The Kugo–Ojima Confinement Criterion
from Dyson–Schwinger Equations[*]

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Prerequisites of confinement in the covariant and local description of QCD are reviewed. In particular, the Kugo–Ojima confinement criterion, the positivity violations of transverse gluon and quark states, and the conditions necessary to avoid the decomposition property for colored clusters are discussed. In Landau gauge QCD, the Kugo–Ojima confinement criterion follows from the ghost Dyson–Schwinger equation if the corresponding Green's functions can be expanded in an asymptotic series. Furthermore, the infrared behaviour of the propagators in Landau gauge QCD as extracted from solutions to truncated Dyson–Schwinger equations and lattice simulations is discussed in the light of these issues.

Prerequisites of a Covariant Description of Confinement

The confinement phenomenon in QCD cannot be accommodated within the standard framework of quantum field theory. Thereby it is known that covariant quantum theories of gauge fields require indefinite metric spaces. Maintaining the much stronger principle of locality, great emphasis has been put on the idea of relating confinement to the violation of positivity in QCD. Just as in QED, where the Gupta-Bleuler prescription is to enforce the Lorentz condition on physical states, a semi-definite *physical subspace* can be defined as the kernel of an operator. The physical states then correspond to equivalence classes of states in this subspace differing by zero norm components. Besides transverse photons covariance implies the existence of longitudinal and scalar photons in QED. The latter two form metric partners in the indefinite space. The Lorentz condition eliminates half of these leaving unpaired states of zero norm which do not contribute to observables. Since the Lorentz condition commutes with the S-Matrix, physical states scatter into physical ones exclusively.

Due to the gluon self-interactions the corresponding mechanism is more complicated in QCD. Here, the Becchi–Rouet–Stora (BRS) symmetry of the gauge fixed action proves to be helpful. Within the framework of BRS algebra, in the simplest version for the BRS-charge Q_B and the ghost number Q_c given by,

$$Q_B^2 = 0 , \quad [iQ_c, Q_B] = Q_B , \tag{1}$$

completeness of the nilpotent BRS-charge Q_B in a state space $\mathcal{V}$ of indefinite metric is assumed. This charge generates the BRS transformations by ghost number graded commutators $\{\,,\,\}$, *i.e.*, by commutators or anticommutators for fields with even or odd ghost number, respectively. The semi-definite subspace $\mathcal{V}_p = \mathrm{Ker}\,Q_B$ is defined on the basis of this algebra by those states which are annihilated by the BRS charge Q_B. Since $Q_B^2 = 0$, this subspace contains the space $\mathrm{Im}\,Q_B$ of so-called daughter states which are images of others, their parent states in $\mathcal{V}$. A *physical* Hilbert space is then obtained as the covariant space of equivalence classes, the BRS-cohomology of states in the kernel modulo those in the image of Q_B,

$$\mathcal{H}(Q_B, \mathcal{V}) = \mathrm{Ker}\,Q_B / \mathrm{Im}\,Q_B \simeq \mathcal{V}_s \,, \tag{2}$$

which is isomorphic to the space $\mathcal{V}_s$ of BRS singlets. Completeness is thereby important in the proof of positivity for physical states [1,2] because it assures the absence of metric partners of BRS-singlets.

With completeness, all states in $\mathcal{V}$ are either BRS singlets in $\mathcal{V}_s$ or belong to quartets which are metric-partner pairs of BRS-doublets (of parent with daughter states). This exhausts all possibilities. The generalization of the Gupta–Bleuler condition on physical states, $Q_B|\psi\rangle = 0$ in $\mathcal{V}_p$, eliminates half of these metric partners leaving unpaired states of zero norm which do not contribute to any observable. This essentially is the quartet mechanism:

> Just as in QED, one such quartet, the elementary quartet, is formed by the massless asymptotic states of longitudinal and timelike gluons together with ghosts and antighosts which are thus all unobservable.

> In contrast to QED, however, one expects the quartet mechanism also to apply to transverse gluon and quark states, as far as they exist asymptotically. A violation of positivity for such states then entails that they have to be unobservable also.

Asymptotic transverse gluon and quark states may or may not exist in the indefinite metric space $\mathcal{V}$. If either of them do exist and the Kugo–Ojima criterion (see below) is realized, they belong to unobservable quartets. In that case, the BRS-transformations of their asymptotic fields entail that they form these quartets together with ghost-gluon and/or ghost-quark bound states, respectively [2]. It is furthermore crucial for confinement, however, to have a mass gap in transverse gluon correlations, *i.e.*, the massless transverse gluon states of perturbation theory have to dissappear even though they should belong to quartets due to color antiscreening [3–5].

The interpretation in terms of transition probabilities holds between physical states. For a local operator A to be observable it is necessary to be BRS-closed, $\{iQ_B, A\} = 0$, which coincides with the requirement of its local gauge invariance. It then follows that all states generated from the vacuum $|\Omega\rangle$ by any such observable fulfill positivity: On the other hand, unobservable, *i.e.*, confined, states violate positivity.

The remaining dynamical aspect of confinement in this formulation resides in the cluster decomposition property [6]. Including the indefinite metric spaces of covariant gauge theories it can be summarized as follows: For the vacuum expectation values of two local operators A and B, translated to a large spacelike separaration R of each other one obtains the following bounds depending on the existence of a finite gap M in the spectrum of the mass operator in $\mathcal{V}$ [2]

$$\left| \langle\Omega|A(x)B(0)|\Omega\rangle - \langle\Omega|A(x)|\Omega\rangle\,\langle\Omega|B(0)|\Omega\rangle \right| \tag{3}$$

$$\leq \begin{cases} \text{Const.} \,\times\, R^{-3/2+2N}\, e^{-MR}, & \text{mass gap } M \,, \\ \text{Const.} \,\times\, R^{-2+2N}, & \text{no mass gap} \,, \end{cases}$$

for $R^2 = -x^2 \to \infty$. Herein, positivity entails that $N = 0$, but a positive integer N is possible for the indefinite inner product structure in $\mathcal{V}$. Therefore, in order to avoid the

decomposition property for products of unobservable operators A and B which together with the Kugo-Ojima criterion (see below) is equivalent to avoiding the decomposition property for colored clusters, there should be no mass gap in the indefinite space $\mathcal{V}$. Of course, this implies nothing on the physical spectrum of the mass operator in $\mathcal{H}$ which certainly should have a mass gap. In fact, if the cluster decomposition property holds for a product $A(x)B(0)$ forming an observable, it can be shown that both A and B are observables themselves. This then eliminates the possibility of scattering a physical state into color singlet states consisting of widely separated colored clusters (the "behind-the-moon" problem) [2].

Confinement depends on the realization of the unfixed global gauge symmetries in this formulation. The identification of the BRS-singlets in the physical Hilbert space $\mathcal{H}$ with color singlets is possible only if the charge of global gauge transformations is BRS-exact *and* unbroken. The sufficent conditions for this are provided by the Kugo-Ojima criterion: Considering the globally conserved current

$$J_\mu^a = \partial_\nu F_{\mu\nu}^a + \{Q_B, D_\mu^{ab}\bar{c}^b\} \qquad (\text{with } \partial_\mu J_\mu^a = 0) , \tag{4}$$

one realizes that the first of its two terms corresponds to a coboundary with respect to the space-time exterior derivative while the second term is a BRS-coboundary with charges denoted by G^a and N^a, respectively,

$$Q^a = \int d^3x \, \partial_i F_{0i}^a + \int d^3x \, \{Q_B, D_0^{ab}\bar{c}^b\} = G^a + N^a . \tag{5}$$

For the first term herein there are only two options, it is either ill-defined due to massless states in the spectrum of $\partial_\nu F_{\mu\nu}^a$, or else it vanishes.

In QED massless photon states contribute to the analogues of both currents in (4), and both charges on the r.h.s. in (5) are separately ill-defined. One can employ an arbitrariness in the definition of the generator of the global gauge transformations (5), however, to multiply the first term by a suitable constant so chosen that these massless contributions cancel. This way one obtains a well-defined and unbroken global gauge charge which replaces the naive definition in (5) above [7]. Roughly speaking, there are two independent structures in the globally conserved gauge currents in QED which both contain massless photon contributions. These can be combined to yield one well-defined charge as the generator of global gauge transformations leaving the other independent combination (the displacement symmetry) spontaneously broken which lead to the identification of photons with massless Goldstone bosons [2,8].

If $\partial_\nu F_{\mu\nu}^a$ contains no massless discrete spectrum on the other hand, *i.e.*, if there is no massless particle pole in the Fourier transform of transverse gluon correlations, then $G^a \equiv 0$. In particular, this is the case for channels containing massive vector fields in theories with Higgs mechanism, and it is expected to be also the case in any color channel for QCD with confinement for which it actually represents one of the two conditions formulated by Kugo and Ojima. In both these situations one has

$$Q^a = N^a = \left\{ Q_B , \int d^3x \, D_0^{ab}\bar{c}^b \right\} , \tag{6}$$

which is BRS-exact. The second of the two conditions for confinement is that this charge be well-defined in the whole of the indefinite metric space $\mathcal{V}$. Together these conditions are sufficient to establish that all BRS-singlet physical states in $\mathcal{H}$ are also color singlets, and that all colored states are thus subject to the quartet mechanism. The second condition thereby provides the essential difference between Higgs mechanism and confinement. The operator $D_\mu^{ab}\bar{c}^b$ determining the charge N^a will in general contain a *massless* contribution from the elementary quartet due to the asymptotic field $\bar{\gamma}^a(x)$ in the antighost field, $\bar{c}^a \overset{x_0 \to \pm\infty}{\longrightarrow} \bar{\gamma}^a + \cdots$ (in the weak asymptotic limit),

$$D_\mu^{ab} \bar{c}^b \overset{x_0 \to \pm\infty}{\longrightarrow} (\delta^{ab} + u^{ab}) \, \partial_\mu \bar{\gamma}^b(x) + \cdots . \tag{7}$$

Here, the dynamical parameters u^{ab} determine the contribution of the massless asymptotic state to the composite field $g f^{abc} A_\mu^c \bar{c}^b \overset{x_0 \to \pm\infty}{\longrightarrow} u^{ab} \partial_\mu \bar{\gamma}^b + \cdots$. These parameters can be obtained in the limit $p^2 \to 0$ from the Euclidean correlation functions of this composite field, *e.g.*,

$$\int d^4x \, e^{ip(x-y)} \langle \, D_\mu^{ae} c^e(x) \, g f^{bcd} A_\nu^d(y) \bar{c}^c(y) \, \rangle =: \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) u^{ab}(p^2) . \tag{8}$$

The theorem by Kugo and Ojima asserts that all $Q^a = N^a$ do not suffer from spontaneous breakdown (and are thus well-defined), if and only if

$$u^{ab} \equiv u^{ab}(0) \overset{!}{=} -\delta^{ab} . \tag{9}$$

Then the massless states from the elementary quartet do not contribute to the spectrum of the current in N^a, and the equivalence between physical BRS-singlet states and color singlets is established. [1,2,7]

In contrast, if $\det(\mathbb{1} + u) \neq 0$, the global gauge symmetry generated by the charges Q^a in eq. (5) is spontaneuosly broken in each channel in which the gauge potential contains an asymptotic massive vector field [1,2]. While this massive vector state results to be a BRS-singlet, the massless Goldstone boson states which usually occur in some components of the Higgs field, replace the third component of the vector field in the elementary quartet and are thus unphysical. Since the broken charges are BRS-exact, this symmetry breaking is not directly observable in the Hilbert space of physical states $\mathcal{H}$.

The condition $u = -\mathbb{1}$ Landau gauge be shown by standard arguments employing Dyson–Schwinger equations and Slavnov–Taylor identities to be equivalent to an infrared enhanced ghost propagator [7]. In momentum space the non-perturbative ghost propagator of Landau gauge QCD is related to the form factor occuring in the correlations of eq. (8),

$$D_G(p) = \frac{-1}{p^2} \frac{1}{1 + u(p^2)} , \quad \text{with } u^{ab}(p^2) = \delta^{ab} u(p^2) . \tag{10}$$

The Kugo–Ojima confinement criterion, $u(0) = -1$, thus entails that the Landau gauge ghost propagator should be more singular than a massless particle pole in the infrared. Indeed, we will present evidence for this exact infrared enhancement of ghosts in Landau gauge.

The necessity for the absence of the massless particle pole in $\partial_\nu F_{\mu\nu}^a$ in the Kugo-Ojima criterion shows that the (unphysical) massless correlations to avoid the cluster decomposition property are *not* the transverse gluon correlations. An infrared suppressed propagator for the transverse gluons in Landau gauge confirms this condition. This holds equally well for the infrared vanishing propagator obtained from Dyson–Schwinger Equations [9,10] and conjectured in the studies of the implications of the Gribov horizon [11,12], as for the infrared suppressed but possibly finite ones extraced from improved lattice actions for quite large volumes [13]. The infrared enhanced correlations responsible for the failure of the cluster decomposition can be identified with the ghost correlations which at the same time provide for the realization of the Kugo–Ojima criterion in Landau gauge.

Verifying the Kugo–Ojima Confinement Criterion from the Dyson–Schwinger Equation for the Ghost Propagator

In Landau gauge the gluon and ghost propagators are parametrized by the two invariant functions $Z(k^2)$ and $G(k^2)$, respectively (with $G(k^2) = 1/(1 + u(k^2))$, *c.f.*, eq. (10)). In Euclidean momentum space one has

$$D_{\mu\nu}(k) = \frac{Z(k^2)}{k^2} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) , \quad D_G(k) = -\frac{G(k^2)}{k^2} . \tag{11}$$

The non-perturbative infrared behaviour of these functions can be studied with employing their Dyson–Schwinger equations [5,14].

The equation for the ghost propagator is the simplest of all QCD Dyson–Schwinger equations. Besides the ghost and gluon propagators it contains the ghost-gluon vertex function. In Landau gauge this 3-point function needs not to be renormalized. Furthermore, it becomes bare whenever the out-ghost momentum vanishes. This has the important consequence that it cannot be singular for vanishing ghost momenta.

Furthermore assuming that the QCD Green's functions can be expanded in asymptotic series[1], $e.g.$,

$$G(p^2; \mu^2) = \sum_n d_n \left(\frac{p^2}{\mu^2} \right)^{\delta_n},$$
(12)

the integral in the ghost Dyson–Schwinger equation can be split up in three pieces. The infrared integral is complicated, and we have not treated it analytically yet (see, however, ref. [15]). The ultraviolet integral, on the other hand, does not contribute to the infrared behaviour. As a matter of fact, it is the resulting equation for the ghost wave function renormalization constant $\widetilde{Z}_3$ which allows one to extract definite information [16] without using any truncation or specific ansatz beyond the underlying assumption for the existence of asymptotic infrared series for QCD Green's functions.

The results are that the infrared behaviour of the gluon and ghost propagators are uniquely related: The gluon propagator is infrared suppressed as compared to the one for a free particle, the ghost propagator is infrared enhanced. This implies that the Kugo–Ojima confinement criterion is satisfied.

A Truncation Scheme for Gluon and Ghost Propagators

The known structures in the 3-point vertex functions, most importantly from their Slavnov-Taylor identities and exchange symmtries, have been employed to establish closed systems of non-linear integral equations that are complete on the level of the gluon, ghost and quark propagators in Landau gauge QCD. This is possible with systematically neglecting contributions from explicit 4-point vertices to the propagator Dyson–Schwinger Equations (DSEs) as well as non-trivial 4-point scattering kernels in the constructions of the 3-point vertices [10,5]. For the pure gauge theory this leads to the propagators DSEs diagrammatically represented in Fig. 1 with the 3-gluon and ghost-gluon vertices (the open circles) expressed in terms of the two functions Z and G. Employing a one-dimensional approximation one obtains the numerical solutions sketched in Fig. 2 [10,17].
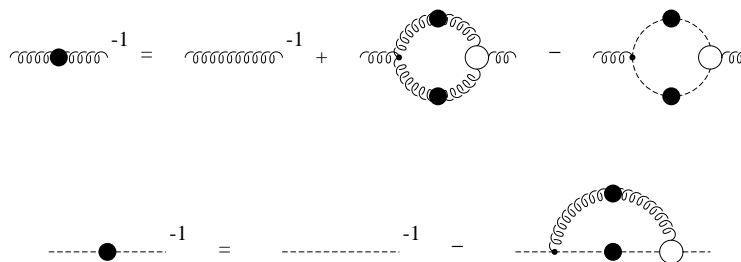

FIG. 1. Diagrammatic representation of the truncated system of gluon and ghost DSEs.

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[1]Note that this is not possible if the infrared slavery picture is correct. An infinite β-function for vanishing scales prohibits such an expansion.

Asymptotic expansions of the solutions in the infrared yield the leading infrared behaviour analytically. It is thereby uniquely determined by one exponent $\kappa = (61 - \sqrt{1897})/19 \approx 0.92$,

$$Z(k^2) \overset{k^2 \to 0}{\sim} \left(\frac{k^2}{\sigma^2}\right)^{2\kappa} \quad \text{and} \quad G(k^2) \overset{k^2 \to 0}{\sim} \left(\frac{\sigma^2}{k^2}\right)^{\kappa} , \tag{13}$$

for which the bounds $0 < \kappa < 1$ can be established requiring consistency with Slavnov–Taylor identities [10]. The renormalization group invariant momentum scale σ represents a free parameter at this point which is later on fixed by choosing a definite value for the strong coupling constant at some scale. The qualitative infrared behavior in eqs. (13) has been also found by studies of further truncated DSEs [18]. Neither does it thus seem to depend on the particular 3-point vertices nor on employed approximations for angular integrals. All these solutions agree qualitatively and confirm the Kugo–Ojima confinement criterion.

There are also recent lattice simulations which test this criterion directly [19]. Instead of $u^{ab} = -\delta^{ab}$ they obtain numerical values of around $u = -0.7$ for the unrenormalized diagonal parts and zero (within statistical errors) for the off-diagonal parts. Taking into account the finite size effects on the lattices employed in the simulations, these preliminary results might still comply with the Kugo-Ojima confinement criterion.
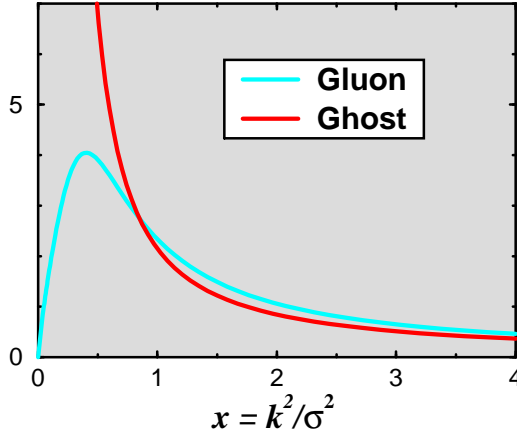

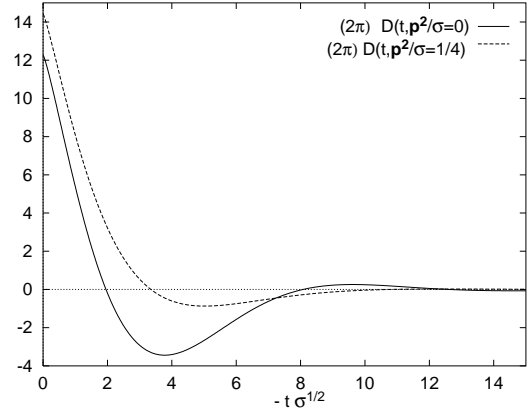
Fig. 2: DSE solutions for $Z(x)$ and $G(x)$ [10].

Fig. 3: $D(t, \mathbf{p}^2)$ from DSEs for the gluon renormalization function Z in Fig. 2.

Positivity violations of transverse gluon states are manifest in the spectral representation of the gluon propagator,

$$D(p^2) := \frac{Z(p^2)}{p^2} = \int_0^\infty dm^2 \frac{\rho(m^2)}{p^2 + m^2} . \tag{14}$$

¿From color antiscreening and unbroken global gauge symmetry in QCD it follows that the spectral density asymptotically is negative and *superconvergent* [3–5]

$$\rho(k^2) \overset{k^2 \to \infty}{\sim} -\frac{\gamma g^2}{k^2}\left(g^2 \ln \frac{k^2}{\Lambda^2}\right)^{-\gamma-1}, \quad \text{and} \quad \int_0^\infty dm^2 \rho(m^2) = \left(\frac{g_0^2}{g^2}\right)^\gamma \to 0 , \tag{15}$$

since $\gamma > 0$ for $N_f \leq 9$ in Landau gauge. This implies that it contains contributions from quartet states (and does therefore not need to be gauge invariant unlike in QED). Here, we consider the one-dimensional Fourier transform

$$D(t, \mathbf{p}^2) = \int \frac{dp_0}{2\pi} \frac{Z(p_0^2 + \mathbf{p}^2)}{p_0^2 + \mathbf{p}^2} e^{ip_0 t} = \int_{\sqrt{\mathbf{p}^2}}^\infty d\omega \, \rho(\omega^2 - \mathbf{p}^2) e^{-\omega t} , \tag{16}$$

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which for $\rho \geq 0$ had to be positive definite (and one had $\frac{d^2}{dt^2} \ln D(t, \mathbf{p}) \geq 0$). This is clearly not the case for the DSE solution shown in Fig. 3 which violates reflection positivity [5,10]. Even though no negative $D(t, \mathbf{p}^2)$ have been reported in lattice calculations yet, the available results [20] agree in indicating that $\ln D(t, \mathbf{p}^2)$ is not the convex function of the Euclidean time it should be for positive ρ [21,22]. These are non-perturbative verifications of the positivity violation for transverse gluon states which already occur in perturbation theory. More significant for confinement is the fact that no massless single transverse gluon contribution to ρ exists for $Z(0) = 0$.

Confirmation of the important result that the gluon renormalization function vanishes in the infrared and no massless asymptotic transverse gluon states occur, *i.e.*, $Z(0) = 0$, is seen in Fig. 4, where the DSE solution of Fig. 2 is compared to lattice data [23] and it was further verified recently with improved lattice actions for large volumes [13]. This infrared suppression as seen in lattice calculations thereby seems to be weaker than the DSE result, apparently giving rise to an infrared finite gluon propagator $D(k) \sim Z(k^2)/k^2$ (corresponding to an exponent $\kappa = 1/2$ in (13)), but a vanishing one does not seem to be ruled out for the infinite volume limit [24]. Similar results with finite $D(0)$ are also reported from the Laplacian gauge which practically avoids Gribov copies [25].
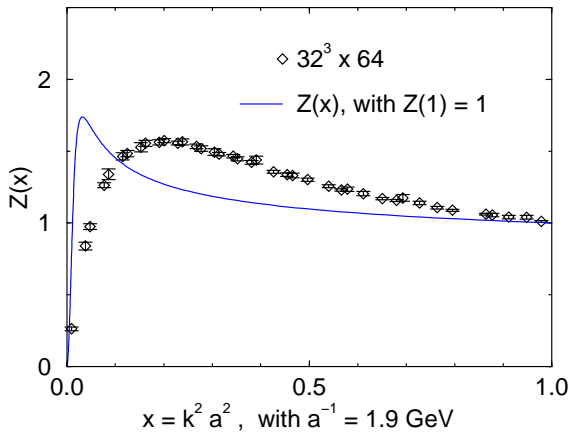

Fig. 4: The gluon renormalization function from the DSE solutions of Ref. [10] (solid line) and from the lattice data of Ref. [23].
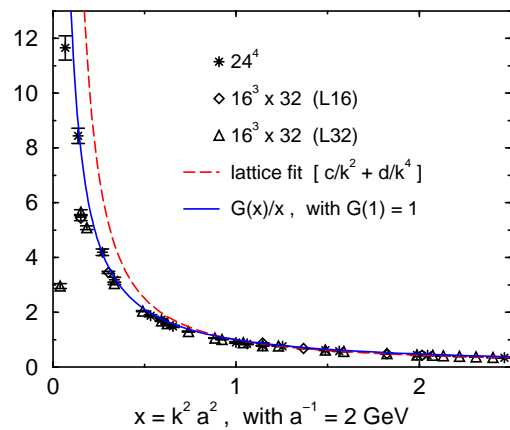
Fig. 5: The ghost propagator from DSEs in Ref. [10] (solid line) compared to data and fit (dashed with $ca^2 = 0.744$, $da^4 = 0.256$ for $x \geq 2$) from Ref. [26].

The infrared enhanced DSE solution for ghost propagator is compared to lattice data in Fig. 5. One observes quite compelling agreement, the numerical DSE solution fits the lattice data at low momenta ($x \leq 1$) significantly better than the fit to an infrared singular form with integer exponents, $D_G(k^2) = c/k^2 + d/k^4$. Though low momenta ($x < 2$) were excluded in this fit, the authors concluded that no reasonable fit of such a form was otherwise possible [26]. Therefore, apart from the question about a possible maximum at the very lowest momenta, the lattice calculation seems to confirm the infrared enhanced ghost propagator with a non-integer exponent $0 < \kappa < 1$. The same qualitative conclusion has in fact been obtained in a more recent lattice calculation of the ghost propagator in $SU(2)$ [24], where its infrared dominant part was fitted best by $D_G \sim 1/(k^2)^{1+\kappa}$ for an exponent κ of roughly 0.3 (for $\beta = 2.7$).

To summarize, the qualitative infrared behavior in eqs. (13), an infrared suppression of the gluon propagator together with an infrared enhanced ghost propagator as predicted by the Kugo-Ojima criterion for the Landau gauge, is confirmed by the presently availabe lattice calculations. The exponents obtained therein ($0.3 < \kappa \leq 0.5$) both seem to be consistently smaller than the one obtained in solving their DSEs. Whether also the lattice data is thereby determined by one unique exponent $0 < \kappa < 1$ for the infrared behavior of both propagators,

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has not yet been investigated to our knowledge. An independent confirmation of this combined infrared behavior which is indicative of an infrared fixed point would support the existence of the unphysical massless states that are necessary to circumvent the decomposition property for colored clusters.

Acknowledgements

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