

The gluon and ghost propagator and the influence of Gribov copies *

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The dependence of the Landau gauge gluon and ghost propagators on the choice of Gribov copies is studied in pure $SU(3)$ lattice gauge theory. Whereas the influence on the gluon propagator is small, the ghost propagator becomes clearly affected by the copies in the infrared region. We compare our data with the infrared exponents predicted by the Dyson-Schwinger equation approach.

The non-perturbative behaviour of the gluon and ghost propagators in Yang-Mills theories is of interest for the understanding of the mechanism of confinement of gluons and quarks. In particular the infrared behaviour of the ghost propagator in the Landau gauge is related to the so-called Kugo-Ojima confinement criterion [1], which expresses the absence of coloured massless asymptotic states from the spectrum of physical states in terms of the ghost propagator at vanishing momentum. On the other hand the suppression of the gluon propagator in the infrared was argued to be related to the gluon confinement [2].

Zwanziger [3] has suggested that the behaviour of both propagators in Landau gauge results from the restriction of the gauge fields to the Gribov region, where the Faddeev-Popov operator is non-negative. Generically, one gauge orbit has more than one intersection (Gribov copies) within the Gribov region. In this contribution we assess the importance of this ambiguity for the ghost and gluon propagator in $SU(3)$ gauge theory on a finite lattice.

In the continuum the gluon and ghost propagators have been computed from a coupled and truncated set of Dyson-Schwinger (DS) equations [4]. Representing the gluon and ghost propagators in the Landau gauge as

$$\begin{aligned} D_{\mu\nu}^{ab}(q^2) &= \delta^{ab} (\delta_{\mu\nu} - q_\mu q_\nu / q^2) Z_{gl}(q^2) / q^2, \\ G^{ab}(q^2) &= \delta^{ab} Z_{gh}(q^2) / q^2 \end{aligned} \quad (1)$$

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the low-momentum behaviour of the corresponding dressing functions was found to be closely constrained by the respective infrared exponents, $Z_{gl} \propto (q^2)^{2\kappa}$, $Z_{gh} \propto (q^2)^{-\kappa}$ with $0.5 < \kappa < 1$.

There have been only a few numerical lattice investigations of the $SU(3)$ ghost propagator in the past [5] contrary to the gluon propagator. Here we present data for both propagators in Landau gauge measured at the same ensemble of $SU(3)$ gauge-fixed configurations and concentrate on the infrared behaviour inside the Gribov region. The strategy of the investigation is similar to a previous analysis of the $SU(2)$ ghost propagator [6].

The Landau gauge condition is implemented by searching for a gauge transformation ${}^gU_{x,\mu} = g_x U_{x,\mu} g_{x+\hat{\mu}}^\dagger$ which maximises the gauge functional $F_U[g] \propto \sum_{x,\mu} \Re \text{Tr } {}^gU_{x,\mu}$ where the gauge field $U_{x,\mu}$ is provided by Monte Carlo simulations. This functional has many local extrema whose number increases with the lattice size and decreasing $\beta \equiv 6/g^2$. All Gribov copies $\{{}^gU\}$ belong to the gauge orbit created by U and satisfy the lattice Landau gauge condition $\partial_\mu {}^gA_{x,\mu} = 0$ with

$${}^gA_{x+\hat{\mu}/2,\mu} = \frac{1}{2i} ({}^gU_{x,\mu} - {}^gU_{x,\mu}^\dagger) \Big|_{\text{traceless}}. \quad (2)$$

Before investigating the infrared behaviour of both propagators we first check the influence of different ways to select Gribov copies on the average propagators taken at some low momenta.

As it was previously found the Gribov copy ambiguity affects the measurements of the ghost propagator in the case of $SU(2)$ [6, 7]. Also from a

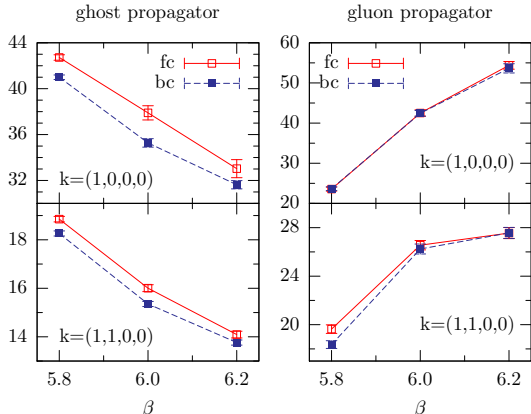


Figure 1. The ghost and the gluon propagator $Z(q^2)/q^2$ in lattice units at the two lowest momenta $q(k)$ as a function of β for the first (fc) and best (bc) gauge copies (lattice size 24^4).

recent study the $SU(3)$ gluon propagator is seen to depend systematically on the choice of Gribov copies [8] contrary to common belief. We report on first measurements on 16^4 and 24^4 lattices at $\beta = 5.8, 6.0$, and 6.2 as well as 32^4 at $\beta = 5.8$. In the update the usual heatbath algorithm with microcanonical overrelaxation steps is used. Then for each configuration $N_{cp} = 30$ random copies on a 16^4 lattice ($N_{cp} = 40$ on 24^4 and $N_{cp} = 10$ on 32^4) have been gauge-fixed using the standard overrelaxation method until $\max_x (\partial_\mu \mathcal{A}_{x,\mu})^2 < 10^{-14}$ has been reached.

The propagators were evaluated on the first (fc) and the best (bc) (with respect to the gauge functional) gauge-fixed copies. The gluon propagator was calculated for all momenta using the fast Fourier transformation of the gauge potentials (2). The ghost propagator was obtained using the conjugate gradient method with a plane wave source only for the lowest non-vanishing momenta. For each momentum squared, $q^2(k) = (4/a^2) \sum_\mu \sin^2(\pi k_\mu/L_\mu)$, the final propagator is an average over the respective ensemble of gauge-fixed copies (fc) or (bc) and over different k giving rise to the same $q^2(k)$. Following Ref. [9], only momenta with k satisfying $\sum_\mu k_\mu^2 - 1/4 \cdot (\sum_\mu k_\mu)^2 \leq 1$ have been used in the analysis of the gluon propagator.

The results from a 24^4 lattice at the chosen values of β are shown in Fig. 1 for some lowest mo-

menta. The ghost propagator is clearly affected by the Gribov copy problem. The effect is bigger than the statistical error and increases with decreasing momenta. On the other hand, for the lowest momentum of the gluon propagator the impact of Gribov copies is completely below the statistical error. For other momenta there are some effects visible as shown in the lower half of Fig. 2

The upper half of Fig. 2 shows both propagators for the best copy as a function of the momentum q scaled with an appropriate lattice spacing a . In order to compare to other studies [8, 9] we have used $a^{-1} = 1.53, 1.885$ and 2.637 GeV for $\beta = 5.8, 6.0$ and 6.2 , respectively. The lower half shows the ratio between the mean value on the first and best copy of the respective propagators dressing function, $\langle Z^{(fc)} \rangle / \langle Z^{(bc)} \rangle$. Due to the low number N_{cp} of gauge copies such ratios on a 32^4 lattice are not shown there. For the lowest momenta the ghost propagator is systematically overestimated if measured on an arbitrary (first) gauge copy, while for the gluon propagator the statistical noise is dominant. However, so far we have only measured the ghost propagator for a single k without permutations of components.

As in [6] we also found outliers in the data of the ghost propagator at the lowest momenta $k = (1, 0, 0, 0)$, shown in Fig. 3, which may make statistical analyses difficult, if arbitrary (first) gauge copies are studied only. However, the reason is unknown yet.

To parametrise the infrared behaviour, we fitted the measured dressing function in the form $Z_{gl} = C_{gl}(q^2)^{2\kappa}$ and $Z_{gh} = C_{gh}(q^2)^{-\kappa}$ with common κ to the data at some low q values. For the best fit with $q_{\max} = 0.43$ we found $\kappa = 0.23(1)$ for the best copy, far away from the DS prediction. However, despite κ tends to increase as q_{\max} decrease a reasonable statement cannot be made from the lattice sizes L^4 used.

In summary, we have reported first results studying the influence of the $SU(3)$ ghost and gluon propagators on Gribov copies. We have found that the ghost propagator is more affected than the gluon propagator. Measuring the ghost propagator on an arbitrary first gauge-fixed configuration the propagator is systematically over-

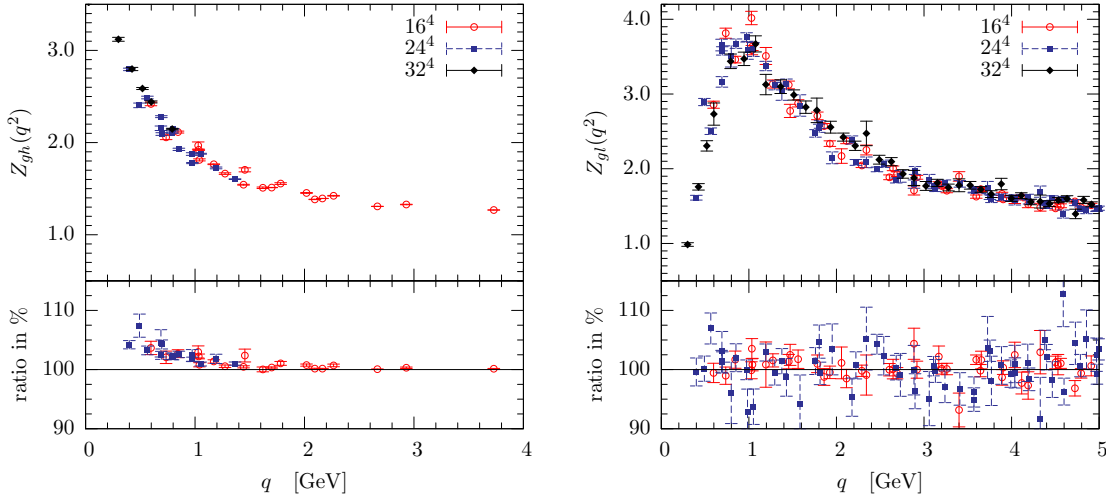


Figure 2. The upper figures show the dressing functions of the ghost and gluon propagator measured on the best copies as functions of the momentum q scaled to physical units for $\beta = 5.8, 6.0$ and 6.2 and various lattice sizes. The lower ones show the ratio $\langle Z^{(\text{fc})} \rangle / \langle Z^{(\text{bc})} \rangle$ determined from the first (fc) and best (bc) gauge copies.

estimated, the effect is largest for the lowest momenta. On the other side there is an irregular impact on the gluon propagator. Concerning the infrared behaviour the results from the lattice sizes used so far do not allow to confirm the proposed scaling behaviour using the ansatz discussed with an infrared exponent $\kappa > 0.5$ simultaneously describing both propagators.

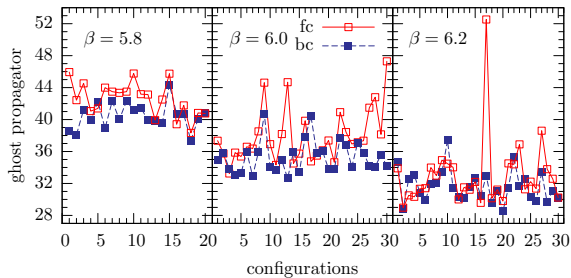


Figure 3. The ghost propagator at $k = (1, 0, 0, 0)$ vs. the first (fc) and best (bc) gauge copy on a 24^4 lattice.

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