Virtual sensors for active noise control

Jacqueline M. Munn

Department of Mechanical Engineering

The University of Adelaide

South Australia 5005

Australia

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Abstract

The need to attenuate noise transmitted into enclosed spaces such as aircraft cabins, automobiles and mining cabins has provided the impetus for many active noise control studies. Studies into active interior noise control began with a pressure squared cost function utilising multiple error sensors and control sources in an attempt to produce global control of the interior sound field. This work found problems with observability of the primary disturbances and a large number of error sensors and control sources were required to produce global control. Since this early work in the 1980's, many new acoustic based cost functions have been developed to improve on the performance of the pressure squared cost function.

This thesis will focus on one novel acoustic cost function, virtual error sensing. Virtual error sensing is a relatively new technique which produces localised zones of attenuation at a location remote to the physical sensors. The practical advantage of this method is the people within these enclosed spaces are able to observe a reduction in sound pressure level without their movement being restricted by error sensors located close to their ears.

The aim of this thesis is to further investigate the performance of forward-difference virtual error sensors in order to understand the factors that affect the accuracy of the pressure prediction at the virtual location and use this information to develop more accurate and efficient forward-difference virtual sensors.

These virtual sensors use linear arrays of microphones containing two or more microphone elements and a linear or quadratic approximation is used to predict the sound at the virtual location. The prediction method determines the weights applied to each microphone signal to predict the sound pressure level at the virtual location. This study investigates susceptibility of the sensors to corruption as a result of phase and sensitivity mismatch between the microphones, as well as in the location of the elements in the error sensing array. A thorough error analysis of the forward-difference virtual microphones was performed in a one-dimensional sound field and in a plane wave sound field. The accuracy of the quadratic virtual microphone was found to be strongly affected by the presence of short wavelength extraneous noise.

From this study, two novel virtual error sensing techniques were developed, namely; higher-order virtual sensors and adaptive virtual sensors. The higher-order virtual error sensors still employ the linear and quadratic prediction method but extra microphone elements are added to the array. The aim of these higher-order virtual microphones is to produce a more accurate prediction of the pressure at the virtual location by spatially filtering out any short wavelength extraneous noise that may corrupt the prediction. These virtual sensors were tested in a real-time control scenario in both a one-dimensional reactive sound field and in a free field. This work found that the higher-order virtual microphones can improve the prediction accuracy of the original virtual sensors but are still prone to problems of phase, sensitivity and position errors.

Finally, the adaptive LMS virtual sensors were investigated in a SIMULINK simulation and tested experimentally using real-time control in a one-dimensional sound field. It was hoped that an adaptive LMS algorithm could overcome previous difficulties arising from inherent and transducer errors by adapting the weights of the signals from the sensing elements which form the array. The algorithm adapts the sensing microphone signals to produce the same signal as the microphone at the virtual location. Once this has been achieved, the sensing microphone weights are fixed and the microphone at the virtual location is removed, thus creating a virtual microphone. The SIMULINK simulation allowed the performance of the fixed weight and virtual microphones to be investigated in the presence of only phase errors, sensitivity errors and position errors and in the presence of all three combined. This work showed that the adaptive virtual sensors had the ability to compensate for the errors. The number of modes used in the simulations was varied to observe the performance of all virtual sensors in the presence of higher-order modes. The prediction accuracy of the fixed weight virtual sensors was found to be greatly affected by the presence of higher-order modes.

The use of the adaptive virtual microphones to produce localised zones of quiet was examined experimentally using real-time control. The study found the real-time control performance is superior to that of the fixed weight higher-order virtual microphones and the original forward-difference virtual microphones.

Statement of originality

To the best of my knowledge, except where otherwise referenced and cited, everything that is presented in this thesis is my own original work and has not been presented previously for the award of any other degree or diploma in any University. If accepted for the award of the degree of Ph.D. in Mechanical Engineering, I consent that this thesis be made available for loan and photocopying.

Jacqueline M. Munn

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