

FACTORIAL LINEAR MODEL ANALYSIS

BY

Christopher J. Brien
B. Sc. Agric. (Sydney)
M. Agr. Sc. (Adelaide)

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Summary

This thesis develops a general strategy for factorial linear model analysis for experimental and observational studies. It satisfactorily deals with a number of issues that have previously caused problems in such analyses. The strategy developed here is an iterative, four-stage, model comparison procedure as described in Brien (1989); it is a generalization of the approach of Nelder (1965a,b).

The approach is applicable to studies characterized as being structure-balanced, multitiered and based on Tjur structures unless the structure involves variation factors when it must be a regular Tjur structure. It covers a wide range of experiments including multiple-error, change-over, two-phase, superimposed and unbalanced experiments. Examples illustrating this are presented. Inference from the approach is based on linear expectation and variation models and employs an analysis of variance. The sources included in the analysis of variance table is based on the division of the factors, on the basis of the randomization employed in the study, into sets called tiers. The factors are also subdivided into expectation factors and variation factors. From this subdivision models appropriate to the study can be formulated and the expected mean squares based on these models obtained. The terms in the expectation model may be nonorthogonal and the terms in the variation model may exhibit a certain kind of nonorthogonal variation structure. Rules are derived for obtaining the sums of squares, degrees of freedom and expected mean squares for the class of studies covered.

The models used in the approach make it clear that the expected mean squares depend on the subdivision into expectation and variation factors. The approach clarifies the appropriate mean square comparisons for model selection. The analysis of variance table produced with the approach has the advantage that it will reflect all the relevant physical features of the study. A consequence of this is that studies, in which the randomization is such that their confounding patterns differ, will have different analysis of variance tables.

Signed statement

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university and, to the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis. The material in chapters 2 and 6, except sections 6.6 and 6.7, is a revised version of that which I have previously published in Brien (1983) and Brien (1989); copies of these two papers are contained in appendices B and C. The material in section 5.2.4 and some of that in section 6.7 is the subject of an unpublished manuscript by Brien and Payne (1989). The analysis for change-over experiments presented in section 4.3.2.4 was originally developed jointly by Mr W B Hall and the author; my contribution to the joint work is detailed in the text.

I consent to the thesis being made available for photocopying and loan if accepted for the award of the degree.

C.J. Brien

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