Statistical Analysis of Trading Volumes on the Energy Market using a Local Parametric Approach

Master Thesis Submitted to

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María de Lourdes Alavez Estevez

Abstract

International electricity trading in the European Union (EU) is the result of a liberalized energy market. Having access to inter-regional and international energy markets grants electric power producers and industrial consumers the possibility to hedge against diverse financial and non-financial risks. Therefore, a precise forecast of the energy demand becomes imminent to improve the risk management process of the electricity market players.

A local adaptive multiplicative error model (MEM) is used to analyze and forecast German electricity demand traded at the European Power Exchange (EPEX SPOT). In order to assess the adaptive forecasts' performance, we compare them against ad hoc fixed window forecasts. We find that in the relatively short-term both methods perform equally, while in the long-term the local adaptive forecasts outperform the ad hoc fixed window forecasts.

Keywords: Multiplicative error model, local adaptive modeling, trading volume, electricity load, forecasting
JEL classification: C51, C53 ,G12, Q47

Zusammenfassung

Der liberalisierte Energiemarkt stellt die Voraussetzung für den internationalen Stromhandel in der Europäischen Union (EU) dar. Erst der Zugang zu interregionalen und internationalen Energiemärkten eröffnet den Stromerzeugern und den industriellen Stromabnehmern die Möglichkeit sich gegen die verschiedenen finanziellen und nicht-finanziellen Risiken abzusichern. Für die optimale Prozessgestaltung des Risikomanagement der Marktteilnehmer ist eine möglichst präzise Prognose des zukünftigen Energiebedarfs entscheidend. Zur Analyse und Vorhersage des deutschen Strombedarfs, der an der European Power Exchange (EPEX SPOT) gehandelt wird, soll hier ein local adaptive multiplicative error model (MEM) angewandt werden. Dabei werden die adaptive forecasts' performance (adaptive Prognose Performance) zur Einschätzung einem ad hoc fixed window forecasts gegenübergestellt und verglichen. Als Ergebnis ist festzustellen, dass beide Modelle bei einem relativ kurzen (zeitlichen) Intervall gleiche Resultate. Bei einem (zeitlich) längeren Intervall hingegen, übertreffen die Werte des local adaptive forecasts die des ad hoc fixed window forecasts.

Schlagwörter: Multiplicative error model, adaptive Schätzverfahren, Energiebedarfs, Vorhersage
JEL Klassifikation: C51, C53 ,G12, Q47

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1 Introduction

Electricity flows nowadays across regional and international borders, but it was until the European energy market liberalization that electricity first became a tradable commodity. With the energy market liberalization, the European Union aimed to achieve an integrated energy market that would boost competition and enhance efficiency. As a result, electricity trading is now possible at national and international energy exchanges.

The possibility of accessing inter-regional and international energy exchanges provides a set of hedging advantages for electricity producers and consumers. EGL (2010) explains that electricity producers and bulk consumers can protect themselves against production disruptions due to geographic and economic limitations as well from price fluctuations derived from weather conditions and changes in fuel prices. In addition, energy exchanges offer a regulated trading process that ensures a transparent and fair price. Nevertheless, the only way to fully take advantage of the liberalized electricity market is through an accurate forecast of the electricity demand.

Electricity demand (load) is essential to the whole power industry for several reasons. First of all, the costs of generating electricity have soared due to the increasing fuel prices and the high costs of nuclear, hydro and wind power generation. Second, since electricity is our main energy source, a small change in electricity production can cause significant price changes. Third, accurate electricity demand forecasts lead to efficient management of power generation in the short, mid and long term, and it ensures the availability of sufficient resources to meet future demand. In general, precise electricity forecasts can help improve exisiting risk management strategies for electricity producers and industrial consumers.

The start point in successfully forecasting the German electricity demand is understanding its dynamics. For instance, it is well known that electricity demand - as many other economic or financial variables - possess a seasonal component that has to be taken into account in the model estimation. Another important issue regarding model estimation, is the selected time interval to compute the model parameters. In econometrics, it is common to assume large time intervals to increase the efficiency of the model estimates. Nevertheless, this time-homogeneity assumption is rather restrictive and might smooth away the microstructure components of the data set. Therefore, in order to see whether a local homogeneity assumption is appropriate, we explore the dynamics over time of the electricity demand.

We use a multiplicative error model (MEM), proposed by Engle (2002), that is able to analyze serially dependent high frequency data with non-negative values. This model has been already successfully applied to other financial time series such as such as trading volumes, asset volatility and trade durations. However, in order to capture the microstructure components within the data set over the sample period, we implement the local parametric approach (LPA) proposed by Spokoiny (1998). We apply a local multiplicative error model (MEM) to analyze and forecast the German electricity load traded at the European Power Exchange (EPEX SPOT). The advantage of locally estimating a model stems from the fact that it allows model parameters to vary over time and it identifies possible structural breaks within the data set.

In chapter 2 we analyze the dynamics of the German electricity demand and the seasonal adjustment procedure that we followed in order to apply the MEM to the data set. In chapter 3 we discuss the statistical properties of the MEM and its estimation method. In chapter 4 we study the LPA set up to locally estimate the MEM. Finally, in chapter 5 we forecast the electricity demand based on the LPA estimates and we compared them to standard fixed-window forecasts.

2 Data

We analyze the electricity load [1] time series for the German market traded at the European Power Exchange [2] (EPEX SPOT). The data consists of 2904 hourly observations, ranging from January 1 to April 30, 2012 (121 trading days with continuous trading activity). Each trading day is composed of 24 observations representing each hour from 01:00 to 24:00. Descriptive statistics of hourly cumulated trading volume of the German electricity load are shown in Table 2.1.

	German Electricity Load (MW)
Minimum	2.98
25%-quantile	745.00
Median	1147.00
75%-quantile	1600.50
Maximum	5093.00
Mean	1258.50
Standard deviation	714.84
LB(10)	5059.47

Table 2.1: Descriptive statistics and Ljung-Box statistics based on intra-day traded electricity load (in Megawatts) in Germany at EPEX SPOT from January 1 to April 30, 2012 (121 trading days, 2904 observations). Descriptive

From Table 2.1 we can see that the average demanded electricity load in 2012's first quarter amounts to 1258.50 Megawatts (MW), while 50% of the trades demanded more than 1147 MW. Furthermore, we observe a right-skewed distribution and a high dispersion level. The null hypothesis of no autocorrelations within the first 10 lags is rejected given the Ljung-Box (LB) test statistic and any pertinent significance level.

Modeling the electricity demand is a daunting task. First of all, as Nowicka-Zagrajek and Weron (2002) pointed out, electricity demand is driven by many exogenous variables such as weather conditions and economic factors. Second, electricity demand time series is composed of a deterministic (seasonal) and stochastic component at different time scales that has to be

[1] Electricity load in this context is defined as the current electricity demand

[2] The EPEX SPOT is a trading platform for the German, French, Austrian and Swiss Intraday and Day Ahead power spot markets. EPEX SPOT publishes prices and traded electricity volumes in €/MWh (Euro per Megawatt-Hour) and MW (Megawatt) respectively for each segment on `www.epexspot.com`, giving transparent and reliable information regarding short-term electricity prices for the Intraday and Day Ahead trading.

removed before a model can be applied to the data.

Following Engle and Russell (1998), we assume that the intraday seasonal component acts in a multiplicative way on the electricity demand (load) $\breve{y}_i$,

$$y_i = \breve{y}_i {s_i}^{-1}, \tag{2.1}$$

where $s_i = s_{i-24}$ for $i > 24$ represents the intraday periodicity component from the twenty-four hours prior to point i, and y_i indicates the seasonally adjusted load volumes. As Härdle et al. (2012) explain, by using exclusively previous observations to compute the intraday seasonal factor, we avoid introducing forward-looking biases into the forecasting procedure. We follow Engle and Rangel (2008) and Härdle et al. (2012) rolling windows technique to capture the slowly moving components of a given high-frequency time series and compute a 16-days rolling windows average electricity load. In the spirit of Veredas et al. (2002), the intraday seasonal factor is then obtained by fitting a cubic spline with three nodes to the 16-days rolling windows mean electricity demand.

The estimated seasonal components correspond to 105 days (2520 observations), ranging from January 17 to April 29, 2012. Figure 2.1 shows the typical behaviour of the electricity load intraday pattern and it corresponds to the eleventh week of 2012 (March 12 - 18, 2012). We observe a stable seasonal component shape from Monday to Sunday and an inverted U-shaped pattern. It is evident from Figure 2.1 that a lower amount of electricity load is demanded for the first 7 hours of the day, while from 8:00 AM on, the demanded electricity load starts to rise - reaching its highest level at 15:00 hours. From then on, the demand slowly declines until 20:00 hours - when it starts to rise again but not as much as during day time.

It is particularly interesting to see how seasonal patterns change across financial markets. For instance, trading volumes in the stock market exhibit a U-shaped seasonal pattern that explains the high volumes at the opening and closing of the market. In the case of the electricity load it is the opposite effect. The electricity demand seasonal component clearly shows a higher load (MW) after mid-day and at night. Additionally, we can cannot observe any significant difference between week days and weekend days.

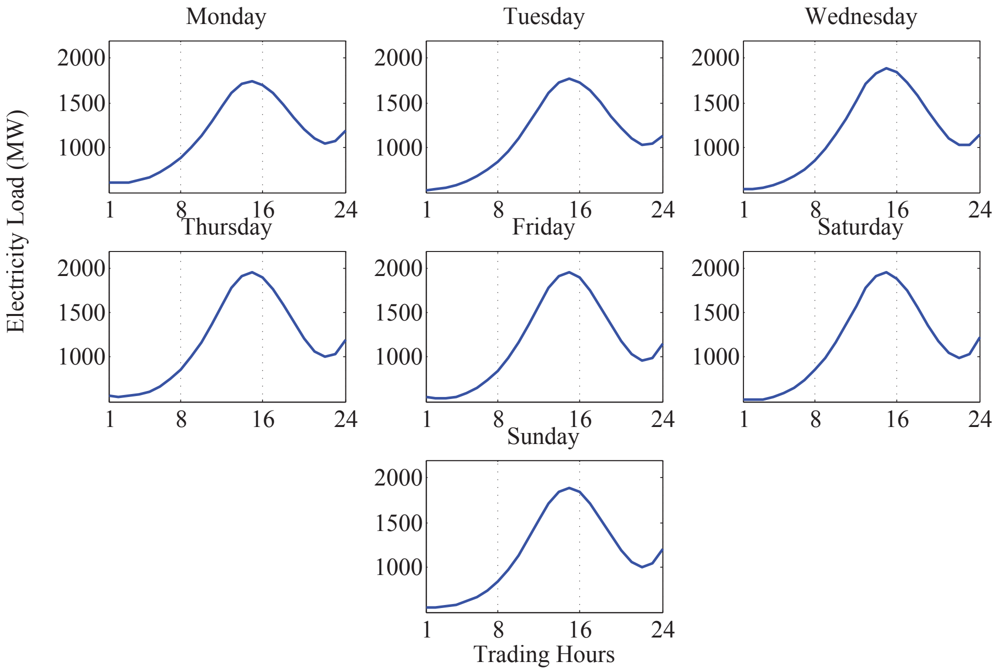

Figure 2.1: Estimated intraday seasonal component for the electricity demand load (in Megawatts) from 20120312-20120318. Seasonality_Graph

3 Multiplicative Error Model (MEM)

Engle and Russell (1998) proposed the autoregressive conditional duration (ACD) model to analyze the the expected durations of financial transaction data. The motivation behind this model was to investigate high frequency data that arrive at irregularly-spaced time intervals - a characteristic that is common in time series of financial markets. Zhang et al. (2001) pointed out that the conditional duration mean described in Engle and Russell (1998), may not be necessarily linear and propose a threshold ACD (TACD) model that was able to identify structural breaks in trade durations. Although the TACD model was an improvement of the original model of Engle and Russell (1998), it did not efficiently addressed the issue of modeling non-negative time series. Therefore, Engle (2002) proposed the Multiplicative Error Model (MEM), with the advantage of modeling any non-negative time series - such as trading volumes, financial durations, bid-ask spreads and asset volatility. Since the MEM has its background on the ACD model, it has the additional advantage of modeling non-negative high frequency data without introducing heteroscedasticity into the data by assuming a short estimation time interval, or by smoothing the microstructure characteristics of the data set over long estimation intervals.

3.1 Model Structure

The MEM(p,q) specifies

$$y_i = \mu_i \varepsilon_i, \tag{3.1}$$

where $y = \{y_i\}_{i=1}^{n}$ is a non-negative process and μ_i represents its conditional mean. The error term is defined as $\mathrm{E}[\varepsilon_i | \mathscr{F}_{i-1}] = 1$, where $\mathscr{F}_{i-1}$ is the information set up to observation i. The conditional mean process of order (p,q) follows an ARMA specification

$$\mu_i = \omega + \sum_{j=1}^{p} \alpha_j y_{i-j} + \sum_{j=1}^{q} \beta_j \mu_{i-j}, \tag{3.2}$$

with parameter $\theta = (\omega, \alpha, \beta)^\top$, such that $\alpha = (\alpha_1, \ldots, \alpha_p)^\top$ and $\beta = (\beta_1, \ldots, \beta_q)^\top$.

The estimation of the model described by equations 3.1 and 3.2 can be done by quasi-maximum likelihood estimation (QMLE) once a distribution with positive support has been selected for the error term over a fixed interval $I = [i_0 - n, i_0]$ of $(n+1)$ observations at observation i_0. There are several distribution choices with non-negative support. Nevertheless, we focus on the (standard) Exponential and the Weibull distribution.

If the error term follows an exponential distribution, $y = \{y_i\}_{i=1}^{n}$ represents a realization of the EACD(p,q) model and its vector parameter $\theta_E = (\omega, \alpha, \beta)^\top$ can be estimated using the exponential log likelihood function:

$$L_I(y;\theta_E) = \sum_{i=max(p,q)+1}^{n} \left(-\log \mu_i - \frac{y_i}{\mu_i} \right) \mathbf{I}\{i \in I\} \tag{3.3}$$

If the error term follows the Weibull distribution, $y = \{y_i\}_{i=1}^{n}$ represents a realization of the WACD(p,q) model and its vector parameter $\theta_W = (\omega, \alpha, \beta, s)^\top$ can be estimated using the Weibull log likelihood function:

$$L_I(y;\theta_W) = \sum_{i \in I} \left[\log \frac{s}{y_i} + s \log \frac{\Gamma(1+1/s)\, y_i}{\mu_i} - \left\{ \frac{\Gamma(1+1/s)\, y_i}{\mu_i} \right\}^s \right] \mathbf{I}\{i \in I\} \tag{3.4}$$

Finally, the quasi maximum likelihood estimates (QMLEs) of θ_E and θ_W over the data interval I are given by

$$\widetilde{\theta}_I = \arg\max_{\theta \in \Theta} L_I(y;\theta) \tag{3.5}$$

3.2 Local Parametric Dynamics

Most econometric models in financial time series analysis are based on the assumption of *parameter homogeneity*. Such an assumption becomes restrictive when the time interval used to estimate the model is large and the structural changes caused by market changes cannot be identified. An alternative technique is the Local Parametric Approach (LPA), which assumes that for every point in our series, there exists an *interval of homogeneity* in which our model can be locally approximated by a constant parametric specification.

The LPA adaptively selects the *interval of homogeneity* by estimating our model at each point in time, using a sequential testing procedure. The procedure starts with the smallest candidate interval and looks for the longest possible interval in which the local approximation statistically

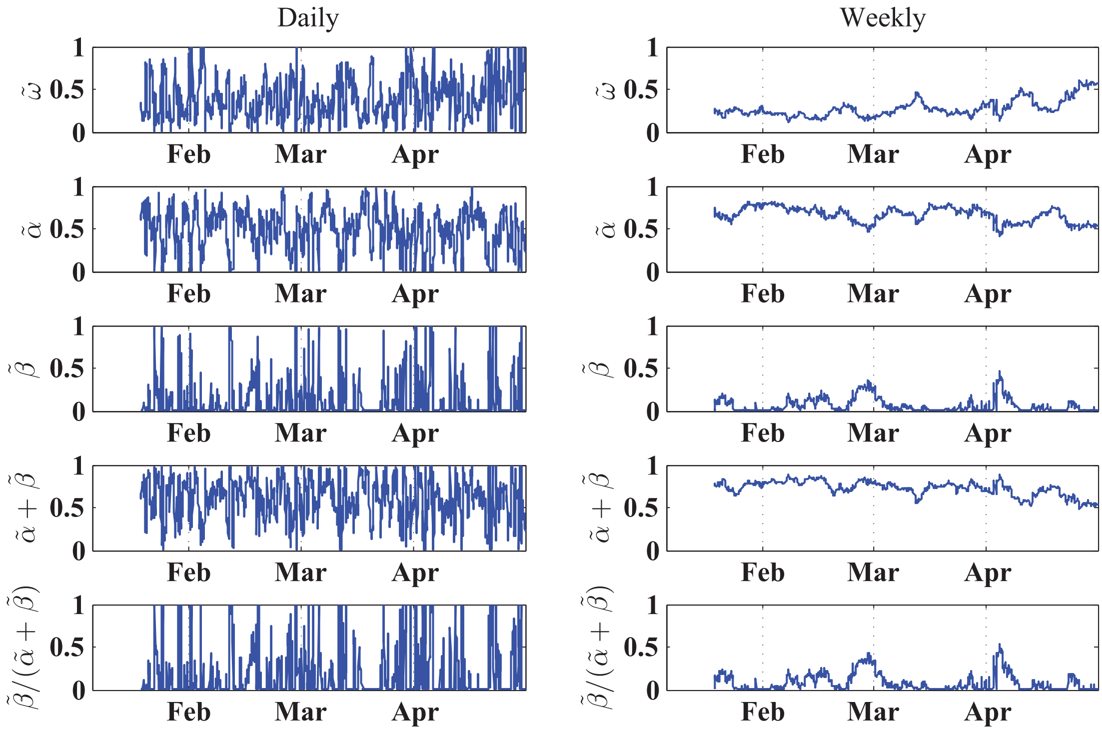

Figure 3.1: The panel shows the estimated parameters based on the seasonally adjusted electricity demand at each hour for the EACD(1,1) model. The left panel shows the 'Daily' estimated parameters, while the right panel displays the 'Weekly' estimated parameters. We can see that the estimated parameters not only vary with respect to time, but also with respect to the selected estimation window length. EACD_Parameter_Dynamics

holds. This technique has two advantages. First, it gives the model parameters the flexibility to vary over time, and second, it identifies structural changes in the data set.

To further investigate the parameter dynamics of our model based on the electricity demand for the first quarter of the year, we estimate an EACD(1,1) and a WACD(1,1) model over four different window lengths: (i) 1 day (24 observations); (ii) 7 days (168 observations); (iii) 15 days (360 observations); and (iv) 30 days (720 observations). Figure 3.1 shows the dynamics of the EACD parameters using a daily and weekly window (interval) length. From the plot we can observe that the estimated parameters ($\tilde{\omega}$,$\tilde{\alpha}$ and $\tilde{\beta}$) and the persistence levels ($\tilde{\alpha}+\tilde{\beta}$) vary not only over time, but also vary upon the estimation window length. For shorter intervals, the parameters appear to be more volatile than those estimated with larger intervals. Härdle et al. (2012) attribute this behavior to a loss of estimation efficiency when using a shorter data window (less observations) or to the possibility that local variations are being smoothed away if data windows become relatively large.

3.3 Estimation Quality

Spokoiny (2009) and Čížek et al. (2009) provide a non-asymptotic risk bound to assess the quality of estimating the true process of equation 3.2 - which describes the observed y_i - through a parametric specification with vector parameter $\widetilde{\theta}_I$, given any I interval. The accuracy of estimating the true vector parameter θ^* is given by

$$\mathrm{E}_{\theta^*}\left|L_I(\widetilde{\theta}_I) - L_I(\theta^*)\right|^r \leq \mathscr{R}_r(\theta^*) \tag{3.6}$$

where $L_I(\widetilde{\theta}_I) - L_I(\theta^*)$ is the deviation of the fitted likelihood function with parameter $\widetilde{\theta}_I$ from the fitted likelihood function with true parameter θ^*. The log likelihood expressions can be decribed either by equation 3.3 or 3.4. Hence, the left side of equation 3.6 represents the loss function to the r-th power of estimating the true vector parameter θ^* via the QMLE $\widetilde{\theta}_I$. $\mathscr{R}_r(\theta^*)$ is a risk bound restricting the loss function, and it depends on r -the risk level or tightness of the risk bound- and θ^*.

Using the parametric risk bound defined in equation 3.6 we can construct non-asymptotc confidence sets and test the accuracy of a local parametric model by simulating a set of critical values to test the local homogeneity assumption required in the LPA. As Härdle et al. (2012) explain, different values of r lead to different risk bounds, critical values and model estimates. By choosing a high risk level r, it is expected that the LPA selects longer intervals of homogeneity. Hence, the QMLE would become more efficient but at the cost of increasing the modeling bias. We follow Čížek et al. (2009) and Härdle et al. (2012) and choose $r = 0.50$ and $r = 1$, considered to be a modest and conservative risk level respectively.

4 Local Parametric Approach

The Local Parametric Approach (LPA) presents an innovative approach to model non-stationary time series based on an adaptive selection of the so-called *interval of homogeneity*. The LPA assumes that at each point in time there exists a historical interval over which equation 3.1 can be well approximated by a local parametric model. Therefore, at each point in time we estimate the model and we obtain a new set of model parameters based on a time-homogeneous interval.

The LPA was proposed by Spokoiny (1998) and since then many useful financial applications have been found, see, e.g., Mercurio and Spokoiny (2004) for an application on daily exchange rates, Čížek et al. (2009) for an adaptation to variant-coefficient parametric models such as GARCH, Chen et al. (2010) for realized volatility modeling, and more recently, Härdle et al. (2012) for localizing multiplicative error models.

4.1 Test of Homogeneity

The selection of the *interval of homogeneity* over which y_i is well approximated by a constant parametric model is done by considering a set of $K+1$ nested intervals with starting point i_0, e.g $I_0 \subset I_1 \subset \ldots \subset I_K$, $I_k = [i_0 - n_k, i_0]$ with $0 < n_k < i_0$. The lengths of the candidate intervals are assumed to geometrically increase with initial length n_0 and a multiplier $c > 1$, $n_k = [n_0 c^k]$. We choose a scheme with $K = 13$, $c = 1.25$ and $n_0 = 60$ (observations), such that the interval lengths (in days) are the following: $n_0 = 2.5$, $n_1 = 3$, $n_2 = 4$, $n_3 = 5$, $n_4 = 6$, $n_5 = 8$, $n_6 = 10$, $n_7 = 12$, $n_8 = 15$, $n_9 = 19$, $n_10 = 24$, $n_{11} = 29$, $n_{12} = 37$ and $n_{13} = 75$.

Our aim is to test the assumption of local homogeneity at each point in time for every I_k interval, until a change point is detected, or until data is exhausted. We use the likelihood ratio (LR) test statistic to test the null hypothesis of parameter homogeneity against the alternative of a change point at an unknown location τ within I_k. The LR test statistic is specified by

$$T_{I_k,J_k} = \sup_{\tau \in J_k} \left\{ L_{A_{k,\tau}}\left(\widetilde{\theta}_{A_{k,\tau}}\right) + L_{B_{k,\tau}}\left(\widetilde{\theta}_{B_{k,\tau}}\right) - L_{I_{k+1}}\left(\widetilde{\theta}_{I_{k+1}}\right) \right\} \tag{4.1}$$

where $J_k = I_k \backslash I_{k-1}$, $A_{k,\tau} = [i_0 - n_{k+1}, \tau]$, and $B_{k,\tau} = (\tau, i_0]$. The testing procedure consists in adding the likelihood ratios of the estimated parameters in intervals $A_{k,\tau}$ and $B_{k,\tau}$, and then substracting the likelihood over interval I_{k+1}. Since the location of the change point τ is unknown,

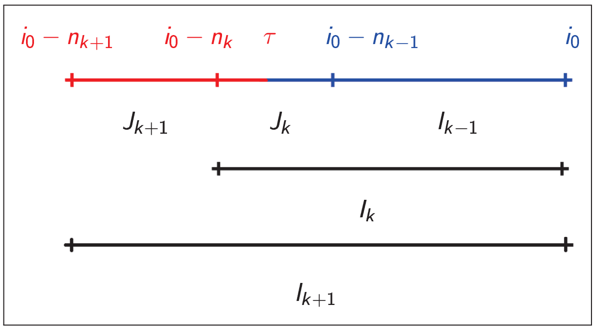

Figure 4.1: Illustration of how data intervals are constructed to test the assumption of local homogeneity within interval I_k with length $n_k = |I_k|$ ending at fixed data point i_0. A local change point is indicated by τ, and depending on its position, splits interval I_{k+1} into two subintervals: $A_{k,\tau}$ (red) and $B_{k,\tau}$ (blue). The sequential testing searchs within interval $J_k = I_k \backslash I_{k-1}$ for a possible change point τ. Figure taken from Härdle et al. (2012).

we consider the supremum of the LR statistics T_{I_k,J_k}.

For a better description of the data-driven intervals involved in the change point detection test, please refer to figure 4.1. We assume that I_0 is homogeneous, and afterwards we test at each step k whether the null can be or not rejected. The unknown location of the change point τ divides interval I_{k+1} into two subintervals: interval $A_{k,\tau} = [i_0 - n_{k+1}, \tau]$ and interval $B_{k,\tau} = (\tau, i_0]$, with parameters $\widetilde{\theta}_{A_k,\tau}$ and $\widetilde{\theta}_{B_k,\tau}$ respectively. The assumption of local homogeneity means that our QMLE parameters $\widetilde{\theta}_{A_k,\tau}$ and $\widetilde{\theta}_{B_k,\tau}$ are statistically homogeneous. Otherwise, if $\widetilde{\theta}_{A_k,\tau} \neq \widetilde{\theta}_{B_k,\tau}$, it means that a change point has been detected and the underlying interval is no longer time homogeneous.

Given the test statistic T_{I_k,J_k}, we select the longest time homogeneous interval that satisfies

$$T_{I_k,J_k} \leq \mathfrak{z}_k \tag{4.2}$$

and $T_{I_{k+1},J_{k+1}} > \mathfrak{z}_{k+1}$, where $\mathfrak{z}_k$ and $\mathfrak{z}_{k+1}$ denote the critical values used to assess whether the null is accepted or not. If an interval I_k fulfills equation 4.2, it means that we have found the longest time-homogeneous interval. In the next section we discuss an adaptive algorithm that optimally selects the longest interval of homogeneity.

4.2 Adaptive Estimation

It is clear by now that the selection of the *interval of homogeneity* plays an important role in the LPA. The goal is to search for this interval with the help of an adaptive estimation procedure that was originally proposed by Spokoiny (1998) to estimate a given function with discontinuities through a local polynomial estimator. The adaptive estimation algorithm basically tests every point $i \in I_k$ against a possible change point τ. The procedure compares the log-likelihood test statistic versus its respective critical value at step k in order to decide whether the tested interval is time-homogeneous. As long as the null is not rejected, the algorithm will extend the search to the next largest interval I_{k+1} and proceed with the local change point detection. On the other hand, if a change point is detected at step k, we set our local homogeneous interval to $\widehat{I}_k = I_{k-1}$ and its respective adaptive estimate as $\widehat{\theta} = \widetilde{\theta}_{I_{k-1}}$.

Čížek et al. (2009) describe this adaptive procedure in three steps: (1) Change Point Detection Test; (2) Adaptive Estimation; and (3) Selection of the Interval of Homogeneity.

1. **Change Point Detection Test**: At each step k of the procedure, the interval I_k is tested against a local change point. Interval I_k is homogeneous if it fulfills the relation described in equation 4.2. Therefore, if the alternative hypothesis is not accepted, we proceed to step 2. Otherwise, we jump to step 3.
2. **Iteration of the Algorithm**: The adaptive procedure will sequentially search for the time-homogeneous interval throughout the K candidate intervals. If the adaptive procedures finds the interval of homogeneity, it will set the adaptive QML estimate as $\widehat{\theta} = \widetilde{\theta}_{I_k}$ and it will continue to step 3. Otherwise, the algorithn will re-set k to $k = k+1$ and proceed with the modeling and the local change point detection.
3. **Selection of the Interval of Homogeneity**: Being at this step means that the null hypothesis within interval I_k has been rejected, and therefore, we have found a change point. Hence, we define the last accepted interval as $\widehat{I} = I_{k-1}$ and the adaptive estimate becomes $\widehat{\theta} = \widetilde{\theta}_{\hat{I}}$.

4.3 Choice of Critical Values

In the LPA hypothesis testing framework, the performance of a statistical model is measured by the probability of committing two types of errors during the estimation of the model. The first one is referred as a false alarm of rejecting the null before the ideal (oracle) interval of homogeneity I_{k^*} has been found. Hence, the algorithm would select an adaptive estimate $\widehat{\theta}_{I_k}$ with a larger variation than $\widehat{\theta}_{I_{k^*}}$. The second type of error occurs when the adaptive estimation algorithm incorrectly selects the interval of homogeneity at a larger stage $\widehat{k} > k^*$.

As a result, we need to select a set of critical values that jointly considers both type of errors and allows us to measure the performance of the LPA under the null hypothesis. Spokoiny (2009) presents a propagation condition - similar to equation 3.6 - that is imposed at every step of the adaptive procedure, and takes into account both, the occurrence of a false alarm and the distance between the underlying model parameters with respect to the adaptive estimate. The propagation condition is defined as

$$\mathrm{E}_{\theta^*} |L_{I_k}(\widetilde{\theta}_{I_k}, \widehat{\theta}_{I_k})|^r \leq \rho_k \mathscr{R}_r(\theta^*) \tag{4.3}$$

where $\mathscr{R}_r$ is the parametric risk bound. The selection of the critical values will depend on the global parameters ρ and r. The role of ρ_k at each step k is similar to the level of significance in hypothesis testing theory, and r represents the power of the loss function. For this reason, the propagation condition will ensure that at each k step we select the minimal critical values that guarantee a small probability of a false alarm. If for example, we take a large r and small ρ_k values, we would increase the value of the critical values and therefore we would improve the performance of the adaptive estimation algorithm. However, by doing this, we would incur in a loss of sensitivity with respect to parameter changes.

Spokoiny (2009) shows that if the critical values are computed by $\mathfrak{z}_k = C + D\log(n_k)$, with C and D constant parameters and $k = 1, \ldots, K$, then equation 4.3 is satisfied. The constant parameters C and D have to be computed via Monte Carlo simulations based on the assumption of parameter homogeneity within $\{I_k\}_{k=1}^K$, and on the vector parameter of equation 3.2. Our critical values for the EACD(1,1) and WACD(1,1) models were provided by Härdle et al. (2012). These sets of critical values were computed based on an estimation window consisting of 1800 observations and on quartiles of persistence $\widetilde{\alpha} + \widetilde{\beta}$ and ratio $\widetilde{\beta}/(\widetilde{\alpha} + \widetilde{\beta})$ levels. They describe nine groups of parameter constellations that are used to simulate $\{\mathfrak{z}_k\}_{k=1}^K$ for the sequential homogeneity testing.

Given the quartiles of estimated persistence and ratio levels - see, e.g. Härdle et al. (2012) - we can select the set of critical values that are appropriate for the sequential test procedure. The first step in selecting the adequate set of critical values consists of computing the persistence level $\widetilde{\alpha} + \widetilde{\beta}$ of our QMLE and classify them according to the persistence levels quartiles. Based on the persistence level we can classify the QMLE into three levels: low, moderate and high. The low persistence level is associated to the first quartile (25% quantile), while the moderate and high persistence level are associated to the second (50% quantile) and third quartile (75% quantile). Then, within each persistence level, we compute the ratio $\widetilde{\beta}/(\widetilde{\alpha} + \widetilde{\beta})$ to further classify each persistence level into low, medium and high. We end up with 9 groups of parameter constellations, for each significance and risk level.

Figure 4.2 displays the critical values for the EACD(1,1) model for $K = 13$, $r = 0.5$ ('moderate risk level') and $\rho = 0.25$. As Härdle et al. (2012) pointed out, the critical values seem to be invariant with respect θ^* across the nine parameter constellations. Furthermore, the largest differences of the critical values across the nine parameter constellations seem to to appear for interval lengths of 3 or 4 days.

For our LPA applications, we use the set of critical values for the EACD(1,1) and WACD(1,1)

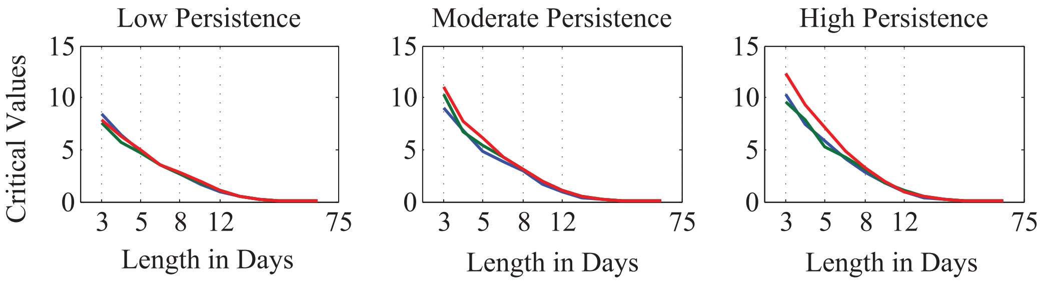

Figure 4.2: Set of simulated critical values of an EACD(1,1) model for the 'moderate risk case' ($\rho = 0.25$ and $r = 0.50$), $K = 13$ and chosen parameter constellations according to persistence and ratio levels. The low (blue), moderate (green) and high (red) curves are associated to the ratio level. Critical_Values

with values $r \in \{0.5, 1\}$ and $\rho \in \{0.25, 0.50\}$. In figure 4.3 and 4.4 we can see the resulting adaptive choice of intervals of homogeneity during the month of April 2012 for the moderate and conservative risk. Although the adaptive choice of intervals of homogeneity for both models were also computed for $r \in \{0.50, 1\}$ and $\rho = 0.50$, we do not display them here since they present similar a similar pattern as the conservative risk.

Nonetheless, we can say that the adaptive estimated length of intervals of homogeneity can give us a better insight on what is the optimal estimation local window. For instance, we can see that the largest candidate interval ($K = 13$ with 1800 observations) is never selected. Thus, such a large estimation window is not appropriate and we should rather select shorter estimation windows that capture the dynamics of high frequency data.

Moreover, the lengths of the intervals of homogeneity vary over time and depend on the model. For instance, in figure 4.3 the estimated length of intervals of homogeneity $n_{\hat{k}}$ for the conservative risk level in the EACD(1,1) ranges from 8 to 10 days, whereas for the moderate risk level it ranges from 6 to 8 days. On the other hand, in figure 4.4 we observe that for the WACD(1,1) the length of the intervals of homogenenity ranges from 2.5 to 6 days in the modest risk case and 6 10 days for the conservative risk case. We can see that for both models, choosing a conservative risk level selects larger intervals with smaller variability but at the cost of increasing the modeling bias.

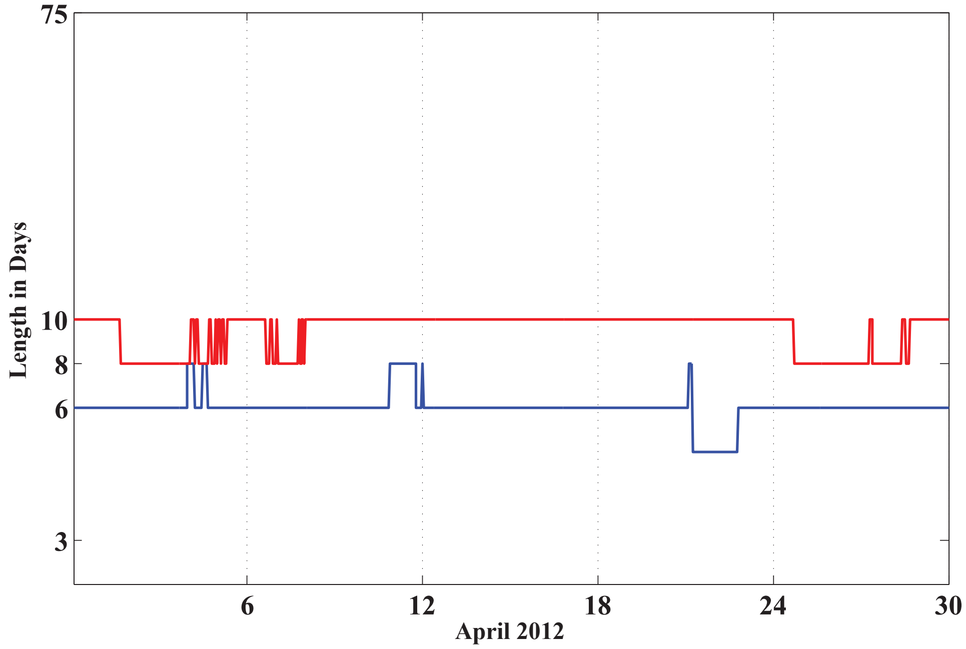

Figure 4.3: Estimated length of intervals of homogeneity $n_{\widehat{k}}$ given the conservative $r = 1$ (red) and modest $r = 0.50$ (blue) risk case for April 2012 using the EACD model with $\rho = 0.25$. EACD_k_hat_Graphs_c

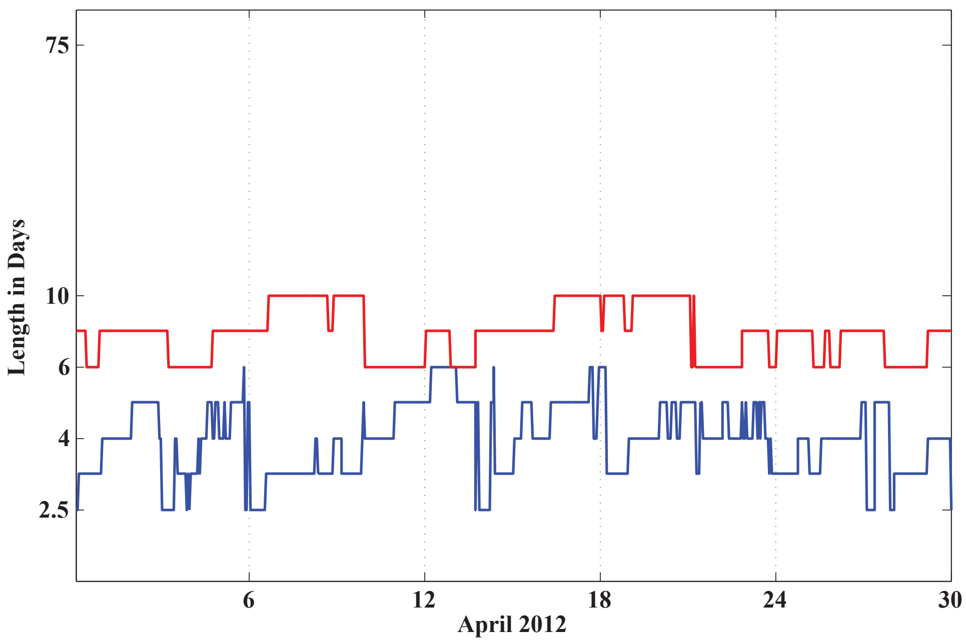

Figure 4.4: Estimated length of intervals of homogeneity $n_{\widehat{k}}$ given the conservative $r = 1$ (red) and modest $r = 0.50$ (blue) risk case for April 2012 using the WACD model with $\rho = 0.25$. WACD_k_hat_Graphs_c

5 Forecasting the Electricity Demand

Accurate forecasts of the electricity demand can have an impact on the risk management strategies of the power market players. Deng and Oren (2006) point out that with a liberalized and competitive electricity market, power companies, load serving entities (LSEs) and power marketers, seek certainty in their costs and revenues through hedging practices and active trading. In the end, all they want is to maximize the company's value by mitigating the market risks. In this chapter, we briefly discuss the usefulness of incorporating precise forecasting techniques into reducing the risk exposure that power generators bear.

In Germany, the biggest five electricity producers are E.ON, RWE, EnBW, Vattenfall Europe and EWE AG. For these companies, balancing their exposure to financial and non-financial risks is crucial to their risk management strategy. Vattenfall (2011) gives a closer insight on this matter and describes the most common risks faced by power companies, electricity suppliers and industrial consumers, and remarks the importance of precise electricity demand predictions. Vattenfall classifies these risks in six different categories: Market & Financial, Technology, Infrastructure, Politics & Society, Laws & Regulations and Personnel & Organization. Nevertheless, we briefly discuss two specific Market & Financial risks that can be directly related to the electricity demand: Volume risk and Price risk.

Volume risk, in terms of electricity load (MW), is defined as the deviation of the forecast with respect to the actual observed volume. On the other hand, price risk (€/MWh) refers to all the exogenous factors that influence the electricity price. And since the price for electricity is the main source of earnings for a power generator and LSEs, all factors that have an impact on electricity prices - such as water levels, generation capacity, electricity demand, and weather and economic conditions - have to be accurately forecasted in order to mitigate the price risk. Therefore, by developing better forecast, the electricity market players can estimate a precise electricity consumption prediction that would reduce the volume risk and price risks.

Deng and Oren (2006) gives a good example that illustrates the importance of electricity demand and its relation to price risk and volume risk. Deng and Oren (2006) supposes that an electricity supplier, agrees on a full-requirement contract with an industrial consumer, like Siemens. It is natural to think, that Siemens would be more interested in having a flexible electricity load contract at a fixed rate per unit of energy regardless if its electricity consumption is high or low. In order to reduce this risk, the power supplier would use future contracts to secure a fixed amount of electricity supply at a fixed cost for hedging the expected electricity demand of Siemens. Nevertheless, the electricity supplier is then at the risk of either under- or over-hedging, as the electricity demand of Siemens will for sure deviate from the amount hedged by futures contracts.

If the electricity spot price is high, the total electricity demand is likely to be high as well. Thus, if the electricity price is higher than the fixed contract rate for serving electricity, there is a possibility that Siemen's electricity demand is significantly higher than the hedged quantity. This would mean that the LSE is under-hedged with respect to the load it has to deliver and must buy electricity in the spot market at a loss due to the fact that the spot price might exceed the contracted price paid by Siemens. In the case of a lower spot price, the LSE bears the risk of being over-hedged, which means it has to sell the electricity surplus in the market at a probably lower price than that of the settled in the full load contract. This is a clear example of the financial risks that a LSE or a power company bear due to the volume and price risk, and to the positive correlation between price and electricity demand.

Hence, the challenge is to develop better forecast techniques in order to efficiently manage electricity financial risks. Our forecast set up predicts the German electricity demand traded at the EPEX SPOT over all $h = 1, 2, \ldots, 24$ hours from April 1, to April 29, 2012. We apply two forecasting methods. The first one, based on the standard fixed estimation window - in which we compute the forecasts on a 3-months (i.e., 1800 observations) window basis. And the second, based on a multi-step ahead forecasting technique that uses the resulting MEM parameters of an adaptively-selected local window for $r \in \{0.5, 1\}$ and $\rho \in \{0.25, 0.50\}$.

Let us denote the adaptive h-step forecast by $\widehat{y}_{i+h}$ and the resulting forecast error as $\widehat{\varepsilon}_{i+h} = \breve{y}_{i+h} - \widehat{y}_{i+h}$, where $\breve{y}_{i+h}$ is the observed electricity load. Conversely, we denote the 3-months forecast as $\widetilde{y}_{i+h}$ and the prediction error $\widetilde{\varepsilon}_{i+h} = \breve{y}_{i+h} - \widetilde{y}_{i+h}$. To account for the intraday periodicity effects explained in Chapter 2, we multiply the corresponding forecasts by the estimated seasonal factor associated with the previous 24 hours.

We evaluate both forecasting methods by means of the root mean square prediction error (RMSPE) in terms of Megawatts and percentages. We define

$$RMSPE_{i+h} = \{n^{-1} \sum_{i=1}^{n} (\breve{y}_{i+h} - \widehat{y}_{i+h})^2\}^{1/2} \tag{5.1}$$

for the adaptive forecast and $RMSPE_h = \{n^{-1} \sum_{i=1}^{n} (\breve{y}_{i+h} - \widetilde{y}_{i+h})^2\}^{1/2}$ for the fixed-window forecast respectively. The RMSPE in percentages is calculated by dividing the RMSPE value at each point by the data sample mean.

Table 5.1 compares the overall RMSPE results obtained for the two forecasting techniques. We observe that for both, the EACD(1,1) and the WACD(1,1) model, the LPA forecasts have

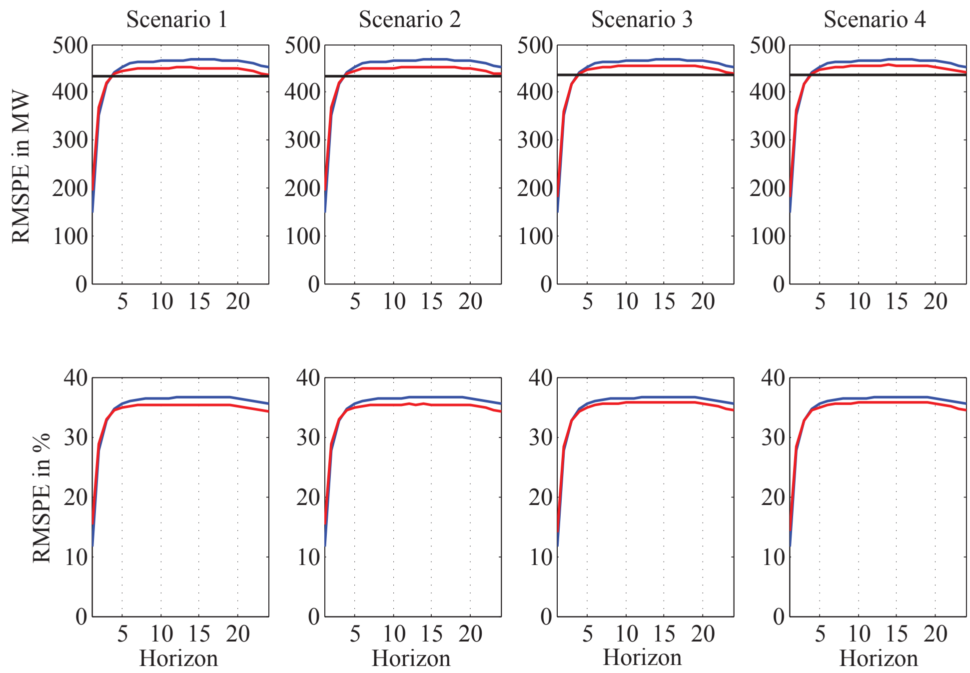

Figure 5.1: Estimated RMSPEs for the(local) EACD(1,1) model for the four possible scenarios. The graphs plot the LPA RMSPEs (red) against the fixed-window RMSPEs (blue) over the sample from April 1 to April 29, 2012. The upper pannel shows the results in terms of Megawatts. The lower pannel shows the RMSPE in percentage terms. EACD_RMSPE

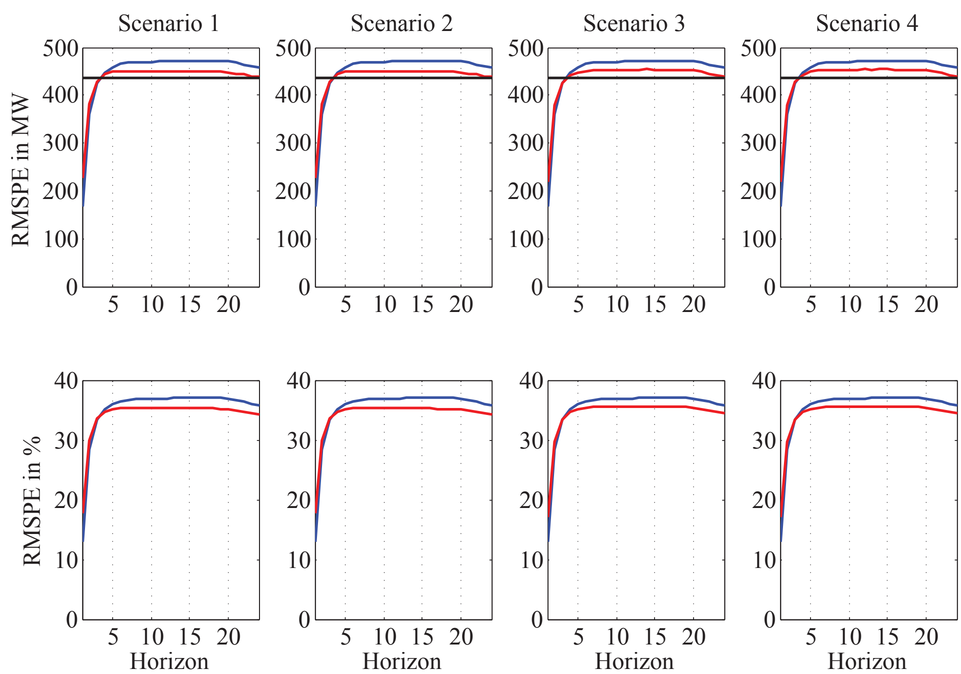

Figure 5.2: Estimated RMSPEs for the(local) WACD(1,1) model for the four possible scenarios. The graphs plot the LPA RMSPEs (red) against the fixed-window RMSPEs (blue) over the sample from April 1 to April 29, 2012. The upper pannel shows the results in terms of Megawatts. The lower pannel shows the RMSPE in percentage terms. WACD_RMSPE

Overall RMSPE (%)	EACD(1,1)	WACD(1,1)
Standard Approach	34.72%	35.18%
LCP Scenario 1, $\rho = 0.25$ & $r = 0.50$	33.93%	34.12%
LCP Scenario 2, $\rho = 0.50$ & $r = 0.50$	33.98%	34.11%
LCP Scenario 3, $\rho = 0.25$ & $r = 1$	34.11%	34.24%
LCP Scenario 4, $\rho = 0.50$ & $r = 1$	34.15%	34.26%

Table 5.1: Comparison of the overall root mean square prediction error (ORMSPE) in percentage terms for the EACD and WACD models for the LPA and fixed-window forecasts. LCP_Forecasting& Standard_Forecasting

a lower coefficient of variation with respect to the sample mean and there are no significant differences across risk and significance levels. A graphical illustration of these results, is shown in figure 5.1 for the EACD(1,1)model and in figure 5.2 for the WACD(1,1) model. For both models, we can see that for the first hours in of the forecast horizon, the two methods perform similarly. However, after the fourth hour, the LPA forecasts outperforms the standard fixed-window forecasts.

In general, by analyzing the RMSPE graphs for the EACD(1,1) and the WACD(1,1) models we can conclude that for up to 4 hours, both forecasting methods perform equally. Hence, the LPA can be safely used to forecast relatively short time horizons. However, after 4 hours, the LPA outperforms the standard fixed-window forecasting approach. Therefore, in the relatively long run, the LPA becomes a better forecast estimate. Moreover, we observe that the forecasting results are robust with respect of the tuning parameters and across the four different LPA scenarios.

6 Conclusion

The German electricity demand traded at EPEX SPOT exhibits an inverted U-shape intraday seasonal component - unlike the typical U-shape observed in stock's seasonal component. This shape is explained by the fact that electricity demand is relatively low in the morning, then it slowly increases around midday, decreases during the evening and finally, it rises in the night. Moreover, we applied a local multiplicative error model (MEM) to analyze and forecast the German electricity load. By doing this, we observed that model parameters vary not only with respect to time, but also with respect to the selected estimation window. For instance, longer estimation windows yield less variable model parameters than those estimated over short intervals. The LPA is a more flexible procedure that allows parameters to vary with time and at the same time detects possible structural breaks. Therefore, LPA estimates become more precise and helpful in forecasting procedures.

For each model, EACD(1,1) and the WACD(1,1), the local parametric approach yielded different estimated lengths of intervals of homogeneity. The average length of intervals of homogeneity for the EACD(1,1) ranges from 6 to 8 days in the modest risk case, whereas for the conservative risk this range lies between 8 and 10 days. For the WACD(1,1) these intervals of homogeneity range from 2.5 to 6 days for the modest risk case and from 6 to 10 days. Hence, we can say that we obtain rebust results concerning the tuning parameters, risk levels and significance levels over the four different LPA scenarios. In addition, by observing the RMSPE results and the ORMSPE table, we can conclude that for up to 4 hours, both forecasting methods perform equally. This means that the LPA procedure can be safely used to estimate and forecast time series over a relatively short time horizon. For this reason, the LPA provides better forecast estimates that will help to further improve - through a precise electricity demand estimation - better risk management techniques in the Energy Market.

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Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated sources. Where I have consulted the published work of others, in any form (e.g. ideas, equations, figures, text, tables),this is always explicitly attributed.

Berlin, August 31st, 2012 María de Lourdes Alavez Estevez

Selbständigkeitserklärung

Ich erkläre, dass ich die vorliegende Arbeit selbständig und nur unter Verwendung der angegebenen Literatur und Hilfsmittel angefertigt habe.

Berlin, den 31.08.2012 María de Lourdes Alavez Estevez