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Estimating inflation expectation co-movement across countries

Shi Chen*
Wolfgang Karl Härdle*
Weining Wang*

* Humboldt-Universität zu Berlin, Germany

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Estimating inflation expectation co-movement across countries*

Shi Chen †, Wolfgang Karl Härdle‡, Weining Wang§

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Abstract

Inflation expectation is an important indicator for policy makers and financial investors. To capture a more accurate real-time estimate of inflation expectation on the basis of financial markets, we propose an arbitrage-free term structure model across different countries. We first estimate inflation expectation by modeling the nominal and the inflation-indexed bond yields jointly for each country. The joint dynamic model for inflation expectation is a cross sectional state space model combined with a GeoCopula model, which accounts for the default risk and the non Gaussian dependency structure over countries. We discover that the extracted common trend for inflation expectation is an important driver for each country of interest. Moreover, the model extracts informative estimates of inflation expectations and will provide good implications for monetary policies.

Keywords: inflation expectation, arbitrage free, yield curve modelling, inflation risk

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†Corresponding author. Humboldt-Universität zu Berlin, C.A.S.E.-Center of Applied Statistics and Economics, Unter den Linden 6, 10099 Berlin, Germany. Email: chenshiq@hu-berlin.de

‡Humboldt-Universität zu Berlin, C.A.S.E.-Center of Applied Statistics and Economics, Unter den Linden 6, 10099 Berlin, Germany. Research fellow in Sim Kee Boon Institute for Financial Economics, Singapore Management University, 90 Stamford Road, 6th Level, School of Economics, Singapore 178903.

§Humboldt-Universität zu Berlin, C.A.S.E.-Center of Applied Statistics and Economics, Unter den Linden 6, 10099 Berlin, Germany.

JEL classification: G12, E43, E31

1 Introduction

Today most economists favour a low and steady rate of inflation because it facilitates real wage adjustments in the presence of downward nominal wage rigidity. Hence one of the major objectives of modern monetary policy is to bring inflation expectation under control, which is considered to be the first step in controlling inflation. Meanwhile, hedging the risk around the inflation forecast becomes more attractive in financial markets, as many investors rely on the stability and predictability of future inflation levels. Moreover, price stability is of immense importance to sustain social welfare, job opportunities and economic upturn. The objective of price stability refers to the general level of prices in the economy which implies avoiding both prolonged inflation and deflation.

Inflation expectation that is involved in a contemporary macroeconomic framework anticipates future economic trends, will further affect monetary decisions. Since there is large demand on having reasonable estimates of inflation expectation levels, a large amount of literature has focused on analysing the government conventional and inflation-indexed bonds, which can implicitly provide a vast amount of information about the expectations of nominal and real interest rates obtained from the market. Such estimates are known to be an important complement to the estimates provided from the survey data.

Despite the fact that inflation indexed bonds have been more frequently and widely issued in recent times, one would still have great difficulties in integrating the market information from multiple countries to get individual level estimates of the inflation expectation. The major problems lie in the relative short period of data availability and the existence of a lot of missing values. While the existing literature's focus is mainly on specific country, we would like to consider an estimation framework that allows us to analyse the co-movement of inflation expectation for multiple countries, and also provide country specific estimates of inflation expectation(IE) and the inflation risk premium (IRP).

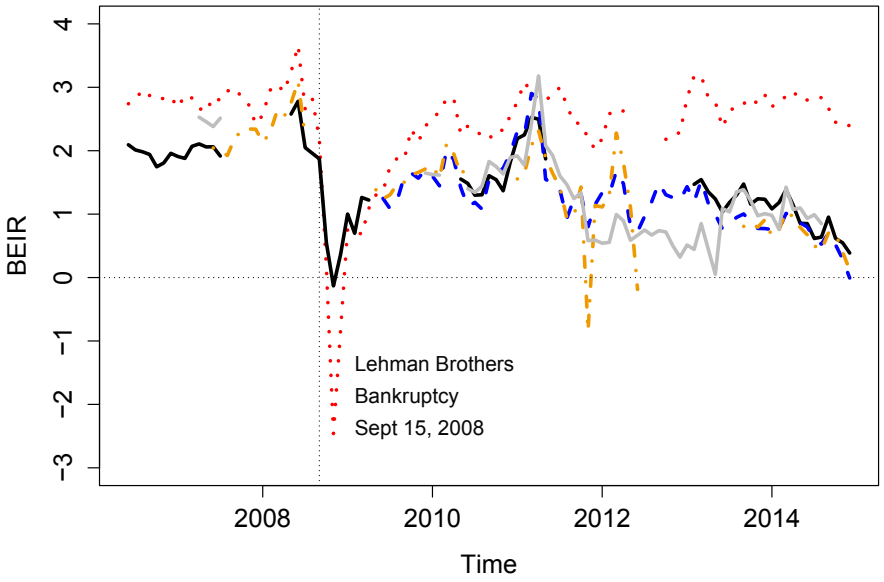

Figure 1: BEIR for five industrialized European countries - U.K.(red dotted line), Germany(blue dashed line), France(black line), Italy(orange dot-dashed line) and Sweden(grey line) with maturity of 3 years.

MTS_BEIR

The starting point of our research is to analyse the break-even inflation rate (BEIR), which is known to be the difference between the yield on a nominal fixed-rate bond and the real yield on an inflation-linked bond of the same maturity and similar credit quality. The BEIR can generally indicate how the inflation expectations are priced into the market. However they are not a perfect measure for IEs, as they may also encompass inflation risk premium, liquidity premium and technical market factors. In Figure 1 we observe the BEIR for five European countries - U.K., Germany, France, Italy and Sweden with maturity of three years. A fall in consumer prices appears since September 2009 due to a drop in energy costs, which exhibits some degree of co-movement. It is known that the euro-zone annual inflation rate was recorded at -0.2 percent in December of 2014 which matches, but is slightly higher than the overall BEIR shown in the Figure. This motivates us to extract a joint time-varying structure of IEs estimated from individual (country-specific) BEIR in a multiple country framework.

The modelling of BEIR requires a model for the joint dynamics of the nominal and the

real yields. For instance, Härdle and Majer (2014) investigated the yield curves using a Dynamic Semiparametric Factor Model (DSFM). To adopt a real time approach to access the term structure of nominal and inflation-linked yields, we consider a three-factor term structure model motivated by Nelson and Siegel (1987). The attractiveness of Nelson-Siegel factor models is due to its convenient affine function structure and good empirical performance. Diebold and Li (2006) extend the original Nelson-Siegel model to a dynamic environment. Theoretically, the Nelson-Siegel (NS) model does not ensure the absence of arbitrage opportunities, as shown by Bjork and Christensen (1999). Christensen, Diebold and Rudebusch (2011) further develop the NS model to an AFNS model by imposing the arbitrage-free hypothesis, which reflects most of the real activities of financial markets. The standard approaches for pricing forwards, swaps are all derived from such arbitrage arguments for both complete and incomplete markets. In our paper, we will use an AFNS model for the dynamics of the nominal and the real yield respectively, and combine the two models afterwards.

Based on the joint dynamics of the nominal and the real yields, a sizable amount of literature has analysed how to isolate IE and IRP from BEIR. Earlier work mainly focuses on U.K. data because the U.K. was one of the first developed economies to issue inflation-indexed bonds for institutional investors. With the first U.K. index-linked gilts issued in 1981, various developments have occured in the international markets. Barr and Campbell (1997) estimate market expectations of real interest rates and inflation from observed prices of U.K. government nominal and inflation-linked bonds. Joyce, Lildholdt and Sorensen (2010) develope an affine term structure model to decompose forward rates to obtain IRP. Notably, Christensen, Lopez and Rudebusch (2010) use an affine arbitrage-free model of the term structure to decompose BEIR that captures the pricing of both nominal and inflation-indexed securities. A four-factor joint AFNS model was achieved by combining the AFNS models for nominal and inflation-linked yields, which proved to be efficient for fitting and forecasting analysis. Unlike Christensen et al. (2010), we align the four factor models over different maturities to make the factors consistent over maturities.

With the AFNS model for the joint dynamics on hand, we proceed with our European

country analysis. Most of the existing literature mentions little about the story of multiple countries. Diebold, Li and Yue (2008) are the first to consider a global multiple country model for nominal yield curves. There are a few European central bank reports, which focus on household and expert inflation expectation and the anchoring of inflation expectations in the two currency areas before and during the 2008 crisis, examples are Pflueger and Viceira (2011).

Here we would like to look into five industrialised European countries by constructing a joint model of country-specific IEs. We construct an AFNS model in multi-maturity term structure for modelling nominal and inflation-indexed bonds simultaneously, and we also propose a joint model of IE dynamics over European countries, which discovers the extracted common trend for IE as an important driver for each country of interest. The GeoCopula model allows us to further understand the non-Gaussian dependency structures across countries. Then we conduct an analysis to explore the estimated common factor by decomposing the variation into parts driven by common effect variation and macroeconomic effect variation.

The rest of the paper proceeds as follows. Section 2 presents the joint AFNS model in a multi-maturity term structure for estimating yields of nominal and inflation-linked bonds, and also introduces the decomposition method of BEIR. In section 3, we explain GeoCopula model and discuss the econometric methodology used in the joint modelling of IE dynamics. The technical details are in the Appendix. The empirical results are shown in Section 4. Finally section 5 concludes and introduces further works.

2 Preliminary Analysis

In this section, we introduce a methodology to obtain the model-implied BEIR. Subsection 2.1 briefly introduces the Nelson-Siegel model, and subsection 2.2 constructs the joint AFNS structure for modelling nominal and inflation-indexed bonds. Subsection 2.3 introduces the joint AFNS model across countries in a multi-maturity term structure. Finally

in the subsection 2.4, we describe the decomposition method of BEIR.

2.1 A factor model representation

The classic Nelson-Siegel (NS) yield curve model for fitting static yield curves is with simple functional form,

$$y(\tau) = \beta_0 + \beta_1 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_2 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) \tag{1}$$

where $y(\tau)$ is a zero-coupon yield with τ months to maturity, and $\beta_0, \beta_1, \beta_2$ and λ are parameters. This model is popular because it is simple and tractable. For a fixed value of parameter λ, β_i with $i = 1, 2, 3$ can be estimated by the ordinary least square method. Maturity τ determines the decay speed of parameters.

The aforementioned dynamic version of Nelson-Siegel (DNS) model enables institutional investors and policy makers to understand the evolution of the bond market over time, the DNS model can be written as,

$$y_t(\tau) = L_t + S_t \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + C_t \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) \tag{2}$$

where $y_t(\tau)$ denotes continuously zero-coupon yields of maturity τ at time t. The time-varying factors are defined as level L_t, slope S_t and curvature C_t. Such choice of the latent factors is motivated by principal component analysis, which gives us three principal components corresponding to the latent factors. For instance, the most variation of yields is accounted by the first principal component - level factor L_t.

By incorporating the arbitrage-free assumption over τ, the AFNS model brings the best of the Nelson-Siegel model and the Arbitrage-Free model. Thus, the AFNS model consists of two equations by taking the structure of the DNS model and the real-world dynamics

(under P-measure) derived from the AF model respectively,

$$
\begin{aligned}
y_t(\tau) &= X_t^1 + X_t^2 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + X_t^3 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) - \frac{A(\tau)}{\tau} \\
dX_t &= K^P(\theta^P - X_t)dt + \Sigma dW_t^P
\end{aligned}
\tag{3}
$$

where $X_t^\top = (X_t^1, X_t^2, X_t^3)$ is a vector of latent factors, $\frac{A(\tau)}{\tau}$ is an unavoidable yield-adjustment term depending on maturity τ. K^P and θ^P correspond to drifts and dynamics terms, and are both allowed to vary freely. Σ is identified as a diagonal volatility matrix.

2.2 A joint factor model

The AFNS structure is a useful representation for term structure research. Christensen et al. (2010) employ and conduct a separate AFNS model estimation of nominal and inflation-linked Treasury bonds respectively. In this Subsection, we construct an extended AFNS structure for modelling nominal and inflation-indexed bonds simultaneously.

The AFNS model of nominal and inflation-indexed bond yields for a specific country i can be written respectively as,

$$
\begin{aligned}
y_{it}^N(\tau) &= L_{it}^N + S_{it}^N \left(\frac{1 - e^{-\lambda_i\tau}}{\lambda_i\tau} \right) + C_{it}^N \left(\frac{1 - e^{-\lambda_i\tau}}{\lambda_i\tau} - e^{-\lambda_i\tau} \right) - \frac{A_i^N(\tau)}{\tau} \\
y_{it}^R(\tau) &= L_{it}^R + S_{it}^R \left(\frac{1 - e^{-\lambda_i\tau}}{\lambda_i\tau} \right) + C_{it}^R \left(\frac{1 - e^{-\lambda_i\tau}}{\lambda_i\tau} - e^{-\lambda_i\tau} \right) - \frac{A_i^R(\tau)}{\tau}
\end{aligned}
$$

where y_{it}^N and y_{it}^R represent the nominal and inflation-linked yields for country i at time t. To explore the relationship between nominal and inflation-indexed bond yields within a country, we need to combine two types and model them jointly.

To work with a simplified version of the yield curves, we impose an assumption on the correlation between the latent factors of nominal and inflation-indexed bonds. The empirical evidence can be found in Christensen et al. (2010). Furthermore, this assumption will be justified by the performance of the joint model illustrated in subsection 4.2. The

assumption takes the form of,

$$\begin{aligned} S_{it}^R &= \alpha_i^S S_{it}^N \\ C_{it}^R &= \alpha_i^C C_{it}^N \end{aligned} \tag{4}$$

Therefore the yield curve of the joint AFNS model can be written as,

$$\begin{aligned} \begin{pmatrix} y_{it}^N(\tau) \\ y_{it}^R(\tau) \end{pmatrix} &= \begin{pmatrix} 1 & \frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau} & \frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau} - e^{-\lambda_i\tau} & 0 \\ 0 & \alpha_i^S \frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau} & \alpha_i^C(\frac{1-e^{-\lambda_i\tau}}{\lambda_i\tau} - e^{-\lambda_i\tau}) & 1 \end{pmatrix} \begin{pmatrix} L_{it}^N \\ S_{it}^N \\ C_{it}^N \\ L_{it}^R \end{pmatrix} \\ &+ \begin{pmatrix} \varepsilon_{it}^N(\tau) \\ \varepsilon_{it}^R(\tau) \end{pmatrix} - \begin{pmatrix} \frac{A_i^N(\tau)}{\tau} \\ \frac{A_i^R(\tau)}{\tau} \end{pmatrix} \end{aligned} \tag{5}$$

The real-world dynamics (under P-measure) is,

$$dX_t = K^P(\theta^P - X_t)dt + \Sigma dW_t^P$$

where the vector of state variables $X_{it}^\top = \left(L_{it}^N, S_{it}^N, C_{it}^N, L_{it}^R\right)$ evolves dynamically.

2.3 Multiple Yield Curve Modelling

Diebold et al. (2008) extend the DNS model to a global version by modelling a potentially large set of yield curves for the countries around the world. This framework allows for both global and country-specific factors. Here we employ the joint AFNS model introduced in equation 5 and further extend it to a multiple-maturity case.

For a specific country i, we first assume the vector of state variables $X_{it}^\top$ is a common factor for the yield curves across different maturities. Therefore the joint AFNS yield

curve in multi-maturity term structure is,

$$
\begin{pmatrix} y_{it}^N(\tau_1) \\ y_{it}^R(\tau_1) \\ y_{it}^N(\tau_2) \\ y_{it}^R(\tau_2) \\ \vdots \\ y_{it}^N(\tau_n) \\ y_{it}^R(\tau_n) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda_i\tau_1}}{\lambda_i\tau_1} & \frac{1-e^{-\lambda_i\tau_1}}{\lambda_i\tau_1} - e^{-\lambda_i\tau_1} & 0 \\ 0 & \alpha_i^S\frac{1-e^{-\lambda_i\tau_1}}{\lambda_i\tau_1} & \alpha_i^C(\frac{1-e^{-\lambda_i\tau_1}}{\lambda_i\tau_1} - e^{-\lambda_i\tau_1}) & 1 \\ 1 & \frac{1-e^{-\lambda_i\tau_2}}{\lambda_i\tau_2} & \frac{1-e^{-\lambda_i\tau_2}}{\lambda_i\tau_2} - e^{-\lambda_i\tau_2} & 0 \\ 0 & \alpha_i^S\frac{1-e^{-\lambda_i\tau_2}}{\lambda_i\tau_2} & \alpha_i^C(\frac{1-e^{-\lambda_i\tau_2}}{\lambda_i\tau_2} - e^{-\lambda_i\tau_2}) & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-\lambda_i\tau_n}}{\lambda_i\tau_n} & \frac{1-e^{-\lambda_i\tau_n}}{\lambda_i\tau_n} - e^{-\lambda_i\tau_n} & 0 \\ 0 & \alpha_i^S\frac{1-e^{-\lambda_i\tau_n}}{\lambda_i\tau_n} & \alpha_i^C(\frac{1-e^{-\lambda_i\tau_n}}{\lambda_i\tau_n} - e^{-\lambda_i\tau_n}) & 1 \end{pmatrix} \begin{pmatrix} L_{it}^N \\ S_{it}^N \\ C_{it}^N \\ L_{it}^R \end{pmatrix}
$$

$$
+ \begin{pmatrix} \varepsilon_{it}^N(\tau_1) \\ \varepsilon_{it}^R(\tau_1) \\ \varepsilon_{it}^N(\tau_2) \\ \varepsilon_{it}^R(\tau_2) \\ \vdots \\ \varepsilon_{it}^N(\tau_n) \\ \varepsilon_{it}^R(\tau_n) \end{pmatrix} - \begin{pmatrix} \frac{A_i^N(\tau_1)}{\tau_1} \\ \frac{A_i^R(\tau_1)}{\tau_1} \\ \frac{A_i^N(\tau_2)}{\tau_2} \\ \frac{A_i^R(\tau_2)}{\tau_2} \\ \vdots \\ \frac{A_i^N(\tau_n)}{\tau_n} \\ \frac{A_i^R(\tau_n)}{\tau_n} \end{pmatrix} \tag{6}
$$

where we recall that $y_{it}^N(\tau_n)$ and $y_{it}^R(\tau_n)$ represent the nominal and inflation-linked yields for country i at time t with maturity τ_n. The real-world dynamics equation is in the same form as before,

$$dX_t^P = K^P(\theta^P - X_t)dt + \Sigma dW_t^P$$

where the vector of state variables $X_{it}^\top = \left(L_{it}^N, S_{it}^N, C_{it}^N, L_{it}^R\right)$ evolves dynamically.

The joint AFNS yield curve model across countries will lead to a more efficient estimation. We demonstrate the goodness of fit of our model by showing the model residuals in subsection 4.4. We use the Kalman filter and maximum likelihood estimation for the state variables and parameters, the technical details are in Appendix A.

2.4 BEIR decomposition

In order to find a more appropriate measure of expected inflation, it is necessary to understand the components of the bond yields. A large literature has adopted a parametric approach to estimate the IE and risk premia using nominal and indexed bonds data, such as Adrian and Wu (2009), Campbell and Viceira (2009), Pflueger and Viceira (2011). They decompose the yield of an inflation-linked bond into current expectation of a future real interest rate and a real interest rate premium. The yield on a nominal bond can then be decomposed into parts of the yield on a real bond, expectations of future inflation and IRP. Therefore the spread between both yields, the BEIR, reflects the level of IE and IRP.

In the context of an arbitrage-free model, it is assumed that investors have no opportunities to make risk-free profits. Thus the bonds can be priced by basic pricing equations according to Cochrane (2005),

$$P_t = \mathsf{E}_t \left\{ M_{t+1} x_{t+1} \right\}, \tag{7}$$

where M_{t+1} is a stochastic discount factor. Define

$$\mathbf{M}^N_{1:\tau} \overset{\text{def}}{=} \left(M^N_{t+1} M^N_{t+2} \cdots M^N_{t+\tau} \right) \tag{8}$$

$$\mathbf{M}^R_{1:\tau} \overset{\text{def}}{=} \left(M^R_{t+1} M^R_{t+2} \cdots M^R_{t+\tau} \right), \tag{9}$$

where the nominal and the real (for inflation-linked bond) SDFs at time t are denoted by M^N_t and M^R_t.

Then the prices of the zero-coupon bonds at time t, which pay one unit measured by the consumption basket at the time of maturity $t+\tau$, are formed as follows,

$$\begin{aligned} P^N_t(\tau) &= \mathsf{E}_t \left(\mathbf{M}^N_{1:\tau} \right) \\ P^R_t(\tau) &= \mathsf{E}_t \left(\mathbf{M}^R_{1:\tau} \right) \end{aligned} \tag{10}$$

where $P^N_t(\tau)$ and $P^R_t(\tau)$ represent the prices of nominal and real bonds respectively. The

price of the consumption basket, which is known as the overall price level Q_t, has the following link with SDFs given the no arbitrage assumption,

$$\frac{M_t^N}{M_t^R} = \frac{Q_{t-1}}{Q_t} \tag{11}$$

Converting the price into the yield by the equation of $y_t(\tau) = -\frac{1}{\tau}\log P_t(\tau)$,

$$\begin{aligned} y_t^N(\tau) &= -\frac{1}{\tau}\log \mathsf{E}_t\left(\mathbf{M}_{1:\tau}^N\right) \\ &\approx -\frac{1}{\tau}\mathsf{E}_t\left(\log \mathbf{M}_{1:\tau}^N\right) - \frac{1}{2\tau}\mathsf{Var}_t\left(\log \mathbf{M}_{1:\tau}^N\right) \\ y_t^R(\tau) &= -\frac{1}{\tau}\log \mathsf{E}_t\left(\mathbf{M}_{1:\tau}^R\right) \\ &\approx -\frac{1}{\tau}\mathsf{E}_t\left(\mathbf{M}_{1:\tau}^R\right) - \frac{1}{2\tau}\mathsf{Var}_t\left(\log \mathbf{M}_{1:\tau}^R\right), \end{aligned}$$

where more details can be found in Appendix.

Therefore,

$$\begin{aligned} y_t^N(\tau) - y_t^R(\tau) &\approx -\frac{1}{\tau}\mathsf{E}_t\left(\log\frac{\mathbf{M}_{1:\tau}^N}{\mathbf{M}_{1:\tau}^R}\right) + \frac{1}{2\tau}\mathsf{Var}_t\left(\log\frac{\mathbf{M}_{1:\tau}^N}{\mathbf{M}_{1:\tau}^R}\right) \\ &- \frac{1}{\tau}\mathsf{Cov}_t\left(\log\frac{\mathbf{M}_{1:\tau}^N}{\mathbf{M}_{1:\tau}^R}, \log\mathbf{M}_{1:\tau}^R\right). \end{aligned}$$

Given the log inflation is $\pi_{t+1} = \log\frac{Q_{t+1}}{Q_t}$ and the relationship between SDFs according to equation (10), the $BEIR$ can be decomposed as,

$$\begin{aligned} y_t^N(\tau) - y_t^R(\tau) &\approx \frac{1}{\tau}\mathsf{E}_t\left(\log \pi_{t+1}\pi_{t+2}\cdots\pi_{t+\tau}\right) - \frac{1}{2\tau}\mathsf{Var}_t\left(\log \pi_{t+1}\pi_{t+2}\cdots\pi_{t+\tau}\right) \\ &+ \frac{1}{\tau}\mathsf{Cov}_t\left(\log \pi_{t+1}\pi_{t+2}\cdots\pi_{t+\tau}, \log\mathbf{M}_{1:\tau}^R\right) \end{aligned} \tag{12}$$

that is,

$$BEIR_t(\tau) = y_t^N(\tau) - y_t^R(\tau) = \pi_t(\tau) + \eta_t(\tau) + \phi_t(\tau), \tag{13}$$

where $\pi_t(\tau)$ is the IE, $\eta_t(\tau)$ is the corresponding convexity effect and $\phi_t(\tau)$ is IRP.

To link the $BEIR_t(\tau)$ with the estimated state variables in equation 6, we assume that

the P-dynamics of the SDFs are,

$$\begin{aligned}\frac{dM_t^N}{M_t^N} &= -(r_t^N - r_{t-1}^N)dt - (\Gamma_t^N - \Gamma_{t-1}^N)dW_t^P \\ \frac{dM_t^R}{M_t^R} &= -(r_t^R - r_{t-1}^R)dt - (\Gamma_t^R - \Gamma_{t-1}^R)dW_t^P,\end{aligned} \tag{14}$$

where $r_t^N = L_t^N + S_t$, $r_t^R = L_t^R + \alpha^R S_t$, Γ_t^N and Γ_t^R represent the corresponding risk premium; their dynamics is connected to the underling state variables X_t in equation (6). More details are given in Appendix B. Hence the dynamics of the overall price level is,

$$\begin{aligned}d\log\left(\frac{Q_{t-1}}{Q_t}\right) &= -(r_t^N - r_t^R)dt + (r_{t-1}^N - r_{t-1}^R)dt \\ d\log(Q_t) &= (r_t^N - r_t^R)dt\end{aligned} \tag{15}$$

The IE is given by

$$\pi_t(\tau) = -\frac{1}{\tau}\log \mathsf{E}_t^P\left[\exp\left\{-\int_t^{t+\tau}(r_s^N - r_s^R)ds\right\}\right] \tag{16}$$

which can be solved by a system of ODEs with a Runge-Kutta method, see Appendix B. The convexity effect can be written as

$$\eta_t(\tau) = -\frac{1}{\tau}\mathsf{E}_t^P\left[\log\exp\left\{-\int_t^{t+\tau}(r_s^N - r_s^R)ds\right\}\right] \tag{17}$$

Then the IRP can be easily calculated out by equation 13.

3 Econometric Modelling of Inflation Expectation

3.1 Global model for inflation expectation

Motivated by Diebold et al. (2008), the model can be extended to a framework that allows for both global and country-specific factors. As far as we have obtained the country-specific estimates of IE, we build a model for multiple countries in this subsection. In particular, we allow the country-specific idiosyncratic factors to load on a common time-

varying factor and country-specific factors. The dynamics of an extracted common trend is also evaluated.

The model without a macroeconomic factor is structured as follows: the idiosyncratic factors $\hat{\pi}^e_{it}$ for each country i at time t is loading on a common time-varying latent factor Π_t,

$$\hat{\pi}^e_{it} = m_i + n_i \Pi_t + u_{it} \tag{18}$$

The dynamics of common factor is set as follows,

$$\Pi_t = p + q\Pi_{t-1} + \nu_t \tag{19}$$

where m, n, p and q are unknown parameters. The errors ν_t are assumed to be i.i.d., and the u_{it} are modeled further in section 3.2.

Since there is a dynamic interaction between macro-economy and the yield curve as evidenced by Diebold, Rudebusch and Aruoba (2006), in Figure 1 we can observe that the decrease of the BEIR appears around 2012 due to the European sovereign debt crisis. A straightforward extension of the joint modelling equation (16) is adding a proxy of the macroeconomic factor - default risk factor. The model with a macroeconomic factor is,

$$\hat{\pi}^e_{it} = m_i + n_i \Pi_t + l_i d_{it} + u_{it} \tag{20}$$

where d_{it} is the default risk factor varying over time and m, n, p, l and q are all unknown parameters. In addition, to capture the joint dependency of the inflation expecation process across countries, u_{it} is assumed to follow a non Gaussian GeoCopula process. And ν_{it} is assumed to to be i.i.d white noise.

3.2 Spatial-temporal Copula

In this subsection, we explain the spatial-temporal copulae model adopted accounting for the non-Gaussian dependency across countries. A Spatial-temporal Copula process for the error term u_{it} in equation (18) with $t = 1, \cdots, T$, $i = 1, \ldots, N$ is described as follows,

$$u_{it} = \alpha_{it} + \xi_{it}, \tag{21}$$

where α_{it} captures majorly the spatial temporal variation and ξ_{it} is the Gaussian noise with mean 0 and variance σ_ξ, see Bai, Kang and Song (2014), and Xu (2015). Our Spatial-temporal Copula model is then formulated as,

$$F_{it}(\alpha) = \Phi_{NT}\{\Phi^{-1}(F_{11}(\alpha_{11})), \cdots, F_{N,T}(\alpha_{NT})|\Sigma\}, \tag{22}$$

wherefor $\Phi_{NT}(\cdot)$ is the cumulative distribution function (c.d.f.) of a multivariate Gaussian distribution with a variance covariance matrix Σ, which models the spatiotemporal dependence. Note that one can also generalize the framework to t distribution. According to the marginal closure property of Gaussian copulae, one can marginalize the high-dimensional c.d.f. in equation (22) into 2-dimensional marginals. Thus for any spatial temporal coordinate t_1, n_1 and t_2, n_2, the dependency can be expressed via a pairwise copulae model:

$$F_{t,n}(\alpha_{t_1,n_1}, \alpha_{t_2,n_2}) = \Phi_2(\Phi^{-1}(\alpha_{t_1,n_1}), \Phi^{-1}(\alpha_{t_2,n_2})|\Sigma_{t_1,n_1,t_2,n_2}), \tag{23}$$

where Σ_{t_1,n_1,t_2,n_2} is a submatrix of Σ.

In this paper, we further parameterize Σ_{t_1,n_1,t_2,n_2} by the function:

$$\sigma(n_2 - n_1, t_2 - t_1) = \sigma(v, u) \stackrel{\text{def}}{=} \begin{cases} \frac{2\sigma^2\beta}{(a^2u^2+1)^\eta(a^2u^2+\beta)\gamma(\eta)}\left(\frac{b}{2}\left(\frac{a^2u^2+1}{a^2u^2+\beta}\right)^{1/2}v\right)^\eta K_\eta\left(b\left(\frac{a^2u^2+1}{a^2u^2+\beta}\right)^{1/2}v\right) \text{ if } v > 0\ , \\ \frac{\sigma^2\beta}{(a^2u^2+1)^\eta(a^2u^2+\beta)\gamma(\eta)} \text{ if } v = 0, \end{cases}$$

where a, b, β, η are parameters, $\gamma(\eta)$ is the gamma function and $K_\eta(\cdot)$ is the Bessel function of the second kind.

Denote $f_{t,n}(\alpha_{t_1,n_1}, \alpha_{t_2,n_2})$ as the joint density of the two random variables. The estimator can be attained by maximizing joint composite log likelihood, see Varin (2008) .

$$l(\theta, d_1, d_2) = \sum_{t_1,t_2,n_1,n_2:\|t_1-t_2\|\leq d_1,\|n_1-n_2\|\leq d_2} \log f_{\alpha_{t_1 n_1},\alpha_{t_2 n_2}}, \tag{24}$$

where $\theta \stackrel{\text{def}}{=} (a, b, \beta, \eta)^\top$. Note that one still needs to choose the cut-off points d_1, d_2 for this methodology. In particular the density is

$$f_{\alpha_{t_1,n_1},\alpha_{t_2,n_2}} \stackrel{\text{def}}{=} c_\Phi\{F(\alpha_{t_1,n_1}), F(\alpha_{t_2,n_2})\} f(\alpha_{t_1,n_1}) f(\alpha_{t_2,n_2})$$

with

$$c_\Phi\{F(\alpha_{t_1,n_1}), F(\alpha_{t_2,n_2})\} = |\Sigma_{t_1,n_1,t_2,n_2}|^{-1/2} \exp\{q^\top (I_2 - \Sigma^{-1}_{t_1,n_1,t_2,n_2}) q\},$$

$$q \stackrel{\text{def}}{=} (q_{t_1,n_1}, q_{t_2,n_2})$$

and

$$q_{t_i,n_i} = \Phi^{-1}\{\hat{F}(x_{t_i,n_i})\}.$$

4 Empirical Results

Subsection 4.1 describes the data. We then show the results for the fitting of the joint AFNS model in (6) for each country, and also evaluate the fitting performance by showing residuals of the fitting in Subsection 4.2. Subsection 4.3 analyzes the estimated country specific IEs. Subsequently subsection 4.4 jointly models the estimated IEs in previous step for each country, and also discusses the common trend extracted. The GeoCopula fitting results are showed in Subsection 4.5. The forecasting results are presented in Subsection 4.6.

4.1 Data

We take monthly nominal and inflation-linked yield data of zero-coupon government bonds from Bloomberg and Datastream. The research databases are supported by the Research Data Center (RDC) from the Collaborative Research Center 649, Humboldt Universität zu Berlin. We consider data from the developed European countries - United Kingdom (U.K.), France, Germany, Italy and Sweden, which are all member states of European Union (EU). The motivation for us to select these countries are the availability of frequently traded inflation indexed bond data. It should be noted that two of the selected five European countries are outside the euro-zone - U.K. and Sweden, and thus with their own currencies therefore independent central banks and monetary policy. While the Swedish krona is somehow tied to EUR, the exchange rate EUR/GPB is more flexible. Treasury bonds data are obtained for each country considered. Namely, we collect the Gilts bonds for U.K., OATs for France, Bunds for Germany, BTPs for Italy and the index-linked bonds for Sweden. Moreover, the real Gilts bonds in UK is linked to UK Retail Prices Index (RPI); the real OATi bonds in France is linked to France consumer price index excluding tobacco (France CPI ex Tobacco); the real bonds - Bundei in Germany, and BT Pei in Italy are both linked to European Harmonised Index of Consumer Prices excluding tobacco (EU HICP ex Tobacco); the real SGBi bonds in Sweden linked to Sweden consumer price index (Sweden CPI). The inflation co-movement can be observed across the selected countries in subsection 4.4. This also motivates our analysis to extract a joint time-varying structure of country-specific IEs.

The lack of short-maturity inflation-linked bonds of the sample countries indicates that inflation-linked yield at short-maturity tends to be less reliable. We therefore select three maturities for each country to ensure that enough observations are available. The sample period covers the subprime crisis in 2008 and European sovereign debt crisis in 2011. Monthly data is collected, and the sample period is sightly different for each country. The surfaces of the yield data are plotted in Figure 2. The blank areas in the Figure represents a small proportion of missing values. Summary statistics are shown in Table 1. The mean

level for nominal bonds are higher than the inflation indexed ones. While Germany has smallest standard deviations, UK can be seen with the largest level of standard deviations.

	Period	Maturity	Type	Min	Mean	Max	SD
UK	30.06.2006	3	nominal	0.16	2.20	5.69	1.73
			inflation-indexed	-2.87	-0.14	5.35	1.81
	—	4	nominal	0.35	2.44	5.62	1.59
			inflation-indexed	-2.62	-0.04	4.74	1.64
	31.12.2014	5	nominal	0.57	2.66	5.56	1.48
			inflation-indexed	-2.37	0.11	4.27	1.49
Sweden	30.04.2007	3	nominal	0.18	1.80	4.67	1.27
			inflation-indexed	-0.71	0.15	1.83	0.54
	—	5	nominal	0.58	2.33	4.71	1.11
			inflation-indexed	-0.84	0.51	2.33	0.79
	29.08.2014	10	nominal	0.88	2.59	4.61	1.03
			inflation-indexed	-0.30	0.98	2 29	0.64
France	30.06.2006	3	nominal	-0.02	1.86	4.74	1.46
			inflation-indexed	-1.19	0.43	2.75	1.29
	—	5	nominal	0.06	2.10	4.80	1.37
			inflation-indexed	-1.29	-0.40	1.06	0.60
	31.12.2014	10	nominal	0.18	2.34	4.80	1.28
			inflation-indexed	-1.09	1.03	2.66	1.03
Germany	30.06.2009	5	nominal	-0.07	0.90	2.38	0.77
			inflation-indexed	-1.39	-0.35	1.00	0.54
	—	7	nominal	0.05	1.37	2.85	0.86
			inflation-indexed	-1.16	0.05	1.36	0.65
	31.12.2014	10	nominal	0.39	1.94	3.29	0.84
			inflation-indexed	-0.53	0.33	1.67	0.67
Italy	30.06.2007	3	nominal	0.55	2.94	7.37	1.33
			inflation-indexed	-0.34	1.51	8.21	1.46
	—	5	nominal	0.95	3.53	7.54	1.20
			inflation-indexed	0.20	2.00	7.84	1.29
	31.12.2014	10	nominal	1.89	4.45	7.11	0.93
			inflation-indexed	1.02	2.77	6.72	1.06

Table 1: Descriptive statistics of the monthly bond yields data. SD is standard deviation.

4.2 Yield Curve Modelling - Single Country

In this subsection, we fit the model in equation (6) and evaluate the model performance. Figure 3 shows the model residuals over different maturities for all five European countries.

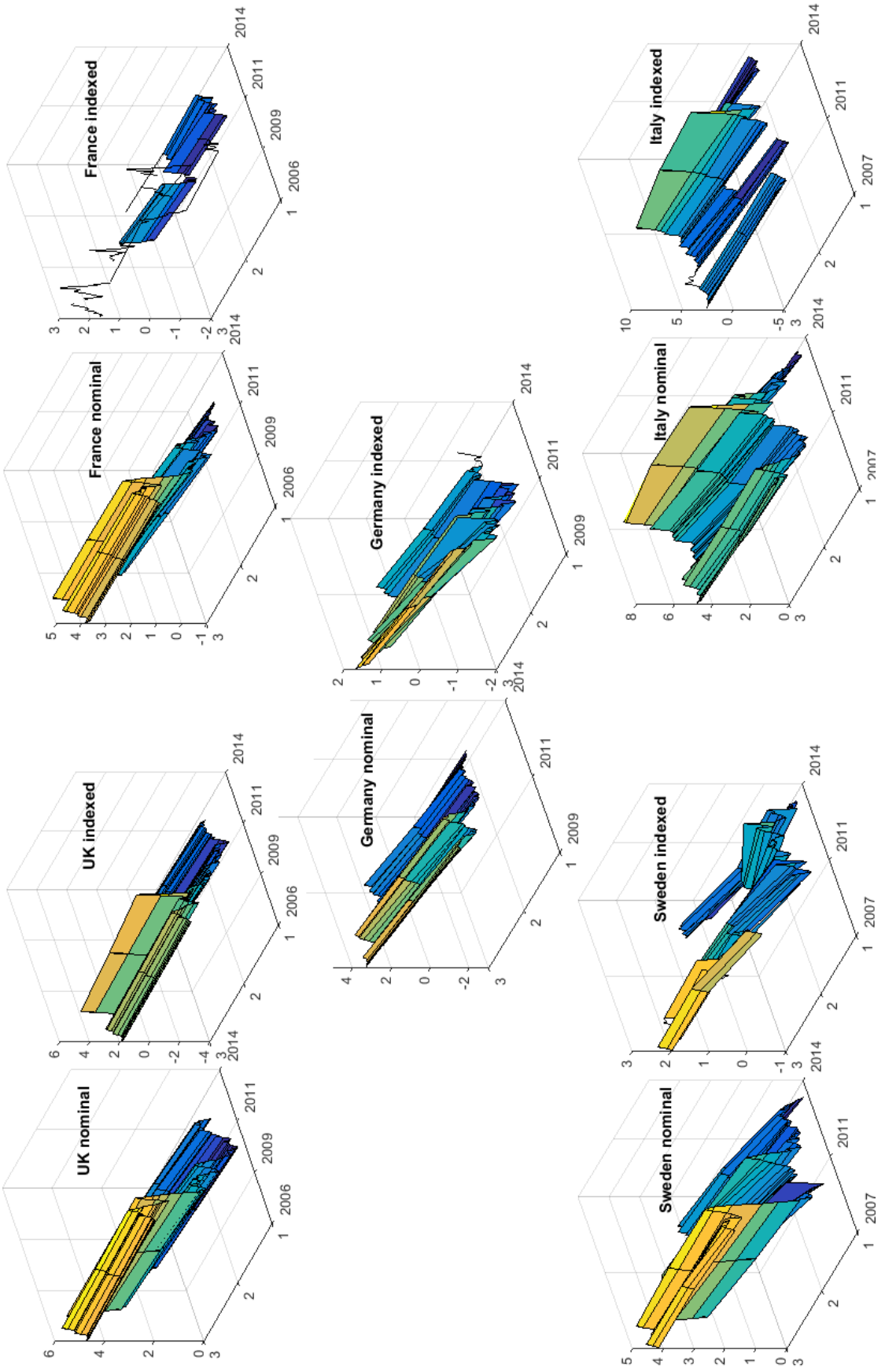

Figure 2: Term structures of nominal and inflation-linked bond yields across five European countries.

The summary statistics of the model fit is presented in Table 2. The overall level of the the residuals is small (average absolute value at around 0.09), and also averaged RMSE is around 0.1, which indicates the goodness of fit for the country specific joint multiple yield curve model. However we still notice that the model residuals have small jumps for short periods. Specifically the outliers observed in Italy happened to be during sovereign default crisis in 2012. We can also observe larger residuals at around September 2008 for the U.K. and Sweden due to the well documented subprime crisis.

	Maturity	Type	Mean	RMSE
UK	3	nominal	0.12	0.10
		inflation-indexed	0.10	0.22
	4	nominal	-0.10	0.08
		inflation-indexed	-0.09	0.18
	5	nominal	0.13	0.08
		inflation-indexed	0.12	0.17
Sweden	3	nominal	0.06	0.12
		inflation-indexed	0.06	0.04
	5	nominal	-0.07	0.11
		inflation-indexed	-0.20	0.51
	10	nominal	0.02	0.12
		inflation-indexed	0.18	0.03
France	3	nominal	0.01	0.08
		inflation-indexed	0.01	0.05
	5	nominal	-0.15	0.07
		inflation-indexed	-0.12	-0.06
	10	nominal	0.02	0.06
		inflation-indexed	0.04	0.05
Germany	5	nominal	0.14	0.08
		inflation-indexed	0.02	0.08
	7	nominal	-0.22	0.05
		inflation-indexed	-0.25	0.09
	10	nominal	0.23	0.08
		inflation-indexed	0.12	0.14
Italy	3	nominal	0.07	0.40
		inflation-indexed	0.01	0.50
	5	nominal	-0.25	0.27
		inflation-indexed	-0.18	0.40
	10	nominal	-0.02	0.13
		inflation-indexed	0.24	0.22

Table 2: Summary statistics of the model fit using a multiple yield curve model. RMSE is a root mean square error.

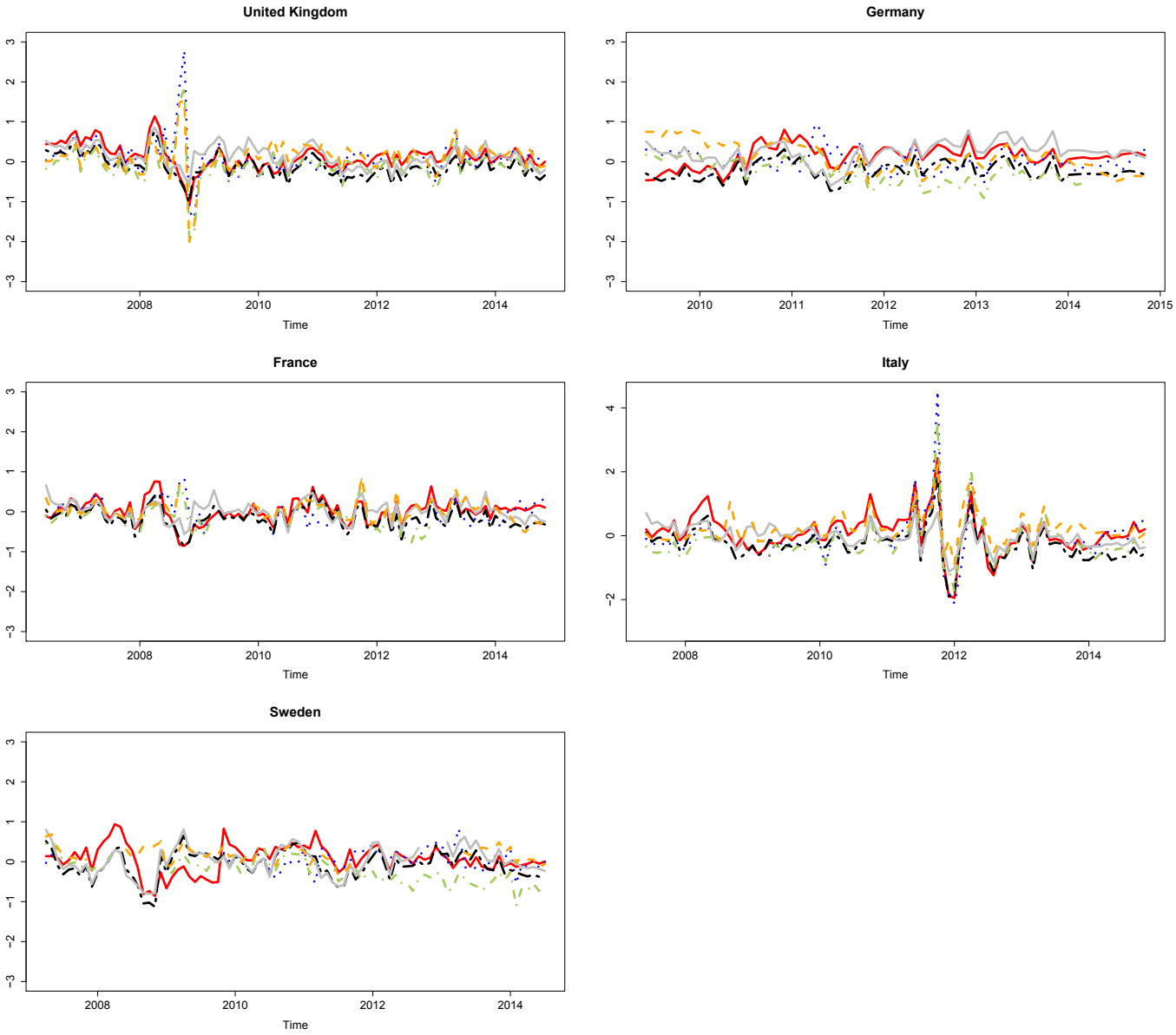

Figure 3: The model residuals of multiple yield curve modelling over different maturities ($\tau_1 < \tau_2 < \tau_3$ according to Table 2). The nominal type with τ_1 is the red line and the real type is the blue dotted line. The nominal and real types with τ_2 are the black long-dashed and green dot-dashed lines. For maturity of τ_3, the nominal type is grey and real type is an orange dashed line.

MTS_multi_modelres

The filtered four country-specific state variables are plotted in Figure 4, with $L_{it}^N, S_{it}^N, C_{it}^N, L_{it}^R$. We observe that the level factors L_{it}^N, L_{it}^R are significantly different, which justifies the factor choices in (6). To be more specific, although the slope and curvature factors can be combined for nominal and real bonds, the level factors needs to be treated as two different ones.

4.3 Decomposition of BEIR - Single Country

We have already obtained the model-implied BEIR in previous subsection. In this subsection, we conduct the decomposition of BEIR, into IE, the convexity effect and IRP for each country as in subsection 2.4.

Figure 5 compares the IE estimates for each country with three-year and five-year maturity respectively. We observe a decrease of the expected inflation for the U.K., which is also seemingly present in the other countries. To illustrates the presence of a similar trend among five countries, we plot the country-specific three-year IE in Figure 6 (IE estimates with other maturities have similar patterns). We find out that our model-implied IE with three year maturity would track the realized inflation level closely. For instance, the realized inflation level of Sweden has two small peaks at around the third quarter of 2008 and 2011 respectively, which can be also observed by our IE estimates for Sweden.

4.4 Joint Model of IE - Multiple Countries

We use model in (18), (19) and (20) to extract a common trend of IE across countries, it also allows us to analyze the country specific deviation from the common trend. Recall that we have the joint model either with or without a default proxy factor, and we start by showing results without a default proxy. The extracted common IE factor is shown in Figure 7. Also, the estimated parameters for the joint model of IE dynamics are presented in the Table 3.

We decompose the variation in the model-implied IEs into parts driven by the estimated

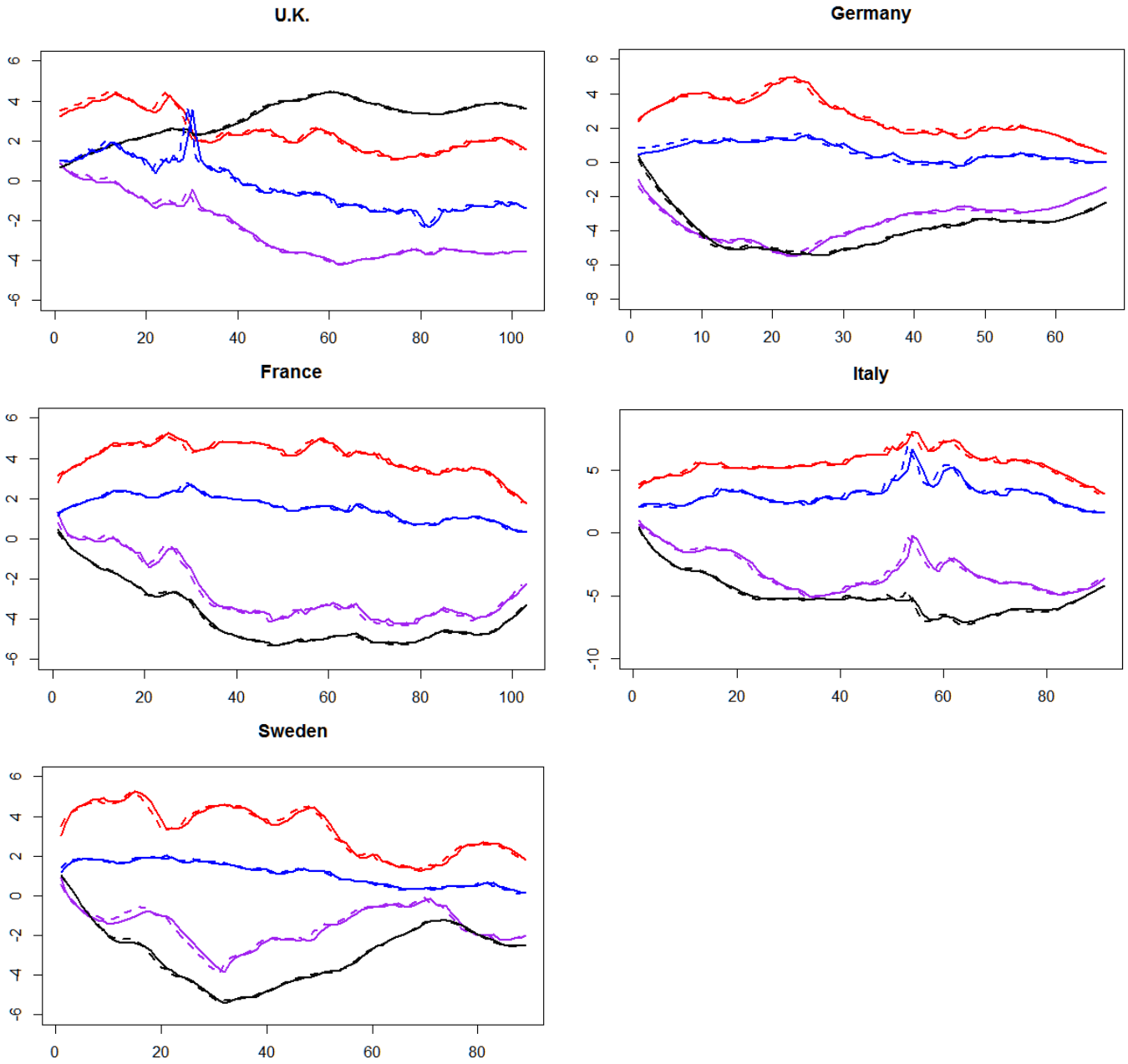

Figure 4: The estimated four latent factors of state variable $X_t = (L_{it}^N, S_{it}^N, C_{it}^N, L_{it}^R)$ for each European country - the nominal level factor L_{it}^N (red), the real level factor L_{it}^R (blue), the nominal slope factor S_{it}^N (purple) and the nominal curvature factor C_{it}^N (black). The preidicted state variables are in solid lines and the filtered state variables are in dashed lines.

MTS_afns_uk, MTS_afns_de, MTS_afns_fr, MTS_afns_it, MTS_afns_sw

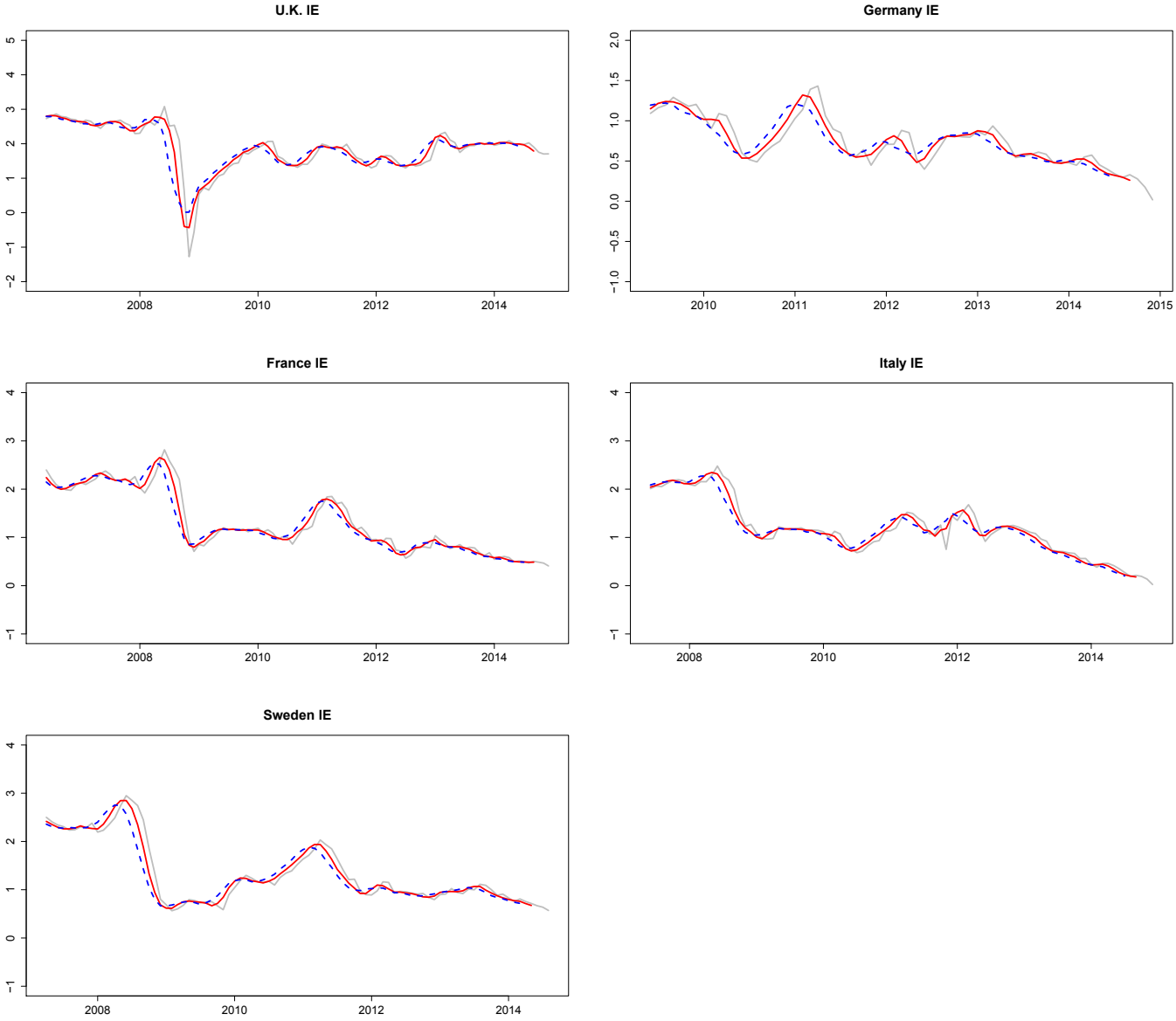

Figure 5: The model-implied IE for each European country. The 3-year IE is the red line and the 5-year IE is dashed blue.

MTS_expinf

Country-specific equations		
UK	$\pi^e_{1t}(\tau) =$	$0.166 + 0.576\Pi_t$
France	$\pi^e_{2t}(\tau) =$	$-0.022 + 0.665\Pi_t$
Italy	$\pi^e_{3t}(\tau) =$	$-0.347 + 0.822\Pi_t$
Sweden	$\pi^e_{4t}(\tau) =$	$-0.057 + 0.665\Pi_t$
Germany	$\pi^e_{5t}(\tau) =$	$0.008 + 0.644\Pi_t$
Common Effect equation	$\Pi_t =$	$0.588 + 0.651\Pi_{t-1}$

Table 3: Estimates for the dynamics of IE without a default proxy.

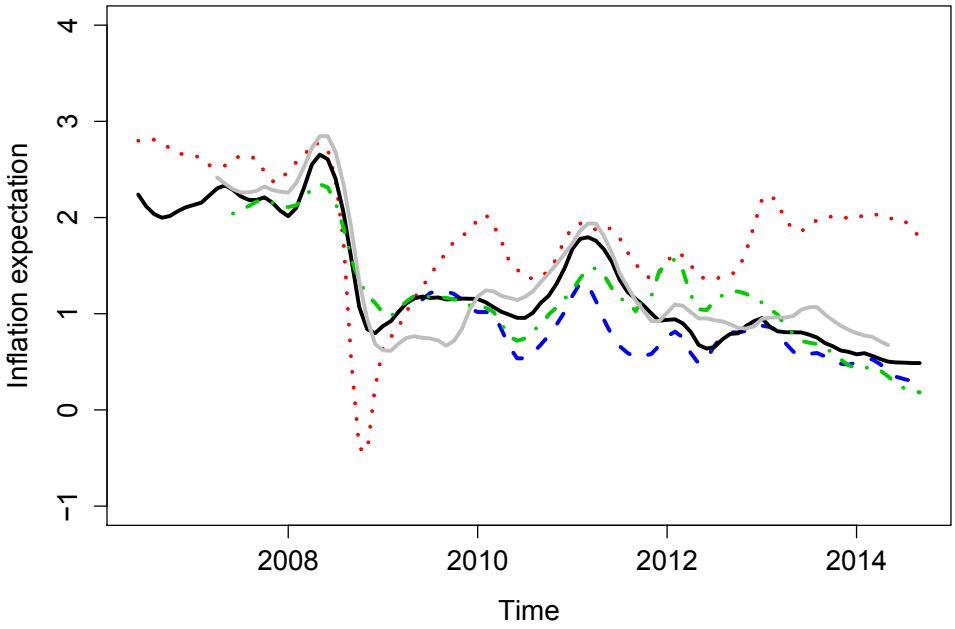

Figure 6: Model-implied inflation expectation for different countries - U.K.(red dotted line), Germany(blue dashed line), France(black line), Italy(green dot-dashed line) and Sweden(grey line)

MTS_expinf

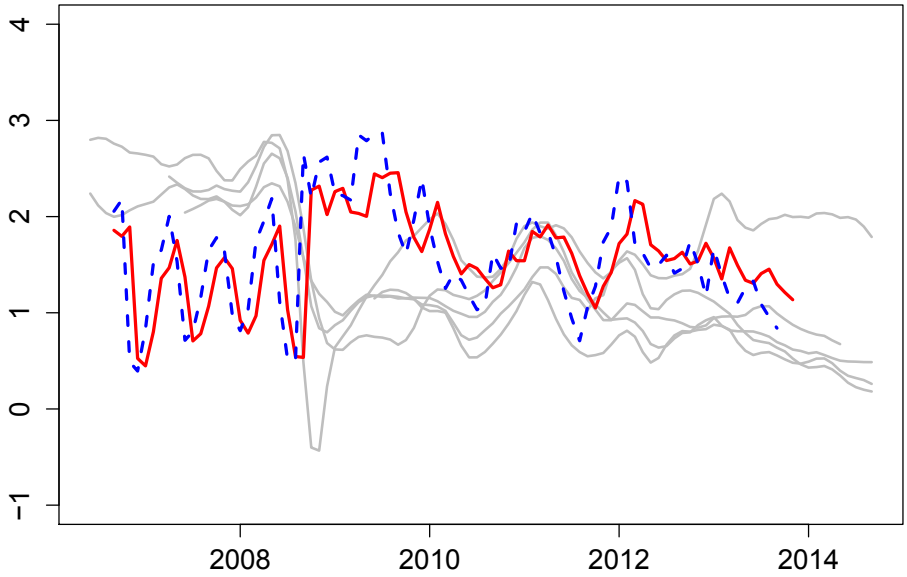

Figure 7: The country-specific IEs (grey). The predicted Π_t (solid red) and the filtered Π_t (dashed blue), three year maturity.

MTS_comexpinf

	U.K.	France	Italy	Sweden	Germany
Common effect	24.91	30.66	40.32	30.65	29.32
Country-specific effect	69.34	50.69	69.35	58.50	70.68

Table 4: Variance decompostion (in percentage)

common factor and the idiosyncratic factors for each country. The variance equation is,

$$\mathsf{Var}(\pi_{it}^e) = \beta_i^2 \mathsf{Var}\left(\Pi_t\right) + \mathsf{Var}\left(u_{it}\right) \tag{25}$$

The variance decomposition is shown in the Table 4. The common effect explains roughly 30% of the total variation of each country. And it explains the least variation of U.K., which is due to the U.K. being outside the euro-zone and having its own currency. Even though Sweden is also outside the euro-zone, it has closer relationships with other European countries compared with the U.K.. The international interaction among countries can also be observed through the estimation results.

To analyze the goodness of fit of our joint IE model, the model residuals are reported in Figure 8. The small size of the model residuals show the overall good performance. However the model residuals are relatively larger at around 2012, due to the European sovereign debt crisis including Italy's default. To eradicate this, we also incorporate

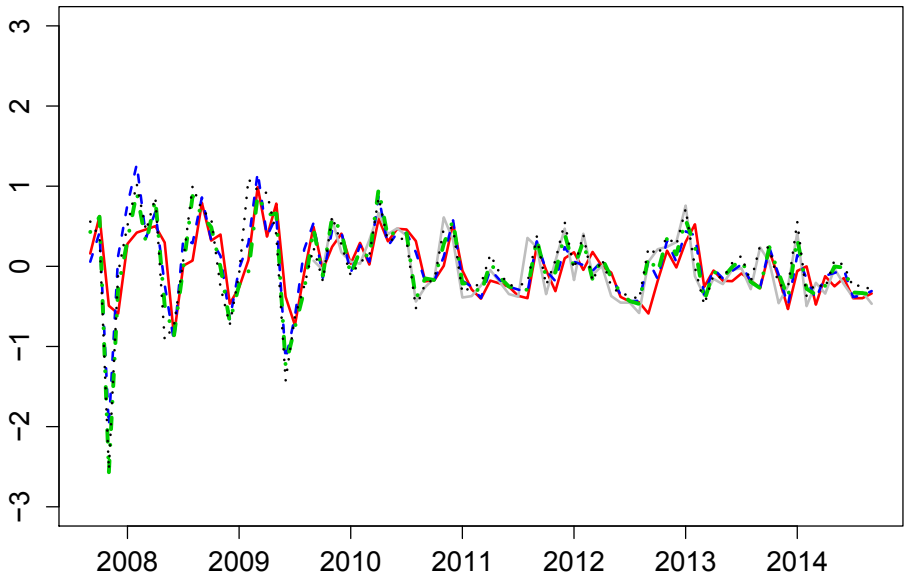

Figure 8: Model residual for modelling of inflation expectation dynamics over different countries - U.K.(red line), Germany(grey line), France(blue dashed line), Italy(black dotted) and Sweden(green dot-dashed).

MTS_comexpinf

one more macroeconomic factor- default risk proxy to improve the model performance by applying the method in (19) and (20) proposed in section 3. Furthermore, we assess model fitting performance by showing the model residuals in Figure 10 and the estimation results listed in Table 6.

The default proxy we adopt is the three-year Credit Default Swap (CDS) of Italy from Bloomberg, which is recognized as a important indicator of sovereign risk. We present in Figure 9 the extracted common inflation factor derived from the joint model of IE dynamics with default proxy. It can seen that the common trend successfully captures the decrease of IE caused by the subprime crisis. The estimated parameters for the joint

Country-specific equations		
UK	$\pi^e_{1t}(\tau) =$	$-0.358d_t + 0.798\Pi_t$
France	$\pi^e_{2t}(\tau) =$	$0.085d_t + 0.714\Pi_t$
Italy	$\pi^e_{3t}(\tau) =$	$1.078d_t + 0.531\Pi_t$
Sweden	$\pi^e_{4t}(\tau) =$	$-0.621d_t + 0.805\Pi_t$
Germany	$\pi^e_{5t}(\tau) =$	$0.045d_t + 0.700\Pi_t$
Common Effect equation	$\Pi_t =$	$0.382 + 0.976\Pi_{t-1}$

Table 5: Estimates for the dynamics of IE with a default proxy.

modelling of inflation dynamics with macroeconomic factors - default proxy are presented in Table 5. The estimated common trend captures the joint movement of the country specific IEs, and also with some degree of persistence.

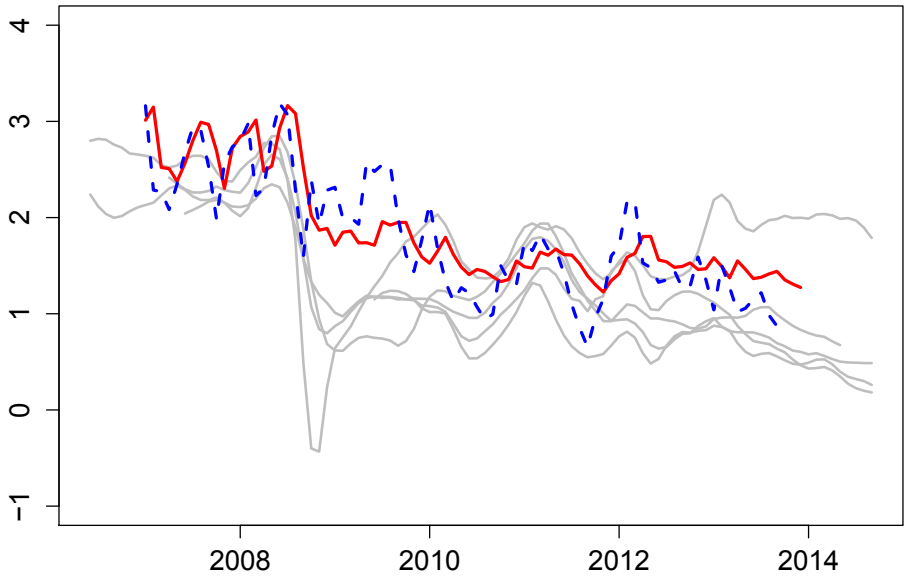

Figure 9: The predicted estimation of common factor Π_t(solid red), the filtered Π_t (blue dashed).

MTS_comexpinf_cds

The variations explained by the estimated common inflation factor Π_t and the estimated default factor d_t are reported in Table 6. The default factor explains the most variation in Italy and the least variation in Germany. This can be explained by the stability of German economy and therefore Germany is usually considered to be the benchmark in the European financial system. It is well known that the European sovereign debt crisis

	U.K.	France	Italy	Sweden	Germany
Common effect	36.08	33.59	11.54	31.87	32.84
Country-specific effect	56.66	65.88	40.92	49.17	67.02
Default risk effect	7.26	0.53	47.55	18.96	0.14

Table 6: Variance decomposition in percentage

including Italy's default happened around 2012, which corresponds to our finding that the default factor for Italy accounts for the 47.55% variation. Moreover, the extracted common trend for IE remains to be an important driver for each country of interest. This can be seen as well from Table 6, which shows that the variation explained by the common inflation factor accounts for more than 30% of total variations.

The model residuals are presented in Figure 10. The model residuals stay at the same level as the previous model without the default proxy.

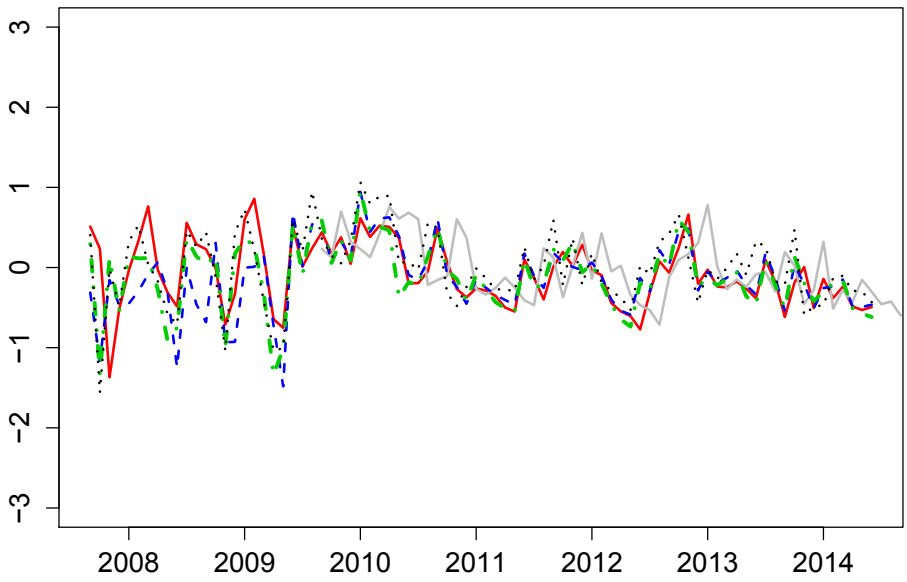

Figure 10: Model residuals for inflation expectation dynamics with a default proxy factor over different countries - U.K.(red solid line), Germany(grey solid line), France(blue dashed line), Italy(black dotted line) and Sweden(green dot-dashed line).

MTS_comexpinf_cds

4.5 Spatial-temporal Copula

In this subsection, we show results of a spatial temporal Copula model fitting for residuals from the aforementioned models. Recall the spatial-temporal Copula model in subsection 3.2. Let the marginal distribution functions be estimated nonparametrically as empirical distribution functions

$$\hat{F}_{\alpha_{i\cdot}}(u) = (T+1)^{-1} \sum_{t}^{T} \mathbf{1}\{\alpha_{it} \leq u\}, \tag{26}$$

and the marginal p.d.f.s are estimated as kernel densities estimators

$$\hat{f}_{\alpha_{i\cdot}}(u) = T^{-1} \sum_{t}^{T} K_h(\alpha_{it} - u), \tag{27}$$

where $K_h(\cdot)$ are kernel density functions. For spatiotemporal covariance function, we have four parameters to estimate, namely, a, b, β, σ. As for the parameter v (characterizing the behavior of the correlation function near the origin), we conduct a grid search method for a pre-estimation of the parameters, and the optimal η is estimated as $\eta = 0.5$, which means that the spatial correlation is an exponential function of u. As a is the scaling parameter of time, b is the spatial scaling parameter and β corresponds to the spatial temporal interaction. We have then the following interpretation for the estimated parameters: given $\hat{\beta} = 0.1958$, $\hat{a} = 0.02277$ means that the marginal temporal correlation decreases by around 2% with 1 month increase in time, and $\hat{b} = 0.00137$ indicates that the marginal space correlation decays by around 1% with a 100-km increase in space. Moreover, we adopt a cross validation method for selecting the cutoff points d_1, d_2, and we have also observed that the estimated cutoff points correspond to the points where the fitted variogram shows the correlation starts to vanish.

The longitude and latitude of the capital cities are used to measure the geographical distance, see Table 7.

Define a variagram as,

$$\Gamma(t_1 - t_2, n_1 - n_2) \stackrel{\text{def}}{=} \frac{1}{2} E(\alpha_{t_1 n_1} - \alpha_{t_2 n_2})^2 \tag{28}$$

	France	Germany	Italy	Sweden	UK
longitude	2.2	13.3	12.3	18.0	0.09
latitude	48.5	52.3	41.5	59.1	51.3

Table 7: The longitude and latitude of countries in degree.

It is worth noting that the concept is closely related to covariance function. For stationary process with σ^2 being the variance, we have

$$\hat{\Gamma}(t_1 - t_2, n_1 - n_2) \stackrel{\text{def}}{=} \sigma^2 - Cov(\alpha_{t_1 n_1}, \alpha_{t_2 n_2}) \tag{29}$$

The empirical variagram is defined as

$$\Gamma(d_1, d_2) \stackrel{\text{def}}{=} \frac{1}{N_{d_1,d_2}} \sum_{t_1,t_2,n_1,n_2:\|t_1-t_2\|\leq d_1, \|n_1-n_2\|\leq d_2} (\alpha_{t_1 n_1} - \alpha_{t_2 n_2})^2, \tag{30}$$

where N_{d_1,d_2} are number of observations within the local region indexed by d_1 and d_2.

The parametrically fitted variagram is defined as $\Gamma_{\hat{\theta}}(d_1, d_2)$, with the estimates of the parameters $\hat{\theta}$ plugged in,

$$\hat{\theta} \stackrel{\text{def}}{=} \arg\min_\theta (\hat{\Gamma}(t_1 - t_2, n_1 - n_2) - \hat{\Gamma}_\theta(t_1 - t_2, n_1 - n_2))^2. \tag{31}$$

The fitted empirical and parameterized variograms on the transformed data are shown in Figure 11 (transformed data are $\Phi(\hat{F}^{-1}(u_{it}))$, with $\Phi(\cdot)$ as the c.d.f. of a standard normal distribution). One axis represents the time lags, and another represents the spatial lags. We observe that the (vertical axis) is increasing in with both time lag and distance, which means a decrease in the variance-covariance matrix in time and in distance.

The one step ahead forecast for five countries are shown in Figure 12, where the blue solid line corresponds to the prediction of residuals coming from our Copula model. The forecast is later integrated to forecast IEs.

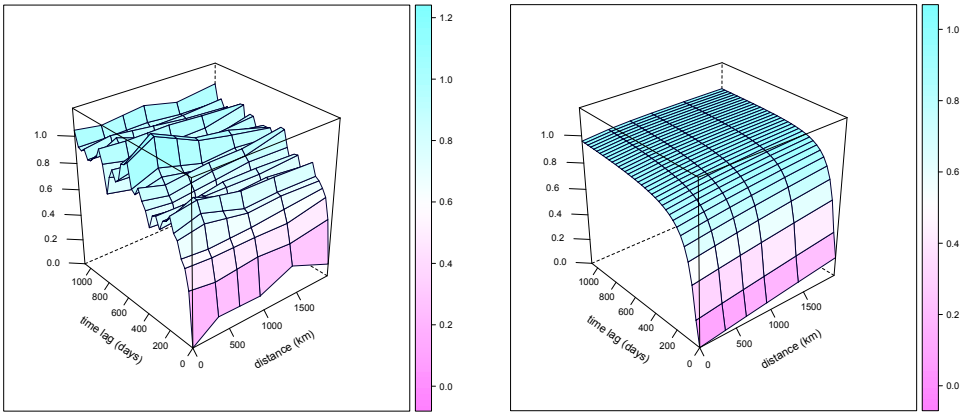

Figure 11: The empirical fitted variogram (left) and the parametrically fitted variogram (right).

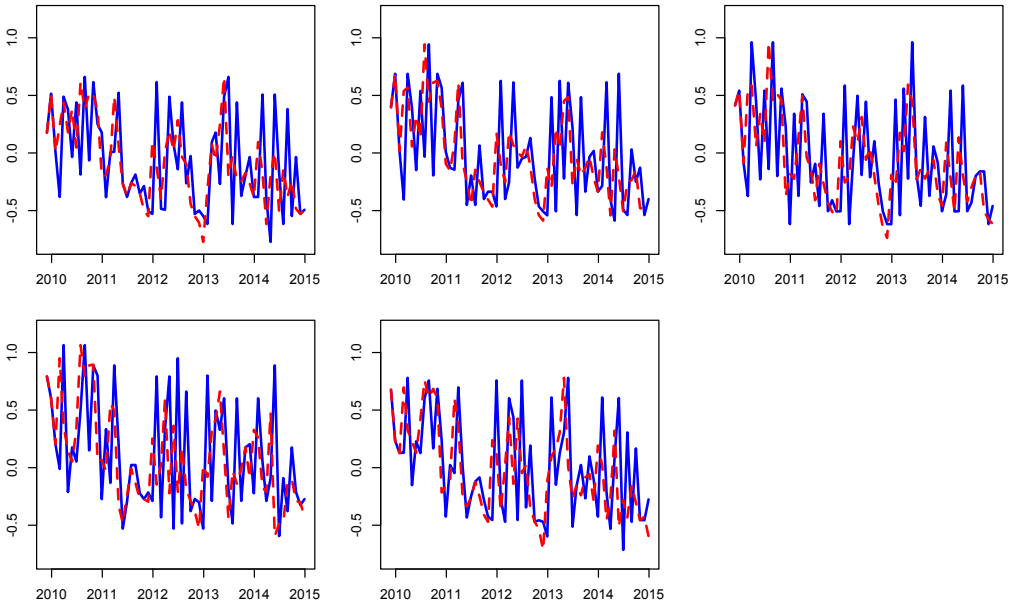

Figure 12: The one month ahead forecast for the residuals without Spatial-temporal Copula Model (red dotted), and the residual with Spatial-temporal Copula model (blue solid).

	Without Spatial-temporal Copula	With Spatial-temporal Copula
U.K.	0.209(0.068)	0.072(0.076)
France	0.196(0.102)	0.099(0.107)
Italy	0.264(0.074)	0.218(0.225)
Sweden	0.273(0.080)	0.099(0.095)
Germany	0.275(0.074)	0.104(0.097)

Table 8: Out-of-sample month-ahead averaged mean squared forecast errors. Variance appears in parentheses.

	Without Spatial-temporal Copula	With Spatial-temporal Copula
U.K.	0.236(0.067)	0.038(0.034)
France	0.235(0.042)	0.565(0.020)
Italy	0.830(0.078)	0.345(0.041)
Sweden	0.209(0.116)	0.050(0.045)
Germany	0.442(0.092)	0.133(0.058)

Table 9: Cross-sectional averaged mean squared forecast errors. Variance appears in parentheses.

4.6 Forecast

In this section, we evaluate the forecast performance of our model 20. The averaged one step ahead forecast errors with and without Spatial-temporal Copula approach are shown in term of mean squared errors. The mean squared one month ahead forecast errors are listed in Table 8. The pre-sample period we select for measuring MSEs is from 06.2009 to 07.2013. The sample period used to calculate MSE is from 08.2013 to 07.2014, where we consider one step ahead forecast for IEs within this period with a moving window. We also conduct cross-sectional forecasts to examine the forecast accuracy, the results are shown in Table 9. We find out that our model with Spatial-temporal Copula model would have IE closer to the realized inflation level.

Figure 13 displays 30 month ahead forecast for the common trend factor (in blue), which is, a two and a half year forecast horizon. The confidence intervals are presented graphically and are shown at confidence levels of 80% and 95%. Figure 14 compares the forecasting behavior with and without the Spatial-temporal Copula approach. The approach with Spatial-temporal Copula is plotted as dotted line with the out-of-sample forecast marked

in blue. We can observe that the two forecast results follow a similar pattern. The forecast result displayed by Spatial-temporal Copula approach is more sensitive to the market with inflation expectation tends to zero, which successfully captures the decrease of inflation rate in euro area.

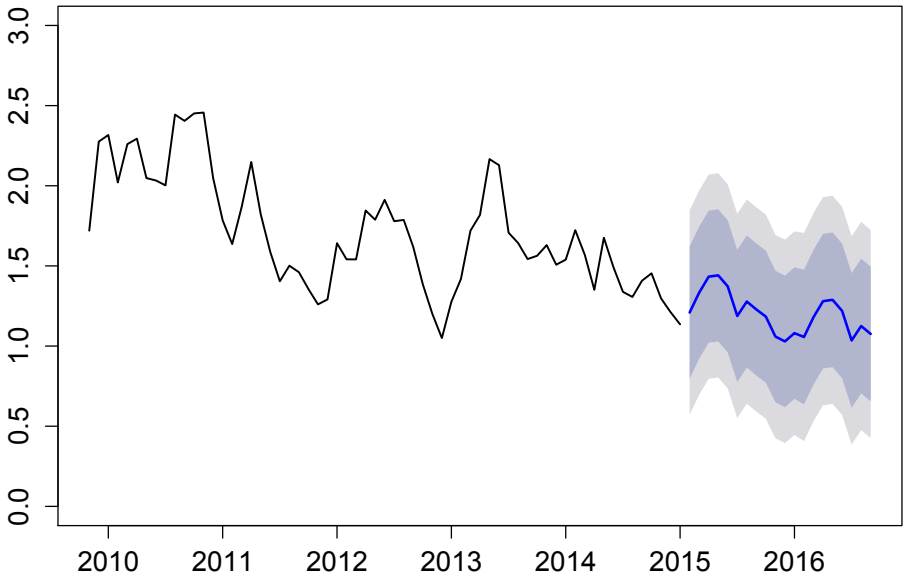

Figure 13: The forecasts derived from the joint model of IE dynamics with default factor. The 80% and 95% confidence intervals are marked in the shaded area.

Figure 15 clearly shows the difference among different measures of inflation. The real-time approach to measure IE proposed previously performs better than the other measures. A similar co-movement exits between the realized inflation level and the three-year IE estimated derived from our model. The 1 year and 2 year SPF (Survey Professional Forecast) data plotted in Figure 15 vary slightly over time, and therefore contains limited information of financial markets.

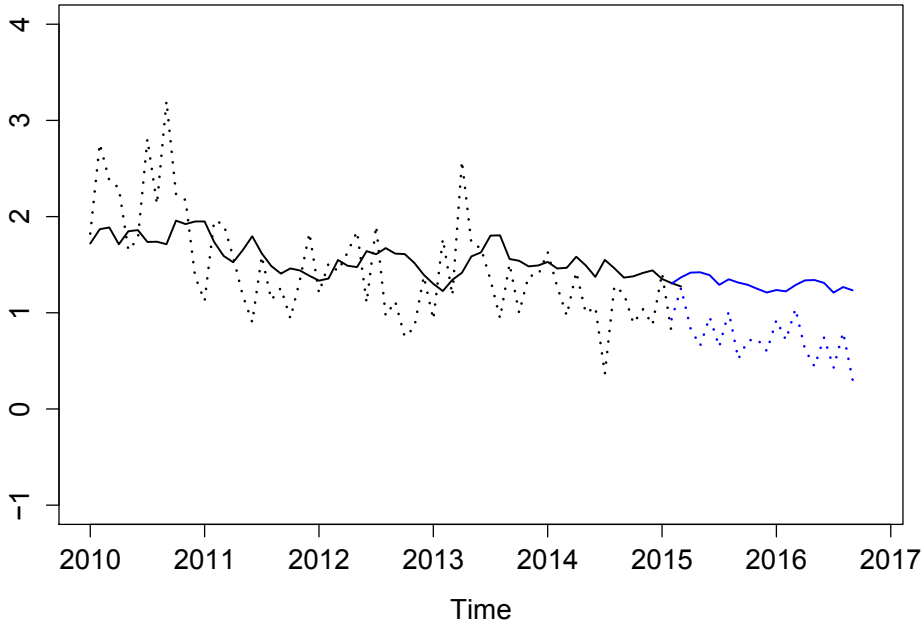

Figure 14: The comparison of two forecasts with (dotted) and without(solid) Spatial-temporal Copula, the estimation results are derived from the joint model of IE dynamics with default factor.

5 Conclusion

This study provide a joint modeling approach of IE across countries. We firstly construct an AFNS model in multi-maturity term structure of modelling nominal and inflation-indexed bonds simultaneously. Then we adopt the decomposition of model-implied BEIR into parts of IE, convexity effect and IRP to facilitate the modelling of joint structure of IE dynamics. A spatial-temporal Copula model is fitted to account for the non-Gaussian dependency among coutries. The joint models of IE dynamics with, and without macroeconomic factors indicate the extracted common inflation factor and was an important driver for each country of interest. Finally, our model provide informative forecast of IEs and provide good implications to monetary policies.

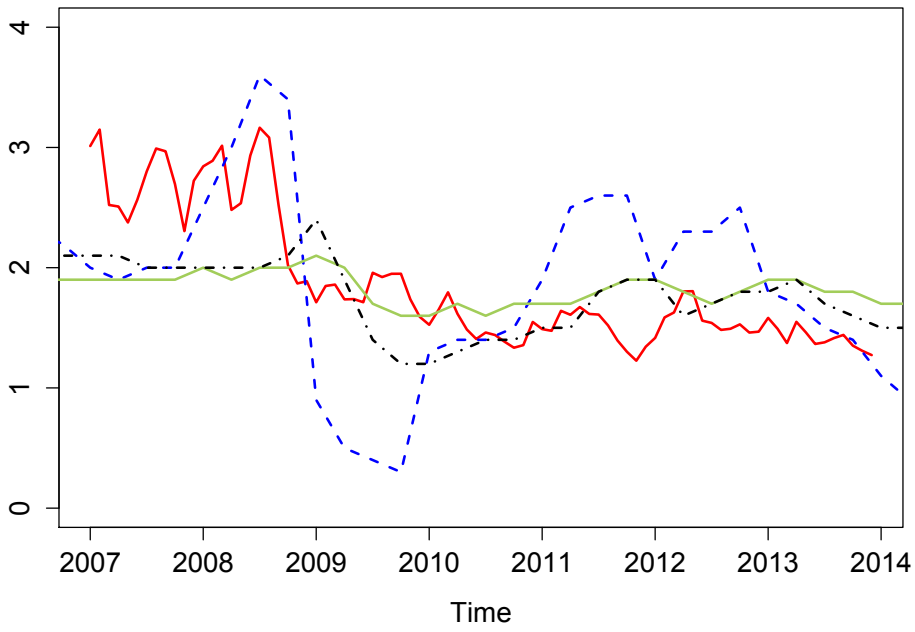

Figure 15: The comparison of different measures of inflation - the model-implied common inflation level (in red line), the observed inflation level (blue dashed line), the 1 year SPF forecast level of inflation (black dot-dashed) and the 2 year SPF forecast (in green).

6 Appendix

A Estimation of multiple yield curve modelling

The analysis starts by introducing the yield-adjustment term proposed in the original AFNS model. Derived in an analytical form, the yield-adjustment term $\frac{A(\tau)}{\tau}$ with τ

months to maturity can be written as,

$$\begin{aligned}
\frac{A(\tau)}{\tau} &= \bar{A}\frac{\tau^2}{6} + \bar{B}\left\{\frac{1}{2\lambda^2} - \frac{1}{\lambda^3}\frac{1-\exp(-\lambda\tau)}{\tau} + \frac{1}{4\lambda^3}\frac{1-\exp(-2\lambda\tau)}{\tau}\right\} \\
&+ \bar{C}\left\{\frac{1}{2\lambda^2} + \frac{1}{\lambda^2}\exp(-\lambda\tau) - \frac{1}{4\lambda}\tau\exp(-2\lambda\tau) - \frac{3}{4\lambda^2}\exp(-2\lambda\tau)\right\} \\
&+ \bar{C}\left\{-\frac{2}{\lambda^3}\frac{1-\exp(-\lambda\tau)}{\tau} + \frac{5}{8\lambda^3}\frac{1-\exp(-2\lambda\tau)}{\tau}\right\} \\
&+ \bar{D}\left\{\frac{1}{2\lambda}\tau + \frac{1}{\lambda^2}\exp(-\lambda\tau) - \frac{1}{\lambda^3}\frac{1-\exp(-\lambda\tau)}{\tau}\right\} \\
&+ \bar{E}\left\{\frac{3}{\lambda^2}\exp(-\lambda\tau) + \frac{1}{2\lambda}\tau + \frac{1}{\lambda}\exp(-\lambda\tau) - \frac{3}{\lambda^3}\frac{1-\exp(-\lambda\tau)}{\tau}\right\} \\
&+ \bar{F}\left\{\frac{1}{\lambda^2} + \frac{1}{\lambda^2}\exp(-\lambda\tau) - \frac{1}{2\lambda^2}\exp(-2\lambda\tau) - \frac{3}{\lambda^3}\frac{1-\exp(-\lambda\tau)}{\tau} + \frac{3}{4\lambda^3}\frac{1-\exp(-2\lambda\tau)}{\tau}\right\}
\end{aligned} \tag{A.1}$$

where the six terms $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$, $\bar{E}$ and $\bar{F}$ can be identified by the volatility matrix Σ defined in the dynamics equation under P-measure. The value of the adjustment term is constant in time t, but depends on time to maturity τ, coefficient λ that governs the mean reversion rate of slope and curvature factors, and the volatility parameters $\bar{A}$, $\bar{D}$ and $\bar{F}$.

The four latent factors defined in the state variable $X_{it}^{\top} = \left(L_{it}^N, S_{it}^N, C_{it}^N, L_{it}^R\right)$ evolve dynamically and hence we can identify their shocks accordingly,

$$\begin{pmatrix} dL_{it}^N \\ dS_{it}^N \\ dC_{it}^N \\ dL_{it}^R \end{pmatrix} = \begin{pmatrix} \kappa_{11} & \kappa_{12} & \kappa_{13} & \kappa_{14} \\ \kappa_{21} & \kappa_{22} & \kappa_{23} & \kappa_{24} \\ \kappa_{31} & \kappa_{32} & \kappa_{33} & \kappa_{34} \\ \kappa_{41} & \kappa_{42} & \kappa_{43} & \kappa_{44} \end{pmatrix} \begin{pmatrix} L_{it}^N \\ S_{it}^N \\ C_{it}^N \\ L_{it}^R \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L^N} \\ dW_t^{S^N} \\ dW_t^{C^N} \\ dW_t^{L^R} \end{pmatrix} \tag{A.2}$$

where $W_t^{L^N}$, $W_t^{S^N}$, $W_t^{C^N}$ and $W_t^{L^R}$ are independent Brownian motions.

We estimate the parameters in (A.2) using the Kalman filter technique. The Kalman filter recursion is a set of equations which allow for an estimator to be updated once a new observation y_t becomes available. It first forms an optimal predictor of the unobserved state variable vector given its previously estimated value. This prediction is obtained using the distribution of unobserved state variables, conditional on the previous estimated values. These estimates for unobserved state variables are then updated using the information

provided by the observed variables.

By rewriting the yield equation (6) of the joint AFNS model in multi-maturity term structure proposed in subsection 2.3, we obtain the measurement equation as,

$$\begin{pmatrix} y_{it}^{N}(\tau_1) \\ y_{it}^{R}(\tau_1) \\ \vdots \\ y_{it}^{R}(\tau_n) \end{pmatrix} = AX_{it} + \begin{pmatrix} \varepsilon_{it}^{N}(\tau_1) \\ \varepsilon_{it}^{R}(\tau_1) \\ \vdots \\ \varepsilon_{it}^{R}(\tau_n) \end{pmatrix} - \begin{pmatrix} \frac{A_i^N(\tau_1)}{\tau_1} \\ \frac{A_i^R(\tau_1)}{\tau_1} \\ \vdots \\ \frac{A_i^R(\tau_n)}{\tau_n} \end{pmatrix} \tag{A.3}$$

The transition equation derived from Christensen et al. (2011) takes the form of,

$$X_{i,t} = \left[I - expm\left(-K^P \Delta t\right)\right] \theta^P + expm\left(-K^p \Delta t\right) X_{i,t-1} + \eta_t \tag{A.4}$$

where *expm* is a matrix exponential. The measurement and transition equations are assumed to have the error structure as,

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} = N\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix} \right\}$$

where Q has a special structure,

$$Q = \int_0^{\Delta t} e^{-K^P s} \Sigma \Sigma^\top e^{-(K^P)^\top s} ds$$

For estimation, the transition and measurement errors are assumed orthogonal to the initial state. The initial value of the filter is given by the unconditional mean and variance of the state variable $X_t^\top$ under the P measure,

$$\begin{aligned} X_{\text{o}} &= \theta^P \\ \Sigma_{\text{o}} &= \int_{\text{o}}^{\infty} e^{-K^P s} \Sigma \Sigma^\top e^{-(K^P)^\top s} ds \end{aligned}$$

which can be calculated using the analytical solution provided in Fisher and Gilles (1996).

B BEIR Decomposition

In the environment of an AF model, there are no opportunities to make risk-free profits. Based on the pricing equation from Cochrane (2005), the bond can be priced by the equation,

$$P_t = \mathsf{E}_t \left\{ M_{t+1} x_{t+1} \right\}. \tag{B.1}$$

To estimate the expected value of inflation using the stochastic discount factor (SDF) M_t. Firstly we use the Taylor series to approximate the moments of the logarithm. Assuming that M_t, in a sense, significant from 0, so the yield for a nominal bond can be extended as follows,

$$\log \left(M_{t+1}^N M_{t+2}^N \cdots M_{t+\tau}^N \right) = \log \left\{ \left(\mu_M + M_{t+1}^N M_{t+2}^N \cdots M_{t+\tau}^N - \mu_M \right) \right\} \tag{B.2}$$

where

$$\begin{aligned} \mu_M &\overset{\text{def}}{=} \mathsf{E}_t \left(M_{t+1}^N M_{t+2}^N \cdots M_{t+\tau}^N \right) \\ \mathbf{M}_{1:\tau}^N &\overset{\text{def}}{=} \left(M_{t+1}^N M_{t+2}^N \cdots M_{t+\tau}^N \right). \end{aligned}$$

Take a Taylor expansion of equation (B.3) and a conditional expectation E_t on both sides,

$$\mathsf{E}_t \left(\mathbf{M}_{1:\tau}^N \right) = \log \mu_M - (2\mu_M^2)^{-1} \mathsf{Var}_t \left(\mathbf{M}_{1:\tau}^N \right) + \mathcal{O}_p(\mathsf{Var}_t \mathbf{M}_{1:\tau}^N), \tag{B.3}$$

further as

$$\mathsf{Var} \log \mathbf{M}_{1:\tau}^N = (\mu_M^2)^{-1} \mathsf{Var} \, \mathbf{M}_{1:\tau}^N + \mathcal{O}_p(\mathsf{Var}_t \mathbf{M}_{1:\tau}^N), \tag{B.4}$$

then we have

$$\mathsf{E}_t \left(\log \mathbf{M}_{1:\tau}^N \right) = \log \mu_M - 2^{-1} \mathsf{Var}_t \left(\log \mathbf{M}_{1:\tau}^N \right) + \mathcal{O}_p(\mathsf{Var}_t \mathbf{M}_{1:\tau}^N). \tag{B.5}$$

Therefore,

$$\begin{aligned} y_t^N(\tau) &= -\frac{1}{\tau}\log \mathsf{E}_t\left(\mathbf{M}_{1:\tau}^N\right) \\ &= -\frac{1}{\tau}\mathsf{E}_t\left(\log \mathbf{M}_{1:\tau}^N\right) - \frac{1}{2\tau}\mathsf{Var}_t\left(\log \mathbf{M}_{1:\tau}^N\right) + \mathcal{O}_p(\mathsf{Var}_t \mathbf{M}_{1:\tau}^N) \end{aligned} \tag{B.6}$$

Similar solution could be obtained for the inflation-indexed bonds by the same logic.

To facilitate the calculation of equation (12), the instantaneous risk-free rate r_t and the risk premium Γ_t are given, more details can be found in Christensen et al. (2011) and Christensen et al. (2010),

$$r_t = \rho_0(t) + \rho_1(t)X_t \tag{B.7}$$

$$\Gamma_t = \gamma_0 + \gamma_1 X_t \tag{B.8}$$

where $\rho_0(t)$, $\rho_1(t)$, γ_0 and γ_1 are bounded, continuous functions. X_t is the state variable and Y_t is the realized observations.

The estimation of the inflation expectation can be calculated by

$$\pi_t(\tau) = -\frac{1}{\tau}\log \mathsf{E}_t^P\left[\exp\left\{-\int_t^{t+\tau}(r_s^N - r_s^R)ds\right\}\right] \tag{B.9}$$

which are the solutions to a system of ordinary differential equations using the fourth-order Runge Kutta method.

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