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# A case study of two-echelon multi-depot vehicle routing problem

Tianqi Yu

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**PURDUE UNIVERSITY**  
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By Tianqi Yu

Entitled

A CASE STUDY OF TWO-ECHELON MULTI-DEPOT VEHICLE ROUTING PROBLEM

For the degree of Master of Science

Is approved by the final examining committee:

Dr. Edie Schmidt

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Date



A CASE STUDY OF TWO-ECHELON  
MULTI-DEPOT VEHICLE ROUTING PROBLEM  
COLLEGE OF TECHNOLOGY

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of

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## TABLE OF CONTENTS

	Page
LIST OF TABLES . . . . .	iv
LIST OF FIGURES . . . . .	v
SYMBOLS . . . . .	vii
ABBREVIATIONS . . . . .	viii
ABSTRACT . . . . .	ix
CHAPTER 1. INTRODUCTION . . . . .	1
1.1 Background . . . . .	1
1.2 Significance . . . . .	3
1.3 Statement of purpose . . . . .	4
1.4 Research Question . . . . .	4
1.5 Assumptions . . . . .	6
1.6 Limitations . . . . .	7
1.7 Delimitations . . . . .	7
1.8 Definitions . . . . .	7
1.9 Summary . . . . .	8
CHAPTER 2. LITERATURE REVIEW . . . . .	9
2.1 Basic Model of VRP . . . . .	10
2.2 Variants of CVRP . . . . .	12
2.2.1 Elements of CVRP . . . . .	12
2.2.2 Extension Problem . . . . .	14
2.2.3 Combination of Inventory and VRP . . . . .	18
2.3 Combination of Location and VRP . . . . .	22
2.4 Summary . . . . .	24
CHAPTER 3. FRAMEWORK, METHODOLOGY AND FINDINGS . . . . .	25
3.1 Theoretical Framework . . . . .	25
3.1.1 Develop Routes Between Local Warehouses and Customers . . . . .	25
3.1.2 Relationship Between Dependent Variables and SA Algorithm Parameters . . . . .	27
3.1.3 Determine Number of DC and Routes Between Local Warehouses and DC . . . . .	28
3.2 Methodology Overview . . . . .	30
3.3 Solving MDVRP between Local warehouses and Customers . . . . .	31

	Page
3.3.1 Step 1: Generate Location Coordinates and Demand . . . .	32
3.3.2 Step 2: Determine the Value of Independent Variables and Key Parameters . . . . .	34
3.3.3 Step 3: Determine Simulated Annealing Parameters . . . .	36
3.3.4 Step 4: Implement SA Algorithm in MDVRP . . . . .	36
3.3.4.1 Start solving with a random initial solution . . . .	38
3.3.4.2 Start SA iterations . . . . .	43
3.3.5 Step 5: Obtain routes for MDVRP . . . . .	50
3.3.6 Matlab coding logic . . . . .	50
3.4 Location-routing Problem between DC and Local Warehouse . . . .	52
3.4.1 Step 1: Determination of basic parameters and independent variables . . . . .	52
3.4.2 Step 2: Trial-and-error . . . . .	54
CHAPTER 4. COMPUTATIONAL RESULTS AND CONCLUSIONS . . .	59
4.1 Results illustration for Multi-depot VRP Problem . . . . .	59
4.2 Trial-and-error Results . . . . .	62
4.3 Results for 2E-MDVRP . . . . .	64
CHAPTER 5. CONCLUSIONS . . . . .	65
5.1 Study Conclusion . . . . .	65
5.1.1 Summary for Simulated Annealing . . . . .	65
5.1.2 Summary of Trial-and-error . . . . .	66
5.2 Discussion . . . . .	66
5.3 Recommendations for Future Studies . . . . .	68
LIST OF REFERENCES . . . . .	69
APPENDIX A. SIMULATED ANNEALING MAIN CODE . . . . .	73
APPENDIX B. COST FUNCTION CODE . . . . .	78

## LIST OF TABLES

Table	Page
2.1 Classification of VRP (Min, Jayaraman, & Srivastava, 1998) . . . . .	15
3.1 Generated customer daily demand . . . . .	33
3.2 Demand of three local warehouses . . . . .	53
3.3 Distance Matrix among 3 local warehouses . . . . .	54
3.4 Distance Matrix between local warehouses and DCs . . . . .	54

## LIST OF FIGURES

Figure	Page
1.1 2E-LRP Transportation Network . . . . .	5
2.1 Logistics decisions classification for different time span . . . . .	9
2.2 Capacitated vehicle routing problem . . . . .	11
3.1 Theoretical framework between the local warehouse and the customer .	26
3.2 Theoretical framework between local warehouse and DC . . . . .	29
3.3 Network between local warehouses and customers . . . . .	30
3.4 Network between DCs and local warehouses . . . . .	31
3.5 The location of the generated local warehouses . . . . .	33
3.6 The location of the generated customers . . . . .	34
3.7 Flow chart of SA . . . . .	37
3.8 Flow chart index 1 . . . . .	38
3.9 Random sequence of 19 customers . . . . .	38
3.10 First route of 19 customers . . . . .	39
3.11 Demand of Customer 7, 1 and 13 . . . . .	39
3.12 8 circles of 19 customers . . . . .	40
3.13 Pictorial presentation of 8 circles . . . . .	40
3.14 Route allocation criteria . . . . .	41
3.15 Allocation of 8 circles to three warehouses . . . . .	42
3.16 Pictorial presentation of allocation of 8 circles to three warehouses . . .	42
3.17 Trucking cost for 8 routes . . . . .	42
3.18 Total cost of initial solution . . . . .	43
3.19 Flow chart index 2 . . . . .	44
3.20 Demonstration of 2-opt swap . . . . .	44
3.21 Flow chart index 3 . . . . .	45



Figure	Page
3.22 Flow chart index 4 . . . . .	46
3.23 Flow chart index 5 . . . . .	47
3.24 Flow chart index 6 . . . . .	48
3.25 Flow chart index 7 . . . . .	49
3.26 Difference between global optimal and local optimal . . . . .	49
3.27 Coding flow chart for cutting customers . . . . .	51
3.28 The routes of Depot A serving three warehouses . . . . .	55
3.29 The routes of Depot B serving three warehouses . . . . .	56
3.30 The routes of Depot B serving Warehouse 1 . . . . .	56
3.31 The routes of Depot B serving Warehouse 2 . . . . .	57
3.32 The routes of Depot B serving Warehouse 3 . . . . .	57
4.1 Routes allocation results in Matlab . . . . .	59
4.2 Pictorial view of convergence procedure with 300 iterations . . . . .	61
4.3 Pictorial view of convergence procedure with 400 iterations . . . . .	62
4.4 Routes allocation between local warehouses and customers . . . . .	63
4.5 The routes of Depot A serving three warehouses . . . . .	63
4.6 Pictorial view of the results for 2E-MDVRP . . . . .	64

## SYMBOLS

$G$	Graph
$V$	Vertex (All points)
$V_0$	All points except depot
$E$	Edge
$c_{ij}$	Distance between $i$ and $j$
$Q$	Vehicle capacity
$K$	Number of routes
$S$	Subset of $V_0$

## ABBREVIATIONS

ID	Intermediate depot
SA	Simulated Annealing
CVRP	Capacitated vehicle routing problem
IRP	Inventory routing problem
LRP	Location routing problem
2E	Two echelon
LAP	Location allocation problem
SCVRP	VRP with symmetric distance matrix
ACVRP	VRP with asymmetric distance matrix
VRPB	VRP with backhauls
VRPSPD	VRP with simultaneous pickup and delivery
MDVRP	Multi-depot VRP
VRPTW	VRP with time window
VRPHTW	VRP with a hard time window
VRPSTW	VRP with a soft time window
VRPDT	Vehicle routing problem with due times

## ABSTRACT

Yu, Tianqi M.S., Purdue University, May 2016. A case study of Two-echelon Multi-depot Vehicle Routing Problem College of Technology. Major Professor: Edie K. Schmidt.

The Vehicle Routing Problem (VRP) is a classic combinatorial optimization problem and a topic still studied for practical applications. Current research focuses on single echelon distribution systems such as distribution centers serving customers. However, in typical distribution, goods flows among regional distribution centers, local warehouses and customers, defined as a two-echelon network.

The two-echelon multiple depot VRP problem is documented and applied to two stages illustrated by a small scale computational example. In the first stage, the simulated annealing algorithm is employed to determine the routes between local warehouses and final customers. For the second stage, trial-and-error is applied to obtain the number and location of regional distribution centers and the routes between regional distribution centers and local warehouses. Matlab is utilized to simulate annealing iterations and cost functions are analyzed. The convergence tendency of simulated annealing is depicted in figures by Matlab coding. Contributions include demonstration between the SA algorithm and a specific combinatorial optimization problem, and an application of the algorithm.

## CHAPTER 1. INTRODUCTION

This chapter provides some background information about the two-echelon problem and motivation for the problem, as well as the definition, the significance and problem statement. Limitations and delimitations are also discussed in this chapter.

### 1.1 Background

Logistics is significant for modern society because it gives economic globalization an impetus. Logistics industry is also growing as a pillar of industry for global economies. From a macroeconomic point of view, the impact of logistics cost for the country's economy plays a decisive role. According to data from China National Bureau of Statistics, since the mid-1990s, the domestic logistics cost of China increased annually. Between 16-18% of China Gross Domestic Product (GDP) is generated from logistics related activities. This ratio was almost twice the U.S GDP (CFLP, 2013). In 2012, this ratio was 8.5% for the US (CSCMP, 2014). From a corporate perspective, logistics significantly affects corporate profit, which is a relatively large component of the cost of final products.

Logistics defined as a management process delivering goods in order to meet customer demand. Logistics cost are generated by seven related activities which include transportation, warehousing, packaging, handling, distribution processing, distribution, and related logistics information.

Transportation costs, accounting for about 50% to 54% of total logistics costs, is a significant factor of logistics (Establish, Inc. Grubb& Ellis Global Logistics). Transportation accounts for 5.5% of the U.S. GDP in 2013 (Bureau of Transportation Statistics, 2014). Therefore, improving the organization and

management of the transportation systems and reducing costs have become an important research area for companies.

Over the past few decades, scholars began to focus on using operations research, mathematical programming and other optimization techniques to effectively determine vehicles routes and full-truck-load strategies. There is a classic optimization problem called the Vehicle Routing Problem (VRP) (Dantzig & Ramser, 1959), which has now become a core of transportation and distribution management modeling. The objective is minimizing cost under the constraints of time, the number of vehicles or length of routes. It is a more general version of the Travelling Salesman Problem (TSP).

Graph theory is used to describe vehicle routing problem as follows: Assume that  $G = \{V, E\}$  is a complete undirected graph, where  $V = \{0, 1, 2, \dots, n\}$  is a set of vertices and  $E = \{(i, j), i, j \in V, i \neq j\}$  is a set of edges.  $V_0 = V \setminus \{0\}$  is a set of customers. 0 is the depot (local warehouse). A fleet of vehicles with identical capacities  $Q$  start from a local warehouse serving customers to fulfill their demand. Each customer has a fixed demand  $q_i$  and fixed service time  $\delta_i$ . Every edge  $(i, j)$  has a weight which denotes distance between  $i$  and  $j$  or cost per mile  $c_{ij}$ . Usually, assume that  $C = (c_{ij})$  satisfies triangle inequality, that is  $c_{ij} \leq c_{ik} + c_{kj}, i, j, k \in V$  (Toth & Vigo, 2011). The objective for a standard VRP is: determine a minimal number of vehicles and their corresponding minimized travel distance and routes, which satisfy the following constraints:

- Every vehicle starts from one local warehouse and ends the route at the same local warehouse
- Each customer can only be served by one vehicle
- Total demand of customers in one route should not exceed vehicle capacity  $Q$

Historical research could date back to two seminal papers. Dantzig and Ramser (1959) published *The Truck Dispatching Problem*, the first study on the optimal vehicle routing problem of gasoline distribution. The proposed solution was

based on linear programming. As an improvement to the Dantzig and Ramer's algorithm, Clarke and Wright (1964) proposed an efficient greedy heuristic in 1964.

Since that time, scholars in the field of operations research have proposed many alternative mathematical models and algorithms to obtain the optimal or approximate optimal solution for many different types of vehicle routing problems. Constraints were added to the basic vehicle routing problem so that different types of variants reflect the actual production problem. These constraints include the vehicle routing problem with time windows, different vehicles capacities, open route and VRP with backhauls. An overview of research on VRP is presented in the literature review chapter.

This problem attracts much attention due to its computational complexity. An efficient vehicle routing algorithm is always a challenging research topic. Currently, the exact algorithm for VRP can only solve routes for around 50 customers. For larger scale VRP, the algorithm computation time increases at an exponential growth rate relative to the size of the problem, such as the number of customers.

Optimization is of great significance for improving the quality of a transportation system, customer satisfaction and enhancing the competitiveness of businesses. Vehicle routing problems can be used to manage the distribution of goods collection and for many applications in the transportation system, such as solid waste collection, school bus routing problems and the Dial-a-Ride problem (See Definition section). In summary, optimization technology is increasingly applied to various fields of production based on operations research, applied mathematics, computer science and management science to reduce operating costs.

## 1.2 Significance

This research makes it useful for solving small case city logistics problem (parcel delivery). This research is a new model for the two-echelon problem. The

two-echelon could be divided to multi-depot(MDVRP) between customers and local warehouses and the location-routing problem between local warehouses and distribution centers which cover larger areas than local warehouses. In addition, heuristics method, the simulated annealing (SA) is applied to solve MDVRP. There is a demonstration showing how SA could be applied to specific optimization problem and programmed by Matlab.

### 1.3 Statement of purpose

The purpose of this research was to generate an easier method to solve 2E-VRP. In previous research, all methods for the two-echelon problem are complex to program and very unique from problem to problem. One subproblem produces routing decisions between decisions customers and multiple local warehouses. The other one is location decision of number and location of DC(s) and after that, routing decisions between DC(s) and local warehouses. By separating the problem into two separate echelons, each subproblem can be used as a small independent problem combining with other new variants of VRP. The calculation speed improves significantly by separating the problem.

Visualized results for a simple case study produces convenience for both drivers and supervisors. The research problem is divided into two subproblems.

Additionally, a link between Simulated Annealing (SA) and programming in Matlab is presented, with a clear demonstration of the application of the SA algorithm.

### 1.4 Research Question

This research explains how to optimize costs for an integrated logistics system — a two-echelon logistics network design considering location and transportation routing decisions, which are foundations for all logistics activities. A sample network is shown in Figure 1.1:



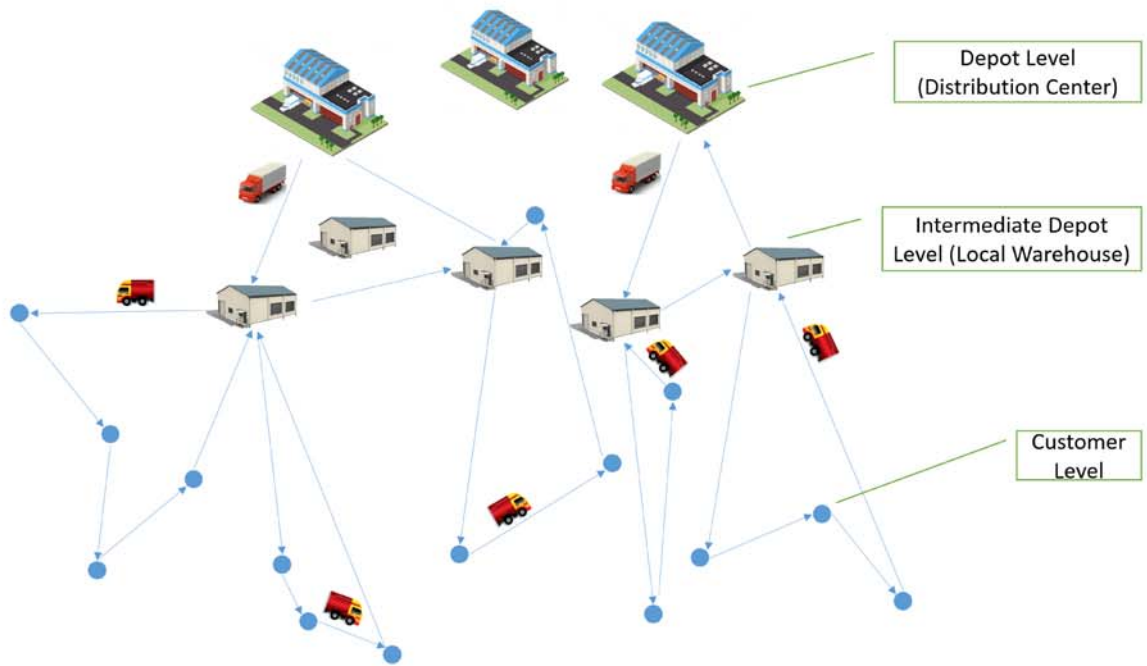


Figure 1.1. 2E-LRP Transportation Network

There are three levels in this network: the depot, the intermediate depot, and the customer. For industrial applications, the levels could be regional distribution centers (DC), local warehouses and final customers.

This problem will be divided into two parts:

- The routes between local warehouses and customers will be solved first using simulated annealing with Matlab. The customers will be allocated to a fixed number of local warehouses.
- The number and location of DC and routes between DC and local warehouses.

The objective function in this research has little difference with Perl and Daskin (1985)'s model. The cost of establishing depot and vehicle dispatch cost are not included in the objective function, but penalty cost is added.

There has been a lot of research for the individual parts - location and routing. Location-allocation Problem (LAP) determines the locations of warehouses

and its appropriate allocation assignment. Vehicle Routing Problem (VRP) designs the optimal routes of vehicles to minimize the transportation cost serving a set of customers.

Companies sometimes need to group both location and routing decisions together to reduce cost. As a result, some research has defined a new problem which combines LAP and VRP, which evolved in the Location-routing problem (LRP). Location and routing problems are closely related because they affect each other. Also, a logistics network contains many echelons, with one-echelon problems including routes between local warehouses and final customers. However, there are also routing problems between regional distribution centers and local warehouses.

In this study, a new model covering these factors will be created to optimize the total cost.

### 1.5 Assumptions

The assumptions of this study follow:

- Both DCs and local warehouses have their own capacity and are limited by the number of trucks or other vehicles which can be sent from depots.
- The location coordinates of all potential DCs, local warehouses, and final customers are given.
- Product being delivered is homogeneous among two echelons.
- Product demand from each customer is given.
- At one level, vehicles are all the same and speed is the same regardless of carrying capacity.
- Each customer can only be served by one vehicle at one time.

### 1.6 Limitations

This study is based on the two-echelon multi-depot network results in:

- This study is limited to a small case (within 50 customers).
- MDVRP uses heuristics method to solve.

### 1.7 Delimitations

The delimitations of this research study include:

- There is no length limit for each delivery route.
- Each route must start from one depot and come back to the same depot.
- This study was limited to routes between depots (distribution centers), intermediate depots (local warehouse) and customers.
- For the location problem, there is a minimum of two depots.
- The distance for each pair of two depots will not change for different directions.

### 1.8 Definitions

*Gross Domestic Product:* The Organisation for Economic Co-operation and Development (OECD) defines GDP as "an aggregate measure of production equal to the sum of the gross values added of all resident, institutional units engaged in production (plus any taxes, and minus any subsidies, on products not included in the value of their outputs).

*Vehicle routing problem:* The vehicle routing problem (VRP) is a combinatorial optimization and integer programming problem seeking to service a number of customers with a fleet of vehicles (Dantzig & Ramser, 1959).

*Echelon:* Each pair of stages represents one level of the distribution network and is usually referred to as an echelon (Cuda, Guastaroba, & Speranza, 2015).

*The Dial-a-Ride Problem:* The Dial-a-Ride Problem (DARP) consists of designing vehicle routes and schedules for  $n$  users who specify pickup and delivery requests between origins and destinations (Cordeau & Laporte, 2007).

*Combinatorial optimization:* Typical optimization problems can be divided into two categories, one is the real-valued variable, one is discrete variables (Papadimitriou & Steiglitz, 1982). The combinatorial optimization problem is one of the important branches in dealing with discrete variables problem. According to the definition (Cook, Cunningham, Pulleyblank, & Schrijver, 1998), combinatorial optimization is a topic that consists of finding an optimal object from a finite set of objects over a discrete structure and this objective could normally be a subset, sort of items, grouping or a graphic structure etc. In other words, combinatorial optimization problems find a global optimal solution rather than a local optimal solution.

*Heuristic algorithm:* The term heuristic is used for algorithms which guide solutions among all possible solutions, but do not guarantee that the optima will be found. Therefore, they are considered approximate and not accurate algorithms. These algorithms usually find a good solution, but quicker than exact searching method (Yang, Bekdaş, & Nigdeli, 2015).

### 1.9 Summary

This chapter provided an overview including background information, the significance of the study, research questions, limitations and delimitations. A detailed literature review is provided in the next chapter.

## CHAPTER 2. LITERATURE REVIEW

There are three decisions needed to be made during logistics process and according to time span, they could be divided into three levels shown in Figure 2.1 (Xia, 2013).

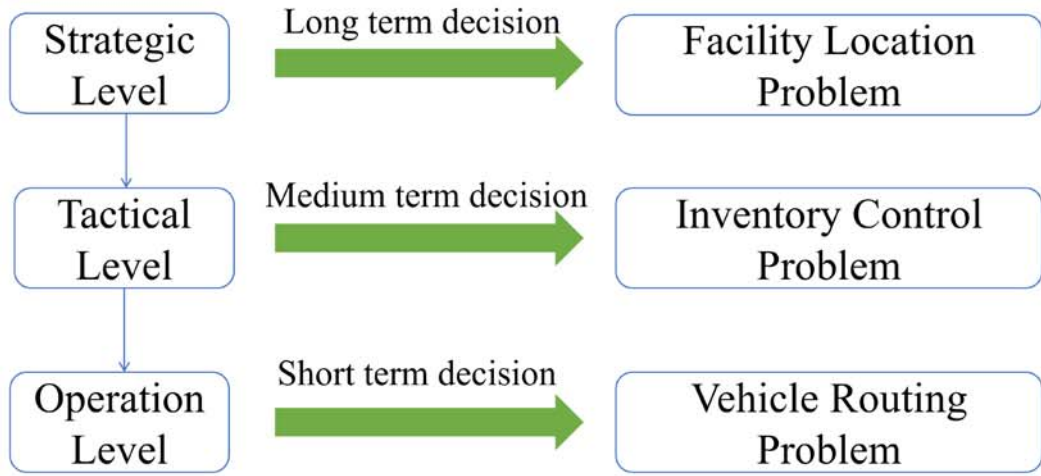


Figure 2.1. Logistics decisions classification for different time span

Topics covered in this research is operation level decision which is a variant of VRP-Multi-depot VRP between local warehouses and customers. In addition, combinations between the strategic level which is the facility location problem with operation level which is the VRP for the echelon between local warehouses and distribution centers (DCs).

The original vehicle routing problem was first proposed by Dantzig and Ramer (Dantzig & Ramser, 1959). The problem is defined as followed: vehicles depart from one or more depots to serve customers which require that each customer must be satisfied and only served once. The route of the vehicles begins at the depot and end at the same depot.

Vehicle routing problems have many applications in the real world with the advance of computing technologies addressing problems, such as city garbage collection problems, parcel distribution problems, school bus arrangement problem, newspaper distribution, milk distribution. VRP have become an important research topic in the field of logistics and distribution management. A large number of real-world applications have widely shown that the use of computerized procedures for the distribution process planning produces substantial savings (generally from 5% to 20%) in global transportation costs (Toth & Vigo, 2011).

The chapter is organized in the following sections according to the three-level decisions: In Section 2.1, a standard mathematical model and some reviews are given. Section 2.2 shows elements and variations of the problem. Section 2.3 reviews the combination of location and routing problems. Section 2.4 is a summary of current research, guiding the research question being proposed in this study.

### 2.1 Basic Model of VRP

A standard VRP is a vehicle routing problem with the limit of loading capacity which is also called *capacitated vehicle routing problem* (CVRP). For each vehicle, it has a maximum capacity. It is the most basic vehicle routing problem as shown in Figure 2.2 below. The middle depot could be the local warehouse and each vehicle has capacity in real-life applications.

Other vehicle routing problems are based on CVRP. This section gives a standard mathematical model for vehicle routing problem. VRP could be presented in three different models from different perspectives including vehicle flow, commodity flow, and set-partitioning problem (Toth & Vigo, 2011). Vehicle flow formulations is a more common one, which uses integer variables to denote whether one edge is included in an optimal solution.

$$\min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \quad (2.1)$$

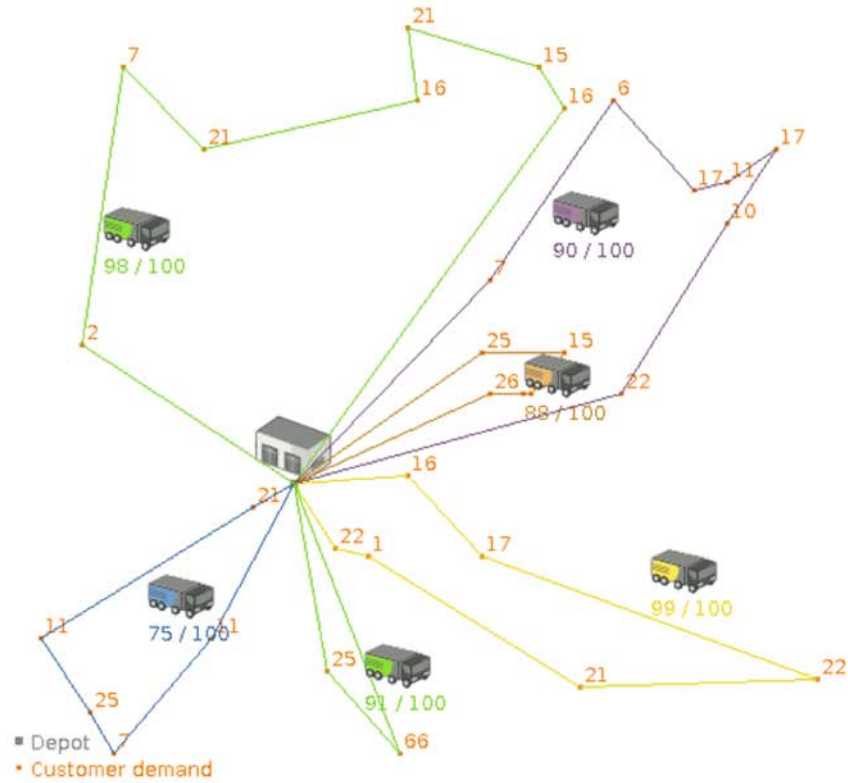


Figure 2.2. Capacitated vehicle routing problem

subject to

$$\sum_{i \in V} x_{ij} = 1 \quad \forall j \in V \setminus \{0\}, \quad (2.2)$$

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V \setminus \{0\}, \quad (2.3)$$

$$\sum_{j \in V} x_{i0} = K, \quad (2.4)$$

$$\sum_{j \in V} x_{0j} = K, \quad (2.5)$$

$$\sum_{i \notin S} \sum_{j \in S} x_{ij} \geq r(S) \quad \forall S \subseteq V \setminus \{0\}, S \neq \emptyset, \quad (2.6)$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in V \quad (2.7)$$

This model is a *two-index vehicle flow formulation*, where  $x$  is a binary variable to indicate if a vehicle traverses an arc in the optimal solution. If  $x_{ij} = 1$ ,

arc  $(i, j)$  (from customer  $i$  to  $j$ ) belongs to the optimal solution and  $x_{ij} = 0$  otherwise (Toth & Vigo, 2014).

(2.2) and (2.3) mean that only one edge enters one customer and only one edge leaves one customer, where 0 is the depot, and  $K$  is the number of routes. If  $K = 3$ , it means with the limit of vehicles, all the demand from customers could be delivered with 3 vehicles and therefore, there are 3 routes. (2.4) and (2.5), analogously, mean that  $K$  vehicles leave and return the depot. (2.6) is the so-called *capacity-cut constraints* (CCCs).  $r(S)$  is the minimum number of vehicles needed to serve all customers in  $S$  ( $S \subseteq V \setminus \{0\}$ ,  $V$  is all points including depot and customers). (2.6) satisfies the capacity requirement and also promises that any route must include a depot.

## 2.2 Variants of CVRP

This section first gives the main characteristics and elements of the CVRP. Based on this, variants of CVRP are discussed. The related research on this problem and its extension problems were widespread, where there are more research questions developing.

### 2.2.1 Elements of CVRP

Elements of a classic CVRP are network, customer, depot, vehicle and objective function or operational objective. Definitions and characteristics are being discussed below.

- Network

A network is a foundation of transportation. Network is a weighted graph containing vertex and arc or edge. Vertex could be customer or depot. Arc is connectivity between customer and depot. According to different characteristics of real situations, arcs could be directed or undirected. A



typical example of directed arc is some one-way street in city while the undirected arc is a two-way road. If the distance matrix among all vertex is symmetric, all arcs are undirected. If not, at least one arc is directed. Therefore, there are symmetric and asymmetric (SCVRP and ACVRP). For each arc, a non-negative weight is given to denote travel distance or time spent. Normally, calculation of distance in VRP problem satisfies *Triangle Inequality*.

- Customer

Customer is a general term, which may represent any type of depot in an actual CVRP, such as retail stores, distribution centers, and individual families. The customer is one vertex in network graph and its demand could be the quantity that delivers from depots to customers and collects from customers. According to this two opposite process, there are extensions such as VRP with backhauls (VRPB) or VRP with simultaneous pickup and delivery. (VRPSPD).

In addition, there could be time limits for customers. Customer service time, which represents a moment that customer requires the products to be delivered. However, time window (VRPTW) refers to a period. Customers could only be served within this time interval each day, otherwise, there will be penalty cost.

- Depot

In the network graph, another important type of vertex is depot. Depot is the start and end of one route. There is a fleet of vehicles at depots to serve the distribution or collection of customer service. Usually, there is only one depot in VRP problems unless the author points out that it is a multi-depot VRP (MDVRP).

- Vehicle

Vehicles are the tool to finish a delivery. Several aspects that will affect the problem are as follows:

- Types of vehicle: Typically, if there is no special statement of type, homogeneous vehicles will be used in papers, which includes same capacity, same fixed cost and variable cost. However, in real cases, companies will use different vehicles to fulfill full-truck-load as much as possible.
- Duration: There is a maximum travel distance or time constraint per day which corresponds to the maximum working time for a driver.
- Objective function

There could be multiple objectives in one problem. Most of the articles now are solving a single objective which includes:

- Minimized distance
- Minimized number of vehicles
- Minimized cost which contains fixed and variable cost of vehicles
- Two-phase objective: Optimize number of vehicle first and then optimize distance based on the first phase

Multiple objective means to optimize two or more objectives simultaneously.

At present, there are many classifications of VRP, the most representative proposed by Min et al. (1998). This classification contains more details and includes almost all aspects as shown in Table 2.1.

### 2.2.2 Extension Problem

According to the analysis of basic elements of CVRP, there are a lot of extension problems which are listed below:

Table 2.1  
Classification of VRP (Min et al., 1998)

	Standard of classification	Types of problems	
1	Direction of goods	One direction	Bi-direction
2	Demand type	Deterministic	Dynamic
3	Number of facility	Single	Multiple
4	Number & Type of vehicles	Single	Multiple
5	Capacity of vehicle	Determinate	Undetermined
6	Capacity of depot	Determinate	Undetermined
7	Number of echelon	Single	Multiple
8	Time windows	Hard & Soft	None
9	Objective function	Single	Multiple

- VRP with Time Window

Vehicle Routing Problem with Time Window (VRPTW) has a great significance in real life situations, which are widely used in practical applications. Serving period  $[e_i, l_i]$  is fixed for customer  $i$  in addition to constraints of basic VRP. Customer  $i$  could only be serviced in a fixed period. VRPTW can be divided into a vehicle routing problem with a hard time window (VRPHTW) and a vehicle routing problem with a soft time window (VRPSTW). Hard time window is strict, hen customers are served out of period, which makes it an infeasible solution. While in soft time window problem, it is still feasible, but just some penalty cost will be added to objective function.

Vehicle routing problems with due times (VRPDT) are a special case of VRPTW, with slack in the lower bound on the time window. VRPDT has a requirement that the service should be done before a predetermined deadline.

If it is violated, then a penalty fee is generated. The goal of VRPDT is not only the shortest travel time, but requires a minimum penalty fee due to the violation of the deadline.

Early work on VRPTW focused on a case study. Pullen and Webb (1967) scheduled van drivers' duty for the bulk conveyance of mail in the Central London area. Knight and Hofer (1968) developed a manual method to schedule a vehicle fleet to save 12%. Later research focused more on developing effective algorithms to solve realistic-size problems (Cordeau & Laporte, 2001). Park (2001) studied a school bus routing problem against this background and proposed a Bi-criteria algorithm depending on the difference in time and regional speed.

- Multi-depot VRP

In a standard VRP, there is only one depot which all vehicles start from and end at. If there are several depots in one area, customers can be served by any of the depots.

Customers are clustered together to be served by different depots and then the sub-problem becomes a standard VRP. The objective function will be realized by two phase. The first objective is to achieve a minimum number of vehicles and the second one is to minimize total distance.

- VRP with Backhauls

VRP with backhauls (VRPB) divided customers into two subsets, one is *Linehaul customers*, each requiring a given quantity of product to be delivered. The other set is *Backhaul customers*, where a given quantity of inbound product must be picked up (Toth & Vigo, 2011). All outbound customers must be served before inbound and all demand is fixed and given.

- VRP with Pickup and Delivery

VRP with pickup and delivery (VRPPD) is more complex than VRPB. It focused on three aspects:

- The distribution and collection of goods occur at the same time, i.e., a client can be both a client receiving delivery and the delivery generator.
- It is about the mixed distribution and collection of goods, which means there is no limit in route order for any client who can be distributors or in charge of goods collection.
- Distribution goes first, and then follows the collecting process which distribution arrangement customer should come after the good collection clients.

Min (1989) first conducted some research to solve the library book shipping problem. In this problem, Min (1989) developed a scenario of one collector-distributor point, two trucks and 22 nodes. First, the author divided the customers into different groups and then in each group figured it out exploiting traveling salesman problem (TSP) method. For the two nodes with no direct links in the real situation, the author assumed that the distance between the points was infinite and then solving as the traveling salesman problem. Halse (1992) did a lot of research on the VRP, including VRPB and VRPPD (Desrosiers, Dumas, Solomon, & Soumis, 1995). Halse solved this types of problems by grouping first and then determining the vehicle routes. First, each of the groups was served by only one truck and then the routes were determined by 3-opt after dividing the clients into various groups. The research tried the theory in both a VRPPD with 100 customers and a VRPB problem with 150 customers. Gendreau (1999) extended his study to the traveling salesman problem with pickup and delivery (TSPPD). First, the author solved the basic TSP without considering the distribution and collection request and then determined the order of the routes of distribution and pickup request.

Gendreau, Laporte, and Vigo (1999) determine the route of each distribution customer, and then inserted each delivery and client pickup to this route. This insertion method took into account the customers who hadnt been served on the current delivery routes using a penalty coefficient. Casco (1988) presented a method based on the insertion path which, for the cost of insertion, considered the loading of trucks on the delivery route (Jacobs-Blecha & Goetschalckx, 1992). Salhi and Nagy (1999) extended Cascos approach allowing multiple pickup clients to be inserted to the existing distribution groups. The quality of this method was improved with a slight increase in computational time, which can be used to solve the first type of VRPPD questions as well (Chen & Wu, 2006).

Toth and Vigo (1996) grouped the clients at first and matched the distribution and pickup nodes before exploiting TSP to improve the quality of the solution. This was a VRPPD problem with one distribution center and 150 clients. In the following year, they used the Lagrangian branch and bound algorithm for solving 100 clients and one distribution center. Osman and Wassan (2002) used saving insertion method to obtain the original feasible solution in the study of such problems, then using tabu search algorithm to solve a 150-clients and one distribution problem.

### 2.2.3 Combination of Inventory and VRP

Thomas and Griffin (1996) proposed that logistics costs account for 30% of the total supply chain cost. According to the research of Buffa and Munn (1989), logistics cost is mainly made up of transportation cost and inventory cost, where transportation cost accounts for one-third of total logistics cost and inventory is one-fifth. In the traditional logistics system research, most transportation and inventory problems are studied alone. But in recent years, more and more researchers have incorporated the two parts and developed a comprehensive problem

to meet the practical needs in the development of a supply chain management system.

Incorporating the transportation and inventory decisions is what researchers called dynamic routing and inventory problems (DRAI).

Baita, Ukovich, Pesenti, and Favaretto (1998) categorized the DRAI problem into two kinds.

The first kind is related to frequency. In this type of study, the decision variable is replenishment frequency. Anily and Federgruen (1990) proposed a method of fixed-partition policies (FPPs) which fix the DRAI problem in the variable of frequency. In the FPPs method, customers are grouped into different regions. The regions and each customer within the region are independent of others. If one client was touched by one truck, then this truck was also in charge of visiting other clients in this region. The zoning method defined in the FPPs problem enables a minimized total cost which includes the transportation and inventory parts taking the load of vehicles into account. Anily and Federgruen (1993) studied the possible replenishment strategies to minimize the total cost of long-term transportation costs and stocking costs. First, the retailers were divided according to their locations. All the retailers in the same region were replenished by the same truck. Bramel and Simchi-Levi (1995) proposed a problem of capacitated vehicle routing problem (CVRP) and the basic framework of inventory routing problem and used numerical experiments to show that the solution of inventory routing problem was better than the vehicle routing problem.

The second kind is to schedule transportation order within the time framework. The transportation quantity and route were determined by operating discrete-time model within a fixed time interval. The most representative example is inventory routing problem (IRP) of natural gas transportation issues. Such issues like keeping a constant inventory level to avoid being out of stock should be considered in this type of question. In an IRP problem, the researchers assumed

that each client had a fixed demand rate that could promise the minimized transportation cost while inventory cost was not the main concern.

Bell et al. (1983) researched the storage and transportation of natural gas with a specific goal and developed the corresponding support system for calculating the distance and time between clients. An optimized mathematical model was designed to calculate the quantity delivered every day and the Lagrangian relaxation method was used to solve a mixed integer programming problem with 200,000 constraints and 800,000 variables. Golden, Assad, and Dahl (1984) researched on traditional large-scale vehicle routing problems with inventory cost constraints, as well as conducted a corresponding analysis. Dror, Ball, and Golden (1985) proposed several different algorithms for the inventory routing problem and the attributes of these algorithms were analyzed and tested by numerical experiments. Campbell, Clark, and Savelsbergh (2002) proposed an inventory routing problem based on a two-stage decision algorithm. The first stage used integer programming to generate a delivery plan and in the second stage, routing was generated by heuristic algorithms. Adelman (2003) figured the natural gas issue out based on price manipulation. The stock replenishment and the penalty cost of losing sales were compared using a linear programming model to obtain optimal decisions.

Researchers took a number of different strategies for IRP. Chien, Balakrishnan, and Wong (1989) studied this issue in a single time period and got a feasible solution using mixed integer programming models and also produced a better upper bound based on Lagrangian relaxation. Federgruen and Zipkin (1990) evaluates the allocation of inventory quantities included in the vehicle routing problem, where not all the customers were generating inventory cost. The authors assign the stocks to certain customers with the premise of ensuring a minimum total cost. Dror and Ball (1985) studied the difference between the transportation and inventory cost within short and long period time frames respectively. Trudeau and Dror (1992) studied the stochastic inventory routing problem which treated the customers' needs as a random variable and considered the possible loss of trucks'



transportation route at the same time, for example, the situation that the customer demand exceeds the capacity of a truck. Viswanathan and Mathur (1997) researched the inventory routing problem with one distribution center, multiple customers, and multiple products. Herer and Levy (2008) focused on a multi-stage decision-making or simplifying the multiple stages to a single time-phase decision. Constable and Whybark (1978) were earlier researchers on making joint decisions for transportation and inventory. Baumol and Vinod (1970) developed a measurement of the transportation performance. The process of making transportation decisions consists of selecting a transportation mode which could be rail, truck, pipe or air and shipping to minimize cost and time spent. After the mode was chosen, the policy maker should evaluate the transit, such as the variability of the transit time. They also summarized the process into three attributes which are: the transportation cost, the expected time in transit, and the variability of transit time. If one transportation alternative was lower in cost, speed and variability, it would dominate other alternatives.

The research defined the transportation alternative and inventory factors (reorder level and order quantity for each cycle) leading to the minimized total cost (transportation cost and inventory cost). The three attributes affect each other, such as the shipping speed can affect the inventory quantity and the cost of transportation could have an impact on the ordering quantity. The solution method was to present a mathematical model adding the two parts cost together and then enumeration solution. Finally, a heuristic procedure can identify decisions by estimating the lowest cost or the relatively lower cost close to the extreme value.

Blumenfeld et al. (1985) modeled a trade-off between transportation cost presented in the form of three types of networks and inventory cost and production cost. The three types of networks are direct networks, network with consolidation terminal(s) and the combination of the previous networks. The main contributions were showing how transportation cost affects production set-up cost and how these two costs affected the inventory decision. Second, the authors divided the network

link by link and then calculated the optimal quantity respectively. Finally, they built a concave cost function to solve the problem. In the same year, Burns et al. (1985) developed the analytical methods to minimize distribution costs where trucks travel from suppliers to several customers. This compares the two allocation strategies: direct shipping and peddling (i.e., send one truck to deliver products to multiple clients each load). The trade-off made was related to the shipment size. For the direct mode, the optimal shipment size was decided by the economic order quantity (EOQ) model, the optimal shipment size was to ensure the truck has a full truckload.

Benjamin (1989) added more real life situations into the problem, such as production constraints and demand requirements. It was a heuristic problem which was solved by exploiting the linear network algorithm. A hypothetical corporate sourcing which used the reduced gradient algorithm and a heuristic solution was defined. A reduced gradient algorithm solution got a 21% improvement compared to a separate optimization problem. Also, the heuristic method got similar results as the reduced gradient algorithm with an even higher efficiency which is good news for researchers.

### 2.3 Combination of Location and VRP

The decision problem to simultaneously determine facility locations and delivery routes is commonly known as location routing problem (LRP, (Min et al., 1998)) . Normally, there are 3 decisions which LRP are making:

- facilities selection which includes number and location
- allocation to facilities after first decision
- optimize routes

LRP has many applications among different industries including retailing, transportation, product distribution, postal service, disaster relief, and so on (Liu &

Kachitvichyanukul, 2013). LRP could date back to 1961, Böventer (1961) proposed the relationship between transportation cost and facility cost. Interdependence between location and routing was realized until the 1970s. Gandy and Dohrn are the first to do research in LRP (Tuzun & Burke, 1999). However, due to the difficulty of solving the LRP, very little progress was made. At the beginning of the 1980s, with the development of integrating logistics, LRP attracted more attention. Laporte, Mercure, and Nobert (1986) proposed the exact algorithm for solving LRP. LRP could be divided into single and multiple echelons and this study focuses on a two-echelon problem.

From the perspective of a whole transportation network, two-echelon (2E) problem was first initiated by Jacobsen and Madsen (1980), Madsen (1983). LRP has at least three levels. The current research assumed that the locations of the first echelon are fixed when finding the solution of the second echelon. These studies used real applications of a newspaper distribution with 4500 customers (Rahmani, Oulamara, & Ramdane Cherif, 2013). In the newspaper distribution system, newspapers were delivered from the printing factories (depots) to transfer points (processing centers) and from these points to customers. Boccia, Crainic, Sforza, and Sterle (2011), Sterle (2009) and Boccia, Crainic, Sforza, and Sterle (2010) proposed the mixed integer programming model (MIP) in which both echelons have capacities and establishing cost. Four MIP are given. The first one is a three-index model based on Ambrosino and Scutella (2005). The second one is a two-index MIP model referring to multi-depot VRP. The third one is a variant of two-index LRP. The last one is a path variable model and small-scale cases were given and solved by commercial software. Those cases were 3 depots, 8 intermediate depots, and 10 customers at most and the results showed that three-index model is better. Boccia et al. (2010) applied Tabu Search to solve 2E problem with capacity and establishing cost. First, 2E-LRP is divided to two 1E-LRP and 1E-LRP is then separated to capacitated facility location problem and multi-depot VRP. The results showed that for small-scale problems, Tabu Search could cost less time to find the

global optimal solution that found by branch-and-cut. For larger scale problem, the different combination of parameters will get a good solution.

Lin and Lei (2009) separated the customers based on their demand: customers with small demand and with big demand. They proposed to use the Genetic algorithm. In the GA, the chromosome represents the open distribution centers and customers with large demand in the first level. The algorithm based on local path search of clustering methods. When using a small scale case to test the effectiveness of GA, the gap between LINGO and GA is within 1%. When using the case of Tuzun and Burke (1999), it took a long time to get the result and the gap is 3.5% comparing to well-known optimal solution internationally.

## 2.4 Summary

From the perspective of time span, all the decisions made in logistics are divided to three levels, which are long, medium and short terms and the corresponding decisions can be called the strategic, tactical and operational decisions. Long term typical questions could be site selection, which is called facility location problem. The medium term decisions relate to warehouses, which is called inventory control problem and the strategy will be kept for a period of time, people would not change it every day. The short term can be vehicle routing problem. This study combines the long and short term problems.

Specifically speaking, this study focuses on making the integration decision on strategic and operational level because there is limited research on applying simulated annealing algorithm to solve 2E-LRP and some of the factors are ignored due to the problem complexity.

## CHAPTER 3. FRAMEWORK, METHODOLOGY AND FINDINGS

This chapter provides a theoretical framework and the methodology to gather case study data and solve the two-echelon multi-depot VRP.

### 3.1 Theoretical Framework

This study is an optimization problem divided into two subproblems between two echelons. One echelon consists of intermediate depots (local warehouses) and customers. The other echelon is depots (distribution centers) and intermediate depots (local warehouses).

In this section, the relationship between the independent variables and dependent variables are identified in respectively and shown and described.

#### 3.1.1 Develop Routes Between Local Warehouses and Customers

The framework of this subproblem is shown as Figure 3.1.

In this figure, there are two sets of independent variables which are the independent variables for sample company generation and the independent variables of SA algorithm. The dependent variables are total distance, routes between local warehouses and customers and time spent to achieve routes.

The independent variables for sample company generation are specifically for this case study which affects the routes and total distance which include:

- Vehicle capacity

Vehicle capacity determines the number of customers in one route. The larger the vehicle capacity, the fewer the number of routes. And then, the number of routes affects the total distance. The more routes, the more connections

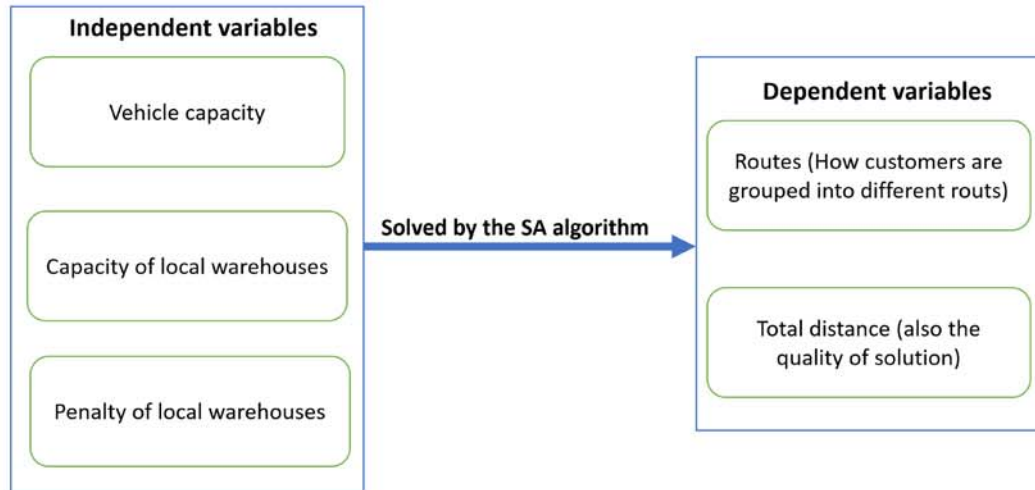


Figure 3.1. Theoretical framework between the local warehouse and the customer

between local warehouses and customers. Consequently, the total distance increases.

- Capacity of local warehouse

The capacity of local warehouse has an effect on fixed cost which is not considered in this research. If the capacity of local warehouses increases, the local warehouse could serve more customers around comparing to a small capacity warehouse. With a small capacity, some customers have to be allocated to a distant warehouse. Hence, the total distance will decrease.

- Penalty of warehouse

The penalty should be large enough (at least increase one order of magnitude) to limit when the delivery from one local warehouse exceeds capacity.

### 3.1.2 Relationship Between Dependent Variables and SA Algorithm Parameters

SA algorithm, one of searching methods will be used to solve this case study. Searching strategy to get the global optimal solution is generally a combination of random search within a large range and a narrow search in a small range.

The value of algorithm parameters affect time spent to achieve routes, and also the routes and the total distance of all the routes, which are the quality of a solution. The selection criteria for algorithm variables would be discussed below:

- Initial temperature

In general, searching requirements of the algorithm could be met only when initial temperature is high enough. However, for different problems, the standard of 'high enough' is different. In large-scale problems, if  $T_0$  is too low, it will be very difficult to jump out of the range of local best solution.

Alternatively, to reduce the amount of calculation,  $T_0$  should not be too large. For this problem, 100 is already high enough to achieve a good solution and convergence process. Higher initial temperature is also implemented, however the result is no better than 100.

- Temperature decreasing function of  $T$

Temperature decreasing function has many forms, a common one is as followed:

$$T_{(k+1)} = T_{(k)}, k = 0, 1, 2, \dots \quad (3.1)$$

Where,  $\alpha$  is a constant which ranges from 0.5 to 0.99. 0.95 is picked in this research. Its value determines the cooling process.

Large  $\alpha$  leads to an increasing number of iterations and time spent to achieve solution will increase. However, there will be more transformations accepted, larger search space and more range would be visited during the process so that better final solution would be returned.

- Terminated temperature

Lower terminated temperature leads a trend to achieve a solution of better quality.

According to acceptable probability  $e^{\frac{-(E_j - E_i)}{kT}}$  of Metropolis criterion, when the temperature  $T$  is high the denominator is relatively large within the exponent, which is a negative exponential, so the value of the whole function tends to 1 (in fact, it is the probability to jump out of current solution). The worse solution will also be accepted, therefore, it is possible to escape from local minima and continue a new wide-area search in the solution space.

While with temperature cooling down, when  $T$  reduced to a relatively small value, the denominator is relatively small for exponent part and so is the value of the whole function, which means a small probability to accept the worse solution. If random search within a large range has been implemented at high temperatures and the range containing global optimal solution has been found, also, if enough narrow searches in a small range have been implemented, then it is possible to get the global optimal solution.

### 3.1.3 Determine Number of DC and Routes Between Local Warehouses and DC

The framework of this subproblem is showed as Figure 3.2.

Independent variables including vehicle capability, the capacity of DC, the number of potential DC(s) have an effect on dependent variables which contain number of DC selected, routes between DC and local warehouses and the total distance of these routes.

- Vehicle capacity

It has the same impact to routes and total distance as in the first echelon.

The larger the vehicle capacity, the fewer the number of routes and the less the total distance.

- Capacity of DC



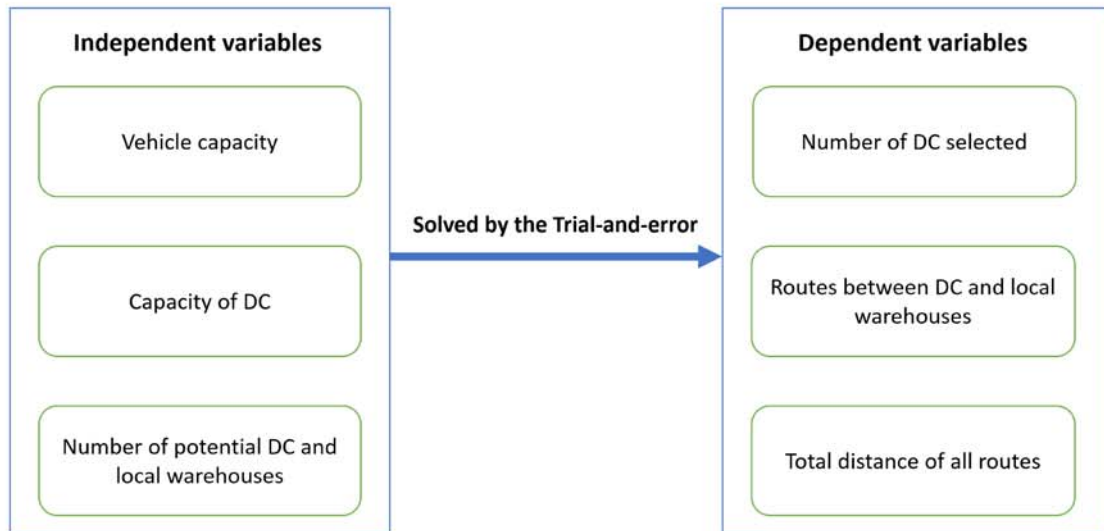


Figure 3.2. Theoretical framework between local warehouse and DC

The capacity of DC has little impact on the total distance. Comparing to the first echelon, the location problem itself in the second echelon is to pick from potential DC to minimize total distance. Therefore, no matter the capacity increases or decreases, DC(s) picked are the ones promising the minimized cost.

- Number of potential DC

Number of DC should be smaller than the number of local warehouses. After satisfying this condition, the more potential DC, the smaller the total distance. Since there is no fixed cost for DC, the more DC in the potential set, the more feasible solutions, it is more possible to achieve a better solution which is smaller total distance.

### 3.2 Methodology Overview

This research applies the two-echelon multi-depot VRP to a small scale case of a company's city logistics for product delivery. A sample company is created. The two echelons are separated into two stages.

- First stage (MDVRP): Develop optimal routes between local warehouses and customers using simulated annealing heuristic.

In Figure 3.3, the dashed red box highlights the scope of this first stage. The dashed yellow lines in Figure 3.3 are 'stem'. 'Stem' means connections between local warehouses and customers.

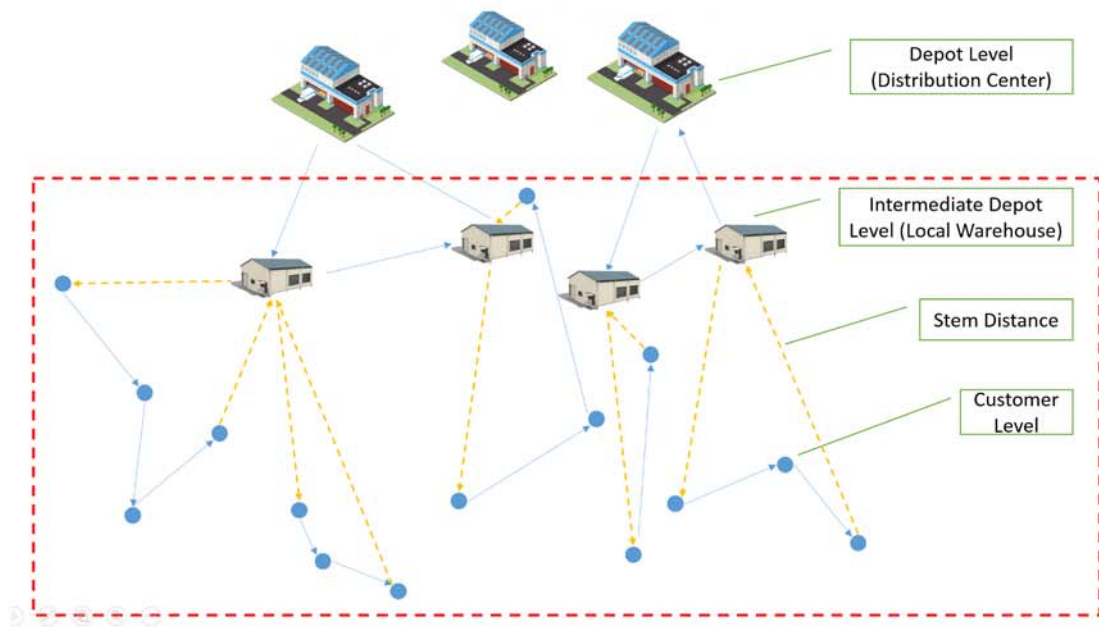


Figure 3.3. Network between local warehouses and customers

- Second stage (LRP) using Trial-and-error: it determines
  - The number of distribution centers  $n$
  - Location(s) of the picked distribution center(s) among potential DC set
  - Routes between the DCs and local warehouses

In Figure 3.4, the dashed red box highlights the scope of this second stage.

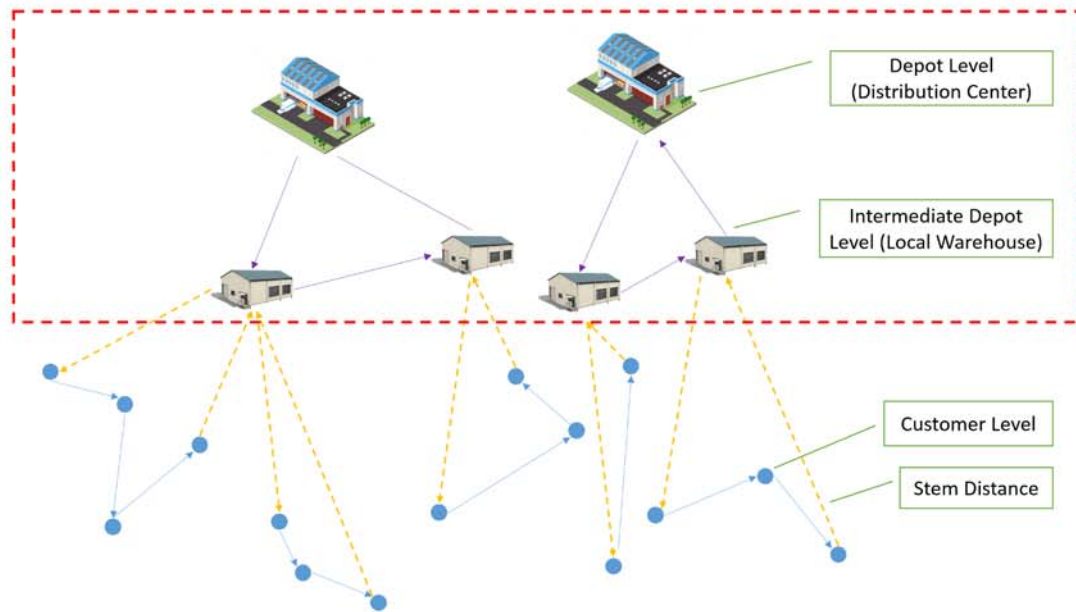


Figure 3.4. Network between DCs and local warehouses

The potential set of locations of distribution centers, local warehouses, and customers are demonstrated for clarity in understanding the problem and methodology.

### 3.3 Solving MDVRP between Local warehouses and Customers

The problem between the local warehouse and the customer belongs to multi-deopt VRP. The simulated annealing algorithm, a heuristic method is applied and coded in Matlab to determine the routes in order to minimize transportation and penalty costs.

The heuristic method is applied in this problem to improve calculation time and solution quality as there are hundreds of combination of routes and allocating routes to three local warehouses for enumeration.

Kirkpatrick, Vecchi, et al. (1983) applied simulated annealing to combinatorial optimization problems because of a series of similarities. The math

model is not necessary when heuristics are applied. The reason that heuristics could achieve a good solution in limited time is because it is a searching process with limited times. SA starts with a random solution and does random iterations, so there is no guarantee that the good solution is the global optimal solution.

SA is applied in 5 steps to solve this MDVRP. They are:

- Step 1:
  - Generate location coordinates of customers
  - Generate demand of customers
  - Generate coordinates of local warehouses
- Step 2: Generate Key Parameters
- Step 3: Determine Simulated Annealing Parameters
- Step 4: Implement SA Algorithm in MDVRP
  - Start solving this problem with a random initial solution
  - Start SA iterations
- Step 5: Obtain routes for MDVRP

### 3.3.1 Step 1: Generate Location Coordinates and Demand

Step 1 is to develop coordinates and demand of customers and coordinates of local warehouses.

- Generate location coordinates for the company local warehouses using 'RAND' and 'ROUND' functions in Matlab and save the coordinates in Excel. In this research, the sample company has three existing local warehouses. The location generated is shown in Figure 3.5:

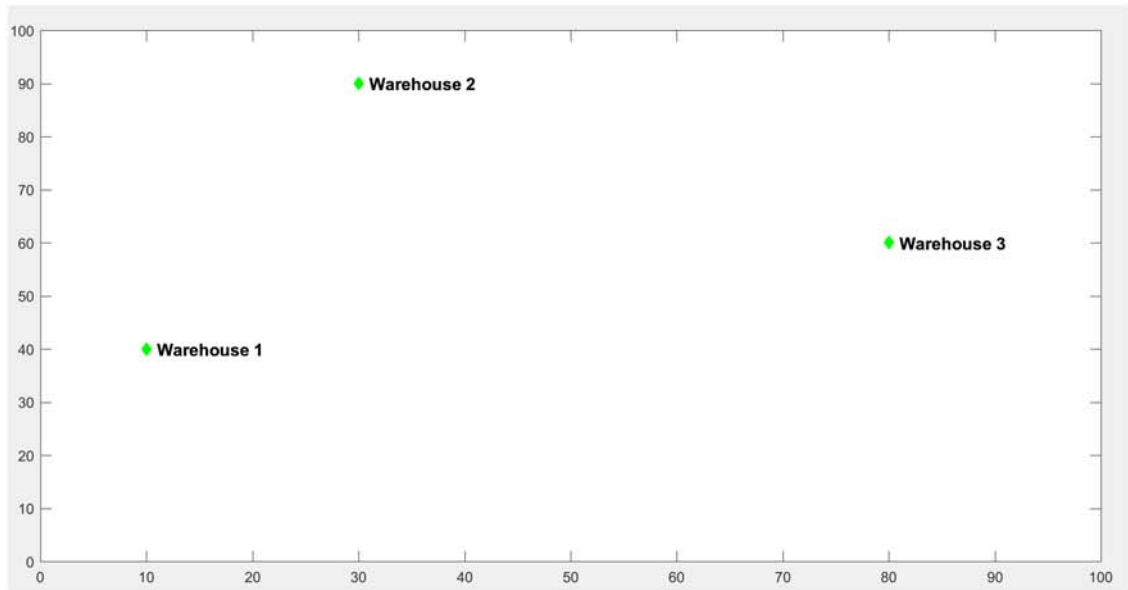


Figure 3.5. The location of the generated local warehouses

- Generate location coordinates for the customers and customer daily demand applying the 'RAND' and 'ROUND' functions in Matlab and export to Excel. The coordinates are shown in Figure 3.6. Assume the sample company has 19 customers with daily demand shown in Table 3.1.

Table 3.1  
Generated customer daily demand

customer number	1	2	3	4	5	6	7	8	9	10
demand	1.64	1.31	0.43	3.38	1.13	3.77	3.48	0.39	0.24	1.03
customer number	11	12	13	14	15	16	17	18	19	
demand	2.35	2.6	1	0.65	2.56	1.27	2.69	3.26	2.97	

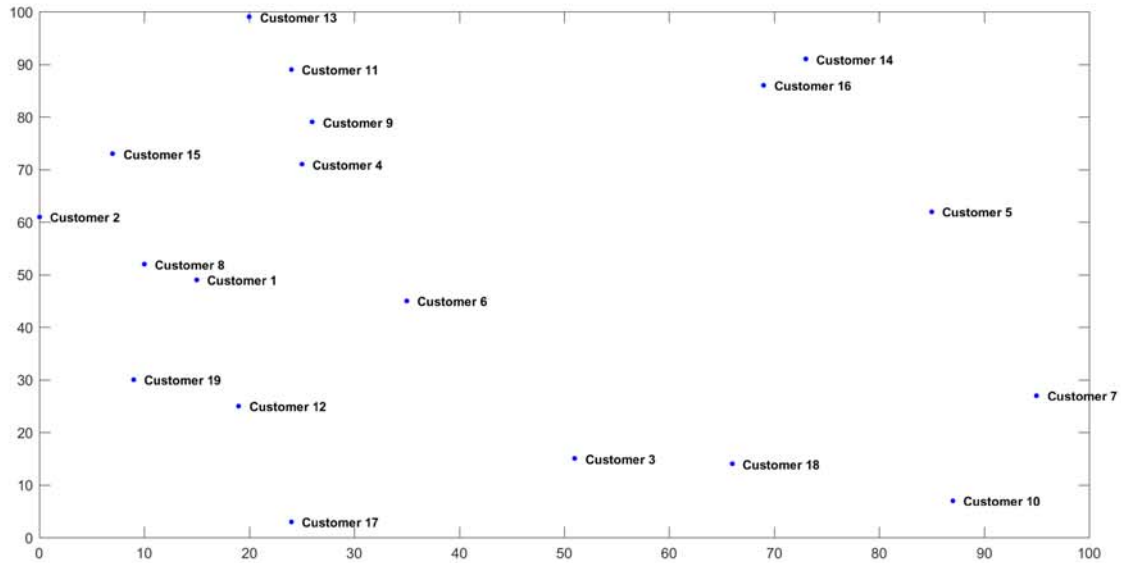


Figure 3.6. The location of the generated customers

### 3.3.2 Step 2: Determine the Value of Independent Variables and Key Parameters

After obtaining the data in Step 1, the independent variables are calculated. Theoretically, independent variables could be any value in the domain of definition. However, SA is a function of objective value and iteration times which is a representation of the whole searching process instead of a function of different independent variables and objective function, there is no iteration for independent variables, consequently, one set of independent variables values is selected to implement. The criteria is as follows:

- Vehicle capacity in units

Daily demand has been shown in Table 3.1 and the average is 1.90 units. It would be more economical if there are more customers served in one route since the distance connecting local warehouse to customers are saved. In this case study, 6 units are set for vehicle capacity which two or more customers

will be served averagely. 5 units or 7 units are also reasonable values for this independent variable and will lead to different optimal solutions.

- Capacity of local warehouses

Capacity is limited by the number of vehicles that send local warehouses. There are 3 local warehouses and total daily demand is 36.15. In case of the seasonal demand, each local warehouse could send at most 3 vehicles, which means maximum capacity of all three local warehouses is 54.

The penalty of local warehouses and trucking cost per mile are two key parameters needed to be generated.

- Penalty of local warehouses

A cost penalty to local warehouses occurs if the quantity that a local warehouse served exceeds its capacity. A high penalty cost would prevent warehouse overload.

Normally, there are two methods to exclude a bad solution for a minimized cost problem. One is a strict constraint if the quantity (demand) served by one local warehouse exceeds its capacity, it is not a feasible solution. The other is to add a cost penalty to the objective function, which is also called the fitness function. Overload is not acceptable, so even if there exists a shorter route, the total cost would be high, but the goal is to minimize cost. Because heuristics algorithms are not like enumeration, which could list all possibilities, it is more common to add penalty costs. Strict constraints sometimes limit feasible solution with a limited number of iterations.

- Trucking cost per mile

Total cost equals trucking cost per mile times total distance and cost of the penalty. According to the data from RTSFinancial, the total trucking cost per mile is \$1.098, so \$1.0 is assumed in this problem.

### 3.3.3 Step 3: Determine Simulated Annealing Parameters

The annealing process is controlled by a set of initial parameters, which is the cooling schedule, including necessary parameters for SA algorithm. The essential is to achieve equilibrium so that the algorithm could approach global optima in a limited time. The cooling schedule includes:

- Initial temperature  $T_0$
- Temperature decreasing function of  $T$
- $T_f$  is a temperature set in advance to make loop stop
- Length of Markov Chain  $L_k$ : Iteration times at one temperature  $T$

$T_0$  should generally be set to a sufficiently large positive number. In large-scale problems, if  $T_0$  is too low, it will be very difficult to jump out of the range of local best solutions. Alternatively, to reduce the amount of calculations,  $T_0$  should not be too large.

$T_f$  should be set to a sufficiently small positive number, such as 0.01 to 5, but this is only a rough estimate with a more sophisticated set of final value and other criteria can be found in Aarts and Korst (1988) and Johnson, Aragon, McGeoch, and Schevon (1989).

### 3.3.4 Step 4: Implement SA Algorithm in MDVRP

Apply the SA algorithm to MDVRP and the corresponding procedure are as Figure 3.7.

SA starts with a random initial solution and then record the initial solution in Matlab. There are outer and inner loops. Initial temperature equals to 100,  $\alpha = 0.95$  after several trials for this case study.

Outer loop: When  $T = T_i$ , inner iteration runs 50 times and then it jumps to next  $T : T_j = 0.95T_i$ . Outer loops runs 50 times which means  $T$  decreases 50 times. Every time,  $T$  is 0.95 times of last time's  $T$ .



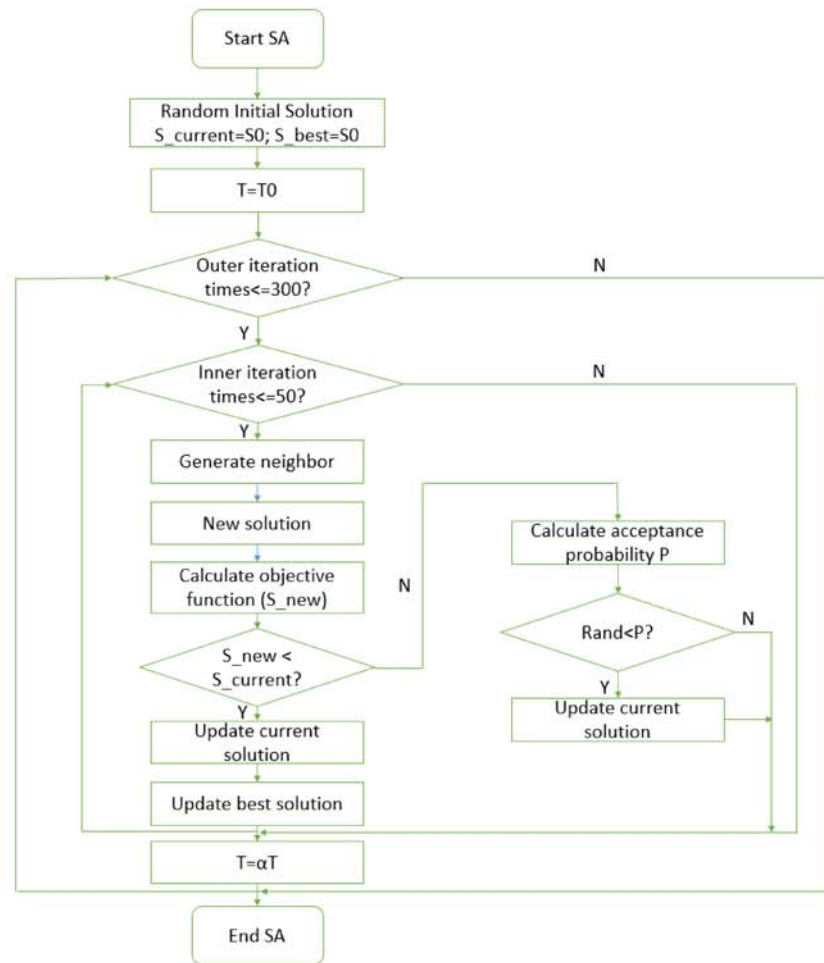


Figure 3.7. Flow chart of SA

Inner loop: Every inner iteration first generates a new neighbor and then runs the same process as achieving an initial solution. After this, the new objective function is judged based on SA acceptance rules. Details are discussed in the following section.

### 3.3.4.1. Start solving with a random initial solution

To demonstrate for clarity, a series of index figures are used to show the corresponding step in the flow chart in Figure 3.8. It means more details of generating random initial solution will be presented as follows.

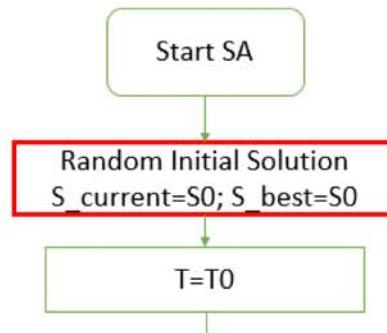


Figure 3.8. Flow chart index 1

1. Number all 19 customers from 1 to 19
2. Use 'RANDPERM' function in Matlab to generate a random sequence of these 19 numbers as Figure 3.9



Figure 3.9. Random sequence of 19 customers

3. Allocate all 19 customers into several circles

The first vehicle serves only Customer 7 and Customer 1. The starting point in this route is Customer 7 and the ending point of this route is Customer 1. Total demand is  $3.48 + 1.64 = 5.12$  (See Figure 3.10)

The following Customer 13 is not in this circle because the demand of Customer 13 plus total customer demand for Customer 7 and 1 ( $5.12 + 1 = 6.12$ ) will exceed the vehicle capacity of 6 units (See Figure 3.11).

sol.routing							
1	2	3	4	5	6	7	8
1 [7,1]	[13,4]	[11,9,18]	[8,2,15]	[6,3]	[19,17]	[10,14,5,12]	16

Daily demand of customers

customer number	1	2	3	4	5	6	7	8	9	10
demand	1.64	1.31	0.43	3.38	1.13	3.77	3.48	0.39	0.24	1.03
customer number	11	12	13	14	15	16	17	18	19	
demand	2.35	2.6	1	0.65	2.56	1.27	2.69	3.26	2.97	

Figure 3.10. First route of 19 customers

Daily demand of customers

customer number	1	2	3	4	5	6	7	8	9	10
demand	1.64	1.31	0.43	3.38	1.13	3.77	3.48	0.39	0.24	1.03
customer number	11	12	13	14	15	16	17	18	19	
demand	2.35	2.6	1	0.65	2.56	1.27	2.69	3.26	2.97	

Figure 3.11. Demand of Customer 7, 1 and 13

According to this sequence, from the first customer in the queue, group all these customers into several circles (It has been assumed in Chapter 1 that each circle could only be served by one vehicle). Therefore, the queue is cut into several small pieces where each route delivers as much product without exceeding vehicle capacity 6 units. The result is shown in Figure 3.12. Figure 3.13 shows the pictorial 8 circles.

#### 4. Allocate circles to different local warehouses

The 8 circles should be all connected to one of the three warehouses and each warehouse could at most connect three circles. The logic is as follows:

- For each route, connect it with the three local warehouses. Route 7 is a circle of 10-14-5-12 in blue lines as shown in Figure 3.14.

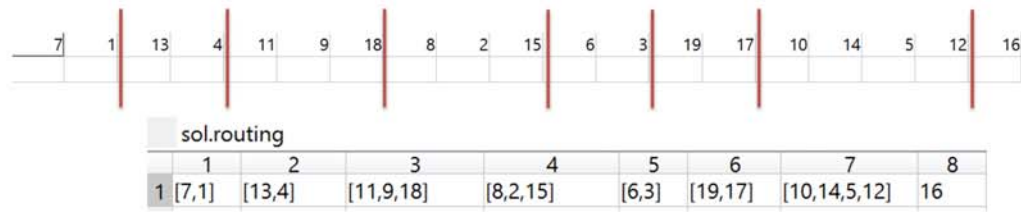


Figure 3.12. 8 circles of 19 customers

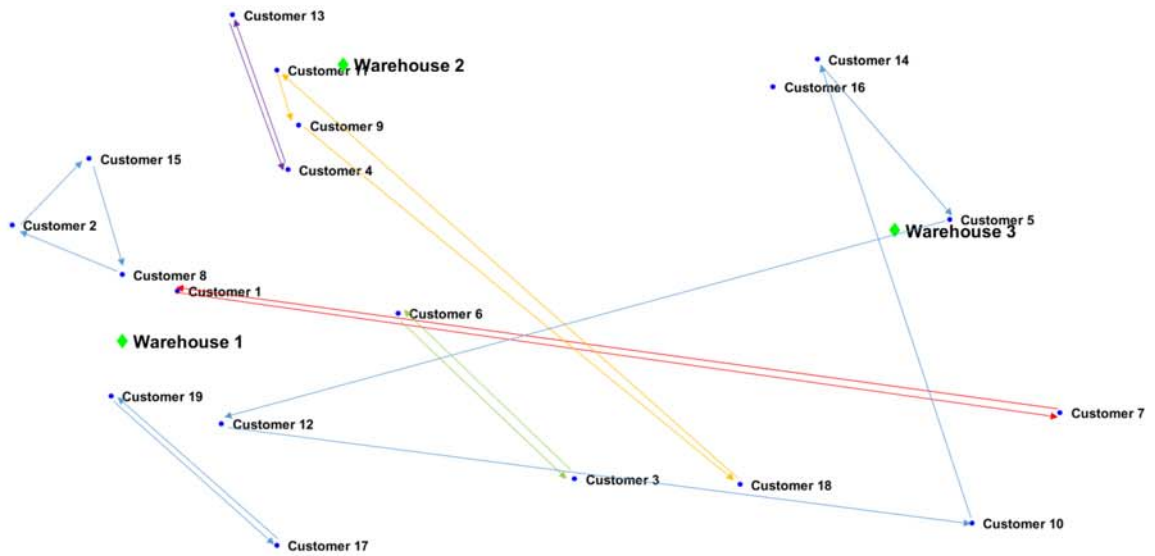


Figure 3.13. Pictorial presentation of 8 circles

The connection between Customer 12 and 10 is replaced by two yellow dashed lines when Route 7 connecting to one warehouse. The yellow dashed lines are stem distance. In Figure 3.14, it shows three pairs of stem which are Route 7 connecting to three local warehouses respectively. For each route, three pairs of the stem distance need to be calculated and shortest stem distance should be picked.

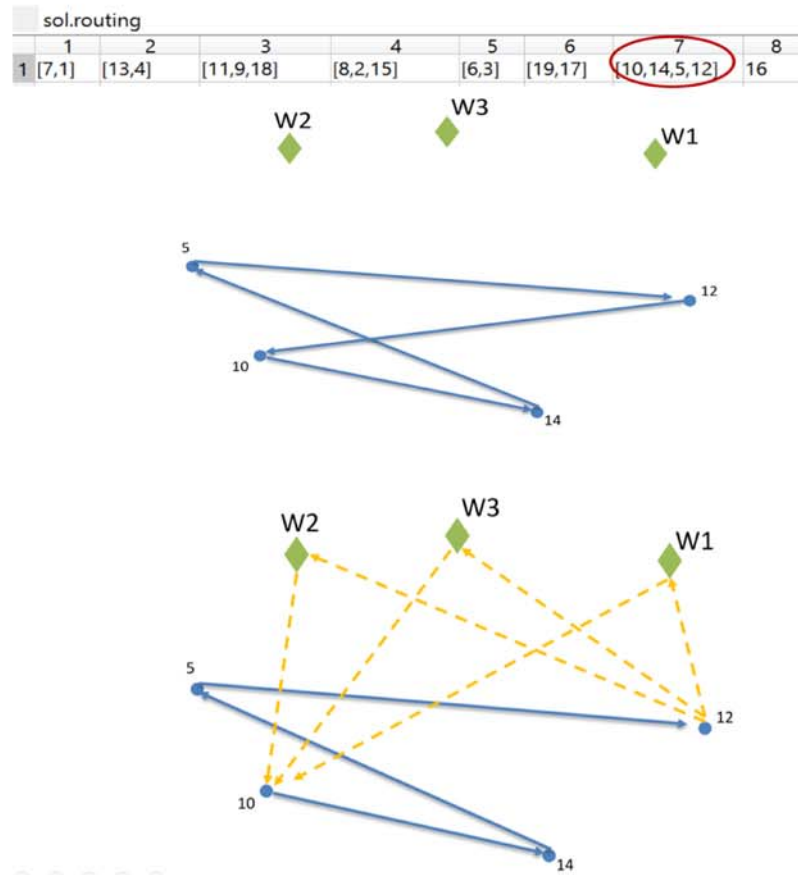


Figure 3.14. Route allocation criteria

- (b) All 8 circles are allocated to three local warehouses as shown in Figure 3.15. Route 1: Customer 7 and 1 connect to Local warehouse 1, etc. Additionally, pictorial routes are shown in Figure 3.16.

##### 5. Calculate total transportation cost

Add the distance of each circle together to achieve the total distance. Since the trucking cost is one dollar per mile, total trucking equals total distance (See Figure 3.17).

##### 6. Calculate penalty cost

sol.routing								
	1	2	3	4	5	6	7	8
1	[7,1]	[13,4]	[11,9,18]	[8,2,15]	[6,3]	[19,17]	[10,14,5,12]	16

sol.allocation								
	1	2	3	4	5	6	7	8
1	1	2	2	1	1	1	1	3

Figure 3.15. Allocation of 8 circles to three warehouses

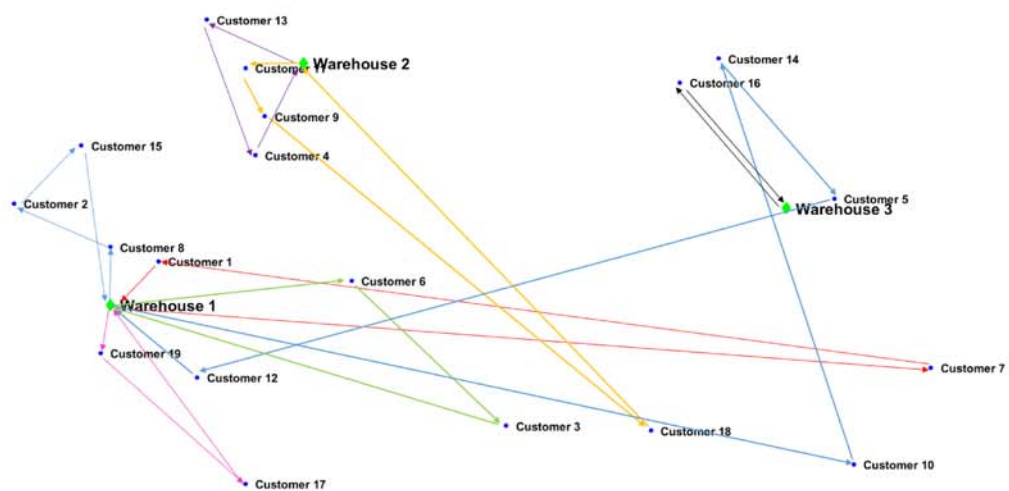


Figure 3.16. Pictorial presentation of allocation of 8 circles to three warehouses

sol.Objfun								
	1	2	3	4	5	6	7	8
1	545.5532							
2								

Figure 3.17. Trucking cost for 8 routes

There are 5 routes connected to Warehouse 1 which exceeds the warehouse capacity (3 vehicles). The penalty should also be added to the objective



function. The penalty for one route is 800, so in this random initial solution, \$1,600 is the penalty cost. The total cost is as Figure 3.18.

sol x sol.Objfun x sol.Objfunfit x								
sol.Objfunfit								
	1	2	3	4	5	6	7	8
1	2145.553							
2								

Figure 3.18. Total cost of initial solution

3.3.4.2. Start SA iterations

The SA algorithm iterations iteratively solve the VRP by checking whether there is a better solution, lower cost solution by changing route. When the lowest possible cost is achieved and no iteration provides a better cost, the algorithm is complete and the lowest cost solution is achieved.

1. When  $T = T_0 = 100$ ,
  - (a) 2-opt swap to generate neighbor (a new sequence of 19 customers)  $\longrightarrow$

Figure 3.19

Assume  $a$  and  $b$  are two random numbers ( $a < b$ ). 2-opt will swap the sections between  $a$  and  $b$ . The original sequence, for example, is  $\pi_1, \pi_2, \pi_3, \dots, \pi_n$ . After swap, the new sequence is  $\pi_1, \pi_2, \dots, \pi_{a-1}, \pi_b, \pi_{b-1}, \dots, \pi_{a+1}, \pi_a, \pi_{b+1}, \dots, \pi_n$ . For example, if  $n = 100, a = 20, b = 70$ , the new sequence is  $\pi_1, \pi_2, \dots, \pi_{19}, \pi_{70}, \pi_{69}, \dots, \pi_{21}, \pi_{20}, \pi_{71}, \dots, \pi_{100}$ . The segment between 20 and 70 has swapped. If this sequence change process is expressed by color spectrum, it is shown as below (Figure 3.20):

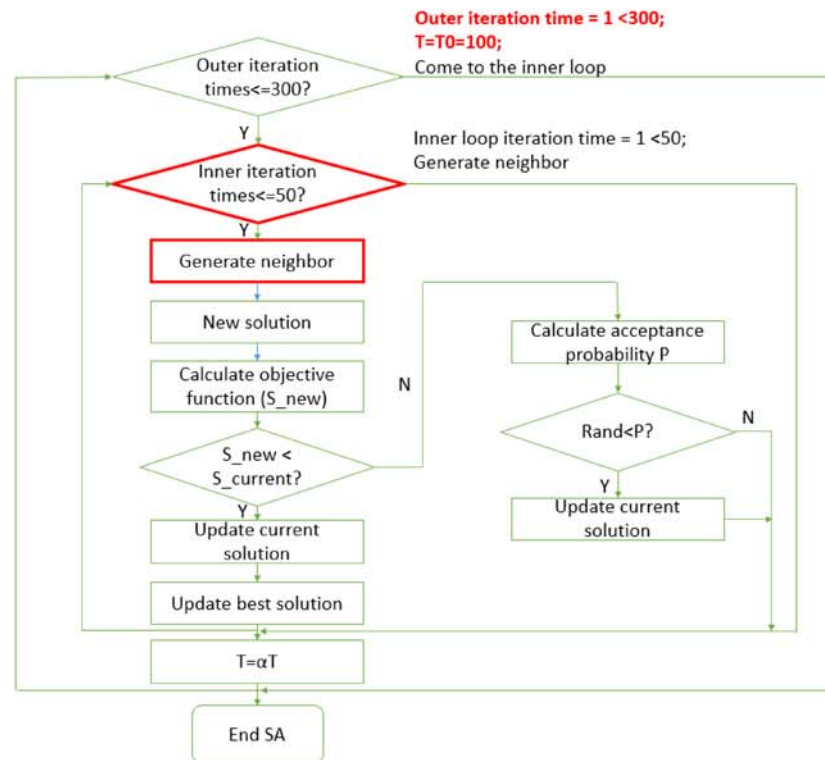


Figure 3.19. Flow chart index 2

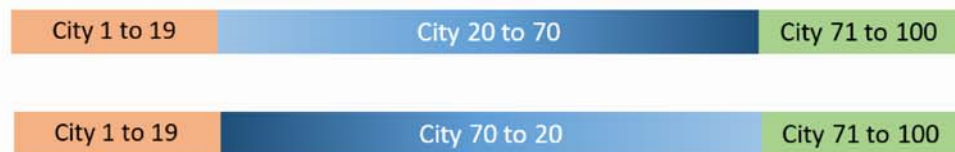


Figure 3.20. Demonstration of 2-opt swap

19 customers have been numbered from 1 to 19 in Section 3.3.4.1, after this generating neighbor step, a new sequence of these 19 customers is created.

- (b) Calculated new objective function (cost of transportation and penalty of new neighbor) → Figure 3.21



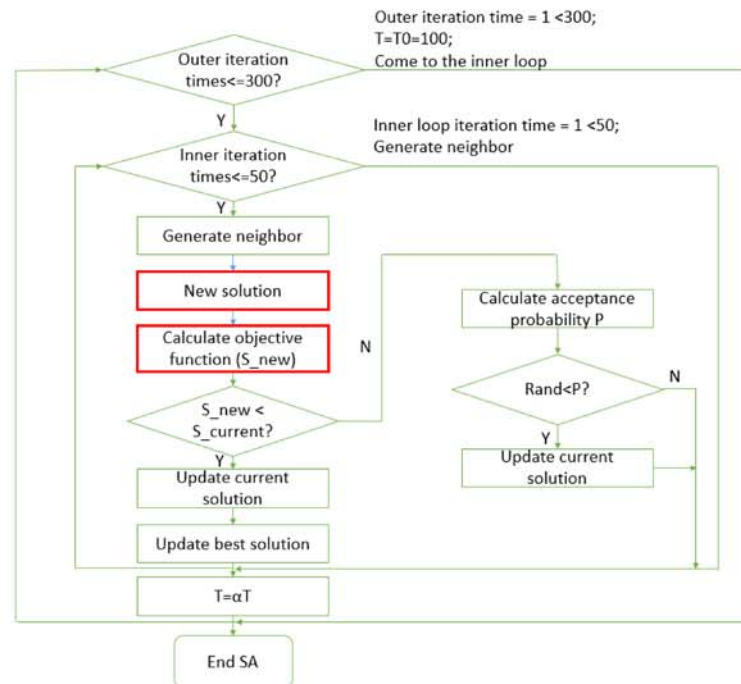


Figure 3.21. Flow chart index 3

Repeat the steps of generating initial solutions as Section 3.3.4.1 after achieving a new sequence of 19 customers to generate new solution and calculate new cost:

- i. Allocate all 19 customers into several circles
- ii. Allocate circles to different local warehouses
- iii. Calculate total transportation cost
- iv. Calculate penalty cost

(c) Accept current solution? → Figure 3.22

- i. Decision route 'Y'

If the current total cost is smaller than initial solution, update the solution to current solution;

- ii. Decision route 'N'

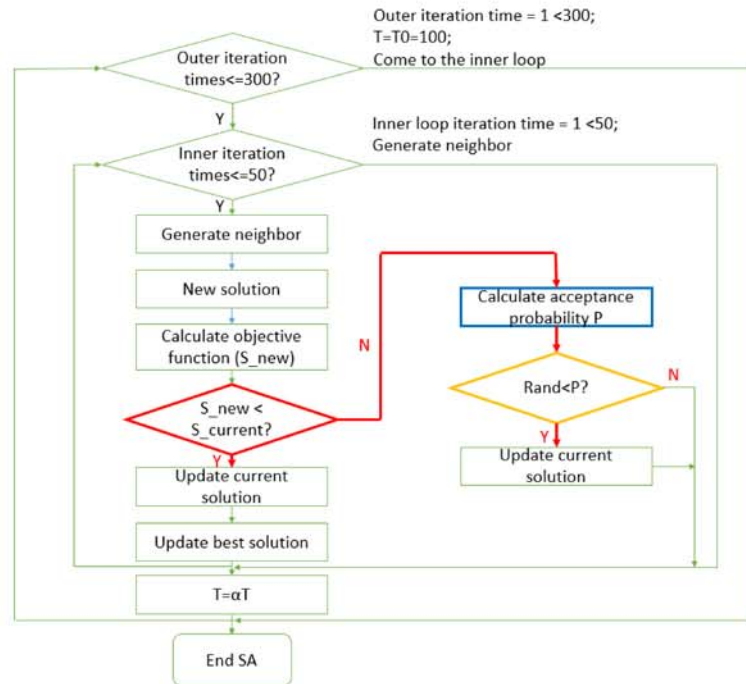


Figure 3.22. Flow chart index 4

Process blue box→Calculate acceptance probability: If the current cost is larger than initial solution, calculate  $e^{\frac{-\Delta E}{T_i}}$  which is the probability to accept this worse solution;

- iii. Decision orange box→Compare  $e^{\frac{-\Delta E}{T_i}}$  with a random positive number between 0 and 1 generated by Matlab 'RAND' function. If  $\text{rand} < \text{probability}$ , accept the worse solution as current solution; or refuse this solution.

(d) Update current solution → Figure 3.23

After determination whether to accept the new cost, there are two results: 1) Keep the cost calculated from previous iteration inner loop; 2) Update this new cost as a better solution.

(e) Repeat the inner loop for 50 times→ Figure 3.24

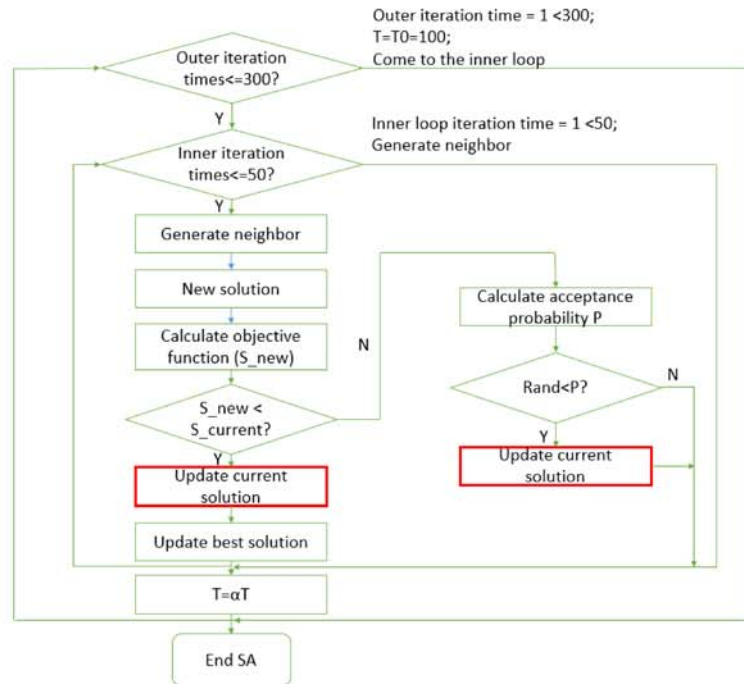


Figure 3.23. Flow chart index 5

The inner loop contains steps (a), (b), (c), (d) above and is shown in the dashed red box in Figure 3.24. When the inner loop runs 50 times, it will jump to the second outer loop.

- (f) Jump to next temperature to calculate cost → Figure 3.25. Next temperature is  $T = 0.95 \times 100$  (current temperature) = 95, outer loop = 2.

The worse solution is accepted to avoid trapping in local searching in SA process. As Figure 3.26, there are global optimal solution and local optimal solution. Global optimal solution is the best solution, which in this problem, is the lowest total cost. Local optimal is the lowest point within a small area. Worse solution assists jumping out of local searching. At the beginning of this temperature decreasing, there is a larger possibility have a global searching.

2. When  $T = 95$ ,

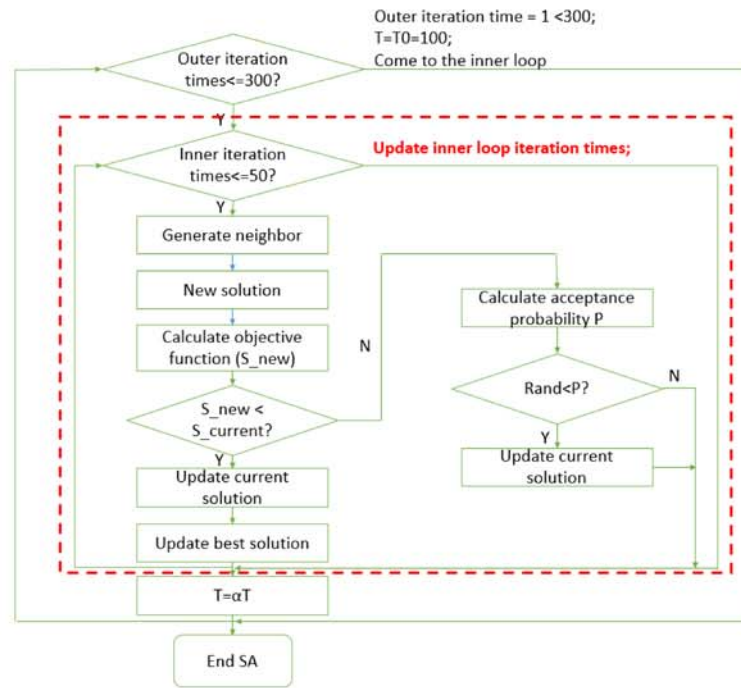


Figure 3.24. Flow chart index 6

- (a) 2-opt swap to generate neighbor
- (b) Calculated new objective function (cost of transportation and penalty of new neighbor)

Repeat the steps of generating initial solutions as Section 3.3.4.1 after achieving a new sequence of 19 customers to generate new solution and calculate new cost:

- i. Allocate all 19 customers into several circles
- ii. Allocate circles to different local warehouses
- iii. Calculate total transportation cost
- iv. Calculate penalty cost

- (c) Accept current solution?

- i. If the current cost is smaller than the previous solution, update this solution to current solution;

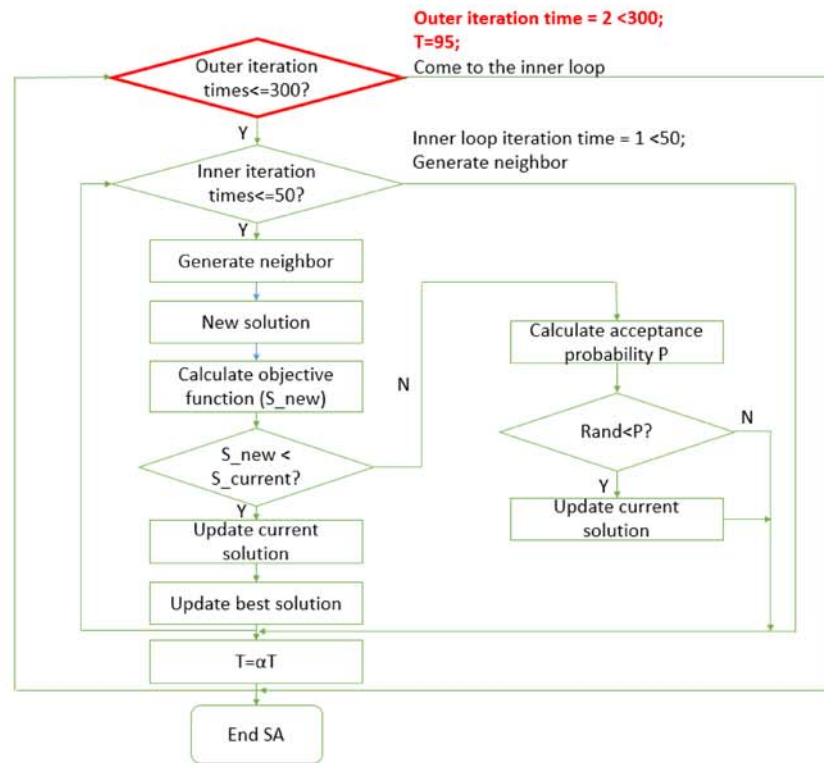


Figure 3.25. Flow chart index 7

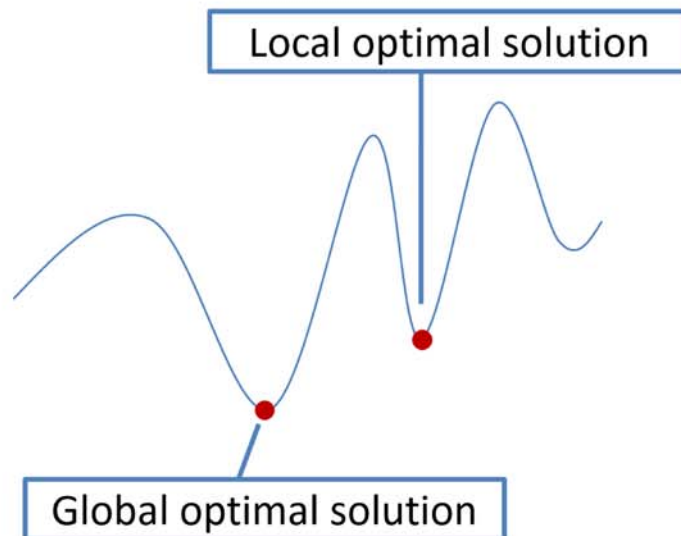


Figure 3.26. Difference between global optimal and local optimal

- ii. If the current cost is larger than initial solution, calculate  $e^{\frac{-\Delta E}{T_i}}$  which is the probability to accept this worse solution;
  - iii. Compare  $e^{\frac{-\Delta E}{T_i}}$  with a random positive number between 0 and 1 generated by Matlab 'RAND' function. If  $\text{rand} < \text{probability}$ , accept the worse solution as current solution; or refuse this solution.
- (d) Update current solution
  - (e) Repeat the inner loop for 50 times
  - (f) Jump to next temperature to calculate cost. Next temperature is  $T = 0.95 \times 95$  (current temperature) = 90.25

The outer loop is a process of temperature decreasing. It terminates when temperature decreases for 300 times, at which time  $T = T_0 \times 0.95^{300}$

### 3.3.5 Step 5: Obtain routes for MDVRP

The detailed results will be shown in Chapter 4 which includes the routing between warehouses and customers.

### 3.3.6 Matlab coding logic

The whole SA process is in the main script in which other steps including neighbor generation, cutting 19 customers to different routes, routes allocation are made in separate M-files as functions and called in the main script. Functions are discussed in this section.

- Cutting 19 customers to different routes

Loop structure is used here to cut 19 customers to different routes. The flow chart is as Figure 3.27. In the flow chart,  $a$  is used to represent the number of the first city in one route,  $j$  is the number of cities in one route,  $T_{\text{load}}$  is temporary load,  $i$  runs from 1 to 19.

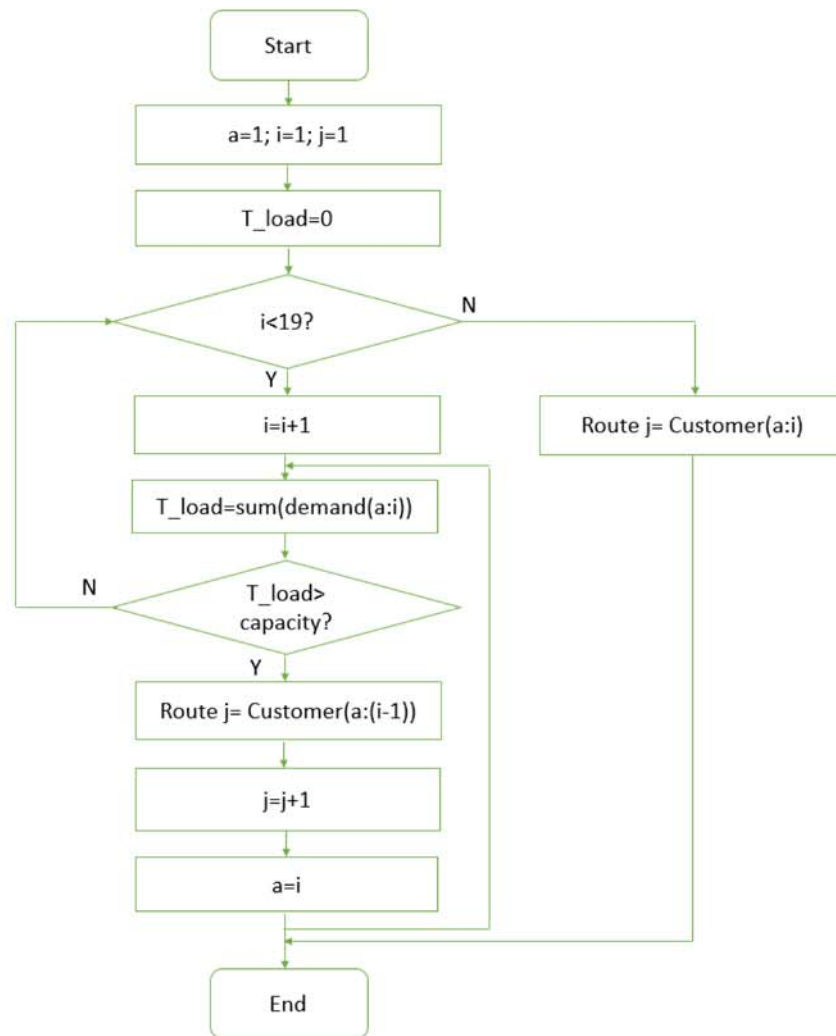


Figure 3.27. Coding flow chart for cutting customers

- Routes allocation

For each route, calculate all stem distance (three for this problem because there are three local warehouses) using a loop structure. The warehouse with shortest stem distance is picked using 'Min' function.

These two steps are included in 'costfun' M-file.

- Neighbor generation

Neighbor generation is called in the main file as a separate function. The random sequence of 19 cities is a  $1 \times 19$  matrix and 'flipfr' function is used to flip a matrix, which means the sequence of 19 cities is changed.

Both SA and cost function code are attached in Appendices.

### 3.4 Location-routing Problem between DC and Local Warehouse

Trial-and-error solutions are time-consuming. The reason for selecting trial-and-error is first, this location routing problem is repeated every two or more years to minimizing total cost because location decisions are relatively a long-term decision. In addition, it is only suitable for the small case. For this problem, there are only two DCs in the potential set and three local warehouses.

This location-routing problem is to determine:

1. The number of DC(s)
2. The location of the picked DC(s)
3. The routes between local warehouses and the picked DC(s)

Since the number of DC and local warehouses is small, trial-and-error is applied to solve this problem.

#### 3.4.1 Step 1: Determination of basic parameters and independent variables

- Number of DC(s)

In this case, there are three local warehouses, and DC's normally cover larger areas than local warehouses, so the number of DC should be less than three.

In this example, there should be one or two DC(s).

- Vehicle capacity for echelon of DC and local warehouse



Vehicles running in this echelon have a larger capacity than vehicles between local warehouses and customers. Five times large as the vehicle is picked, which is vehicle capacity is 30 units. Similarly, smaller or larger vehicles of 4 times or 6 times are also reasonable.

- Capacity of DCs

The capacity of DCs is an independent variable. Based on the previous assumption that maximum service one local warehouse could provide is  $6 \times 3 =$  capacity of each vehicle  $\times$  number of vehicles  $= 18$  and there are three local warehouses. The total service capability is  $18 \times 3 = 54$ . From the DC perspective, this is the total demand of local warehouses. To avoid seasonal problems, assume that larger vehicles can be used to deliver goods between DCs and local warehouses. The vehicle capacity is 30. The number of vehicles that can be sent from each DC is 3. The total demand of 54 is smaller than the delivering capacity of one depot 90, therefore, all three local warehouses could be served by one DC.

- Generate demand of local warehouses

Develop demand using 'RAND' function in Matlab and it is shown in Table 3.2 .

Table 3.2  
Demand of three local warehouses

	Warehouse 1	Warehouse 2	Warehouse 3
demand	10	30	20

- Generate coordinates of the two DCs using 'RAND' function in Matlab.
- Calculate distance

Number the Depot  $A$  (2,94) and Depot  $B$ (85,7) and the local warehouses 1(10,40), 2 (30,90) and 3 (80,60). The distance matrix among local warehouses is calculated as Table 3.3. The distance matrix between DC and local warehouses is showed in Table 3.4.

Table 3.3  
Distance Matrix among 3 local warehouses

	Warehouse 1	Warehouse 2	Warehouse 3
Warehouse 1	0	53.58	72.80
Warehouse 2	53.85	0	58.31
Warehouse 3	72.80	58.31	0

Table 3.4  
Distance Matrix between local warehouses and DCs

	Depot A	Depot B
Warehouse 1	54.59	81.94
Warehouse 2	28.28	99.57
Warehouse 3	85.10	53.24

#### 3.4.2 Step 2: Trial-and-error

All possible situations with three local warehouses served by either one or two DCs will be discussed in this section.

- Depot A is picked

Route 1: Depot A- warehouse 1-warehouse 3-Depot A

Route 2: Depot A-2-Depot A

The total distance is:  $54.59 + 72.80 + 85.10 + 28.28 \times 2 = 268.31$ . The routes are shown in Figure 3.28.

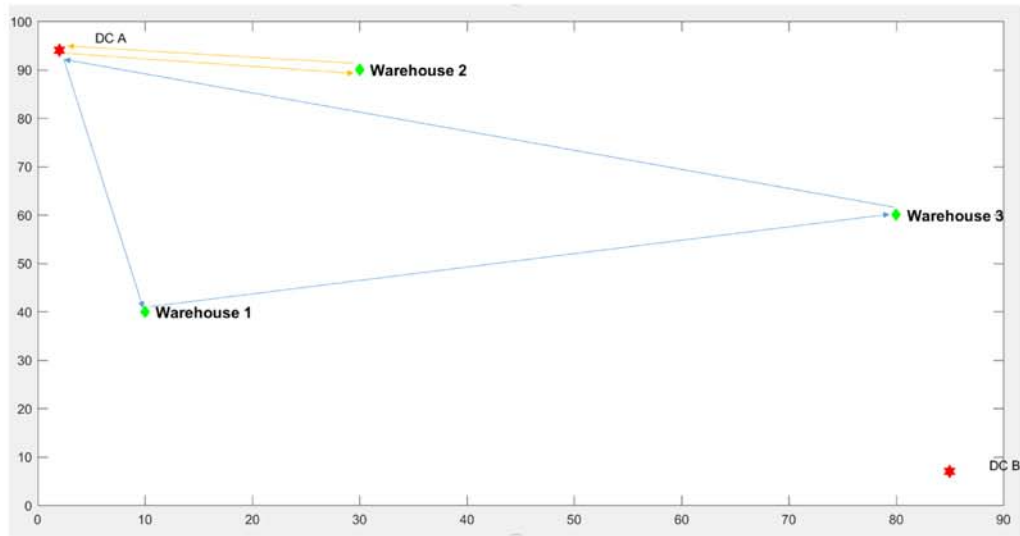


Figure 3.28. The routes of Depot A serving three warehouses

- Depot B is picked

Route 1: Depot B-Warehouse 1- Warehouse 3-Depot B

Route 2: Depot B-2-Depot B

The total distance is:  $81.94 + 72.80 + 53.24 + 99.57 \times 2 = 407.12$ . The routes are shown in Figure 3.29.

- Both Depot 1 and 2 are selected, there are three combinations as follows.
  1. Warehouse 1 is served by Depot B, Warehouse 2 and 3 are served by Depot A (See Figure 3.30)
  2. Warehouse 2 is served by Depot B, Warehouse 1 and 3 are served by Depot A (See Figure 3.31)
  3. Warehouse 3 is served by Depot B, Warehouse 1 and 2 are served by Depot A (See Figure 3.32)

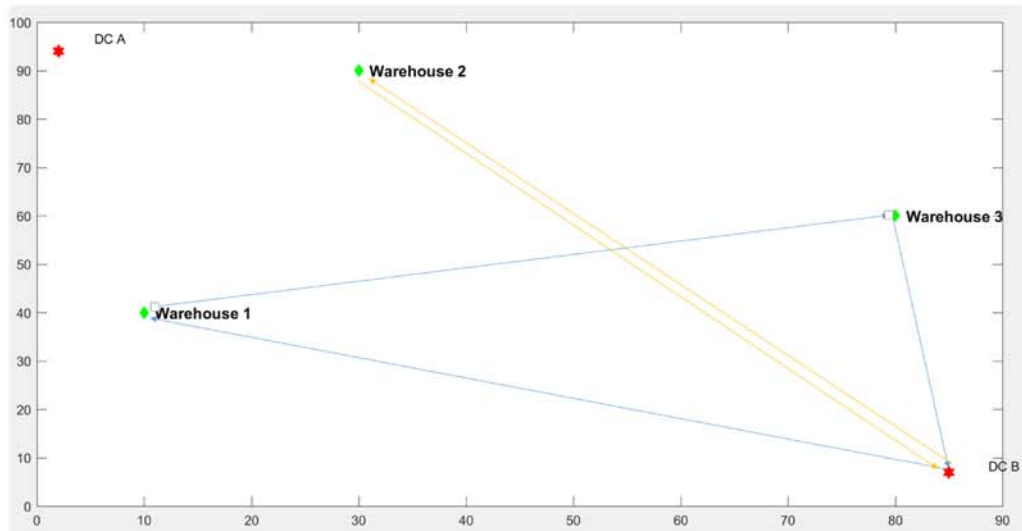


Figure 3.29. The routes of Depot B serving three warehouses

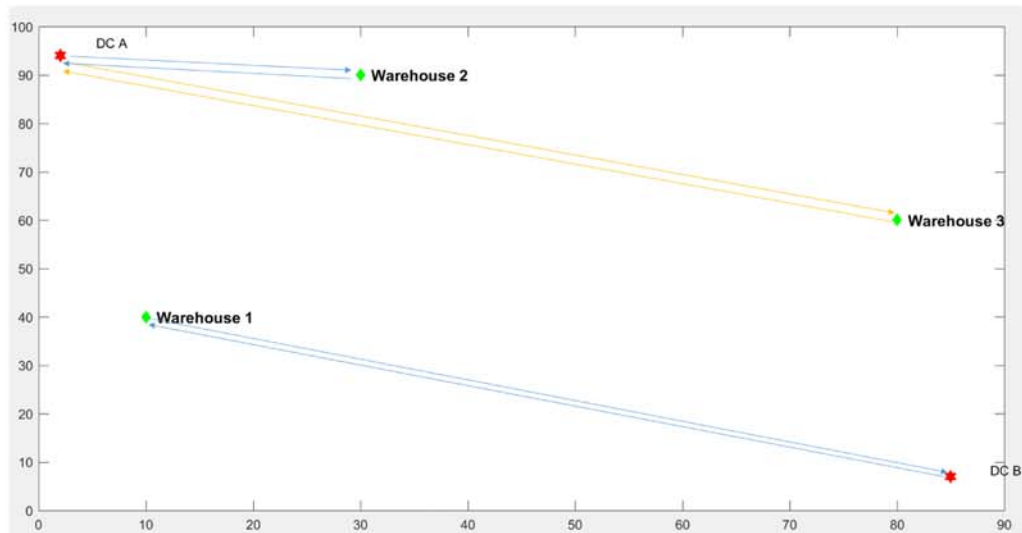


Figure 3.30. The routes of Depot B serving Warehouse 1

4. Warehouse 1 is served by Depot A, Warehouse 2 and 3 are served by Depot B
5. Warehouse 2 is served by Depot A, Warehouse 1 and 3 are served by Depot B

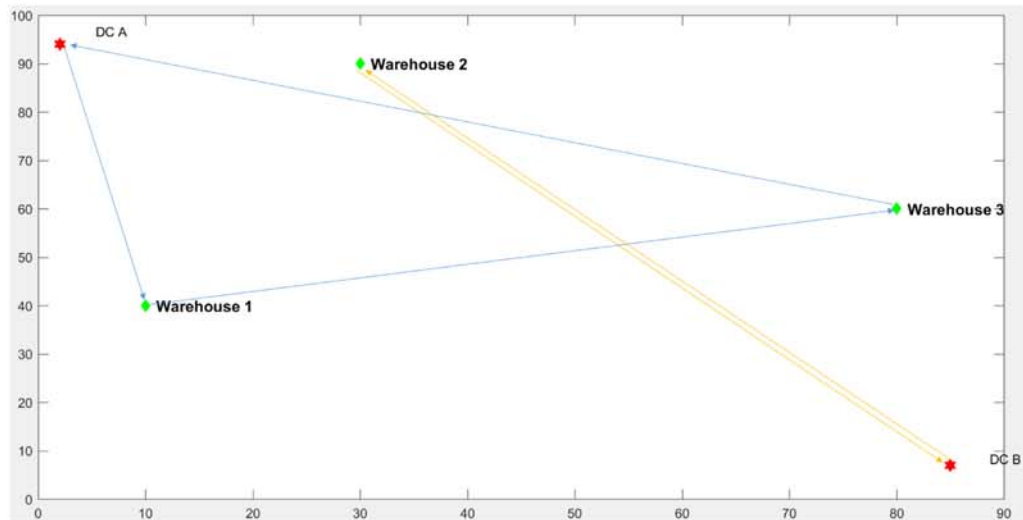


Figure 3.31. The routes of Depot B serving Warehouse 2

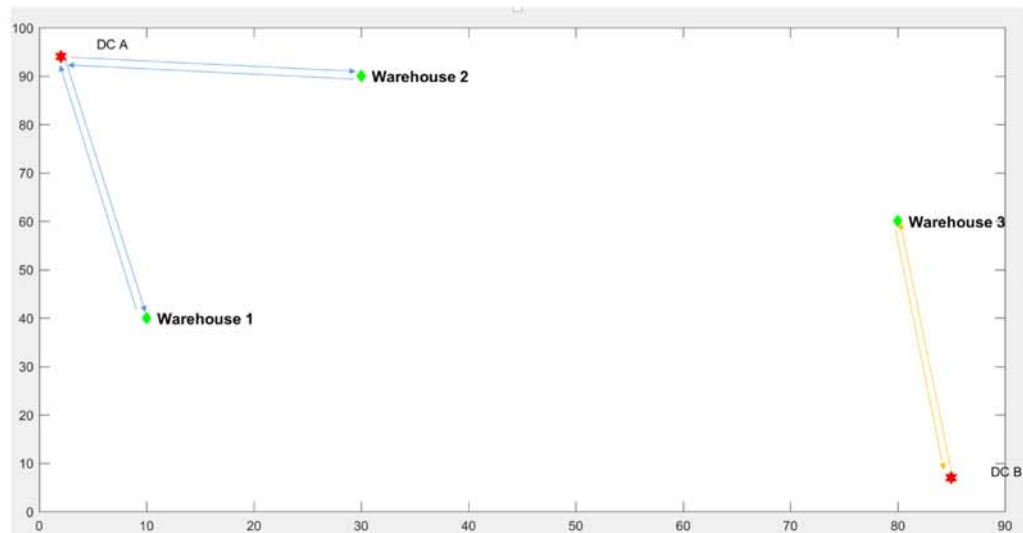


Figure 3.32. The routes of Depot B serving Warehouse 3

6. Warehouse 3 is served by Depot A, Warehouse 1 and 2 are served by Depot B

The corresponding distance is:

1.  $(28.28 + 85.10 + 81.94) \times 2 = 390.64$

$$2. 54.59 + 72.80 + 85.10 + 99.57 \times 2 = 411.63$$

$$3. 54.59 \times 2 + 28.28 \times 2 + 53.24 \times 2 = 272.22$$

$$4. (54.59 + 99.57 + 53.24) \times 2 = 414.8$$

$$5. 28.28 \times 2 + 81.94 + 72.8 + 53.24 = 264.54$$

$$6. (85.10 + 81.94 + 99.57) \times 2 = 533.22$$

The shortest path is Warehouse 2 is served by Depot A, Warehouse 1 and 3 are served by Depot B and the total distance is 264.54.

In Chapter 3, case study company data generation and selection of key SA parameters were explained. This chapter has also explained the specific methodology employed in this sample company case study for both the two echelons among DCs, local warehouses and customers. Chapter 4 presents the results from the case study and covers the sample company case study results.

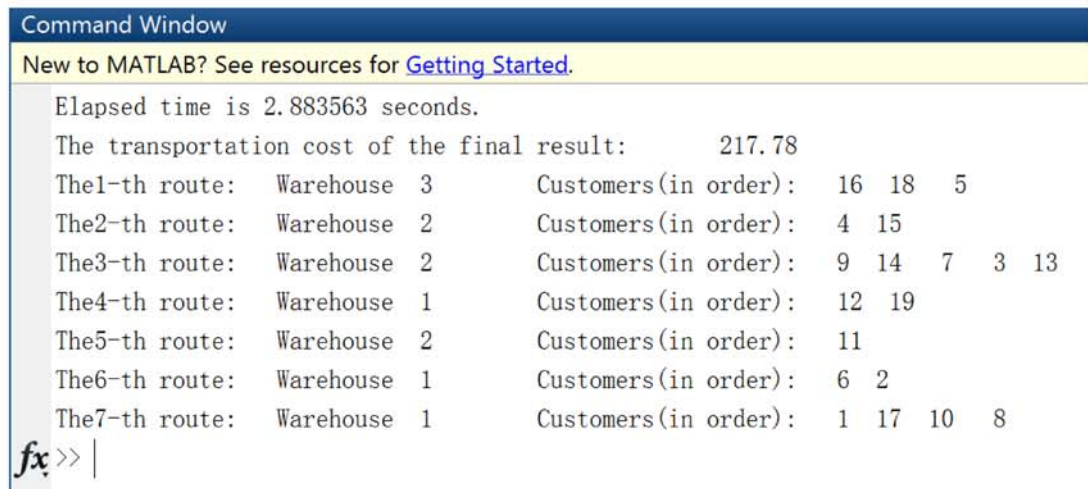
## CHAPTER 4. COMPUTATIONAL RESULTS AND CONCLUSIONS

The small case is conducted and results are analyzed in the following subsections. Section 4.1 illustrates the time spent achieving the results, convergence curve and final routes between local warehouses and customers. Section 4.2 shows the optimal solution of trial-and-error method. Section 4.3 gives the routes of two echelons from a pictorial view.

### 4.1 Results illustration for Multi-depot VRP Problem

The results of this case study are as follows when : the vehicle capacity is 6 units, each warehouse could only send at most 3 vehicles, penalty cost is 800, trucking cost per mile is 1, outer iteration is 300, inner iteration is 50, the initial temperature is 100 and decreasing rate is 0.95.

Figure 4.1 shows the time spent to get the result, total travel distance and allocation of seven total routes to 3 local warehouses.



```

Command Window
New to MATLAB? See resources for Getting Started.

Elapsed time is 2.883563 seconds.
The transportation cost of the final result:      217.78
The1-th route: Warehouse 3      Customers(in order): 16 18 5
The2-th route: Warehouse 2      Customers(in order): 4 15
The3-th route: Warehouse 2      Customers(in order): 9 14 7 3 13
The4-th route: Warehouse 1      Customers(in order): 12 19
The5-th route: Warehouse 2      Customers(in order): 11
The6-th route: Warehouse 1      Customers(in order): 6 2
The7-th route: Warehouse 1      Customers(in order): 1 17 10 8
fx >> |
  
```

Figure 4.1. Routes allocation results in Matlab

The SA algorithms were implemented in MATLAB<sup>®</sup> 2015b platform and carried out on a personal computer with common specifications of 2.30GHz Intel Core i5, 8 MB RAM. Time spent to solve the MDVRP is 2.88 seconds. There are seven total routes. The longest route contains five customers and the shortest contains only one customer (Customer 11), whose demand coupled with the following customer demand (Customer 6, see Figure 4.1) would exceed the vehicle capacity.

The total distance is 217.78 which is smaller than 800 (penalty cost). When the penalty is high, the shorter total distance has less impact on total cost. There is no penalty cost in this solution because the total cost is smaller than 800.

The convergence shape in Figure 4.2. is stepwise, which shows the process of combination of local search and global search in the predetermined iteration times.

After a small plateau which is a local search, there is a deep decrease for the first 20 times outer iterations. The searching jumps out of this small area which the objective function is around 365 and enters a global search. That is the greatest strength of SA, the worse solution will be accepted with a probability.

From around 25<sup>th</sup> iteration, it is a plateau and there is almost no change in objective function until 75<sup>th</sup> iteration and global search causes a significant decline in the objective function. And then another plateau repeated. Objective function value keeps at around 210 for a long time until 300 iteration terminates, but it can not be inferred that more iteration times will obtain a better solution (See Figure 4.3). As showed in Figure 4.3, when iteration times increases from 300 to 400, the objective function increases to 237.92. Time spent increases from 2.88 to 3.7 seconds.

General conditions for the convergent global optimum are: (1) the initial temperature is high enough; (2) time of thermal equilibrium is long enough; (3) the temperature to terminate the process is low enough; (4) the cooling process is slow enough. However, it is very difficult to meet the above conditions simultaneously.



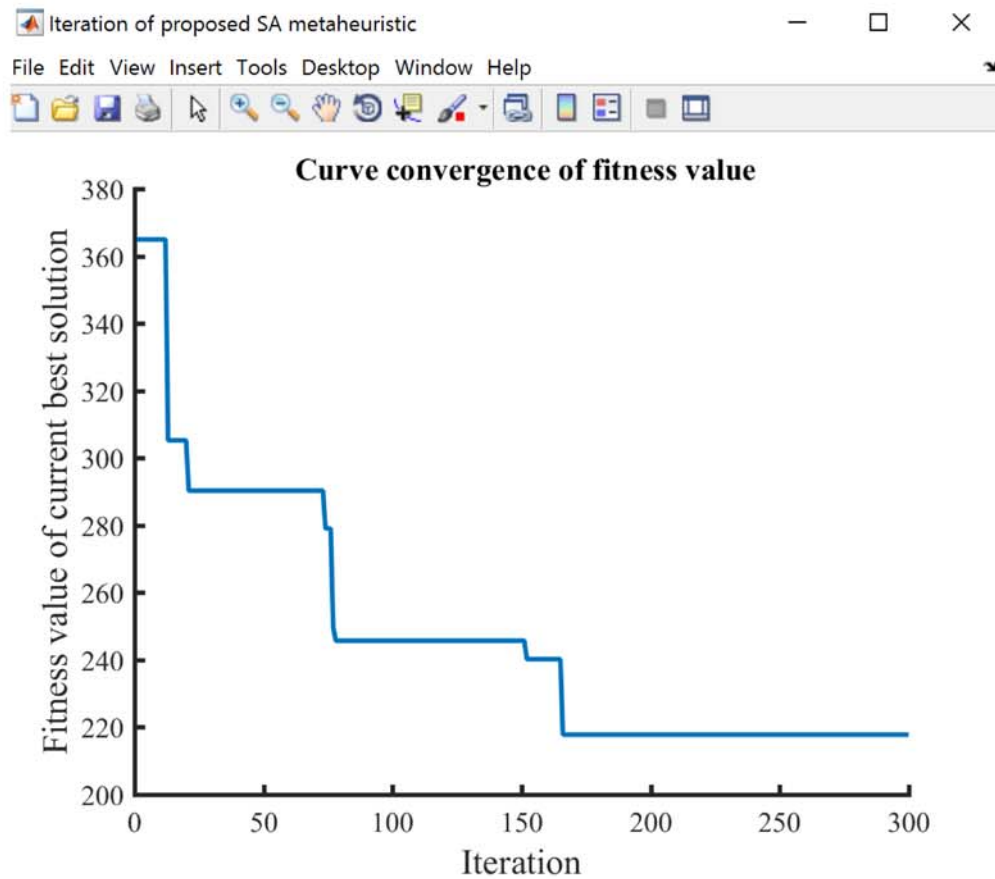


Figure 4.2. Pictorial view of convergence procedure with 300 iterations

Figure 4.4 depicts the routes on the map. It could be concluded that some of the routes are not reasonable, however, some small revision could be done in real life. For example, with the constraint of generated neighbor sequence, the route starting from Warehouse 1 - Customer 1- Customer 17- Customer 10- Customer 8- Warehouse 1. If this route changes to Warehouse 1 - Customer 17- Customer 10- Customer 1- Customer 8- Warehouse 1, the total distance must be smaller than the current solution.

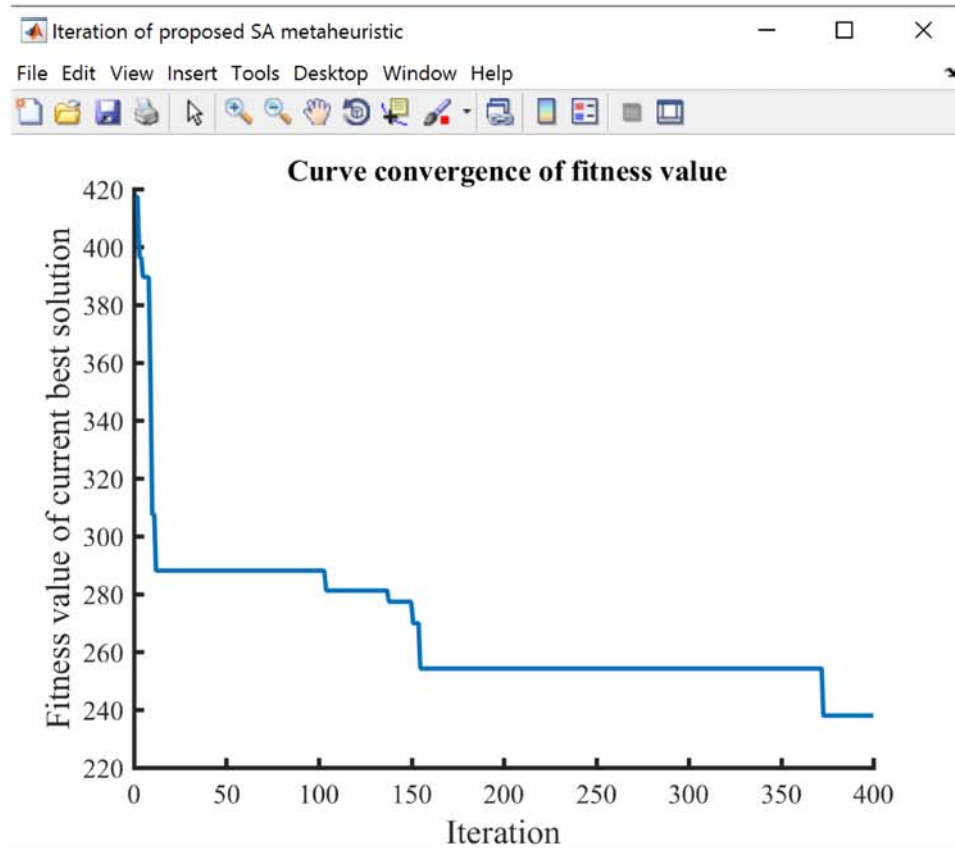


Figure 4.3. Pictorial view of convergence procedure with 400 iterations

#### 4.2 Trial-and-error Results

The routing decisions between local warehouses and customers change everyday with customer demand. However, the location between DC(s) and local warehouses is a relatively long-term decision, which could be one or two months if there is no big change.

The methodology describes the steps to model, solve and apply to this echelon. The shortest routes are as following Figure 4.5. The shortest distance is 268.31. This is an exact global optimal solution because all possibilities are enumerated and best solution is selected.

Depot A is picked to serve Warehouse 2.

Route 1: Depot A-W2-Depot A

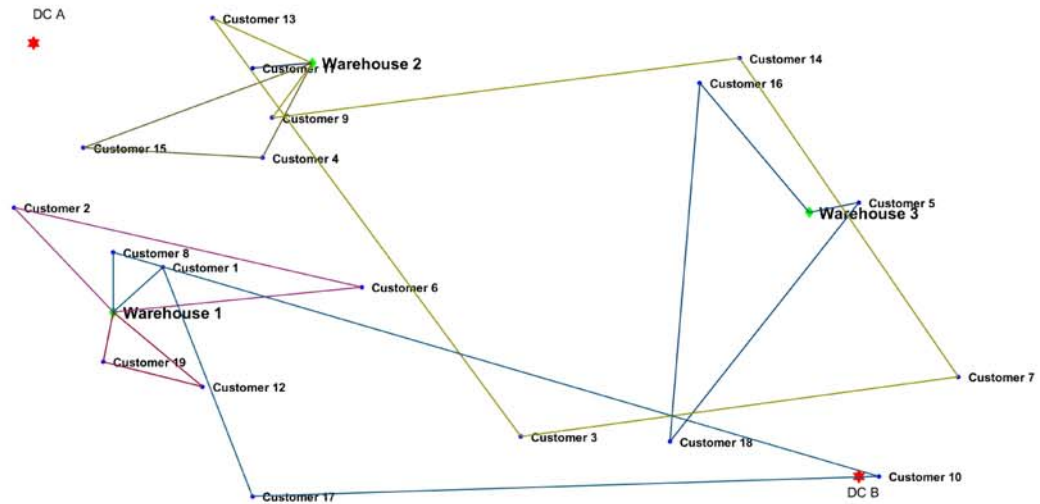


Figure 4.4. Routes allocation between local warehouses and customers

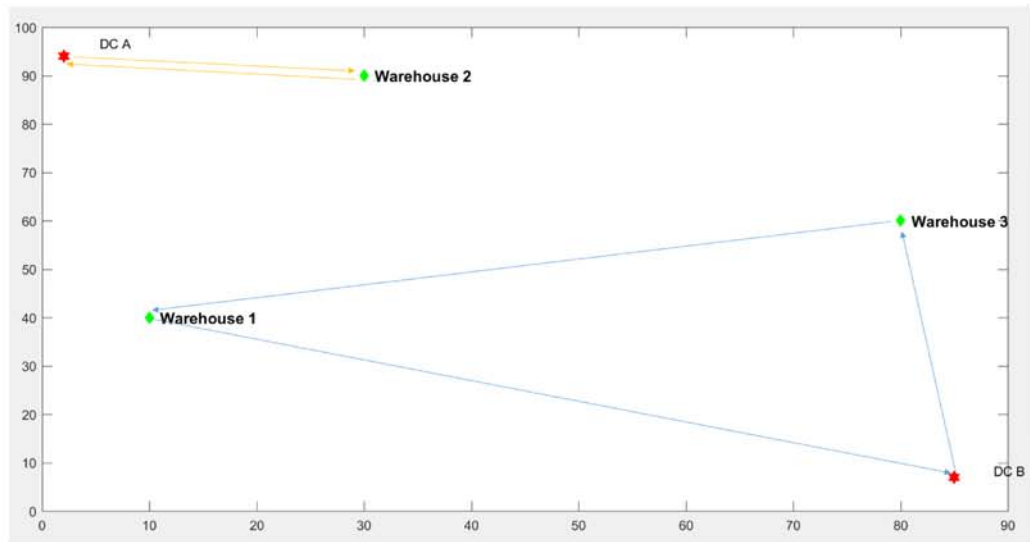


Figure 4.5. The routes of Depot A serving three warehouses

Route 2: Depot B-warehouse 1-warehouse 3-Depot B

The total distance is:  $28.28 \times 2 + 81.94 + 72.8 + 53.24 = 264.54$ .

It can be concluded that usually depot is built in places far from the city for the sake of traffic congestion, so if there are more customers on one route, it would save money because the distance from the DC to the local warehouse is saved.

### 4.3 Results for 2E-MDVRP

The 2E-MDVRP result is showed in Figure 4.6.

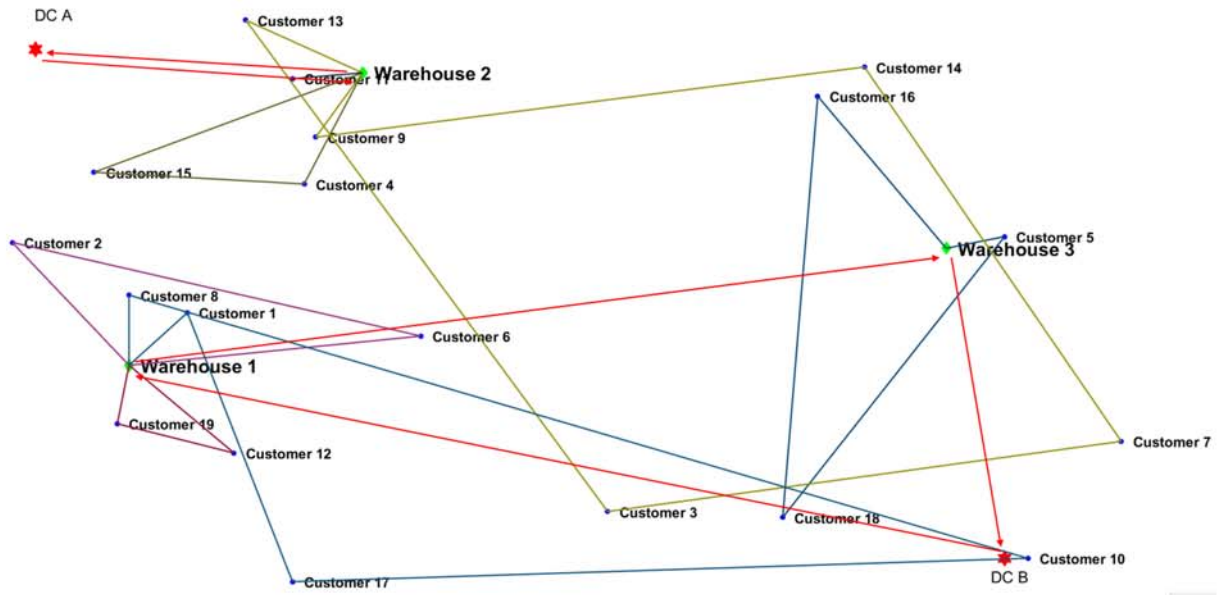


Figure 4.6. Pictorial view of the results for 2E-MDVRP

In summary, for the location routing problem between DC and local warehouses, only one DC is selected among all potential DCs which is DC A. DC B does not need to be considered anymore. For the MDVRP problem between local warehouses and customers, routing is obtained employed SA in Matlab.

## CHAPTER 5. CONCLUSIONS

This chapter presents the conclusions for results of both the two echelons, application and benefits from a company perspective and potential improvements for future research.

### 5.1 Study Conclusion

It is of great significance to study different types of vehicle routing problems and efficient algorithms to meet the actual needs. This thesis focused on a two-echelon location-routing problem, which is a combinatorial optimization and heuristic algorithms with trial-and-error given for a small scale example. The two-echelon is divided into two subproblems, one is the multiple depot VRP and the other one is the location-routing problem. Multi-depot VRP is coded in Matlab and the location-routing problem is solved by trial-and-error.

#### 5.1.1 Summary for Simulated Annealing

Essentially, Multi-depot VRP is still a routing problem with an additional allocation process. Each customer should be allocated to only one of the potential depots.

As a matter of fact, SA algorithm is a two-tier circulation, which includes inner loop and outer loop. A new solution is generated under perturbation at any temperature and the change is calculated in the objective function to determine whether to accept the new solution. Because of a high initial temperature, new solutions with higher  $E$  may also be accepted so that the algorithm can escape from the local minimum, then by slowly reducing the temperature, the algorithm may

eventually converge to a global optimal solution. Further, when the accepting function value has been very small at low temperatures, but there is still the possibility to accept a worse solution, so usually the best feasible solution would be recorded during annealing process and as an output with the last accepted feasible solution before the algorithm stops. Initial solution and parameter selection all have an effect on the time and quality of the solution.

### 5.1.2 Summary of Trial-and-error

The advantage of trial-and-error is that exact global optimal solution will be achieved. Since the location decision is a long-term strategic decision, an exact solution is necessary.

The stem distance should be avoided. One way is to group more customer in one route keeping vehicle full-truck-load. The other way is to have more distribution center. In this problem, fixed cost of the distribution center is not included, which is also necessary in real world case.

In the end, the routing and decisions are visualized which makes this problem more convenient in application.

## 5.2 Discussion

Two-echelon network optimization problem is derived from real life situation. This research covers topics including operation level and also the combination of strategic and operation level.

This research focused on developed a user-friendly way to help companies make routing and location decisions under the limitation of small-scale case in the following three aspects:

- Multi-depot VRP is an everyday routing decision based on the changing demand. It is simple to set up and applied to a real-life case. Input is location

coordinates of all customers, local warehouses which will not change very often. Demand is another input. Data including location coordinates and demand are exported to Excel, which will be read into Matlab directly to reach the final routes decision. When demand changes, the company could just change the data in Excel and it is very easy for front-end user.

- In this problem, there is only one set of value of independent variables including vehicle capacity and warehouse capacity. Other different combinations could also be implemented to determine the best vehicle capacity like a control experiment. For example, 6 units are picked in this research as vehicle capacity between local warehouses and customers. Keep all other independent variables and key parameters involved in the problem the same except for the vehicle capacity. Then, different capacities could be tried to optimize the total cost. Warehouse capacity could also be tested to achieve an optimized solution.
- There is a clear demonstration between MDVRP and the SA algorithm. The SA algorithm itself is already a fixed structure. The only difference is how to calculate the objective function. If the company in the future needs to add more constraints and change it to another variant of VRP, for example, VRP with simultaneous pickup and delivery, only the coding for objective function needed to be changed. Also, for this research, it is a combination of LRP and MDVRP. MDVRP could be combined with other problem for a new two-echelon problem. The idea of separation two echelons increases the calculation speed significantly and also makes each echelon an independent module which could be added to other problem conveniently for companies.

### 5.3 Recommendations for Future Studies

In Chapter 4, a small scale case is given. It takes around 3 seconds to solve the local warehouse-customer echelon of the network and the time will increase exponentially. However, in real situations, there will be a larger scale problem.

Also, from an algorithmic perspective, new heuristic algorithms could be used in future studies for the location-routing problem. For example, Intelligent Water Drops (IWD) is a relatively new algorithm which is a simulation of the formation of rivers and waterways. Kamkar, Akbarzadeh-T, and Yaghoobi (2010) applied IWD to solve VRP in 2010. 14 VRP problems were tested. Other heuristics algorithms including (simulated annealing, tabu search algorithm, ant colony algorithm) are compared together and it turns out that IDW can quickly converge to the optimal solution and get better results).

Additionally, there are some factors that are ignored in this thesis due to complexity. More parameters such as fixed cost of establishing depots, dispatching vehicles could be added to the objective function which makes it closer to real-world situations in the future. The time window, asymmetrical distance matrix and limit of route length are also very meaningful considerations. In modern logistics industry, especially express, delivery time and fixed delivery period have become a more and more important factor. Some of the routes are one-way so that asymmetrical distance matrix should be taken into consideration. For route length, it is usually a factor to measure the labor time of drivers. Fatigue in driving is not only dangerous, but also will reduce the customer satisfaction.

The other research direction is to combine inventory into routing and location so that more benefits may be obtained from this more integrated system. Especially with the development of driverless cars, distribution could be achieved with less faults.



## LIST OF REFERENCES

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- Aarts, E., & Korst, J. (1988). *Simulated annealing and boltzmann machines*. New York, NY; John Wiley and Sons Inc.
- Ambrosino, D., & Scutella, M. G. (2005). Distribution network design: new problems and related models. *European Journal of Operational Research*, 165(3), 610–624.
- Anily, S., & Federgruen, A. (1990). One warehouse multiple retailer systems with vehicle routing costs. *Management Science*, 36(1), 92–114.
- Anily, S., & Federgruen, A. (1993). Two-echelon distribution systems with vehicle routing costs and central inventories. *Operations Research*, 41(1), 37–47.
- Baita, F., Ukovich, W., Pesenti, R., & Favaretto, D. (1998). Dynamic routing-and-inventory problems: a review. *Transportation Research Part A: Policy and Practice*, 32(8), 585–598.
- Bell, W. J., Dalberto, L. M., Fisher, M. L., Greenfield, A. J., Jaikumar, R., & Kedia, P. (1983). Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer. *Interfaces*, 13(6), 4–23.
- Boccia, M., Crainic, T. G., Sforza, A., & Sterle, C. (2010). A metaheuristic for a two echelon location-routing problem. In *Experimental algorithms* (pp. 288–301). Springer.
- Boccia, M., Crainic, T. G., Sforza, A., & Sterle, C. (2011). *Location-routing models for designing a two-echelon freight distribution system*. Cirrelt Montreal.
- Böventer, E. V. (1961). The relationship between transportation costs and location rent in location problems. *Journal of Regional Science*, 3, 27–40.
- Bramel, J., & Simchi-Levi, D. (1995). A location based heuristic for general routing problems. *Operations Research*, 43(4), 649–660.
- Buffa, F., & Munn, J. (1989). A recursive algorithm for order cycle that minimizes logistic cost. *Journal of Operational Research Society*, 40, 357.
- Chen, J.-F., & Wu, T.-H. (2006). Vehicle routing problem with simultaneous deliveries and pickups. *Journal of the Operational Research Society*, 579–587.
- Clarke, G., & Wright, J. W. (1964). Scheduling of vehicles from a central depot to a number of delivery points. *Operations Research*, 12(4), 568–581.
- Cook, W. J., Cunningham, W. H., Pulleyblank, W. R., & Schrijver, A. (1998). *Combinatorial optimization* (Vol. 605). Springer.

- Cordeau, J.-F., & Laporte, G. (2001). A tabu search algorithm for the site dependent vehicle routing problem with time windows. *Infor-Information Systems and Operational Research*, 39(3), 292–298.
- Cordeau, J.-F., & Laporte, G. (2007). The dial-a-ride problem: models and algorithms. *Annals of Operations Research*, 153(1), 29–46.
- Cuda, R., Guastaroba, G., & Speranza, M. G. (2015). A survey on two-echelon routing problems. *Computers & Operations Research*, 55, 185–199.
- Dantzig, G. B., & Ramser, J. H. (1959). The truck dispatching problem. *Management Science*, 6(1), 80–91.
- Desrosiers, J., Dumas, Y., Solomon, M. M., & Soumis, F. (1995). Time constrained routing and scheduling. *Handbooks in Operations Research and Management Science*, 8, 35–139.
- Dror, M., Ball, M., & Golden, B. (1985). A computational comparison of algorithms for the inventory routing problem. *Annals of Operations Research*, 4(1), 1–23.
- Gendreau, M., Laporte, G., & Vigo, D. (1999). Heuristics for the traveling salesman problem with pickup and delivery. *Computers & Operations Research*, 26(7), 699–714.
- Golden, B., Assad, A., & Dahl, R. (1984). Analysis of a large scale vehicle routing problem with an inventory component. *Large Scale Systems*, 7(2-3), 181–190.
- Jacobs-Blecha, C. D., & Goetschalckx, M. (1992). *The vehicle routing problem with backhauls: Properties and solution algorithms*. Material Handling Research Center, Georgia Institute of Technology.
- Jacobsen, S. K., & Madsen, O. B. (1980). A comparative study of heuristics for a two-level routing-location problem. *European Journal of Operational Research*, 5(6), 378–387.
- Johnson, D. S., Aragon, C. R., McGeoch, L. A., & Schevon, C. (1989). Optimization by simulated annealing: an experimental evaluation; part i, graph partitioning. *Operations Research*, 37(6), 865–892.
- Kamkar, I., Akbarzadeh-T, M.-R., & Yaghoobi, M. (2010). Intelligent water drops a new optimization algorithm for solving the vehicle routing problem. In *Systems man and cybernetics (smc), 2010 ieee international conference on* (pp. 4142–4146).
- Kirkpatrick, S., Vecchi, M. P., et al. (1983). Optimization by simulated annealing. *Science*, 220(4598), 671–680.
- Knight, K., & Hofer, J. (1968). Vehicle scheduling with timed and connected calls: A case study. *OR*, 299–310.
- Laporte, G., Mercure, H., & Nobert, Y. (1986). An exact algorithm for the asymmetrical capacitated vehicle routing problem. *Networks*, 16(1), 33–46.

- Lin, J.-R., & Lei, H.-C. (2009). Distribution systems design with two-level routing considerations. *Annals of Operations research*, 172(1), 329–347.
- Liu, J., & Kachitvichyanukul, V. (2013). A new solution representation for solving location routing problem via particle swarm optimization. In *Proceedings of the institute of industrial engineers asian conference 2013* (pp. 103–110).
- Madsen, O. B. (1983). Methods for solving combined two level location-routing problems of realistic dimensions. *European Journal of Operational Research*, 12(3), 295–301.
- Min, H. (1989). The multiple vehicle routing problem with simultaneous delivery and pick-up points. *Transportation Research Part A: General*, 23(5), 377–386.
- Min, H., Jayaraman, V., & Srivastava, R. (1998). Combined location-routing problems: A synthesis and future research directions. *European Journal of Operational Research*, 108(1), 1–15.
- Osman, I. H., & Wassan, N. A. (2002). A reactive tabu search meta-heuristic for the vehicle routing problem with back-hauls. *Journal of Scheduling*, 5(4), 263–285.
- Papadimitriou, C. H., & Steiglitz, K. (1982). *Combinatorial optimization: algorithms and complexity*. Courier Corporation.
- Park, Y.-B. (2001). A hybrid genetic algorithm for the vehicle scheduling problem with due times and time deadlines. *International Journal of Production Economics*, 73(2), 175–188.
- Perl, J., & Daskin, M. S. (1985). A warehouse location-routing problem. *Transportation Research Part B: Methodological*, 19(5), 381–396.
- Pullen, H., & Webb, M. (1967). A computer application to a transport scheduling problem. *The Computer Journal*, 10(1), 10–13.
- Rahmani, Y., Oulamara, A., & Ramdane Cherif, W. (2013). Multi products location routing problem with pickup and delivery: Two-echelon model. In *11th ifac workshop on intelligent manufacturing-ims 2013, sao paulo, brazil* (pp. 124–129).
- Salhi, S., & Nagy, G. (1999). A cluster insertion heuristic for single and multiple depot vehicle routing problems with backhauling. *Journal of the Operational Research Society*, 1034–1042.
- Sterle, C. (2009). *Location-routing models and methods for freight distribution and infomobility in city logistics*. Unpublished doctoral dissertation, Università degli Studi di Napoli Federico II.
- Thomas, D. J., & Griffin, P. M. (1996). Coordinated supply chain management. *European Journal of Operational Research*, 94(1), 1–15.
- Toth, P., & Vigo, D. (1996). A heuristic algorithm for the vehicle routing problem with backhauls. In *Advanced methods in transportation analysis* (pp. 585–608). Springer.

- Toth, P., & Vigo, D. (2011). 1. an overview of vehicle routing problems. In *The vehicle routing problem* (p. 1-26). Retrieved from <http://epubs.siam.org/doi/abs/10.1137/1.9780898718515.ch1> doi: 10.1137/1.9780898718515.ch1
- Toth, P., & Vigo, D. (2014). *Vehicle routing: problems, methods, and applications* (Vol. 18). Siam.
- Tuzun, D., & Burke, L. I. (1999). A two-phase tabu search approach to the location routing problem. *European Journal of Operational Research*, 116(1), 87–99.
- Xia, M. (2013). *Integrated supply chain network design: Location, transportation, routing and inventory decisions*. Doctoral dissertation. ProQuest Dissertations and Theses. Retrieved from <http://search.proquest.com/docview/1353398893?accountid=13360>
- Yang, X.-S., Bekdaş, G., & Nigdeli, S. M. (2015). *Metaheuristics and optimization in civil engineering* (Vol. 7). Springer.

## APPENDICES

## APPENDIX A. SIMULATED ANNEALING MAIN CODE

```

1 %% Multi-Depot VRP
2 clc, clear, close all;
3 feature jit off
4 tic
5 %% Parameters
6 capacity=6; % Capacity of vehicles
7 capacityW=3; % Capacity of intermediate depot
8 penalcoef=800;
9 % Penalty of Warehouse (Constance in fitness function)
10
11 percost =1 ; % cost per mile
12 Whouse.position= xlsread('warehouse.xlsx');
13 % coordinates of warehouse
14 Whouse.number= size(Whouse.position, 1); % # of warehouse
15 B= xlsread('customer.xlsx'); % demand & coordinates of customer
16 customer.position=B(:,1:2); % coordinates of customer
17 customer.demand=B(:,3); % demand of customer
18 customer.number=size( customer.position ,1); % # of customer
19 [customer.distance ]=customdist(customer.position);
20 % distance matrix among customers
21 [customer.CWd] =CWdist( Whouse.position , customer.position );
22 %distance between customer and ID(warehouses)
23
24 %% SA Parameters
25 MaxIt=300; % Maximum Number of Iterations
26 MaxIt2=50; % Maximum Number of Inner Iterations
27 T0=100; % Initial Temperature
28 alpha=0.95; % Temperature Damping Rate

```

```

29
30 %% Create initial solution randomly, calculate obj fun
31
32 [sol.chrom ]=CreateRandomSolution(customer.number );
33
34 % [sol.routing, sol.allocation, sol.Objfun]=
35 %Costfun(sol.chrom, percost, capacity, customer.CWd,
36 %customer.demand, customer.distance);
37
38 [sol.routing, sol.allocation, sol.Objfunfit, sol.Objfun]
39 =Costfun...
40 ( sol.chrom, percost , capacity , customer.CWd,
41 customer.demand, customer.distance , capacityW, penalcoef);
42
43 %% Update Best Solution Ever Found
44 BestSol=sol;
45 % Array to Hold Best Cost Values
46 BestCost=zeros(MaxIt,1);
47 % Set Initial Temperature
48 T=T0;
49
50 %% SA loop
51
52
53 for it=1:MaxIt
54     for it2=1:MaxIt2
55         %% neighbourhood generation
56         [ newchrom ] =CreateNeighbor( sol.chrom );
57         [ newrouting, newallocation, newObjfunfit ,
58         newObjfun ]=Costfun...
59         ( newchrom, percost, capacity ,
60         customer.CWd, customer.demand, customer.distance,...
61         capacityW, penalcoef);
62
63         if newObjfunfit ≤ sol.Objfunfit

```



```

64         % xnew is better, so it is accepted
65         [ sol.chrom ]=newchrom;
66         [ sol.routing, sol.allocation, sol.Objfunfit,
67 sol.Objfun ]=Costfun...
68         (sol.chrom, percost , capacity ,
69         customer.CWd, customer.demand, customer.distance ,
70         capacityW, penalcoef);
71     else
72         Δ=newObjfunfit -sol.Objfunfit;
73         p=exp(-Δ/T);
74         if rand≤p
75             [ sol.chrom ]=newchrom;
76             [ sol.routing, sol.allocation,
77 sol.Objfunfit, sol.Objfun ]=Costfun...
78             (sol.chrom, percost , capacity ,
79             customer.CWd, customer.demand, customer.distance ,
80             capacityW, penalcoef);
81
82         end
83     end
84
85     % Update Best Solution
86     if sol.Objfun≤BestSol.Objfun
87         BestSol=sol;
88     end
89
90 end
91 BestCost(it)=BestSol.Objfun;
92
93 % Reduce Temperature
94 T=alpha*T;
95
96 end
97 toc
98

```

```

99
100 %% results display
101 disp( sprintf('The transportation cost of the final result:
102
103 %12.2f',BestSol.Objfun) );
104 for i=1: length(BestSol.routing)
105     str=['The' num2str(i) '-th route:' ' Depot '
106         num2str( BestSol.allocation(i)) ' Customers(in order): '
107         num2str( BestSol.routing{i}) ];
108     disp(str)
109 end
110
111 %% results illustration
112 %% curve convergence of Transportation cost
113 figure('NumberTitle', 'off', 'Name', ...
114 'Iteration of proposed SA metaheuristic', 'Color',[1 1 1]);
115 plot(BestCost,'LineWidth',2);
116 title(' Curve convergence of fitness value','fontsize',13)
117 set(gca, 'FontName','Times New Roman',
118 'FontSize',12,'LineWidth',2 );
119 xlabel('Iteration','fontsize',15,'fontname','Times new roman');
120 ylabel('Fitness value of current best solution','fontsize',15,
121 ...'fontname','Times new roman')
122 grid off ;
123 box off
124
125 %% location
126 % customers
127 figure('NumberTitle', 'off', 'Name', ' Schematic diagram of
128 the final solution', 'Color',[1 1 1]);
129 for i=1:length( customer.position)
130     plot( customer.position(i,1) , customer.position(i,2)
131         , 'o', 'MarkerEdgeColor','b', ...
132         'MarkerFaceColor','b', 'MarkerSize',4);
133     hold on

```

```

134     str=[ 'Customer ' num2str(i) ];
135     text( customer.position(i,1)+1 ,
136         customer.position(i,2),str,'FontWeight','Bold','FontSize',9);
137 end
138 % Warehouse
139 for i=1:Whouse.number
140     plot( Whouse.position(i,1) , Whouse.position(i,2)
141         , 'o', 'MarkerEdgeColor','r', ...
142         'MarkerFaceColor','r', 'MarkerSize',7); hold on
143     str=[ 'Warehouse ' num2str(i) ];
144     text( Whouse.position(i,1)+1 ,
145         Whouse.position(i,2),str,'FontWeight','Bold','FontSize',12);
146     %label the id of every task Coordinate fonts and other features
147 end
148 box off
149 axis off; %
150 %% route
151 for i=1: length(BestSol.routing )
152     depott=BestSol.allocation(i);
153     routt=BestSol.routing{i} ;
154     routposition=[ Whouse.position( depott,:) ;
155         customer.position( routt,:) ; Whouse.position( depott,:) ];
156     linecol=0.6-0.6*rand(1,3);
157     for j=2:length( routposition )
158         lineh=plot([ routposition(j-1,1), routposition(j,1) ], ...
159             [ routposition(j-1,2), routposition(j,2) ],...
160             '-','LineWidth',1) ; hold on
161         set( lineh,'color', linecol);
162     end
163 end

```

## APPENDIX B. COST FUNCTION CODE

```

1 function [ routing, allocation, Objfunfit, Objfun ]=...
2 Costfun( chrom, percost , capacity , CWd ,demand, distance,
3 capacityW, penalcoef)
4 % routing
5 % allocation
6 % Objfun
7 % Warehouse capacity constraints
8 %% Allocate routes
9 fir=1; j=1;
10 for i=1: length( chrom )
11     tempload=sum( demand( chrom( fir :i ) ) );
12     if tempload > capacity
13         routing{j}= chrom( fir :i-1 );
14         fir=i;
15         j=j+1;
16     end
17     if i== length( chrom )
18         routing{j}= chrom( fir : i );
19     end
20 end
21
22 %% Warehouse allocation for each route
23 allocation=zeros(1, length( routing ) );
24 Drout=zeros(1, length( routing ) );
25 % shortest distance to all warehouse
26 W2rout=zeros( size(CWd ,1) ,1 );
27
28 for i=1: length( routing )

```

```

29     rout=routing{i}; % Route i
30     for j=1:size(CWd ,1)
31         W2rout(j)= CWd(j,      rout(1) )  +      CWd(j, rout(end)) ;
32     end
33     [ val, ind]=min(      W2rout );
34     allocation(i)=ind;
35
36     if length(rout )==1
37         Drout(i)=      W2rout( ind );
38     else
39         kk=0;
40         for j=2:length( rout )
41             kk=kk+distance( rout(j-1), rout(j-1) );
42         end
43         Drout(i)=W2rout( ind )+kk;
44     end
45
46 end
47
48 %% Obj Function
49 Objfun= percost * sum(Drout );
50 kk=zeros(1, length( routing ) );
51 for i=1:length( routing )
52     kk( allocation(i) )=kk( allocation(i) )+1;
53 end
54 penalcost=penalcoef*...
55     sum( max(      kk- capacityW,      zeros(1, length( routing ) ) ) );
56     % penalty cost
57 Objfunfit= Objfun+penalcost; % fitness value

```