Solution strategies to the stochastic design of mineral flotation plants

Nathalie Jamett,[a] Luis A. Cisternas,[a,b,1] Juan P. Vielma, [c]

[a]Universidad de Antofagasta, Av. Universidad de Antofagasta s/n, Antofagasta, Chile
[b]CICITEM, Avda. José Miguel Carrera Nº170, Antofagasta, Chile
[c]Sloan School of Management, Massachusetts Institute of Technology, USA

ABSTRACT

The aim of this study is two-fold: first, to analyze the effect of stochastic uncertainty in the design of flotation circuits and second, to analyze different strategies for the solution of a two-stage stochastic problem applied to a copper flotation circuit. The paper begins by introducing a stochastic optimization problem whose aim is to find the best configuration of superstructure, equipment design and operational conditions, such as residence time and stream flows. Variability is considered in the copper price and ore grade. This variability is represented by scenarios with their respective probability of occurrence. The resulting optimization problem is a two-stage stochastic mixed integer nonlinear program (TS-MINLP), which can be extremely challenging to solve. For this reason, several solvers for this problem are compared and two stochastic programming methodologies are applied. The combination of these techniques allows the production of high quality solutions and an analysis of their sensitivity to epistemic uncertainty. The results show that the stochastic problem gives better designs because it allows operational parameters to adapt to the uncertainty of the parameters. The results also show that the flotation circuit structure can vary with the feed grade and copper price. The sensitivity analysis shows small to moderate variability with epistemically uncertain parameters.

Keywords: Flotation circuits, uncertainty, stochastic programming, copper, process design.

1. Introduction

Flotation is a physicochemical process that allows the separation of minerals, such as copper sulfide minerals and molybdenum, from the remaining minerals that form most of the parent rock substrate, including contaminants, such as arsenic minerals (Bruckard et al., 2010). The separation is performed on milled aqueous mineral suspensions (pulp) subjected to forceful air bubbling, which produces the separation (flotation) of valuable metals from tailings based on the hydrophobic and hydrophilic properties of the minerals. The flotation process is performed using equipment, such as flotation cells, which are usually grouped into banks, or flotation columns. This equipment is interconnected in predetermined arrangements that allow the outputs of the systems to be divided into metal concentrate and tailing flows. Because the desired separation cannot be achieved in a single stage, various coupled stages are used, which are referred to as a "flotation circuit." The behavior of the entire process, therefore, depends on the configuration of the circuit and the chemical and physical nature of the treated pulp.

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[1] *Corresponding author-* Tel:+56552637323
E-mail address: luis.cisternas@uantof.cl (L.A. Cisternas)

Mineral processing presents several challenges, such as the use of energy and water with minerals that are increasingly complex in composition. In addition, environmental regulations have led smelters (Gálvez et al., 2005) to consider penalties for polluting or to include pretreatment of the concentrates. As such, the preliminary design of a flotation circuit is important because the concentrate quality, metal recovery, costs and revenues depend on it.

The methods employed in flotation circuit design can be generally classified into heuristic, rigorous, or hybrid types. Heuristic designs are based on the application of rules that permit feasible solutions to be attained and where the experience of the designer is employed (Chan and Prince, 1989). Rigorous designs describe the phenomenon or relations among the design parameters and propose a superstructure that represents a universe of alternatives for circuits, which, after an optimization technique has been applied, allow the optimal alternative to be determined (Cisternas et al., 2004, 2006; Méndez et al., 2009, Hu et al., 2013). Hybrid designs are a combination of the two preceding methods, in which computational tools and the experience of the designer are employed to solve the problem (Loveday and Brouckaert, 1995; Gálvez, 1998; Sepúlveda et al. 2014). None of the preceding methods for the design of flotation circuits consider that the design might contain parameters that cannot be completely defined or values that might be subject to degrees of uncertainty.

Large numbers of variables are handled in the design of flotation circuits, some of which may involve uncertainty. These uncertainties usually involve external factors, such as product price, economic factors, environmental factors, or internal plant conditions, including feed grade and kinetic constants, which might lead to an inefficient process design (Kraslawski, 1989). Simonsen and Perry (1999) indicated that the type of uncertainty in mining operations includes market characteristics (especially price), mineral reserves and their composition, the functioning of the process (grades, efficiency), operational and capital costs, and the lengths of the planning phases. The study by Xiao and Vien (2003) showed that the previously mentioned uncertainty affects the design of the flotation process.

In general uncertainty can be classified as epistemic or stochastic. Epistemic uncertainty (or subjective, reducible) arises from the lack of knowledge about the appropriate value to use for a quantity that is assumed to have a fixed value in the context of a particular analysis. Stochastic uncertainty (or irreducible) derives from an inherent randomness in the behavior of the system under study. In the design of a flotation circuit the metal price and the feed grade present stochastic uncertainties. On the other hand, kinetic constants have epistemic uncertainty because these values change with equipment size, pulp chemistry, particle size, and particle composition among several other variables. Therefore, the exact value for the kinetic constants is not known before the circuit is designed.

A few studies have been conducted in which stochastic or epistemic uncertainty is included in the design and/or simulation of the processes. Some of these studies have focused on the representation of parameter uncertainty, either through specific probability functions, such as the Gaussian distribution (Bensal et al., 1998), or through the Poisson distribution (Orbán-Mihalyco et al., 2005) to solve these optimization problems. Other studies have used stochastic optimization to design process under uncertainty (Sacco et al. 2006; Poplewski et al., 2011). Jamett et al. (2012) applied stochastic programming for flotation

design using a simple flotation model. Montenegro et al. (2013) presented a methodology to analyze and/or design processes in which it is not possible to accurately define the operating conditions. The methodology considered the construction of a superstructure that includes alternatives to be analyzed, the stage recoveries were represented by distribution functions, and Monte Carlo simulation was used to evaluate the alternatives under uncertainty. Other examples can be found in Galvez and Cisternas, 2014 and its references.

The present study aims to analyze the effect of stochastic uncertainty in the process design of flotation circuits and to show the feasibility of conducting this analysis through a stochastic optimization problem. The rest of this study is organized as follows. Section 2 introduces the stochastic optimization problem used for the analysis. Section 3 then considers the solution of this optimization problem. This section shows that a careful selection of a commercial solver and the application of standard stochastic programming techniques can significantly reduce the time required to obtain good quality solutions. Section 4 shows that these solutions already can provide better designs that those obtained by optimization problems that do not consider uncertainty. Section 4 also shows that there is at most a moderate sensitivity to epistemically uncertain parameters. Finally, Section 5 presents some conclusions and future research directions.

2. Optimization Problem

The flotation circuit is represented through a superstructure of stages that represents all alternative flotation-circuit configurations. The superstructure is based on a generic representation of a stage, which is used to describe all stages that are to be included in the design. In the superstructure, discrete variables are used to represent the different alternatives for the circuit configuration, and continuous variables are used to represent the design specifications and operational conditions. Fig. 1 shows the structure that is used to represent a flotation stage, where the triangle in the input stream represents a feed mixer that enables concentrate and tail streams from other stages to be fed to the stage. The triangles in the output streams represent splitters that allow the concentrate and tail streams from one stage to be sent to other stages.

The generic representation of a stage is complemented with an origin-destination matrix where the allowed concentrate and tail streams are identified (Cisternas et al., 2014).
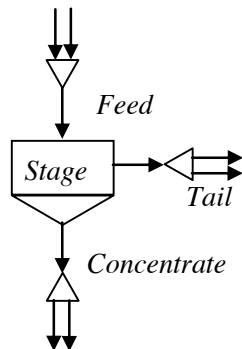

Fig. 1. Flotation stage representation

While expert analysis can be used to significantly reduce the number of alternatives (e.g. see the discussion on origin-destination matrices in Section 2.4), the large number of parameters and their possible combinations can still lead to an astronomically large number of possible configurations. For this reason, an optimization problem is introduced to implicitly describe all these alternatives. Appropriate optimization software and techniques are then used to obtain the desired design without the need to enumerate all alternatives.

Because of the presence of stochastic uncertainty, a class of optimization problems known as two-stage stochastic (non-convex) mixed integer nonlinear programs (TS-MINLP) is used. A TS-MINLP is divided into two stages, which, in our context, can be identified with the design and operational stages. The first stage (i.e. the design stage) includes all decisions that need to be made before the uncertainty is resolved (e.g. physical characteristics of the circuit stages). The second stage (i.e. the operational stage) includes all decisions that can be made after the uncertainty is resolved (e.g. after feed grades are known). A common simplification in TS-MINLP is to assume that all possible ways in which uncertainty can be resolved can be summarized in a finite number of scenarios with given realization probabilities. Using this simplification, a generic TS-MINLP can be written as (Steimel et al., 2014):

$$\min_{x,y} \ G(x) + \sum_{s=1}^{\Omega} P_s F_s(y_s) \tag{1}$$

$$s.t. \quad f_{s,i}(x,y_s) \leq 0 \qquad \forall s = 1, \dots, \Omega; i = 1, \dots, n_s \tag{2}$$

$$x \in Z, \ y_s \in Z \ \forall s = 1, \dots, \Omega \tag{3}$$

The objective function in (1) consists of two parts. The first contribution is from the first-stage costs that are only determined by the values of the first-stage decisions x. The second contribution arises from the first and second stage decisions y and can be different for each scenario s. The expected value of the second-stage cost function is calculated by a weighted sum of all of the scenario costs multiplied by the scenario probability P_s. The constraints are separated into two sets. Constraints (2) include general nonlinear inequalities and constraints (3) specify the domain of the decisions, which can take either integral or continuous values. Finally, note that in addition to the design and operational parameters that describe the flotation circuit, both first and second stage variables can also include auxiliary variables that are used to specify the operation of the circuit or are used to impose appropriate requirements.

Following is a description of the constraints, objective function, specific problems and instances used throughout the paper. Note that indices are represented as lowercase letters (e.g., s, f, e), sets are represented as uppercase bold letters (e.g., S, F, E), variables are represented as lowercase bold letters ($e.g. \ w_{s,l,e}, \ r_{s,f}$), certain constants are represented as uppercase letters ($e.g. K_s$, M, B), and potentially uncertain constants are represented as Greek letters ($e.g. \ \pi_s, \gamma_{s,e}$). More details are found in the nomenclature section.

2.1 Constraints

The first set of constraints specifies feed flows of specie e under scenario s to the flotation circuit. These constraints correspond to

$$w_{s,1,e} = T \cdot \gamma_{s,e} \qquad\qquad\qquad s \in S, e \in E, \tag{4}$$

where

- $w_{s,1,e}$ is the feed flows of specie e under scenario s,

- T is the total mass-flow rate of input to the system, and

- $\gamma_{s,e}$ is the feed grade corresponding to specie e in scenario s.

The next set of constraints enforces mass balance in the flotation stage. To describe these constraints a recovery model is needed. There are several models for the kinetic of flotation (Méndez et al., 2009) for both cell and bank. For simplicity, this work considers a bank model that assumes first-order kinetics considering a rectangular, rate-constant distribution function to account for the rate-constant change along the bank, while keeping a constant residence time for each cell (Yianatos and Henríquez, 2006). More elaborate bank and cell models can easily be incorporated. However, such models can significantly increase the solution times of the optimization problem. Our selected model enforces a simple proportional recovery with a complicated proportionality constraint, which corresponds to

$$w_{s,lc(f),e} = \text{Rmax}_{e,f} \cdot \left(1 - \frac{1-(1+K_{e,f} \cdot r_{s,f})^{1-n_f}}{(n_f-1) \cdot K_{e,f} \cdot r_{s,f}} \right) \cdot w_{s,li(f),e} \qquad s \in S. f \in F. e \in E, (5)$$

- where $w_{s,l,e}$ is the flow rate of specie e on stream l under scenario s,

- $lc(f)$ and $li(f)$ are the concentrate and feed streams in the flotation stage f respectively,

- $Rmax_{e,f}$ is the maximum recovery of specie e for flotation stage f,

- $K_{e,f}$ is the flotation rate of specie e for flotation stage f,

- $r_{s,f}$ is the operating retention time (one cell) in flotation stage f under scenario s, and

- n_f is the number of cells in flotation stage f.

The kinetic parameters, $Rmax_{e,f}$ and $K_{e,f}$, are considered constants, but the values of these parameters can change with the equipment size and/or circuit design. The potential effects of this assumption will be discussed later.

The next set of constraints enforce mass balance in flotation stage, mixers and splitters, which corresponds to

$$\sum_{l \in M^{in}(m)} w_{s,l,e} - \sum_{l \in M^{out}(m)} w_{s,l,e} = 0 \qquad m \in F \cup X \cup P. s \in S. e \in E, (6)$$

where

- $M^{in}(m)$ and $M^{out}(m)$ are the input and output streams of the flotation stage, splitter or mixer m,

- F is the set of flotation stages,

- X is the set of mixers, and

- P is the set of splitters.

The next set of constraints enforce the requirement that the residence times and flows be compatible with the cell volumes, which corresponds to

$$\frac{1}{A_f \cdot D_f} \cdot r_{s,f} \cdot \sum_{e \in E} w_{s,li(f),e} = v_f \qquad\qquad\qquad s \in S, f \in F, \quad (7)$$

where

- v_f is the cell volume in flotation stage f,

- A_f is the pulp aeration factor for flotation stage f, and

- D_f is the corrected density for flotation stage f.

The next set of constraints enforces the splitter selection, which can be described through the disjunctions

$$\begin{bmatrix} y_{s,m} \\ w_{s,l_1(m),e} \geq 0 \\ w_{s,l_1(m),e} \leq M \\ w_{s,l_2(m),e} = 0 \end{bmatrix} \lor \begin{bmatrix} 1 - y_{s,m} \\ w_{s,l_2(m),e} \geq 0 \\ w_{s,l_2(m),e} \leq M \\ w_{s,l_1(m),e} = 0 \end{bmatrix} \qquad s \in S, m \in P, e \in E, \qquad (8)$$

where

- $l_1(m)$ and $l_2(m)$ are the first and second possible outputs for splitter m,

- M is a constant that is larger than the mass-flow rates in any stream for any specie under any scenario, and

- $y_{s,m}$ is a binary variable that indicates the choice in the disjunction (i.e. $y_{s,m}$ is 1 if the mass flow is sent to stream $l_1(m)$ and 0 if it is sent to stream $l_2(m)$ under scenario s).

The final set of constraints adds penalties for the presence of arsenic in the concentrate. In this study, three possible situations are considered: a) if the As concentration is below 0.2%, then penalties and additional costs are not considered; b) if the arsenic concentration is between 0.2 and 0.5%, then it is considered a penalty payment; and c) if the arsenic concentration exceeds 0.5%, then an additional cost of treatment of the concentrate by roasting to partially remove arsenic at a level below 0.2% is considered. This can be described through the disjunctions

$$\begin{bmatrix} zas_{1,s} \\ \dfrac{\sum_{e\in E} R_e \cdot w_{s,P,e}}{\sum_{e\in E} w_{s,P,e}} \geq 0 \\ \dfrac{\sum_{e\in E} R_e \cdot w_{s,P,e}}{\sum_{e\in E} w_{s,P,e}} \leq 0.2 \\ c_{As,s} = 0 \end{bmatrix} \vee \begin{bmatrix} zas_{2,s} \\ \dfrac{\sum_{e\in E} R_e \cdot w_{s,P,e}}{\sum_{e\in E} w_{s,P,e}} \geq 0.2 \\ \dfrac{\sum_{e\in E} R_e \cdot w_{s,P,e}}{\sum_{e\in E} w_{s,P,e}} \leq 0.5 \\ c_{As,s} = A_{As} \cdot \left(\dfrac{\sum_{e\in E} R_e \cdot w_{s,P,e}}{\sum_{e\in E} w_{s,P,e}} - B_{As} \right)^+ \end{bmatrix} \vee$$

$$\begin{bmatrix} zas_{3,s} \\ \dfrac{\sum_{e\in E} R_e \cdot w_{s,P,e}}{\sum_{e\in E} w_{s,P,e}} \geq 0.5 \\ \dfrac{\sum_{e\in E} R_e \cdot w_{s,P,e}}{\sum_{e\in E} w_{s,P,e}} \leq 10.0 \\ c_{As,s} = RC_{As} \cdot \sum_{e\in E} was_{3,s,P,e} \end{bmatrix}$$

$$s \in S, \qquad (9)$$

where

- $w_{s,P,e}$ is the final concentrate flowrate for specie e for scenario s,

- R_e is the amount of arsenic in specie e,

- A_{As}, B_{As} and RC_{As} are constants used to define penalty payments and roasting costs,

- $(x)^+ = \max\{x, 0\}$,

- $c_{As,s}$ is a variable that captures the costs associated with the presence of arsenic in scenario s, and

- $zas_{1,s}$, $zas_{2,s}$ and $zas_{3,s}$ are binary variables that indicate the choice in the disjunction (e.g. $zas_{1,s} = 1$, $zas_{2,s} = 0$ and $zas_{3,s} = 0$ if arsenic concentration is bellow 0.2% for scenario s).

Disjunctions (8) and (9) are transformed to MINLP constraints through standard techniques (Grossmann and Ruiz, 2012).

2.2 Objective Function Components

Besides the possible arsenic penalty $c_{As,s}$ the objective function also includes an amortized fixed equipment cost and a revenue component for each scenario.

The amortized fixed equipment cost fc corresponds to

$$fc = Z \sum_{f\in F} n_f \left(P_1 + P_2 \cdot v_f + P_3 \cdot v_f{}^Q \right), \qquad (10)$$

where

- Z is an amortization constant for fixed cost, and

- P_1, P_2, P_3, Q are economic parameters for the equipment costs.

The revenue of selling concentrate is calculated using the net-smelter-return formula (Shena et al., 1996), which corresponds to

$$rev_s = PE \cdot (\pi_s - RFC) \cdot \sum_{e\in E} G_e \cdot w_{s,P,e}$$

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$$-(TRC + PE \cdot U \cdot (\pi_s - RFC)) \cdot \sum_{e \in E} w_{s,P,e} \qquad\qquad s \in S \qquad (11)$$

where

- PE is the fraction of metal paid,
- π_s is the copper price under scenario s,
- RFC is the refinery charge,
- G_e is the copper grade of specie e,
- U is the grade deduction, and
- TRC is the treatment charge.

2.3 Problems

Deterministic Problem

For the deterministic problem, there is only a single scenario; hence, $S = \{1\}$. The problem simply maximizes the total profit subject to the operational, design and economic constraints and is given by

Maximize $rev_1 - c_{As,1} - fc$

subject to

$Eq. (4)$ to (10)

This problem is denoted the *deterministic* problem. In this problem, the uncertain parameters are replaced by its weighted average.

Stochastic Problem

For the stochastic problems, it is assumed that there is more than one scenario (s > 1). Consequently, the objective considers the maximization of the expected total income, which requires knowledge of the probability of a given scenario s, denoted by P_s. The most adaptable version of this problem is the TS-MINLP given by

Maximize $-fc + \sum_{s \in S} P_s \cdot (rev_s - c_{As,s})$

subject to

$Eq. (4)$ to (10)

This problem is denoted the *stochastic problem*.

2.4 Instances

The computational experiments in this paper consider a flotation plant that has four mineralogical species: fully liberated chalcopyrite ($CuFeS_2$), partially liberated chalcopyrite

(70% liberated), tennantite ($Cu_{12}As_4S_{13}$), and gangue. Each of these species has a specific floatability. Therefore three species of interest are identified: copper, arsenic, and gangue. The treatment plant capacity is 28,800 t/d.

The stochastic parameters are the feed-copper grade and the copper price. Because of the variability of the processed ore, three levels of feed-copper grade are considered (low, medium, and high), and because of the variability of the world market, the price of copper on the London Metal Exchange is highly uncertain, which is why three levels of metal price are considered (low, medium, and high). The combination of these parameters is represented in 9 stochastic scenarios, as shown in Fig. 2. For example, scenario 1 has a probability of 5% and corresponds to 0.45% chalcopyrite, 0.3% partially liberated chalcopyrite, and 0.45% tennantite, and a copper price of 2,025 US$/t.

Total Copper Grade — Low, Medium, High

Copper Price — Low, Medium, High

Scenario 1: E1 = 0.0045, E2 = 0.003, E3 = 0.0045, P = 2205 — Scenario Probability 5 %

Scenario 2: E1 = 0.0045, E2 = 0.003, E3 = 0.0045, P = 4409 — Scenario Probability 15 %

Scenario 3: E1 = 0.0045, E2 = 0.003, E3 = 0.0045, P = 8300 — Scenario Probability 10 %

Scenario 4: E1 = 0.002, E2 = 0.03, E3 = 0.0045, P = 2205 — Scenario Probability 10 %

Scenario 5: E1 = 0.002, E2 = 0.03, E3 = 0.0045, P = 4409 — Scenario Probability 35 %

Scenario 6: E1 = 0.002, E2 = 0.03, E3 = 0.0045, P = 8300 — Scenario Probability 15 %

Scenario 7: E1 = 0.03, E2 = 0.05, E3 = 0.0045, P = 2205 — Scenario Probability 5 %

Scenario 8: E1 = 0.03, E2 = 0.02, E3 = 0.0045, P = 4409 — Scenario Probability 3 %

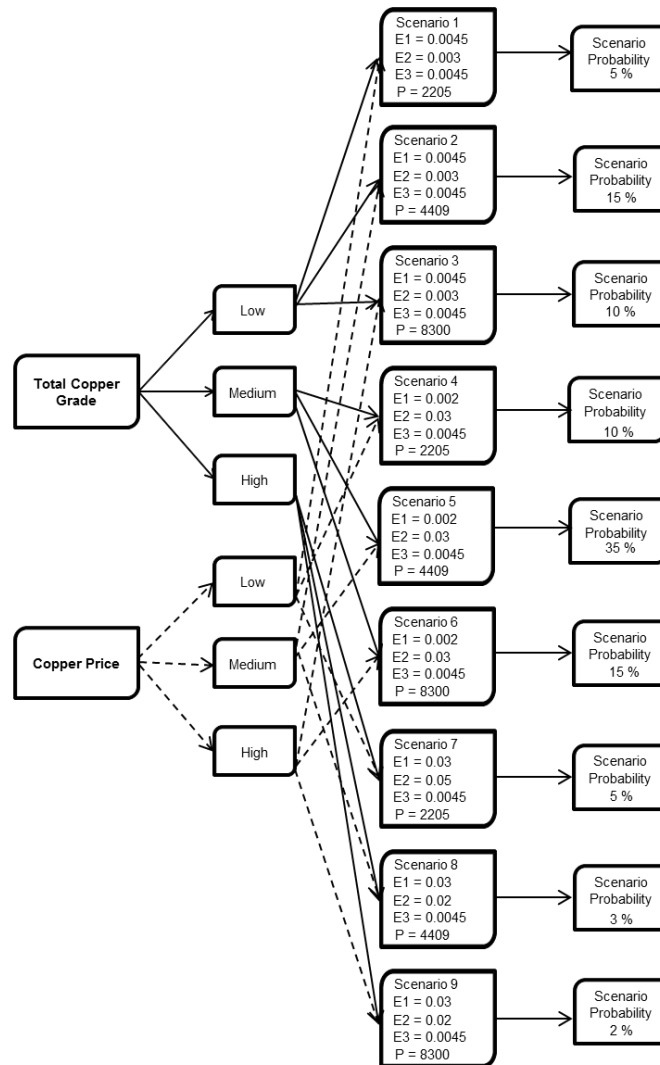Scenario 9: E1 = 0.03, E2 = 0.02, E3 = 0.0045, P = 8300 — Scenario Probability 2 %

Fig. 2: Variability of uncertain parameters represented in nine scenarios. E1, E2 and E3 are the chalcopyrite fully liberated, chalcopyrite partially liberated and tennantite feed grade, respectively. P is the copper price in US$/t.

For the superstructure, a flotation plant with five flotation stages was considered: rougher (R), scavenger (S), cleaner-scavenger (CS), cleaner (C), and re-cleaner stages (CC). The numbers of cells by bank are fixed according to the notes made by Bourke (2002), that is, 6 cells for the rougher, scavenger, and cleaner-scavenger stages; for the cleaner and re-cleaner stages the number of cells is fixed to 4. The superstructure is represented by the origin-destination matrix (see Table 1). For example, for the cleaner-scavenger stage (CS), the concentrate can be sent to the C stage, CC stage or sent directly to the final concentrate stream (P). However, the CS tail can be sent to the S stage or sent directly to the final tail stream (W). From Table 1, it is possible to identify 64 paths for the concentrate and 48 paths for the tail, and thus this superstructure has 3,072 circuit alternatives. Some advantages of using the origin-destination matrix are: 1) equivalent circuit alternatives are not considered, therefore the 3,072 circuit alternatives are all different, 2) Senseless alternatives are not included (e.g. sending the cleaner concentrate to the scavenger stage), 3) the designer can choose which interconnections to include in the design problem. Using the origin-destination matrix reduces the total number of structures from 60 million to 3,072. However, the non-convexities of the optimization problem can still make it extremely hard to solve. Effective solution methodologies are explored in the following section.

Table 1
Origin-Destination matrix for concentrates and tails of the flotation superstructure. X represents the concentrate of the stage, and o represents the tail of the flotation stage. P and W are final concentrate and final tail streams, respectively.

	R	S	CS	C	CC	P	W
R		o		x		x	o
S	x		x	x		x	o
CS		o		x	x	x	o
C	o	o	o		x	x	o
CC	o		o	o		x	o

3   Solution Methodology

Finding optimal solutions for the target TS-MINLP can be extremely challenging even with state of the art solvers. However, as shown in this section, using simple stochastic programming techniques it is possible to obtain high-quality (and possible near-optimal) solutions. The section is divided into two parts. The first considers the selection of an appropriate MINLP solver and the second describes two simple stochastic programming techniques.

3.1 Solvers analysis.

All problems described in Section 2.3 are so-called non-convex or general MINLP problems. Such problems are extremely challenging as they include both integer variables and non-convex constraints (e.g. Bussieck and Vigerske, 2014). In particular, finding solutions that are guaranteed to be globally optimal requires specialized solvers such as BARON or LINDOGLOBAL. Unfortunately, the convergence of such solvers can be extremely slow even for moderately small instances. An alternative to such global optimization solvers is to utilize so-called convex MINLP solvers. These solvers only

guarantee global optimality for convex MINLP problems; that is, for problems whose only non-convexities are the requirements on integer variables. However, convex MINLP solvers can have significantly faster convergence and provide locally optimal solutions that can often be close to being globally optimal. This subsection compares the performance of several global and convex MINLP solvers including BONMIN, DICOPT, KNITRO, LINDOGLOBAL, OQNLP, and SBB. Table 2 shows the main characteristic of these solvers. More information about them can be found in the user manuals (see BONMIN, DICOPT, KNITRO, LINDOGLOBAL, OQNLP, and SBB manuals).

Table 2. Characteristic of solvers used in the study.

Solver	Algorithms	Optimality Guarantee for General MINLP	Advantage	Disadvantage
BARON	Specialized branch-and-reduce algorithm.	Global.	Global optimality up to numerical accuracy.	Slow convergence for large problems.
DICOPT	MILP/NLP outer-approximation algorithm.	Only local.	Fast convergence. Global optimality for convex MINLP.	No global optimality for general MINLP.
BONMIN	Branch-and-bound and outer-approximation based algorithms.	Only local.	Fast convergence. Global optimality for convex MINLP	No global optimality for general MINLP.
KNITRO	Various branch-and-bound and outer-approximation based.	Only local.	Fast convergence and numerical stability. Global optimality for convex MINLP.	No global optimality for general MINLP.
LINDO GLOBAL	Branch-and-bound and outer-approximation based algorithm.	Global.	Global optimality up to numerical accuracy.	Extremely slow convergence.
OQNLP	Multi-start heuristic algorithms.	Only local.	Very fast convergence time.	No global optimality for general or convex MINLP.
SBB	NLP based branch-and-bound algorithm.	Only local.	Fast convergence. Global optimality for convex MINLP.	No global optimality for general MINLP.

Because the complete problem is too challenging for some of the solvers, this subsection considers a subset of the superstructure that involves three flotation stages: rougher, scavenger, and cleaner. In addition, the numbers of cells by bank are fixed according to the notes made by Bourke (2002) and each scenario was run separately. The total elapsed time

reported corresponds to the sum of all elapsed time and the profit recovery and grade are the weighted sum of the values for each scenario. All experiments were executed in a computer with an Intel core i7 processor (2.7 GHz) and 6 GB of memory RAM. All problems were coded in GAMS, a high-level modeling system for mathematical programming.

The results for the optimization problem (total income, recovery and grade) and execution time are shown in Table 3. Note that KNITRO was not able to find the integer solution of the binary variable for some scenarios. For those scenarios the best upper bound returned by the solver was used, which lead to a higher optimal solution. LINDOGLOBAL, after 30 days of execution, was not able to provide a result for the optimization problem. Because the behavior of SBB and BONMIN were similar, their functioning was compared with the full five-stage problem. BARON was also included to compare the optimal solution; however BARON, like the case with three stages, requires very high computation times. SBB was unable to converge to a solution, while BONMIN and BARON obtained the same optimal solution, but BONMIN used less time than BARON. As a conclusion, the chosen solver for the subsequent analysis of the stochastic method is BONMIN due to the quality of the results compared to BARON and the low execution times.

Table 3
Results and execution times for different MINLP solvers. Profit is in MMUS$/y, overall recovery is the weighted average recovery for all of the scenarios, and Cu Grade is the weighted average copper grade in the concentrate for all of the scenarios.

Variable	MINLP Solver						
	BARON	DICOPT	BONMIN	KNITRO	LINDOGLOBAL	OQNLP	SBB
Profit	2.771	2.674	2.770	2.851	-	2.765	2.770
Recovery	0.633	0.599	0.632	0.622	-	0.628	0.632
Cu Grade	0.254	0.244	0.254	0.253	-	0.255	0.254
Time (sec)	2,360,007.5	11.5	19.4	55.9	-	705.3	11.8

3.2 Strategies to solve the stochastic problem

Even with an appropriate solver selection, the stochastic problem is still too hard to solve directly. For this reason, two methodologies are analyzed to facilitate the convergence of the stochastic optimization problem.

The first methodology begins by independently solving each scenario. As described in the previous section this can be done extremely fast using Bonmin. However, this solution has the first stage design variables (i.e. volumes) varying with the scenarios, which is not feasible. For this reason, in the second step of the methodology, the volume is fixed to the maximum value of all of the volumes found in each of the scenarios. This ensures that there is enough capacity of each flotation stage for all scenario-specific operational selections in the first step. The methodology ends by resolving the problem for each scenario with the volumes fixed to the common volumes determined in the previous step. This is denoted as Solution Methodology A.

The second methodology again begins by independently solving each scenario. The binary variables of stream directions of the superstructure in the stochastic problem are then fixed to the values obtained in the previous step. In this manner, the majority of the binary

variables in the stochastic problem become parameters making it much easier to solve. The final step of this methodology is to simply solve the stochastic problem with fixed binary variables. This is denoted as Solution Methodology B.

The results of the two strategies for the full 5-stage instance are presented in Table 4. By employing methodology A, the profits are approximately 250,000 US$/y better than with methodology B. The increase in the volume in the re-cleaner stage allows increasing the recovery of the intermediate scenarios (4 to 6), which corresponds to scenarios with higher probabilities. Although the difference between the two methodologies in this study case is not high, this is not true in all cases. For example, if the feed stream decreases from 28,800 to 16,800 t/d, the optimal solution strategy is methodology B, with a difference between both methodologies of approximately 1 MMUS$/y. In this case, by employing methodology A, the volume is fixed to the maximum found for all scenarios, and therefore there are scenarios (low price and grade) in which this sizing is too large and the amount of mineral entering to the flotation circuit is small. This result means low revenues and higher capital costs, which directly affects profits.

Table 4
Results of the comparison between solution methodologies A and B.

	Solution Methodology A	Solution Methodology B
Total Income MMUS$/y	63.346	63.097
Volume Rougher Cell m^3	200	200
Volume Scavenger Cell m^3	200	200
Volume Cleaner - Scavenger Cell m^3	200	200
Volume Cleaner Cell m^3	200	200
Volume Recleaner Cell m^3	43	21

4. Results and Discussion

This section compares solutions for the stochastic and deterministic average problems. The solution for the stochastic problem is that obtained by solution methodology A and the solution of the deterministic average problem was obtained by solving the appropriate MINLP with BONMIN. This section also presents a sensitivity analysis for the epistemically uncertain parameters.

4.1 Comparison between the stochastic and the deterministic average problems.

This section compares the design and operational variables plus the profits obtained by the stochastic and deterministic average problems. As noted in section 2.3, the deterministic average problem utilizes the weighted averages of the stochastic parameters. In this case, the feed grade corresponds to 0.56% of chalcopyrite, 2.39% partially liberated chalcopyrite, and 0.45% of tennantite. The price of the metal corresponds to 5,018 US$/t.

Fig. 3 shows the optimal configuration for the deterministic average problem, which considers the recirculation of the C tail to the R stage and the CC tail and the concentrate of the CS stage to the C stage. Finally, the tail of the CS stage is sent to the S stage. The real profit is determined by fixing the direction of the streams and the volumes determined

previously and recalculating the real operating variables, such as the residence time, flow rates, recoveries, and grades for each scenario. In this way, the real profit is determined based on the profit of each scenario. In Table 5, the recalculated results for each scenario are shown. For first three scenarios, an arsenic concentration of 2.2 % is obtained, and therefore, an arsenic refining cost must be paid. The configuration of Fig. 3 maximizes the overall recovery due to the good average values of feed grade and copper price, but the limited size of the design cells produces a decrease in the residence time, thereby affecting the recovery of the valuable specie in each stage for all scenarios.
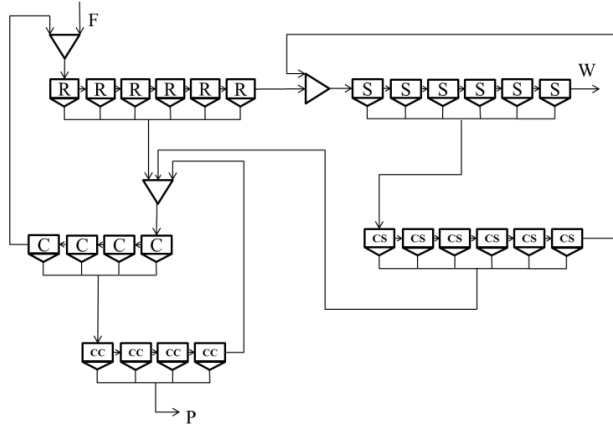

Fig. 3: Optimal configuration of the deterministic average problem.

Table 5
Results for the deterministic average problem.

Variable	Scenario								
	1	2	3	4	5	6	7	8	9
Volume Cell Rougher (m^3)					200				
Volume Cell Scavenger (m^3)					200				
Volume Cell Cleaner-Scavenger (m^3)					43				
Volume Cell Cleaner (m^3)					30				
Volume Cell Re-cleaner (m^3)					5				
Copper grade Concentrate %	31.0	31.0	31.0	25.4	25.4	25.4	29.0	29.0	29.0
Arsenic grade in concentrate %	2.2	2.2	2.2	0.45	0.45	0.45	0.073	0.073	0.073
Copper Recovery %	52.9	52.9	52.9	62.1	62.1	62.1	62.8	62.8	62.8
Total real Income MMUS$/y					59.448				

In contrast, the stochastic problem can adjust its operational variables to the uncertainty of the design parameters. These adjustments increase the final profits by 3,900 MMUS$/y. In this case, the volume increases in the CS, C, and CC stages, corresponding to the optimal results for the most favorable scenario of grade and price. Once the first level variables are defined, the second level variables to be optimized are the stream directions and operating time of each stage. The results are shown in Table 6 and Fig. 4. The overall recovery

changes from 67% for the most unfavorable scenario to 86% for most favorable scenario, and these values are significantly higher than those obtained by the deterministic average problem. The residence times are approximately constant for all scenarios, 7 to 10 minutes per cell for the R, S, CS, and C stages. Only in the CC stage the residence time varies from 8 to 24 minutes depending on each scenario. The difference between all possible configurations is the recirculation of the S concentrate. For scenarios with a low copper price, except for the combination of low copper price and the highest copper feed grade, the optimal configuration considers the recirculation of the S concentrate to the CS stage (Fig. 4a and 4d). This configuration allows higher grades in the concentrate due to the recirculation being sent to early stages. However, due to the low feed grade, the optimal design of the process considers an increment in the residence time in the CC stage from 8 minutes for the most favorable scenario to 24 minutes for the most unfavorable one. This increased time improves the overall recovery as well as the arsenic concentration in the concentrate.

Table 6
Results for the stochastic problem.

Variable	Scenario								
	1	2	3	4	5	6	7	8	9
Volume Cell Rougher (m^3)					200				
Volume Cell Scavenger (m^3)					200				
Volume Cell Cleaner-Scavenger (m^3)					200				
Volume Cell Cleaner (m^3)					200				
Volume Cell ReCleaner (m^3)					43				
Copper grade Concentrate %	26.71	26.03	26.03	22.84	24.49	24.49	28.25	28.25	28.25
Arsenic grade in concentrate %	3.32	3.50	3.50	0.86	1.26	1.26	0.47	0.47	0.47
Copper Recovery %	67.06	70.44	70.44	37.20	79.63	79.63	85.72	85.72	85.72
Total real Income MMUS$/y					63.346				

The optimal configuration changes as the values of copper grade and price change. For example, for scenarios of low and medium levels of feed grade combined with favorable levels of price, the configuration changes, sending the S concentrate to the C feed (Fig. 4b) increasing the overall recovery. This configuration (from the combinations found for the different scenario) delivers a greater recovery because the streams are sent directly to the final concentrate through a few flotation stages. Therefore, the amount of concentrate is maximized but contains a greater amount of impurities, decreasing the final concentrate grade. However, for the scenarios with the highest-value copper grade and price parameters, the recirculation of the S concentrate is sent to the R stage (Fig. 4c). With these conditions of price and grade, the plant configuration should be the same or change to continue increasing the overall recovery, but the results of the optimization problem are different from those expected. The optimal configuration changes the recirculation to the earlier stage of the plant to reprocess the concentrate streams many times over, which is necessary to improve the final concentrate grade. It is also worth mentioning that this improvement in the configuration also decreases the arsenic concentration to 0.5%, and thus the total cost is considered a penalization and not an additional refining cost.

a) Scenario 1

b) Scenarios 2, 3 and 5

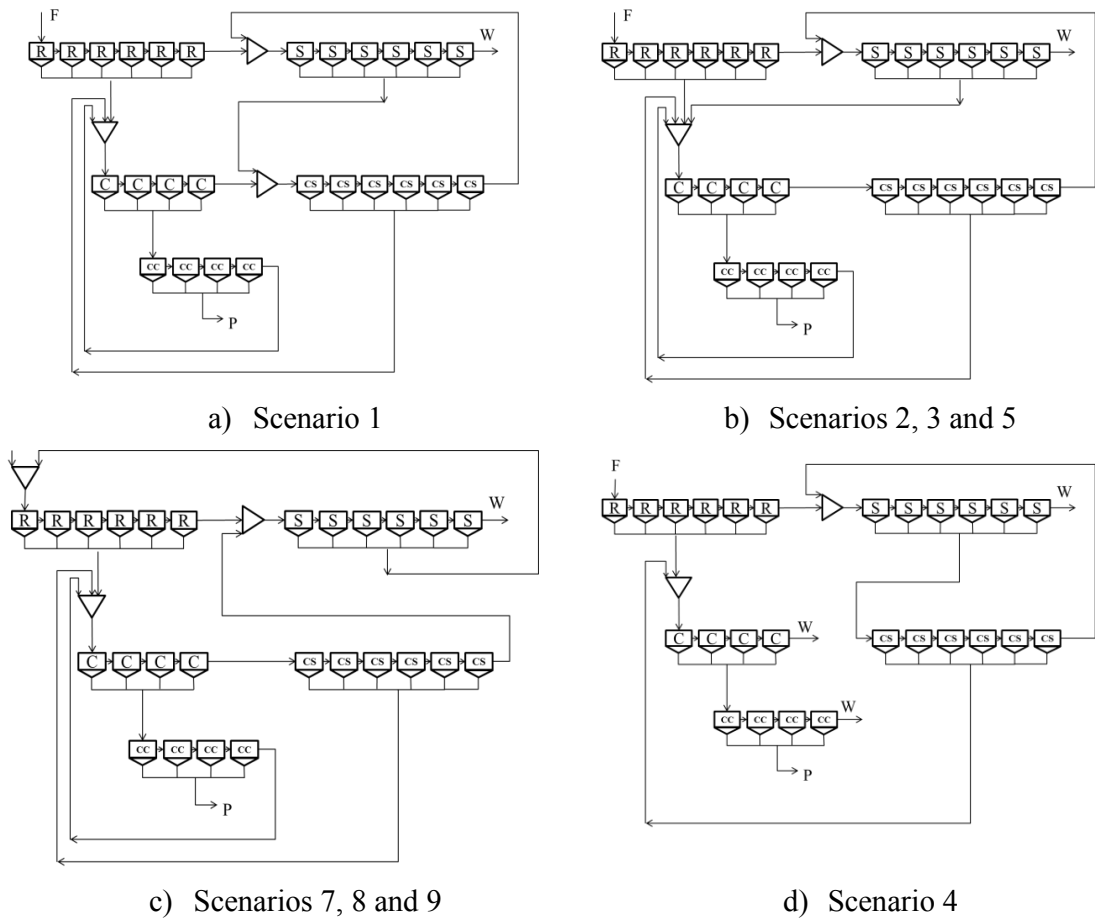c) Scenarios 7, 8 and 9

d) Scenario 4

Fig. 4. Optimal configurations for the stochastic problem.

The stochastic problem sacrifices some scenarios to obtain higher profits. Thus, scenarios 1, 4 and 5 give worse results than the deterministic average problem. However, the greatest profit obtained in other scenarios justifies this choice. These differences between the profits for each scenario are shown in Fig. 5.
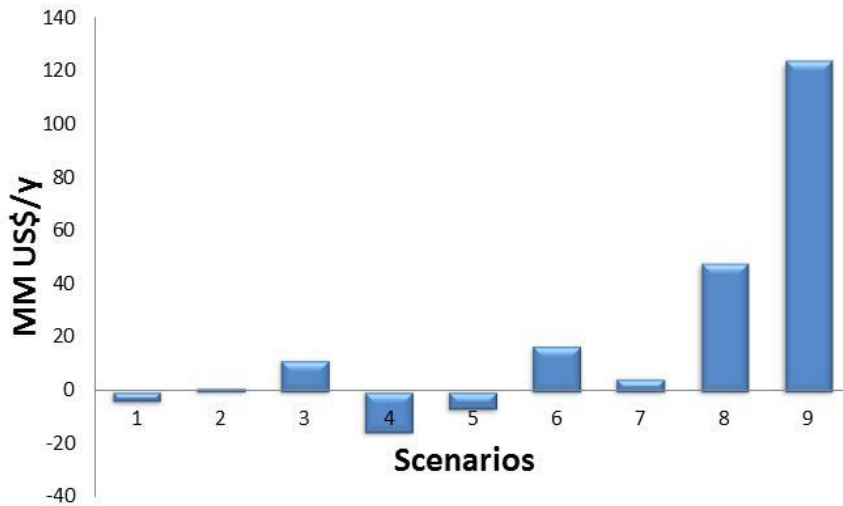
Fig. 5. Difference in profit between the stochastic and deterministic equivalent problems in MMUS$/y.

4.3 On the epistemic uncertainty of kinetic parameters

In this work the stage recoveries were calculated using the kinetic model of Yianatos and Henríquez, (2006) for flotation banks. The kinetic parameters were assumed constants, but these parameters can change with the flotation circuit design. For example, kinetic parameters can change with the cell size and chemical composition of the pulp. Also the design can affect other phenomena such as the water recovery, which can affect the entrainment. Also, particle size distribution was not considered and the flotation rate is particle size dependent. This means that a change in one stage affects the flotation rate of minerals in the others stages and therefore the kinetic parameters are not constants. This uncertainty in the kinetic parameters and stage recovery, as was indicated previously, corresponds to epistemic uncertainty because there is a lack of knowledge on the values of these parameters and the behavior of recovery with circuit and stage design. This uncertainty was not the focus of this work because previous studies have shown that the best flotation circuit is not highly sensitive to the flotation stage recovery (e.g. see Galvez and Cisternas, 2014 and its references). However, this section provides a concise analysis of the effect of this source of uncertainty.

To obtain insight of the effect of the stage recovery uncertainties three numerical experiments were performed: First, the cell volume was assumed constant and the kinetic parameter $Rmax_{s,f}$ was varied by 5, 10 and 15% and the model was applied for each of the scenarios. The results show that two, one and three of the nine scenarios changed when the value of Rmax changed to 5, 10 and 15% respectively. The scenarios with high and low values of metal prices were unchanged, indicating that the stochastic uncertainty in the price of metal has an important effect on the optimal configuration of the circuit. Second, the volume of the cells were assumed variables and the kinetic parameter $K_{s,f}$ was varied by 10 and 20% and the model was applied for each of the scenarios. Three scenarios

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changed for a 10% change in the kinetic parameter, and only two scenarios changed for the case of 20%. Again, scenarios with high values and low values in metal prices were unchanged. In both experiments the circuits changed in some scenarios, but they changed to circuits already observed in other scenarios. Finally, the volume of cells were considered variables and the value of $Rmax_{s,f}$ was changed in 15%, then five of the nine scenarios changed, indicating that in these situations the structure is more sensitive to changes in the recovery. This case is an important change in recovery. The scenarios with high and low values in the metal price and in the feed grade were unchanged. Also, some circuits obtained had not been obtained in previous studies. It can be concluded that the kinetics may influence the results, though clearly the price of the metal and the feed grade are dominant in the circuit obtained.

5. Conclusions

A stochastic optimization problem for the design of flotation circuits under stochastic uncertainty has been presented. Uncertainty is represented by scenarios that include changes in the feed grade and in the metal price. The problem allows the operating conditions (residence time and mass flows of each stream) and flow structure (tail and concentrate) to be changed for each scenario while the fixed design (size of cells in flotation stages) for all scenarios is maintained. The problem can be modified to include other uncertainties and other adaptive variables.

While the problem is extremely flexible, it can be extremely hard to solve with standard software. For this reason several solution approaches were explored. The first approach was an appropriate selection of a MINLP solver. The optimization problem developed requires the solution by a specialized solver to guarantee a global optimal solution is found. However, standard global optimization solvers such as BARON and LINDOGLOBAL cannot currently handle the instance sizes considered in this paper. This lead to the computational evaluation of solvers that are only guaranteed to provide a local optimal solution. These experiments showed that utilizing the solver BONMIN provided an excellent compromise between solution time and quality. Unfortunately, solver selection alone was not enough to handle the target instances. For this reason two simple stochastic programming methodologies were used to solve the optimization problem. These methodologies delivered high quality solutions in reasonable times, which allowed an analysis of the advantages of the stochastic optimization problem and a sensitivity analysis of the parameters that have some epistemic uncertainty.

The analysis showed that the stochastic problem gives better results than the average deterministic problem by allowing the optimal to adapt to the feed composition and the price of the metal. The sensitivity analysis to the epistemic uncertainty in the kinetic parameters showed small to moderate variability.

Finally, it can be concluded that the use of stochastic optimization can be a beneficial tool in the design of a metallurgical process, specifically the copper flotation process. The optimal configuration is capable of adapting to uncertainty, leading to an increase of the profits of the company. Further studies are still needed to incorporate the epistemic uncertainty in solving flotation design problems. Fortunately, the flexibility of the optimization problems and the effectiveness of standard methodologies should allow these issues to be resolved in the near future.

Acknowledgments

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Nomenclature

Sets and indices:

s: Scenarios. The set of all scenarios is $S=\{1.....sn\}$, where sn is the number of scenarios.

f: Flotation stages. The set of all flotation stages is $F=\{$rougher, scavenger, cleaner-scavenger cleaner, recleaner$\}$.

e: Species. The set of all species is $E=\{$fully liberated chalcopyrite, partially liberated chalcopyrite, tennantite, gangue$\}$.

l: Streams. The set of all streams is $L=\{1....16\}$.

li(f), lc(f), and lt(f): The feed, concentrate output, and tail output streams of stage f, respectively (e.g., li(Rougher)=2). These are subsets of L.

m: Mixers and splitters. The set of all mixers is X, and the set of all splitters is P.

$l_1(m), l_2(m)$: The first and second possible outputs for splitter m, respectively

$M^{in}(m), M^{out}(m)$: The input and output streams of the splitter or mixer m.

Variables

$w_{s,l,e}$: Flow of specie e on stream l under scenario s.

$r_{s,f}$: Operating retention time (one cell) in flotation stage f under scenario s.

v_f : Cell volume in flotation stage f.

$y_{s,m} = \begin{cases} 1 \text{ if the flow is sent to stream } l_1(m) \text{ of splitter under scenario s.} \\ 0 \text{ if the flow is sent to stream } l_2(m) \text{ of splitter under scenario s.} \end{cases}$

$zas_{as,s}$: Binary variable, for arsenic level decision as in scenario s.

$c_{As,s}$: Cost associated to arsenic under scenario s.

c_s : Operation cost under scenario s.

rev_s : Income under scenario s.

fc : Amortised fixed equipment cost.

Certain Constants

$K_{e,f}$: Flotation rate of specie e for flotation stage f

$Rmax_{e,f}$: Maximum Recovery of specie e for flotation stage f.

$M: Bi$g-M constant that is larger than the flow in any stream for any specie under any scenario.

A_{As}, B_{As}: Constants for arsenic penalization.

A_f, D_f : Pulp aeration factor and corrected density for flotation stage f.

P_1, P_2, P_3, Q : Economic parameter for fixed costs.

R_e : Arsenic grade of specie e.

G_e : Copper grade of specie e.

TRC, PE, U, RFC : Processing and economic parameters for income and operational costs.

T : Total mass of input to the system.

Z : Amortization constant for fixed cost.

n_f : Number of cells in flotation stage f.

Potentially Uncertain Constants

π_s : Copper price under scenario s.

$Y_{s,e}$: Feed grade corresponding to specie e in scenario s.