2), making use of Karman-Howarth equation, derived the so called linear decay law. He assumed partially self preserving solutions for the correlation functions near r = o at decay times which are not large. The results can be presented as

$$u'^2 \sim t^{-1}$$
$$\lambda^2 = 10 \, \nu t$$
$$Re_\lambda = u'\lambda/\nu = \text{const.}$$

Experiments with the grid turbulence were carried out by Batchelor and Townsend (6,7). The results verified the above linear decay law for the values of Re_M at least down to 600, for distances upto about 150 mesh lengths from the grid. They suggested a division of the life history of the turbulence into an initial, a transition and a final period. This is considered with respect to time but for turbulence produced by a grid it applies to consecutive regions downstream of the grid (as $\frac{\partial}{\partial t}$ may be substituted with $U \frac{\partial}{\partial x}$, Taylor's hypothesis (11)). In the initial period, inertial

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effects predominate over viscous effects and the reverse
is the case in the final period. According to Batchelor
and Townsend the phase which may be considered the initial
period of decay was characterised by the energy decay law

$$\overline{u'^2} \sim t$$

or

$$\overline{U}^2/\overline{u'^2} = \alpha \left(\frac{x}{M} - \frac{x_0}{M}\right) \quad \text{where } \alpha \text{ and } x_0 \text{ are constants.}$$

By using different types of grids they attempted to relate
the characteristics of turbulence to those of the grids. The
grid characteristics comprise the geometry of the grid and
its specific drag.

Frenkiel (14) made use of Karman Howarth equation to
give a relation different from the linear decay law. He
suggested the following equation,

$$\frac{\overline{u'^2}}{\overline{u_0'^2}} = \left[\frac{7\nu}{\lambda_0^2} (t-t_0) + 1\right]^{-10/7}$$

or approximately

$$\frac{\overline{u'^2}}{\overline{u_0'^2}} = \left(\frac{x}{x_0}\right)^{-10/7}$$

On comparison with the experimental results reported by
previous investigators, he found good agreement for the decay
of longitudenal turbulent energy. Later experimental values
obtained by Baines and Peterson (1) agreed well with Frenkiel's
equation rather than with the linear decay law.

Lin (21) put forward the decay law making assumption
different from Batchelor which he claimed to be in accordance
with the Kolmogoroff's concept of turbulence at high Reynolds
numbers. The assumption made was that the actual deviation
from the similarity of the spectrum will be limited only to
small values of wave numbers and hence the effect of deviat-
ion will enter only in the calculation of energy but be neg-

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ligible in the computation of the rate of energy dissipation. He suggested the following relations,

$$\dot{u'}^2 \sim t^{-1}$$

$$\lambda^2 = 10\nu t \left(1 \div \frac{\beta}{\alpha} t\right) \qquad \begin{array}{l}\alpha \text{ and } \beta \text{ are} \\ \text{constants}\end{array}$$

$$R_\lambda = \frac{u'\lambda}{\nu} = \text{not constant}$$

Goldstein (15) also derived the same law. He commented that the law does not follow from Kolmogoroff's original hypothesis. He stated that the eddies corresponding to the lower wave numbers are permanent; these eddies are not changed either by dissipation or energy transfer. But he justified the law on the assumption that the permanent eddies contain only an infinitesimal fraction of the energy and hence the eddies corresponding to all but the lowest wave numbers practically contain all the energy and are responsible for the whole of the dissipation.

Tsuji and Hamma (30) measured decay of turbulence behind a single grid and behind two grids. Lin's decay law was confirmed unless, in the two grid case, the large scale turbulence produced by the first grid was overwhelmingly predominant.

Later Tsuji (29) criticized Batchelor's linear decay law as well as Lin's decay law and found the points of self contradiction, inherent in the two laws. Tsuji derived the energy decay equation for the initial period from Karman Howarth equation,

$$V \frac{d^2 V}{dX^2} + C \left(\frac{dV}{dX}\right)^2 - E \left(\frac{dV}{dX}\right)^{3/2} = 0$$

and suggested the solution of this equation by numerical integration. In the above equation $V = \bar{U}^2/u'^2$, $X = x/M$,

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$C = (7/15)G - 2$, $E = (7/3)S \sqrt{Rm/10}$, $S = \left[\overline{(\partial^u/\partial x)^3} / \overline{(\partial^u/\partial x)^2}\right]^{3/2}$

and $G = \overline{u^2 \ \overline{(\partial^2 u/\partial x^2)}} / \left[\overline{(\partial u/\partial x)^2}\right]^2$ Tsuji claims that the

assumptions made by him were based on the experimental

findings of previous investigations.

The different assumptions made in the three

decay laws can be summarized as under

	S	G	Re	S-(2G/Re)
Linear Decay Law	Const.	Const.	Const.	Const.
Lin's Decay Law	-	-	Not Const.	Const.
Tsuji's Decay Law	Const.	Const.	Not Const.	Not Const.

Deissler (12) considered correlation between fluctuat-

quantities at two, three and four points respectively. The

set of equation, so obtained was made determinate by neglect-

ing highest order (quintuple) correlations. The colution re-

presented the turbulence for times between the initial and

final periods. The results reproduced most of the trends

observed experimentally in previous investigations. Deissler

suggest that the analysis of turbulence in the initial per-

iod can be made possible by utilizing correlation equations

involving a sufficient number of points in the fluid. Each

times the set of equation would be made determinate by neg-

lecting the highest order correlation involved.

Webb (37) carried out the study of the decay of tur-

bulent energy downstream of a grid in air, argon and helium.

He found that above a grid Reynolds number of about 2,000

and below a turbulence Knudsen number (Ratio of molecular

mean free path to the Kolmogorroff microscale) of 0.01 an

identical initial period decay law occurs independent of the

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gas employed or the ambient pressure level. The rate of decay was found to be approximately linear with x/M.

Uberoi (34) reported the measurements of $\overline{u^2}$ and $\overline{v^2}$ in the turbulence behind a round rod square mesh grid. He found that the ratio u'/v' had a nearly constant value of 1.2 indicating that grid generated turbulence is not strictly isotropic and there is no strong tendency for it to become so further downstream. He suggested the following equation for decay

$$q^2 \sim (x-x_0)^{-1.2}$$
$$\text{or} \quad q^2 \sim t^{-1.2}$$

Uberoi and Wallis (35) attempted to obtain isotropic turbulence by passing grid turbulence through small axisymmetric contraction. It was found that while passing through contraction v' and u' tend to equalize but a clear tendency of strained isotropic turbulence to return to the precontraction anisotropic condition was observed. The same attempt was made by Comte-Bellot and Corrsin (10) and they found that the isotropy persists downstream of the small contraction. For the isotropic turbulence so obtained, they found that both u' and v' satisfied the power law of the form of equation (2.5) with an index of 1.28. For the turbulence behind square rod and round rod grids they found u'/v' $\simeq$ 1.05 to 1.15 but a slow tendency to isotropy was observed. For the decay law indices varied from 1.16 to 1.33 with different grid geometries and mean velocities.

Very recently Uberoi and Wallis (36) reported the study of the effect of grid geometry on turbulence decay.

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The ratio u'/v' was found to be essentially constant during decay with a value between 1.1 and 1.16 with different grids. The turbulence intensity was found to decay according to $\overline{u'^2} \sim t^{-n}$ with n between 1.22 and 1.48 depending on the grid geometry.

2.2 Anisotropic Turbulence:

In 1944 MacPhail (24) obtained strong anisotropic turbulence by passing nearly isotropic grid turbulence through a distortion. He allowed the resulting anisotropic turbulence to flow through a constant area section of the wind tunnel. He observed the recovery of isotropy taking place in an oscillatory fashion.

Townsend (26) obtained anisotropic axisymmetric turbulence by passing grid turbulence through a fine mesh gauze. He measured $\overline{u^2}$ and $\overline{v^2}$ for few mesh lengths downstream of the mesh gauze. He observed that after a short period of comparatively rapid interchange of energy between the velocity components, the ratio of intensities become very nearly constant, and the once powerful tendency towards isotropy becomes a weak drift barely detectable. He expressed the view that while the turbulence is in the zone of influence of the gauze, its configuration is such that interchange of energy between velocity components is very rapid. According to this, the initial rapid readjustment occurs only in the zone of influence of the gauze, and the subsequent slow approach to isotropy of the energy containing eddies is typical of free anisotropic turbulence. Townsend (27)

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also obtained anisotropic turbulence by passing grid turbulence through a distorted passage. Again he observed that the tendency to isotropy was very weak. From measurement of the velocity derivatives he infered that the dissipating structure was isotropic in both of the above cases.

From Kolmogoroff's hypothesis Uberoi (33) infered that if we deform a fluid in isotropic turbulent motion then it should become rapidly isotropic once the deformation has ceased. Kolmogorroff's hypothesis (33) may be briefly stated as follows. At sufficiently high Reynolds number the turbulence is made up of a hierarchy of eddies. It is assumed that there is a transfer of energy from larger to smaller eddies in the same way as the transfer of energy from the mean to the turbulent motion in a shear flow. The large scale eddies dissipate little energy by viscosity and pass on most of it to eddies of smaller scale, and so on to the smallest scale eddies which are responsible for most of the viscous dissipation. Uberoi produced anisotropic axisymmetric turbulence by passing grid turbulence through an axisymmetric contraction. Measurements in the uniform section after the contraction indicated a strong tendency toward isotropy. It was found that the larger components were losing more energy due to viscosity than by transfer to the smaller component. Approximately one third of the turbulent energy was left by the time $\overline{u^2}$ and $\overline{v^2}$ became nearly equal. It was also shown that the turbulence was becoming locally isotropic at a faster rate than the equippartition of energy was taking place.

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In the course of study of large eddies of turbulent motion, Grant's (16) observations indicated very slow decrease of the anisotropy after distortion.

Tucker and Reynolds (31) observed a marked tendency towards isotropy for the turbulence in the parallel flow following the distortion similar to that used by Townsend. They concluded that the return to isotropy was rapid for highly anisotropic turbulence and slower as the turbulence approaches the isotropic condition. They presented this as one of the reasons of the discrepency between their results and those of Townsend and of Grant.

With the exception of Uberoi's study, the above mentioned works were not carried out with prime interest in the study of the decay of anisotropic turbulence and the inferences were based on the observations for very small distances allowed for the decay in the parallel section.

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3. THEORY

The current literature on the theoretical treatment of anisotropic turbulence does not give very clear picture about the behaviour of homogeneous anisotropic turbulence during decay. A rough idea concerning the tendency of anisotropic turbulence towards isotropy can be formed by considering the energy equation. The particular type of field considered here is an incompressible, anisotropic turbulent field under a steady unidirectional mean flow with symmetry about the direction of mean flow and homogeneity in planes normal to mean flow direction. Tensor notation is used in this chapter and the coordinate axes x,y,z are refered to here as x_1, x_2, x_3 respectively, the mean velocities along the three axes are denoted by $\overline{U_1}$, $\overline{U_2}$, $\overline{U_3}$ and the fluctuating velocities u, v, w by u_1, u_2, u_3 respectively.

For incompressible flow with steady main motion and with statistical average values in steady state, the following turbulent energy equation can be obtained from the Navier Stokes equation.

$$\overline{U_k} \frac{\partial}{\partial x_k} \overline{u_i u_i} = -2 \overline{u_k u_i \frac{\partial U_i}{\partial x_k}} - \frac{\partial}{\partial x_k} \overline{u_i u_i u_k} - \frac{1}{\rho} \overline{u_i \frac{\partial p}{\partial x_i}} + \nu \frac{\partial^2}{\partial x_1 \partial x_1} \overline{u_i u_i}$$

$$-2\nu \overline{\frac{\partial u_i}{\partial x_1} \frac{\partial u_i}{\partial x_1}} \tag{3.1}$$

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The derivation of the above equation is shown in appendix D. From the above equation we can write two different equations for u_1 and u_2

$$\overline{U}_k \frac{\partial}{\partial x_k} \overline{u_i^2} = -2\overline{u_k u_1} \frac{\partial \overline{U_1}}{\partial x_k} - \frac{\partial}{\partial x_k} \overline{u_1^2 u_k} - \frac{1}{\rho} \overline{u_1 \frac{\partial p}{\partial x_1}} + \nu \frac{\partial^2}{\partial x_1 \partial x_1} \overline{u_1^2}$$

and

$$-2\nu \overline{\frac{\partial u_1}{\partial x_1} \frac{\partial u_1}{\partial x_1}}$$

$$\overline{U}_k \frac{\partial}{\partial x_k} \overline{u_2^2} = -2\overline{u_k u_2} \frac{\partial \overline{U_2}}{\partial x_k} - \frac{\partial}{\partial x_k} \overline{u_2^2 u_k} - \frac{1}{\rho} \overline{u_2 \frac{\partial p}{\partial x_2}}$$

$$+ \nu \frac{\partial^2}{\partial x_l \partial x_l} \overline{u_2^2} - 2\nu \overline{\frac{\partial u_2}{\partial x_l} \frac{\partial u_2}{\partial x_l}}$$

For the special case with mean flow in x_1 direction and homogeneity in x_2 and x_3 directions, the equations reduce to

$$\overline{U}_1 \frac{\partial}{\partial x_1} \overline{u_1^2} = -\frac{1}{\rho} \overline{u_1 \frac{\partial p}{\partial x_1}} - 2\nu \overline{\frac{\partial u_1}{\partial x_l} \frac{\partial u_1}{\partial x_l}} + \nu \frac{\partial^2}{\partial x_1^2} \overline{u_1^2} - \frac{\partial}{\partial x_1} \overline{u_1^3}$$

$$\qquad\quad \text{I} \qquad\qquad \text{II} \qquad\qquad \text{III} \qquad\quad \text{IV} \qquad\quad \text{V}$$

$$\overline{U}_1 \frac{\partial}{\partial x_1} \overline{u_2^2} = -\frac{1}{\rho} \overline{u_2 \frac{\partial p}{\partial x_2}} - 2\nu \overline{\frac{\partial u_2}{\partial x_l} \frac{\partial u_2}{\partial x_l}} + \nu \frac{\partial^2}{\partial x_1^2} \overline{u_2^2} - \frac{\partial}{\partial x_1} \overline{u_2^2 u_1}$$

The various terms in the above two equations can be interpreted as follows:

I represents change in turbulent energy component in the direction of main flow.

II are the correlation between the velocity fluctuation and derivative of the pressure fluctuation.

III is the viscous dissipation.

IV is the transport of energy due to viscous shear stresses.

V represents turbulent transport of component energies.

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Comte-Bellot and Corrsin (10) in dealing with the
energy equations have presented various points about the
tendency of homogeneous turbulence towards isotropy or
anisotropy. Because of their importance in the present
experimental study, some of the points are reproduced here
with reference to the above two equations.

The static pressure terms exchange energy among
velocity directional components. It is generally believed,
though possibly not theoretically deducible from the equat-
ions of motion, that they tend to transfer energy from more
energetic to less energetic components. The viscous dis-
sipation destroys component energy at a rate proportional
to that energy, hence an approach toward isotropy may be
present. The transport due to viscosity will be negligible
compared with the turbulent transport. For isotropic tur-
bulence $\overline{u_1^3}$ and $\overline{u_1 u_2^2}$ are both identically zero. However,
the growth of $\dfrac{\overline{u_1^3}}{(\overline{u_1^2})^{3/2}}$ has been observed in contraction stud-
ies. In a 4:1 contraction this grew to 0.08, and it re-
turned rapidly to zero in the straight section following
the contraction. The authors (Comte-Bellot and Corrsin)
carried out an approximate theoretical analysis for $\overline{u_1^3}$
due to strain, and on similar basis an approximate express-
ion was found for $\overline{u_1 u_2^2}$. Then to estimate the effect of
only the turbulence transport of component energies on the
tendency towards isotropy, the approximate expression was
obtained

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$$\frac{\overline{u_{1s}^2} - \overline{u_{2s}^2}}{\overline{u_{1e}^2}} \simeq \frac{u'_{1e}}{U_{1e}} \left\{ \frac{\overline{u_{1e}^3}}{(\overline{u_{1e}^2})^{3/2}} - \frac{\overline{u_{1e}u_{2e}^2}}{u'_{1ee}\overline{u_{2e}^2}} \right\}$$

$$-- (3.7)$$

where subscript e refers to the position just after the contraction and s refers to the position where $\overline{u_1^3}$ and $\overline{u_1 u_2^2}$ become sensibly zero.

In the above expression

$$\frac{\overline{u_1^3}}{(\overline{u_1^2})^{3/2}} \simeq 3\,\frac{u'_{1p}}{U_{1p}}\left(1 - \frac{\overline{U_{1p}^2}}{\overline{U^2(x)}}\right)$$

and

$$\frac{\overline{u_1 u_2^2}}{u'_1\,\overline{u_2^2}} \simeq \frac{-1}{6}\,\frac{\overline{u_1^3}}{(\overline{u_1^2})^{3/2}}$$

Substituting these values in equation (3.7) and making use of equation (1.1) for $\overline{u_{1e}^2}/\overline{u_{1p}^2}$, we obtain

$$\frac{\overline{u_{1s}^2} - \overline{u_{2s}^2}}{\overline{u_{1e}^2}} \simeq \frac{10}{3}\left(\frac{c-1}{c}\right)\frac{c^2}{\log 4c^3 - 1}\,\frac{\overline{u_{1e}^2}}{U_{1e}^2}$$

(3.8)

We may conclude the above discussion with the following remarks:

The velocity-pressure-gradient correlations tend to decrease the anisotropy.

The viscous dissipation of component energies is higher with the higher intensity components, this also tends to equalize the turbulent components.

A tendency towards energy inequality may be present due to turbulent transport of component energies (equation 3.8). But this effect is quite negligible in comparison to the other effects.

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4. APPARATUS AND INSTRUMENTATION

4.1 Wind Tunnel:

A closed circuit low speed wind tunnel of Mechanical Engineering Department of the University of Windsor was used for the experiments. A short description of the tunnel is included here, detailed description and calibration are given in appendix A.

The dimensions of the test section were $2\frac{1}{2}$ ft. square by 20 ft. long. Over the speed range used in the experiments, the free stream turbulence levels (without the turbulence generating grid) were of considerable magnitude. On the centreline at the upstream end of the tunnel $u'/\bar{U}$ was found to lie between 0.20% and 0.25% and $v'/\bar{U}$ (=$w'/\bar{U}$) between 0.30% and 0.60%. At the downstream end these values were found to lie between 0.23% and 0.30% and between 0.30 and 0.65% respectively.

The coordinate system with respect to the test section is shown in figure 1.

4.2 Grid:

A perforated steel sheet was used as a grid for turbulence generation. The sheet was 0.03 in thick having 11/16 in. (±.0045) square mesh with bars 3/16 in. (±.0025) wide.

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The solidity, which is defined as projected solid area per unit total area, was 0.47. This value lies in the range which is recommended for obtaining approximately homogeneous turbulence in planes parallel to the grid (11). This type of grid has not been used in studying the decay of grid turbulence by previous investigators. Tucker and Reynolds (31) have reported its use for the study of deformation effects on grid turbulence. Cheapness, light weight, and dimensional homogeneity were some of the advantages of this grid. The turbulence level obtained by this grid was nearly equal to that obtained by previous workers using square rod grids of the same solidity (11). For studying decay of isotropic turbulence the grid was inserted at the entrance of the test section. For studying decay of anisotropic axisymmetric turbulence the grid was inserted in the wind tunnel so that it was followed by a 5 3/4 :1 axisymmetric contraction leading to the test section.

Grid was made in two sizes, one to fit upstream of the contraction for anisotropic turbulence and the other to fit at the starting of the parallel section for isotropic turbulence. Because of large crossectional area upstream of the contraction a single manufactured grid was not available and three pieces were spot-welded together as shown in figure 2. With this, however, axisymmetry of the turbulence was slightly affected and it necessitated recording the fluctuating components of velocity u', v' and w'.

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4.3 Instrumentation:

All turbulence measurements were made with constant temperature hot wire anemometers. The following instruments were used.

DISA 55 A 01 constant temperature hot wire anemometers (two units).

DISA 55 A 06 random signal indicator and correlator.

Bruel Kjaer audio frequency spectrometer type 2112.

Bruel Kjaer random noise voltmeter type 2417.

The signal transducers were the following DISA manufactured constant temperature hot wire probes.

Type 55 A 25 miniature straight hot wire probe with wire of 5μ diameter and 1 mm length mounted normal to the probe axis.

Type 55 A 29 miniature probe with slanting wire of 5μ diameter and 1 mm length forming an angle of $45°$ with the probe axis.

Type 55 A 32 hot wire X-probe with the wire planes parallel to themselves and to the probe axis and both wires forming an angle of $45°$ with the probe axis, the diameter and length of each wire being 5μ and 1 mm respectively.

All probes were made of platinum plated tungsten wire. The magnified views of the probes are shown in fig. 37b.

The two miniature probes with 55 A 21 miniature probe support were used to measure turbulence intensities u', v', and w'. For this purpose each of the probes was calibrated in the same wind tunnel using a pitot static probe

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with a Wilhelm Lambrecht slanting tube micromanometer No. 655. The calibration of hot wire is described in detail in appendix B. The calibration of other instruments is reported in appendix C. For measuring turbulence energy spectra miniature straight probe was used for u' and X-probe with 55 A 30 probe support for v' and w'. Figure 35 gives the block diagrams showing the arrangements of instruments for the different measurements.

4.4   Probing Technique:

Provision was made in the test section of the tunnel for inserting ¼" diameter steel rods in vertical position at intervals of 18" along the test section length. Each inserted rod was provided with a brass block drilled to slide on the vertical rod and having a slot to support a square bar horizontally. Each block carried two screws one to fix its own position on the vertical rod and the other to hold the supported horizontal bar. Three rods could be inserted at a time for supporting a ½" square x 4 ft. long steel bar at any desired position in the central vertical plane in the test section and parallel to the direction of mean flow. (Similar arrangements were made to support the square bar horizontally in vertical planes at 6" on either side of the central vertical plane. This arrangement was made so that transverse homogeneity of turbulence could be checked in planes normal to the direction of flow at two different stations in the

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test section.) The square bar was bored and slotted at
one end so that the miniature probe support could be fixed
in line with the bar axis, while the 55 A 30 probe support
required allignment. The long bar could be given any num-
ber of 90° rotations because of its square section. The
round ends were properly marked to enable the square bar
to be supported along the centre line of the test section.

With the three vertical rods fixed at their positions,
the probe could be traversed through any distance with-
in 18" along the test section length. After every 18"
traverse, out of the three vertical rods the upstream one
could be removed and other vertical rod could be introd-
uced further downstream. In this way three vertical rods
could again support the horizontal bar for further 18"
traverse of the probe. Figure 36 gives the schematic view
of the arrangement.

The ceiling of the test section was not a plane sur-
face, the ramaining wall surfaces were approximately plane.
It was estimated that the axis of the test section could be
located within ± 1/8" horizontally and ± ¼"vertically.

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5. EXPERIMENTS

5.1 Types of Field:

The present investigation was carried out to study the decay of an anisotropic axisymmetric homogeneous field of turbulence in an incompressible steady mean flow. As mentioned previously a perforated sheet grid was used for generating the turbulence. Previous investigators have used biplane square mesh grids with round or square rods and not the perforated sheet grid used here. For the regular type of biplane grids extensive experimental research information is available for the decay of the turbulence in a constant area section behind the grid. Similar information for the perforated grid was not available. Therefore it was felt necessary to conduct experiments to obtain this information so that a comparison could be made with the turbulence produced by the more commonly used biplane grids. By this study a comparison could also be made between the nearly isotropic turbulence in a parallel section downstream of the grid and the strongly anisotropic turbulence produced by using a contraction between the grid and the paralled section. We will refer to the nearly isotropic grid turbulence and the strongly anisotropic turbulence by the terms isotropic turbulence and anisotropic turbulence respectively.

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5.2 Properties and their measurement:

The distance downstream of the grid taken along the axis of the test section was considered to be the independent variable. The varying quantities measured along this distance were the three fluctuating components of velocity u', v' and w' when the mean velocity of flow was maintained constant. In the isotropic case the axisymmetry of the turbulence was confirmed within the accuracy of measurement, therefore only u' and v' were recorded. In case of anisotropic turbulence v' and w' were different, therefore all the three components were measured.

In addition to the turbulence intensities, energy spectra were obtained at two different stations at the axis of the test section downstream of the grid. Energy spectra for u',v' and w' were measured in both the cases of isotropic and anisotropic turbulence.

The experiments were carried out at the mean speeds of 39 ft/sec and 54 ft/sec for the anisotropic turbulence and 27 ft/sec and 36 ft/sec for the isotropic turbulence.

5.3 Homogeneity Check:

Homogeneity of turbulence was checked in planes normal to the working section axis. For this purpose the turbulence intensities were measured at different points covering an area of 1 foot square around the axis. (see figs. 19-21) Check of homogeneity was also carried out for free stream surbulence (see fig. 34)

5.4 Effects of acoustical and free stream turbulence
and other inaccuracies:

From the sprectrum measurements it was infered that
the waves at the frequencies above 10 kc/s were mainly due
to instrument noise. Filters provided in the anemometer
were used to eliminate frequencies above 10 kc/sec while
recording the r.m.s. voltages of the turbulence intensities.
The filters were not used for some of the spectrum read-
ings.

For acoustical and free stream turbulence 0.1% is
considered as the upper limit which can be allowed in ex-
perimental studies similar to the present one. In the wind
tunnel used for the present work, as mentioned in the prev-
ious chapter, the turbulence was found to have very high
values. The effect of this was more pronounced in the ani-
sotropic field because of very low intensities of turbulence
in the test section. To correct for free stream turbulence
and instrument noise at any point the mean square voltages
in the empty wind tunnel were subtracted from the corres-
ponding voltages at the same operating conditions and mean
velocity of flow with the grid in position. This method
of correction was used in the decay study of both isotrop-
ic as well as the anisotropic turbulence. A similar meth-
od was reported by Uberoi and Wallis (36). In the measure-
ment of energy spectra also similar type of subtraction was
made by Uberoi (32). In the measurement of the energy

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spectra, in the present study the same type of correct-
ion was applied for isotropic turbulence. During the mea-
surement of spectra in anisotropic turbulence, at some of
the frequencies it was noticed that the r.m.s. voltage was
higher in the free stream turbulence than in the anisot-
ropic turbulence. The exact effect of free stream tur-
bulence spectra upon the grid turbulence spectra could
not be pointed out. Therefore the energy spectra obtain-
ed for anisotropic turbulence were not corrected for free
stream turbulence. It may be mentioned here that the
spectrum readings were taken beyond 20 c/sec and large part
of free stream turbulence was found to consist of the
eddies in frequency range below 20 c/sec. (see Appendix A)

In each run, after starting the motor and fan, 15
to 20 minutes were allowed, before beginning the measure-
ment of the turbulence quantities in order to insure low
increase in temperature during these measurements. A diff-
erence of $2°$ to $4°$ F could be observed during the run, the
effect of which on the anemometer readings could be neg-
lected (11).

All of the electronic instruments were provided with
the r.m.s. voltmeters and on comparison with each other
some of them showed small differences (see appendix C).
All of the r.m.s. readings were recorded from the random
noise voltmeter to have minimum possible noise effect and
to avoid discrepencies.

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6. RESULTS AND DISCUSSION

Present experimental work was carried out mainly to study the decay of anisotropic turbulence in steady mean flow. As explained in the previous chapter nearly isotropic turbulence obtained with the same type of grid was also studied mainly for comparison. The decay results will be described under the seperate headings of isotropic and anisotropic turbulence after mentioning the computation procedure and the homogeneity of the turbulence.

6.1 Computation:

The three fluctuating components of velocity were calculated from the hot wire data using the following equations

$$u'^2 = e'^2_x / Sn^2$$

$$v'^2 = \frac{1}{C^2}\left\{\left[(e'^2_{y_1} + e'^2_{y_2})/2S_s^2\right] - u'^2\right\}$$

$$w' = \frac{1}{C^2}\left\{\left[(e'^2_{z_1} + e'^2_{z_2})/2S_s^2\right] - u'^2\right\}$$

where $S = a\ Y^{1/a}/2\overline{E}\ (\overline{E}^2 - X)^{1-a/a}$

values of a, Y, X and $\overline{E}$ are taken for normal and for slanting wires respectively to find S_n and S_s .

and $C = \sin Q \cos Q\ (1-k^2)/(\sin^2 Q + k^2 \cos^2 Q)$

The subscripts x, y_1, y_2 , z_1 and z_2 correspond to the diff-

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erent positions of the probe which are shown in figure
37. The constants a, X and Y were determined by calib-
ration of the hot wire probes as outlined in appendix B2.
The value of the factor k, which represents the direction
sensitivity of the wire, is discussed in appendix B5. The
procedure for computing energy spectra is given in appendix
B6. IBM 1620 II digital computer was used for major arith-
matical work.

6.2 Homogeneity of Turbulence:

Homogeneity is one of the most important factors
which affect the experimental findings in this type of
turbulence. As stated by Grant and Nisbet (17), inhomo-
geneity is believed to be the cause of the discrepancies
between various sets of measurement of energy decay in the
initial period.

Of necessity decaying grid turbulence is inhomogeneous
in the direction of the mean flow. According to Corrsin
(11) it may be considered effectively homogeneous if the
following conditions are met:

$$dL_x/dx \ll 1 \qquad (a)$$

$$(Lx/\lambda)(d\lambda/dx) \ll 1 \qquad (b)$$

$$-(Lx/\overline{u^2})(d\overline{u^2}/dx) \ll 1 \qquad (c)$$

Approximate values of Lx could be found from the measured
energy spectra as described in a later section in this
chapter. To find its derivative Lx was assumed to vary

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linearly with $\sqrt{x}$ (19). Conditions (a) and (c) were approx-
imately checked in case of the isotropic turbulence and the
values of the two quantities were found to be below 0.04.
Similar check was not made for anisotropic turbulence but
effective homogeneity could be assumed because of the same
order of turbulence energy decay as for isotropic turbulence.

To check the homogeneity in planes normal to the
direction of flow the turbulence intensities were recorded
at different points covering an area of one foot square
around the axis of the test section. Figures 19, 20 and
21 show the location of the points and values of the fluct-
uating quantities in relation to the values at the centre.
Table 6.1 gives the extent of homogeneity obtained in the
present experiments and a comparison with other results
in a nearly isotropic turbulent field. Similar measure-
ments for anisotropic turbulence obtained in the present
study are also included at the end of the table. In order
to facilitate comparison, the table is based on the mean
square values of fluctuating quantities, though the figures
17, 18 and 19 are based on root mean square values. In-
homogeneity in present experiments appears quite large but
more clear picture is obtained when consideration is given
to the area covered, the value of x/M and the grid solidity.

6.3   Isotropic Turbulence

6.3.1 Decay Rate:

The turbulence level in a homogeneous turbulent field

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is considered to be a function of time. The usual proced-
ure for analytical treatment of experimental decay data is
to fit power law of the form

$$f'^2/\overline{U}^2 \sim t^{-n}$$

where the time is measured from the virtual origin. Accord-
ing to Taylor's hypothesis (11) $\frac{\partial}{\partial t}$ may be substituted with
$\overline{U}\frac{\partial}{\partial x}$. Making use of this, the following relation for the
decay of grid turbulence may be written in terms of distance
downstream of the grid.

$$\frac{\overline{U}^2}{f'^2} = A\left(\frac{x}{M} - \frac{x_0}{M}\right)^n \qquad (6.1)$$

We try to find the best values of A, n, and x_0/M to fit
the experimental data. In the present case different values
of n were assumed and values of A and x_0/M were determined
by using the least square regression method. The set of
values giving best correlation was selected. For the diff-
erent components of turbulence the values of x_0/M were
slightly different. In order to avoid complications a
common value of $(x_0/M)=5$ was considered suitable for all
sets of data. After fixing the value of $(x_0/M)=5$, each
set of readings was again alalysed to find values of n
and A by least square regression method. The date could
be presented to fit straight line on logarithmic plot.
Figures 1, 2 and 3 show such plots. The results are sum-
marized in table 6.2. These results are quite in agree-
ment with those obtained by several earlier investigators.
Some of the previous results, most of which have been sum-

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marized by Comte-Bellot and Corrsin (10) are presented in table 6.4.

6.3.2 Isotropy:

It is remarkable to note that u'/v' was found to lie between 1.1 and 0.9. This means that the flow was more near to isotropy than reported in most of the earlier investigations. The reasons of this may be the following

1. The grids used in previous investigations were biplane round rod or square rod periodic grids and not the perforated sheet grid which has been used in the present study.

2. The direction sensitivity of the hot wire probe affects the computation of transverse components of the fluctuating velocity. As pointed out by Champagne (8) there appears considerable disagreement as to the directional sensitivity of a hot wire and, therefore, to the accuracy of measurements made with wire oblique to the flow. This has been discussed in detail in appendix B5. Some of the previous investigators have used X-wire probe for determining transverse components of fluctuating velocity.

It should be pointed out that MacPhail (24) found $\overline{u^2}/\overline{v^2}$ almost equal to unity for grid turbulence.

Considering the present results to be accurate the use of perforated sheet grid may be encouraged for an experimental study of isotropic turbulence.

6.3.3 Energy Spectra:

One dimensional u'^2, v'^2 and w'^2 spectra are shown in figures 11 to 14. Measured spectra are compared with the theoretical curves. The following equations for the isotropic turbulence (19, p. 174) were used to draw theoretical

curves
$$E_1(k_1,t) = \frac{2}{\pi} u'^2 \frac{Lx}{1+k_1^2 L_x^2} \qquad (6.8)$$

$$E_2(k_1,t) = E_3(k_1,t) = \frac{1}{\pi} u'^2 L_x \frac{1+3k_1^2 \cdot L_x^2}{(1+k_1^2 L_x^2)^2} \qquad (6.9)$$

As in the present case the turbulence was approximately isotropic, v' and w' were used in place of u' in the above equations for E_2 and E_3 respectively. To determine the theoretical curve the integral scale was not measured directly. From the measured values of E_1 at different values of k_1, Lx was calculated at each point using equation (6.8) and then arithmetic average was taken over the whole spectrum. That average value of Lx was used in equations (6.8) and (6.9) to plot the theoretical curves for longitudenal and lateral components of fluctuating velocity respectively. At the mean velocity of 36 ft/sec and at x/M = 262 the measured spectrum was found to be considerably affected probably by the noise of fan or by unnoticed grounding of the electronic circuit. Therefore measured values of E_2 and E_3 were considered to calculate average Lx from equation (6.9) by trial and error. It can be seen that good agreement between the measured and the theoretical spectra is obtained except at high value numbers. At higher wave

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numbers we see that the measured values lie below the
theoretical line. The same tendency was observed in case
of the spectra measured by A. Favre (19, p. 61), by Simmons
and Salter (13) and by Dryden (13). It is worthwhile to
point out that theoretical equations are based on the ass-
umption that the coefficient of longitudenal correlation
can be approximated by an exponential function.

6.4   Anisotropic Turbulence

6.4.1 Decay Rate:

An attempt was made to find a suitable power law of
the form of equation (6.1) to fit the experimental data in
anisotropic turbulence also. High scatter was observed in
case of the individual components of the fluctuating vel-
ocity but good correlation could be obtained for the total
trubulence kinetic energy. The results are summarized in
table 6.3 and the plots are shown in figures 6, 7 and 8.
It should be noticed that the exponent n for the q^2 decay
is nearly the same as for isotropic turbulence.

6.4.2 Isotropy:

Though the correlation coefficients for the decay
data were quite low for the three individual components of
fluctuating velocity, it can be concluded from the best
fitting curves, shown in figures 6 & 7, that a clear
tendency of anisotropic turbulence towards isotropy is
present. The same tendency can be seen in the plots of

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the ratios (u'/v') and (u'/w') against x/M, in figure 10.
This conclusion is supported by the results of Uberoi (33)
and of Tucker and Reynolds (31). But the observations of
Townsend (28) and those of Grant are on the contrary.

It can be seen that the ratios u'/v' and u'/w',
after being equal to unity after initial period of decay
start increasing further downstream. This is in accordance
with the findings of Batchelor and Stewart (5). They gave
the view that close behind the grid, the system of rods
parallel to y axis produces large contributions to $\overline{u^2}$ and
$\overline{w^2}$ while the system parallel to the z axis produces large
contributions to $\overline{u^2}$ and $\overline{v^2}$, the net result being that the
contribution to $\overline{u^2}$ from the largest eddies is permanent-
ly greater than the contributions to $\overline{v^2}$ and $\overline{w^2}$. These
large eddies, according to Batchelor (3), which contain
the negligible portion of the total energy in the beginning,
are permanent and become dominate in the final period when
the smaller eddies have decayed.

6.4.3 Energy Spectra:

Measured one dimensional energy spectra for u', v'
and w' are shown in figures 15 to 18.When u' spectra are
compared with v' and w' spectra they show different trend
in the energy containing range. Any significant change in
the shape of the spectra is not observed with the downstream
distance or with different mean velocities of flow. For
the measured spectra the tendency towards different power

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laws (19, p.260) in different ranges of wave number can be seen by matching the measured values with the straight lines drawn with the slopes -1, - 5/3 and -7 according to the power laws. Uberoi (32) measured precontraction and post contraction u' and v' energy spectra to study the effect of contraction on grid turbulence. When the present results are compared with Uberoi's post contraction results the same tendency can be seen for v' and w' spectra but u' spectra from the two results seem to have different trends. The reason may be slightly different wave number range studied in the two investigations.

Table 6.1 Homogeneity of Turbulence in Planes Parallel to the Grid

Reference	Grid type & solidity	Re_M	Area or length covered normal to flow direction	x/M	quantity	Extreme variation (% of the centre line value)
Nearly Isotropic Turbulence						
Present Study						
Fig. 17	Perforated Steel sheet (0.47)	8,900	12 in. sq. area	52	u'^2, v'^2, w'^2	22
Fig. 18	"	11,400	"	262	u'^2, v'^2	38
Uberoi & Wallis (36)	Rd. steel rod		18 in. length	95	'"	10
"	Rd. wooden rod		"	95	"	23
Grant & Nisbet (17)	Rd. brass rod (0.34)		6 in. sq. area	80	"	30
"	"		6 in. length	30	u'^2	6
Shafer & Goldrick (11)	Sq. rod (0.34)	16,000	6 in. length	40	u'^2	14
Anisotropic Turbulence						
Present Study						
Fig. 19	Perforated steel sheet (0.47)	22,000	12 in. sq. area	157	u'^2, v'^2, w'^2	65

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Table 6.2   <u>Decay Characteristics of Nearly Isotropic Turbulence</u> (ref. equ. 6.1)

$\bar{U}$(ft/sec)	Re_M ($\times 10^{-3}$)	quantity	n	x_0/M	A	Corr. coeff.	No. of exptl. pts.
35.7	12	u'^2	1.178	5	13.92	0.998	24
27.0	9	u'^2	1.230	5	12.11	0.999	28
35.7	12	v'^2	1.163	5	15.34	0.998	24
27.0	9	v'^2	1.195	5	13.59	0.998	28
35.7	12	q^2	1.168	5	14.84	0.999	24
27.0	9	q^2	1.207	5	13.07	0.999	28

Table 6.3   <u>Decay Characteristics of Anisotropic Turbulence</u> (ref. equ. 6.1)

$\bar{U}$(ft/sec)	Re_M ($\times 10^{-3}$)	quantity	n	x_0/M	A	Corr. coeff.	No. of exptl. pts.
53.3	18	u'^2	0.485	5	6.3×10^3	0.800	15
39.2	13	u'^2	0.192	5	2.95×10^4	0.743	10
53.3	18	v'^2	1.52	5	16.8	0.926	15
39.2	13	v'^2	1.78	5	5.34	0.945	10
53.3	18	w'^2	1.73	5	6.8	0.950	15
39.2	13	w'^2	2.02	5	1.70	0.973	10
53.3	18	q^2	1.31	5	60.0	0.969	15
39.2	13	q^2	1.31	5	69.5	0.987	10

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Table 6.4 Comparison of Turbulence Decay Characteristics
for different Grids (ref. equ. 6.1)

Reference	Re_M (x10^{-3})	Biplane grids solidity	Rods	u' decay n	$\frac{x_o}{M}$	A	v' decay n	$\frac{x_o}{M}$	A
Corrsin (1942)	8.5	0.44	Rd.	1.30	1	20	1.22	1.5	59
	17	0.44	Rd.	1.28	3	23	1.14	2.5	62
	26	0.44	Rd.	1.35	1	17	1.16	1	58
Batchelor & Townsend (1947, 1948)	5.5	0.34	Rd.	1.13	5	70			
	11	0.34	Rd.	1.25	8	48			
Baines & Peterson (1951)	24	0.44	Sq.	1.37	3	8.5			
Tsuji & Hama. (1953)	33	0.36	Rd.	1.35	0	10			
Wyatt (1955)	11	0.34	Rd.	1.27	5	31	1.27	5	40
	22	0.34	Rd.	1.27	3.5	35	1.27	3.5	45
	44	0.34	Rd.	1.25	4	35	1.25	4	45
Kistler & Vrebalovich (1961)	2420 (p=4atm)	0.34	Rd.	1.0	9	71	1.0	7	110
Uberoi (1963)	29	0.44	Rd.	1.20	4	24	1.20	4	35
Corrsin (1966)	17	0.34	Sq.	1.33	1.5	13	1.27	1.5	20
	34	0.34	Sq.	1.26	3	17	1.20	3	25
	135	0.34	Sq.	1.18	3.5	22	1.16	3	27
	68	0.44	Rd.	1.27	2	27	1.26	2	33
Uberoi & Wallis (36)		0.23	Rd.	1.39			1.39		
		0.34	Rd. (wooden grid roughened with sand grains)	1.22			1.22		

7. CONCLUSIONS

The conclusions which are of specific interest in the present study of anisotropic homogeneous turbulence under steady mean flow are:

1. Anisotropic turbulence shows a strong tendency towards isotropy in the initial period of decay.

2. The rate of decay of the total turbulent energy follows nearly the same power law as isotropic turbulent flow.

3. One dimensional energy spectra of longitudenal and lateral components of turbulence show considerably different trends in the energy containing range. No appreciable change with distance downstream of the grid could be noticed in the energy spectra.

On comparison with other investigators, the characteristics (transverse homogeneity, isotropy and the decay rates) of the turbulence obtained in a parallel section behind a perforated sheet grid were found comparable with those obtained by commonly used biplane grids. For the nearly isotropic grid turbulence the decay rate for the three individual components of turbulence and for the total turbulent energy was found to satisfy the commonly accepted equation.

$$\bar{U}^2/f^2 = A\left(\frac{x}{M} - \frac{x_0}{M}\right)^n$$

with n between 1.16 and 1.23 when $(x_0/M) = 5$

It has to be pointed out that the free stream turbulence (0.2 to 0.65%) in the present wind tunnel had a pronounced effect on the results for anisotropic turbulence even though a correction was applied. This was because of low grid turbulence intensity existing after the contraction.

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APPENDICES.

WIND TUNNEL AND FREE STREAM TURBULENCE

A1. Description of the Tunnel

A closed circuit low speed wind tunnel of Mechanical Engineering Department of the University of Windsor was used for the experiments. It was designed and constructed by the Department of Civil Engineering of Colorado State University. A general view of the tunnel is given in figure 22; some of the important features are presented here.

Test section dimensions:

crossection   30"(horizontal), 30"-31 3/4" (vertical)

length   20'

The divergence of the vertical dimension of the test section was provided to avoid axial gradient in mean speed.

Contraction upstream of test section:

length	75"
upstream dimensions	72" square
downstream dimensions	30" sqaure
contraction ratio	5.76

Screens upstream of the contraction:

size	80" x 80"

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square mesh	24 x 24
wire diameter	0.0075"
wire material	stainless steel
number of screens	four
spacings between screens	8"

Electric motor:

3 phase,   60 cycles, 20 h.p.

550 volts,   19 amperes, 1760 r.p.m.

Belt driven axial flow fan:

Maximum allowable speed:1200 r.p.m.

Maximum velocity of flow without inserting any extra screen:

80 ft/sec.

Maximum velocity of flow was reduced to about 40 ft/sec when the grid, to obtain nearly isotropic turbulence, was inserted at the entrance of the working section. The velocity was not much affected by inserting the grid upstream of the contraction to obtain strongly anisotropic turbulence; a maximum velocity of about 70 ft/sec could then be obtained.

The test section was provided with eight glass windows at each of the side walls. As the platform for the measuring instruments and for working was located on one side only, the windows on the other side were sealed to minimise infiltration of air. In studying anisotropic turbulence the intensities were low and to avoid additional turbulence due to infiltration effects the windows up-

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stream of the point at which the measurement were being taken including those on the working side were properly sealed.

Various measurements of the mean velocity and the turbulence in the empty tunnel (without inserting the turbulence producing grid) will be described in the following paragraphs.

A2? Mean Velocity Variation in the Test Section:

Measurements of the mean velocity of flow were carried out using a pitot static probe with the Wilhelm Lambrecht slanting tube micromanometer. The mean velocity at a fixed fan speed was measured at various points along the centre line of the test section. The measurements are presented in figure 23.

A3? Turbulence Level:

The wind tunnel was used for the fundamental study of grid generated turbulence which is necessarily of low relative intensity. In such studies, one of the main requirements of a wind tunnel is that it should produce an air stream with turbulence negligible in comparison with the turbulence being examined. Corrsin (11) mentions that, in the study of grid turbulence, each of the three components of the root-mean square upstream turbulence velocity (plus sound) should be well under $0.001\ \bar{U}$.

Figures 24 and 25 give the measurement of longitud-

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enal and transverse component of turbulence along the test section axis at different mean velocities. The values indicated in these figures include the fan noise and the anemometer noise. Filters to eliminate frequencies above 10 kc/sec were used because, on measuring the energy spectra, these waves were found to be mainly due to noise. The same filters were used during the measurement of turbulence decay. Figures 26 and 27 indicate the free stream turbulence at different velocities in different frequency ranges. The values in these figures do not include the anemometer noise. This data was specially obtained to understand the effect of free stream turbulence on the energy spectrum measurements after inserting the grid. From figures 24 to 27 we infer that, using the 10 kc/sec filters, the maximum value of $\frac{u'}{U}$ is 0.003 and the maximum value of $\frac{v'}{U}$ is 0.0065. These values are high and therefore the decay measurements after inserting the grid had to be corrected for free stream turbulence as mentioned in Chapter 5.

A4. Energy Spectra:

Figures 28 to 33 indicate the measurement of the one dimensional energy spectra of the logitudenal and transverse components of turbulence with different mean velocities at two different points on the test section axis. It may be pointed out here that the areas under the different spectrum curves are not same because the points on the graphs do not cover the range of wave numbers which was used for measuring

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the values of u', v' or w'. The oscillating nature of the spectrum curves after a value of k ≅ 10 to 12 in. indicates that the noise is predominant at higher wave numbers. The effect of noise seems to be more pronounced in u' spectra than in v' or w'. In both the longitudenal and transverse component spectra no definite change with the mean velocity of flow is observed except that the noise effect is comparatively less at higher velocities. When the spectra of longitudenal component are compared with those of transverse components, significant difference can be seen in the energy containing range.

A5.   Homogeneity of Turbulence:

    In the study of the gird turbulence homogeneity of the turbulence was checked in planes normal to the direction of flow. The same type of measurements were carried out with the free stream turbulence as well. The turbulence intensities were recorded at different points covering an area of one foot square around the axis. Figure 34 shows the location of the points and values of the fluctuating quantities in relation to the values at the centre. Figure shows that the variation in the fluctuating components is as high as ±40% of the respective values at the test section axis.

HOT WIRE ANEMOMETER

B1. Heat Transfer relations:

The development of the hot wire anemometer is one of the major factors in the advancement of understanding of the behaviour of the turbulent flow. An electrically heated wire is introduced into the flow to be studied. The flowing fluid causes the temperature of the wire to drop and consequently the electric resistance of the wire diminishes. The circuit used to take care of this variation measures voltage or current which is related to the flow velocity.

The following sections deal with the various relations which are used to measure mean velocity of the flow and the three fluctuating components of turbulence.

For the hot wire placed normal to the direction of the flow the form of the heat loss relation has conventionally been assumed to be that of the empirical King's law (19, p. 79) which can be written as

$$Nu = A + B \ Re^{0.50} \qquad (B1)$$

where A and B are considered as constants. While the values of A and B reported by various authors are diverse,

they generally agree on the value of Reynolds number
exponent as 0.50. The empirical relation given by
Kramers (19, p.76) also agrees with King's equation when
the values of A and B are taken as

$$A = 0.42 \ (Pr)^{0.2}$$

$$B = 0.57 \ (Pr)^{0.33}$$

For air and diatomic gases it has been assumed to be valid
in the range $0.01 < Re < 10,000$.

A notable exception is the series of tests reported
by Collis and Williams (9). They carried out the measure-
ments in the range $0.01 < Re < 140$. They suggested the
relation

$$Nu = A + B \ Re^a$$

which is the same as equation (B1) when $a = 0.5$. But the
values of a found by Collis and Williams are different.
According to them

$$a = 0.45 \quad \text{when} \quad 0.02 < Re < 44$$

$$a = 0.51 \quad \text{when} \quad 44 < Re < 140$$

More recently Webb (37) has found different values of a
to fit his experimental data in air and argon at various
ambient pressures. For air at atmospheric temperature
and pressure in the Reynolds number range of 0.4 to 4 he
found

$$Nu = 0.3 + 0.44 \ Re^{0.52}$$

and under the same conditions for argon he found

$$Nu = 0.23 + 0.50 \ Re^{0.45}$$

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As explained by Grant and Kronauer [18] the discrepancies in the values of Reynolds number exponent may be due to different values of length to diameter ratio of the wires used by different investigators.

In the hot wire anemometry, as pointed out earlier, proper electronic circuits are used to enable measurement of heat transfer rate in terms of electric current or voltage. Hinze [19, p.76] has derived the following relation from Kramer's equation

$$\frac{I^2 R_w}{R_w - R_g} = A + B\, U^{0.50} \qquad (B3)$$

For the present investigation, constant temperature (which means R_w = constant) hot wire anemometer has been used in a flow at nearly room temperature. R_g could be assumed constant because of very small variation of temperature of the air during a run as mentioned in Chapter 4. Moreover equation B3 can be written with voltage and velocity as the variables since the anemometer, which is used for present work, measures voltage and not current corresponding to the heat transfer rate. In view of the divergence of opinion over the proper value of the exponent of velocity, a more general equation has to be considered. Therefore instead of equation (B3) we can write

$$E^2 = X + Y\, U^a \qquad (B4)$$

In the above equation X, Y and a are to be determined by calibration.

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If the wire is inclined to the direction of velocity by an angle Q then, in the above equation, U can be replaced by the effective velocity (19, p.103) which is defined as

$$U_{eff} = U^2 (\sin^2 Q + k^2 \cos^2 Q) \tag{B2}$$

where k is a factor depending on direction sensitivity of the wire. In other words for inclined wire we can write the equation as

$$E^2 = X_1 + Y_1 (U_{eff})^a \tag{B5}$$

$$\text{or} \quad E^2 = X + Y U^a$$

where $Y = Y_1 (\sin^2 Q + k \cos^2 Q)^{\frac{1}{2}}$ and $X = X_1$

Calibration of the hot wire anemometer with different probes to determine the voltage-velocity curves and the discussion and experimentation for the value of k will be reported in different sections.

B2.   Calibration of the Hot Wire Anemometer:

DISA constant temperature hot wire anemometer 55 A 01 was used in the present study. The DC voltmeter incorporated in the anemometer had an accuracy of ±1%. For the calibration purposes a pitot static probe with Wilhelm Lambrecht micromanometer was used as reference. The DC voltmeters and RMS voltmeters on both of the anemometers in use were precalibrated against other instruments as reported in Appendix C.

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The hot wire probe was exposed to the flow in the same wind tunnel which was used for the main experimental work. It was mounted such that the probe axis was parallel to the direction of mean flow as shown in figure 36. A pitot static probe was stationed close (3/4") to the hot wire. Flow velocity was varied with the fan regulator and the anemometer DC voltages and the corresponding manometer readings were recorded simultaneously at various velocities. Mean velocities were calculated from the manometer readings.

In equation (B4) various values of 'a' were assumed in the range 0.20 to 0.60 with an interval of 0.01. For each value of a, X and Y were determined to fit the experimental data. Least square regression method was used for curve fitting. The set of values of a, X and Y giving highest correlation coefficient (or least RMS error) was selected to represent the best fitting curve to the experimental data. This analysis was carried out for the experimental points in the range of interest only that is nearly 15 ft/sec to about 60 ft/sec. To be more clear the voltmeter reading with the wire unexposed to flow, refered to as E_o, was not included for determining the best fitting curve. A value of a = 0.35 (or slightly different) was found to give the best correlation with each hot wire probe. But this value was very low when compared to the values recommended by previous investig-

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ators. But the same value was found to be the most appropriate even on repeating the experiments with some of the probes. Such a clear contradiction with previous investigators compelled the present author to try the above method of curve fitting after including E_0 as one of the experimental points. And it was found that the best fitting curve was obtained with a = 0.50 (or slightly different) which was in accordance with the conventional King's law. Though the correlation coefficient in this case was not as high as obtained on excluding E_0 but it was high enough from the statistical point of view. On the other hand very small but a systematic deviation of experimental data from the best fitting straight line could be appreciated. Same type of deviation was observed in a typical calibration curve supplied by DISA, the manufacturers. Therefore a = 0.35 and hence the use of the curve fitting in the range of interest was considered to be more suitable for computing turbulence quantities. The calibration curves can be seen in figures 38 and 39. The detail of the probes and the calibration constants is as under.

| | Right Angle Probe | | Slanting Probe | | X-Probe |
	No. 1	No. 2	No. 1	No. 2	(two wires)
Type specified by DISA	55A25	55A25	55A29	55A29	55A32
Material	Platinum plated tungsten for all				
Wire diameter (micron)	5	5	5	5	5,5
Wire length (mm)	1	1	1	1	1,1
Cold resistance at the temperature indicated (ohm)	3.49 (82°F)	3.30 (77°F)	3.53 (84°F)	3.31 (78°F)	3.36, 3.47 (85°F)
Operating resistance (ohm)	6.40	6.40	6.40	6.40	6.40, 6.40
Velocity range for calibration (ft/sec)	7-50	11-60	7-50	11-60	
Barometric pressure (in. mercury)	29.53	29.67	29.57	29.42	

Calibration constants (excluding E_o)

a	0.35	0.35	0.35	0.35	
X	21.5569	21.9211	20.0074	21.2872	
Y	13.9339	13.0812	11.4625	11.5825	

Calibration constants (including E_o)

a	0.50	0.50	0.50	0.50	
X	33.9571	33.4865	30.1121	32.4955	
Y	6.0528	5.6635	4.9982	4.8584	

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The following table will give an idea about the sensitivity of equation (B4) towards the value of a.

Analysis of experimental data disregarding the value of Eo

	a	X	Y	RMS error (inE)	Corr. coeff.	No. of exptl. points
Right Angle Probe						
No. 1	0.36	22.6767	13.1338	0.0813	0.99995	14
	0.35	21.5569	13.9339	0.0750	0.99996	
	0.34	20.3715	14.7943	0.0753	0.99995	
No. 2	0.36	23.1004	12.2915	0.1050	0.99990	14
	0.35	21.9211	13.0812	0.1036	0.99990	
	0.34	20.6722	13.9326	0.1050	0.99989	
Slanting Probe						
No. 1	0.36	20.9286	10.8043	0.1210	0.99983	16
	0.35	20.0074	11.4625	0.1129	0.99985	
	0.32	16.8989	13.7545	0.1031	0.99988	
	0.31	15.7292	14.6427	0.1054	0.99987	
No. 2	0.36	22.3255	10.8847	0.1065	0.99987	13
	0.35	21.2872	11.5825	0.1047	0.99987	
	0.34	20.1877	12.3349	0.1050	0.99987	

Analysis of experimental data including the value of Eo

	a	X	Y	RMS error	Corr. coeff.	No. of exptl. points
Right Angle Probe						
No. 1	0.51	34.2220	5.8008	0.3628	0.99945	15
	0.50	33.9571	6.0528	0.3560	0.99947	
	0.49	33.6911	6.3152	0.3785	0.99941	
No. 2	0.51	33.7370	5.4182	0.5003	0.99899	15
	0.50	33.4865	5.6635	0.4121	0.99931	
	0.47	32.7374	6.4644	0.2203	0.99981	
	0.46	32.4895	6.7544	0.2282	0.99979	
Slanting Probe						
No. 1	0.51	30.3348	4.7888	0.3262	0.99931	17
	0.50	30.1121	4.9982	0.3131	0.99937	
	0.49	29.8882	5.2166	0.3203	0.99934	
No. 2	0.51	32.6937	4.6499	0.2697	0.99963	14
	0.50	32.6955	4.8584	0.2421	0.99970	
	0.49	32.2976	5.0758	0.2443	0.99969	

B3.   Equations to Compute Turbulence Quantities

The equations obtained for turbulence quantities are based on the assumption that fluctuating components of velocity are very small compared to the mean velocity, i.e.

$$\frac{u'}{\overline{U}} \ll 1, \quad \frac{v'}{\overline{U}} \ll 1, \quad \frac{w'}{\overline{U}} \ll 1 \text{ and therefore } \frac{e'}{\overline{E}} \ll 1.$$

Figure 37 shows a calibration curve and various positions of the hot wire probe with respect to the flow.

Let us start from equation (B4) which reads

$$E^2 = X + Y U^a \qquad (B4)$$

where $\quad U = \overline{U} + u$

and $\quad E = \overline{E} + e$

We can use mean quantities in place of total quantities (19, p.81) in equation (B4), hence we can write

$$\overline{E}^2 = X + Y \overline{U}^a \qquad (B5)$$

Substituting the values of U and E in equation (B4), we get, for a probe at right angles to the direction of flow

$$(\overline{E} + e)^2 = X + Y(\overline{U} + u)^a$$

$$\text{or} \quad \overline{E} + 2e\overline{E} + e^2 = X + Y \overline{U} \left(1 + a\frac{u}{\overline{U}} + \frac{a(a-1)}{2!} \left(\frac{u}{\overline{U}}\right)^2 + \cdots\right)$$

$$\text{or} \quad 1 + \frac{2e}{\overline{E}} = \frac{1}{\overline{E}^2}\left[X + Y \overline{U}^a \left(1 + a\frac{u}{\overline{U}} + \frac{a(a-1)}{2}\left(\frac{u}{\overline{U}}\right)^2 + \cdots\right)\right] - \left(\frac{e}{\overline{E}}\right)^2$$

as $\quad \frac{u}{\overline{U}} \ll 1$ and $\frac{e}{\overline{E}} \ll 1$, the quantities $\left(\frac{e}{\overline{E}}\right)^2$ and $\left(\frac{u}{\overline{U}}\right)^2$

and higher order terms can be neglected.

$$\therefore \quad 1 + 2\frac{e}{\bar{E}} = \frac{1}{\bar{E}^2}\left[X + Y\bar{U}^a + Ya\frac{u}{U}\bar{U}^a \right]$$

Substituting the value of $\bar{E}^2$ from equation (B6), we get

$$1 + \frac{2e}{\bar{E}} = 1 + \frac{1}{\bar{E}^2}\ Ya\frac{u}{U}\bar{U}^a$$

or $\quad e = \dfrac{Ya\bar{U}^a}{2\bar{E}}\ \dfrac{u}{U}$

or $\quad e = S\ u$

where $\quad S = $ sensitivity

$$= \frac{Ya\bar{U}^{(a-1)}}{2\bar{E}}$$

Again making use of equation (B4a), we get

$$\bar{U}^{(a-1)} = \left(\frac{\bar{E}^2 - X}{Y}\right)^{(a-1)/a}$$

$$\hat{S} = \frac{(a/2)\ Y^{1/a}}{\bar{E}\ (\bar{E} - X)^{(1-a)/a}} \tag{B27}$$

where a, X, Y are determined from the calibration

curve and $\bar{E} = $ dc voltage reading from the anemometer

$\therefore \qquad e = Su$

or $\qquad e^2 = S^2 u^2$

or $\qquad \overline{e^2} = S^2\overline{u^2}$ $\tag{B7}$

where $\qquad \overline{e^2} = e'^2$ and

$\qquad e' = $ RMS voltmeter reading with the right

angle probe.

Now take the case of the slanting probe (refer to figure 37 for the explanation of various quantities in use).

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Rewrite equations (B4) and (B6)

$$E^2 = X + Y \, U^a \qquad (B4)$$

$$\overline{E}^2 = X + Y \, \overline{U}^a \qquad (B6)$$

Using effective velocity for the slanting probe in the above equations, we get

$$E^2 = X_1 + Y_1 \, (U_{eff})^a \qquad (B8)$$

$$\overline{E}^2 = X_1 + Y_1 \, (\overline{U_{eff}})^a \qquad (B9)$$

where

$$U_{eff}^2 = U^2 \sin^2 Q + k^2 \, U^2 \cos^2 Q$$
$$= U_n^2 + k^2 \, U_t^2$$

and therefore

$$Y = \text{slope} = Y_1 \, (\sin^2 Q + k^2 \cos^2 Q)^{a/2} \qquad (B10)$$

Now

$$U_t = (\overline{U} + u)\cos Q - v \sin Q$$

$$U_n = (\overline{U} + u)\sin Q + v \cos Q$$

$$\begin{aligned}
U_{eff}^2 &= \left[(\overline{U}+u)\sin Q + v\cos Q\right]^2 + k^2 \left[(\overline{U}+u)\cos Q - v \sin Q\right]^2 \\
&= (\overline{U}+u)^2 \sin^2 Q + v^2 \cos^2 Q + 2v(\overline{U}+u)\sin Q \cos Q \\
&= + k^2(\overline{U}+u)^2\cos^2 Q + k^2 v^2 \sin^2 Q - k^2 2v(\overline{U}+u)\sin Q \cos Q \\
&= (\overline{U}+u)^2(\sin^2 Q + k^2\cos^2 Q) + v^2(\cos^2 Q + k^2 \sin^2 Q) \\
&\quad + 2v(\overline{U}+u)\sin Q \cos Q(1-k^2)
\end{aligned}$$

or

$$\begin{aligned}
U_{eff}^2 &= \overline{U}^2(\sin^2 Q + k^2\cos^2 Q) + 2u\overline{U}(\sin^2 Q + k^2\cos^2 Q) \\
&\quad + 2v\overline{U}\sin Q \cos Q(1-k^2) + u^2(\sin^2 Q + k^2\cos^2 Q) \\
&\quad + v^2(\cos^2 Q + k^2\sin^2 Q) + 2uv \sin Q \cos Q(1-k^2) \\
&= \overline{U}^2(\sin^2 Q + k^2\cos^2 Q)\left[1 + \frac{2u}{\overline{U}} + \frac{2v}{\overline{U}}\frac{\sin Q \cos Q(1-k^2)}{\sin^2 Q + k^2\cos^2 Q}\right. \\
&\quad \left. + \left(\frac{u}{\overline{U}}\right)^2 + \left(\frac{v}{\overline{U}}\right)^2 \frac{\cos^2 Q + k^2\sin^2 Q}{\sin^2 Q + k^2\cos^2 Q} + \frac{2uv}{\overline{U}^2}\frac{\sin Q \cos Q(1-k^2)}{\sin^2 Q + k^2\cos^2 Q}\right]
\end{aligned}$$

$$U_{eff}^a = (\bar{U}\sqrt{\sin^2Q+k^2\cos^2Q})^a \left[1+2\,\frac{u}{\bar{U}} + \frac{2v}{\bar{U}}\,\frac{\sin Q\cos Q(1-k^2)}{\sin^2Q+k^2\cos^2Q} \right.$$

$$\left. + \frac{u}{\bar{U}}^2 + \left(\frac{v}{\bar{U}}\right)^2 \frac{\cos^2Q+k^2\sin^2Q}{\sin^2Q+k^2\cos^2Q} + \frac{2uv}{\bar{U}^2}\frac{\sin Q\cos Q(1-k^2)}{\sin^2Q+k^2\cos^2Q} \right. \qquad (B11)$$

$$= (\bar{U}\sqrt{\sin^2Q+k^2\cos^2Q})^a \left[1+ \frac{a}{2}\left\{ \frac{2u}{\bar{U}} + \frac{2v}{\bar{U}}\,\frac{\sin Q\cos Q(1-k^2)}{\sin^2Q+k^2\cos^2Q} \right.\right.$$

$$\left. + \left(\frac{u}{\bar{U}}\right)^2 + \left(\frac{v}{\bar{U}}\right)^2 \frac{\cos^2Q+k^2\sin^2Q}{\sin^2Q+k^2\cos^2Q} + \frac{2uv}{\bar{U}^2}\frac{\sin Q\cos Q(1-k^2)}{\sin^2Q+k^2\cos^2Q} \right\}$$

$$+ \frac{\frac{a}{2}\left(\frac{a}{2}-1\right)}{2!}\left\{ \frac{2u}{\bar{U}} + \frac{2v}{\bar{U}}\,\frac{\sin Q\cos Q(1-k^2)}{\sin^2Q+k^2\cos^2Q} + \left(\frac{u}{\bar{U}}\right)^2 \right.$$

$$\left. + \left(\frac{v}{\bar{U}}\right)^2 \frac{\cos^2Q+k^2\sin^2Q}{\sin^2Q+k^2\cos^2Q} + \frac{2uv}{\bar{U}^2}\frac{\sin Q\cos Q(1-k^2)}{\sin^2Q+k^2\cos^2Q} \right\}^2$$

$$+ \quad \text{cubic and higher order terms} \Big]$$

$$\text{or } U_{eff}^a = (\bar{U}\sqrt{\sin^2Q+k^2\cos^2Q})^a \left[1+a\,\frac{u}{\bar{U}} + a\,\frac{\sin Q\cos Q(1-k^2)}{\sin^2Q+k^2\cos^2Q}\,\frac{v}{\bar{U}} \right.$$

$$+ \frac{a(a-1)}{2}\left(\frac{u}{\bar{U}}\right)^2 + \frac{a}{2}\left\{ \frac{(a-1)\sin^2Q\cos^2Q(1-k^2)^2+k^2}{(\sin^2Q+k^2\cos^2Q)^2} \right\}\left(\frac{v}{\bar{U}}\right)^2$$

$$\left. + a(a-1)\left\{ \frac{\sin Q\cos Q(1-k^2)}{\sin^2Q+k^2\cos^2Q}\,\frac{uv}{\bar{U}^2} \right\} + \begin{array}{l}\text{cubic and higher}\\\text{order terms.}\end{array} \right]$$

Neglecting the terms associated with

$$\left(\frac{u}{\bar{U}}\right)^2, \; \left(\frac{v}{\bar{U}}\right)^2 \text{ and } \left(\frac{uv}{\bar{U}}\right)^2 \text{ , we get}$$

$$U_{eff}^a = (\bar{U}\sqrt{\sin^2Q+k^2\cos^2Q})^a \left[1 + a\,\frac{u}{\bar{U}} + a\,\frac{\sin Q\cos Q(1-k^2)}{\sin^2Q+k^2\cos^2Q}\,\frac{v}{\bar{U}} \right]$$

$$(B12)$$

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Rewrite equations (B8) and (B9)

$$E^2 = X_1 + Y_1 \, (U_{eff})^a \qquad\qquad (B8)$$

$$\overline{E}^2 = X_1 + Y_1 \, (\overline{U}_{eff})^a \qquad\qquad (B9)$$

Substituting $E = \overline{E} + e$ in (B8), we get

$$(\overline{E} + e)^2 = X_1 + Y_1 \, (U_{eff})^a$$

or $\qquad \overline{E}^2 + 2e\overline{E} + e^2 = X_1 + Y_1 \, (U_{eff})^a$

Substituting the value of $(U_{eff})^a$ from above and dividing the equation by $\overline{E}^2$, we get

$$1 + \frac{2e}{\overline{E}} + \left(\frac{e}{\overline{E}}\right)^2 = \frac{1}{\overline{E}^2}\left[X_1 + Y_1 \, (\overline{U} \sqrt{\sin^2 Q + k^2 \cos^2 Q})^a \right.$$

$$\left. + Y_1 a (\overline{U} \sqrt{\sin^2 Q + k^2 \cos^2 Q})^a \left(\frac{u}{\overline{U}} + \frac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q} \frac{v}{\overline{U}}\right) \right]$$

But from equation (B10)

$$Y = Y_1 \, (\sqrt{\sin^2 Q + k^2 \cos^2 Q})^a \quad .$$

and $X = X_1$

we get

$$1 + \frac{2e}{\overline{E}} + \left(\frac{e}{\overline{E}}\right)^2 = \frac{1}{\overline{E}^2}\left[X + Y \, \overline{U}^a + Y \, a \frac{\overline{U}^a}{\overline{U}}\left(u + \frac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q} \cdot v\right) \right]$$

Substituting $X + Y \, \overline{U}^a = \overline{E}^2$ according to equation

(B6) and neglecting $\left(\frac{e}{\overline{E}}\right)^2$, we get

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$$1 + 2\frac{e}{\overline{E}} = 1 + \frac{Y\,a\,\overline{U}^{(a-1)}}{\overline{E}^2}\left(u + \frac{\sin Q\cos Q(1-k^2)}{\sin^2 Q + k^2\cos^2 Q}\,v\right)$$

Again from equation (B6), we get

$$\overline{U}^{(a-1)} = \left(\frac{\overline{E}^2-X}{Y}\right)^{(a-1)/a}$$

$$e = \frac{a\,Y^{1/a}}{2\overline{E}\,(\overline{E}^2-X)^{(1-a)/a}}\left(u + \frac{\sin Q\cos Q(1-k^2)}{\sin^2 Q + k^2\cos^2 Q}\,v\right)$$

or $\quad e = \left(S_1\right)_y u + \left(S_2\right)_y v$

or $\quad e^2 = (S_1)_y^2\,u^2 + 2(S_1)_y(S_2)_y\,uv + (S_2)_y^2\,v^2$

or $\quad \overline{e^2} = (S_1)_y^2\,\overline{u^2} + 2(S_1)_y(S_2)_y\,\overline{uv} + (S_2)_y^2\,\overline{v^2}$ $\qquad$ (B13)

where

$$(S_1)_y = \frac{a\,Y^{1/a}}{2\,\overline{E}\,(\overline{E}^2-X)^{(1-a)/a}}$$

and

$$(S_2)_y = \frac{\sin Q\cos Q(1-k^2)}{\sin^2 Q + k^2\cos^2 Q}\,(S_1)_y = C(S_1)_y$$

where $C = \left(\sin Q\cos Q(1-k^2)\right)\big/(\sin^2 Q + k^2\cos^2 Q)$

After deriving equations (B7) and (B13), we can write the following equations for different positions of the probes as shown in figure 37b

x position: $\quad e'^2_x = S_x^2 u'^2$

y_1 position: $\quad e'^2_{y1} = (S_1)^2_{y1} u'^2 + (S_2)^2_{y1} v'^2 + 2(S_1 S_2)_{y1} \overline{uv}$

y_2 position: $\quad e'^2_{y2} = (S_1)^2_{y2} u'^2 + (S_2)^2_{y2} v'^2 + 2(S_1 S_2)_{y2} \overline{uv}$

z_1 position: $\quad e'^2_{z1} = (S_1)^2_{z1} u'^2 + (S_2)^2_{z1} w'^2 + 2(S_1 S_2)_{z1} \overline{uw}$

z_2 position: $\quad e'^2_{z2} = (S_1)^2_{z2} u'^2 + (S_2)^2_{z2} w'^2 + 2(S_1 S_2)_{z2} \overline{uw}$

As the values of a, X, Y and $\overline{E}$ for the slanting probe are same in the four positions, we have

$$(S_1)_{y1} = (S_1)_{y2} = (S_1)_{z1} = (S_1)_{z2} = S_s, \text{ where sub-}$$

script s represents slanting probe.

For convenience, let us use S_n for S_x where n stands for normal (or right angle) probe.

Now, we have

$\quad$ Q in y_2 position = 180° - Q in y_1 position.

$\quad (\sin Q)_{y2} = (\sin Q)_{y1}$

$\quad$ and $(\cos Q)_{y2} = -(\cos Q)_{y1}$

But $\quad (S_2)_y = \dfrac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q} S_1$

$\therefore \quad (S_2)_{y1} = \left(\dfrac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q}\right)_{y1} (S_1)_{y1}$

and $\quad (S_2)_{y2} = \left(\dfrac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q}\right)_{y2} (S_1)_{y2}$

$\qquad\qquad = -\left(\dfrac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q}\right)_{y1} (S_1)_{y2}$

$\qquad\qquad = -\left(\dfrac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q}\right)_{y1} (S_1)_{y1}$

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$$\therefore \quad (S_2)_{y2} = -(S_2)_{y1}$$

which means

$$(S_2)^2_{y2} = (S_2)^2_{y1}$$

Similarly it can be shown that

$$(S_2)^2_{z2} = (S_2)^2_{z1}$$

But $\quad (Q)_{y1} = (Q)_{z1}$

$$\therefore \quad (S_2)^2_{y1} = (S_2)^2_{y2} = (S_2)^2_{z1} = (S_2)^2_{z2} = S^2_{s2} = C^2 S^2_s$$

Now, write the equations for x, y_1 and y_2 positions

$$e'^2_x = S^2_n u'^2 \qquad\qquad (B14)$$

$$e'^2_{y1} = S^2_s u'^2 + C^2 S^2_s v'^2 + 2 C S_s \overline{uv} \qquad (B15)$$

$$e'^2_{y2} = S^2_s u'^2 + C^2 S^2_s v'^2 - 2 C S_s \overline{uv} \qquad (B16)$$

Solution of these equations gives

$$u'^2 = e'^2_x / S^2_n \qquad\qquad (B17)$$

$$v'^2 = \frac{1}{C^2}\left[\left\{(e'^2_{y1} + e'^2_{y2}) / 2 S^2_s\right\} - u'^2\right] \qquad (B18)$$

$$\overline{uv} = \left\{(e')^2_{y1} - (e')^2_{y2}\right\} \Big/ 4C S^2_s \qquad (B20)$$

Similarly, making use of the equations for the x, z_1 and z_2 positions, we will get

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$$w'^2 = \frac{1}{C^2}\left[\left\{(e'^2_{z1} + e'^2_{z2}) / 2S_s^2\right\} - u'^2\right] \quad (B19)$$

$$\overline{uw} = \left\{(e')^2_{z1} - (e')^2_{z2}\right\} / 4C\,S_s^2 \quad (B21)$$

where $C = \dfrac{\sin Q \cos Q(1-k^2)}{\sin^2 Q + k^2 \cos^2 Q} = \dfrac{1-k^2}{1+k^2}$ when $Q = 45^o$

as in the present case. (B26)

This may be mentioned that the above expressions for the turbulence quantities are considerably simpler than the ones obtained by Champagne (8), and on comparison of some numerical values the results were found to agree upto three significent figures.

B4.   Hot wire Anemometer with Linearizer:

As we have seen the output voltage of constant temperature anemometer is non linear function of the flow velocity under measurement. An electric circuit can be used which gives output voltage linearly related to velocity. The use of linearizer is preferred in the measurement of high degrees of turbulence. In the course of present investigation, experimentation was carried out with known turbulent shear flow in a pipe. This was done in an attempt to find an appropriate value of the direction sensitivity of slanting hot wire probe. DISA type 55D10 linearizer was used with DISA constant temperature hot wire anemometer. Only the slanting hot wire probe was needed in this study. The probe was calibrated without linearizer, it had to be recalibrated with the linearizer assuming an appropriate value of the velocity exponent 'a' in equation (B4). Being influenced by the experimental findings of Collis and Williams (9) to take a = 0.45 and by the calibration without linearizer in the present investigation which gave a = 0.35, two values of a that is 0.45 and 0.40 were tried first. a = 0.45 gave higher correlation coefficient than the one obtained with a = 0.40. For the linear relation

$$E_{11} = X_{11} + Y_{11}U \qquad (B22)$$

$$E_1 = X_1 + Y_1 U_{eff} \qquad (B23)$$

the following values of constants were obtained to fit

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the experimental data with the two values of the index a

a	X_{1_1}	Y_{1_1}	RMS error	Corr. Coeff.,	No. of exptl. pts.
1/2.5+(.40)	-2.0702	0.2162	0.0451	0.9997	13
1/2.22+(.45)	-1.0313	0.1867	0.0206	0.9999	13

Before carrying out this calibration, it was not realized that a = 0.50 gives best results with the non linearized circuit when the voltage reading (E_o) at zero velocity is included in the analysis, as it is pointed out in the previous section on calibration. Therefore a = 0.45 was selected as the best value with linearizer, otherwise a = 0.50 could also be tried. The calibration curve obtained with a = 0.45 has been shown in figure 40.

The expressions for the turbulence quantities will be different in this case than the ones obtained with a non linear relation between voltage output and the velocity.

From equation (B11), we can write

$$U_{eff} = \bar{U} \sqrt{\sin^2Q + k^2\cos^2Q} \left[1 + 2\frac{u}{\bar{U}} + 2\frac{v}{\bar{U}} \frac{\sin Q \cos Q(1-k^2)}{\sin^2Q + k^2\cos^2Q} + \left(\frac{u}{\bar{U}}\right)^2 \right.$$

$$\left. + \left(\frac{v}{\bar{U}}\right)^2 \frac{\cos^2Q + k^2\sin^2Q}{\sin^2Q + k^2\cos^2Q} + 2\frac{uv}{\bar{U}^2} \frac{\sin Q \cos Q(1-k^2)}{\sin^2Q + k^2\cos^2Q} \right]^{\frac{1}{2}}$$

As equation (B12) was obtained from equation (B11), we can obtain the following approximate expression for U_{eff}

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$$U_{eff} = \bar{U}\sqrt{\sin^2 Q + k^2 \cos^2 Q}\left[1 + \frac{u}{\bar{U}} + \frac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q} \frac{v}{\bar{U}}\right]$$

Now $\quad E_1 = X_1 + Y_1 U_{eff}$

$$= X_1 + Y_1 \bar{U}\sqrt{\sin^2 Q + k^2 \cos^2 Q}\left[1 + \frac{u}{\bar{U}} + \frac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q}\frac{v}{\bar{U}}\right]$$

But $\quad \bar{E}_1 = X_1 + Y_1 \bar{U}\sqrt{\sin^2 Q + k^2 \cos^2 Q}$

where
$$E_1 = \bar{E}_1 + e_1$$

$$e_1 = Y_1 \bar{U}\sqrt{\sin^2 Q + k^2 \cos^2 Q}\left[\frac{u}{\bar{U}} + \frac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q}\frac{v}{\bar{U}}\right]$$

$$= Y_1 \sqrt{\sin^2 Q + k^2 \cos^2 Q}\left[u + \frac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q} v\right]$$

or $\quad e_1' = S_{1_1} u + S_{1_2} v$

where
$$S_{1_1} = Y_1\sqrt{\sin^2 Q + k^2 \cos^2 Q}$$

and
$$S_{1_2} = \frac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q} S_{1_1} = C S_{1_1}$$

where
$$C = \frac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q} = \frac{1-k^2}{1+k^2} \quad \text{when } Q = 45° \text{ as in the present case.}$$

Referring to equation (B20), for y_1 and y_2 positions of the probe in figure 37a, we can write

$$\overline{uv} = (e'_1)^2_{y_1} - (e'_1)^2_{y_2} \; / \; 4C\, S^2_{1_1} \qquad (B24)$$

This equation was used in Reynolds shear stress calculations as appearing in the preceding section.

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B5. Direction Sensitivity of the slanting hot wire:

If a hot wire is placed so that it makes an angle Q with respect to the mean flow direction as shown in figure 37a then the output voltage is given by

$$E^2 = X_1 + Y_1 U_{eff}^a \qquad (B5)$$

$$\text{where } U_{eff}^2 = U^2 \sin^2 Q + k^2 U^2 \cos^2 Q. \qquad (B2)$$

This is based on the assumption that the cooling of the wire is accomplished essentially by the component of the velocity which is normal to the wire that is $U \sin Q$. In the above expression $k^2 U^2 \cos^2 Q$ indicates that the tangential component of the incident velocity also have some effect on cooling. In this connection the literature review presented by Champagne (8) can be summarized as follows.

Prandtl, Struminsky, Jones and Sears established for infinitely long cylinder that the heat loss rate per unit length depends only on the normal velocity components. Schubauer and Klebanoff concluded from their experimental study that $k \approx 0$ for finite wires when $90° > Q > 20°$. Newman and Leary concluded that

$$U_{eff}^{1/2} = U^{1/2} (\cos Q)^n \quad \text{with } n = 0.457$$

although Sandborn and Laurence found that n varied with each wire tested. Kronauer's experimental results indicate that the direction sensitivity of the wire is dependent on the length to diameter ratio of the wire and

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substantially independent of the Reynolds number. He expressed his results in the form

$$U_{eff} = U \cos Q + 1.2 \left(\frac{d}{1}\right)^{\frac{1}{2}} \sin^2 Q$$

Hinze gave the factor k, in equation B2, a value between 0.1 and 0.3 depending upon the magnitude of the velocity (k increasing with decreasing velocity). Webster determined k to be 0.20. Delleur stated that for $90^{\circ} > Q > 20^{\circ}$, $k \approx 0$ but for $Q < 20^{\circ}$ he recommended k = 0.14.

According to Champagne (8), for the wires with finite length the non uniform wire temperature indicates that the tangential component of velocity has considerable effect on the heat transfer from the wire and it must be taken into account. He carried out measurements of heat transfer from wires inclined and normal to the flow. He found that the value of k depends primarily upon the length to diameter ratio (1/d) of the wire. He determined that for platinum wires $k \approx 0.20$ for 1/d = 200, decreases with increasing 1/d, and becomes effectively zero at 1/d = 600.

Tucker and Reynolds (31) reported the measurement of turbulence quantities with a slanting hot wire probe. They used k = o and in justification they presented the measurements of shear stress in pipe flow and found good agreement with theory. Same type of shear stress measurements were carried out by Schredl (25) in an attempt to predict k for the slanting hot wire probes which were

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used in the turbulence decay study. He used DISA constant temperature hot wire anemometer without linearizer. His results indicate that $0.20 < k < 0.35$ would give the values of shear stress as predicted by theory. The present author used the same experimental facilities as reported by Schredl (25) but DISA type 55D10 linearizer was included in the measuring equipment.

The probe which has been refered to as slanting probe No. 1 in the previous sections was used in this study of pipe flow. The calibration of the probe with and without linearizer is presented in Appendix B2 and B4 and figures 38, 39 and 40.

For an incompressible fully developed turbulent pipe flow the theoretical relation as derived by Laufer (20) is

$$\overline{uv} = \nu \frac{d\overline{U}}{dr} + \frac{r}{a} \cdot U_*^2 \qquad\qquad (B25)$$

where $\qquad U_*^2 = \dfrac{\tau_o}{\rho_{air}}$

and $\qquad 2\pi a L \tau_o = \pi a^2 (p_1 - p_2)/g_o$

where $\qquad$ L = length of the pipe between the points for
$\qquad\qquad\qquad$ which static pressure difference $p_1 - p_2$
$\qquad\qquad\qquad$ was measured.

and $\qquad$ a = radius of the pipe.

The experiment was carried out at a Reynolds number

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(based on centre line velocity) of 88,700. The experimental procedure may be summarized as under. $\overline{uv}$ and $\overline{U}$ were determined from the hot wire readings at a number of points along the radius of the pipe. At each point the slanting wire was placed in two positions y_1 and y_2 as shown in figure 37 and corresponding d.c. voltages ($\overline{E}_{11}$) and r.m.s. voltages (e_{1y1} and e_{1y2}) were recorded. $\overline{U}$ was calculated from equation (B22) and $\overline{uv}$ from equation (B24). For U_*^2, static pressure drop was measured for 10 ft. length of pipe using wall taps leading to the micromanometer.

The theoretical curve in figure 42 was drawn according to equation (B25) in which the quantities at right hand side were found experimentally. The experimental points were obtained using equation (B24) for $\overline{uv}$ and assuming different values of k. The values of k which would give the values of shear stress in exact agreement with equation B25 were also found from the same experimental data. For this purpose the value of $\overline{uv}$ in equation B24 can be substituted from equation B25 and the combination gives the following equation with k as the only unknown

$$\left\{(e_1')_{y1}^2 - (e_1')_{y2}^2\right\} \Big/ 4C\, s_{1_1}^2 = \nu \frac{d\overline{U}}{dr} + \frac{r}{a} U_*^2$$

$$C = \left\{(e_1')_{y1}^2 - (e_1')_{y2}^2\right\} \Big/ \left\{4 s_{1_1}^2 \left(\nu \frac{d\overline{U}}{dr} + \frac{r}{a} U_*^2\right)\right\}$$

In this equation all the quantities on RHS were known

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from the experiment.

Now $C = \dfrac{\sin Q \cos Q (1-k^2)}{\sin^2 Q + k^2 \cos^2 Q} = \dfrac{1-k^2}{1+k^2}$ when $Q = 45^{\circ}$ as in the

present case.

$\therefore \quad k = \sqrt{(1-C)/(1+C)}$

The values of k determined in this way are presented in figure 41. Figures 41 and 42 indicate that k lies between 0.2 and 0.4. After the experimentation the probe holder was examined for its axial movement by observing a magnified projection. It was found that, while rotating the probe through 180°, it experienced a shift from its original point. Therefore the values of k obtained from the above experiment were not used in turbulence calculation. As the wires had a length to diameter ratio of 200, k = 0.2 was assumed, as suggested by Champagne (8).

B6. Measurement and Computation of one dimensional
Energy Spectra:

Consider, for instance, the component u of the turbulence fluctuation of velocity in a fixed point of the flow field. We assume that this field is quasi steady so that it is statistically homogeneous with respect to time. There then exists a constant average value of u^2 which can be considered to consist of the sum of the contributions of all the frequencies n. Let E_1 (n) dn be the contribution to $\overline{u^2}$ of the frequencies between n and n + dn, the distribution function E_1 (n) then has to satisfy the condition (19, p.54)

$$\int_0^\infty dn E_1(n) = \overline{u^2}$$

It is the general practice to consider the wave number $k_1 = 2\pi n/\overline{U}$ instead of n and to use the energy spectrum function $E_1(k_1)$ instead of $E_1(n)$, so that $E_1(k_1)$ will be defined as (19, p.166)

$$E_1(k_1) = E_1(n)\ \overline{U}/2\pi$$

and therefore $\int_0^\infty dk_1 E_1(k_1) = \overline{u^2}$

Similarly $\int_0^\infty dk_1 E_2(k_1) = \overline{v^2}$

and $\int_0^\infty dk_1 E_3(k_1) = \overline{w^2}$

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We can write

$$d u^2 = dk_1 E_1(k_1)$$

but $dk_1 = 2\pi dn/\bar{U}$

$\therefore$ $d u^2 = E_1(k_1) . 2\pi dn/\bar{U}$

$\therefore$ $E_1(k_1)/\overline{u^2} = \bar{U} \, d\overline{u^2}/\overline{u^2} . 2\pi dn$

where $d\overline{u^2}/\overline{u^2} = d\overline{e^2}/\overline{e^2}$

$$= \frac{\text{RMS voltage at frequency n for a band dn}}{\text{total RMS voltage}}$$

The spectrum curves are plotted with $E_1(k_1) /\overline{u^2}$,

$E_2(k_1) /\overline{v^2}$ and $E_3(k_1) /\overline{w^2}$ against k_1 for u', v' and w'

respectively.

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APPENDIX C

CALIBRATION OF THE INSTRUMENTS

C1. Micromanometer:

Wilhelm Lambrecht slanting tube micromanometer
No. 655 with pitot static probe was used for measuring
the dynamic pressure to give the mean velocity of flow
to calibrate the hot wire anemometer. The micromano-
meter was filled with normal butyl alcohol the specific
gravity of which was given to be 0.81 at normal temper-
ature and pressure. The micromanometer tube was grad-
uated from 0 to 200 mm with divisions of 1 mm each. The
tube could be fixed in four slanting positions with the
slopes of 1:2, 1:5, 1:10 and 1:25. With the graduated
tube at 1:10 and at 1:25 this micromanometer was calib-
rated against Flow Corporation model MM3 micromanometer.
Flow Corporation micromanometer was filled with normal
butyl alcohol. The alcohol was supplied by Flow Corp-
oration and, according to them, in the temperature range
$50^\circ F$ to $100^\circ F$, its density could be given to 0.03% by

$$\text{density (gm/cc)} = 0.8176 - 0.0004(t^\circ F - 50)$$

The micromanometer could be read to $\pm.0002''$ of the liquid
column over a 2" range.

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For calibration purposes both of the manometers
were connected to the same pitot static probe. The pitot
static probe was exposed to the flow in the wind tunnel.
The flow velocity could be controlled by regulating the
wind tunnel fan. The readings of the two meters were re-
corded similtaniously at a number of velocities. The dyn-
amic pressure in inches of water was calculated from Flow
Corporation manometer readings and it was plotted against
the Wilhelm Lambrecht micromanometer readings in mm. The
calibration data is plotted in figure 43. The best fitt-
ing lines were determined by least square regression method.

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C2. D.C. Voltmeters on the hot wire anemometers:

Two sets of DISA Constant temperature hot wire anemometers (numbered as MEC97 and MEC116) were used in the experimentation. The d.c. voltmeters on these anemometers were accurate to ±1%. These voltmeters were caliberated against Hewlett Packard digital voltmeter 3440A which could read the d.c. voltage upto 4 significant figures in various ranges from 100 mV to 1000 volts. For caliberation, connections were provided between anemometer and the digital voltmeter so that both of them could simultaneously receive the signal from the hot wire probe exposed to the regulated flow in the wind tunnel. The two anemometer voltmeters were caliberated seperately. Because of observable fluctuations in the digital voltmeter readings, lowest and highest observed values were recorded from that against a mean value on anemometer voltmeter. The following tables give some of the readings obtained for calibration.

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DC Volts from Anemometer	DC Volts from digital voltmeter		$\frac{(B+C)/2 - A}{A} \times 100$
	highest	lowest	
(A)	(B)	(C)	

Anemometer MEC116

5.53	5.679	5.675	2.66
5.86	6.022	6.011	2.67
6.44	6.626	6.604	2.72
6.79	6.980	6.967	2.70
7.05	7.234	7.227	2.56
7.19	7.374	7.358	2.45
7.30	7.494	7.464	2.45
7.50	7.704	7.686	2.60
7.68	7.868	7.851	2.34

Anemometer MEC97

5.41	5.577	5.566	2.99
5.51	5.698	5.666	3.12
5.97	6.187	6.143	3.27
6.24	6.444	6.406	2.96
6.49	6.704	6.661	2.97
6.70	6.906	6.855	2.69
6.82	7.051	7.013	3.11
7.12	7.345	7.321	2.99

The above tables show that both of the anemometer volt-meters were showing lower values when compared to the digital voltmeter which was considered to be more accurate and reliable. From the tables it could also be infered that both of the anemometer voltmeters were quite in agreement with each other. Therefore the digital voltmeter was checked with the other available meters. This checking could be made in lower ranges than that used in the above tables. The digital voltmeter was

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found to give higher readings upto 2%. Therefore the readings on anemometer voltmeters were directly used for determining various quantities.

C3. RMS Voltmeters on various instruments:

As mentioned in the section on instrumentation, the following instruments were needed for the measurement of various turbulence quantities.

DISA constant temperature hot wire anemometer
55 A 01 (two sets numbered as MEC97 and MEC116)

DISA random signal indicator and correlator
55 A 06

Bruel and Kjaer audio frequency spectrometer
Type 2112

Every one of the above instruments was provided with RMS voltmeters which could be used in various ranges with an accuracy of ±1% at full scale deflection. All RMS voltmeters were checked with each other and with Bruel and Kjaer random noise voltmeter type 2417. Some of the calibration readings are being presented here.

RMS mV from Anemometer (B)	RMS mV from random noise voltmeter (A)	$\dfrac{B-A}{A} \times 100$
Anemometer MEC116		
7.2	7.0	2.86
9.5	9.6	-1.04
14.3	14.4	-0.69
20.9	20.9	0.00
24.8	24.9	-0.40
30.0	30.5	-1.16
34.2	34.6	-1.16
Anemometer MEC97		
11.0	14.0	-21.43
14.0	17.0	-17.65
15.9	19.3	-17.62
18.1	21.7	-16.59
21.2	25.8	-17.83
23.0	29.5	-22.03
26.0	32.1	-19.00
28.4	35.5	-25.00

RMS mV from Correlator (B)	RMS mV from Anemometer MEC116 (A)	$\dfrac{B-A}{A} \times 100$
8.8	8.8	0.00
10.7	10.6	0.94

RMS mV from Spectrometer (B)	RMS mV from Anemometer MEC116 (A)	$\dfrac{B-A}{A} \times 100$
4.10	3.72	10.22
4.20	3.95	6.33
4.90	4.40	11.36

From the above values we can infer that the readings given by the random noise voltmeter, the anemometer MEC116 and the correlator are in agreement. The anemometer MEC97 and

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the spectrometer give considerably different values. To avoid the discrepencies and because of high fluctuations in RMS milli volts, especially in the spectrometer, all RMS signals were measured on the random noise voltmeter in which sufficient damping could be achieved.

APPENDIX D

ENERGY EQUATION

The Navier Stokes equation of an imcompressible fluid for the case of steady main motion (19, p.250) reads

$$\frac{\partial u_i}{\partial t} + (\bar{U}_k + u_k) \frac{\partial}{\partial x_k} (\bar{U}_i + u_i) = -\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x_i} - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2}{\partial x_l \partial x_l} (\bar{U}_i + u_i) \qquad (3.1)$$

The equation of continuity for incompressible flow reads

$$\frac{\partial U_k}{\partial x_k} = 0 \qquad (3.2)$$

But

$$\frac{\partial U_k}{\partial x_k} = \frac{\partial \bar{U}_k}{\partial x_k} + \frac{\partial u_k}{\partial x_k} = \frac{\partial u_k}{\partial x_k}$$

Adding $u_i \dfrac{\partial u_k}{\partial x_k} = 0$ to the left-hand side of equation (3.1), we get

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial \bar{U}_i}{\partial x_k} + \bar{U}_k \frac{\partial u_i}{\partial x_k} + \frac{\partial}{\partial x_k} u_i u_k$$

$$= -\frac{1}{\rho} \frac{\partial P}{\partial x_i} - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2}{\partial x_l \partial x_l} (\bar{U}_i + u_i)$$

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Subtracting from this its average value, we obtain

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial \bar{U}_i}{\partial x_k} + \bar{U}_k \frac{\partial u_i}{\partial x_k} + \frac{\partial}{\partial x_k}(u_i u_k - \overline{u_i u_k})$$

$$= -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial u_i}{\partial x_1 \partial x_1} \qquad (3.3)$$

For the velocity component u_j, we can write the similar equation

$$\frac{\partial u_j}{\partial t} + u_k \frac{\partial \bar{U}_j}{\partial x_k} + \bar{U}_k \frac{\partial u_j}{\partial x_k} + \frac{\partial}{\partial x_k}(u_j u_k - \overline{u_j u_k})$$

$$= -\frac{1}{\rho}\frac{\partial p}{\partial x_j} + V \frac{\partial^2 u_j}{\partial x_1 \partial x_1} \qquad (3.4)$$

Multiplying (3.3) by u_j and (3.4) by u_i and adding the two equations, we get

$$\frac{\partial}{\partial t} u_i u_j + u_k u_j \frac{\partial \bar{U}_i}{\partial x_k} + u_k u_i \frac{\partial \bar{U}_j}{\partial x_k} + \bar{U}_k \frac{\partial u_i u_j}{\partial x_k} + \frac{\partial u_i u_j u_k}{\partial x_k}$$

$$- u_j \frac{\partial}{\partial x_k} u_i u_k - u_i \frac{\partial}{\partial x_k}\overline{u_j u_k} = -\frac{1}{\rho}\left\{ u_j \frac{\partial p}{\partial x_i} + u_i \frac{\partial p}{\partial x_j}\right\}$$

$$+ \nu \frac{\partial^2}{\partial x_1 \partial x_1}\overline{u_i u_j} - 2\nu \frac{\partial u_i \partial u_j}{\partial x_1 \partial x_1}$$

After averaging all the quantities, we obtain

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$$\frac{\partial}{\partial t}\,\overline{u_i u_j} + \overline{u_k u_j}\,\frac{\partial \bar{U}_i}{\partial x_k} + \overline{u_k u_i}\,\frac{\partial \bar{U}_j}{\partial x_k} + \bar{U}_k\,\frac{\partial}{\partial x_k}\,\overline{u_i u_j} + \frac{\partial}{\partial x_k}\,\overline{u_i u_j u_k}$$

$$= -\frac{1}{\rho}\left(\overline{u_j\,\frac{\partial p}{\partial x_i}} + \overline{u_i\,\frac{\partial p}{\partial x_j}}\right) + \nu\frac{\partial^2}{\partial x_1 \partial x_1}\,\overline{u_i u_j} - 2\nu\,\overline{\frac{\partial u_i}{\partial x_1}\frac{\partial u_j}{\partial x_1}}$$

A contraction gives

$$\frac{\partial}{\partial t}\,\overline{u_i u_i} + 2\overline{u_k u_i}\,\frac{\partial \bar{U}_i}{\partial x_k} + \bar{U}_k\,\frac{\partial}{\partial x_k}\,\overline{u_i u_i} + \frac{\partial}{\partial x_k}\,\overline{u_i u_i u_k}$$

$$= -\frac{1}{\rho}\,\overline{u_i\,\frac{\partial p}{\partial x_i}} + \nu\,\frac{\partial^2}{\partial x_1 \partial x_1}\,\overline{u_i u_i} - 2\nu\,\overline{\frac{\partial u_i}{\partial x_1}\frac{\partial u_i}{\partial x_1}}$$

The statistical average values are assumed to be in
steady state, therefore

$$\bar{U}_k\,\frac{\partial}{\partial x_k}\,\overline{u_i u_i} = -2\,\overline{u_k u_i}\,\frac{\partial \bar{U}_i}{\partial x_k} - \frac{\partial}{\partial x_k}\,\overline{u_i u_i u_k} - \frac{1}{\rho}\,\overline{u_i\frac{\partial p}{\partial x_i}}$$

$$+ \nu\,\frac{\partial^2}{\partial x_1 \partial x_1}\,\overline{u_i u_i} - 2\nu\,\overline{\frac{\partial u_i}{\partial x_1}\frac{\partial u_i}{\partial x_1}}$$

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APPENDIX E

SUMMARY OF ERRORS

The following lines give an account of the analysis of the appreciable errors in the measurement of mean velocity and the turbulence intensities.

(i) Mean Velocity: The accuracies of the instruments were as follows:

Wilhelm Lambrecht slanting tube micromanometer, $\pm 0.5mm = dh$

D.C. voltmeter with the hot wire anemometer, $\pm 1\% = d\bar{E}/\bar{E}$

The relation used to find the velocity from the manometer readings was

$$\tfrac{1}{2} \rho_{air} U^2 = \triangle P$$

for a given temperature and pressure

$$U = a \text{ constant } x\sqrt{h}$$

$$dU/U = (\text{constant}/2)(dh/U^2)$$

Assuming h as the readings in mm. with the manometer tube having a slope of 1:5, at a temperature of 80^0F and a pressure of 29.9 in. mercury,

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constant $\simeq 5.4$

$$dU/U = 2.7\ ^{dh}/U^2$$

In the experiments $U > 25$ ft/sec

$$\therefore \quad dU/U < 0.0028 \text{ or } 0.28\%$$

Hot wire response is given by the equation (B6)

$$\bar{E}^2 = X + Y\ \bar{U}^a$$

$$d\bar{U}/\bar{U} = d\bar{E}^2 / a(\bar{E}^2 - X).$$

To find the mean velocity by hot wire, $\bar{E}$ was measured by the anemometer and then the above equation (obtained by calibration) was used to find $\bar{U}$. Therefore maximum error in $\bar{U}$ is given by

$$\left| d\bar{U}/\bar{U} \right| = \left\{ \left| d\bar{E}^2_{reading} \right| + \left| d\bar{E}^2_{calibration} \right| \right\} / a(\bar{E}^2 - X)$$

In calibration of the four hot wire probes used in experiments the r.m.s. error in $\bar{E}^2 < 0.105$, Y had values between 11.4 and 14.0, X between 20 and 22 and $a = 0.35$. Considering a value of $\bar{E} = 8$ volts which correspond to $\bar{U} \simeq 25$ ft/sec, error in reading is given by

$$d\bar{E}^2 = 2\bar{E}^2 \left| d\bar{E}/\bar{E} \right| = 128 \times 0.01 = 1.28$$

$$\therefore \left| d\bar{U}/\bar{U} \right| < (1.28 + 0.105)/\{0.35(64 - 22)\} < 0.0942, \text{ say } \pm 10\%$$

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Similarly considering a value of $\bar{E} = 9$ volts which correspond to $\bar{U} \simeq 60$ ft/sec, we get

$$\left| d\bar{U}/\bar{U} \right| < 0.089, \text{ say } \pm 9\%$$

(ii) Longitudenal Component of Fluctuating Velocity:
The accuracy of the r.m.s. voltmeter was $\pm 1\%$ at full scale deflection. Assume a value of $\pm 4\%$ as the accuracy for normal deflections and to take into account the effect of fluctuations during observations.

$$de'^2/e'^2 = \pm 0.08$$

Consider the equation (ref. eqn. B17 and B27)

$$u'^2/\bar{U}^2 = e'_x{}^2/(S_n^2 . \bar{U}^2)$$

$$d(u'^2/\bar{U}^2)/(u'^2/\bar{U}^2) = (de'_x{}^2/e'_x{}^2) - d(S_n^2.\bar{U}^2)/S_n^2\bar{U}^2$$

Now $\qquad S_n = aY^{1/a} \Big/ 2\bar{E}(\bar{E}^2-X)^{(1-a)/a}$

and $\qquad \bar{U}^2 = \left\{ (\bar{E}^2-X)/.Y \right\}^{2/a}$

$$S_n^2\bar{U}^2 = (a^2/4)\left\{ (\bar{E}^2-X)^2/\bar{E}^2 \right\}$$

$\therefore$ error in $S_n^2\bar{U}^2$ is given by

$$d(S_n^2\bar{U}^2)/S_n^2\bar{U}^2 = (a^2/4)\left\{ \left| d(\bar{E}^2-X)^2/(\bar{E}^2-X)^2 \right| + \left| d\bar{E}^2/\underline{E}^2 \right| \right\}$$

$$= (0.35^2/4)\left\{ \left| d\bar{E}^2/(\bar{E}^2-X)^2 \right| + 0.02 \right\}$$

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Considering a value of $\overline{E} = 8$ volts, and taking $X = 22$, the above expression is negligible.

$$\left| d \ (u'^2/\overline{U}^2)\Big/(u'^2/\overline{U}^2) \right| = \left| d \ e'^2_x\Big/e'^2_x \right|$$

(iii) Lateral Components of Fluctuating Velocity: Considering equations (B18), (B19) and (B27) we can see that

$$\left| d(v'^2/\overline{U}^2)/(v'^2/\overline{U}^2) \right| = \left| d(v'^2/\overline{U}^2)/(w'^2/\overline{U}^2) \right|$$

$$= 2 \left| d(u'^2/\overline{U}^2)/(u'^2/\overline{U}^2) \right| = 0.16 \text{ or } 16\%$$

when C is assumed to be without any error.

The grid turbulence was corrected for the free stream turbulence as mentioned in chapter 5. In free stream turbulence the intensity values were less than 20% of the values in nearly isotropic grid turbulence and less than 50% of the values in strongly anisotropic grid turbulence. Therefore in isotropic grid turbulence

$$\left| d(u'^2/\overline{U}^2)/(u'^2/\overline{U}^2) \right| = 0.08 + (0.08 \times 20/100)$$
$$= 0.096, \text{ say } 10\%$$

similarly
$$\left| d(v'^2/\overline{U}^2)/(v'^2/\overline{U}^2) \right| = 0.192, \text{ say } 20\%$$

In the same way for strongly anisotropic turbulence, the above values will be 12% and 24% respectively.

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(iv) Effect of the Value of k on Measurement of v'^2:
A value of k = 0.20 was used for the slanting hot wire in
the present study as mentioned in Appendix B5. Assuming that
k has got a value between 0.15 and 0.25, let us find the
effect on the value of C.

$$C = SinQcosQ(1-k^2)/sin^2Q+k^2cos^2Q$$

$$= (1-k^2)/(1+k^2) \text{ as } Q = 45° \text{ in the present case}$$

$$\left| dC/C \right| = \left| dk^2/(1-k^2) \right| + \left| dk^2/(1+k^2) \right|$$

$$= (0.02/0.96) + (0.02/1.04)$$

$$\simeq 0.04$$

$$\left| dC^2/C^2 \right| \simeq 0.08 \text{ or } 8\%.$$

In the equations (B18) and (B19) we can see that v'^2 and
w'^2 will have an error of ±8% due to uncertainty in the
value of k by ±25%.

(v) Effect of inaccuracy in the angle Q: The
slanting hot wire was inclined to the mean flow by an
angle, Q = 45°. Assuming that Q = 45°±¼ let us find the
effect on C

$$C = sinQcosQ(1-k^2)/(sin^2Q+k^2cos^2Q)$$

$$\left| dC/C \right| = d \ sinQ/sinQ + d \ cosQ/cosQ + \frac{d \ (sin^2Q+k^2cos^2Q)}{(sin^2Q+k^2cos^2Q)}$$

In the present case Q = 45°

$$\therefore \ dC/C \simeq 4 \ d \ sinQ/sinQ = 4(sin45.25°-sin44.75°)/sin45°$$

$$\simeq 0.0308 \text{ or } 3.1\%$$

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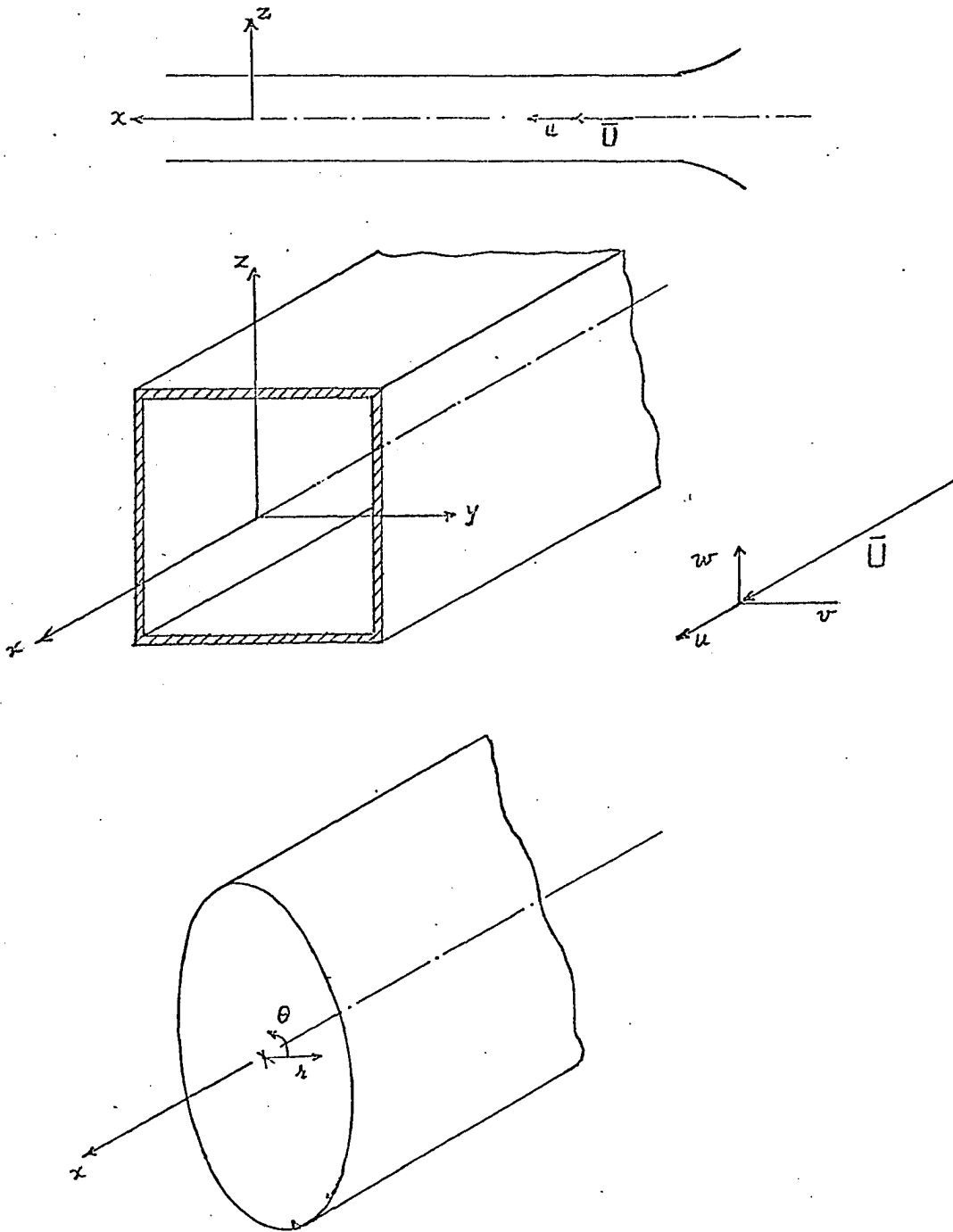
Fig.1 Coordinate system w.r.t. the wind tunnel test
section and the pipe.

Spot welding at the vertical bars(each 3/16 in.wide)

(ii) Dimensions of the grid used at the begining of the contractionfor anisotropic turbulence.

(i) Small section of perforated sheet grid used in present experiments.

Fig.2a
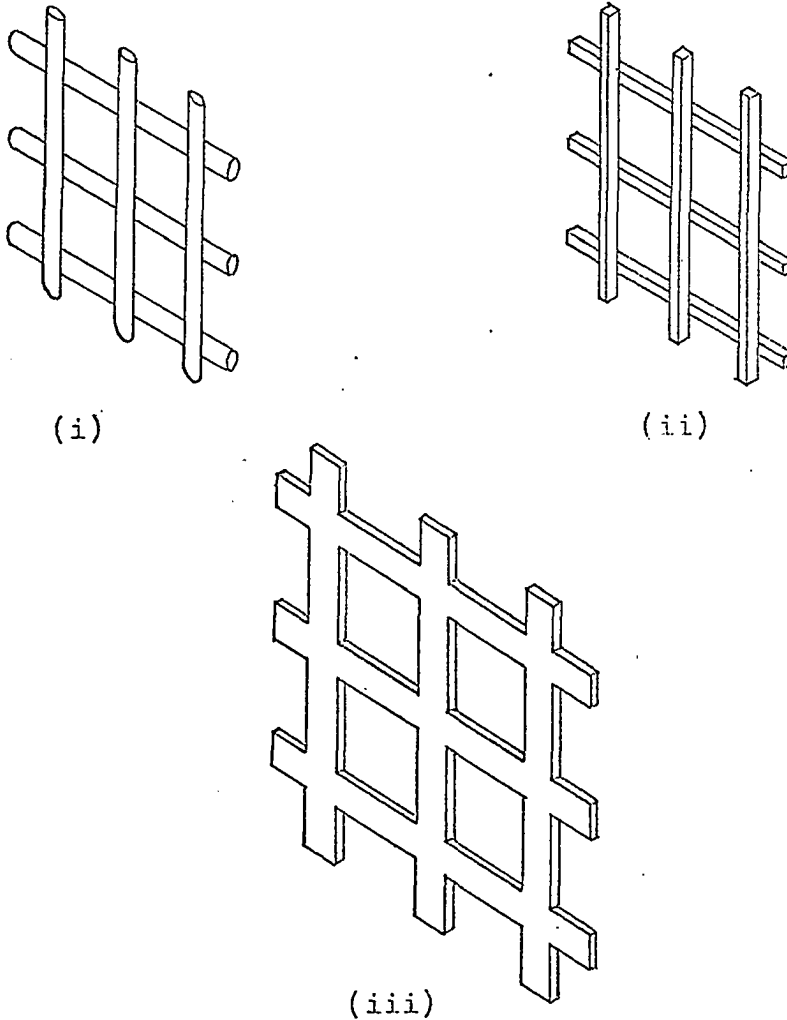
Fig.2b Typical grids for generating approximately
        isotropic turbulence
        (i) Commonly used round rod bi-plane grid
        (ii)Commonly used square rod bi-plane grid
        (iii)Perforated sheet grid used in present
             experiments.

Fig.3

Fig.4

Isotropic turbulence decay
$$\frac{3(\bar{U})^2}{(\bar{q})^2} \text{ vs } \frac{(x)}{(M)}$$

mean velocity	experimental data	least sq. reg. line
27.0ft./sec.	O	-----
35.7ft./sec.	△	———

$.3(\bar{U}/q)^2 \times 10^{-2}$

(x/M)

Fig.5

Fig.6

Anisotropic turbulence decay
Mean velocity=53.3ft./sec.
$(\overline{U}/f)^2$ vs (x/M)

quantity	experimental data	least sq. reg.line
$(\overline{U}/u')^2$	O	——
$(\overline{U}/v')^2$	△	—·—·—
$(\overline{U}/w')^2$	□	——·——·——
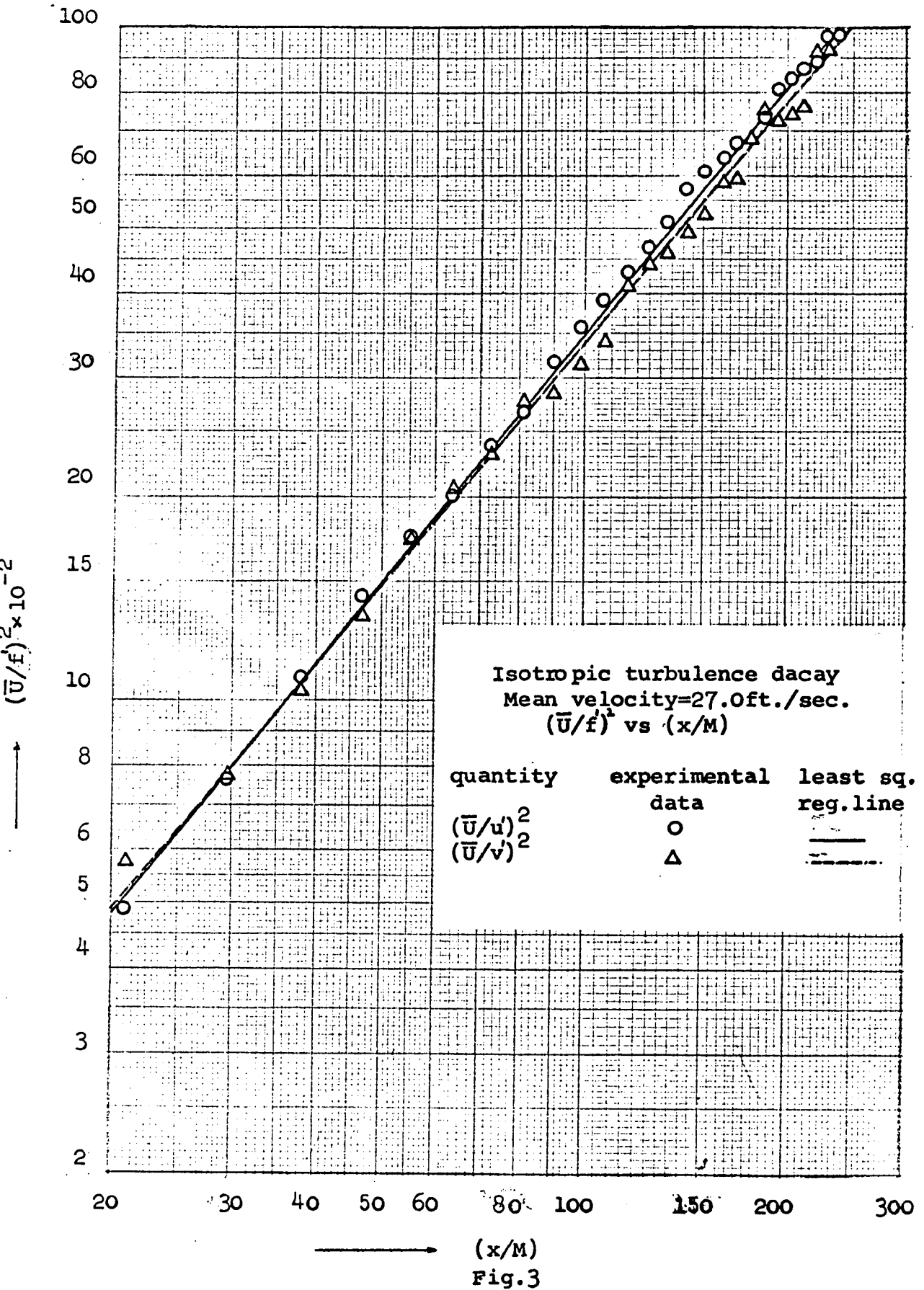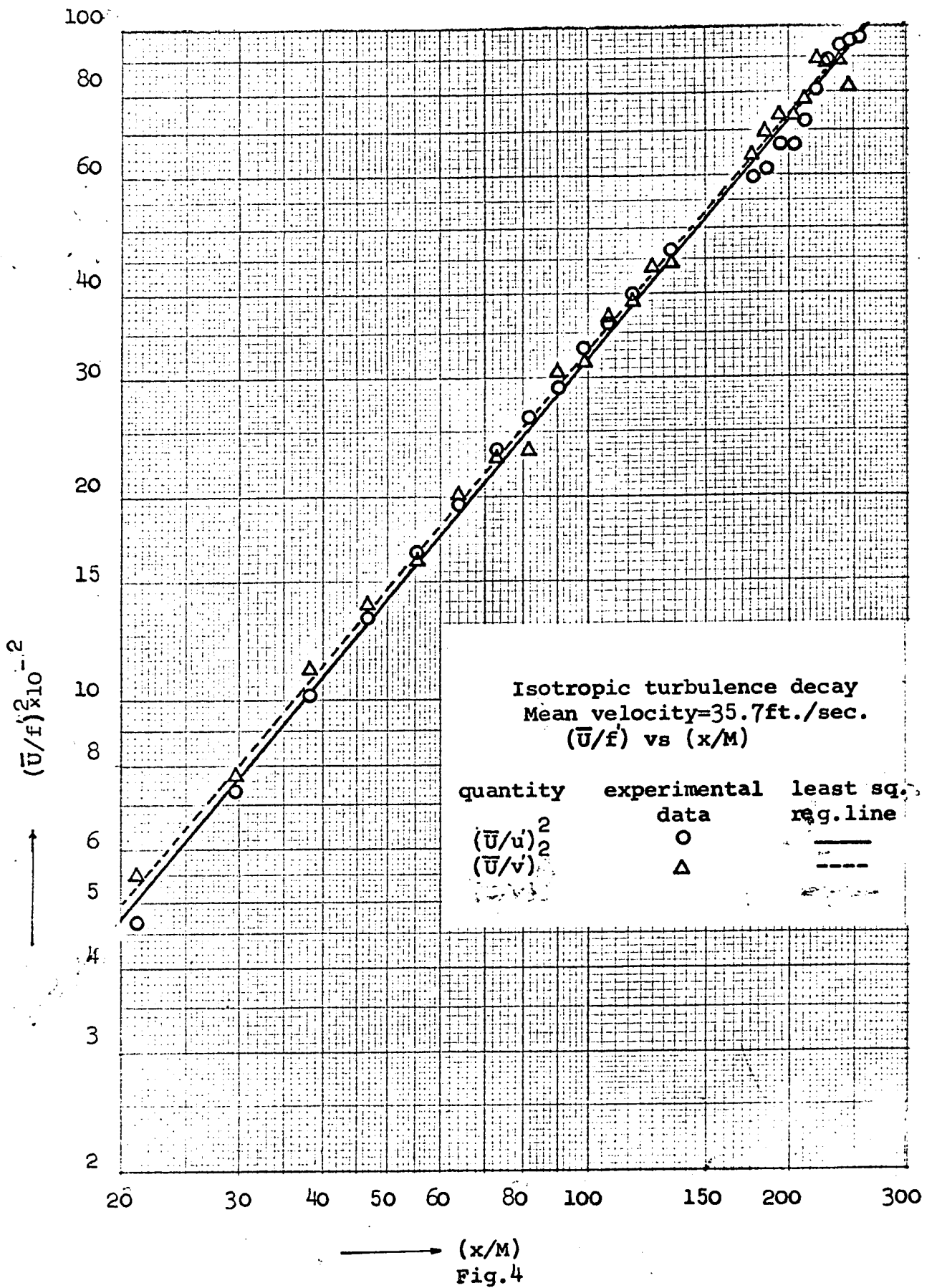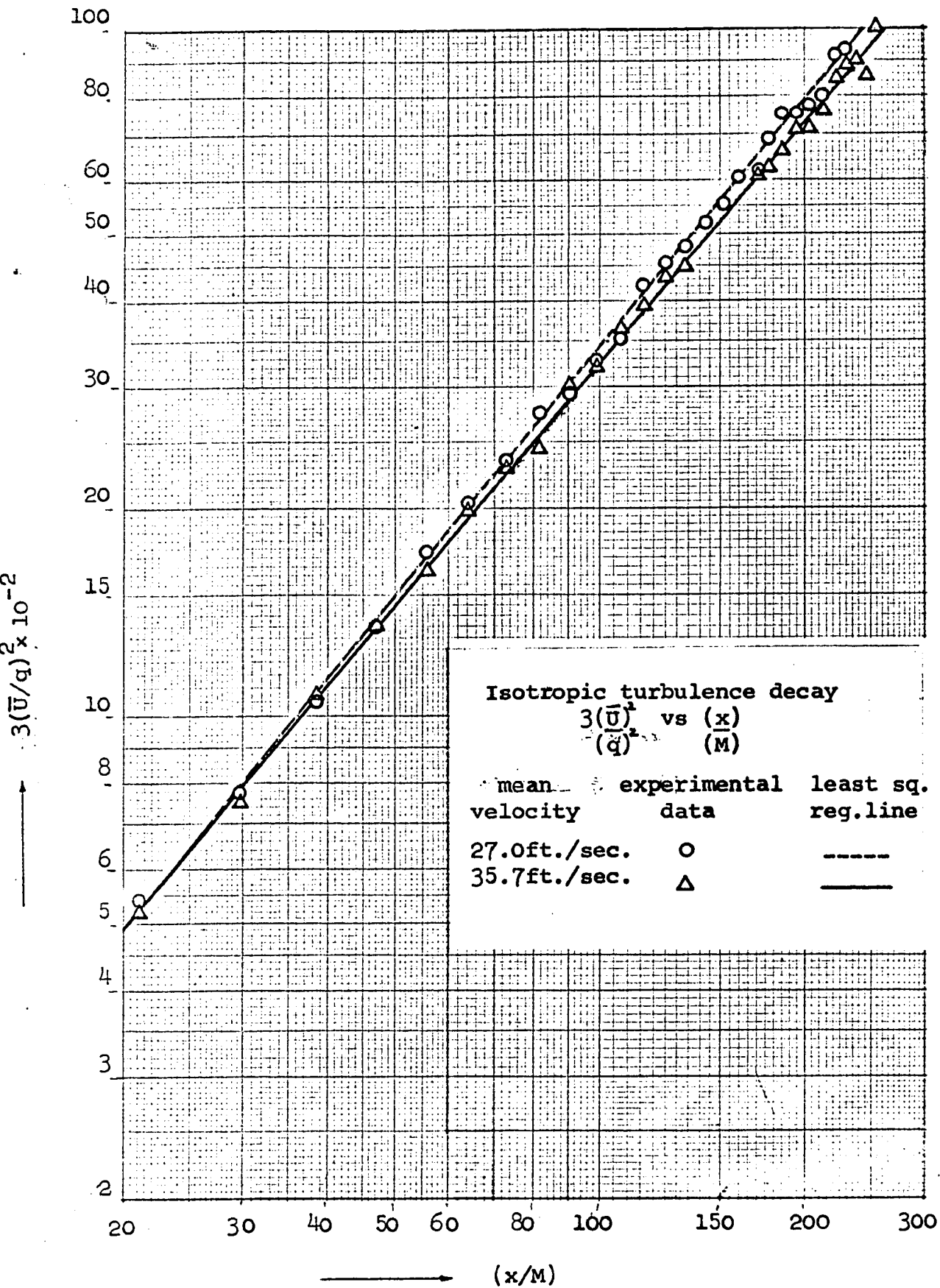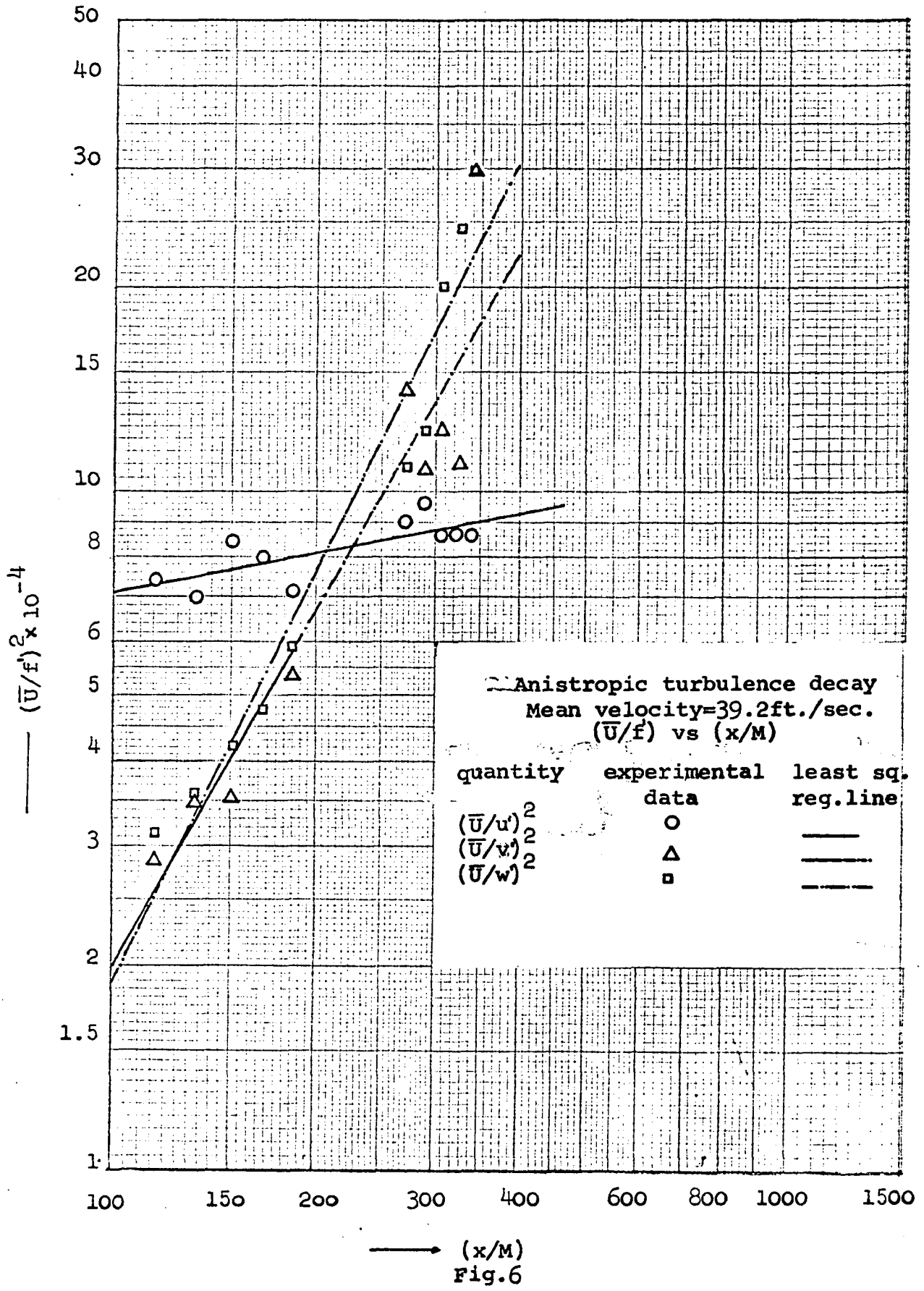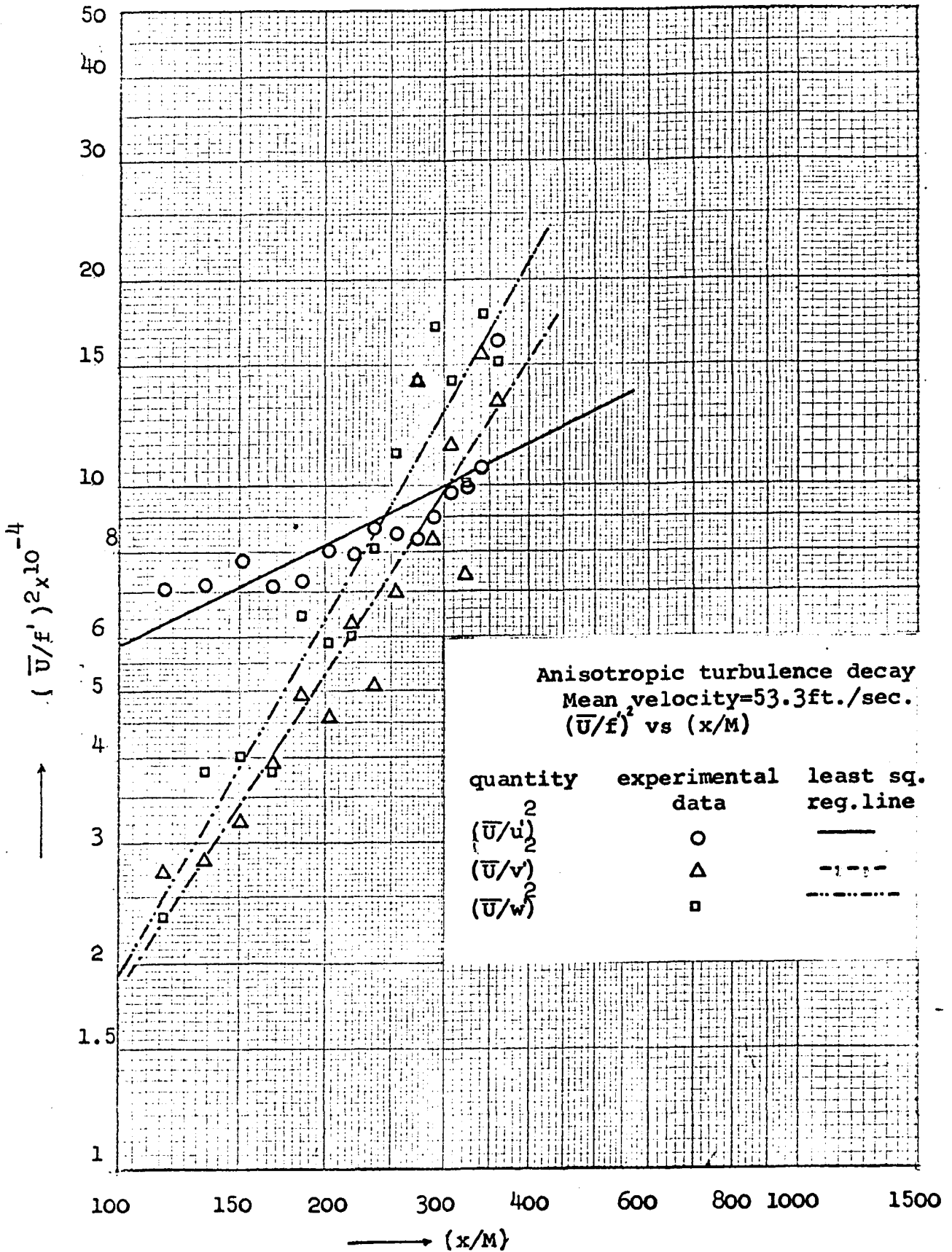
Fig.7
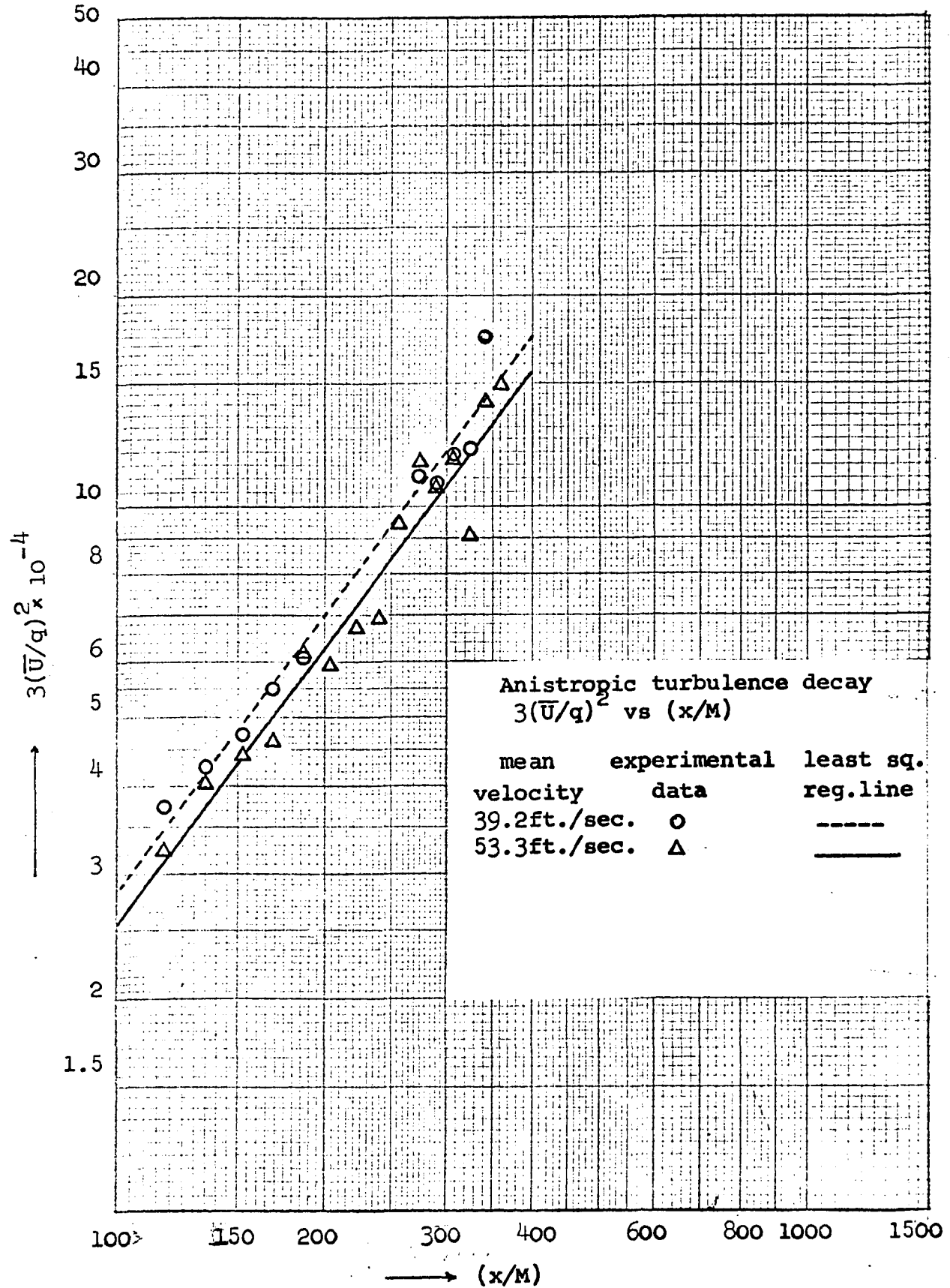
Fig.8
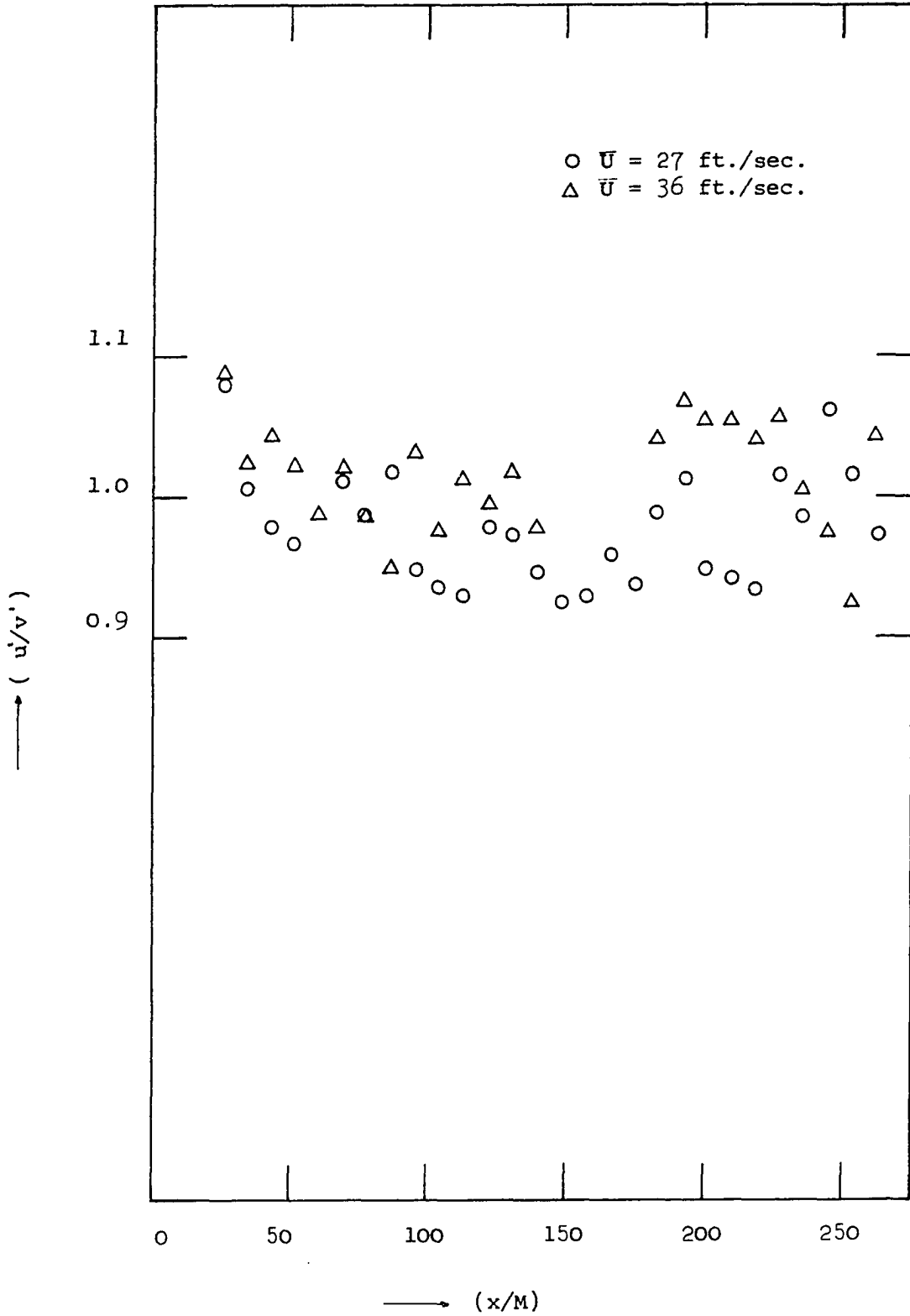
Fig.9 Change in u'/v' in nearly isotropic turbulence
during decay.
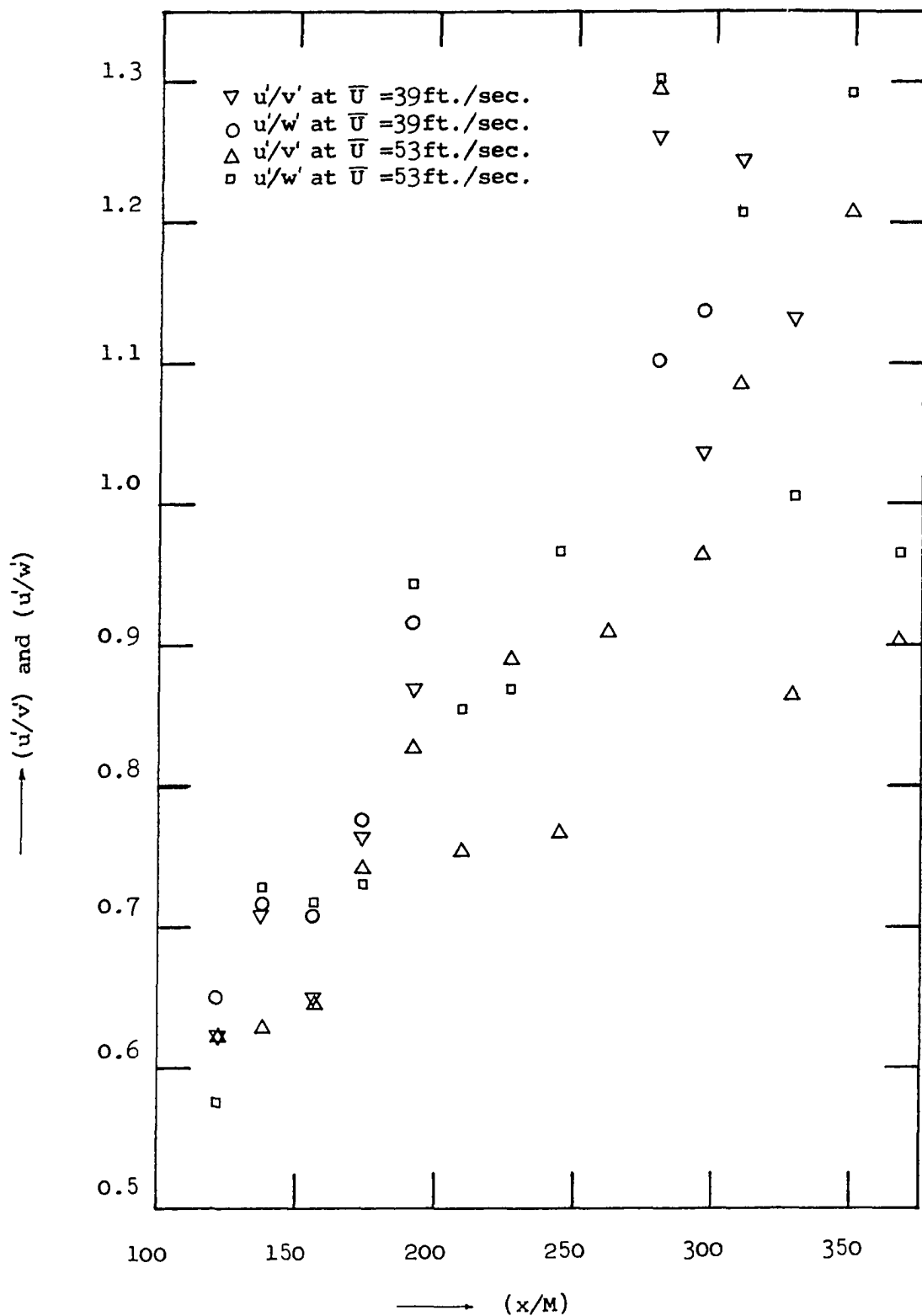
Fig.10 Change in u'/v' and u'/w' in anisotropic Turbulence during decay.
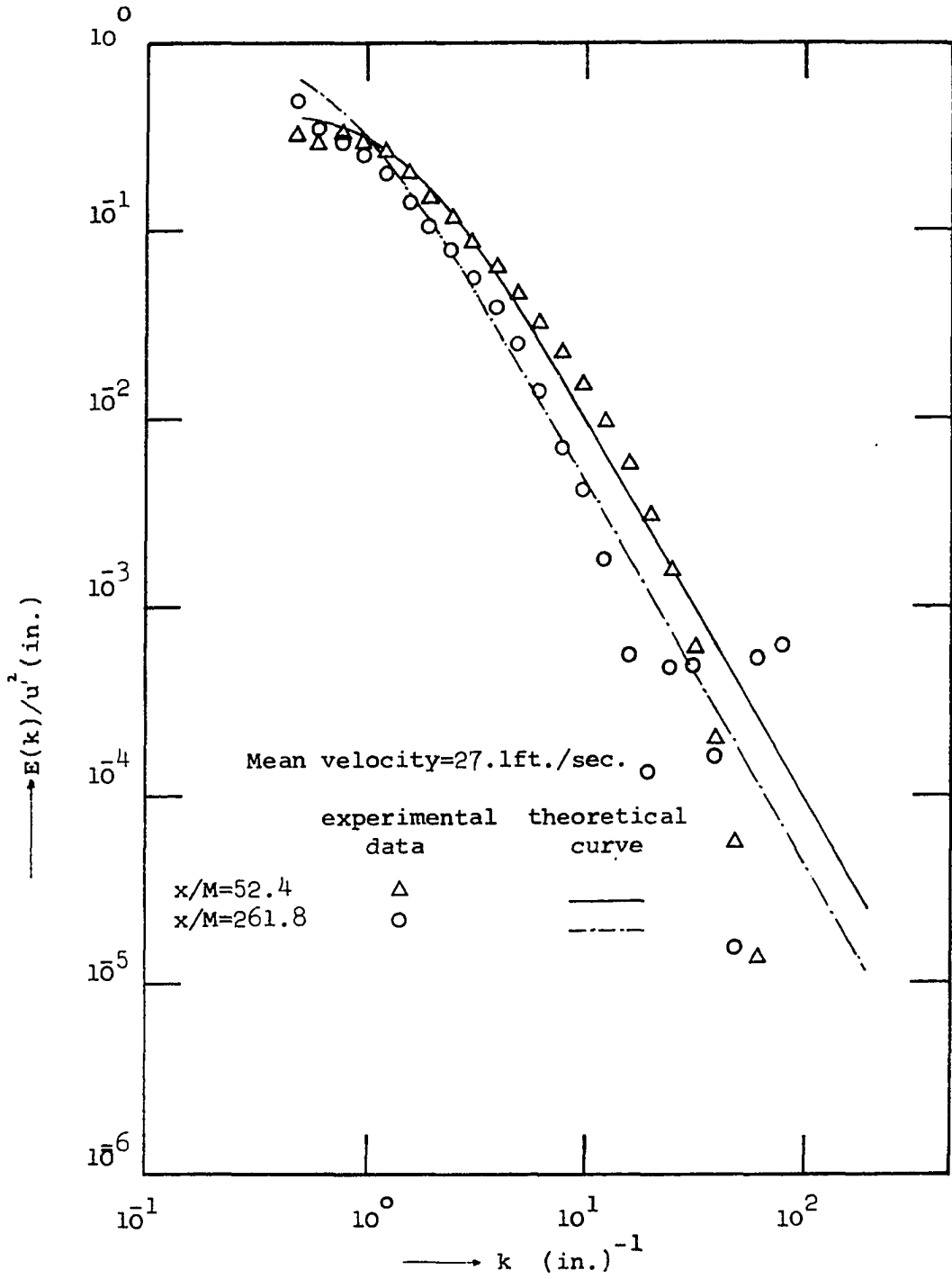
Fig.11 Spectra of u' in nearly isotropic turbulence.
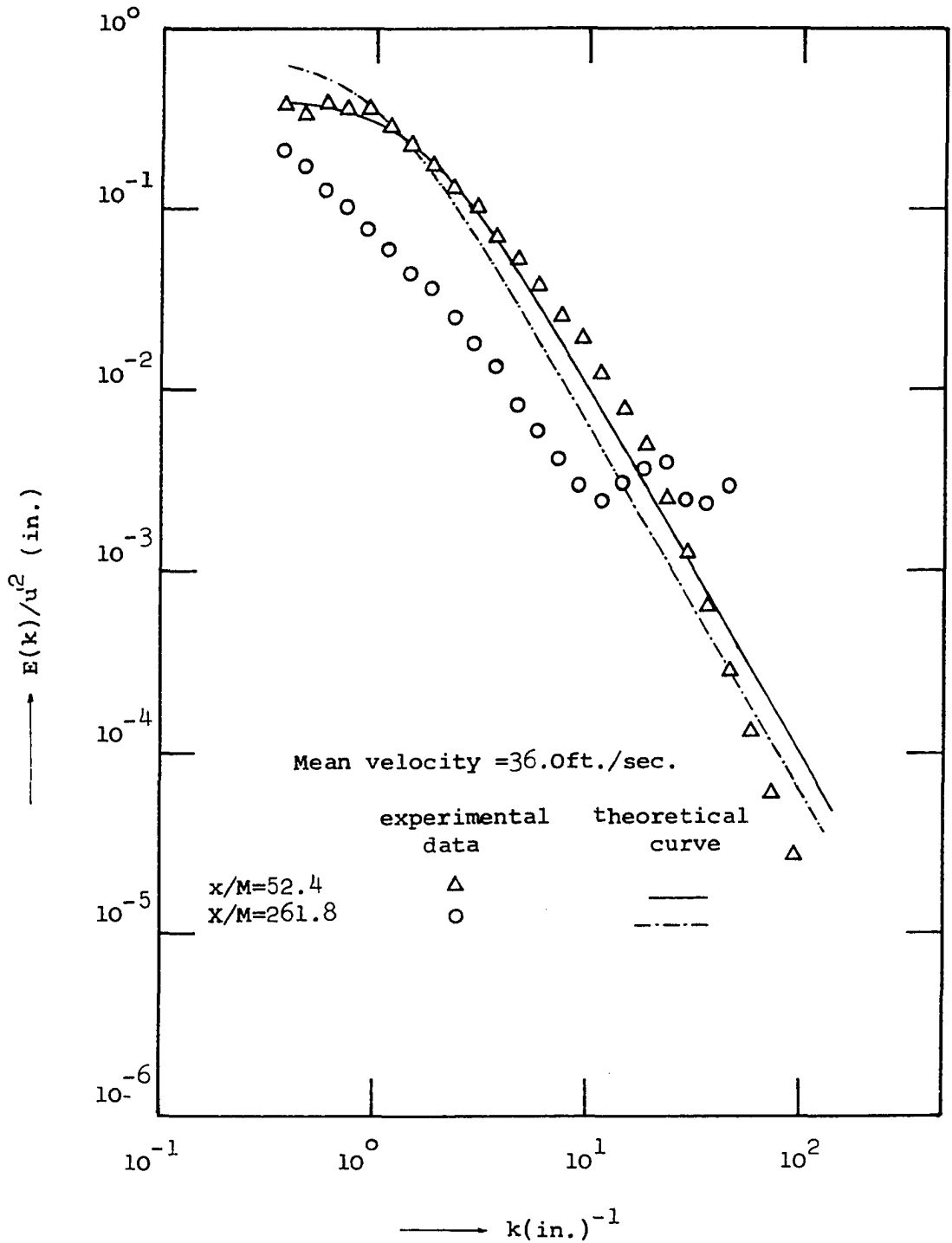
Fig.12 Energy spectra of u′ in isotropic turbulence
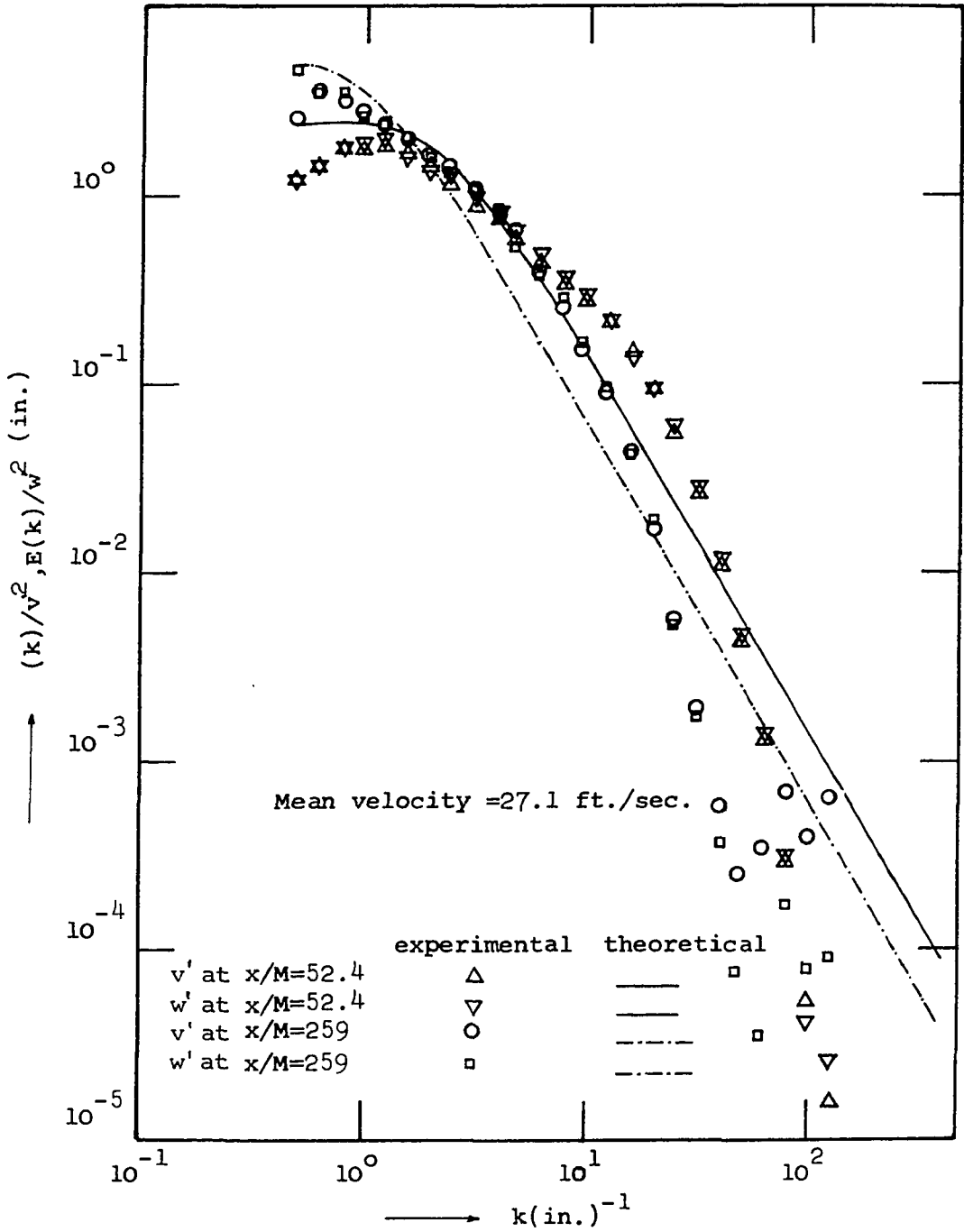
Fig.13 Energy spectra of v' and w' in nearly isotropic turbulence.
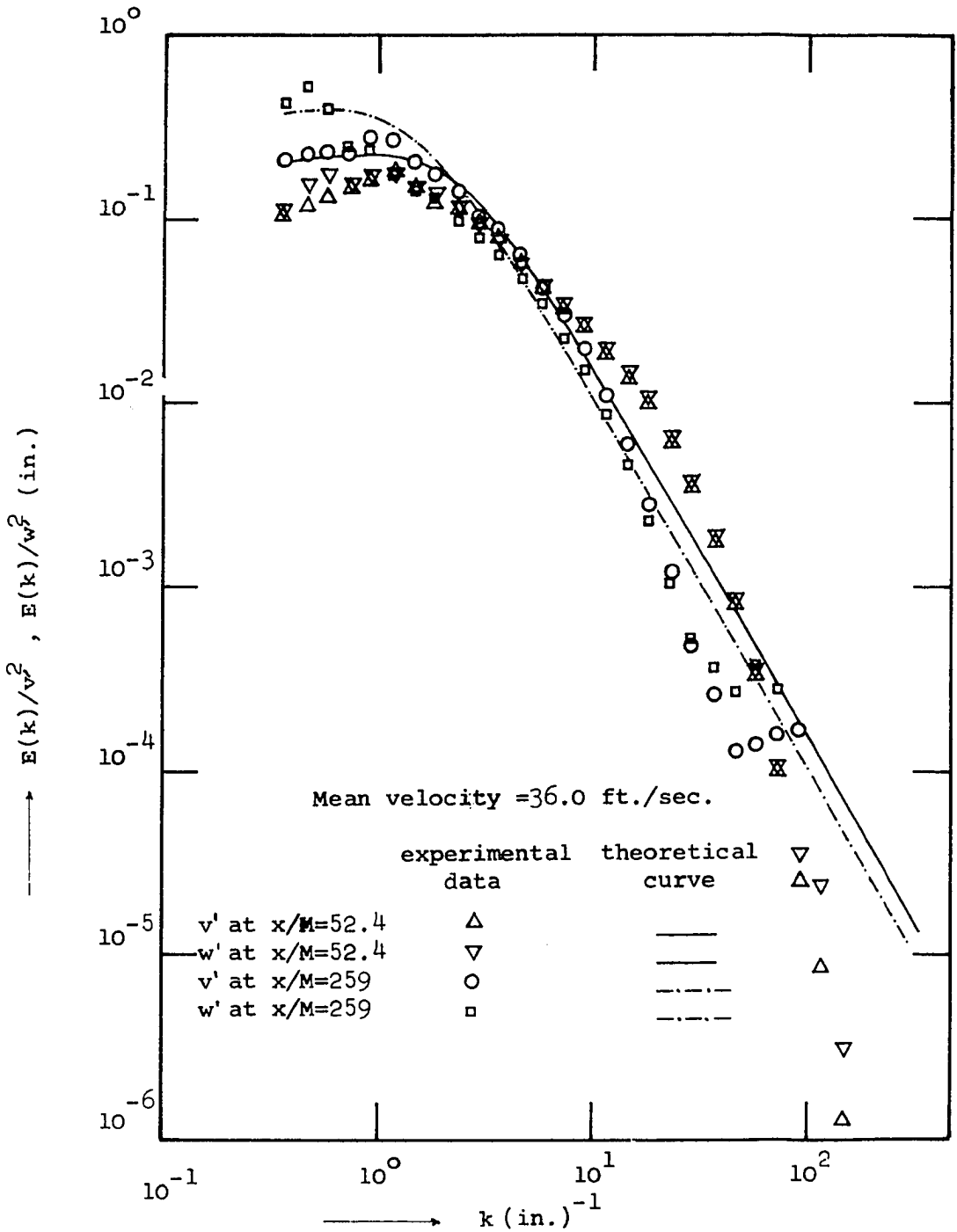
Fig.14 Energy spectra of v and w in nearly isotropic turbulence.

Fig.15 Energy spectra of u' in anisotropic turbulence

Mean velocity = 39.2 ft./sec.

△ x/M =52.4
O x/M =261.8
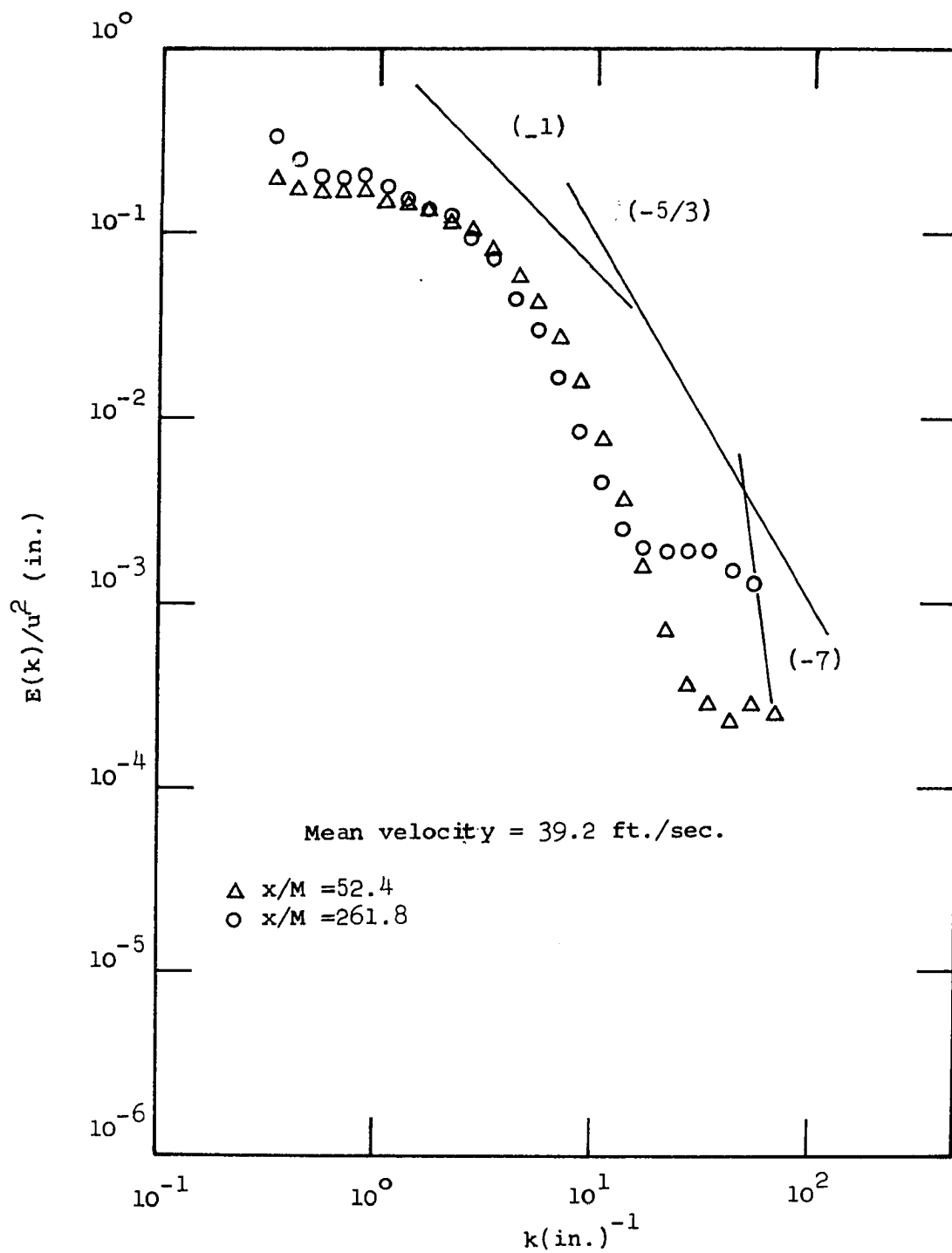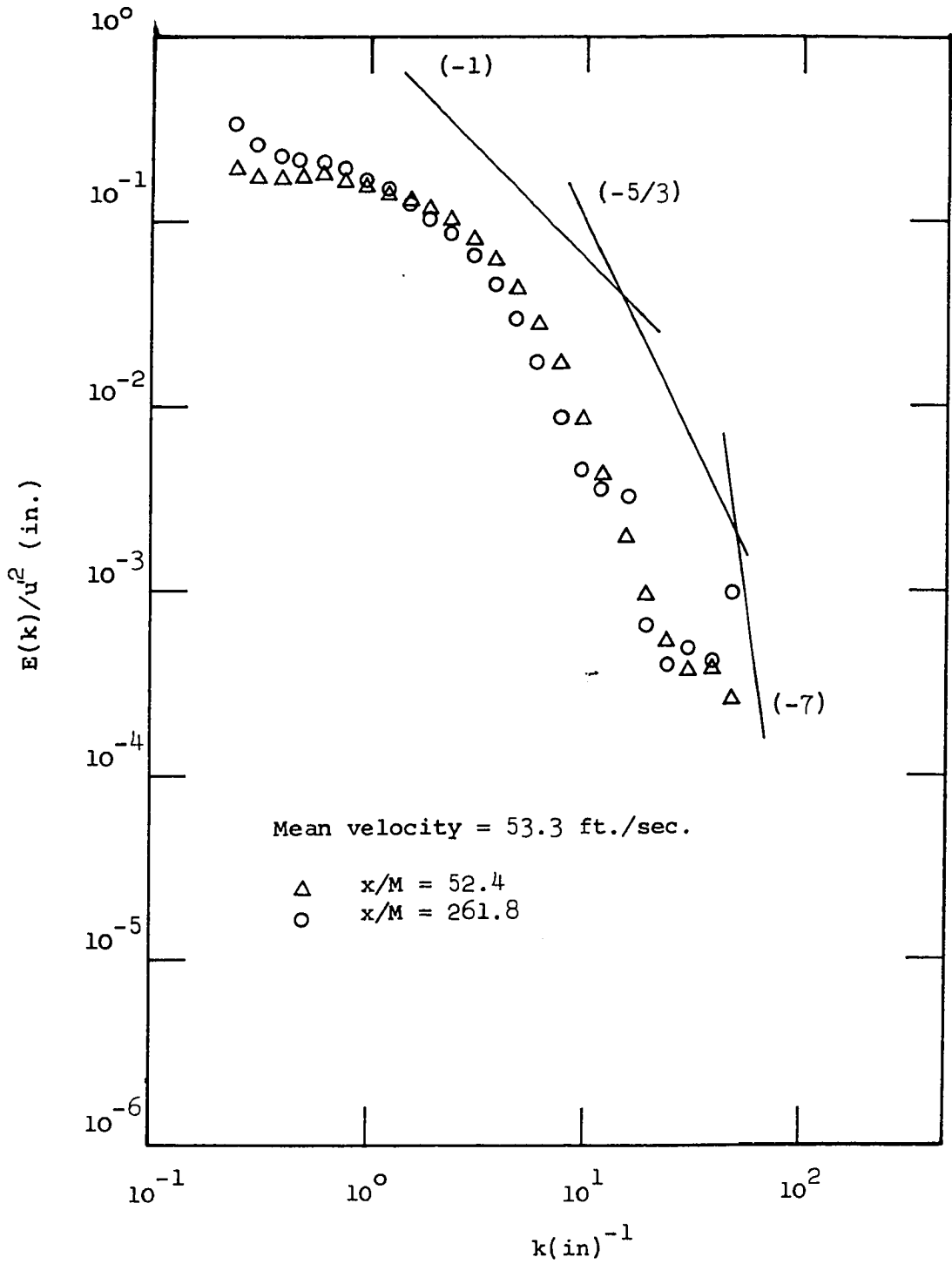
Fig.16 Energy spectra of u′ in anisotropic turbulence.
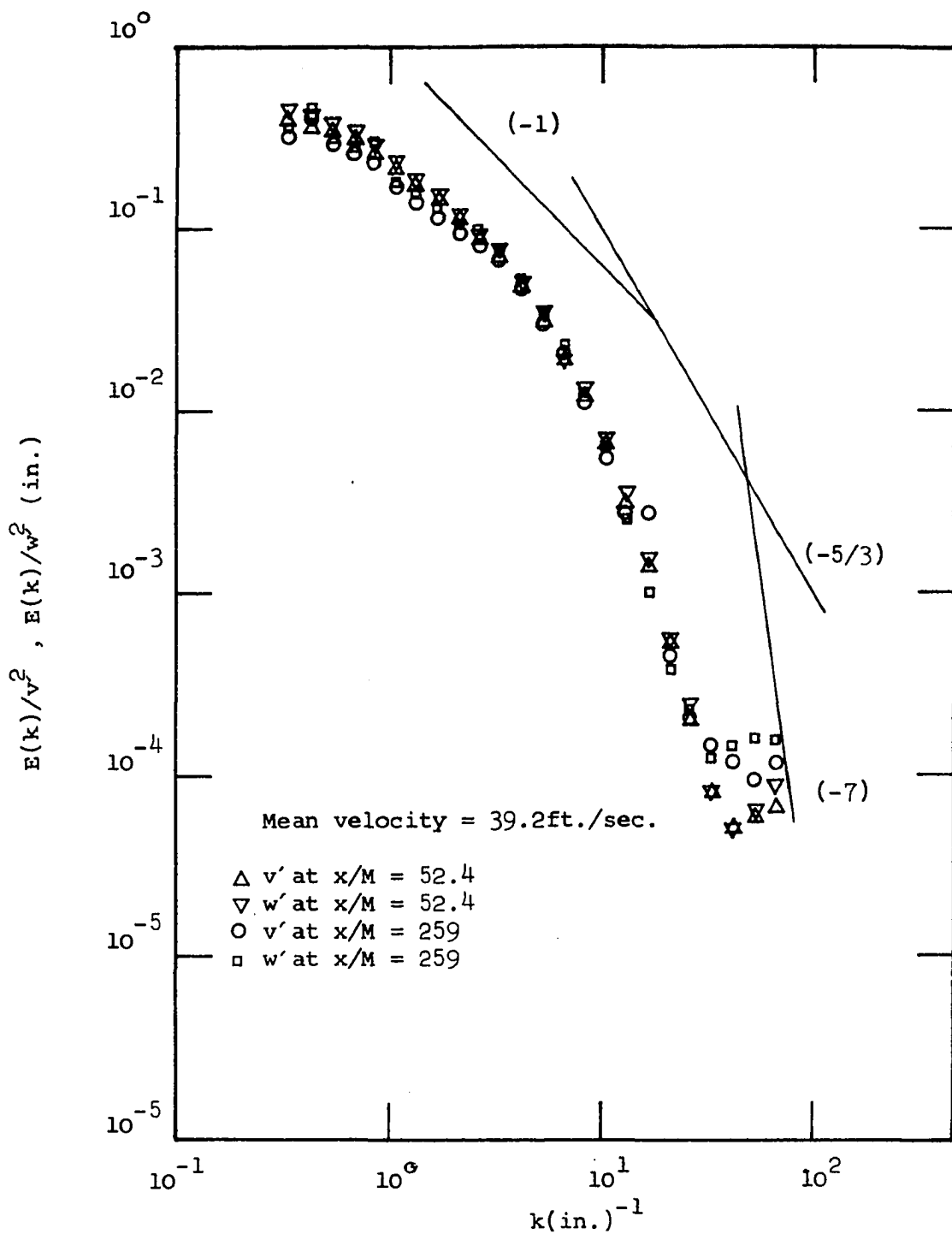
Fig.17 Energy spectra of v´and w´in anistropic turbulence

Fig.18 Energy spectra of v′and w′in anisotropic
turbulence.
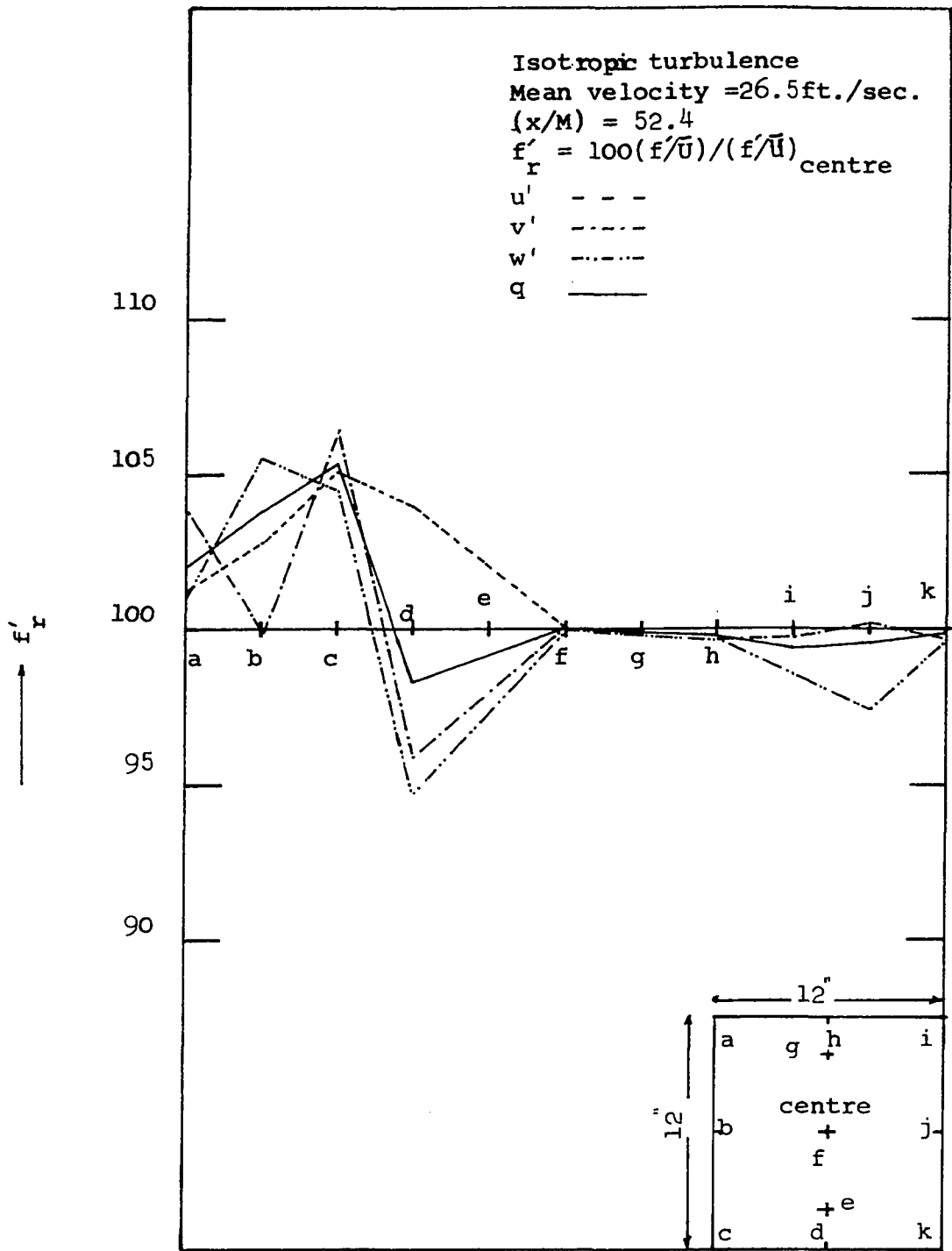
Fig.19 Homogeneity of turbulence in a plane normal
to the direction of flow.
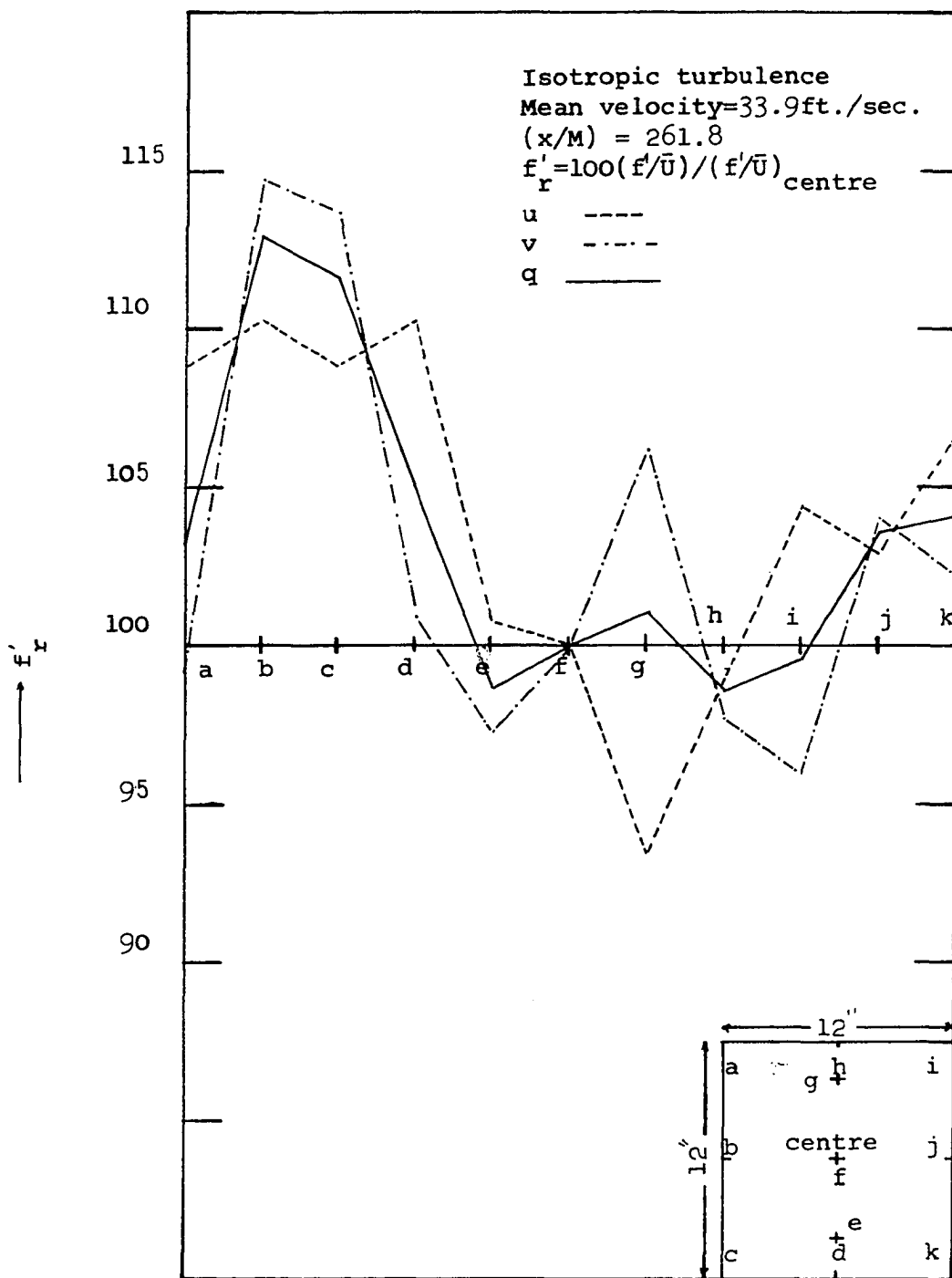
Fig.20 Homogeneity of turbulence in a plane normal
to the direction of flow.
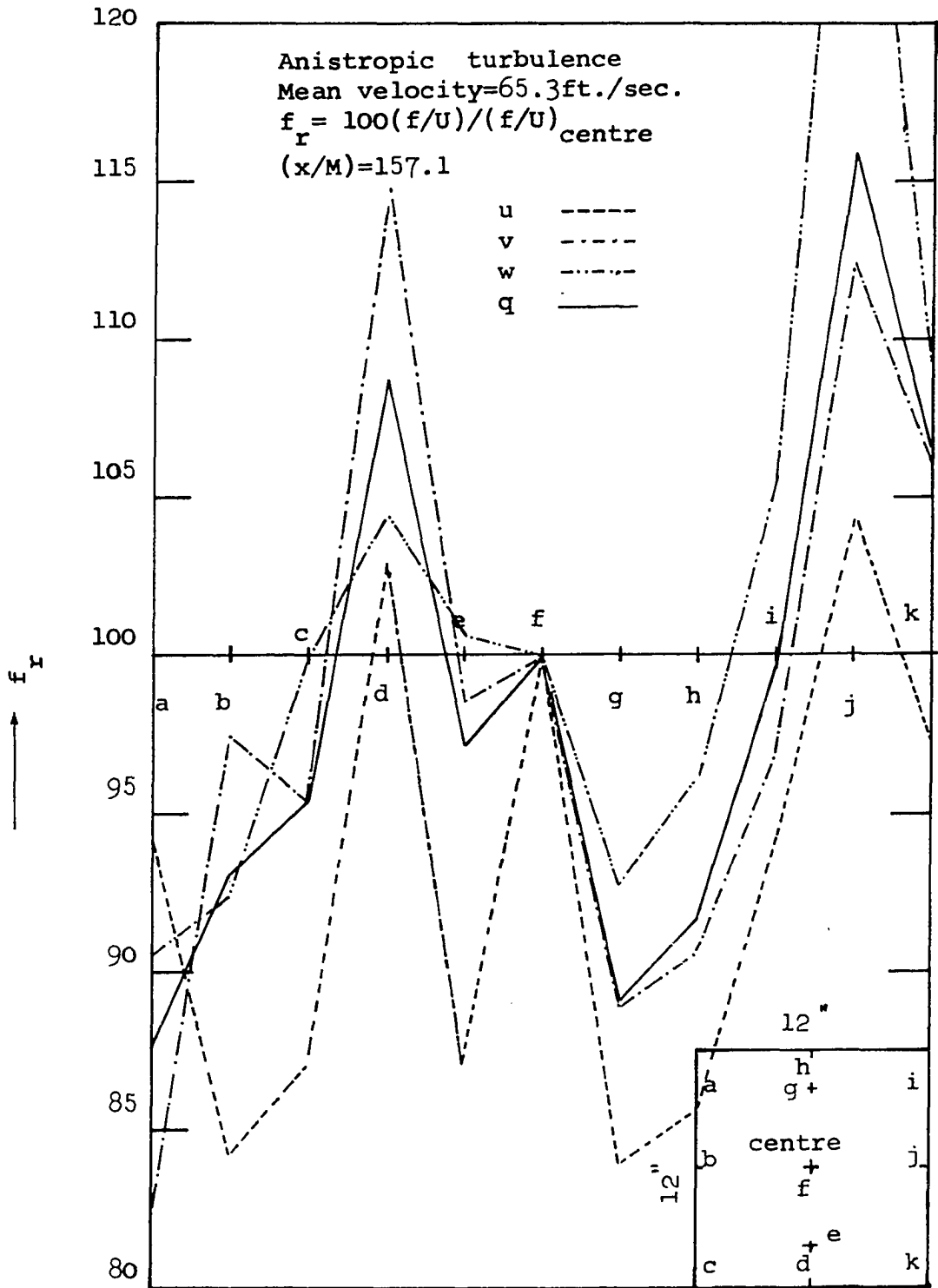
Fig.21 Homogeneity of turbulence in a plane normal to the direcion of flow.

FIG.22 GENERAL VIEW OF THE WIND TUNNEL

1 TEST SECTION
2 SMALL BEND
3 TRANSITION SQUARE TO ROUND
4 CIRCULAR BEND
5 MOTOR
6 DRIVING PULLEY FOR FAN
7a RETURN DUCT
   TRANSITION ROUND TO SQUARE
7b RETURN DUCT
   DIVERGING SECTION

8a LARGE BEND: BOTTOM
8b LARGE BEND: TOP
9 SCREENS
10 CONTRACTION LEADING TO TEST SECTION
11 PLATFORM
G1 GRID POSTION TO GET
   NEARLY ISOTROPIC TURBULENCE
G2 GRID POSTION TO GET
   STRONGLY ANISOTROPIC TURBULENCE

Fig.23 Mean velocity variation of flow along the centre line of the test section
(U_1 = velocity at the starting point.)
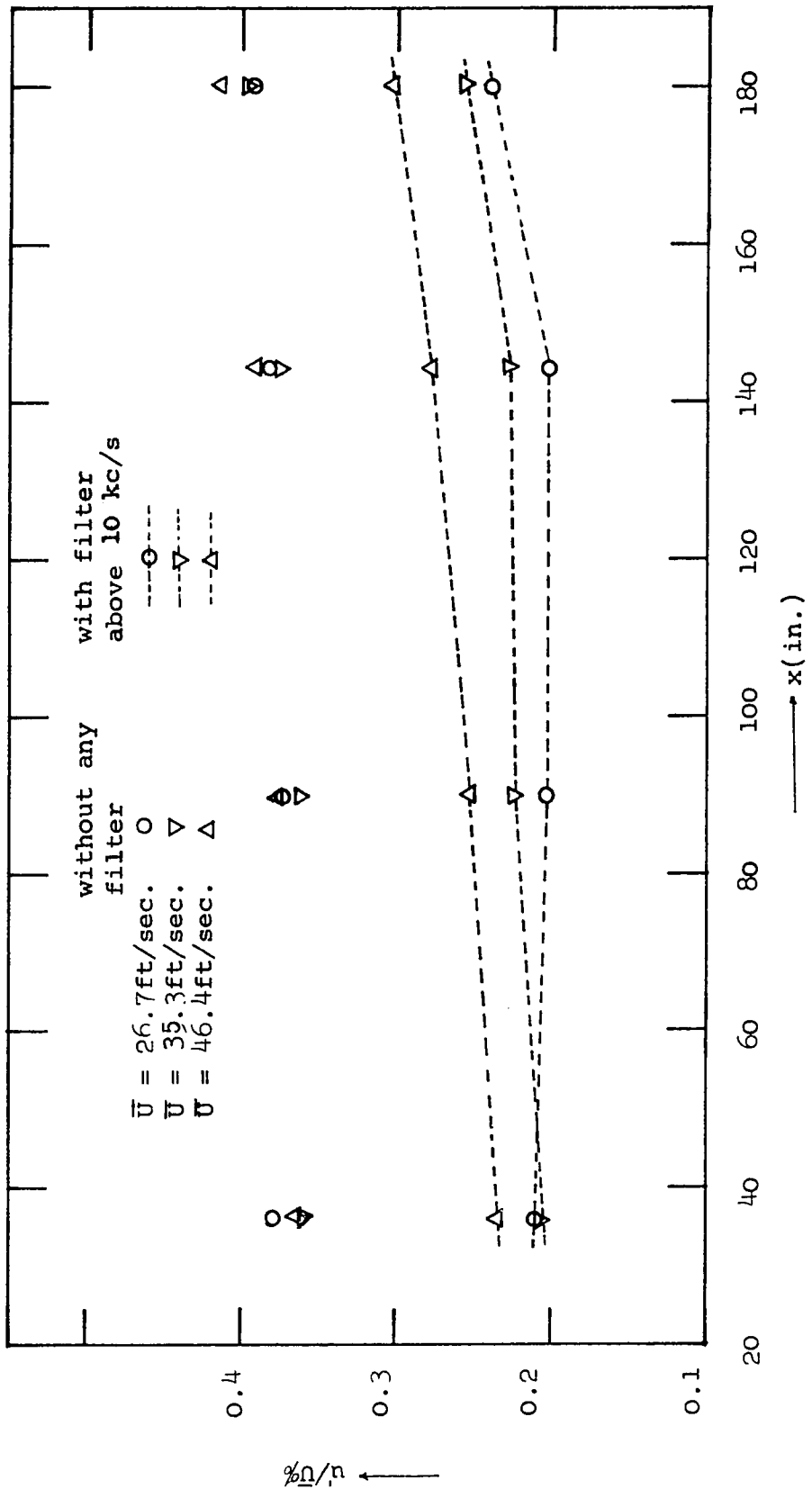
Fig.24 Longitudenal compenent of free stream turbulence(plus instrument noise) at different velocities and frequency ranges.
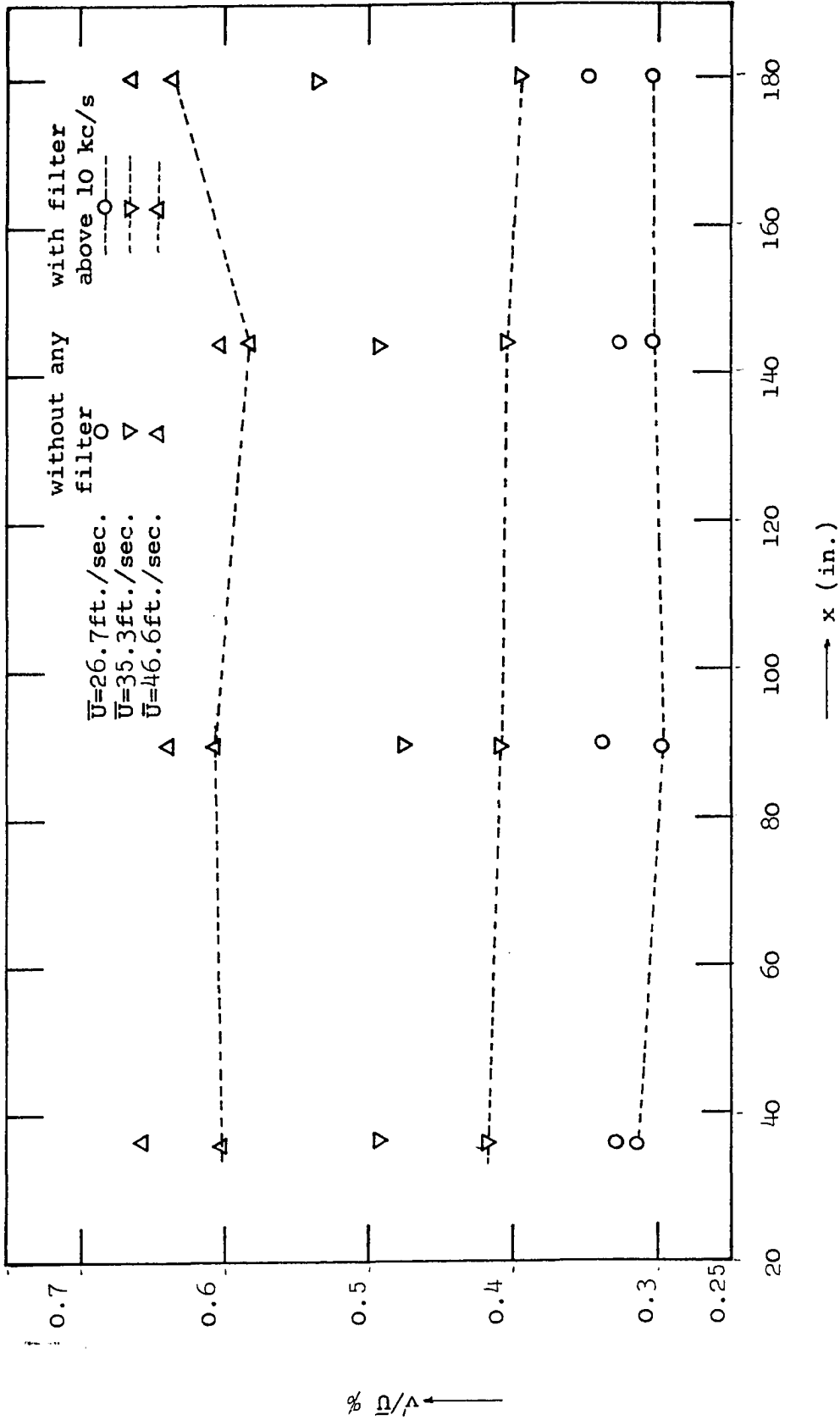
Fig.25 Transverse component of free stream turbulence (plus instrument noise) at different velocities and frequency ranges.
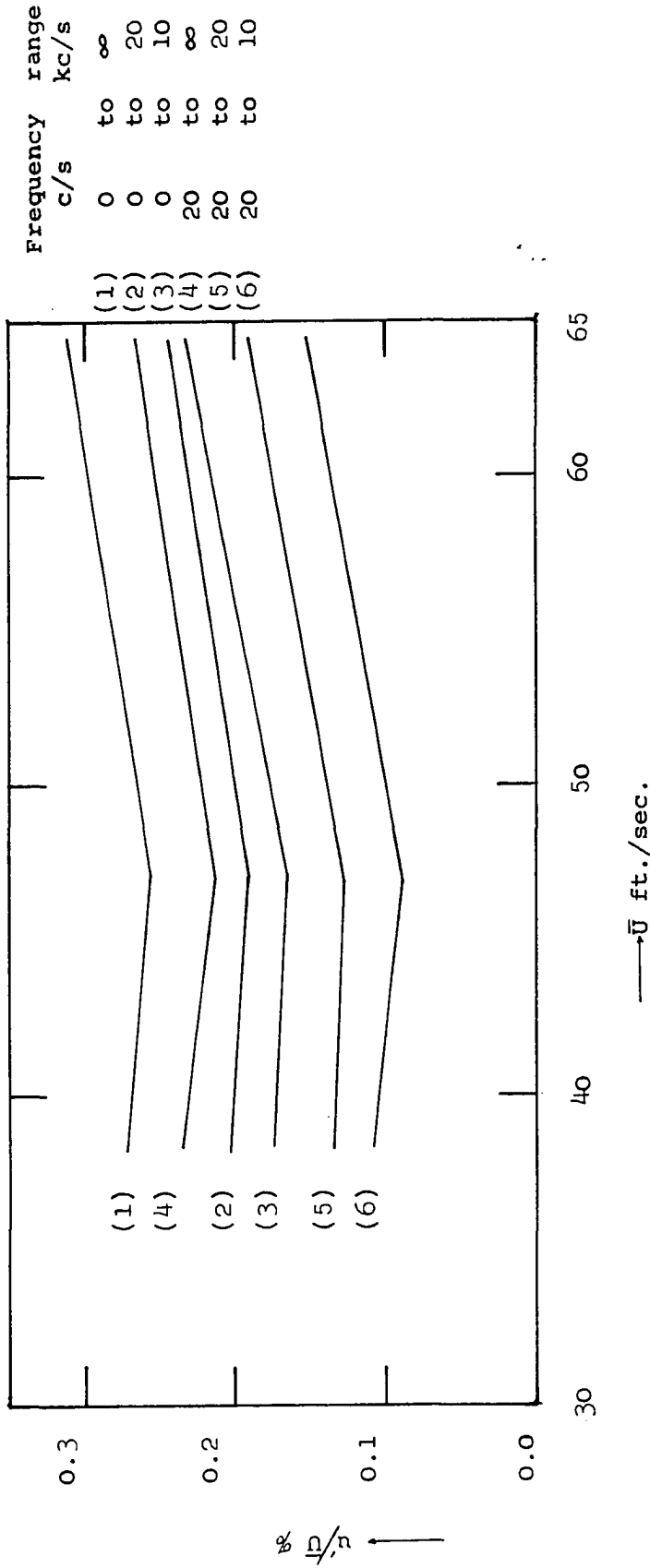
Fig.26 Longitudenal component of free stream turbulence in the wind tunnel in different frequency ranges measured at the centre line of the test section at x = 36 in.
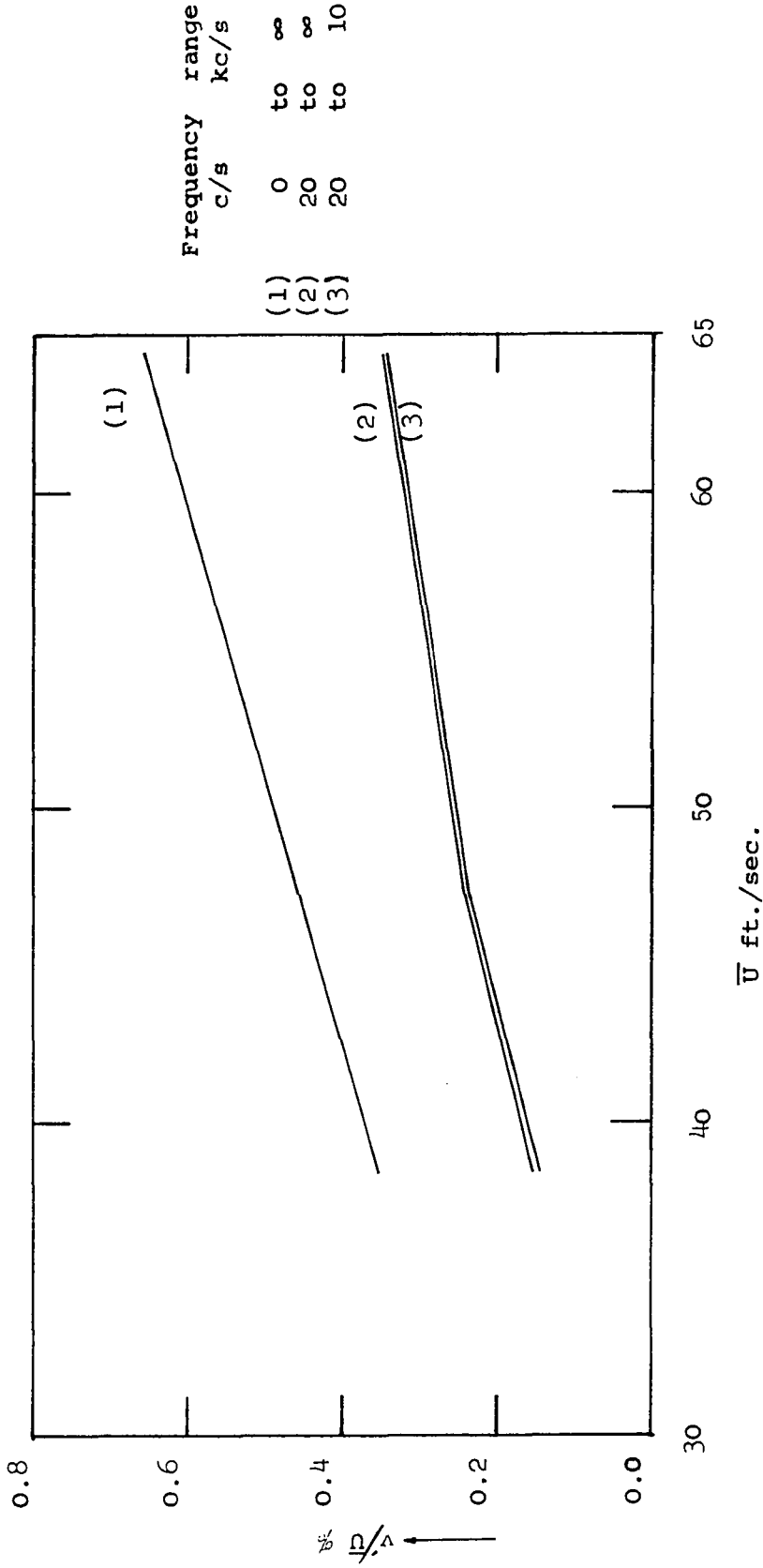
Fig.27 Transverse component of free stream turbulence in the wind tunnel in different frequency ranges measured at the centre line of the test section at x = 36 in.
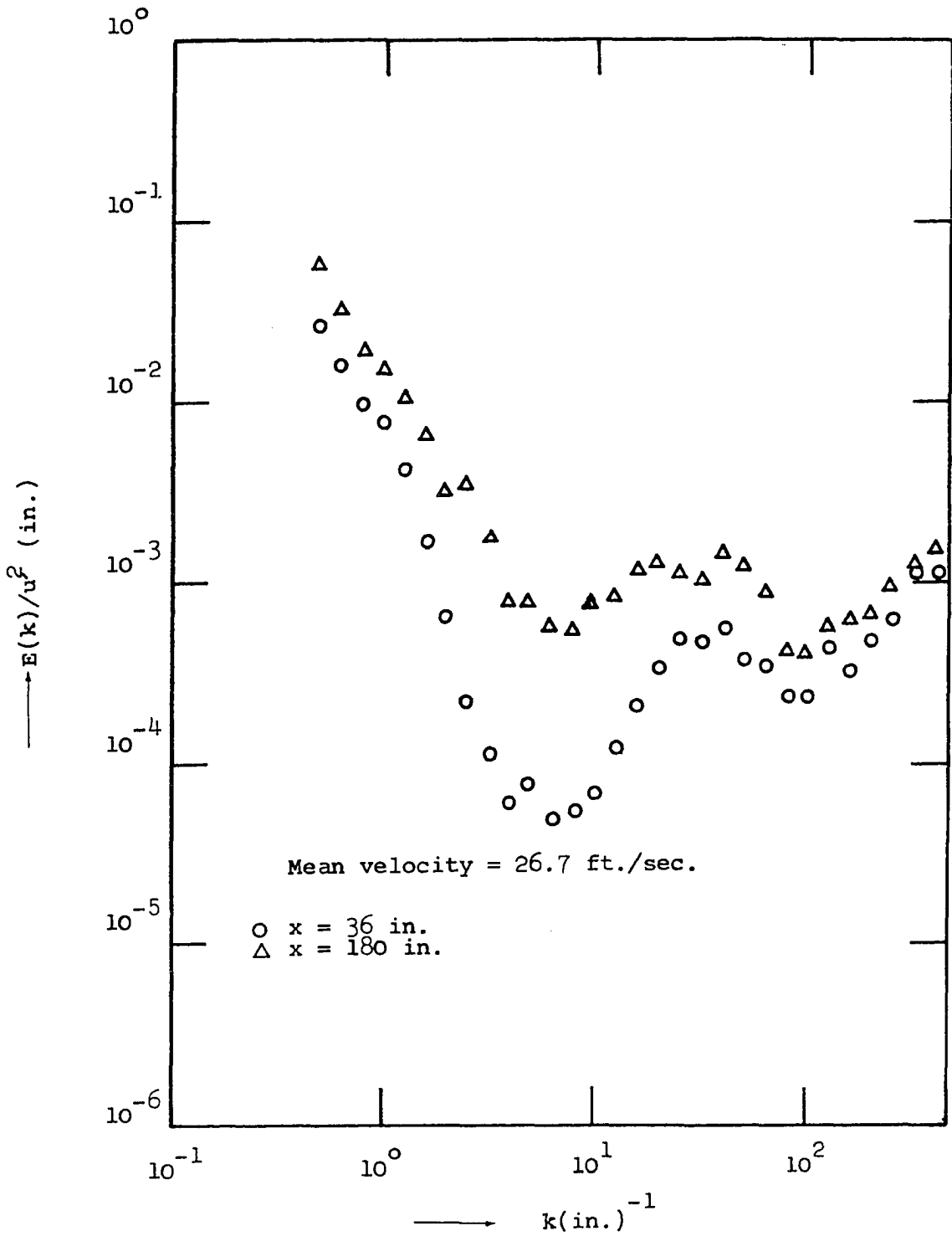
Fig.28 Energy spectra of u' in free stream turbulence
in the wind tunnel.
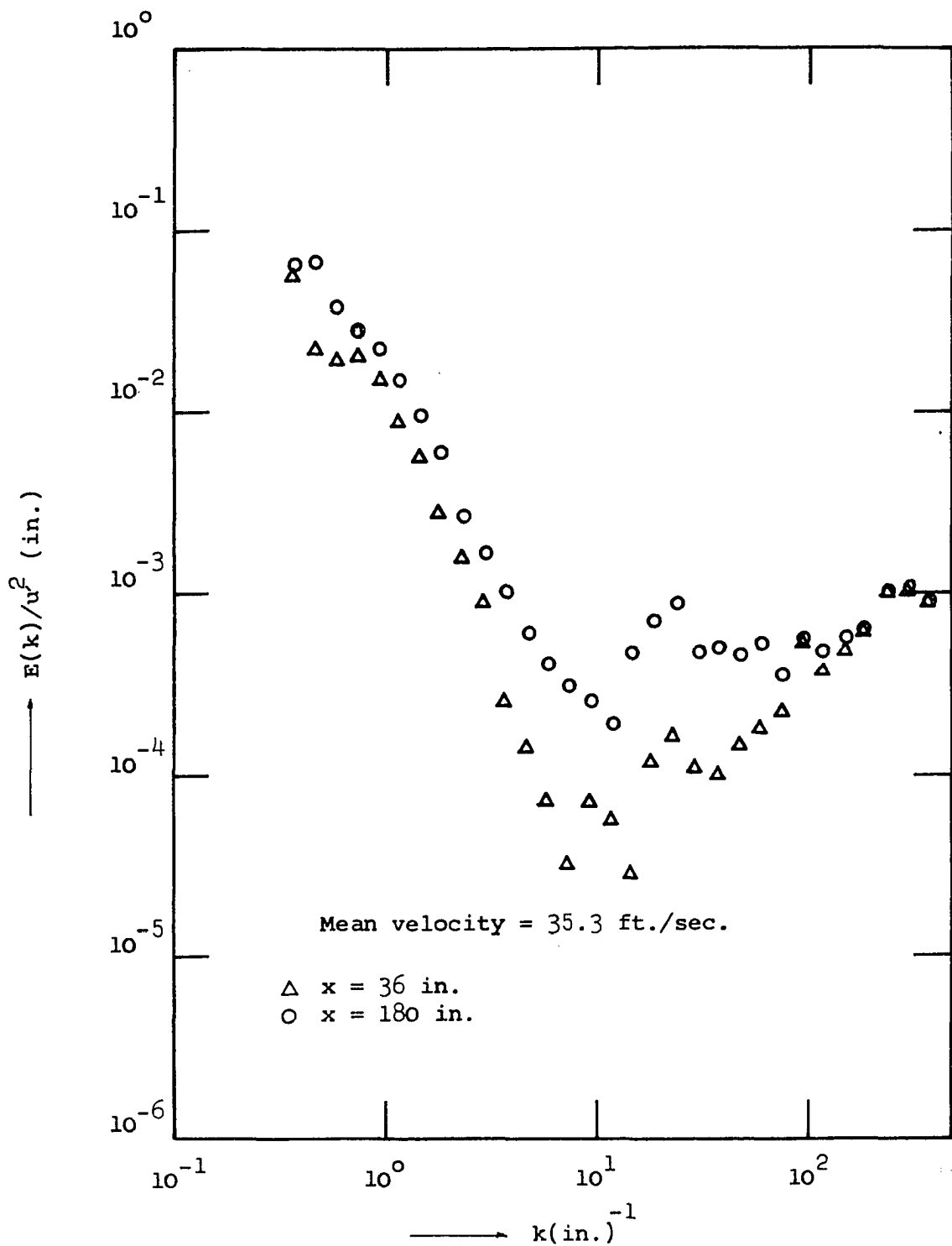
Fig. 29 Energy spectra of u' in free stream turbulence in the wind tunnel.
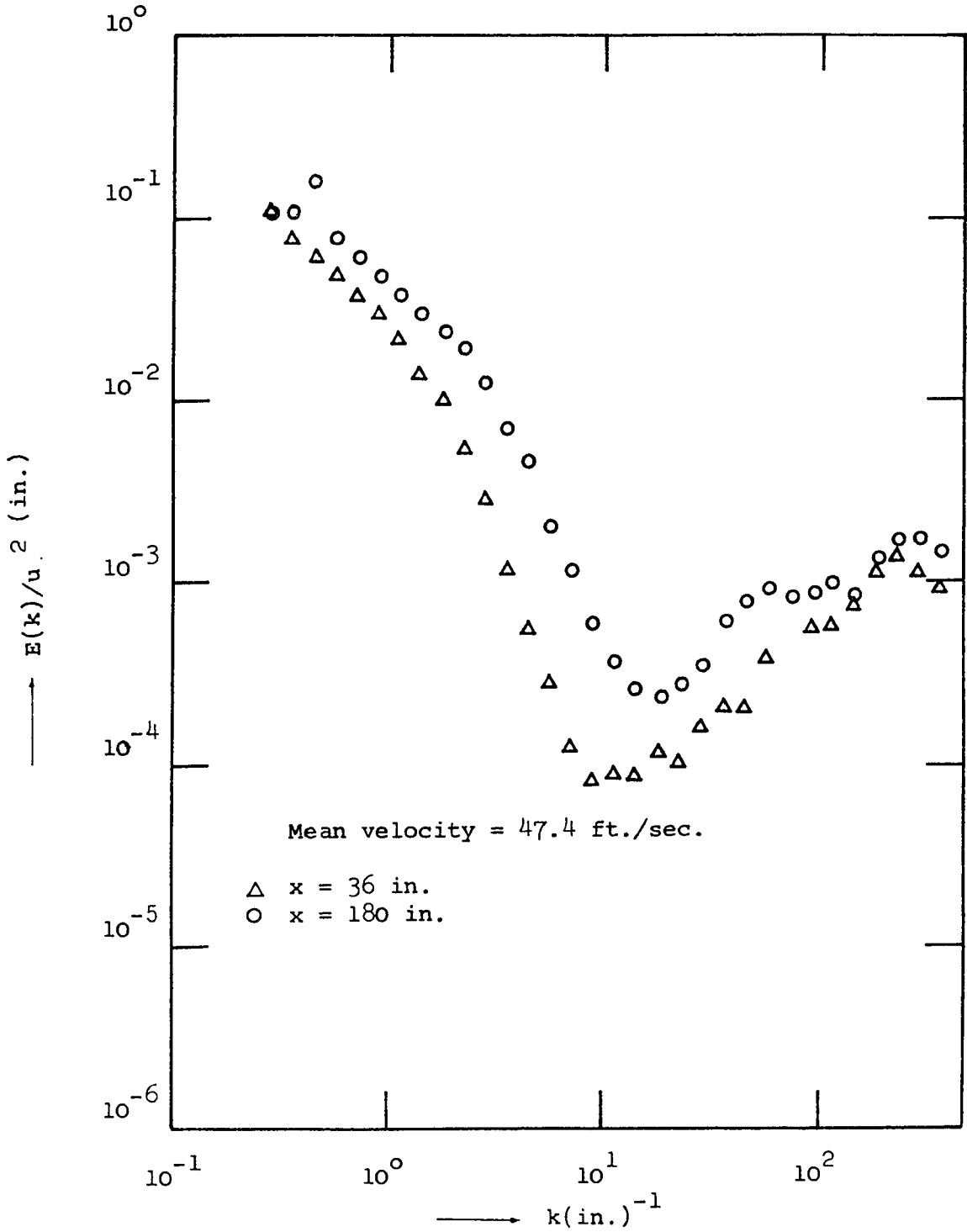
Fig.30 Energy spectra of u′ in free stream turbulence
in the wind tunnel.
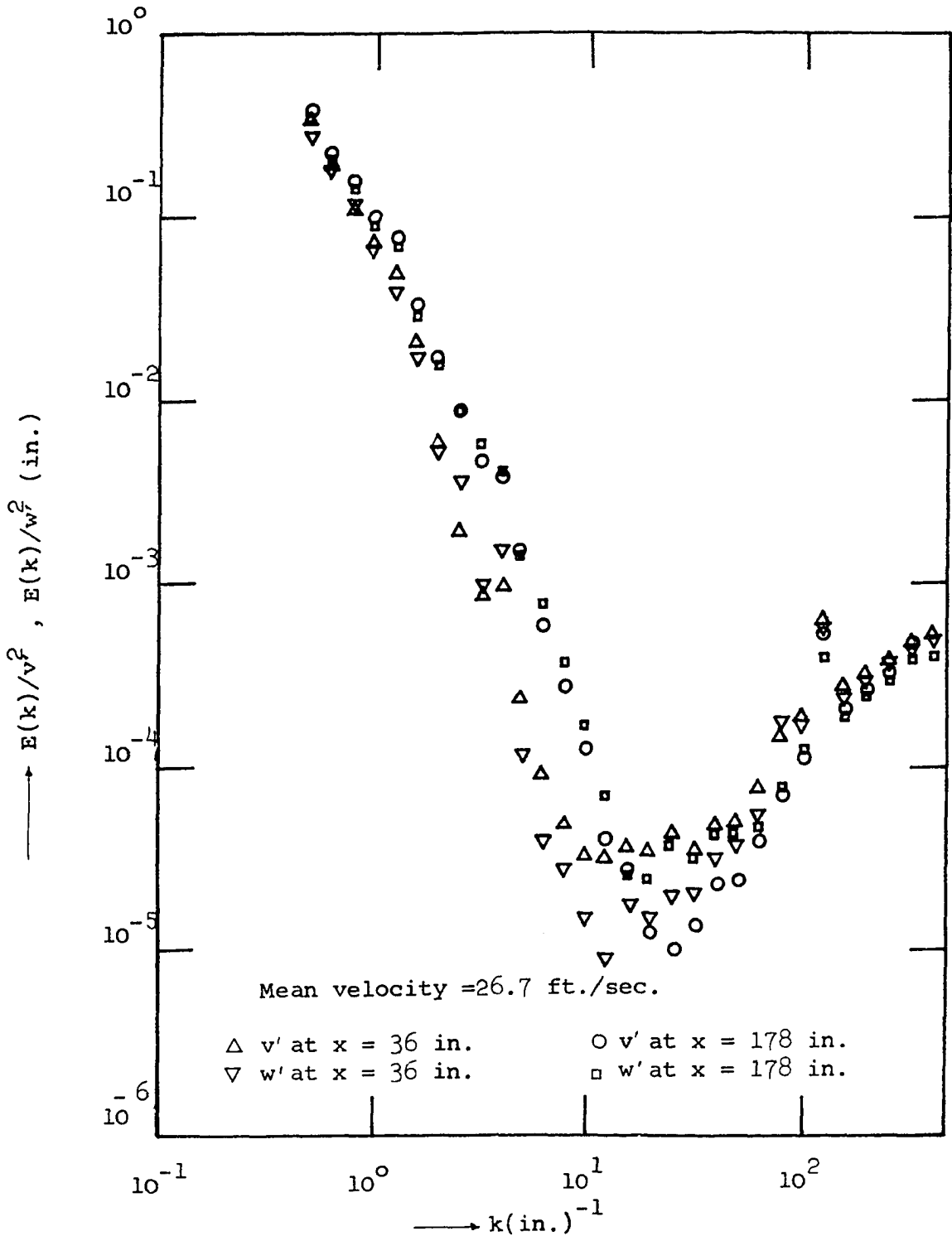
Fig.31 Energy spectra of v and w in free stream turbulence in the wind tunnel.
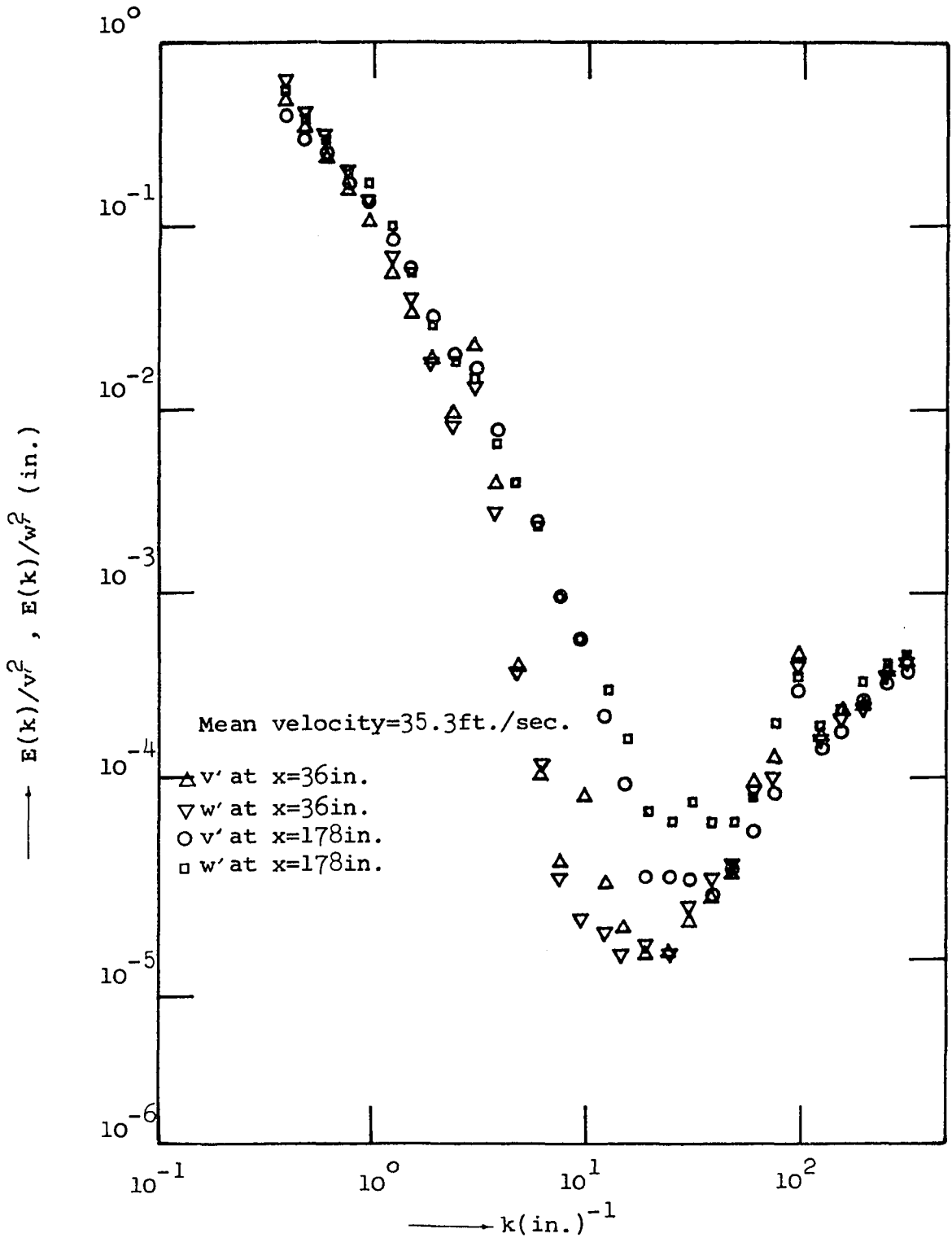
Fig.32 Energy spectra of v and w in free stream
turbulence in the wind tunnel.
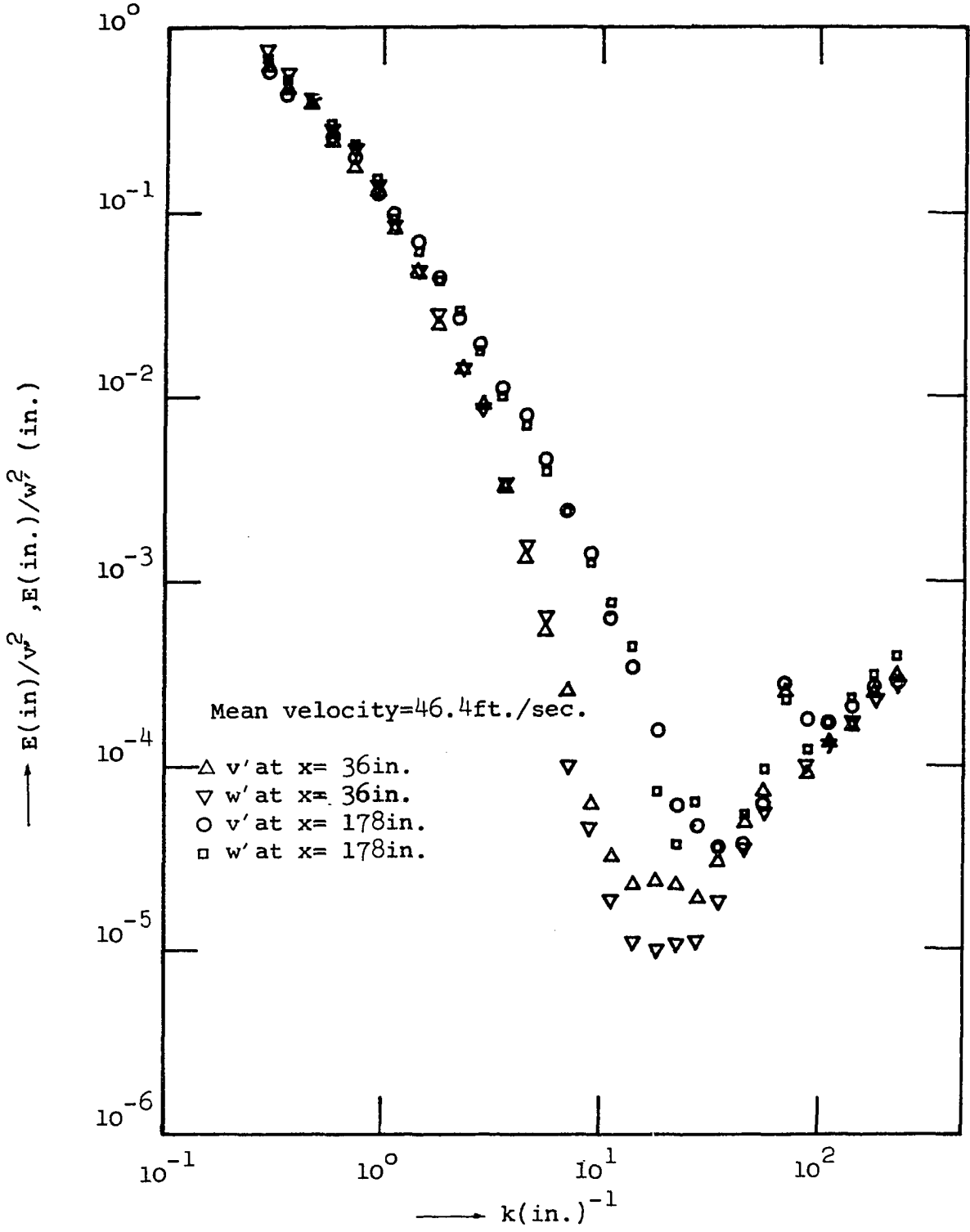
Fig.33 Energy spectra of v' and w' in free stream
turbulence in the wind tunnel.
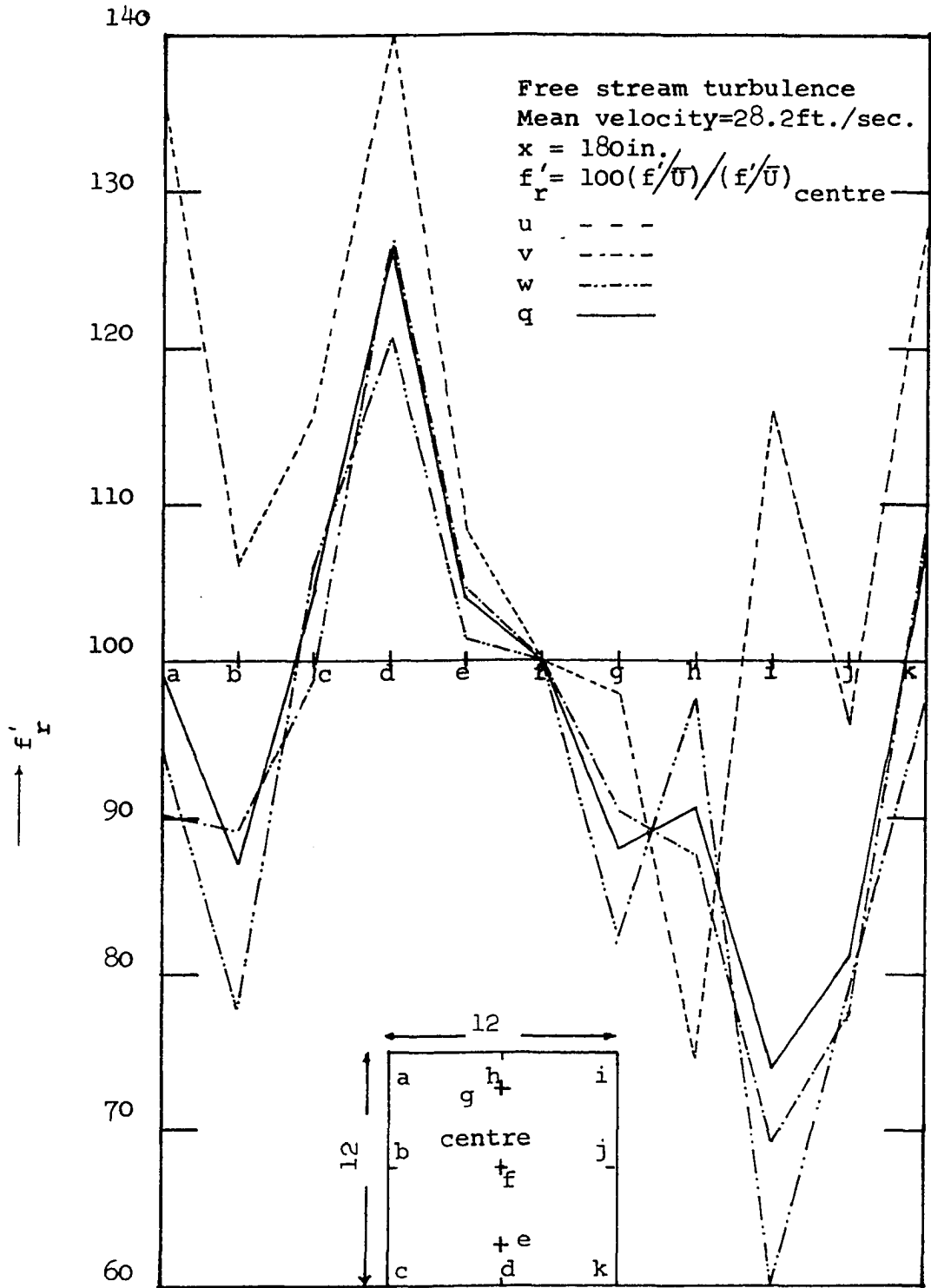
Fig. 34 Homogeneity of turbulence in a plane normal to the direction of flow.

PR Hot wire probe
AN Hot wire anemometer
RN Random noise voltmeter
SP Spectrometer
CR Random signal indicator and
   correlator.
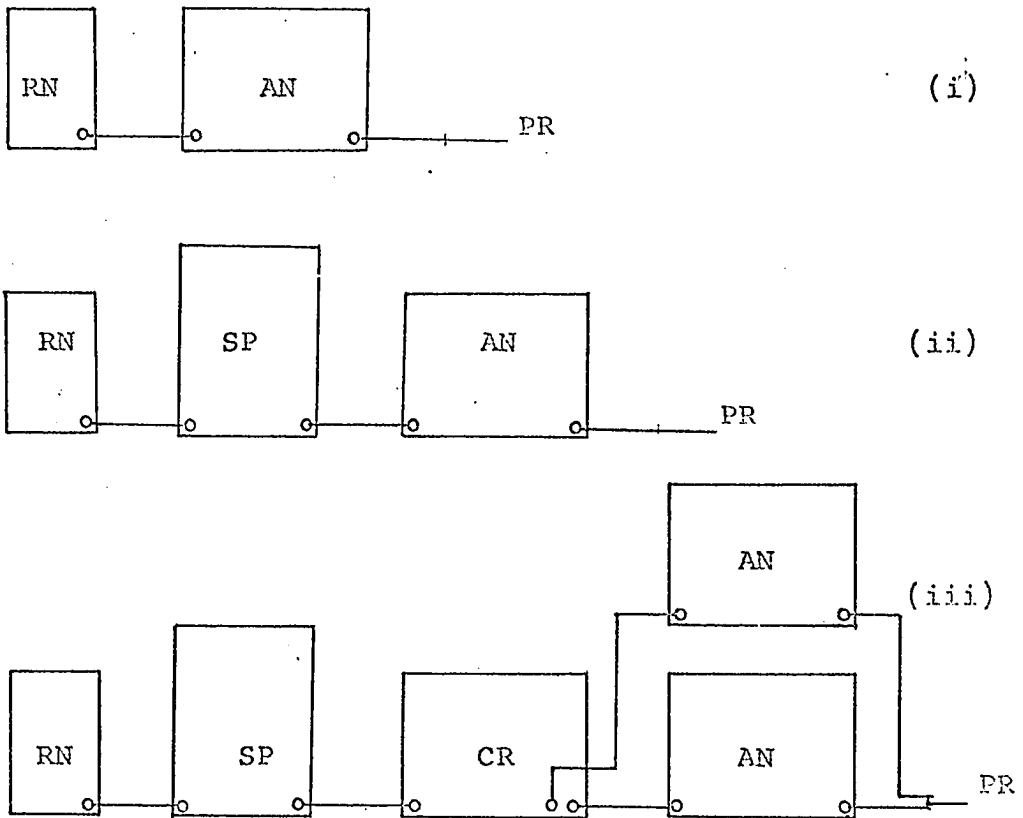
(i) Measurement of u,v, and w
(ii) Measurement of energy spectra for u
(iii) Measurement of energy spectra for v and w

Fig.35 Arrangement of measuring instruements for
       different turbulence quantities.
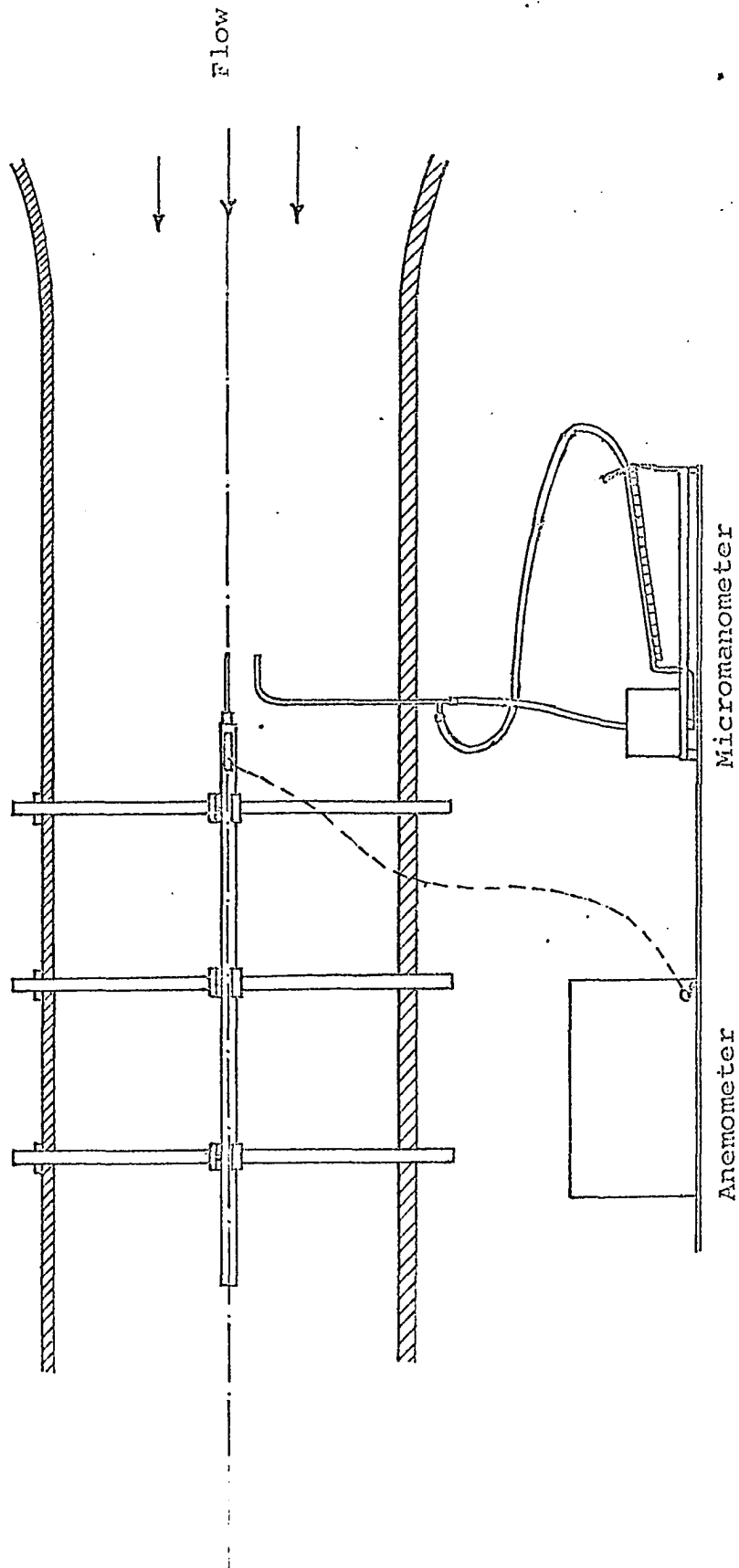
Fig.36 Schematic view of the probing arrangement in the test section(pitot static probe is inserted only for calibration of hot wire.)
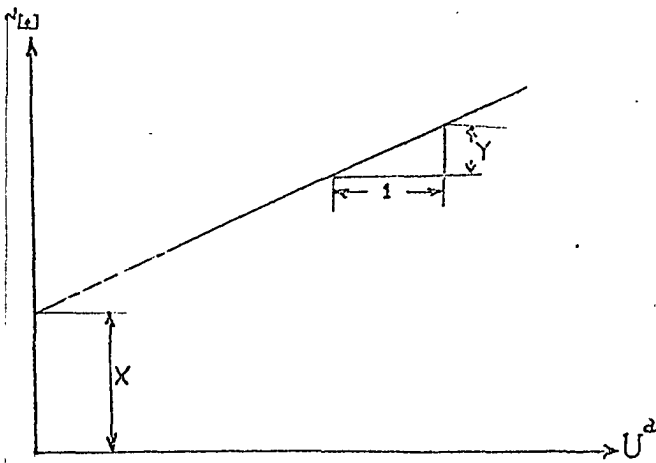
(i) Linear relationship between E^2 and U^a for a hot wire.

Normal wire (x position)   Slanting wire
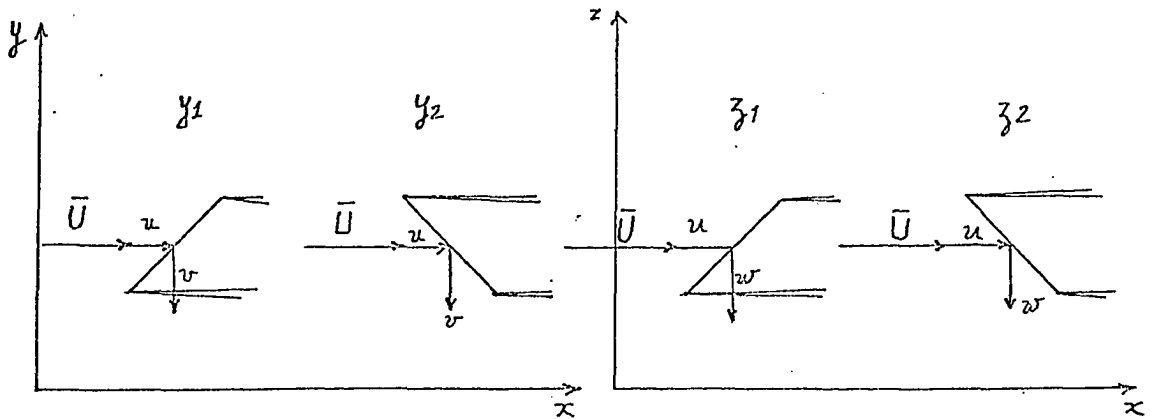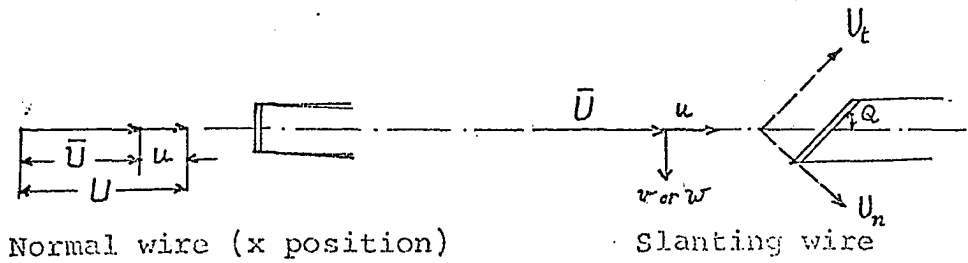
(ii) Positions of normal and slanting wires w.r.t. the flow direction for measuring the three turbulence intensities.

fig.37a

(a) Magnified view of normal hot wire probe.

(b) Magnified view of slanting hot wire probe.

Fig.37 b

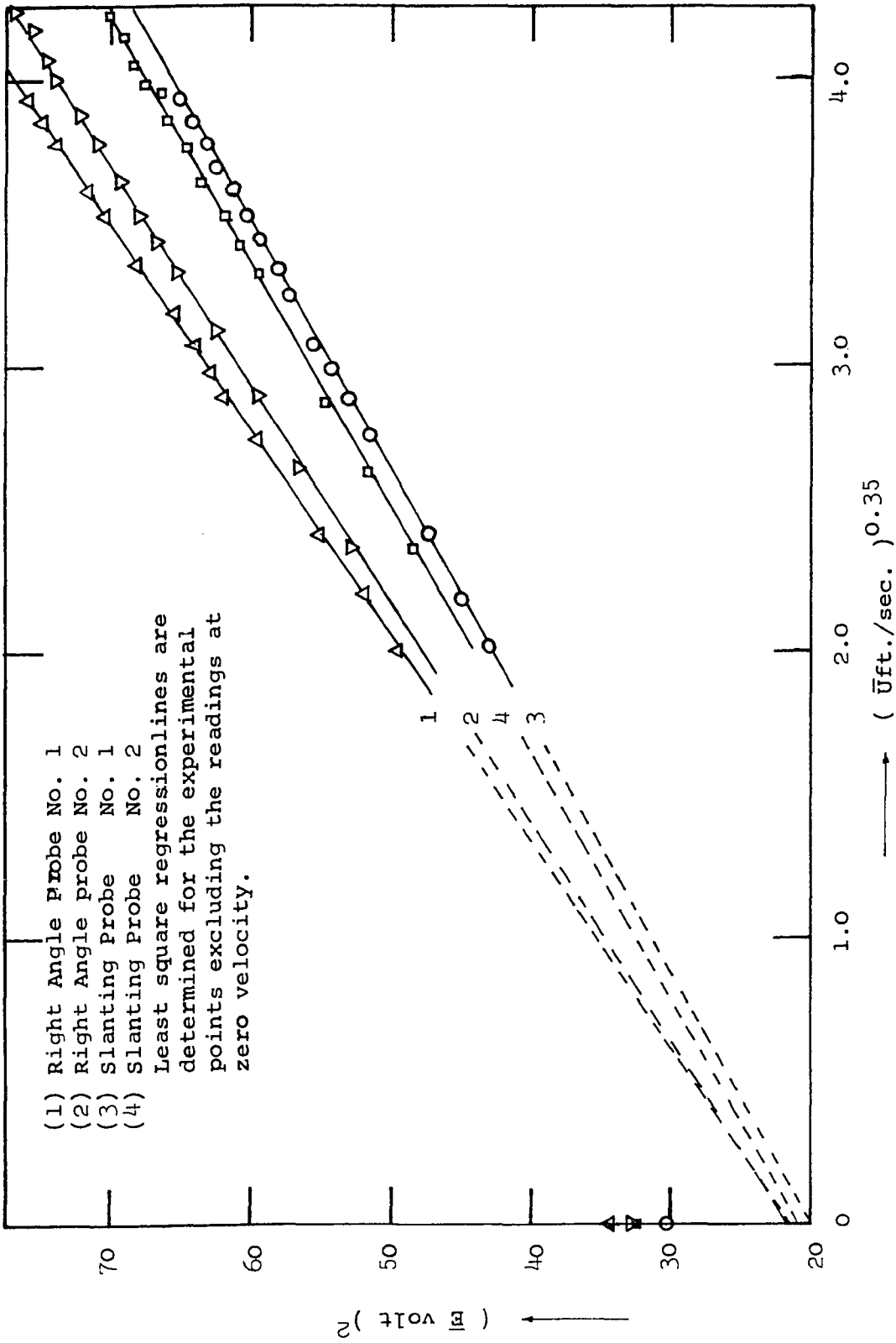(c) Magnified view of X hot wire probe.

Fig. 37 b

(1) Right Angle Probe No. 1
(2) Right Angle probe No. 2
(3) Slanting Probe No. 1
(4) Slanting Probe No. 2

Least square regressionlines are
determined for the experimental
points excluding the readings at
zero velocity.

Fig.38 Calibration of the right angle and slanting hot wire probes used for
experimentation.
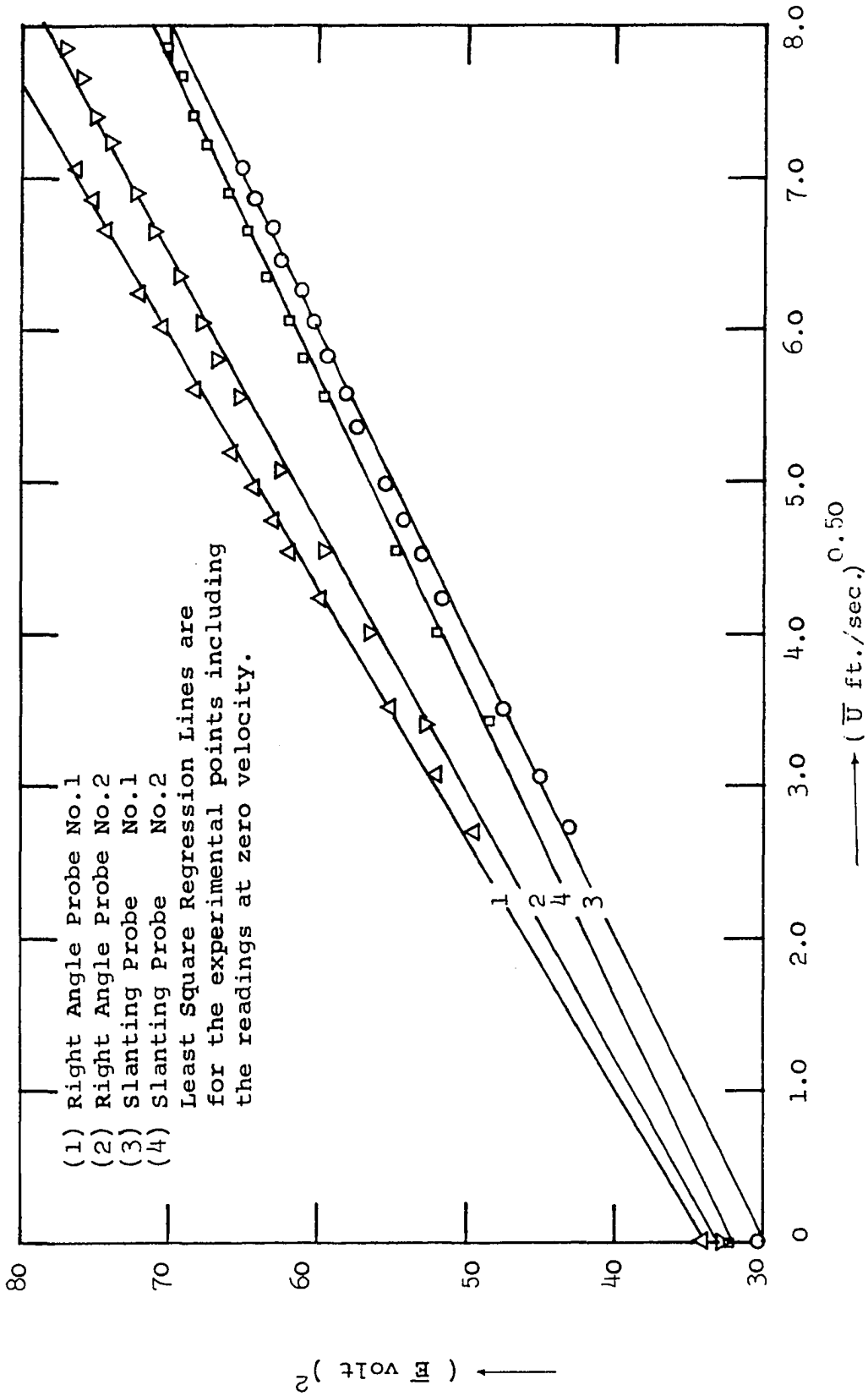
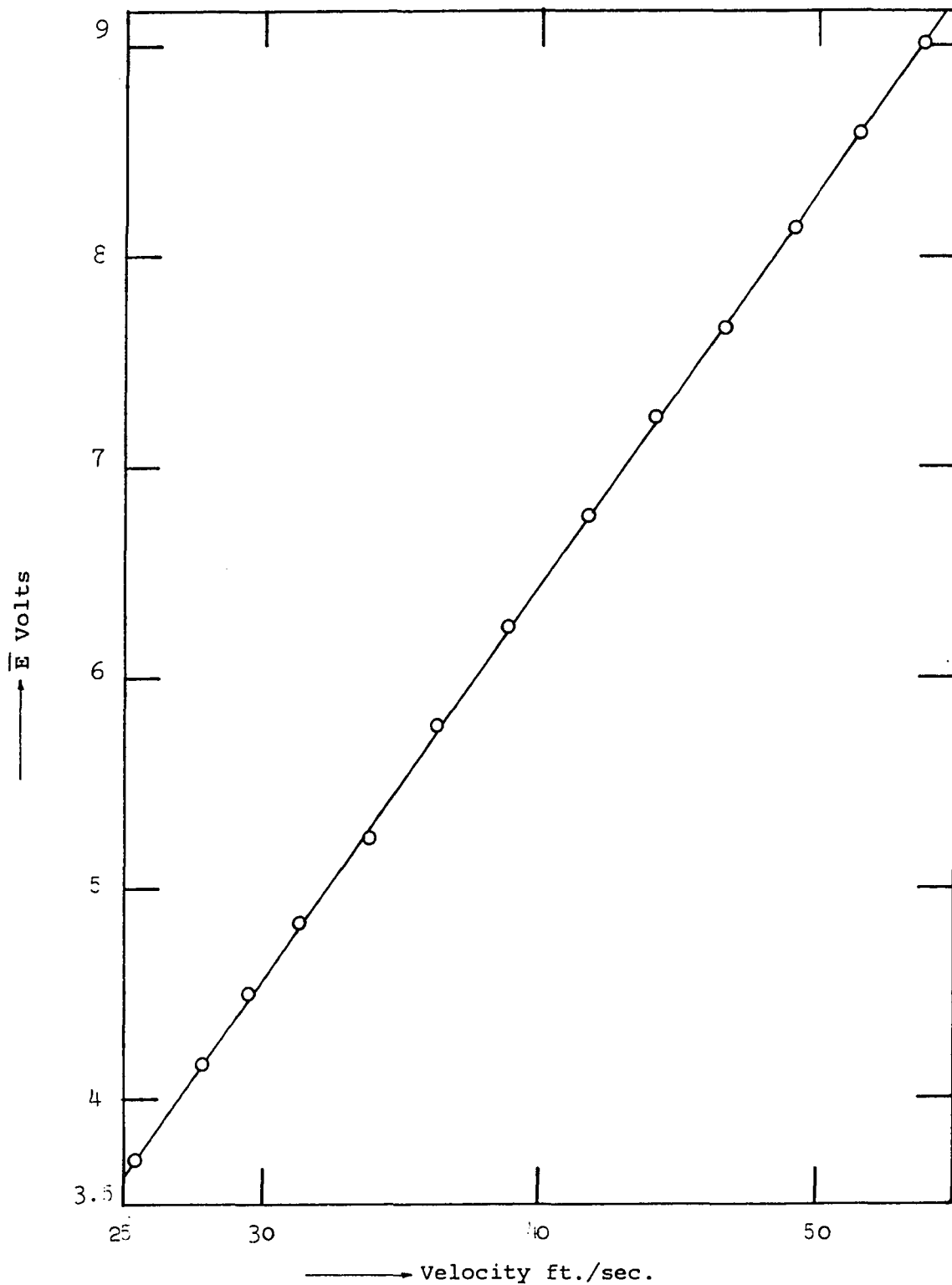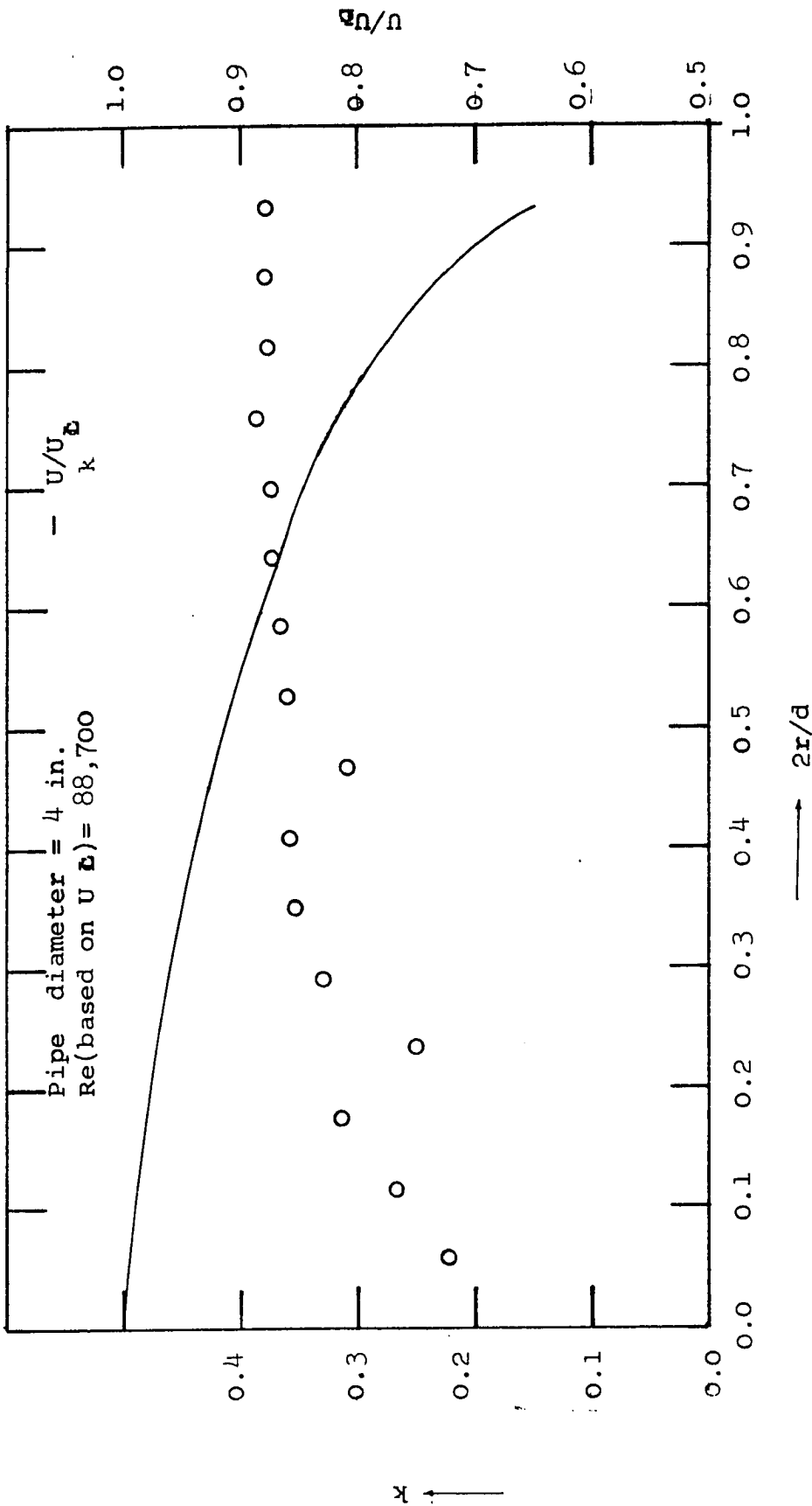Fig.39 Calibration of the right angle and slanting hot wire probes used for experimentation.

Fig.40 Calibration of a slanting hot wire probe using linearizerin the circuit, assuming velocity exponent =1/2.22

Mean velocity profile in turbulent flow in a pipe and values of k of slanting hot wire determined from the known Reynolds shear stress distribution.
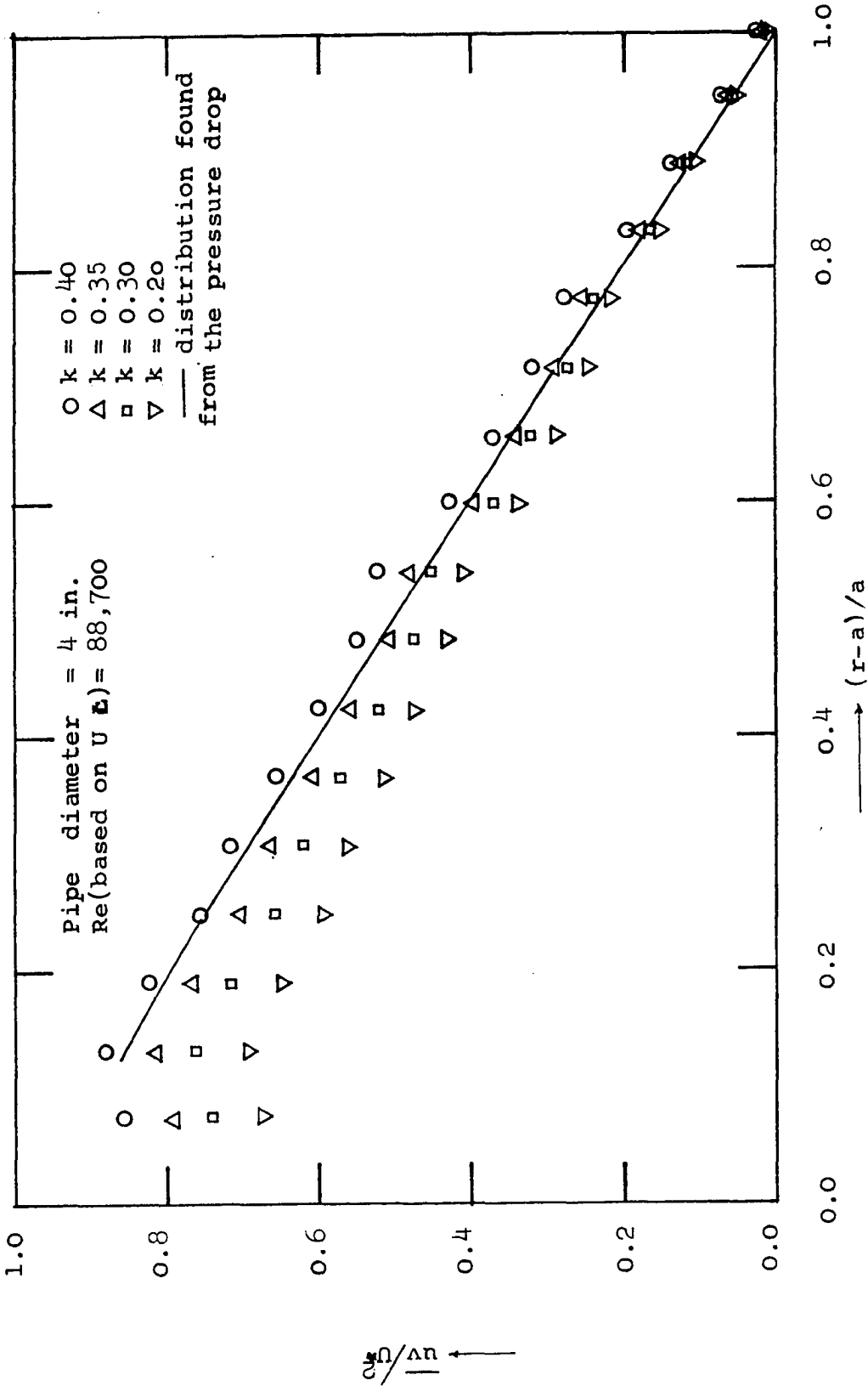Fig.41

Fig.42 Comparison of radial Reynolds shear stress distribution by hot wire anemometer assuming different values of k, with that obtained from pressure drop along the pipe.
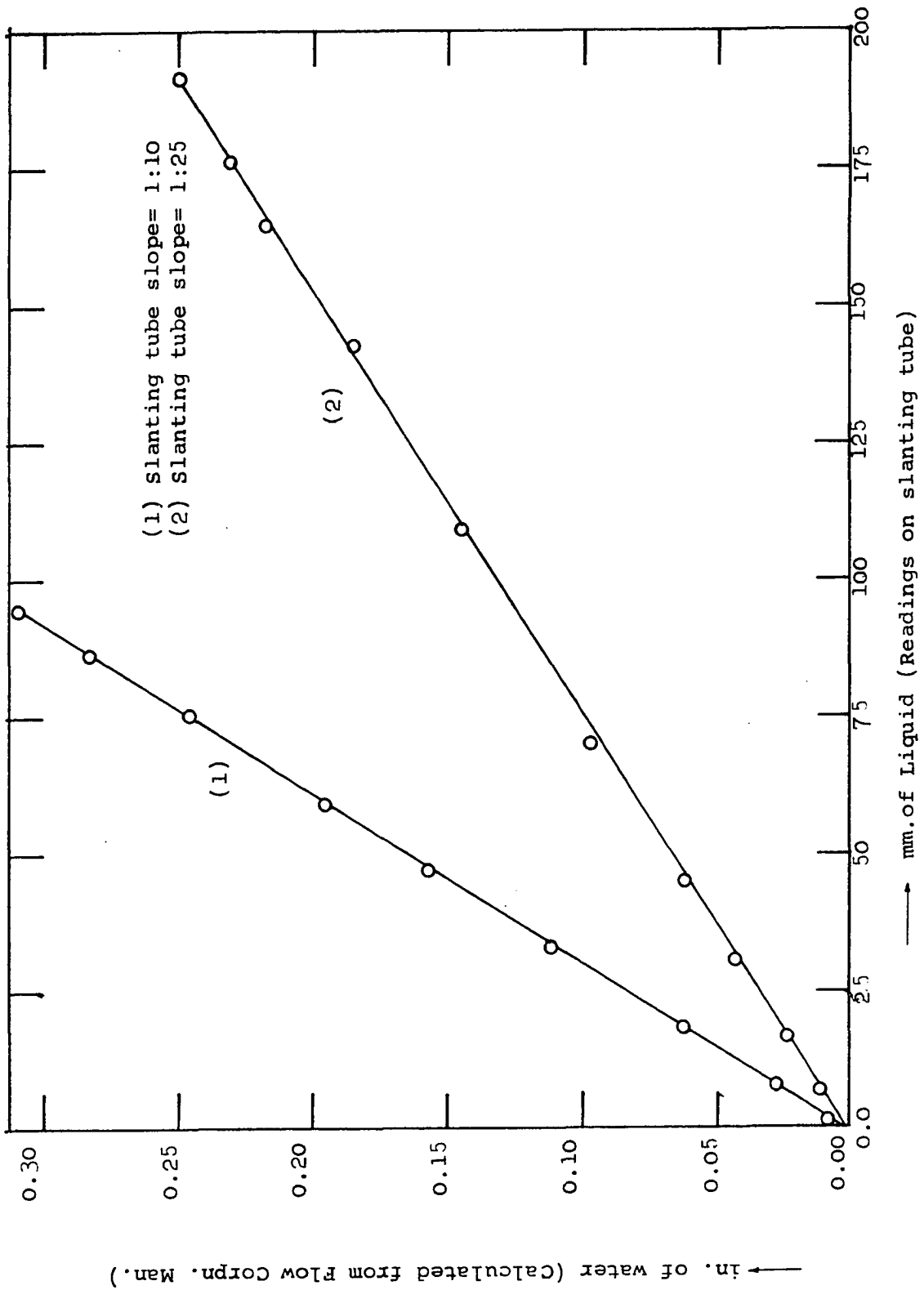
Fig.43 Calibration of WL slanting tube micromanometer against Flow Corportion
Micromanometer.

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