Bidding Behavior in Asymmetric Auctions: An Experimental Study*

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Abstract

We review an asymmetric auction experiment. Based on Plum (1992) private valuations of the two bidders are independently drawn from distinct but commonly known distributions, one of which stochastically dominating the other. We test the qualitative properties of that model of asymmetric auctions, in particular whether the weak bidder behaves more aggressively than the strong and then test bidders' preference for first- vs. second–price auctions.

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1. Introduction

Like most of the theoretical literature, auction experiments typically assume that bidders' valuations or signals are drawn from the same probability distribution (see the survey by Kagel, 1995). However, this symmetry assumption is violated in many real-life auction environments, because bidders often know that and how they differ, for example, in the light of earlier experiences or due to collusion between subsets of otherwise symmetric bidders. The limited relevance of the symmetric auction framework is aggravated by the fact that many of the celebrated results of symmetric auctions, such as the revenue equivalence of a larger class of auction games, do not extend to asymmetric auctions.

There is a small theoretical literature on asymmetric auctions, which deals with various kinds of asymmetries: asymmetries between commonly known distribution functions from which valuations or signals are independently drawn, asymmetries induced by a known ranking of valuations (which involves a particular stochastic dependency), and asymmetries between valuation functions while maintaining the symmetry of the distribution from which private signals are drawn.

Several authors have followed the first approach to model bidder asymmetry, and assumed that bidders' valuations are independently drawn from different probability distributions, which are common knowledge among them. In this spirit, Vickrey (1961) already considers an auction with two bidders in which the one bidder's valuation is known to the other bidder with certainty, and Griesmer et al.(1967) analyze first-price auctions with two bidders whose valuations are uniformly distributed over different supports.

More recently, Plum (1992) analyzes the two bidder case for arbitrary continuous distributions, and proves that the first-price auction has a unique pure strategy equilibrium with strictly monotone increasing bid functions. In addition, he explicitly solves the first-price auction game for a parametric class of distribution functions.

In a similar framework, Maskin and Riley (2000a) explain the properties of several asymmetric two bidder examples, where a stochastic order stronger than first-order stochastic dominance is assumed, Maskin and Riley (2000b), Reny (1999), and Simon and Zame (2000) analyze existence of pure strategy equilibria in first-price auctions for the general n bidder case, and Lebrun (1999) proves uniqueness of pure strategy equilibria in the general n bidder case.[1]

Asymmetries induced by a known ranking of valuations are analyzed by Landsberger et al.(2000). This asymmetry cannot be subsumed under the approach that assumes that valuations are independently drawn from commonly known distribution functions. And indeed, it generates distinct results.

Another branch of the literature has also begun to analyze asymmetries between valuations functions in the affiliated and (almost) common-value framework, while maintaining the symmetry of the distribution from which private signals are drawn (see the example by Bikchandani (1988), and the associated experiment by Avery and Kagel (1998); see also the example by Bulow, Huang, and Klemperer (1999)).

In the literature there are only a few asymmetric auction experiments. Most of them concern the common-value case (see the survey by Kagel, 1995). Employing the private-value framework Pezanis-Christou (1999) tests one variant of the Maskin and Riley model (2000a) and rejects the predicted revenue ranking.

The present paper reports on a laboratory experiment of bidding behavior in asymmetric auctions, which assume that valuations are private information and are independently drawn from distinct but commonly known distribution functions. The experiment employs the functional specification used by Plum (1992) and Kalkofen and Plum (1996) for which explicit equilibrium solutions of bidding strategies are available. We explore whether actual bidding exhibits similar qualitative properties as the game-theoretic solution. In particular, we ask:

[1] Incidentally, these proofs of existence employ very different methods. Maskin and Riley (2000b) use topological methods developed by Dasgupta and Maskin (1986). Plum (1992) and Lebrun (1999) establish directly that a solution to a suitable set of differential equations exists. And Reny (1999) employs his concept of "payoff secure" games. Finally, Simon and Zame (2000) view the tie-breaking rule as part of the solution of the game, and show that there is always some tie–breaking rule for which an equilibrium exists.

- Does the weak bidder bid more aggressively than the strong bidder?
- Does the first–price auction raise more expected revenue for the seller?
- Does the second–price auction generate higher payoffs to the strong bidder and the first-price auction to the weak bidder?
- Do bidders rank the two auctions accordingly?
- Can deviations from equilibrium strategies be explained by caution or risk aversion?

In the experiment each subject played either the role of the weak or the strong bidder, and this for 6 rounds of bidding in a first–price auction and another 6 rounds in a second–price auction. The 1st phase consisted of 4 cycles of such 12 bidding rounds. Subjects confronted randomly selected partners in matching groups of 14 participants, namely 7 bidders 1 and 7 bidders 2. Thus each participant confronted altogether 7 co-bidders in an irregular fashion.

Do both bidders understand which auction yields higher payoffs, as unambiguously predicted by theory? To explore this question, the second phase of an experimental session introduces an additional stage. Before learning their private values participants must determine a price limit for dictating the pricing rule. The second phase consists of 16 successive such auction games. Due to incentive compatibility of random pricing (see Becker, De Groot, and Marschak, 1963) and of random dictatorship our mechanism should induce participants to reveal their true willingness to pay for choosing the pricing rule.

At the end of the experiment participants play another 16 rounds (third phase) where they choose their price limits after learning their private values.

The remainder of the paper is organized as follows. The theoretical background and the experimental design are explained in Sections 2 and 3. In Section 4 we present the main experimental results. We close with the summary of the results in Section 5.

2. Theoretical background

We consider a slightly simplified version of Plum (1992). Specifically, two risk-neutral bidders ($i = 1, 2$) compete for the purchase of a single item in either a first– or a second–price sealed–bid auction. Bidders' valuations are private information and independently drawn from uniform distributions with supports $[\alpha, \beta_1]$ and $[\alpha, \beta_2]$, with

$$\beta_2 = 200 > \beta_1 = 150 > \alpha = 50, \tag{2.1}$$

as illustrated in Figure 1. Therefore, the random valuation V_2 is obtained by "stretching" the valuation V_1. Obviously, V_2 is more favorable than V_1, in the strong sense of first–order stochastic dominance.

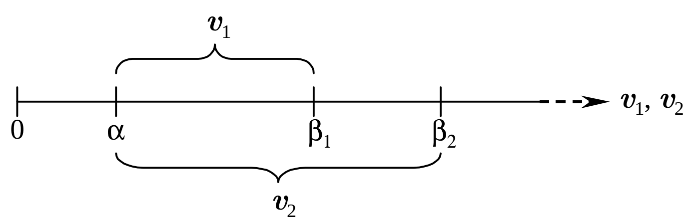

Figure 1: Support of random valuations

Equilibrium bid functions As shown by Plum (1992, pp. 401–403), the first–price sealed–bid auction has the following equilibrium bid functions:

$$b_i^f(v_i) = \alpha + \frac{v_i - \alpha}{1 + \sqrt{1 + \gamma_i c (v_i - \alpha)^2}} \quad \text{for } i = 1, 2 \tag{2.2}$$

$$\text{with} \quad c := \frac{1}{(\beta_1 - \alpha)^2} - \frac{1}{(\beta_2 - \alpha)^2} \tag{2.3}$$

$$\gamma_1 := -1, \quad \gamma_2 := 1. \tag{2.4}$$

These are plotted in Figure 2, which shows that the bidder with the more favorable valuation (bidder 2) bids pointwise less than bidder 1. An immediate implication is that the first-price auction gives rise to inefficiency, because the bidder with the lower valuation wins the auction with positive probability. (For example, if $v_2 = 150$, the bidder with the lower valuation wins the auction for all V_1 in (130.18,150), as illustrated in Figure 2).

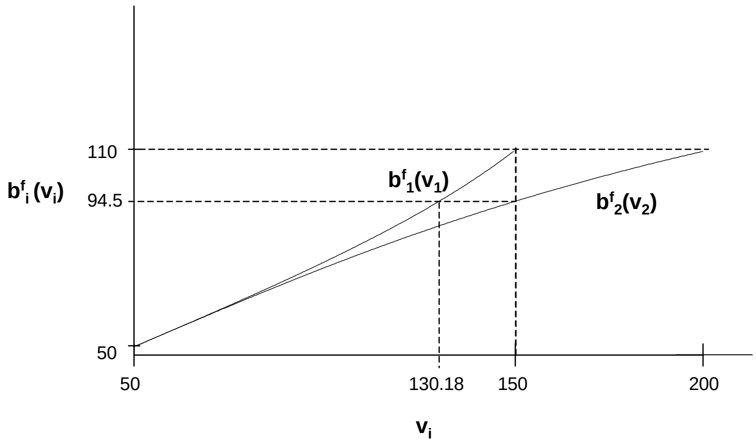

Figure 2: Equilibrium bid functions in first-price sealed-bid auction

Of course, truthful bidding is an equilibrium in dominant strategies in the second–price sealed–bid auction, and so is the following plausible modification of truthful bidding which assumes that bidder 2 will never bid more than his rival's maximum valuation β_1:

$$b_i^s(v_i) = \begin{cases} v_i & \text{if } v_i \le \beta_1 \\ \beta_1 & \text{if } v_i \in [\beta_1, \beta_2] \end{cases} \tag{2.5}$$

Bidders' equilibrium payoffs Using Plum's solution of the two auction games, one can compute bidders' equilibrium payoffs, denoted by $u_i(v_i)$. These are needed as a benchmark in our experiment.

If the auction is second–price sealed–bid, the computation of $u_i^s(v_i)$ is straightforward. By a well–known result $u_i^{s\prime}(v_i) = F_j(v_i)$, therefore:

$$u_1^s(v_1) = \int_\alpha^{v_1} F_2(x)dx \quad \text{and} \tag{2.6}$$

$$u_2^s(v_2) = \begin{cases} \int_\alpha^{v_2} F_1(x)dx & \text{if } v_2 \le \beta_1 \\ \int_\alpha^{\beta_1} F_1(x)dx + v_2 - \beta_1 & \text{if } v_2 \in [\beta_1, \beta_2] \end{cases} \tag{2.7}$$

If the auction is a first–price sealed–bid auction, bidders i's equilibrium probability of winning the auction is

$$\Pr\left(b_i^f(v_i) > b_j^f(V_j)\right) = \Pr\left(V_j < b_j^{f^{-1}}(b_i^f(v_i))\right) \tag{2.8}$$
$$= F_j\left(b_j^{f^{-1}}(b_i^f(v_i))\right) \tag{2.9}$$

Therefore, in order to compute $u_i^f(v_i)$, one needs to find the inverse $\phi_i(x)$ of the equilibrium bid function which is defined on bids x, as follows:[2]

$$\phi_i(x) := b_i^{f^{-1}}(x) = \alpha + \frac{2(x-\alpha)}{1-\gamma_i(x-\alpha)^2 c}. \tag{2.10}$$

Therefore, one obtains:

$$u_i^f(v_i) := F_j(\phi_j(b_i^f(v_i)))(v_i - b_i^f(v_i)) \tag{2.11}$$
$$= \frac{v_i - b_i^f(v_i)}{\beta_j - \alpha}\left(\frac{2(b_i^f(v_i)-\alpha)}{1-\gamma_j(b_i^f(v_i)-\alpha)^2 c}\right). \tag{2.12}$$

Bidders' equilibrium payoffs are plotted for both auction rules in Figure 3 which illustrates that the strong bidder 2 strictly prefers the second-price auction, whereas the weak bidder 1 strictly prefers the first-price auction:

$$u_1^f(v_1) > u_1^s(v_1) \qquad \text{for all } v_1 \in (\alpha, \beta_1] \tag{2.13}$$
$$u_2^s(v_2) > u_2^f(v_2) \qquad \text{for all } v_2 \in (\alpha, \beta_2]. \tag{2.14}$$

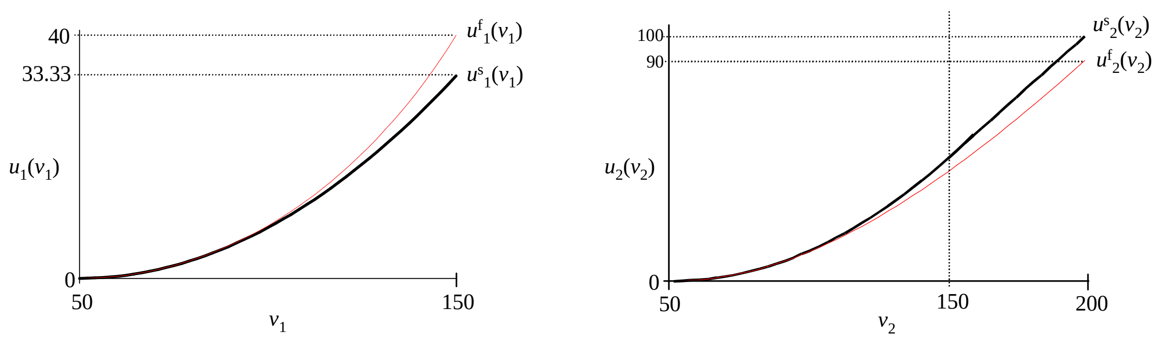

Figure 3: Bidders' equilibrium payoffs

[2] In order to find the inverse of equilibrium bid functions, employ the following transformation: $B_i := b_i - \alpha$, $x_i := v_i - \alpha$, $d_i := \sqrt{1+\gamma_i c x_i^2}$, and one obtains, after a few manipulations, $x_i = 2B_i/(1-\gamma_i B_i^2 c)$.

These completely opposite rankings are intuitively appealing, and they are consistent with a key result in Maskin and Riley (2000a).

Bidders' equilibrium payoffs behind the veil of ignorance From the $u_i(v_i)$'s one can compute bidders' equilibrium payoffs behind the veil of ignorance, i.e., the expected payoffs determined before valuations are drawn:

$$U_i^s := E\left[u_i^s(V_i)\right] = \frac{1}{\beta_i - \alpha}\int_{\alpha}^{\beta_i} u_i^s(x)dx \tag{2.15}$$

$$U_i^f := E\left[u_i^f(V_i)\right] = \frac{1}{\beta_i - \alpha}\int_{\alpha}^{\beta_i} u_i^f(x)dx. \tag{2.16}$$

For the assumed values of the parameters α, β_i this implies:

$$U_1^s = 11.111 < U_1^f = 12.295 \tag{2.17}$$

$$U_2^s = 36.111 > U_2^f = 32.474. \tag{2.18}$$

Of course, this ranking is implied by (2.13)-(2.14).

Bidders' value of dictatorship behind the veil of ignorance In the experiment (2nd phase) we give bidders the chance to select the auction rule, before they know their valuation. Specifically, we let them bid for the right to choose the favored auction against a random price generator (Becker, De Groot, Marschak, 1963) If the bid is at or above that random price, the bidder must pay the random price and dictatorially chooses his favored auction rule, otherwise the auctioneer selects the auction rule by the flip of a fair coin.

Obviously, truthful bidding is the optimal strategy and the value of the right to choose the auction, which we call value of dictatorship, L, is defined as follows:

$$L_i = \max\left\{U_i^s, U_i^f\right\} - \frac{1}{2}(U_i^s + U_i^f) \tag{2.19}$$

$$L_1 = 0.592 < 1.8185 = L_2. \tag{2.20}$$

Of course, if the bidder gains „dictatorship", he chooses the auction that gives the higher U_i. As one can see from (2.17)-(2.18), the weak bidder 1 prefers the first-price auction whereas the strong bidder 2 favors the second–price auction. In view of $L_2 > L_1$ the same random price generator applied to bidder 1 and 2 will less often induce dictatorial rule choices in case of bidder 1.

Value of dictatorship if bidders know their valuation In the experiment (3rd phase) we also give bidders the chance to select the auction rule after learning their valuation, i.e., after the veil of ignorance has been removed, again using the random price mechanism. In the spirit of L_i one may define the value of dictatorship as follows:

$$l_i(v_i) = \max\left\{u_i^s(v_i), u_i^f(v_i)\right\} - \frac{1}{2}(u_i^s(v_i) + u_i^f(v_i)). \tag{2.21}$$

However, this ignores some subtle updating and strategic signalling issues. For example, in the event when the strong bidder 2 fails to dictate the preferred second-price format, player 1 should update his probability assessment of v_2, which in turn affects the equilibrium strategies of the subsequent first-price auction game and hence the equilibrium payoff $u_i^f(v_i)$ in (2.21). Therefore, (2.21) can only serve as a rough benchmark, if at all.

3. Experimental design

In the experiment we neither excluded over- nor underbidding by letting v_1 vary from 50 to 150 and v_2 from 50 to 200 whereas bids $b_i(v_i)$ could vary from 0 to 250. The random price p of the 2nd (3rd) phase was chosen from the range $0 \leq p \leq 30$ since the value of the right to dictate the pricing rule should always be non-negative. Of course, neither v_i nor $b_i(v_i)$ can vary continuously in a computerized setup. Actually both, v_i and b_i had to be integers. The two private values v_1 and v_2 were independently drawn form a uniform distribution with support $\{50, 51, ..., 150\}$, respectively $\{50, 51, ..., 200\}$..

A session was subdivided into three phases:

The 1st Phase consisted of 4 cycles of 12 bidding rounds, first 6 rounds with the first-price rule, then 6 rounds of the second-price rule. In each round first the private values v_1 and v_2 were randomly selected. Then the two bidders determined their bid $b_1(v_1)$, respectively $b_2(v_2)$.

The 2nd Phase consisted of 16 bidding rounds with endogenous price rule. First both, bidder $i = 1$ and bidder $i = 2$, had to state their price limit l_i for being able to dictate the pricing rule (F or S). It was then determined by an unbiased chance move whether 1 or 2 became the potential dictator. If this was bidder i, he could dictate the pricing rule F or S whenever the randomly chosen price d of dictatorship with $0 \leq d \leq 30$ satisfied $d \leq l_i$. After choosing F or S the auction round was played as in the 1st phase.

The 3rd Phase differed from the 2nd phase only by the timing of events. Here first the private values v_1 and v_2 were randomly selected, i.e., all decisions (l_i, F or S, b_i) were made knowing one's private value v_i.

The bidders were informed whether or not they have bought, about the price to be paid by the buyer, their private (reselling) value, their own bid, the bid of the other bidder, their own payoff in the current round, their total profit up to this time and their average profit in each auction type. At the end of each auction in phase 2, respectively 3, the dictator also learned about his costs for choosing the pricing rule.

Participants were invited to register for the experiment (mainly by distributing leaflets in undergraduate courses of the economics faculty of Humboldt University in Berlin). After entering the computer laboratory participants were seated at visually separated terminals where they could read the instructions (see Appendix B). They could privately ask for clarifications but not for advice. We have performed 8 sessions, 7 with 14 participants and 1 with 12 participants each (due to a technical problem only the results of the 1st phase could be saved for one of the 8 sessions). All sessions lasted about two hours.

4. Results

In the following, we use the theoretical results of Section 2.2 as a benchmark to assess the actual bidding behavior in phases 1 and 2. The analysis of phase 3 is primarily of an exploratory nature since the associated theoretical benchmark is debatable.

4.1. Efficiency, prices and bidders' profits

Despite the assumed asymmetry, the second-price auction is efficient, since truthful bidding is an equilibrium in dominant strategies. However, the first-price auction give rise to inefficiency, because the bidder with the lower valuation wins with positive probability (see Figure 2). In the experiment we measure efficiency (I) as follows:

$$I = \frac{v_{\text{buyer}}}{\max\{v_1, v_2\}}$$

Although there is a slight inefficiency of second-price auctions, it is, however, lower than the one of the first-price auction (see Figure A.1 and Table "Summary Statistics" in Appendix A).

price rule		first-price	second-price
phase	1	.97 (84%)	.98 (88%)
	2	.97 (85%)	.99 (94%)
	3	.98 (87%)	.99 (94%)

Table 1: Average degree I of efficiency

(in brackets: Percentage of Pareto-efficient allocations)

Learning to induce efficient allocations is more pronounced in second-price auction than in first-price auction when measured by phases (see Table 1). As shown in Figure 4 we observe on average a stable increase of efficiency in second-price auctions over time. The greater inefficiency of first-price auctions results from the need to coordinate how far one should underbid one's private value, i.e., how large the positive difference $v_i - b_i(v_i)$ should be.

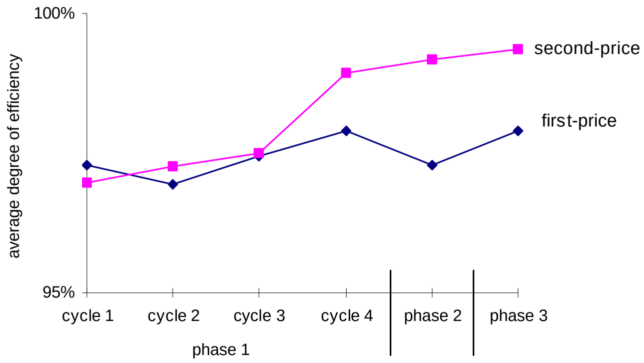

Figure 4: Average degree I of efficiency over time

Let us assume the perspective of the seller who is interested in a large expected revenue. Table 2 lists in the top row the expected equilibrium prices p^* (see Kalkofen and Plum, 1996) and in the lower rows the average prices of the first- and second-price auction, separately for phase 1, 2, and 3.[3] A Binomial test ($p = .004$, one-tailed, $N = 8$, phase 1)[4] for all three phases reveals that the seller's revenues from first-price auctions are higher than those from second-price auctions (see Table "Summary Statistics" in Appendix A). The cumulative distribution function for second-price auction is in all three phases above that one for first-price auction (see Figure A.1 in Appendix A). Note that according to the realized prices in phases 1 and 2 the incentive of the seller for the first-price rule is substantially larger than indicated by the solution prices for risk-neutral bidders, i.e., the theoretic solution underestimates the actual advantage of relying on first- rather than on second-price auctions.

price rule		first-price	second-price
normative		*90.46*	*88.89*
phase	1	103.87	87.06
	2	104.70	90.69
	3	98.74	89.97

Table 2: Expected/average prices

[3] Of course, these benchmark prices can only be used as a rough approximation in the analysis of phase 3.

[4] For phases 2 and 3: $p = .008$ ($N = 7$).

According to Table 3 the mean profits for both bidders are higher for the second-price auction than for the first-price auction. Whereas the payoffs in phases 1 and 2 of the second-price auction are close to their benchmarks in all sessions[5], in the first-price auction they are invariantly below their benchmarks (2.17)-(2.18). The second-price auction generates significantly higher payoffs to the bidders, both weak and strong, and in all three phases.[6] The possibility to endogenously select the auction type in the 2nd (3rd) phase leads to no essential earning effects.[7]

bidder		1		2	
price rule		first-price	second-price	first-price	second-price
phase	1	6.37	11.23	22.75	35.36
	2	6.65	11.12	21.68	34.42
	3	7.35	11.72	22.15	33.83

Table 3: Average bidders' profits

4.2. Bidding behavior

In Table 4 we compare the average level of under/over-bidding $[v_i - b_i(v_i)]/v_i$ for first- and second-price auction as well as for the different phases and the two bidders, for $v_i \in \{50, 51, ..., 150\}$.

price rule		first-price		second-price	
bidder		1	2	1	2
phase	1	.19	.22	-.03	-.02
	2	.19	.23	-.04	-.01
	3	.20	.24	-.05	-.01

Table 4: Average under/overbidding

[5] We have confirmed that the distributions of actually selected values v_1 and v_2 do not differ significantly from the a priori-distributions.

[6] One-tailed Binomial test; for phase 1: $p = .004$ ($N = 8$), phase 2: $p = .008$ ($N = 7$), phase 3: $p = .062$ ($N = 7$).

[7] In phase 2 and 3 this includes, of course, the costs resulting from the random price mechanism, i.e., the earnings from bidding are slightly higher when being able to dictate the auction type.

According to Table 4 under/overbidding ratios are rather stable (over phases). The average tendency in second-price auctions seems to be truthful bidding, i.e., $b_i(v_i) = v_i$ for $i = 1, 2$. Nearly half of all bids in all three phases are equal to subjects' reselling values (39% in phase 1, 47% in phase 2, and 48% in phase 3). Possible reasons for the revealed slight overbidding could be that:

- the dominant strategy is not transparent;
- spite on behalf of bidder 1 (who overbids more often and who might feel underprivileged);
- sealed-bid procedures mays slow down learning, since they frequently do not provide the feedback that tends to reduce overbidding.

As to be expected, first-price auctions rely on substantial underbidding (see Table "Summary Statistics" in Appendix A). The mean degree of underbidding in the range $50 \leq v_i \leq 150$ is significantly larger for bidder 2 than for bidder 1 ($p = .035$, one-tailed Binomial test, $N = 8$, phase 1).[8]

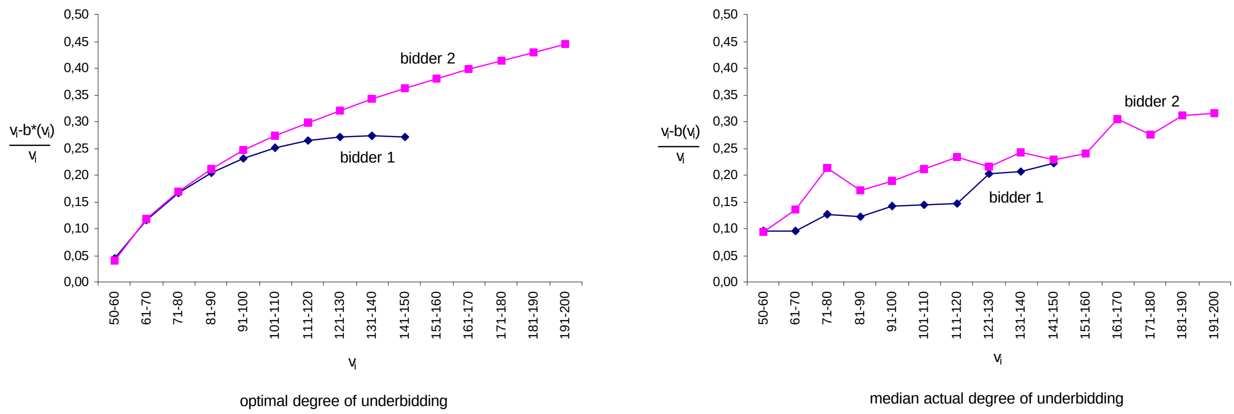

Figure 5: Degree of underbidding in first-price auction (phase 1)

For first-price auction it is interesting to analyze how the degree of underbidding depends on v_i. In Figure 5 the left diagram illustrates the optimal degree of

[8]For phases 2 and 3 in 5, respectively 6, out of 7 sessions we observe larger underbidding for bidder 2.

underbidding $[v_i - b_i^*(v_i)]/v_i$ for $i = 1, 2$ and the right diagram the median actual (underbidding) results for both bidders in phase 1.[9] The equilibrium as well as the observed bids are computed as the average bids for the respective v_i-brackets. Two qualitative aspects of the left diagram seem to hold also for the right one, namely the same underbidding degree by bidder 1 and 2 for $v_1 = v_2 = 50$ and a general tendency of bidder 1 to underbid less.

Let us now investigate the curvature of the observed first-price bid functions for both bidder types. To evaluate whether the bid functions were increasing and convex, concave or linear we estimated the following piecewise linear regression model:

$$b_i = \rho_i + \delta_i v_i^{low} + \gamma_i v_i^{high} \quad (i = 1, 2), \qquad \text{with} \tag{4.1}$$

$$v_1^{low} = \begin{cases} v_1 & \text{if } v_1 < 100 \\ 100 & \text{otherwise} \end{cases}, \qquad v_2^{low} = \begin{cases} v_2 & \text{if } v_2 < 125 \\ 125 & \text{otherwise} \end{cases}$$

$$v_1^{high} = \begin{cases} v_1 - 50 & \text{if } v_1 \geq 100 \\ 50 & \text{otherwise} \end{cases}, \qquad v_2^{high} = \begin{cases} v_2 - 75 & \text{if } v_2 \geq 125 \\ 50 & \text{otherwise} \end{cases}$$

Thus we allow for a kink at $v_1 = 100$, respectively $v_2 = 125$ (the mean values of v_i, $i = 1, 2$).[10] This can provide piecewise-linear approximations of convex or concave bid functions. Model (4.1) was estimated for each session and each phase separately, giving us a total of 22 estimated bid functions per bidder's type. The fit of these piecewise-linear approximations is reasonable: in 75% (33 out of 44 observed bid functions) the estimated individual bid function explained 70% or more of the variance ($R^2 \geq 0.7$).

In all cases the bid functions were strictly increasing, i.e., $\delta_i > 0$ and $\gamma_i > 0$ $(i = 1, 2)$. To determine the curvature of the bid functions we computed the

[9] The corresponding mean degrees of underbidding are similar but, due to outliers, slightly less regular.

[10] A slightly simpler formulation of a piecewise–linear regression model may be: $b_i = \sigma_i + \lambda_i v_i + \mu_i v_i^{high}$ with all variables as defined above. However, this model is equivalent to model 1, and the latter is better suited to compare δ_i and γ_i below. For instance, if a bid function is perfectly linear, this results in $\delta_i = \gamma_i$.

difference between the two slope coefficients $\Delta_i \equiv \gamma_i - \delta_i$, $i = 1, 2$. We consider a bid function as linear if $|\Delta_i| \leq \pm 0.01$.[11] If $\Delta_i < -0.01$ ($\Delta_i > +0.01$), the respective bid function is classified as concave (convex). Due to these criteria in all three phases there are no linear bid functions (see Table 5). For both bidders most of them are concave (except for bidder 1 in phase 3).

curvature		concave		linear		convex	
bidder		1	2	1	2	1	2
phase	1	75%	100%	0%	0%	25%	0%
	2	57%	71%	0%	0%	43%	23%
	3	43%	86%	0%	0%	57%	14%

Table 5: Curvature of the estimated first-price bid functions

We also analyze for first-price auctions the relative bid deviations from the risk neutral equilibrium (RNE) bidding strategy $b_i^*(v_i)$, namely $[b_i(v_i) - b_i^*(v_i)]/v_i$ for $i = 1, 2$ and both phases, 1 and 2. There are no significant differences for both bidders: For bidder 1 65% of all bids in phase 1 (66% in phase 2) are above the benchmark, for bidder 2 the share is 67% (respectively 67%). A Binomial test rejects the null hypothesis of no deviations in favor of the weak overbidding prediction, i.e., there is a positive relative deviation from RNE ($p = .035$, one-tailed, $N = 8$, phase 1).[12]

Most bids below the benchmark result from low reselling values, i.e., if $v_i < 100$ (see Figures A.2 and A.3 in Appendix A).[13] The share of negative deviations in phase 1 decreases nearly by half when increasing private values. A Spearman correlation analysis reveals no clear-cut correlation between the absolute relative deviations from RNE and the rounds.

[11] Admittedly, the chosen criterion for linearity is ad hoc. It could have been set less (or more) restrictively. Even if the criterion for linearity is set less restrictively, e.g. $|\Delta_i| \leq \pm 0.02$, i.e. the prediction area has been doubled compared to the criterion we report here, only one bid function for bidder 2 in the 2nd phase would be considered as linear.

[12] For phase 2 in 6 out of 7 sessions the (weak) overbidding prediction is valid.

[13] The coexistence of negative (mainly for low v_i-values) and positive (mainly for high v_i-values) deviations from risk-neutral bidding indicates that such deviations are not only due to risk aversion.

4.3. Price limits

Table 6 gives the average price limits l_1 and l_2 for phases 2 and 3 first for all participants, then separately for those who, as actual dictator, have chosen the first- (game F) or the second-price rule (game S). The number in brackets are the number of observations on which these averages rely. The equilibrium price limits of risk-neutral bidders behind the veil of ignorance, i.e., before bidders know their private valuations (see the results in Section 2.2) are

$$L_1 = 0.592 \quad \text{and} \quad L_2 = 1.8185$$

In phase 3 where bidders 1 and 2 know their private values v_1 and v_2, respectively, the actual and observed price limits should, of course, depend on these values.

	2. phase		3. phase	
	l_1	l_2	l_1	l_2
all	4.92 (768)	7.55 (768)	4.10 (768)	7.88 (768)
first-price (F)	15.53 (15)	11.67 (15)	10.93 (14)	7.82 (11)
second-price (S)	9.00 (60)	15.87 (89)	13.49 (43)	19.64 (101)

Table 6: Average price limits separately for phase 2/3 and for F/S-dictators[14]

According to Table 6 in the second phase bidders 2 on average choose higher price limits (l_2) than bidders 1 (l_1), as suggested by L_1 and L_2. We have checked for phase 2 that bidders 1 and 2 had nearly equal chances to become the random dictator. Therefore, the observed higher frequency with which bidder 2 dictates the auction mechanism (104/75) can be attributed to the fact that the higher average price limit of bidder 2 translates into a higher chance to win against the random price generator.

[14]For phase 3 the price limits are (simply) the averages of all price limits $l_i(v_i)$ for all 16 rounds and randomly chosen v_i-levels.

Both bidders reveal considerably higher willingness to pay for dictating the price rule than predicted by theory (see also Table "Summary Statistics" in Appendix A). Remarkable is the fact that for the weak bidders in phase 2 the average price limit of F-voters exceeds the one of S-voters. Although this is consistent with the theoretical result that the weak bidder should prefer the first-price auction, it is, however, puzzling that the average price limits are so much higher than the theoretical predictions.

Also both bidders have predominantly chosen game S which may be due to the fact that it makes bidding easier. Besides, the second-price auction yielded higher average profits than the first-price auction for 75% of the weak bidders and 88% of the strong bidders in the first phase of the experiment.

bidder	1		2	
value v_i	$v_1 \leq 100$	$v_1 > 100$	$v_2 \leq 125$	$v_2 > 125$
first-price (F)	50%	50%	45%	55%
second-price (S)	19%	81%	18%	82%

Table 7: Frequency of F/S, chosen in phase 3, depending on the mean value of v_i

By Table 7 we can investigate in more detail how choosing the pricing rule during the 3rd phase depends on whether v_i is above its mean value 100, respectively 125, or not. The large majority of S-voters have private values in the upper v_i-range, while for F-voters the two v_i-ranges have (nearly) equal frequencies.

Whereas during phase 2 both price limits, l_1 and l_2, on average decrease over time (Figure A.4 in Appendix A), this effect is much weaker for phase 3 (see Figure A.5 in Appendix A where, for the sake of comparability, we only look at price limits $l_i(v_i)$ for v_i in the 30 width-interval around its mean). This smaller decline in phase 3 seems to be mainly due to lower starting values l_1 and l_2. At the beginning of phase 2 participants were apparently too enthusiastic about the possibility of dictating the pricing rule.

For phase 3 we have split up the price limits l_1 and l_2 into its two, respectively three 50 width-intervals $[50, 100]$, $(100, 150]$, $(150, 200]$. The data reveal that larger values v_i trigger on average larger price limits l_i (see Figures A.6 and A.7 in Appendix A). As shown by Figures A.8 and A.9 (see Appendix A) for the range $[50, 100]$, respectively $(100, 150]$ the price limits of bidders 1 and 2 with nearly equal values are very close. Actually comparing the l_1- and l_2-distribution for the two intervals reveals no significant difference.

5. Concluding remarks

Although auctions are a familiar topic in experimental economics, most experiments deal with the case of symmetry. Our experiment studies auctions where private valuations are independently drawn from distinct but commonly known distributions, one of which first-order stochastically dominates the other. Our results (phase 1 and 2) support the theoretical prediction that a seller should be better off choosing the first-price than the second-price auction as well as other qualitative properties of the game-theoretic solution. More specifically we observe:

- prices are significantly higher in a first-price auction
- in first-price auctions the strong bidders bid far less than weak bidders
- in first-price auctions, bid functions are nonlinear
- in second-price auctions, bids are close to bidders' valuations
- strong bidders set, on average, higher price limits
- the weak bidders who opt for the first-price auction set, on average, higher price limits than those who opt for the second-price format.

These results confirm that the predictive power of the asymmetric auction model is reasonably good, despite the complexity of the bidding problem. Given the failure of revenue equivalence in experimental studies of the symmetric independent private values model, the world of asymmetric auctions seems to be an interesting and promising area for further research.

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Appendix A:

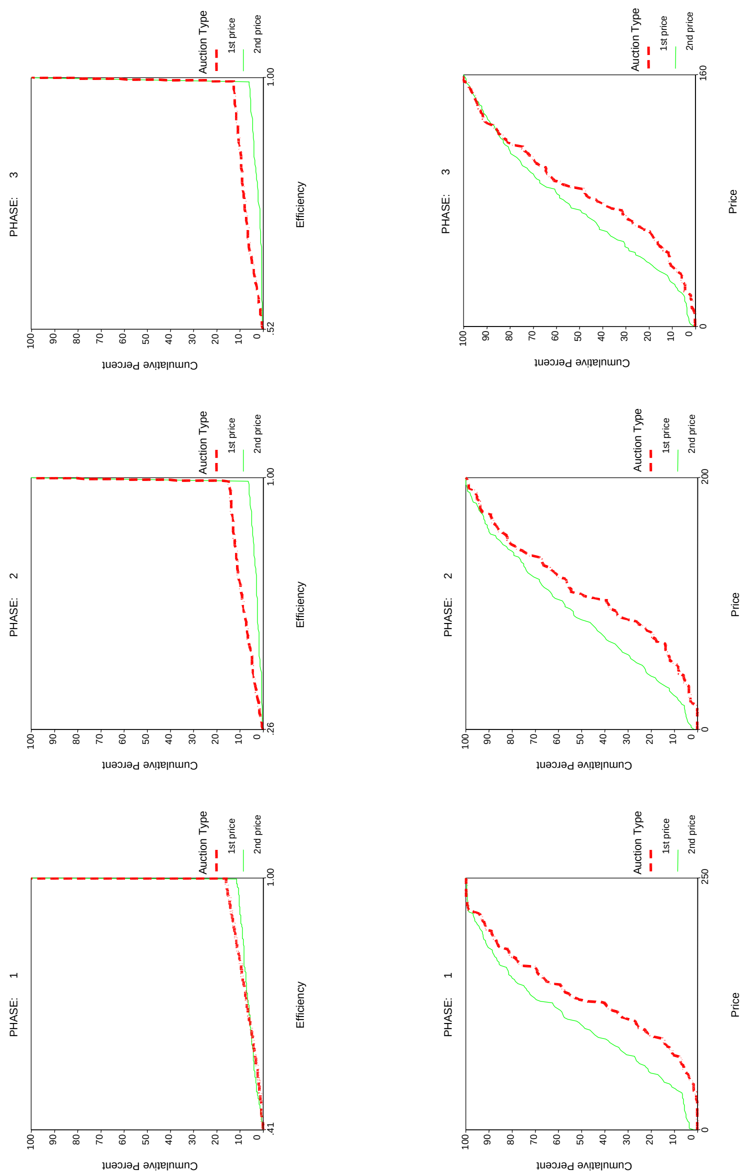

Figure A.1: Cumulative distributions of the observed efficiency and prices (for both auction types)

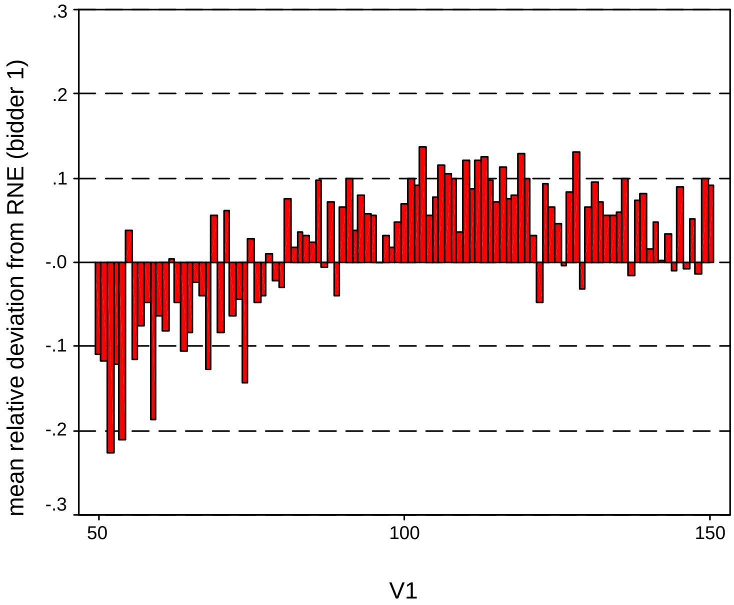

Figure A.2: Relative bid deviation from RNE
(bidder 1, first-price auction, phase 1)

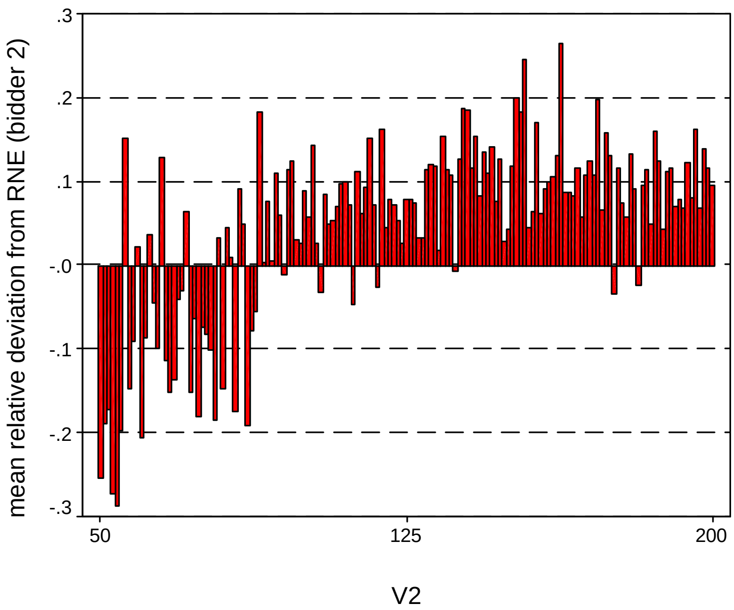

Figure A.3: Relative bid deviation from RNE

(bidder 2, first-price auction, phase 1)

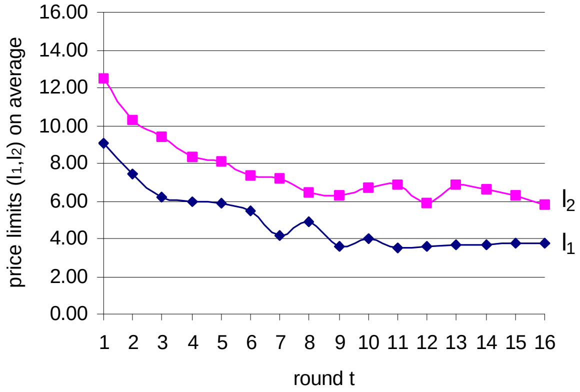

Figure A.4: Time paths of price limits (l_1, l_2) in phase 2

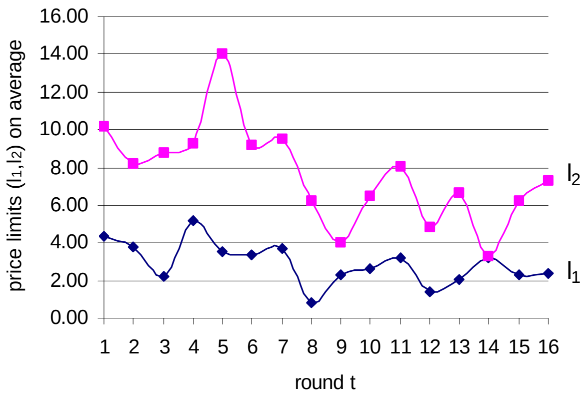

Figure A.5: Time paths of price limits (l_1, l_2) in phase 3
(price limits $l_i(v_i)$ only for v_i in 30 width-interval around its mean)

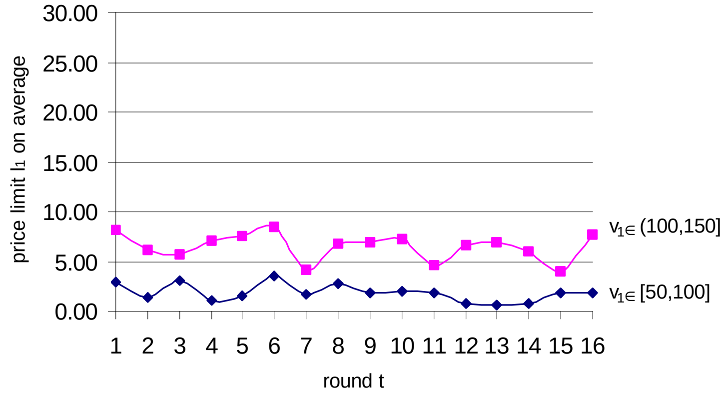

Figure A.6: Time paths of price limit l_1 for $v_1 \in [50, 150]$ in phase 3

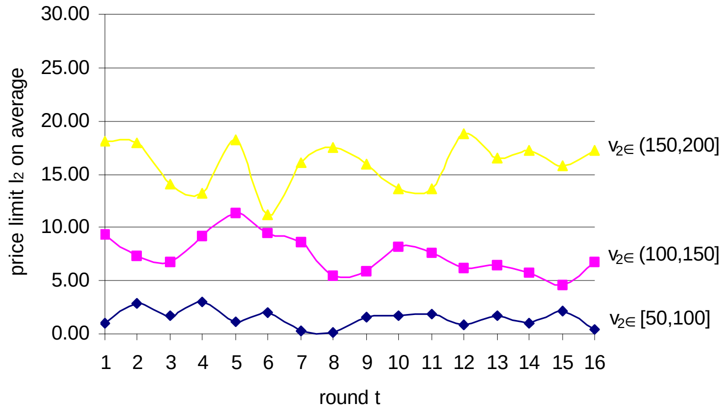

Figure A.7: Time paths of price limit l_2 for $v_2 \in [50, 200]$ in phase 3

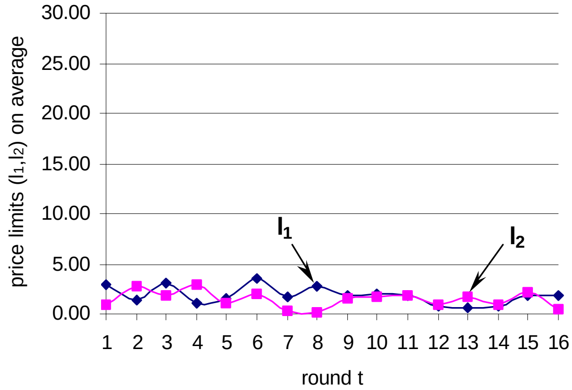

Figure A.8: Time paths of price limits (l_1, l_2) for $v_1, v_2 \in [50, 100]$ in phase 3

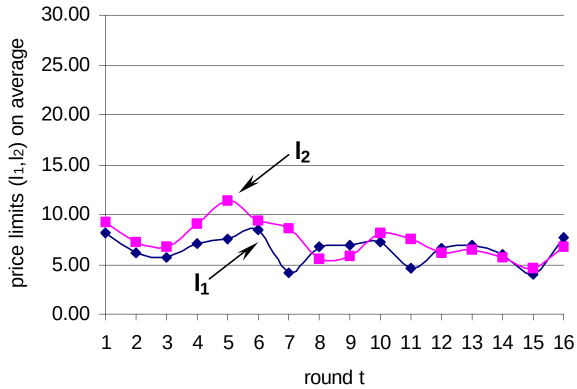

Figure A.9: Time paths of price limits (l_1, l_2) for $v_1, v_2 \in (100, 150]$ in phase 3

	auction type		phase	All	Session							
					1	2	3	4	5	6	7	8
mean efficiency rate	first price		1	0.97	0.97	0.97	0.98	0.96	0.98	0.98	0.98	0.97
			2	0.97	0.97	0.98	0.98	0.97	-	0.96	0.97	0.98
			3	0.98	0.97	0.98	0.99	0.96	-	0.97	0.99	0.99
	second price		1	0.98	0.97	0.97	0.97	0.98	0.98	0.98	0.99	0.97
			2	0.99	0.99	0.99	0.99	1.00	-	1.00	1.00	0.98
			3	0.99	0.99	1.00	0.99	1.00	-	1.00	1.00	0.99
percentage of	first price		1	84.1%	84.5%	81.0%	85.4%	80.4%	85.1%	87.5%	86.3%	82.7%
pareto opt. allocations			2	85.1%	80.7%	92.9%	84.4%	86.1%	-	86.7%	81.4%	85.4%
			3	87.2%	84.1%	85.5%	95.9%	79.4%	-	83.3%	91.7%	88.0%
	second price		1	88.4%	87.5%	86.9%	85.4%	90.5%	89.3%	90.5%	92.9%	83.9%
			2	93.5%	92.7%	92.9%	92.2%	93.4%	-	95.5%	97.1%	90.6%
			3	93.9%	91.2%	96.5%	93.6%	97.4%	-	95.3%	93.8%	88.1%
mean price	first price		1	103.87	103.80	108.86	101.93	92.80	107.93	106.89	107.84	100.65
			2	104.70	103.51	103.79	103.25	95.00	-	108.02	106.47	110.48
			3	98.74	98.61	104.27	102.63	77.21	-	100.15	102.21	98.92
	second price		1	87.06	83.82	93.65	90.61	70.18	88.68	89.74	89.02	91.29
			2	90.69	88.49	95.19	94.23	72.58	-	90.00	94.62	102.13
			3	89.97	96.75	92.32	96.19	74.76	-	88.16	92.33	94.26
std. dev.	first price		1	25.80	27.62	28.52	21.06	22.20	26.35	24.86	23.66	27.04
(mean price)			2	26.52	28.40	23.36	27.98	27.84	-	29.39	23.65	23.62
			3	26.25	27.79	24.89	21.57	25.58	-	25.98	24.05	27.48
	second price		1	33.79	33.01	29.95	33.67	46.36	30.72	29.35	26.14	32.22
			2	33.78	32.75	30.07	29.51	45.16	-	28.80	23.89	33.59
			3	32.09	27.87	24.37	27.55	41.09	-	28.17	27.61	35.91
mean under/overbidding	first price	bidder 1	1	0.19	0.21	0.12	0.18	0.36	0.14	0.11	0.14	0.22
(v-b)/v for v $\in$ [50,150]			2	0.19	0.18	0.11	0.16	0.51	-	0.06	0.12	0.21
			3	0.20	0.27	0.15	0.19	0.38	-	0.13	0.14	0.21
		bidder 2	1	0.22	0.25	0.17	0.23	0.33	0.18	0.15	0.20	0.27
			2	0.23	0.29	0.17	0.24	0.37	-	0.19	0.13	0.21
			3	0.24	0.23	0.18	0.21	0.51	-	0.22	0.17	0.25
	second price	bidder 1	1	-0.03	-0.03	-0.06	-0.10	0.20	-0.03	-0.07	-0.04	-0.10
			2	-0.04	-0.07	-0.06	-0.09	0.14	-	-0.05	-0.05	-0.17
			3	-0.05	-0.10	-0.06	-0.07	0.09	-	-0.03	-0.09	-0.16
		bidder 2	1	-0.02	0.10	-0.12	0.00	0.09	-0.16	-0.05	-0.05	0.02
			2	-0.01	0.05	-0.11	-0.07	0.15	-	-0.03	-0.08	0.02
			3	-0.01	-0.03	-0.10	-0.07	0.14	-	0.01	-0.04	-0.05
std. dev.	first price	bidder 1	1	0.20	0.24	0.15	0.11	0.27	0.12	0.19	0.13	0.19
(mean und./overbid.)			2	0.27	0.14	0.09	0.10	0.32	-	0.47	0.09	0.18
			3	0.17	0.23	0.10	0.12	0.25	-	0.07	0.10	0.18
		bidder 2	1	0.20	0.20	0.21	0.12	0.24	0.15	0.18	0.13	0.23
			2	0.20	0.20	0.20	0.19	0.20	-	0.11	0.11	0.26
			3	0.20	0.17	0.16	0.16	0.20	-	0.13	0.09	0.26
	second price	bidder 1	1	0.30	0.26	0.29	0.33	0.41	0.30	0.29	0.12	0.24
			2	0.24	0.10	0.14	0.12	0.35	-	0.23	0.08	0.32
			3	0.23	0.24	0.16	0.11	0.29	-	0.15	0.11	0.33
		bidder 2	1	0.31	0.30	0.35	0.21	0.31	0.35	0.23	0.20	0.39
			2	0.28	0.23	0.32	0.26	0.40	-	0.15	0.14	0.27
			3	0.23	0.13	0.24	0.24	0.31	-	0.11	0.11	0.26
mean price limits		bidder 1	2	4.92	8.35	4.67	2.78	3.48	-	7.40	4.15	3.28
			3	4.10	4.10	3.08	3.53	6.02	-	5.64	2.97	3.28
		bidder 2	2	7.55	7.79	6.79	4.83	11.29	-	4.64	9.89	7.23
			3	7.88	8.63	9.51	6.24	9.69	-	7.00	8.69	5.16
median price limits		bidder 1	2	2.00	4.00	3.00	1.00	4.00	-	5.00	0.50	1.00
			3	1.00	3.00	0.00	0.50	4.00	-	1.00	0.00	0.00
		bidder 2	2	5.00	10.00	5.00	3.00	13.50	-	2.00	8.00	5.00
			3	4.00	5.00	3.00	1.00	6.00	-	4.50	6.50	0.00
std. dev.		bidder 1	2	6.62	8.84	6.20	5.06	3.71	-	8.06	5.71	5.02
(price limits)			3	6.49	5.18	6.07	5.47	7.04	-	8.02	5.03	7.28
		bidder 2	2	7.39	6.64	7.41	5.42	8.00	-	5.73	7.40	8.18
			3	9.53	9.71	10.86	9.08	10.49	-	9.00	8.88	7.57

Summary Statistics

Appendix B:

Instructions[15]:

Phase 1:

Please, read these instructions carefully. They are identical for all participants.

During the experiment you will take part in several auctions. In every auction a fictitious commodity is for sale which you can resell to the experimenters. You are one of two bidders. In each auction there are two bidders, 1 and 2, with different value ranges $[50, 150]$ and $[50, 200]$. Each participant belongs either to type 1 or to type 2 and keeps his own type during the entire experiment. Both bidders know for sure the both value ranges. In every auction the private reselling value v of each bidder is independently drawn from the interval $50 \leq v_1 \leq 150$ for bidder 1 and from the interval $50 \leq v_2 \leq 200$ for bidder 2, respectively, with every integer number between 50 and 150 for bidder 1 and between 50 and 200 for bidder 2, respectively, being equally likely. Each bidder may place integer bids in the range from 0 to 250. The bidder with the highest bid buys the commodity and pays a price according to the pricing rule. Then he sells the commodity to the experimenter and receives his reselling value. The other bidder does not pay anything and does not receive anything. If the both bids are equal, the buyer is chosen randomly (by the flip of a fair coin).

There are two different types of auction. In first-price auction the price is the highest bid. In second-price auction the price which has to be paid is the second highest bid. We denote by b_w the highest and by b_{2w} the second highest bid.

First-Price Auction:

- Price = highest bid ($p = b_w$)

[15] This is a translated version of the instructions. For the original instructions (in German), please contact one of the authors.

- Bidder with highest bid becomes buyer. He pays p.
- Profit of buyer: $v_w - p$
- Profit of non-buyer: 0

Second-Price Auction:
- Price = second highest bid ($p = b_{2w}$)
- Bidder with highest bid becomes buyer. He pays p.
- Profit of buyer: $v_w - p$
- Profit of non-buyer: 0

In each auction the bidder groups are formed randomly. After you have placed your bid you are informed whether or not you are the buyer, about the price, which has to be paid by the buyer, your private reselling value, your own bid, the bid of the other bidder, how much you have earned in this auction, your total profit up to this time and your average profit in each auction type (per auction type). Altogether, you will play 48 successive auctions, which consist of 4 cycles of 12 bidding rounds. In one cycle each participant plays first 6 times the first-price auction and then 6 times the second-price auction.

All values are denoted in a fictitious currency termed ECU for Experimental Currency Unit. The exchange rate from ECU to DM is: 100 ECU = DM 1.50. You receive an initial endowment of DM 10.50 (700 ECU) to cover possible losses.

Any decision you make is anonymous and cannot be related to you personally by your co-bidders. If you have questions, please, raise your hand. We will then clarify your problems privately.

Phase 2 and 3:

You now will play another 32 auctions. By the first 16 auctions (phase 2) the bidders can choose the auction type according to the following rules: Both bidders submit their price limits stating how much they are willing to pay at most for the right to choose the price rule, i.e., the auction type. It is determined by chance who of the two bidders is selected as the potential dictator of the auction type. If this is bidder i, his price limit p_i determines the result as follows: If the randomly chosen price p of dictatorship exceeds p_i, the bidder i does not become the dictator, and the auction type is randomly chosen (by the flip of a fair coin). If $p \leq p_i$, the bidder i has bought the right to dictate the auction type at cost p and can freely choose between first-price and second-price auction. After the auction type is determined, either by chance or by a dictator, the auction is played as in the first phase. At the end of each auction in this phase the dictator also gets information about his costs for choosing the auction type. By the rest 16 auctions (phase 3) before you choose the auction type as described above, you are informed about your private reselling value v.