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# Mortality modeling: Lee-Carter and the macroeconomy

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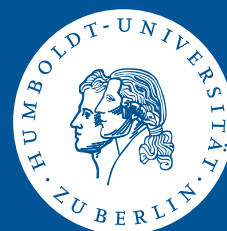


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# Mortality modeling: Lee-Carter and the macroeconomy

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## Abstract

Using data for six OECD countries, this paper studies the effect of macroeconomic conditions on the mortality index  $k_t$  in the well-known Lee-Carter model. Significant correlations are found with real GDP growth rates in Australia, Canada, and the United States, and with unemployment rate changes in Japan, for the period 1950–2005. In recent years, the relationship between the state of the economy and mortality is found to change from procyclical to countercyclical in all six countries. Based on these findings, variants of the Lee-Carter model are proposed that capture a substantial fraction of the variation in the mortality index. *Keywords:* Demography, Lee-Carter, business cycle, time series model, *JEL classification:* C32, E32, I12, J11.

## Introduction

Mortality modeling and forecasting has made considerable progress in the last few years. On the one hand, various stochastic mortality models have been proposed that allow demographers and actuaries to quantify the uncertainty associated with long-run mortality forecasts. On the other hand, several recent epidemiological studies find that mortality rates react to changes in macroeconomic conditions—even in industrialized countries. This article combines the results of both these domains by analyzing the relation between the mortality index  $k_t$  in the widely applied Lee-Carter model and changes in real GDP or unemployment rates. Using data for six OECD countries (Australia, Canada, France, Japan, Spain, and the United States) over the period 1950–2005, the paper makes the following contributions to the demographic literature. First, it shows that the mortality index in the Lee-Carter model is not merely an unobserved, latent variable that fluctuates erratically, but is significantly driven by

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external factors. Second, it documents that the link between economic conditions and aggregate mortality as established in several previous studies is subject to a structural change, even leading to a change of sign in recent years. Finally, an elegant way is proposed to incorporate the relationship between mortality and the state of the economy into an established forecasting model.

The paper is set out as follows. In the first two sections, the relevant demographic literature is reviewed and estimation of the mortality index  $k_t$  is described. Then, correlations between changes in the mortality index and macroeconomic fluctuations are analyzed across the entire sample period, as well over three subperiods, 1951–1970, 1971–1990, and 1991–2005. To better understand the relation between mortality and the state of the economy, “moving correlations” are studied in a next step. Based on this insight, time series models for the mortality index are developed that account for the time-varying effect of macroeconomic changes on the mortality index. The discussion part of the paper shows that the observed reversal of this effect is present for age-specific death rates as well, and suggests possible explanations for this pattern. A summary of all results is provided in the last section of the paper.

### **Literature review**

Cairns, Blake, and Dowd (2008), Booth (2006), and Booth and Tickle (2008) provide excellent systematic overviews of the development and status quo of stochastic mortality modeling. Existing models can be classified by the number of factors used to describe their dynamics, for example, or by whether cohort effects are included in addition to random period effects. “The earliest model and still the most popular” (Cairns et al., 2008) was proposed by Lee and Carter (1992) for U.S. data. This model is widely employed both in the academic literature and by practitioners working for pension funds, life insurance companies, and statutory pension systems. The original approach has seen several extensions (see, e.g., Lee and Miller, 2001; Brouhns, Denuit and Vermunt, 2002; or Renshaw and Haberman, 2006), and has been applied to mortality rates of many countries, including the G7 countries (Tuljapurkar, Li, and Boe, 2000), Spain (Debón, Montes, and Puig, 2008), Australia (Booth, Maindonald and Smith, 2002), and China and South Korea (Li, Lee, and Tuljapurkar, 2004). Besides, variations of the Lee-Carter model have been employed to forecast other demographic variables such as fertility rates or migration flows (Giroso and King, 2008; Härdle and Myšičková, 2009). For mortality projections, however, the Lee-Miller variant has

been widely adopted as the standard Lee-Carter method (Booth and Tickle, 2008). It performs well in a ten-population comparison study of five variants or extensions of the Lee-Carter method by Booth, Hyndman, Tickle and de Jong (2006).

The key driver of mortality dynamics in the Lee-Carter model is the “index of the level of mortality”  $k_t$  (Lee and Carter, 1992). Other authors characterize this variable as the “dominant temporal pattern in the decline of mortality” (Tuljapurkar et al., 2000), “a random period effect” (Cairns et al., 2008), or simply as a latent variable (Hári, De Waegenare, Melenberg, and Nijman, 2008). Statistical modeling of the mortality index  $k_t$  has already attracted a certain amount of research interest. Li and Chan (2007) propose an outlier-adjusted model for  $k_t$  that results in a good fit of the Lee-Carter model over long periods for Canada (1921–2000) and the United States (1900–2000). Chan, Li, and Cheung (2008) analyze the dynamics of  $k_t$  for Canada, England and Wales, and the United States. Their results reveal that the mortality index is better explained by a trend-stationary model that allows for a break in the rate of mortality decline in the 1970s than by a random walk model.

Another branch of demographic literature analyzes the effect of changes in macroeconomic conditions on mortality rates. Ruhm (2000) was the first to discover that total mortality, as well as several cause-specific mortality rates, fluctuate procyclically in the United States over the period 1972–1991. More precisely, he proved that mortality rates attributable to heart or liver disease, influenza and pneumonia, and vehicle crashes and other accidents tend to increase when economic activity accelerates, whereas cancer deaths and suicides are acyclical or even countercyclical. A similar pro-cyclical pattern was observed by Tapia Granados (2005a, 2005b, 2008) for mortality rates in the United States, Spain, and Japan, by Tapia Granados and Ionides (2008) for Sweden, and by Gerdtham and Ruhm (2006) for 23 OECD countries over the 1960–1997 period. Neumayer (2004) and Hanewald (2008) further corroborate Ruhm’s results using German data for 1980–2000 and 1956–2004, respectively. A summary of the theories offered to explain the adverse effect of economic upturns on health and mortality rates can be found in Tapia Granados (2008).

Recently, two frameworks for incorporating the effects of external (macroeconomic) factors into the stochastic modeling of age-specific mortality rates have been proposed, both following the Bayesian paradigm. Girosi and King (2008) develop a general Bayesian hierarchical framework for forecasting different demographic variables that includes

covariates as proxies for systematic causes or as predictors. They illustrate their method by studying mortality from transportation accidents in Argentina and Chile using GDP as a proxy for the level of infrastructure and effort invested in transportation safety. Reichmuth and Sarferaz (2008a) develop a Bayesian vector autoregressive (VAR) model that allows studying the short-run interaction between a latent mortality variable and several covariates. In a companion article, Reichmuth and Sarferaz (2008b) provide estimation results for the United States (1933–1969 and 1956–2004) and for France and Japan (1956–2004 for both) using unemployment rates and GDP growth as economic indicators. The chief tool employed in their analysis is impulse response functions for single age groups that illustrate how mortality rates react to shocks in the economic variables. In contrast to Girosi and King (2008) and Reichmuth and Sarferaz (2008a), we follow a more direct approach by analyzing the relationship between the mortality index  $k_t$  in the original Lee-Carter model and macroeconomic conditions. Since  $k_t$  is the main driver in the Lee-Carter model, any identified relationship will translate directly into age-specific mortality rates.

### **Extracting the mortality index $k_t$**

#### *The Lee-Carter model*

Lee and Carter (1992) build their model on the insight that log mortality rates in the United States quite accurately follow a common linear trend over the last decades, and propose the following parsimonious parameterization:

$$\ln(m_{x,t}) = a_x + b_x \cdot k_t + \varepsilon_{x,t}, \quad (1)$$

where  $m_{x,t}$  is the central death rate at age  $x$  in year  $t$ . The model consists of an age-specific constant  $a_x$ , the product of a time-varying index  $k_t$  and a factor  $b_x$  describing the sensitivity of age-specific mortality rates to changes in  $k_t$ , and an age-specific error term  $\varepsilon_{x,t}$ . In a first step, the unobservable index  $k_t$  is filtered from the matrix of age-specific mortality rates with singular value decomposition. Then, the obtained time series for  $k_t$  is usually fitted either to the observed number of deaths (Lee and Carter, 1992) or to life expectancy (Lee and Miller, 2001). Next, time series methods, such as the Box-Jenkins procedure, are applied to model  $k_t$ . In most cases,  $k_t$  is found to follow a random walk with drift, which allows forecasting the matrix of log mortality rates  $\ln(m_{x,t})$  as a one-dimensional random walk:

$$k_t = k_{t-1} + \theta + \varepsilon_{k,t}. \quad (2)$$

Model (2) is equivalent to  $\Delta k_t = \theta + \varepsilon_{k,t}$ ; therefore, the drift term  $\theta$  can be estimated by regressing  $\Delta k_t$  on a constant only, which gives the OLS-estimator  $\hat{\theta} = \frac{1}{T-1}(k_T - k_1)$ , where  $T$  denotes the final year of the estimation period.

### *The data*

Annual period death rates and population size for Australia, Canada, France, Japan, Spain and the United States were obtained from the Human Mortality Database (HMD) (University of California and Max Planck Institute, 2008), which contains comparable data for 34 countries or areas. Death rates were available for 1950–2005 for all selected countries except Australia (1950–2004)<sup>1</sup>. Real GDP levels for 1950–2005 were taken from Maddison (2008). Standardized unemployment rates were retrieved from the OECD Main Economic Indicators (2008); Table 1 reports their availability.

### *Extracting the mortality index $k_t$*

The Lee-Miller (2001) variant of the Lee-Carter model was estimated by employing the R package “demography” by Hyndman, Booth, Tickle and Maindonald (2008),<sup>2</sup> a program developed for the (1x1) data format of the Human Mortality Database. Lee and Carter (1992) originally proposed their model for total population mortality data. However, since we expect the mortality rates of males and females to react differently to economic conditions, male and female forecasts are treated as two separate applications of the basic Lee-Carter approach as described by Lee (2000). Lee and Carter (1992) estimated their model for the five-year age groups 0, 1–4, 5–9, ... up to age 84, and the open age group 85 and over. The same age spectrum is used to estimate variants of the Lee-Carter model (see, e.g., Lee and Miller, 2001; Booth, Maindonald and Smith, 2002). Other authors extend the age range to 99 (Renshaw and Haberman, 2006) or 105 (Tuljapurkar et al., 2000). Brouhns, Denuit, and Vermunt (2002) estimate their modification of the Lee-Carter model only for the one-year age groups between 60–98. Here, the model is estimated with the same upper age limit as in the original article and a minimum age of 30.

The model setup is summarized as follows:

- Estimation period: 1950–2005,
- Separate estimation for males and females,
- One-year age groups: 30, 31, ... , 84, 85 and older (85+), and
- Adjustment of the mortality index  $k_t$  to life expectancy  $e_0$ .

The overall fit of the estimated models is very good; for nearly all of them the proportion of explained temporal variance exceeds 93 percent for (see Table 1), the exception being Spain, where the adjusted  $R^2$  for the mortality index of males is 82.7 percent—a finding that accords with the results of Debón et al. (2008). Fig. 1 plots the extracted mortality indices for Australia and the United States. As true for Canada, France, Japan and Spain, a downward trend can be detected in the series for males and females. The ADF test confirms that all twelve series are nonstationary for the period 1950–2005, even when a linear time trend is included in the test equations. Only for Spanish males was  $k_t$  found to be trend stationary. As a solution, series for the mortality index were transformed to first differences  $\Delta k_t$ , which can be interpreted as percentage changes since the mortality index was extracted from the matrix of log mortality rates. Accordingly, time series for real GDP and unemployment rates were transformed to relative changes by taking the first difference of their logarithm so as to make them stationary.

*[Insert Fig. 1 here.]*

## **Correlation analysis**

### *1951–2005 and three subperiods*

In this section, correlations between macroeconomic changes and fluctuations in the mortality index  $k_t$  for males and females are studied. The analysis is performed with real GDP growth rates as well as with relative changes in unemployment rates as economic indicators. Correlations are calculated for the entire sample period (1951–2005), as well as for the three subperiods (1951–1970, 1971–1990, and 1991–2005). In cases of limited data availability, the actual sample period is given in parentheses. All results are set out in Table 1.

There are two main results. First, significant correlations between changes in the mortality index  $k_t$  and macroeconomic fluctuations can be observed for four of the six countries for the sample period 1951–2005. For the United States and Australia, significant positive correlations are found between real GDP growth rates and the mortality indices for both males and females, whereas for Canada this is the case only for males. Significant negative correlations with unemployment rate changes were observed for Japan (males and females) and Australia (females only). In all cases, the sign of the observed relationship indicates

procyclical behavior of aggregate mortality rates, a finding that agrees with those of Ruhm (2000) and others in regard to age-specific mortality rates.

Second, and even more interesting, for all countries a structural change can be observed in the relationship between economic conditions and the mortality index when looking at the three subperiods, 1951-1970, 1971-1990, and 1991-2005. In several cases, the relation is even reversed. For 1951-1970, all correlations except one between  $\Delta k_t$  and real GDP growth rates are positive, with values of around 40 percent for the United States and 41-52 percent for Canada. By contrast, we find that 10 of 12 correlations are negative for 1991-2005. For Australia, Canada (males only), France, and the United States, a clear downward trend can be observed from the first to the second to the third subperiod. For example, correlations for  $\Delta k_t$  of French males decrease from 52 percent to 37 percent and then to -22 percent. The change in sign most often occurs after the second subperiod; however, for France, negative correlations already occur in the second period. In Japan and Spain, the reversal tendency can be more clearly seen when looking at unemployment rates. For the mortality index of Japanese males (females) corresponding correlations increase from -54 (-49) percent for 1956-1970 to -35 (-31) percent and finally to 7 (-8) percent during 1991-2005. Similarly, absolute correlations for Spain are lower in the third period than they are in the second. In general, the results obtained for unemployment rates support the findings for real GDP growth rates.

### *Moving correlations*

Let us now investigate whether the relation between the changes in the mortality index  $\Delta k_t$  and macroeconomic fluctuations ( $\Delta \ln(\text{real GDP}_t)$  or  $\Delta \ln(\text{unemployment rate}_t)$ ) is subject to a gradual change or, rather, to an abrupt shift. Thus, “moving correlations” over 20-year subperiods with different starting points are plotted in Fig. 2 for Australia, Canada, Japan, and the United States. For each period, the corresponding correlation is plotted at the starting year, e.g., for 1951-1970 at 1951, for 1952-1971 at 1952, and so on. In accordance with the analysis presented above, the last starting point is set at 1991 making this period only 15 years long, instead of 20. An abrupt shift is not observed in any of the four countries; instead, correlations appear to decrease since the 1960s in the United States and Canada (males only) and even earlier in Japan. The reversal pattern is most prominent for the mortality index of males in the United States. In Australia, a downward tendency in the correlations does not appear to start until 1985.



[Insert Fig. 2 here.]

### *Cross-correlations*

In their study on Sweden in the 19<sup>th</sup> and 20<sup>th</sup> century, Tapia Granados and Ionides (2008) find that economic growth has a negative effect on mortality rates with a short lag of about one or two years in the second half of 20<sup>th</sup> century. For the United States, 1900–1996, Tapia Granados (2005a) observes the highest cross-correlations between changes in cause-specific death rates and changes in real GDP growth as well as unemployment rates “almost always” at zero lag or at a lag of one or two years. In a further study, Tapia Granados (2008) finds that there is no evidence of a delayed impact of economic change on mortality in post-World War II Japan. Here, cross-correlations between  $\Delta k_t$  and changes in real GDP or unemployment rates up to a five-year lag were calculated for the entire sample period (1951–2005). With respect to real GDP, significant effects are noted at a lag of one year for Australia (mortality indices of males and females) and Canadian (males), at a lag of four years for Canada (males), and at a lag of five years for France and Spain (both males). As these effects do not follow a clear pattern, they will be ignored in the following regression analysis. In case of unemployment rates, there are no significant effects for any of the six countries.

### **Regression analysis**

The results reported in the last section indicate that the mortality index  $k_t$  in the Lee-Carter model is significantly correlated with macroeconomic fluctuations for four of the six countries studied. The usual random walk model for  $k_t$  does not incorporate this relationship and instead assumes that  $\Delta k_t = \theta + \varepsilon_{k,t}$ . This model will now be extended by including changes in real GDP or unemployment rates as explanatory variables. Based on the results of the correlation analysis, the following countries and economic indicators were chosen: Japan—unemployment rates; and Australia, Canada, and the United States—real GDP. In a first step, the following model was estimated separately for males and females for each country for the entire sample period (1951–2005):

$$\Delta k_t = \theta + \beta \cdot \Delta \ln(X_t) + \varepsilon_{k,t}, \quad (3)$$

where  $X$  denotes the corresponding economic indicator. In accordance with the correlation analysis, we find that the estimated coefficient  $\beta$  is significant for the mortality indices of males and females for all four countries, with the exception of females in Canada (see Table 2). However, when analyzing the residuals of the estimated models, significant

autocorrelations were detected for Australia, Canada, and Japan. Therefore, Model (3) was extended by autoregressive terms for  $\Delta k_t$  and then reestimated for all countries:

$$\Delta k_t = \theta + \beta \cdot \Delta \ln(X_t) + \alpha_1 \cdot \Delta k_{t-1} + \alpha_2 \cdot \Delta k_{t-2} + \varepsilon_{k,t} \quad (4)$$

For Australia (males and females) and Japan females, only the AR(1) term was found to be significant, whereas for Japanese males, both  $\Delta k_{t-1}$  and  $\Delta k_{t-2}$  appear to have an impact on current changes in the mortality index. In all cases, the estimated coefficient  $\alpha$  for the autoregressive terms is negative, indicating that the process for  $\Delta k_t$  is less persistent than a random walk. All in all, the goodness of fit of the models for Australia and Japan has increased considerably, leading to adjusted R<sup>2</sup> values of up to 29.2 percent for the mortality index of males in Japan. In case of Canada and the United States, a significant autoregressive term was found only at Lag 1 for changes in the mortality index of Canadian females. However, since the effect of real GDP is not significant in this case, the result will not be considered further.

In a next step, we want to account for the observed change in the relationship between  $\Delta k_t$  and real GDP growth rates or unemployment rate changes. Although the analysis of “moving correlations” showed that this is a gradual change, I refrain from modeling  $\beta$  as a time-dependent parameter since this would lead to implausibly high (absolute) correlations for long-run forecasts of the Lee-Carter model. Instead, the change is accounted for by including a dummy variable  $D_{x,t}$  indicating the years after a break-point year  $x$ :

$$\Delta k_t = \theta + (\beta + \delta_\beta \cdot D_{x,t}) \cdot \Delta \ln(X_t) + \alpha_1 \cdot \Delta k_{t-1} + \alpha_2 \cdot \Delta k_{t-2} + \varepsilon_{k,t} \quad (5)$$

$$D_{x,t} = \begin{cases} 0, & t = 1950, K, x-1 \\ 1, & t = x, K, 2005 \end{cases}$$

Model (5) was estimated several times for each country and the corresponding economic factor using different dummy variables  $D_{x,t}$  and a varying number of autoregressive terms. Because the relation between the mortality index and the state of the economy changes gradually, significant coefficients were found for a number of years. For example, for the mortality index of Australian males,  $\delta_\beta$  was significant for all years between 1986 and 1996. For every country, the best specifications were selected on the basis of the adjusted R<sup>2</sup> measure; these are summarized in Table 3. For Canada, Japan, and the United States, 1989 was an important turning point for the mortality rates of males. In all three countries, the resulting coefficient on real GDP growth rates, or unemployment rate in the case of Japan, changes sign at this date. For example, estimates for the United States translate into:

$$\begin{aligned}
\Delta k_t &= \theta + (\beta + \delta_\beta \cdot D_{1989,t}) \cdot \Delta \ln(GDP_t) + \varepsilon_{k,t} \\
&= -0.871 + (12.775 - 19.049 \cdot D_{1989,t}) \cdot \Delta \ln(GDP_t) + \varepsilon_{k,t} \\
&= \begin{cases} -0.871 + 12.775 \cdot \Delta \ln(GDP_t) + \varepsilon_{k,t}, & t = 1950, K, 1988 \\ -0.871 - 6.274 \cdot \Delta \ln(GDP_t) + \varepsilon_{k,t} & t = 1989, K, 2005 \end{cases}
\end{aligned}$$

For Australia, the year 1990 was selected, leading to a positive but small coefficient  $\beta$  for 1990–2004 in the basic model without autoregressive factors. Including a dummy for 1995 resulted in an improvement in terms of the adjusted  $R^2$  and led to  $\beta = -4.454$  for 1995–2004. Similarly, results for Canada are improved by using a dummy for 1996 instead of for 1989. All in all, accounting for a change in the relation between the mortality index and macroeconomic fluctuations at the beginning of the 1990s increases the model fit substantially in comparison to Models (3) and (4)<sup>3</sup>. The highest adjusted  $R^2$  values are found for changes in the mortality index of males in Australia ( $\bar{R}^2 = 0.267$ ,  $D_{1995,t}$ , AR(1)), Canada, ( $\bar{R}^2 = 0.331$ ,  $D_{1996,t}$ ), and Japan ( $\bar{R}^2 = 0.336$ ,  $D_{1989,t}$ , AR(2)). No significant breakpoint could be identified for females in any of the four countries; instead, Model (4), with a constant coefficient  $\beta$  for real GDP or unemployment rate and up to two autoregressive terms, is best.

## Discussion

### *Examination of age-specific death rates*

Let us now investigate whether the observed reversal in the relationship between the mortality index  $k_t$  and macroeconomic fluctuations are found in age-specific death rates as well. Again, all time series were transformed to relative changes by calculating the first differences of the log series. As an example, the correlation between real GDP growth rates and changes in the death rates of 30- to 85-year-old U.S. males is chosen. In accordance with the above-cited epidemiological findings, these correlations are positive for all but one of the 56 age groups ( $\rho = -0.031$  for 67-year-olds) for 1951–2005. Furthermore, Fig. 3 shows that considerable correlations with real GDP growth rates are also found for higher age groups. For instance, significant correlations can be detected for death rates of males aged 38, 45, 59, 65, 66, 76, and 84. A similar observation was made by Tapia Granados (2008) for the death rates of 65–84-year-olds and unemployment rates in Japan which he explains with the dependency of elderly people on the care and attention of their working-age relatives. These findings support our idea of incorporating the effects of macroeconomic changes at the level of the general mortality index into the Lee-Carter model.

*[Insert Fig. 3 here.]*

In a next step, the sample is split into the three subperiods: 1951–1970, 1971–1990, and 1991–2005. The resulting correlations, which are plotted Fig. 4, support the finding of a change in the relationship between the mortality index  $\Delta k_t$  and real GDP growth rates. Average correlations for all age groups 30–85 decrease from the first (26.3 percent) to the second period (20.3 percent), and turn negative in the third period (–19.2 percent). Comparing the correlations in the first period (1951–1970) with those in the third (1991–2005), a reversal in sign from positive to negative can be observed for 40 of 56 age groups. High negative correlations with real GDP growth rates are observed in the third period for working-age males in particular.

*[Insert Fig. 4 here.]*

#### *Comparison with other studies, possible explanations*

In the following, the above-presented results will be compared with findings from other studies that relate age-specific mortality rates directly to changes in the macroeconomic conditions.

The period analyzed by Ruhm (2000), 1972–1991, is too short to include the reversal that occurs at the beginning of the 1990s, and Neumayer (2004) does not split up his sample period (1980–2000). Similarly, Tapia Granados (2005b) and Gerdtham and Ruhm (2006) do not consider subperiods when estimating their panel models. However, Tapia Granados's (2005a) analysis of U.S. data is more useful for purposes of comparison. He analyzes four periods of roughly similar length (1900–1919, 1920–1944, 1945–1970, and 1971–1996) and finds that the year-to-year percentage changes of age-adjusted mortality rates associated with a one percent point increase in GDP growth or unemployment rates are smallest in absolute terms for the period 1971–1996. In his study on Japan, Tapia Granados (2008) analyzes the subperiods 1960–1977 and 1978–1996, finding that “the impact of macroeconomic change seems to be clearly weaker in the years 1978–2002 compared with the earlier years of the study period.” Analyzing results of the Chow breakpoint test, he concludes that “significant evidence exists of a structural change in the effect of business cycles on mortality at ages 45–64 before 1978 and the most recent years.” The estimated coefficient for changes in unemployment rates is –0.36 for the period 1960–1977; however, it is only –0.13 for 1978–

1996. As a possible explanation, he mentions the increased share of acyclical/countercyclical cancer deaths and countercyclical suicides in Japan.

Indeed, there is further evidence supporting Tapia Granados's hypothesis that changes in the causes of death lead to an altered reaction of aggregate mortality to economic conditions. Mokdad, Marks, Stroup, and Geberding (2004) summarize a large number of studies on actual (external) causes of death in the United States in 2000 and compare their results with those of a similar study by McGinnis and Foege (1993) for the year 1990. The comparison reveals that several causes of deaths that are known to behave procyclically have become less common since 1990. For example, the share of deaths attributable to tobacco and alcohol consumption has decreased from 24 to 21.6 percent of all deaths. Furthermore, Mokdad et al. (2004) find that mortality from infectious and parasitic diseases has declined since 1990 from 4.1 to 3 percent, and identify the decrease in influenza and pneumonia deaths as a major cause. They also document a reduction in the number of deaths due to motor vehicle crashes from 47,000 to 43,354, which they attribute to successful public health efforts in motor-vehicle safety (e.g., promoting the use of child safety seats and safety belts). These tendencies in the causes of death—in combination with the results by Ruhm (2000), who found highest positive elasticity toward real GDP for mortality from motor vehicle crashes, accidents, and homicides, followed by flu/pneumonia and heart and liver disease—explain why reaction of aggregate mortality to economic fluctuations decreased over the last year. They furthermore provide a possible explanation for the observed reversal in the effect from procyclical to countercyclical as (i) Mokdad et al. (2004) report a substantial increase in the number of deaths attributable to poor diet and lack of physical activity (rising from 14 to 16.6 percent between 1990 and 2000), and (ii) Tapia Granados (2008) shows mortality attributable diabetes mellitus correlates negatively with GDP.

### **Summary**

By combining work from two demographic research fields, this paper studies the effect of macroeconomic conditions on the mortality index  $k_t$  in the Lee-Carter model using data for six OECD countries from four continents. We find that changes in  $k_t$  are significantly correlated with real GDP growth rates in Australia, Canada, and the United States, and with unemployment rate changes in Japan, for the period 1950–2005. In all cases, accelerated economic activity is associated with increased mortality rates—a finding in accordance with results for age-specific mortality rates provided by Ruhm (2000), Tapia Granados (2005), and

others. However, comparing the subperiods 1951–1970, 1971–1990, and 1991–2005, we find that the relationship between the state of the economy and mortality changes over time from procyclical to countercyclical in all six countries. This change occurs gradually, as shown by the analysis of “moving correlation,” and is also observed for age-specific mortality rates. Using dummy variables, a switch in sign is identified for males in Australia, Canada, Japan, and the United States at the beginning of the 1990s. Based on our findings, we derive sex-specific time series models for  $k_t$  that appropriately incorporate the effect of macroeconomic fluctuations and account for a substantial fraction of the variation in the mortality indices—as can be seen, for example, for the mortality index of Canadian males in Fig. 5.

*[Insert Fig. 5 here.]*

The paper makes the following contributions to the demographic literature. First, it shows that the mortality index in the Lee-Carter model is not merely an unobserved, latent variable that fluctuates erratically, but is driven to some extent by external factors. Second, it documents that the link between economic conditions and aggregate mortality as established in several previous studies is subject to a structural change, even leading to a change of sign in recent years. A possible explanation for this tendency is that procyclical causes of death have become less common over the last decades, whereas the opposite is the case for acyclical/countercyclical fatalities such as cancer deaths. As a consequence, mortality rates now seem to increase during economic downturns instead of responding to accelerated economic growth as in previous decades.

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<sup>1</sup> The Human Mortality Database (HMD) provides two numbers for the U.S. population in 1959: 1959+ and 1959-. After consulting the HMD team, 1959+ data were chosen. Similarly, 1972+ data were chosen for Japan.

<sup>2</sup> The package can be downloaded at no charge from <http://www.robjhyndman.com>.

<sup>3</sup> I also examined the stability of the trend  $\theta$  in  $\Delta k_t$  in Model (5) at the identified breakpoints, but did not find a break for any of the specifications presented in Table 3.

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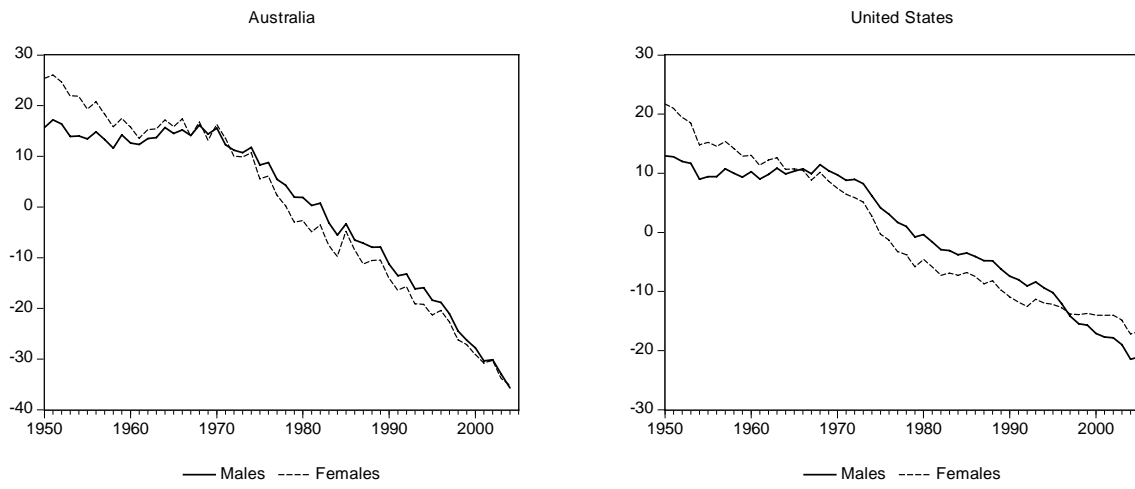
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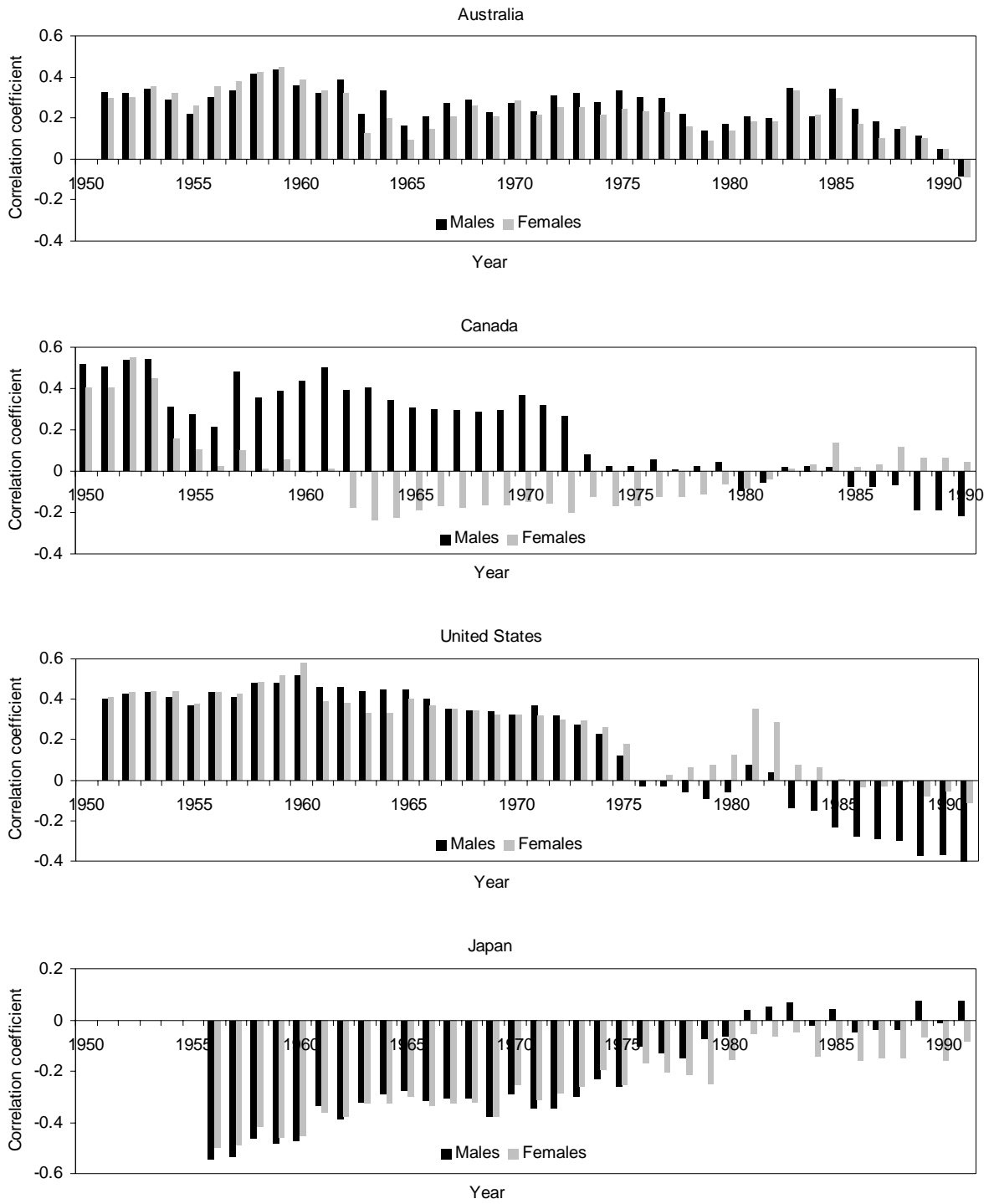


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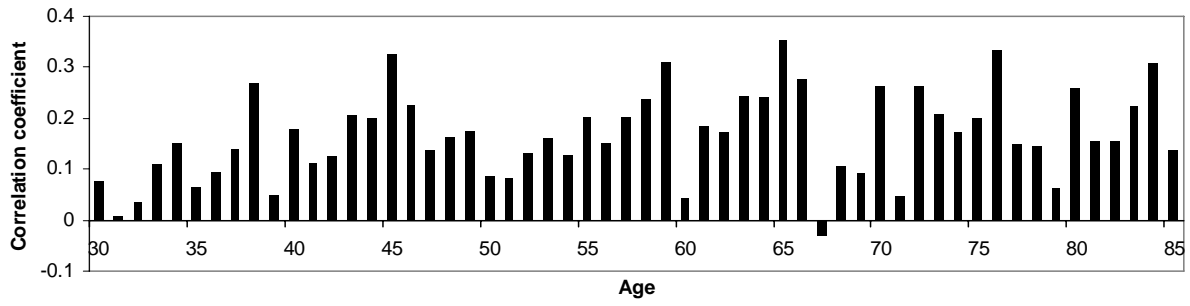
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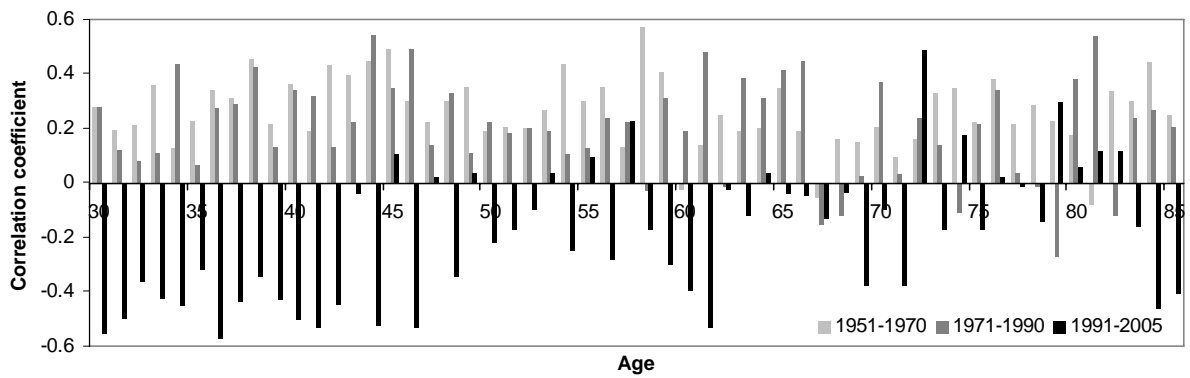
**Fig. 1.** Mortality indices  $k_t$  for males and females.



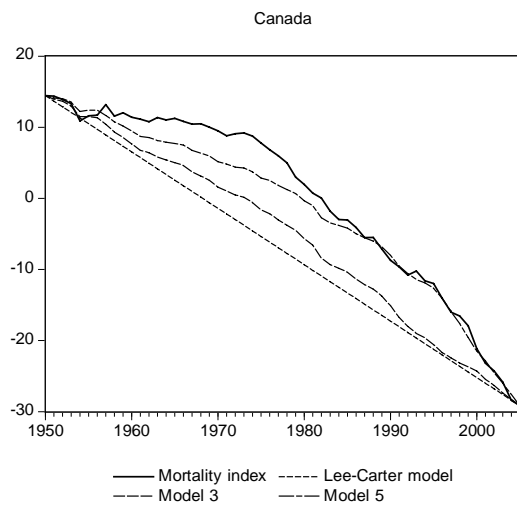
**Fig. 2.** Moving correlations between  $\Delta k_t$  and relative changes in real GDP (Australia, Canada, United States) or in unemployment rates (Japan).



**Fig. 3.** Correlation between relative changes in real GDP and relative changes in death rates of U.S. males, 1951–2005.



**Fig. 4.** Correlation between relative changes in real GDP and relative changes in death rates of U.S. males.



**Fig. 5.** Mortality indices  $k_t$ , males.

**Table 1.** Data availability, correlation results (actual sample periods in parentheses)

	Australia	Canada	France	Japan	Spain	USA
Data availability						
- mortality rates	1950-2004	1950-2005	1950-2005	1950-2005	1950-2005	1950-2005
- real GDP	1950-2005	1950-2005	1950-2005	1950-2005	1950-2005	1950-2005
- unemployment rates	1967-2005	1956-2005	1978-2005	1955-2005	1978-2005	1955-2005
Percentage of variation explained by LC model						
- males	94.8	94.5	93.3	96.8	82.7	93.3
- females	95.1	95.5	96.8	96.9	95.0	96.4
Cor ( $\Delta k_t, \Delta \ln(\text{real GDP}_t)$ )						
1951-2005	(1951-2004)					
- males	0.326*	0.384**	0.109	-0.076	0.000	0.285*
- females	0.269*	0.061	0.000	0.051	0.036	0.286*
1951-1970						
- males	0.326	0.520*	0.229	-0.028	0.013	0.400 <sup>+</sup>
- females	0.295	0.409 <sup>+</sup>	0.166	0.095	0.004	0.406 <sup>+</sup>
1971-1990						
- males	0.235	0.369	-0.176	0.017	0.073	0.367
- females	0.217	-0.150	-0.175	0.036	0.102	0.321
1991-2005	(1991-2004)					
- males	-0.081	-0.217	-0.271	-0.357	-0.161	-0.400
- females	-0.092	0.046	-0.171	-0.222	0.031	-0.113
Cor ( $\Delta k_t, \Delta \text{unemployment rate}_t$ )						
1951-2005	(1968-2004)	(1957-2005)	(1979-2005)	(1956-2005)	(1979-2005)	(1956-2005)
- males	-0.297 <sup>+</sup>	0.014	0.010	-0.338*	-0.141	-0.185
- females	-0.400*	-0.093	-0.051	-0.351*	-0.164	-0.271 <sup>+</sup>
1951-1970				(1956-1970)		(1956-1970)
- males	n.a.	-0.211	n.a.	-0.537*	n.a.	-0.353
- females	n.a.	-0.265	n.a.	-0.494 <sup>+</sup>	n.a.	-0.312
1971-1990			(1979-1990)		(1979-1990)	
- males	-0.424 <sup>+</sup>	-0.086	-0.120	-0.345	-0.365	-0.348
- females	-0.518*	0.024	-0.172	-0.314	-0.245	-0.344
1991-2005	(1991-2004)					
- males	0.014	0.169	0.071	0.074	-0.030	0.212
- females	0.004	-0.050	0.013	-0.083	-0.100	-0.019

\*\* P < 0.01, \* P < 0.05, <sup>+</sup> P < 0.1.

**Table 2.** Regression results, 1951–2005.

Country	Indicator	Dependent Variable	$\theta$	$\beta$	$\alpha_1$	$\alpha_2$	Adj. R <sup>2</sup>	BIC
Australia	$\Delta \ln(\text{real GDP})$	$\Delta k_t$ males	-2.135**	31.258*			0.089	3.923
			(0.525)	(12.573)				
		$\Delta k_t$ females	-2.392**	30.671*	-0.256*		0.140	3.899
			(0.518)	(12.087)	(0.127)			
Canada	$\Delta \ln(\text{real GDP})$	$\Delta k_t$ males	-2.364**	32.815*			0.054	4.444
			(0.681)	(16.317)				
		$\Delta k_t$ females	-2.675**	28.008 <sup>+</sup>	-0.412**		0.211	4.324
			(0.630)	(15.004)	(0.123)			
Japan	$\Delta \ln(\text{unempl. rate})$	$\Delta k_t$ males	-1.359**	15.173**			0.132	2.601
			(0.218)	(5.004)				
		$\Delta k_t$ females	-1.063**	3.041			-0.015	3.232
			(0.299)	(6.858)				
United States	$\Delta \ln(\text{real GDP})$	$\Delta k_t$ males	-1.113**	-5.503*			0.096	3.932
			(0.232)	(2.214)				
		$\Delta k_t$ females	-1.576**	-5.979**	-0.374**		0.216	3.845
			(0.269)	(2.067)	(0.129)			
		$\Delta k_t$ males	-2.115**	-4.699*	-0.466**	-0.321*	0.292	3.801
			(0.337)	(2.034)	(0.128)	(0.131)		
$\Delta k_t$ females	-1.738**	-6.865*			0.105	4.288		
	(0.277)	(2.647)						
United States	$\Delta \ln(\text{real GDP})$	$\Delta k_t$ males	-2.349**	-0.494**	-7.493**		0.255	4.162
			(0.315)	(0.151)	(2.422)			
		$\Delta k_t$ females	-3.216**	-6.968**	-0.536**	-0.236 <sup>+</sup>	0.319	4.129
			(0.476)	(2.381)	(0.130)	(0.128)		
		$\Delta k_t$ males	-1.027**	12.326*			0.064	2.755
			(0.225)	(5.687)				
$\Delta k_t$ females	-1.160**	13.876*			0.064	2.988		
	(0.253)	(6.390)						

Standard errors in parentheses. \*\* P < 0.01, \* P < 0.05, <sup>+</sup> P < 0.1.

**Table 3.** Regression results, 1951–2005, Model (5).

Country	Indicator	Dependent Variable	Break Year	$\theta$	$\beta$	$\delta_\beta$	$\alpha_1$	$\alpha_2$	Adj. R <sup>2</sup>	BIC	
Australia	$\Delta \ln(\text{real GDP})$	$\Delta k_t$ males	1990	-1.856** (0.517)	31.397* (12.057)	-30.720* (13.041)			0.162	3.893	
				-2.126** (0.490)	30.918** (11.250)	-36.928** (12.505)	-0.335** (0.121)		0.255	3.810	
			1995	-1.921** (0.505)	32.180** (11.952)	-36.634* (14.260)			0.178	3.875	
				-2.206** (0.482)	31.801** (11.164)	-42.216** (13.575)	-0.325** (0.120)		0.267	3.794	
		$\Delta k_t$ females	1990	-2.205** (0.699)	32.894* (16.320)	-17.522 (17.652)			0.054	4.499	
	1995		-2.244** (0.689)	33.332* (16.307)	-20.543 (19.456)			0.056	4.496		
Canada	$\Delta \ln(\text{real GDP})$	$\Delta k_t$ males	1989	-1.193** (0.201)	15.850** (4.503)	-23.855** (6.475)			0.298	2.442	
			1996	-1.196** (0.196)	15.514** (4.392)	-29.403** (7.171)			0.331	2.394	
			$\Delta k_t$ females	1989	-1.104** (0.309)	2.873 (6.906)	5.939 (9.932)			-0.028	3.298
		1996		-1.072** (0.308)	3.021 (6.923)	1.780 (11.303)			-0.034	3.304	
Japan	$\Delta \ln(\text{unempl. rate})$	$\Delta k_t$ males	1989	-1.194** (0.229)	-8.079** (2.501)	9.796 <sup>+</sup> (4.876)			0.149	3.928	
				-1.693** (0.260)	-8.863** (2.297)	10.864* (4.465)	-0.396** (0.123)		0.291	3.802	
				-2.116** (0.326)	-7.292** (2.352)	8.917* (4.424)	-0.468** (0.124)	-0.265* (0.130)	0.336	3.793	
			$\Delta k_t$ females	1989	-1.788** (0.282)	-8.460** (3.082)	6.064 (6.009)			0.105	4.345
United States	$\Delta \ln(\text{real GDP})$	$\Delta k_t$ males	1989	-0.871** (0.224)	12.775* (5.440)	-19.049* (7.776)			0.145	2.718	
		$\Delta k_t$ females	1989	-1.208** (0.265)	13.737* (6.430)	5.892 (9.191)			0.054	3.053	

Standard errors in parentheses. \*\* P &lt; 0.01, \* P &lt; 0.05, + P &lt; 0.1.

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