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Transient analysis of synchronous machines.

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TRANSIENT ANALYSIS OF SYNCHRONOUS MACHINES

BY

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ABSTRACT

The primary object of this thesis is to present a complete transient analysis of a synchronous machine. Starting with the fundamental equations of induced voltage, armature reaction, and torque, the complete performance equations of a synchronous machine, valid for both steady state and transient conditions, are developed. No complex reactance transformations are involved. Kron's invariant transformation is used to derive a reciprocal system representing the actual synchronous machine. Under this transformation the power formula remains invariant both in form and in magnitude.

Rigorous expressions for currents, field excitation, and torque in the three-phase short circuit case are derived. The Laplace transform calculus is used in the mathematical treatment. Approximate solutions which neglect and include the effect of armature resistance are also derived.

The moving reference axes, and the α-, β-, and zero-axis quantities are introduced to solve a line-to-line short circuit case in detail. An approximate

solution for the non-linear differential equations with variable coefficients is also proposed.

Oscillographic records for both the three-phase short circuit and the line-to-line short circuit cases are taken experimentally and then compared to the digital computer solution of the performance equations derived.

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CHAPTER I

DEVELOPMENT OF THE COMPLETE TRANSIENT EQUATIONS FOR A SYNCHRONOUS MACHINE

Assumptions

The following assumptions are made in the analysis of synchronous machines:

1) Negligible saturation and hysteresis.

2) The distribution of armature phase mmf is assumed to be sinusoidal. The effects of space harmonics in the distribution of the air-gap flux density, therefore, is neglected. This assumption is practically true of most machines so that no appreciable error is involved on this score in applying the results to practical cases.

3) The machine is assumed to have only one rotor circuit, that is, the main field winding in the direct axis. The additional short-circuited windings, where applicable, are referred to as amortisseur or damper windings.

The General Equations of Induced Voltage and Armature Reaction

The general equations of induced voltage and armature reaction, first introduced by Professor L. V. Bewley, will be used as tools to develop the complete performance equations of a synchronous machine. For a group of q-coils with center at x_0, and taking skew, pitch, and distribution into account, the fundamental induced voltage of the coil group is given by

$$e_1 = -\frac{qNK_1}{10^8}\left\{\frac{\partial\phi_1}{\partial t}\sin\left(\frac{\pi x_0}{\tau}\pm\gamma_1\right) + \phi_1\left(\frac{\pi}{\tau}\frac{dx_0}{dt}\pm\frac{d\gamma_1}{dt}\right)\cos\left(\frac{\pi x_0}{\tau}\pm\gamma_1\right)\right\} \text{------} (1)$$

if the flux density is specified by

$$\beta_1(x) = \beta_1\sin\left(\frac{\pi}{\tau}x\pm\gamma_1\right) \text{-----------} (2)$$

The fundamental component of armature reaction for the group of q-coils is given by

$$A_1(x) = 0.8\,qNiK_1\cos\frac{\pi}{\tau}(x-x_0) \text{---------} (3)$$

in which, q = number of coils per phase group.

N = number of turns per coil.

τ = pole pitch.

K_1 = combined reduction factor including skew, pitch, and distribution effects.

x_0 = location of the center of the group of coils measured from an arbitrary stationary reference axis.

x = distance measured from the same reference axis.

γ_1 = phase angle of the flux density wave.

$\phi_1 = \frac{2}{\pi}\tau l\beta_1$ = total flux included by the coil.

l = effective length of coil.

The complete derivations of equations (1) and (3) will be found in Appendices II and III respectively.

Direct-, Quadrature-, and Zero-Sequence Components of Current

If i_a, i_b, and i_c are the instantaneous phase currents in the three symmetrical windings of the three-phase machine in Fig. 1, the resultant fundamental component of armature reaction due to these currents is

$$
\begin{aligned}
A_1(x) &= 0{\cdot}8\,g N K_1 \left\{ i_a \cos\left(\theta - \frac{\pi x}{\tau}\right) + i_b \cos\left(\theta - \frac{2\pi}{3} - \frac{\pi x}{\tau}\right) + i_c \cos\left(\theta - \frac{4\pi}{3} - \frac{\pi x}{\tau}\right) \right\} \\
&= 0{\cdot}8\,g N K_1 \left\{ i_a \cos\theta + i_b \cos\left(\theta - \frac{2\pi}{3}\right) + i_c \cos\left(\theta - \frac{4\pi}{3}\right) \right\} \cos\frac{\pi x}{\tau} \\
&\quad + 0{\cdot}8\,g N K_1 \left\{ i_a \sin\theta + i_b \sin\left(\theta - \frac{2\pi}{3}\right) + i_c \sin\left(\theta - \frac{4\pi}{3}\right) \right\} \sin\frac{\pi x}{\tau} \quad \text{-----(4)}
\end{aligned}
$$

in which $\theta = \frac{\pi x_0}{\tau}$ = angle between the field pole axis (reference axis) and the axis of phase a.

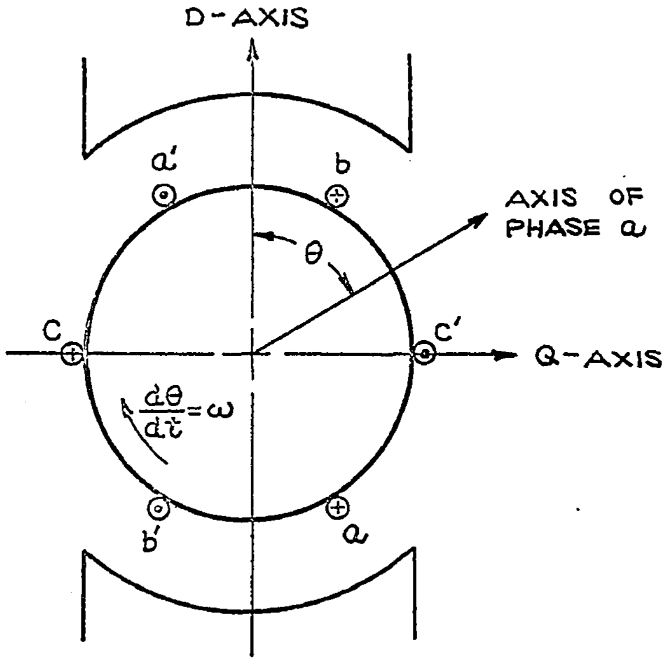

Fig. 1. Elementary diagram of a three-phase machine.

In equation (4), the armature reaction acts entirely on the pole axis if x = 0, and entirely on the interpolar axis if $x = \frac{\tau}{2}$. If the polar axis and the interpolar axis are designated as the direct and quadrature axis respectively, then the first term in equation (4) will be the direct axis component of armature reaction, and the second term the quadrature component of armature reaction. Therefore, the form of equation (4) suggests that it may be simplified by the substitution of new variables i_d and i_q, defined by the following relations:

$$i_d = \sqrt{\frac{2}{3}}\left\{ i_a \cos\theta + i_b \cos\left(\theta - \frac{2\pi}{3}\right) + i_c \cos\left(\theta - \frac{4\pi}{3}\right)\right\} \text{---------} (5)$$

$$i_q = \sqrt{\frac{2}{3}}\left\{ i_a \sin\theta + i_b \sin\left(\theta - \frac{2\pi}{3}\right) + i_c \sin\left(\theta - \frac{4\pi}{3}\right)\right\} \text{-------} (6)$$

Then, $$A_d = 0.8\sqrt{\frac{3}{2}}\, q N K_1 i_d \text{-------------} (7)$$

= direct-axis component of armature reaction.

$$A_q = 0.8\sqrt{\frac{3}{2}}\, q N K_1 i_q \text{-------------} (8)$$

= quadrature-axis component of armature reaction.

and, $$A_1(x) = A_d \cos\frac{\pi x}{\tau} + A_q \sin\frac{\pi x}{\tau}$$
$$= 0.8\sqrt{\frac{3}{2}}\, q N K_1 \left(i_d \cos\frac{\pi x}{\tau} + i_q \sin\frac{\pi x}{\tau}\right) \text{------} (9)$$

The factor $\sqrt{\frac{2}{3}}$ was first introduced by G. Kron so that the armature circuit and the field circuit will have reciprocal mutual inductances. This fact will be shown later.

Inspection of equation (9) shows that the armature reactions due to i_d and i_q are both sinusoidally distributed, and build up fluxes in the direct and quadrature axis respectively. During transient conditions these fluxes in general are not constant, but will vary with time. Thus these fluxes together with the field flux will produce transformer emf in the armature windings.

To allow for the current flow during unbalanced conditions, a zero-sequence component of current is defined as follows:

$$i_o = \frac{1}{\sqrt{3}}(i_a + i_b + i_c) \text{..............} (10)$$

The factor $\frac{1}{\sqrt{3}}$ is introduced so that the power will remain invariant both in form and in magnitude.

Equations (5), (6), and (10) can now be solved to give

$$i_a = \sqrt{\frac{2}{3}}\, i_d \cos\theta + \sqrt{\frac{2}{3}}\, i_q \sin\theta + \frac{1}{\sqrt{3}}\, i_o \text{......................} (11)$$

$$i_b = \sqrt{\frac{2}{3}}\, i_d \cos(\theta - \tfrac{2\pi}{3}) + \sqrt{\frac{2}{3}}\, i_q \sin(\theta - \tfrac{2\pi}{3}) + \frac{1}{\sqrt{3}}\, i_o \text{.........} (12)$$

$$i_c = \sqrt{\frac{2}{3}}\, i_d \cos(\theta - \tfrac{4\pi}{3}) + \sqrt{\frac{2}{3}}\, i_q \sin(\theta - \tfrac{4\pi}{3}) + \frac{1}{\sqrt{3}}\, i_o \text{.....} (13)$$

Air-gap Flux Density

For the time being, assume that the machine has no amortisseur windings in either the direct or quadrature axis. The flux distributions due to the field and armature reaction are:

1) Flux distribution due to field excitation, which consists of the field leakage flux $\Phi_{f\ell}$, and the useful flux Φ_f which crosses the air gap and enters the armature. When the field excitation is constant, and if the effects of armature slots is neglected, the space flux density distribution is in general expressible as a Fourier series:

$$\beta_f = \Sigma \beta_{2K-1} \cos(2K-1)\frac{\pi X}{\tau} \text{ ---------------- } (14)$$

2) Flux distribution due to armature reaction.

Let $\quad P(X) = P_0 + P_2 \cos\frac{2\pi X}{\tau}$ ------------ (15)

= permeance for the fundamental component of armature reaction.
Then, if the armature reaction is

$$A_1(X) = A_d \cos\frac{\pi X}{\tau} + A_q \sin\frac{\pi X}{\tau} \text{ .... } (16)$$

the flux density is

$$\beta_a = (P_0 + P_2 \cos\frac{2\pi X}{\tau})(A_d \cos\frac{\pi X}{\tau} + A_q \sin\frac{\pi X}{\tau})$$

$$= (P_0 + P_2/2) A_d \cos\frac{\pi X}{\tau} + (P_0 - P_2/2) A_q \sin\frac{\pi X}{\tau}$$

$$+ \frac{P_2}{2}(A_d \cos\frac{3\pi X}{\tau} + A_q \sin\frac{3\pi X}{\tau}) \text{ .... } (17)$$

The total fundamental flux density with respect to the direct axis is obtained, therefore, by adding equations (14) to (17), that is,

$$\beta(X) = \beta_{1f} \cos\frac{\pi X}{\tau} + (P_0 + \frac{P_2}{2}) A_d \cos\frac{\pi X}{\tau} + (P_0 - \frac{P_2}{2}) A_q \sin\frac{\pi X}{\tau}$$

$$= \beta_{1f} \sin(\frac{\pi X}{\tau} + \frac{\pi}{2}) + 0{\cdot}8\sqrt{\frac{3}{2}}\, q N K_1 (P_0 + \frac{P_2}{2}) i_d \sin(\frac{\pi X}{\tau} + \frac{\pi}{2})$$

$$+ 0{\cdot}8\sqrt{\frac{3}{2}}\, q N K_1 (P_0 - \frac{P_2}{2}) i_q \sin\frac{\pi X}{\tau} \text{ .... } (18)$$

The phase angle in each term in equation (18) is constant, and it follows that the air-gap flux is stationary in space with respect to the direct axis, so that $\frac{d\gamma_1}{dt}=0$. Equation (1) then reduces to

$$e_1 = -\frac{qNK_1}{10^8}\left\{\frac{\partial\phi_1}{\partial t}\sin\left(\frac{\pi x_0}{\tau}\pm\gamma_1\right)+\phi_1\frac{\pi}{\tau}\frac{dx_0}{dt}\cos\left(\frac{\pi x_0}{\tau}\pm\gamma_1\right)\right\} \cdots\cdots\cdots (19)$$

if $$\beta = \beta_1 \sin\left(\frac{\pi x_0}{\tau}\pm\gamma_1\right) \cdots\cdots\cdots\cdots (20)$$

Induced Voltage in Armature Windings

Let $\theta = \frac{\pi}{\tau}x_0$

= center of a coil group of q-coils moving at some speed $\frac{d\theta}{dt}=\frac{\pi}{\tau}\frac{dx_0}{dt}$

the fundamental induced voltage of the coil group becomes

$$e_1 = \sqrt{\frac{2}{3}}\,\frac{qNK_1}{10^8}\left\{\sqrt{\frac{3}{2}}\,\phi_f\frac{d\theta}{dt}+\phi_{ad}\frac{d\theta}{dt}-\frac{d\phi_{aq}}{dt}\right\}\sin\theta$$

$$-\sqrt{\frac{2}{3}}\,\frac{qNK_1}{10^8}\left\{\sqrt{\frac{3}{2}}\,\frac{d\phi_f}{dt}+\frac{d\phi_{ad}}{dt}+\phi_{aq}\frac{d\theta}{dt}\right\}\cos\theta \cdots\cdots\cdots (21)$$

in which $$\phi_f = \frac{2}{\pi}\tau l \beta_{1f} \cdots\cdots\cdots (22)$$

= field flux per pole, proportional to the field current i_f.

$$\phi_{ad} = \frac{2.4}{\pi}\tau l\, qNK_1\left(P_0+\frac{P_2}{2}\right)i_d \cdots\cdots\cdots (23)$$

= direct-axis armature reaction flux

$$\phi_{aq} = \frac{2.4}{\pi}\tau l\, qNK_1\left(P_0-\frac{P_2}{2}\right)i_q \cdots\cdots\cdots (24)$$

= quadrature-axis armature reaction flux

ϕ_f , ϕ_{ad} , and ϕ_{aq} are proportional to i_f, i_d, and i_q respectively, and the voltages due to them may be accounted for by the proportionality factors L_{af}, L_{ad}, and L_{aq} defined as follows:

$$L_{afm} = \frac{gNK_1\Phi_f}{10^8 i_f} \text{.................. (25)}$$

= maximum mutual inductance between field and armature.

$$L_{ad} = \frac{2.4\tau l\,(gNK_1)^2}{10^8\pi}\left(P_0 + \frac{P_2}{2}\right) \text{.................. (26)}$$

= direct-axis inductance of armature reaction.

$$L_{aq} = \frac{2.4\tau l\,(gNK_1)^2}{10^8\pi}\left(P_0 - \frac{P_2}{2}\right) \text{.................. (27)}$$

= quadrature-axis inductance of armature reaction.

Equation (21) can then be rewritten as

$$e_a = \sqrt{\frac{2}{3}}\left\{\sqrt{\frac{3}{2}}L_{af}i_f\frac{d\theta}{dt} + L_{ad}i_d\frac{d\theta}{dt} - L_{aq}\frac{di_q}{dt}\right\}\sin\theta$$
$$-\sqrt{\frac{2}{3}}\left\{\sqrt{\frac{3}{2}}L_{af}\frac{di_f}{dt} + L_{ad}\frac{di_d}{dt} + L_{aq}i_q\frac{d\theta}{dt}\right\}\cos\theta$$

$$= \sqrt{\frac{2}{3}}\left\{M_{af}i_f\frac{d\theta}{dt} + L_{ad}i_d\frac{d\theta}{dt} - L_{aq}\frac{di_q}{dt}\right\}\sin\theta$$
$$-\sqrt{\frac{2}{3}}\left\{M_{af}\frac{di_f}{dt} + L_{ad}\frac{di_d}{dt} + L_{aq}i_q\frac{d\theta}{dt}\right\}\cos\theta \text{.......... (28)}$$

in which a new mutual inductance between the field and the armature is defined by $M_{af} = \sqrt{\frac{3}{2}}L_{afm}$

The first two terms in equation (28) $M_{af}i_f\frac{d\theta}{dt}$ and $L_{ad}i_d\frac{d\theta}{dt}$ are speed emf's due to the movement of armature conductors cutting the flux. The third term, $L_{aq}\frac{di_q}{dt}$ is a transformer emf due to the variation of flux.

Armature Terminal Voltage

The armature terminal voltage is equal to the induced voltage less the leakage reactance, zero-sequence reactance, and the resistance drops.

Let R_a = armature resistance, per phase.

L_ℓ = armature leakage inductance, per phase.

L_o = zero-sequence inductance, per phase.

The armature leakage flux linkage of phase a is then,

$$\psi_\ell = \sqrt{\tfrac{2}{3}}\, L_\ell i_d \cos\theta + \sqrt{\tfrac{2}{3}}\, L_\ell i_q \sin\theta$$

and the leakage reactance drop is

$$\frac{d\psi_\ell}{dt} = \sqrt{\tfrac{2}{3}}\Big\{\Big(L_\ell \frac{di_q}{dt} - L_\ell i_d \frac{d\theta}{dt}\Big)\sin\theta + \Big(L_\ell \frac{di_d}{dt} + L_\ell i_q \frac{d\theta}{dt}\Big)\cos\theta\Big\} \text{-------------}(30)$$

The zero-sequence flux linkage is defined as $\psi_o = L_o i_o$, $\sqrt{3}$ times the conventional one used in symmetrical components.

The corresponding zero-sequence reactance drop is,

$$\frac{1}{\sqrt{3}}\frac{d\psi_o}{dt} = \frac{1}{\sqrt{3}} L_o \frac{di_o}{dt} \text{---------------}(31)$$

and the resistance drop is,

$$R_a i_a = R_a \left\{ \sqrt{\frac{2}{3}}\, i_d \cos\theta + \sqrt{\frac{2}{3}}\, i_q \sin\theta + \frac{1}{\sqrt{3}}\, i_o \right\} \text{------------} (32)$$

The terminal voltage of phase a is then,

$$V_a = e_a - \frac{d\psi_l}{dt} - \frac{d\psi_o}{dt} - R_a i_o$$

$$= -\sqrt{\frac{2}{3}}\left(M_{af}\frac{di_f}{dt} + L_d\frac{di_d}{dt} + L_q i_q\frac{d\theta}{dt} + R_a i_d \right)\cos\theta$$

$$- \sqrt{\frac{2}{3}}\left(L_q\frac{di_q}{dt} - M_{af} i_f\frac{d\theta}{dt} - L_d i_d\frac{d\theta}{dt} + R_a i_q \right)\sin\theta$$

$$- \frac{1}{\sqrt{3}}\left(L_o\frac{di_o}{dt} + R_a i_o \right) \cdots\cdots\cdots\cdots (33)$$

in which $L_d = L_{ad} + L_l$

= direct axis inductance.

$L_q = L_{aq} + L_l$

= quadrature axis inductance.

Equation (33) shows that the terminal voltage consists of a direct-axis component, a quadrature-axis component, and a zero-sequence component voltage. That is,

$$e_d = -\left(M_{af}\frac{di_f}{dt} + L_d\frac{di_d}{dt} + L_q i_q\frac{d\theta}{dt} + R_a i_d \right) \cdots\cdots\cdots (34)$$

= direct-axis component of armature voltage.

$$e_q = -\left(L_q\frac{di_q}{dt} - M_{af} i_f\frac{d\theta}{dt} - L_d i_d\frac{d\theta}{dt} + R_a i_q \right) \cdots\cdots\cdots (35)$$

=quadrature-axis component of armature voltage.

$$e_o = -\left(L_o\frac{di_o}{dt} + R_a i_o \right) \cdots\cdots\cdots\cdots (36)$$

= zero-sequence component of armature voltage.

Therefore,

$$V_a = \sqrt{\frac{2}{3}}\, e_d \cos\theta + \sqrt{\frac{2}{3}}\, e_q \sin\theta + \frac{1}{\sqrt{3}}\, e_o \cdots\cdots\cdots (37)$$

The armature terminal voltage for phase b and c can be found by replacing θ by $(\theta-\frac{2\pi}{3})$ and $(\theta-\frac{4\pi}{3})$ respectively.

Equations (34), (35), and (36) are set up for a generator with rotating armature, clock-wise rotation, and with the quadrature axis ahead of the direct axis. For a motor, the signs of all the terms in these three equations must by changed. If the field is the rotating element, as is in practical machines, and rotating in the clock-wise direction, then relative to the field the armature is rotating counter-clockwise so that it is only necessary to substitute $-\frac{d\theta}{dt}$ for $\frac{d\theta}{dt}$ in all the equations.

Flux Linkage Relations in Armature Circuit

The direct-axis flux linkage is

$$\psi_d = M_{af}\, i_f + L_d\, i_d$$

$$= \sqrt{\frac{3}{2}}\, L_{afm}\, i_f + L_d\, i_d \quad \text{------------------}(38)$$

and,

$$\psi_q = L_q\, i_q \quad \text{----------------------------------}(39)$$

$$\psi_o = L_o\, i_o \quad \text{----------------------------------}(40)$$

By substituting equations (38), (39), (40) into equations (34), (35), and (36), there results the voltage equations in terms of the flux linkages.

$$e_d = -\frac{d\psi_d}{dt} - \psi_q \frac{d\theta}{dt} - R_a i_d \text{ -------------------- } (41)$$

$$e_q = -\frac{d\psi_q}{dt} + \psi_d \frac{d\theta}{dt} - R_a i_q \text{ ------------------ } (42)$$

$$e_o = -\frac{d\psi_o}{dt} - R_a i_o \text{ ---------------------------- } (43)$$

The armature terminal voltage in terms of the flux linkages in phase a is then,

$$V_a = -\frac{d}{dt}\left(\sqrt{\frac{2}{3}}\psi_d \cos\theta + \sqrt{\frac{2}{3}}\psi_q \sin\theta + \frac{1}{\sqrt{3}}\psi_o\right) - R_a i_a$$

$$= -\frac{d\psi_a}{dt} - R_a i_a \text{ ------------------------ } (44)$$

in which $\psi_a = \sqrt{\frac{2}{3}}\psi_d \cos\theta + \sqrt{\frac{2}{3}}\psi_q \sin\theta + \sqrt{\frac{1}{3}}\psi_o$ ------------(45)

= flux linkage of phase a.

Corresponding equations can be written for phases b and c similarly.

Performance in the Field Circuit

The flux linkage of the field is due to:

1) The flux produced by i_f, and

2) The mutual flux produced by the armature currents.

The mutual inductance between phase a and the field is,

$$L_{af} = L_{afm} \cos\theta \text{ -------------------- } (46)$$

Similarly, for phase b and phase c,

$$L_{bf} = L_{afm} \cos\left(\theta - \frac{2\pi}{3}\right) \text{-----------------} (47)$$

$$L_{cf} = L_{afm} \cos\left(\theta - \frac{4\pi}{3}\right) \text{---------------} (48)$$

The total flux linkages in the field is then

$$\psi_f = L_{ff} i_f + L_{afm}\left\{ i_a \cos\theta + i_b \cos\left(\theta - \frac{2\pi}{3}\right) + i_c \cos\left(\theta - \frac{4\pi}{3}\right)\right\}$$

$$= L_{ff} i_f + \sqrt{\frac{3}{2}}\, L_{afm}\, i_d$$

$$= L_{ff} i_f + M_{af} i_d \text{--------------------} (49)$$

where L_{ff} = self-inductance of the field.

The voltage equation of the field therefore, is,

$$e_f = R_f i_f + \frac{d\psi_f}{dt}$$

$$= R_f i_f + L_{ff}\frac{di_f}{dt} + M_{af}\frac{di_d}{dt} \text{---------------} (50)$$

where e_f = field terminal voltage being treated as a voltage drop.

R_f = field resistance.

Equations (38) and (50) shows evidently that they may be considered as the expressions for the flux linkages of two coupled circuits, the field circuit and the direct-axis armature circuit, having a fixed coupling with the field circuit, in which the current is i_d. When the factor $\sqrt{\frac{2}{3}}$ is used for i_d, instead of $\frac{2}{3}$, as was originally proposed by Park, the mutual inductance becomes $M_{af} = \sqrt{\frac{3}{2}} L_{afm}$ in both directions, so that the system is reciprocal.

Representation of a Synchronous Machine by D-Axis, Q-Axis, and Zero-Sequence Windings

According to the machine performance equations developed, the actual machine can be replaced by a set of imaginary circuits as shown in Fig. 2.

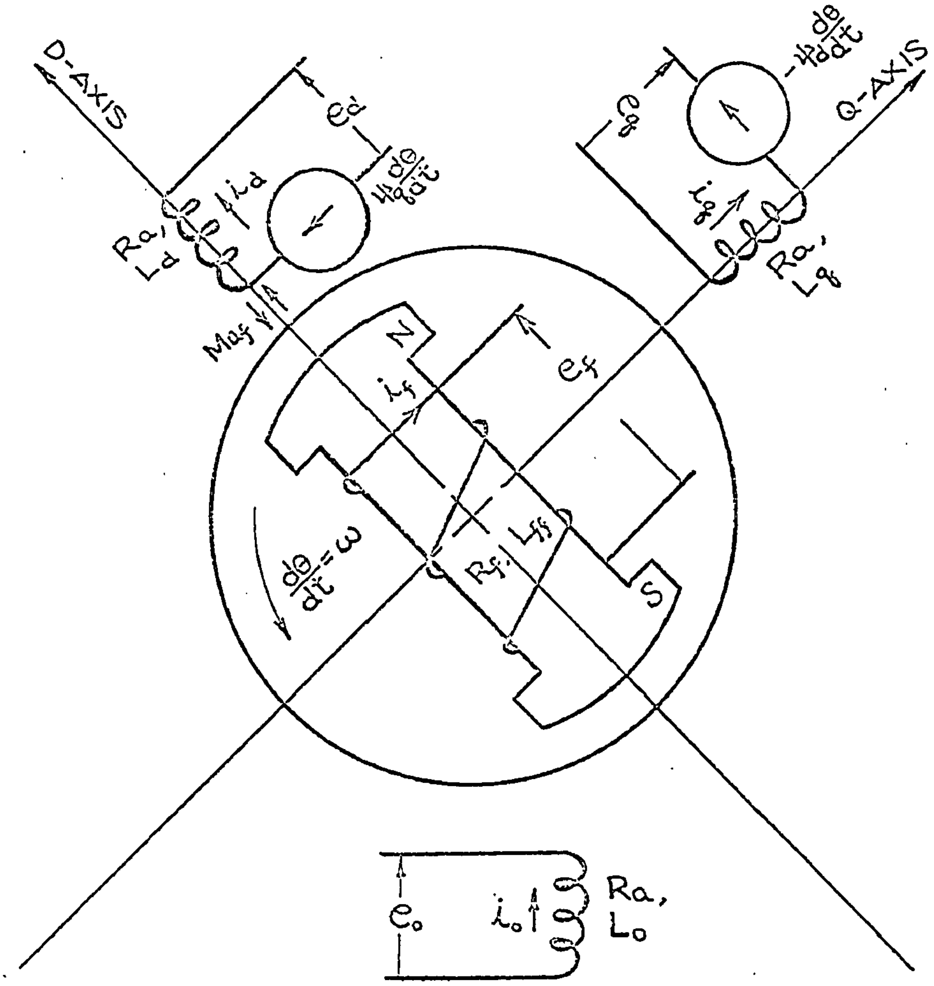

Fig. 2. Representation of a synchronous machine with reciprocal mutual inductance by direct-axis, quadrature-axis, and zero-sequence windings.

In Fig. 2, the field on the rotor is the same as in the actual machine. The actual armature circuits are now represented by the d-axis, q-axis, and the zero-sequence circuits. The direct and quadrature axis circuits are centered on the d-axis and the q-axis respectively, and rotate synchronously with the field in order to maintain their relative positions. The d-axis circuit has a constant mutual inductance, M_{af}, with the field, and this inductance is the same in both directions. The q-axis circuit has no mutual inductance with the field or the d-axis circuit, but will have constant mutual inductance with any field circuit in the q-axis. The zero-sequence circuit is stationary and independent.

Extension to Damper Circuits

Additional rotor circuits are provided in synchronous machines for several purposes, of which the following are the most important:

1) To damp mechanical oscillations or hunting of the rotor,

2) To minimize armature harmonics during unbalanced or single-phase operating conditions, and

3) To provide starting torque, if the machine is operated as a self-starting motor, either normally or under emergency conditions.

The effects of these short-circuited damper windings which have been neglected in the analysis of machine performance thus far will now be taken into consideration.

The additional rotor circuits in the direct axis will have mutual inductances with the field winding because both are symmetrical about the direct axis. However, the additional rotor circuits in the quadrature axis will have no coupling with the field, because they are symmetrical about different axes.

Under normal balanced steady-state conditions at synchronous speed the currents in all additional rotor circuits are zero. However, under transient or unbalanced conditions, or with operation at non-synchronous speed, currents may be induced may be induced in the additional rotor circuits and their effects must be taken into account in all machines which have these additional circuits. To simplify our analysis, assume that the machine has only one damper circuit in each axis.

Let, R_{11d} = resistance of direct-axis damper circuit.

R_{11q} = resistance of quadrature-axis damper circuit.

L_{11d} = self-inductance of direct-axis damper circuit.

L_{11q} = self-inductance of quadrature-axis damper circuit.

L_{a1d} = mutual inductance between direct-axis armature circuit and direct-axis damper circuit.

L_{a1q} = mutual inductance between quadrature-axis armature circuit and quadrature-axis damper circuit.

M_{f1d} = mutual inductance between the field and the direct-axis damper circuit.

i_{11d} = direct-axis damper circuit current.

i_{11q} = quadrature-axis damper circuit current.

The equations for flux linkages (38), (39), and (49) may now be modified as,

$$\psi_d = \sqrt{\tfrac{3}{2}}\, L_{afm} i_f + L_d i_d + \sqrt{\tfrac{3}{2}}\, L_{a1d}\, i_{11d}$$

$$= M_{af} i_f + L_d i_d + M_{a1d} i_{11d} \text{ ----------------------- } (51)$$

$$\psi_q = L_q i_q + \sqrt{\tfrac{3}{2}}\, L_{a1q}\, i_{11q}$$

$$= L_q i_q + M_{a1q} i_{11q} \text{ ------------------------- } (52)$$

$$\psi_f = L_{ff} i_f + \sqrt{\tfrac{3}{2}}\, L_{afm} i_d + M_{f1d} i_{11d}$$

$$= L_{ff} i_f + M_{af} i_d + M_{f1d} i_{11d} \text{ --------------- } (53)$$

in which we define,

$$M_{a1d} = \sqrt{\tfrac{3}{2}}\, L_{a1d}$$

$$M_{a1q} = \sqrt{\tfrac{3}{2}}\, L_{a1q}$$

The voltage equations for the damper circuits in the direct and quadrature axes are respectively,

$$0 = -\frac{d\psi_{1d}}{dt} - i_{11d} R_{11d} \text{ ------------------ } (54)$$

$$0 = -\frac{d\psi_{1q}}{dt} - i_{11q} R_{11q} \text{ ------------------ } (55)$$

where

$$\psi_{1d} = L_{11d} i_{11d} + M_{f1d} i_f + \sqrt{\frac{3}{2}} L_{a1d} i_d$$

$$= L_{11d} i_{11d} + M_{f1d} i_f + M_{a1d} i_d \text{ ------------------ } (56)$$

$$\psi_{1q} = L_{11q} i_{11q} + \sqrt{\frac{3}{2}} L_{a1q} i_q$$

$$= L_{11q} i_{11q} + M_{a1q} i_q \text{ ------------------ } (57)$$

Substituting equations (51), (52), (53) into equations (41), (42), and (50), and also substituting equations (56) and (57) into equations (54) and (55), the following six equations are obtained, by putting

$\frac{d}{dt} = D$, $\frac{d\theta}{dt} = \dot{\theta}$.

$$\left.\begin{aligned} e_d &= -(R_a + L_d D) i_d - L_q i_q \dot{\theta} - M_{af} D i_f - M_{a1d} D i_{11d} \\ &\quad - M_{a1q} i_{11q} \dot{\theta} \\ e_q &= L_d i_d \dot{\theta} - (R_a + L_q D) i_q + M_{af} i_f \dot{\theta} + M_{a1d} i_{11d} \dot{\theta} \\ &\quad - M_{a1q} D i_{11q} \\ e_f &= M_{af} D i_d + (R_f + L_{ff} D) i_f + M_{f1d} D i_{11d} \end{aligned}\right\} (58)$$

$$
\left.\begin{aligned}
0 &= -M_{a1d} D i_d - M_{f1d} D i_f - (R_{11d} + L_{11d} D) i_{11d} \\
0 &= -M_{a1q} D i_q - (R_{11q} + L_{11q} D) i_{11q} \\
e_o &= -(R_a + L_o D) i_o
\end{aligned}\right\} \quad (58)
$$

The six linear differential equations above may be solved simultaneously to obtain the six unknown currents, viz., i_d, i_q, i_f, i_{11d}, i_{11q}, and i_o. Once i_d, i_q, and i_o are known, the phase currents i_a, i_b, and i_c can be obtained from equations (11), (12), and (13).

Torque

The torque is found by the following equation:

$$
\begin{aligned}
T_{3\phi} &= K \frac{3P}{4} (\psi_q i_d - \psi_d i_q) \\
&= -K \frac{3P}{4} (M_{af} i_f i_q + (L_d - L_q) i_d i_q \\
&\qquad + M_{a1d} i_{11d} i_q - M_{a1q} i_{11q} i_d) \quad (59)
\end{aligned}
$$

in which P = number of poles,

K = 550/746 if the torque is expressed in lb-ft.,
= 1, if the torque is expressed in newton-meter.

Equations (58) and (59) are valid for the case where there is only one damper circuit in each axis. The extension to any number of damper circuits may be made in a similar manner.

CHAPTER II

THREE-PHASE SHORT CIRCUIT OF A SYNCHRONOUS MACHINE WITH NO DAMPER CIRCUITS

Introduction

The differential equations of machine performance which determine the transient currents are the simultaneous equations (34), (35), (36), and (50). At constant machine speed these voltage-current equations are linear differential equations with constant coefficients. These equations will now be solved for a three-phase short circuit at the terminals of a synchronous machine with no damper circuits. Rigorous expressions as well as approximate ones for currents and torque will be derived.

Initial Operating Conditions

Suppose a synchronous generator is operating with a balanced load at synchronous speed ω , and a torque angle δ . By putting $i_d = i_q = i_o = 0$ and $i_f = i_{fo}$ in equations (34), (35), (36) and (37), the open circuit induced voltage is obtained,

$$e_a = \sqrt{\frac{2}{3}} M_{af} i_{fo} \frac{d\theta}{dt} \sin\theta$$

$$= \sqrt{\frac{2}{3}}\, \omega \left(\sqrt{\frac{3}{2}} L_{afm}\right) i_{fo} \sin\theta$$

$$= \omega L_{afm} i_{fo} \sin\theta$$

$$= E_f \sin(\omega t + \theta_o)$$

where $E_f = \omega L_{afm} i_o$

i_{fo} = constant field current before short circuit.

θ_o = angle between direct axis and axis of phase a at t = 0.

The armature terminal voltage for phase a is then,

$$v_a = V_a \sin(\omega t + \theta_o - \delta)$$

$$= V_a \sin(\omega t + \theta_o)\cos\delta - V_a \cos(\omega t + \theta_o)\sin\delta$$

$$= -V_a \sin\delta \cos\theta + V_a \cos\delta \sin\theta$$

Hence, by comparing with equation (37), we see that

$$\left.\begin{aligned} e_{do} &= -\sqrt{\frac{3}{2}}\, V_a \sin\delta \\ e_{qo} &= \sqrt{\frac{3}{2}}\, V_a \cos\delta \\ e_{oo} &= 0 \end{aligned}\right\} \quad (60)$$

Since the machine is operating under steady state initially, equation (34), (35), and (36) become,

$$\left.\begin{aligned}
e_{do} &= -\sqrt{\tfrac{3}{2}}\, V_a \sin\delta \\
&= -L_q i_{qo} \frac{d\theta}{dt} - R_a i_{do} \\
&= -X_q i_{qo} - R_a i_{do} \\
e_{qo} &= \sqrt{\tfrac{3}{2}}\, V_a \cos\delta \\
&= M_{af} i_{fo} \frac{d\theta}{dt} + L_d i_{do} \frac{d\theta}{dt} - R_a i_{qo} \\
&= \sqrt{\tfrac{3}{2}}\, L_{afm} i_{fo}\,\omega + L_d i_{do}\,\omega - R_a i_{qo} \\
&= \sqrt{\tfrac{3}{2}}\, E_f + X_d i_{do} - R_a i_{qo} \\
e_{oo} &= 0 = -R_a i_{oo}
\end{aligned}\right\} \quad (61)$$

By solving equation (61), the initially load currents are obtained,

$$i_{do} = -\frac{\sqrt{\tfrac{3}{2}}\, X_q (E_f - V_a\cos\delta) - \sqrt{\tfrac{3}{2}}\, R_a V_a \sin\delta}{R_a^2 + X_d X_q} \text{------------} (62)$$

$$i_{qo} = \frac{\sqrt{\tfrac{3}{2}}\, R_a (E_f - V_a\cos\delta) + \sqrt{\tfrac{3}{2}}\, X_d V_a \sin\delta}{R_a^2 + X_d X_q} \text{---------} (63)$$

$$i_{oo} = 0 \text{--} (64)$$

Rigorous Solution of Short Circuit Current

When the armature winding is short-circuited, the magnetic flux linked with the closed field circuit cannot change significantly in the first moment, yet, since it cannot now enter the short-circuited armature winding, it is forced by the demagnetizing action of the armature currents to pass through paths (the leakage paths) of greater reluctance; that is, the short-circuited armature is equivalent to an increased magnetic reluctance. Hence, an additional current must appear in the field circuit in order to sustain the flux in those new paths, and this spontaneous additional direct-current, being unsupported by the exciter voltage, is, of course, transient in character; that is, the voltage which supports it through the resistance of the field circuit is generated by the decay of the flux through that circuit.

Since a three-phase short circuit suddenly reduces the terminal voltage to zero, the net effect of this on the circuit currents is equivalent to applying $-e_{do}$, $-e_{qo}$, and $-e_{oo}$ in the voltage-current differential equations where e_{do}, e_{qo}, and e_{oo} are the values of e_d, e_q, and e_o before short circuit respectively. Assuming that constant synchronous speed is maintained, and that the field excitation is constant, the voltage-current differential equations (34) and (35) become, by putting $\frac{d}{dt} = D$, $\frac{d\theta}{dt} = \omega$, and U(t) for the unit function,

$$\left.\begin{aligned} M_{af} D i_f + (R_a + L_d D) i_d + \omega L_q i_q &= -\sqrt{\tfrac{3}{2}}\, V_a \sin\delta\, U(t) \\ \omega M_{af} i_f + \omega L_d i_d - (R_a + L_q D) i_q &= \sqrt{\tfrac{3}{2}}\, V_a \cos\delta\, U(t) \\ &\text{and also by equations (36) and (50),} \\ -(R_a + L_0 D) i_0 &= 0 \\ (R_f + L_{ff} D) i_f + M_{af} D i_d &= 0 \end{aligned}\right\} \quad (65)$$

The currents in equation (65) are only the components caused by the fault, and the currents found from these equations must be added to the initial currents existing before the short circuit in order to obtain the resultant current after the fault has occurred.

Equation (65) shows that no zero sequence effects are produced by simultaneous short circuit of all three phases. Therefore, it is only necessary to consider the direct and quadrature equations. Applying the Laplace transform to these differential equations, and letting I(S) denote the transform of i, the following algebraic equations are obtained,

$$\left.\begin{aligned}\frac{-\sqrt{\frac{3}{2}}\,V_a \sin\delta}{S} &= M_{af} S I_f(S) + (R_a + L_d S) I_d(S) + \omega L_q I_q(S) \\ \frac{-\sqrt{\frac{3}{2}}\,V_a \cos\delta}{S} &= \omega M_{af} I_f(S) + \omega L_d I_d(S) - (R_a + L_q S) I_q(S) \\ 0 &= (R_f + L_{ff} S) I_f(S) + M_{af} S I_d(S)\end{aligned}\right\} \quad (66)$$

The three equations above are solved simultaneously to obtain the following operational expressions for $I_d(S)$, $I_q(S)$, and $I_f(S)$.

$$I_d(S) = -\frac{\sqrt{\frac{3}{2}}\,V_a (R_f + L_{ff} S)\left\{(R_a + L_q S)\sin\delta + \omega L_q \cos\delta\right\}}{S\,D(S)} \quad \text{--------}(67)$$

$$I_q(S) = \frac{\sqrt{\frac{3}{2}}\,V_a (R_f + L_{ff} S)\left\{(R_a + L_d(S) S^2)\cos\delta - \omega L_d(S)\sin\delta\right\}}{S\,D(S)} \quad \text{----}(68)$$

$$I_f(S) = \frac{\sqrt{\frac{3}{2}}\,V_a M_{af}\left\{(R_a + L_q S)\sin\delta + \omega L_q \cos\delta\right\}}{D(S)} \quad \text{----------------}(69)$$

where $L_d(S) = \frac{L_d}{S} - \frac{M_{af}^2}{(R_f + L_{ff} S)}$ -------------------- (70)

= direct-axis Laplace transform inductance.

$D(S)$ = determinental equation

$$= S^3 L_{ff} L_q L_d' + S^2\left\{R_f L_d L_q + R_a L_{ff}(L_d' + L_q)\right\}$$
$$+ S\left\{R_a R_f (L_d + L_q) + L_{ff}(R_a^2 + \omega^2 L_q L_d')\right\}$$
$$+ R_f (R_a^2 + \omega^2 L_d L_q) \quad \text{--------------------------}(71)$$

Equation (71) can be factorized to yield $D(S) = L_{ff}L_qL_d'(S - S_1)(S - S_2)(S - S_3)$, in which S_1, S_2, and S_3 are the roots of $D(S) = 0$ found by numerical methods. L_d' is defined as,

$$L_d' = \lim_{S \to \infty} S\,L_d(S) = \lim_{S \to \infty} \left(L_d - \frac{M_{af}^2 S}{(R_f + L_{ff}S)} \right)$$

$$= L_d - \frac{M_{af}^2}{L_{ff}}$$

which in the notation of Laplace transform calculus is equivalent to computing the initial value of this function. Thus L_d' is very appropriately called the transient inductance of the direct axis.

The inverse transforms of equations (67), (68), and (69) can be found by means of the Heaviside expansion,

$$i = \sum_{k=0}^{n} \frac{P(S_k)}{q'(S_k)} \varepsilon^{S_k t}$$

$$i_d = -\frac{\sqrt{\frac{3}{2}}\,V_a(R_a \sin\delta + X_q \cos\delta)}{R_a^2 + X_d X_q} - \sum_{K=1}^{3} \frac{\sqrt{\frac{3}{2}}\,V_a(R_f + L_{ff}S_k)\left\{(R_a + L_q S_k)\sin\delta + \omega L_q \cos\delta\right\}}{S_k D'(S_k)} \varepsilon^{S_k t}$$

$$i_q = \frac{\sqrt{\frac{3}{2}} V_a (R_a \cos\delta - X_d \sin\delta)}{R_a^2 + X_d X_q}$$

$$+ \sum_{K=1}^{3} \frac{\sqrt{\frac{3}{2}} V_a (R_f + L_{ff} S_K) \{ (R_a + L_d(S_K) S_K^2) \cos\delta - \omega L_d(S_K) S_K \sin\delta}{S_K D'(S_K)} \varepsilon^{S_K t}$$

$$i_f = \sum_{K=1}^{3} \frac{\sqrt{\frac{3}{2}} M_{af} V_a \{ (R_a + L_q S_K) \sin\delta + \omega L_q \cos\delta \}}{D'(S_K)} \varepsilon^{S_K t}$$

where $$D'(S) = \frac{d}{dS} D(S)$$

The resultant current after the short circuit is obtained by adding i_{do} to i_d, etc.; therefore,

$$i_d(t) = -\frac{\sqrt{\frac{3}{2}} X_q E_f}{R_a^2 + X_d X_q}$$

$$+ \sum_{K=1}^{3} \frac{\sqrt{\frac{3}{2}} V_a (R_f + L_{ff} S_K) \{ (R_a + L_q S_K) \sin\delta + X_q \cos\delta \}}{S_K D'(S_K)} \varepsilon^{S_K t} \quad \text{--------(72)}$$

$$i_q(t) = \frac{\sqrt{\frac{3}{2}} R_a E_f}{R_a^2 + X_d X_q}$$

$$+ \sum_{K=1}^{3} \frac{\sqrt{\frac{3}{2}} V_a (R_f + L_{ff} S_K) \{ (R_a + L_d(S_K) S_K^2) \cos\delta - \omega L_d(S_K) S_K \sin\delta}{S_K D'(S_K)} \varepsilon^{S_K t}$$

$$\text{------------- (73)}$$

$$i_f(t) = i_{fo} + \frac{\sqrt{\frac{3}{2}} M_{af} V_a \{ (R_a + L_q S_K) \sin\delta + X_q \cos\delta \}}{D'(S_K)} \varepsilon^{S_K t} \quad \text{------------(74)}$$

Equations (72), (73), and (74) are the rigorous solutions for i_d, i_q, and i_f after a three-phase short circuit, with initial operating torque angle δ and armature terminal voltage V_a. The phase currents may, of course, be found from these equations by the application of equations (11), (12), and (13). The short circuit torque may also be found from the fundamental equation of torque.

$$T_{3\phi} = K \frac{3P}{4}(\psi_q i_d - \psi_d i_q)$$

$$= -K\frac{3P}{4\omega}\left\{X_{af} i_f i_q + (X_d - X_q) i_d i_q\right\} \qquad (75)$$

The roots of the determinental equation, D(S) = 0, for each particular case must be found, in actual cases, by numerical methods.

Approximate Solution with
all Resistances Neglected

The effect of the internal resistance of the machine windings in determining the magnitude of the short circuit currents, in most cases, is negligible. An approximate solution with these resistances neglected, therefore, is justified and permissible.

Recalling the determinental equation D(S) and letting,

$$\frac{R_a}{L_d} = C_d \ ,$$

$$\frac{R_a}{L_q} = C_q \,,$$

$$\frac{R_f}{L_{ff}} = C_f \,, \text{ and,}$$

$$1 - \frac{M_{af}^2}{L_{ff} L_d} = \tau$$

= leakage coefficient in the direct axis.

Dividing the determinental equation by $L_d' L_{ff} L_q$, we get,

$$S^3 + S^2 (C_f + C_d + \tau C_q)\frac{1}{\tau} + S(\omega^2 \tau + C_d C_q + C_f C_d + C_f C_q)\frac{1}{\tau}$$

$$+ \frac{1}{\tau} C_f (\omega^2 + C_d C_q) = 0 \text{ ------------------- } (76)$$

It has, in most cases, one real negative and a pair of conjugate complex roots. As ω is large as compared with the C's, the determinental equation is approximately equal to,

$$S^3 + S^2 \left(\frac{C_f + C_d}{\tau} + C_q \right) + \omega^2 S + \omega \frac{C_f}{\tau} = 0 \text{ ----------------- } (77)$$

The root of S having the smaller magnitude is closely approximated by taking the ratio of the last two coefficients.

$$S_1 = -\frac{C_f}{\tau} = -\frac{L_d}{L_d'} \frac{R_f}{L_{ff}} \text{ ------------------ } (78)$$

The time constant for thsi transient is,

$$T_{ds}' = \frac{L_d'}{L_d} T_{do}' \text{ ---------------------- } (79)$$

in which $T_{do}' = \frac{L_{ff}}{R_f}$ ....(80)

The time constants T_{d3}' and T_{do}' involve only the parameters of the field. They are thus appropriately called the transient field time constant and the open-circuit field time constant, respectively. In terms of i_d and i_q the transient having the time constant T_{d3}' is a damped direct current. But as the phase currents are related to i_d and i_q by equations (11), (12), and (13), the corresponding armature transient is a damped sinusoidal current of fundamental frequency. (See Fig. 15 on page 79.) The decay of this transient part of the phase currents correspond to the decay of flux linking the field and would be governed by the field time constant.

Removing S_1 from equation (77) by factoring, the resultant quadratic equation is approximately,

$$S^2 + \left(\frac{C_d}{\tau} + C_q\right)S + \omega^2 = 0 \quad \text{..................(81)}$$

The roots of which are,

$$S_{2,3} = -\left(\frac{C_d + \tau C_q}{2\tau}\right) \pm j\omega$$

$$= -m \pm j\omega \quad \text{..................(82)}$$

The time constant of these oscillatory transients is,

$$T_{as}' = \frac{1}{m} = \frac{2\tau}{C_d + \tau C_q} = \frac{2 L_d' L_q}{R_a (L_d + L_q)} \text{------------------}(83)$$

which involves only the parameters of the armature, and is called the transient armature time constant. Equation (81) and (82) show that the oscillatory transients for i_d and i_q have an angular velocity nearly the same as the synchronous value, unless R_a is very large. With these values of i_d and i_q, the corresponding phase currents may have both dc and double-frequency components. Both of these components arise from the flux which is trapped in the armature circuits at the instant of short circuit. This flux gradually decays to zero and generates fundamental frequency current and flux in the field. Therefore, the oscillatory transients for i_d and i_q are damped according to the armature time constant.

The approximate expression for D(S) is therefore given by:

$$D(S) = L_{ff} L_d' L_q (S - S_1)(S - S_2)(S - S_3) \text{--------------------}(84)$$

The approximate expressions for i_d, i_q, and i_f can now be found by substituting the values of S_1, S_2, and S_3 into equations (72), (73), and (74). Since, however, the results will be approximate, it is desirable to make other simplifications involving approximations of the same order. As R_a and R_f are normally small in comparison to their reactances at fundamental frequency, a satisfactory value of transient current is obtained if the armature and field resistances are neglected. As an illustration, we are going to derive the approximate solutions for these currents as follows.

Differentiating equation (84) with respect to S, and substituting S_1 and S_2 for S respectively, and with all resistances neglected, we have,

$$D'(S_1) = L_{ff} L_d' L_q (S_1 - S_2)(S_1 - S_3)$$

$$\cong L_{ff} X_d' X_q \quad \text{.........(85)}$$

$$D'(S_2) = L_{ff} L_d' L_q (S_2 - S_1)(S_2 - S_3)$$

$$\cong -2 L_{ff} X_d' X_q \quad \text{.........(86)}$$

$$\frac{R_f + L_{ff} S_1}{S_1 D'(S_1)} \cong \frac{1}{X_q}\left(\frac{1}{X_d'} - \frac{1}{X_d}\right) \quad \text{.........(87)}$$

$$\frac{R_f + L_{ff} S_2}{S_2 D'(S_2)} \cong -\frac{1}{2}\left(\frac{1}{X_q} \frac{1}{X_d'}\right) \quad \text{.........(88)}$$

Substituting these approximations in equation (72) and neglecting all the terms that involve the resistances, we have the following transient terms for

i_a. A damped direct current

$$i_d(S_1) \cong \left(\frac{1}{X_d'} - \frac{1}{X_d}\right) \varepsilon^{-\frac{t}{T_d'}} \sqrt{\frac{3}{2}}\, V_a \cos\delta \text{ ....} (89)$$

An oscillatory transient term,

$$i_d(S_2) \cong -\frac{1}{2}\frac{1}{X_d'}\, \varepsilon^{-\frac{t}{T_a'}} \sqrt{\frac{3}{2}}\, V_a (j\sin\delta + \cos\delta)\, \varepsilon^{j\omega t} \text{ ....} (90)$$

and since S_2 and S_3 are conjugate, a second oscillatory transient term $i_d(S_3)$ can be combined with $i_d(S_2)$ to give,

$$i_d(S_2) + i_d(S_3) = 2\,R_e\, i_d(S_2)$$

$$\cong -\frac{1}{X_d'}\, \varepsilon^{-\frac{t}{T_a'}} \sqrt{\frac{3}{2}}\, V_a \cos(\omega t + \delta) \text{ ....} (91)$$

The complete expression for i_d, including the initial load current, therefore, is

$$i_d(t) = -\frac{\frac{3}{2}E_f}{X_d} - \left\{ i_d(S_1) + i_d(S_2) + i_d(S_3) \right\}$$

$$= -\frac{\frac{3}{2}E_f}{X_d} - \left(\frac{1}{X_d'} - \frac{1}{X_d}\right) \varepsilon^{-\frac{t}{T_d'}} \sqrt{\frac{3}{2}}\, V_a \cos\delta$$

$$+ \frac{1}{X_d'}\, \varepsilon^{-\frac{t}{T_a'}} \sqrt{\frac{3}{2}}\, V_a \cos(\omega t + \delta) \text{ ....} (92)$$

Similarly, from equation (73) and (74), the complete expressions for i_q and i_f are,

$$i_q(t) = \frac{1}{X_q}\, \varepsilon^{-\frac{t}{T_a'}} \sqrt{\frac{3}{2}}\, V_a \sin(\omega t + \delta) \text{ ....} (93)$$

$$i_f(t) = i_{f0} + \frac{\sqrt{\frac{3}{2}}\,M_{af}}{L_{ff}} \frac{V_a}{X_d'} \left\{ \varepsilon^{-\frac{t}{T_{d3}'}} \cos\delta - \varepsilon^{-\frac{t}{T_{a3}'}} \cos(\omega t + \delta) \right\} \quad \text{......(94)}$$

Equations (92), (93), and (94) are the approximate solutions for i_d, i_q, and i_f with all resistances neglected, except in the decremental factors. They are valid only for the cases in which the armature and field resistances are relatively small.

Approximate Solution with Armature Resistance

The effect of resistance is to cause the dc component and the second harmonic component which appear in the short circuit current to decay so rapidly that they have negligible effect on the total transient current.

When the resistance is negligible the results obtained in the last section will be accurate. It remains, however, to consider the case of a short circuit occurring through an external resistance which is not negligible. When the resistance is large, the transient armature time constant, as given by equation (83), is so small that the oscillatory transients for i_d and i_q will disappear almost instantly. Therefore, for the present case the oscillatory transients have negligible effect on the total transient current, and may be neglected. The derivation of the approximate solution is very similar to that of the preceding analysis, except that values of total resistance should be used instead of just the armature resistance.

Let r_e = external resistance of the armature circuit.

$r = r_e + R_a$

= total resistance of the armature circuit.

The determinental equation (76) is then approximately

$$S^3 + \left(\frac{C_f + C_d}{\tau} + C_q\right)S^2 + \left(\omega^2 + \frac{C_d C_q}{\tau}\right)S + \frac{1}{\tau}C_f(\omega^2 + C_d C_q) = 0 \quad \text{--------(95)}$$

The single root corresponding to the effective field decrement factor as obtained approximately from the last two terms is

$$S_1 \cong -\frac{C_f(\omega^2 + C_d C_q)}{\omega^2 \tau + C_d C_q}$$

$$= -\frac{1}{T_{do}'}\frac{r^2 + X_d X_q}{r^2 + X_d' X_q} \quad \text{------------------------(96)}$$

The effective field time constant T_{d2}' therefore is,

$$T_{d2}' = T_{do}' \frac{r^2 + X_d' X_q}{r^2 + X_d X_q} \quad \text{----------------------(97)}$$

This is a more general expression than equation (79) to which it reduces if r = 0.

The expression for D(S), as far as the root S_1 is concerned, may be approximated by,

$$D(S) \cong L_{ff}(r^2 + X_d' X_q)S + R_f(r^2 + X_d X_q) \text{.................} (98)$$

Then, $D'(S_1) \cong L_{ff}(r^2 + X_d' X_q)$

$$\frac{R_f + L_{ff} S_1}{S_1 D'(S_1)} \cong \frac{(X_d - X_d')X_q}{(r^2 + X_d' X_q)(r^2 + X_d X_q)} \text{..............} (99)$$

Applying the approximations of equations (96) and (99) and neglecting all terms containing R_f and the oscillatory transient terms, the new forms of equation (72), (73) and (74) are,

$$i_d(t) = -\frac{\frac{3}{2} X_q E_f}{r^2 + X_d X_q} - \frac{X_q(X_d - X_d')(r \sin\delta + X_q \cos\delta)}{(r^2 + X_d X_q)(r^2 + X_d' X_q)} \sqrt{\frac{3}{2}}\, V_a \varepsilon^{-\frac{t}{T_{d2}'}} \text{.........} (100)$$

$$i_q(t) = \frac{\frac{3}{2} r E_f}{r^2 + X_d X_q} + \frac{r(X_d - X_d')(r \sin\delta + X_q \cos\delta)}{(r^2 + X_d X_q)(r^2 + X_d' X_q)} \sqrt{\frac{3}{2}}\, V_a \varepsilon^{-\frac{t}{T_{d2}'}} \text{.........} (101)$$

$$i_f(t) = i_{fo} + \sqrt{\frac{3}{2}} \frac{M_{af}}{L_{ff}} \frac{(r \sin\delta + X_q \cos\delta)}{r^2 + X_d' X_q} V_a \varepsilon^{-\frac{t}{T_{d2}'}} \text{...........} (102)$$

The phase current in the three-phase short circuit is therefore, by equations (11), (12), and (13),

$$i_a = i_b = i_c = -\sqrt{r^2 + x_q^2}\left\{\frac{E_f}{r^2 + x_d x_q} + \frac{(x_d - x_d')(r\sin\delta + x_q\cos\delta)}{(r^2 + x_d x_q)(r^2 + x_d' x_q)} V_a \varepsilon^{-\frac{t}{T_{d2}'}}\right\}$$

$$\cos\left(\omega t + \theta_0 + \tan^{-1}\frac{r}{x_q}\right) \quad \text{..................} (103)$$

CHAPTER III

UNBALANCED SHORT CIRCUIT

Moving Reference Axes

A set of orthogonal moving reference axes (α , β) will be introduced here to solve the unsymmetrical short circuit cases. The α-axis is rigidly attached to phase a as shown in Fig. 3. The displacement of the new axes from the stationary axes is $\theta(t)$, a function of time. Considering only the fundamental in the space distribution of armature reaction and air-gap flux density, the transformations from the stationary to moving axis quantities are,

$$\left.\begin{aligned} f_\alpha &= f_d \operatorname{Cos}\theta + f_q \operatorname{Sin}\theta \\ f_\beta &= -f_d \operatorname{Sin}\theta + f_q \operatorname{Cos}\theta \end{aligned}\right\} \text{..........} (104)$$

or conversely,

$$\left.\begin{aligned} f_d &= f_\alpha \operatorname{Cos}\theta - f_\beta \operatorname{Sin}\theta \\ f_q &= f_\alpha \operatorname{Sin}\theta + f_\beta \operatorname{Cos}\theta \end{aligned}\right\} \text{..........} (105)$$

where f may stand for i, e, or ψ.

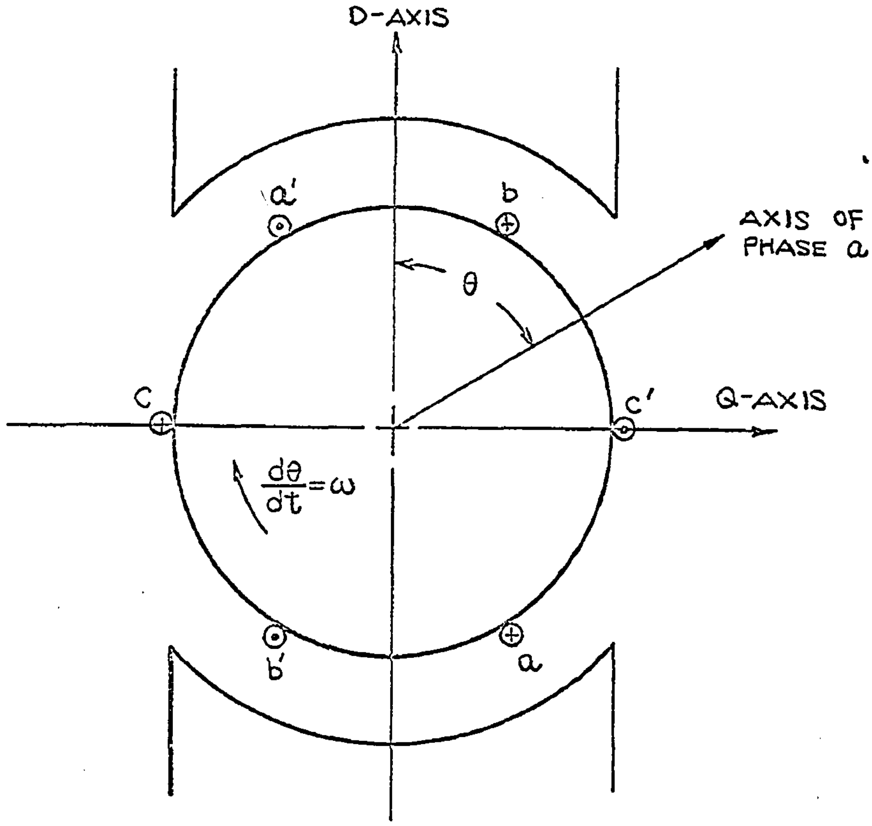

Fig. 3. Elementary diagram of a three-phase machine with moving reference axes.

Current Relations

It is always possible to resolve the armature reaction of the space fundamental into two orthogonal components regardless of the reference axes being chosen. Thus projecting the phase currents on the moving axes and defining the zero-sequence current in the usual way,

$$\left.\begin{aligned} i_\alpha &= \sqrt{\frac{2}{3}}\left\{i_a + i_b \cos\left(-\frac{2\pi}{3}\right) + i_c \cos\left(-\frac{4\pi}{3}\right)\right\} \\ &= \sqrt{\frac{2}{3}}\left(i_a - \frac{1}{2} i_b - \frac{1}{2} i_c\right) \\ i_\beta &= \sqrt{\frac{2}{3}}\left\{i_b \sin\left(-\frac{2\pi}{3}\right) + i_c \sin\left(-\frac{4\pi}{3}\right)\right\} \\ &= -\frac{1}{2}(i_b - i_c) \\ i_0 &= \frac{1}{\sqrt{3}}(i_a + i_b + i_c) \end{aligned}\right\} \cdots\cdots (106)$$

or conversely,

$$\left.\begin{aligned} i_a &= \left(\sqrt{\frac{2}{3}}\, i_\alpha + \frac{1}{\sqrt{3}} i_0\right) \\ i_b &= \left(-\frac{1}{\sqrt{2}\sqrt{3}} i_\alpha - \frac{1}{\sqrt{2}} i_\beta + \frac{1}{\sqrt{3}} i_0\right) \\ i_c &= \left(-\frac{1}{\sqrt{2}\sqrt{3}} i_\alpha + \frac{1}{\sqrt{2}} i_\beta + \frac{1}{\sqrt{3}} i_0\right) \end{aligned}\right\} \cdots\cdots (107)$$

Substituting i_d, i_q for i_a, i_b, and i_c, the relation in equations (104) and (105) can be verified:

$$\left.\begin{aligned} i_\alpha &= i_d \cos\theta + i_q \sin\theta \\ i_\beta &= -i_d \sin\theta + i_q \cos\theta \end{aligned}\right\} \cdots\cdots (108)$$

or conversely,

$$\left.\begin{aligned} i_d &= i_\alpha \cos\theta - i_\beta \sin\theta \\ i_q &= i_\alpha \sin\theta + i_\beta \cos\theta \end{aligned}\right\} \cdots\cdots (109)$$

Voltage Relations

It has been shown that ψ_d and ψ_q are sinusoidally distributed in space and centered on the d- and q-axes respectively; and ψ_0 is constant and stationary in spcae. With respect to the α-axis, the total flux linked with the armature of a synchronous machine may be represented by the following expression which simply represents a shift of θ degrees to α, β axes from the d, q axes of equation (18) and (40).

$$\psi(x) = \psi_d \operatorname{Sin}\left(\frac{\pi X}{\tau} + \theta + \frac{\pi}{2}\right) + \psi_q \sin\left(\frac{\pi X}{\tau} + \theta\right)$$

$$+ \psi_0 \operatorname{Sin}\left(\frac{\pi X}{\tau} + \frac{\pi}{2}\right)_{\tau \to \infty} \quad \text{------------------------(110)}$$

where, assuming one damper circuit in each d- and q-axis,

$$\left.\begin{aligned} \psi &= \frac{qN}{10^8} K_1 \phi \\ \psi_d &= M_{af} i_f + L_d i_d + M_{a1d} i_{11d} \\ \psi_q &= L_q i_q + M_{a1q} i_{11q} \\ \psi_0 &= L_0 i_0 \end{aligned}\right\} \text{----------------(111)}$$

or, in terms of i_α and i_β ,

$$\left.\begin{aligned} \psi_d &= M_{af} i_f + M_{a1d} i_{11d} + L_d i_\alpha \operatorname{Cos}\theta - L_d i_\beta \operatorname{Sin}\theta \\ \psi_q &= M_{a1q} i_{11q} + L_q i_\alpha \operatorname{Sin}\theta + L_q i_\beta \operatorname{Cos}\theta \\ \psi_0 &= L_0 i_0 \end{aligned}\right\} \text{--------(112)}$$

Equation (110) shows that the first two fluxes, with respect to the α-axis, are travelling waves with a velocity $-\frac{d\theta}{dt}$. The zero-sequence component of the flux is stationary in space and may be considered as a sine wave with infinite wave-length, that is, $\tau \to \infty$.

Since the reference axes are attached to the moving armature, the center of each armature phase winding as measured from the α-axis is constant, and is equal to zero for phase a. Therefore, the general equation of induced voltage reduces to the following form,

$$e = -\left\{\frac{d\psi}{dt}\sin\left(\frac{\pi x_0}{\tau}+\gamma\right) + \psi\frac{d\gamma}{dt}\cos\left(\frac{\pi x_0}{\tau}+\gamma\right)\right\} \text{------------} (113)$$

where $\psi(x) = \psi \sin\left(\frac{\pi x}{\tau}+\gamma\right)$

Applying equations (110) and (113), and putting , and rearranging, there results the expression for the induced voltage, including the reactance drops in phase a. ($\gamma = \theta + \frac{\pi}{2}$ for ψ_d , $\gamma = \theta$ for ψ_q).

$$e_a = +\left\{\psi_d\frac{d\theta}{dt} - \frac{d\psi_q}{dt}\right\}\sin\theta$$
$$-\left\{\frac{d\psi_d}{dt} + \psi_q\frac{d\theta}{dt}\right\}\cos\theta - \frac{d\psi_0}{dt} \text{------------} (114)$$

Substituting equation (112) into (114), we get,

$$e_a = -\frac{d}{dt}(M_{af}\, i_f \cos\theta) - \frac{d}{dt}\, i_\alpha (A + B\cos 2\theta)$$

$$+ \frac{d}{dt}(i_\beta B \sin 2\theta) - \frac{d}{dt}\, i_{11d} M_{a1d} \cos\theta$$

$$- \frac{d}{dt}\, i_{11q} M_{a1q} \sin\theta - \frac{d}{dt} L_o\, i_o \text{ ------------------(115)}$$

$$\left.\begin{aligned} A &= \frac{L_d + L_q}{2} \\ B &= \frac{L_d - L_q}{2} \end{aligned}\right\} \text{--------------(116)}$$

The resistance drop is $R_a i_a = \sqrt{\frac{2}{3}} R_a i_\alpha + \frac{R_a i_o}{\sqrt{3}}$.

Therefore,

$$V_a = e_a - R_a i_a$$

$$= -D(M_{af}\cos\theta) i_f - \left\{ \sqrt{\frac{2}{3}} R_a + D(A + B\cos 2\theta) \right\} i_\alpha$$

$$+ D(B \sin 2\theta) i_\beta - D(M_{a1d}\cos\theta)\, i_{11d}$$

$$- D(M_{a1q}\sin\theta)\, i_{11q} - \left(\frac{R_a}{\sqrt{3}} + DL_o\right) i_o \text{ --------------(117)}$$

Similarly,

$$V_b = -DM_{af}\cos(\theta - 120^\circ) i_f + DM_{a1d}\cos(\theta - 120^\circ)\, i_{11d}$$

$$- DM_{a1q}\sin(\theta - 120^\circ)\, i_{11q}$$

$$+ \left\{ \frac{R_a}{\sqrt{2}\sqrt{3}} + D\left[\frac{A}{2} - B\cos(2\theta - 120^\circ)\right] \right\} i_\alpha$$

$$+ \frac{\sqrt{3}}{2} \left\{ \frac{\sqrt{2}}{3} R_a + D\left[A + \frac{2}{\sqrt{3}} B \sin(2\theta - 120^\circ)\right] \right\} i_\beta$$

$$- \left(\frac{R_a}{\sqrt{3}} + DL_o\right) i_o \text{ ------------------------------------(118)}$$

$$V_c = -DM_{af}\cos(\theta+120°)i_f - DM_{a1d}\cos(\theta+120°)i_{11d}$$

$$- DM_{a1q}\sin(\theta+120°)i_{11q}$$

$$+ \left\{\frac{1}{\sqrt{2}\sqrt{3}}R_a + D\left[\frac{A}{2} - B\cos(2\theta+120°)\right]\right\}i_\alpha$$

$$- \frac{\sqrt{3}}{2}\left\{\frac{\sqrt{2}}{3}R_a + D\left[A - \frac{2}{\sqrt{3}}B\sin(2\theta+120°)\right]\right\}i_\beta$$

$$-\left(\frac{R_a}{\sqrt{3}} + DL_o\right)i_o \text{ ------------------------------------ } (119)$$

and,

$$V_{bc} = V_b - V_c$$

$$= -\sqrt{3}\left\{DM_{af}\sin\theta\, i_f + DM_{a1d}\sin\theta\, i_{11d}\right.$$

$$- DM_{a1q}\cos\theta\, i_{11q} + DB\sin 2\theta\, i_\alpha$$

$$\left. - \left[\frac{\sqrt{2}}{3}R_a + D(A - B\cos 2\theta)\right]i_\beta\right\} \text{ --------------- } (120)$$

Equations (117) and (120) can be rewritten as follows:

$$V_a = e_\alpha + e_o \text{ -------------------- } (121)$$

$$V_{bc} = -\sqrt{3}\, e_\beta \text{ -------------------- } (122)$$

where

$$e_\alpha = -DM_{af}\cos\theta\, i_f - DM_{a1d}\cos\theta\, i_{11d}$$

$$-\left[\sqrt{\frac{2}{3}}R_a + D(A + B\cos 2\theta)\right]i_\alpha$$

$$+ DB\sin 2\theta\, i_\beta - DM_{a1q}\sin\theta\, i_{11q} \text{ --------- } (123)$$

$$e_o = -\left(\frac{R_a}{\sqrt{3}} + DL_o\right)i_o \text{ ------------------------------ } (124)$$

$$e_\beta = DM_{af}\sin\theta\, i_f + DM_{a1d}\sin\theta\, i_{1d}$$

$$+ DB\sin 2\theta\, i_\alpha - \left[\frac{\sqrt{2}}{3}R_a + D(A - B\cos 2\theta)\right] i_\beta$$

$$- DM_{a1q}\cos\theta\, i_{1q} \text{ ------------------(125)}$$

e_α and e_β are the voltage components in the α- and β-axis respectively. When referred to the d- and q-axis,

$$V_a = e_d\cos\theta + e_q\sin\theta + e_0 \text{ ---------------(126)}$$

$$V_{bc} = \sqrt{3}\,(e_d\sin\theta - e_q\cos\theta) \text{ ----------------(127)}$$

Again, the relation in equations (104) and (105) can be verified by comparing equations (121) and (122) with equations (126) and (127). That is,

$$\left.\begin{aligned} e_\alpha &= e_d\cos\theta + e_q\sin\theta \\ e_\beta &= -e_d\sin\theta + e_q\cos\theta \end{aligned}\right\} \text{ -----------------(128)}$$

$$\left.\begin{aligned} e_d &= e_\alpha\cos\theta - e_\beta\sin\theta \\ e_q &= e_\alpha\sin\theta + e_\beta\cos\theta \end{aligned}\right\} \text{ -----------------(129)}$$

Flux Linkage Relations

The α-and β-axis components of voltages can be expressed as,

$$\left.\begin{aligned} e_\alpha &= -\frac{d\psi_\alpha}{dt} - \sqrt{\frac{2}{3}}\,R_a\,i_\alpha \\ e_\beta &= -\frac{d\psi_\beta}{dt} - \frac{\sqrt{2}}{3}\,R_a\,i_\beta \end{aligned}\right\} \text{------------------(130)}$$

in which we define,

ψ_α = α-axis flux linkage

$$= M_{af}\cos\theta\, i_f + M_{a1d}\cos\theta\, i_{11d} + (A + B\cos 2\theta)\, i_\alpha - B\sin 2\theta\, i_\beta + M_{a1q}\sin\theta\, i_{11q} \text{ ----------(131)}$$

ψ_β = β-axis flux linkage

$$= -M_{af}\sin\theta\, i_f - M_{a1d}\sin\theta\, i_{11d} - B\sin 2\theta\, i_\alpha - (A - B\cos 2\theta)\, i_\beta + M_{a1q}\cos\theta\, i_{11q} \text{ -----------(132)}$$

These equations show that when the moving reference axes are used, all the inductances are not constant, but are functions of time because of the movement of fluxes with respect to the reference axes.

Performance in the Field Circuit

The flux linkage of the main field circuit is,

$$\psi_f = L_{ff}\, i_f + \sqrt{\frac{2}{3}}\, M_{af}\left[i_a\cos\theta + i_b\cos\left(\theta - \frac{2\pi}{3}\right) + i_c\cos\left(\theta - \frac{4\pi}{3}\right)\right] + M_{f1d}\, i_{11d}$$

$$= L_{ff}\, i_f + \sqrt{\frac{2}{3}}\, M_{af}\left[\left(i_a - \frac{1}{2} i_b - \frac{1}{2} i_c\right)\cos\theta + \frac{\sqrt{3}}{2}(i_b - i_c)\sin\theta\right] + M_{f1d}\, i_{11d}$$

$$= L_{ff}\, i_f - M_{af}\sin\theta\, i_\beta + M_{af} + M_{af}\cos\theta\, i_\alpha + M_{f1d}\, i_{11d} \text{ ---------(133)}$$

Similarly, the flux linkages of the d- and q-axis damper circuits are,

$$\psi_{1d} = M_{f1d}\, i_f + \sqrt{\tfrac{3}{2}}\, M_{a1d} \cos\theta\, i_\alpha - \sqrt{\tfrac{3}{2}}\, M_{a1d} \sin\theta\, i_\beta$$

$$+ L_{11d}\, i_{11d} \text{ ------------------------------------ } (134)$$

$$\psi_{1q} = \tfrac{3}{2}\, M_{a1q} \sin\theta\, i_\alpha + \tfrac{3}{2}\, M_{a1q} \cos\theta\, i_\beta + L_{11q}\, i_{11q} \text{ --------- } (135)$$

The corresponding voltage equations are,

$$e_f = \frac{d\psi_f}{dt} + R_f\, i_f$$

$$= (R_f + DL_{ff})\, i_f + DM_{af} \cos\theta\, i_\alpha - DM_{af} \sin\theta\, i_\beta$$

$$+ DM_{a1d}\, i_{11d} \text{ ------------------------------------ } (136)$$

$$0 = \frac{d\psi_{1d}}{dt} + R_{1d}\, i_{11d}$$

$$= DM_{f1d}\, i_f + \sqrt{\tfrac{3}{2}}\, DM_{a1d} \cos\theta\, i_\alpha - \sqrt{\tfrac{3}{2}}\, DM_{a1d} \sin\theta\, i_\beta$$

$$+ (R_{1d} + DL_{11d})\, i_{11d} \text{ ------------------------------ } (137)$$

$$0 = \frac{d\psi_{1q}}{dt} + R_{1q}\, i_{11q}$$

$$= \sqrt{\tfrac{3}{2}}\, M_{a1q} \sin\theta\, i_\alpha + \sqrt{\tfrac{3}{2}}\, M_{a1q} \cos\theta\, i_\beta$$

$$+ (R_{1q} + DL_{11q})\, i_{11q} \text{ ------------------------------ } (138)$$

Torque Equation

The general torque equation is,

$$T \propto \vec{\psi} \times \vec{i}$$

$\propto$ vector product of ψ and i .

Therefore,

$$T_{3\phi} = K\frac{3P}{4}(\psi_\beta i_\alpha - \psi_\alpha i_\beta) \text{ ----------------(139)}$$

in which ψ_α and ψ_β are given by equations (131) and (132) respectively.

CHAPTER IV

LINE-TO-LINE SHORT CIRCUIT OF A SYNCHRONOUS MACHINE

Initial Conditions

For a short circuit between phase b and phase c on an unloaded synchronous machine running at synchronous speed $\frac{d\theta}{dt} = \omega$, as shown in Fig. 4, the following conditions are true,

$$\left.\begin{aligned} i_a &= 0 \\ i_b &= -i_c \\ V_{bc} &= 0 \end{aligned}\right\} \text{------------------}(140)$$

When referred to the α- and β-axis components,

$$\left.\begin{aligned} i_\alpha &= 0 \\ i_o &= 0 \\ e_\beta &= 0 \end{aligned}\right\} \text{------------------}(141)$$

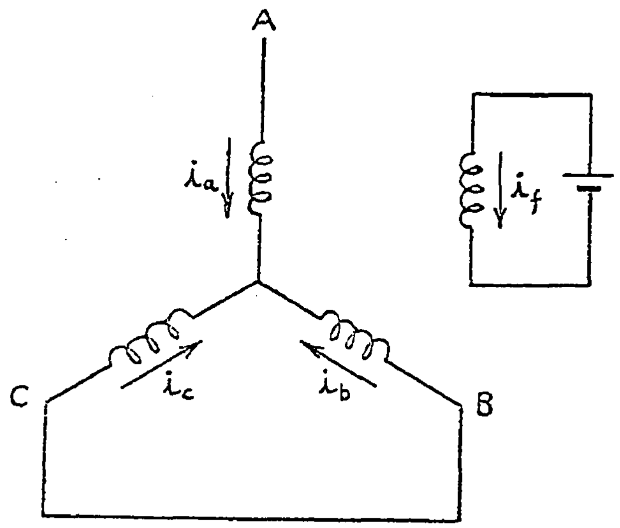

Fig. 4. Line-to-line short circuit of a synchronous machine.

The open-circuit voltages before short circuit are,

$$\left.\begin{aligned} V_a &= \omega M_{af} i_{fo} \sin(\omega t + \theta_o) \\ &= \sqrt{\tfrac{3}{2}}\, E_f \sin\theta \\ V_b &= \sqrt{\tfrac{3}{2}}\, E_f \sin\left(\theta - \tfrac{2\pi}{3}\right) \\ V_c &= \sqrt{\tfrac{3}{2}}\, E_f \sin\left(\theta - \tfrac{4\pi}{3}\right) \end{aligned}\right\} \text{------------------} (142)$$

Then, by equation (125),

$$e_{\beta o} = \frac{3}{2} E_f \cos\theta \text{ ------------------------------------ } (143)$$

where i_{fo} = constant field current before short circuit.

θ_o = angle between α- and d-axis at t = 0.

Short Circuit Currents

By the superposition principle the effect of short circuit on phases b and c is simulated by applying $-e_{\beta o}$ to the armature with the field voltage equal to zero. Assuming that the machine has no damper circuits in either axis except the main field winding in the d-axis,

$$M_{a1d} = M_{a1q} = M_{f1d} = 0$$

$$R_{1d} = R_{1q} = \infty$$

$$i_\alpha = 0$$

Equations (125) and (136) become,

$$\left.\begin{aligned} -\sqrt{\tfrac{3}{2}}\, E_f \cos\theta &= D M_{af} \sin\theta\, i_f - \left[\tfrac{\sqrt{2}}{3} R_a + D(A - B\cos 2\theta)\right] i_\beta \\ 0 &= -D M_{af} \sin\theta\, i_\beta + (R_f + D L_{ff})\, i_f \end{aligned}\right\} \quad (144)$$

The coefficients in equation (144) are not constant and an exact solution is not possible. However, an approximate solution can be obtained by successive approximations. As a first step, all the resistances in equation (144) are neglected, and when integrated between the limits θ_o and θ ,

$$\left.\begin{aligned} M_{af} \sin\theta\, i_f - (A - B\cos 2\theta)\, i_\beta &= -\sqrt{\tfrac{3}{2}}\, E_f \,\frac{\sin\theta - \sin\theta_o}{\omega} \\ 0 &= L_{ff}\, i_f - M_{af} \sin\theta\, i_\beta \end{aligned}\right\} \quad (145)$$

These simultaneous equations are then solved to give the short circuit currents due to the fault,

$$\left.\begin{aligned}
i_\beta &\cong \frac{2\sqrt{\frac{3}{2}}\, E_f\,(\sin\theta - \sin\theta_o)}{(X_d' + X_q) - (X_d' - X_q)\cos 2\theta} \\
i_f &\cong 2\sqrt{\frac{3}{2}}\,\frac{M_{af}}{L_{ff}}\,\frac{E_f(\sin\theta - \sin\theta_o)\sin\theta}{(X_d' + X_q) - (X_d' - X_q)\cos 2\theta} \\
&\cong \frac{i_{fo}\,(X_d - X_d')(1 - \cos 2\theta - 2\sin\theta\sin\theta_o)}{(X_d' + X_q) - (X_d' - X_q)\cos 2\theta}
\end{aligned}\right\} \quad (146)$$

By Fourier series expansion, the current equaions (146) may be resolved into the harmonic series,

$$\left.\begin{aligned}
i_\beta &= \frac{\sqrt{2}\sqrt{3}\,E_f}{X_d' + \sqrt{X_d' X_q}}\left\{\sin\theta + \sum_{n=1}^{\infty}(-b)^n \sin(2n+1)\theta\right\} \\
&\quad - \frac{\sqrt{\frac{3}{2}}\,E_f \sin\theta_o}{\sqrt{X_d' X_q}}\left\{1 + 2\sum_{n=1}^{\infty}(-b)^n \cos 2n\theta\right\} \\
i_f &= \sqrt{\frac{3}{2}}\,\frac{M_{af}}{L_{ff}}\,\frac{E_f}{X_d' + \sqrt{X_d' X_q}}\left\{1 + \frac{1+b}{b}\sum_{n=1}^{\infty}(-b)^n \cos 2n\theta\right\} \\
&\quad - \sqrt{\frac{3}{2}}\,\frac{M_{af}}{L_{ff}}\,\frac{E_f \sin\theta_o}{\sqrt{X_d' X_q}}(1+b)\left\{\sin\theta + \sum_{n=1}^{\infty}(-b)^n \sin(2n+1)\theta\right\}
\end{aligned}\right\} \quad (147)$$

where $$b = \frac{\sqrt{X_q} - \sqrt{X_d'}}{\sqrt{X_q} + \sqrt{X_d'}} = \frac{\sqrt{X_d' X_q} - X_d'}{\sqrt{X_d' X_q} + X_d'}$$

The currents calculated from equation (148) are the initial short circuit currents. It also shows that there is an unending series of reflections between the armature and the field. The odd harmonic series for the armature current corresponds to an even harmonic series for the field current components, whereas the even harmonic series corresponds to an odd harmonic series of field current components.

Correction for Small Resistances

If the resistances are not neglected, we may substitute for equation (147) the following approximations, in which $i_{f(2c)}$ is taken as different from $i_{f(dc)}$.

$$\left.\begin{aligned} i_\beta &= i_{\beta(dc)}\left\{1 + 2\sum_{n=1}^{\infty}(-b)^n \cos 2n\theta\right\} \\ &\quad + i_{\beta(s)}\left\{\sin\theta + \sum_{n=1}^{\infty}(-b)^n \sin(2n+1)\theta\right\} \\ i_f &= i_{f(dc)} + i_{f(2c)}\frac{1+b}{b}\sum_{n=1}^{\infty}(-b)^n \cos 2n\theta \\ &\quad + i_{f(s)}\left\{\sin\theta + \sum_{n=1}^{\infty}(-b)^n \sin(2n+1)\theta\right\} \end{aligned}\right\} \quad (149)$$

in which $i_{\beta(dc)}$, $i_{f(dc)}$, etc. are the undetermined coefficients except at t = 0. At t = 0, these coefficients can be found by equation corresponding coefficients with equation (147). In writing these expressions it is assumed that all the harmonic terms of the same series are subject to a decrement with the same time constant. This assumption is generally justified by tests.

The five current components, $i_{\beta(dc)}$, $i_{\beta(s)}$, $i_{f(dc)}$, $i_{f(2c)}$, and $i_{f(s)}$ are found by substituting equation (149) into equation (144), expanding the trigonometric expressions, and equating coefficients of corresponding terms on both sides of the equation. The relatively small resistances in the coefficients of the harmonic terms are neglected, but the resistance in the dc terms are retained. This process yields thirteen additional equations for five unknown current components. However, only five are found to be non-redundant. They are:

$$\left.\begin{aligned}
&(R_f + DL_{ff})\, i_{f(dc)} - \tfrac{1}{2} M_{af} D\, i_{\beta(s)} = 0 \\
&L_{ff}\, i_{f(s)} - M_{af}(1+b)\, i_{\beta(dc)} = 0 \\
&L_{ff}\, i_{f(2c)} - \tfrac{1}{2} M_{af}\, i_{\beta(s)} = 0 \\
&\frac{M_{af}}{2} D\, i_{f(s)} - \left\{ \frac{\sqrt{2}}{3} R_a + (A + bB) D \right\} i_{\beta(dc)} = 0 \\
&- \left\{ A + \frac{(1+b)}{2} B \right\} i_{\beta(s)} + M_{af} \left\{ i_{f(dc)} + \frac{1+b}{2} i_{f(2c)} \right\} \\
&\qquad\qquad = - \frac{\sqrt{\frac{3}{2}}\, E_f}{\omega}
\end{aligned}\right\} \quad (150)$$

The Laplace transforms of equation (150) are:

$$\left.\begin{aligned}
&(R_f + SL_{ff})I_{f(dc)}(S) - \tfrac{1}{2} M_{af} S I_{\beta(s)}(S) \\
&\qquad = L_{ff}\, i_{f(dc)}(0) - \tfrac{1}{2} M_{af}\, i_{\beta(s)}(0) \\
&L_{ff} I_{f(s)}(S) - M_{af}(1+b) I_{\beta(dc)}(S) = 0 \\
&L_{ff} I_{f(2c)}(S) - \tfrac{1}{2} M_{af} I_{\beta(s)}(S) = 0 \\
&\frac{M_{af}}{2} S I_{f(s)}(S) - \left\{ \frac{\sqrt{2}}{3} R_a + (A + bB) S \right\} I_{\beta(dc)}(S) \\
&\qquad = \frac{M_{af}}{2}\, i_{f(s)}(0) - (A + bB)\, i_{\beta(dc)}(0) \\
&- \left\{ A + \frac{(1+b)B}{2} \right\} I_{\beta(s)}(S) + M_{af} \left\{ I_{f(dc)}(S) + \frac{1+b}{2} I_{f(2c)}(S) \right\} \\
&\qquad = - \frac{\sqrt{\frac{3}{2}}\, E_f}{S\omega}
\end{aligned}\right\} \quad (151)$$

From equation (147), the initial short circuit currents are,

$$\left.\begin{aligned}
i_{\beta(dc)}(0) &= - \frac{\sqrt{\frac{3}{2}}\, E_f \sin\theta_0}{\sqrt{X_d' X_q}} \\
i_{\beta(s)}(0) &= \frac{\sqrt{2}\,\sqrt{3}\, E_f}{X_d' + \sqrt{X_d' X_q}} \\
i_{f(dc)}(0) &= \sqrt{\frac{3}{2}}\, \frac{M_{af}}{L_{ff}}\, \frac{E_f}{X_d' + \sqrt{X_d' X_q}} \\
i_{f(2c)}(0) &= \sqrt{\frac{3}{2}}\, \frac{M_{af}}{L_{ff}}\, \frac{E_f}{X_d' + \sqrt{X_d' X_q}} \\
i_{f(s)}(0) &= - \sqrt{\frac{3}{2}}\, \frac{M_{af}}{L_{ff}}\, \frac{E_f \sin\theta_0}{\sqrt{X_d' X_q}} (1+b)
\end{aligned}\right\} \quad (152)$$

Then,

$$\frac{M_{af}}{2} i_{f(s)}(0) - (A + bB) i_{\beta(dc)}(0) = \sqrt{\frac{3}{2}} \frac{E_f \operatorname{Sin} \theta_o}{\omega}$$

$$L_{ff} i_{f(dc)}(0) - \frac{1}{2} M_{af} i_{\beta(s)}(0) = 0$$

Applying these conditions and solving equations (151) simultaneously, we get,

$$\left.\begin{aligned}
I_{\beta(dc)}(S) &= -\frac{\sqrt{\frac{3}{2}} E_f \operatorname{Sin} \theta_o}{X_2 (S + \frac{1}{T_a'})} \\
I_{f(dc)}(S) &= \sqrt{\frac{3}{2}} \frac{M_{af}}{L_{ff}} \frac{E_f}{(X_d' + X_2)(S + \frac{1}{T_d'})} \\
I_{\beta(s)}(S) &= \frac{\sqrt{2}\sqrt{3} E_f}{S(X_d + X_2)} + \frac{(X_d - X_d')\sqrt{2}\sqrt{3} E_f}{(X_d' + X_2)(X_d + X_2)(S + \frac{1}{T_d'})} \\
I_{f(s)}(S) &= -\sqrt{\frac{3}{2}} \frac{M_{af}}{L_{ff}} \frac{E_f \operatorname{Sin} \theta_o (1 + b)}{X_2 (S + \frac{1}{T_a'})} \\
I_{f(2c)}(S) &= \sqrt{\frac{3}{2}} \frac{M_{af}}{L_{ff}} \frac{E_f}{S(X_d + X_2)} \\
&\quad + \sqrt{\frac{3}{2}} \frac{M_{af}}{L_{ff}} \frac{(X_d - X_d') E_f}{(X_d' + X_2)(X_d + X_2)(S + \frac{1}{T_d'})}
\end{aligned}\right\} (153)$$

in which $T_a' = \frac{X_2}{\omega R_a} = \frac{\sqrt{X_d' X_q}}{\omega R_a}$

= armature time constant.

$$T_d' = \frac{X_d' + X_2}{X_d + X_2} T_{do}'$$

$$= \frac{X_d' + \sqrt{X_d' X_q}}{X_d + \sqrt{X_d' X_q}} \frac{L_{ff}}{R_f}$$

= field time constant.

$$X_2 = \sqrt{X_d' X_q}$$

By inverse transform,

$$\left.\begin{aligned}
i_{\beta(dc)} &= -\frac{\sqrt{\frac{3}{2}} E_f \sin\theta_0}{X_2} \varepsilon^{-t/T_a} \\
i_{f(dc)} &= \sqrt{\frac{3}{2}} \frac{M_{af}}{L_{ff}} \frac{E_f}{X_d' + X_2} \varepsilon^{-t/T_d'} \\
i_{\beta(s)} &= \sqrt{2}\sqrt{3} E_f \left\{ \frac{1}{X_d + X_2} + \left(\frac{1}{X_d' + X_2} - \frac{1}{X_d + X_2} \right) \varepsilon^{-t/T_d'} \right\} \\
i_{f(s)} &= -\sqrt{\frac{3}{2}} \frac{M_{af}}{L_{ff}} (1+b) \frac{E_f \sin\theta_0}{X_2} \varepsilon^{-t/T_a} \\
i_{f(2c)} &= \sqrt{\frac{3}{2}} \frac{M_{af}}{L_{ff}} E_f \left\{ \frac{1}{X_d + X_2} + \left(\frac{1}{X_d' + X_2} - \frac{1}{X_d + X_2} \right) \varepsilon^{-t/T_d'} \right\}
\end{aligned}\right\} \quad (154)$$

The final approximate current expressions are,

$$i_\beta = \sqrt{2}\sqrt{3} E_f \left\{ \frac{1}{X_d + X_2} + \left(\frac{1}{X_d' + X_2} - \frac{1}{X_d + X_2} \right) \varepsilon^{-t/T_d'} \right\}$$

$$\left\{ \sin\theta + \sum_{n=1}^{\infty} (-b)^n \sin(2n+1)\theta \right\}$$

$$- \frac{\sqrt{\frac{3}{2}} E_f \sin\theta_0}{X_2} \varepsilon^{-t/T_a} \left\{ 1 + 2\sum_{n=1}^{\infty} (-b)^n \cos 2n\theta \right\} \text{------------} (155)$$

$$i_f = i_{fo} + \sqrt{\frac{3}{2}}\,\frac{M_{af}}{L_{ff}}\,\frac{E_f}{X_d'+X_2}\,\varepsilon^{-t/T_d'}$$

$$+\sqrt{\frac{3}{2}}\,\frac{M_{af}}{L_{ff}}\,E_f\left\{\frac{1}{X_d+X_2}+\left(\frac{1}{X_d'+X_2}-\frac{1}{X_d+X_2}\right)\varepsilon^{-t/T_d'}\right\}\cdot$$

$$\left\{\frac{1+b}{b}\sum_{n=1}^{\infty}(-b)^n \cos 2n\theta\right\}$$

$$-\sqrt{\frac{3}{2}}\,\frac{M_{af}}{L_{ff}}(1+b)\,\frac{E_f \sin\theta_o}{X_2}\,\varepsilon^{-t/T_d'}\left\{\sin\theta+\sum_{n=1}^{\infty}(-b)^n \sin(2n+1)\theta\right\}$$

------------------(156)

By the Fourier series expansion, equations (155) and (156) become,

$$i_\beta = \frac{\sqrt{2}\sqrt{3}\,E_f\,(F\sin\theta - G\sin\theta_o)}{X_d'+X_q-(X_d'-X_q)\cos 2\theta} \qquad \text{------------------(157)}$$

$$i_f = i_{fo} + \sqrt{\frac{3}{2}}\,\frac{M_{af}}{L_{ff}}\,E_f\left\{\frac{\varepsilon^{-t/T_d'}-1}{X_d+X_2}\right.$$

$$\left.+\frac{2(F\sin\theta - G\sin\theta_o)\sin\theta}{X_d'+X_q-(X_d'-X_q)\cos 2\theta}\right\} \qquad \text{--------(158)}$$

Hence, for a line-to-line short circuit, all the phase currents may be obtained by equation (107) since $i_\alpha = i_o = 0$. That is,

$$i_a = 0$$

$$i_b = -\frac{1}{\sqrt{2}}\,i_\beta$$

$$i_c = \frac{1}{\sqrt{2}}\,i_\beta$$

Open-phase Voltage

The α-axis flux linkage for the line-to-line short circuit case is, by equation (131),

$$\psi_\alpha = M_{af} i_f \cos\theta - B i_\beta \sin 2\theta \text{ ---------------(159)}$$

Substituting equations (157) and (158) into equation (159), and rearranging, there results,

$$\psi_\alpha = \frac{\sqrt{\frac{3}{2}} E_f F \cos\theta}{\omega} + \frac{\sqrt{\frac{3}{2}}(X_q - X_d') E_f (F \sin\theta - G \sin\theta_0) \sin 2\theta}{\omega\{X_d' + X_q - (X_d' - X_q)\cos 2\theta\}} \text{ ------(160)}$$

Comparing with equation (157), we see that,

$$\psi_\alpha = \frac{\sqrt{\frac{3}{2}} E_f F \cos\theta}{\omega} - \frac{1}{2}\frac{(X_d' - X_q)}{\omega} i_\beta \sin 2\theta$$

or

$$\psi_\alpha = \frac{\sqrt{\frac{3}{2}} E_f F \cos\theta}{\omega} - \frac{1}{2}\frac{(X_d' - X_q)}{\omega} \sin 2\theta \left\{ \frac{\sqrt{2}\sqrt{3} E_f F}{X_d' + X_2} (\sin\theta - b\sin 3\theta + b^2 \sin 5\theta - \cdots) \right.$$

$$\left. - \frac{\sqrt{2}\sqrt{3} E_f G \sin\theta_0}{X_2} (0.5 - b\cos 2\theta + b^2 \cos 4\theta - \cdots) \right\}$$

$$\text{---------------(161)}$$

Simplifying, the above equation becomes,

$$\psi_\alpha = \frac{\sqrt{\frac{3}{2}} E_f F (1+b)}{\omega} (\cos\theta - b\cos 3\theta + b^2 \cos 5\theta - \cdots\cdots)$$

$$- \frac{\sqrt{2}\sqrt{3} E_f G \sin\theta_0}{\omega} (\sin 2\theta - b\sin 4\theta + b^2 \sin 6\theta - \cdots\cdots)$$

$$\text{---------------(162)}$$

By differentiation, the open-phase voltage is obtained,

$$e_a = \sqrt{\tfrac{2}{3}}\, e_\alpha$$

$$= -\sqrt{\tfrac{2}{3}}\, \frac{d\psi_\alpha}{dt}$$

$$= \frac{E_f(X_d - X_d')(1+b)}{\omega T_d'(X_d + X_2)}\, \varepsilon^{-t/T_d'}(\cos\theta - b\cos 3\theta + b^2\cos 5\theta - \cdots)$$

$$+ E_f F(1+b)(\sin\theta - 3b\sin 3\theta + 5b^2 \sin 5\theta)$$

$$- \frac{2bE_f\sin\theta_0}{\omega T_a'}\, \varepsilon^{-t/T_a'}(\sin 2\theta - b\sin 4\theta + b^2\sin 6\theta - \cdots)$$

$$+ 4bE_f G\sin\theta_0(\cos 2\theta - 2b\cos 4\theta + 3b^2\cos 6\theta - \cdots)$$

$$\text{----------}(163)$$

Sustained Currents and Voltages

From equations (155) and (156), the sustained armature and field currents are,

$$i_c = -i_b = \frac{\sqrt{3}E_f}{X_d + X_2}(\sin\theta - b\sin 3\theta + b^2\sin 5\theta - \cdots)\text{-----}(164)$$

$$i_f = i_{fo} + \sqrt{\tfrac{3}{2}}\,\frac{M_{af}}{L_{ff}}\,\frac{E_f}{X_d + X_2}(1+b)(-\cos 2\theta + b\cos 4\theta - b^2\cos 6\theta + \cdots)\text{--------}(165)$$

The sustained voltage across the open-phase, from equation (163) is,

$$e_a = E_f\frac{2X_2}{X_d + X_2}(\sin\theta - 3b\sin 3\theta + 5b^2\sin\theta - \cdots)\text{---------}(166)$$

From equations (164), (165), and (166), it can be seen that the armature current and voltage contain a fundamental frequency component and odd harmonics. The field current contains even harmonics only. As the absolute value of b is less than unity, each succeeding harmonic is less than the preceeding one, and when b is small, the higher harmonics are negligible.

Short-circuit Torque

The torque equation for the line-to-line short circuit case is,

$$T = -K\frac{3P}{4\omega}\,\omega\,\psi_{\alpha}\,i_{\beta} \text{ ----------------- (167)}$$

By substituting equations (157) and (160) into the above equation, we have,

$$T = -K\frac{3P}{4\omega}\left\{\frac{3E_f^2F(F\sin\theta - G\sin\theta_o)\cos\theta}{X_d' + X_q - (X_d' - X_q)\cos 2\theta} + \frac{3(X_q - X_d')[E_f(F\sin\theta - G\sin\theta_o)]^2\sin 2\theta}{[X_d' + X_q - (X_d' - X_q)\cos 2\theta]\cos 2\theta}\right\} \text{ ----- (168)}$$

CHAPTER V

DETERMINATION OF SYNCHRONOUS MACHINE CONSTANTS

In obtaining the short circuit currents by analytical methods in both the three-phase short circuit and the line-to-line short circuit cases, the following machine constants are used:

R_a = armature resistance per phase.

R_f = field resistance.

X_d = direct component of synchronous reactance.

X_q = quadrature component of synchronous reactance.

L_{afm} = maximum value of mutual inductance between armature and field.

X_d' = direct-axis transient reactance.

L_{ff} = self inductance of field circuit.

The test methods for measuring these machine constants will now be discussed briefly. (The machine used in the experimental work is specified in Appendix I.)

R_a and R_f are obtained experimentally by the ordinary dc voltmeter-ammeter method. R_a is found to be 0.38 ohms and R_f to be 17.12 ohms.

X_d and X_q are measured by the slip-test. The alternator under investigation is driven at slightly less than synchronous speed with its field circuit open. Balanced, reduced voltage is applied to the armature terminals. Applied armature volts, armature current, and the voltage induced in the field are read. Variation will occur as shown in Fig. 5.

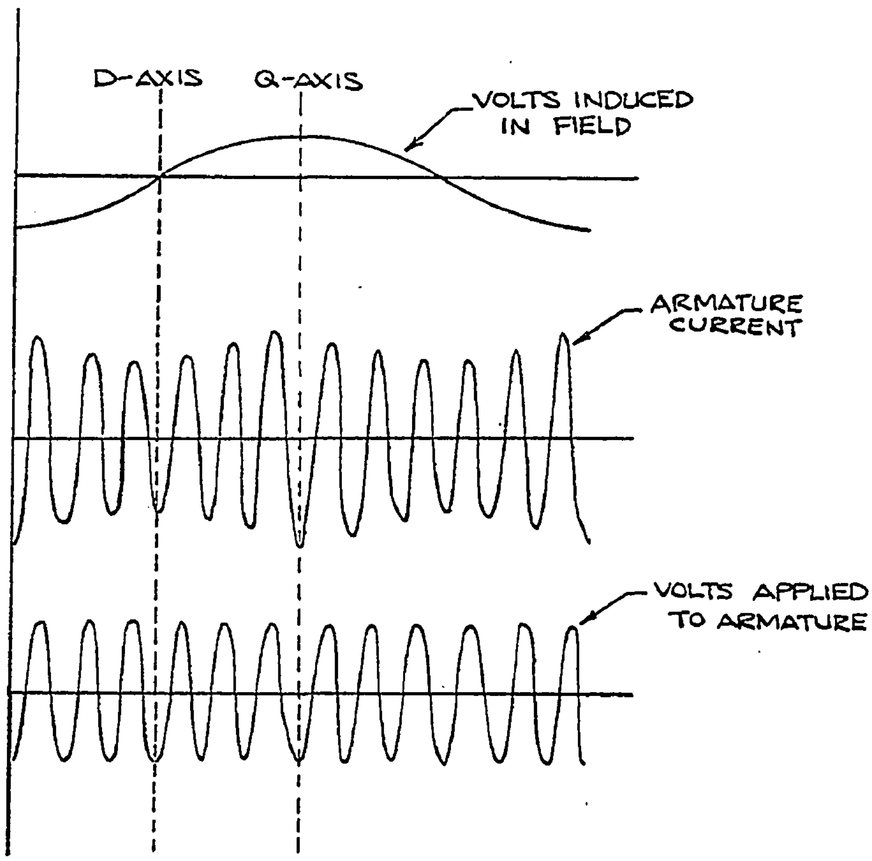

Fig. 5. Slip-test for determining X_d and X_q.

At the instant when the voltage across the field is zero,

$$X_d = \frac{\text{Armature volts per phase}}{\text{Armature current}}$$

As the field structure rotates through the gap, at slightly greater or lesser speed than synchronous, it is exposed to the rotating mmf of armature reaction. The physical poles and the armature-reaction mmf are alternately in phase and out, the change occuring at slip frequency. When the axis of the poles and the axis of the armature-reaction mmf wave coincide, the armature mmf acts through what is ordinarily the field magnetic circuit. The voltage applied to the armature is then equal to the drop caused by the direct component of armature reaction and leakage reactance. The entire armature current is in the position of I_d, being completely wattless except for the effect of armature resistance. Continued rotation brings the armature mmf in quadrature with the field poles. Under this condition the applied voltage is equal to the leakage-reactance drop plus the equivalent voltage drop of the cross-magnetizing field. It follows then that the fluctuation in current as the field slip in and out of step is a measure of the two components of synchronous machine, containing as they do, by definition, component effects of leakage reactance as well.

Because of the applied voltage may vary slightly with change in current and the swing of the armature may be influenced by inertia, an oscillograph record is usually taken for reading the values of armature current and voltage. The voltage induced in the field is a measure of the rate of change of flux through the field circuit. The flux is a maximum and the rate of change is zero at the instant the mmf's coincide. This point serves as an indication for taking the readings. It corresponds to the instant of minimum current.

Experimentally, the following results are obtained:

$$X_d = \frac{\text{maximum voltage}}{\text{minimum current}}$$

$$= \frac{23.0}{\sqrt{3} \times 1.28} = 10.36 \text{ ohms}$$

$$X_q = \frac{\text{minimum voltage}}{\text{maximum current}}$$

$$= \frac{23.0}{\sqrt{3} \times 2.10} = 6.33 \text{ ohms}$$

The direct-axis reactance X_d can also be found by using the open-circuit and short-circuit characteristics of the machine. It equals the open-circuit phase emf divided by the short-circuit phase current, both for the same field current. The open-circuit emf should be taken from a point on the air-gap line so that the condition of the magnetic circuit will be taken as the same on open circuit as on short circuit. This method is not disturbed by having induced currents in any closed circuits on the field poles. The average value of X_d found by this method, as shown in Fig. 6, is 10.25 ohms, which compares favourably with that obtained by the slip-test.

The quadrature-axis reactance X_q is always less than the direct-axis reactance, and in the absence of more specific data may be estimated as 0.65 times the value of X_d.

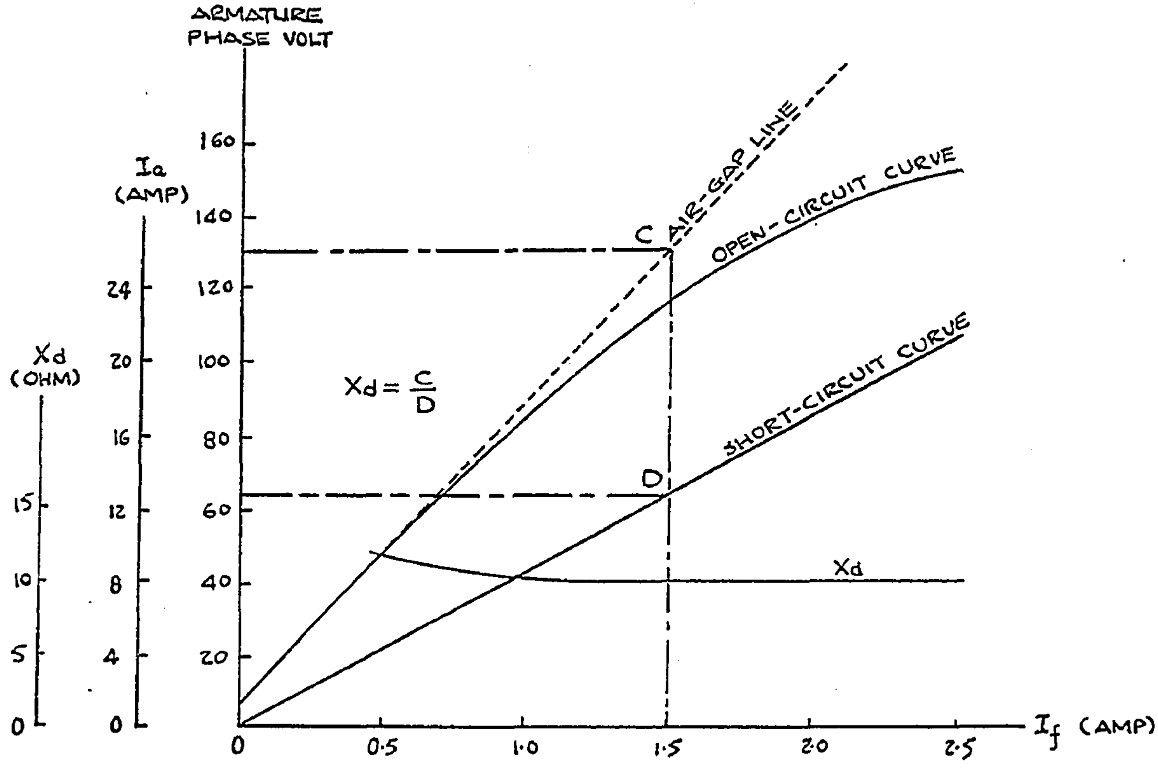

Fig. 6. Determination of X_d by the open-circuit and short-circuit characteristics.

L_{afm}, the maximum value of inductance between the armature and the field, can be calculated from the open-circuit characteristics of the alternator. It equals the maximum value of the open-circuit armature emf per phase divided by the direct field current read for any point on the air-gap line. An average value for L_{afm} found by this method is 0.325 henries.

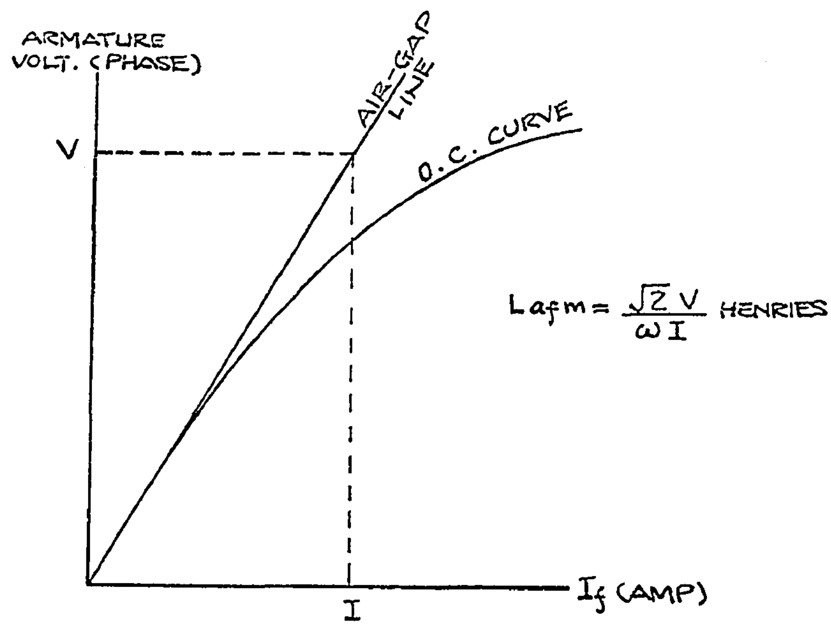

Fig. 7. Calculation of L_{afm} from open-circuit characteristics.

If the machine is now operated at no load and its field winding is short-circuited, the variation of armature voltage, shortly after the beginning of the transient, will follow a decrement having a time constant T_{do}, which is called the open-circuit time constant and is larger than the short-circuit time constant T_d'. These time constants are related by the simple expression:

$$T_d' = \frac{X_d'}{X_d} T_{do}$$

The above relation provides an easy means of determining X_d', the direct-axis transient reactance, when the other quantities are known.

The method for measureing the open-circuit time constant is discussed first. With the machine running at synchronous speed, and with its armature winding open circuit, build up the armature voltage to about half the rated voltage of the machine. Let this voltage be E_o (maximum value).

When the voltage in the armature becomes constant, suddenly short circuit the field winding and take an oscillographic record of the change in the armature voltage. A resistance is placed between the field winding and the exciter to prevent short-circuiting the field voltage source. The function of the envelope of armature voltage will be,

$$E = E_o \varepsilon^{-t/T_{do}} + E_r$$

or,

$$(E - E_r) = E_o \varepsilon^{-t/T_{do}}$$

in which E_r is the residual armature voltage when $t = T_{do}$.

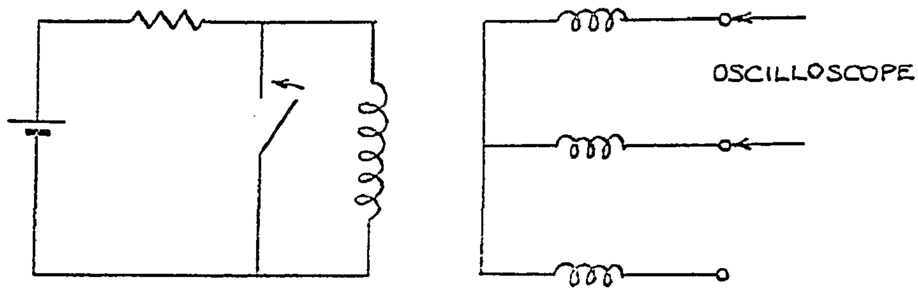

Fig. 8. Circuit diagram for determining the open-circuit time constant T_{do}.

The time in seconds required for $(E - E_r)$ to drop from E_o to 0.368 E_o is T_{do}.

T_{do} is then found by plotting the values of $(E - E_r)$ as measured from the oscillograph record against time on a semi-logrithmic paper. Thus the following table is constructed from the values obtained from the oscillograph.

Table 1

Determination of T_{do}

Time in Cycles	0	1	2	3	4	5	6	7	∞
E in scale divisions	2.65	2.50	2.32	2.20	2.06	1.92	1.80	1.66	----
E in volts	26.5	25.0	23.2	22.0	20.6	19.2	18.0	16.6	0.19
$(E - E_r)$ volts	26.3	24.8	23.0	21.8	20.4	19.0	17.8	16.4	0

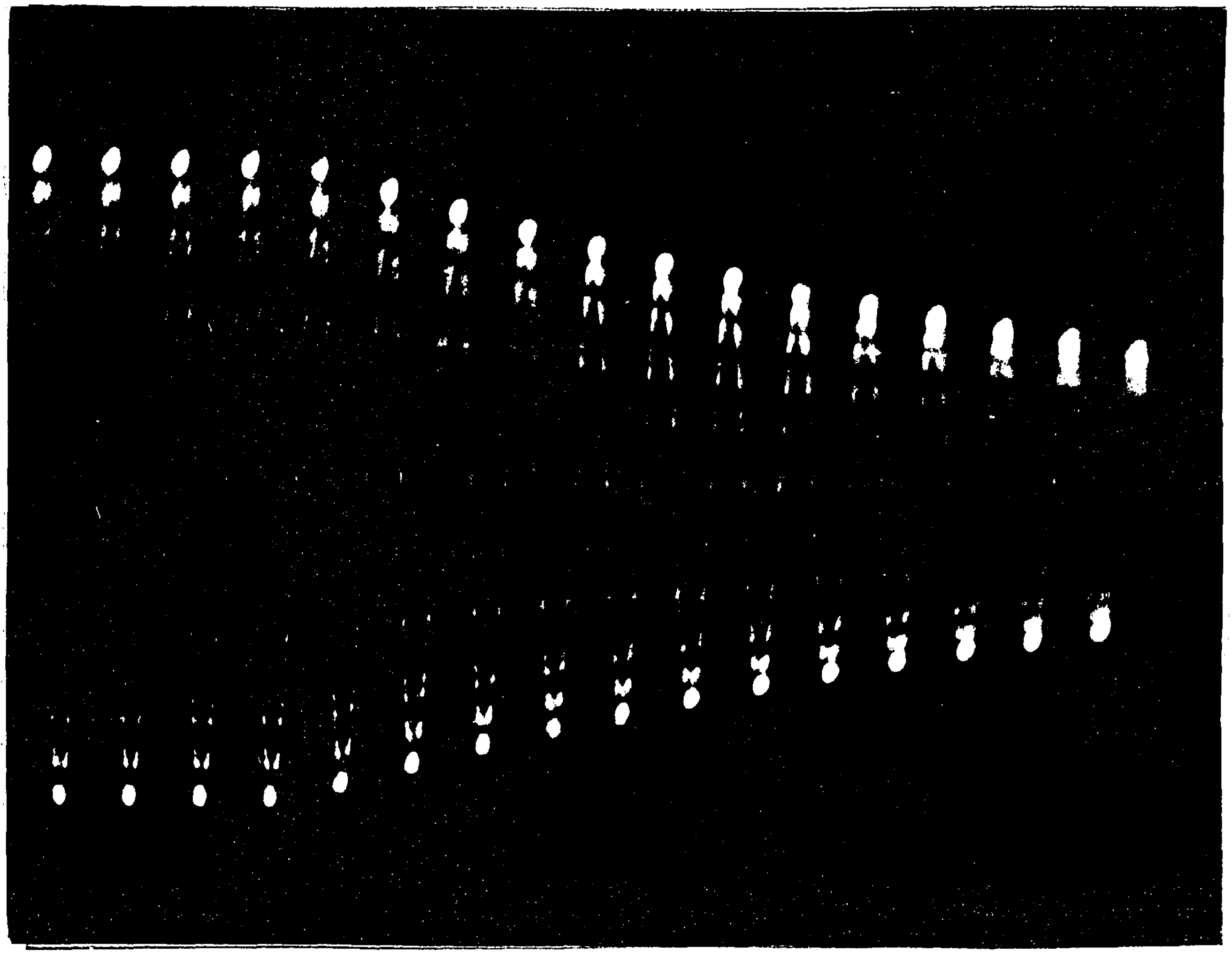

Fig. 9. Oscillograph for measuring T_{do}.
Scale: 10 volts/div.

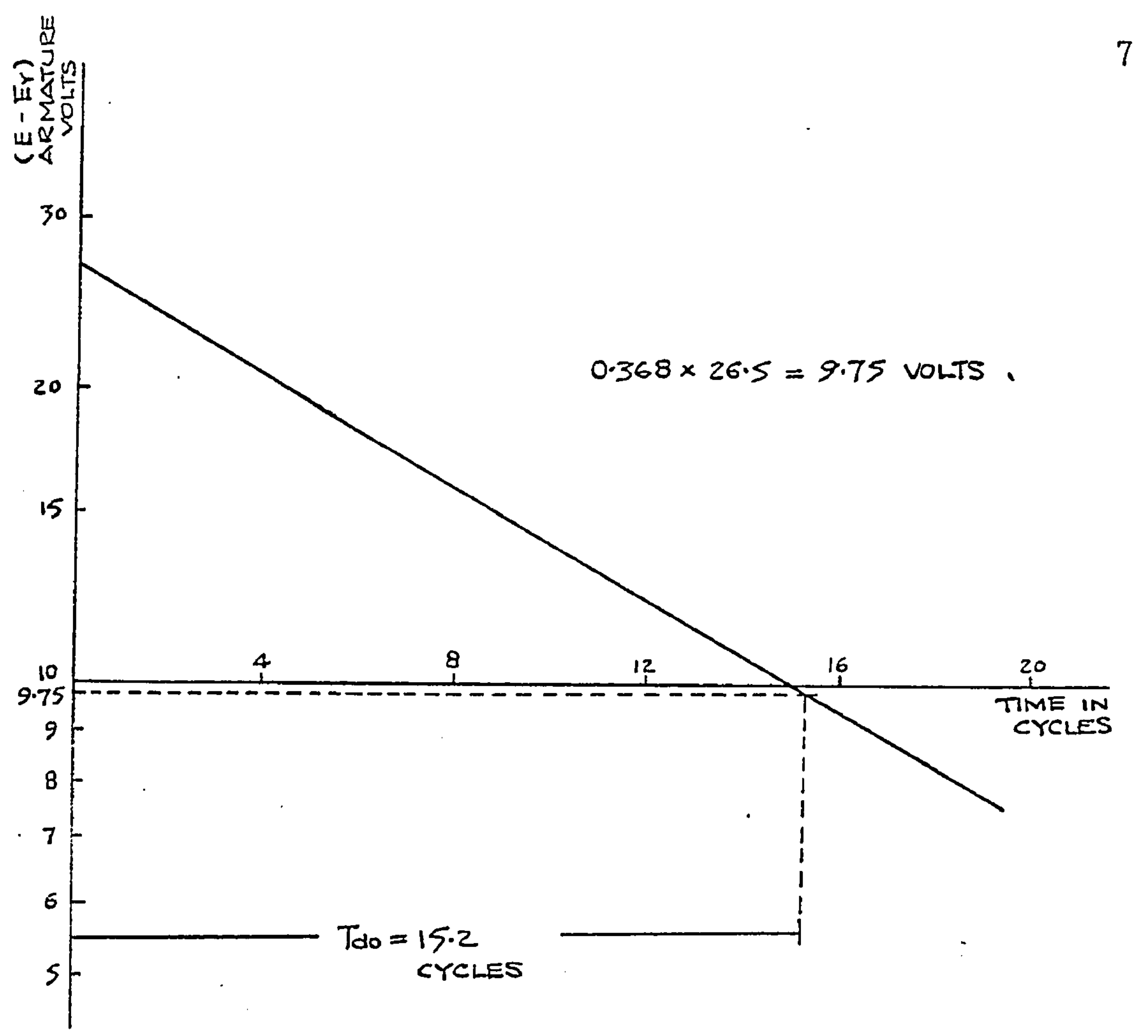

Fig. 10. Determination of T_{do} graphically.

From Fig. 10, the open-circuit time constant is found to be 15.2 cycles.

To measure the short-circuit time constant T_d', the machine is run at synchronous speed, and with the armature terminals short-circuited. The excitation of the machine is suddenly removed by short circuiting the field winding. The armature current, therefore, decays in an exponential manner, and is recorded by the oscillograph. This transient current may be expressed in the form:

$$I = I_o \varepsilon^{-t/T_d'} + I_r$$

or, $$(I - I_r) = I_o \varepsilon^{-t/T_d'}$$

in which I_r is the armature current produced by the residual excitation of the machine.

At $t = T_d'$, $(I - I_r) = I_o \varepsilon^{-t/T_d'}$. Therefore, the time T_d' required for the transient armature current to decay to 0.368 of its initial value is called the short-circuit time constant. By plotting $(I - I_r)$ as measured from the oscillograph record against time in a semi-logrithmic paper, T_d' can be found.

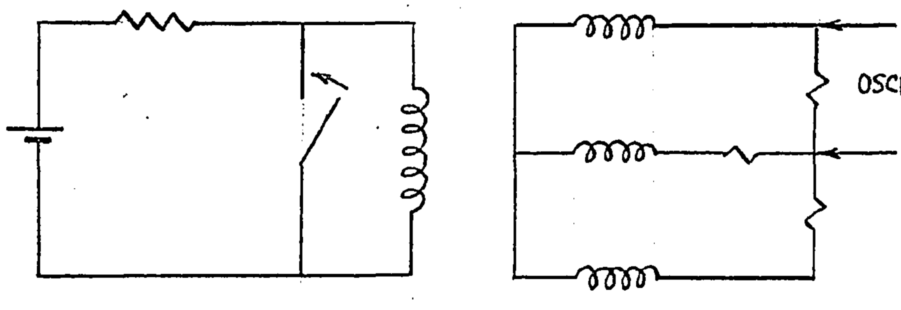

Fig. 11. Circuit diagram for determining the short-circuit time constant T_d'.

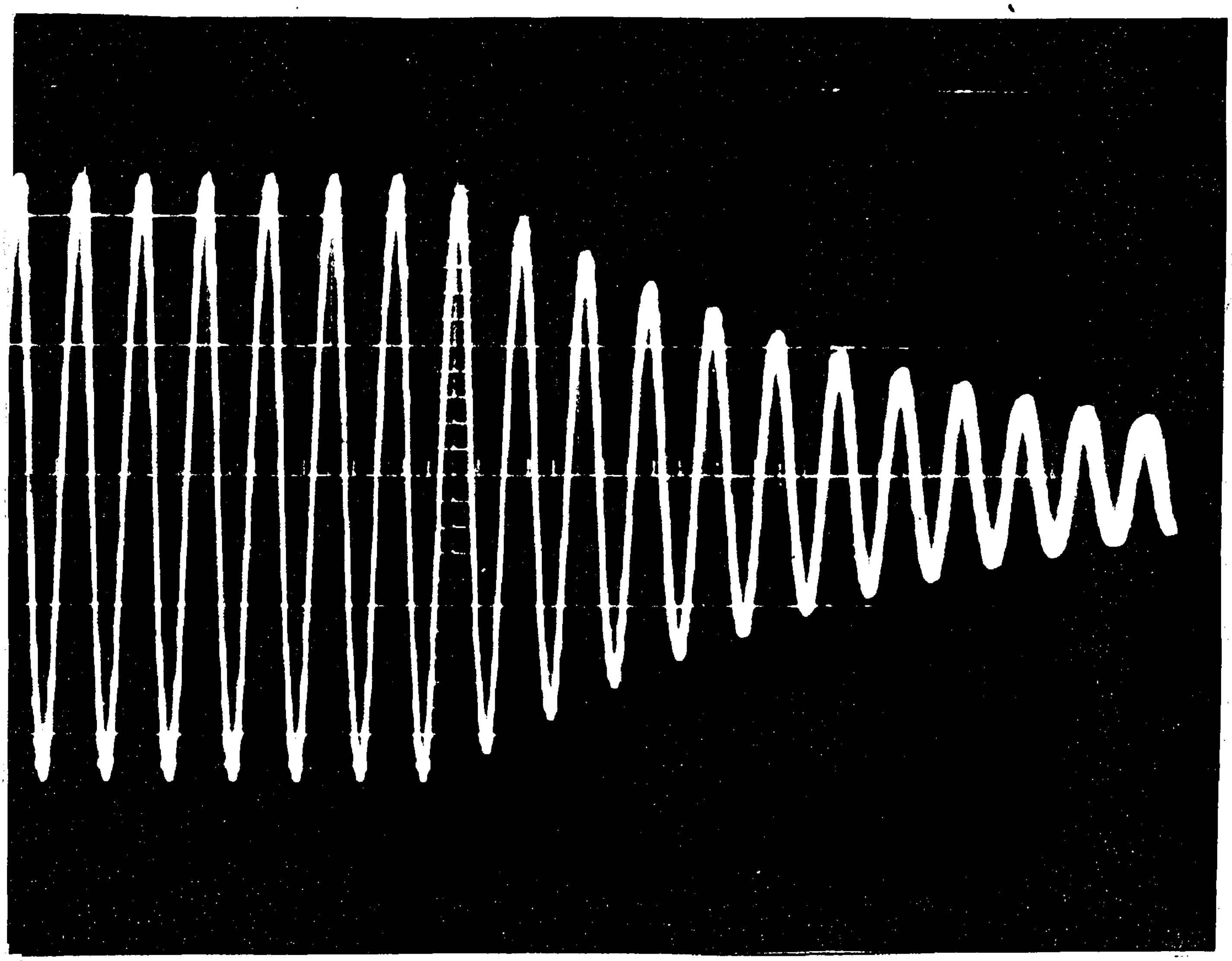

Fig. 12. Oscillograph for measuring T_d'.

Scale: 1 amp/div.

Table 2

Determination of T_d'

Time in cycles	0	1	2	3	4	5	6	7	∞
I in scale divisions	2.30	2.10	1.82	1.60	1.38	1.20	1.00	0.84	----
I in amps	2.30	2.10	1.82	1.60	1.38	1.20	1.00	0.84	0.06
$(I - I_r)$ amps	2.24	2.04	1.76	1.54	1.32	1.14	0.94	0.78	0

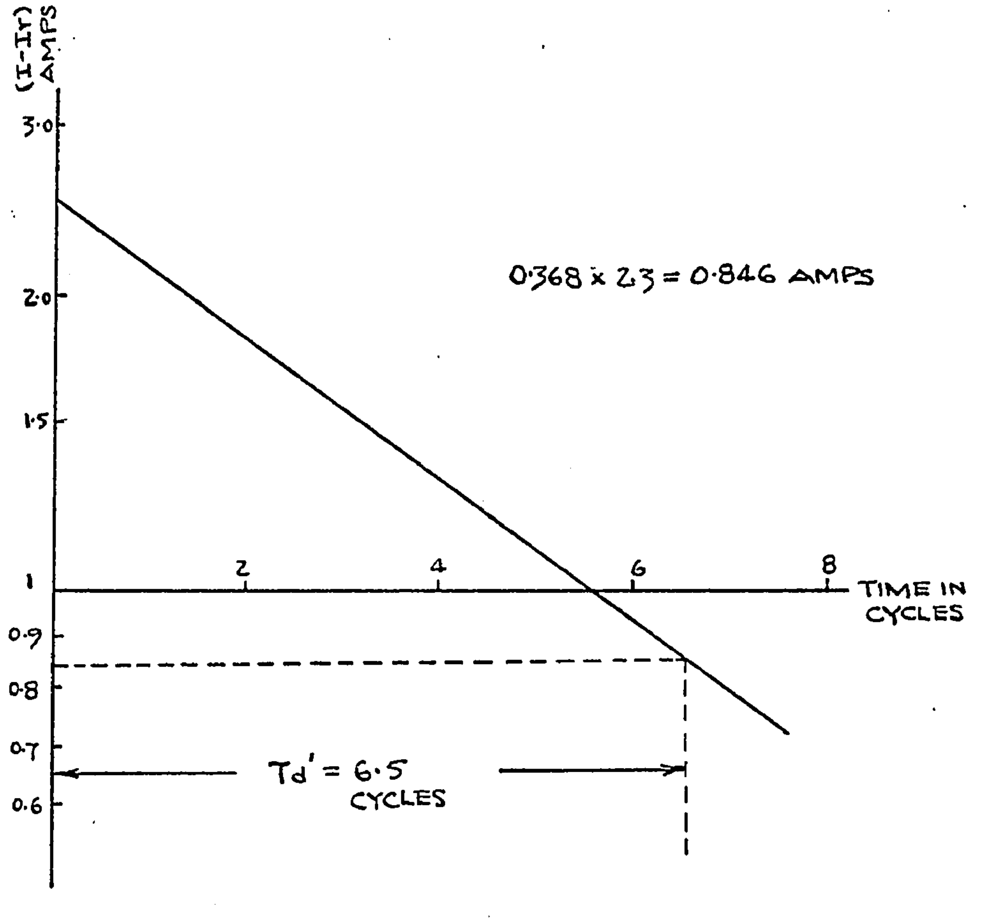

Fig. 13. Determination of T_d'.

When the open-circuit and short-circuit time constants are known, the direct transient reactance X_d' can be calculated from the following modidified expression:

$$T_d' = T_{do} \frac{r^2 + X_d' X_q}{r^2 + X_d X_q}$$, where r is the

total resistance in the armature phase.

The transient reactance arises from the physical fact that, at the moment of transition, the field flux linkage cannot change, and a transient field current is induced to oppose the effect of the new armature currents. As a result, the magnetizing effect of the armature currents is partially modified and the apparent reactance is substantially reduced. The field voltage is insufficient to maintain the transient field current, and as the field transient dies out, the direct-axis reactance is restored to full effectiveness.

We have defined previously that,

$$X_d' = X_d - \frac{X_{af}^2}{X_{ff}}$$

X_{ff}, or L_{ff}, can therefore be calculated when X_d', X_d, and X_{af} are known. Furthermore, from the definition of the open-circuit field time constant, we have,

$$T_{do} = \frac{L_{ff}}{R_f}$$, which provides another means for calculating L_{ff}.

If the effects of slots are neglected, the permeance of the magnetic circuit of the field winding alone is unchanged with rotor position, so that the self inductance of the field, L_{ff}, is a constant.

CHAPTER VI

DIGITAL COMPUTER SOLUTION OF THE TRANSIENT EQUATIONS

Experimental Results for the Three-Phase Short Circuit of a Synchronous Machine

The alternator with a balanced resistive load of ten ohms per phase is driven at synchronous speed by a shunt-connected dc motor. A five-ohm resistance is inserted between the actual terminals of the machine and the load to limit the short circuit currents. The armature resistance per phase is thus modified to 5.38 ohms instead of 0.38 ogms. The machine is then suddenly short circuited at the new terminals, and oscillographs for the armature current and the field current are taken. Experimental data are recorded as follows:

Field supply voltage = 24.2 volts dc.

Field current = 0.224 amps. dc.

Open-circuit terminal voltage $= \dfrac{24.7}{\sqrt{3}}$

$= 14.25$ volts rms.

Terminal voltage before short circuit $= \frac{14.6}{\sqrt{3}}$

$= 8.43$ volts.

Armature current before short circuit = 0.75 amps.

Armature current after short circuit = 1.31 amps.

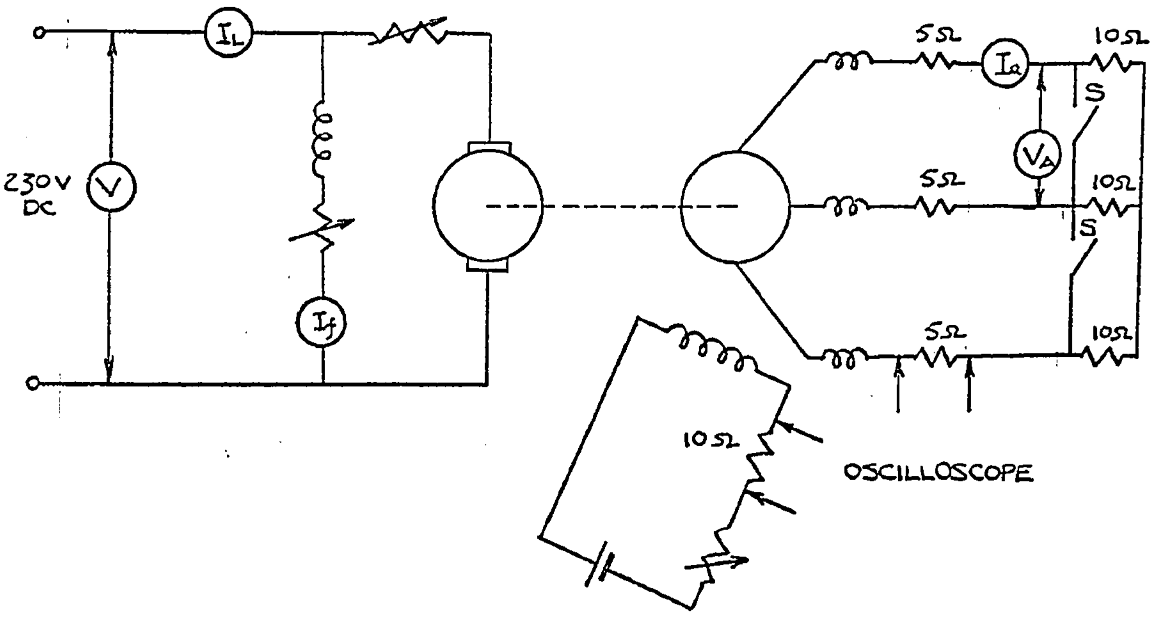

Fig. 14. Wiring diagram for a three-phase short circuit.

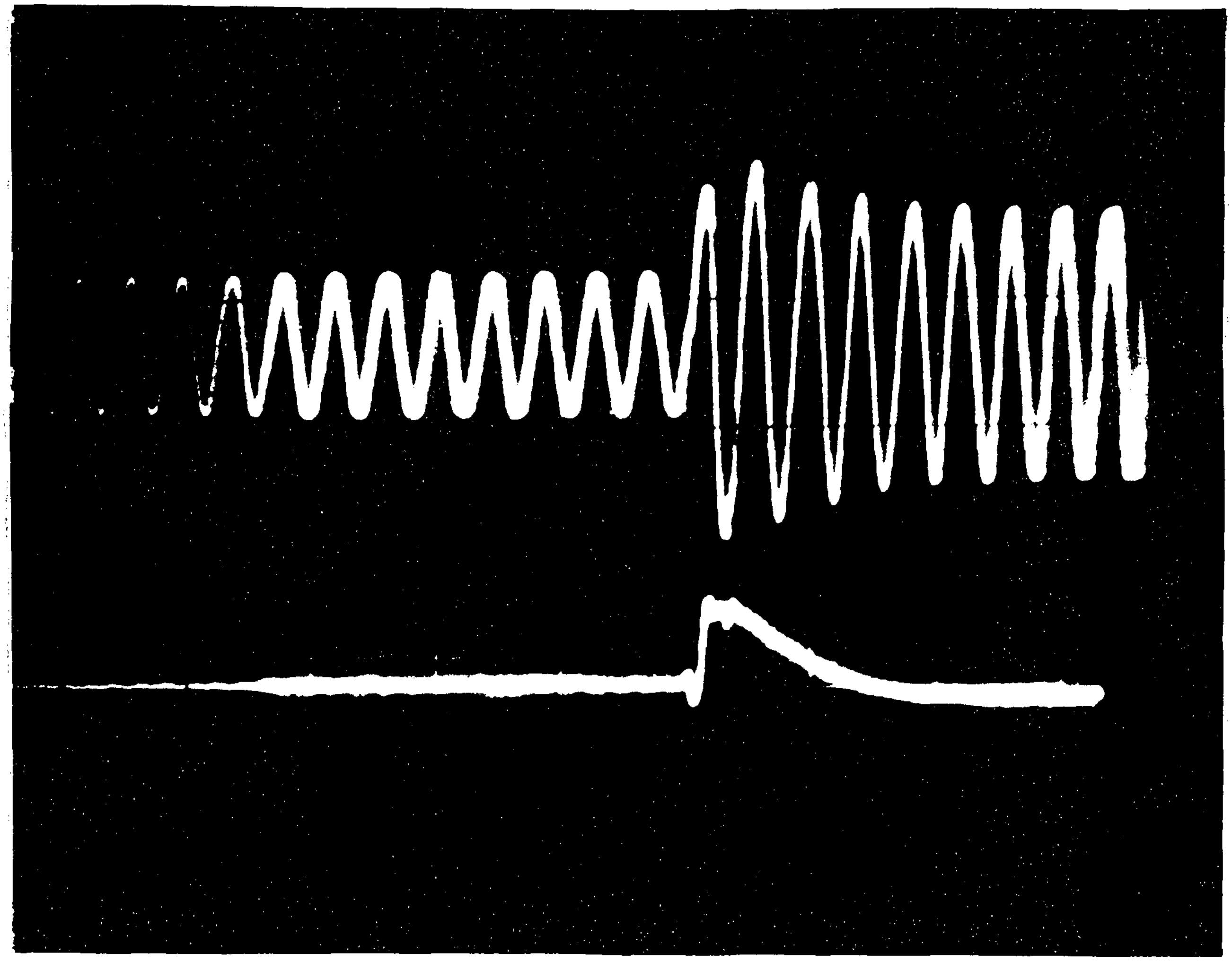

Fig. 15. Oscillographs for armature current (upper) and field current (lower) for a three-phase short circuit of a loaded alternator.

Scale: Armature current, 2 amps/div.
Field current, 0.2 amps/div.

Experimental Results for the Line-to-Line Short Circuit of a Synchronous Machine

The same alternator with a total resistance of 5.38 ohms per phase is driven at synchronous speed by a shunt-connected dc motor. Phase b and phase c of the alternator are then short circuited, and oscillographs for the voltage in phase a, the short-circuit current in phase b or c, and the field current of the alternator are taken. The experimental data are recorded as follows:

Speed = 1200 rpm.

Field supply voltage = 24.2 volts dc.

Field current = 0.224 amps dc.

Open-circuit terminal voltage $= \frac{24.7}{\sqrt{3}}$

$= 14.25$ volts rms.

Short-circuit armature current = 1.67 amps.

Open-phase voltage = 12.4 volts rms.

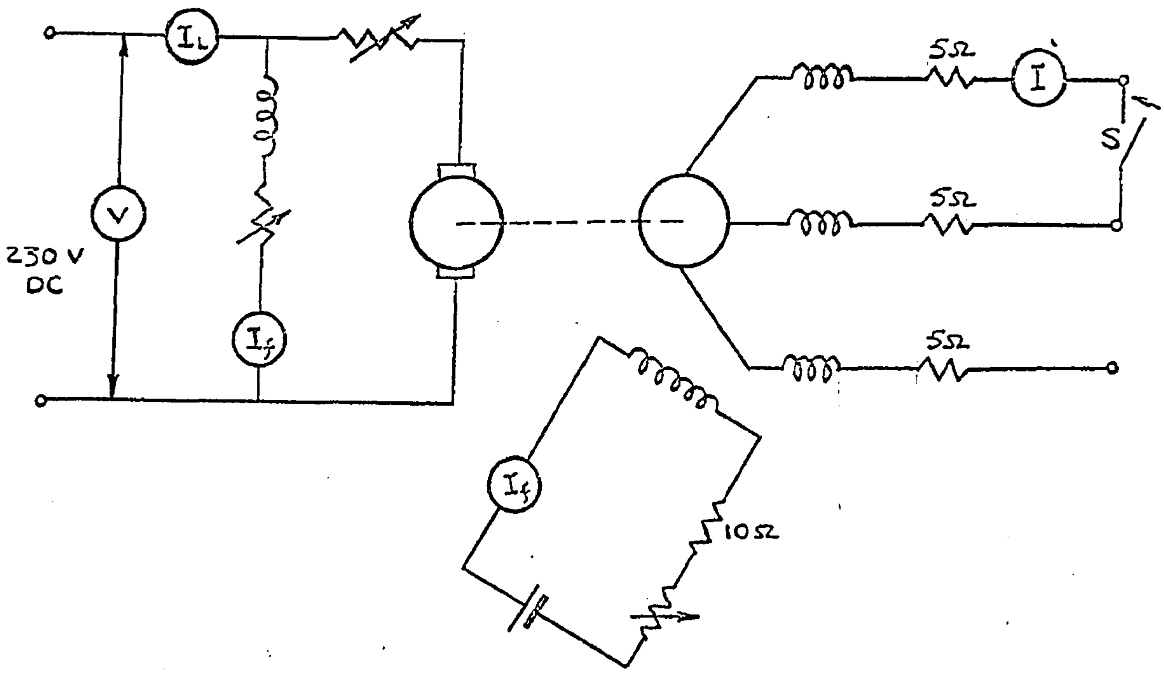

Fig. 16. Wiring diagram for a line-to-line short circuit.

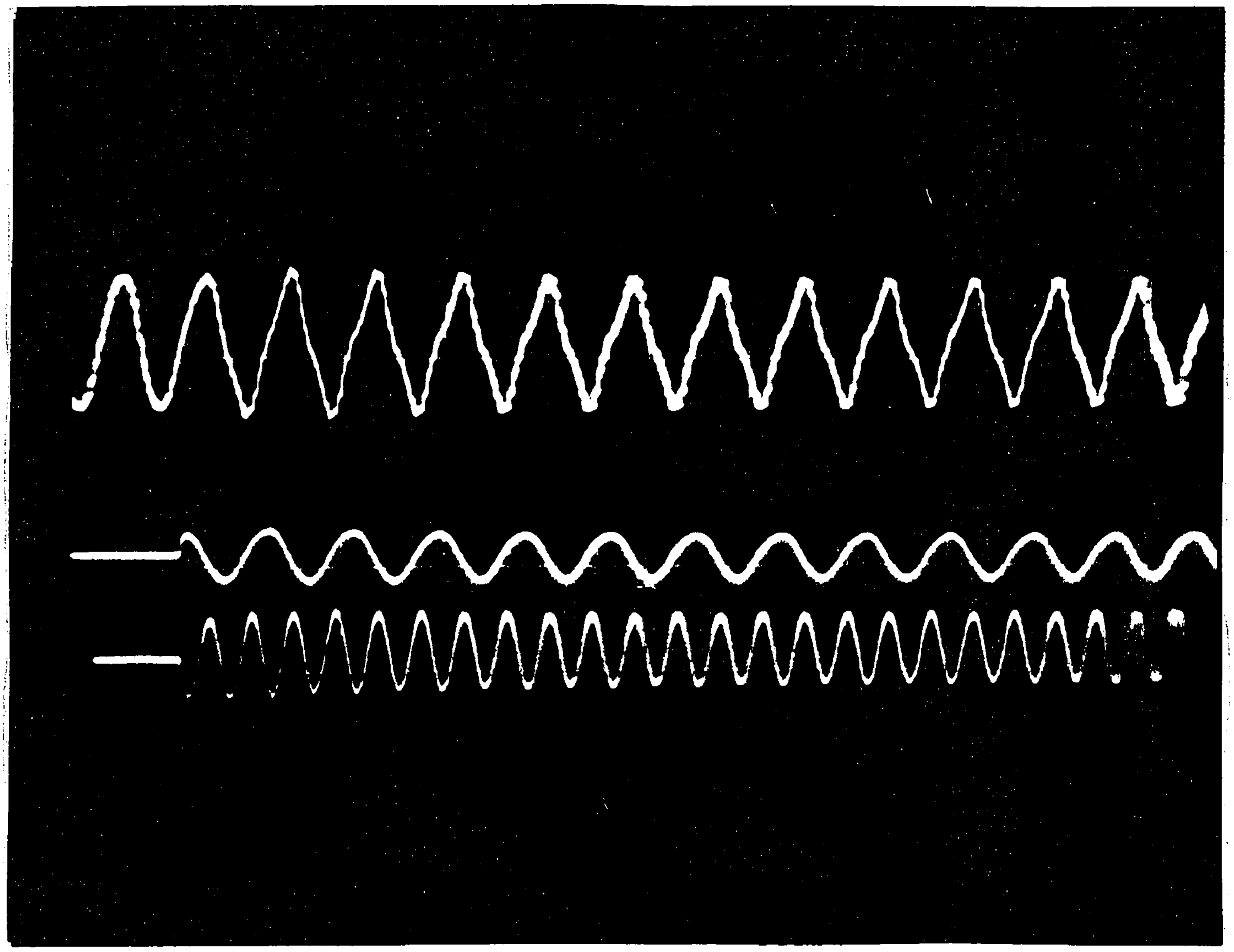

Fig. 17. Oscillographs for open-phase voltage (upper), short-circuit armature current (middle), and field current (lower) for a line-to-line short circuit of an alternator.

Scales: Voltage, 30 volts/div.
Armature current, 15 amps/div.
Field current, 3.4 amps/div.

Digital Computer Solution of the Transient Equations for a Three-Phase Short Circuit

The approximate solutions for the short-circuit armature current and the short-circuit field current for a three-phase short circuit as developed in Chapter II will now be solved using a Royal-McBee LGP-30 digital computer. Recalling,

$$i_a = -\sqrt{r^2 + x_q^2}\left\{\frac{E_f}{r^2 + X_d X_q} + \frac{(X_d - X_d')(r \sin\delta + X_q \cos\delta)}{(r^2 + X_d X_q)(r^2 + X_d' X_q)} V_a \varepsilon^{-t/T_{d2}'}\right\}$$

$$\cos\left(\omega t + \theta_0 + \tan^{-1}\frac{r}{X_q}\right)$$

$$i_f = i_{f0} + \frac{3}{2}\frac{M_{af}}{L_{ff}}\frac{(r \sin\delta + X_q \cos\delta)}{r^2 + X_d' X_q} V_a \varepsilon^{-t/T_{d2}'}$$

E_f = 14.25 volts, rms.

V_a = 8.43 volts.

$r = 5.0 + 0.38 = 5.38$ ohms.

X_d = 10.36 ohms.

X_q = 6.33 ohms.

$L_{afm} = \dfrac{\sqrt{2} \times 14.25}{0.224 \quad 377} = 0.238$ henries.

$L_{ff} = \dfrac{17.12 \times 15.2}{60} = 4.34$ henries.

X_d' = 2.98 ohms.

$T_{d2}' = \dfrac{6.5}{60}$ sec.

The torque angle δ is found graphically using the experimental data in the three-phase short circuit.

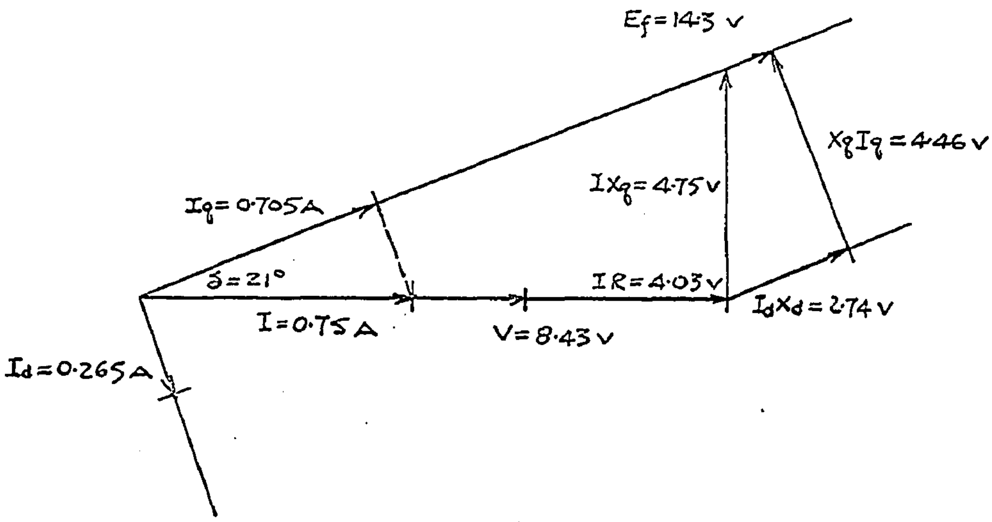

Fig. 18. To determine the torque angle graphically.

From Fig. 15, the switching angle is found to be,

$$\theta_0 = 36° + 21°$$

$$= 57°$$

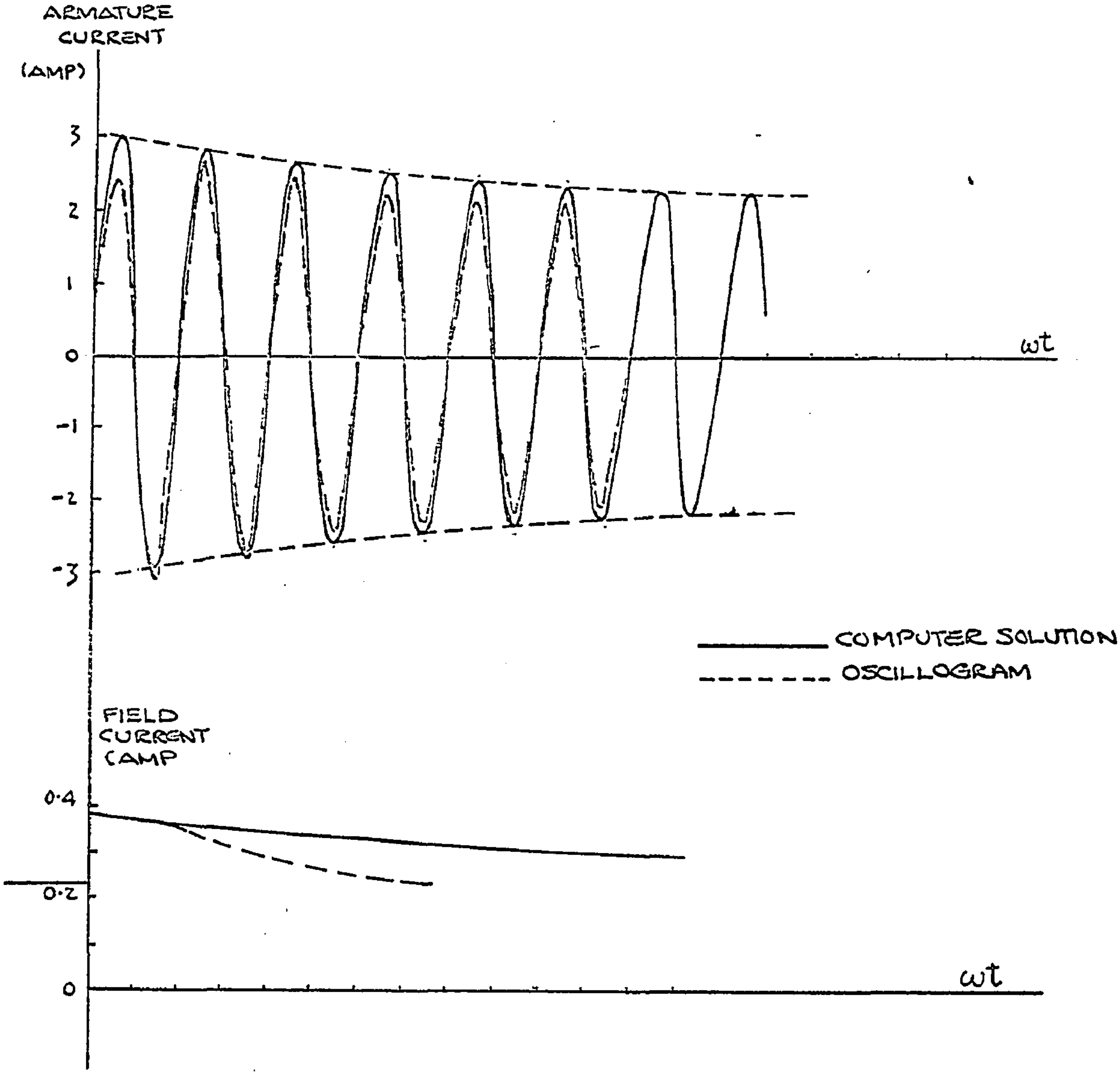

Fig. 19. Computer solution of transient armature current (upper) and transient field current (lower) for a three-phase short circuit of a loaded alternator.

Computer Solution of Armature Current
in Three-Phase Short Circuit

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80000000.	06-	13962667.	07-	29844450.	07-
10000000.	05-	17453333.	07-	28418891.	07-
12000000.	05-	20944000.	07-	23587413.	07-
14000000.	05-	24434666.	07-	15954042.	07-
16000000.	05-	27925333.	07-	64552295.	08-
17999999.	05-	31416000.	07-	37548769.-	08-
20000000.	05-	34906666.	07-	13444535.-	07-
21999999.	05-	38397333.	07-	21452867.-	07-
23999999.	05-	41887999.	07-	26828797.-	07-
25999999.	05-	45378665.	07-	28943875.-	07-
27999999.	05-	48869332.	07-	27565600.-	07-
29999999.	05-	52359998.	07-	22882712.-	07-
31999999.	05-	55850665.	07-	15479734.-	07-
33999999.	05-	59341331.	07-	62641670.-	08-
35999999.	05-	62831998.	07-	36446308.	08-
37999999.	05-	66322664.	07-	13051279.	07-
39999998.	05-	69813331.	07-	20828457.	07-
41999998.	05-	73303997.	07-	26051868.	07-
43999998.	05-	76794664.	07-	28109980.	07-
45999998.	05-	80285331.	07-	26775488.	07-
47999998.	05-	83775998.	07-	22230185.	07-
49999998.	05-	87266664.	07-	15040543.	07-
51999999.	05-	90757331.	07-	60872500.-	08-
53999999.	05-	94247997.	07-	35425484.-	08-
55999999.	05-	97738664.	07-	12687145.-	07-
57999998.	05-	10122933.	06-	20250282.-	07-
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61999999.	05-	10821066.	06-	27337826.-	07-
63999998.	05-	11170133.	06-	26043875.-	07-
65999998.	05-	11519199.	06-	21625970.-	07-
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69999999.	05-	12217333.	06-	59234290.-	08-
71999998.	05-	12566399.	06-	34480283.	08-
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75999997.	05-	13264533.	06-	19714915.	07-
77999997.	05-	13613599.	06-	24666326.	07-
79999998.	05-	13962666.	06-	26622843.	07-
81999998.	05-	14311732.	06-	25366429.	07-
83999997.	05-	14660799.	06-	21066491.	07-
85999997.	05-	15009866.	06-	14257304.	07-
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97999996.	05-	17104265.	06-	25960797.-	07-
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37999999.	05-	66322664.	07-	13051279. 07-
39999998.	05-	69813331.	07-	20828457. 07-
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45999998.	05-	80285331.	07-	26775488. 07-
47999998.	05-	83775998.	07-	22230185. 07-
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23799996.	04-	41538926.	06-	16370076.-	07-
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30399993.	04-	53058122.	06-	46281153.	08-
30599993.	04-	53407188.	06-	27010323.-	08-
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32599992.	04-	56897853.	06-	95698526.	08-
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32999992.	04-	57595986.	06-	19173377.	07-
33199992.	04-	57945053.	06-	20727015.	07-
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33599992.	04-	58643186.	06-	16452769.	07-
33799992.	04-	58992252.	06-	11151837.	07-
33999992.	04-	59341319.	06-	45205666.	08-
34199992.	04-	59690386.	06-	26391059.-	08-
34399992.	04-	60039452.	06-	94635394.-	08-
34599992.	04-	60388519.	06-	15131581.-	07-
34799991.	04-	60737585.	06-	18963286.-	07-
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35199991.	04-	61435718.	06-	19566355.-	07-
35399991.	04-	61784785.	06-	16276275.-	07-
35599991.	04-	62133851.	06-	11033027.-	07-
35799991.	04-	62482918.	06-	44726859.-	08-
35999991.	04-	62831984.	06-	26115361.	08-

Computer Solution of Field Current in Three-Phase Short Circuit

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59999999.	06-	37994951.	08-
80000000.	06-	37862226.	08-
10000000.	05-	37730631.	08-
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14000000.	05-	37470792.	08-
16000000.	05-	37342528.	08-
17999999.	05-	37215356.	08-
20000000.	05-	37089266.	08-
21999999.	05-	36964249.	08-
23999999.	05-	36840297.	08-
25999999.	05-	36717399.	08-
27999999.	05-	36595547.	08-
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31999999.	05-	36354946.	08-
33999999.	05-	36236179.	08-
35999999.	05-	36118423.	08-
37999999.	05-	36001669.	08-
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41999998.	05-	35771133.	08-
43999998.	05-	35657335.	08-
45999998.	05-	35544505.	08-
47999998.	05-	35432636.	08-
49999998.	05-	35321718.	08-
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59999999.	05-	34781131.	08-
61999999.	05-	34675759.	08-
63999998.	05-	34571283.	08-
65999998.	05-	34467696.	08-
67999998.	05-	34364991.	08-
69999999.	05-	34263160.	08-
71999998.	05-	34162196.	08-
73999998.	05-	34062090.	08-
75999997.	05-	33962837.	08-
77999997.	05-	33864429.	08-
79999998.	05-	33766858.	08-
81999998.	05-	33670118.	08-
83999997.	05-	33574201.	08-
85999997.	05-	33479100.	08-
87999996.	05-	33384808.	08-
8999997.	05-	33291320.	08-
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29799993.	04-	26877481.	08-
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30199993.	04-	26801592.	08-
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30599993.	04-	26726989.	08-
30799993.	04-	26690163.	08-
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[illegible]	[illegible]	[illegible]	08-

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33399992.	04-	26238999.	08-
33599992.	04-	26206326.	08-
33799992.	04-	26173931.	08-
33999992.	04-	26141812.	08-
34199992.	04-	26109967.	08-
34399992.	04-	26078392.	08-
34599992.	04-	26047087.	08-
34799991.	04-	26016047.	08-
34999991.	04-	25985272.	08-
35199991.	04-	25954758.	08-
35399991.	04-	25924505.	08-
35599991.	04-	25894509.	08-
35799991.	04-	25864768.	08-
35999991.	04-	25835280.	08-

Digital Computer Solution of the Transient Equations for a Line-to-Line Short Circuit

Recalling from Chapter IV the following transient equations:

$$i_c = \frac{1}{\sqrt{2}} i_\beta$$

$$= \sqrt{3}\, E_f \left\{ \frac{1}{X_d + X_2} + \left(\frac{1}{X_d' + X_2} - \frac{1}{X_d + X_2} \right) \varepsilon^{-t/T_d'} \right\}$$

$$\left\{ \sin\theta + \sum_{n=1}^{\infty} (-b)^n \sin(2n+1)\theta \right\}$$

$$- \frac{\sqrt{\frac{3}{2}}\, E_f \sin\theta_0}{X_2}\, \varepsilon^{-t/T_a'} \left\{ 1 + 2 \sum_{n=1}^{\infty} (-b)^n \cos 2n\theta \right\}$$

$$i_f = i_{f0} + \sqrt{\frac{3}{2}}\, \frac{M_{af}}{L_{ff}}\, \frac{E_f}{X_d' + X_2}\, \varepsilon^{-t/T_d'}$$

$$+ \sqrt{\frac{3}{2}}\, \frac{M_{af}}{L_{ff}}\, E_f \left\{ \frac{1}{X_d + X_2} + \left(\frac{1}{X_d' + X_2} - \frac{1}{X_d + X_2} \right) \varepsilon^{-t/T_d'} \right\}$$

$$\left\{ \frac{1+b}{b} \sum_{n=1}^{\infty} (-b)^n \cos 2n\theta \right\}$$

$$- \sqrt{\frac{3}{2}}\, \frac{M_{af}}{L_{ff}} (1+b)\, \frac{E_f \sin\theta_0}{X_2}\, \varepsilon^{-t/T_a'} \left\{ \sin\theta + \sum_{n=1}^{\infty} (-b)^n \sin(2n+1)\theta \right\}$$

$$e_a = \frac{E_f(X_d - X_d')(1+b)}{\omega T_d'(X_d + X_2)} \varepsilon^{-t/T_d'}(\cos\theta - b\cos 3\theta + b^2\cos 5\theta - \cdots)$$

$$+ E_f F (1+b)(\sin\theta - 3b\sin 3\theta + 5b^2\sin 5\theta)$$

$$- \frac{2bE_f\sin\theta_0}{\omega T_a'} \varepsilon^{-t/T_a'}(\sin 2\theta - b\sin 4\theta + b^2\sin 6\theta - \cdots)$$

$$+ 4bE_f G\sin\theta_0(\cos 2\theta - 2b\cos 4\theta + 3b^2\cos 6\theta - \cdots)$$

in which,

$$F = \frac{X_d' + X_2}{X_d + X_2} + \left(1 - \frac{X_d' + X_2}{X_d + X_2}\right)\varepsilon^{-t/T_d'}$$

$$G = \varepsilon^{-t/T_a'}$$

Since the vlaue of b is much less than unity, the higher harmonics can be neglected, and in computing the transient equations, the value of n is taken to 2 only.

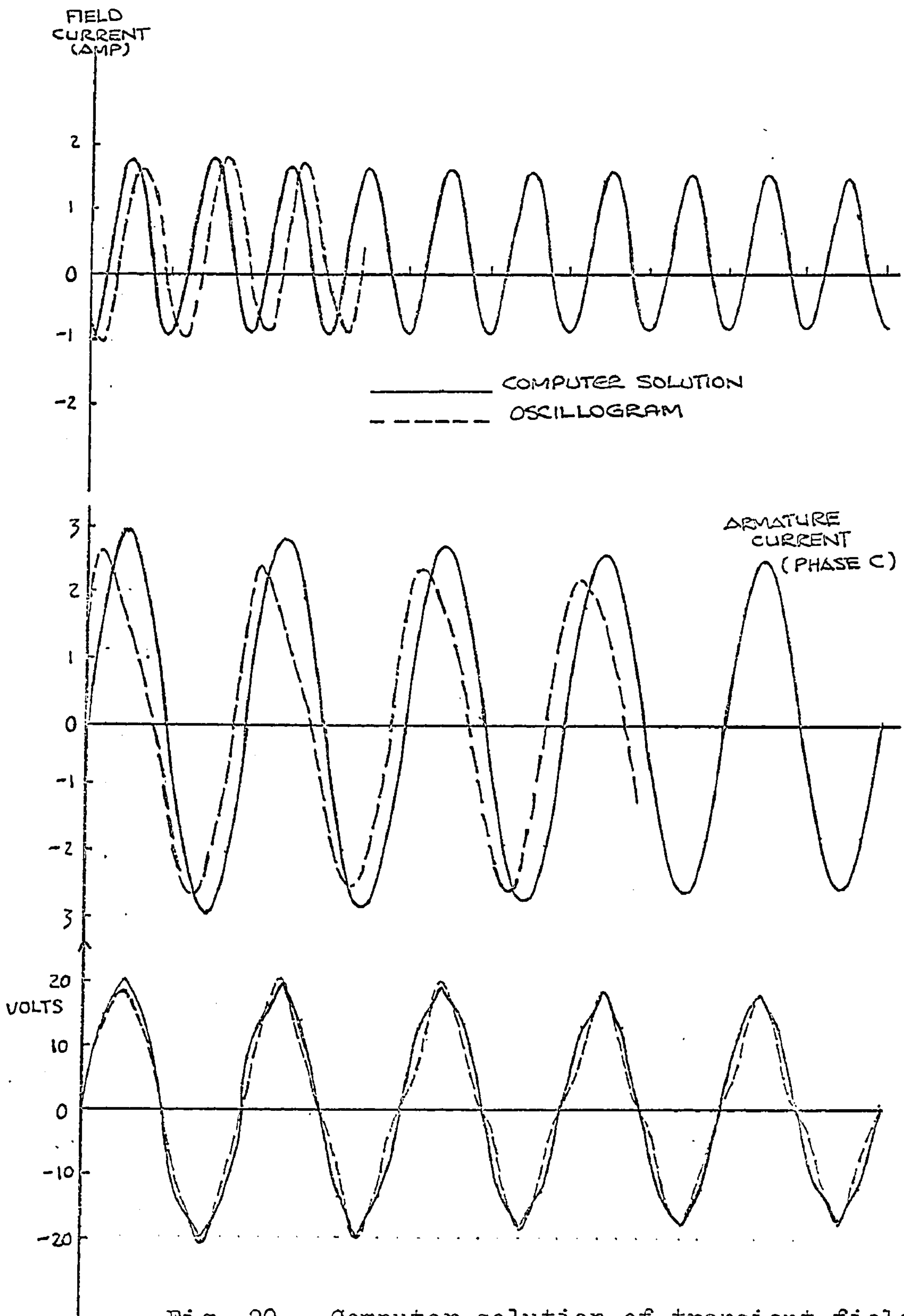

Fig. 20. Computer solution of transient field current (upper), armature current (middle), and open-phase voltage (lower) for a line-to-line short circuit of an alternator.

The results obtained experimentally and the solution of the transient equations by the digital computer agree closely and they are in accordance with the results expected. The chief source of error arises from the difficulty involved in evaluating the switching angle accurately from the oscillographs. It must also not be forgotten that certain approximations are introduced in order to arrive at the final equations for short-circuit currents. The percentage difference in the results by the two methods, however, are certainly within the ten-percent margin.

It is felt that should sufficient appropriate equipments be available, the short-circuits of the machine should be done at normal voltage instead of at reduced voltage. Saturation effect will then of course be more appreciable. The study and the experimental investigation of the effects of saturation on machines will be, in the opinion of the author, a good and valuable extension to this thesis.

COMPUTER SOLUTION OF THE TRANSIENT EQUATIONS FOR A LINE-TO-LINE SHORT CIRCUIT

44999998.	06-	14409543.	06-	20564887.	07-	34147796.	08-
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13499999.	05-	14112327.	06-	20352825.	07-	33812477.	08-
17999999.	05-	12215964.-	08-	20344200.-	12-	97752914.-	08-
22499999.	05-	14165325.-	06-	20146750.-	07-	33491659.	08-
26999999.	05-	19804737.-	06-	29054519.-	07-	17002131.	07-
31499999.	05-	13866371.-	06-	19945283.-	07-	33169679.	08-
35999999.	05-	11615835.	08-	39859480.	12-	95918840.-	08-
40499999.	05-	13880453.	06-	19750119.	07-	32869287.	08-
44999999.	05-	19432750.	06-	28486956.	07-	16688230.	07-
49499999.	05-	13591164.	06-	19558698.	07-	32559924.	08-
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62999999.	05-	19078322.-	06-	27948583.-	07-	16390474.	07-
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11699999.	04-	18166830.	06-	26493973.	07-	15585977.	07-
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19799999.	04-			18663702.-	11-	83124027.-	08-
20249999.	04-			16983238.-	07-	28528728.	08-
20699999.	04-			24527541.-	07-	14498408.	07-
21149999.	04-			16861740.-	07-	28304983.	08-
21599999.	04-			20055361.	11-	82042321.-	08-
22049998.	04-			16749323.	07-	29161861.	08-
22499998.	04-			24192802.	07-	14313275.	07-
22949998.	04-			16633727.	07-	27945238.	08-
23399998.	04-			21329491.-	11-	81016247.-	08-
23849998.	04-			16527442.-	07-	27813903.	08-
24299998.	04-			23875278.-	07-	14137663.	07-
24749998.	04-			16417437.-	07-	27603791.	08-
25199998.	04-			22577275.	11-	80042944.-	08-
25649998.	04-			16316981.	07-	27483882.	08-
26099998.	04-			23574085.	07-	13971083.	07-
26549998.	04-			16212262.	07-	27279943.	08-
26999998.	04-			23799831.-	11-	79119699.-	08-
27449998.	04-			16117341.-	07-	27170877.	08-
27899997.	04-			23288383.-	07-	13813070.	07-
28349997.	04-			16017642.-	07-	26972707.	08-
28799997.	04-			25002763.	11-	78243936.-	08-
29249997.	04-			15927973.	07-	26874013.	08-

31049997.	04-	15748353.-	07-	26592
31499997.	04-	22760303.-	07-	13521
31949997.	04-	15657906.-	07-	26404
32399997.	04-	27434955.	11-	76625
32849997.	04-	15577968.	07-	26325
33299997.	04-	22516453.	07-	13386
33749996.	04-	15491781.	07-	26142
34199996.	04-	28582842.-	11-	75877
34649996.	04-	15416350.-	07-	26072
35099996.	04-	22285145.-	07-	13258
35549996.	04-	15334202.-	07-	25893
35999996.	04-	29796631.	11-	75168
36449996.	04-	15263047.	07-	25832
36899996.	04-	22065732.	07-	13136
37349996.	04-	15184723.	07-	25657
37799996.	04-	30839415.-	11-	74496
38249996.	04-	15117626.-	07-	25604
38699996.	04-	21857605.-	07-	13021
39149996.	04-	15042930.-	07-	25433
39599995.	04-	31950931.	11-	73858
40049995.	04-	14979693.	07-	25388
40499995.	04-	21660181.	07-	12912
40949995.	04-	14908424.	07-	25220
41399995.	04-	33050829.-	11-	73253
41849994.	04-	14848858.-	07-	25183
42299994.	04-	21472911.-	07-	12808
42749994.	04-	14780845.-	07-	250191

APPENDIX I

Physical Construction of the Machine and Name Plate Data

Manufacturer: Westinghouse Electric Corporation.

Serial Number: DC Generator-Motor---EEMG1
Six-Phase Alternator-Synchronous Motor---EEMG2.

DC Motor	AC Alternator
Type - SK	KVA - 4
Frame - 326	Volts - 110/220
HP - 7.5	Amps - 21/10.5
Volts - 230	Phase - 1-3-6
Amps - 29	Style - 16B3518
RPM - 950-1200	Serial - 1
Compound Wound	Hours - 24
Style - 16B3517	RPM - 1200
Serial - 1S-58	Exc. Amp. - 3.2
% Load - 100	Exc. Volts - 125
Hours - 24	Cycles - 60
°C Rise Open - 50	% PF - 90
	% Load - 100
	°C Rise - 50

APPENDIX II

The General Equation of Induced Voltage

Faraday's law states that the voltage induced in a closed circuit is proportional to the time rate of increase of the flux linked with that circuit.

$$e = -\frac{N}{10^8}\frac{d\phi}{dt}$$

The negative sign is occasioned by Lenz's law, which recognizes that any current which is permitted to flow as a result of this voltage will oppose the original flux change.

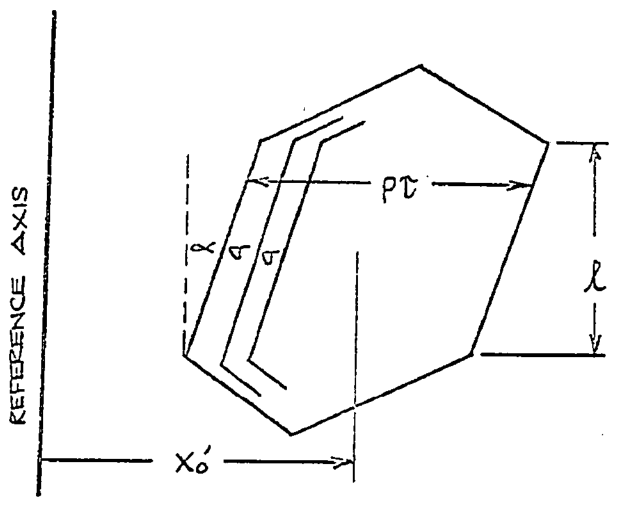

Fig. 1. Fractional-pitch coil with skew coil sides.

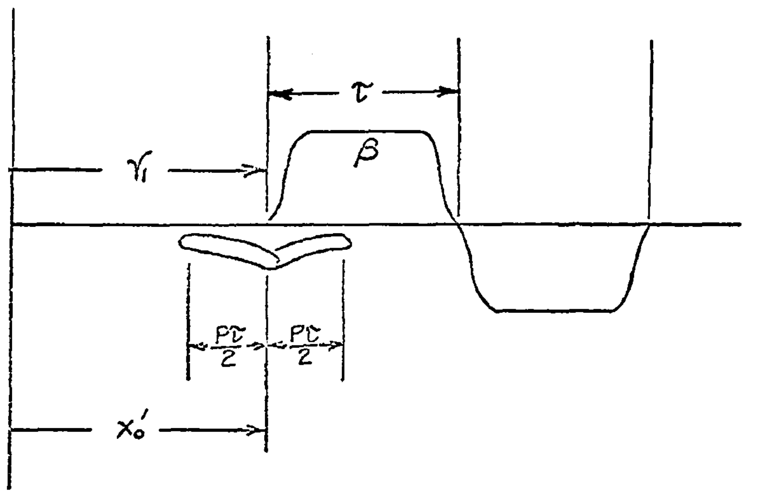

Fig. 2. A single coil in the field.

Referring to Fig. 2, let the flux density distribution be specified by the Fourier series,

$$\beta = \Sigma \beta_K \operatorname{Sin} K \left(\frac{\pi x}{\tau} \pm \gamma_K\right)$$

in which

k = order of the space harmonic.

β_K = amplitude of the k^{th} harmonic (a function of time).

γ_K = phase angle (position) of the k^{th} harmonic (a function of time).

2τ = pitch (wave-length) of the fundamental.

x = distance measured from an arbitrary stationary reference axis.

Fig. 1 shows a coil of N turns with effective coil length ℓ, coil pitch p, and with center located at x_o' measured from an arbitrary reference axis. Then its trailing and leading coil sides are located at

$$x' = (x_o' - p\frac{\pi}{2} + y\tan\alpha)$$

$$x'' = (x_o' + p\frac{\pi}{2} + y\tan\alpha)$$

in which α = angle of skew.

The flux linked with the coil at any particular instant is,

$$\phi = \int_{x_o' - \frac{p\tau}{2}}^{x_o' + \frac{p\tau}{2}} l\beta\, dx = \frac{2}{\pi}\tau l \sum K_{pk}K_{sk} \frac{\beta_K}{K} \sin K\left(\frac{\pi x_o'}{\tau} \pm \gamma_k\right)$$

in which $K_{pk} = \sin\frac{kp\pi}{2}$

= pitch coefficient of the k^{th} harmonic.

$K_{sk} = \dfrac{2\tau}{k\pi l\tan\alpha}\sin\dfrac{k\pi l\tan\alpha}{2\tau}$

= skew coefficient of the k^{th} harmonic.

It will be remembered that the average value of a sine wave is $\frac{2}{\pi}$ times its amplitude, and the half-wavelength of a k^{th} harmonic is $\frac{\tau}{k}$, since by definition it has k wavelengths in a fundamental wavelength. Therefore, the flux in a half-wavelength is,

$$\phi_k = \frac{2}{\pi}\beta_k\frac{\tau}{k}l$$

Hence ϕ may be rewritten as,

$$\phi = \Sigma K_{pk} K_{sk} \phi_k \sin k \left(\frac{\pi x_o'}{\tau} \pm \gamma_k\right)$$

$$= f\,(\phi_k,\ x_o',\ \gamma_k)$$

Bt applying Faraday's law, the induced voltage in the coil is then,

$$e = -\frac{N}{10^8}\frac{d\phi}{dt} = -\frac{N}{10^8}\left\{\frac{\partial\phi}{\partial\phi_k}\frac{d\phi_k}{dt} + \frac{\partial\phi}{\partial x_o'}\frac{dx_o'}{dt} + \frac{\partial\phi}{\partial\gamma_k}\frac{d\gamma_k}{dt}\right\}$$

$$= -\Sigma \frac{N}{10^8} K_{pk} K_{sk} \left\{\frac{d\phi_k}{dt}\sin\left(\frac{\pi x_o'}{\tau} \pm \gamma_k\right)\right.$$

$$\left. + k\phi_k \left(\frac{\pi}{\tau}\frac{dx_o'}{dt} \pm \frac{d\gamma_k}{dt}\right)\cos k\left(\frac{\pi x_o'}{\tau} \pm \gamma_k\right)\right\}$$

From the above equation, the three process of voltage induction are easily identified.

$\frac{d\phi_k}{dt}$, variation of the flux.

$\frac{dx_o'}{dt}$, movement of the conductors.

$\frac{d\gamma_k}{dt}$, movement of the field.

For a group of q-coils, separated by the slot pitch σ , the center of the h^{th} coil from the end is at,

$$x_o' = x_o - \frac{q+1}{2}\sigma + h\sigma$$

Summing up e over q coils of the group by means of the Distribution summation, there results,

$$e = \sum_{h=1}^{q} e_n = -\sum \frac{qN}{10^8} K_{pk} K_{sk} K_{dk} \left\{ \frac{d\phi_k}{dt} \sin k \left(\frac{\pi x_0}{\tau} \pm \gamma_k \right) + k\phi_k \left(\frac{\pi}{\tau} \frac{dx_0}{dt} \pm \frac{d\gamma_k}{dt} \right) \cos \left(\frac{\pi x_0}{\tau} \pm \gamma_k \right) \right\}$$

in which $$K_{dk} = \frac{\sin(k\pi q\sigma/2\tau)}{q \sin(k\pi\sigma/2\tau)}$$

= distribution coefficient of the k^{th} harmonic.

APPENDIX III

The General Equation of
Armature Reaction

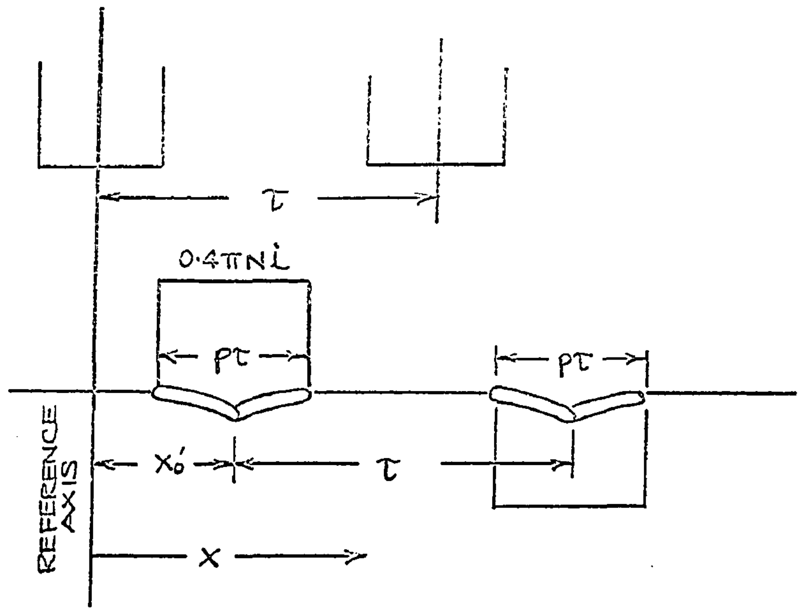

Fig. 3. A single coil of N-turns and pitch $p\tau$ carrying an instantaneous current i.

In Fig. 3, the center of the coil is located at a distance x_o' from the arbitrary reference axis. Assuming the winding is of such a nature that a similar coil a pole pitch away carries the same current (reversed), it is permissible to assume the magnetomotive force due to the two coils to be the block wave shown in Fig. 4. This wave has a magnetomotive force magnitude of $0{\cdot}4\pi Ni$ and the magnetomotive force per coil, therefore, may be expressed by the Fourier series,

$$F(x) = 0.8 N i \sum \frac{K_{pk} K_{sk}}{k} \cos \frac{k\pi (x - x_0')}{\tau}$$

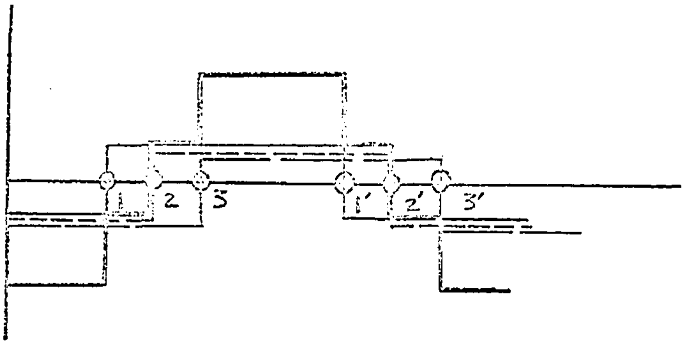

Fig. 4. Resultant magnetomotive force of coils.

The total mmf of q such coils, displaced by the slot pitch σ and carrying equal current i, may be found in a similar way as in the case of the induced voltage. For the h^{th} coil, $x_0' = x_0 - \frac{(q+1)\sigma}{2} + h\sigma$ holds and the mmf for the group of q coils therefore is,

$$A(x) = 0.8 N i \sum_{k} \sum_{h=1}^{q} \frac{K_{pk} K_{sk}}{k} \cos \frac{k\pi}{\tau} \left(x - x_0 + \frac{(q+1)\sigma}{2} - h\sigma \right)$$

$$= 0.8\, q N i \sum_{k} \frac{K_{pk} K_{sk} K_{dk}}{k} \cos \frac{k\pi}{\tau} (x - x_0)$$

in which K_{pk}, K_{sk}, and K_{dk} are defined as in Appendix II.

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The author was born and received his high school education in Hong Kong, from where he came to Canada in 1956. He studied Pre-Engineering at Essex College, Assumption University of Windsor, from 1956 to 1957. He transferred to McGill University in Montreal in 1957, and graduated in 1961 with a Bachelor of Engineering degree in Electrical Engineering. He was admitted to the faculty of Graduate Studies for M.A.Sc. degree in Electrical Engineering at Assumption University of Windsor in 1962. He was also a full-time instructor in Electrical Engineering at Essex College from 1961 to 1963.