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Dynamic response of a viscously damped two adjacent degree of freedom system linked by inerter subjected to base harmonic excitation

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Abstract

The study investigates the dynamic response of a viscously damped two adjacent single degree-of-freedom (2-ASDOF) system coupled by a connection that includes an inerter element. The dynamical model of a pair of simple oscillators coupled with various connection elements is synthetic but also representative to describe different classes of structures (i.e. contiguous buildings, adjacent walls and frames and so on). The specific kind of connection fundamentally alters the dynamic behavior of the entire system. Coupling elements typically studied are springs, dampers, linear or non-linear, passive, semi-active or active, e.g. [1,2]. The inerter is a novel device able to generate a resisting force, proportional to the relative acceleration of its terminals, equivalent to a force produced with an apparent (inertial) mass two orders of magnitude greater than its own physical (gravitational) mass [3]. In this study, a non-conservative connection, realized with a spring-inerter-viscous damper elements, adjusted in parallel, is considered as linking scheme for the 2-ASDOF system. In order to perform modal analysis, the first order state-space representation is adopted and the modal equations for the viscously damped system are derived. By solving the eigenvalue problem, the attention is focused on how modal parameters, i.e. the natural frequencies, the modal damping ratios and modes are affected by the connection. The system is then subject to harmonic base excitation and frequency response functions are depicted showing the influence of the link (through spring stiffness, inertance and damping coefficient) on the dynamic response. From the analysis with the different linking schemes, it emerges that the specific kind of connection influences the system dynamic characteristics.

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Keywords: adjacent structures; two-degree-of-freedom system; inerter; viscous damping; modal analysis; harmonic base excitation.

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1. Introduction

In recent decades, many researchers have studied, in the context of vibrations control, the dynamics of a pair of simple oscillators coupled with various connection elements [4-6], since such model is a synthetic but also representative to describe different classes of structures (i.e. contiguous buildings, adjacent walls and frames and so on). The specific kind of connection fundamentally alters the dynamic behavior of the entire system. In early 2000s, Smith [3] proposed a novel device, named inerter, able to generate a resisting force, proportional to the relative acceleration of its terminals, equivalent to a force produced with an apparent (inertial) mass two orders of magnitude greater than its own physical (gravitational) mass. In the same context, the authors already studied the structural configuration represented by a primary single degree of freedom (SDOF) structure equipped with a classical linear tuned mass damper (TMD) with mass linked to the primary system via a spring-damper and linked to the ground via an inerter [7].

This paper studies the dynamics of a novel two degree-of-freedom (2-DOF) system consisting of two adjacent single degree-of-freedom (2-ASDOF) systems coupled by a connection that includes an inerter. The first order state-space representation for a viscously damped system linked by a spring-damper-inerter elements adjusted in parallel is derived. By solving the eigenvalue problem, the system's frequencies, modal damping ratios and complex modes are obtained. Moreover, by considering a dynamic action represented by harmonic base excitation, frequency response functions are depicted showing the influence of the link parameters on the dynamic response.

2. Equations of motion

The system shown in Fig.1 is defined by stiffness k_i, mass m_i, and viscous damping coefficient c_i with, for the uncoupled case, natural frequencies $\omega_i = \sqrt{k_i/m_i}$ and damping ratio $\xi_i = c_i/(2m_i\omega_i)$ $i = 1,2$. The linking element is defined by stiffness k, inertance b and viscous damping coefficient c.

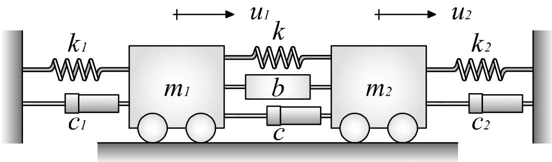

Fig. 1. Mechanical model.

The following non-dimensional parameters are introduced for the structures, i.e. frequency ratio $\nu = \omega_2/\omega_1$ and mass ratio $\mu = m_2/m_1$ and for the link, i.e. stiffness ratio $\lambda = k/k_1$, inertance ratio $\beta = b/m_1$ and damping ratio $\xi = c/(2m_1\omega_1)$.

The governing equations of motion for the system subject to base excitation can be written into the first-order state space form as

$$\begin{aligned}\dot{\boldsymbol{z}}(t) &= \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{B}e(t)\\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{z}(t) + \boldsymbol{D}e(t)\end{aligned} \tag{1}$$

where $\boldsymbol{z}(t) = [u_1(t) \quad u_2(t) \quad \dot{u}_1(t) \quad \dot{u}_2(t)]^T$ is the state vector, $e(t)$ is the applied input and $\boldsymbol{y}(t)$ is the output vector; the state space or system matrix $\boldsymbol{A}$ and the input influence matrix $\boldsymbol{B}$ are estimated as:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{L} \end{bmatrix} \qquad \boldsymbol{B} = [\boldsymbol{0} \quad -\boldsymbol{M}^{-1}\widetilde{\boldsymbol{M}}\boldsymbol{\tau}]^T \tag{2}$$

The mass $\boldsymbol{M}$ and $\widetilde{\boldsymbol{M}}$, damping $\boldsymbol{L}$ and stiffness $\boldsymbol{K}$ matrices are respectively:

$$\boldsymbol{M} = m_1\begin{bmatrix} 1+\beta & -\beta \\ -\beta & \mu+\beta \end{bmatrix} \qquad \widetilde{\boldsymbol{M}} = m_1\begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix} \tag{3a}$$

$$\boldsymbol{L} = m_1\omega_1\begin{bmatrix} 2(\xi_1+\xi) & -2\xi \\ -2\xi & 2(\xi_2\nu\mu+\xi) \end{bmatrix} \qquad \boldsymbol{K} = m_1\omega_1^2\begin{bmatrix} (1+\lambda) & -\lambda \\ -\lambda & (\nu^2\mu+\lambda) \end{bmatrix} \tag{3b}$$

$\boldsymbol{\tau} = [1 \quad 1]^T$ is the influence vector. Finally, $\boldsymbol{C}$ is the influence output matrix and $\boldsymbol{D}$ is the feed-through matrix which can vary depending on the assumed output vector.

From this general position, different sub-cases can be obtained if in the link (Fig. 1) we consider de-activated one or more elements. In this study, we will consider the 2-ASDOF system in which the damping is located only in the connection ($c_1 = c_2 = 0, c \neq 0$). Two cases will be considered: undamped connection (spring-inerter elements) and damped connection (spring-inerter-dashpot elements).

3. Modal analysis

By considering null the force vector in the Eq. (1), the solution has the exponential form $\boldsymbol{z}(\boldsymbol{t}) = \boldsymbol{e}^{\boldsymbol{\lambda t}}\boldsymbol{z}$. For non-conservative systems, the algebraic eigenvalue problem is $\boldsymbol{Az} = \boldsymbol{\lambda z}$ with eigenvalues $\boldsymbol{\lambda_i}$ and eigenvectors $\boldsymbol{z_i}$, $\boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}$. The eigenvalues that come in complex conjugate pairs $\boldsymbol{\lambda_i}, \bar{\boldsymbol{\lambda}}_i = \boldsymbol{\alpha_i} \pm \boldsymbol{j\omega_i}$ with $\boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}$ indicate sub-critically damped oscillatory modes. The pseudo modal frequency $\boldsymbol{\Omega_i}$ and the pseudo modal damping factor $\boldsymbol{\eta_i}$ associated to the i-th mode are evaluated as $\boldsymbol{\Omega_i} = \sqrt{\boldsymbol{\lambda_i}\bar{\boldsymbol{\lambda}}_i}$ and $\boldsymbol{\eta_i} = -\boldsymbol{Re}(\boldsymbol{\lambda_i})/\sqrt{\boldsymbol{\lambda_i}\,\bar{\boldsymbol{\lambda}}_i}$ with $\boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}$. Concerning the eigenvectors, they have been normalized as shown in [8]. In the following figures, the eigenvectors with the introduced normalization have been reported. Modal frequencies are ordered from the lowest to the highest and the modal damping factors are ordered as the modal frequencies. Results are presented at first considering the undamped connection and then the damped one.

***Undamped connection* ($\boldsymbol{\xi = 0}$)**

The maps for the dimensionless frequencies defined as $\gamma_i = \Omega_i/\omega_1$ with $i = 1, 2$, versus the frequency ratio ν are depicted in Fig. 2, given the values of μ= 0.2, β =0.1, and two assumed values for the elastic element $\lambda = \nu^2\beta$ and $\lambda = \beta$. Each map indicates admissible and not-admissible zones (grey painted background) for the frequencies. These zones, for each case, do not vary irrespectively of the values assumed by the system and connection parameters. In each plot, in the admissible zones, three regions are indicated, which are representative of a behavior observed also for the 2-ASDOF system considering activated in the connection only one element at once (the spring or the inerter only). Region I indicates the admissible domain for the frequencies associated to a system that behaves as connected with an inerter element only. Region III indicates the admissible domain for the frequencies associated to a system that behaves as connected with a spring element only. Between Region I and III there are transition curves that separate the two subdomains, obtained as special solutions for particular values of the connection coefficients which are in turn $\lambda = \nu^2\beta$ and $\lambda = \beta$ and define another region, named Region II. This latter region indicates the admissible domain for the frequencies associated to a system that exhibits a behavior mixed associated to a connection with both spring and inerter elements. Each region for frequency γ_1 matches with the corresponding one relative to frequency γ_2.

Figure 3 (a) illustrates the plots of the mode shapes for assigned parameters of the 2-ASDOF system in case of connection with spring-inerter elements. Since the system is conservative, the modes are natural and real. The first natural mode has both masses in phase whereas the second one has masses out of phase.

***Damped connection* ($\boldsymbol{\xi \neq 0}$)**

The frequencies maps versus the frequency ratio ν are depicted in Figure 2 having assumed $\xi = 0.1$ in case of connection with spring-inerter-viscous damper elements, with the other parameters as indicated for the undamped connection. The curves that delimitate the three regions are overlapped to those obtained for the undamped system, this means that, in the parameters range examined, modal frequencies are not significantly affected by the value assumed by the viscous damper element and these only depend on the spring and the inerter coefficients.

Figure 3 (a)-(b) illustrates the real and imaginary part respectively of the complex mode shapes, for assigned parameters of the 2-ASDOF system. The real component of the complex mode shows the first one with masses out of phase and the second one with masses in phase. The imaginary part has non-null components for both the modes only in correspondence of the first DOF.

Figure 4 illustrates the maps of the pseudo modal damping factors versus the frequency ratio ν, given the values of $\mu = 0.2$, $\beta = 0.1$, $\xi = 0.1$ and two assumed values for the elastic element $\lambda = \nu^2\beta$ and $\lambda = \beta$. Both pseudo modal damping factors assume value smaller than the unity denoting oscillatory motion.

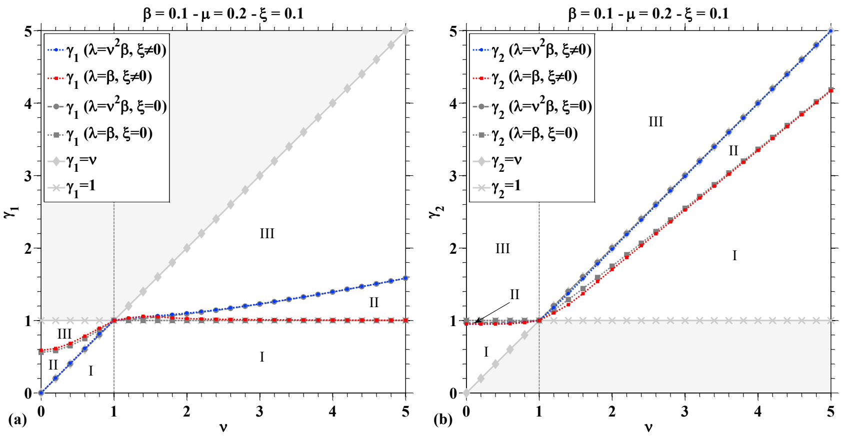

Fig. 2. Domain (a) $\gamma_1 - \nu$ and (b) $\gamma_2 - \nu$, for connection with spring-inerter-viscous damper elements.

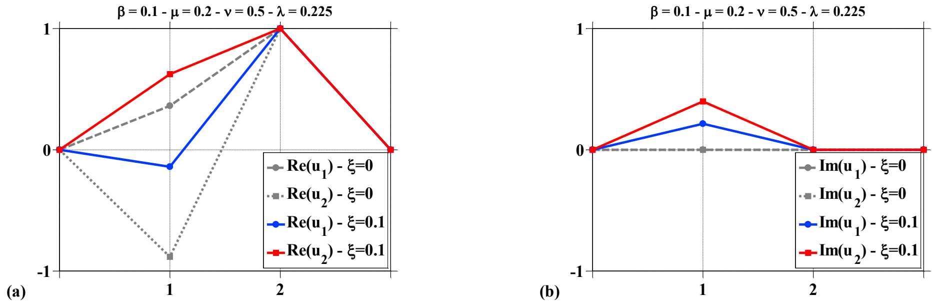

Fig. 3. Natural modes for conservative system (ξ =0) and complex modes for non-conservative system ($\xi \neq 0$) (a) real part (b) imaginary part.

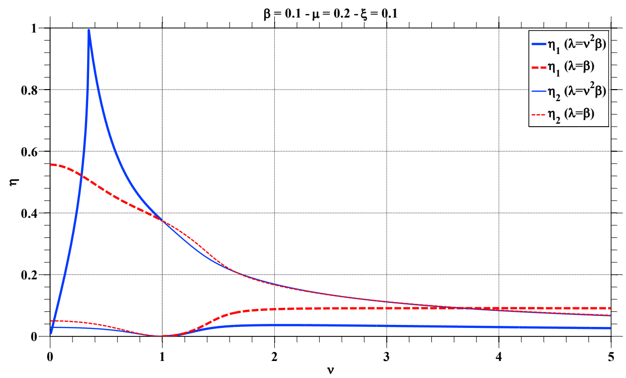

Fig. 4. Domain $\eta - \nu$ for connection with spring-inerter-viscous damper elements.

For $0 < \nu < 1$, η_1 assumes relative high values, for $\lambda = \nu^2\beta$ it increases until the unity with a cusp and then decreases, for $\lambda = \beta$ it decreases with the frequency ratio ν. η_1 curves in the region $0 < \nu < 1$ have a continuous behavior with the corresponding $\eta_2$curves in the region ν > 1, and these latter two curves are almost overlapped in this region, decreasing with the frequency ratio monotonically. In the range $0 < \nu < 1$, η_2 curves are almost similar for $\lambda = \nu^2\beta$ and $\lambda = \beta$, with low values, decreasing with ν until zero around ν = 1. In terms of frequencies, in this point, in fact the two coupled frequencies become one, equal to the first uncoupled one. η_2 curves in the region $0 < \nu < 1$ have a continuous behavior with the corresponding η_1 curves in the region $\nu > 1$, and these latter two curves in this region become different, increasing until a constant value. At the end of the frequency range examined, all curves tend to have a similar value.

4. Dynamic response

When the system is subject to harmonic base excitation with frequency ω_f, it is possible to represent the response in the frequency domain by means of the frequency response function (FRF). For the state-space model defined in Eq. (1), the frequency response function $\boldsymbol{H}(j\omega)$ is evaluated as:

$$\boldsymbol{H}(j\omega) = \frac{\boldsymbol{y}(j\omega)}{e(j\omega)} = \boldsymbol{D} + \boldsymbol{C}(j\omega\boldsymbol{I} - \boldsymbol{A})^{-\boldsymbol{1}}\boldsymbol{B} \tag{4}$$

where $\boldsymbol{y}(j\omega)$ and $e(j\omega)$ are the Fourier transform of the output vector and the input respectively.

The attention is focused on one SDOF response only and the FRF of the relative displacement $|H_{u1}|$ as a function of the frequency ratio ρ defined as $\rho = \omega_f/\omega_1$ is plotted.

The objective is to observe how the response varies with the parameters of the connection. In all the figures, FRF estimated for the system without considering coupling (SDOF) and in case of a rigid connection (RC) are reported as well for comparison purposes. Results are presented firstly for the undamped connection and then for the damped one.

***Undamped connection* ($\boldsymbol{\xi = 0}$)**

For the 2-ASDOF linked with spring-inerter elements it was observed that the FRF shows an anti-resonance point in correspondence of the first SDOF system frequency ($\rho = 1$) for specific values of the connection coefficients (β and λ) related to $\mu - \nu$ couples. The analytic expression of the values to be assumed by the link parameters in order to obtain the anti-resonance point is given as:

$$\lambda = \beta - \frac{\mu}{1+\mu}(\nu^2 - 1); \forall \nu \neq 1 \tag{5}$$

By observing Eq. (5) it can be noticed that, the value to assign to the spring coefficient in order to obtain anti-resonance, for $0 < \nu < 1$ is always $\lambda > 0$ for each assumed β and μ; differently, for $\nu > 1$ we obtain $\lambda \geq 0$ only if $\beta \geq \mu(\nu^2 - 1)/(1+\mu)$. As a result, there exists a minimum value to be assumed by the inerter coefficient in order to have the anti-resonance condition. This is $\beta_{min} = 0$ for $0 < \nu < 1$, and $\beta_{min} = \mu(\nu^2 - 1)/(1+\mu)$ for $\nu > 1$.

Figure 5 (a) shows $|H_{u1}|$ in the range $0 < \nu < 1$, having assumed $\mu = 0.2$, $\nu = 0.5$, $\beta = \beta_{min} = 0$ and $\beta = 0.5$ and setting λ according to Eq. (5). Figure 5 (b) shows $|H_{u1}|$ in the range $\nu > 1$, having assumed $\mu = 0.2$, $\nu = 0.5$, $\beta = \beta_{min} = 0.5$ and $\beta = 1$ and setting λ according to Eq. (5). In the figures, the response in case of no-connection (SDOF) shows the well-known amplification curve for an undamped SDOF system with maximum at $\rho = 1$. When a rigid connection is considered (RC), the maximum amplification of the FRF moves to left in the range $0 < \nu < 1$, to right in the range $\nu > 1$. When the connection with spring-inerter elements is considered, the FRFs show two amplifications and the anti-resonance point at $\rho = 1$. Generally, when coupling is assumed, the FRF always shows fixed points: they are located by intersection of the FRF of the RC case (green line) with the FRF of the other cases (blue or red lines). By observing Fig. 5a and 5b, it can be noticed that the two fixed points move close by increasing β.

***Damped connection* ($\boldsymbol{\xi \neq 0}$)**

Figure 5 (c)-(d) shows $|H_{u1}|$ when the connection element includes also viscous damping ($\xi \neq 0$), in the range $0 < \nu < 1$ and $\nu > 1$, respectively. The fixed points are present also in case of viscous damping. All curves are below the $\xi = 0$ curve outside the two fixed points. The anti-resonance point disappears, even if the response in the neighborhoods of $\rho = 1$ is still very low. The two maximum amplifications of the 2-ASDOF system are reduced if compared with the undamped case. By increasing the damping ratio ξ, the first resonance frequency moves towards right for $0 < \nu < 1$ and toward left for $\nu > 1$ and the amplification decreases. After a certain value of ξ, the FRF shows one amplification only (see Fig. 5 (c)); for this value of the damping ratio, the maximum amplification is observed in correspondence of the first fixed point. After that value, the amplification of the response starts to increase again. As a result, it is possible to select a suitable value for the damping ratio in order to obtain a very small amplification of the response. In the cases reported a good choice could be $\xi = 0.1$.

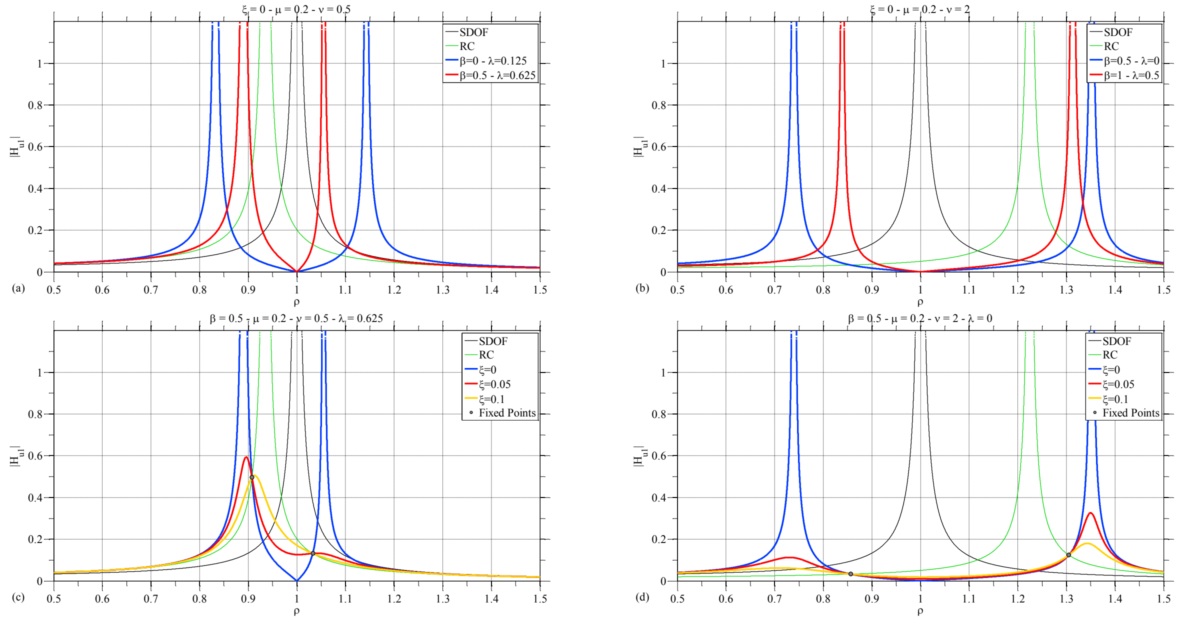

Fig. 5. Frequency response plots for structure 1: (a)-(b) for the undamped system, (c)-(d) for the damped system.

5. Conclusions

The study investigated the dynamic response of the structural configuration represented by a 2-ASDOF system coupled by a link that includes an inerter element. A non-conservative connection, realized with a spring-dashpot-inerter elements adjusted in parallel is considered as linking scheme. The attention focused on how modal parameters or the 2-ASDOF system, i.e. natural frequencies, modal damping ratios and modes varied with the connection coefficients. From the analysis with the different linking schemes, it emerged that the specific kind of connection influences the system dynamic characteristics. The system is then subject to base harmonic excitation and frequency response functions are depicted showing the influence of the link parameters on the dynamic response. By assuming an undamped connection, it is possible to obtain the anti-resonance point by properly selecting the spring and inerter coefficients. When coupling is assumed (RC, undamped and damped connection), there exist fixed points. By conveniently choosing the connection coefficients, it is possible to obtain a very small response in the neighborhoods of the resonance of the primary structure considered uncoupled.

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