Some Unpleasant Bargaining Arithmetic?*

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Abstract

It is commonly believed that voting rules that are relatively more inclusive (e.g., unanimity or supermajority), are likely to yield relatively more equitable outcomes than simple-majority rule. We show that this is not necessarily the case in bargaining environments. We study a multilateral bargaining model à la Baron and Ferejohn (1989), where players are heterogeneous with respect to the potential surplus they bring to the bargaining table. We show that unanimity rule may generate equilibrium outcomes that are more unequal (or less equitable) than under majority rule. In fact, as players become relatively more patient, we show that the more inclusive the voting rule, the less equitable the equilibrium allocations.

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1 Introduction

Multilateral bargaining is a staple of political economy, as many political negotiations entail bargaining among several players over the allocation of some surplus (e.g., legislative bargaining, government formation, domestic and international policy negotiations). Whenever negotiations involve more than two players, the voting rule that is used to specify how agreements are reached plays a fundamental role in determining the allocations that are ultimately agreed upon. Starting with the seminal contribution of Baron and Ferejohn (1989), several articles have studied the relative performance of alternative voting rules in multilateral bargaining models (e.g., Banks and Duggan (2000), Baron and Kalai (1993), Eraslan (2002), Eraslan and Merlo (2002), Harrington (1990), Yildirim (2007)). The emphasis of those papers, however, has been primarily on efficiency. In this paper, we focus on the distributional consequences (or equity properties) of different voting rules.

It is commonly believed that voting rules that are relatively more inclusive (e.g., unanimity or supermajority), are likely to yield relatively more equitable outcomes than simple-majority rule. For example, Buchanan and Tullock (1962; p. 190) argue that: "Majority decision-making (or any decision-making with less-than-unanimity rules for choice) will tend to produce some asymmetry in gain-sharing among the individual members of the group for which the choices are made. The members of the effective coalition will receive differentially larger shares of the benefits expected to result from collective action and/or they will bear differentially smaller shares of the costs of collective action providing general benefits for the whole group." Their argument postulates that since unanimity safeguards the rights of each individual, it protects minorities from the possibility of expropriation and yields more equitable outcomes than majority rule. Similarly, in her analysis of the advantages and disadvantages of supermajority rules, Schwartzberg (2013; pp. 3-8) argues that: "Those who believe their right have been violated under majority rule turn to the promise of supermajority rule as a means of guaranteeing their protection. [...] Because the majority could act without the support of a minority, a simple-majority rule would not protect vulnerable minorities from abuse or neglect. [...] Majority rule may usher in injustice or exacerbate distributive inequalities." This general sentiment is also prevalent among constitutional scholars. In their defense of the original interpretation of the U.S. Constitution as "a supermajoritarian constitution," McGinnis and Rappaport (2013) maintain the superiority of supermajority rules over simple-majority rule on equity grounds. In a different context, the same authors also argue (McGinnis and Rappaport (1999; pp. 372-373)) that supermajority rules "may promote a more harmonious political existence by making it harder for interest groups to acquire other people's resources for themselves. [...] We maintain that supermajority rules can be preferable to majority rules for categories of legislation over which special interests have particular leverage. [...] Spending laws tend to be disproportionately favored by special interests and therefore a supermajority rule should be required to offset the effects of these special interests."[1]

We show that, in bargaining environments, it is not necessarily the case that relatively more inclusive voting rules lead to relatively more equitable outcomes. We study a multilateral bargaining model à la Baron and Ferejohn (1989),where players are heterogeneous with respect to the potential surplus they bring to the bargaining table. We show that unanimity rule may generate equilibrium outcomes that are more unequal (or less equitable) than equilibrium outcomes under majority rule. In fact, as players become relatively more patient, the more inclusive the voting rule with respect to the number of votes required to induce agreement, the less equitable the equilibrium allocations. These results are a direct implication of basic insights from bargaining theory (some *unpleasant bargaining arithmetic*?).

Like in Baron and Ferejohn (1989), we study an infinite-horizon, n-player bargaining model where in every period each player is randomly selected with equal probability to make a proposal on how to allocate some surplus. The players' payoffs are linear in the amount of surplus they receive and they evaluate future payoffs using a common discount factor. The voting rule specifies the minimum number of players who have to vote in favor of a proposal for it to be implemented. This number varies from one (dictatorship) to n (unanimity). Our point of

[1] In a broader context, Acemoglu and Robinson (2012) also argue that concentration of political power leads to economic inequality. For example, in their interpretation of Egyptian poverty they write (p. 3): "Egypt is poor precisely because it has been ruled by a narrow elite that have organized society for their own banefit at the expense of the vast mass of people. Political power has been narrowly concentrated, and has been used to create great wealth for those who possess it, such as the $70 billion fortune apparently accumulated by ex-president Mubarak. The losers have been the Egyptian people, as they only too well understand."

departure from the canonical model is to assume that players differ with respect to the amount of surplus they have available for distribution in the event they are selected as proposers. This represents the only source of heterogeneity among players, and is taken to be an inalienable characteristic of each individual. For example, different players may have different (innate) abilities in formulating and implementing proposals, which make them inherently more or less "productive" and cannot be transferred across players or mimicked by others.[2] One of the most striking results we obtain is that under the unanimity rule it is possible for the most productive player to appropriate the entire surplus in equilibrium. On the other hand, under any other voting rule, it is always the case that some other players also have positive equilibrium payoffs. Consequently, unanimity rule may generate equilibrium outcomes that are more unequal (or less equitable) than equilibrium outcomes under majority rule. At the same time, equilibrium outcomes under majority rule need not be efficient even though equilibrium outcomes under unanimity rule are always efficient (Merlo and Wilson (1998) and Eraslan and Merlo (2002)). Thus our results highlight a trade-off between fairness and efficiency of agreement rules.

There is a recent, related literature that studies the implications of alternative voting rules in a variety of economic and political environments. Dixit, Grossman and Gül (2000) analyze the extent to which political compromise arises in a dynamic environment where two parties interact repeatedly and their political strength changes stochastically over time according to a Markov process. The party that is in power at any given time (i.e., the party whose political strength exceeds a given threshold determined by the voting rule), decides to what extent it is willing to share the available surplus with the opposition (i.e., the political compromise). They show that, depending on the degree of persistence in the parties' political strength, there may be less political compromise, and hence more inequality, under supermajority (where if neither party's strength exceeds the designated threshold, then both parties must agree to any policy change), than under simple majority rule.

Compte and Jehiel (2010) compare the performance of alternative voting rules in a collective search environment where exogenously specified proposals, which may differ along many

[2]Baccara and Razin (2007) study a model where firms bargain over the implementation of new ideas and the distribution of the rents they generate. Like in our model, people can only propose their own ideas.

dimensions, are drawn independently from a known distribution. A committee considers new proposals sequentially and search stops when the current proposal receives the support of a given number of committee members specified by the voting rule. They show that, depending on the voting rule and on the number of dimensions of the policy space, some committee members may have no real voting power, "in the sense that small changes in the objectives or preferences of such members would not affect at all the set of possible agreements" (p. 190).

Our paper analyzes a bargaining game with exogenous status quo in which bargaining ends after an agreement is reached. There is an active literature on bargaining with endogenous status quo where the game continues after agreement is reached, and the default allocation in future negotiations is given by the most recent agreed upon allocation. Several papers in this literature have shown the existence of equilibria in which one player extracts all the surplus,[3] but to our knowledge, there are no studies comparing distributional properties of different agreement rules.[4]

The rest of the paper is organized as follows. In Section 2, we present the model and in Section 3, we characterize the equilibrium payoffs for different voting rules. In Section 4, we use the characterization result from Section 3 to present an example to illustrate our main result. We compare the equilibrium implications of different voting rules with respect to their equity properties in Section 5 and conclude in Section 6. The proofs of the formal results stated in the main text are in the appendix.

2 The Model

Consider a situation where $n > 2$ players have to collectively decide how to allocate some surplus. Each player i is endowed with a *potential surplus* $y_i > 0$, denoting the amount of

[3]See Kalandrakis (2004), Bernheim, Rangel and Rayo (2006), Kalandrakis (2010), Ali, Bernheim, Fan (2014), Nunnari (2016). Winter (1996) obtains the same result with exogenous status quo in the presence of a veto player as the players become perfectly patient.

[4]Yıldırım (2010) studies the distributional consequences of alternative bargaining protocols. He considers bargaining environments where players can contest the right to make proposals and compares an environment where proposal rights are determined once-and-for-all before the beginning of a negotiation, to one where a contest determining the identity of the proposer takes place prior to each bargaining round. He shows that when agreement requires unanimous consent and players differ with respect to their discount factor, equilibrium allocations are relatively more unequal in the former environment than in the latter.

surplus she would have available for distribution if selected as the proposer. As mentioned in the Introduction, this situation would arise, for example, in an environment where players are heterogeneous with respect to their ability in formulating and implementing proposals. These (innate) abilities make the players inherently more or less "productive" and cannot be transferred across players or mimicked by others (e.g., Baccara and Razin (2007)). We enumerate the players from the least productive to the most productive, i.e., $y_1 \leq y_2 \leq \ldots \leq y_n$.

In each period, a player is randomly offered the possibility of submitting a proposal with probability $1/n$.[5] The selected player i may then make a proposal specifying the way she would distribute surplus y_i among the players, or forego the opportunity and pass. If a proposal is submitted, all players then vote (sequentially) on whether or not to approve it. If at least $q \in \{1, ..., n\}$ people including the proposer accept the proposal, the game ends and the surplus is shared according to the accepted proposal. Otherwise, a new player is selected as the proposer and the process repeats itself (possibly *ad infinitum*).

Players have an identical, single date, von Neumann-Morgenstern payoff function that is linear in their own share of the surplus, and discount the future with a common discount factor $\delta \in (0, 1)$. In the event that agreement is never reached, all players receive a payoff of zero.

If $q = n$, then the agreement rule is unanimity and the game is a special case of the stochastic bargaining model of Merlo and Wilson (1995, 1998) in which the "cake" process and the "proposer" process are perfectly correlated. If n is odd and $q = (n + 1)/2$, then the agreement rule is majority rule as in Eraslan and Merlo (2002). If, in addition, the surplus available for distribution is the same for all players, i.e., $y_1 = ... = y_n$, then the game reduces to the one studied by Baron and Ferejohn (1989). For any $q \in \{1, ..., n\}$, we refer to the voting rule as the q-quota rule.[6]

Let h^t denote the past history of the game up to time t (i.e, the identity of the previous proposers, whether they made proposals, the proposals they made if they made any, and how

[5] Since the focus of our analysis is to study the equity properties of alternative voting rules, we deliberately assume an egalitarian bargaining protocol. Eraslan (2002) and Merlo and Wilson (1995, 1998) study bargaining environments where players differ with respect to the probability of being selected as proposer and show that equilibrium payoffs are non-decreasing in such probability, for any voting rule.

[6] In what follows, when we refer to q-quota, we omit the quantifier with the understanding that $q \in \{1, ..., n\}$. Likewise, we omit the quantifiers on generic players i and j with the understanding that $i, j \in \{1, ..., n\}$.

each player voted for these proposals), together with the identity of the current proposer and the proposal she made if she made one. A (behavior) strategy ψ_i for player i is a probability distribution over feasible actions for each date t and history at date t. A strategy profile ψ is an n-tuple of strategies, one for each player. Let $G(h^t)$ denote the game from date t on with history h^t. Let $\psi|h^t$ denote the restriction of ψ to the histories consistent with h^t. Then $\psi|h^t$ is a strategy profile on $G(h^t)$. A strategy profile ψ is subgame perfect if, for every history h^t, $\psi|h^t$ is a Nash equilibrium of $G(h^t)$. An SSP strategy profile is a subgame perfect strategy profile with the property that the actions prescribed at any history depend only on the proposer and offer. A stationary, subgame perfect (SSP) outcome and payoff are the outcome and payoff generated by an SSP strategy profile.

It is well known that in multilateral bargaining games like the one considered here there is multiplicity of subgame perfect equilibria even under unanimity rule (e.g., Sutton (1986)). However, it has also been recognized that stationarity is typically able to select a unique equilibrium (e.g., Baron and Ferejohn (1989), Merlo and Wilson (1995)). Thus, we restrict attention to SSP equilibria.

3 Characterization of SSP Payoffs

In this section, we characterize the set of SSP payoffs, i.e. the set of continuation payoff vectors, and study their properties. Given an SSP payoff vector v, player i accepts a proposal x if $x_i \geq v_i$ and rejects it if $x_i < v_i$.[7] Let $r_{ij}(v)$ denote the probability that i includes j in her coalition when she is selected as the proposer and when the payoff vector is v (i.e., the probability that i offers j a payoff equal to v_j), and $r_i(v) = (r_{i1}(v), ..., r_{in}(v))$. Let $w_i(v)$ denote the *total cost* to player i of her coalition partners (i.e., the total amount of surplus player i has to offer to her

[7]We assume without loss of generality that player i accepts an offer when indifferent. To see that this assumption is without loss of generality, suppose to the contrary that player i rejects an offer x with $x_i = v_i$ with positive probability. If there is no player j who makes an offer x with $x_i = v_i$ with positive probability in equilibrium, then clearly player i's decision when indifferent is irrelevant, and hence, there is another equilibrium which is payoff equivalent to the original equilibrium in which player i accepts any offer x with $x_i = v_i$ with probability one. If instead there is some player j who makes an offer x with $x_i = v_i$ with positive probability, then player j can increase his payoff by slightly increasing x_i and decreasing x_j.

coalition partners to induce them to support her proposal), that is,

$$w_i(v) = \sum_{j=1}^{n} r_{ij}(v)v_j. \tag{1}$$

We maintain the convention that each player i includes herself as a coalition partner (i.e., $r_{ii}(v) = 1$). This is without loss of generality, since the payments made to self cancel out with the distribution received from self.

Since the game we consider is a stochastic bargaining game, agreement need not be reached immediately (Merlo and Wilson (1995, 1998)). In particular, agreement is not reached in any given period if the surplus available in that period is sufficiently small relative to the expected surplus next period. Given a payoff vector v, let $\alpha_i(v) \in [0, 1]$ denote the probability that agreement is reached when i is the proposer. In equilibrium, we must have

$$\alpha_i(v) = \begin{cases} 1 & \text{if } y_i - w_i(v) > 0, \\ 0 & \text{if } y_i - w_i(v) < 0. \end{cases} \tag{2}$$

If $y_i = w_i(v)$, when selected as the proposer player i is indifferent between passing and proposing and therefore $\alpha_i(v)$ can take any value in $[0, 1]$.

If player i is not the proposer, then she receives her continuation payoff when player j is the proposer in one of two ways: either player j includes player i in his coalition, or player j passes and the bargaining game continues to the next period. Let $\mu_{ji}(v; r(v))$ denote the probability that i receives her continuation payoff v_i when j is the proposer and the payoff vector is v, given the offer probabilities $r(v) = [r_i(v)]_{i=1}^{n}$. Formally,

$$\mu_{ji}(v; r(v)) = \alpha_j(v)r_{ji}(v) + (1 - \alpha_j(v)). \tag{3}$$

Finally, let

$$\mu_i(v; r(v)) = \sum_{j=1}^{n} \frac{1}{n} \mu_{ji}(v; r(v)) \tag{4}$$

denote the total probability that i receives her continuation payoff v_i either because of delay or because she is included in the winning coalition.

The generic definition of SSP equilibria stated in the previous section is not very useful for analysis. The following proposition provides an alternative definition of SSP payoff vectors for any q-quota game. Proposition A.1, stated and proved in the appendix, establishes their existence for any q-quota game.

Proposition 1. *v is an SSP payoff vector for the q-quota game if and only if for all i*

$$v_i = \delta[\frac{1}{n}\alpha_i(v)(y_i - w_i(v)) + \mu_i(v; r(v))v_i] \tag{5}$$

and

$$r_i(v) \in \arg\min_{z\in[0,1]^n} \sum_k z_k v_k \quad \text{subject to} \quad \sum_{k\neq i} z_k \geq q-1 \text{ and } z_i = 1. \tag{6}$$

The expression in brackets on the right hand side of equation (5) is the expected payoff to player i. With probability $1/n$ player i is the proposer. If she decides to propose an allocation that would be accepted, then she receives her surplus y_i net of the cost of her coalition partners (including herself). Otherwise, she either passes or proposes an allocation that would be rejected, and receives her continuation payoff in either case. Our convention that $r_{ii}(v) = 1$ implies that player i receives her continuation payoff (over and above the proposer's surplus $y_i - w_i(v)$ if $\alpha_i(v) > 0$) with probability one when she is the proposer. This happens with probability $1/n$. With probability $(n-1)/n$, on the other hand, someone else is the proposer and player i receives her continuation payoff with probability $\sum_{j\neq i} r_{ji}(v)$. Since $\mu_i(v; r(v)) = \sum_{j=1}^{n} \frac{1}{n}\mu_{ji}(v; r(v))$, the expression on the right hand side of equation (5) is the discounted expected payoff of player i. In equilibrium, this must equal her SSP payoff.

Before discussing uniqueness of SSP equilibrium payoffs, we present some properties of SSP equilibria.

3.1 Properties of SSP Equilibria

In this section, we establish some properties of SSP equilibria which are needed in proving our main results. In what follows, we let $v = (v_1, \ldots, v_n)$ denote an arbitrary SSP equilibrium payoff vector for any q-quota game.

Our first result establishes that if a delay is possible when a player is the proposer, then his or her payoff must be zero.

Lemma 1. *For all i, if $\alpha_i(v) < 1$, then $v_i = 0$.*

In the class of bargaining games we consider, players derive their bargaining power mainly from two sources: their ability to propose and their ability to vote against a proposal. As shown by Kalandrakis (2006), the proposal power is in general much more significant in determining a player's bargaining power. Our analysis confirms and strengthens this result. In particular, if an equilibrium admits the possibility of delay when a player is the proposer, then that player cannot make use of his or her proposal power at all, which in turn results in a complete loss of bargaining power and hence an equilibrium payoff of zero.

The above lemma implies that if $v_i > 0$, then we must have $\alpha_i(v) = 1$, which is possible when $y_i \geq w_i(v)$. The following lemma strengthens this result and shows that the inequality must be strict.

Lemma 2. *For all i, $v_i > 0$ if and only if $y_i > w_i(v)$.*

Equation (2) states that if the surplus available when player i is the proposer exceeds the cost of her coalition, then there is agreement when i is the proposer. Recall that we maintain the convention that each player i includes herself in her coalition. The next lemma shows that agreement is also reached if the surplus available when player i is the proposer exceeds the cost of her coalition net of the cost of including herself as a coalition partner.

Lemma 3. *For all i, if $y_i > w_i(v) - v_i$, then $\alpha_i(v) = 1$.*

From equation (5), it can be observed that a player's payoff depends on three endogenous factors: the probability of agreement when she is the proposer, the total cost of his or her coalition partners, and the probability that she receives his or her continuation payoff when she is not the proposer. Given that a player's payoff is decreasing in the total cost of his or her coalition, one might expect that "cheaper" players have higher total costs. The next results shows that this is not the case. To the contrary, if player i has a lower continuation payoff than player j, then the total cost of player i's coalition cannot exceed the total cost of player j's coalition.

Lemma 4. *For all i, j, if $v_i \leq v_j$, then $w_i(v) \leq w_j(v)$.*

This result seems counterintuitive at first since for any two players i and j with $v_i \leq v_j$, the cost of coalition partners for player i is weakly higher than the cost of coalition partners for player j because they have access to the same set of coalition partners except for each other. But notice that the cost of coalition for a player includes the cost of his own vote. If players i and j do not include each other in their coalitions, then the total cost of their coalition partners excluding themselves are the same, and since $v_i \leq v_j$, the total cost of coalition for player i is lower. If instead player j includes player i with some positive probability, then we can obtain an upper bound on player i's total coalition cost by artificially restricting player i to include player j in his coalition with the same probability. In the proof of Lemma 4, we show that this upper bound is lower than player j's total coalition cost.

Note that if agreement is ever reached when player i is the proposer, then the surplus available net of the payments to her coalition partners must not be smaller than her own SSP payoff. Since the same coalition partners are also potentially available to any other player j when he is the proposer, it follows that if player j is more productive than player i, then agreement must always be reached when j is the proposer.

Lemma 5. *For all i, if $y_i > w_i(v)$, then $\alpha_j(v) = 1$ for all $j \geq i$.*

The next result establishes that if an equilibrium admits the possibility of delay when player i is the proposer, which implies that her payoff is equal to zero, then the equilibrium payoff of all less productive players $j < i$ is also zero.

Lemma 6. *For all i, if $\alpha_i(v) < 1$, then $v_j = 0$ for all $j \leq i$.*

When player i is "cheaper" than player j, player j cannot be included in other players' coalitions more often than player i. In addition, if agreement is reached with certainty when one of these players is the proposer, then it is also the case that the probability that player j receives his continuation payoff when he is not the proposer cannot be higher than the probability that player i receives her continuation payoff when she is not the proposer. This is the intuition for our next result.

Lemma 7. *For all i, j, if $v_i < v_j$ and $\alpha_i(v) = \alpha_j(v) = 1$, then $\mu_i(v; r(v)) \geq \mu_j(v; r(v))$.*

Since the only source of asymmetry among players is their productivity, if player i is less productive than player j, then one would expect j to fare no worse than i in equilibrium. The following lemma shows that this is indeed the case.

Lemma 8. *SSP payoffs are monotone: that is,* $v_i \leq v_j$ *for all* $i < j$, $i, j = 1, ..., n$.

From Lemma 1, if there is a possibility of no agreement when player q is the proposer, then $v_q = 0$ for any q-quota game. By Lemma 8, it follows that $v_i = 0$ for all $i \leq q$. But then the cost of a winning coalition for player q is zero as she needs $q - 1$ votes in addition to his vote in order to secure acceptance of her proposal. This argument implies the following result:

Lemma 9. *In any* q*-quota game agreement is always reached when player* q *is the proposer and player* q *always receives a positive payoff: that is,* $\alpha_q(v) = 1$ *and* $v_q > 0$.

For the class of games we consider here, when $q = n$ (i.e., in the unanimity game), the results of Merlo and Wilson (1998) imply that the SSP payoff vector is unique. It is also straightforward to see that equilibrium is unique when $q = 1$ (i.e. under random dictatorship). In what follows, we characterize the unique equilibria for these special cases.

3.2 Special Cases: Unanimity Rule and Random Dictatorship

Let $v^n(\delta) = (v_1^n(\delta), \ldots, v_n^n(\delta))$ denote the unique equilibrium under unanimity rule when the discount factor is δ. Let κ denote the player with the lowest index such that the equilibrium probability of an agreement being reached when she is the proposer is positive: that is, κ is the smallest i such that $\alpha_i(v^n(\delta)) > 0$. Under unanimity rule, all players receive their continuation payoffs regardless of the identity of the proposer, and regardless of whether agreement is reached or not. Hence, $r_{ij}(v^n(\delta)) = 1$ for all $\delta \in (0, 1)$ and for all i, j. It follows that $\mu_i(v^n(\delta); r(v^n(\delta))) = 1$ for all $\delta \in (0, 1)$ and for all i, and equation (5) reduces to:

$$v_i^n(\delta) = \begin{cases} 0 & \text{if } i < \kappa, \\ \delta[\frac{1}{n}(y_i - \sum_{j=1}^n v_j^n(\delta)) + v_i^n(\delta)] & \text{if } i \geq \kappa. \end{cases} \tag{7}$$

Summing over all i and rearranging, we obtain:

$$\sum_{i=1}^n v_i^n(\delta) = \sum_{i=\kappa}^n v_i^n(\delta) = \frac{\delta \sum_{i=\kappa}^n y_i}{(1-\delta)n + \delta(n - \kappa + 1)}. \tag{8}$$

Substituting back in equation (7) allows us to characterize the unique equilibrium under unanimity rule.

Proposition 2. *The payoffs in the unique equilibrium under unanimity rule are given by*

$$v_i^n(\delta) = \begin{cases} 0 & \text{if } i < \kappa, \\ \frac{\delta}{n(1-\delta)}\left(y_i - \frac{\delta \sum_{i=\kappa}^n y_i}{(1-\delta)n+\delta(n-\kappa+1)}\right) & \text{if } i \geq \kappa. \end{cases} \tag{9}$$

where

$$\kappa = \min\{j : y_j \geq \frac{\delta \sum_{i=j}^n y_i}{(1-\delta)n + \delta(n-j+1)}\}.$$

When $y_{n-1} < y_n$ and players are sufficiently patient, we have a sharper characterization as a corollary of Proposition 2. Let

$$\delta_n = \frac{n y_{n-1}}{y_n + (n-1)y_{n-1}}. \tag{10}$$

Note that $\delta_n < 1$ when $y_{n-1} < y_n$.

Corollary 1. *If $y_{n-1} < y_n$ and $\delta > \delta_n$, then $v_n^n(\delta) = \frac{\delta y_n}{n-\delta(n-1)}$ and $v_i^n(\delta) = 0$ for all $i < n$.*

This means that under the unanimity rule, if $y_{n-1} < y_n$ and players are sufficiently patient, then agreement is reached only when player n is the proposer in which case she receives the entire surplus under the agreed upon allocation. Intuitively, this is a consequence of the result in Merlo and Wilson (1998) that the unique equilibrium under unanimity rule is efficient. If players are sufficiently patient, efficiency requires agreement only to occur when the largest surplus is available for distribution.

We next characterize the equilibrium under random dictatorship. Let $v^1(\delta) = (v_1^1(\delta), \ldots, v_n^1(\delta))$ denote the unique equilibrium under random dictatorship when the discount factor is δ. By Lemma 9, agreement is reached all the time, and since no player ever needs the vote of another player, which immediately implies the following result.

Proposition 3. *The payoffs in the unique equilibrium under random dictatoship are given by*

$$v_i^1(\delta) = \frac{\delta}{n} y_i \tag{11}$$

for all i.

For general q, uniqueness of the SSP equilibrium payoff vector is guaranteed when the surplus to be divided is the same for all players, i.e., $y_1 = \ldots = y_n$ (Baron and Ferejohn (1989), Eraslan (2002)). However, the equilibrium payoffs need not be unique for general agreement rules when the surplus to be divided is stochastic (Eraslan and Merlo (2002)). In the next section, we first illustrate multiplicity of equilibria with an example, and then characterize the unique SSP equilibrium payoffs associated with the most efficient equilibrium for any q-quota game.

3.3 General q-quota Games

To see that equilibria need not be unique for q-quota games when $q \in \{2, \ldots, n-1\}$, consider the following example. There are three players with a common discount factor δ equal to 0.9. The surplus available for distribution is $y_1 = 0.4$ when player 1 is the proposer, and $y_2 = y_3 = 1$ when either player 2 or player 3 are the proposer. The agreement rule is majority rule (i.e., $q = 2$).

By Lemma 9, agreement always occurs when player 2 is the proposer and $v_2 > 0$. Then, by Lemma 2, $y_2 > w_2(v)$, and so, by Lemma 5, agreement always occurs when player 3 is the proposer as well. Let p denote the probability that agreement occurs when player 1 is the proposer. Given p, let $v_i(p)$ denote the payoff to player i and $v(p) = (v_1(p), v_2(p), v_3(p))$. In equilibrium, $\alpha_1(v(p)) = p$. Lemma 8 implies that $v_2(p) = v_3(p)$ for all p.

First, consider the possibility that $p = 0$. By Lemma 1, $v_1(0) = 0$. Since $p = 0$, player 2 and player 3 receive their continuation payoffs with probability one when player 1 is the proposer, i.e., $\mu_{12}(v(0)) = \mu_{13}(v(0)) = 1$, and the cost of including player 1 in the winning coalition when either player 2 or player 3 are the proposer is zero.[8] Hence, $\mu_{23}(v(0)) = \mu_{32}(v(0)) = 0$. Substituting in equation (5), we have:

$$\begin{aligned} v_2(0) &= 0.9\left(\frac{1}{3} + \frac{1}{3}v_2(0)\right), \\ v_3(0) &= 0.9\left(\frac{1}{3} + \frac{1}{3}v_3(0)\right). \end{aligned}$$

Since $v_2(0) = v_3(0) = 0.429 > y_1 = 0.4$, it is indeed optimal for player 1 to pass when she is the proposer, which is consistent with $p = 0$. Therefore, there is an equilibrium with $v = (0, 0.429, 0.429)$, $\alpha(v) = (0, 1, 1)$, $r_2(v) = (1, 1, 0)$ and $r_3(v) = (1, 0, 1)$.

Next, consider the possibility that $p = 1$, with $v_1(1) < v_2(1) = v_3(1)$. Since player 1 has the lowest SSP payoff, she is included with probability one in the winning coalition when either

[8] To simplify notation, in the example, we omit r as an argument of μ.

player 2 or player 3 are the proposer, i.e., $r_{21}(v(1)) = r_{31}(v(1)) = 1$, and the cost of including player 1 in the winning coalition is $v_1(1)$. In this case, however, when player 1 is the proposer, players 2 and 3 are no longer guaranteed to receive their SSP payoffs. Since agreement is reached with probability one, and agreement requires the consent of only one additional player in addition to the proposer, we have that $\mu_{12}(v(1)) = r_{12}(v(1))$ and $\mu_{13}(v(1)) = r_{13}(v(1))$. Substituting in equation (5), we have:

$$v_1(1) = 0.9\left(\frac{0.4}{3} - \frac{1}{3}v_j(1) + \frac{2}{3}v_1(1)\right),$$

and

$$v_j(1) = 0.9\left(\frac{1}{3} - \frac{1}{3}v_1(1) + \frac{1}{3}r_{1j}v_j(1)\right),$$

for $j = 2, 3$. Since $r_{12}(v(1)) + r_{13}(v(1)) = 1$, we obtain that $v_1(1) = 0.048$, $v_2(1) = v_3(1) = 0.336 < 0.4 = y_1$, and $r_{12}(v(1)) = r_{13}(v(1)) = \frac{1}{2}$. This, in turn, implies that it is optimal for player 1 to make a proposal when she is the proposer, which is consistent with $p = 1$. Therefore, there is an equilibrium with $v = (0.048, 0.336, 0.336)$, $\alpha(v) = (1, 1, 1)$, $r_1(v) = \left(1, \frac{1}{2}, \frac{1}{2}\right)$, $r_2(v) = (1, 1, 0)$ and $r_3(v) = (1, 0, 1)$.

In addition, there is a third equilibrium in which player 1 is indifferent between proposing and passing when she is the proposer. This implies that $v_2(p) = v_3(p) = 0.4$, $v_1(p) = 0$, and the cost of including player 1 in the winning coalition when either player 2 or player 3 are the proposer is zero. Notice that by symmetry $r_{12}(v(p)) = r_{13}(v(p)) = \frac{1}{2}$, otherwise Lemma 8 would be violated, possibly after re-enumerating the players. Hence, for $j = 2, 3$, $\mu_{1j}(v(p)) = pr_{1j}(v(p)) + (1 - p) = 1 - \frac{p}{2}$. Substituting in equation (5), we have:

$$v_j(p) = 0.9\left(\frac{1}{3} + \frac{1}{3}(1 - \frac{p}{2})v_j(p)\right)$$

for $j = 2, 3$, which in turn implies that $p = 1/3$.

Note that $v_1(p)$ is increasing in $r_{21}(v(p))$ and $r_{31}(v(p))$, but $r_{21}(v(p))$ and $r_{31}(v(p))$ are decreasing in $v_1(p)$. This implies that there is no equilibrium in which $p > 0$ and $v_1(p) = v_2(p) = v_3(p)$. Thus, the three equilibria characterized above are the only equilibria.

The intuition for this multiplicity result is as follows. When player 1 is the proposer, she needs the vote of only one other player, whereas if she passes, both players 2 and 3 receive their SSP payoff. This means the payoffs of players 2 and 3 are increasing in the probability that player 1 passes when she is the proposer. At the same time, as the payoffs of players 2 and 3 increase, it is more likely that player 1 cannot afford to have his proposal accepted, decreasing the probability that his proposal passes. This reinforcement effect makes the payoffs of players 2 and 3 even higher.

The following result uniquely characterizes the most efficient equilibria in any q-quota game when the players are sufficiently patient.

Proposition 4. *Fix a q-quota game with* $q \in \{2, \ldots, n-1\}$*. If* $y_i < y_{i+1}$ *for all* $i < q$ *and* $y_q > (n-q+1)y_{q-1}$*, then there exists* $\delta_q \in (0,1)$ *such that, for all* $\delta > \delta_q$*, the most efficient equilibrium of the q-quota game satisfies*

$$v_i = \begin{cases} 0 & \textit{if } i < q, \\ \frac{\delta y_i}{n - \delta(q-1)} & \textit{otherwise.} \end{cases} \tag{12}$$

Intuitively, when the discount factor is sufficiently high and when the surplus of player q is sufficiently larger than the surplus of player $q-1$, efficiency requires that agreement is not reached when player $q-1$ is the proposer. If that is the case, then by Lemma 1, we must have $v_i = 0$ for all players $i < q$. This in turn implies that the cost of winning coalition partners for players $i \geq q$ is zero, and therefore they receive strictly positive payoffs. Consequently, they are never included in each other's coalitions, and they receive their continuation payoffs only when there is no agreement as a result of one of the players $i < q$ being chosen as the proposer.

Notice that even though Proposition 4 focuses on the case when $q \neq 1$ and $q \neq n$, the equation (12) characterizing the most efficient equilibrium payoffs is also satisfied by the unique equilibrium when $q = n$ and the players are sufficiently patient (see Corollary 1) and the unique equilibrium when $q = 1$ regardless of the discount factor (see Proposition 3).

Before moving on to establishing our main result formally, we provide an example in the next section.

4 An Example

In this section, we illustrate our main result with an example. Suppose there are three players with $y_1 = 0.25$, $y_2 = 0.8$, $y_3 = 1$, and the discount factor is $\delta = 0.95$.

If the agreement rule is unanimity rule, then player 3 receives his continuation payoff when any other player is the proposer regardless of whether that player passes or makes an offer that is accepted. This in turn makes player 3 so expensive that even player 2 is unable to make an offer that would be accepted by player 3. In equilibrium $v_1 = v_2 = 0$ and $v_3 = 0.86$. To see these are indeed equilibrium continuation payoffs, note that player 3 is the proposer with 1/3 probability, in which case she receives y_3, and with probability 2/3, another player is the proposer, and player 3 receives her continuation payoff v_3. In equilibrium, we must have $v_3 = \delta(\frac{1}{3}y_3 + \frac{2}{3}v_3)$ which is satisfied only when $v_3 = 0.86$. Since the surplus player $i \neq 3$ has when he is the proposer is lower than the continuation payoff of player 3, it is indeed the case that player i passes and obtains his continuation payoff. Since he also receives his continuation payoff when any other player is the proposer, we must have $v_i = \delta v_i$, i.e. $v_i = 0$ for all $i \neq 3$.

If the agreement threshold is $q = 2$, then when player 2 is the proposer, he is now able to offer a proposal that would be accepted. This is because he no longer needs the consent of player 3 for his offer to be accepted. This in turn makes player 2 relatively expensive, and player 1 is still not able to make an offer that would be accepted by player 2. Since player 3's vote is even more expensive than player 2's, player 1 passes when he is the proposer. In equilibrium, $v_1 = 0$, $v_2 = 0.37$, and $v_5 = 0.46$. To see, note that when either player 2 or player 3 is the proposer, it is optimal to exclude each other since the vote of player 1 is sufficient to pass a proposal. Thus, players 2 and 3 receive their continuation payoffs only when player 1 is the proposer which happens with probability 1/3. As such, $v_i = \delta(\frac{1}{3}y_i + \frac{1}{3}v_i)$ for $i = 2, 3$ which are satisfied only when $v_2 = 0.37$, and $v_3 = 0.46$. Since the surplus players 1 has when he is the proposer is lower than the continuation payoff of player 2, it is indeed the case that he passes and obtains a payoff of zero when he is the proposer, and receives his continuation payoff when any other player is the proposer. As before, this implies that the payoff of player 1 is zero. The parameters of this example satisfy the conditions in the statement of Proposition 4, and the discount factor is higher than the lower bound constructed in the proof of Proposition 4 when $q = 2$. Thus, the above construction characterizes the most efficient equilibrium when $q = 2$.

Finally if the agreement threshold is $q = 1$, i.e. if the agreement rule is random dictatorship, then agreement is reached all the time, and the continuation payoffs are given by $v_1 = 0.08, v_2 = 0.25, v_3 = 0.32$. It can be easily seen that as the agreement rule becomes more inclusive, the inequality increases in the most efficient equilibrium. In the next section, we generalize this observation.

5 The Equity Properties of Alternative Voting Rules

In this section, we compare the equity of equilibrium payoffs (i.e., the relative inequality of the distribution of equilibrium payoffs), under alternative voting rules. The measure of inequality we use is the Gini coefficent. As is well known, as a distribution becomes more unequal, its Gini coefficient increases. At the extremes, a Gini coefficient of 0 corresponds to complete equality, while a Gini coefficient of 1 corresponds to complete inequality with only one element in the distribution taking a strictly positive value and all others being zero.

Formally, we let G_y denote the Gini coefficient for the distribution of potential surplus across players which summarizes the fundamental heterogeneity in the bargaining environment we consider:

$$G_y = \frac{2\sum_{i=1}^{n} i y_i}{n\sum_{i=1}^{n} y_i} - \frac{n+1}{n}. \tag{13}$$

By Lemma 8, any equilibrium payoff vector $v = (v_1, \ldots, v_n)$ is monotone, and so we may define the Gini coefficient associated with that payoff vector as

$$G(v) = \frac{2\sum_{i=1}^{n} i v_i}{n\sum_{i=1}^{n} v_i} - \frac{n+1}{n}. \tag{14}$$

We first show that unanimity rule always induces equilibrium outcomes that are at least as unequal as the fundamentals. Specifically, we show that for any $\delta \in (0,1)$, the Gini coefficient associated with the unique SSP payoff vector $v^n(\delta)$ of the unanimity rule game with discount factor δ is at least as large as the Gini coefficient for the distribution of potential surplus across players, that is, $G(v^n(\delta)) \geq G_y$ for all $\delta \in (0,1)$. We also show that random dictatorship always induces equilibrium outcomes as unequal as the fundamentals.

Proposition 5. *$G(v^n(\delta)) \geq G_y = G(v^1(\delta))$ for all $\delta \in (0,1)$, and the inequality is strict if and only if $y_i \neq y_j$ for some $i, j = 1, ..., n$.*

Now consider any q-quota rule and let $v^q(\delta)$ denote an equilibrium payoff vector under the q-quota rule when the discount factor is δ. Let $\tilde{\kappa}$ denote the player with the lowest index such that $y_i = y_n$, and let

$$\tilde{\delta} = \frac{n y_{\tilde{\kappa}-1}}{(n-\tilde{\kappa}+1)y_n + (\tilde{\kappa}-1)y_{\tilde{\kappa}-1}}. \tag{15}$$

Note that $\tilde{\delta} = \delta_n$ (recall equation (10)) when $y_{n-1} < y_n$.

By Lemma 8, all players $i > \tilde{\kappa}$ receive the same payoff. This implies that $\mu_i(v^q(\delta); r(v^q(\delta))) = \mu_n(v^q(\delta); r(v^q(\delta)))$ for all $i > \tilde{\kappa}$. If agreement is not reached when $q-1$ is the proposer, i.e. if $\alpha_{q-1}(v^q(\delta)) = 0$, then we must have $\tilde{\kappa} \geq q$ and $\mu_i(v^q(\delta); r(v^q(\delta))) = \frac{q}{n}$ for all $i > \tilde{\kappa}$ since players $1, \ldots, q-1$ constitute the cheapest coalition partners, so each player $i > \tilde{\kappa}$ receives her continuation payoff only when she is the proposer or one of the players $1, \ldots, q-1$ are the proposers (in which case there is no agreement). If instead agreement is reached with positive probability when player $q-1$ is the proposer, then not all players receive their continuation payoff when player $q-1$ proposes, and by Lemma 8, players $\tilde{\kappa}, \ldots, n$ are the more expensive coalition partners and are therefore excluded with positive probability from the winning coalition when another player is the proposer. Thus, $\mu_i(v^q(\delta); r(v^q(\delta))) \leq \frac{q}{n}$. Furthermore, in this case, the cost of coalition partners for players $i > \tilde{\kappa}$ is at least $v_n^q(\delta)$ (recall that i's coalition partners include player i herself), and $v_i^q(\delta) = v_n^q(\delta)$. Consequently, $v_n^q(\delta) \leq \frac{\delta y_n}{n-\delta(q-1)}$. Thus, if $q \leq \tilde{\kappa}$ and $\delta > \tilde{\delta}$, then

$$v_i^q(\delta) \leq \frac{\delta y_n}{n-\delta(q-1)} \leq \frac{\delta y_n}{(1-\delta)n + \delta(n-\tilde{\kappa}+1)} = v_i^n(\delta)$$

for all $i \geq \tilde{\kappa}$, and if $q < \tilde{\kappa}$, then $v_i^q(\delta) < v_i^n(\delta)$ for all $i \geq \tilde{\kappa}$. This means that if players are sufficiently patient, then all players with the highest potential surplus fare better under unanimity rule than under the q-quota rule. Conversely, since $v_i^n(\delta) = 0$ for all $i < \tilde{\kappa}$ when δ is sufficiently large, all players except those with the highest potential surplus (weakly) prefer the q-quota rule to the unanimity rule.

More generally, the next proposition states that if δ exceeds the bound defined in (15), then the unique equilibrium outcome under unanimity rule is always more unequal than any equilibrium outcome under any q-quota rule with $q < n$ as long as $q \leq \tilde{\kappa}$.

Proposition 6. *If $\delta > \tilde{\delta}$, then $G(v^n(\delta)) \geq G(v^q(\delta))$ for any $q \leq \tilde{\kappa}$ and for any equilibrium payoff vector $v^q(\delta)$ of the q-quota game. The inequality is strict if $q < \tilde{\kappa}$.*

An immediate implication of Proposition 6 is that when $y_n > y_{n-1}$, the equilibrium outcome under unanimity rule is strictly more unequal than any equilibrium outcome under any q-quota rule with $q < n$ when players are sufficiently patient.

For the next result, let v^{*q} denote the most efficient equilibrium of the q-quota game characterized in Proposition 4 and let

$$\delta^* = \max\{\delta_2, \ldots, \delta_n\}$$

where δ_q is defined in the proof of Proposition 4 when $2 < q < n$, and δ_n is defined in equation (10). Note that, since the equilibrium payoffs are unique when $q = 1$ and $q = n$, we have $v^{*1} = v^1$ and $v^{*n} = v^n$ where v^1 is characterized in Proposition 3 and v^n is characterized in Proposition 2. We now show that if no two players have the same potential surplus (i.e., $y_1 < y_2 < \cdots < y_n$), and the potential surpluses are sufficiently different from each other, then as q increases (i.e., as the voting rule becomes increasingly more inclusive), the most efficient equilibrium of the game becomes relatively more inequitable.

Proposition 7. *If $y_q > (n - q + 1)\, y_{q-1}$ for every $q = 2, ..., n,$ then*

$$G(v^{*1}(\delta)) < G(v^{*2}(\delta)) < ... < G(v^{*n-1}(\delta)) < G(v^{*n}(\delta)$$

for all $\delta > \delta^*$.

6 Discussion and Concluding Remarks

In this paper, we have studied the equity properties of different voting rules in a multilateral bargaining environment where players are heterogeneous with respect to the potential surplus they bring to the bargaining table. We have shown that unanimity rule may generate equilibrium outcomes that are more unequal (or less equitable) than equilibrium outcomes under majority rule. In fact, as players become perfectly patient, if there is enough heterogeneity, then the more inclusive the voting rule with respect to the number of votes required to induce agreement, the less equitable the equilibrium allocations.

These results follow naturally from basic insights of bargaining theory in distributive settings. Unanimity rule protects the rights of every player, including the most productive one. As players become perfectly patient, no other player has a potential surplus that is large enough to satisfy the demands of the most productive player (i.e., her reservation payoff), in order to induce her to accept a proposal when she is not the proposer. Hence, under unanimity rule, agreement occurs only when the most productive player is the proposer and every other player receives an equilibrium payoff of zero, since they never make a proposal that is accepted in equilibrium. In fact, one of the fundamental insights of noncooperative bargaining theory is that there is a benefit to proposing only when agreements occur. If agreement does not occur, the payoff associated with proposing in such an instance is just the expected discounted payoff of future agreements. Similarly, if a player is not the proposer, she will never be offered more than the expected value of her future payoffs. Since future payoffs are discounted, if follows that if she ever earns a positive payoff, her highest payoff must be when she makes an acceptable offer. If none of her proposals are ever accepted, her SSP payoff must be zero.

On the other hand, under majority rule (in fact, under any q-quota rule with $q < n$), the vote of the most productive player is no longer required to reach an agreement when she is not the proposer. Hence, under a q-quota voting rule, agreement always occurs whenever a player $i \geq q$ is selected as proposer, and the most productive player loses her advantage. This "egalitarian" force makes the payoff distribution relatively more equitable.

Baron and Ferejohn (1989) present an example with a deterministic surplus of size 1, three players ($n = 3$) and majority rule ($q = 2$) in which there is an equilibrium with players receiving equal SSP payoffs even though they have different probabilities of being selected as the proposer, i.e. $v^2 = (1/3, 1/3, 1/3)$.[9] Although they do not show it, for the example they present, the SSP payoff of each player under unanimity rule is equal to his probability of being selected as the proposer, i.e. $v^3 = (p_1, p_2, p_3)$ where p_i is the probability with which player i is selected as the proposer. Obviously v^3 is less equal than v^2. In that sense, this example implicitly illustrates the possibility that players with strong bargaining powers may lose their powers under less inclusive voting rules even when the surplus to be divided is deterministic. However, for this example, the equilibrium payoffs under random dictatorship is identical to the equilibrium payoffs under unanimity rule, i.e. $v^1 = v^3$. Hence, unlike in our model, the payoff distribution does not necessarily become more unequal as the voting rule become more inclusive with a deterministic surplus.[10]

[9]By Eraslan (2002), $(1/3, 1/3, 1/3)$ is the only SSP payoff vector for the example they present.

[10]Note that there is no heterogeneity in the original Baron and Ferejohn (1989) model, and therefore all players receive identical equilibrium payoffs regardless of the voting rule. Eraslan (2002) allows for heterogeneity in the probability of being the proposer and the discount factor, and Eraslan and McLennan (2013) further

Given the trade-off between fairness and efficiency of threshold voting rules we find, it would be interesting to explore whether there are other institutional designs, such as amendments and deadlines, that can reduce inequality at the same time preserving efficiency. We leave this question to future work.

We conclude by discussing two other possible extensions of our results.

Heterogeneous disagreement payoffs:

If disagreement leads to an even worse outcome, then veto right is not valuable. Conversely, if the disagreement payoffs are sufficiently high but not equitable, then more equitable agreements could potentially be vetoed, in which case unanimity safeguards the rights of those already well-off. We provide an example to illustrate that our results could be robust to formulations that allow for negative correlation between surpluses and disagreement payoffs. To do so requires extending our model to a setup where the players have heterogeneous disagreement payoffs. Let d_i denote the disagreement payoff of player i. It is straightforward to show that equation (5) that characterizes equilibrium payoffs now becomes

$$v_i = (1-\delta)d_i + \delta[\frac{1}{n}\alpha_i(v)(y_i - w_i(v)) + \mu_i(v; r\,(v))v_i].$$

Reconsider now the example in Section 4 and assume that the payoffs from indefinite disagreement are given by $d_1 = 0.75$, $d_2 = 0.2$ and $d_3 = 0$. Note that the surpluses and disagreement payoffs are perfectly negatively correlated. The Gini coefficient for the disagreement payoffs and the surpluses are given by 0.52 and 0.2439 respectively.

Under random dictatorship, there is always agreement, the payoffs are given by

$$v_i = (1-\delta)d_i + \frac{\delta}{3}y_i$$

for all i. Under majority rule, there is a unique pure strategy equilibrium in which agreement only when players 2 and 3 propose and player 1 is included in any winning coalition. The payoffs are given by

$$v_1 = (1-\delta)d_1 + \frac{2}{3}v_1,$$

and

$$v_i = (1-\delta)d_i + \frac{\delta}{3}(y_i - v_1) + \frac{\delta}{3}v_i$$

allow for heterogeneity in voting power. Neither of these papers discusses the equity properties of different voting rules.

for $i = 2, 3$. Under unanimity rule, there is agreement only when player 3 proposes and the payoffs are given by

$$v_i = (1-\delta)d_i + \frac{\delta}{3}(y_i - \sum_{j \neq i} v_j) + \delta \frac{2}{3} v_i$$

for all i. It follows that for the above equilibria, the Gini coefficient for the unanimity, majority and random dictatorship are 0.47, 0.2442 and 0.19 respectively. Consequently, as the voting rules become more inclusive, equilibrium payoffs become less equitable in this example.

Spatial bargaining:

We provide an example with a single period bargaining in a spatial setting to illustrate that our main result may extend beyond the distributive setting. There are three players who decide on a single dimensional policy. Player i's payoff from implementing policy y is given by $10 - (y - x_i)^2$ where x_i is the ideal policy of player i.[11] Each player is selected with equal probability to make a proposal. If the offer is accepted by q players, then it is implemented, otherwise default policy d is implemented. As before we compare unanimity ($q = 3$), majority ($q = 2$) and random dictatorship ($q = 1$).

Suppose $d = -1$, $x_1 = 0$, $x_2 = 1$ and $x_3 = 3.5$. Then under unanimity rule, when selected as the proposer, player 1 proposes 0 while players 2 and 3 propose 1, and all these proposals are accepted. Under majority rule, when selected as the proposer, player 1 proposes 0, player 2 proposes 1 and player 3 proposes 3, and all these proposals are accepted. Under random dictatorship rule, each player proposes her own ideal, and all these proposals are accepted. It is straightforward to verify that for this example, Gini coefficient for the unanimity, majority and random dictatorship are 0.254, 0.163 and 0.147 respectively. Consequently, the more inclusive the voting rule, the less equitable the equilibrium payoffs.

We leave it to future research to explore whether these two examples can be generalized.

A Proofs

Proof of Proposition 1: Suppose the SSP payoff vector is given by v. Let i denote the proposer, and consider an SSP response to a proposal x by player j. Player j accepts the

[11] Adding the constant 10 to the quadratic loss utility functions ensures that the payoffs are positive in equilibrium for this example.

proposal if $x_j \geq v_j$ and rejects it if $x_j < v_j$. Note that the proposer needs only $q-1$ votes in addition to her vote for a proposal to be accepted. Then, if the proposer decides to make an offer that will be accepted, she will solve the program:

$$r_i(v) \in \arg \min_{z\in\{0,1\}^n} \quad \sum_k z_k v_k \tag{16}$$
$$\text{subject to} \quad \sum_{k\neq i} z_k \geq q-1 \text{ and } z_i = 1.$$

Let Γ_i denote the set of minimizers of (16). Each $\gamma_i = (\gamma_{ij})_{j=1}^n \in \Gamma_i$ corresponds to a pure proposal, since an SSP proposal in pure strategies by player i can be identified by the $(n-1)$-dimensional vector which specifies the players to whom player i offers their continuation payoff. A minimizer of (6), however, does not necessarily correspond to a pure proposal. Rather, it corresponds to a mixed proposal, where player i randomizes over the proposals corresponding to the elements in Γ_i (possibly with degenerate probabilities). In equilibrium, player i randomizes over the proposals corresponding to the elements in Γ_i since any proposal corresponding to an element in Γ_i yields the lowest possible payoff to player i. It is straightforward to verify that $r_{ij}(v)$ is a minimizer of (6) if and only if there exists a probability distribution $\pi_i(.)$ over Γ_i such that

$$r_{ij}(v) = \sum_{\gamma_i\in\Gamma_i} \gamma_{ij}\pi_i(\gamma_i).$$

In other words, randomizing over pure proposals is payoff equivalent to offering mixed proposals. Intuitively, $r_{ij}(v)$ denotes the probability that player j is offered his continuation payoff when player i is the proposer who proposes an allocation that will be accepted.

If player i offers an allocation that is accepted, this allocation yields the payoff $y_i - \sum_{j\neq i} r_{ij}(v)v_j$ to the proposer and it yields the expected payoff $r_{ij}(v)v_j$ to player j. If no proposal is accepted, then all the players receive their continuation payoffs. Given our convention that $r_{ii}(v) = 1$, a payoff maximizing proposer i offers an allocation that will be accepted if $y_i - \sum_j r_{ij}(v)v_j > 0$, passes if $y_i - \sum_j r_{ij}(v)v_j < 0$, and is indifferent between proposing an allocation that will be accepted and passing if $y_i - \sum_j r_{ij}(v)v_j = 0$. Recall that $\alpha_i(v)$ denotes the probability that player i proposes an allocation that will be accepted. Then $\alpha_i(v)$ must satisfy the restrictions imposed in equation (2).

In equilibrium, the offer probabilities $r_{ij}(v)$ and proposal probabilities $\alpha_i(v)$, $i, j = 1, ..., n$, must induce the continuation payoffs v, that is $v = \delta E[v]$ where the expectation is taken over the proposer selection probabilities. Next, we show that this is satisfied by equation (5).

If agreement is not reached in the current period, then next period player i is the proposer with probability $\frac{1}{n}$. With probability $\alpha_i(v)$ she proposes an allocation that will be accepted in which case her payoff is $y_i - \sum_{j\neq i} r_{ji}(v)v_j$. With probability $1 - \alpha_i(v)$ player i passes and receives her continuation payoff v_i. Thus, conditional on being the proposer, next period's expected payoff for player i discounted back to the current period is

$$\delta \frac{1}{n}[\alpha_i(v)(y_i - \sum_{j\neq i} r_{ij}(v)v_j) + (1 - \alpha_i(v))v_i]. \tag{17}$$

Now consider the case when player i is not the proposer next period. With probability $\frac{1}{n}$ player $j \neq i$ is the proposer. Player j proposes an allocation that will be accepted with probability $\alpha_j(v)$ in which case the expected payoff to player i is $r_{ji}(v)v_i$. With probability $1 - \alpha_j(v)$ player j passes in which case player i receives her continuation payoff v_i. Thus, conditional on not being the proposer, next period's expected payoff for player i discounted back to the current period is

$$\delta \sum_{j\neq i} \frac{1}{n}[\alpha_j(v)r_{ji}(v) + (1 - \alpha_j(v))]v_i. \tag{18}$$

Combining (17) and (18) and rearranging, the continuation payoff for player i is given by equation (5).

To complete the proof consider the following strategy. When player i is not the proposer, she accepts any proposal if and only if the proposal gives her at least v_i. When player i is the proposer, she proposes an allocation with probability $\alpha_i(v)$ and passes with probability $1 - \alpha_i(v)$. If she proposes an allocation, the allocation she proposes is x with probability $\pi(\gamma_i)$, where $x_i = y_i - \sum_{j\neq i} \gamma_{ij}v_j$, $x_j = \gamma_{ij}v_j$ for all $j \neq i$, and $\pi_i(.)$ is the probability distribution on Γ_i that induces the offer probabilities $r_{ij}(v)$. Clearly, this strategy implements the payoffs given by (5) and no player has an incentive to unilaterally deviate from it. ∎

Proposition A.1. *There exists an SSP payoff vector for the q-quota game, for any $q \in \{1, ..., n\}$.*

Proof: Given $\alpha = (\alpha_1, \ldots, \alpha_n)$ and $r = [r_i]_{i=1}^n$, define the mapping $A(.; \alpha, r) : \mathbb{R}^n \to \mathbb{R}^n$ as

$A(v;\alpha,r) = (A_1(v;\alpha,r),\ldots,A_n(v;\alpha,r))$ where

$$A_i(v;\alpha,r) = \delta\bigg(\frac{1}{n}\alpha_i(y_i - \sum_j r_{ij}v_j) + \sum_j \frac{1}{n}[\alpha_j r_{ji} + (1-\alpha_j)]v_i\bigg), \tag{19}$$

for all $i = 1,...,n$. Define the set-valued mapping $T(.)$ on $[0, y_n]^n$ as

$$T(v) = \{g \in I\!R^n : \exists r \in r(v), \exists \alpha \in \alpha(v) \text{ such that } g = A(v;\alpha,r)\}. \tag{20}$$

By Proposition 1 and the definition of $T(.)$, v is an SSP payoff vector for the q-quota game if and only if it is a fixed point of the set-valued mapping $T(.)$, that is $v \in T(v)$.

Note that $T(.)$ maps $[0, y_n]^n$ to non-empty subsets of $[0, y_n]^n$. It is easily seen that $T(v)$ is convex for all v since $r(.)$ and $\alpha(.)$ are convex valued. Furthermore, $T(.)$ is upper semi-continuous since $r(.)$ and $\alpha(.)$ are upper semi-continuous and A is continuous in v, α and r. Finally, for all $v \in [0, y_n]^n$, $T(v)$ is a closed subset of the compact set $[0, y_n]^n$ and hence, $T(v)$ is compact. Thus, the result follows from Kakutani Fixed Point Theorem. ∎

Proof of Lemma 1: If $\alpha_i(v) = 0$, then from (5) we obtain $v_i = \delta\mu_i(v; r(v))v_i \leq \delta v_i$. Since $v_i \geq 0$, we must have $v_i = 0$. If $0 < \alpha_i(v) < 1$, then the proposer must be indifferent between proposing and passing, and hence $y_i = w_i(v)$. Again, plugging in (5), we obtain $v_i \leq \delta v_i$, and the result follows since $v_i \geq 0$. ∎

Proof of Lemma 2: First we show that if $v_i > 0$, then $y_i > w_i(v)$. Suppose not. Then, $v_i > 0$, but $y_i \leq w_i(v)$. If the inequality is strict, then $\alpha_i(v) = 0$. Thus, $\alpha_i(v)(y_i - w_i(v)) = 0$ whether or not the inequality is strict. Plugging this in (5) and rearranging we obtain $v_i = 0$ which is a contradiction.

Next we show that if $y_i > w_i(v)$, then $v_i > 0$. Since $y_i > w_i(v)$, by (2) we have $\alpha_i(v) = 1$, and thus (5) implies $v_i > 0$. ∎

Proof of Lemma 3: Suppose not. Then $y_i > w_i(v) - v_i$, but $\alpha_i(v) < 1$. By Lemma 1, $v_i = 0$, and so $y_i > w_i(v)$. But then (2) implies that $\alpha_i(v) = 1$ which is a contradiction. ∎

Proof of Lemma 4: If $v_i = v_j$, then $w_i(v) = w_j(v)$ because otherwise one of the players would not be maximizing their payoff. If $v_i < v_j$, given the probability $r_{ji}(v)$ that j includes i in his coalition when he is the proposer in equilibrium, let

$$\tilde{w}_j = \min_{z\in[0,1]^{n-2}} \sum_{k\neq i,j} z_k v_k + r_{ji}(v)v_i + v_j \qquad (21)$$
$$\text{subject to} \quad \sum_{k\neq i,j} z_k \geq q - 1 - r_{ji}(v),$$

and

$$\tilde{w}_i = \min_{z\in[0,1]^{n-2}} \sum_{k\neq i,j} z_k v_k + r_{ji}(v)v_j + v_i \qquad (22)$$
$$\text{subject to} \quad \sum_{k\neq i,j} z_k \geq q - 1 - r_{ji}(v).$$

Notice that $w_i(v) \leq \tilde{w}_i \leq \tilde{w}_j = w_j(v)$, where the first inequality follows from the fact that the vector $r_i(v)$ that defines $w_i(v)$ is a minimizer for a less restrictive program than (22), the second inequality follows from the fact that $r_{ji}(v)v_j + v_i \leq r_{ji}(v)v_i + v_j$, and the equality follows from the definition of $r_{ji}(v)$ and $w_j(v)$. ∎

Proof of Lemma 5: Suppose to the contrary that $y_i > w_i(v)$, but $\alpha_j(v) < 1$ for some $j > i$. Since $y_i > w_i(v)$, by (2) we have $\alpha_i(v) = 1$ and by (5) we have $v_i > 0$. Since $\alpha_j(v) < 1$, Lemma 1 implies $v_j = 0$. Consequently, by Lemma 4, we have $w_j(v) \leq w_i(v)$. Thus, $y_j \geq y_i > w_i(v) \geq w_j(v)$. But then (2) implies that $\alpha_j(v) = 1$ which is a contradiction. ∎

Proof of Lemma 6: Suppose that $\alpha_i(v) < 1$. By Lemma 1 we must have $v_i = 0$. Now suppose to the contrary of the claim that there exists a $j < i$ such that $v_j > 0$. By Lemma 2, it must be the case that $y_j > w_j(v)$. But then, by Lemma 5, it must be the case that $\alpha_i(v) = 1$, which is a contradiction. ∎

Proof of Lemma 7: Since $v_i < v_j$, any player $k \neq i, j$ includes player i in his coalition at least as often as he includes player j, and so $r_{ki}(v) \geq r_{kj}(v)$. Furthermore, $r_{ji}(v) \geq r_{ij}(v)$. If not, then either player i or player j are not maximizing their payoff. Then, $\mu_{ji}(v; r(v)) = \alpha_j(v)r_{ji}(v) + (1 - \alpha_j(v)) \geq \alpha_i(v)r_{ij}(v) + (1 - \alpha_i(v)) = \mu_{ij}(v; r(v))$ and the result follows from equation (3). ∎

Proof of Lemma 8: If $y_i \leq w_i(v)$, then $v_i = 0$ and the proof is immediate. So suppose that $y_i > w_i(v)$ and suppose to the contrary that $v_j < v_i$. Since $y_i > w_i(v)$, we have by

(2) that $\alpha_i(v) = 1$, and, by Lemma 5, we also have $\alpha_j(v) = 1$. Then by Lemma 7, we have $\mu_j(v; r(v)) \geq \mu_i(v; r(v))$. In addition, by Lemma 4, we have $w_i(v) \geq w_j(v)$.

Since $\alpha_i(v) = \alpha_j(v) = 1$, from (5) we have

$$v_i = \delta[\frac{1}{n}(y_i - w_i(v)) + \mu_i(v; r(v))v_i], \tag{23}$$

$$v_j = \delta[\frac{1}{n}(y_j - w_j(v)) + \mu_j(v; r(v))v_j]. \tag{24}$$

Subtracting (23) from (24), we obtain

$$v_j - v_i = \delta[\frac{1}{n}(y_j - y_i) + \frac{1}{n}(w_i(v) - w_j(v)) + \mu_j(v; r(v))v_j - \mu_i(v; r(v))v_i].$$

Since $\mu_j(v; r(v)) \geq \mu_i(v; r(v))$, we have

$$v_j - v_i \geq \delta[\frac{1}{n}(y_j - y_i) + \frac{1}{n}(w_i(v) - w_j(v)) + \mu_i(v; r(v))(v_j - v_i).$$

Rearranging, we have that

$$v_j - v_i \geq \delta\frac{1}{n}\frac{(y_j - y_i) + (w_i(v) - w_j(v))}{1 - \delta\mu_i(v; r(v))} \geq 0.$$

This contradicts the assumption that $v_j < v_i$. ∎

Proof of Lemma 9: If $y_{q-1} > w_{q-1}(v)$, then the result follows from Lemma 5. If $y_{q-1} < w_{q-1}(v)$, there is no agreement when $q-1$ is the proposer and $v_{q-1} = 0$ by Lemma 1. If $y_{q-1} = w_{q-1}(v)$, then by equation (5), it is also the case that $v_{q-1} = 0$. Hence, $v_i = 0$ for all $i \leq q-1$ by Lemma 8. This implies that $w_q(v) - v_q = 0 < y_q$, and hence $\alpha_q(v) = 1$ by Lemma 3. Then, $v_q > 0$ by equation (5). ∎

Proof of Proposition 4: Fix $q \in \{2, \ldots, n-1\}$ and for any $i < q$, let

$$\underline{\delta}_i = \frac{n\sum_{j=i}^{q-1} y_j}{(q-1)\sum_{j=i}^{q-1} y_j + (q-i)\sum_{j=q}^{n} y_j}.$$

Note that $\underline{\delta}_i \in (0,1)$ for all $i < q$ since $y_i < y_{i+1} < \ldots < y_n$. Let

$$\delta_q = \max\{\frac{ny_{q-1}}{y_q + y_{q-1}(q-1)}, \max_{i<q} \underline{\delta}_i\}.$$

Clearly $\delta_q > 0$. In addition, we have $\delta_q < 1$ since $y_{q-1} < \frac{y_q}{n-q+1}$.

First we show that if $\delta \geq \delta_q$, then there is an equilibrium with $v_i = 0$ for all $i < q$ and $v_i = \frac{\delta y_i}{n - \delta(q-1)}$ for all $i \geq q$. To verify that there is an equilibrium with these payoffs, note that when $\delta > \delta_q$, we have $y_{q-1} < \frac{\delta y_q}{n-\delta(q-1)}$ which in turn implies that $y_{q-1} < w_{q-1}(v)$ and hence $\alpha_{q-1}(v) = 0$ by equation (2). Therefore, by Lemma 6, $v_i = 0$ for all $i < q$. This, in turn, implies that $\mu_i(v; r(v)) = \frac{q}{n}$ and $w_i(v) = v_i$ for all $i \geq q$. Plugging in (5), we obtain $v_i = \frac{\delta y_i}{n-\delta(q-1)}$ for all $i \geq q$.

We next show that the SSP payoff vector for the most efficient equilibrium satisfies $v_i = 0$ for all $i < q$ and $v_i = \frac{\delta y_i}{n-\delta(q-1)}$ for all $i \geq q$. Given these SSP payoffs, denote the aggregate SSP payoff by V, that is,

$$V = \frac{\delta \sum_{i=q}^{n} y_i}{n - \delta(q-1)}.$$

Fix any other equilibrium payoff vector v' and let $\kappa = \min\{i : \alpha_i(v') = 1\}$ denote the player with the lowest index such that there is agreement with probability one when that player is the proposer when the equilibrium payoff vector is v'. Note that by Lemma 9, $\alpha_q(v') = 1$, so we must have $\kappa \leq q$. If $\kappa = q$, then $v' = v$, so assume $\kappa < q$. Let $V' = \sum_{i=1}^{n} v'_i$ denote the aggregate SSP payoff when the equilibrium payoff vector is v'. To complete the proof, we show that $V > V'$.

By equations (1)-(5), suppressing the dependency of $\alpha_{\kappa-1}$ on v', we have

$$V' = \delta \frac{1}{n}[\sum_{i=\kappa}^{n} y_i + \alpha_{\kappa-1} y_{\kappa-1} + ((1 - \alpha_{\kappa-1}) + \kappa - 2)V']. \tag{25}$$

Rearranging, we have

$$V' = \frac{\delta(\sum_{i=\kappa}^{n} y_i + \alpha_{\kappa-1} y_{\kappa-1})}{n - \delta(\kappa - \alpha_{\kappa-1} - 1)}. \tag{26}$$

Note that $V > V'$ if and only if

$$\frac{\delta \sum_{i=q}^{n} y_i}{n - \delta(q-1)} > \frac{\delta(\sum_{i=\kappa}^{n} y_i + \alpha_{\kappa-1} y_{\kappa-1})}{n - \delta(\kappa - \alpha_{\kappa-1} - 1)}. \tag{27}$$

Canceling terms and rearranging, (27) holds whenever

$$\delta > \frac{n(\sum_{i=\kappa}^{q-1} y_i + \alpha_{\kappa-1} y_{\kappa-1})}{(q-1)(\sum_{i=\kappa}^{q-1} y_i + \alpha_{\kappa-1} y_{\kappa-1}) + (q - \kappa + \alpha_{\kappa-1}) \sum_{i=q}^{n} y_i}. \tag{28}$$

Since right hand side is decreasing in $\alpha_{\kappa-1}$, and $\delta > \delta_q \geq \underline{\delta}_\kappa$, the desired result follows. ∎

Proof of Proposition 5: First note that by equations (13), (14), and (11), it is immediate that $G_y = G(v^1(\delta))$. In the rest of the proof, we suppress δ and let v^n and denote the equilibrium payoffs for the unanimity rule game characterized in Proposition 2. Recall κ is the player with the lowest index such that agreement occurs with positive probability under unanimity rule. We first show that

$$\frac{\sum_{i=1}^{n} i y_i}{\sum_{i=1}^{n} y_i} < \frac{\sum_{i=\kappa}^{n} i y_i}{\sum_{i=\kappa}^{n} y_i} \text{ if and only if } \kappa > 1.$$

Clearly, the inequality is not satisfied if $\kappa = 1$. Thus, to establish the claim, it is sufficient to

show
$$\frac{\sum_{i=1}^{\kappa-1} iy_i + \sum_{i=\kappa}^{n} iy_i}{\sum_{i=1}^{\kappa-1} y_i + \sum_{i=\kappa}^{n} y_i} < \frac{\sum_{i=\kappa}^{n} iy_i}{\sum_{i=\kappa}^{n} y_i} \text{ if } \kappa > 1.$$
This inequality holds if and only if
$$\sum_{i=1}^{\kappa-1} iy_i \sum_{i=\kappa}^{n} y_i < \sum_{i=\kappa}^{n} iy_i \sum_{i=1}^{\kappa-1} y_i,$$
which is satisfied when $\kappa > 1$ because $\sum_{i=1}^{\kappa-1} iy_i < (\kappa-1)\sum_{i=1}^{\kappa-1} y_i$ and $\kappa \sum_{i=\kappa}^{n} y_i < \sum_{i=\kappa}^{n} iy_i$.

Since $v_i^n = 0$ for all $i < \kappa$, we have
$$G(v^n) = \frac{2\sum_{i=1}^{n} iv_i^n}{n\sum_{i=1}^{n} v_i^n} - \frac{n+1}{n} = \frac{2\sum_{i=\kappa}^{n} iv_i^n}{n\sum_{i=\kappa}^{n} v_i^n} - \frac{n+1}{n}.$$
Hence, the proof is complete if we show that
$$\frac{\sum_{i=\kappa}^{n} iv_i^n}{\sum_{i=\kappa}^{n} v_i^n} \geq \frac{\sum_{i=\kappa}^{n} iy_i}{\sum_{i=\kappa}^{n} y_i},$$
with equality if $\kappa = 1$. Letting $Y = \sum_{i=\kappa}^{n} y_i$, the left hand side is equal to
$$\frac{\sum_{i=\kappa}^{n} i\left(y_i - \frac{\delta Y}{(1-\delta)n+\delta(n-\kappa+1)}\right)}{\sum_{i=\kappa}^{n}\left(y_i - \frac{\delta Y}{(1-\delta)n+\delta(n-\kappa+1)}\right)} = \frac{\sum_{i=\kappa}^{n} iy_i - \frac{\delta}{(1-\delta)n+\delta(n-\kappa+1)} Y \sum_{i=\kappa}^{n} i}{Y - \frac{\delta}{(1-\delta)n+\delta(n-\kappa+1)} Y(n-\kappa+1)}.$$
The proof follows since $(\sum_{i=\kappa}^{n} iy_i)(n-\kappa+1) \geq (\sum_{i=\kappa}^{n} i)(\sum_{i=\kappa}^{n} y_i)$, with strict inequality if $y_i \neq y_j$ for some $i, j = 1, ..., n$. ■

Proof of Proposition 6: Fix $q \leq \tilde{\kappa}$. Since $y_{\tilde{\kappa}} = \ldots = y_n$, in any equilibrium of any q-quota game, players $\tilde{\kappa}, \ldots, n$ receive the same payoff by Lemma 8. That is, for any δ, we have $v_i^q(\delta) = v_n^q(\delta)$ for all $i \geq \tilde{\kappa}$. Fix $\delta > \tilde{\delta}$. By Proposition 2, we have $v_i^n(\delta) = 0$ for all $i < \tilde{\kappa}$, and therefore
$$G(v^n(\delta)) = \frac{2\sum_{i=\tilde{\kappa}}^{n} iv_i^n(\delta)}{\sum_{i=\tilde{\kappa}}^{n} v_i^n(\delta)} - \frac{n+1}{n} = \frac{2\sum_{i=\tilde{\kappa}}^{n} i}{n-\tilde{\kappa}+1} - \frac{n+1}{n}$$
for any equilibrium payoff vector $v^q(\delta)$ for the q-quota rule. Furthermore, as in the proof of Proposition 5,
$$\frac{\sum_{i=1}^{n} iv_i^q(\delta)}{\sum_{i=1}^{n} v_i^q(\delta)} \leq \frac{\sum_{i=\tilde{\kappa}}^{n} iv_i^q(\delta)}{\sum_{i=\tilde{\kappa}}^{n} v_i^q(\delta)}$$
with strict inequality if and only if $\tilde{\kappa} > 1$. Thus $G(v^n(\delta)) \geq G(v^q(\delta))$. ■

Proof of Proposition 7: From Proposition 4 and Corollary 1, we know that if $\delta > \delta^*$ and $y_{q-1} < \frac{y_q}{n-q+1}$ for every $q = 2, ..., n$, then each q-quota game has the most efficient equilibrium of any q-quota game satisfies $v_i^{*q} = 0$ for all $i < q$, and $v_i^{*q} = \frac{\delta y_i}{n-\delta(q-1)}$ for $i \geq q$. Thus,

$$G(v^{*q}(\delta)) = \frac{2\sum_{i=q}^{n} iy_i}{n\sum_{i=q}^{n} y_i} - \frac{n+1}{n} < \frac{2\sum_{i=q+1}^{n} iy_i}{n\sum_{i=q+1}^{n} y_i} - \frac{n+1}{n} = G(v^{*q+1}(\delta))$$

for any $q = 1, \ldots, n-1$. ∎

REFERENCES

Acemoglu, D. and J. Robinson (2012), *Why Nations Fail: The Origins of Power, Prosperity, and Poverty*, New York: Crown Publishing.

Ali, N., D. Bernheim and X. Fan (2014), "Predictability and Power in Legislative Bargaining", Working Paper.

Baccara, M. and R. Razin (2007), "Bargaining over New Ideas: The Distribution of Rents and the Stability of Innovative Firms," *Journal of the European Economic Association*, **5**, 1095-1129.

Banks, J. and J. Duggan (2000), "A Bargaining Model of Collective Choice," *American Political Science Review*, **94**, 73-88.

Baron, D. and J. Ferejohn (1989), "Bargaining in Legislatures," *American Political Science Review*, **83**, 1181-1206.

Baron, D. and E. Kalai (1993), "The Simplest Equilibrium of a Majority Rule Division Game," *Journal of Economic Theory*, **61**, 290-301.

Bernheim, D., A. Rangel and L. Rayo (2006), "The Power of the Last Word in Legislative Policy Making." *Econometrica*, **74**, 1161-1190.

Buchanan, J. M. and G. Tullock (1962), *The Calculus of Consent*, Ann Arbor: University of Michigan Press.

Compte, O. and P. Jehiel (2010), "Bargaining and Majority Rules: A Collective Search Perspective," *Journal of Political Economy*, **118**, 189-221.

Dixit, A., Grossman, G. M. and F. Gül (2000), "The Dynamics of Political Compromise," *Journal of Political Economy*, **108**, 531-568.

Eraslan, H. (2002), "Uniqueness of Stationary Equilibrium Payoffs in the Baron-Ferejohn Model," *Journal of Economic Theory*, **103**, 11-30.

Eraslan, H. and A. McLennan (2013), "Uniqueness of Stationary Equilibrium Payoffs in Coalitional Bargaining," *Journal of Economic Theory*, **148**, 2191-2222.

Eraslan, H. and A. Merlo (2002), "Majority Rule in a Stochastic Model of Bargaining," *Journal of Economic Theory*, **103**, 31-48.

Harrington, J. (1990), "The Role of Risk Preferences in Bargaining when Acceptance of a Proposal Requires less than Unanimous Approval," *Journal of Risk and Uncertainty*, **3**, 135-154.

Kalandrakis, T. (2004), "A Three-Player Dynamic Majoritarian Bargaining Game," *Journal of Economic Theory*, **116**, 294-322.

Kalandrakis, T. (2006), "Proposal Rights and Political Power," *American Journal of Political Science*, **50**, 441-448.

Kalandrakis, T. (2010), "Minimum Winning Coalitions and Endogenous Status Quo," *International Journal of Game Theory*, **50**, 441-448.

McGinnis, J. O. and M. B. Rappaport (1999), "Supermajority Rules as a Constitutional Solution," *William & Mary Law Review*, **40**, 365-470.

McGinnis, J. O. and M. B. Rappaport (2013), *Originalism and the Good Constitution*, Cambridge: Harvard University Press.

Merlo, A. and C. Wilson (1995), "A Stochastic Model of Sequential Bargaining with Complete Information," *Econometrica*, **39**, 617-643.

Merlo, A. and C. Wilson (1998), "Efficient Delays in a Stochastic Model of Bargaining," *Economic Theory*, **11**, 39-55.

Nunnari, S. (2016), "Dynamic Legislative Bargaining with Veto Power: Theory and Experiments" Working Paper.

Schwartzberg, M. (2014), *Counting the Many: The Origins and Limits of Supermajority Rule*, New York: Cambridge University Press.

Sutton, J. (1986), "Non-Cooperative Bargaining Theory: An Introduction", *Review of Economic Studies*, **53**, 709-724.

Yıldırım, H. (2007), "Proposal Power and Majority Rule in Multilateral Bargaining with Costly Recognition," *Journal of Economic Theory*, **136**, 167-196.

Yıldırım, H. (2010), "Distribution of Surplus in Sequential Bargaining with Endogenous Recognition," *Public Choice*, **142**, 41-57.