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Multistage Accelerated Reliability Growth Testing Model and Data Analysis

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Abstract

Accelerated reliability growth testing has recently received a renewed interest in reliability engineering. The concepts of accelerated testing and reliability growth individually have been used in a variety of applications, either for hardware systems or software systems. The advantage of using a combined strategy is that it could shorten the testing time while maximizing the reliability. In the literature, there are many references related to optimal test design for reliability from either a component level or a system level. In this research, we suggest an approach which conducts accelerated testing at the component level while supporting estimates of reliability at the system level. Our approach helps one decide where and at what level to conduct accelerated test during the system design and testing process. Our approach is designed to reduce testing cost while still demonstrating that system level requirements are met. We do this testing at lower levels in an accelerated environment, where costs are lower, and minimize the amount of testing at the higher integrated system level where it tends to be more expensive.

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Chapter 1. Introduction

1.1. Background

Reliability engineering as a discipline became popular since just after World War II in the manufacturing field. As a critical approach in reliability improvement, the goal of accelerated tests was to improve a product's reliability with confidence and therefore decrease maintenance costs (Klyatis & Verbitsky. 2010). While the original notion of reliability and maintainability originated primarily in the US defense sector, it was the Japanese who first utilized it extensively in the commercial sector. Today's thriving Japanese car industry is a result of a strong focus on reliability and quality engineering. Lately, many companies have placed a renewed emphasis on reliability. Two tools, reliability growth testing, and accelerated testing have been used to help organizations measure reliability and improve their designs and products through the use of a test, analyze, and fix approach. Originally, reliability growth testing and accelerated testing have been applied separately. The possible reason for it might because "accelerated testing is usually done at the component level while reliability growth planning is more applicable at the subsystem/system level." (Feinberg, 1994)

Traditional reliability engineering approaches alone can no longer fully satisfy the ever increasing demands on product quality and reliability. In order to economically enhance quality and reliability, to further decrease the testing time and marginal cost, accelerated reliability growth testing (ARGT) has emerged as an area of interest. In this research, we use ARGT to help shorten the testing time and operational cost for a system during testing. Mathematical models on how to determine the optimal accelerated test strategy and optimize reliability for the system will be proposed in this plan.

Reliability growth is an approach that is used to eliminate failure mechanism of devices through testing (Andonova & Atanasova, 2004). Gurunatha and Siegel (2003) developed a 12-step Six-Sigma testing policy in reliability growth. Reliability growth testing has been applied in many fields, such as life time tests (Krasich, 2007, Krasich, 2011, Xing & Wu, 2011) and step stress screening (Wong, 1990, Pohl & Dietrich, 1999), to improve the effectiveness of new products. The growth of reliability comes from eliminating failure modes (FMs) through increased testing. The trade-offs between reliability and cost is determined by testing goals (Quigley & Walls, 2003). There are several famous reliability growth models that have been formulated during the past decades, for example, Duane's model, the AMSAA (Crow) model, the IBM model, the Goel-Okumoto (G-O) model, etc. Some of these models, e.g., the G-O model, focus on software reliability growth, while others, e.g., Duane's model and AMSAA (Crow) model, can be applied to both software and hardware systems. According to our review of the literature on accelerated reliability growth testing, while small, there has been a slight increase in the number of research papers being published in this area over the last 24 years (see below).

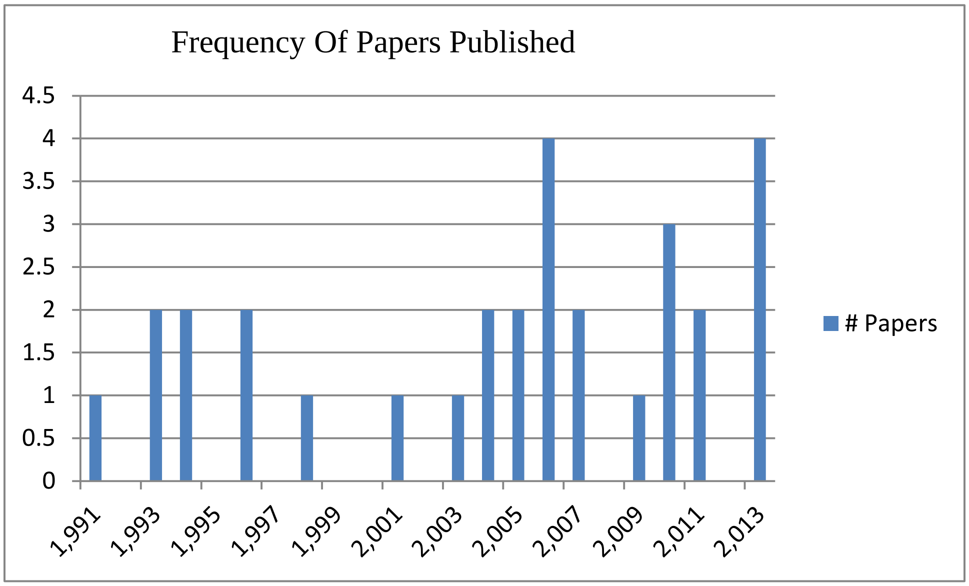

Figure 1. Frequency of Papers Published

A variety of reliability growth approaches have been proposed during the past decade. For example, a common approach is to maintain a predetermined reliability level using a test, analyze and fix process, which is known as TAAF. Most of these approaches test in the nominal use environment. In this research, accelerated testing during the reliability growth process will be studied.

ARGT is designed to detect failure modes and the time to system failures in a shorter time through the use of acceleration factors (AFs) (Hu, et al., 1993, Jayatilleka & Appliances, 2006, Ye, Jiang, et al., 2013). In the literature, Yuanquan Zhou and his research team have published several ARGT papers. Their research efforts covers the introduction of ARGT (Zhou & Zhu, 2000), the definition of acceleration factors (Zhou & Zhu, 2001a), data analysis (Zhou & Zhu, 2001b, Zhou & Zhu, 2001c, Zhou & Zhu, 2001d), step stress analysis (Zhou & Zhu, 2003a), and structure and performance analysis (Zhou & Zhu, 2003b). The acceleration factors can usually be divided into two main groups; environmental and operational factors (Krasich, 2004, Acevedo, 2006). The fundamental element of an ARGT approach is the requirement to determine the acceleration factors and the stress levels. Acceleration factors are defined as the influences that affect a product's reliability most. Stress levels are determined by the ratio of normal lifetime to accelerated lifetime. The more step stress levels are implemented, the faster failures will occur. Due to the complexity of multi steps, most of the research was limited to single step stress or two step stresses (Bai & Kim, 1993, Xiong, 1999, Watkins, 2001, Alhadeed & Yang, 2002, Hassan, 2013, Kamal, Zarrin & Islam, 2013). Further, there are two common types of step stress strategies, time step stress and failure step stress. In the literature, time step stress is more broadly used since it is easier to control.

1.2. Motivation

Our two research objectives capture our contribution to the accelerated reliability growth test literature. First, an overview will be presented on what has been done during the past 20+ years in the area of ARGT. Based on our initial examination of the literature, there has not been a systematic literature review conducted on this topic for this period. This research will fill the gap in this area. In this review, we will cover the application fields, research methods, research tendency etc., which could provide a systematic overview of this topic.

Secondly, we will come up with a new mathematical acceleration model. Most of the research considers the situation purely at the system level (Hu, et al., 1993, Feinberg, 1994) or exclusively at the component level (Crown & Feinberg, 1998, Krasich, 2004, Zanoff & Ekwaro-Osire, 2010). According to what we have reviewed, we noticed that there is little research that has explicitly modeled the accelerated tests on component level while measuring the reliability on system level. Since it costs less and is easier to conduct accelerated tests and reliability tests at the component level, we think it is easier to consider implementing accelerated test at the component level and nominal testing of the reliability at the system level.

Chapter 2. Literature review

2.1. Protocol Research

In this chapter, a systematic literature review is summarized. Though ARGT does not have a long history, there are multiple models proposed in the literature that can be used to estimate reliability in this area. In addition, there are a couple of papers that focus specifically on data analysis for these types of models.

The importance of this literature review can be explained in several aspects. First, ARGT is an important part of reliability engineering yet there is little system-level literature written in this field. This literature review will help identify the void and outline opportunities to fill it. By reviewing and examining the main models and comparing their assumptions, strengths and weaknesses, we can identify areas that need further research. Second, by analyzing the existing models and applications, it can identify the possible research opportunities for future researchers. Third, since ARGT models are presented, test designers and test executers can easily find the most appropriate model to meet their demands.

Most of the ARGT models are based on existing reliability growth models. The acceleration approaches are derived from the accelerated testing literature. It is the merging of these two areas that has created the AGRT research area. In this paper, a systematic review will be presented to show the development of the ARGT field and the remaining open areas related to this topic.

The literature review protocol used for this research was carried out in the Compendex Database, using the following protocols for key words, year, language etc. The search items used are listed below:

Criteria	Protocol Description
Search Term	Accelerated Reliability Growth Test
Database	Name: Compendex Search field: All Fields Date: 1990 to 2013 Academic papers
Exclusion Criteria	Duplicate papers Papers with incomplete information (titles, author, publisher, year, etc.) Papers written in language other than English

Table 1. Protocol Research Criteria

Using this protocol, 352 papers were identified that related to this topic. There are up to nine unique publications that have ARGT papers published in them. Among the papers found in the Compendex database, there are 93 of them from Institute of Electrical and Electronics Engineering Inc. It accounts for more than half of the total number of papers. The diversity of the research field also includes but is not limited to IEEE Computer Society, Spie, and Elesevier LTD.

However, after distinguishing the pure acceleration papers, reliability papers, or specific case study papers, there were only 29 papers which were classified as ARGT papers, including ARGT applications in software systems (Okamura, Dohi, & Osaki, 2001, Wu, Zhang & Lu, 2010, Feng, Liu & Zheng, 2011, Wang, Wu & Li, 2011, Li, Luo & Wang, 2013). The number is relatively small for the period studied. Therefore, the average publishing rate for this topic is 0.8/year, or less than one paper per year.

There are some other papers published in languages other than English. For example, Zhou Y. and his research team published 8 ARGT papers in Chinese. Their topics covered the theoretic foundation of ARGT, data analysis of ARGT, graphic analysis of ARGT etc. Since ARGT is an extension of Accelerated Test and Reliability Growth Test, it is not surprising to us that there are few fresh and unique authors who focused only on ARGT. After an authorship study, there were only 15 distinct new authors identified. The rest of them had published previously in either the Accelerated Test literature or the Reliability Growth Testing areas previously to publishing on ARGT. Among the researchers, Alec A. Feinberg contributed 4 papers and Milena Kasich contributed 3 papers respectively.

Within the literature, the use of acceleration factors can be divided into two areas, namely environmental and operational factors. Depending on step stress modes, Acceleration Testing is divided into failure step stress tests and time step stress tests. The acceleration factors in step stress tests could be divided into Iso (isogenous) or non-Isoapproaches. Depending on testing level, Reliability Growth Testing is usually conducted at the board level or system level, or on occasion both levels. Despite the fact that there could be a significant number of possible research combinations for ARGT, there is only a handful of ARGT topics that have been researched. In the next section, the existing research directions and models are examined in detail.

2.2. Existing Models

2.2.1. ISO acceleration factor

Based on MIL-KDBK-189 document, Feinberg (1994) proposed an ISO-ARGT model. The assumptions associated with this model include:

I. An effective acceleration factor, *A*, exists and can be estimated.

II. Time is linearly compressed by the factor A.

III. Equal reliability growth is possible in an uncompressed time period *t*, as in the accelerated compressed time period (given as *A* divided by t).

$$\boldsymbol{M(t) = M_I\,, t \le t_1,} \quad (1)$$

$$\boldsymbol{M^A(t) = \frac{M_I}{1-\alpha}\left(\frac{t}{t_1}\right)^{\alpha} A^{\alpha}, t > t_1.} \quad (2)$$

M_{I} is the initial MTBF, t_1 is the testing time length of stage 1, α is the growth parameter, and $A > 1$ is acceleration factor.

This is a single step acceleration model which emphasizes measuring the mean time between failures (MTBF). In the first stage, the MTBF is the initial value. In the second stage, an ISO acceleration factor is used to account for testing in an accelerated environment. Instead of considering all the acceleration factors separately, the effect of acceleration factors was integrated to a single one. In addition, a growth parameter α was considered. The original reliability growth model is

$$\boldsymbol{M^A(t) = \frac{M_I}{1-\alpha}(\frac{t}{t_1})^{\alpha},\ t > t_1}. \tag{3}$$

In practice, there would be multiple acceleration factors that affect a products' reliability. However, in this modeling approach, there is only one acceleration factor that is considered, which is *A*. Therefore, the model is more appropriate for a simple system or subsystem.

2.2.2. Environmental and operational acceleration factors approach

Krasich M. (2004) proposed an ARGT model that utilizes both operational and environmental stresses which is denoted as follows:

$$\boldsymbol{R(T) = R_u(T) \times \prod_i R_{e\_i}(T) \times \prod_k R_{o\_k}}. \tag{4}$$

It is assumed that the lifetime of a product is *T*. In this model, the reliability under environmental stress is denoted as R_{e_i}. The reliability under operational stress is denoted as R_{o_k}. The interaction effect of individual stresses is denoted as $R_u(T)$. The subscripts i and k are indices on the number of environmental acceleration factors and operation acceleration factors respectively.

The individual reliability is obtained from the equation (IEC 60300-3-1)

$$\boldsymbol{R_i} = \emptyset\left[\frac{\boldsymbol{\mu_{S\_i}}-\boldsymbol{\mu_{L\_i}}}{\sqrt{\boldsymbol{\sigma^2_{S\_i}}+\boldsymbol{\sigma^2_{L\_i}}}}\right]. \tag{5}$$

The mean demonstrated strength is μ_{S_i} while μ_{L_i} is the mean accumulated load. The corresponding variances are $\sigma^2_{S\_i}$ and $\sigma^2_{L\_i}$ respectively. According to different types of acceleration factors, the value of the acceleration factor is determined by the corresponding acceleration mode. In this model, they used thermal cycling, thermal exposure, humidity, vibration, and power cycling as possible acceleration factors. Arrhenius and power law or inverse power law models are applied in order to estimate the acceleration levels.

In the test, the stresses were applied to the system in some prescribed sequence. Although it is relatively easy to obtain the reliability under only one stress, it is often very difficult to find out the interaction index for an individual system.

Based upon this model, Krasich (2006, 2011) further discussed the data analysis and test design associated with this model. Additionally, Krasich (2014) discussed the possible errors associated with failure rate estimation due to failure modes ignorance. Specifically, she introduces extra failure modes, design defects and random failure modes, to her model. The two new introduced failure modes are assumed to have a constant failure rate.

2.2.3. ARGT through critical parts

Acevedo, Jackson and Kotlowitz (2006) proposed an ARGT model for electronics that focuses on critical parts. Only environmental stress factors were considered in this model. Three assumptions were made in their paper:

I. The product is repairable.

II. The product has multiple systems.

III. Repair interval is neglected in the MTBF prediction since the repair interval is assumed to be small relative to the MTBF.

They utilized a two-parameter Weibull distribution in their model. Therefore, the expected number of failures in a system is given by:

$$\boldsymbol{E}[\boldsymbol{N}(\boldsymbol{t})] = \int_{\mathbf{0}}^{\boldsymbol{t}} \boldsymbol{\lambda\beta v^{\beta-1} dv}. \quad (6)$$

Parameters estimation for λ and β were given by

$$\boldsymbol{\lambda} = \frac{\sum_{q=1}^{k} N_q}{k(A_F T)^{\beta}}, \quad (7)$$

$$\boldsymbol{\beta} = \frac{\sum_{q=1}^{k} N_q}{\sum_{q=1}^{k} \sum_{i=1}^{N_q} \ln(\frac{T}{X_{iq}})}. \quad (8)$$

The scale and shape parameters of the Weibull distribution are λ and β respectively. The number of failures that occur in the system is represented by N_q. The test truncation time is T and X_{iq} is the time of i^{th} failure in system q. The acceleration factor is noted as A_F.

2.2.4. Chi-Squared accelerated reliability growth testing method

Feinberg (2013) proposed a Chi-Squared accelerated reliability growth model for three test scenarios. They are a single accelerates stress test, single stress test with multiple test groups, and multiple stress test types and multiple test groups.

The corresponding failure intensity growth models are

$$\Delta\boldsymbol{\lambda}_{\boldsymbol{1Growth}}^{\boldsymbol{1}} = \boldsymbol{\lambda}_{\boldsymbol{1initial}}^{\boldsymbol{1}}\left(\boldsymbol{\mathcal{X}}^{2}\left(\boldsymbol{\gamma}, \boldsymbol{Y}_{\boldsymbol{1}}^{\boldsymbol{1}}\right), \boldsymbol{N}_{\boldsymbol{1}}^{\boldsymbol{1}}\boldsymbol{A}_{\boldsymbol{1}}^{\boldsymbol{1}}\boldsymbol{t}_{\boldsymbol{1}}^{\boldsymbol{1}}\right) - \boldsymbol{\lambda}_{\boldsymbol{1final}}^{\boldsymbol{1}}\left(\boldsymbol{\mathcal{X}}^{2}\left(\boldsymbol{\gamma}, \boldsymbol{f}_{\boldsymbol{1}}^{\boldsymbol{1}}\boldsymbol{Y}_{\boldsymbol{1}}^{\boldsymbol{1}}\right), \boldsymbol{N}_{\boldsymbol{1}}^{\boldsymbol{1}}\boldsymbol{A}_{\boldsymbol{1}}^{\boldsymbol{1}}\boldsymbol{t}_{\boldsymbol{1}}^{\boldsymbol{1}}\right) \quad (9)$$

$$\Delta\boldsymbol{\lambda}_{\boldsymbol{StressTGrowth}}^{\boldsymbol{1}} =$$

$$\boldsymbol{\lambda}_{\boldsymbol{initial}}^{\boldsymbol{1}}\left(\boldsymbol{\mathcal{X}}^{2}\left(\boldsymbol{\gamma}, \sum_{\boldsymbol{T=1}}^{\boldsymbol{n}} \boldsymbol{Y}_{\boldsymbol{T}}^{\boldsymbol{1}}\right), \sum_{\boldsymbol{T=1}}^{\boldsymbol{n}} \boldsymbol{N}_{\boldsymbol{T}}^{\boldsymbol{1}}\boldsymbol{A}_{\boldsymbol{T}}^{\boldsymbol{1}}\boldsymbol{t}_{\boldsymbol{T}}^{\boldsymbol{1}}\right) - \boldsymbol{\lambda}_{\boldsymbol{final}}^{\boldsymbol{1}}\left(\boldsymbol{\mathcal{X}}^{2}\left(\boldsymbol{\gamma}, \sum_{\boldsymbol{T=1}}^{\boldsymbol{n}} \boldsymbol{f}_{\boldsymbol{T}}^{\boldsymbol{1}}\boldsymbol{Y}_{\boldsymbol{T}}^{\boldsymbol{1}}\right), \boldsymbol{N}_{\boldsymbol{T}}^{\boldsymbol{1}}\boldsymbol{A}_{\boldsymbol{T}}^{\boldsymbol{1}}\boldsymbol{t}_{\boldsymbol{T}}^{\boldsymbol{1}}\right) \quad (10)$$

$$\Delta\boldsymbol{\lambda}_{\boldsymbol{StressGrowth}}^{\boldsymbol{All}} = \sum_{\boldsymbol{S=1}}^{\boldsymbol{K}} \boldsymbol{\lambda}_{\boldsymbol{initial}}^{\boldsymbol{S}} - \sum_{\boldsymbol{S=1}}^{\boldsymbol{K}} \boldsymbol{\lambda}_{\boldsymbol{final}}^{\boldsymbol{S}}. \quad (11)$$

Modified from Duane's growth model, Feinberg presented his growth model as

$$\boldsymbol{\lambda}_{\boldsymbol{I}} = \boldsymbol{\lambda}_{\boldsymbol{initial}}(\boldsymbol{\mathcal{X}}^{\boldsymbol{2}}(\boldsymbol{\gamma}, \boldsymbol{Y}), \boldsymbol{N}\ \boldsymbol{A}\ \boldsymbol{t}), \quad (12)$$

$$\boldsymbol{\lambda}_{\boldsymbol{F}} = \boldsymbol{\lambda}_{\boldsymbol{I}}\boldsymbol{t}^{\boldsymbol{\beta-1}}. \quad (13)$$

The superscript is stress type and the subscript is test number. The Chi-squared alpha value is γ. The number of failures for the stress test is Y. The fix effectiveness factor is f which has a range between 0 and 1. There are N units of components tested. The acceleration factor is given by A for the test and t is the test time.

The model was demonstrated using actual manufacturing data. Since the model concept is relatively simple, it is easy to understand and implement. Once the desired growth rate is fixed, the testing time can be determined from the failure intensity growth model. However, since there is only one stress considered each time, the model could be limited when multiple stresses need to be considered. In addition, the calculation process for the Chi-Square function is computationally complex, especially when trying to automate the implementation of the model.

2.2.5. Multiple environmental stresses model

Based on the AMSAA model, Ye et al. (2013) proposed an ARGT model that includes double-stress. There are two different types of stress, temperature and non-temperature, which are considered. In this model, it assumed that the two stresses are accelerated at the same time. The other assumptions were:

I. The products experience reliability growth at normal and accelerated stress levels.

II. Stress level does not change the failure distribution but change the distribution parameters.

III. Condition of product failure mechanism does not change with stresses.

IV. Can define the relationship of reliability growth distribution and stress levels before test.

V. Give time-equivalent formula under different stress levels.

Based on the Eyring Model, the model is modified to

$$\boldsymbol{\xi} = \frac{\boldsymbol{A}}{\boldsymbol{T}} \cdot \boldsymbol{exp}\left(\frac{\boldsymbol{B}}{\boldsymbol{KT}}\right) \cdot \boldsymbol{exp}\left(\boldsymbol{V}\left(\boldsymbol{C} + \frac{\boldsymbol{D}}{\boldsymbol{KT}}\right)\right). \quad (14)$$

Reaction rate is denoted as ξ. Absolute temperature in equation (14) is represented by *T, A, B, C, D,* and *V* are corresponding coefficients in this model. Taking the natural log of both sides, produces

$$\boldsymbol{ln\,\xi = a + b\varphi_1(T) + c\varphi_2(V) + d\varphi_1(T)\varphi_2(V)}, \qquad (15)$$

where, $a = lnA,\ b = \frac{B}{K},\ c = \frac{D}{K},\ \varphi_1(T) = \frac{1}{T},\ \varphi_2(V) = lnV.$

Assuming the lifetime of the product follows an exponential distribution, and then the reliability is given by:

$$\boldsymbol{R = e^{-\lambda t} = e^{-t/M_{i0}}}. \qquad (16)$$

Where M_{i0} is the normal MTBF. In this model, the value of acceleration factors was determined from the Eyring model. This model was tested for aerospace electronic products.

2.2.6. Accelerated Testing Based on the Duane Model

Wang, Zhang and Li (2013) put forward a new ARGT model based on the Duane Model. It assumes that the failures are exponentially distributed over a period of time. The cumulative number of failures is log-linear related. The traditional cumulative failure intensity for the Duane Model is given by:

$$\boldsymbol{\lambda_{sum} = lna - mlnt}. \qquad (17)$$

Thus, the number of failures over time t is

$$\boldsymbol{N(t) = at^{1-m}}. \tag{18}$$

Since $\boldsymbol{a}$ and m are undetermined variables in equation (18), the estimation of $\boldsymbol{a}$ and m increases the difficulty to use the model. In this paper, in order to simplify the objective equation, the cumulative failure number is modified to

$$\boldsymbol{N(t) = at^{b} + c}. \tag{19}$$

The modified model only has one variable *t* with three parameters *a*, *b*, and *c*. The author suggested least square method to determine the value of *a*, *b*, and *c* in the content.

Therefore, the instantaneous failure rate becomes

$$\boldsymbol{\lambda(t) = abt^{b-1}}. \tag{20}$$

There are three parameters, *a*, *b* and *c*, which are included in the model. Compared with other acceleration models, this one is relatively easy to implement. This approach is applied in the temperature acceleration case, which has demonstrated good applications. However, due to its simplicity, this model could only be applied to temperature related acceleration tests. One drawback with this model is the difficulty in obtaining valid data.

These six models presented are the main ones found in the literature and represent the current state of the art in ARGT. Although there is a large variety among reliability growth models proposed in the literature, the number of ARGT models is significantly less. There are two key elements in ARGT literature. The first one is to determine the acceleration stress. The second one is to determine the reliability growth policy.

In the literature, there are acceleration models for either environmental or operational factors. The relationship between acceleration factors and stress can vary considerably. It could be linear, Arrhenius, Power Law or inverse power law, Eyring, or even a mixed relationship (Krasich, 2004). The most typical one is the Arrhenius model since temperature related stresses are often the critical ones in accelerated testing. This model was used in all of the six models presented previously.

2.3. Potential Research Directions

Among the six models presented, (1), (2), (3), (5) and (6) are presented from the system level, considering acceleration testing and reliability estimation only at the system level. Model (4) considers acceleration testing and reliability evaluation both from component level and at the system level. In all of the models, the common assumption is that all the stresses work independently. However, this might not be true in practice. In Krasich’s model, she assumed an interaction index between stresses. This makes more sense than the other models though it is difficult to determine the value of this interaction index. Based on the previous analysis, it is suggested that an approach that focuses on component acceleration testing and system reliability evaluation.

2.4. Source for model modification

Reviewing the literature, there are a couple of approaches to accelerated reliability growth testing. According to the literature, acceleration factors can be divided into environmental and operational factors. Krasich (2004) suggested that environmental factors include but not be limited to temperature, thermal cycles, thermal dwell, humidity, and vibration levels. Operation factors include but are not limited to power stresses, voltage variations and pressure. In this kind of accelerated test, a fixed acceleration index is attached to each factor.

Krasich's model (2004) is presented below:

Thermal cycling acceleration:

$$\boldsymbol{A_{TC} = (\frac{\Delta T_{Test}}{\Delta T_{Use}})^{m}} \tag{21}$$

Thermal Dwell Acceleration:

$$\boldsymbol{A_{TD} = \exp \text{^} \left[\frac{E}{k_B} \times \left(\frac{1}{T_{Use}} - \frac{1}{T_{Test}}\right)\right]} \tag{22}$$

Humidity Acceleration:

$$\boldsymbol{A_{H} = (\frac{RH_{Test}}{RH_{Use}})^{n} \times \exp \left[\frac{E}{k_B} \times \left(\frac{1}{T_{Use_{OFF}}} - \frac{1}{T_{H_{Test}}}\right)\right]} \tag{23}$$

Vibration Acceleration:

$$\boldsymbol{A_{Vib} = (\frac{W_{Test}}{W_{Use}})^{M}} \tag{24}$$

Another alternative approach to accelerated testing is the use of step stress testing. There are two approaches to step stress tests in the literature: time step stress and failure step stress. Time step stress is used more often in the literature than the other due to its simplicity. To simplify the analysis process, the majority of papers in the literature use only two steps. Kamal Mustafa, Shazia Zarrin and Arif-UI-Islam (2013) proposed a two-step failure time testing plan. The optimal testing time in the first stage is determined from minimizing "the asymptotic variance of the MLE of the p^{th} percentile of the lifetime distribution at normal stress condition".

Additionally, Ye et al. (2013) proposed a model, which considered both acceleration factors and step stress acceleration in their paper. They considered a two stage accelerated test with multi environmental stresses in their example. According to their results, "The accelerated reliability growth program under multiple stresses, not only can accelerate product reliability growth, effectively shorten the product development cycle, but also can get the conversion relationship between the product life and stresses, which has a wide range of applications in engineering."

McLaren, A. E. (2011) outlines a multistage accelerated test model in her dissertation. She assumes that there are two kinds of failure modes in each stage. The weighted failure intensity of each stage is closely related to the current failure intensity and the previous failure intensity from previous stage. In her method, the known information has been used comprehensively. Our research approach builds off of this idea to estimate the failure intensity as well.

McLaren's model is given as

$$\boldsymbol{\lambda_1} = \sum \boldsymbol{\lambda_{1j}}, \tag{25}$$

$$\boldsymbol{\lambda_2} = \sum \boldsymbol{\lambda_{2j}} = \sum_{\boldsymbol{seen}} \boldsymbol{\lambda_{1j}}\left(\boldsymbol{1} - \boldsymbol{d_{1j}}\right) + \sum_{\boldsymbol{unseen}} \boldsymbol{\lambda_{1j}}, \tag{26}$$

where j is the failure mode, d is effectiveness factor and λ is failure intensity.

Because

$$\sum_{\boldsymbol{seen}} \boldsymbol{\lambda_{1j}} + \sum_{\boldsymbol{unseen}} \boldsymbol{\lambda_{1j}} = \boldsymbol{\lambda_1}, \tag{27}$$

equation (27) becomes

$$\boldsymbol{\lambda_2} = \boldsymbol{\lambda_1} - \sum_{\boldsymbol{seen}} \boldsymbol{\lambda_{1j}}\, \boldsymbol{d_{1j}}. \tag{28}$$

When it is assumed that $d_{1j} = \bar{d}$, which is a constant, and then substituting the observed modes with the notation ρ, the above equation becomes

$$\boldsymbol{\lambda_2} = \boldsymbol{\lambda_1} - \boldsymbol{\bar{d}\rho_1}. \tag{29}$$

The weighted average of failure intensity of stage 2 is based upon both stage 1 and stage 2. Therefore, the length of testing time is $t_1 + t_2$, the weighted failure intensity is

$$\boldsymbol{\lambda_2^M(1,2)} = \frac{\boldsymbol{t_1 * \lambda_2(1) + t_2 * \lambda_2}}{\boldsymbol{t_1 + t_2}} = \frac{\boldsymbol{t_1 * (\lambda_1 - \bar{d}_1\rho_1) + t_2 * \lambda_2}}{\boldsymbol{t_1 + t_2}} = \frac{\boldsymbol{t_1 * \lambda_1 + t_2 * \lambda_2 - t_1\bar{d}_1\rho_1}}{\boldsymbol{t_1 + t_2}}. \tag{30}$$

Following the same pattern,

$$\boldsymbol{\lambda_3^M(1,2,3)} = \frac{\boldsymbol{t_1 * \lambda_1 + t_2 * \lambda_2 + t_3 * \lambda_3 - t_1 \bar{d}_1 \rho_1 - (t_1 + t_2) \bar{d}_2 \rho_2}}{\boldsymbol{t_1 + t_2 + t_3}}, \tag{31}$$

$$\boldsymbol{\lambda_r^M(1, \dots, r)} = \frac{\boldsymbol{\sum_{i=1}^{r} t_i \lambda_i - \sum_{i=1}^{r-1} \rho_i * \bar{d}_i * \sum_{o=1}^{i} t_o}}{\boldsymbol{\sum_{i=1}^{r} t_i}}. \tag{32}$$

In her research, she developed the failure intensity model iteratively. We utilize this concept to estimate failure intensity in our model.

Despite the fact that accelerated test has been researched at the component level or system level, we noticed that there is little research conducted in which it is conducted at both component level and system level. We developed our research approach to take this into consideration in order to allow us to develop cost effective optimal test plans with multiple stages.

Chapter 3. Proposed Modeling Framework

3.1. Problem statement

When there are new products to release, reliability testing should be conducted on them before they are placed into the market. Once the products reliability satisfies the predetermined goal with a specified level of confidence, then the tests stop. Testing costs and time are usually the two constraints which determine the number of tests and test stages. Suppose there are $n \geq 2$ stages and $m \geq 2$ components in the system. Additionally, let there be a failure mode for each component in every stage. Thus, there could be $m \times n$ times to failures occurrences. When a failure is recognized, it is not fixed until the end of the stage. Once an improvement action is implanted, it is applied the whole system. Meanwhile, engineers are working on finding the root causes to avoid similar mistakes in updated products that are produced and tested in the next testing stage. The number of failure modes and the weighting factors for each acceleration factor should be determined using the expert judgment of the design engineers or based off of experience with similar products or technologies. Figure 2 is an ARGT testing flow chart:

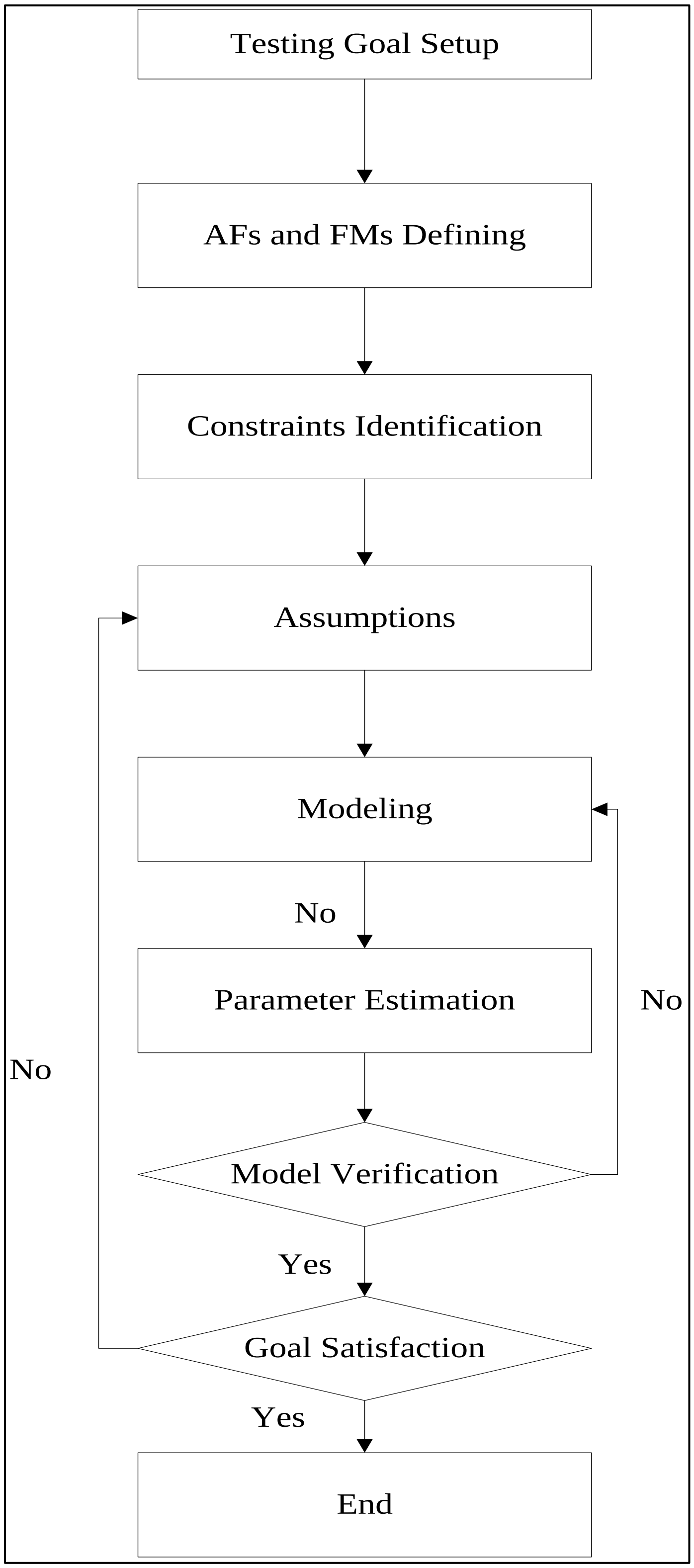

Figure 2: Modeling Process

3.2. Notation

There are a number of models proposed in the literature. Some of them are trying to maximize the reliability while the others are focusing on minimizing cost. In this paper, we emphasize on proposing an accelerated testing plan. Reliability, cost and time are considered as constraints in the model. We assume the occurrence of failures follows a Weibull distribution, which is often used to model reliability in the literature.

Currently, there is no paper in the literature that incorporate acceleration factors, failure modes, fix effectiveness factors and weighted factors all together to analyze the reliability growth process for a multi-stage test. Since the failure intensity is iterative, we obtain the failure intensity in real time, which allows timely corrective actions in the production process. We use the following notation in our model:

η_i: Weibull scale parameter at testing stage i

β_i: Weibull shape parameter at testing stage i

k: acceleration factors

m: failure modes

m': design induced failure modes

n: number of testing stages

NumSys: the number of samples in testing

C: total cost

T: product release time

c_i: fixed cost to test system level reliability at stage *i*

c_{ij}^{l}: fixed cost of per unit time for each component at stage i, mode j, with acceleration factor l

α_{ij}^{l}: acceleration factor at stage i, mode j, with acceleration factor l

f_{ij}^{l}: effectiveness factor at stage i, mode j, with acceleration factor l

N_{ij}^{l}: number of failures of failure mode j with acceleration factor l at stage i

d_i: the acceleration ratio of acceleration factor at stage *i*

γ: reliability threshold

$\lambda\_Ini_{ij}^{l}$: failure intensity in normal condition at stage i, mode j, with acceleration factor l

$\lambda\_IniD_{ij'}^{l}$: failure modes induced by redesign in normal condition at stage i, mode j', with acceleration factor l

$\lambda\_D_{ij'}^{l}$: failure modes induced by redesign in acceleration condition at stage i, mode j', with acceleration factor l

λ_{ij}^{l}: failure intensity at stage i, mode j, with acceleration factor l

λ_{1j}: failure intensity at stage 1, mode j

λ_i: failure intensity at phase *i* without acceleration

λ_i^{α}: failure intensity at phase *i* with acceleration

$E[\lambda_{ij}^{l}(t_i)]$: expected individual failure intensity in time interval t_i

I_{ij}^{l}: indicator variable at stage i, mode j, with acceleration factor l

$R_i(t)$: reliability at time t, stage *i*

t_i: testing time interval between phase *i* and *i*+1

3.3. Assumptions

To develop our model, there are 6 fundamental assumptions we make:

1) All acceleration factors work independently.

2) The failure intensity decreases after each testing stage.

3) For all censored faults and failures, a fix is implemented at the end of each phase. The fix is perfect and instantaneous.

4) A product's life distribution does not change after a fix redesign, but the parameters can change.

5) The maximum number of acceleration stages is predetermined.

6) Estimates for all possible failure modes and corresponding failure intensities are available before testing.

Although the acceleration factors may influence each other in practice, in this research plan, like the majority of the papers in the literature, the assumption has been made that all the factors works independently. Due to the independence of acceleration factors, it is assumed that failure modes are independent and identically distributed as well. To simplify the problem, a TFT (test-fix-test) approach is modeled. Specifically, repair or replacement is conducted at the end of each phase instead of right after finding a failure mode. Furthermore, it is assumed that after each fix, the product's life distribution doesn't change. To reach the testing goal, either from the failure intensity aspect or testing cost aspect, the number of acceleration stages is then determined.

3.4. Testing Policy

Before starting the test, there is a set of potential failure modes that are believed to exist in the product based on engineering analysis or from data collected from similar systems. The failures modes that have not occurred in previous data are not considered in this test planning model. Meanwhile, the failure intensity of each failure mode is evaluated according to prior statistical analysis. If the number of failures of a failure mode is greater than 1 in a testing time interval, it is assumed that the failure mode has occurred in that testing stage. Repair is implemented at the end of each testing stage. Design improvement is devised based on the information collected from the previous stage. There is an effectiveness factor associated with each failure mode which models the reduction in the likelihood of failures of the same mode in the next testing stage. The value of effectiveness factors varies from 0 to 1. The testing time of each failure mode that did not occur in current stage is accumulated until the appearance of that mode. For example, if failure mode 1 with acceleration factor 1 does not occur in the first stage, the testing time is accumulated from stage 1 until it appears or reaches the end of the testing stage. The stopping rule of each testing stage depends on the predetermined reliability threshold. Once the reliability under the accelerated conditions reaches the threshold, testing is terminated. Based on testing goal, the number of testing levels is then determined. There is also testing costs associated with each level. Per unit of time cost varies along with the value of acceleration factors at each level.

3.5. Proposed Model

The objective of this testing plan is to maximize the reliability at completion of the last phase within a specified cost budget and a specified product release time. In other words, if more failure modes could be detected during testing and corrected, there will be less potential failures in the future due to preventive actions. Different from most research papers, which set a specific MTTF (mean time to fail) as a testing standard (McLinn, 1998), we focus on evaluating the system performance by using system reliability. Since the reliability is tested at the system level, the reliability function in testing stage *i* is written as

$$\boldsymbol{R_i(t) = e^{-\left(\frac{t-\sum_{i'=1}^{i-1} t_{i'}}{\eta_i}\right)^{\beta_i}} \geq \gamma, \sum_{i'=1}^{i-1} t_{i'} < t \leq \sum_{i'=1}^{i} t_{i'}}\ , \tag{33}$$

where

$$\boldsymbol{\eta_i = h(\lambda_i^a)}, \tag{34}$$

$$\boldsymbol{\beta_i = g(\lambda_i^a)}. \tag{35}$$

The characteristic life of product is η_i. The shape parameter β_i and scale parameter η_i, are functions, denoted as *h(•)* and *g(•)*, of the failure intensity. When designing a certain product, the characteristic life is usually assumed. It is assumed that η_i is changing along with the change of failure intensity at each phase. Since the maintenance or replacement that occurs at the end of each stage is perfect, each stage could be regarded as a renewal process. Therefore, the age of the equipment is 0 at the beginning of each testing stage.

Before testing, the total testing cost is determined. At the component level, the testing cost is associated with the length of testing time in that stage. At the system level, there is a fixed cost which includes the cost of component replacement. Thus, the total cost for all tests should be within the cost budget.

$$\sum_{j=1}^{m}\sum_{l=1}^{k}\sum_{i=1}^{n} \boldsymbol{t_i c_{ij}^l} + \sum_{i=1}^{n} \boldsymbol{c_i} \leq \boldsymbol{C}\ . \tag{36}$$

In today's competitive global market, time to market is valued almost as much as cost in most aspects, especially in new product research. Thus, it is required that the total testing time be within a specified time limit in order to ensure the product release date is met.

$$\sum_{i=1}^{n} \boldsymbol{t_i} \leq \boldsymbol{T}. \tag{37}$$

When there is a single acceleration factor, it is assumed that the acceleration index at each stage is the sum of all the acceleration indices of all the factors at each stage. It is denoted as

$$\sum_{l=1}^{k} \boldsymbol{\lambda_{ij}^l} = \boldsymbol{\lambda_{ij}}. \tag{38}$$

In this test plan, multiple stages and multiple acceleration factors are considered. Following the previous pattern, the acceleration index at each stage is

$$\boldsymbol{\lambda_i^a} = \sum_{j=1}^{m}\sum_{l=1}^{k} \boldsymbol{\alpha_{ij}^l \lambda_{ij}^l (1 - f_{ij}^l\ )}. \tag{39}$$

Our proposed model is an iterative process that takes previous stages into account in order to estimate the current acceleration index. To estimate the current acceleration intensity, an effectiveness factor, which represents the likelihood that a failure model identified is eliminated in future stages, is subtracted. Since multiple stages are considered in this research, we assume $i \geq 2$. Meanwhile, the effectiveness factor is restricted to be between 0 and 1.

Because in the assumed model, the failure intensity of the next phase is dependent on the previous stages, the model could be used as a tool in reliability prediction. Since it is assumed that testing is not ended upon component failures but reliability threshold, the testing procedures is indicated in Figure 3.

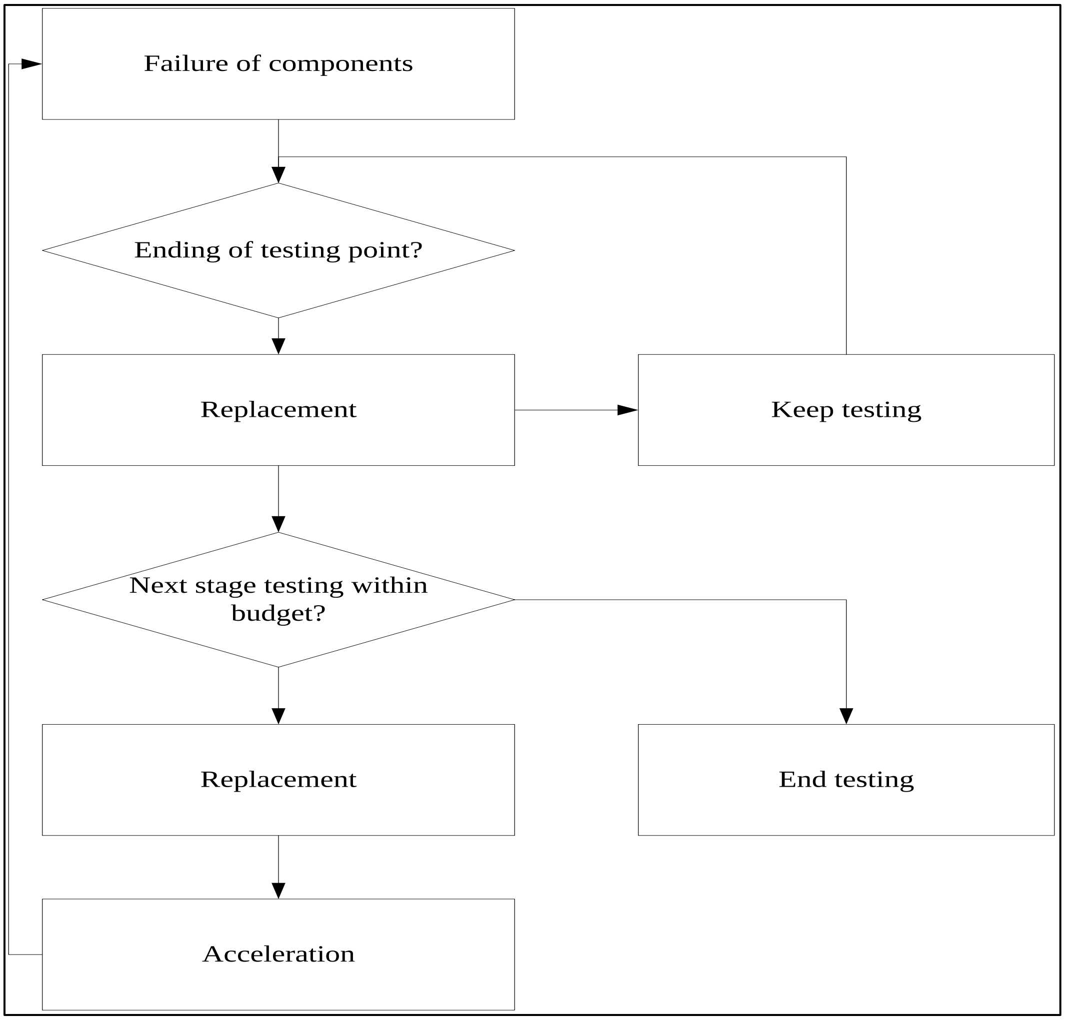

Figure 3. Testing Procedure

3.6. Implementation of Proposed Model

Accelerated life tests aim to find failure modes in a short time which helps predict lifetime reliability of products. In this modeling approach, we proposed an accelerated reliability growth test based on McLaren's (2011) model and assumed an underlying Weibull distribution. In this model, we are able to predict failure intensity and reliability based on the initial failure intensity.

Depending on the value of the shape parameter, this model could be applied for a decreasing failure rate, increasing failure rate or constant failure rate product.

When the shape parameter $\beta = 1$, it represents a constant failure rate and the reliability of the product follows an exponential distribution. It has been shown that, for complex systems, the combination of different failure modes will often exhibit a constant failure rate during its useful life period. In the literature, in order to simplify the modeling approach process, it is common to assume the failure rate is constant.

The hazard function for the Weibull distribution, for either normal condition or an accelerated condition, is

$$\boldsymbol{\lambda_t = \frac{\beta}{\eta^{\beta}} t^{\beta-1}}. \tag{40}$$

When $\beta = 1$, λ_t is a constant, the reliability function then becomes

$$\boldsymbol{R(t) = e^{-\frac{t}{\eta}} = e^{-\lambda t}}. \tag{41}$$

When shape parameter $\beta < 1$, the product has a decreasing failure rate. Products, such as electronics often fall into this group. In order to maintain high reliability after selling products, accelerated burn-in testing is a common strategy to deal with this group of products.

When shape parameter $\beta > 1$, the product has an increasing failure rate. Products that physically wear out are included in this group.

In the cases when $\beta \neq 1$, according to probability theory,

$$\boldsymbol{R}(\boldsymbol{t}) = \boldsymbol{e}^{-\int_{0}^{t} \lambda_u du}. \tag{42}$$

It becomes more difficult to estimate the Weibull scale and shape parameters when the hazard function is unknown as well. Future research is needed in this area for this modeling framework.

Chapter 4. Multiple Cases Consideration

Depending on the value of the Weibull shape parameters, there are two different modeling approaches presented, one assumes $\beta=1$ and the other assumes $\beta\neq1$. First, the exponential distribution model is presented. Then the more general case of the Weibull distribution with parameter estimation is proposed.

4.1. Exponential distribution

In this testing plan, it is assumed that acceleration starts in the first stage. The estimation of the failure intensity is an iterative process which incorporates information from current and previous stages. Suppose the initial failure intensity is $\lambda\_Ini_{1j}^{l}$, which is known before testing, for each mode without acceleration, therefore, the failure intensity under accelerated conditions in the first stage is given by:

$$\boldsymbol{\lambda_{1j}^{l} = \lambda\_Ini_{1j}^{l}\alpha_{1j}^{l}}. \qquad (43)$$

Since it is assumed that the times-to-failure are exponentially distributed during testing at each level, then the number of failures that occur in time interval t_1 is

$$\boldsymbol{N_{1j}^{l} = \lambda_{1j}^{l}t_1 NumSys}. \qquad (44)$$

In practice, technicians know which components to fix and redesign only when failures occur. An indicator variable is assigned to each failure mode.

$$I_{ij}^{l} = \begin{cases} 1 & N_{ij}^{l} \geq 1 \\ 0 & otherwise \end{cases}, \qquad (45)$$

is used to indicate which modes receive a fix.

Therefore the accelerated failure intensity in the first stage is

$$\lambda_1^a = \sum_{j=1}^{m} \sum_{l=1}^{k} \lambda_{1j}^{l} I_{1j}^{l}. \qquad (46)$$

Once a failure mode occurs, which has $N_{1j}^{l} \geq 1$, redesign is conducted on a system to improve the performance of the system. If failure modes do not occur in the current stage, then no effective action is implemented to improve those modes. Depending on if there is corrective action in first stage, normal condition failure intensity in the second stage is given by:

$$\lambda\_Ini_{2j}^{l} = \begin{cases} \lambda\_Ini_{1j}^{l}(1 - f_{1j}^{l}) & N_{1j}^{l} \geq 1 \\ \lambda\_Ini_{1j}^{l} & otherwise \end{cases}. \qquad (47)$$

Therefore, the accelerated condition failure intensity in the second stage should be

$$\lambda_{2j}^{l} = \lambda\_Ini_{2j}^{l} \alpha_{2j}^{l}. \qquad (48)$$

Then the number of failures in the second stage, when a failure mode occurs in first stage, is given by:

$$N_{2j}^{l} = \lambda_{2j}^{l} t_2 NumSys. \qquad (49)$$

If not, the number of failures should be

$$N_{2j}^{l} = \lambda_{1j}^{l} t_1 NumSys + \lambda_{2j}^{l} t_2 NumSys. \qquad (50)$$

Since only the failure modes that occur during testing are considered in determining testing time, the accumulated failure intensity should be

$$\boldsymbol{\lambda_2^a = \sum_{j=1}^{m}\sum_{l=1}^{k}\lambda_{2j}^l\, I_{2j}^l}. \tag{51}$$

According to the indicator matrix generated in stage 2, the individual normal condition failure intensity in stage 3 is

$$\boldsymbol{\lambda\_Ini_{3j}^l = \begin{cases}\lambda\_Ini_{2j}^l(1-f_{2j}^l) & N_{2j}^l \geq 1\\ \lambda\_Ini_{2j}^l & otherwise\end{cases}}. \tag{52}$$

The corresponding accelerated failure intensity in the third stage is given by:

$$\boldsymbol{\lambda_{3j}^l = \lambda\_Ini_{3j}^l\alpha_{3j}^l}. \tag{53}$$

Therefore, the corresponding number of failures should be

$$\boldsymbol{N_{3j}^l = \lambda_{3j}^l \mathrm{t}_3 NumSys + \cdots + \lambda_{i'j}^l \mathrm{t}_{i'} NumSys}, \tag{54}$$

where i' is the testing stage where failure mode j with acceleration factor l occurred after last testing stage.

Based on if failure modes occur or not, the cumulative failure intensity in stage 3 is given by:

$$\boldsymbol{\lambda_3^a = \sum_{j=1}^{m}\sum_{l=1}^{k}\lambda_{3j}^l\, I_{3j}^l}. \tag{55}$$

Following this same approach, when $n \geq 2$, the individual failure intensity under normal condition at stage n is

$$\boldsymbol{\lambda\_Ini_{nj}^{l}} = \begin{cases} \boldsymbol{\lambda\_Ini_{(n-1)j}^{l}(1 - f_{(n-)j}^{l})} & \boldsymbol{N_{(n-1)j}^{l} \geq 1} \\ \boldsymbol{\lambda\_Ini_{(n-1)j}^{l}} & \boldsymbol{otherwise} \end{cases}. \quad (56)$$

Then the individual failure intensity under normal conditions should be:

$$\boldsymbol{\lambda_{nj}^{l} = \lambda\_Ini_{nj}^{l}\alpha_{nj}^{l}}. \quad (57)$$

The number of failures for each failure mode is:

$$\boldsymbol{N_{nj}^{l} = \lambda_{nj}^{l}\mathbf{t_n}NumSys + \lambda_{(n-1)j}^{l}\mathbf{t_{(n-1)}}NumSys + \cdots + \lambda_{i'j}^{l}\mathbf{t}_{i'}NumSys}. \quad (58)$$

Where i' is derived as in equation (54).

If we sum over all the failure intensities in stage *n*, then the cumulative failure intensity should be:

$$\boldsymbol{\lambda_n^a = \sum_{j=1}^{m}\sum_{l=1}^{k}\lambda_{nj}^{l}\,I_{nj}^{l}}. \quad (59)$$

It is not difficult to prove that when accelerating for more time, the final failure intensity is lower. In other words, products have better performance when they go through accelerated testing and specific failure modes are mitigated. In this model, which simulates the behavior of the product in normal and accelerated environments, we assume products are tested at 5 different levels. The red curves in Figure 4 represent the reliability under normal conditions after each fix and redesign. The blue curves characterize reliability in the accelerated condition of the product during that specific stage. Red curves are normal condition reliability and blue curves represent acceleration condition reliability for all following plots. When the reliability in the testing stage reaches a threshold value of (0.6), testing in the current stage is terminated. Due to the effect of product redesign, the reliability under normal conditions becomes flatter. Therefore, the reliability of product has been improved. Correspondingly, the testing time under accelerated conditions has been extended because of the increasing of the acceleration factors

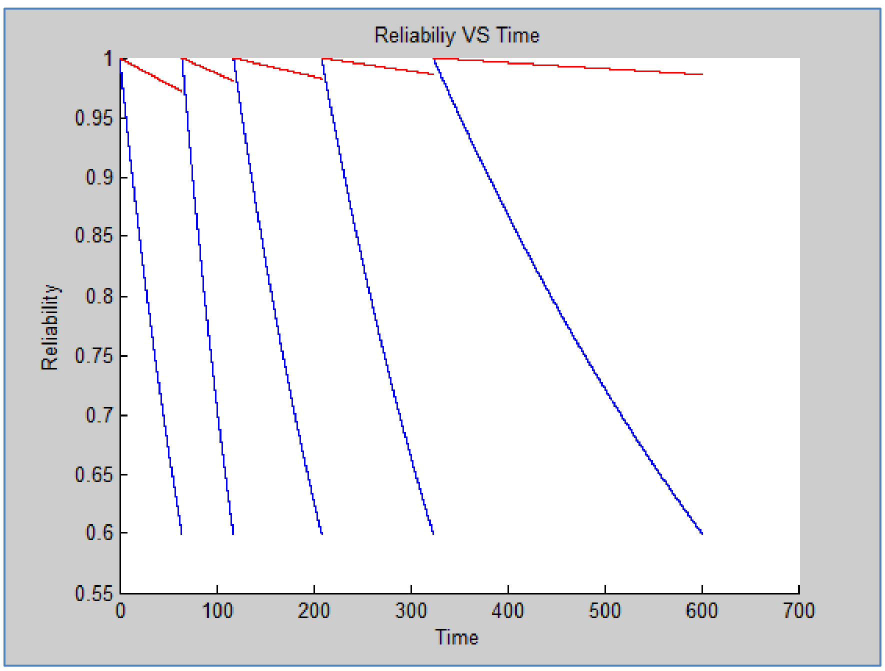

Figure 4. Reliability Tendency Plot

4.2. Weibull Distribution

In the previous two cases, we assumed that the Weibull shape parameter, $\beta = 1$, under this condition, the reliability function is exponentially distributed with a constant hazard function at each stress level. However, when $\beta \neq 1$, the hazard function is changes with time. Assume the reliability function follows a Weibull distribution over all the stages. In each stage, the shape parameter is fixed while the scale parameter varies over time. Therefore, in the first stage, the reliability is

$$\boldsymbol{R(t) = e^{-(\frac{t}{\eta_1})^{\beta}}}. \quad (60)$$

The reliability function in the second stage is given by:

$$\boldsymbol{R(t) = e^{-(\frac{t-t_1}{\eta_2})^{\beta}}}. \quad (61)$$

Unlike the exponential distribution, where the hazard function is a constant, the Weibull distribution has hazard function that varies with time, which is given as:

$$\boldsymbol{\lambda(t) = \frac{\beta}{\eta^{\beta}} t^{\beta-1}}. \quad (62)$$

Assume we have some understanding of the distribution of failure modes before testing, denoted as $\lambda\_Ini_{1j}^{l}(t)$, the corresponding failure intensity distribution under accelerated conditions should be

$$\boldsymbol{\lambda_{1j}^{l}(t) = \lambda\_Ini_{1j}^{l}(t) * \alpha_{1j}^{l}}. \quad (63)$$

Therefore, the expected failure intensity for a specific time interval is:

$$\boldsymbol{E[\lambda_{1j}^{l}(t_1)] = \int_0^{t_1} \lambda_{Ini}{}_{1j}^{l}(t) * \alpha_{1j}^{l} * t dt / t_1}. \quad (64)$$

Follow the same approach used earlier for the Exponential distribution, if a failure mode occurs, it is included in estimating the testing time. Also, fix or redesign is implemented to improve performance of products. If not, no maintenance or fix performs. The number of failures that occur for each failure mode is denoted as N_{ij}^{l}. Therefore,

$$\boldsymbol{N_{1j}^{l} = E\left[\lambda_{1j}^{l}(t_1)\right] * t_1}. \tag{65}$$

As before an indicator

$$\boldsymbol{I_{ij}^{l} = \begin{cases} 1 & N_{ij}^{l} \geq 1 \\ 0 & otherwise \end{cases}}, \tag{66}$$

is associated with each failure mode, which has. Therefore, the failure intensity in level one should be

$$\boldsymbol{\lambda_1^{a}(t) = \sum_{j=1}^{m} \sum_{l=1}^{k} \lambda_{1j}^{l}(t)\, I_{1j}^{l}}. \tag{67}$$

Combing (62) and (67) yields the expression

$$\boldsymbol{\frac{\beta}{\eta_1{}^{\beta}} t^{\beta-1} = \sum_{j=1}^{m} \sum_{l=1}^{k} \lambda_{1j}^{l}(t)\, I_{1j}^{l}}. \tag{68}$$

Integrating both sides with small $t, t \leq t_1$, then we can estimate the value of η_1 and β. Since it is assumed that β does not change over time, only η_2 should update to η_2 in the second stage.

In the second stage, which depends on the value of the indicator function, the normal condition failure intensity distribution is similar to equation (47). Once a failure mode occurs, then $N_{1j}^{l} \geq 1$, when the reliability reaches the threshold, perfect replacement is carried out on the associated component. If failure modes do not occur in the current stage, then no effective action is implemented to improve those modes. Depending on if there is corrective action in first stage, normal condition failure intensity in the second stage is given by:

$$\boldsymbol{\lambda\_Ini_{2j}^{l}}(\mathbf{t}) = \begin{cases} \boldsymbol{\lambda\_Ini_{1j}^{l}(t)(1-f_{1j}^{l})} & \boldsymbol{N_{1j}^{l} \geq 1} \\ \boldsymbol{\lambda\_Ini_{1j}^{l}(t)} & \boldsymbol{otherwise} \end{cases}. \tag{69}$$

Therefore, the accelerated condition failure intensity is:

$$\boldsymbol{\lambda_2^a(t) = \lambda\_Ini_{2j}^{l}(t) * \alpha_{1j}^{l}}. \tag{70}$$

To calculate the number of failures in the second stage, we need to find the expected value of the failure intensity function. Then, we have

$$\boldsymbol{E[\lambda_{2j}^{l}(t_2)] = \int_0^{t_2} \lambda_{Ini_{2j}}^{\ \ l}(t) * \alpha_{2j}^{l} * tdt\,/t_2}. \tag{71}$$

Thus, the number of failures of each failure mode is

$$\boldsymbol{N_{2j}^{l} = E\left[\lambda_{2j}^{l}(t_2)\right] * t_2}. \tag{72}$$

Depending on the number of failures which occur during testing, the indicators are

$$\boldsymbol{I_{2j}^{l}} = \begin{cases} \mathbf{1} & \boldsymbol{N_{2j}^{l} \geq 1} \\ \mathbf{0} & \boldsymbol{otherwise} \end{cases}. \tag{73}$$

Similarly, the failure intensity in stage 2 is expressed as

$$\boldsymbol{\lambda_2^a(t) = \sum_{j=1}^{m} \sum_{l=1}^{k} \lambda_{2j}^{l}\,(t)\, I_{2j}^{l}}. \tag{74}$$

Also, combining equation (62) and equation (74), then we have

$$\frac{\boldsymbol{\beta}}{\boldsymbol{\eta_2}^{\boldsymbol{\beta}}}\boldsymbol{t}^{\boldsymbol{\beta}-\mathbf{1}} = \sum_{j=1}^{m}\sum_{l=1}^{k}\boldsymbol{\lambda}_{\mathbf{2}j}^{l}\,(\boldsymbol{t})\,\boldsymbol{I}_{\mathbf{2}j}^{l}, \tag{75}$$

which could be used to update the scale parameter in stage 2.

Following this same pattern until stage n, then we can update η_n as

$$\frac{\boldsymbol{\beta}}{\boldsymbol{\eta_n}^{\boldsymbol{\beta}}}\boldsymbol{t}^{\boldsymbol{\beta}-\mathbf{1}} = \sum_{j=1}^{m}\sum_{l=1}^{k}\boldsymbol{\lambda}_{nj}^{l}\,(\boldsymbol{t})\,\boldsymbol{I}_{nj}^{l}. \tag{76}$$

Then the corresponding reliability function is

$$\boldsymbol{R}(\boldsymbol{t}) = \boldsymbol{e}^{-\left(\frac{t-\sum_{i'=1}^{i-1} t_{i'}}{\eta_n}\right)^{\beta}}. \tag{77}$$

Chapter 5. Modeling Analysis and Sensitivity Analysis

In this chapter, we simulate the behavior of our ARGT model. We use our modeling framework to identify the most significant failure modes with acceleration factors, and then assess the effect of the various model parameters on the system reliability behavior by using sensitivity. The parameters investigated include: acceleration factors, effectiveness factors, potential failure modes, and initial failure intensity.

5.1. Failure intensity analysis

In Table 2, sensitivity analysis associated with failure intensity is presented. The analysis is based on decreasing the baseline failure intensity of each failure mode by 50%. For example, if the baseline failure intensity in the first stage was decreased by 50% for failure mode 1, the overall failure intensity under normal condition (λ_Ratio) decreases by -0.94%. The difference of failure intensity under acceleration and normal condition are denoted as $\Delta\lambda(A)$ and $\Delta\lambda(N)$ respectively. Corresponding to this test, the testing time has increased by approximately 47 units of testing time and cost has increased by 500. Also, it is seen that when the ratio of failure intensity is positive, which means the final failure intensity increases, the cumulative time and cost are decreased. Therefore, it is not difficult to notice that decreasing the failure intensity results in increasing the accumulative testing time (ΔAccT) and testing cost (ΔAccC). Correspondingly, testing time ratio (T_Ratio) and testing financial cost ratio (C_Ratio) are increased.When decreasing failure modes 6 and 7, the final failure intensity has decreased the most. Comparing the results from failure modes 6 (FM6) and 2 (FM2), when decreasing failure mode 2 by 50%, it costs more time and money to complete 5 acceleration stages then decreasing failure mode 6. Also, the final failure intensity is higher than the case when decreasing failure mode 6. Then in this case, decreasing failure mode 6 is profitable then decreasing failure mode 2.

	ΔAccT	ΔAccC	T_Ratio	C_Ratio	Δλ(A)	Δλ(N)	λ_Ratio
FM1	38.40	410	6.83%	5.08%	-1.68E-04	-1.21E-06	-2.35%
FM2	50.71	558	9.02%	6.91%	-2.64E-04	-1.27E-06	-2.47%
FM3	22.50	224	4.00%	2.77%	-1.70E-05	-1.68E-06	-3.27%
FM4	32.82	352	5.84%	4.36%	-1.49E-04	-1.58E-06	-3.06%
FM5	32.23	329	5.73%	4.07%	-1.09E-04	-1.18E-06	-2.29%
FM6	30.04	312	5.34%	3.86%	-1.48E-04	-3.11E-06	-6.03%
FM7	43.68	485	7.77%	6.00%	-2.66E-04	-3.42E-06	-6.64%
FM8	27.54	267	4.90%	3.30%	-6.24E-05	-2.35E-07	-0.46%

Table 2. Decrease Failure Intensity by 50%

Similar results are obtained from Table 3, which shows the impact of increasing of each mode's intensity by 50%. When increasing the failure intensity, the testing time and testing cost are decreased greatly if the failure modes are significant in the testing period. Examining the ratios, failure modes 4 and 6 affect the failure intensity most. Failure mode 8 decreases the testing time as much as failure mode 6, while the failure intensity is not increased as much as failure mode 6.

	ΔAccT	ΔAccC	T_Ratio	C_Ratio	Δλ(A)	Δλ(N)	λ_Ratio
FM1	-27.13	-269	-4.82%	-3.33%	-1.83E-05	1.90E-06	3.69%
FM2	-32.13	-331	-5.71%	-4.10%	5.24E-07	3.23E-07	0.63%
FM3	-29.68	-322	-5.28%	-3.98%	4.49E-05	2.99E-06	5.80%
FM4	-42.12	-478	-7.49%	-5.92%	2.01E-04	4.86E-06	9.43%

Table 3. Increase Failure Intensity by 50%

	ΔAccT	ΔAccC	T_Ratio	C_Ratio	Δλ(A)	Δλ(N)	λ_Ratio
FM5	-42.51	-467	-7.56%	-5.78%	1.49E-04	2.98E-06	5.78%
FM6	-48.81	-568	-8.68%	-7.03%	3.39E-04	5.25E-06	10.19%
FM7	-38.68	-440	-6.88%	-5.45%	1.81E-04	3.72E-06	7.22%
FM8	-45.67	-516	-8.12%	-6.39%	2.09E-04	3.11E-06	6.03%

Table 4. Increase Failure Intensity by 50% (Cont.)

5.2. Acceleration factors analysis

In Table 4 and Table 5, sensitivity analysis of changes in the acceleration factors (AF) is presented. The value of acceleration factors 1(AF1), 2 (AF2), 3 (AF3) and 4 (AF4) are derived from inverse power law model but with different index, which will be introduced in Chapter 7. Aacceleration factors 5 (AF5) and 6 (AF6) are derived from Arrhenius relationship, as shown in equation (89). Acceleration factors 7 (AF7) and 8 (AF8) are derived from equation (91). Specific introduction of the three-parameter estimation equation will be introduced in Chapter 7. When decreasing acceleration factor 1, which is the AF associated with failure mode 1 and is modeled by an acceleration model, one finds it affects the final failure intensity the most. This is due to the reduction of testing time that is achieved due to this high acceleration rate. When increasing acceleration factors by 50%, acceleration factor 2 (AF2) affects the failure intensity most. As we can see, though failure intensity and testing time decrease in this case, the testing cost increases. Since it is assumed that testing cost of component level is associated with acceleration factors, increasing acceleration factors results in an increase of testing cost. The cost is even higher than the cost saving from the shorter testing time. Depending on financial and product release time constraints, a testing strategy could be determined based on the following two tables.

	ΔAccT	ΔAccC	T_Ratio	C_Ratio	Δλ(A)	Δλ(N)	λ_Ratio
AF1	24.55	-287	4.36%	-3.55%	2.40E-05	2.36E-05	46.27%
AF2	42.25	-224	7.51%	-2.77%	-2.74E-04	-4.15E-06	-8.15%
AF3	28.53	-525	5.07%	-6.50%	-7.54E-05	9.47E-06	18.59%
AF4	41.42	-526	7.37%	-6.51%	-2.48E-04	2.41E-07	0.47%
AF5	37.64	316	6.69%	3.91%	-4.38E-05	-2.63E-06	-5.16%
AF6	40.41	373	7.19%	4.62%	-1.57E-04	2.66E-06	5.22%
AF7	32.22	307	5.73%	3.80%	-1.79E-04	2.69E-07	0.53%
AF8	26.89	197	4.78%	2.44%	-2.12E-05	4.58E-07	0.90%

Table 5. Decrease Acceleration Factors by 50%

	ΔAccT	ΔAccC	T_Ratio	C_Ratio	Δλ(A)	Δλ(N)	λ_Ratio
AF1	-42.09	34	-7.48%	0.42%	2.11E-04	1.97E-06	3.83%
AF2	-40.40	134	-7.18%	1.66%	2.75E-04	-5.08E-07	-0.99%
AF3	-22.53	524	-4.01%	6.49%	2.76E-05	-1.64E-06	-3.18%
AF4	-46.12	313	-8.20%	3.88%	2.07E-04	-2.38E-07	-0.46%
AF5	-50.17	-509	-8.92%	-6.30%	1.39E-04	-9.36E-07	-1.82%
AF6	-41.49	-410	-7.38%	-5.08%	1.11E-04	-1.17E-06	-2.27%
AF7	-31.58	-305	-5.61%	-3.78%	5.98E-05	-5.90E-08	-0.11%
AF8	-34.15	-309	-6.07%	-3.83%	-2.12E-05	-1.23E-06	-2.38%

Table 6. Increase Acceleration Factors by 50%

Also we could see in Table 4 that when decreasing the acceleration factor 2 and 5, the last stage failure intensity is decreasing. This is because the due to the extension of testing time, more failure modes could occur and associated failure modes are well fixed. Therefore, there is lower failure intensity after that stage.

5.3. Effectiveness factors analysis

Table 7 and Table 8 examine the sensitivity of the model to changes in the fix effectiveness factor (EF). It is not difficult to see that the EFs affect the failure intensity the most when compared to all of the other model parameters. The failure intensity increased as much as 74% when effectiveness factor of failure mode 1(FM1) was decreased by 50%. Correspondingly, testing time and cost are reduced significantly. Similarly, increasing the effectiveness factor of failure mode 2 by 50%, decreases the failure intensity by as much as 22%. Therefore, improving the value of effectiveness factor should be explored as part of the reliability growth process as it has the most significant impact on cost, time and the final system reliability of the products that will be delivered to the field. In practice, this requires that the testers correctly find the root causes of the failures and make design changes that eliminate or mitigate the occurrence of the failure mode.

	ΔAccT	ΔAccC	T_Ratio	C_Ratio	Δλ(A)	Δλ(N)	λ_Ratio
FM1	-147.92	-1851	-24.62%	-21.61%	0.0016	3.80E-05	74.67%
FM2	-138.10	-1711	-22.98%	-19.98%	0.0013	2.46E-05	48.37%
FM3	-126.89	-1578	-21.12%	-18.43%	0.0012	2.45E-05	48.05%
FM4	-139.82	-1719	-23.27%	-20.07%	0.0012	1.12E-05	21.98%
FM5	-92.30	-1103	-15.36%	-12.88%	0.0005	5.31E-06	10.43%
FM6	-100.66	-1209	-16.75%	-14.12%	0.0005	4.30E-06	8.44%
FM7	-52.28	-629	-8.70%	-7.34%	0.0002	-9.26E-08	-0.18%
FM8	-95.00	-1171	-15.81%	-13.68%	0.0006	1.47E-08	0.03%

Table 7. Decrease Effectiveness Factors by 50%

	ΔAccT	ΔAccC	T_Ratio	C_Ratio	Δλ(A)	Δλ(N)	λ_Ratio
FM1	28.19	310	4.69%	3.62%	0.0001	-1.09E-05	-21.41%
FM2	66.64	814	11.09%	9.50%	-0.0003	-1.13E-05	-22.13%
FM3	22.27	244	3.71%	2.85%	0.0000	-3.85E-06	-7.56%
FM4	27.49	299	4.58%	3.49%	0.0000	-2.95E-06	-5.79%
FM5	97.78	1179	16.27%	13.77%	-0.0004	-3.35E-06	-6.58%
FM6	75.59	909	12.58%	10.62%	-0.0003	-2.12E-06	-4.17%
FM7	41.56	506	6.92%	5.91%	-0.0002	-1.37E-07	-0.27%
FM8	38.09	444	6.34%	5.18%	-0.0001	-1.06E-07	-0.21%

Table 8. Decrease Effectiveness Factors by 50%

Table 8 illustrates the effect of the effectiveness function on the modeling framework when EF equals to 0 (EF-0), 0.5 (EF-0.5), 0.9 (EF-0.9), and 1(EF-1). It is clear that the larger EF the lower the corresponding failure intensity is in subsequent stages. However, the testing time and cost are much higher to maintain a high level of EF. Therefore, we should choose appropriate EFs according to some specified testing budget in practice.

	AccT	AccC	T_Ratio	C_Ratio	λ(A)	λ(N)
Baseline	600.871	8562	0.0018	5.09E-05	0.0018	5.09E-05
EF-0	149.866	3244	0.0294	4.45E-04	0.0294	4.45E-04
EF-0.5	271.061	5617	0.0057	1.09E-04	0.0057	1.09E-04
EF-0.9	1409.336	17891	0.0008	1.77E-05	0.0008	1.77E-05
EF-1	9614.779	120444	0.0001	4.52E-07	0.0001	4.52E-07

Table 9. Effectiveness Factor Analysis

5.4. Failure modes elimination analysis

Table 10 illustrates the sensitivity analysis associated with eliminating failure modes due to increased levels of accelerated testing. If failures associated with a specific AF are eliminated, the most significant acceleration factor produces the largest decrease of failure intensity. When eliminating failure modes associated with AF2, the subsequent failure intensity decreases the most while the testing cost increases. Similarly, when eliminating failure modes associated with AF1, the failure intensity decreases up by 28% while testing cost increases by less than half of the increase for AF2. There are two interesting cases that warrant further discussion. For AF7 and AF8, in this experiment, no failures occur during testing and the resulting failure intensities go up after the testing. This is because the increased accelerated of testing time increases the likelihood of failure modes in subsequent stages of testing. Based on this information, careful selection of an improvement plan should be made to reach the reliability growth goal.

	ΔAccT	ΔAccC	T_Ratio	C_Ratio	Δλ(A)	Δλ(N)	λ_Ratio
AF1	65.53	722	10.91%	8.43%	-0.0001	-1.44E-05	-28.22%
AF2	136.21	1581	22.67%	18.47%	-0.0004	-1.72E-05	-33.67%
AF3	57.05	630	9.49%	7.36%	-0.0001	-5.03E-06	-9.88%
AF4	75.25	786	12.52%	9.18%	-0.0001	-5.04E-06	-9.89%
AF5	214.01	2391	35.62%	27.93%	-0.0006	-7.17E-06	-14.07%
AF6	135.63	1488	22.57%	17.38%	-0.0004	-2.90E-06	-5.70%
AF7	82.75	935	13.77%	10.92%	-0.0003	5.42E-07	1.06%
AF8	68.88	705	11.46%	8.24%	-0.0001	5.66E-07	1.11%

Table 10. Failure Modes Elimination Analysis

5.5. Cost analysis

Testing cost is one of the main constraints in our model. Accessing the cost sensitivity of each failure mode is meaningful when a decision is needed to improve the economics associated with the test plan. In our testing model, the testing cost is influenced by the level of acceleration as well as total testing time. Since the failure intensity of each failure mode doesn’t change, the testing time remains the same in each testing level. Therefore, testing cost is only associated with acceleration factors at this point. Table 11 presents the cost analysis associated with each acceleration factor.

	AF1	AF2	AF3	AF4	AF5	AF6	AF7	AF8
ΔAccC	-552	-709	-858	-980	-34	-39	-53	-57
C_Ratio	-6.44%	-8.28%	-10.02%	-11.44%	-0.40%	-0.46%	-0.62%	-0.67%

Table 11. Cost Analysis

In Table 10, we can see that the cost of AF4 affects testing cost the most while AF 5, 6, 7, and 8 has very little influence. Therefore, in real testing, we should try to minimize the effect of acceleration factors which are most sensitive to tests. Instead of saving cost, testing cost goes up by the same amount when increasing testing cost of each acceleration factor by 50%.

Chapter 6. Design Induced Failure Modes Model

6.1. Modified model

In the previous proposed model, it is assumed that all possible failure modes could be predicted before testing. Therefore, no new failure modes are induced by redesigning the product after failures have occurred. However, in reality, design/maintenance induced failure modes are very common. Building on the model that was discussed in Chapter 4, design induced failure modes were incorporated into the model presented below.

The first stage of the revised model is the same since there is no design modification. Therefore, the testing time at the first level is determined based on the failure modes that occurred. After testing phase 1, design improvement is made according to the failures that appeared in stage 1. Different from the previous model, design induced failure modes are introduced after each redesign change at stage i. The failure intensity of a redesign induced failure mode is $\lambda\_D_{ij'}^{l}$. Since most of the time designers can predict new failure modes based upon design changes, effectiveness actions are taken before starting a new testing level. Therefore, the failure intensity of design induced failure modes are:

$$\boldsymbol{\lambda_{ij'}^{l} = \lambda\_D_{ij'}^{l}(1 - f_{(i-1)j'}^{l})\alpha_{ij'}^{l}}. \tag{78}$$

The corresponding number of failures is given by:

$$\boldsymbol{N_{ij}^{l} = \lambda_{ij}^{l}\mathrm{t}_{i}NumSys + \cdots + \lambda\_D_{i'j}^{l}\mathrm{t}_{i'}NumSys}. \tag{79}$$

Where i' has the same definition as equation (54).

In stage 2, the failure times of the initial failure modes, which occurred in stage 1, is

$$\boldsymbol{N_{2j}^{l} = \lambda_{2j}^{l} t_2 NumSys}. \qquad (80)$$

If there is a new failure mode which has not been censored in stage 2, the number of failures will be given by:

$$\boldsymbol{N_{2j}^{l} = \lambda_{1j}^{l} \mathrm{t}_1 NumSys + \lambda_{2j}^{l} \mathrm{t}_2 NumSys}. \qquad (81)$$

Since we know the kinds of potential failure modes that will occur due to redesign, preventive fixing measures are implemented to decrease the probability of potential failure modes appearance. Since improvement of potential failure modes induced after each redesign is not depends on the occurrence of failure mode, we do not have to calculate the number of failures. Therefore, based on indicator variable, the failure intensity of stage 2 is

$$\boldsymbol{\lambda_2^a = \sum_{j=1}^{m} \sum_{l=1}^{k} \lambda_{2j}^{l} I_{2j}^{l} + \sum_{j'=1}^{m'} \sum_{l=1}^{k} \lambda\_D_{2j'}^{l}}. \qquad (82)$$

Similar to stage 2, when we move to testing level n, the failure intensity in normal conditions is given by equation (56). The design induced failure intensity at stage *n* is

$$\boldsymbol{\lambda\_D_{nj'}^{l} = \lambda\_D_{(n-1)j'}^{l} (1 - f_{(n-1)j'}^{l}) \alpha_{nj'}^{l}}. \qquad (83)$$

The number of failures of each individual failure mode initially included is

$$N_{nj}^{l} = \lambda_{nj}^{l} \mathrm{t_n} NumSys + \lambda_{(n-1)j}^{l} \mathrm{t_{(n-1)}} NumSys + \cdots + \lambda_{i'j}^{l} \mathrm{t}_{i'} NumSys. \quad (84)$$

The associated indicator variables are

$$I_{nj}^{l} = \begin{cases} 1 & N_{nj}^{l} \geq 1 \\ 0 & otherwise \end{cases}. \quad (85)$$

Therefore the failure intensity at stage *n* is given by

$$\lambda_{n}^{a} = \sum_{j=1}^{m} \sum_{l=1}^{k} \lambda_{nj}^{l} I_{nj}^{l} + \sum_{j=1}^{m'} \sum_{l=1}^{k} \lambda_{nj'}^{l}. \quad (86)$$

Similar to the original model, testing time is determined by the specified reliability threshold. Once the system reliability goes down to the threshold, the current testing stops. Based on the failure information, a corresponding fix and design are implemented to improve system performance.

6.2. Modified model results

Using this model, we simulated the test performance when five acceleration levels are and four common failure modes are possible during tests. After each redesign, a new failure mode is possibly introduced. The resulting indicator matrix for the first stage is shown in

FM1	0	1	0	1	1	1	0	1
FM2	0	0	1	1	0	1	1	1
FM3	1	0	0	0	1	1	0	1
FM4	0	1	1	0	1	0	1	0

Table 12. Stage 1 Indicator Matrix

In the first stage, the only failure modes that can occur are four failure modes assumed to exist before testing. After stage one, the design is improved and the resulting, indicator matrix becomes

FM1	1	1	1	1	1	0	0	0
FM2	1	1	0	1	1	1	1	1
FM3	0	1	1	1	0	0	1	0
FM4	1	1	0	1	0	1	1	1
FM5	0	0	0	0	0	0	0	0

Table 13. Stage 2 Indicator Matrix

Comparing Table 11 and Table 12, we see that more failure modes occur due to the increase in acceleration rates and the accumulation testing time. Meanwhile, some failure modes are mitigated or eliminated at stage 2 depending on the effectiveness factor associated with that mode once the failure has occurred during testing.

Failure mode 5 is introduced during stage 2 as a result of the redesign, but it does not reveal itself during stage 2. This could be result of low failure intensity, or short testing time. As mentioned earlier, preventive measures are implemented to prevent the occurrence of design induced failures.

At testing stage 5, failure mode 8 is introduced after failure mode 6 and 7 are introduced in previous testing stages. The associated indicator matrix is given by:

FM1	1	1	0	0	1	1	1	1
FM2	1	0	0	0	0	1	0	0
FM3	0	0	0	0	1	1	1	1

Table 14. Stage 5 Indicator Matrix

FM4	0	0	0	1	1	0	0	0
FM5	1	0	0	1	0	0	0	0
FM6	1	0	1	0	1	1	0	1
FM7	0	0	0	1	1	0	0	0
FM8	0	0	0	0	0	0	0	0

Table 15. Stage 5 Indicator Matrix (Cont.)

In Table 13, the redesign/repair induced failure modes, 5, 6, and 7 occur at this testing level. This is mostly because of the accumulation of testing time on the system. Though the total number of failure mode occurrences for modes 1, 2, 3 and 4 has declined significantly, the testing time is extended greatly. As shown in Table 14, testing times for stages 1 (t_1) to 3 (t_3) do not change since there are no failures for repair/redesign induced failure modes. In testing level 4(t_4), due to the occurrence of some repair/redesign failure modes, testing times become shorter. When the number of testing levels becomes larger, testing time will shrink significantly.

	t_1	t_2	t_3	t_4	t_5
Without	81.89	113.24	229.87	367.64	921.02
With	81.89	113.24	229.87	351.73	674.66

Table 16. Testing Time With or Without Repair/Redesign Induced Failure Modes

Similar sensitivity experiments have been conducted for the model proposed in this chapter. Results suggested that the fix effectiveness index is the most influential factor that affects testing time. This is consistent with the conclusion we had in Chapter 6.

	ΔAccT	ΔAccumC	T_Ratio	C_Ratio	Δλ(A)	Δλ(N)	λ_Ratio
FM1	-536.34	-6375	-36.95%	-34.82%	0.0006	1.18E-05	62.61%
FM2	-596.15	-7109	-65.15%	-59.57%	0.0009	1.07E-05	56.87%
FM3	-434.72	-5196	-50.83%	-46.39%	0.0005	8.36E-06	44.36%
FM4	-507.68	-6074	-49.94%	-46.32%	0.0007	9.00E-06	47.75%

Table 17. Decrease Effectiveness Factors by 50% of Modified Model

	ΔAccT	ΔAccC	T_Ratio	C_Ratio	Δλ(A)	Δλ(N)	λ_Ratio
FM5	-90.63	-1086	-9.60%	-8.88%	0	1.33E-06	7.05%
FM6	0	0	0.00%	0.00%	0	0	0.00%
FM7	-103.09	-1322	-7.10%	-7.22%	0.0001	1.81E-06	9.59%
FM8	-125.47	-1610	-9.31%	-9.48%	0.0002	1.96E-06	10.38%

Table 18. Decrease Effectiveness Factors by 50% of Modified Model (Cont.)

	ΔAccT	ΔAccC	T_Ratio	C_Ratio	Δλ(A)	Δλ(N)	λ_Ratio
FM1	287.40	3411.88	19.80%	18.63%	-0.0002	-3.86E-06	-20.47%
FM2	166.88	1779.83	9.60%	8.19%	0.0000	-3.21E-06	-17.03%
FM3	244.37	2772.86	15.10%	13.80%	-0.0001	-2.39E-06	-12.70%
FM4	193.23	2455.93	11.40%	11.65%	-0.0002	-2.83E-06	-15.01%
FM5	0.00	0.00	0.00%	0.00%	0.0000	0.00E+00	0.00%
FM6	42.57	546.09	2.93%	2.98%	0.0000	0.00E+00	0.00%
FM7	42.57	546.09	2.85%	2.90%	0.0000	-1.40E-06	-7.45%
FM8	0.00	0.00	0.00%	0.00%	0.0000	-1.10E-06	-5.86%

Table 19. Decrease Effectiveness Factors by 50% of Modified Model

As seen in Table 15 and Table 16, changing the fix effectiveness factor could result in up to 62% change in the failure intensity. Since failure modes 5, 6, 7 and 8 are induced later in testing process, the effects of these failure modes are smaller than failure modes 1 to 4. Also, it is not difficult to see that when decreasing the effectiveness factor of failure mode 6, the failure intensity remaining constant. The failure intensity has the same outcome when increasing effectiveness factors of failure modes 5 and 6. Even, when increasing effectiveness factor of failure mode 6 (FM6), the accumulative testing time (ΔAccT) and money (ΔAccC) increases. Therefore, in practice, we should avoid implementing measures which produce results like these.

Figure 5 is a reliability simulation plot for the case where potential failure modes are introduced during redesign. The red curve represents the reliability under normal use conditions and the blue curve represents behavior under accelerated conditions. As shown, the testing time for each level is increasing due to the decrease of the systems total failure intensity. Also, the reliability under normal conditions is improving since there is improvement after each testing stage. The conclusions from Figure 5 and Figure 4 are similar since they both show significant growth in the reliability of the system as the product moves through the different testing stages. Further, for future research, potential failure modes which could not be predicted initially should be incorporated using elements of uncertainty to make our model even more realistic.

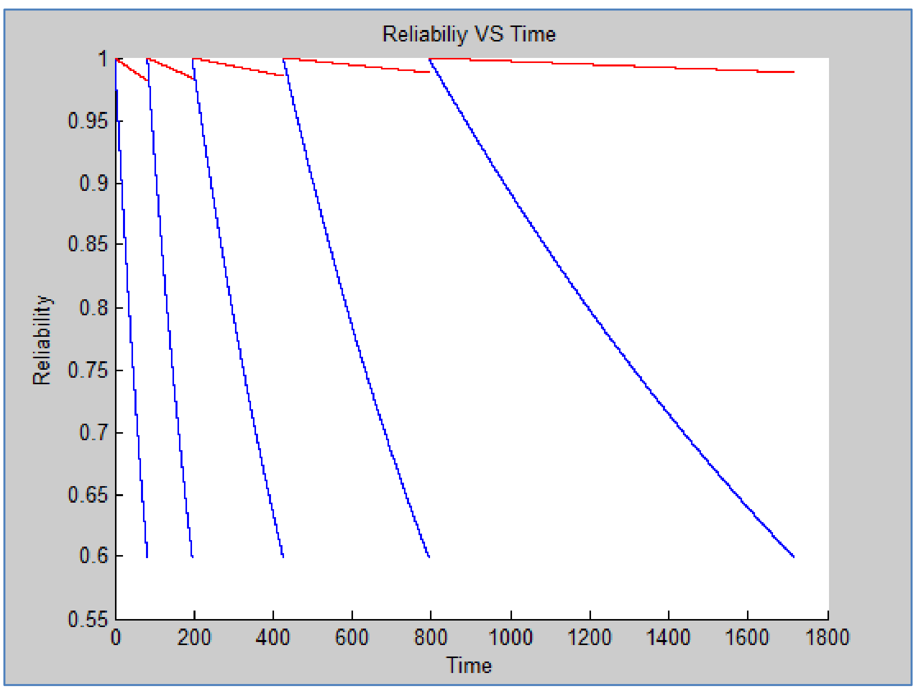

Figure 5. Modified Reliability Plot

Chapter 7. Parameter Estimation

The basic modeling framework was proposed in the previous two chapters, what is left for completing the modeling effort is the development of a strategy to estimate the parameters associated with the models. In our previous simulation example, we assumed all the parameters, initial failure intensity, acceleration factors and effectiveness factors, were known with certainty. However, the actual parameter estimation procedures can be extremely complex. In this section, the specific details associated with estimating the parameters are presented. First, we begin with the estimation of the failure intensity at the end of each phase. Then, an approach to determine the acceleration factors is illustrated. Finally, an approach for estimating the effectiveness factors is provided.

7.1. Estimation of normal condition failure intensity

Before testing, possible failure modes which can occur during test are classified. Those failure modes are separated based on the environmental factors that are required to accelerate those modes to failure. It is assumed that enough previous information exists to cover all the aspect we need for the test planning and analysis effort. Assuming failures occur according to a Poisson process. Therefore, the estimated failure intensity is given as:

$$\boldsymbol{\lambda_{1j}^{l} = N'^{l}_{1j}/t_i} \tag{87}$$

The normal condition failure number of each mode is denoted as N'^{l}_{1j}. In this modeling approach, we predict the failure intensity based on the first stage; therefore, we only need to detect the failure number at stage 1.

7.2. Estimation of acceleration factors

We assumed that there are k acceleration factors in our system. Additionally, if it is assumed that the acceleration ratio is predicted, then it has

$$\boldsymbol{\alpha^{l}_{ij} = d_i \alpha^{l}_{(i+1)j}} \tag{88}$$

where d_i is the acceleration ratio of acceleration factors.

In the testing plan, we assume that there are multiple acceleration factors that affect the product during testing. The estimation of acceleration factors follows the Arrhenius relationship (89), power law model (90), or the combination of the previous two models.

$$\boldsymbol{\alpha^{l}_{1j} = \exp \wedge (\frac{E}{k_B}(\frac{1}{T_{Test}} - \frac{1}{T_{Use}}))} \tag{89}$$

$$\boldsymbol{\alpha^{l}_{1j} = (\frac{\Delta P_{Test}}{\Delta P_{Normal}})^{r}} \tag{90}$$

$$\boldsymbol{\alpha^{l}_{1j} = (\frac{\Delta P_{Test}}{\Delta P_{Normal}})^{r} \exp \wedge (\frac{E}{k_B}(\frac{1}{T_{Test}} - \frac{1}{T_{Use}}))} \tag{91}$$

Where E is the activation energy and k_B is a constant multiplier of E. Testing temperature is denoted as T_{Test} and expected temperature in use is denoted as T_{Use}. The value span of acceleration factors under test condition is represented as ΔP_{Test} and under normal condition is represented as ΔP_{Normal}. Depends on the attribute of acceleration factors, the three equations are chosen.

7.3. Estimation of effectiveness factors

According to previous analysis, we noticed that effectiveness factors are the most sensitive parameters in testing strategy. Thus, the accuracy of effectiveness factors has the biggest impact on the testing framework. Therefore, neither underestimation nor overestimation of effectiveness factors affects testing time or testing cost significantly. The value of effectiveness factors ranges from 0 to 1 and larger values represents better fixes which yields more significant improvements in product reliability. Based on different systems, the environmental and operational factors affect systems in different ways and levels. To get the most comprehensive information about effectiveness factors, subject matter specialist assessment is suggested for this testing plan. This is because the effect of improvement after each redesign is difficult to obtain from observation alone.

Chapter 8. Conclusion

8.1. Conclusions

In this paper, we proposed an accelerated reliability growth model based on component level and system level testing. We incorporate previous and current stage failure intensity to estimate testing time in accelerated conditions. The decision is also based on an established reliability threshold. This work is important because it helps manufactures improve reliability in compressed time scales and simultaneously reduce development and testing costs. The simulation result illustrated that the proposed model improves products' reliability through this testing framework and modeling paradigm.

The model considers multiple levels of stress testing based on multiple types of acceleration factors, which broaden the existing models in the literature. In ARGT field, most of the models consider only one factor or testing in a single stage. This is due to the difficulty associated with estimating the potential failure modes. In this model, we come up with an indicator matrix to estimate whether potential failure modes will appear or not based upon the previous stages failure intensity and the associated testing time. In addition, we accumulate the testing time until the occurrence of specific failure mode. The approach is more comprehensive than incorporating only the current stage information.

8.2. Future work

Since the simulation is not based on real industrial data, the result presented can vary significantly from practice. Therefore, to test the practical aspects of our model, real product data would be beneficial to test the efficacy our model.

Another element that needs additional research is parameter estimation. Before testing, we assume that the information of possible failure modes and failure intensity could be gathered through previous data or through similar systems in the field. One way to combine actual data and subjective data based on previous experience is the use of Bayesian statistics. Further research could explore how a Bayesian framework could be incorporated into our modeling paradigm.

Additionally, in our model, we only consider the most common failure modes and repair/redesign induced failure modes. However, there could be new failure modes that occur during testing especially when acceleration factor grows rapidly. Due to a lack of information, our model might underestimate the true failure intensity if these random failure modes are not accounted for. In future extensions, we will explore how to model the random occurrence of new failure modes into our modeling framework.

Finally, in this paper we consider only the case where the failure intensity is assumed to be constant during each test level. The case is much easier than the scenario where the failure intensity varies with time at each test stage. Future work will investigate the implications of using a time varying hazard rate within each stage of testing.

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Appendix

Appendix 1. Protocol Research Details

	Year	Author	Title	Source
1	2013	Ye etc.	Accelerated reliability growth test under multiple environment stresses	QR2MSE 2013 - Proceedings of 2013 International Conference on Quality, Reliability, Risk, Maintenance, and Safety Engineering
2	1994	Feinberg	Accelerated Reliabiltiy Growth Models	Journal of the IES
3	1993	Feinberg etc.	Accelerted reliabilty Growth	Proceedings, Annual Technical Meeting - Institute of Environmental Sciences
4	2006	Acevedo etc.	Reliability growth and forecasting for critical hardware through accelerated life testing	Bell Labs Technical Journal
5	2005	Hajime etc.	Markovian operational software reliability measurement based on accelerated life testing model	2005 Proceedings - 11th ISSAT International Conference on Reliability and Quality in Design
6	2001	Hiroyuki etc.	A reliability assessment method for software products in operational phase-proposal of an accelerated life testing model	Electronics and Communications in Japan

Table 20. Protocol Research

	Year	Author	Title	Source
7	2013	Li etc.	Accelerated reliability testing approach for high-reliablity software based on the reinforced operational profile	2013 IEEE International Symposium on Software Reliability Engineering Workshops
8	1996	Granlund etc.	Method of reliability improvement using accelerated testing methodologies	National Electronic Packaging and Production Conference-Proceedings of the Technical Program (West and East)
9	2013	Wang etc.	Accelerated testing of equipment based on the duane model	Journal of Applied Sciences
10	2006	Jun etc.	Research on reliability growth technology of electromechanical-hydraulic system	ICAS-Secretariat - 25th Congress of the International Council of the Aeronautical Sciences 2006
11	1998	Crowe etc.	Stage-gating accelerated reliability growth in an industrial environment	Institute of Environmental Sciences - Proceedings, Annual Technical Meeting
12	2007	Krasich	Accelerated reliability growth testing and data analysis method	Journal of the IEST
13	2005	Andonova etc.	Accelerated reliability growth of electronic devices	27th International Spring Seminar on Electronics Technology
14	1993	Donovan	Accelerated environmental stress screening & reliability growth testing of the B-52 infrared camera	Annual Reliability and Maintainability Symposium. 1993 Proceedings
15	2011	Wang; etc.	Software reliability accelerated testing method based on test coverage	Annual Reliability and Maintainability Symposium

Table 21. Protocol Research (Cont.)

	Year	Author	Title	Source
16	2004	Krasich	Accelerated testing for demonstration of product lifetime reliability	Journal of the IEST
17	2013	Feinberg	Chi-squared accelerated reliability growth model	Annual Reliability and Maintainability Symposium, 2013, 59th
18	2011	Feng etc.	Software reliability accelerated testing based on the combined testing method	Safety First, Reliability Primary: Proceedings of 2011 9th International Conference on Reliability, Maintainability and Safety
19	2009	Krasich	Realistic Reliability Requirements for Stresses in Use	Journal of the IEST
20	2004	Krasich	Test design and acceleration for product lifetime reliability demonstration	SAE Technical Papers
21	1991	Bubel etc.	Optical fiber reliability implications of uncertainty in the fatigue crack growth model	Optical Engineering
22	2003	Gurunatha etc.	Applying quality tools to reliability: a 12-step Six-sigma process to accelerate reliability growth in product design	Annual Reliability and Maintainability Symposium
23	2010	Wu etc.	Software reliability accelerated testing method based on mixed testing	Annual Reliability and Maintainability Symposium

Table 22. Protocol Research (Cont.)

	Year	Author	Title	Source
24	2007	Krasich	Realistic reliability requirements for the stresses in use	2007 Annual Reliability and Maintainability Symposium
25	2010	Janamanchietc.	Reliability growth vs. HASS cost for product manufacturing with fast-to-market requirement	International Journal of Productivity and Quality Management
26	1996	McAfee	Time compressing reliability development tests	National Electronic Packaging and Production Conference-Proceedings of the Technical Program
27	2006	Kim etc.	Reliability assessment of indium tin oxide thin films by accelerated degradation test	Journal of Electroceramics
28	2010	Klyatis etc.	Accelerated reliability/durability testing as a key factor for accelerated development and improvement of product/process reliability, durability, and maintainability	SAE Technical Papers
29	2006	Jayatilleekaetc.	Accelerated life testing for speedier product development: Problems and strategies	Annual Reliability and Maintainability Symposium

Table 23. Protocol Research (Cont.)

Appendix 2. Sensitivity Analysis

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
t_1	66.56	66.77	59.52	62.73	66.61	66.82	67.17	66.95
c1	683	684	642	661	683	684	687	685
t_2	64.23	63.04	70.95	68.08	64.47	61.07	59.45	62.36
c2	858	849	912	889	860	833	821	844
AccT2	130.78	129.81	130.46	130.81	131.08	127.89	126.62	129.31
AccC2	1541	1533	1554	1550	1544	1518	1507	1529
t_3	81.14	90.34	78.56	80.87	80.89	87.49	92.88	85.27
c3	1206	1297	1180	1203	1203	1269	1322	1247
AccT3	211.93	220.14	209.03	211.68	211.97	215.38	219.50	214.58
AccC3	2747	2830	2734	2753	2747	2787	2829	2776
t_4	136.89	128.52	141.37	133.91	137.78	127.54	121.86	135.87
c4	2008	1913	2059	1974	2018	1902	1837	1997
AccT4	348.82	348.67	350.40	345.59	349.75	342.92	341.36	350.46
AccC4	4756	4743	4794	4727	4765	4689	4667	4772
t_5	251.98	264.43	234.49	249.62	244.87	249.51	264.71	239.48
c5	3732	3892	3508	3702	3641	3701	3896	3572
AccT5	600.80	613.10	584.89	595.21	594.62	592.43	606.07	589.93
AccC5	8488	8636	8302	8430	8407	8390	8563	8345
λ(A)	0.0020	0.0019	0.0022	0.0020	0.0021	0.0020	0.0019	0.0021
λ(N)	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05

Table 24. Full Sensitivity Experiment with Decreasing Failure Intensity By 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
Δt_1	7.04	7.25	0.00	3.22	7.10	7.30	7.66	7.43
Δc1	41	42	0	19	41	42	44	43
Δt_2	3.85	2.66	10.57	7.70	4.10	0.70	-0.93	1.98
Δc2	30	21	84	61	32	6	-7	16
ΔAccT2	10.89	9.92	10.57	10.92	11.19	8.00	6.73	9.42
ΔAccC2	71	63	84	79	73	48	37	58
Δt_3	1.30	10.50	-1.28	1.03	1.05	7.65	13.04	5.43
Δc3	13	104	-13	10	10	76	129	54
ΔAccT3	12.20	20.41	9.30	11.94	12.24	15.65	19.77	14.85
ΔAccC3	84	167	71	90	84	123	166	112
Δt_4	6.91	-1.46	11.39	3.93	7.79	-2.44	-8.13	5.89
Δc4	79	-17	130	45	89	-28	-92	67
ΔAccT4	19.10	18.95	20.68	15.87	20.03	13.20	11.64	20.74
ΔAccC4	163	150	201	134	172	96	74	179
Δt_5	19.30	31.76	1.81	16.95	12.20	16.84	32.04	6.80
Δc5	248	407	23	217	156	216	411	87
ΔAccT5	38.40	50.71	22.50	32.82	32.23	30.04	43.68	27.54
ΔAccC5	410	558	224	352	329	312	485	267
Δλ(A)	-2E-04	-3E-04	-2E-05	-1E-04	-1E-04	-1E-04	-3E-04	-6E-05
Δλ(N)	-1E-06	-1E-06	-2E-06	-2E-06	-1E-06	-3E-06	-3E-06	-2E-07
λ_Ratio	-2.35%	-2.47%	-3.27%	-3.06%	-2.29%	-6.03%	-6.64%	-0.46%

Table 25. Full Sensitivity Analysis with Decreasing Failure Intensity by 50%

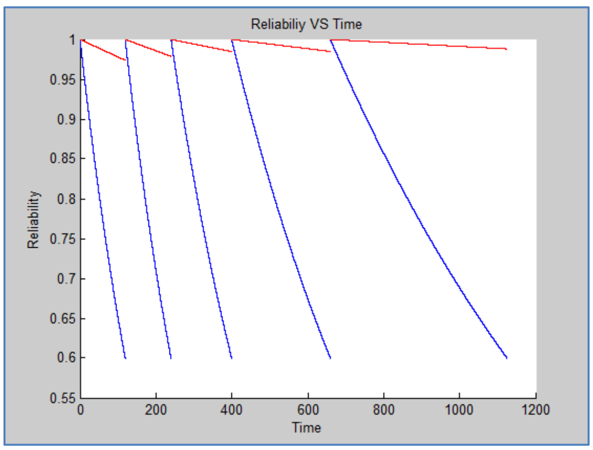

Figure 6. Reliability Plot with All modes Failure Intensity Decreased by 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
t_1	57.09	59.36	59.85	58.78	57.08	58.47	59.17	58.35
c1	628	642	644	638	628	636	640	636
	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
t_2	52.66	49.28	52.23	54.82	53.61	56.01	53.52	54.92
c2	767	740	763	784	774	793	774	785
AccT2	109.75	108.64	112.08	113.61	110.69	114.48	112.69	113.27
AccC2	1395	1382	1408	1422	1403	1430	1414	1420
t_3	82.13	83.22	82.11	75.56	78.80	72.05	79.84	74.78
c3	1216	1227	1215	1150	1183	1116	1193	1143
AccT3	191.88	191.86	194.19	189.16	189.49	186.53	192.53	188.05
AccC3	2611	2608	2623	2573	2585	2545	2607	2563
t_4	108.75	105.78	110.50	117.96	112.49	125.46	116.26	116.18
c4	1688	1654	1708	1793	1731	1878	1773	1773
AccT4	300.63	297.64	304.69	307.12	301.97	311.99	308.79	304.23
AccC4	4299	4262	4331	4365	4316	4424	4380	4336
t_5	234.63	232.62	228.01	213.15	217.91	201.59	214.92	212.49
c5	3510	3484	3425	3234	3295	3086	3257	3226
AccT5	535.26	530.26	532.71	520.27	519.88	513.58	523.71	516.72
AccC5	7809	7746	7756	7600	7611	7510	7638	7562
λ(A)	0.0022	0.0022	0.0022	0.0024	0.0023	0.0025	0.0024	0.0024
λ(N)	5E-05	5E-05	5E-05	6E-05	5E-05	6E-05	6E-05	5E-05

Table 26. Full Sensitivity Experiment with Increasing Failure Intensity by 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
Δt_1	-2.43	-0.15	0.33	-0.73	-2.44	-1.04	-0.34	-1.16
Δc1	-14	-1	2	-4	-14	-6	-2	-7
Δt_2	-7.71	-11.10	-8.14	-5.55	-6.76	-4.37	-6.86	-5.46
Δc2	-61	-88	-64	-44	-54	-35	-54	-43
ΔAccT2	-10.14	-11.25	-7.81	-6.28	-9.20	-5.41	-7.20	-6.62
ΔAccC2	-75	-89	-63	-48	-68	-41	-56	-50
Δt_3	2.29	3.38	2.27	-4.28	-1.05	-7.79	0.00	-5.06
Δc3	23	34	23	-43	-10	-77	0	-50
ΔAccT3	-7.85	-7.87	-5.54	-10.57	-10.25	-13.20	-7.20	-11.68
ΔAccC3	-52	-55	-40	-91	-78	-118	-56	-100
Δt_4	-21.24	-24.21	-19.48	-12.03	-17.50	-4.53	-13.72	-13.81
Δc4	-242	-276	-222	-137	-199	-52	-156	-157
ΔAccT4	-29.09	-32.07	-25.02	-22.59	-27.74	-17.72	-20.93	-25.49
ΔAccC4	-294	-331	-262	-228	-277	-169	-213	-257
Δt_5	1.95	-0.06	-4.66	-19.53	-14.76	-31.09	-17.76	-20.18
Δc5	25	-1	-60	-250	-189	-399	-228	-259
ΔAccT5	-27.13	-32.13	-29.68	-42.12	-42.51	-48.81	-38.68	-45.67
ΔAccC5	-269	-331	-322	-478	-467	-568	-440	-516
Δλ(A)	-2E-05	5E-07	4E-05	2E-04	1E-04	3E-04	2E-04	2E-04
Δλ(N)	2E-06	3E-07	3E-06	5E-06	3E-06	5E-06	4E-06	3E-06
λ_Ratio	3.69%	0.63%	5.80%	9.43%	5.78%	10.19%	7.22%	6.03%

Table 27. Full Sensitivity Analysis with Increasing Failure Intensity by 50%

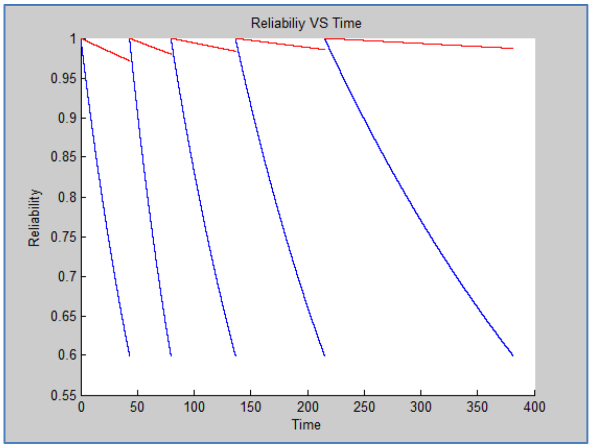

Figure 7. Reliability Plot with All modes Failure Intensity Increased by 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
t_1	66.56	66.77	59.52	62.73	66.61	66.82	67.17	66.95
c1	683	684	642	661	683	684	687	685
t_2	64.23	63.04	70.95	68.08	64.47	61.07	59.45	62.36
c2	858	849	912	889	860	833	821	844
AccT2	130.78	129.81	130.46	130.81	131.08	127.89	126.62	129.31
AccC2	1541	1533	1554	1550	1544	1518	1507	1529
t_3	81.14	90.34	78.56	80.87	80.89	87.49	92.88	85.27
c3	1206	1297	1180	1203	1203	1269	1322	1247
AccT3	211.93	220.14	209.03	211.68	211.97	215.38	219.50	214.58
AccC3	2747	2830	2734	2753	2747	2787	2829	2776
t_4	136.89	128.52	141.37	133.91	137.78	127.54	121.86	135.87
c4	2008	1913	2059	1974	2018	1902	1837	1997
AccT4	348.82	348.67	350.40	345.59	349.75	342.92	341.36	350.46
AccC4	4756	4743	4794	4727	4765	4689	4667	4772
t_5	251.98	264.43	234.49	249.62	244.87	249.51	264.71	239.48
c5	3732	3892	3508	3702	3641	3701	3896	3572
AccT5	600.80	613.10	584.89	595.21	594.62	592.43	606.07	589.93
AccC5	8488	8636	8302	8430	8407	8390	8563	8345
λ(A)	0.0020	0.0019	0.0022	0.0020	0.0021	0.0020	0.0019	0.0021
λ(N)	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05

Table 28. Full Sensitivity Experiment with Decreasing Acceleration Factor by 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
Δt_1	7.04	7.25	0.00	3.22	7.10	7.30	7.66	7.43
Δc1	41	42	0	19	41	42	44	43
Δt_2	3.85	2.66	10.57	7.70	4.10	0.70	-0.93	1.98
Δc2	30	21	84	61	32	6	-7	16
ΔAccT2	10.89	9.92	10.57	10.92	11.19	8.00	6.73	9.42
ΔAccC2	71	63	84	79	73	48	37	58
Δt_3	1.30	10.50	-1.28	1.03	1.05	7.65	13.04	5.43
Δc3	13	104	-13	10	10	76	129	54
ΔAccT3	12.20	20.41	9.30	11.94	12.24	15.65	19.77	14.85
ΔAccC3	84	167	71	90	84	123	166	112
Δt_4	6.91	-1.46	11.39	3.93	7.79	-2.44	-8.13	5.89
Δc4	79	-17	130	45	89	-28	-92	67
ΔAccT4	19.10	18.95	20.68	15.87	20.03	13.20	11.64	20.74
ΔAccC4	163	150	201	134	172	96	74	179
Δt_5	19.30	31.76	1.81	16.95	12.20	16.84	32.04	6.80
Δc5	248	407	23	217	156	216	411	87
ΔAccT5	38.40	50.71	22.50	32.82	32.23	30.04	43.68	27.54
ΔAccC5	410	558	224	352	329	312	485	267
Δλ(A)	-2E-04	-3E-04	-2E-05	-1E-04	-1E-04	-1E-04	-3E-04	-6E-05
Δλ(N)	-1E-06	-1E-06	-2E-06	-2E-06	-1E-06	-3E-06	-3E-06	-2E-07
λ_Ratio	-2.35%	-2.47%	-3.27%	-3.06%	-2.29%	-6.03%	-6.64%	-0.46%

Table 29. Full Sensitivity Analysis with Decreasing Acceleration Factor by 50%

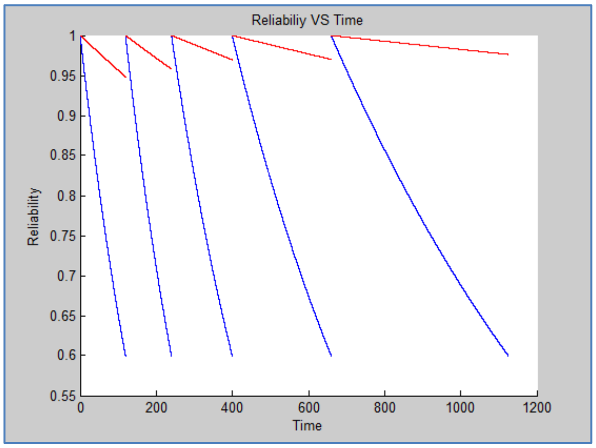

Figure 8. Reliability Plot with All AFs Decreased by 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
t_1	57.09	59.36	59.85	58.78	57.08	58.47	59.17	58.35
c1	628	642	644	638	628	636	640	636
t_2	52.66	49.28	52.23	54.82	53.61	56.01	53.52	54.92
c2	767	740	763	784	774	793	774	785
AccT2	109.75	108.64	112.08	113.61	110.69	114.48	112.69	113.27
AccC2	1395	1382	1408	1422	1403	1430	1414	1420
t_3	82.13	83.22	82.11	75.56	78.80	72.05	79.84	74.78
c3	1216	1227	1215	1150	1183	1116	1193	1143
AccT3	191.88	191.86	194.19	189.16	189.49	186.53	192.53	188.05
AccC3	2611	2608	2623	2573	2585	2545	2607	2563
t_4	108.75	105.78	110.50	117.96	112.49	125.46	116.26	116.18
c4	1688	1654	1708	1793	1731	1878	1773	1773
AccT4	300.63	297.64	304.69	307.12	301.97	311.99	308.79	304.23
AccC4	4299	4262	4331	4365	4316	4424	4380	4336
t_5	234.63	232.62	228.01	213.15	217.91	201.59	214.92	212.49
c5	3510	3484	3425	3234	3295	3086	3257	3226
AccT5	535.26	530.26	532.71	520.27	519.88	513.58	523.71	516.72
AccC5	7809	7746	7756	7600	7611	7510	7638	7562
λ(A)	0.0022	0.0022	0.0022	0.0024	0.0023	0.0025	0.0024	0.0024
λ(N)	5E-05	5E-05	5E-05	6E-05	5E-05	6E-05	6E-05	5E-05

Table 30. Full Sensitivity Experiment with Increasing Acceleration Factor by 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
Δt_1	-2.43	-0.15	0.33	-0.73	-2.44	-1.04	-0.34	-1.16
Δc1	-14	-1	2	-4	-14	-6	-2	-7
Δt_2	-7.71	-11.10	-8.14	-5.55	-6.76	-4.37	-6.86	-5.46
Δc2	-61	-88	-64	-44	-54	-35	-54	-43
ΔAccT2	-10.14	-11.25	-7.81	-6.28	-9.20	-5.41	-7.20	-6.62
ΔAccC2	-75	-89	-63	-48	-68	-41	-56	-50
Δt_3	2.29	3.38	2.27	-4.28	-1.05	-7.79	0.00	-5.06
Δc3	23	34	23	-43	-10	-77	0	-50
ΔAccT3	-7.85	-7.87	-5.54	-10.57	-10.25	-13.20	-7.20	-11.68
ΔAccC3	-52	-55	-40	-91	-78	-118	-56	-100
Δt_4	-21.24	-24.21	-19.48	-12.03	-17.50	-4.53	-13.72	-13.81
Δc4	-242	-276	-222	-137	-199	-52	-156	-157
ΔAccT4	-29.09	-32.07	-25.02	-22.59	-27.74	-17.72	-20.93	-25.49
ΔAccC4	-294	-331	-262	-228	-277	-169	-213	-257
Δt_5	1.95	-0.06	-4.66	-19.53	-14.76	-31.09	-17.76	-20.18
Δc5	25	-1	-60	-250	-189	-399	-228	-259
ΔAccT5	-27.13	-32.13	-29.68	-42.12	-42.51	-48.81	-38.68	-45.67
ΔAccC5	-269	-331	-322	-478	-467	-568	-440	-516
Δλ(A)	-2E-05	5E-07	4E-05	2E-04	1E-04	3E-04	2E-04	2E-04
Δλ(N)	2E-06	3E-07	3E-06	5E-06	3E-06	5E-06	4E-06	3E-06
λ_Ratio	3.69%	0.63%	5.80%	9.43%	5.78%	10.19%	7.22%	6.03%

Table 31. Full Sensitivity Analysis with Increasing Acceleration Factor by 50%

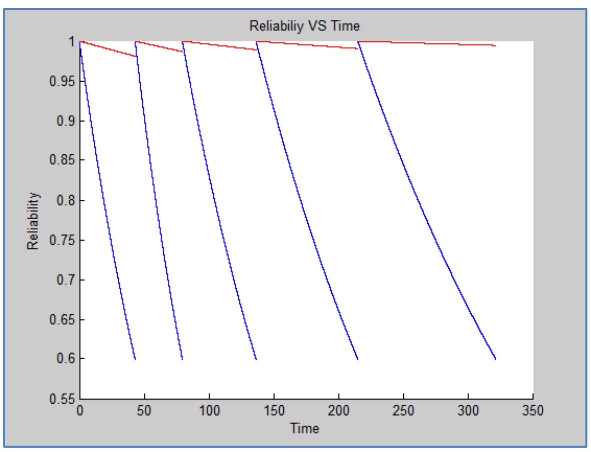

Figure 9. Reliability Plot with All AFs Increased by 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
t_1	64.49	64.49	64.49	64.49	64.49	64.49	64.49	64.49
c1	671	671	671	671	671	671	671	671
t_2	52.87	48.85	52.87	49.08	43.75	47.03	49.21	49.39
c2	768	737	768	738	696	722	739	741
AccT2	117.36	113.34	117.36	113.58	108.24	111.52	113.70	113.88
AccC2	1440	1408	1440	1410	1367	1393	1411	1412
t_3	77.37	81.17	77.33	77.09	93.09	84.96	91.73	93.45
c3	1168	1206	1168	1166	1324	1244	1311	1328
AccT3	194.73	194.51	194.69	190.66	201.33	196.48	205.43	207.34
AccC3	2608	2614	2607	2575	2692	2637	2722	2740
t_4	109.83	106.51	108.07	103.97	85.66	90.12	96.98	89.03
c4	1700	1662	1680	1634	1425	1476	1554	1463
AccT4	304.55	301.02	302.75	294.64	286.99	286.60	302.41	296.37
AccC4	4308	4276	4288	4209	4117	4113	4276	4204
t_5	148.39	161.75	171.23	166.41	221.58	213.61	246.18	209.51
c5	2404	2575	2697	2635	3343	3240	3658	3188
AccT5	452.95	462.77	473.98	461.05	508.57	500.21	548.59	505.87
AccC5	6712	6851	6984	6844	7460	7353	7934	7391
λ(A)	0.0034	0.0032	0.0030	0.0031	0.0023	0.0024	0.0021	0.0024
λ(N)	9E-05	8E-05	8E-05	6E-05	6E-05	6E-05	5E-05	5E-05

Table 32. Full Sensitivity Experiment with Decreasing Effectiveness Factor by 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
Δt_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Δc1	0	0	0	0	0	0	0	0
Δt_2	0.00	-4.02	0.00	-3.78	-9.12	-5.84	-3.66	-3.48
Δc2	0	-32	0	-30	-72	-46	-29	-28
ΔAccT2	0.00	-4.02	0.00	-3.78	-9.12	-5.84	-3.66	-3.48
ΔAccC2	0	-32	0	-30	-72	-46	-29	-28
Δt_3	-13.50	-9.70	-13.54	-13.78	2.22	-5.91	0.86	2.58
Δc3	-134	-96	-134	-137	22	-59	9	26
ΔAccT3	-13.50	-13.72	-13.54	-17.56	-6.90	-11.75	-2.80	-0.89
ΔAccC3	-134	-128	-134	-167	-50	-105	-20	-2
Δt_4	-5.45	-8.77	-7.21	-11.31	-29.61	-25.15	-18.29	-26.25
Δc4	-62	-100	-82	-129	-337	-286	-208	-299
ΔAccT4	-18.95	-22.49	-20.75	-28.87	-36.51	-36.90	-21.10	-27.14
ΔAccC4	-196	-228	-217	-295	-387	-391	-229	-301
Δt_5	-128.97	-115.61	-106.14	-110.96	-55.79	-63.76	-31.18	-67.86
Δc5	-1655	-1483	-1362	-1423	-716	-818	-400	-871
ΔAccT5	-147.92	-138.10	-126.89	-139.82	-92.30	-100.66	-52.28	-95.00
ΔAccC5	-1851	-1711	-1578	-1719	-1103	-1209	-629	-1171
Δλ(A)	0.0016	0.0013	0.0012	0.0012	0.0005	0.0005	0.0002	0.0006
Δλ(N)	3E-05	2E-05	2E-05	1E-05	5E-06	4E-06	-9E-08	1E-08
λ_Ratio	74.67%	48.37%	48.05%	21.98%	10.43%	8.44%	-0.18%	0.03%

Table 33. Full Sensitivity Analysis with Decreasing Effectiveness Factor by 50%

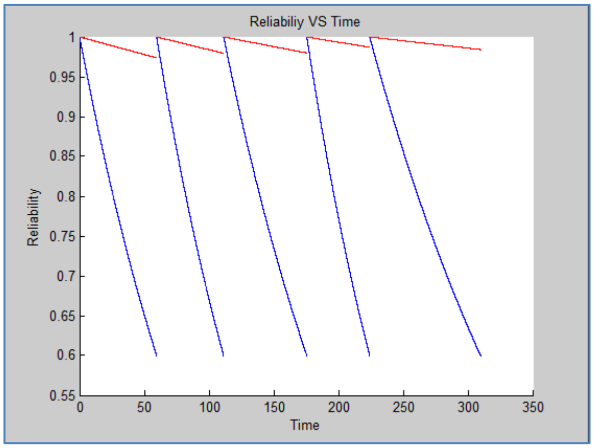

Figure 10. Reliability Plot with All EFs Decreased by 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
t_1	64.49	64.49	64.49	64.49	64.49	64.49	64.49	64.49
c1	671	671	671	671	671	671	671	671
t_2	52.87	52.87	52.87	52.87	52.87	52.87	52.87	52.87
c2	768	768	768	768	768	768	768	768
AccT2	117.36	117.36	117.36	117.36	117.36	117.36	117.36	117.36
AccC2	1440	1440	1440	1440	1440	1440	1440	1440
t_3	90.87	100.62	90.87	99.95	117.62	108.90	98.58	101.49
c3	1302	1399	1302	1393	1568	1482	1379	1408
AccT3	208.23	217.99	208.23	217.31	234.98	226.26	215.94	218.85
AccC3	2742	2839	2742	2832	3008	2921	2819	2847
t_4	150.91	124.31	144.18	134.04	113.78	120.83	118.52	125.23
c4	2168	1865	2091	1976	1745	1825	1799	1876
AccT4	359.14	342.29	352.41	351.35	348.76	347.09	334.47	344.08
AccC4	4910	4704	4833	4808	4753	4747	4618	4723
t_5	269.93	325.22	270.73	277.01	349.90	329.37	307.97	294.88
c5	3963	4672	3973	4054	4989	4725	4451	4283
AccT5	629.07	667.51	623.14	628.36	698.66	676.46	642.43	638.96
AccC5	8873	9376	8806	8862	9742	9472	9069	9006
λ(A)	0.0019	0.0016	0.0019	0.0018	0.0015	0.0016	0.0017	0.0017
λ(N)	4E-05	4E-05	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05

Table 34. Full Sensitivity Experiment with Increasing Effectiveness Factor by 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
Δt_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Δc1	0	0	0	0	0	0	0	0
Δt_2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Δc2	0	0	0	0	0	0	0	0
ΔAccT2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ΔAccC2	0	0	0	0	0	0	0	0
Δt_3	0.00	9.76	0.00	9.08	26.75	18.04	7.72	10.62
Δc3	0	97	0	90	266	179	77	105
ΔAccT3	0.00	9.76	0.00	9.08	26.75	18.04	7.72	10.62
ΔAccC3	0	97	0	90	266	179	77	105
Δt_4	35.63	9.03	28.91	18.77	-1.50	5.55	3.24	9.96
Δc4	406	103	329	214	-17	63	37	113
ΔAccT4	35.63	18.79	28.91	27.85	25.25	23.58	10.96	20.58
ΔAccC4	406	200	329	304	249	242	114	219
Δt_5	-7.44	47.86	-6.64	-0.36	72.53	52.00	30.60	17.52
Δc5	-95	614	-85	-5	930	667	393	225
ΔAccT5	28.19	66.64	22.27	27.49	97.78	75.59	41.56	38.09
ΔAccC5	310	814	244	299	1179	909	506	444
Δλ(A)	0.0001	-0.0003	0.0000	0.0000	-0.0004	-0.0003	-0.0002	-0.0001
Δλ(N)	-1E-05	-1E-05	-4E-06	-3E-06	-3E-06	-2E-06	-1E-07	-1E-07
λ_Ratio	-21%	-22%	-8%	-6%	-7%	-4%	-0.2%	-0.2%

Table 35. Full Sensitivity Analysis with Increasing Effectiveness Factor by 50%

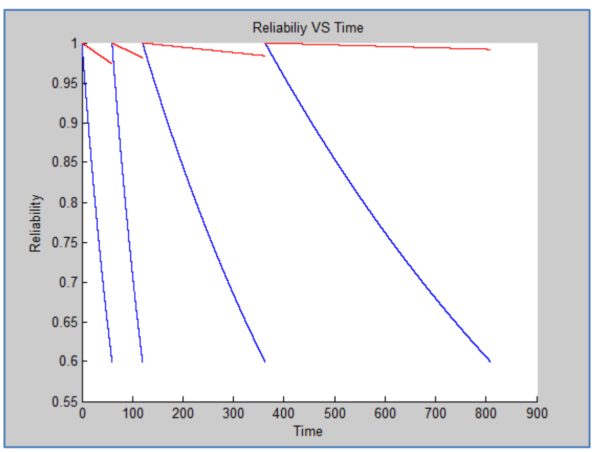

Figure 11. Sensitivity Plot with All EFs Decreased by 50%

	AF1	AF2	AF3	AF4	AF5	AF6	AF7	AF8
t_1	64.49	70.56	64.49	71.83	95.24	83.44	72.55	75.95
c1	671	706	671	713	848	780	717	737
t_2	66.01	57.12	64.04	60.45	54.64	57.32	55.04	57.81
c2	872	802	857	828	782	804	786	808
AccT2	130.50	127.68	128.53	132.27	149.87	140.76	127.59	133.76
AccC2	1544	1508	1528	1542	1630	1584	1503	1545
t_3	93.97	121.73	94.57	112.53	135.00	120.02	107.60	111.07
c3	1333	1609	1339	1518	1741	1592	1469	1503
AccT3	224.47	249.40	223.10	244.81	284.88	260.78	235.20	244.82
AccC3	2877	3117	2867	3059	3371	3176	2972	3048
t_4	146.73	124.31	140.37	134.04	115.28	123.21	122.64	125.23
c4	2120	1865	2048	1976	1762	1853	1846	1876
AccT4	371.20	373.71	363.47	378.85	400.15	383.98	357.84	370.06
AccC4	4997	4982	4915	5035	5133	5028	4818	4923
t_5	295.20	363.37	294.45	297.27	414.73	352.52	325.79	299.69
c5	4287	5162	4277	4314	5820	5022	4679	4345
AccT5	666.40	737.09	657.92	676.12	814.88	736.50	683.62	669.75
AccC5	9284	10144	9192	9349	10954	10051	9497	9268
λ(A)	0.0017	0.0014	0.0017	0.0017	0.0012	0.0014	0.0016	0.0017
λ(N)	4E-05	3E-05	5E-05	5E-05	4E-05	5E-05	5E-05	5E-05

Table 36. Full Sensitivity Experiment When No Failure Mode by Acceleration Factor

	AF1	AF2	AF3	AF4	AF5	AF6	AF7	AF8
Δt_1	0.00	6.07	0.00	7.33	30.75	18.95	8.06	11.46
Δc1	0	35	0	42	177	109	46	66
Δt_2	13.14	4.25	11.17	7.58	1.77	4.45	2.17	4.94
Δc2	104	34	88	60	14	35	17	39
ΔAccT2	13.14	10.32	11.17	14.91	32.51	23.40	10.23	16.40
ΔAccC2	104	69	88	102	191	144	64	105
Δt_3	3.10	30.86	3.70	21.66	44.14	29.15	16.73	20.20
Δc3	31	306	37	215	438	289	166	201
ΔAccT3	16.24	41.18	14.87	36.58	76.65	52.55	26.97	36.60
ΔAccC3	135	375	125	317	629	434	230	306
Δt_4	31.46	9.03	25.09	18.77	0.00	7.93	7.36	9.96
Δc4	358	103	286	214	0	90	84	113
ΔAccT4	47.70	50.21	39.96	55.34	76.65	60.48	34.33	46.55
ΔAccC4	493	478	411	531	629	524	314	419
Δt_5	17.83	86.01	17.08	19.90	137.36	75.15	48.42	22.32
Δc5	229	1103	219	255	1762	964	621	286
ΔAccT5	65.53	136.21	57.05	75.25	214.01	135.63	82.75	68.88
ΔAccC5	722	1581	630	786	2391	1488	935	705
Δλ(A)	-0.0001	-0.0004	-0.0001	-0.0001	-0.0006	-0.0004	-0.0003	-0.0001
Δλ(N)	-1E-05	-2E-05	-5E-06	-5E-06	-7E-06	-3E-06	5E-07	6E-07
λ_Ratio	-28.22%	-33.67%	-9.88%	-9.89%	-14.07%	-5.70%	1.06%	1.11%

Table 37. Full Sensitivity Analysis When No Failure Mode by Acceleration Factor

	AF1	AF2	AF3	AF4	AF5	AF6	AF7	AF8
t_1	64.49	64.49	64.49	64.49	64.49	64.49	64.49	64.49
c1	645	637	622	610	668	668	667	666
t_2	52.87	52.87	52.87	52.87	52.87	52.87	52.87	52.87
c2	737	722	718	702	766	766	764	764
AccT2	117.36	117.36	117.36	117.36	117.36	117.36	117.36	117.36
AccC2	1382	1359	1339	1312	1434	1434	1431	1430
t_3	90.87	90.87	90.87	90.87	90.87	90.87	90.87	90.87
c3	1230	1206	1182	1166	1298	1297	1295	1294
AccT3	208.23	208.23	208.23	208.23	208.23	208.23	208.23	208.23
AccC3	2612	2565	2522	2478	2732	2731	2726	2724
t_4	115.28	115.28	115.28	115.28	115.28	115.28	115.28	115.28
c4	1652	1620	1591	1566	1756	1755	1752	1751
AccT4	323.51	323.51	323.51	323.51	323.51	323.51	323.51	323.51
AccC4	4263	4186	4112	4045	4487	4485	4478	4476
t_5	277.37	277.37	277.37	277.37	277.37	277.37	277.37	277.37
c5	3748	3668	3592	3538	4041	4038	4032	4030
AccT5	600.87	600.87	600.87	600.87	600.87	600.87	600.87	600.87
AccC5	8011	7853	7705	7583	8528	8523	8510	8505
λ(A)	0.0018	0.0018	0.0018	0.0018	0.0018	0.0018	0.0018	0.0018
λ(N)	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05	5E-05

Table 38. Full Sensitivity Experiment with Decreasing Test Cost by 50%

	AF1	AF2	AF3	AF4	AF5	AF6	AF7	AF8
Δt_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Δc1	-26	-34	-50	-61	-3	-3	-4	-5
Δt_2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Δc2	-32	-47	-51	-66	-3	-3	-4	-5
ΔAccT2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ΔAccC2	-58	-81	-100	-127	-5	-6	-8	-9
Δt_3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Δc3	-73	-96	-120	-136	-5	-6	-8	-8
ΔAccT3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ΔAccC3	-130	-177	-220	-264	-10	-11	-16	-18
Δt_4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Δc4	-111	-142	-172	-196	-7	-8	-10	-11
ΔAccT4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ΔAccC4	-241	-319	-392	-460	-17	-19	-26	-29
Δt_5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Δc5	-311	-391	-466	-520	-17	-20	-26	-28
ΔAccT5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ΔAccC5	-552	-709	-858	-980	-34	-39	-53	-57
Δλ(A)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Δλ(N)	0	0	0	0	0	0	0	0

Table 39. Full Sensitivity Analysis with Decreasing Test Cost by 50%

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
t_1	81.89	81.89	81.89	81.89	81.89	81.89	81.89	81.89
c1	771	771	771	771	771	771	771	771
t_2	94.31	94.03	93.69	97.14	113.24	113.24	113.24	113.24
c2	1096	1094	1092	1119	1246	1246	1246	1246
AccT2	176.20	175.92	175.58	179.03	195.13	195.13	195.13	195.13
AccC2	1868	1865	1863	1890	2017	2017	2017	2017
t_3	156.04	142.34	181.73	154.37	229.87	229.87	229.87	229.87
c3	1950	1814	2205	1933	2683	2683	2683	2683
AccT3	332.24	318.25	357.31	333.40	425.00	425.00	425.00	425.00
AccC3	3817	3679	4068	3823	4700	4700	4700	4700
t_4	214.68	219.62	251.51	254.10	299.00	351.73	351.73	351.73
c4	2894	2950	3313	3343	3854	4454	4454	4454
AccT4	546.91	537.87	608.82	587.50	724.00	776.73	776.73	776.73
AccC4	6711	6629	7381	7166	8554	9154	9154	9154
t_5	368.14	317.38	407.86	356.22	636.77	674.66	571.58	549.19
c5	5223	4572	5732	5070	8669	9155	7833	7545
AccT5	915.06	855.25	1016.68	943.71	1360.77	1451.40	1348.31	1325.92
AccC5	11934	11201	13113	12235	17223	18309	16987	16700
λ(A)	0.0014	0.0016	0.0013	0.0014	0.0008	0.0008	0.0009	0.0009
λ(N)	3E-05	3E-05	3E-05	3E-05	2E-05	2E-05	2E-05	2E-05

Table 40. Full Sensitivity Experiment with Decreasing Effectiveness Factor by 50% (Redesign Induced FM Model).

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
Δt_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Δc1	0	0	0	0	0	0	0	0
Δt_2	-18.93	-19.21	-19.55	-16.09	0.00	0.00	0.00	0.00
Δc2	-150	-152	-155	-127	0	0	0	0
ΔAccT2	-18.93	-19.21	-19.55	-16.09	0.00	0.00	0.00	0.00
ΔAccC2	-150	-152	-155	-127	0	0	0	0
Δt_3	-73.83	-87.54	-48.14	-75.51	0.00	0.00	0.00	0.00
Δc3	-733	-869	-478	-750	0	0	0	0
ΔAccT3	-92.76	-106.74	-67.69	-91.60	0.00	0.00	0.00	0.00
ΔAccC3	-883	-1021	-633	-877	0	0	0	0
Δt_4	-137.06	-132.12	-100.22	-97.63	-52.73	0.00	0.00	0.00
Δc4	-1560	-1504	-1141	-1111	-600	0	0	0
ΔAccT4	-229.82	-238.86	-167.91	-189.24	-52.73	0.00	0.00	0.00
ΔAccC4	-2443	-2525	-1774	-1989	-600	0	0	0
Δt_5	-306.52	-357.29	-266.80	-318.45	-37.90	0.00	-103.09	-125.47
Δc5	-3932	-4583	-3423	-4085	-486	0	-1322	-1610
ΔAccT5	-536.34	-596.15	-434.72	-507.68	-90.63	0.00	-103.09	-125.47
ΔAccC5	-6375	-7109	-5196	-6074	-1086	0	-1322	-1610
Δλ(A)	0.0006	0.0009	0.0005	0.0007	0.0000	0.0000	0.0001	0.0002
Δλ(N)	1E-05	1E-05	8E-06	9E-06	1E-06	0E+00	2E-06	2E-06
λ_Ratio	62.61%	56.86%	44.36%	47.75%	7.05%	0.00%	9.58%	10.38%

Table 41. Full Sensitivity Analysis with Decreasing Effectiveness Factor by 50% (Redesign Induced FM Model)

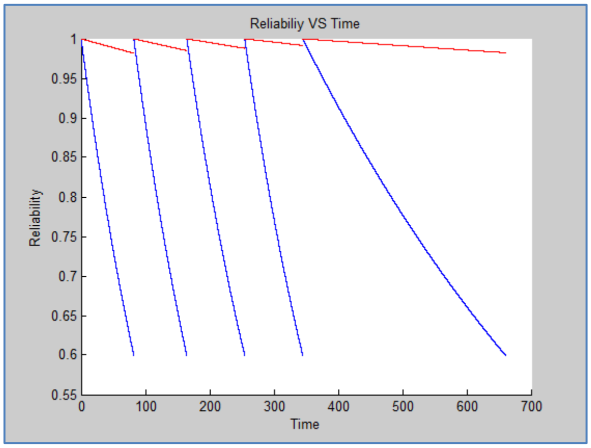

Figure 12. Reliability Plot with All EFs Decreased by 50% (Redesign Induced FM Model

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
t_1	81.89	81.89	81.89	81.89	81.89	81.89	81.89	81.89
c1	771	771	771	771	771	771	771	771
t_2	132.43	140.97	113.24	113.24	113.24	113.24	113.24	113.24
c2	1398	1466	1246	1246	1246	1246	1246	1246
AccT2	214.32	222.86	195.13	195.13	195.13	195.13	195.13	195.13
AccC2	2169	2237	2017	2017	2017	2017	2017	2017
t_3	285.48	266.24	339.23	229.87	229.87	229.87	229.87	229.87
c3	3235	3044	3769	2683	2683	2683	2683	2683
AccT3	499.80	489.10	534.35	425.00	425.00	425.00	425.00	425.00
AccC3	5405	5281	5786	4700	4700	4700	4700	4700
t_4	365.33	434.35	383.06	367.64	351.73	351.73	351.73	351.73
c4	4609	5395	4811	4635	4454	4454	4454	4454
AccT4	865.13	923.45	917.42	792.64	776.73	776.73	776.73	776.73
AccC4	10013	10676	10597	9336	9154	9154	9154	9154
t_5	873.67	694.83	778.35	851.99	674.66	717.23	717.23	674.66
c5	11708	9414	10485	11430	9155	9701	9701	9155
AccT5	1738.80	1618.28	1695.77	1644.63	1451.40	1493.96	1493.96	1451.40
AccC5	21721	20089	21082	20765	18309	18855	18855	18309
λ(A)	0.0006	0.0007	0.0007	0.0006	0.0008	0.0008	0.0007	0.0008
λ(N)	2E-05	2E-05	2E-05	2E-05	2E-05	2E-05	2E-05	2E-05

Table 42. . Full Sensitivity Experiment with Increasing Effectiveness Factor by 50% (Redesign Induced FM Model)

	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8
Δt_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Δc1	0	0	0	0	0	0	0	0
Δt_2	-18.93	-19.21	-19.55	-16.09	0.00	0.00	0.00	0.00
Δc2	-150	-152	-155	-127	0	0	0	0
ΔAccT2	-18.93	-19.21	-19.55	-16.09	0.00	0.00	0.00	0.00
ΔAccC2	-150	-152	-155	-127	0	0	0	0
Δt_3	-73.83	-87.54	-48.14	-75.51	0.00	0.00	0.00	0.00
Δc3	-733	-869	-478	-750	0	0	0	0
ΔAccT3	-92.76	-106.74	-67.69	-91.60	0.00	0.00	0.00	0.00
ΔAccC3	-883	-1021	-633	-877	0	0	0	0
Δt_4	-137.06	-132.12	-100.22	-97.63	-52.73	0.00	0.00	0.00
Δc4	-1560	-1504	-1141	-1111	-600	0	0	0
ΔAccT4	-229.82	-238.86	-167.91	-189.24	-52.73	0.00	0.00	0.00
ΔAccC4	-2443	-2525	-1774	-1989	-600	0	0	0
Δt_5	-306.52	-357.29	-266.80	-318.45	-37.90	0.00	-103.09	-125.47
Δc5	-3932	-4583	-3423	-4085	-486	0	-1322	-1610
ΔAccT5	-536.34	-596.15	-434.72	-507.68	-90.63	0.00	-103.09	-125.47
ΔAccC5	-6375	-7109	-5196	-6074	-1086	0	-1322	-1610
Δλ(A)	0.0006	0.0009	0.0005	0.0007	0.0000	0.0000	0.0001	0.0002
Δλ(N)	1E-05	1E-05	8E-06	9E-06	1E-06	0E+00	2E-06	2E-06
λ_Ratio	62.61%	56.86%	44.36%	47.75%	7.05%	0.00%	9.58%	10.38%

Table 43. Full Sensitivity Analysis with Increasing Effectiveness Factor by 50% (Redesign Induced FM Model)

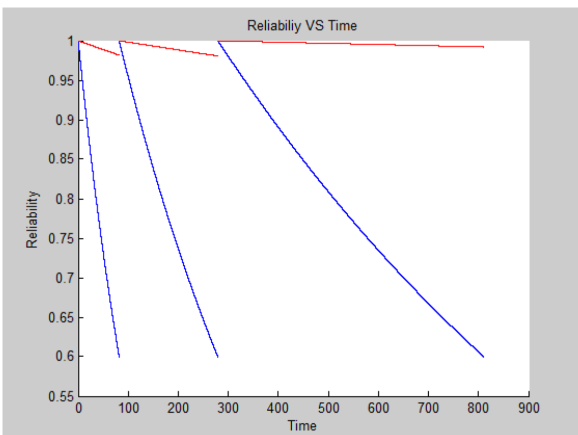

Figure 13. Sensitivity Plot with All EFs Increased by 50% (Redesign Induced FM Model)