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# Extended Double Lattice BRST, Curci-Ferrari Mass and the Neuberger Problem

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**Abstract.** We present Extended Double BRST on the lattice and extend the Neuberger problem to include the ghost/anti-ghost symmetric formulation of the non-linear covariant Curci-Ferrari (CF) gauges. We then show how a CF mass regulates the 0/0 indeterminate form of physical observables, as observed by Neuberger, and discuss the gauge parameter and mass dependence of the model.

**Keywords:** Lattice BRST, TFT, Neuberger Problem, Curci-Ferrari Mass

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## INTRODUCTION

In the covariant continuum formulation of gauge theories one has to deal with the redundant degrees of freedom due to gauge invariance. Within the language of local quantum field theory, the machinery for this is based on the Becchi-Rouet-Stora-Tyutin (BRST) symmetry which can be considered the quantum version of local gauge invariance. Beyond perturbation theory one faces the famous Gribov ambiguity: the existence copies of gauge-configurations that satisfy the Lorentz condition (or any other local gauge fixing condition) but are related by gauge transformations, and are thus physically equivalent. As a result, the usual definitions of a BRST charge fail to be globally valid. A rigorous non-perturbative framework is provided by lattice gauge theory. Its strength and beauty derives from the fact that gauge-fixing is not required. However, in order to arrive at a non-perturbative definition of non-Abelian gauge theories in the continuum, from a lattice formulation, we need to be able to perform the continuum limit in a formally watertight way. The same ambiguity then shows in another form when attempting to fix a gauge via BRST formulations on the lattice. There it is known as the Neuberger problem which asserts that the expectation value of any gauge invariant (and thus physical) observable in a lattice BRST formulation will always be of the indefinite form 0/0 [1].

In this talk we present the ghost/anti-ghost symmetric Curci-Ferrari gauges with double BRST on the lattice. We show how Neuberger's argument can be extended to include these non-linear covariant gauges, and how the indeterminate form 0/0 of expectation values is regulated by CF mass term [2] thereby decontracting the double BRST algebra to its extended version. Finally, we discuss how the gauge-parameter  $\xi$  dependence of the model can be compensated by adjusting the CF mass with  $\xi$ .

In pure  $SU(N)$  lattice gauge theory, the gauge transformation of link  $U_{ij}$  is defined as  $U_{ij}^g = g_i U_{ij} g_j^\dagger$ . BRST and anti-BRST transformations  $s$  and  $\bar{s}$  in the topological setting do not act on the link variables  $U$  directly, but on the gauge transformations  $g_i$  like infinitesimal left translations in the gauge group with real ghost and anti-ghost

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Grassmann fields  $c_i^a, \bar{c}_i^a$  as parameters,  $sg_i = c_i g_i$  and  $\bar{s}g_i = \bar{c}_i g_i$ , where  $c_i \equiv c_i^a X^a$  and  $\bar{c}_i \equiv \bar{c}_i^a X^a$ . For the normalization of the anti-Hermitian generators  $X^a$  in the fundamental representation we use  $\text{tr} X^a X^b = -\delta^{ab}/2$ . The action of the topological lattice model for gauge fixing *à la* Faddeev-Popov with double BRST invariance can then be written as

$$S_{\text{cf}} = i s \bar{s} \left( V[U^g] + i \xi \sum_i \text{tr} \bar{c}_i c_i \right) = \sum_i \left( i b_i^a F_i^a[U^g] - \frac{i}{2} \bar{c}_i^a M_i^a[U^g, c] + \frac{\xi}{2} (b_i^a)^2 + \frac{\xi}{8} (\bar{c}_i \times c_i)^2 \right),$$

where  $V[U^g] = -\sum_i \sum_{j \sim i} \text{tr} U_{ij}^g = -2 \sum_{x, \mu} \text{Re} \text{tr} U_{x, \mu}^g$  is the gauge fixing functional of covariant gauges which here assumes the role of a Morse potential on a gauge orbit.  $F_i^a[U^g] = 0$  is the gauge-fixing condition and  $M_i^a[U^g, c] = \sum_j M_{ij}^{ab} c_j^b$  defines the Faddeev-Popov operator of the ghost/anti-ghost symmetric gauges. Note the occurrence of quartic ghost self-interactions  $\propto (\bar{c}_i \times c_i)^2 \equiv (f^{abc} \bar{c}_i^b c_i^c)^2$  which make the Neuberger problem somewhat less obvious in these gauges. Details will be presented elsewhere.

## REGULARISATION OF 0/0 AND $\xi$ INDEPENDENCE

Following Neuberger, we introduce an auxiliary parameter  $t$  upfront the Morse potential, to write the Euclidean partition function used as the gauge-fixing device, with double BRST,

$$Z_{\text{cf}}(t) = \int d[g, b, \bar{c}, c] \exp \left\{ -i s \bar{s} \left( t V[U^g] + i \xi \sum_i \text{tr} \bar{c}_i c_i \right) \right\} \quad (1)$$

which is independent of  $\{U\}$  and  $\xi$ . Because a derivative w.r.t.  $t$  produces a BRST-exact operator in the integrand, it is in fact independent of  $t$  also, *i.e.*,  $Z_{\text{cf}}'(t) = 0$ . For  $t = 0$  on the other hand we find that  $Z_{\text{cf}}(0) = 0$ ; and this is the reason for the indeterminate form of 0/0 for all observables first derived for the standard linear covariant gauges in [1]. The fact that this conclusion holds also in the ghost/anti-ghost symmetric formulation with its quartic self-interactions directly relates to the topological interpretation [3] of the Neuberger zero:  $Z_{\text{cf}}$  can be viewed as the partition function of a Witten-type TQFT which computes the Euler character  $\chi$  of the gauge group. On the lattice the gauge group is a direct product of  $SU(N)$ 's per site, and  $Z_{\text{cf}} = \chi(SU(N)^{\#\text{sites}}) = \chi(SU(N))^{\#\text{sites}} = 0^{\#\text{sites}}$ . For  $t = 0$  the action decouples from the link-field configuration and  $Z_{\text{cf}}(0)$ , albeit computing the same topological invariant, has no effect in terms of fixing a gauge. In the present formulation,  $Z_{\text{cf}}(0)$  factorises into independent Grassmann integrations per site of the quartic term containing the curvature of  $SU(N)$ , each of which computes the vanishing Euler character of  $SU(N)$  via the Gauss-Bonnet theorem [4].

As proposed in [5], this zero can be regularised, however, by introducing a Curci-Ferrari mass  $m$ , such that the gauge-fixing action  $S_{\text{cf}}$  is replaced by

$$S_{\text{mcf}}(t) = i (s \bar{s} - im^2) \left( t V_U[g] + i \xi \sum_i \text{tr} \bar{c}_i c_i \right). \quad (2)$$

The corresponding partition function  $Z_{\text{mcf}}(t)$  no-longer vanishes at  $t = 0$ , and this part in Neuberger's disastrous conclusion is thus avoided. We have explicitly calculated  $Z_{\text{mcf}}(0)$ , which is polynomial in  $\xi m^4$ , for  $SU(2)$  and  $SU(3)$ . The original zero is obtained

for  $m^2 \rightarrow 0$  which corresponds to a Wigner-Ionu contraction of the so-called extended double BRST superalgebra. While a non-vanishing  $m^2$  thereby breaks the nilpotency of BRST and anti-BRST transformations, which is known to result in a loss of unitarity, it also serves to regulate the  $0/0$  indeterminate form of expectation values in lattice BRST formulations, and to obtain finite results for  $m^2 \rightarrow 0$  via l'Hospital's rule.

For gauge fixing we need to have  $t \neq 0$ . The partition function  $Z_{\text{mCF}}(t)$  of the massive CF model is no-longer  $t$ -independent because  $s$  and  $\bar{s}$  are no-longer nilpotent and the simple argument above fails, *i.e.*,  $Z_{\text{mCF}}'(t) \neq 0$  for  $m^2 \neq 0$ . However, the existence of 3 independent parameters  $t$ ,  $\xi$  and  $m^2$  is an illusion. A change in  $t$  can always be compensated by changing the gauge parameter  $\xi$  and  $m^2$ . In fact, simple scaling arguments and explicit calculations show that  $Z_{\text{mCF}}$  only depends on 2 combinations of the 3, we can parametrise  $Z_{\text{mCF}} = f(t^2/\xi, \xi m^4) \equiv f(x^2, \hat{m}^4)$ , where we defined  $x^2 \equiv t^2/\xi$  and  $\hat{m}^4 \equiv \xi m^4$ . Our explicit calculations for  $t = 0$  yield  $f(0, \hat{m}^4)$ . Independence of  $t$  then comes together with gauge parameter  $\xi$  independence. To achieve this, we allow  $\hat{m}^2 \equiv \hat{m}^2(x)$  so that  $Z_{\text{mCF}} = f(x, \hat{m}^4(x))$ . This means that we adjust the CF mass  $\hat{m}^2$  with  $x$  such that our  $x = 0$  results remain unchanged. In particular, we must have

$$\frac{d}{dx} Z_{\text{mCF}} = \left( \frac{\partial}{\partial x} + \frac{d\hat{m}^2}{dx} \frac{\partial}{\partial \hat{m}^2} \right) Z_{\text{mCF}} = 0 \quad (3)$$

which can be used to determine the derivative of  $\hat{m}^2(x)$ . This is always possible. The crucial question at this point is whether it can be done independent of the link configuration  $\{U\}$ . As our explicit calculations are restricted to  $x = 0$  we have explicitly verified that  $\hat{m}^{2'}(0)$  is finite and independent of  $\{U\}$ . While this is merely necessary, but not sufficient, it demonstrates that we can get away from  $x = 0$ , at least infinitesimally. This is of qualitative importance as a non-zero value of  $x = t/\sqrt{\xi}$ , no matter how small, corresponds to a large but finite  $\xi$  at  $t = 1$  and thus eliminates the gauge freedom.

## CONCLUSIONS

The massive Curci-Ferrari model with extended double BRST symmetry can be formulated on the lattice without the  $0/0$  problem. The parameter  $m^2$  is not interpreted as a physical mass but to meaningfully define a limit  $m^2 \rightarrow 0$  in the spirit of l'Hospital's rule. At finite  $m^2$  the topological nature of the gauge-fixing partition function seems lost. It is possible, however, to tune the CF mass with the gauge parameter  $\xi$  so that the limit  $m^2 \rightarrow 0$  can be defined along a certain trajectory in parameter space independent of  $\xi$ . An interesting open question might then be the topological interpretation of the model within the extended double BRST superalgebra framework.

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